DE GRUYTER

Paul Weingartner (Ed.)

KNOWLEDGE AND SCIENTIFIC AND RELIGIOUS BELIEF

PHILOSOPHICAL ANALYSIS

Paul Weingartner

Knowledge and Scientific and Religious Belief

Philosophische Analyse/ Philosophical Analysis

Herausgegeben von/Edited by
Katherine Dormandy, Rafael Hüntelmann,
Christian Kanzian, Uwe Meixner, Richard Schantz,
Erwin Tegtmeier

Band/Volume 77

Paul Weingartner

Knowledge and Scientific and Religious Belief

DE GRUYTER

ISBN 978-3-11-058443-1
e-ISBN (PDF) 978-3-11-058579-7
e-ISBN (EPUB) 978-3-11-058451-6
ISSN 2198-2066

Library of Congress Control Number: 2018949259.

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at http://dnb.dnb.de.

Printing: CPI books GmbH, Leck

www.degruyter.com

To Mandy Stake

Preface

This is a book on epistemology.

The first three chapters show the possibility of knowledge against skepticism. In chapter 4, I defend a pluralism for the concept of knowledge by distinguishing seven types of knowledge. In chapter 5, a new type of knowledge is proposed and defined with the help of the concepts of *epistemic entropy* and *epistemic information*. Chapter 6 deals with properties common to all seven types of knowledge. It is stressed in several parts of the book that *logical omniscience* or *deductive infallibility* are not properties of human knowledge. Chapter 7 shows the necessity for belief. The chapter 8, 9 and 11 discuss important features of both scientific and religious belief. Chapter 10 and 12 discuss special questions of religious belief. Chapter 13 offers a decidable deductive theory of knowledge, belief, and assumption without logical omniscience and deductive infallibility. It includes three types of knowledge, two types of belief and one type of assumption.

Acknowledgments: I want to thank Edgar Morscher, Mandy Stake and the members of the July-2016 Bergseminar for valuable suggestions concerning improvements. Moreover, my thanks go to Mandy Stake for typing and the layout of the book as well as to Kathryn Bishop for improving my English style.

https://doi.org/10.1515/9783110585797-201

Contents

1 Whether it is Possible to Know Something

1.1 Arguments Contra

1.11 As Georgias of Leontini (483 B. C.) says we cannot know something since:

(1) Suppose there is nothing; then there is no knowledge because to know means to know *something*.

(2) Suppose there is something; then it could not be known. For if there is knowledge of being, then what is thought must be and not-being could not be thought at all; in which case there could be no error, which is absurd.[1]

Thus it seems doubtful whether it is possible to know something.

1.12 To know something about real things means to judge them as they are in reality. As Sextus Empiricus says, it is impossible to judge things as they really are.

> The Mode based upon relativity as we have already said is that whereby the object has such and such an appearance in relation to the subject judging and to the concomitant percepts, but as to its real nature, we suspend judgement.[2]

Therefore it seems doubtful whether it is possible to know something about real things.

1.13 An assertion can be called knowledge only if the assertion is true. As Sextus Empiricus says, if someone claims that his assertion is true he ought to give a proof for the truth of his assertion. And then again a proof for this proof and so on.

> The Mode based upon regress *ad infinitum* is that whereby we assert that the thing adduced as a proof of the matter proposed, needs a further proof, and this again another, and so on *ad infinitum* so that the consequence is suspension, as we posses no starting point for our argument.[3]

1 Diels (1960) II Fragment 1,3. It is not clear whether Gorgias believed these statements himself. By claim (2) he may have pointed to the fallibility of human knowledge. Cf. Copleston (1947) Vol. I, p. 93 and Freeman (1959), p. 360

2 Sextus Empiricus (OPr) I, p. 167. He deals here with 5 modes leading to suspension defended by the later Skeptics. The one quoted above is the third mode. Cf. (OPr) I, p. 135.

3 Sextus Empiricus (OPr) I, p. 166.

https://doi.org/10.1515/9783110585797-001

This leads to an infinite regress without terminus. Thus the original claim cannot be proven to be true, and consequently an assertion cannot be called knowledge.

1.2 Arguments Pro

1.21 "Bodily Awareness" is one type of direct, immediate knowledge.

> …the body is originally constituted in a double way: first it is a physical thing … secondly I find on it, and I *sense* 'on' it and 'in' it: warmth on the back of the hand, coldness in the feet, sensations of touch in the fingertips.[4]

> 'My body' is the body of which, when I am conscious, I have self-conscious knowledge.[5]

Therefore it is possible to know something.

1.22 To be able to calculate with the help of the multiplication table implies to have knowledge within a small part of finite number theory. Every educated child manages to calculate with the calculation table. Therefore it is possible to know something.

1.3 Proposed Answer

The statement "it is possible to know something" can have different meanings. For a more accurate analysis, we shall understand "something" as some state of affairs, which can be represented by some statement or proposition p. Then to know something can be understood as to know that p is the case or to know that p is true. Under this interpretation, the statement "it is possible to know something" can have three meanings which differ in strength:

1.3.1 Three Meanings, Different in Stength

(1) It is possible that someone (i. e. some human being) has knowledge concerning some proposition p (i. e. knows that some proposition p is true or that the respective state of affairs obtains). Symbolically: $\Diamond(\exists x_{\in \mathrm{H}},\ \exists p)\, xKp$

4 Husserl (1989), section 40
5 Hamilton (2005), p. 271

(2) It is possible that everyone (i. e. every human being) has knowledge concerning some (or other) proposition p. Symbolically: $\Diamond(\forall x{\in}\mathrm{H}, \exists p)\, xKp$
(3) It is possible that there is some proposition p such that everyone has knowledge concerning p (i. e. knows that p is true or that the respective state of affairs obtains). Symbolically: $\Diamond(\exists p, \forall x{\in}\mathrm{H})\, xKp$

In the arguments contra, the possibility to know something is denied. This means that these arguments claim the negation of (1) or (2) or (3). The respective negations are the following:
(1') It is not possible that someone has knowledge concerning some proposition. Symbolically: $\neg\Diamond(\exists x{\in}\mathrm{H}, \exists p)\, xKp$
(2') It is not possible that everyone has knowledge concerning some (or other) proposition. Symbolically: $\neg\Diamond(\forall x{\in}\mathrm{H}, \exists p)\, xKp$
(3') It is not possible that concerning some proposition p, everyone has knowledge of p. Symbolically: $\neg\Diamond(\exists p, \forall x{\in}\mathrm{H})\, xKp$

Statement (1) is the weakest claim concerning human knowledge and consequently claim (1') is the strongest kind of skepticism concerning human knowledge. Such a claim can be refuted by a single example of human knowledge possessed by a single person as the self-conscious knowledge ("bodily awareness") of the own body or as a calculation according to the multiplication table. On a higher level (1) also claims that there is expert-knowledge, some selected (by education and training) people know some specific things. To deny this, i. e. to claim (1') is certainly absurd. We need experts all the time (physicians, technicians, pilots, bus drivers, chemists, peasants, …). Statement (2) is stronger than (1) claiming that it is possible that everyone has some knowledge in some domain or other. If someone denies this possibility, i. e. if he claims (2') then he denies that all people have knowledge in some domain. It is clearly absurd to deny this since very simple things like self-conscious knowledge or E1–E4 below everyone will know. Statement (3) is even stronger than (2) claiming that it is possible that there are some propositions with respect to which everyone has knowledge. A statement of the form "it is possible that p" is proved if one can give an example for an actual instance of p, since if p is the case, then possible p holds. Several kinds of such examples can be given which justify statement (3). Some famous ones from the philosophical tradition are these:

1.3.2 Famous Examples E 1–E 5

E 1 "If there is only one sun then there are not two."; "Either the world is finite, or the world is infinite."[6]

E 2 "If he doubts he knows that he doubts … if he doubts he knows that he does not know."[7]

E 3 "Si enim fallor, sum." Since he who does not exist cannot err. Therefore I am if I err.[8]

E 4 "Cogito ergo sum." I think, therefore I am.[9]

Examples E 2 – E 4 are concerned with self-reflecting knowledge. E 1 is concerned with knowledge of simple logical propositions. In this respect Aristotle has proposed his principle of non-contradiction as the most general and the most basic logical principle which implies and justifies statement (3) above: "Let this, then, suffice to show that the most indisputable of all beliefs is this:

E 5 Contradictory statements are not at the same time true."[10]

The principle E 5 is the most tolerant formulation of the principle of non-contradiction which is invariant w. r. t different types of systems of logic like classical two-valued logic, intuitionistic logic, minimal logic, different types of many-valued logic.[11]

E 1–E 5 refute the claim that knowledge is not possible, expressed in different strength by (1'), (2') and (3'). By this, they justify the statements (1) and (2) and (3).

1.4 Answers to the Objections

1.41 (To 1.11:) There are at least three possibilities to interpret Gorgias' claims. First, he did not take them serious himself, but, as a Sophist, wanted just to provoke. Second, he took them serious because he gave some arguments of defence, although these arguments contain untenable premises like "being can neither be

6 Augustin (Ord) II, 13, 38.
7 Augustin (Trin) X, 10, n. 14.
8 Augustin (Civ) XI, 26.
9 Descartes (Med) II.
10 Aristotle (Met) 1011b13
11 Cf. Rescher (1969), p. 143ff., where he discusses 6 versions of the principle of non-contradiction of which E 5 is the most tolerant one.

one nor be many."[12] Third, he wanted to stress the fallibility of human knowledge. In this case, he starts with the experience that there is error in human thinking. Moreover, if knowledge is not compatible with that kind of error we have to give up knowledge. "What is thought must be, and not-being could not be thought of at all" is not a literal quotation from the fragments but seems to be a reasonable interpretation. If correct, it shows that Gorgias did not differentiate between *being* and *being so*, and not between existing and non-existing objects to which our thoughts are directed.[13] It is correct that thinking is intentionally directed to some object. However, such objects may exist (in space-time like Venus or independent of space-time like the prime number between 5 and 11) or not exist (in space-time like a perpetuum mobile or independent of space-time like the prime number between 31 and 37). Thus there can be error concerning the existence or non-existence (being or not-being). However, errors occur much more frequently concerning being so or not being so, i. e. concerning the question whether the object has certain properties or hasn't. As has been pointed out correctly by Heller,

> the postulate of the existence of the world is not a presupposition of science even though it is a natural and frequently imposed 'presupposition' of the majority of people who are concerned with science. Their conviction is so natural that the successes of science contribute little to it being strengthened.[14]

From these considerations, it is manifest that we need not give up knowledge despite the occurrence of errors.

1.42 (To 1.12:) The third skeptical mode of Sextus Empiricus quoted in 1.1.2 is the mode of relativism. It is concerned with three points: (1) the relativity of the judging subject, (2) the relativity of the object's appearance, and (3) the inaccessibility of the real nature of things.

(1) Although all human thinking and judging is done by a human subject, the content of the thoughts and judgments is something objective. Similarly, although thinking and judging is something psychological, the content of thinking and judging (that what is thought or judged about) is something objective. This has been underlined already in Stoic Logic and was emphasized strongly by Bolzano and Frege:

12 For Gorgias' argumentation see Brentano (1963), p. 150f.
13 For the distinction between being and being so cf. Lambert (1983).
14 Heller (2010), p. 32

> Some, and above all those of the Stoa, think that truth is distinguished in three ways from what is true, … truth is a body, but what is true is immaterial; and this is shown, they say, by the fact that what is true is a proposition (axioma), while a proposition is a lecton, and lecta are immaterial.[15]

> But the proposition in itself (Satz an sich), that makes up the content of the thought or judgement, …[16]

> I understand by 'laws or logic' not psychological laws of takings-to-be-true ("des Fürwahrhaltens") but laws of truth ("Gesetze des Wahrseins").[17]

> I understand by a thought (Gedanke) not the subjective act of thinking but its objective content.[18]

(2) The second point in the mode of relativism described by Sextus Empiricus is that objects as they appear to a human subject are not the objects as they really are. In support of that one can put forward many situations both in everyday life and in scientific problem situations: optical illusions, constraints of our sense organs, for example the optical and acoustical spectrum (i.e. the sensitivity for only a small part of the electromagnetic or acoustic waves), the constraints of our sense organs concerning microscopic things and macroscopic things or things far away or even unobservable things, like black holes, … etc. Since these constraints are facts it follows that Sextus Empiricus's point is correct in this sense. However, man was not an inactive onlooker (spectator) but actively and successfully managed to overcome these constraints to a great extent: by microscopes, by telescopes and by all sorts of technical design and scientific instruments. Although it is correct to say that human thinking can never be freed completely from errors, mistaken hypotheses, and conjectures or erroneous experimental results, there is the permanent task to discover and correct them and to learn from mistakes.[19] In addition, today's science, technology, and medicine are built on neither of them. Both everyday knowledge and scientific knowledge are not concentrated on a few subjects (human beings). There is division of labour with the help of many experts.[20]

15 Sextus Empiricus (AMt) VII, 38, transl. in Bochenski (1961), 19.06. It should be noted that for the Stoics, mental activities like thinking, judging etc. are corporal. Although Sextus Empiricus knew the Stoic distinction he apparently made no use of it in his modes leading to suspension.

16 Bolzano (1929) I, § 19.

17 Frege (1967) Introduction, p. XVI.

18 Frege (1892), p. 32.

19 Cf. Popper (1972) chs. 6 (Of Clouds and Clocks) and 7 (Evolution and the Tree of Knowledge).

20 For the concept of "expert" and "epistemic authority" see Bochenski (1965) and (1974). Scholz (2009).

(3) The third critical claim in Sextus' mode of relativism is the inaccessibility of the real nature (or essence) of things. The doctrine of essence was invented by Aristotle. According to him, only natural individual beings (individual substances)[21] can have an essence. Aristotle's examples are taken from the biological domain only. Thus, for example, an individual man like Socrates has an essence: rational animality. Similarly, an individual horse or an individual tree has an essence (nature). The essence (nature) is always common to the respective species: to all the individual men of humankind or all individual horses of some species horse and all individual trees of the respective species. It appears as the definiens of a real definition[22] in the Aristotelian sense: man = rational animal; living thing = thing possessing growth, nutrition, and propagation. This latter demarcation of living things in contra-distinction to non-living things is roughly correct also today, even if growth (not meant permanently through life) and nutrition is united in *metabolism* and propagation is replaced by *DNA-replication* (because actual propagation is not always realized).

Today the essence (nature) of man or a horse species or a plant species can also be represented by the DNA of the respective species. The discovery of the DNA is, therefore, an important scientific progress with respect to the original idea of Aristotle. In this sense Sextus Empiricus is not correct: the essence of living things became accessible by the discovery of the DNA and by research on metabolism, DNA-duplication and heredity.

It has already been said that Aristotle does not give good examples for the essence of non-living things. Although he gives examples of non-living substances, for example, the celestial bodies, which are eternal according to him.[23] According to today's knowledge, one could propose as substances those chemical elements which have been found in nature. These are roughly 81 elements up to Bismuth (Nr. 83) since Technetium (Nr. 43), and Promethium (Nr. 61), and those beyond Nr. 83 are not (or only vestiges) to be found in nature and are usually unstable and are by-products of certain non-natural nuclear reactions. However, this selection is somewhat arbitrary and one could also include stable compounds like water or glucose which are both essential for life. The essence of them could be the atomic structure + necessary properties and relations for the chemical elements and the molecular structure for the compounds water and glucose. If such structures are acceptable to be essences, Sextus Empiricus is not right that they are inaccessi-

21 Aristotle (Cat) ch. 5

22 Aristotle (APo) ch. 3–6; cf. Weingartner (1991)

23 Aristotle (Met) 1069a, p. 30f.

ble. On the other hand, there is no clear criterion for essences in the domain of non-living things.

Classical mechanics viewed mass and shape (form) of solid bodies as substantial properties, whereas place and velocity as accidental. It was, therefore, some surprise that every rigid body can be uniquely described by its position and momentum, i.e. by accidental properties. Mass was interpreted to be constant, and time and simultaneity to be universal. Length, time, and mass were "rigid"basic magnitudes. The three were backed up by Newton's absolute space and time.[24] However, Special Relativity showed that neither of the three is rigid but can be stretched and compressed or can increase or decrease. Such scientific developments revised our knowledge of non-living things and made it more and more difficult to give reasonable demarcations for essences or to give definitions where the definientia describe the essence.

However, from what has been said about living things (DNA-structure of species and individual DNA-structure) it follows that a reasonable and scientific interpretation of the essence of a species and even of an individual is possible. Therefore Popper's general criticism seems to be too much of an exaggeration:

> … I reject all *what-is questions*: questions asking what a thing is, what its essence is or its true nature… We must give up the view, …that it is the essential properties inherent *in each individual or singular thing* which may be appealed to as the explanation of this thing's behaviour.[25]

We may critically remark that the answer given by the definiens of a correct definition is an answer to a *what-is question*, e. g.: What is a circle? What is a chemical element? What is heredity? To the first question there is a definite and true answer, already given by the Greeks, the present answer to the second and third question, though correct in the main points, may still be improved in the future.

1.43 (To 1.1.3:) Infinite Regress: This epistemological (or skeptical) type of regress has been used by several philosophers in the tradition and also in the 20*th*

24 Newton (Princ) Scholium

25 Popper (1972), p. 195. However, Popper's criticism applies correctly to some "inflationary essentialists": There are philosophers defending essentialism who advocate an inflation of essences such that not only natural objects but also their parts and artefacts have essences; even space and time are substances having an essence. Cf. Oderberg (2007), p. 80, 167f., 81. An inflation w. r. t. the traditional Aristotelian view with a general criterion is due to Husserl: all properties which necessarily belong to the object belong essentially to it and vice versa; whereas according to Aristotle only one direction (from essential to necessary) holds. We do not share such inflationary views.

century. Although it looks somewhat plausible at first, it is seen to be problematic at a closer look. Because it contains presuppositions or strong hidden assumptions which cannot be satisfied on logical grounds. These assumptions are roughly four:

(1a) That every proposition that is true can be proved to be true.
(1b) That every proposition that is true should be proved to be true.
(2) That for every proposition that is true, it is possible to prove it.
(3a) That for every proof of (proposition) p there is a further proof of the proof of p.
(3b) That for every proof of (proposition) p there should be a further proof of the proof of p.
(4) That every proof with infinitely many steps leads to a *regressus ad infinitum* and consequently to *no proof*.

Ad (1a)/(1b): The claim (1a) was never defended in this generality by any philosopher or scientist. Moreover, in the texts of Sextus Empiricus it seems to be rather a hidden assumption which he attributed to the academic philosophers. However, he seems to have assumed the normative form of (1a) namely (1b). In addition, he wanted to show that (1b) cannot be satisfied and therefore one has to suspend judgement.

However, there was a strong hope even at the beginning of the 20^{th} century (Hilbert) that (1a) is correct for mathematics; i.e. that every mathematical truth can be rigorously proved. It has been shown by Gödel (1931) that this hope cannot be satisfied. Already in the theory of natural numbers built up axiomatically, there are mathematical statements which are undecidable (i. e. statements which cannot be proved or disproved). Although such statements can be decided in a richer mathematical system than that of natural numbers, there will be then new statements which are undecidable in the richer system and so on.

Since claim (1a) cannot be satisfied also (1b) cannot be satisfied. Because it is not reasonable to require something normatively if it cannot be satisfied on logical grounds.

Ad (2): The claim (2) was defended by several philosophers in the sense that the human intellect is such as to be possibly open to every truth. The idea was that although the human intellect does not, in fact, know everything and cannot actually know everything since it is imperfect and not omniscient, it may still hold true that for any given true proposition it is at least *possible* to know it or to prove it. Simply put: Whatever is true is possibly known.

Although this assumption sounds plausible, it is much too strong. It has been proved that this assumption leads—even with a very weak interpretation of possibility—to the thesis that man knows everything that is true, that is to: whatever is true is known.[26] The weak interpretation of possibility is that of the system of modality S 0.5. As additional axioms we only need:

(1) Necessarily: Whatever is known is true.
(2) Necessarily: Each conjunct of what is known is known.

Since the absurd thesis "whatever is true is known" follows from "whatever is true is possibly known" even with very weak assumptions on modality (and specifically on possibility), it will also follow from stronger assumptions. This shows unambiguously that claim (2) is false, i. e. too strong as a general principle.

Ad (3a)/(3b): The claim (3a) is a theorem of some logics for the concept provable (B) in the sense: If p is provable ($B(p)$) then it is provable that p is provable ($B(B(p))$) : $B(p) \to B(B(p))$.[27]

The main point to notice here is that $B(B(p))$ does not make the proof $B(p)$ of p stronger or does not give some further evidence of or information about the proof of p. This however—to provide more evidence or make the proof $B(p)$ of p more reliable—seems to be the hidden assumption in the requirement (3b) for the further proof. However, this hidden assumption is not satisfied by the above theorem which only iterates the provability predicate.

On the other hand, what really provides further evidence and reliability for a proof of p is a new proof of p with other methods or where the proof of p can be received in another way. This frequently happens in mathematics: If a proposition in a branch of mathematics can be proved in one way, it may also be proved in another way. For example, in number theory one often asks the question whether a proposition proved analytically can also be proved by elementary methods (prime number theorem). In algebra, one asks whether a theorem about real numbers which has been proved by use of complex numbers can also be proved without using the latter. In physics it frequently happens in the following way: If some result has been proved experimentally it may follow independently from some theory which extends far beyond all the experimental results in that domain. For example, the perihel deviation of Mercury was a well-known astronomical result long before it appeared to be a consequence of Einstein's General Theory of Relativity (of 1916). In these cases where more evidence and more reliability is achieved by a new (way of) proof, there is no question or problem of a regressus, since to ask for a proof of the proof is much weaker and less informative than to ask for a new proof from a different angle.

26 Routley (1981)

27 See for example Hilbert/Bernays (1939), p. 289–294; Löb (1955), p. 116, V.

Ad (4): This claim is just a mistake. An important counterexample is the mathematical proof by induction: If 1 has the property F and if it holds true that if n has F, then so has $n + 1$, then every n has the property F. There are certainly infinite steps assumed in the proof but there is no question of regressus or the impossibility of this proof because of the infinite number of steps.

2 Whether it is Possible to Know Something with Certainty

2.1 Arguments Contra

2.11 Certainty concerning knowledge requires most precise concepts. Most precise concepts require definitions. In every definition, the definiendum is defined with the help of the definiens. However, each definiens contains again undefined terms. Since they have to be defined, again, this leads to a regressus ad infinitum. Thus nothing can be definitely or most precisely defined, and we cannot have most precise concepts. Therefore certainty concerning knowledge does not seem to be possible.

2.12 To achieve certain knowledge there are three possibilities according to Fries[1]: dogmatism, infinite regress, psychologism. If we want to avoid dogmatism, we have to rely on proof. However, proof leads to an infinite regress. If we want to avoid both dogmatism and infinite regress we have to rely on psychologism, i. e. on grounding knowledge on psychological phenomena like sense perception, introspection etc. However, none of the three provides certain knowledge. Therefore certain knowledge does not seem to be possible.

2.13 In order to know we have to think in a specific way, that is to judge, i. e. to assert or to deny. Consequently, in order to know with certainty we have to assert or to deny with certainty. Asserting and denying are psychological activities, and they obey psychological laws, not logical laws.

> … the only thing essential to our inquiries is that sentences signify something other than themselves, which can be the same when sentences differ … this something must be psychological.[2]

However, to assert or to deny with certainty requires logical laws, not psychological laws: "I understand by 'laws of logic' not psychological laws of takings-to-be-true (das Fürwahrhalten) but laws of truth (Gesetze des Wahrseins)"[3] Therefore it does not seem possible to know something with certainty.

1 Fries (1828)

2 Russell (1940) p. 189

3 Frege (1967) Introduction p. XVI.

https://doi.org/10.1515/9783110585797-002

2.14 According to the traditional view stemming from Plato, knowledge is defined as *justified true belief*. Consequently, certain knowledge must be very well or very firmly justified true belief. However, there are examples of justified true belief, which are not knowledge. Two more artificial examples are due to Gettier.[4] Scientific types of examples are the famous mathematical or physical conjectures which were very well justified and true but were never called 'knowledge' before they have been rigorously proved. Therefore: Even in the case of very well justified true belief we cannot speak of certain knowledge, or to know with certainty.

2.15 Rules of inference of a system of deductive logic are usually understood as transmitters of truth: if the premises are true then also the conclusion drawn with the help of the rules of inference must be true. Thus it must also hold: if there is certain knowledge of the premises there must be certain knowledge of the conclusion derived by the rules of inference.

However, as Wittgenstein says, rules of inference are (meta-liguistic) advice (information) of an author who thereby states: in this book there are such and such relations between the sentences.

> Now Russell wants to say: "*This* is how I am going to infer, and it is *right*". So he means to tell us how he means to infer: this is done by a *rule* of inference. How does it run? That this proposition implies that one? —Presumably that in the proofs in this book a proposition like this is to come after a proposition like this. —But it is supposed to be a fundamental law of logic that it is *correct* to infer in this way! — Then the fundamental law would have to run: "It is correct to infer… from …"; and this fundamental law should presumably be self-evident—in which case the rule itself will self-evidently be correct, or justified. "But after all this rule deals with sentences in a book, and that isn't part of logic!"—Quite correct, the rule is really only a piece of information that in this book only *this* move from one proposition to another will be used (as if it were a piece of information in the index); for the correctness of the move must be evident in its own place; and the expression of the 'fundamental law of logic' is then the *sequence of propositions* itself.[5]

However, if this is correct, the rule of inference relies on an advice of an author instead of being a principle (or instance of a principle) of logic which can be translated into a theorem of logic. Consequently the certainty of the conclusion would rely on a subjective advice of an author. Therefore, it seems there cannot be certainty (certain knowledge) of the conclusion even if there is certain knowledge of the premises.

4 Gettier (1963)

5 Wittgenstein (1956) 20.

2.2 Arguments Pro

The examples E 1–E 5 of ch. 1 show that knowledge is possible. Since they are very simple and transparent, they are also instances of knowledge with certainty. Therefore it is possible to know something with certainty.

2.3 Proposed Answer

The question "Is it possible to know something with certainty?" can be answered with Yes, provided that "certainty" is not interpreted in an unrealistic sense. Absolute certainty might be a heroic philosophical idea, but in science, it does not exist. What is important is the more modest concept of *certain enough* for the present purpose or task. This holds for all experimental results and for all scientific, historical data. Concerning hypotheses, laws, and theories we have to distinguish different levels of certainty. This also holds for the laws and theorems of logic and mathematics. Before we go into a more accurate elaboration of these different levels and of the experimental and historical data we apply the question to those results of ch. 1 which are relevant and also presupposed here.

2.3.1 Specific Cases of Knowledge with Certainty

(1) If *knowledge with certainty* refers to *all men* (all human beings) then the statements E 1–E 5 (ch. 1) are good examples for knowledge with certainty.

(2) If *knowledge with certainty* refers to each human being separately, then judgements of *introspection* like "I am now reflecting about introspection", "I am now thinking of my beloved", "I am now considering to go to the dentist" … etc. are good examples for certain knowledge. This can also be seen from the following fact: In all such cases, we can place "I know that" before such statements; for example: "I know that I am now reflecting about introspection." Observe that to place the knowledge-operator before judgements of introspection is not the same as the so-called K-K-thesis.[6] The K-K-thesis says that one can iterate the knowledge-operator; i. e. to every statement of the form "I know that…" one can add "I know that…" in front of the original statement. In this case, the original statement "I know that…" may concern any domain. For example, from "I know

6 Cf. Hintikka (1962) ch. V.

that he owns a car ” one can get “I know that I know that he owns a car”... etc. But in our case above the scope of the knowledge operator is restricted to judgements of introspection. It does not apply to any judgement (statement) whatsoever. On the other hand, one could iterate the knowledge operator also here, but it does not make much sense: “I know that I know that I am now reflecting on introspection” is only a complication of “I know that I am now reflecting on introspection”.

2.3.2 Reduction to Subjective Certainty?

A judgement of introspection can hardly be doubted or criticized. A question of a friend “Are you sure that you are now reflecting about introspection?” does not make much sense. The answer can be: yes, or yes I am sure, or yes it appears to me that way, or something similar. There has been a philosophical program which tried to reduce every judgement of sense perception to a respective judgement of introspection in order to achieve certainty for the judgement of sense perception. This philosophical program was called *phaenomenalism* since, in general, it reduced statements about things to statements about the psychic phaenomena of things. Thus, for example “this is red” is transferred to “this appears to me red”, or “the measuring instrument indicates 3 Ampere” to “the measuring instrument appears to me indicating 3 Ampere”... etc. The more sophisticated version of phaenomenalism is incorporated in the “Protokollsatz-Theorie”[7] of the Vienna Circle (mainly defended by Neurath and Carnap) in order to be most accurate. Thus the last example is developed into a long story of the kind: It appears to the observer A, on October 10 in the physical laboratory L, standing in such and such an angle in the direction towards the measuring instrument, which is illuminated by a bulb of 100 Watt... etc. that the pointer indicates 3 Ampere.[8] What can be said of such a transformation or reduction from a methodological point of view? First: What is gained is certainty in the sense of incontestability for the price of objectivity. Second: This type of phaenomenalistic subjectivity makes the statements immune to criticism. Third: There is an accumulation of complications without a selection of those factors which are relevant for the accuracy of the measurement report. Fourth: It neglects the aim of science. Science tries to find out what is the case, but not what appears to be the case. And this leads to the fifth point: “From its

7 Cf. Neurath (1932) p.393. The term “Protokollsatz” is due to Neurath. However according to Neurath the “Protokollsätze” are revisable, whereas for Schlick they were unrevisable affirmations about phenomenal states and he considered them as unrevisable foundations. Carnap (1932) 2, p. 432 ff., and 3, p. 107 ff. For a critical review see Popper (1934, 1959) p. 95ff. and (1972 p. 78 f.)

8 For a detailed elaboration of phenomenalism see Stegmüller (1958).

seeming to me—or to everyone—to be so, it doesn't follow that it *is* so."[9] This shows that the certainty of introspection cannot be used to provide certainty of sense perception of scientific observation. However it is important for the description of the psychic phenomena. But if this is done in a scientific way, obeying scientific methodological rules, the kind of certainty is also not absolute, since even in psychic experience by introspection deception is possible.

2.3.3 Certainty of Observational Results

> 'We are quite sure of it' does not mean just that every single person is certain of it, but that we belong to a community which is bound together by science and education."[10]

This is a quite adequate description of a situation in which many observational results are reported. One may think of a team in a research laboratory. There are two important points included in the above quotation: First: that every single person of the team is quite sure of the result. However, this individual certainty is not all that is at stake here; in a sense, it does not even exist as separately belonging to an individual of the team. Thus, secondly, there is an additional type of a *strengthened certainty* the members of the team have, because they are "bound together" by working according to a series of scientific methodological rules and sophisticated practices which are necessary for receiving the respective result.

2.3.3.1 The Certainty of Historical Facts

In the first of the following examples, the historical certainty relies on the memoirs of that politician who was essentially involved in the matter; in the second examples, it relies heavily on a conjecture about the character of the involved person.

Example 1: "Emser Depesche". Fact 1: Bismarck abbreviated and published (July 13, 1870) a telegram which he received from King Wilhelm of Prussia in such a way that it should offend France. The telegram was written by Privy Councillor Abeken (and sent to Bismarck from Bad Ems) containing a report about the requirement of the French ambassador that King Wilhelm should definitely renounce any claim to the succession of the Spanish throne (by the German nobility). This requirement was refused by King Wilhelm, and Bismarck aggravated the refusal. Fact 2: It did offend France. Fact 3: Bismarck says in his memoirs that he wanted the war between France and Germany but in such a way that France

9 Wittgenstein (1969) 2.

10 Wittgenstein (1969) 298.

(offended by the changed offending telegram) declares the war (on Germany). Fact 4. France did declare the war (July 19, 1870).

Example 2: Fact 1: Melanchton claims that Luther has advertised his theses on the church of Wittenberg on October 31, 1517. Fact 2: A letter from Luther on October 31, 1517, to the archbishop of Magdeburg-Mainz including provoking theses ("Streitsätze"). In this letter, Luther asks the archbishop to read his theses and to cancel the "instructio" of the archbishop concerning indulgence. Fact 3: One letter to Pope Leo X (May 1518) and to Kurfürst (Elector) Friedrich (May 21, 1518), both by Luther, saying that he first wrote to some high dignitaries of the church asking for an opinion about his theses before he announced them publicly. The historical hypothesis of Iserloh[11]: The public announcement of Luther's theses did not happen on October 31, 1517 but later. Reason: If he had his theses publicly announced on October 31, 1517, he would not have given time to the archbishop to think about them as he says in the letter and he would have been lying to the Pope and the Elector Friedrich. This, says Iserloh, is not compatible with Luther's honest character. Therefore, says Iserloh, the public announcement of the theses by Luther must have happened later, but before May 21, 1518.

In the first example, the historical documents lead to a rather *historically sufficient* certainty concerning some essential reasons for the war of 1870. Although also memoirs have to be examined concerning their reliability.

In the second example this is not the case; the historical documents are too incomplete. Therefore they have to be supplemented by a hypothesis; in this case a hypothesis about the character of Luther. Observe however that for methodological reasons of reliability such a hypothesis has to be tested and confirmed independently with the historical facts; i. e. it has to be substantiated with the help of different (independent) historical documents that Luther never lied in such situations. And this is a rather difficult task which shows that the certainty is much weaker here.

2.3.3.2 Certainty of Experimental Facts in Physics

Both of the following experiments show: (1) experiments are guided by theoretical models, (2) the actual execution of an experiment requires some or other idealisation which means abstracting or dispensing of some aspects or parameter, (3) as especially example 2 shows, sometimes one has to use and to rely additionally on laws of connected domains in order to execute the experiment.

11 Iserloh (1962)

Example 1: The spherical pendulum[12]: Consider the investigation of a spherical pendulum with small oscillations. We might start by listing what we suspect to be relevant parameters (variables) and their dimensions: Period of oscillation T (s); length of pendulum l (cm); mass of pendulum m (g), acceleration of gravity g ($cm \cdot s^{-2}$), angle of swing γ. This selection of parameters is already guided by a conjectural theoretical model which allows us to dispense (at least provisionally) with the following parameters. air (damping), air (draught), suspension (assumed to be rigid), thread (assumed to be inextensible) other environment (assumed to be not disturbing). In this sense, the theoretical model describes an "ideal" pendulum. The general relation among the selected parameters is $R\,(T,\ l,\ m,\ g,\ \gamma)$ and since we are looking for a law, we are looking for an invariance under changes (of units) of one or several of these five parameters. Suppose we want to find a solution for T as a function of l, m, g and γ. Then we might proceed as follows.

First discovery: a change of the (unit of) mass does not change any of the four other parameters Thus mass is not a relevant parameter. Result: $T = F\,(l, g, \gamma)$.

Second discovery: changes of l affect g if T does not change. Result: $T = f\,(l/g, \gamma)$.
Since γ is dimensionless and the dimension l/g is T^2 the result is $T = \sqrt{l}/g \cdot f(\gamma)$.

Third discovery by experiment: $f(\gamma)$ is close to 2π. Result: $T = 2\pi\sqrt{l}/g$.

For the certainty of the experimental result that T depends only on the length of the (spherical) pendulum, the mentioned restricted conditions about the angle, air, etc. are important. Small changes in these parameters may lead to effects which lie somewhere between small perturbations and weaker or stronger cases of chaotic behaviour.[13]

Observe that even if repeated experiments with the spherical pendulum lead to the conjecture that the result is a law, every single experiment has its own certainty although this certainty is strengthened by the fact that each experiment is embedded in a sequence of experiments with the same result.

Example 2: The charge of the electron: The idea of the experiment goes back to a proposal of F. R. Ehrenhaft (1907) for a method with charged oil-drops and was performed and improved by Millikan (from 1911 on) who got the Nobel-price for it in 1923. The theoretical guiding idea for measuring the charge of the electron was to establish an equilibrium between two forces: On the one hand the gravitational force $m \cdot g$ (m = mass of the oil-drop on which electrons are accumulated which are produced by a photo-electric effect; g = earth acceleration); on the other hand the

12 The example is due to Bunge (1967) I, p. 322.
13 CF. Lighthill (1986), Weingartner (1996) p. 51ff.

electrical force $e \cdot E$ (e = elementary charge, i. e. charge of the electron; E = electric field strength). The oil-drops are sprayed between the two plates of a capacitor: the gravitational force pulls them down, the electric force pulls them up when the upper plate is positively charged. If the field strength of the capacitor is adjusted in such a way that the charged oil-drops could be kept in suspension then the experimental situation could be simply described by the equation $m \cdot g = e \cdot E$ and e could be calculated by knowing m, g and E.

However, in the actual performance it was not possible to adjust the field strength of the capacitor in such a way that the charged oil-drops could be kept in suspension. Therefore it was necessary to observe two velocities: the velocity of falling down (by gravitational force) of the oil-drops when the capacitor is discharged and the velocity of going down slower than with gravity alone, when the capacitor was charged. To accumulate these velocities, it was necessary to use Stoke's law which incorporates the friction in the air. From measuring the two different velocities and the two respective equations, one could calculate the diameter of the oil-drops and the elementary charge.

The famous result was twofold: it gave the magnitude of the charge of the electron ($1,6 \cdot 10^{-19}$ Coulomb) in a rather direct way and it showed that any charge equals the charge of the electron multiplied by a factor which is an integer; i. e. the Millikan experiment was an important direct proof to show that charge comes in quanta. The first important hint for it was the photo-electric effect of Albert Einstein (1905) which showed that light comes in quanta but suggested strongly that also charge might come in (quanta of) electrons.[14]

The certainty of this result depends on several factors. First on an exact measurement (done with the help of a microscope) of the two velocities of the oil-drops between the plates. The exactness of this measurement has been improved by a series of this experiment from 1911 on. Secondly, the result depends on the reliability and accuracy of Stoke's law. This law is very well confirmed and is sufficiently accurate for Reynold-Numbers <0,1 where the Reynold-Number measures the proportion of inertia (of the oil-drops) to the viscosity (force of friction of the oil-drops in air). The latter is, in this case, more than ten times greater than the former which shows that Stoke's law is reliable in this domain. Stoke's law has been very well confirmed already before Millikan applied it to his experiment.

This means that despite of depending on several factors the result has a high degree of certainty. This is independently corroborated by the fact that the numerical value of the elementary charge (charge of the electron) leads to the same result when calculated with other more indirect methods.

14 Einstein (1905 a) §8 and 9.

2.3.3.3 Certainty of Experimental Facts in Psychology

The so-called “False Belief Test” is a type of experimental test with children which has been very frequently repeated always leading to the same result. The test was invented by Wimmer and Perner in 1983 at the department of psychology of the University of Salzburg and has been performed since in many other departments of psychology. In the false belief task (FB) children (between 3 and 5 years) are shown the following happenings either on a TV-screen or by playing with puppets:

Maxi interrupts his play with the ball to get a drink. He puts his ball into the blue wardrobe and leaves. Lena comes in, takes out Maxi’s ball and puts it into the green drawer. Control Question 1: Where did Maxi put the ball? Then Lena leaves for the playground. CQ 2: Where is the ball now? CQ 3: Who put it there? CQ 4: Did Maxi see that? CQ 5: Where did Maxi put the ball in the beginning? If children give a wrong answer to any of these questions, the story has to be repeated. If they answer all the control questions correctly, they are asked the test question: Where will Maxi look for his ball when he comes in again?

The result is this: Most of the children up to 3,5 years have difficulties to give the right answer, i. e. to understand that Maxi has a false belief concerning the place of his ball. From 4 years on, however, most of the children understand the false belief of Maxi and give the right answer. This result has been confirmed very well in the following sense:

(1) The FB was repeated in the same way as previously described above many times with the same result in different departments of psychology.
(2) There was a problem that 3 and 3,5 years old children did not pass the original FB. Rubio-Fernandez/Geurts (2013) therefore designed an experiment preserving essential points of the FB but having a reduced complexity with Duplo toys (girl, bananas, fridge with blue and fridge with red door). The girl figure puts bananas into the blue door fridge, goes on a walk (to be seen); meanwhile, the experimenter switches the bananas to the red door fridge. When the girl comes back looking for bananas, the child is asked where she will go. This FB* with reduced complexity was passed by 80 % of the 3 year-olds.
(3) Surprisingly, Onishi/Baillargeon (2005) showed in a non-verbal FB^+ which used observation of the kind of *looking* (in the direction, or shorter or longer) of 13–15-month-old infants that they significantly pass the FB^+. Further, Buttelmann/Carpenter/Tomasello (2009) showed that 18-month-old try to help (lead) an agent retrieve a toy while taking into account the fact that the agent hasn’t been in a position to see the toy being switched from box A to B; but the infant is not ready to help (lead) when the agent has seen the toy switched.[15]

15 For a survey of these and other related experiments see Gallagher (2015)

The results show an interesting methodological aspect concerning certainty: The certainty of a result (in this case FB) can increase with quantity of repeated tests on one and the same level (same age, same type of FB). It can also get support from "lower" levels: by experiments in which the complexity is reduced, but the gist of the original FB is preserved. It can get further support by experiments which preserve the original idea of FB but are completely non-verbal: either using interacting help-behaviour of the child or using the kind of looking of the infant.

2.3.4 Certainty in Logic and Mathematics

The certainty in logic and in mathematics is twofold. One is the certainty of proofs, the other the certainty of the principles or axioms. Both are already discussed in Aristotle's Posterior Analytics:

> But we say now that we do know through demonstration. By demonstration I mean a scientific deduction; and by scientific I mean one in virtue of which, by having it, we understand something... It is necessary for demonstrative understanding in particular to depend on things which are true and primitive and immediate and more familiar than and prior to and explanatory of the conclusion (for in this way the principles will also be appropriate to what is being proved)[16]

2.3.4.1 The Certainty of Proofs

There have been general investigations about proofs which developed (within the 20th century) into a domain called *Proof Theory*. A classic is Schütte's Proof Theory of 1960 (1977), a contemporary survey is Kreisel's (1968) and (1971) and Kohlenbach's Applied Proof Theory (2008). The actual proofs carried out in logic and in mathematics are of two types.

(1) One type uses an idea of Hilbert although it does not satisfy Hilbert's conditions and requirement. The idea was to completely formalize the proof in the metalanguage which proofs some general property of the mathematical system in the object language like consistency, completeness, and decidability. Hilbert's condition was that the proof in the metalanguage has to be finitistic in a rather strong sense such that every step was transparent like in a mechanical syntactic procedure. It was shown by Gödel (1931) that this condition cannot be fulfilled. On the contrary, such a proof in the metalanguage needs to go beyond such strong finitistic means. An example of a proof completely formalized in the metalanguage

16 Aristotle (APost) 71b 18–24.

is Gödel's incompleteness proof of 1931. The mathematical system in the object language was that part of the system *Principia Mathematica* (of Whitehead and Russell) required to establish the theorems of the arithmetic of integers. About this system (and similar systems) it is proven that there are (meaningful mathematical) propositions of this system which are neither provable nor disprovable within this system; i. e. the system is *incomplete* (i. e. some true arithmetical propositions do not follow from the axioms) and *undecidable* (some arithmetical propositions are neither provable nor disprovable). For the formalization of the proof in the metalanguage Gödel constructed a metamathematical coding system which assigns numbers to the signs and to the sequences of signs of the object system in a definite way.[17] On a first level of coding natural numbers 1–10 are assigned to the constants $\neg \vee \rightarrow \exists = 0\ s\ (\ )$, where '$s$' means successor (of a number). These and the following coding numbers are called *Gödel numbers*. On a next level Gödel numbers are assigned to variables: to variables for numbers $x, y, z \ldots 11, 13, 17 \ldots$ (prime numbers ≥ 11); to sentence variables $p, q, r \ldots 11^2, 13^2, 17^2 \ldots$ (squares of such prime numbers); to predicate variables $P, Q, R \ldots 11^3, 13^3, 17^3 \ldots$ (cubes of such prime numbers). By this procedure of coding, every sentence of the object system gets a certain Gödel number. Take as an example the sentence: Every natural number has an (immediate) successor: $\neg(\exists y)\neg(\exists x)(x = sy)$. According to the first and second level of coding, this sentence (which has 16 digits) gets the number $1, 8, 4, 13, \ldots 13, 9$. In order to assign to this sequence, one single unique number one takes the first 16 prime numbers with the power of the above Gödel numbers: $2^1 \cdot 3^8 \cdot 5^4 \cdot 7^{13} \ldots 47^{13} \cdot 51^9$. Take this Gödel number to be m. Coding a proof: Take as an example $\neg(\exists y)\neg(\exists x)(x = sy)$; from this it follows as a consequence by proof that: $(\exists x)(x = s1)$. Take the Gödel number of this conclusion to be n. Then to the proof sequence the Gödel number $k = 2^m \cdot 3^n$ is assigned. One can easily see that by this method of *arithmetization* every expression, every sentence and every sequence of sentences and therefore also every proof, can be represented by a unique number, i. e. Gödel number.

The whole idea is a full realization of what Leibniz and Descartes realized partially. It is roughly the idea to decide an argument or proof by a mathematical calculation with numbers in order to give the decision a maximum of certainty. Leibniz in addition to his characteristica universalis[18], constructed a coding system for Syllogistics which assigned to each of the three predicates (subject-term, object-term, middle-term) an ordered pair of a positive and a negative number[19], where the

17 The coding described here differs a bit from that Gödel used (for reasons of transparency) but only in an inessential way. Cf. Nagel and Newman (1959).

18 Cf. Leibniz GP7, p. 205.

19 Leibniz, OF, pp. 49, 50, 78, 79.

pairs of numbers have to be relatively prime to each other (i. e. having no common divisor). The relation that the predicate is included in the subject is represented by a mathematical function. The relation of derivation (inferring) of the conclusion from the premises is represented by the mathematical function of division. The following passages of Leibniz describe these two representations:

> Regulae usus characterum in propositionibus categoricis sunt sequentes: *Si propositio Universalis Affirmative est vera*, necesse est ut numerus subjecti dividi possit exactè seu sine residuo per numerum praedicati.[20]

> I come now to the numbers by which terms are to be expressed and to that end I shall give the following rules and definitions:
> (I) Any term of any proposition, whether a subject or a predicate is to be written as two numbers, the one being modified by a plus sign and the other by a minus sign…
> (II) A true universal affirmative proposition, for example
> Every wise man is pious
> +20, -21 +10, -3
> is one in which any symbolic number of the subject (e. g. +20 and -21) can be divided exactly (i. e. in such a way that no remainder is left) by the symbolic number of the same sign belonging to the predicate (+20 by +10, -21 by -3)… Conversely when this does not hold the proposition is false…[21]
> (IV) A true universal negative proposition, for example
> No pious is unhappy
> +10, -3 +5, -14
> is one in which two numbers of different signs and different terms have a common divisor. (For example -10 and -14, since the former has the sign + and the latter the sign -; the former is taken from the subject, the latter from the predicate…)[22]

By this method, all laws of conversion, of the square of opposition, and all the valid moods of assertoric syllogism can be proved to be valid. Moreover, the method exhibits as valid Łukasiewicz' four axioms of assertion and also his axiom of rejection.[23]

Leibniz in fact found by this method of arithmetization a decision method for syllogistics. Another decision method has been found for Syllogitics by the scholastic method of mnemonics aid, although this was found with good luck but without rigorous proof. However, Leibniz' mathematatization provided a rigorous

20 Leibniz (OF) p. 42

21 Leibniz (OF) p. 78

22 Leibniz (OF) p. 79. Translation by G. H. R. Parkinson in: Parkinson (1966). These rules are a selection concerning universal propositions. Rule III and V concern the particular negative and affirmative propositions. For a detailed discussion see Weingartner (1983) p. 172ff.

23 Cf. Łukasiewicz (1957) p. 126–129 proof in the sense of "calculemus" as he said himself. The certainty of the proof is reduced here to the certainty of calculating by the multiplication table.

Descartes invented another method of arithmetization. He invented co-ordinate geometry by assigning ordered pairs of numbers to the points of plane Euclidean Geometry. Descartes proved geometrical theorems about points by proving algebraic theorems about numbers. Also here the certainty is that of relatively simple and transparent mathematical calculating.

Still another type of formalization of a proof is by constructing a formalized metalogic. An example for it is Tarski's essay on "The concept of truth in formalized languages".[24] In § 2 of this essay Tarski constructs a formalized metalanguage for application to some object theory as the theory of classes (§3) or that of finite order (§4). In this metalanguage the concept of statement and that of logical consequence is defined and with the help of these that of deductive system, consistency, and completeness. Another feature of this metalanguage is the following:

> As we know from § 2, to every sentence of the language of the calculus of classes there corresponds in the metalanguage not only a name of this sentence of the structural-descriptive kind, but also a sentence having the same meaning.

In that metalanguage, Tarski is able to give an adequate definition of truth applicable to the object theory in question (theory of classes or that for languages of finite order). One of the important theorems proved there says that an adequate (i. e. obeying Tarski's truth condition or convention T or biconditional) and consistent definition of truth for the language $L1$ cannot be given in $L1$ but only in a richer metalanguage $L2$. This is called the non-definability theorem of Tarski. In Tarski's words:

> It is impossible to give an adequate definition of truth for a language in which the arithmetic of the natural numbers can be constructed, if the order of the metalanguage in which the investigations are carried out does not exceed the order of the language investigated.

Some results can achieve certainty also by the fact that several trials, to refute the result or to relativize it, fail. This has been the case with the theorems of Tarski concerning the definability of truth. There have been successful proposals to define truth in the object language contrary to Tarski's definability theorem. But these proposals do not refute Tarski's theorem, since they all deviate essentially from Tarski's requirements for a theory of truth which is formally correct (avoids the semantic paradoxes) and materially adequate (is an interpretation of the traditional correspondence theory of truth going back to Aristotle) and uses Classical Logic.

24 Tarski (1935) § 2. Engl. Transl. (1956).

Thus Myhill's (1950) object language system is *without negation*. Kripke's (1975) proposal accepts *truth value gaps*, is based on Kleene's three valued logic and accepts predicates which are only *partially defined*. Hintikka's (1991, 1996) IF-Logic is weaker than classical logic, neither complete nor finitely axiomatisable and has *independent quantifiers*. Deflationist theories which use Tarski's truth condition (biconditional) and its substitution instances (not his truth definition!) are not sufficient for a general theory of truth, as Tarski anticipated in § 1 of his essay and as Ketland (1999) and others have shown.[25]

(2) A second type of proofs in logic and mathematics is much more frequently used than the one which provides a complete formalization. It is formalized only partially or not at all. But this does not mean that the proof is not rigorous. Its method is what Kreisel described as *informal rigour*:

> It is a commonplace that formal rigour consists in setting out formal rules and checking that a given derivation follow these rules; one of the more important achievements of mathematical logic is Turing's analysis of what a formal rule is. Formal rigour does *not* apply to the discovery or choice of formal rules nor of notions; neither of basic notions such as *set* in so-called classical mathematics, nor of technical notions such as *group* or *tensor product* (technical, because formulated in terms of an already existing basic framework).
> The 'old fashioned' idea is that one obtains rules and definitions by analysing intuitive notions and putting down their properties. This is certainly what mathematicians thought they were doing when defining length or area or, for that matter, logicians when finding rules of inference or axioms (properties) of mathematical structures such as the continuum. What the 'old fashioned' idea assumes is quite simply that the intuitive notions are *significant*, be it in the external world or in thought (and a *precise* formulation of what is significant in a subject is the result, not a starting point of research into that subject).
> Informal rigour wants (i) to make this analysis as precise as possible (with the means available), in particular to eliminate doubtful properties of the intuitive notions when drawing conclusions about them; and (ii) to extend this analysis, in particular not to leave undecided questions which can be decided by full use of evident properties of these intuitive notions.[26]

In fact, many of the famous proofs in logic and most of the famous proofs in mathematics are proofs with informal rigour.

An example on the border between logic and mathematics is Zermelo's restriction of the axiom of comprehension in Set Theory.[27] There was no need of formalization for understanding that a two arbitrary use of the concepts of set or predicate leads to inconsistency. One example is Russell's paradox w. r. t. Frege's

25 For a detailed account see Weingartner (2000) chs. 3 and 7.

26 Kreisel (1967) p. 138f.

27 Zermelo (1908)

axiom 5 of *Grundgesetze*, others are the well known paradoxes of Set Theory like that of Cantor or Burali-Forti.[28] Zermelo could show in a way of informal rigour (i. e. without formalizing a proof for it in the meta-theory) that his *Axiom der Aussonderung* (axiom of separation, or of subsets) is a restricted form of the axiom of comprehension which—together with other axioms of pairing, union, and power set—provides sufficient means for constructing sets. In addition to that, Zermelo's set theory has been controlled and used by logicians and mathematicians since more than hundred years without leading to some inconsistency; therefore the degree of certainty is very high. We have here two supporting factors: The axiom of separation is intuitively understood and established by informal rigour, and it is additionally supported by frequent control and practice. Take as comparison an arithmetic theorem proved by Zermelo's set theory and one proved by the Riemann hypothesis. The first has both supports, the second only that of frequent control and practice.

A famous proof in mathematics, although its roots can be traced back to the incompleteness proof of Gödel (1931), is not based on complete formalization or formal rigour but uses informal rigour. I mean Matijasevic's answer to Hilbert's 10^{th} problem. The question is whether a diophantic equation (i. e. an algebraic equation containing only integers) has a solution in integers. Matijasevic proved that this problem is equivalent to Turing's *halting problem* of the computer (Turing machine) which Turing (1936) proved to be undecidable. Matijasevic's (1970) result is: To a given diophantic equation one can construct a computer program which halts if and only if the diophantic equation has a solution in integers, and vice versa.

The degree of certainty of such or similar results is very high for the following reasons:

(1) First the paper with the proof undergoes a severe referee committee before it is published.
(2) Many mathematicians are attracted to look at the proof and to control the proof for two reasons: (a) Since it is an answer to one of the most difficult problems in the famous list of Hilbert's 23 problems offered on the international congress of mathematicians in 1900. (b) Since the answer connects two apparently quite different areas, number theory, and theory of ideal computers.

The other two famous proofs in mathematics of the last decades are that of Fermat's theorem, by Wiles 1994, and that of the Poincaré-conjecture by Perelman 2004. Also, these proofs are examples for proofs by informal rigour rather than by for-

28 Cf. Fraenkel, Bar-Hillel, Levy (1973) p. 5ff.

malization of the metalanguage. The reason for the high degree of certainty are analogous to those given above concerning the proof of Matijasevic.

2.3.4.2 The Certainty of Axioms

The modern use of axiom also includes Aristotle's postulate and sometimes has even been shifted to postulate.

> An immediate deductive principle I call a postulate if one cannot prove it, but it is not necessary for anyone who is to learn anything to grasp it; and one which is necessary for anyone who is going to learn anything whatever to grasp, I call axiom.[29]

Aristotle's understanding of axiom fits very well to the examples E 1–E 5 of ch. 1, especially to E 5 of which he says that it is "the most indisputable of all beliefs."[30]

(1) Axioms of Logic: There is a detailed discussion and an important defence of E 5, the principle of non-contradiction, already in Aristotle in (Met) book IV sections 3 to 6. Especially the defence with the help of showing many absurd consequences when denying this principle. With such a method the degree of certainty for some knowledge can be considerably increased as in this case with the principle of non-contradiction.

Since this principle is only one principle of logic this leads to the question whether there are further principles of logic which can be viewed as first evident axioms with a very high degree of certainty.

It is manifest from Aristotle's metaphysics that he does not give that much credit to the principle of *tertium non datur* as to that of non-contradiction and he views them as different (not equivalent) principles. This is so for four reasons:

(i) He treats the principle of non-contradiction in sections 3 to 6 whereas he spends for the *tertium non datur* the separate section 7.

(ii) He seems to have understood that the validity of the *tertium non datur* is dependent in a non-trivial sense on the definiteness of the concepts or predicates used in the principle.[31]

(iii) As Paul Bernays says, it is important to notice that the *tertium non datur* is not presupposed as a principle for building up the 19 Aristotelian syllogisms:

> Historisch ist bemerkenswert, daß in der Aristotelischen Logik bei den bekannten Schlußfiguren das *tertium non datur* nirgends erfordert wird, weil das allgemein be-

---

29 Aristotle (APo) 72a15

30 Aristotle (Met) 1011b12

31 Cf. Aristotle (Met) 1012b20–24

jahende Urteil so interpretiert wird, daß es die Existenz unter den Subjektsbegriff fallender Gegenstände behauptet.[32]

(iv) As can be seen from section 9–13 of *De Interpretatione*, Aristotle is aware of the difficulties if the *tertium non datur* is applied to future contingent propositions. Although his solution there is by applying modal operators without giving up the principle.[33]

A principle which certainly counts as an axiom for Aristotle is the modus *Barbara* of his syllogistic: "Among the [syllogistic] figures the first is the most scientific one. For those sciences which are mathematical formulate their proofs in this figure, as arithmetic, geometry, optics…"[34]

This can also be defended from the standpoint of modern logic. The more general principle underlying *Barbara* is just the principle (or property) of the transitivity of implication or more general the transitivity of deduction. It is, in fact, a valid principle also in most of the well-known alternative logic-systems. Therefore it can be called a kind of invariance principle for logical systems:

$A \to B,\ B \to C / : A \to C \qquad A \vdash B,\ B \vdash C / : A \vdash C$

Further fundamental and simple principles which can take the role of axioms or of inference rules are:

Modus ponens:	$A \to B,\ A / :\ B$	$A \vdash B,\ A / :\ B$
Modus tollens:	$A \to B,\ \neg B / :\ \neg A$	$A \vdash B,\ \neg B / :\ \neg A$
Simplification:	$A \wedge B / :\ A$	$A \wedge B / :\ B$

There are at least two further principles which are very simple and transparent and which are also invariant w. r. t. most logical systems. It is the principle of identity $x = x$ and the so-called *dictum de omni*, the inference from *all* to *some* (belonging to the same domain); for example: if all material things are changeable, then some (at least one) material thing is changeable. In symbols:

$(\forall x)Fx \to Fa;\ (\forall x)Fx \vdash Fa$

32 Bernays (1976) p. 82

33 Cf. Hintikka (1964) and Weingartner (1964). Both authors independently defend Aristotle's solution with the help of modalities and without giving up the *tertium non datur*.

34 Aristotle (APo) 79a17.

The requirement of Aristotle that an axiom is evident and can be grasped by everyone will hold for the principles of non-contradiction[35] for that of identity, for the dictum de omni, for Barbara, for modus ponens, and for simplification. Modus tollens might be an exception and may need some logical consideration.

The degree of certainty of these axioms or inference rules is very high. Not only that they are constantly used in everyday language communication, but they are also permanently used in scientific argumentation. That *modus tollens* is an exception, can be seen from the fact that a similar form is a fallacy very frequently used in everyday language and sometimes even in sciences, except mathematics: $A \rightarrow B,\ \neg A / :\ \neg B$. It can be refuted by very simple and transparent counterexamples such as: If its a mouse it is grey. It is not a mouse. Therefore it is not grey.

This leads to the question for a general criterion or principle which distinguishes valid logical inferences from fallacies which are invalid moves. This principle is a kind of axiom or better meta-axiom which tells us what a valid logical inference is or which are the logical consequences of some premises. The principle is also called the principle of logical consequence or of logical deduction. It says: An inference (deduction) is logically valid if it always leads from true premises to true conclusions. Or: An inference or an argument-form is valid if there is no instance with true premises and a false conclusion. Moreover: A valid inference permits to conclude from a false conclusion that at least one of the premises must be false.[36] The principle of logical consequence is an invariance principle concerning deductive logical systems; i. e. it is satisfied by most if not by all different deductive logical systems. Note that "deductive" has to be inserted

35 The law of non-contradiction is not accepted as universally true by so-called *paraconsistent* logics. The purpose of these logics is to be allegedly better applicable to scientific theories which contain hidden contradictions. Although this is very questionable, we agree with a minimum criterion of paraconsistent logics which is also adopted by every *relevant* logic: The principle, that from a contradiction every (arbitrary) proposition follows (which is valid in Classical Logic) has to be rejected. It is also invalid in syllogistic because it cannot even be formulated as a syllogistic mode. Cf. Batens et al. (2000), Weingartner (2000) p. 316ff. Although one should observe that a case where an explicite contradiction is used to derive something completely arbitrary is rather seldom or does not occur at all in science and scientific discourse. Hidden contradictions may occur, but these (since unknown) are never used to derive anything whatsoever. Cf. the following footnote about Bolzano.

36 A model-theoretic formulation of the principle of logical consequence goes back to Bolzano (1929) §155, and independently to Tarski (1936). It is interesting to notice that Bolzano's concept of logical consequence (cf. Morscher (2016)) is narrower than Tarski's in the sense that it has a relevance condition inbuilt: The premises of a proof have to be consistent. Otherwise, we do not have a genuine proof available.

here since this principle is not satisfied by types of inductive logic. The *certainty* consists here in a highly transparent understanding of what deductive logic is or could and should be. It is rather an evident insight of understanding something similar of which what Aristotle would require of an axiom or of a simple real definition like that of man = rational animal.

(2) Axioms of Mathematics: If we take the Aristotelian conditions for an axiom *immediate, first, unprovable, true* and necessary for anyone who is going to learn anything whatever to grasp then the multiplication table up to ten is certainly an example. One which is better for scientific use are the axioms of Peano (1889) for the natural numbers viz. for the derivation of theorems about positive natural numbers (positive integers) w. r. t. addition:

(i) 0 is a natural number (NN).
(ii) If n is a NN then also $n + 1$ (n').
(n' is called the successor of n.)
(iii) If $n' = m'$ then $n = m$.
(iv) Not: $n' = 0$
(v) If 0 has the property F and if it holds: if n has the property F then also $n + 1$ has the property F. Then it holds: every natural number has the property F.

The first four axioms are as easy to grasp as the multiplication table up to 10. Axiom (v) is the axiom of mathematical induction. In general, induction is an inference to a generalization from its instances. It certainly does not satisfy the Aristotelian conditions for an axiom.

An important type of axioms of mathematics are the axioms of set theory. This is so because great parts of mathematics can be built on set theory. For example, with the help of set theory one can define ordinal and cardinal numbers and its principles can be derived from set theoretical principles.[37] The axioms of set theory, however, satisfy only partially the Aristotelian conditions for axioms. They are certainly not “necessary for anyone who is going to learn anything whatever to grasp.” On the other hand, the characteristics *immediate, first, unprovable, true* fit very well. For some of the axioms of set theory, it also holds that they can be grasped as insights into simple structures. This is so for example for the axioms of pairing (1), union (sum-set) (2) and power set (3):

37 Cf. Bernays (1958)

(1) For any two sets x and y, there exists a set z such that z contains just x and y.
(2) For any non-empty set x, there exists the set whose members are just the members of the members of x.
(3) For any set x, there exists the set whose members are just all subsets of x.

For the other axioms, this does not hold in the same way. Although the original axiom of comprehension: Every property (represented by a predicate) defines a class is very simple and intuitive, it is inconsistent as Russell showed first in a letter to Frege. The restricted form of this axiom (Zermelo's axiom of Aussonderung) does not have such a simple intuitive form anymore. The axioms of infinity, of replacement, of choice are of still more complex structure.

This shows more clearly that from the point of view of certainty, mathematics is not a homogeneous discipline: the multiplication table has a different type of certainty from the axiom of induction and from the axiom of choice.

2.3.5 Certainty of Laws of Nature

The certainty of laws of nature can be understood as their necessity. This is not the type of necessity which Leibniz had in mind when he tried to define the necessity of the laws of logic or the logical necessity: to be valid in all possible worlds; i. e. in all the worlds which differ from ours but are possible in the sense of not self-contradictory. The laws of nature are valid only in a subset of all possible worlds; their necessity is not logical necessity but the narrower natural necessity. What is this necessity? This necessity of laws of nature can be best characterized as their invariance. It can be explained and justified as follows:

(i) From a comparison with its opposition: Statements which are not necessary, i. e. statements which describe contingent facts, like initial conditions, do not express an invariance or symmetry. On the other hand, the invariance structure 'in the laws' implies that they do not hold contingently, but necessarily: "Nevertheless, there is a structure in the laws of nature which we call the laws of invariance."[38]
(ii) From the universal status of the core of a theory which consists of laws: not the contingent facts are contained in the core of a theory, but necessary correlations: "The irrelevant initial conditions must not enter in a relevant fashion into the results of the theory."[39]

38 Wigner (1967), p. 29. The special view of Wigner of "laws of invariance" as laws about (or in) laws was discussed in detail in sect. 5.3.1 (3) of Mittelstaedt/Weingartner (2005)
39 Wigner (1967), p. 8.

(iii) From a definition of a natural necessity. Such a definition was proposed by Popper: "A statement may be said to be naturally or physically necessary iff it is deducible from a statement function which is satisfied in all worlds that differ from our world (if at all) only with respect to initial conditions."[40] Popper's definition is concerned with dynamical laws (expressed by law statements). It can also be formulated with the concept of invariance: A dynamical law may be said to be naturally or physically necessary if it is invariant under a change of initial conditions.

The connection with a certain type of certainty can be understood as follows: The more changes of initial conditions are possible without affecting the law, the more certain and the more confirmed the law is.

An analogous definition for statistical laws may be formulated as follows: A statistical law, governing a huge number of individual things, may be said to be naturally or physically necessary if it is invariant under a change of "individual laws" governing the behaviour of these individual things.[41]

(iv) From a consideration of the "symmetry group of nature". We understand by "symmetry group of nature" the set of all changes which do not change laws of nature.[42] If we think in terms of models or possible worlds in which the laws are satisfied, then the symmetry group of nature is the set of all models or possible worlds in which the laws of nature are satisfied.

The above points should not be misunderstood in the sense that the laws of nature as we know them at present and as they are formulated at present are final and ultimate truths. All laws of nature as we know them at present are open to revision. Although not every law is on the same level of generality and some laws presuppose some others which are more fundamental. Thus the law of conversation od energy and the law of entropy are more fundamental than the laws of Classical Mechanics. However, concerning certainty we can assume that all fundamental laws of nature are good approximations to the truths, i. e. have a high degree of *verisimilitude*.[43]

40 Popper (1959), p. 433. Here "statement" is to be understood as a law statement which is neither a law of logic nor a law of mathematics.

41 Cf. Mittelstaedt/Weingartner (2005), ch. 12.3

42 Cf. Weinberg (1987), p. 73 and Mittelstaedt/Weingartner (2005) pp. 71ff.

43 The original idea is due Popper (1963), p. 391–397. For a revised definition which is not affected by Tichy's and Miller's objection see Schurz/Weingartner (1987), Weingartner (2000) ch. 9 and Schurz/Weingartner (2010).

2.4 Answers to the Objections

2.41 (To 2.11:) That not everything can be defined is a fact which has been pointed out already by Aristotle.[44] Pascal formulated an important condition for correct definitions, the condition of eliminability: definitions should enable us to replace, in every context, the defined term (the *definiendum*) by its *definiens*.[45] Behmann observed[46] that the definitions which give rise to the logical paradoxes do not satisfy Pascal's condition. The modern theory of definition contains two criteria, necessary for correct definitions: eliminability and non-creativity. However, eliminability makes sense only if the process of replacement (of *definiendum* by *definiens*) comes to a stop after a finite (usually small) number of steps of replacement. In this case, one arrives at basic concepts. This is usually the case in scientific application of explicit or implicit or recursive definitions. As a case of application consider the recursive definition of well-formed formula (wff) in propositional logic:
(1) $p, q, r\ldots$ are wffs.
(2) If p is a wff then so is $\neg p$.
(3) If $p,\ q$ are wffs then so are $p \wedge q$, $p \vee q$, $p \rightarrow q$, $p \leftrightarrow q$.
(4) Nothing is a wff except by conditions (1)–(3).

Here the basic concepts presupposed are only a rudimentary conception of one-place (negation), and two-place operators (the other logical connectives) applied to elementary wffs defined by (1). For the recursive definition of wff, it is not necessary that the other logical connectives have been introduced by truth tables or by natural deduction rules, although they can be further defined in this way.

A further application is the reduction of physical concepts to the three basic concepts *c* (centimetre), *g* (gram) and *s* (second). Here the definitions are explicit. This so-called *cgs*-system was later (1954) replaced by an international system SI of 7 basic concepts (basic units): *m* (meter), *kg* (kilogram), *s* (second), *A* (ampere), *K* (kelvin), *mol* (Mol) and *cd* (candela). Theoretically, all physical concepts can be reduced to these 7 basic concepts with the help of explicit definitions. Thus velocity (v) as length pro time-unit can be reduced to *m* (metre) and *s* (second). On the other hand, this does not mean that *m* (metre) is most basic concerning the possibility of exact measurement: Since 1983 *m* (metre) is defined as that length (distance) which is covered by light in vacuum during the time of 1/299792458 seconds. That means that the unit of length (meter) is based via the constant of light velocity on

44 Aristotle (APost) II, 7
45 Pascal (EGA)
46 Behmann (1931)

the more exact measurement of seconds with the help of atomic clocks. This shows that the basic concepts of a discipline are not absolute in a twofold sense: First, in the sense that they can become *defined* concepts by others since the values of the latter can be measured more exactly. Secondly, in the sense that their units are not observer-invariant constants since for example length, mass, and time depend on the velocity of the reference system relative to the observer according to the theory of Special Relativity. Consequently, also measurement rods and clocks are not absolutely rigid when they are freely moved in space.

The history of the basic concept of length (and its units: metre)also shows another point taken up in objection 2.11: Concepts in science are never absolutely precise. They are *precise enough* for a certain purpose at a certain time. Thus the meter was first defined (1795) as the meridian circle (of the earth) divided by 40 million. This distance was later realized as a measurement rod made of platinum-iridium (1889) in Paris. In 1960, the meter was defined as a factor $(1,65 \cdot 10^6)$ of the wave length of the electromagnetic emission of a stable Krypton isotope. Since 1983, it has been defined with the help of light velocity in vacuum (as stated above). The unit *one metre* is now much more precise than it was when it was introduced in 1795. Therefore it is an illusion and a sign of ignorance concerning scientific research, if one requires most precise or absolute precise concepts as said in objection 2.11. Moreover, the definitional regressus ad infinitum is more a theoretical fantasy than a real process occurring in science. One does not stop at some definiens absolutely. If one finds a better one later, the former will be replaced as the definiens with the wave-length of Krypton was replaced as the definiens with the help of second and velocity of light in vacuum. Nevertheless, there is no regressus. One is satisfied with the so far best definiens at a certain time as long as there is no better available and as long as it is precise enough for the present purpose. It can always be revised or improved later with the help of new knowledge.

2.42 (To 2.12:) The so-called “trilemma of Fries” assumes that the threefold alternative dogmatism or infinite regress or psychologism is complete in the sense that every type of knowledge has to be subsumed under one of the three alternatives. However, this assumption is wrong even if it might seem plausible at first sight. That it is wrong can be seen by an application of the three alternatives to different types of knowledge:

(1) First application: the knowledge of the multiplication table. Children begin to grasp it with about 7 to 8 years or earlier. It would be absurd to call this insight dogmatic. There is no regressus ad infinitum either; although one can build up number theory from basic axioms and definitions as Leibniz already proposed. Finally, the thought processes, which are psychological, are particular and individual, whereas the knowledge of the multiplication table is not.

(2) Second application: the knowledge of an observational result. Take the deviation of light rays by great masses (when they pass close to them) which was predicted by the theory of General Relativity. It was first observed and confirmed by the expedition of the Royal Society 1919 to South-Africa and has been confirmed with very precise instruments many times since. This type of knowledge never has been a dogma, it is not identical with some psychological actions of thought (of which scientist?) and it would be absurd to claim that it leads to a regressus ad infinitum.
(3) Third application: the knowledge of laws of nature. Take as an example Newton's law of gravitation: $F = G \cdot m_1 \cdot m_2/r^2$. It is certainly not a dogma and has not been one by Newton or any other physicist later. But it could be experimentally confirmed only 111 years later by Cavendish's famous experiment with the torsion balance. It has been confirmed many times since with severe tests. There is no regressus ad infinitum, however, and the knowledge of laws of nature is not identical with the psychological thoughts of anybody thinking about laws of nature.

From the considerations (1)–(3) it is evident that the trilemma of Fries is a rather illusionary idea not applicable to real examples of knowledge. Concerning the *infinite regress*, what has been said in section 1.43 above, also holds here.

2.43 (To 2.13:) This objection has two aspects: The first is that knowing, in contradiction to knowledge, is a psychological phenomena. And so knowing with certainty means a certainty of a psychological kind. To this it can be said, that it is correct that knowing (and also believing, asserting, accepting) are psychic phenomena. However, knowing as a psychic phenomena is a particular individual action of a human being at a certain time; whereas knowledge does not have these properties but is a conceptual entity. Therefore, what holds for psychic phenomena like knowing does not in general hold for knowledge. Therefore the conclusion of 2.13 is not proved. The second aspect is the reduction of phenomenalism or the reduction to knowing by introspection. This point has already been elaborated in section 2.32 above and need not be repeated here.

2.44 (To 2.14:) It is correct as the objection says that there are serious cases of well justified true belief which are not knowledge. The examples of Gettier (1963) are first of all very artificial and secondly either based on irrelevant inferential moves (counterexample II) or not a real counterexample (example I).[47] On the

47 Cf. Weingartner (1996a) p. 227ff.

other hand, there are many examples in the history of science that show clearly that well justified true beliefis not always knowledge. A recent example is Fermat's conjecture that there are no solutions for $x^n + y^n = z^n$ where $n > 2$. Fermat claimed he had a proof for it; later Euler showed that there is no solution for $n = 3$. Many mathematicians of the 20^{th} century like Tanijama, Faltings, and Frey contributed important pre-conditional results. These results contributed strongly to the justification of that belief. Still one could not speak of knowledge. Only after Wiles in a second attempt finished his proof in 1994, could one speak of knowledge. Another case was Poincaré's conjecture before it was proved by Perelman. A case in the history of physics was Newton's law of gravitation which was experimentally confirmed only 111 years later by Cavendish's famous experiment with the torsion balance. Another case was the predictions of Einstein's General Theory of Relativity (deviation of light rays and red shift) before they were observationally confirmed. And Friedmann's conjecture (1922) about the expansion of the universe, later (1929) confirmed by Hubble.

Coming back to the definition of knowledge as *justified true belief*, we may say that it is appropriate in a great many cases. In these cases, it is acceptable that *well* or *severe justified true belief* can be equated with *certain knowledge*. But there are interesting and severely exceptions—as has been stated above—and then one cannot speak of certain knowledge before the proof or the experimental test or the observation established the fact. After the proof, experimental, or observational test have been completed and controlled, it is appropriate to speak of *certain knowledge*.[48]

2.45 (To 2.15:) That inference rules are information to the reader—as Wittgenstein says—is correct but only in a rather extrinsic sense: it is a side effect and moreover connected with the accidental fact that inference rules are used in textbooks. The main question is whether the inference rules are valid or not. And they are valid if they satisfy the general *principle of logical consequence* (cf. section 2.3.4.2 (1) above). Therefore there can be certainty (certain knowledge) of the conclusion if there is certain knowledge of the premises, provided the inference rules are valid.

In addition, we may mention that there are inference rules which are very specific and appropriate to some logical system and others which are very general in the sense that they are invariant w. r. t. many or most (deductive) logical systems. An example for the latter is modus ponens. However, independently of being specific or generally applicable, these rules all have to satisfy the *principle of logical consequence*.

48 (Cf. ch. 4 for more on that.)

3 Whether it is Possible to Know Something Universal

3.1 Arguments Contra

3.11 Man knows something universal if what he knows does not hold only for himself but holds for everyone. However, as Protagoras says: “Man is the measure of all things, of those that are that they are and of those that are not that they are not.”[1] According to Plato, the meaning of this dictum is “what appears to you to be true is true for you, and what appears to me to be true is true for me.”[2] Therefore: It does not seem to be possible to know something universal.

3.12 Universal scientific hypotheses cannot be verified. Since one cannot test all instances by observation or experiment.

> A set of singular observation statements (‘basic statements’, as I call them) may at times falsify of refute a universal law; but it cannot possibly verify a law, in the sense of establishing it.[3]

Therefore one can know only single observational or experimental results but one cannot have knowledge of a universal hypothesis.

3.13 A universal hypothesis or law or theory surpasses all possible observations and experiments:

> The following, however, appears to me to be correct in Kant’s statement of the problem: in thinking we use, with a certain ‘right’, concepts to which there is no access from the materials of sensory experience, if the situation is viewed from the logical point of view.[4]

> Thus we may assert that at least some of the objects of which the theory treats are abstract and unobservable objects.[5]

Therefore: Since knowledge does not seem to go beyond sense experience and observations to such an extent, it seems hardly possible to know something universal.

1 Diels (1960) Protagoras, Fragment 1.
2 Plato (The) 152 a.
3 Popper (1983) p. 181
4 Einstein (1944) p. 287
5 Popper (1963) p. 186

https://doi.org/10.1515/9783110585797-003

3.2 Arguments Pro

Every more sophisticated technical success is based and relies on universal laws. This is true of technical instruments beginning from household technology via medicine and scientific laboratories to space shuttles. However, in order to use and apply the universal laws in these domains, they have to be known. Therefore it is possible to know something universal.

3.3 Proposed Answer

To “know something universal” has at least four different meanings. In the first case “universal” refers to *what is known*. The question then is whether it is possible to know some universal truths. In the second case “universal” refers to *whom who knows*. The question then is whether it is possible that something is known to everyone. In the third case “universal” refers to both, to *what is known* and to *whom who knows*. The question then is whether it is possible that everyone knows some universal truths. In the fourth case “universal” refers to the *reasons* or *motives* for knowledge. The question then is whether there are types of reasons for knowledge which are accessible to everyone. As will be shown subsequently, there is knowledge in all four senses.

3.3.1 Knowledge of Universal Truths

Man’s intellect seems to be constructed in such a way that it is much easier to grasp particular truths than to comprehend universal truths. This holds certainly for empirical truths. But it also holds of logical and mathematical truths with the exception of very simple cases like $p \to p$, the law of non-contradiction or the multiplication table. Thus it is impossible to grasp without proof that all true propositions of the 2-valued classical Propositional Calculus can be derived from the 3 axioms of Łukasiewicz and Tarski[6] with the help of the derivation rules of substitution and *modus ponens*.

Similarly, it is impossible to grasp without proof[7] that $x^n + y^n = z^n$ is not satisfied for $n > 2$. Man’s intellect cannot comprehend all the consequences of a universal proposition which is used as an axiom. A case in point is Frege’s axiom

6 Tarski (1956) p. 43

7 For proof in logic and mathematics recall section 2.341 above and the references given there.

of comprehension in his *Grundgesetze der Arithmetik* of which Russell showed him an inconsistent consequence.[8]

As an important type of knowledge of empirical universal truths, we might mention our knowledge of laws of nature. Take first as an example Kepler's laws of planetary motion:

(1) The orbits of the planets are ellipses (not circles) with the sun in one focus.
(2) The radius vector (from the sun to a planet) coats equal surfaces in equal times.
(3) $a^3/T^2 = const$ (where a is the big semiaxis of the ellipse of some planet, and T is the time of one orbital period of that planet).

Man cannot have an intellectual insight in any of the three laws and consequently cannot grasp its universality. However, man can invent them as conjectures with the help of intuition. Moreover, such a conjecture may have been based on a series of observations; in this case it was based on the accurate astronomical measurements by Tycho Brahe, who appointed Kepler as his assistant in Prague (in 1600).

Newton's laws of motion in his Principia and his law of gravitation are on a higher level of universality than those of Kepler. And also here no direct knowledge is possible. Concerning discovery, Newton's intuition might have played an important role; but his conjectures were certainly based on many observations of astronomers and of himself and certainly on his acquaintance with the theory of Copernicus and the laws of Kepler, which were already corroborated to a high degree then. Moreover, it can be objectively proved that Newton's theory is closer to the truth than Kepler's. This can be done with the help of Popper's theory of verisimilitude: Newton's theory has more true consequences and less false ones than Kepler's.[9]

On a still higher level of universality is Einstein's theory of Special Relativity (SR). It unifies the theory of Classical Mechanics with Maxwell's theory of electromagnetism. Again on a higher level of universality is Einstein's Theory of General Relativity (GR). It drops the restriction to inertial reference frames, and so incorporates accelerated movements and gravitation. It further drops the restriction to Galilean coordinates because the geometry of light rays in a three-dimensional reference frame, which moves with acceleration, is not Euclidean. Thus, the world

8 Frege (1893) Vol II, Nachwort.

9 Here "consequence" has to be replaced by "relevant informative consequence-element" (see Schurz/Weingartner (1987), (2010) and Weingartner (2000) ch. 9.) in order to escape the objections by Tichy and Miller's proof of 1974 which is based on irrelevant consequences allowed by classical 2-valued Propositional Calculus. Popper's theory of verisimilitude has been developed since he invented it 1963 in his *Conjectures and Refutations*, pp. 391–397.

line of light rays and of physical systems is geodesic, which means “as straight as possible”.[10]

How can we have knowledge of such general theories and of the universal laws which constitute them? As has been said already this is only possible by deriving consequences, making predictions and testing them. Both Einstein’s SR and GR have withstood all severe tests so far, and its famous predictions have been experimentally and observationally confirmed: dilatation of time and increasing mass for SR and perihelion of Mercury, deviation of light rays and red shift of galaxies for GR and several more. Since such particular tests with all the particular test-situations are accessible to our (human) knowledge, we trust the universal laws and theories invented and conjectured by ingenious scientists. And the main reason why we trust them is that they have been confirmed by reality or nature via their consequences which have been tested. This also holds of further fundamental laws as these of Quantum Mechanics, Thermodynamics and the Law of Entropy.

That the experimental and observational results are accessible to our knowledge does not mean that they are absolutely certain. As is evident from chapter 2 (cf. section 2.33) observational and experimental results can have a high degree of certainty although “absolute certainty” exists nowhere in science.

Further, one should not forget that without knowledge of the above-mentioned laws and theories the present technology and its works (beginning from household facilities to power stations, aeroplanes and rockets) would be impossible. Moreover, the fact that these many technological achievements work quite well shows the truth or at least the approximate truth of these laws.

3.3.2 There are Truths Known by Everyone

Examples which prove that are E 1–E 5 given in ch. 1. However, to analyse this question more accurately, a distinction made by Thomas Aquinas[11] seems suitable. Knowledge, he says, is of two sorts: One can be found by research, investigation and comparison only, the other is available for man without investigation, whenever needed and without error (under normal conditions). The first type of knowledge is not appropriate for considering that some truths are known by Everyone since different people are investigating different domains. Examples E 1–E 5 are examples for the second type of knowledge being based on a minimum of investigation and available when needed. Such examples could be extended not only in the domain

10 Cf. Mittelstaedt/Weingartner (2005) pp. 121f. and 124f.

11 Thomas Aquinas (STh) I, 79, 12 and 13; Ver 16,1.

of theoretical knowledge but also in that of practical knowledge as for example: an action which is not forbidden, is permitted, or promises should be kept.

If we add a further condition to the three above (without investigation, whenever needed, without error) there are only very few principles left of practical knowledge. The condition makes the principle independent of any system of ethics or system of religion. Like the law of non-contradiction when expressed in its most tolerant form is invariant w. r. t. most of the different types of logical systems the practical principle in question should be invariant w. r. t. different types of ethical and religious systems. An example is the following principle which was proposed by Thomas Aquinas as the first principle of natural law on which all ethics is based:

> Hence this is the first precept of law, that good is to be done and pursued, and evil is to be avoided. All other precepts of the natural law are based upon this.[12]

The "good" which is referred to is the natural good concerning which all men (in fact all living things) agree. This is manifest from the sentence before the quotation above:

> The first principle in the practical reason is one founded on the notion of good, viz. that good is that which all [things][13] seek after.

Another principle, which seems to satisfy the above four conditions, is the so-called *Golden Rule* (Mt 7,12): Do to others what you would have them do to you.

The above principles show that truths known by everyone are not restricted to statements which have an indicative form as those of E 1-E 5. However, also value statements and norms can be "known" by everyone; i. e. in such cases we may say that everyone has knowledge of the correctness or validity of such value-statements and norms.[14]

A sign for the widespread agreement in jurisprudence that everyone (also the men of the street) has knowledge of such basic principles of ethics is the court of lay accessors.

12 Thomas Aquinas (STh) I–II, 94,2.

13 "Things" is absent in the Latin original which reads: Bonum est quod omnia appetunt. This principle originates in the first sentences of Aristotle's Nicomachean Ethics.

14 For the question how value statements and norms can be introduced into scientific discourse see: Weingartner (1983), (1984), (2015) and Schurz (1997).

3.3.3 Everyone Knows Some Universal Truths

This is already manifest by the examples E 1–E 5 (of ch. 1) and by the two basic principles of ethics discussed in 3.3.2 above. There are similar further examples of simple universal truths in logic and mathematics and of universal norms in the domain of ethics and jurisprudence.

3.3.4 General Reasons and Motives

They can be understood in a twofold way: In one way they may be understood as first premises which are reasons and motives for deriving informative consequences (conclusions). In this sense, they obey the three (or four) criteria given in 3.3.2: available without investigation, available whenever needed, without error (under normal conditions; in addition, some of them may be invariant w. r. t. different domains of knowledge.

In another way "general reasons or motives" for knowledge may be understood in dispositional or habitual sense: those dispositions and habits as human abilities to perform actions of knowing without investigation, whenever needed and without error (under normal conditions).

3.4 Answers to the Objections

3.41 (To 3.11:) The so-called "homo mensura" statement of Protagoras can be interpreted in different ways.

(1) Platon's first interpretation in his dialogue Theaitetos understands "man" as *individual man* and "thing" as those things which we *know* by sense-perception. Accordingly, Theaitetos' definition of knowledge as perception[15] is referred to by Socrates as the well-known view of Protagoras. This definition is then shown to be inadequate by Socrates with several objections; one of them is the following inconsistency: after accepting that to remember something which was known (=perceived) before means also to know, it follows that one knows and does not know at the same time since when remembering one does not perceive (which is: not know).

15 Platon, Theaitetos, 151e, 2–3.

(2) Platon's additional interpretation in Theaitetos is that according to Protagoras "perception" means appearance. And his dictum is: " What appears to you to be true is true for you and what appears to me to be true is true for me."[16] Also, this statement is refuted by Socrates. At first it is not clear how Protagoras can claim that he is a teacher of wisdom (sophist) if it is impossible that he can teach. That this is impossible follows from his view that what appears to the pupil to be true is true for him such that a teacher can hardly convince him to accept what is true for the teacher; since the teacher cannot be wiser than the pupil because everyone is the best judge of his (her) own perception. Secondly, and independent of sense perception, the doctrine of Protagoras leads to the following paradoxical situation: Protagoras' friends discussing with him and, holding his doctrine to be false, hold the truth—according to Protagoras himself—although this "truth" contradicts Protagoras' doctrine.

If the doctrine of Protagoras is interpreted according to Plato as in (1) and (2) above, it can be refuted along the lines of Plato in his dialogue Theaitetos. In this case, the objection of 3.11 is not proved because the premises are not true.

However, there are two other interpretations possible of Protagoras' doctrine. The first is that "thing" is not understood as thing known by sense perception but by introspection. In this case man can easily be interpreted as individual man and it also would be correct to say "what appears to you to be true is true for you". This then would mean a reduction of knowledge to subjective certainty as it was described in section 2.3.2 of ch. 2 as a program of phenomenalism.[17]

The second interpretation is concentrated on ethical judgements and understands "man" as humankind. It is connected with the historical fact that Protagoras became a friend of Perikles and was invited by him to draw up a legal code for the colony of Thurii in 444 B. C.[18] According to this interpretation, the law is founded on certain ethical tendencies implanted in all men, although the individual varieties of law (as in particular states) are relative, one being sounder than another one.[19]

If the interpretation with *introspection* is meant, what has been said in section 2.3.2 also holds here. Such a reduction—although leading to a kind of subjective

16 Platon, Theaitetos, 152a. Cf. also Copleston (1947) pp. 87f. and 142f.

17 We do not claim that this interpretation can be substantiated historically. We didn't find a hint for it in the usual histories of Greek Philosophy. However, we want to propose this interpretation as an interesting possibility for further investigation.

18 Cf. Audi (1995) p. 752.

19 Cf. Copleston (1947 p. 88.)

certainty—is neither a serious objection against the possibility of certain knowledge nor of knowledge of universal truths.

If the interpretation understands "man" as humankind and "thing" as ethical or juridical or moral states of affairs then the respective relativism is rather weak and acceptable to a considerable extent. In this case, the Theaitetos interpretation is restricted to sense perception and not applicable to ethical judgements. But then, it is not a general objection against knowledge of universal truths because ethical rules and legal codes can be universal at least to some degree.

3.42 (To 3.12:) It is correct that universal hypotheses or laws and consequently also laws of nature can never be verified. Already a simple hypothesis like "all metals expand when heated" can never be tested with *all* metals, neither on the earth and much less in the universe. However, they can be criticized and revised, for example it has been shown that the above hypothesis is not true for every level of temperature: on very low temperatures the metal does not expand within a small range of the increasing heating temperature.

However, from all this, it does not follow that there cannot be knowledge of universal hypotheses. As has been outlined in section 2.3.5 there is knowledge of important properties of universal hypotheses or laws: of their natural or empirical necessity, of their verisimilitude. And in this sense there is also knowledge of something universal, i. e. of universal states of affairs described by universal hypotheses and laws.

3.43 (To 3.13:) What has been said in this objection and in the two quotations of Einstein and Popper is true and important. It can be stressed more and extended. We make such an extension first by continuing Einstein's passage and then give more reasons by Popper.

> As a matter of fact, I am convinced that even much more is to be asserted: the concepts which arise in our thought and in our linguistic expressions are all—when viewed logically—the free creations of thought which can not inductively be gained from sense-experiences. This is not so easily noticed only because we have the habit of combining certain concepts and conceptual relations (propositions) so definitely with certain sense-experiences that we do not become conscious of the gulf—logically unbridgeable—which separates the world sensory experiences from the world of concepts and propositions.[20]

20 Einstein (1944) p. 287

Popper lists the following important differences between observations, experiments, and measurements on the one hand and hypotheses, laws, and theories on the other[21]:

(a) Observations are always inexact, while the theory makes exact assertions.
(b) Each observed situation is always highly specific while the theory claims to apply in all possible circumstances, for example not only to the planets of our solar system but to all planetary motion and to all solar systems.
(c) Observations are always concrete, while the theory is abstract. For example we never observe mass points but concrete bodies.
(d) We can never observe anything like Newtonian forces, although we can—indirectly—measure forces via measuring masses and accelerations. This, however, is only possible by presupposing Newton's second law of motion.

As we have said, all this is true and important. How is it then possible to have knowledge of abstract universal hypotheses, laws and theories?

The answer to this question has been given mainly in section 3.3.1 above. We can have knowledge of them by deriving their consequences and investigating and testing them; and by investigating their presuppositions or restrictions and criticizing and dropping or relaxing them. Moreover, this type of knowledge can grow. Since we can distinguish theories which are nearer to the truth than other theories in one domain where a respective comparison is possible with the help of a revised idea of verisimilitude (see note 9 above).

21 Popper (1963) p. 186

4 Whether Knowledge is Justified True Belief

4.1 Arguments Contra

4.11 To know is more than to believe. Thus if I have achieved knowledge of something I do not and need not still have belief of it. Therefore: Knowledge cannot be justified true belief.

4.12 Famous mathematical conjectures like Fermat's conjecture or Poincaré's conjecture have been justified true beliefs before the proof was given. However, before the proof was established (by Wiles in 1994 and by Perelman in 2004) they could not be called knowledge. Therefore: Knowledge cannot be justified true belief.

4.13 Einstein's Theory of General Relativity made three famous predictions: (1) The correct perihelion of Mercury, (2) the deviation of light rays passing great masses, (3) the red-shift of galaxies. The first position was known by astronomical observations before Einstein published his theory of General Relativity in 1916 and therefore was a first observational confirmation of it since Newton's Theory gave the wrong result for the perihelion. However, the second and third prediction were justified true beliefs before the observational proofs were given. Neither (2) nor (3) could be called knowledge before the observational proofs were given. Therefore: Knowledge cannot be justified true belief.

4.14 According to Gettier there are counterexamples against the definition of knowledge as justified true belief. Suppose that Smith (who together with Jones applied for a certain job) has justified belief that

(a) Jones will get the job, and Jones has ten coins in his pocket and consequently has justified belief that:
(b) The man who will get the job has ten coins in his pocket.
Assume in addition the following:
(c) Smith will get the job and Smith has ten coins in his pocket but does not know this.

Then: Smith has justified true belief that (b) is true but does not know that (b) is true.[1]

1 Gettier (1963). The artificiality of that "counterexample" is rather astonishing; and so is the ignorance of such simple counterexamples as given in 4.12 and 4.13, since they are frequent in

https://doi.org/10.1515/9783110585797-004

4.15 As Hintikka says "to know that S is to be in a position to verify that S."[2] However, to have justified true belief that S, does not generally imply to be in a position to verify (that S). This is shown by the examples given in 4.12 and 4.13. Therefore: If Hintikka is right then knowledge cannot be justified true belief.

4.16 To answer question 4 above is to assume that to *know* some state of affairs *p* means
(1) that *p* is the case (is true),
(2) that it is believed that *p* is the case,
(3) it is justified that it is believed that *p* (is the case.)

However, according to Thomas Aquinas, "the proper object of the human intellect is the quiddity of a material thing."[3] To know the quiddity means to know *what something is* in contradiction to knowing *that something is the case* as it is described by the above conditions (1)–(3) for knowledge as justified true belief. Therefore: It seems that knowledge is not justified true belief.

4.17 A disjunction of an arbitrary statement *q* with a logical truth is true, and one can have justified true belief in it. However, this cannot be called knowledge since the statement *q* is completely arbitrary and can be false.
Therefore, knowledge is not justified true belief.

4.18 The most important statements on which our science is based are the law statements (the laws) forming the kernel of the scientific theories. However, as Bunge says "according to the definition of knowledge in terms of belief, most scientific statements would not qualify as bits of knowledge because they are at best partially true."[4] Therefore, knowledge cannot be justified true belief.

4.2 Argument Pro

We can say that we (all educated people) know that the spatial form of the DNA of living systems has the structure of a double helix. However, this does not mean that all educated people would be in a position to verify it; rather it means that we

the history of science. Nevertheless, Gettier's "counterexamples" (he offers a second one not less artificial) received widespread attention. Cf. the commentary in 4.44 below.

2 Hintikka (2013) p. 7

3 Thomas Aquinas STh, I, 85,5 ad 3.

4 Bunge (1983) TBP, Vol. V, p. 86.

(all educated people) believe those experts in molecular biology who in fact can verify it. This belief in experts is justified belief because it is based on controllable methodological rules for reliable investigations and results. Moreover, the double-helix structure of the DNA has been confirmed again and again to such an extend that we can speak of a true belief. Therefore such kind of knowledge can be defined as *justified true belief*.

4.3 Proposed Answer

4.3.1 Seven Types of Knowledge

Justified true belief is one important type of knowledge. But it is not the only type, and it is not applicable to all situations where we reasonably have to speak of knowledge; i. e. there are cases of knowledge which are not justified true belief. On the other hand there are cases of justified true belief which are not knowledge as the arguments in 4.12 and 4.13 show. In what follows, we shall distinguish seven important types of knowledge:

(1) Knowledge as true immediate objective understanding
(2) Knowledge as true immediate intrasubjective understanding
(3) Knowledge as ability to prove
(4) Knowledge as ability to verify
(5) Knowledge as justified corroboration and explanation
(6) Knowledge as justified true belief
(7) Knowledge as possessing epistemic entropy and epistemic information (ch. 5)

Concerning these seven types, one has to observe that they differ not only by the description given in (1)–(7) but also through other very general aspects.

Aspect 1: Actual versus potential (viz. dispositional): Concepts like knowledge, belief, doubt, perception, assumption, opinion, etc. can be understood in a twofold way: as activities or as capabilities; or in other words as actual or as dispositional. Thus, knowledge as *true understanding* when exemplified by the examples E 1–E 5 of ch. 1 is *usually* understood as actual, as an actual activity of the person. It may, however, be interpreted dispositionally too, as a capability to perform such activities described by E 1–E 5. Knowledge as *ability to prove* suggests by its description that it is *usually* understood as dispositional, as a capability. Also, in this case, it may exceptionally be understood as an activity of proving or verifying. Knowledge as *justified true belief* may be understood in both ways, as actual believing in a justified way or dispositional as a capability to perform such actual justified true belief whenever needed. Knowledge as *possessing information* is *usually* understood as dispositional, i. e. as possessing justified information ready to be used when needed.

Aspect 2: Objective versus subjective: One sense of *objective* versus subjective applied to knowledge is that knowledge is independent of individual and personal circumstances, but not independent of human reason in general as Frege says.[5] This holds for all types of knowledge to be discussed here.

A further sense of *objective* means that all individual subjects (persons) come to the same result when having knowledge of a certain type or about a certain state of affairs. This is also the concept of knowledge used in Classical Mechanics, i. e. the knowledge possessed by an observer: the facts of Classical Mechanics are assumed to be observer-invariant. This second sense of *objective* versus *subjective* generally holds for the first type of knowledge, to be discussed here: knowledge as true understanding.

For the other types (2)–(7) of knowledge it depends on the domain to which knowledge is applied. We certainly have to assume that when the domain is logic or mathematics everyone who is able to make the proof will come to the same result. This is not so clear when the domain is empirical science and when "to prove" means to verify some singular facts. For example, the statement "the tree is before the rock" might be correct from viewpoint A but incorrect from viewpoint B. Thus, in many situations of empirical sciences, the inclusion of or relativization to the point of reference (or reference system) is important. That is, we do not generally have observer-invariant knowledge in the empirical sciences. Moreover, the observer-invariance holding in Classical Mechanics has to be given up in relativistic physics (Theory of Special and General Relativity). This does not mean that different observers cannot come to an agreement; they can if they relativize their statements to a reference system.

Still, a further sense of *objective* versus subjective is due Kant and means: universally valid and necessary.[6] This does not hold for all propositions known by everyone or by someone. It holds for fundamental laws of logic as for E 5 but it does not hold for a singular fact known by direct or indirect observation. If universality is applied to those who know, then again this holds only for some propositions known by everyone like E 1–E 5 or similar; and some of them are not necessary but contingent as in "I exist" for example.

5 Frege (1884) p. 72

6 Kant (1783) § 19

Eventually, a sense of *objective* versus subjective is due to Popper when he speaks of "epistemology without a knowing subject".[7] Popper wants to avoid speaking of the act of knowing which belongs to his world 2, the psychological world of states of consciousness, of mental states and of dispositions to act. However, Popper wants rather to speak of *that what is known*: of the objective contents of thought, the contents of journals, books, and libraries; further of theories, hypotheses, problem situations, states of discussion, critical arguments, etc. they belong to the inhabitants of world 3. This shows that much more belongs to world 3 than *what is known*. But knowledge is understood here in this more abstract sense and not as a type of belief.

This abstract sense of knowledge understood as that *what is known* can be used as an interpretation of all seven types of knowledge. Thus, we may ask what is it that is truly understood w. r. t. the principles E 1–E 5. Alternatively, what is proved, or what is justified truly believed or what is received by getting informed.

4.3.2 Four Modes of Knowledge

Before we try to give a more differentiated answer to question 4, we want to make a fourfold distinction:
(A) Knowledge *that* something is the case
(B) Knowledge *why* something is the case
(C) Knowledge *how* something is
(D) Knowledge *what* something is

Books on knowledge and systems of epistemic logic (i. e. of the logic of knowledge, belief, assumption etc.) treat knowledge usually almost exclusively as (A). This can be justified with two reasons. First, because many examples of knowledge in everyday life and scientific discourse can be treated as mode (A). Thus, examples E 1, E 2, E 5 of ch. 1 and the examples of observational results and of the axioms (ch. 2) can very well be interpreted as knowledge that something is the case. Secondly, because in many cases, modes (B)–(D) can be reduced in some good sense to mode (A). Thus, knowledge *why* some state of affairs p obtains, can be translated to the knowledge that p follows from some premises q and r, or from some law l, or that p is the effect of some cause c. Similarly, knowledge *how* something is, for example, how the chemical element carbon is built up, might be reduced to the knowledge *that* it has 6 electrons, *that* 2 are on the inner shell and 4 are on the outer shell,

7 Popper (1968)

etc. Finally, knowledge *what* something is, for example, what a circle is, can be formulated by the knowledge *that* a circle is the set of all points equidistant from one point.

However, modes (B)–(D) need not be reduced to type (A) but can get their own appropriate answer. Thus, knowing *why* something is the case can mean to know the proof for it as it is with some theorems of mathematics or logic, for which a proof has been found (recall section 2.341); or it can mean to know the (an) explanation for it as it is when some event, say an eclipse or the mutual attraction of two bodies is explained with the help of laws of nature plus some initial conditions (recall section 2.35). Further, knowing *how* something is, can mean to know the structure of the molecule DNA, or know to design a reliable experiment or know to give a proof.[8] Similarly, knowing *what* something is may mean knowing its definition or a unique description of it. It should be mentioned that according to Aristotle an answer to a *what is* question by a real definition describes the essence or nature of a thing (where 'thing' means mainly a biological species the members of which are concrete individuals or substances). This holds also for many medieval thinkers such as Thomas Aquinas. The question for a correct definition, since it tells us the nature of things, was important for these thinkers.[9] Independent of the fact that definitions describe sometimes the nature of things or come close to it, the question for a correct or true definition has been neglected frequently because of the widespread wrong view that all definitions are only useful conveniences or abbreviations which are neither true nor false.[10] That some (e. g. abbreviations) are, is trivial. Otherwise, this view is refuted not only by the Greek definition of a circle but also by important definitions in empirical science like that of chemical element or that of living organism. The latter two have been improved and revised which also shows that the above view is wrong.[11]

There is a further sense of knowing *what* which cannot be easily reduced to knowing *that*. In this sense to know means to be acquainted with it. This can be understood in a more loose or in a more accurate way. In a more loose way when I say "I know his sister". In a more scientific way when I say I know the main theses of Marxism or I know the symmetry principle of parity. That these cannot be directly reduced to *knowing that* is shown by the following example which expresses an independence of "knowing p" from "knowing that p": I know the symmetry principle of parity (p) and I know that it (p) was refuted by Lee and Wang in 1956. However, in an indirect and more clumsy way also this example of

8 Cf. Hintikka's discussion of knowing how, that and what in Hintikka (1974) ch. 2.

9 Cf. section 1.42 (3) above and Weingartner (2000) ch. 5.

10 Cf. Whitehead and Russell (1927) p. 11

11 For a detailed discussion cf. Weingartner (2000) ch. 5.

knowing what it is may be reduced to *knowing that*, thus: I know that the symmetry principle of parity says that… (means that…) and that this principle was refuted by Lee and Wang in 1956.

4.3.3 Knowledge as *True Immediate Objective Understanding*

In his Posterior Analytics Aristotle says at the beginning that "all teaching and all intellectual learning come about from already existing knowledge".[12]

Also any scientific demonstration (proof) needs such pre-existent knowledge. These are principles "which are true and primitive and immediate and more familiar than and prior to and explanatory of the conclusion".[13]

Of these principles which cannot be proved, some are not necessary for anyone who is going to learn anything .. and these he calls *postulates*; but those others which are necessary to grasp for anyone who is going to learn anything he calls *axioms*.

An example of an axiom which satisfies the above characteristics best is the principle of non-contradiction according to Aristotle (cf. E5 of ch. 1). In addition, we could list very simple principles of propositional calculus like modus ponens and the principle *dictum de omni* (what is true for all members is true for one particular member) of First Order Predicate Logic. For mathematics $1 + 1 = 2$, $2 + 1 = 3$ would be examples from which (together with definitions) Leibniz built up the natural numbers.[14]

For all these examples of axioms or first primitive truths, the characteristics of Aristotle fit very well: true primitive, immediate, more familiar than and prior to the conclusion. We drop here Aristotle's last characteristic "explanatory" since this may fit to some but not to all of our examples.

Leibniz calls such *axioms* ''primitive truths of reason" and characterizes them as follows:

> The primitive truths of reason are those which I call by the general name of identicals… Those which are affirmative are such as the following: everything is what it is, and in as many examples as we may desire, A is A, B is B.[15]

12 71 a 1

13 71 b 21

14 Leibniz (NE) 4, 7 10

15 Leibniz (GP) 5, p. 343; (NE 4, 2, 1. Leibniz also counts the principle of non-contradiction to the primitive truths. On some passages, Leibniz says that the principle of non-contradiction asserts that every proposition is true or false (NE) 4, 2, 1. This is a rather restricted formulation which

These examples exemplify the type of knowledge that shall be described as *true immediate and objective understanding*.

Knowledge 1 (K1) $=_{df}$ true immediate objective understanding of primitive logical or mathematical principles.

$aK1p$ for a knows in the sense of Knowledge 1 that p.

$aK1p$ iff one of the following two conditions are satisfied:
(1) p is a simple logical principle, i. e. an axiom in the sense of Aristotle, such as the principle of non-contradiction or the principle of identity $x = x$ or similar.
(2) p is a simple mathematical statement (e. g. a statement of the multiplication table).

4.3.4 Knowledge as *True Immediate Intrasubjective Understanding*

Knowledge 1 is such that (3) of section 1.3 above is satisfied: it holds for those propositions (or states of affairs) p such that *everyone* has knowledge concerning p. However, as Descartes points out correctly, this condition (3) is satisfied also for primitive and immediate empirical propositions:

> Thus each individual can perceive by intellectual intuition that he exists, that he thinks, that a triangel is bounded by three lines only, a sphere by a single surface, and so on.[16]

This quotation by Descartes contains two types of principles: the first two examples are *empirical* and belong to K 2 (true immediate intrasubjective understanding). The third (triangle) to K 1 above (4.3.3) and the fourth might need some additional definition to be understood or to be proved. In this latter case it would belong to K 3 (below).

The true immediate understanding of such empirical propositions—as the *fallor ergo sum* of Augustine, the *cogito ergo sum* (of Descartes) and those mentioned above by Leibniz—is not a completely "subjective" one; because everyone understands them in the same way although he (she) understands them for himself

rules out those many-valued logics which have some other values besides true or false; although it does not rule out such many-valued logics which distinguish more than one value for true and more than one for false without assuming values different from true or false. However, Leibniz also knows other more tolerant (or general) forms of the principle. Cf. Rescher (1969) p. 143ff. and Weingartner (1983a), sections 2.2 and 2.4.

16 Descartes (RD) 3, (AT) 10, 368

(herself). We might call this understanding "intrasubjective". And consequently we might define a second type of knowledge thus:

Knowledge 2 (K2) $=_{df}$ true immediate intrasubjective understanding of primitive empirical statements by introspection.

$aK2p$ iff one of the following two conditions are satisfied:
(1) p is a simple empirical statement about one's own existence (like the *fallor ergo sum* of Augustine or the *cogito ergo sum* of Descartes).
(2) p is a simple statement of introspection asserting one's own present psychic action (like: I now wish to see my friend).

4.3.5 Knowledge as *Ability to Prove*

"To prove" can be understood in two ways: First, as to prove in the strong sense of a logical or mathematical proof. And second in the sense of verifying empirical individual facts (cf. 4.3.6 below). Accordingly, we distinguish two types of *ability to prove*.

Ability to prove in the strong sense of a proof in logic or in mathematics: This type of knowledge is discussed in many places by Wittgenstein, especially in the *Remarks* on the *Foundations of Mathematics*, part II:

> A mathematical proof must be perspicuous. Only a structure whose reproduction is an easy task is called a 'proof'. It must be possible to decide with certainty whether we really have the same proof twice over, or not.
>
> A proof ought to show not merely that this is how it is, but this is how it has to be.[17]

If we look at famous and difficult mathematical or logical proofs, it is very questionable whether they are perspicuous; and their reproduction is certainly not an easy task. Take as examples the proofs of Matjasiewich (1970, 10^{th} *Problem of Hilbert*), Wiles (1994, *Fermat's Conjecture*), and Perelman (2004, *Poincaré conjecture*). Even the last claim that it is decidable whether we have the same proof twice is not guaranteed viz. can be very complicated to decide (provided that the *same* does not mean syntactically the same signs).

The second quotation stresses two aspects of the proof correctly: First, the difference between factual truths on the one hand and between logical and mathematical truths on the other, pointing out that a proof is not just a factual truth.

17 Remarks II, § 1, 9.

Second, that the rules or principles for a proof have a necessary and/or normative character; they say that it has to be that way.

Gödel knew in the sense of this type of knowledge K3 that there are statements of arithmetic which are undecidable by arithmetic. And he knew that the consistency of arithmetic is not provable in arithmetic, i. e. by purely arithmetical means. In the same sense Matijasevic knows (since 1970) that the question whether a diophantic equation (that is an equation which contains only integers) has a solution in integers (this is Hilbert's 10^{th} problem) is equivalent to the question whether a computer program comes to a stop (the *halting problem*): both questions are undecidable. Alternatively, Wiles knows (since 1994) that Fermat's famous conjecture was right, i. e. that the equation $x^n + y^n = z^n$ has no solutions for $n > 2$.

In all the three examples "knows" is in the first place restricted to the three persons who made the proof. However, we can surely extend this type of knowledge to some other persons: those who have proven the same result with similar or other methods of proof, and alternatively, those who have controlled the respective proof step by step. In the last case (Fermat's conjecture) a gap in the proof was discovered by Ribet in the earlier version of 1993 and solved by Wiles in 1994. Moreover, several people have done preparatory work for this proof in the sense that they proved related important results (Frey, Tanijama, Faltings). However, for such difficult proofs as the three mentioned, there will be still very few people who *know* in the sense of being able to do such a proof. Other scientists in or outside this domain have knowledge in the sense of justified true belief the knowledge of the experts. But there are many other examples which show that the *ability to prove* in the sense of a mathematical proof can be satisfied by every educated person. Thus, for example, a calculation that $2^{10} = 1024$ or that $8 \cdot 31 \cdot 0,14159 = 35,11432$ can be done easily by every high school student (see the examples in section 2.341). It is an important fact that such logical or mathematical proofs can be controlled by colleagues working in the same or a similar domain. This fact is a sufficient reason to call these proofs or the respective knowledge "objective". Accordingly, we arrive at a further type of knowledge thus:

Knowledge 3 (K3) $=_{df}$ ability to make logical or mathematical proofs.

$aK3p$ iff one of the following two conditions are satisfied:
(1) a is able to carry out a logical proof for p.
(2) a is able to carry out a mathematical proof for p.

4.3.6 Knowledge as *Ability to Verify*

This type of knowledge is a further kind of proving: the *ability to prove* in the sense of verifying it. There are many ways in which we can have knowledge in the sense of being able to verify it.

First by *direct observation*: to notice a car accident, to verify the claim that there are Alpine Ibex in the zoo of Hellbrunn (Salzburg), to find out that it is snowing today, and so on. This type of knowledge is available for all human adults under normal conditions.

Second by *indirect observation*: To find a tubercle bacillus in the sputum of some patients (with the help of a microscope), to identify a criminal by checking his fingerprints, to identify a star with a telescope, and so on. This second type of knowledge by verifying is available only for specially educated people within the respective domain. Third, by *scientific experiment* which uses direct and indirect observation but transcends both. This type is available only to experts (see the examples in section 2.33 of different scientific domains). The type of knowledge described in the above way can be defined as follows:

Knowledge 4 (K4) $=_{df}$ ability to verify by direct or indirect observation or by experiment.

$aK4p$ iff p_d is a statement describing a possible real state or event at space time interval d (cf. 5.322), and one of the following three conditions are satisfied:
(1) a is able to verify p_d by direct observation.
(2) a is able to verify p_d by indirect observation.
(3) a is able to verify p_d by scientific experiment.

Concerning indirect observation and scientific experiment we have to make an important reservation: All severe scientific observations or experiments have to be interpreted in the light of scientific theories to establish a scientific result, say a measurement result. Recall the description of the famous Millikan-experiment for the charge of the electron or the inspection of Melanchton's claim about Luther's theses in sections 2.3.3.2 and 2.3.3.1 above. A further simple example is obtaining a measurement result on a voltameter which presupposes laws of electric current, of magnetism, of elasticity, etc. A more complicated case exists where the observed movement of a star together with the process of its losing mass has to be explained by the effects of a black hole. This is described below in section 12.32,1.
These considerations lead to two general questions:

Q 1: Are the different types of knowledge K 1–K 7 disjunct?

More specifically here: If K 4 needs K 5 (corroboration and explanation of hypotheses, laws, and theories), then they seem not to be disjunct. However, since K 5 needs K 4 for corroboration this leads to a second question:

Q 2: Is there a certain circularity among some types of knowledge, especially in K 4 and K 5?

Concerning Q 1 we do not claim that the different types of knowledge are disjunct in a strong or sharp sense. Observe that K 2 as introspective awareness can *accompany* all other types of knowledge. Further, *belief* as in K 6 is involved in K 1–K 4 as a *consequence of understanding*, *proving* and *verifying*, even if not as its central activity. Further, *possessing information* K 7 is present in all other types of knowledge K 1–K 6, again not as their central activity but as the *final result* of the activity.

Concerning Q 2 we admit that there is, in fact, an important sense of circularity—where K 4 and K 5 depend on each other—involved in every serious scientific corroboration and confirmation of scientific hypotheses, laws, and theories. However, this does not destroy the severity of scientific corroboration and confirmation. On the contrary, this is the only serious way. That this is so can be shown by the following considerations:

(a) No scientific measurement can be done relative to an independent or absolute external fixed point. This we have learned from the Theory of Relativity. Newton thought that the "real"—in contradistinction to the "relative"—movement of the coach before the house or the planet can be calculated *in principle* with reference to a fixed point, i. e. absolute space. But this is not possible as Leibniz and Mach pointed out already.

(b) Take as an example measuring the strength of an earthquake with the help of a seismograph. The seismograph has to be mounted on the earth which then shakes. The big mass hanging on the rope which records the shakes is not an external fixed point, but a part of the seismograph that is only relative independent of it; i. e. it is not at (absolute) rest but is dragged to some extent by the shaking seismograph. The so emerging measurement-mistakes have to be corrected by rather complicated theoretical calculations that have to use geological and physical hypotheses and laws.

(c) What were the conditions under which the Special Theory of Relativity (SR) has been tested? The basic conditions, mainly due to Einstein were these: (i) Physical measurement instruments (rods and clocks) are real physical objects. (ii) Therefore, they have to obey physical laws; but which ones? The laws of Classical Mechanics? (iii) Of course not: The measurement instruments applied to test SR have to obey the laws of SR.[18] Condition (iii) involves certainly a circularity. However, this does not mean that the test is not reliable. The time-dilatation and mass-increase predicted by SR was in fact tested that way by Hafele and Keating (1972) with atomic clocks in orbiting aeroplanes and by huge particle accelerators.

The considerations (a)–(c) show that the circularity involved when knowledge K 4 is used for the corroboration and confirmation of hypotheses, laws, and theories (K 5) does not lead to non-reliability but is the only way to receive severe tests.

4.3.7 Knowledge as *Ability for Corroboration and Explanation*

(1) Corroboration: A scientific theory cannot be proved in the sense of verifying it. Verification can only be done concerning singular facts or states: that something happens at a certain space-time point. Even this (recall the examples of 4.3.6 and 2.3.3) has to be taken with care since such verifications cannot be taken to be absolute but until further notice such that the results can be revised or made more precise on further investigation.

On the other hand, a scientific theory or even a scientific hypothesis usually refers to indefinitely many objects. Thus it could be verified only by investigating indefinitely many observational data. Since this is impossible, a verification of a scientific hypothesis or theory is impossible too. There are mainly two views for confirming or corroborating theories:

One is that of Hempel and Carnap understood as an inductive concept of confirmation. According to Hempel and Carnap, a hypothesis H is confirmed by the finite set of observation statements B iff B partially (or inductively) implies H. The quantitative degree of confirmation is expressed by $c(He)$ which represents a conditional probability (of H given e). The different proposals to interpret this probability by Carnap (understood mainly as subjective probability)[19] and others, later did not lead to an acceptable concept of confirmation especially when well-known hypotheses, laws, or theories of natural science are taken as examples. One simple reason is that a universal hypothesis, law or theory refers to indefinitely many observational statements and moreover, it refers to thus far unknown objects and observations. Consequently, the degree of confirmation interpreted as a probability

18 This principle is also an explanation for Einstein's critical view towards the *Copenhagen-Interpretation* of Quantum Mechanics: Since this interpretation defends that measurement instruments which test QM-phenomena have to be instruments obeying the laws of *Classical Mechanics*
19 Carnap (1950), (1971/1980)

is zero or close to zero for the best of our scientific hypotheses, laws, or theories. This important difficulty has been realized by Carnap (1950) p. 571ff. His proposed way out with the help of *instance confirmation* is not a solution as has been shown by Hintikka[20].

The second view is due Popper and originated in his *Logik der Forschung* of 1935, ch. 10.[21] Popper speaks of "corroboration" since "confirmation" seems to suggest a final (firm) certainty which is never available concerning a universal hypothesis, law, or theory. Corroboration tells how a hypothesis, law, or theory has withstood tests and how severe these tests were. One of the most important points here is that corroboration of a hypothesis, law, or theory cannot be interpreted as a probability of that hypothesis, law, or theory:

> It is possible that a hypothesis *h* should have a very high probability although (its evidence) *e* is based on few observations… Then *h* is not acceptable, however, for a high probability is not a sufficient condition of acceptability.[22]

> Thus that part of our approach … is related very closely to Popper's ideas of the inverse relation of *a priori* probability on one hand and on the other hand the acceptability of theories and the ease in falsifying them.[23]

A further reason stems from the universal character of hypotheses, laws, and theories, already mentioned above:

> This may be supported by the remark that every universal hypothesis *h* goes far beyond any empirical evidence *e* that its probability $p(h,\ e)$ will always remain zero, because the universal hypothesis makes assertions about an infinite number of cases, while the number of observed cases can only be finite.[24]

The type of knowledge described above means to have knowledge of hypotheses, laws or theories in the sense of being able to prove that they have withstood severe tests by making predictions which could be verified by direct or mostly by indirect observation and experiment.

(2) Explanation: The same hypothesis, law, or theory which is corroborated is also used as the *explanans* in a scientific explanation. According to a so-called *covering law model* events and phenomena are explained by dynamical or sta-

20 Hintikka (1968)
21 Popper (1935) First Engl. Ed. 1959.
22 Hintikka/Hilpinen (1966a) p. 13
23 Hintikka (1966b) p. 130
24 Popper (1983) p. 219

tistical laws together with initial and boundary conditions. Thus, the statement describing the event or phenomena is derivable from the law plus initial and boundary conditions.

Explanation by dynamical laws: The present state of sun, earth, and moon and their masses and mean distances (initial and boundary conditions) plus Newton's second law of motion and his law of gravitation provide an explanation for a future eclipse.

Explanation by statistical laws: The change of the present volume of a gas (in a certain state) plus the General Gas Law provide an explanation for the respective change of the pressure of that gas. Or: The entropy E of the macrostate M of the isolated system at time t_1 plus the law of entropy provide an explanation for the higher entropy E' of M at time t_2.[25]

This type (5) of knowledge is as type (3) and partially (4) only available for scientists working in the respective domain.

Knowledge 5 (K5) $=_{df}$ ability to corroborate and explain.

$aK5p$ iff the following conditions are satisfied:

(1) p is a law statement representing a universal law of nature or a scientific hypothesis.
(2a) a corroborates p himself by
  (i) having knowledge K3 of consequences of p + initial and random conditions;
  (ii) having knowledge K4 of these consequences; or
(2b) a believes the experts b, c, d who satisfy (i) and (ii) above.
(3a) a explains events e_1, e_2,... e_n by showing (K 3) that the statements that describe them follow from laws and initial and boundary conditions; or
(3b) a believes experts b, c, d who satisfy (3a).
(4a) a proves that p has a high degree of verisimilitude.
(4b) a believes the experts b, c, d who satisfy (4a).

Knowledge K 5 is the first type of knowledge considered here that does not generally satisfy the condition $aKp \rightarrow p$ (K), i. e. that truth is a necessary condition for knowledge. On the other hand we agree with Hintikka (1962) and Chisholm (1963, 1966) that a strong concept of knowledge that satisfies this condition is defensible. In fact, the types of knowledge K 1–K 4 satisfy this condition (cf. ch. 6 below). It

25 For a detailed discussion of such explanations in science see Weingartner (2016) section 4.

is a widespread view—especially under philosophers who are not familiar with science—that every kind of knowledge has to satisfy condition K. This is in conflict with what we have in mind if we speak of the huge amount of scientific knowledge available at present in the 21*st* century. This consists of a huge amount of data from the domain of physics via chemistry, biology, psychology, and sociology to history on the one hand and of a considerable amount of laws and theories on the other. The maximum of information is contained in the laws and theories; think of the laws of Archimedes, Galileo, Kepler, Newton, Ohm, Boltzmann, Planck, Einstein, Heisenberg and Schrödinger for example. These are the best corroborated laws, although they do not satisfy K because they might have some false consequences despite their enormous amount of true consequences and an enormous amount of information; i. e. despite their high degree of verisimilitude. Also, the huge amount of data only partially satisfies K since they have to be interpreted in the light of laws and theories in order to become scientific results.

Should we now say that all this most important part of today's scientific information cannot be called "knowledge" because it only partially satisfies condition K, whereas grasping simple logical (K 1) or empirical facts (K 2) can be called so? Moreover, even if we add K 3 and K 4: to speak of human knowledge only in the restricted sense of K 1–K 4 but forget or rule out most of the scientific knowledge in the empirical domain of today would be rather ridiculous. Therefore, we accept the huge amount of scientific information in all empirical domains *as knowledge* satisfying the necessary condition of having a high degree of *verisimilitude*.

The basic idea of verisimilitude is due to Karl Popper. Provided that the true and false consequences of two theories T_1 and T_2 are comparable: A theory T_2 has a higher verisimilitude (is closer to the truth) than a theory T_1 iff either T_2 has more true consequences than T_1 and not more false consequences than T_1, or T_2 has fewer false consequences than T_1 and not fewer true consequences than T_1. Popper's original definition[26] had the consequence that no false theory T_2 could be closer to the truth than any other false theory T_1.[27]

Though this possibility was Popper's main intention, since a theory, as a whole, is false even if it has many important true consequences but a few false ones. And this is the real situation with scientific theories. The reason for this unintended consequences lies in the property of the concept of consequence in Classical Logic that allows redundancies and irrelevances in the consequence class. Although scientists never utilize such irrelevant consequences when they apply logic.

26 Popper (1963), p. 233 and (1972), p. 52.

27 This was shown by Tichy (1974) and Miller (1974).

Popper's original idea can be rehabilitated if one restricts the consequence class of Classical Logic to a class of relevant, non-redundant and most informative consequence elements as has been shown in Schurz-Weingartner (1987) and (2010). This so restricted consequence class is classically logically equivalent (though not relevantly equivalent) to the classical consequence class.[28] There are other approaches to verisimilitude with the help of possible worlds, models and constituents[29] in contradistinction to the above relevant consequence-approach. In Schurz (2011) it is shown that the relevant consequence-approach is also suitable for theory revision and expansion.

4.3.8 Knowledge as *Justified True Belief*

4.3.8.1 Justified True Belief Cannot Be Used as a General Definiens for Knowledge

It is a widespread opinion among philosophers that knowledge can be adequately defined as *justified true belief*. Although this view can certainly be defended for many cases of knowledge, both in everyday practice and in science, it cannot be accepted as a general definition of knowledge.

First, because there are cases of justified true belief that are not knowledge. Examples are famous mathematical conjectures that have a history of preliminary and auxiliary proofs with increasing the degree of certainty of the conjecture like in the case of Fermat's conjecture. Before the final proof by Wiles the conjecture was justified true belief, but it was knowledge only after the proof had been given in 1994. Other examples are famous scientific (true) predictions well justified by best confirmed scientific theories; however they could not be called knowledge before the experimental test established the result (recall 4.12 and 4.13 above).

Second, because there are cases of knowledge that are not justified true beliefs. Realizing that $2 + 3 = 5$ is not *believing that*, even if the belief that it is so (is true) might be a consequence of it; it is first of all *understanding* or *realizing* or *grasping*. Imagine that a child (elementary school) says to the teacher: "I *believe* you that $2 + 3 = 5$." This would be justified (since it is the *teacher* who taught so) true belief. But in this case the teacher would not be satisfied. He would say to the child: "That you believe it is not sufficient. You have to *understand* it; you have to grasp why it is so." Similarly, being aware of feeling pain or realizing that I must exist if I err are actions of *understanding* or *realizing* or *grasping* although *believing* that it is that way may be a later consequence of it. Therefore, knowledge in the sense of K 1

28 For the proof see Schurz and Weingartner (1987), lemma 7.2.

29 For a short overview see Schurz (2011), p. 204ff.

(true immediate objective understanding) and K 2 (true immediate intrasubjective understanding) are cases of *understanding*, not of belief, although they are usually followed by beliefs. Thus we might say knowledge K 1 implies belief and knowledge K 2 implies belief.

Third, because, as has been elaborated at the end of section 4.3.7 above, most of scientific knowledge is not justified *true* belief, since it does not satisfy conditions K. It is justified *approximately true* belief or justified belief possessing a high degree of verisimilitude.

4.3.8.2 The Type of Knowledge as Justified True Belief Occurs Most Frequently as a Belief in Experts

Most of the things (propositions, states of affairs) of which we say that we know them, we, in fact, believe others, because we are unable to prove them ourselves. This begins in childhood where we believe our parents and other adults, and it continues through school and university. On all these levels, the belief is *justified* because parents, teachers, and professors are trustworthy under normal conditions; where "normal conditions" refer to both to the reliability of the person and his (her) education and experience. The respective belief is usually or in many cases also true.

However, we cannot guarantee truth such that we could generally assume the principle K: $aK6p \rightarrow p$. Therefore we put "true" into parenthesis in the definition of K 6 below. In many cases where we speak of knowledge, we mean *justified true belief*. With the exception of knowledge 1 (knowledge as true understanding, 4.3.3 and 4.3.4) most of the cases of knowledge 3, 4, 5 and 7 (sections 4.3.5–4.3.7 and ch. 5) are justified true belief. We cannot prove difficult mathematical or logical theorems ourselves, we believe them to experts; even if we can make simple calculations or prove some simple logically valid arguments ourselves. Although we can verify some facts by direct observation, for many others we believe reliable persons. Moreover, although we might verify some facts by indirect observation if we have studied some special field, most of them we believe experts. Again, we may be able to corroborate some hypotheses if we are experts in the field but most of such tasks are done by others, and we believe them, since we trust their reliability.

In a scientific community, such as the Max Planck Institute or CERN, scientists believe each other in a justified sense. If the head of such an institution says to a larger scientific community "We are quite sure of it" it means we have justified true belief in the sense described by Wittgenstein:

> 'We are quite sure of it' does not mean just that every single person is certain of it, but that we belong to a community which is bound together by science and education.[30]

"Bound together by science and education" is an abbreviated characteristic referring to the components of the scientific background of a research team: to the methodological rules of the respective domain which guide observation and experiment and the severe testing procedures for hypotheses and their predictions (cf. the description of reliable observations or experiments in section 2.33 above).

4.3.8.3 "Justified" and "Justified Belief" as Superimposed Concepts For Many Types of Knowledge

The view that *justification* has to be contained in any type of knowledge seems to be defensible: Justification in the sense of an argument with providing reasons or premises is certainly present in K 3, K 5, K 6, and K 7; it might be present in K 4, but it is not present in K 1 and K 2. In K 1 and K 2 the kind of "understanding" or "realizing" or "grasping" the respective state of affairs is itself a special kind of justification as "insight", without argument-structure.

"Justified belief" and "belief" is a consequence of the "insight" in the activity of "understanding", "realizing" or "grasping". On the other hand "justifed belief" is contained in all other types of knowledge K 3–K 7 as has been said above (4.3.8.2).

Knowledge 6 (K6) $=_{df}$ justified (true) belief.

$aK6p$ iff one of the following two conditions is satisfied:
(1) a believes that experts b, c, d… have knowledge K3, K4, K5, or K7 concerning p and therefore aBp.
(2) a has himself knowledge K3, K4, K5, or K7 concerning p and therefore aBp.

4.4 Answers to the Objections

4.41 (To 4.11:) There are two concepts of belief: knowledge-inclusive and knowledge-exclusive. According to the first believing that p means assuming that p is true (is the case). And since this is also implied by knowledge, to know implies to believe, or knowledge implies belief in this first sense (belief 1).

30 Wittgenstein (1969) 298.

belief 1 = knowledge-inclusive belief.[31]

However, for knowledge-exclusive belief (belief 2) it holds: believing 2 that *p* implies as necessary conditions (a) assuming that *p* is true (like belief 1) and (b) being aware that one does not know that *p*.

This type of belief 2 is frequent in science and religion: Before the proof is given there is belief 2, but after the proof is established, belief 2 disappears and is replaced by knowledge as the examples in 4.12 and 4.13 show.[32]

In the objection 4.11 belief 2 is meant, and it is correct to say that when knowledge has been achieved one does not and one need not to have belief 2 any more. This shows as the arguments in 4.12 and 4.13 that knowledge $=_{df}$ justified true belief is not suitable as a general definition of knowledge although it is still applicable in many cases (cf. 4.38 above).

4.42 (To 4.12:) As the objection states, it is correct that all cases of famous true mathematical conjectures are counterexamples to the definition: knowledge $=_{df}$ justified true belief. However, from this, it does not follow that this definition is not applicable at all, but only that it cannot be used as a general definition of knowledge. As has been shown in section 4.38 it can be defended as one of several types of knowledge.

4.43 (To 4.13:) The answer to that objection is similar as that to 4.12. Also, cases of empirical conjectures are counterexamples to the definition of knowledge as justified true belief.

To be a little bit more accurate about Einstein's predictions concerning the perihelion of Mercury, we have to add the following facts: Since the orbits of the planets are ellipses, on every orbit there is a point which is closest to the sun; this point is called the perihelion. Because of the gravitational force of the other planets, the elliptical orbits rotate around the sun, which means that the perihelion moves a certain distance per year. This effect is particularly strong in Mercury because it is the closest to the sun and has the smallest mass. According to observations, the movement of the perihelion of Mercury has been 5,74 seconds of arc per year. Newton's Theory predicted 5,32. This discrepancy has been known to astronomers; its amount was 43 seconds of arc per 100 years. Einstein's General

31 This type of belief is treated in detail in Hintikka's *Knowledge and Belief* (1962).

32 For this distinction in the application to scientific and religious belief see Weingartner (1994). For an axiomatic theory containing these two kinds of belief besides a concept of knowledge and one for assumption, see Weingartner (1982) and this book ch. 13.

Theory of Relativity predicted exactly the 43 seconds of arc as the difference, and this was the first immediate confirmation of Einstein's General Theory.

The two other predictions, the deviation of light rays and the red shift of the galaxies were confirmed later. The deviation of light rays was first confirmed by an expedition of the Royal Society to South Africa in 1919, where the effect was observed when there was an eclipse of the sun. The relativistic red shift of the galaxies occurs as a change of the frequency of γ-rays in a decreasing gravitational field. This effect was confirmed within the gravitational field of the earth by the Mössbauer-Effect which enabled to achieve very fine tuning decomposition of spectral lines.

To come back to the objection 4.13: What has been said there is correct: Before the deviation of light rays and the relativistic red shift of the galaxies was confirmed by observation, we cannot speak of knowledge; although, before this experimental confirmation, there were justified true beliefs. Thus, justified true belief is not always knowledge. Therefore, knowledge $=_{df}$ justified true belief cannot be used as a general definition of knowledge since there are other types of knowledge as elaborated in this chapter. However, concerning our numerous beliefs in experts, such a definition is suitable.

4.44 (To 4.14:) As is manifest from 4.12 and 4.13 there are serious counterexamples to the definition: knowledge $=_{df}$ justified true belief from the domain of science: true and justified mathematical conjectures and true and justified empirical (physical, astronomical) conjectures. From this, it follows that Gettier's claim that there are cases in which true justified belief is not knowledge is correct. However, his two examples are hardly serious counterexamples.

Gettier's first case consists of an argument which uses a kind of partial belief (i. e. a part of Smith's belief) which is not really his belief: Smith has strong evidence for proposition:

(d) Jones will get the job (Gj) and Jones has ten coins in his pocket (Pj).
Smith infers from (d) proposition:
(e) The man who will get the job has ten coins in his pocket.

Gettier assumes in case 1 that either Smith or Jones will get the job but Gj is false (i. e. that Smith will get the job, Gs). Gettier claims that Smith believes that (e) and is justified in believing that (e). However, this is only a part of Smith's belief. Since he believes that (e) in virtue of believing that (d) from which he infers (e). Thus Smith, in fact, believes that both (d) and (e) hold and therefore he has a false belief; since (d) is false because Gj is false. Thus Smith does not have justified true belief, and so Gettier's first case is not a counterexample.

The second counterexample (case 2) of Gettier is based on such an irrelevant inference which is classically valid but nevertheless the culprit of many paradoxes in different domains: it is the principle of addition, *p* therefore *p* or *q* (where *q* can be any statement). An example of case 2: Jones owns a Ford, therefore: Jones owns a Ford or Brown is in Boston. The irrelevant inference of addition is the culprit of Hesse's paradox of confirmation, of Goodman's paradox, of the Ross paradox and of the paradox of Free-Choice (the last two belonging to the area of Deontic Logic). Therefore, Gettier's case 2 is based on an irrelevant inference schema which is not used in this way in either science or everyday language: i. e. in such a way that *q* can be any statement whatsoever and can also be replaced by its negation. Consequently, case 2 is not a serious counterexample.[33]

4.45 (To 4.15:) The position of Hintikka "to know that *S* is to be in a position to verify that *S*" is one type of knowledge: Knowledge 4. But it may include knowledge 3. As is clear from section 4.3.5 and 4.3.6, this type of knowledge is different from type K6, knowledge as justified true belief. In fact, both types are widely applicable and frequently used. However, Hintikka's point does not rule out knowledge as justified true belief since Hintikka's definition is also not suitable as a general defintion of knowledge. One reason is that first simple principles (like the law of non-contradiction or the fallor ergo sum) cannot be proved but are still known. A second reason is that we could never speak of the knowledge of an expert or, taking a particular example the student of mathematics, could not speak of knowledge of his professor of mathematics.

4.46 (To 4.16:) The kind of knowledge Thomas Aquinas had in mind is *knowledge what something is* (cf. section 4.3.2). The first observation is that some features of *knowledge what something is* can be reduced to *knowledge that something is* (4.3.2). Secondly, there is a specific aspect concerning the knowledge of definitions when real definitions, which describe the essence of a species, are at stake. This is the case here when Aquinas speaks of the "quiddity of a material thing". According to Aristotle and Thomas Aquinas, such definitions are necessary true equivalences, where "necessity" is usually not *logical* or *mathematical necessity* but a kind of natural necessity since their examples are mainly taken from the biological domain.

33 For a detailed critical comment to Gettier's "counterexamples" see Weingartner (1996). For different irrelevant inference forms which are the culprit of many paradoxes when Classical Logic is applied to empirical sciences see Weingartner (2001) and (2009).

4.47 (To 4.17:) It is correct that there cannot be knowledge of q if q is false.[34] However, also the justified true belief is belief of the disjunction (for example: $(p \rightarrow p) \vee q$) not of q. There can be knowledge of the logical truth in the sense of K 1 (cf. 4.3.3) if the logical truth is simple or in the sense of K 3 if it is more complicated.

However, there is the deeper question concerning the logical closure conditions for the concept of knowledge. In this special case, the question is: If we know that $p \rightarrow p$ and if $(p \rightarrow p) \vee q$ follows logically from $p \rightarrow p$, do we also know that $(p \rightarrow p) \vee q$ holds? To answer with Yes would mean (when generalizing this case) to accept *deductive infallibility* for the concept of knowledge: one knows all the logical consequences of what one knows. We reject this interpretation of knowledge (cf. 6.3.1 (3)). There we defend the weaker forms of modus ponens and distribution MPK and DK. According to these weaker principles, it holds for our special case: If we know that $p \rightarrow p$ and *if we know* that $(p \rightarrow p) \vee q$ follows from $p \rightarrow p$, then we also know that $(p \rightarrow p) \vee q$ holds. Thus, according to MPK or DK, there is knowledge of $(p \rightarrow p) \vee q$.

However, there is also a solution if, as in the objection 4.17 this seems implausible for any concept of knowledge. The reason for the implausibility is the irrelevant deduction in the sense that, in the conclusion, q can be replaced by any arbitrary other proposition (also by its own negation) salva validitate of the deduction.

This relevance criterion (called "replacement criterion") originated in Schurz-Weingartner (1987) and was later further developed.[35] Respective relevance-restricted closure conditions for modus ponens and distribution over $\rightarrow$ are the following:

$$MPK_{RC} \quad [aKp \wedge aK(p \xrightarrow{RC} q)] \rightarrow aKq$$

$$DK \rightarrow_{RC} \quad aK(p \xrightarrow{RC} q) \rightarrow (aKp \rightarrow aKq)$$

4.48 (To 4.18:) It is correct, as Bunge says, that law statements representing laws of nature cannot qualify as knowledge if knowledge is understood as justified true belief. And this shows once more that justified true belief, though suitable as one type of knowledge cannot be used as a general definition of knowledge.

However, since we defend a pluralism concerning concepts of knowledge it can be shown that especially two types of the seven types of knowledge are suitable for a scientific knowledge of law statements: K 5 as the ability to corroborate or to confirm and K 7 as possessing epistemic entropy and epistemic information.

34 Consider, however, the validity of the K-thesis in section 6.3.2 below.
35 Cf. section 8.4.1 and note 4 of 8.4.1.

5 Whether Knowledge is Possessing Information

5.1 Arguments Contra

5.11 Information is the message carried by a signal. But knowledge is not just possessing the message carried by a signal; otherwise, knowledge 1 and knowledge 2 could not be genuine knowledge contrary to what has been established in ch. 4. Therefore, knowledge is not information.

5.12 Information is concerned with the elimination of possibilities represented by the occurrence of a state of affairs. Knowledge, on the contrary, is concerned with the actuality represented by the occurrence of a state of affairs. Therefore, knowledge is not possessing information.

5.13 Knowledge with a degree of probability $Pb = 1$ is knowledge with the highest degree of certainty. On the other hand, information with the probability $Pb = 1$ is information of degree 0. Therefore, knowledge cannot be possessing information.

5.14 Knowledge concerning a number of possible states of affairs seems to be most certain if only one state of affairs can occur. Information, on the contrary, is minimal if only one state of affairs can occur. Therefore, knowledge cannot be possessing information.

5.2 Argument Pro

5.21 A message transmitted by a signal is called a bit of information. However, getting and possessing such a message means to increase one's knowledge. Therefore it seems that knowledge is possessing information.

5.3 Proposed Answer

5.3.1 Different Meanings of "Information"

The importance of information in living systems is described very well by Haken:

> One of the most striking features of any biological system is the enormous degree of coordination among its individual parts… Quite clearly, all these well-coordinated, coherent processes

https://doi.org/10.1515/9783110585797-005

> become possible only through the exchange of information, which must be produced, transmitted, received, processed, transformed into new forms of information, communicated between different parts of the system and at the same time, as we shall see, between different hierarchical levels. We are thus led to the conclusion that information is a crucial element of the very existence of life.[1]

In this passage, "information" may have more than one meaning. Mahner and Bunge list six meanings of "information", which are used in scientific literature[2]:
1. information 1 = signal
2. information 2 = message the bearer of which is a signal
3. information 3 = quantity of order (negentropy)
4. information 4 = meaning (semantic information)
5. information 5 = knowledge
6. information 6 = communication of information 5 by social behavior (speaking) using information 2

For an interpretation of Haken's quotation, all six meanings, perhaps with the exception of information 3, are possible candidates. We shall concentrate on information 5 with a certain analogy-relation to information 3.

5.3.2 Epistemic Entropy

We shall begin with a simple example of two propositions *r* and *s* which have different degrees of information:

r… Hans will arrive in Salzburg during the week of March 3 to March 9, 2015.
s… Hans will arrive in Salzburg on March 3, 2015.

We consider now the number of possible real states satisfying *r* and the number of possible real states satisfying *s* and compare them. It is plain that the number satisfying *r* is much greater than that satisfying *s*. The possible real state that Hans arrives on March 4 satisfies *r*, but not *s*. We may split *r* into a disjunction saying that Hans will arrive on March 3, March 4, March 5, or … March 9. From this, it is also manifest that the number of possible real states satisfying a disjunction is much greater than the number of real states satisfying one disjunct. Moreover, it

1 Haken (1988), p. 23
2 Mahner/Bunge (2000), p. 275

will be clear that the number of possible real states satisfying a conjunction must be smaller than the number of possible states satisfying one conjunct.

The number of possible real states satisfying the proposition *p* we shall call the *epistemic entropy* of *p* (abbreviated as *EE*(*p*)). This leads to the definition:

5 D 1 $EE(p) =_{df}$ the number of possible real states satisfying *p*.

Recalling the above example, the epistemic entropy of *r* is much greater than that of *s*:

$EE(r) > EE(s)$

5.3.3 Possible Real States

Before continuing, it should be said how "possible real states" and "possible states" (see 5.3.4) are understood. Possible real states are understood as possible states of reality which satisfy the following conditions:

(1) Possible real states are compatible (consistent) with our well confirmed laws of nature. For example, a state which violates $E = mc^2$ or the law of entropy is not a possible real state.
(2) Possible real states are compatible (consistent) with the fundamental constants of nature. For example, a state which violates the fine-structure constant 1/137 (beyond the accuracy of measurement) or the gravitational constant *G* (beyond the accuracy of measurement) is not a possible real state.
(3) We assume that there are only finitely many possible real states in the universe. The reasons are the following:
    (a) The universe is finite in extension, i. e. spatially finite, according to the Theory of General Relativity which is very well corroborated.
    (b) The elementary particles and the atoms of the universe seem to be finite. The number of hydrogen atoms of the universe seems to be about 10^{80}.
    (c) The standard Big Bang Theory tells us that the universe has a finite age. This theory is corroborated by the well-known cosmic background radiation.
    (d) Negations and disjunctions are *de dicto*, not *de re*. They belong to the conceptual sphere, not to reality. There are neither negative possible real states nor disjunctive ones. Possible real states cannot be multiplied by logical operations. Even the corresponding state of a conjunction is restricted since it is not always the case that the fusion of two possible real states is again a possible real state.

(e) Possible real states are singular, not compound.
(f) Possible real states are contingent. The statements representing them are neither logical nor mathematical truths.
(g) The continuum belongs to the mathematical and physical theory of space-time, i. e. to the conceptual sphere. It need not be realized in concrete reality, too, even if it is very useful for the description of concrete reality.

(4) We assume that a possible real state lasts a short time interval such that a measurement or an observation is possible. Thus a possible real state is a short event at a certain place (relative to a reference system). A longer lasting event is called a process. Moreover, every measurement or observation of position or time (of some physical, biological systems) refers always to some reference frame (reference system of the world or universe).

(5) Possible real states can be described or represented by *basic statements* in the sense of Popper's[3] satisfying conditions (1) and (2) above. The general form is: state s occurs at space time interval $\Delta(O_1 t_1 / O_2 t_2)$. Or: event e occurs at space-time interval $(O_1 t_1 / O_2 t_2)$.

(6) Possible real states can be described or represented alternatively by the "actual states of reality" in the sense of Weingartner[4], where "actual" is replaced by possible (satisfying conditions (1) and (2) above).

(7) Concerning both possibilities of describing possible real states—(5) and (6)—we have to observe the following restrictions:
(a) Neither basic statements (5), nor propositions representing possible states of reality (6) are invariant w. r. t. transformations of logical equivalence of Classical Logic (two-valid propositional logic or first order predicate logic).
Example: If p is a basic statement (or a proposition representing a possible state of reality), then $[(p \wedge q) \vee (p \wedge \neg q)]$ is (classically) logical equivalent to p, but it is neither a basic statement nor a proposition representing a possible state of reality. One important reason is that q can be replaced on both of its occurrences by any arbitrary other variable (including $\neg q$) salva validitate of the equivalence.
(b) Logical consequences (by Classical Logic) of basic statements (or propositions representing possible states of reality) are not always again basic statements (propositions…). They are for example, if from a conjunction of basic statements (or propositions…) one conjunct is logically derived. But they are not in general. For example, if p is a basic statement (or proposi-

3 Cf. Popper (1935), (1959), §§27-29.
4 Cf. Weingartner (2000), ch. 8, p. 162ff., D 10–D 10.5.

tion...), then $p \vee q$ and $\neg p \rightarrow q$ are logical consequences of p (according to Classical Logic), but $p \vee q$, $\neg p \rightarrow q$ are not basic statements (and not propositions...). One important reason is again that q is replaceable by any arbitrary other variable (including $\neg q$) salva validitate of the inference. To avoid such redundancies allowed by Classical Logic, one may apply relevance criteria. They also help to avoid different types of paradoxes coming up when Classical Logic is applied to empirical science. The usual Relevance Logics, however, are too weak to do the job. A relevant consequence relation strong enough to avoid different types of paradoxes in different domains has been defined in Schurz/Weingartner (1987), (2010). A decidable many valued calculus which approximates largely this relevant consequences relation has been proposed by Weingartner (2009), (2010).

5.3.4 Possible States

We understand "possible states" or "possible states of affairs" as satisfying weaker conditions than possible real states. They are related by inclusion: All possible real states are possible states and not vice versa. In order to be most transparent we refer by comparison to the conditions of possible real states when giving the conditions for possible states:

(1) Possible states are compatible with the laws of logic (First Order Predicate Logic with Identity).

(2) Possible states are compatible with laws and proved results of mathematics.

(3) Possible states are not restricted to finitely many, i. e. there may be infinitely many.

(4) Possible states are singular, i. e. not compound. They are distinguished from possible real states by dropping the specific space-time coordinates. Thus the eclipse at time t and planetary position 0 is a possible real state, whereas the eclipse is a possible state.

(5) Possible states can be described or represented by atomic statements. The general form is: state s occurs.

(6) Possible states are contingent. The statements representing them are neither logical nor mathematical truths or falsehoods.

(7) Similar to 5.3.3 (7) it holds for possible states: The statements representing possible states are not invariant w. r. t. transformations of logical equivalence of Classical Logic (see 5.3.3, 7a).

5.3.5 Epistemic Information

From the example in 5.3.2 and from the definition of *epistemic entropy* it is easy to grasp that the degree of information is higher iff the epistemic entropy is lower. We could, therefore, define the epistemic information as 1/epistemic entropy. However, in this case the epistemic information would always be smaller or equal to 1, whereas the epistemic entropy can be a huge number. For this and other reasons (cf. examples in section 5.4 below) we define the *epistemic information* of p $EI(p)$ by the states which are excluded by p. However, it would not be sufficient to take only possible real states to be excluded since a true singular statement or an approximately true law also excludes possible states which are represented by false statements and which are incompatible with well confirmed laws of nature (5.3.3, 1). Thus statement s of 5.3.2 rules out not only that Hans arrives on March 4 but also that he arrives on March 20 or April 2 (outside of the possible real states restricted by statement r). Similarly, Kepler's first law, which states that the orbits of the planets are ellipses, rules out not only that they are circles, as Galileo thought, because of aesthetic reasons but also perhaps because he understood that circles are the simplest solutions of rationally symmetric laws. However, Kepler's first law includes also that the orbit lies in one plane (which contains the sun in one focus of the ellipse) and that excludes possible states of a planet that would be outside a plane; and these are not compatible with well confirmed laws of nature, i. e. possible states which are not possible *real* states. Thus we define $Ei(p)$ the epistemic information of p as follows:

5 D 2 $EI(p) =_{df}$ the number of possible states which are excluded by p

It has to be observed moreover that epistemic entropy and epistemic information behave differently when applied to singular states of affairs represented by basic statements or to general states of affairs represented by universal laws. In the first case, as with the statement s, or more accurately when a space-time point is added, the epistemic entropy is usually = 1 and the epistemic information is a huge number. In the second case, take the law $E = mc^2$, both the epistemic entropy and the epistemic information can be a huge number. These facts will become clearer when the concepts of EE and EI are applied to cases of empirical science (see below, sections 5.4.3 and 5.4.4).

We shall now use these two definitions to define a seventh type of knowledge: knowledge as possessing information. And we shall say that to possess information is to possess both epistemic information and epistemic entropy. Thus, we arrive at the following definition of type (7) of knowledge:

5 D 3 *Knowledge 7* $=_{df}$ possessing epistemic entropy and epistemic information.

In other words:

5 D 3a $aK7p$ iff both of the following conditions are satisfied:
(1) a possesses epistemic entropy of p
(2) a possesses epistemic information of p

5.4 Application of Knowledge 7

5.4.1 Application to Logic and Mathematics

A first consideration shows that it does not make sense to apply this kind of knowledge to logic or mathematics. Taking a law of logic, for example, the principle of non-contradiction or the modus ponens, reveals that its epistemic entropy consists of all possible real states and its epistemic information—i. e. the possible states excluded—consists of zero members; since that what they exclude are contradictions and possible states are not contradictory. A similar consideration concerns true statements of mathematics. First take arithmetic, for example, with the principles of the multiplication table; they are satisfied by every possible real state, and they exclude no possible state. Thus, in both cases the epistemic information is zero.

5.4.2 Application to Geometry

However, the question of application is not as simple concerning geometry. A decision is only possible if we distinguish sharply between a purely mathematical topology on the one hand and a theory which describes the geometrical structure of the real world (of the real universe) viz. the real physical space on the other. The geometry of real space is not a purely mathematical theory. This can be shown by Helmholtz's theorems:

(1) If we are given sufficiently small, rigid bodies, freely movable in space, then the geometry of the space that is measured with these rigid bodies is Riemannian.

(2) If we are given even rigid bodies of finite extension that are freely movable, then the geometry of the space measured with these bodies is the geometry of a Euclidean, elliptic, or hyperbolic space, i. e. is space with constant curvature.[5]

These theorems show that contingent presuppositions—namely that (a) there are rigid bodies (measurement rods) and (b) that they are freely movable in space without changing their form, length, or mass—determine the decision for a certain geometry.

Therefore, the statement "the real physical space of the universe is Euclidean" has both epistemic entropy and epistemic information. And the more accurate statement "the real physical space of the universe is Euclidean locally where there is no concentration of big masses, but Riemannean—curved in dependence of the concentration of masses—elsewhere" has greater epistemic entropy and greater epistemic information.

5.4.3 Application of Knowledge 7 to Empirical Science

In order to show that this type of knowledge is mainly applicable to the empirical sciences, we shall discuss some examples.

5.4.3.1 Simple examples

(1) p is a basic statement in the sense of Popper[6]; i. e. p says that a particular event occurs at a particular space-time point where "space-time point" has to be understood in the relaxed way explained in 5.3.3 (4) above. For example: a photon hits a detector, a sequence-exchange-repair occurs on a special place of a DNA, two billiard balls strike together… etc. We assume here that these events are not processes, but are relatively short to occur on one space-time point or short space-time interval. Then:

$EE(p) = 1 \qquad EI(p) = \text{a huge number}$

(2) p has the form $q \vee r$. Then:

$EE(q \vee r) > EE(q) \qquad EI(q \vee r) < EI(q)$
$EE(q \vee r) > EE(r) \qquad EI(q \vee r) < EI(r)$

5 Helmholtz (1868). Cf. Mittelstaedt/Weingartner (2005), p. 55.
6 Popper (1959), §§27–29

(3) p has the form $q \wedge r$. Then:

$$EE(q \wedge r) < EE(q) \qquad EI(q \wedge r) > EI(q)$$
$$EE(q \wedge r) < EE(r) \qquad EI(q \wedge r) > EI(r)$$

Example: Take Kepler's first and second law as q and r. It is plain that *EE* of both laws is smaller than the *EE* of one of them; and the *EI* of both laws is greater (carries more information) than that of one law. The following three examples are more complicated.

5.4.3.2 Kepler's Laws of Planetary Motion

We consider the epistemic entropy and the epistemic information of the three laws of planetary motion of Kepler:

1. The planets move on ellipses with the sun in one focus.
2. The straight line from the sun to a planet coats surfaces in equal times.
3. $T_1^2 : T_2^2 = a_1^3 : a_2^3$; where T it the time of one revolution and a is the mean-distance from the sun.

Considering the *EE* of Kepler's laws we understand immediately that there is a huge number of possible real states satisfying the three laws which differ from the actual real state by having different contingent magnitudes—in general called boundary conditions—as for example: different masses of the planets, different distances between the planets, different distances from the sun (different lengths of the orbits), different time for one revolution around the sun... etc.

On the other hand, if we take the three laws of Kepler together with the special magnitudes of the above-mentioned boundary conditions (mass, distances between the planets, distances from the sun, time of revolution) for each planet, then the epistemic entropy (*EE*) is drastically smaller. Consequently, the epistemic information (*EI*) in this case is much higher than the epistemic information of just the three laws of Kepler.

Moreover, observe that the *EE* of the three laws plus the above boundary condition is still greater than 1; since there are other boundary conditions which have not been mentioned so far. For example, the time of one axial rotation of the respective planet or of being oblate.

5.4.3.3 Ideal Gas-Law and Van der Waals Equation

Ideal gases are described by the ideal gas-law. Real gases behave like ideal gases under the conditions of sufficiently high temperatures and sufficiently small densities.

The ideal gas-law is understood universally in the sense that it holds for arbitrary pressures p, volumes V (or densities ρ) and temperatures T:

$p \cdot V = n \cdot R \cdot T$
(where n is the mole-number and R is the gas-constant)

This law gives correct results if the temperatures are sufficiently high and the densities are sufficiently small. These conditions usually hold in normal technologies with air and other gases. However, for low temperatures and the critical temperature for the transition to the aggregate state of fluid, the above law gives incorrect results.

If these states of a gas are included, it is no longer possible to give a universal law, but one has to add specific parameters (depending on the gas) for the volume of the gas-molecules (b) and for the interrelated molecular forces (a). These specific parameters are taken care of by Van der Waals equation of real gases:

$(p + \frac{a}{V^2})(V - b) = n \cdot R \cdot T$

When considering both the *EE* and the *EI* of the ideal gas-law in comparison to Van der Waals' equation, the answer is not so straightforward. On the one hand, it seems that the *EE* of Van der Waals' law is greater than the *EE* of ideal gas law because the former describes also gases on low temperatures and with higher density. If *EI* were defined as 1/*EE* then the epistemic information (*EI*) of Van der Waals' law is smaller than that of the ideal gas law. However, we have defined *EI* of some statement (law) p as the number of possible states that are excluded by p. And according to this definition Van der Waals' law, because it is more concrete by presupposing certain values for the parameters a and b, certainly excludes more possible states than the ideal gas law and thus has more epistemic information.

But does it really have a greater *EE* as well? The answer to this question seems, in a sense, to depend on the interpretation of the ideal gas law. If we stress the point that the variables p and T are restricted to low p and high T, the *EE* of the ideal gas law is smaller than *EE* of Van der Waals law. If on the other hand we say that the ideal gas law is a most abstract general law and consequently leaves open (i. e. does not rule out) the addition of new specific parameters with specific magnitudes, then *EE* of the ideal gas law is much greater than that of Van der Waals' law.

However, independently of this consideration, it could seem that the term "possible state" (and "possible real state") does not have the same meaning in both laws. Because the state for a gas, which is close to the critical temperature (for the transition to the aggregate state of fluid), is quite different from the state of a gas with high temperature and low density. From this point of view, both laws could be hardly comparable w. r. t. *EE* and *EI*. That there need not to be an ambiguity is shown in 5.4.4 below where $EE(p)$ is split up into its components.

5.4.3.4 Newton's Law of Motion

There are two forms of Newton's Law of Motion (his second law of the *Principia*) which are used in textbooks. The most common one is the formulation of Euler:

$$F = m \cdot \frac{d^2x}{dt^2}$$

Here the mass m is separated and viewed as constant. At the time of Newton and later (until experiments with nuclear particle accelerators) one could not have measurement results where the mass was increasing under velocities close to the velocity of light as the theory of Special Relativity tells us. Since there is no restriction for the velocity, the above law of motion is incorrect under such conditions.

Newton's Latin formulation is different. It says that force is the change of the momentum in time. In this formulation, the mass m is incorporated in the momentum and thus in the differential derivation.

$$F = \frac{d(m \cdot v)}{dt}$$

This formulation leaves open whether mass can increase with velocity. Therefore, it cannot be criticized in the way Euler's formulation can, because mass is not constant at velocities close to the velocity of light. On the other hand, velocity v is also unrestricted here, it can be arbitrary, but finite.

Considering *EE* and *EI* of both laws, we are in a different situation than in the thermodynamic case: Newton's law has greater *EE* (epistemic entropy) than Euler's since also those possible real states where mass is increased are satisfying Newton's law. On the other hand, since Newton's formulation leaves it open whether mass depends on velocity it neither rules out an increasing mass (under the condition of high velocity) nor rules out that the mass stays constant independent of the velocity. Thus, in this sense, Euler's formulation excludes more possible states and consequently has the greater epistemic information than Newton's formulation. And this is so although Newton's formulation is closer to the truth than Euler's having less false consequences since lacking those that follow from the constancy of mass. However Einstein's improvement[7] of Newton's law (in either form) by Lorentz-Transformation and light-speed limit has greater epistemic information than Newton's law. And it is also closer to the truth.

However, we have to realize here a similar question to that concerning the gas-laws. Does the term "possible state" (and "possible real state") have the same meaning in both Euler's form and Newton's form of the law and in Einstein's improvement? It does not seem so since a physical system having relativistic mass is conceptually different from a physical system having mass as a constant. Consequently, it seems that the possible real states are conceptually different in both systems. A further point of conceptual difference is the velocity. A physical system in the sense of Classical Physics can have arbitrary finite velocity, whereas velocity of a physical system in the sense of Relativistic Physics has the limit of the velocity of light in vacuum. Thus, concerning velocity, it seems that the *EE* of Newton's law is larger than the *EE* of Einstein's improvement with the help of the Lorentz-Transformation and light-speed limit. However, concerning the mass, *EE* of Newton's law in Euler's form is smaller than *EE* of Einstein's improvement. These difficulties concerning ambiguity of "possible real state" will be resolved below (5.4.4) when we split up $EE(p)$ into its components.

5.4.4 Epistemic Entropy revisited

The above difficulties concerning an ambiguity of "possible real state" suggest the following distinction:

1. $EE1(p)$ = the number of possible real states satisfying p that have been realized up to now (present) in the universe
2. $EE2(p)$ = the number of possible real states satisfying p that will be realized in the future (in the universe)
3. $EE3(p)$ = the number of possible real states satisfying p that will not (never) be realized in the universe

Examples for $EE3(p)$ are: (1) There might be possible constellations of our planetary system which never occur during the lifetime of the planetary system. At some point in time in 1983, all planets were on a straight line on one side of the sun. One might think of some constellation satisfying a specific geodesic that might never be realized. (2) The macrostate "liter of air at room temperature" can be

7 We speak of Einstein's improvement which consists of applying the Lorentz-Transformation and the velocity of light limit to Newton's law of movement. We cannot speak of the Special Theory of Relativity since this is restricted to inertial systems.

realized by a huge number of microstates which differ in their constellation of the molecules. The number is about $10^{5\cdot 10^{22}}$ if the number of molecules in the liter of air is about 10^{22}. It is very improbable that all these possible microstates will be played through during the lifetime of air. (3) Extrapolating example (2) we may ask whether all possible microstates of the whole universe can be played through during its lifetime, assuming it to be finite. This is still much more improbable. The sum of all the three is then the epistemic entropy of p:

5 D 4 $EE(p) = EE1(p) + EE2(p) + EE3(p)$
5 D 5 $EE^{\star}(p) = EE1(p) + EE2(p)$

The restrictions given for possible real states in 5.3.3 above hold also for the three subtypes of possible real states, i. e. for $EE1(p)$, $EE2(p)$ and $EE3(p)$.

With the help of the above threefold distinction we may consider once more the epistemic entropy of the Gas-Law and Newton's Law of Motion.

Concerning the Gas-Law it is already clear from what has been said in section 5.4.3.3 that the epistemic information (EI) of the Van der Waals Law is greater than that of the Ideal Gas-Law. For the epistemic entropy we shall use now the above distinction: If we restrict the epistemic entropy to the realized cases of past and future ($EE1 + EE2 = EE^{\star}$) then this epistemic entropy is greater of the Van der Waals Law:

$$EE1(v.\,d.\,W.) + EE2(V.\,d.\,W.) > EE1(I.\,G.\,L) + EE2(I.\,G.\,L.)$$
$$EE^{\star}(V.\,d.\,W.) > EE^{\star}(I.\,G.\,L.)$$

This is so since the Van der Waals Law can correctly describe more possible real states that are realized (or will be realized) because it also describes gases with states of lower temperature and higher density, whereas the ideal gas-law is restricted to "ideal gases", i. e. gases with high temperature and low density or pressure. On the other hand, the ideal gas law is satisfied also by a huge number of possible real states that will never be realized such that concerning $EE3$ the number of possible real states will be greater for the ideal gas-law than for Van der Waals' law.

$$EE3(I.\,G.\,L.) > EE3(V.\,d.\,W.)$$

With the help of the above distinction, the mentioned ambiguity concerning both gas-laws can be resolved. To calculate the epistemic entropy, it is important to distinguish the possible real states that are and will be realized ($EE^{\star}$) and those which will never be realized ($EE3$) but are possible real states fulfilling all the conditions of 5.3.3. Then we can give a unique (non-ambiguous) answer to the question which law has the greater epistemic entropy in the sense of $EE^{\star}$ or $EE3$.

If we compare $EE^{\star}$ to EE (the sum of all three: $EE1 + EE2 + EE3$) then it is very probable that because of the huge number of $EE3$ it holds that $EE(I.G.L.) > EE^{\star}(V.d.W.)$.

In a similar way we can reconsider Newton's Law of Motion. First of all, it is plain that the epistemic entropy of Newton's formulation of his law is greater than the epistemic entropy of Euler's formulation since Newton's formulation permits also cases of possible real states, where the mass m is increasing.

Further, there is no ambiguity of the term "possible real state" provided one includes the differentiation above into $EE1$, $EE2$, and $EE3$. Thus, both $EE^{\star}$ and EE are greater in Newton's formulation (NForm) than in Euler's (EForm):

$$EE^{\star}(NForm) > EE^{\star}(EForm)$$
$$EE(NForm) > EE(EForm)$$

Comparing the arbitrary finite velocity permitted by Newton's law with the velocity-limit (velocity of light in vacuum) of Einstein's improvement (Ein) there is no difference in the number of possible real states since there are none for velocities beyond that limit. Thus, concerning velocity $EE(NForm) = EE(Ein)$. However, the possible states satisfying Newton's law are much greater than those satisfying Einstein's improvement. Consequently, the possible states excluded by Einstein's improvement (Ein) are much greater than those excluded by Newton's law. That means that the epistemic information of Einstein's improvement is much greater than the one of Newton's law:

$$EI(Ein) > EI(NForm)$$

Concerning mass, we have already mentioned that Newton's formulation has a greater epistemic entropy than Euler's. If we compare the epistemic entropy of Newton's formulation of his law to that of Einstein's improvement, then we have to observe that Newton's formulation leaves open everything about the values of the dependency between mass and velocity; whereas the Lorentz Transformation determines a special dependency between mass and velocity v such that mass increases with the factor $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ (where c is the velocity of light in vacuum). From this, it seems that $EE(NForm) > EE(Ein)$. However, like in the consideration with velocity, there are no possible real states that do not obey Einstein's improvement since by condition (1) of 5.3.3 they have to be compatible with the well confirmed laws. Thus, $EE(NForm) = EE(Ein)$. However, there is a great number of possible states satisfying Newton's law which do not satisfy Einstein's improvement: all those that satisfy other dependencies between mass and velocity are permitted by Newton's formulation. From this, it follows that Einstein's improvement excludes much more possible states than Newton's formulation of his law. In other words, also w. r. t. mass it holds: the epistemic information of Einstein's improvement is greater than that of Newton's law:

$EI(Ein) > EI(NForm)$

5.4.5 Verisimilitude

The idea of verisimilitude was invented by Karl Popper. According to Popper's idea a theory is nearer to the truth (has greater verisimilitude) the more true and the less false consequences it has. Assuming T to be the true, F to be the false sentences, and $Cn(A)$, $Cn(B)$ to be the consequences of A and of B then we may define the true consequences of A as $A_T = Cn(A) \cap T$ and the false consequences of A as $A_F = Cn(A) \cap F$. Then Popper's idea of verisimilitude[8] can be defined as follows. Assuming that theory A and B are comparable concerning their consequences, then:

Theory A is nearer to the truth than theory B ($A \overset{T}{>} B$) iff
either: $B_T \subset A_T$ and $A_F \subseteq B_F$
or: $B_T \subseteq A_T$ and $A_F \subset B_F$

In simple words: A is nearer to the truth than B iff A has more true and not more false consequences than B or B has not more true consequences and more false consequences than A (where the concept of consequence is the classical concept of logical consequence defined by Tarski[9]. The origin of the idea of verisimilitude was seen and stated by Popper very clearly as follows: In all empirical sciences, we are often confronted with the question which of two hypotheses or theories is the better one in the sense that it corresponds better to the facts than the other,

8 It is described first in his book *Conjectures and Refutations* (1963), p. 233 and later in *Objective Knowledge* (1972), p. 52.
9 Tarski (1936; 1956 ch. 16)

although both theories may have some false consequences and hence be, strictly speaking, false. So though we assume that neither Newton's theory nor Kepler's nor Galileo's are, strictly speaking, true, we are sure that Newton's theory is a better approximation to the truth than either of the others or than even the two others together. In other words, we are dealing with false theories from which we have to choose those which are closer to the truth; and this means we have to choose—according to Popper—that theory which has more true consequences and less false ones than the other alternative theories. This approach to verisimilitude—due to Popper—is called the *consequence-approach*.

In 1974 both Tichy and Miller showed[10] (independently) that according to Popper's definition of verisimilitude (given above) no false theory can stand in this relation $\overset{T}{>}$ to another false theory. As has been said already in section 4.3.7 above the reason is the property of Classical Logic to allow redundancies or irrelevant elements in the consequence-class of a valid argument. This property of Classical Logic (CL) is also the culprit of many paradoxes when CL is applied outside logic and mathematics to empirical sciences: Hesse's paradox of confirmation, Goodman's paradox, paradoxes of Deontic Logic and others.[11] As has been shown in Schurz and Weingartner[12] Popper's original idea can be rehabilitated if one restricts the classical consequence class to the most informative relevant consequence elements. Nothing essential is lost with this restriction however since it can be proved that this restricted consequence class is classically logically equivalent (though not relevantly equivalent) to the classical consequence class.

There are other approaches to verisimilitude: The constituent approach of Tichy (1974, 1976), Niiniluoto (1987), Miller's approach (1976) and the possible worlds approach by Hilpinen (1976) and Kuipers (1982). The first has difficulties when applied to non-trivial scientific theories since in this case, the number of possible constituents reaches an astronomical size so that such an application is hardly possible. The second leads to the unacceptable result that if theory A is closer to the truth than theory B and both are false, then B follows logically from A. The third approach was refuted by the impossibility theorem of Zwart and Franssen (2007). This theorem, however, does not hold for the consequence-approach, defended as a rehabilitation of Popper's original idea, by Schurz and Weingartner (1987 and 2010).

The following idea is a new approach based on knowledge K 7, defined above with the help of epistemic entropy and epistemic information. Since these two con-

10 Tichy (1974); Miller (1974)

11 See Weingartner (2001)

12 Schurz and Weingartner (1987, 2010), Weingartner (2000), ch. 9.

cepts are defined quantitatively the so defined concept of verisimilitude becomes a quantitative one too.

5D6 Let A, B be two hypotheses or theories comparable concerning their possible real states. Then: A is nearer to the truth than B iff $EE^{\star}(A) > EE^{\star}(B)$ and $EI(A) > EI(B)$.

For "A is nearer to the truth than B" we may also write "A has greater verisimilitude (VS) than B", i. e. $VS(A) > VS(B)$. We take $EE^{\star}$ instead of EE (5 D 4) since with respect to possible real states which will never be realized ($EE3$) two hypotheses or theories are hardly comparable.

5.4.5.1 Application to Kepler and Newton

When applying 5 D 6 to Kepler's and Newton's theory there are two possibilities of comparing: (1) by assuming the same domain as a reference of the theories. This means that the reference of both is the planetary system which restricts, of course, Newton's theory significantly. (2) by assuming the planetary system as the reference of Kepler's theory and a physical system of arbitrary moving bodies as the reference of Newton's theory. For Kepler's theory we take his three laws (5.4.3.2 above) and for Newton's theory we take his law of motion (5.4.3.4) plus his law of gravitation:

$$F_G = G \cdot \frac{m_1 \cdot m_2}{r^2}$$

Here m_1, m_2 are the masses of the two bodies, r is their distance and G is the gravitational constant which also plays an important role in physics in general and specifically in the theory of General Relativity and cosmology.[13]

Ad (1): For the first four planets Kepler's laws are correct until the $5th$ decimal place even with the present accurate measurements. Therefore, concerning the first four planets $EE^{\star}(Kepler) = EE^{\star}(Newton)$. However concerning the big planets, Jupiter and Saturn, Kepler's third law is not accurate enough since the mass of the planets in proportion to the mass of the sun is neglected in the third law. Therefore, concerning all planets together $EE^{\star}(Newton) > EE^{\star}(Kepler)$. Because of this (even small) inaccuracy of Kepler's third law, also Newton's theory rules out more possible states (as not obtaining) such that $EI(Newton) > EI(Kepler)$. Thus

13 For more information on fundamental constants of nature see Mittelstaedt and Weingartner (2005) section 8.2.

the verisimilitude VS of Newton's theory is greater than that of Kepler's theory with respect to the planetary system: $VS(Newtonrestricted) > VS(Kepler)$.

Ad (2): If we compare Newton's theory referring to moving bodies in general with Kepler's theory referring to bodies of the planetary system, then it is easy to see that both will hold:

$EE^{\star}(Newton) > EE^{\star}(Kepler)$
$EI(Newton) > EI(Kepler)$

Thus also in the general case when Newton's theory is not restricted it has a greater verisimilitude than Kepler's: $VS(Newton) > VS(Kepler)$

5.4.5.2 Application to the Gas-Law

From the consideration in 5.4.3.3 and 5.4.4 it is plain that the epistemic information of Van der Waals' law is greater than that of the General Gas Law. Therefore it holds:

$EI(V.d.W.) > EI(G.G.L.)$

Concerning the epistemic entropy we have to distinguish $EE^{\star}$ from EE, since only $EE^{\star}$ representing the possible real states of the past and future are relevant for verisimilitude (5 D 6). And for $EE^{\star}$ it holds:

$EE^{\star}(V.d.W.) > EE^{\star}(G.G.L.)$

From these two propositions it follows:

$VS(V.d.W) > VS(G.G.L.)$

5.5 Answers to the Objections

5.51 (To 5.11:) It is correct, as stated in the objection that not all types of knowledge are information, that is received as messages carried by a signal: knowledge 1 and knowledge 2 are not. However, both knowledge 1 and knowledge 2 is information in the wider sense that possessing them means—at least partially—possessing epistemic entropy and epistemic information. The phrase "at least partially" refers to the degree of the reflection concerning simple principles of logic or mathematics or one owns inner experience as in the cogito ergo sum.
5.52 (To 5.12:) To be concerned with the actuality represented by the occurrence

of a state of affairs also means to have selected that state of affairs and eliminated all other possibilities. Therefore, the kinds of knowledge 3–7 (except the ones 1 and 2) are all concerned with both epistemic entropy and epistemic information.

5.53 (To 5.13:) A high degree of probability is not always a high value for knowledge. This should be clear from sections 5.3.2: Logical truths which carry no empirical information have probability $Pb = 1$. If a proposition has an extremely high degree of epistemic entropy, then its probability is close to 1. A state of affairs in the vicinity of the equilibrium has a probability close to 1. An informative state or a state of order is improbable. Analogously, non-trivial empirical knowledge is improbable. Also according to Shannon's definition of information, a high degree of information goes together with a low degree of probability. Therefore, possessing information can be a necessary condition for non-trivial empirical knowledge.

5.54 (To 5.14:) The objection in 5.14 can be clarified when "information" is replaced by "measure of information" in the sense of Shannon. Since then it means the amount of (new) information (or knowledge) that is *added* when getting the message or when making the measurement. Thus, if there is only one state of affairs that can occur, a further message or measurement will not add anything such that the measure of information will be minimal or zero. From this consideration it is plain that there is no hindrance to speaking of knowledge as (new) information.

6 Whether there are Properties Common to All Seven Types of Knowledge

6.1 Arguments Contra

6.11 As Chisholm and Hintikka say, a strong concept of knowledge is defensible:

> We consider certain things that we know to be true…[1]

> …whatever is known has to be true.[2]

This can be defended since in a good sense, one can say that “the person *a* knows that *p*, but *p* is false” is contrary to a widespread usage of the concept of knowledge in both everyday language and scientific discourse. Thus, the principle $aKp \rightarrow p$ seems to be defensible as a general principle for knowledge.

However, looking at the seven types of knowledge defined in chs. 4 and 5, only K1–K4 can satisfy this principle, whereas K5, K6, and K7 do not satisfy it: Not K5 because even very well corroborated (confirmed) hypotheses might be false, not K6 because there are cases where justified true belief is not knowledge and not K7 because epistemic entropy and epistemic information might not be sufficient for truth.

Therefore, properties of a strong concept of knowledge as $aKp \rightarrow p$ are not common to all seven types of knowledge.

6.12 As Hintikka says, closure concerning *modus ponens*, or more general, closure concerning logical consequence, is a defensible property of knowledge:

> By means of my rules, it is readily seen that
> (11) $aKp \rightarrow aKq$
> is valid as soon *p* logically implies *q* in our ordinary propositional logic.[3]

Although this property may be defensible for very simple inferences applied to K1 and K2, it is certainly not defensible for logical consequences in difficult proofs of logic and mathematics (K3) and not for consequences of complex experimental results (K4). Otherwise, Frege would have known the inconsistent consequence of

1 Chisholm (1966), p. 24. Cf. chs. 2 and 5.
2 Hintikka (1962), p. 48
3 Hintikka (1962 p. 30)

https://doi.org/10.1515/9783110585797-006

his axiom 5 (comprehension axiom) discovered by Russell[4] and more than only a few physicists would have drawn the right consequences for quantum mechanics from Einstein's experiment of the photo-electric effect.[5]

Therefore, closure concerning logical consequence is not a property common to all seven types of knowledge; and it may even not be a property of any type of human knowledge.

6.13 The distribution of knowledge over the conjunction

$aK(p \wedge q) \rightarrow (aKp \wedge aKq)$

seems to be a property common to all types of knowledge. There is, however, the problem of knowability, that from the premise "whatever is true is possibly known", $p \rightarrow \Diamond aKp$, the false conclusion "whatever is true is known", $p \rightarrow aKp$, follows. The premises needed are only the two necessary implications:
(1) $aKp \Rightarrow p$ as necessary condition for a strong concept of knowledge, and
(2) $aK(p \wedge q) \Rightarrow (aKq \wedge aKq)$ which seems even more harmless.

Here, $p \Rightarrow q =_{df} \Box(p \rightarrow q)$ and the modalities $\Diamond$ and $\Box$ can be interpreted with the weak modal system S0.5.[6]

From this it follows: Since the strong condition for knowledge $aKp \Rightarrow p$ does not seem to be the reason for deriving the false conclusion it seems that the principle of distribution of knowledge over conjunction (2) must be the culprit.

Therefore: Distribution of knowledge over conjunction cannot be a property common to all types of knowledge.

6.14 It seems that a strong concept of knowledge is not defensible at all. According to Hintikka the following principles are acceptable (in an epistemic logic)[7]:
(1) $aKp \rightarrow p$ strong concept of knowledge
(2) $\neg aK\neg p \leftrightarrow aPp$
where aPp = it is possible, for all that a knows, that p.

4 Cf. Frege (1903) Vol. II, Postscript.
5 Einstein (1905a). Cf. Jammer (1966)
6 Cf. Routley (1981). The gist of the argument seems to originate with Fitch (1963) who attributed it to an anonymous referee. See Williamson (2000) p. 170.
7 See Hintikka (1962) p. 34 and 59.

However, from these two assumptions absurd consequences follow:
(3) $aK\neg p \rightarrow \neg p$ from (1)
(4) $p \rightarrow \neg aK\neg p$ from (3)
(5) $p \rightarrow aPp$ from (2) and (4)

Consequence (5) seems totally unacceptable. If we interpret aPp as $aK\Diamond p$ then (5) says: If p is true then (any person) a knows that p is possible. This is refuted by the fact of some (so far) unknown scientific truth p_1 which was not known to be possible—even by scientists working in the respective domain—before p_1 was proved of verified by experiment. If we interpret aPp as $\Diamond aKp$ then we end up with the premise $p \rightarrow \Diamond aKp$ which leads together with (1) and the absurdity of $aK(p \wedge \neg aKp)$, i. e. $\neg\Diamond aK(p \wedge \neg aKp)$ to the non-acceptable absurd consequence: $p \rightarrow aKp$. Therefore (1), i. e. a strong concept of knowledge, seems not to be acceptable.

6.2 Argument Pro

6.21 There must be a common property to all seven types of knowledge, provided they are consistent. If they are, they must satisfy the following principle which might be called the consistency condition for knowledge which seems to hold also for other epistemic notions such as belief:

$$aKp \rightarrow \neg aK\neg p$$

In words: If a knows that p then it is not the case that a knows that not-p. If this principle is violated then both: a knows that p and a knows that not-p which is contradictory. Therefore: There are properties common to all seven types of knowledge.

6.3 Proposed Answer

There are common properties to all seven types of knowledge and other properties that are common to some subgroups of the seven types of knowledge.

6.3.1 Common Properties to All Seven Types of Knowledge

(1) Consistency Condition: For all seven types of knowledge, it would be absurd that someone would know that p and know that non-p or that someone would know both: p and non-p. Since we have not introduced time-indices for the concept of knowledge the reference is always to the same time.

Thus to know that $2 + 3 = 5$ and that $2 + 3 \neq 5$ is impossible (K1). To know that I exist and that I don't exist is also impossible (K2). To know that p is both provable and not provable is impossible (K3). So for the other types of knowledge. Therefore the following consistency conditions for all seven types of knowledge must hold:

6 C 1 $\neg aK(p \wedge \neg p)$

6 C 2 $\neg(aKp \wedge aK\neg p)$

6 C 3 $aKp \rightarrow \neg aK\neg p$

6 C 4 $aK\neg p \rightarrow \neg aKp$

6 C 5 $aKp \rightarrow Cons(p)$

(2) Enlarged Consistency Conditions: Hintikka accepts the following enlarged condition: If a set of statements $\{p_1, p_2 \ldots p_n\}$ is consistent then the set $\{p_1, p_2 \ldots p_n; Kp_1, Kp_2 \ldots Kp_n\}$ is also consistent.[8] This principle can be accepted since a consistent set of statements cannot become inconsistent just by adding that these statements are known.

(3) Modus Ponens and Distribution over $\rightarrow$: *Modus ponens* is frequently used in both scientific discourse and everyday language discourse. It seems impossible that someone knows that p, aKp, and in addition knows that $p \rightarrow q$, i. e. that p implies q or that q follows from p, $K(p \rightarrow q)$ and yet does not know that q. In other words: If someone a knows that p then—provided a knows that q follows from p—a also knows that q. More generally: If one knows that p one also knows those consequences q of which one knows that they follow from p. As should be clear, these are not all the consequences that logically follow from p but only those that are known to follow by the respective person (a). The so described *modus ponens* principle for knowledge holding for all seven types of knowledge is this:

MPK $[aKp \wedge aK(p \rightarrow q)] \rightarrow aKq$

8 Hintikka (1962) p. 24

In the second part $p \rightarrow q$ can be interpreted as a material implication; but the principle holds also if $p \rightarrow q$ is replaced by a strict implication $p \Rightarrow q$ (where $p \Rightarrow q =_{df} \Box(p \rightarrow q)$) or by logical consequence $p \vdash q$.

By 2-valued Classical Propositional Logic (CPC) distribution over → (DK→) follows from MPK and vice versa. The principle used are *exportation* and *importation*. A justification of DK→ is analogous to that of MPK.

DK→ $aK(p \rightarrow q) \rightarrow (aKp \rightarrow aKq)$

It is important to observe that MPK and DK→ must not be confused with the much stronger principles of *deductive infallibility*. They obtain from MPK and DK→ by dropping aK in front of $p \rightarrow q$.

DI 1 $(aKp \wedge p \rightarrow q) \rightarrow aKq$
$(p \rightarrow q) \rightarrow (aKp \rightarrow aKq)$

DI 2 $(aKp \wedge p \Rightarrow q) \rightarrow aKp$
$(p \Rightarrow q) \rightarrow (aKp \rightarrow aKq)$

DI 3 $(aKp \wedge p \vdash q) \rightarrow aKq$
$p \vdash q \rightarrow (aKp \rightarrow aKq)$

We do not accept any DI for knowledge since human knowledge does not satisfy these principles. Some speak of an "ideal" conception of knowledge[9], some others interpret "to know"as "to be knowable". However, both explanations are playing down the facts: there is no such rationality among humans that would make it possible to know all the logical consequences of what one knows. Otherwise, proofs of theorems would be superfluous. Moreover, every difficult logical or mathematical proof (recall section 4.35) refutes principles DI of deductive infallibility.

According to Thomas Aquinas, the knowledge of angels has the property of deductive infallibility. Every angel knows all the logical consequences of what he knows.[10] The replacement by "knowable" is not adequate since not all logical consequences of what one knows are knowable in the sense of human ability. Rescher has proposed to restrict closure of logical consequences (for beliefs) to "obvious consequences". This, although not a sharp demarcation, seems reasonable also for knowledge if such simple principles as *modus ponens, double negation,*

9 Hintikka's system in his *Knowledge and Belief* accepts DI. Cf. Hintikka (1962) p. 34.f.
10 Thomas Aquinas (STh) I, 58, 3

hypothetical syllogism (BARBARA), simplification, modus tollens are involved.[11] Moreover, as will be shown in section 6.43 (cf. 6.13), the thesis that all truths are "knowable" is much too strong and leads to the false claim that all truths are known[12] (even with a very weak system of modality as S0.5).

Deductive Infallibility is also sometimes expressed by rules: If $p \to q$ is provable (valid) then so is $K(p \to q)$ and $Kp \to Kq$:

$$\frac{\vdash(p\to q)}{\vdash K(p\to q)} \qquad \frac{\vdash(p\to q)}{\vdash(Kp\to Kq)}$$

Deductive Infallibility sometimes originates in *logical omniscience*, i. e. in the thesis that all logically true propositions are known.

LO If p is logically true (valid, provable) then p is known (Kp).

The motive of LO often comes from the fact that epistemic systems are constructed out of modal systems (preferably S 4 or S 5 or some intermediate system between). These systems have the so-called "necessitation rule", i. e. if p is logically true (valid, provable) then: necessary p ($\Box p$) holds.[13]

N If p is logically true (valid, provable) then $\Box p$.

In such reconstructions '$\Box$' is interpreted epistemologically as 'K' such that logical omniscience (LO) follows immediately from necessitation (N). Moreover, it is easily seen that the above rules which express deductive infallibility follow from LO.

The mistake of such an interpretation is obvious: it confuses necessity (validity, provability) with knowledge. Whereas all logical consequences of necessary (valid, provable) propositions are necessary (valid, provable), it is not the case that all logical consequences of known propositions are again known. Thus, necessity, validity, provability are different notions from (human) knowledge.[14]

That deductive infallibility (DI) follows from logical omniscience can be easily seen: If every proposition that is logically true, valid or provable is known then also every valid argument leading from premises to conclusions is known, whatever the premises and conclusions are. The realistic epistemic logic below does neither accept logical omniscience nor deductive infallibility; however, it has

11 Rescher (1967) p46f.

12 This is the so-called "paradox of knowability" attributed in Fitch (1963).

13 For a modal logic without necessitation see Weingartner (1968) and (2009).

14 See the criticism in Weingartner (1981) p. 149f., (1982) p. 249f.

MPK and DK→ among its principles. Observe that the epistemic operators K, G, B for knowledge and belief—as they are understood here—do not allow exchange of wffs by logically equivalent wffs inside the scope of the epistemic operator; nor do they obey logical closure of wffs inside the epistemic operator. Otherwise, deductive infallibility and logical omniscience would enter through the back door.

(4) Distribution of knowledge over ∧: What is meant here is the following principle DK∧ which is valid is many systems of epistemic logic:

DK∧ $aK(p \wedge q) \rightarrow (aKp \wedge aKq)$

Some epistemic logics accept even the equivalence. Accepting the equivalence means to also accept the opposite of DK∧, i. e. the fusion of knowledge over conjunction (FK∧).

FK∧ $(aKp \wedge aKq) \rightarrow aK(p \wedge q)$

That FK∧ cannot be accepted as a general principle of epistemic logic is shown by the following counterintuitive applications in different domains.

(a) Physics: Commensurability: CPC makes the presupposition that two arbitrary propositions may be fused into a conjunction. According to Classical Logic the domain of meaningful propositions $p, q, r, \ldots$ is truth-functionally closed under the usual connectives[15] and thus also under conjunction ∧. Thus, if p and q describe physical states, CPC dictates that also $p \wedge q$ must describe a physical state. Or more specifically: If (under conditions r) proposition p describes the physical state P (that the position of a particle has a certain precise value) and (under conditions r) the proposition q describes the physical state Q (that the momentum of that particle has a certain precise value), then it is not the case that (under conditions r) the conjunction $p \wedge q$ describes a measurable magnitude at all. However, according to classical logic it should, because the corresponding principle is a theorem in the underlying classical propositional logic:

$$[(r \rightarrow p) \wedge (r \rightarrow q)] \Rightarrow [r \rightarrow (p \wedge q)]$$

15 This problem of universal logical closure is also stressed as an important problem by Dalla Chiara et. al. (2001) p. 59 in connection with Unsharp Quantum Logic.

If we apply knowledge to this case and assume that p (describing the physical state P) is known and q (describing the physical state Q) is known then $p \wedge q$ cannot be known because it does not exist as a physical state.

(b) Animal Behaviour: Assume the proposition S represents (describes) the observable state of affairs that sexual excitement obtains, the proposition A represents (describes) the observable state of affairs that aggression obtains and the proposition F represents (describes) the observable state of affairs that fear obtains. Then research about animal behaviour shows the following facts: $S \wedge F$ does not represent (describe) an observational state in male animals but does represent (describe) an observable state in female animals. On the other hand: $S \wedge A$ does not represent (describe) an observable state in female animals but does represent (describe) an observable state in male animals. Let K be K_4 (known as verified by observation). Then: If $aK(S) \wedge aK(F)$ then $aK(S \wedge F)$ is not true for male animals. And if $aK(S) \wedge aK(A)$ then $aK(S \wedge A)$ is not true for female animals.

(c) Human Actions: Assume proposition A represents (describes) the (observable) action (state of affairs) of writing an essay on Alternative Logics and the proposition B represents (describes) the (observable) action (state of affairs) of making a ski-tour. Then $A \wedge B$ does not represent (describe) an (observational) action. Therefore: $aK(A) \wedge aK(B)$ may be true though $aK(A \wedge B)$ is not.

The examples (a)–(c) show that FK∧ cannot be accepted as a general principle.[16] On the other hand, there do not seem to be serious counterexamples against DK∧. Support for DK∧ is also provided from investigation on relevance by Gerhard Schurz and Paul Weingartner: To give a solution to Popper's problem of verisimilitude it is necessary to restrict the consequences which follow according to CPC from some premises in two ways: by throwing out those which can be replaced by any other consequence salva validitate of the inference (replacement criterion RC)*replacement criterion* and by throwing out those which are not simplest most informative consequence elements (reduction criterion RD). These relevance restrictions are also necessary when logic is applied to empirical science (and especially to physics) and also to Deontic Logic.[17] DK∧ is supported by the criterion RD which allows splitting up conjunctions into its parts, but does not allow in general to make fusions from separated parts. Furthermore, RD permits only those distribution laws (from disjunctions to conjunctions) which are acceptable in Quantum Logic. For the reasons outlined, we accept DK∧ as a general valid principle of Epistemic Logic (see however 6.43 below).

16 Hintikka (1962) defends FK∧ as self-sustaining, i. e. as a "virtual implication" (in his terminology). However, he does not discuss real counterexamples such that this defence depends very much on his system (cf. p. 58f.) which makes too strong assumptions anyway.

17 Cf. Schurz/Weingartner (1987), Schurz (1991), Weingartner (2001), (2009), (2010), (2015).

(5) Knowledge Whether: We say that someone knows whether (K^0) something (p) is the case iff he (she) knows that p is the case or that non-p is the case. This holds for all seven types of knowledge. Thus the definition is:

$\mathrm{K}^0 \quad aK^0p \leftrightarrow (aKp \vee aK\neg p)$

It is easy to see that K^0 follows from both from aKp (a knows that p) and from $aK\neg p$ (a knows that non-p):

$aKp \rightarrow aK^0p$
$aK\neg p \rightarrow aK^0p$

(6) Ignorance: We say that someone is ignorant (IG) concerning p iff he (she) neither knows that p nor knows that non-p. In other words: a is ignorant concerning p iff it is not the case that a knows whether p.

$\mathrm{IG} \quad aIGp \leftrightarrow (\neg aKp \wedge \neg aK\neg p)$

For human knowledge, we have to assume that everyone is ignorant concerning some proposition. Even the stronger assumption has to be accepted that there are some propositions (representing facts) such that everyone is ignorant about them. These two principles are represented thus:

$(\forall a{\in}\mathrm{H})(\exists p)(\neg aKp \wedge \neg aK\neg p)$
$(\exists p)(\forall a{\in}\mathrm{H})(\neg aKp \wedge \neg aK\neg p)$

Recalling section 1.3, we may say that knowledge (of different) types is not only possible but a fact. And thus, from the fact that $(\exists p)(\forall a{\in}\mathrm{H})aKp$, there are propositions such that everyone knows them, it follows that for some proposition p everyone knows whether p is the case. Consequently it also holds that everyone knows concerning some proposition p whether p is the case.

Summing up, we may say that the principles given in (1)–(6) of section 6.3.1 can be accepted as holding for all seven types of knowledge described in chs. 4 and 5.

6.3.2 Common Properties to the First Four Types of Knowledge

There is the problem whether the strong property of knowledge—that knowledge implies truth—can be defended for all types of knowledge. This property of knowledge is usually defended by considering its negation or denial, i. e. that someone knows that p is the case but it is not so (true) that p is the case. Since this seems to be impossible, the principle K seems to be generally acceptable:

K $\quad aKp \rightarrow p$

Checking through the seven types of knowledge it seems clear that knowledge 1—the axioms in the sense of Aristotle—must fulfil K (cf. 4.3.3). The same holds for knowledge 2, that is for Augustine's *fallor ergo sum* and Déscartes's *cogito ergo sum* (cf. 4.3.4). However, also knowledge by proof, i. e. a proof which can be objectively checked by different scholars satisfies condition K (cf. 4.3.5). Eventually, knowledge 4 is a candidate for a strong concept of knowledge since we suppose that the respective single event can be verified directly, indirectly (by instrument), or by experiment in an objective and reliable way (cf. 4.3.6).

Concerning the remaining 3 types of knowledge, the principle K cannot be claimed as completely satisfied:

There is first the knowledge 5 of a hypothesis that is very well corroborated or confirmed. Although the confirming (position) instances as single events can be known in the sense of knowledge 4, the respective hypothesis is not thereby known; it is only known to be well corroborated if the tests were severe and reliable (cf. 4.3.7).

Second, there is knowledge 6 as justified true belief. Since it is true belief of some fact p it seems to satisfy K. However, the difficulty is that the fact that the belief is true is not known, as the examples of famous conjectures show (cf. section 4.3.8 above). Since before the proof was given we cannot speak of knowledge.[18] However, in many other cases, think of believing experts, justified true belief is sufficient for knowledge including satisfying principle K. Therefore, knowledge 6 seems to satisfy principle K in many or even in most cases. Since there are some serious exceptions, we have to say that knowledge 6 satisfies principle K only partially.

Third, knowledge 7 as possessing both epistemic entropy and epistemic information might satisfy principle K. But there is no guarantee that it generally will do so. First of all, we have to remember that knowledge 7 is applicable only to empirical knowledge; an application to logical or mathematical truths is inappro-

18 Concerning the artificial examples of Gettier see section 4.4.4 above.

priate since their epistemic information is zero. A proposition with high epistemic entropy—if true—has a low degree of informative content; a proposition with high epistemic information—if true—has a very selected (not necessarily low) degree of epistemic entropy. However, in neither of the cases, there is a guarantee for truth. Therefore, principle K can only be partially satisfied by knowledge 7.

6.3.3 Common Properties when Two Persons are Involved

Observe that the following principles and their consequences can be defended without presupposing K, $(aKp \rightarrow p)$, as a general principle. Only specific instances of it are used here and defended independently. We assume that two persons a and b are involved and consider the following statements:

(1) a knows that b knows that p; short: $aK(bKp)$.
(2) a knows whether b knows that p; short: $aK^0(bKp)$.
(3) a knows that b knows whether p; short: $aK(bK^0p)$.
(4) a knows whether b knows whether p; short: $aK^0(bK^0p)$.

Statement (1) can be understood in a twofold way:

(1a) a knows that also b knows that p. In this case, both a and b know that p.
(1b) a knows that certainly b (will) know(s) that p. In this case, it follows only that b knows that p, but not generally that a knows that p.

Example: The student of mathematics, a, knows that the professor of mathematics, b, knows that the essential steps of Perelman's proof (of the Poincaré conjecture) are such and such. From this, it need not—and usually does not—follow that also the student of mathematics knows this.

Checking whether both (1a) and (1b) are applicable to all seven types of knowledge the result is: (1a) is certainly applicable to all seven types of knowledge. (1b) is applicable to K3–K7, but not to K1 and K2. This can be seen from the examples E 1–E 5 for K1 and K2: These truths are so simple that always both know the respective truth, i. e. (1a) is satisfied.

(1a) $aK(bKp) \rightarrow (aKp \wedge bKp)$
(1b) $aK' bKp \rightarrow bKp$

We may define therefore $aK(bKp)$ accordingly:

6 D 1 $\quad aK(bKp) \leftrightarrow (aKp \wedge aK' bKp)$

According to K^0 (section 6.3.2. (5)) (2) can be defined as: a knows that b knows that p or a knows that b does not know that p.

6 D 2 $\quad aK^0(bKp) \leftrightarrow (aK(bKp) \vee aK(\neg bKp))$

In some cases, $aK(\neg bKp)$ can mean: a knows that b does not yet know that p. 6 D 2 (2) can be applied to all seven types of knowledge. However, if applied to K1 and K2 $aK(bKp)$ will be always true and $aK(\neg bKp)$ false such that $aK^0(bKp)$ will always be satisfied. For the other types K3–K7 either of the alternatives may be fulfilled and so $aK^0(bKp)$ is satisfied.

The sense, in which $aK^0(bKp)$ is defined, is best expressed by saying "a knows whether b already knows that p". This phrase is often used in everyday language and scientific discourse. Its meaning presupposes clearly that a knows that p and thus also that a knows whether p. However, it is left open whether b knows that p or whether he (she) does not yet know it. Thus neither bKp nor $\neg bKp$ nor bK^0p follows from $aK^0(bKp)$:

$aK^0(bKp) \rightarrow aKp$
$aK^0(bKp) \rightarrow aK^0p$

With the interpretation of (3) we face a similar distinction as seen in (1). There is a stronger (3a) and a weaker (3b) version of (3) which can be grasped from a similar example:

(3a) If we say the professor of nuclear physics knows that his students know whether an atomic nucleus can have more than 3 quarks (p) then we assume that both the professor and his students know whether…

(3b) However, if we say the student knows that the professor knows whether p then we assume that the professor knows whether p, but not necessarily the student.

(3a) $\quad aK(bK^0p) \rightarrow aK^0p \wedge bK^0p$
(3b) $\quad aK' bK^0p \rightarrow bK^0p$

The respective definitions are these:

6 D 3 $\quad aK(bK^0p) \leftrightarrow (aK(bKp) \vee aK(bK\neg p))$
6 D 4 $\quad aK' bK^0p \leftrightarrow aK(bK^0p) \vee bK^0p$

6 D 4 is one possibility to define a weaker version of $aK(bK^0p)$. An alternative definition is D 9 of ch. 13.

Concerning the applicability to the seven types of knowledge, (3a) is applicable to all seven types whereas (3b) only to K3–K7 since in the case of K1 and K2 also aK^0p must be assumed.

Statement (4) is weaker than all the proceeding three. The meaning seems best expressed by saying "*a* knows whether *b* already knows whether *p*". An example would be: The physician knows whether the parents of the patient already know whether the result of the operation was good or not. In this case, we assume that the physician knows whether the result was good or not. Thus aK^0p must be derivable from $aK^0(bK^0p)$. On the other hand, we do not assume that the parents in fact already know whether the operation had a good result or not. Thus bK^0p must not be derivable from $aK^0(bK^0p)$. Moreover, one would agree to the following: If the physician *knows that* the parents already *know whether* the operation was successful then it holds also that the physician *knows whether* the parents already *know whether*…, i. e. $aK(bK^0p)$ implies $aK^0(bK^0p)$. Further, if the physician *knows whether* the parents already *know that* the operation was successful, then again the physician *knows whether* the parents already *know whether*…, i. e. $aK^0(bK^0p)$ must also be derivable from $aK^0(bKp)$.

(4a) $aK^0(bK^0p) \to aK^0p$

The definition is as follows:

6 D 5 $aK^0(bK^0p) \leftrightarrow [aK(bKp) \vee aK(\neg bKp) \vee aK(bK\neg p)]$

From 6 D 5 it is easy to see that (4) follows from all the three, from (1), (2) and (3). Concerning applicability to the different types of knowledge, we have to say that (4) is applicable to K3–K7. In the cases of K1 and K2 we should require that also the second person knows whether (and that) *p*.

6.4 Answers to the Objections

6.41 (To 6.11:) It is correct as said in 6.1.1 that a strong concept of knowledge which satisfies K ($aKp \to p$) is applicable only to the four types of knowledge K1–K4. However, what has been said to the other three types of knowledge is too incomplete:

(1) The well-corroborated and well-confirmed laws are the most important part of our scientific theories. They are more corroborated and well confirmed than the best corroborated hypotheses. These laws are the gist of such theories as the theory of Special and General Relativity and Quantum Theory. Even if these

laws might have some yet unknown false consequences, they are nearer to the truth than all their scientific precursors. “Nearer to the truth” can be defined with the help of Popper’s definition of verisimilitude: Theory T 1 is nearer to the truth than T 2 iff T 1 has more true and less false consequences than T 2. Under “consequence” we have to understand here a relevance-restricted consequence of Classical Logic in order to avoid the objections of Tychi and Miller and to save Popper’s original idea.[19] The answer to the question whether type K5 can be called knowledge is this: laws as the core of scientific theories are the most comprehensive scientific “knowledge” that we have. They might be not completely universally true, but they are very close to the truth. In this sense, they satisfy the condition K for strong knowledge not completely but almost completely.

(2) Justified true belief (K6) satisfies the condition that it is true and thus satisfies condition K. That it is not called knowledge in cases where the conjecture has not yet been proved is a more accidental factor of finding out within some time. Therefore it can be called knowledge in a strong sense after the proof has been given.

(3) It is correct that epistemic entropy and epistemic information might not be sufficient for truth. However, the situation here is similar to that of corroboration and confirmation. A proposition or a scientific law can have such an epistemic entropy and such an epistemic information that the law (proposition) is very close to the truth, such that condition K is satisfied almost completely.

6.42 (To 6.12:) As the objection says correctly, closure concerning logical consequence (deductive infallibility) is not a property of human knowledge. It may be a property of angels as Thomas Aquinas says.[20] This was also substantiated in section 6.3.1 (3) for specific principles DI, LO and N. N is a strong assumption of most modal logics, and it leads to LO (logical omniscience) if necessity is interpreted as knowledge. That such interpretations are not appropriate to human knowledge has been discussed there and need not be repeated here.

6.43 (To 6.13:) The distribution of knowledge over conjunction (DK$\wedge$) is not the culprit of the knowability paradox. One reason for that is that $p \rightarrow aKp$ is derivable without the principle DK$\wedge$ of 6.13 (distribution over conjunction): Williamson (1990) has shown that $p \rightarrow aKp$ can be derived from $p \Rightarrow \Diamond aKp$ and its converse $\Diamond aKp \Rightarrow p$ with the help of the modal system S 4. Although the converse

19 Such a solution has been offered in Schurz/Weingartner (1987).

20 Thomas Aquinas (STh)I, 58,3 and 4

and S 4 are rather strong assumptions, this still shows the independence of the knowability paradox from distribution over conjunction. As has been shown in 6.3.1 (4) the opposite of DK∧, FK∧ is too strong and cannot generally be accepted as the three counterexamples, given there, show. On the other hand, DK∧ is a basic principle of the weakest systems of different applied logics as modal logic, epistemic logic or deontic logic. DK∧ but not FK∧ can also be independently supported by an important relevance restriction RD, as has been said in 6.3.1 (4) above. The real culprit of the knowability paradox is $p \rightarrow \Diamond aKp$ (whatever is true is possibly known). This can be shown by the following simple argument which does not directly use DK∧:

(1)	$\neg aK(p \wedge \neg aKp)$	This can be derived as *reductio ad absurdum* by assuming the negation of (1) which leads to a contradiction.
(2)	$\Box\neg aK(p \wedge \neg aKp)$	Necessitation
(3)	$\neg\Diamond aK(p \wedge \neg aKp)$	From (2)
(4)	$(p \wedge \neg aKp) \rightarrow \Diamond aK(p \wedge \neg aKp)$	From $p \rightarrow \Diamond aKp$.
(5)	$\neg(p \wedge \neg aKp)$	M. T. (3), (4)
(6)	$p \rightarrow aKp$	From (5)

That $p \rightarrow \Diamond aKp$ is the culprit is also shown by Routley (1981) p.99f., who proves the opposite: There are unknowable truths. For this he uses the axiom: For some proposition q, q is true but not known. If we interpret knowledge (“known”) here as K3 (knowledge in the sense of proof), then the above axiom is a result of Gödel’s theorem: There are true propositions which cannot be proved nor disproved in an axiomatic system like *Principia Mathematica* or one about arithmetic.

6.44 (To 6.14:) As the objection says from the two premises (1) and (2) $p \rightarrow aPp$ follows. And this consequence—which is a consequence of Hintikka’s system—leads to absurd claims for both of its interpretations: aPp—it is possible, for all that a knows that p—as either $aK\Diamond p$ (a knows that it is possible that p) and $\Diamond aKp$ (possibly a knows that p).

However, the culprit is not (1) $aKp \rightarrow p$, i. e. a strong concept of knowledge, but Hintikka’s interchangeability of $\neg aK\neg p$ with aPp and its reading: it is possible, for all that a knows, that p. As said above and in the objection 6.14 this reading leads to absurd consequences when interpreted as either $aK\Diamond p$ or $\Diamond aKp$.

Another interpretation could be: For all propositions q, such hat a knows (them) q, it is possible that p, for some p known by a: $(\forall q)(aKq \rightarrow (\exists p)aK\Diamond p)$. But the difficulty with this interpretation is that it is provable:

(i) $(\forall q)(aKq \rightarrow aK\Diamond q)$ Assuming that $aK(p \rightarrow \Diamond p)$
(ii) $(aK\Diamond q \rightarrow (\exists p)aK\Diamond p)$
(iii) $(\forall q)(aKq \rightarrow (\exists p)aK\Diamond p)$

Therefore, according to this interpretation aPp is true by some weak epistemic logic and so is $p \rightarrow aPp$ trivially. However, it is impossible that Hintikka understood by aPp an epistemologically provable proposition. This is manifest from the fact that aPp is interchangeable with $\neg aK\neg p$ and the latter is not provable in Hintikka's epistemic system.

7 Whether it is Necessary to Believe Something

7.1 Arguments Contra

7.11 If a human action is free then it is not necessary. However, believing and learning something are free human actions. Therefore it is not necessary to believe something.

7.12 To disbelieve something is as free a human action as to believe something. Therefore it is not necessary to believe something.

7.13 If it is necessary to believe something, then the motives for this belief are compulsion, but not reason. However, neither in scientific nor in religious belief are the motives compulsion. Therefore, it is not necessary to believe something.

7.14 It cannot be logically necessary to believe something, say A, since disbelieving A is not contradictory. Nor can it be naturally necessary, i. e. necessary by laws of nature, to believe something since believing and disbelieving are free human actions. Therefore it is not necessary to believe something.

7.15 If that what is believed is not logically necessary but contingent then it is not necessary to believe it. Therefore it is not necessary to believe any contingent truth.

7.16 That what is believed can be logically or mathematically necessary. For example Fermat believed correctly that $x^n + y^n = z^n$ has no solutions for $n > 2$, which was proved by Wiles in 1994. But from the necessity of what is believed, it does not follow that the belief is neccessary. Therefore it does not seem to be necessary to believe something.

7.2 Arguments Pro

Every child learns something. But in order to learn something, it is necessary to believe something. Therefore, it is necessary to believe something.

https://doi.org/10.1515/9783110585797-007

7.3 Proposed Answer

It is necessary to believe something. This thesis, which will be defended subsequently, is understood in the following universal sense: It is necessary that every human being believes something. We want to say from the beginning that we do not restrict the term *belief* but understand it in the widest possible way. That is, we include belief of everyday life, scientific belief and also religious belief.

The "something" is interpreted in the same way as it was done with knowledge (cf. sections 1.3 and 4.3.2). That is, as a state of affairs, usually represented by a statement or proposition: human being a believes that p (is the case, is true): aBp. Symbolically, the above thesis may be written thus:

$$N(\forall a{\in}\mathrm{H},\ \exists p)xBp$$

The sign 'N' stands here for 'necessary'. However, we cannot use the sign '$\Box$' here since this is understood as the necessity-operator of some Modal Logic. Used in this (the usual) way, it means some type of *logical necessity* described by some Modal Logic in which at least all theorems of Classical Propositional Calculus (CPC) and First Order Predicate Calculus are necessary.

It has to be observed that "necessary" or the necessity-operator can be applied here to three things: to what is believed, i. e. to the content of belief, to the action of believing, and to the reasons or motives for believing.

We first apply the term "necessary" to the content of belief. Before doing so, we have to consider the question of what kind the content is: Is it always representable by a statement or can it be also a norm (7.3.1)? Only then, we can apply the term "necessary" to the content of belief (7.3.2).

Secondly, we apply the term "necessary" to the action of believing and ask whether this can be naturally necessary (7.3.3) and then whether it can be necessary for some goal (7.3.4).

Thirdly, we ask whether the reasons for believing can be necessary (7.3.5).

7.3.1 The Content of Belief

The content of belief, i. e. what is believed, can be understood in two ways: (1) as something which is the case (or is not the case), and (2) as something which should or ought to be the case (or should/ought not to be the case). (1) is the usual understanding, but it will become clear from examples below that also (2) is a fact, though often neglected.

(1) The descriptive character of belief: That is, what is believed, is something, that is the case (or is not the case), something that is true (or false). Although this may seem to be trivial at first, it was and still is a problem at least in religious belief. There are many discussions of this question among leading theologians[1]. I agree with Bochenski that in real religious discourse at least a part—and in fact an essential part—of what is believed is propositional, i. e. is something which is (the case) or is not (the case), something which is true or false. When I say “in fact an essential part”, I mean that this part is concerned with the “creed” of that special religion; i. e. with those selected propositions, which characterize a particular religion and which have to be believed by someone in order to be counted as a member of that religious society or as belonging to that special religion. Though there is no big problem concerning the propositional content of what is believed in scientific belief, there is a further similarity concerning the religious creed. Also in a special scientific discipline, there is something like a creed, a selected set of propositions, the elements of which have to be believed by someone who is counted as a member of the physicists, biologists, or mathematicians, for example. Such a creed usually consists of the well confirmed hypotheses and theories and of the well-established laws and theorems in that discipline plus a number of background assumptions which also transcend the discipline. One may also add certain interpretations of crucial experimental results. Great scientists sometimes do not believe everything belonging to the creed of a discipline (at a certain time), especially if it is concerned with some basic assumptions. This usually leads to new theories (think of Newton, Maxwell, Einstein, or Planck, etc.). This also shows that there is an important difference between the religious and the scientific creed: the former does not change in its basic assumptions, the latter can change even in its basic assumptions.

(2) The prescriptive (normative) character of belief: In both scientific and religious belief the content of belief is also of normative character. That is, a part of what is believed is something that ought to be the case (or ought not to be the case), something which is valid (or invalid) (i. e. something which is also evaluated in a two-valued way). Although this claim may seem to be strange or wrong in the first place, it will become more evident with some examples. It is essential for a religious believer that he not only believes in certain states of affairs, but also in the validity of norms. Thus, it is essential for a Christian believer to believe in the validity of the Ten Commandments or more generally in the validity of any norm ordered (and revealed) by God. The same is true of the other great

1 Cf. the discussion in Bochenski (1965) ch. 13, p. 40ff.

religions such as Judaism, Islam, or Buddhism. However, there is also a similarity in science here. It is also essential for a scientist to believe in the validity of certain methodological norms. Otherwise, he or she cannot work. Thus, general norms like "Base your hypothesis on all the information available" or "The new theory should include the correct results of the old theory (as special cases)" are accepted by all scientists. Whereas special ones like "Physical laws should be invariant against coordinate (space-time) transformations" will be believed at least by scientists of that special discipline. Also with respect to norms, there is a similar difference like that concerning the creed. Methodological norms in science are sometimes revised. Thus, the norm to look for continuous dependencies of causes and effects (in order to be able to describe them by differential equations, the so-called lex continuitatis) was a successful methodological norm since the time of Newton, but was severely restricted by quantum theory.

Norms in religion may also be interpreted according to contemporary needs, but it seems that they cannot be revised so radically as some of them have been revised in the development of science. Though to be just, one should compare of course very general and basic norms on both sides on the one hand and more specifically applicable norms on the other. And concerning the belief in the first, the difference in science and religion might not be so great. Also, it has to be observed that a change or a revision of norms can be of two sorts: (a) to take away (or to abandon) something or (b) to add (or to complement) something. In both scientific and religious belief, the revision usually is done in the sense of (b): The lex continuitatis was restricted and supplemented by discrete change. The Ten Commandments were supplemented by the human law (e. g. the Human Rights).

7.3.2 *Necessary* Applied to the Content of Belief

The content of belief or that what is believed can be necessary in all the three senses.

(1) The content of belief can be *logically necessary*. This is the case with theorems of logic or mathematics but also with statements about one's own existence like the *fallor ergo sum*.

It is an interesting question whether the action of believing is naturally necessary or is guided by laws of nature when the content is a very simple and transparent logically necessary statement like E1–E5 of ch. 1. Although we rather speak of knowledge in this sense (ch. 1 and 4). The question is whether in such cases our reason is forced to assent by (laws of) nature as Aquinas[2] says: If, on the other hand, that what is believed is neither simple nor transparent even if logically necessary, or if it is not logically necessary (naturally necessary or contingent), and no case of knowledge in the sense of K1–K4 applies, then the belief in it may need an assent of will.

(2) The content of belief can be *naturally necessary*. We say that a statement (a law) is naturally necessary iff it holds in all possible worlds which are different from our world only with respect in initial conditions.[3] Physicists believe in conservation laws; a famous example is the conservation of energy in closed systems. Other laws are the dynamical laws of Classical Mechanics, Maxwell's Theory, and Special and General Theory of Relativity. Statistical laws like the Law of Entropy, are another example. There are a lot of justifying reasons to believe in these laws: First, they are very well confirmed by observation and experiment (knowledge K4). Second, they are mutually consistent. Third, they complement each other within the respective domain and also from domain to domain. Despite the many justifying reasons, a strong and firm belief in these laws may need an assent of will of the scientist.

(3) The content of belief can be necessary for some goal. This is so for all cases of advice given by parents, by educators, by supervisors, and experts in general. The goal is to gain information and to avoid error concerning basic values like survival and health and concerning higher values like education, increasing knowledge, expertise, etc.

7.3.3 *Naturally Necessary* Applied to the Action of Believing

It has to be observed first that the thesis of 7.3 does not claim that it is logically necessary that every human being believes something. Otherwise, the negation of the thesis, i. e. that possibly not everyone believes something would be a logically contradictory statement. But this is not the case. Rather the thesis claims it is a empirical fact that everyone believes something. However, the necessity involved there can be the necessity of laws of nature. That is, the fact that everyone believes something can be determined by biological laws. This can be supported in more than one way:

(1) Babies cannot but believe and trust their mothers (or substitute mothers) who breastfeed them. This is, of course, a type of belief which is partially unconscious and not reflected upon. However, it is a biological basis for conscious beliefs on a mere mature level. Beliefs of children can be fully conscious at least begin-

2 Aquinas (STR) II–II, 2, 9 ad 2.

3 Cf. Popper (1959) p. 433

ning at about four years old. Beginning at this age they understand false beliefs of others. This has been shown by many investigations starting from the invention of the *false belief test* by Wimmer and Perner in 1983.

(2) Babies and infants learn a lot in the first months and years. Effective learning presupposes belief and trust in parents and other authorities for assuming that imitation and following advice result positively. Also, this is partially unconscious at the beginning and develops as they mature and getting conscious.

7.3.4 Necessary as *Necessary for some Goal*

7.3.4.1 Goal: Health and Survival

A rough argument is as follows:
(1) Belief is necessary for successful learning.
(2) Successful learning is necessary for health and survival.
(3) Therefore: Belief is necessary for health and survival.
(4) Health and survival are important goals of all living systems.
(5) Therefore: Belief is necessary for important goals of all living systems.

Ad (1): Learning is learning from others or learning by oneself. Learning from others presupposes believing others, even if critical learning involves partially believing others until further notice. Learning by oneself presupposes believing or assuming some hypothesis. Also here, critical learning includes believing the hypothesis until further notice and trial to corroborate what is believed (the hypothesis).

Ad (2): Learning occurs via trial and error. Trial involves believing in a hypothesis and applying it when acting. If the hypothesis was correct, the action (searching for food, repulsing an attack, etc.) is successful and serves health and survival. If it was an error, it causes "biological costs" (hunger, thirst, injury, etc.) which are hindrances for achieving health and survival. Therefore: Successful learning is necessary to achieve health and survival.

Ad (4): It is an empirical fact that all living systems strive for health and survival. This is clear from the observation of the behavior of lower and higher animals. And it is especially apparent in humans. However, plants also strive for survival which is seen from the competition of growing trees in a forest to get to light in order to execute photosynthesis necessary for life (survival). A goal of a living system *ls* is a state *S* of a process of becoming *C* of *ls*, such that *S* is on the highest level of order relative to other states of C. In this sense, health and survival are goals for all living systems.[4]

7.3.4.2 Goal: Higher-Level Goals

There are different types of higher-level goals:

(i) Goals connected with human nature.
Examples: Increasing knowledge in accordance with abilities and interests, improving control of natural forces, human life in peace, average welfare.

(ii) Personal or individual goals.
Examples: Choosing a particular profession, a particular academic study, a particular partner, practicing religion.

(iii) Goals beyond humans nature.
Example: Happiness in another life after this life.

Approaching or achieving these goals is not possible without beliefs. It is in very many cases belief in experts or in hypotheses defended by people who know more and specifically better in the respective domains. Modern science is based on reliable scientific beliefs among colleagues. The overwhelming amount of beliefs of every individual scientist is that kind of belief, not knowledge.

There is also belief and trust in individual persons (partner) or supervisors in academic studies. And there is religious belief. The different types of belief will be considered in subsequent chapters.

7.3.5 *Necessary* Applied to the Reasons for Belief

The reasons for belief can be necessary in two ways: First in the sense that the statement which represents (or describes) the reason is necessary in one of the three ways of necessity considered in 7.3.3. Second in the sense that the respective reason is a necessary condition for the belief; just as q is a necessary condition in the valid implication: $p \Rightarrow q$. An example of a reason for belief is the knowledge or the proof (K3) that that, what is believed, is logically consistent. And we might require that for both scientific and religious belief.[5]

4 For more accurate and detailed definitions of order, process, becoming, goal and teleological order see Weingartner (2015), sections 5.33–5.36 and 7.3.2–7.3.6.

5 Thomas Aquinas requires this for religious belief and for theology to prove this ((STh) II–II, 2, 10 ad 2).

In both cases, it might be difficult to prove this. Take a new cosmological hypothesis which can explain many cosmological facts. It will be usually quite difficult to prove its own (internal) consistency and also its external consistency, i. e. its compatibility with well-established physical theories. The compatibility with well-established physical theories can also be called its “empirical-possibility”. If these facts, internal and external consistency, are purely logical or mathematical matters, we can say that the reason for the belief is also logically necessary. However, to prove the compatibility with well established physical theories will usually require to making additional empirical assumptions like boundary conditions and constraints on parameters. In this case, the reasons (as necessary conditions of the belief) may be naturally necessary or necessary for a certain purpose.

7.4 Answer to the Objections

7.41 (To 7.11:) Even a human action which is free can be necessary to achieve some goal although it is not naturally necessary.

7.42 (To 7.12:) Although both belief and disbelief are free actions if they are conscious and reflected upon, they can be necessary for some goal. Belief may be necessary to get new information, disbelief necessary to avoid error and biological costs.

7.43 (To 7.13:) If *necessary* means *necessary for some goal* then the motives for belief are not compulsion but reason: reasons for believing in the right means for achieving the goal.

7.44 (To 7.14:) It is correct that it cannot be logically necessary to believe something. However, it can be both naturally necessary, if the belief is partially unconscious like in infants or animals, and necessary for some goal as has been elaborated in 7.3.3 and 7.3.4.

7.45 (To 7.15:) From the modal status of what is believed (of the content of belief) nothing follows for the modal status of belief (of the action of believing). Therefore even if the modal status of what is believed is contingent (for example that the mother takes care of her baby), the modal status of believing (by the baby) can be both naturally necessary and necessary for some goal.

7.46 (To 7.16:) As it is correctly said in the objection from the necessity of what is believed, it does not follow that the belief is necessary; but from this, it follows that the modal status of the action of believing is independent from the modal status of that what is believed. Therefore, the action of believing can be necessary whether or not that what is believed is necessary.

8 Whether to Believe is the Same as not to Know

8.1 Arguments Pro

8.11 If someone were to ask me: "Do you know that?" I might answer: "No, I do not know it, but I believe it." Thus, to believe seems to be not to know.

8.12 In mathematics knowledge is achieved by proof. However, before the proof is completed, one does not know but believes that the respective conjecture is true. Therefore: to believe implies not to know.

8.13 Einstein's Theory of General Relativity (1916) predicted that big masses cause deviation of light rays. The first empirical test confirming this prediction was done in 1919: The Royal Society sent an expedition to South Africa to observe the effect when there was an eclipse of the sun. Einstein had a strong scientific belief that his prediction would be confirmed but did not have scientific knowledge by observational proof. Therefore: to believe scientifically is not to know scientifically.

8.14 Religious belief can be very strong. However, even a strongly hold religious belief means not to have knowledge. Therefore: To believe is not to know.

8.2 Argument Contra

If to believe is the same as (is equivalent with) not to know, then it follows that not to believe is the same as (is equivalent with) to know. This, however, is absurd. Therefore, it is not true that to believe is the same as not to know.

8.3 Proposed Answer

8.3.1 Two Types of Belief

Belief can be of two sorts: One satisfies the condition that what is known is also believed in the sense that what is known is also assumed to be true. This sort of belief may be called *knowledge-inclusive belief*. We shall abbreviate it as B-Belief (aBp). The other satisfies the condition that what is believed is not (yet) known and what is known is (no more) believed. This sort of belief may be called *knowledge-exclusive belief*. We shall abbreviate it as G-belief (aGp).

https://doi.org/10.1515/9783110585797-008

Neither of the two sorts means that to believe is the same as not to know. It is easy to see that the first sort of belief, B-belief, violates this equivalence of believing and not knowing since what is known is B-believed. The above equivalence does not either follow from G-belief. Since the equivalence not only claims that what is believed is not-known but also the opposite: what is not-known is believed and moreover, what is not-believed is known. And both of the latter are completely untenable if not absurd. Thus, it is wrong to say that to believe is the same as not to know.

Examples of G-belief: Before the proof of the independence of the Continuum Hypothesis (from the axioms of set theory) was given, v. Neumann believed (but didn't know) that the Continuum Hypothesis is independent. After Gödel proved the first part[1], i. e. that the General Continuum Hypothesis (GCH) can be consistently added to the axioms of Neumann-Bernays-Gödel Set Theory (even if very strong axioms of infinity are used), v. Neumann wrote:

> Two surmised theorems of set theory, or rather two principles, the so-called 'Principle of Choice' and the so-called 'Continuum Hypothesis' resisted for about 50 years all attempts of demonstration. Gödel proved, that neither of the two can be disproved with mathematical means. For one of them we know that it can not be proved either, for the other the same seems likely, although it does not seem likely, that a lesser man than Gödel will be able to prove this.[2]

But after the proof of the second part—that also the negation of GCH can be consistently added to the axioms of Set Theory (it holds for both systems, that of Zermelo-Fraenkel and that of Neumann-Bernays-Gödel)—was given by Paul Cohen in 1963[3], von Neumann didn't believe it anymore, but knew that GCH was independent (from the axioms of Set Theory).

The General Theory of Relativity (completed by Einstein in 1915) made three important predictions: (a) The perihelion of Mercury, (b) the deviation of light rays which pass close to big masses, and (c) the red-shift of the light reaching us from distant stars. The first (a) was known as an effect (not explainable by Newton's Theory) before Einstein's Theory was created. And so the prediction (and

1 Gödel (1940)

2 v. Neumann (1951), in: J.J. Bulloff/Th.C. Holyoke/S.W. Hahn (1969). The "Tribute to Dr. Gödel" from which the passage is cited was given by v. Neumann in March 1951 on the occasion of the presentation of the Albert Einstein Award to Gödel. It appeared in print in the volume *Foundations of Mathematics* (ed. Bulloff et al.), a collection of papers given at a symposium commemorating the sixtieth birthday of Kurt Gödel.

3 Cohen (1963/64). Cf. Cohen (1966)

explanation) because of stronger gravitation, since Mercury is much closer to the sun, by General Relativity was a success immediately. In this case, Einstein knew the positive result of the test of his theory. In the cases of (b) and (c), he strongly believed that they were correct and there would be a positive test possible too. In 1919 came the first confirmation of the prediction (b), the deviation of light rays: A British expedition of astronomers observed a total eclipse of the sun in Africa and confirmed the effect that light rays from a star, which run very close to the sun, are deviated towards the sun (in general towards great masses). Later, better and more exact confirmations of (b) were obtained.

In 1922 the Soviet meteorologist Alexander Friedmann believed (but did not know) and predicted (on the basis of Einstein's picture of dynamic space) that the entire universe is in dynamic change. In 1929, the American astronomer Edwin Hubble confirmed this prediction. He found out that the light, reaching us from distant stars, is shifted towards red of the spectrum (red-shift) and that this red-shift is proportional to the distance of the emitting star(s) (galaxy). This was the confirmation of (c) which was later again confirmed many times. Thus, after these positive results of testing predictions, Einstein knew that predictions (b) and (c) were correct and were positively confirmed by tests. And this also means that he did not and needed not to believe (G-believe) that anymore since there is sufficient justification to say that he knows now.

Examples of B-belief: No special examples for B-belief are necessary since B-belief may be interpreted in the following way: To B-believe that something (*p*) is the case means just to think that *p* is true (valid), to hold that *p* is true (valid), to strongly assume that *p* is true (valid) etc. Thus, if someone knows that chromosomes duplicate, then he also B-believes it, and also if someone G-believes that GCH is independent (from the axioms of Set Theory), then he also B-believes it.

Religious belief is always knowledge-exclusive, i. e. is always first of all G-belief. Since if one believes religiously—for instance that Christ came for the salvation of humankind or that there will be some kind of conscious life after death—one does not know it (and knows that one does not know it). And this holds for all religious beliefs even if not necessarily for all the statements of the creed of some special religion. Since first of all the statements of the creed might not be all logically independent of one another, such that some believer may infer one proposition of the creed from some others. And in this case, he knows that one is a consequence of the other. Such inferences may also be done by theological argumentation. Still, the propositions so derived are not known but believed, as known consequences of others which are believed.

What generally holds is that if someone religiously believes something, then he does not know it, but he holds it to be true, he strongly assumes it to be true.

This also holds for religious belief in important norms which are written down in religious texts like the Bible, the Koran or the Sutta Pitaka in Buddhism. Important and ultimate norms like those proposed in religions are not known. The religious believer believes that they hold (are valid) and gives a justification with the help of other beliefs: (1) God has revealed (required) it. (2) And when God reveals something, it is true or it holds. Alternatively: (1) God has given us (by revelation) certain rules for acting in society. (2) When God has given a rule, then it should be obeyed.

8.3.2 Interrelations and Relations to Knowledge

Interpreting B-belief as assuming *p* to be true or holding *p* to be true reveals that both knowledge implies B-belief, and G-belief implies B-belief:

$$\begin{tikzcd} aKp \arrow[dr] & & aGp \arrow[dl] \\ & aBp & \end{tikzcd}$$

Further important propositions are the following ones:

8P1 $aKp \to aBp$ $\qquad \neg aBp \to \neg aKp$
8P2 $aGp \to aBp$ $\qquad \neg aBp \to \neg aGp$
8P3 $aGp \to \neg aKp$ $\qquad aKp \to \neg aGp$
8P4 $aGp \to \neg aK\neg p$ $\qquad aK\neg p \to \neg aGp$
8P5 $aGp \to \neg aKp \wedge \neg aK\neg p$
8P6 $aKp \to \neg aGp \wedge \neg aG\neg p$

Both types of belief have several further relations to knowledge:

(1) Knowledge *that* it is believed: If person *a* believes (B, G) that *p* then *a* knows that *a* believes (B, G) that *p*. This knowledge is a kind of introspective knowledge as described by E2 (section 1.3) and knowledge K2 (section 4.3.4).

8P7 $aBp \to aK2(aBp)$
8P8 $aGp \to aK2(aGp)$

Observe however, that not-believing does not generally imply knowledge of not-believing. It may, but it need not. Therefore, we have the following cases:

8P9 $(\exists p)[\neg aBp \wedge \neg aK(\neg aBp)]$ $(\exists p)[\neg aGp \wedge \neg aK(\neg aGp)]$
8P10 $(\exists p)[\neg aBp \wedge aK(\neg aBp)]$ $(\exists p)[\neg aGp \wedge aK(\neg aGp)]$

Cases 8P9 express not being concerned with p via belief and being ignorant of that. Cases 8P10 express not-belief (disbelief) and knowledge (K2) of that disbelief.

An example from the history of science for 8P10 is the case of Ludwig Mach, a son of Ernst Mach, who did not believe in Einstein's Theory of Relativity (neither special nor general). He changed the preface in Mach's Mechanics (in the 9^{th} edition of 1933) and in Mach's Optics (Ludwig published the first edition in 1921) as if his father would have refused Einstein's theory. The forgery was fully discovered and proved in a painstaking way by Gereon Wolters (1987). In this case, Ludwig Mach's disbelief is even stronger, i. e. is the belief that $\neg p$—($aG\neg p$)—from which $\neg aGp$ follows.

Examples (for 8P10) from religious belief are the infidel (disbeliever) and the heretic. Both know that they do not believe and usually have reasons for not-believing. For the first, the not-belief or the disbelief is concerned with a whole Creed (of some religion) or most of its part, whereas the second disbelieves is some special statements of the Creed.

An example for 8P9 would be a person who has never heard anything of, say the Christian religion and consequently does neither have belief (nor disbelief) nor knowledge of it.

(2) Knowledge *what* is believed: If person a believes (B, G) that p, then a knows *what* a believes or *what it is* that a believes. Or a knows that p is that unique proposition that a believes: $p = (1q)aBq$.

8P11 $aBp \rightarrow aK((1q)(aBq) = p)$
8P12 $aGp \rightarrow aK((1q)(aGq) = p)$

8.4 Answers to the Objections

8.41 (To 8.11:) If "I do not know it, but I believe it" is interpreted as a conjunction—$\neg iKp \wedge iGp$—then by Classical Propositional Logic (CPC) it follows in fact that "'I do not know it' is equivalent with 'I believe it"', i. e. $\neg iKp \leftrightarrow iGp$. This is so because in CPC '$(p \wedge q) \rightarrow (p \leftrightarrow q)$' is a theorem. However, from every equivalence two implicational consequences follow. The two implications which follow from $\neg iKp \leftrightarrow iGp$ are: $\neg iKp \rightarrow iGp$ and $iGp \rightarrow \neg iKp$. That is: "if I do not know that p then I believe that p", and "if I believe that p then I do not know that p".

The second implication is acceptable and says that the respective belief is knowledge-exclusive belief, i. e. G-belief (recall section 8.3). The first implication, however, is absurd, as is also its logically equivalent contraposition, i. e. the statement: if I do not believe that p then I know that p.

We see now that from a meaningful and true claim, i. e. from "I do not know (that p), but I believe (that p)" one can derive by CPC the above absurd consequences. Who is the culprit? The culprits are certain properties of CPC which permit irrelevant parts in the consequence-class of inferences valid in CPC. "Irrelevant part(s)" can be defined more accurately as follows:

8D1 α is an irrelevant consequence of A iff
(1) $A \Rightarrow \alpha$ or $A \vdash \alpha$
(2) some propositional variable is replaceable in α on some (one or more than one) occurrences by an arbitrary propositional variable salve validitate of $A \Rightarrow \alpha$ or $A \vdash \alpha$.

This criterion, called *Replacement Criterion (RC)*, can be extended to Predicate Logic (add 'or predicate' to 'propositional variable') and to irrelevant identity formulas.[4]

8D2 α is a relevant consequence of A iff
(1) $A \Rightarrow \alpha$ or $A \vdash \alpha$
(2) there is no propositional variable or predicate in α of the kind in 8D1 (2) above.

RC (in the form of 8D1 or of 8D2) can now be applied to the theorem $(p \wedge q) \rightarrow (p \leftrightarrow q)$ of CPC: We see that $p \leftrightarrow q$ is an irrelevant consequence of $p \wedge q$ when we split up the equivalence into two implications: $(p \wedge q) \rightarrow (p \rightarrow q)$ and $(p \wedge q) \rightarrow (q \rightarrow p)$. In the first p, in the second q is replaceable by an arbitrary propositional variable salva validitate of the implication.

Conclusion: To solve the objection we have to put, in this case, a relevance restriction on CPC. Then the move from a conjunction to an implication is blocked as irrelevant (though classically valid). Moreover, the statement "I do not know it, but I believe it" then expresses the situation of knowledge-exclusive belief—G-belief—as described by 8P3 in section 8.3.

4 This criterion (for relevant consequences) originates in Schurz/Weingartner (1987). It was completed with a Reduction Criterion (RD) which reduces relevant consequences to simple (shortest) most informative consequence elements. Further developments are in Schurz (1991), Weingartner (2000). A decidable many-valued system which approximates RC and RD is in Weingartner (2009), (2010).

8.42 (To 8.12:) It is correct what the objection says: in this situation believing that p implies not-knowing that p. This is the frequent case in scientific belief and is described by G-belief (cf. 8P3 and 8.3). However, from this the opposite and wrong implication $\neg aKp \rightarrow aGp$ is not derivable; without this, wrong part belief is not the same as not-knowledge.

8.43 (To 8.13:) In this objection everything is true except the last sentence after "therefore". To correct this sentence, we have to replace the word "is" by "implies". Then we have the same situation as in 8.42: G-belief (8P3).

8.44 (To 8.14:) The answer is the same as to 8.13: the words "means" and "is" have to be replaced by "implied". The result then is that religious belief is G-belief.

9 Whether Knowledge of Reasons for Belief is a Necessary Condition of Belief

9.1 Arguments Contra

9.11 As is evident from section 7.3.3, babies have a strong belief in their mothers with natural necessity concerning their actions. However, they cannot have knowledge of reasons for their belief. Therefore: Knowledge of reasons for belief cannot be a necessary condition of belief.

9.12 In mathematics there has sometimes been strong belief in axioms or postulates without sufficient knowledge of the reasons for that belief. This happens especially when it turned out later that the respective axioms were inconsistent or the postulates were not necessary. Therefore: Knowledge of reasons for belief does not seem necessary for belief.

9.13 In physics, there sometimes exists a situation where the knowledge of reasons for a belief is a knowledge of two or more incompatible reasons for the respective belief. For example the knowledge of reasons for Einstein to believe in Maxwell's Theory of Electromagnetism was in conflict with the Galilean invariance of Classical Mechanics. As long as the conflict exists (is not solvable), the inconsistent reasons cannot be a necessary condition of the belief. Therefore: Knowledge of reasons for belief are not always necessary conditions for belief.

9.14 If beliefs are too global, there seems to be no serious knowledge of the reasons for that belief. However, there are such global beliefs, for example when religious belief searches or provides ultimate and global explanations or when belief in a world-view extends the theory of evolution to an evolution of the universe. Therefore: In cases where beliefs are too global, the requirement for knowledge of reason for that belief is not satisfied.

9.2 Argument Pro

If a belief is conscious but does not have reasons for the belief known to the believer, then this belief seems irrational. Therefore it holds for conscious and rational belief that there must be known reasons for the belief.

https://doi.org/10.1515/9783110585797-009

9.3 Proposed Answer

Whereas we can defend that in conscious belief (1) and (2) of section 8.3.2 are generally satisfied, this cannot be so defended for the knowledge of reasons for belief. To satisfy the knowledge of reasons for the belief, it is necessary that the respective person has *considered* and *investigated* the reasons for his (her) belief; and that he (she) has come to a convincing justification that these are the main reasons for his (her) belief, such that we can say that he (she) *knows* the reasons for this belief.

The reasons for the belief are twofold: There are reasons which support the belief, and there are reasons which remove hindrances for the belief. The reasons which support *scientific belief* are essentially the same as the supporting reasons for knowledge in the sense of K4–K7. Such reasons have been discussed in chapters 2 (recall the certainty of historical and psychological facts and of laws of nature), 3 (recall knowledge of universal truths), 4 (recall knowledge as ability to verify and to corroborate and as justified true belief), and 5 (recall the reasons for information). As long as knowledge has not been reached sufficiently in these cases, the supporting reasons are the same as for scientific belief. Thus we need not repeat these different supporting reasons here. However, without supporting reasons, belief cannot be called scientific belief. The supporting reasons for religious belief are dealt with in ch. 10 below. But what holds for supporting reasons holds as least as much for reasons that remove hindrances for belief. Since as long as hindrances like inconsistencies or ambiguities are not removed, neither scientific nor religious belief can take place. Therefore, such reasons for belief and moreover the knowledge of them are necessary conditions for both scientific and religious belief.

9.3.1 Reasons that Remove Hindrances

There might be hindrances of the sort that it is not clearly understood *what* is believed or that the belief is only partially conscious. However, such hindrances are excluded here since we presuppose conditions (1) and (2) of section 8.3.2.

One of the most important hindrances for both scientific and religious belief is that what is believed is inconsistent or even—to the best of our knowledge—known to be inconsistent. This can happen in three ways:

(1a) The statement that is believed—p in aGp/aBp—is itself inconsistent.

(1b) The statement that is believed—p in aGp/aBp—is inconsistent according to many interpretations which seem possible.

(2) p of aGp/aBp is inconsistent with some other statements in that system or set of beliefs to which p belongs and what concerns a certain domain or field. This may be called internal inconsistency of the belief-system.
(3) p of a aGp/aBp is inconsistent with another domain containing both statements known (scientifically established) and statements believed which are scientifically well confirmed or corroborated. This may be called external inconsistency of the statement believed.

Observe, that in most cases of belief involved in (1a)–(3) we have to do with G-belief, i. e. knowledge-exclusive belief such that what is believed is not (or not yet) known.

9.3.2 Inconsistency of *p* in *aGp*

Example for (1a): Frege strongly believed that axiom 5 (the comprehension axiom) of his "Grundgesetze der Arithmetik" is true[1] before Russell discovered an inconsistency in that axiom.

Example for (1b): The statement in the Bible "Who saves his life will lose it and who loses his life will save it" is inconsistent when—according to normal usage of language—(a) to lose means not to save and to save means not to lose, and (b) "life" has one unique meaning, e. g. life in this world. However, if we tolerate dropping condition (b) and interpret the expression "life" as life in this world, but interpret "it" as external life then the inconsistency disappears.

To avoid inconsistent belief described in (1a) and (1b), we can propose the following postulates:

9P1 $aGp \rightarrow Cons(p)$
9P2 $aBp \rightarrow Cons(p)$
9P3 $aGp \rightarrow aK(Cons(p))$
9P4 $aBp \rightarrow aK(Cons(p))$

As the example for (1a) shows, these 4 postulates are not generally satisfied. In the case of Frege, it would be absurd—and contrary to what he says himself in the epilogue—to say that he did not believe in his axiom 5: since this would follow if p is not consistent or not known to be consistent.

1 Cf. Frege (1967) Vol. II, epilogue. However, Frege says there that he was never so fully convinced of that axiom 5 as he was of the other four axioms. But he strongly believed in it since he could not imagine how the theorems of arithmetic could have an axiomatic foundation otherwise.

This shows that 9P1–9P4 are not acceptable as generally valid principles. Therefore, a weaker requirement would be to formulate the above postulates as norms which are satisfied by every rational belief. The normative operator $\mathcal{O}$ is to be understood as “obligatory that”, “ought to be that” or “should be that”. There are two possibilities to express the respective norm: as *de dicto* or as *de re*.

9P5 $\mathcal{O}(aGp \to Cons(p))$
9P6 $\mathcal{O}(aBp \to Cons(p))$
9P7 $aGp \to \mathcal{O}(Cons(p))$
9P8 $aBp \to \mathcal{O}(Cons(p))$

If Gp/Bp is scientific belief, then all four principles are acceptable. From 9P5 and 9P6 follow consequences by distributing the $\mathcal{O}$-operator. The distribution of $\mathcal{O}$ over $\to$ is generally accepted in Deontic logic and has their analogue in Modal Logic for $\Box$ and in Epistemic Logic for K (recall 6DK).

9P9 $\mathcal{O}(aGp) \to \mathcal{O}(Cons(p))$
9P10 $\mathcal{O}(aBp) \to \mathcal{O}(Cons(p))$

9P9 has a special application to religious belief. In religious belief, the belief is commended, as for example in the first of the Ten Commandments. Therefore, the antecedens $\mathcal{O}(aGp)$ gets a special interpretation in religious discourse: It is always satisfied. At least under the condition that p is a part of the *Creed*[2] of the respective religion. This presupposes of course that we have to consider those religions which have a Creed based on an accepted text like the Bible (Old and New Testament) or the Koran. Since $\mathcal{O}(aGp)$ is satisfied, the consequent $\mathcal{O}(Cons(p))$ should also be satisfied, i. e. that what is religiously believed in the Creed should be consistent. Who has the task to investigate this and to show this? According to Thomas Aquinas in some sense, every religious believer has the task to find a consistent way in those things he believes religiously. However, in a more important way and on a higher level this is one of the most important tasks of theology and of theologians.[3] The requirement for theology is then:

9P11 If p belongs to the Creed then theology should show (prove) the consistency of p.

2 Cf. the detailed elaboration about the Creed of a religion in Bochenski (1965) §§ 3.3, 3.4, 13.1.
3 Thomas Aquinas (STH) II–II, 1, 5 ad 2, 10 ad 2.

The expression "should show" or "should prove" can be understood as inserting *knowledge* such that postulates 9P9 and 9P10 could also be expressed in a stronger way thus:

9P12 $\mathcal{O}(aGp) \rightarrow \mathcal{O}[aK(Cons(p))]$
9P13 $\mathcal{O}(aBp) \rightarrow \mathcal{O}[aK(Cons(p))]$

If it is obligatory to believe p, then it is also obligatory to know that p is consistent.

With respect to scientific belief, one may also speak in an analogous way of the Creed of physicists, chemists, or biologists. To this belong certain laws and hypotheses which have been very well corroborated and are permanently used to explain old and newly discovered phenomena with great success. For scientific belief, there is no such obligation as for religious belief and therefore postulate 9P7 and its strengthening by inserting the operator for knowledge 9P14 will be appropriate:

9P14 $aGp \rightarrow \mathcal{O}[aK(Cons(p))]$

If we consider the contraposition of 9P9 $\neg\mathcal{O}(Cons(p)) \rightarrow \neg\mathcal{O}(aGp)$ and replace not obligatory ($\neg\mathcal{O}$) by permitted not ($P\neg$), then we get:

9P15 $P\neg(Cons(p)) \rightarrow P\neg(aGp)$

That is: If it is permitted that p is inconsistent then it is permitted not to believe p. This sounds reasonable for both scientific and religious belief. If the situation with some scientific hypothesis is such that—despite serious attempts to avoid it—an inconsistent interpretation is always allowed as possible, then there is no obligation to believe the hypothesis (i. e. it is permitted not to believe it).

Analogously: If the situation with some statement of the Creed of some religion is such that—despite serious attempts to avoid it—an inconsistent interpretation is always allowed as possible, then there is no obligation to believe the statement (i. e. it is permitted not to believe it).

Both instances of 9P9 hold of course also when an inconsistent interpretation is hardly avoidable for all that one (the interpreter or the scientific community) knows. However, from this it does not follow that the scientific hypothesis is refuted or the statement of the Creed is false. In both cases the difficulty in the interpretation may be an invitation to reconsider the case and investigate the situation more deeply in order to find a consistent interpretation.

9.3.3 Internal Inconsistency

In this case, p of aGp/aBp is inconsistent with some statements in that system or set of beliefs to which p belongs and which concern a certain domain or field of scientific or religious discourse (cf. (2) of 9.3.1 above).

Examples: The internal inconsistency of Quine's Mathematical Logic: In its first edition, it contained an inconsistency within the theory of description. The inconsistency was discovered by Barkeley Rosser and solved by Hao Wang.[4] The domain in this case was the chapter with the theory of description. However, the inconsistency was restricted to that domain, it was isolated concerning the larger domain of the whole book and did not affect this.

The internal inconsistency which causes the *dark-matter-problem*: If the calculation of the mass M of a galaxy is done by Newton's laws of motion and of gravitation then M is much too small to attract the millions of stars of the galaxy enough in order not to fly apart, despite the great rotational velocity observed; i. e. the theoretical predictions are inconsistent with the observation of the rotational velocity. To avoid the inconsistency, three options have been proposed:

1. Newton's laws are incorrect; at least for galaxies thousands of light years away.
2. Since the predicted masses are too small for the observed rotational velocities, the galaxy will fly apart.
3. The mass difference is due to unobserved dark matter.

Milgrom proposed with his "Modified Newtonian Dynamics" a corrected version of Newton's law according to which the gravitational strength is growing at sizeable distances.[5] However, there might be a fourth option that the usual equation for calculating the mass M of galaxies—$M(r) = rv^2(rot)/G$—is not correct (where r is the radius, v the rotational velocity of the galaxy and G the gravitational constant)[6]; since it reduces the million-body problem to a collection of two-body systems.[7]
As long as the inconsistency cannot be removed, it remains a hindrance for scientific belief.

The apparently internal inconsistency of the two (probably incomplete) genealogical trees: According to Matthew 1, 1–17 there are 42 generations from Abraham to Jesus, whereas according to Luke 3, 22–34 there are 76 generations from Jesus to Adam.

4 Cf. Quine (1951) Preface to the third edition.
5 Milgrom (1983)
6 Saari (2015)
7 Saari (2015)

9.3.4 External Inconsistency

Some philosophical doctrines are external inconsistent with well established or corroborated scientific results or theories:

Hegel's doctrine that everything causally influences everything else is external inconsistent with the Special and General Theory of Relativity according to which causal propagation has an upper limit (the speed of light in vacuum).

Kant's doctrine that the real space of the universe is based neither on empirical nor on contingent conditions but on a priori conditions of our mind determining Euclidean Geometry is external inconsistent with two theorems proved by Helmholtz.[8]

Helmholtz proved that the contingent empirical assumption that rigid bodies (measurement rods) are freely moveable in space (and remain rigid) determines the geometry (recall section 5.4.2 above).

That measurement rods (real physical bodies of a certain length) are freely moveable in space is an empirical and contingent condition. Nevertheless, this condition determines the geometry: either one of constant curvature (Euclidean, elliptic or hyperbolic) or one of changing curvature depending on the empirical and contingent distribution of masses. These external inconsistencies of Hegel's and Kant's doctrines are hindrances to believe them; or in other words, the Theory of Relativity and Helmholtz's theorems are justified reasons to disbelieve these philosophical doctrines.

Concerning religion, there is the problem of the external inconsistency of the Creed (or a part of it) either with well-established scientific results or well-corroborated scientific theories or with certain extrapolation of such results or theories. This is a most discussed problem in *theology!contemporary*. Accordingly, there are two main reasons for a conflict in the sense of external inconsistency.

(a) The Creed (or a part of it) is interpreted in such a way that it is inconsistent with well-established scientific results or well-corroborated scientific theories:
  (i) This is certainly so with some doctrines of Creationism for example if it is claimed that humankind (or the earth) is only 6000 years old.
  (ii) The description of God's creation in the Genesis is interpreted in such a way as to exclude any kind of development and evolution.
  (iii) A specific scientific result like "self-organisation" or an evolutionary transition is denied because God is interpreted as "allwilling" or "allcausing"

8 Helmholtz (1868). For a discussion see Mittelstaedt/Weingartner (2005), p. 55f.

which means that for any event E it holds that God wills (causes) that E occurs or that God wills (causes) that E does not occur.

(b) Scientific results or theories are extrapolated to such an extent that they cannot be tested anymore in this form:
   (i) Evolution is extended to the development of the whole universe with the claim: Everything is due to (is a matter of) evolution. Consequently, there cannot be creation.
   (ii) The claim of the "chance hypothesis", i. e. the claim that the emergence of the DNA of all living systems resulted from a random process by pure and blind chance alone: "Chance alone is the source of every innovation, of all creation in the biosphere. Pure chance, absolutely free, but blind… All forms of life are the product of chance."[9] Since every kind of creation requires at least some order, the chance hypothesis is incompatible with creation.
   (iii) The claim of the so-called macro-evolution of all animal species from very primitive animals like amoebae. This is in conflict only with special interpretations of the description of creation in Genesis.

Commentary to the external inconsistencies with a proposal how they can be resolved:

Ad (a)(i): These claims of some creationists are just patently wrong. However, there are hardly justifying reasons for extracting them from any text of the Bible (Old or New Testament).

Ad (a)(ii): If some interpretation of creation in the Genesis is such that it excludes any kind of development and evolution then it must be wrong. However, the text in Genesis does not rule out first, that creation is done by evolution, and second, that God created such creatures who themselves contribute to the development and evolution of the universe as expressed by Gödel: "God created things in such a way that they themselves can create something."[10]

On the other hand, if someone understands by "evolution" a theory which rules out creation as not possible then this theory exceeds its scientific limits by far since it is not testable.

9 Monod (1972) p. 110
10 Gödel MAX PHIL 4X.

Ad (a)(iii): Processes like "self-organisation" or evolutionary transitions are facts. Thus, their denial is wrong.

However, to interpret God as *allwilling* or *allcausing* is also wrong. An allwilling or allcausing God leads immediately to absurd consequences like his willing and causing all moral evil and war-criminality; and further to the consequence that God is inconsistent by giving the Ten Commandments and causing moral evil[11]: If *E* is the event of a moral evil then God neither wills (causes) that *E* occurs nor wills (causes) that *E* does not occur but permits it to occur since he has endowed man with free will.

Ad (b)(i): The claim that everything is due to evolution cannot be correct. It is known that the fundamental constants of nature (m_p/m_e, α, c, G, h) had to have these numerical values (or values very close to it) from the beginning since otherwise, no formation of galaxies, stars, and the solar system would have been possible.

> We have found nature to be constructed upon certain immutable foundation stones, which we call fundamental constants of nature. We have no explanation to the precise numerical values taken by these unchangeable dimensionless numbers. They are not subject to evolution or selection by any known natural or unnatural mechanism.[12]

> Several well-known physicists and astronomers, among them Carter, Dyson, Wheeler, Barrow, Tipler, Hoyle, to cite only a few have all made the point in recent publications—that our type of carbon-based life could only exist in a very special sort of universe and that if the laws of physics had been very slightly different we could not have existed.[13]

By the same reason, fundamental laws of nature cannot have evolved, but must have been the same from the very beginning. Moreover, matter must have been there as the subject of an evolutionary process. If there is nothing there, no evolution is possible or cannot even be defined. Since the claim of universal evolution is false, it is no objection against creation.

Ad (b)(ii): The quotation of Monod contains two claims: One is that there is chance in some absolute sense, the other is the so-called chance hypothesis, i. e. that life and consequently the DNA is a product solely of processes of chance. A

11 Cf. the discussion of the doctrine of the allwilling and allcausing God in Weingartner (2015b) pp. 130–134

12 Cf. the detailed discussion in Barrow/Tipler (1986) p. 31 and chs. 4–5, and in Penrose (2005) chs. 27 and 28.

13 Denton (1998) p. 16. Cf. Weingartner (2015b) pp. 191ff.

reflection on the different types of chance reveals that the first claim is rather scientifically useless. Understood as scientific concepts, chance and randomness are—like most scientific concepts—relative and not absolute. That means that "chance" and "randomness" are only definable relative to order or law or law governed structure. In other words, "pure chance", "absolutely free but blind" do not exist, and the respective concepts are not scientific concepts.

The second claim, the *chance hypothesis*, can be formulated more specifically w. r. t. the DNA as follows: The specific sequence of the nucleotides in the DNA-molecule of the first organism came about by a purely random process in the early history of the earth.[14] Let us see what this means w. r. t. different living systems. For the following examples, we assume that all sequence alternatives (or permutations) are physically equivalent.

(1) The smallest catalytically active protein (enzyme) has $\approx 10^{130}$ sequence alternatives (sqa).
(2) The human genome: $3 \cdot 10^9$ nucleotides, i. e. $10^{600 mill}$ sqa.

A suitable numerical example is this: A Coli bacterium has about the same chance (according to the "chance hypothesis") of arising purely randomly (by pure chance) as a 4-volume textbook of biochemistry (420 pages per volume) has of arising by random-mixing of its letters.

If the "chance hypothesis" i. e. claims for the origin of life in terms of "blind chance", "pure chance" or "chance alone" were correct, an emergence of life would be excluded:

> No matter how large the environment one considers, life cannot have had a random beginning… there are about two thousand enzymes, and the chance of obtaining them all in a random trial is only one part in 10^{40000}, an outrageously small probability that could not be faced even if the whole universe consisted of organic soup.[15]

> Our statistical discussion has shown that we will for fundamental reasons never be able to determine the frequency of appearance of living systems in the universe. This is because a well-founded answer to this question could only be given if we possessed sufficient knowledge of the functional properties of all the alternative sequences of a biological information carrier. However, this extremely large number of sequence alternatives makes it impossible to obtain such knowledge, by theoretical or empirical means, for even the lowest levels of macromolecular complexity.[16]

14 Küppers (1990) ch. 6. Cf. Weingartner (2015b) p. 191ff.
15 Hoyle and Wickramasinghe (1981) p. 148f.
16 Küppers (1990) p. 65f.

The above considerations show that the *chance hypothesis* is no reasonable answer. To split the improbabilities into handy pieces does not help for the explanation of the emergence of living systems.[17] This can be seen as follows. The age of the universe is about 10^{17} seconds.[18] If we grant 10^9 evolutionary steps per second, then we get 10^{36} evolutionary steps since the Big Bang. This number is ridiculously small in comparison to the number of sequence alternatives of the smallest catalytically active protein ($\approx 10^{130}$).

Ad (b)(iii): The concept of evolution according to Darwin was originally concerned with species. And it claimed and defended two evolutionary transitions: That from species A to subspecies A', (today) called *micro-evolution* and the transition from species A to another species B, called *macro-evolution*.

The micro-evolution, the transition from species A to a variation A' of species A, is very well confirmed empirically by fossils, by fast variation in nature and by evolutionary experiments. One example is the different types of so-called "Darwin finches" which can be mutually crossed or hybridized; they spread probably from the west of South America to the Pacific Isles including Galapagos. Another example is the descent of dogs. The analysis of mitochondrial DNA shows that all dog breeds originate from the species wolf (and not from coyote or jackal); they were produced by mutual crossing. Experiments with guppy-populations are another example. An example of fast micro-evolution are the colored perches (cichlidae) in the East-African lakes: there are about 500 different species in the Lake Victoria which developed within about 100.000 years[19], whereas the normal period for building subspecies is 2–3 millions of years. An even faster development seems to have happened in the Lake Malawi.[20]

The second transition from species A to an entirely different species B which is called *macro-transition* or *macro-evolution* still has its problems, although it is believed by most biologists of evolution.

---

17 Cf. Dawkins (1996). Dawkins' view seems to be at least ambiguous if not inconsistent. On the one hand he says that Darwinism cannot be a theory based on chance because of the huge improbabilities, on the other, he thinks he only needs to split them into small pieces.

18 Even if this number would be incorrect for several orders of magnitude (recent research assumes that it is greater)—because a universal cosmological time is hardly definable (cf. Mittelstaedt (2008))—the proportion to the numbers of sequence alternatives mentioned above would be roughly the same.

19 This was shown by DNA sequence analysis by Meyer, see Salzburger and Meyer (2004) and Meyer et al. (1990).

20 For further examples of micro-evolution see Kutschera (2008) p. 216ff. and Junker and Scherer (1998) p. 290ff.

(i) First of all the concept and scientific demarcation of species is a problem that is only partially solved: “We conclude that, despite the intense interest of biologists in the ‘species problem’ for over two centuries the answers are not as clear as they should be.”[21]

Moreover, it should be noted that for the greater part of biomass on earth, for Prokaryotes the concept of species has never been defined. The theory of neo-Darwinism (modern synthesis) concentrated its research on species of Eukaryotes only.

(ii) Secondly, the transition from *A* to an entirely different species of higher order *B*—say from a reptile to a mammal—is a transition which requires a completely new plan or blueprint, i. e. such an evolutionary process must be a process from lower to (essentially) higher complexity. However, there seems to be no general law to support such a transition: “On the theoretical side, there is no reason why evolution by natural selection should lead to an increase in complexity.”[22]

Bacteria, for example, seem to be no more complex today than for several two thousand million years ago; similarly crocodiles for hundred millions of years.

(iii) Thirdly, the fossil record for a transition from *A* to an entirely different species *B* (for example from mammal-like reptiles to mammals) is still incomplete, and the intermediate stages permit different hypotheses:

> And of course, there must have been many intermediate stages in the transition that cannot be reconstructed for want of appropriate fossil representation of those particular grades—The known fossils certainly do not themselves constitute an ancestral-descendant series.[23]

From what has been said, we can draw the following conclusion:

Micro-evolution has been corroborated to such an extent that we can speak of scientific knowledge of well-corroborated facts (K5 of ch. 4). Thus if some interpretation of the text of creation in Genesis denies any kind of evolution, then this interpretation must be wrong.

Concerning macro-evolution one cannot speak of scientific knowledge in the sense of K5 so far, but certainly of scientific belief. Such a belief is supported if the species are not too far from each other and are not too different in their DNA complexity. However, in this case, it is also an open question whether the later

---

21 Maynard-Smith and Szathmary (1999) p. 166

22 Maynard-Smith and Szathmary (1999) p. 4

23 Kemp (2005) p. 88. For general definitions of selection, development and evolution see Mahner and Bunge (2000) pp. 319ff. and Weingarnter (2015) pp. 145–156.

species may be a subspecies as in the case of the dog breeds having the wolf as a basic species.

If species B has a much higher complexity than species A, then fossils do not support such an evolution from A to B, not even for smaller steps which essentially differ in their complexity. Thus, in this case, the respective scientific belief also has its hindrances. A mediating hypothesis is proposed by Junker and Scherer[24] which in fact originates in Aristotle. It has been developed as a criterion for the demarcation between species by Ernst Mayr[25], although Mayr restricted it by geographical barriers which are sometimes too narrow (birds). The more tolerant version of the mediating hypothesis is that all descents which can be obtained by intercrossing or hybridizing belong to one huge species or type. Junker and Scherer call these species *fundamental types*.

Concerning the interpretation of the text of creation in Genesis, the text certainly does not forbid that God created the *fundamental types* and endowed them to develop the huge number of subspecies in the sense of (b)(iii) above (cf. note 13 above). However, even if it should be true that an evolution took place from a reptile to an elephant or even from a procaryote (bacterium) to man—which is no more than a wild hypothesis without scientific corroboration—one can always ask the question how such a bacterium stored with such a huge information for all living systems can emerge; or whether the bacterium was created. An analogous question goes on with the origin of life.

In general, we may say as a conclusion: Ultimately, there cannot be a conflict. Since if God is the author of both, the laws of nature and their consequences on the one hand and the scriptures (Bible) on the other, there cannot be an (external) inconsistency. The inconsistency can arise only through man: either by incorrect interpretation of those essential parts of the Bible which are a basis for the Creed or by incorrect extrapolation of scientific results beyond their limits.

Thus, for instance, it can be shown that chance in the universe is compatible with teleological order in the universe and that both are compatible with God's providence concerning the universe (creation).[26]

24 Junker and Scherer (1998) p. 34ff. and 45ff.

25 Mayr (1942)

26 Cf. Weingartner (2015)

9.4 Answer to the Objections

9.41 (To 9.11:) It is correct what it is said in the objection that if the belief is only partially conscious one cannot speak of knowledge of reasons for that belief. Thus, knowledge of reasons for the belief pertains only to conscious belief of older children or adults. As the so-called "false belief test" shows, children have conscious beliefs from about 4 years on since at that time they can clearly understand the situation of having a false belief of others (recall section 2.3.3.3).

9.42 (To 9.12:) As it is said in the objection in such cases there was not sufficient knowledge of the reasons for the belief. For example in the case of Frege's axiom of his Grundgesetze, he had strong supporting reasons in the sense that without that axiom arithmetic cannot be built up. But he had some doubts concerning the evidence for the very generality of the axiom; and therefore he had not sufficient knowledge concerning possible hindrances for the belief which in fact existed as inconsistent consequences.[27]

Many mathematicians after Euclid strongly believed that the postulate of the parallels was necessary for any geometry, and others like Gauss tried to derive it from the other postulates of Euclid.[28] In both cases, there was not sufficient knowledge of the reasons for the belief. It was not known that without the postulate of the parallels and even by denying it, other geometries were possible and that this postulate was independent of the others. However, insufficient knowledge of reasons for a belief does not mean no knowledge of reasons for it. On the contrary, there were many supporting reasons known for accepting the postulate of the parallels.

9.43 (To 9.13:) It is correct what is said in the objection that if the reasons for a belief are known to be incompatible, they, or the knowledge (in the sense of acquaintance) of them cannot be a necessary condition for the belief. But if the conflict is soluble they can be. Since Maxwell's Theory is not Galilei-invariant there were three options for a way out for Einstein: (i) Assume an ether as the designated

27 Cf. the epilogue in second volume of the Grundgesetze.
28 As a letter of Gauss to W. Bolyai (from 1804) shows that he still tried to prove the parallel postulate. Later letters to Bessel (1829) and Schumacher (1831) show ideas but without proofs. The first fully developed system of "Non-Euclidean Geometry" was given by Lobachevsky in a lecture at the section of Mathematics and Physics 1826 at Kazan University and published in 1829 ("On the foundation of geometry"). It was followed independently by J. Bolyai's "The absolute geometry" of 1831. Cf. Meschkowski (1978) p. 28ff. and Bonola (1955) p. 84ff. For an overview on different non-Euclidean geometries see Mainzer (2004a), (2004b).

inertial frame; (ii) try to adopt Maxwell's Theory to Galilei invariance; (iii) invent a new kind of invariance which is obeyed by Maxwell's Theory and adapt Classical Mechanics. Einstein had chosen (iii), even before experiments had confirmed his choice. A strong reason for Einstein's belief that (iii) was the true solution, was that the velocity of light must be constant and its propagation independent of the emitting source. This was also supported by Einstein's knowledge of Maxwell's Theory.[29] (i) was refuted by the Michelson-Morley-experiment, and (ii) was refuted by experiments showing the independence of the light velocity from the source (from 1913 on by Sitter and others). These refutations were not reasons for Einstein's belief. The main reason was Einstein's knowledge of Maxwell's Theory and connected with it, the constancy of light velocity.

9.44 (To 9.14:) Even if the belief is very global, there can be known reasons for the belief. For example, there are several reasons to believe in developmental processes in cosmology describing the formation of stars and galaxies. To use the term "evolution" here, instead of "development", might be misleading since first of all, this expression was originally used for species (mainly) of animals, and second, the necessary processes for evolution like variation, selection, and adaption cannot directly be applied to cosmological processes. Moreover, there are limits. There is no scientific knowledge of the development of the universe beyond "the first three minutes"[30] and much less beyond Plank time. Beliefs neglecting such limits can at best be mathematical constructions incapable of being tested or loose speculations. Moreover, such global beliefs (claims) as in 9.3.4 (b)(i) that everything is due to evolution are blatantly false and therefore cannot have consistent reasons.

Whereas the global beliefs pertaining to "world views" go beyond science and its well-corroborated laws and theories but still do not surpass the universe, global beliefs of religions are usually transcendent, i. e. surpass the universe. For example, the religious believer knows that the existence of an almighty God as the cause of the universe is a strong explanatory reason for his view (and belief) of the order and beauty of the universe; further for the "natural miracles" on this earth like growth, nutrition, and propagation in plants, animals, and order in non-living things.[31] By such a transcendental reason also known and partially known facts in this world get a positive interpretation in the sense of order, beauty, and meaningfulness.

29 For more details see Mittelstaedt and Weingartner (2005) p. 121f.

30 Cf. Weinberg (1977)

31 Cf. Weingartner (2015)

10 Whether there are Supporting Reasons for Religious Belief

10.1 Arguments Contra

10.11 If there are supporting reasons for a belief then consequences of what is believed have to be testable in a positive way, i. e. confirmable. However, in the case of religious belief there seem to be no consequences which are positively testable (confirmable). Therefore: Religious belief seems not to have supporting reasons.

10.12 If the supporting reasons for religious belief are transcendental, then the transcendental sources have to be believed previously in order to give support. However, in this case we might ask for other supporting reasons of the transcendental sources and so on without end. Therefore: religious belief seems not to have supporting reasons.

10.13 If the supporting reason for religious belief is the religious leader then this supporting reason is rather subjective since this person is not replaceable by any other person; as in science where the scientist (observer) is replaceable by other scientists working in the same fields. Therefore: Religious belief seems to have only subjective supporting reasons.

10.14 The strongest reasons for religious belief are miracles on appeal to God by the prophet or religious leader. But as Hume says, there cannot be miracles:

> A miracle is a violation of the laws of nature; and as a firm and unalterable experience has established these laws, the proof against a miracle, from the very nature of the fact, is as entire as any argument from experience can possibly be imagined.[1]

Therefore: If Hume is right, there are no strongest reasons for religious belief.

1 Hume (EHU) section X.

https://doi.org/10.1515/9783110585797-010

10.2 Argument Pro

As Plantinga says, supporting reasons for religious belief are needed:

> The theist must be able to answer the question ‘How do you know or why do you believe?’ if his belief is to be rational; or at any rate there must *be* a good answer to this question. He needs evidence of some sort or other: he needs some reason for believing.[2]

However, such answers and reasons have been given by philosophers and theologians through the centuries. Therefore, there are supporting reasons for religious belief.

10.3 Proposed Answer

There are two types of supporting reasons for religious belief: non-transcendental ones and transcendental ones. The non-transcendental reasons are the world (universe), religious texts, religious leaders, norms for life (and their confirmation), religious activity, strength and intensity of belief, etc. Transcendental reasons are: the text is revealed, the religious leader is sent by God; he can make miracles; man’s goal of happiness cannot be reached without religious belief (and according to Christianity: grace).

10.3.1 Non-transcendental Reasons

The first non-transcendental reason is certainly the world (universe) which invites many humans to believe in a creator. It is the many miracles of nature, the beauty, the differentiation and the fine tuning of the universe which is a strong motive for many to believe in an almighty omniscient and benevolent creator. As de Broglie says:

> Scientists have come to feel more and more keenly that there exists in nature an order, a harmony, which is at least partially accessible to our intelligence, and that they have devoted all their efforts to discover each day more of the nature and the extent of the harmony.[3]

A further reason is scientific theories about this world (our universe) which are very well corroborated and confirmed. Einstein’s General Theory of Relativity

2 Plantinga (1967) p. 187

3 L. de Broglie (1947) p. 206.

tells us that the universe is spatially finite (constancy of light velocity, finitely many material bodies of finite extension, light rays bent by big masses) which is in agreement with the religious belief that creation is finite. The best confirmed cosmological theory, the Big Bang Theory tells us that the universe has a finite age. It is strongly experimentally confirmed by the cosmic microwave background radiation discovered by Penzias and Wilson in 1964; it is strongly theoretically supported by the singularity theorems of Hawking and Penrose which say that under reasonable assumptions about the universe there must be a beginning.[4] This is again in agreement with the religious belief of a creation with a beginning in contradistinction to the Greek idea of an everlasting material base (materia prima). Other cosmological theories, for example, Hartle-Hawking's "no-boundary" approach which introduces an imaginary time has no experimental confirmation so far and seems to be no more than a genius mathematical construction.[5]

Moreover, it contradicts the singularity-theorems that he himself and Roger Penrose have proven. [6] These theorems also support the Big Bang Theory.
These reasons have been elaborated by many scientists and philosophers in a detailed and painstaking way. Therefore, we shall not go into it here but refer to the excellent literature.[7]

10.3.1.1 Religious Texts

Several religions have a special text as the basis of their religious belief. This is foremost true of the three Abraham religions, Judaism, Christianity, and Islam. This is not to say that a believer in one of those religions has conscious belief of the whole text. However, there is a gist or kernel of such a text which can be called the *Creed* of the respective religion.[8] The Creed contains the most important and relevant propositions of that text in the following sense: a believer is a believer in one of the respective religions (say a Christian believer *ChG*) if

---

4 Cf. Faber (2001), Penzias/Wilson (1965), Hawking/Penrose (1970).

5 As Stephen Hawking says himself in his "A Brief History of Time", p. 144 (Cf. Hartle/Hawking (1983), cf. Weingartner (2010) p. 43ff.):

> I'd like to emphasize that this idea that time and space should be finite without boundary is just a *proposal*: it cannot be deduced from some other principle. Like any other scientific theory, it may initially be put forward for aesthetic or metaphysical reasons, but the real test is whether it makes predictions that agree with observation.

6 Hawking/Penrose (1970). Cf. Penrose (1965) and (1979).

7 Barrow/Tipler (1986), Denton (1998), Dyson (1972), Faber (2001), Weingartner (2015).

8 Cf. Bochenski (1965) 3.3, 3.4, 13.1.

and only if he believes all the propositions of the respective (Christian) Creed (CC). In other words, the propositions of the Creed are such that if he does not believe some or other of the Creed, he could not be called a member of that religion.

10 D 1 $(\forall p)[a \in ChG \leftrightarrow (p \in CC \wedge aGp)]$

For example, the Christian Creed contains the statements that Jesus Christ was born from Virgin Mary, crucified, and resurrected. If someone would deny one of these statements, he could not be called a member of the Christian religion. But how is the belief in these three statements of the Creed supported?

(1) One type of support is *that* part of the whole text which forms the context for these three statements. Thus, for example, the events of Christ's birth reported by Matthew and Luke, the events of Christ's pain and death and that of resurrection (reported by all four Matthew, Mark, Luke, John).

(2) Another type of support are historical sources which are independent of the religious text. This is the case with reports about Herodes and Pilate concerning Christ's martyrdom and death.

(3) A further kind of support in the wider sense is to show that what is believed is *not impossible*. This has been dealt with already in sections 9.3.1–9.3.4 under the heading of removing hindrances for belief. Applying this to the three statements of the Christian Creed there is no question concerning martyrdom and death since these are facts even today and hence possible. Concerning the possibility of the virginity of Holy Mary there are new recent biological facts:
(i) If the question is whether there can be propagation without male sperm in *plants* the answer is yes and this is well-known for centuries.
(ii) If the question is whether there can be propagation without male sperm in *animals* the answer is also yes and it is well-known since decades that there can be just egg-division in female animals: It occurs, for example, often with hen and turkey-hen. Recently, one knows by DNA-analysis that in case the descendant is male, he has only two female chromosomes like a female descendant.
(iii) If the question is whether there can be propagation without male sperm in women (parthenogenesis), the answer seems to be yes too. Established facts are that there are male humans who have XX-chromosomes (like females) but no Y chromosome. This has been shown by DNA analysis. These male humans

are otherwise normal, except that they cannot produce children.[9] Further, it is known that some women became pregnant without having contact with a man. Further: when such women were pregnant having a boy, the DNA-analysis showed that the boy had only XX-chromosomes instead of XY-chromosomes. Different scientific explanations have been proposed for these facts: One is that in some way a spermatozoon was involved and destroyed in the egg on the way to the uterus. Another is that the mother received too much ultraviolet radiation because being in the sun. Again another is that the Y-chromosome was suppressed and is not dominant. Moreover, eventually, another is that there was egg-division in a virgin woman without the involvement of sperm. Thus, the *possibility* of virgin-birth (parthenogenesis) has been proven by today's biology (DNA-analysis). As long as this possibility is in doubt, the doubt is a hindrance for the respective religious belief. However, after the possibility has been shown by science, this becomes a supporting reason for the respective religious belief.[10]

The religious text or a part of it is also a description of the religious leader. This is done either directly or indirectly. Directly in the case of the New Testament for the life, the actions and the words of Christ. In a direct and indirect way in the Old Testament concerning the prophets and kings. For example, the reports of David's life as a direct description and his Psalms as an indirect description. The prophets are sometimes only described by their message (from God) which they have to deliver to the people. Mohammed is described mainly by his Koran.

The religious leader plays an important role also in Buddhism, Hinduism, Brahmanism, and Confucianism. In all the religions the way of life of the religious leader is an important support for believing what he said.

10.3.1.2 Norms for Life, Religious Activity, and Corroboration

As it says in the title of this section, we shall deal first with the belief in norms, secondly with religious activity and thirdly with the question of whether there can be some corroboration for these norms and the religious activity.

(1) As it has been said in 7.3.1 (2) both scientific and religious belief also extend to belief in norms.

9 The frequency according to recent results is 1 in about 25–30.000. For human XX-Maleness cf. Anderson, M. et al. (1986), Balakier, H. et al. (1993), Chapelle, A. (1981), Page, D. (1985), Petit, Ch. et al. (1987), Winston, N. et al (1991).

10 For the scientific explanation of virgin-birth cf. Tipler (2007) ch. VII.

Concerning Judaism and Christianity, the most important norms for life are the Ten Commandments (Old Testament) and the commandment of love (New Testament). They are complemented by the advice and rules in the books of Wisdom, of Proverbs and of Jesus Sirah.

Concerning Islam, there is a similar belief in the norms as guides for life which are contained in the Koran.

Since norms are not true or false like statements as indicative sentences, there is the question how they are connected with epistemic operators like belief or knowledge. The difference is that the truth of a statement is understood as a correspondence to facts: The first statement of Aristotle's Metaphysics, "All men by nature desire to know", is true if all men by nature desire to know. And this can be corroborated and is corroborated. However, the validity of a norm cannot be understood as a correspondence to facts: The norm "Nobody should lie" cannot be understood as valid if nobody lies; since it is valid also if some (many) do not act according to this norm.

However, we can translate norms into *that-clauses* and then say "that nobody should lie" is true (or is a valid norm). Translated into *that-clauses* makes it possible to apply epistemic operators like K, B, G just in the same way as they have been applied to indicative sentences. Thus, that a Christian believer believes in the Ten Commandments (TC) can be formulated thus:

10 P 1 $\quad (a \in ChG \wedge p \in TC) \rightarrow aGp$

(2) For religious belief in Judaism, Christianity, and Islam it is essential that it manifest itself in religious activity as a consequence of belief whereas for scientific belief there is no such strong connection.

More accurately, for the three mentioned religions it is not enough to believe. Acting as a consequence of belief and according to the belief is necessary, according to the passages: "Not those who say Lord, Lord, but those who obey the will of my father…" "What you do to the least of my brothers…".

Thus "living" or "active life" according to the belief in both religious statements and norms (for example Ten Commandments) is a necessary condition for religious belief at least according to the three Abraham-religions.

Though concerning scientific belief there is no such necessary condition. One must not forget, that "activity" is also required here: the proposed new hypothesis has to be exposed to severe tests, it has to be confronted with reality, it has to be used to explain already known and new phenomena, etc.

Concerning religious belief, one might say that the belief is not serious if it is not accompanied by active life according to the religious norms. Moreover, this—the belief and the activity—has to occur in the same person. However, in science it may split up in the sense of division of labor. The theoretician might invent and propose the hypothesis and draw some conclusions from it, but the experimentalist might work on the procedure to test it.

In religion no such “division of labor” is possible. The religious believer cannot leave the religious activity, the religious life to others: “One has to live according to the faith in order to be saved.”[11]

If we try to express this in a postulate, it would be too strong claiming that every religious believer or every ChG acts according to his beliefs in the Creed and the normative rules. Because this is not always satisfied. Therefore, we have to say that a religious believer *should* act according to his beliefs; or for a religious believer it is *obligatory* to act according to his beliefs.

10 P 2 $(a \in ChG \wedge p \in TC) \rightarrow \mathcal{O}aAp$
(‘aAp’ stands for ‘a acts in such a way that p occurs’, $\mathcal{O}$ for obligatory.)

Postulate 10 P 2 does not include the belief in the Creed because *what* is believed concerning the Creed cannot be executed by human action. This however does not say that the statements of the Creed would not be connected with the norms given, for example, the TC. Since the TC have a much higher value if they are revealed or commanded by an almighty authority having created the universe.

(3) Can there be confirmation and corroboration of the religious norms and advice and of the respective human actions which satisfy them?

An important general criterion for confirmation and corroboration of both norms and advice and also human actions is given in Matthew 7, 16 ff. in order to recognize false prophets: “By their fruit you will recognize them… every good tree bears good fruit, but a bad tree bears bad fruit. A good tree cannot bear bad fruit and a bad tree cannot bear good fruit.”

Specific “fruits” as consequences of human actions and style of life are given by St. Paul, Galatians 5, 19:

> The acts of flesh are obvious: sexual immorality, impurity, and debauchery; idolatry and witchcraft; hatred, discord, jealousy, fits of rage, selfish ambitions, dissensions, factions and envy; drunkenness, orgies and the like… But the fruit of the spirit is love, joy, peace, forbearance, kindness, goodness, faithfulness, gentleness, and self-control.

If we consider the kind of confirmation and corroboration proposed here, then it has a similar structure to that of scientific hypothesis: The consequences and

11 Salamucha (2003) p. 277

predictions of a scientific hypothesis are tested and when the tests have a positive outcome they confirm and corroborate the hypothesis. Analogously for a religious hypothesis or advice or commandment for life. The hypothesis that the fruits of "spiritual life" are love, joy, peace, patience, mildness, goodness, faithfulness, gentleness, moderation…, as the consequences and predictions, is confirmed and corroborated when these consequences and predictions really occur as an experience of the execution of that kind of life. And so also the complementary hypothesis that the fruits of the "flesh", that are unjustity, lust, idolatry, magic, hostility, quarrelling, jealousy, anger, intrigue, splitting, hate, murder, drunkenness, and gluttony, is testable and is confirmed and corroborated if experience of this kind of life shows this.

This kind of test—since it is established by experience of real life—may be called *pragmatic*. It is in accordance with what Charles Sanders Peirce called the key principle of pragmatism "By their fruits, ye shall know them."[12]

10.3.1.3 Strength and Intensity of Belief

There is certainly a difference between a strong and intensive belief and an average or weak belief. Accordingly, we can distinguish the case where the action is symmetric with respect to both sides p and $\neg p$, such that there is no definite assertion or inclination to either side. In this case, we may speak of *doubt*. A second case is when there is an average or weak belief or assertion and inclination with respect to one side (p) but with an uneasiness and concern to the other side ($\neg p$) i. e. with a fear that also $\neg p$ could be true. In this case, we may speak of *conjecture* or more disobligingly of *opinion*. The third case is when there is a clear, definite, and strong assertion and an inclination to one side (p) without doubt and without uneasiness and concern to the other side ($\neg p$). In this case, we can speak of *strong* and *intensive belief*.[13]

(1) The cases of scientific belief: All the three cases are present in science; and the last two cases are very frequent and important in scientific belief.

For example, there was Newton's strong belief that his law of gravitation $F = G \cdot \frac{m_1 \cdot m_2}{r^2}$ holds for all masses small or big. Although this was correct, it was proved only 111 years later by the famous experiment of Cavendish.

There was Einstein's conjecture that the velocity of light (in vacuum) is independent of a movement of its source. This conjecture was confirmed by the experiment of Michelson and Morely (1887) and by later experiments of Michelson

12 Peirce (1960) 5.465, cf. Matthew 7, 16 and Luke 6, 43.

13 Cf. Thomas Aquinas (STh) II–II, 2, 1.

and of De Sitter in 1913. Similarly with Einstein's conjecture of the deviation of light rays by big masses which is a consequence of his General Theory of Relativity (GR) of 1916 and was confirmed for the first time by the expeditions of the Royal Society to South Africa and Brazil by observing the effect on the occasion of an eclipse (1919). In both cases one is not quite clear how sure Einstein in fact was; i. e. whether he still had some concern to the opposite as it is appropriate for a *scientific conjecture* or whether it was strong belief with no uneasiness to the other side. It seems that Einstein's certainty developed in stages, although there was a quite strong belief that his intuitive ideas were correct. His sister Maya says that Einstein was worried whether his paper on the Special Theory of Relativity would be accepted by the Journal Annalen der Physik.[14] Concerning the first (velocity of light) Einstein knew the publication of Lorentz (1895) in which also the result of Michelson-Morely is discussed. Further, he knew Poincaré's (1898) doubts concerning simultaneity. From a letter to Grossmann in 1901 it seems clear that at that time he still had beliefs though with doubts concerning the existence of an ether. However, the doubts had disappeared certainly in the beginning of 1905.[15] Einstein did not live to see the experimental tests of his Special Theory of Relativity. Concerning the deviation of light rays by huge masses, the certainty also came in stages: his discovery of the principle of equivalence and its consequence of the deviation of light happened already in 1907. In 1911, he conjectured that the effect should be measurable with light from stars which touch the sun during a total eclipse and calculates a wrong magnitude since he did not incorporate the curvature of space which he discovered in 1912; incorporating the curvature, he calculated a magnitude of 1,74" (in 1915), which was corroborated later (1919). Before that (in March 1914) his belief was already very strong: "Ich zweifle nicht mehr an der Richtigkeit des ganzen Systems, mag die Beobachtung der Sonnenfinsternis gelingen oder nicht."[16]

A great scientist who had a lot of difficulties from his colleagues and from the scientific community of his time was Boltzmann. "The great enterprise of Boltzmann's life was to give an interpretation of thermodynamics in the framework of the 'atomic hypothesis' by doing statistical mechanics"[17] Entropy could then have been explained with the help of the number of possible microstates which can realize a certain macrostate, with the respective probability, and with his H-theorem. However, at this time the "atomic hypothesis" was very controversial under physicists, and they refused Boltzmann's derivation of an irreversible time

14 Cf. Stachel (2001).
15 Cf. Pais (1982) 6d.
16 In a letter to M. Besso. Cf. Besso (1972) p. 52. Cf. Pais (1982) 16 b.
17 Ruelle (1991) p. 109.

evolution from the laws of Classical Mechanics. It seems that despite the attacks by Loschmidt, Zermelo and others, Boltzmann strongly believed in his theory. This appears clearly from his answers to Zermelo's attacks in Wiedemann's Annalen and to Poincaré's recurrence theorem. As Ruelle says, he knew that he was right, but too early.

Concerning the three examples of Newton, Einstein, and Boltzmann, we may ask what their *supporting reasons* for conjecture and for strong belief were.

A first reason might be their personal disposition plus their understanding of their own research. By "understanding their own research" several motives might be involved of which the following are very important: a strong search for comprehensive and informative truth connected with simplicity, unification of conceptual framework and extended applicability. These components are mentioned explicitly by Einstein and seem to be more important for all three thinkers than the often mentioned aesthetic impact: "A theory is the more impressive, the greater the simplicity of its premises is, the more different kinds of things it relates and the more extended is its area of applicability."[18]

A second reason could be the reaction of colleagues and the scientific community in the respective scientific domain. However, in these three examples, the reaction of the respective scientists was not unambiguous and rather controversial. On the contrary, when Schrödinger published his wave-equation (the "Schrödinger-Equation") for Quantum Mechanics (1926) he immediately got a very positive and commending letter from Einstein. This meant great support for his strong belief. Newton, Einstein and Boltzmann did not have this opportunity. Boltzmann would have needed Einstein's view of Boltzmann's theory before 1906:

> Therefore the deep impression which classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of the applicability of its basic concepts, it will never be overthrown... On the basis of the kinetic theory of gases Boltzmann had discovered that, aside from a constant factor, entropy is equivalent to the logarithm of the 'probability' of the state under consideration... This idea appears to be of outstanding importance also because of the fact that its usefulness is not limited to microscopic description on the basis of mechanics.[19]

A third reason is the first corroboration and confirming tests by observation and experiment. Newton did not live to see Cavendish's experiment, Einstein did, concerning the velocity of light and the three important predictions of GR, but not concerning SR. The first tests for the proof of the increase of mass were only possible with sophisticated particle accelerators and the first experimental proof

18 Einstein's Autobiographical Notes in Schilpp (1949) p. 33.

19 Ibid. p. 33 and 43

for time dilatation was the effect proved by Hafele and Keating with atomic clocks in orbiting aeroplanes (1972). After such positive tests, the positive reaction of the scientific community is a further supporting reason. Boltzmann could base his belief certainly on the first reason, but on the second reason only partially in the sense that he could refute some of his opponents. Zermelo, however, at that time Planck's assistant, based his criticism on some basic misunderstandings as Boltzmann had showed him in his replies.[20] Planck hoped for a reduction of the Second Law to Classical Mechanics but did not agree with Zermelo: "Zermelo, however, goes further [than I] and I think that incorrect. He believes that the second law, considered as a law of nature, is incompatible with any mechanical view of nature."[21] Concerning the third reason, Boltzmann did not live to see the universal applicability of his theory far beyond thermodynamics in all domains of living systems and cosmology.

(2) The cases of religious belief: All the three cases which are present in science are also present in religion. There is doubt, weak belief, and strong belief. And the cases of weak belief and strong belief are frequent and important in religious belief. Moreover, there are many further similarities and analogies: there is a development concerning the strength of belief in stages, there is adherence despite of difficulties and attacks from the community, and despite the uncertainty of some presuppositions and reasons. There are strong motives for searching for truth and honesty, for simplicity, for unification, and for extended applicability. Moreover, there is the stubbornness of the disbelievers. These similarities and analogies will be discussed subsequently.

20 Boltzmann (1896), (1897). In Boltzmann (1968) Vol III § 119 and 123. That Zermelo was wrong, was also later proved by Lebowitz (1994) who showed that Boltzmann's theory is concerned with a distribution function (Maxwell-Boltzmann's velocity distribution) whereas Poincaré's recurrence theorem is concerned with simple trajectories. Cf. Prigogine (1985) p. 175, and Mittelstaedt and Weingartner (2005) p. 162. Moreover Loschmidt's objection can be experimentally refuted as Prigogine shows (ibid. p. 176f.).

21 Planck in a letter to his friend Leo Gratz, cited in Kuhn (1978) p. 27. Cf. Mittelstaedt and Weingartner (2005) p. 150.

Zermelo seems to have left physics afterwards and became famous in the domain of foundations of mathematics, especially set theory (1908). But a certain stubbornness in refusing new important results seemed to belong to his character: He rejected very strongly with polemical words Gödel's undecidability results as is clear from a letter to his mathematician friend Reinhold Baer in 1931 after the Bad Elster Conference. Cf. Weingartner and Schmetterer (1987) p. 45 f., where this letter is reprinted.

(a) Definition of religious belief:

10 D 1 Religious Belief (Faith) is "confidence in what we hope for and assurance about what we do not see."[22]

Recalling the situation of Newton, Einstein and Boltzmann, this description fits analogously also well to their scientific belief: There is confidence in the hope that the conjecture or theory will be confirmed by experience and there is assurance and conviction about what scientists do not see; at least about what scientists do not see at the time they invent their theories or even about what technically or in principle is unobservable.[23]

In religious belief "in what we hope" is usually concerned with transcendental goals like eternal life, poetic justice, reward and punishment. "What we do not see" refers to these things of hope just mentioned but also to the reasons for belief; thus concerning the Christian New Testament, that means that we are not eyewitnesses but believe the eyewitness report, not being able to see the eyewitnesses.

(b) Doubt, weak belief, strong belief: Like scientific belief, religious belief comes in different types of strength and develops in stages. *Doubt* and *disbelief* are connected with lack of confidence (cf. 10 D 1). It frequently occurs in the Old and New Testament: "How long will they refuse to believe in me despite all the signs I have performed among them" (Num 14,11); "In spite of this, you did not trust in the Lord your God." (Deut 1,32). Elijah's doubts were so strong that he gave up and wanted to die (1 Kg 19,4ff.).

However, his doubt and depression were not concerned with *what* he religiously believed nor with *what* his message was that he had to transfer from God to the people of Israel. His doubts and depression were concerned with the negative effects in the religious community and his inability to carry out his task. Boltzmann was in a very similar situation. He had no doubt, but rather strong belief, about the truth of what he discovered, which later turned out to be one of the greatest discovery in physics; his doubt and depression were concerned with the negative effects from the scientific community and his inability to communicate his discovery. Elijah was shaken into action from his wish to die by an angel, but Boltzmann committed suicide.

22 St. Paul (Heb) 11

23 This has been described convincingly and in detail by Popper (1963), Ch. 8. Cf. also Polanyi (1958 pp. 5, 15 and 37.)

Thomas: “Unless I see the nail marks in his hands and put my finger where the nails were, and put my hand into his side, I will not believe.” (Joh 20,25)

“The disciples … asked, ‘Why couldn’t we drive it out?’ He replied, ‘Because you have so little faith”’ (Mt 17,19).

“They believe for a while, but in the time of testing they fall away” (Lk 8,13).

Weak belief:

“If you do not stand firm in your faith you will not stand at all” (Is 7,9).
“You of little faith why are you talking among yourselves about having no bread?” (Mt 16,8).

“The boy’s father exclaimed, ‘I do believe; help me overcome my unbelief”’ (MK 9,24).

The synagogue leader was told that his daughter is dead. “Jesus told him; Don’t be afraid just believe” (MK 5,36).

Strong belief:

Abram: “You have given me no children, so a servant in my household will be my heir.” The Lord: “… a son who is your own flesh and blood will be your heir … Look up at the sky and count the stars … so shall your offspring be.” Abram believed the Lord, and he credited it to him as righteousness (Gen 15,2f.).

“When she heard about Jesus, she came up behind him in the crowd and touched his cloak because she thought ‘If I just touch his clothes, I will be healed’. Immediately her bleeding stopped… Jesus said to her: ‘Daughter your faith has healed you’…” (Mk 5,27f.).

“The centurion replied: ‘Lord I do not deserve to have you come under my roof. But just say the word, and my servant will be healed’… When Jesus heard this he was amazed and said to those following him: ‘Truly I tell you, I have not found anyone in Israel with such great faith” (Mt 8,8f.).

(c) Certainty and confidence develop in stages: This is so in many cases of both scientific and religious belief. However, there can also be strong belief and trust spontaneously.

Einstein’s belief in his General Theory of Relativity developed until it was firm around 1914 (recall the quotation of footnote 15).

The disciples of Jesus Christ developed their religious belief (faith) only gradually by steps. One reason for this was their misunderstandings concerning Christ’s mission. Eventually, they reached a strong belief only after Christ’s resurrection.

(d) Adherence despite difficulties: Concerning scientific belief, Newton, Einstein and Boltzmann had to overcome many difficulties. Difficulties and draw backs concerning their own research and its understanding and difficulties and attacks from the scientific community.

In a similar way the prophets and the apostles had to overcome difficulties concerning their own faith and understanding of their task. And they adhered to their religious belief despite the attacks of the then official religious community (Pharisees, Sadducees).

There are similar difficulties from religious or pagan communities for a believer also at the present time. However, there are also communities such as families, villages, cities, and countries which are a considerable support for the believer who has grown up there.

(e) Adherence concerning reasons and motives: Concerning both scientific belief and religious belief there is adherence to:
(i) strong search for truth and honesty: compare the honest search for truth in science with the honest search for the truth about God and the world in religion.
(ii) transparency and simplicity: compare the search for few informative universal principles or laws in science with the few fundamental principles guiding Christian life (Ten Commandments, Principle of Charity) in religion.
(iii) unification: compare the search for unifying and interrelating different domains in science—Maxwell's equations unify electricity magnetism and optics—with the unifying force of religious belief concerning the meaning and goal of human life and the world as created by God.
(iv) extended applicability: compare the search for universal applicability of the laws of a scientific theory with the universal applicability of the rules for human life given in the Christian Bible.

10.3.2 Transcendental Reasons

It is plain that especially the transcendental reasons mentioned need again belief to be reasons at all. It might be thought that this fact already destroys the trustworthiness or reliability of religious belief. However, this need not be the case since there are several similar situations in scientific belief. The situations may be of two sorts:

(1) The *reasons* for the respective belief are ungrounded or not grounded enough. An example is the dark matter problem (recall 9.3.3): We might believe (G 1) that the reason for the respective mass difference is due to unobserved dark matter. In this case, we have to believe (G 2) a group of astronomers who try to justify this. However, another group believes (G 1) that the reason lies in a wrong calculation of

the mass of galaxies. In this case, we have to believe (G 2) those who are experts in the field for the calculation of far distant masses. Although in both cases G 1 is not enough supported by G 2, we do not think that this destroys already the reliability of the respective scientific belief even if it requires further investigation.

(2) The reasons for the respective belief in the hypothesis are in a *circular way* depending on this hypothesis. An example is the experimental tests for corroborating the Special Theory of Relativity. The underlying methodological assumptions were these:

(i) Physical measurement instruments (rods and clocks) are real physical objects, not ideal entities.

(ii) Because of (i) they have to obey physical laws. But which ones? According to the *Copenhagen Interpretation*, the quantum-mechanical phenomena have to be measured by a measurement instrument "outside" of the QM-system which obeys the laws of Classical Mechanics. Einstein refused this view both for his Theory of Relativity and for Quantum Mechanics. Therefore he required (iii).

(iii) The measurement instruments applied to test the Special Theory of Relativity (SR) have to obey the laws of SR.

It is plain that assumption (iii) leads to a kind of circularity: The measurement instruments which are used to test SR presuppose and obey the laws of SR. Does this mean that such a test or such reasons are not reliable? As the facts show this is not the case and moreover reveals that it is the only way to test the predictions of SR, i. e. the time-dilatation and the mass-increase. The first was tested with the help of very accurate atomic clocks in airplanes orbiting the earth (Hafele and Keating 1972) the second in huge particle accelerators.

Concerning religious belief, some of the transcendental reasons are analogous to (1) and (2) in science.

Ad (1): That the religious leader is sent by God is supported by his style of life and by miracles he has done. However, his style of life and his miracles have been observed (in the case of Christ) by the Apostles. Two of them and two of their pupils of them have reported about it in the four gospels. That is, in order to believe in the religious leader (Christ), we have to believe that the supporting reasons (style of life, miracles) took place (G 1); and in order to have a belief in the sense of G 1 we have to believe the Apostles and Evangelists and their reports (G 2) like believing the experts in science.

Ad (2): The text of the gospel says "of itself" that it is revealed by God; for example reporting what John the Baptist says about Christ and what Christ said to the

Apostles and the people of Israel. Thus the situation is somewhat circular: The text is a reason for belief (cf. 10.31). The revealed text is a stronger (transcendental) reason for belief. However, the justification or confirmation for the text being revealed is given by this very text. Thus there is no ultimate external resort that can provide a kind of absolute justification for religious belief based on a revealed text. And this point that there is no ultimate external resort that can provide a kind of absolute justification, has a strong analogy in science; it is this fact we have learned from the Theory of Relativity: We cannot have an ultimate external measurement apparatus outside and separated from the physical system which is to be measured and judged (cf. (2)above) by this apparatus. Every measurement is relative in this sense, *without being unreliable*. In a similar sense, every reason for religious belief is relative and there is no external absolute last resort that could support or judge it—at least as long as we live on this earth.

What has been said holds *in general*. However, there can be exceptions in the sense that saints or prophets have a special religious experience or get a certain message; or that there are miracles of healing in present times (for example in Lourdes or Fatima or somewhere else). Such events are possible and cannot be excluded. Moreover, such events are to be understood as very strong external reasons for religious belief.

Concerning saints, these reasons, especially religious experience are bound to the respective person and can hardly be communicated; therefore Thomas Aquinas says theology has to accept the possibility and seriousness of such special religious experience but theology cannot use it as an argument or premise.[24] Concerning prophets, they may have the additional task to communicate a certain message to people; but although getting such a task will be a very strong external reason for the prophet's belief, this does not hold in general for the people. Miracles of healing are certainly a very strong reason of religious belief for the respective person, and in this case it can also be very strong for those who have watched the process of healing. Others, however, have to rely on the respective reports.

Even granted these exceptions, in the normal and general case it holds that there is no external absolute ultimate resort that could provide sufficiently strong reasons for religious belief.

According to Christianity, there is, however, one exception which holds generally. For religious belief, grace is given by God. However, the effects of this grace are still individually different.

24 Cf. Thomas Aquinas (STh) I, 1,8 ad 2. Cf. (STh) II–II, 174, 6 ad 3.

10.3.2.1 Revealed Text, Leader sent by God

As has been said already in section 10.3 all transcendental reasons need a kind of meta-belief in order to be reasons at all. However, this does not necessarily lead to an insoluble circularity or regressus. It was already said that such situations also occur in science where we have to believe in scientific theories which we cannot prove nor confirm ourselves and therefore as a meta-belief in scientific colleagues and in their reliability who defend them.

The religious text itself is a reason for religious belief, but that the text is ultimately revealed by God—though written by imperfect humans—is a strong supporting reason for religious belief. This reason is called transcendental because it transcends, i. e. goes beyond human experience in this world and human understanding of this world (universe). Since science is concerned with this world (universe), it also goes beyond science. This does not mean, however, that it could contradict science (recall section 9.3.4).

Observe however: that the text is revealed by God does not imply that everything what is said, described, or reported in the text is willed or accepted by God. This holds for some parts of the text but not for others. Thus, that God has created the world and that he has sent his son is certainly willed and caused by God; but that the people of Israel committed idolatry or that they killed Jesus are reported historical facts, but were neither willed nor caused by God; although they were permitted by him who endowed man with free will.

That the religious leader (for example Christ) is sent by God is another strong supporting reason for the belief in everything the religious leader (Christ) said, which is reported by the four gospels. Here the way of life and the actions of the leader (Christ) contribute an additional supporting reason for believing in his reliability.

10.3.2.2 Miracles

According to the usual terminology, there are *miracles* in a very wide sense which we might call miracles of nature (1); and there are *miracles* in very restricted sense which we might call miracles of religion (2).

(1) Miracles of Nature:

10 D 2 A miracle of nature is an event or a process which satisfies conditions (i)–(iii) or (i)–(iv):

(i) it occurs very frequently
(ii) its occurrence is very probable
(iii) it is admirable and amazing

(iv) it is not yet fully or not at all understood because its causes are partially or completely hidden for everyone.

We might distinguish here two kinds, a still wider kind which satisfies only the first three conditions and a special kind which satisfies all four conditions.

Examples for (i)–(iii): the beauty of flowers, trees, animals, mountains, lakes, landscapes, human beings… etc.; or the recently discovered DNA-repair-mechanisms in order to correct mistakes in the DNA-sequence occurring in the human cell either under normal conditions like in replication or which are due to environmental factors like radiation.[25]

Examples for (i)–(iv): how green plants manage in the photosynthesis to split up water into hydrogen and oxygen; how trees and especially high trees manage to transport water to the top.

(2) Miracles of Religion:

10 D 3 A miracle of religion is an event or process which satisfies conditions (i)–(vii):
(i) it occurs as a single unique event
(ii) its occurrence is very improbable and seldom
(iii) it is admirable and amazing
(iv) the possibility of its occurrence is not yet fully or not at all understood because it's causes are hidden from everyone
(v) it is not against nature or its laws but for it to occur, the natural capacity of creatures is not sufficient
(vi) it is done by God (sometimes on appeal to him, sometimes without appeal)
(vii) its purpose is to help man for salvation

Ad (i): Single and unique event: This means that this event as such is historically unique and not repeatable. Repeatable is only the type or kind of the event; for example, there are different types of healing for a blind person or persons with leprosy or different fish-fangs (according to NT). On the other hand, some miracles do not occur in different instances but only once; for example, calming down the storm, or transformation of water into wine. This is already a clear difference in contradistinction to miracles of nature which frequently occur as the same event.

This difference can be represented very well if we apply the concepts of epistemic entropy and epistemic information to the respective events (cf. ch. 5): If *e*

25 Cf. Scientific American 12 (2015), Nobel Prize in Chemistry.

is a miracle of nature (*MN*), its epistemic entropy is a large number but also its epistemic information is a large number. The first is the case since miracles of nature occur frequently and are therefore satisfied by a large number of possible real states. The second is the case since miracles of nature are very specific events and therefore exclude a great number of possible states.

10 P 3 $e \in MN \rightarrow (EE(e) \text{ is large} \wedge EI(e) \text{ is large})$

On the other hand, if *e* is a miracle of religion (MR) then its epistemic entropy equals 1 since it is a unique event satisfied just by one possible real state. However, its epistemic information is a large number because as a single state it excludes all other possible states.

10 P 4 $e \in MR \rightarrow (EE(e) = 1 \;\wedge\; EI(e) \text{ is very large})$

Ad (ii): Very improbable and seldom: In this section we shall consider three things: First the improbability of MR-events, second their compatibility with laws of nature and third their possibility explained with the help of laws of nature.

(a) MR-events are improbable and seldom:
Probability we shall use in an "objective" sense, i. e. we use the frequency interpretation of probability; "improbable" means then *having low frequency*. If a state of a physical system is very improbable, its physical entropy is very low and its order and structure is very high. Analogously, if an MR-event is very improbable, its epistemic entropy is very low, but its epistemic information is very high (cf. ch. 5). This fits also to the concept of information defined in Shannon's theory of information: A very improbable event has a high degree of information[26] From this it also follows that if we are only concerned with that component of the MR-event which can be interpreted as the accompanying physical event—say the calming down of the storm or the disappearance of the leprosy—then this event or process has a low entropy and a high degree of order and information.

(b) Compatibility with laws of nature:
Assume that one litre of hot water and one litre of cold water are poured together. According to the law of entropy, they will mix very quickly. The mixed state of luck-warm water has a much higher probability than the state where the two litres separate again into a cold and a hot part. This latter state is highly improbable. This also means that the entropy of the mixed state is much higher

26 Shannon/Weaver (1949)

than the entropy of the separated state; i. e. the number of microstates which can realize the mixed luck-warm state is much greater than the number of microstates which can realize the state of separation of hot and cold. Although this state of separation is not impossible and is not incompatible with the law of entropy, it is extremely improbable.

This shows that even an extremely improbable event can be compatible with the laws of nature. And thus also an MR-event as a very improbable event can be compatible with laws of nature.

It should be added that the understanding of miracles of religion in the Middle Ages has not been that they violate laws of nature (Hume's view). The theologians of the Middle Ages understand them as events which satisfy the following conditions:

(i) for bringing them about either human (or creature's) capacities are not sufficient; i. e. some miracles which surpass human capacities may be brought about by demons. But genuine miracles surpass the capacities of any creature

(ii) they are effects where their causes are hidden to all humans

(iii) they are not contrary to nature, although they are contrary to the usual course of nature

(iv) they are directed to the confirmation of faith

Some selected quotations are:

> Everything which happens through the power of some creature cannot be called *miracle*.[27]
>
> Now a miracle is so called as being full of wonder; as having a cause absolutely hidden from all; and this cause is God. Wherefore those things which God does outside those causes which we know, are called miracles.[28]
>
> That which is done miraculously by the divine power is not contrary to nature, though it be contrary to the usual course of nature.[29]
>
> The working of miracles is ascribed to faith for two reasons: First, because it is directed to the confirmation of faith; secondly, because it proceeds from God's omnipotence on which faith relies.[30]

(c) The possibility of MR-events explained with the help of laws of nature

Under the assumption that both, the MR-event is done by God and God has created the universe with its laws, we have to admit that God may *use his laws* when doing MR-events; and moreover that he knows in a comprehensive way how his laws can be used. On the other hand, humans have only a very partial

27 Thomas Aquinas (SCG) III, 2, 101

28 Thomas Aquinas (STh) I, 105, 7

29 Thomas Aquinas (STh), II–II, 154, 2 ad 2

30 Thomas Aquinas (STh), II–II, 178, 1 ad 5 knowledge of the laws of nature (of the universe) and a very partial knowledge how to use them.

This is manifest, for example, from scientist's or technician's understanding of Einstein's law: $E = m \cdot c^2$. If this law could be fully used we would not have energy-problems anymore since 1 g of mass would give us $2,5 \cdot 10^7$ kilowatt-hours of energy.

Therefore it follows that, for example, the possibility of the ascent into "heaven" or the one of walking on the water are easily explainable by a transformation of less than a gram of skin into repulsing radiation-energy. Concerning *MR*-events of healing, their possibility may be explained by causing a special DNA-repair-mechanism in the patient's cells which is a law-like process under normal circumstances. Recent discoveries have shown that in tumors and illnesses certain DNA-repair-mechanisms have been eliminated.[31]

Concerning birth from a virgin, it is scientifically known today that there are XX-man, i. e. male humans who possess only the two female XX-chromosomes without possessing a Y-chromosome. This is a rarely occurring fact (according to today's estimation it is 1:25.000). Therefore, with the help of the statistical law for the occurrence of XX-man, one can explain the *possibility* of virgin-birth (cf. 10.311 (3)).

10.3.2.3 Religious belief is connected with the belief and the desire in a state of happiness or reward after this life.

The particular connection of religious belief to the transcendent state of happiness or reward which is conceived as the final goal is this: If a person believes religiously that p and—provided that p belongs to the Creed of that religion, i. e. to the important statements of it—then he also believes that if he would not believe that p, he would not reach the state of happiness or reward (his final goal). Moreover, since the religious believer believes in the existence of such a state of happiness (in the actual existence concerning saints or prophets and in the possible existence for him) and further since he wants to be happy, he wants to believe. To arrive at this consequence, we certainly need a principle concerning human will which was presupposed. It is this: If a wills that p is (will be) the case and a believes (G- or B-) that q is a necessary condition of p, then a also wills that q is (will be) the case. If 'q is a necessary condition of p' is interpreted as '$p \to q$' (of Standard Classical Propositional Logic), then similar paradoxes as the Ross-Paradox in Deontic Logic and others may arise. To avoid them one can add relevance criteria

31 Cf. the discoveries of DNA-repair by Lindahl, Modrich and Sancar (Nobel-Price Chemistry 2015) as I have proposed elsewhere.[32] The principles which are involved to derive the conclusion are these:

10 P 5 $aG_Rp \rightarrow aB(\neg aG_Rp \rightarrow \neg H(a))$
'$H(a)$' for a is (will be) happy (rewarded).

10 P 6 $aW[H(a)]$
For arbitrary religious believers a, a wills (wants) to be happy (rewarded) (in a lasting sense) after this life.

10 P 7 $[aWp \wedge aB(p\underset{\rightarrow}{r}q)] \rightarrow aWq$
'$(p\underset{\rightarrow}{r}q)$' for '$p \rightarrow q$' according to some relevance criteria.

From 10 P 5–10 P 7 one can deduce 10 T 1 which states one important feature of the will in religious belief:

10 T 1 $aG_Rp \rightarrow aW[aG_Rp]$

Principle 10 T 1 states an important point. Many great Christian philosophers, at least from Augustine on stressed the important impact of the will in religious belief. For them, religious belief (faith) was understood from the very beginning as an action of both intellect and will (cf. ch. 11 above). This action belonged to both sides of the highest sphere of the human soul, e. g. to reason which included both intellect and will.

Concerning principle 10 T 1 I want to state an analogy between religious and scientific belief: In science, principle 10 T 1 has some analogon. It does not hold generally as in religious belief but it may hold sometimes for particular cases that if some scientist believes in a scientific hypothesis, then he also wants to believe in it. Think of the inventor of some new interesting hypothesis. Of him, it may be true to say that he also wants and wills to believe in the hypothesis he has proposed.

10.3.2.4 Strength of religious belief revisited

The transcendental reasons discussed in 10.3.2.1–10.3.2.3 are supporting reason to such an extent that religious belief includes the belief that which is religiously believed cannot (in its main part or gist) be false.

32 See section 6.3.1 note 17.

(1) This claim needs some clarifying comments. What is meant here is certainly serious belief in important propositions of a Creed of a religion. The Creed of a religion consists of those propositions by which a believer belonging to that specific religion is defined, i. e. as someone who believes those propositions of the Creed. However, concerning such propositions it holds that if they are believed religiously, this belief includes (implies) the belief (the strong assumption) that those propositions cannot be wrong. Symbolically (G_R for religious belief):

10 P 8 $\quad aG_Rp \rightarrow aB\neg\Diamond(aG_Rp \wedge \neg p)$

For some religions, especially Christianity, Judaism and Islam, the reason for this lies partially also in the fact that the religious believer believes that those propositions (of the Creed) have been revealed by an omniscient and almighty God even if this revelation happened via mediators (cf. 10.3.2.1). The more mature religious believer would certainly confess that there might be theological (exegetical) discussion about the correct interpretation of some of the propositions of the Creed (of a particular religion). However, this does not mean that 10 P 8 is violated. One way of handling this is to understand 10 P 8 conditionally in the sense that the mature religious believer grants to make the presupposition "if p is correctly interpreted". However, independently of that, there are some basic beliefs which do not allow much debate of interpretation: either Christ (more than a prophet) was born or not, either Christ was resurrected or not. Though there may be a debate on the meaning of Christ (son of God etc.), but hardly about "was born" or "was resurrected". Similarly with "there will be reward or punishment after death" etc. Moreover, with respect to such propositions the religious believer also believes that they cannot possibly be (altogether) false, even if he may grant that he might not understand some detail here.

(2) Concerning scientific belief, I do not think that the analogue of 10 P 8 never holds. An example is the law of the conservation of energy which states that in a closed system (and especially also in the universe as a whole) the amount of energy is constant. Concerning this law, most physicists would say that they not only believe in it but also believe that it is impossible that this is false. Another basic belief in physics is the constancy of light velocity with its well-known amount (of

~300.000 km/sec)—at least relative to our universe.[33] Further examples are the other fundamental constants like m_p/m_e (proton mass/electron mass) G, h, α.[34]

There are less good examples in mathematics. Before a proof is given, the analogue of 10 P 8 would not hold for one of some famous conjectures in mathematics, say Goldbach's or Fermat's. On the other hand, after a proof was given, say that the Axiom of Choice or the Generalized Continuum Hypothesis are independent of the axioms of Set Theory, one does not speak of belief anymore but of knowledge. In cases where a proof can hardly be expected, we also cannot apply the analogue of 10 P 8. Thus, only few would claim that the belief in the consistency of Zermelo-Fraenkel or Neumann-Gödel-Bernays Set Theory cannot be false.

The important difference between scientific and religious belief in this respect seems to me to be this: In a great many cases—which admittedly do not concern the most basic beliefs of a certain scientific discipline—scientific belief in a certain scientific hypothesis, law or theory admits that what is believed can be false. This means that one could have the opposite belief in the consequent of 10 P 8 if scientific hypotheses are concerned (G_S for scientific belief):

10 P 9 $aG_Sp \rightarrow aB\Diamond(aG_Sp \wedge \neg p)$

There is the question whether 10 P 9 can be applied to conjectures in mathematics. Since we admit distribution principle over conjunction for B-belief and accept the distribution law $\Diamond(p \wedge q) \rightarrow (\Diamond p \wedge \Diamond q)$, it follows from 10 P 9 that $aG_Sp \rightarrow aB\Diamond\neg p$. Moreover, the latter is problematic whenever "p is proved (provable) in mathematics" is interpreted in such a way that $\Box p$ holds (where '$\Diamond\neg$' is understood as '$\neg\Box$'). Thus let p be the conjecture of v. Neumann that the Generalized Continuum Hypothesis is independent of the axioms of Set Theory. According to the passage cited (cf. 8.31), he believed that p is provable, i. e. that p can be proved or—according to the interpretation—that p be necessarily true. And the latter is incompatible with the belief that it is possibly false (i. e. with: $B\Diamond\neg p$). To this difficulty I want to say three things:

First of all, we might interpret "a believes that p is provable" as "a believes that it is possible that p is proved" and so, if *proved*, is interpreted with *necessary* we get "a believes that it is possible that p is necessary". But this is not incompatible

33 According to an interesting theory of A. Meessen the amount of the light velocity is dependent on the amount of energy distributed in the universe satisfying the equation $c = 2.a.E_U/h$ (where c is the light velocity, a the smallest measurable distance yet known, E_U the energy of the universe and h Planck's constant). If the amount of energy would be greater, the light velocity would be greater too, but still constant in that other universe. Cf. A. Meessen (1989).

34 Cf. Barrow/Tipler (1986) and Mittelstaedt and Weingartner (2005) ch. 8.

anymore with the belief that p is possibly false (i. e. not necessary), provided that not strong Modal Systems are presupposed.

Secondly, Standard Possible World Semantics (Kripke-style) has the necessitation only for truths of logic, i. e. if $\vdash p$ (where '$\vdash$' means provable in Logic, usually in First Order Predicate Logic) then $\Box p$. It is one of the strange consequences of this semantics that even very simple basic truths without which mathematics is impossible like $(\exists x)(\exists y)(x \neq y)$ (there are at least two individuals) are not provable with $\Diamond$ in this semantics, i. e. $\vdash \Diamond(\exists x)(\exists y)x \neq y$ is not a theorem. Needless to say, that also $\Diamond(2 + 3 = 5)$ is not provable in this semantics. Thus if '$\Diamond$' in 10 P 9 is interpreted with the help of the usual Possible-World-Semantics, 10 P 9 is not applicable to propositions (belief, conjectures) of mathematics.

Thirdly, we might interpret $\Diamond$ and $\Box$ intuitively and accept the rule that what is proved (and provable) in mathematics should hold with $\Box$. Then 10 P 9 is not correct for mathematical conjectures but only for conjectures in empirical sciences in the sense that we view the propositions in these disciplines as contingent, at least contingent relative to those of logic and mathematics. Though also in these disciplines one might distinguish between propositions which are necessary like laws ('empirical necessity', 'natural necessity') and others which are not, this kind of necessity is certainly not the same as that of logical and mathematical truths. In order to have an analogous principle of 10 P 9 for conjectures in mathematics (and one may add: in logics) we might interpret '$\Diamond$' in another way, for example epistemically: If a G-believes (in mathematics, logic) that p, then he B-believes that it is compatible with all that he believes and knows that p might be false.

10.4 Answer to the Objections

10.41 (To 10.11:) As has been elaborated in section 10.3.1.2, religious activity and living in accordance of religious norms (Ten Commandments and other advice) has testable consequences. And if these consequences are positive for living in a family, community, and society as St.Paul says then the respective religious activity is corroborated; and this corroboration is a strong reason for religious belief.

An analogous test in the profane domain is the long term study on self-control in New Zealand continuing for more than 40 years with the same sample of more than 1000 persons. It shows that self-control from the age of 3 on leads to more health more stable financial status and less criminality.[35] Moreover, advice for self-control is frequent in religious texts, for example in the books of Proverbs,

35 Cf. Moffitt et al. (2014)

Wisdom and Sirach of the Old Testament, and in many places in the New Testament.

10.42 (To 10.12:) It is correct as it is said in the objection that concerning transcendental reasons there is a need for other supporting reasons. However, as has been shown in section 10.3.2, several such reasons are in fact given: the life of the religious leader, miracles of religion, and further the non-transcendental reasons like historical support for special figures (kings, prophets, emperors) described in the religious text. However, it has to be observed what has been stressed in section 10.3.2: There is no external absolute last resort of support. And there is a certain kind of circularity involved when giving transcendental reasons. However, both do not necessarily lead to non-reliable support. This is proved by the analogous situation in science as has been elaborated with the example of supporting measurement results for the Special Theory of Relativity.

10.43 (To 10.13:) Some religious leaders have been replaced by others, better ones. Some, however, are not replaceable. Thus Saul was replaced by David, and Christ as the son of God is of course not replaceable. Even if the supporting reason is subjective in the sense of concentrating on one person, this does not necessarily mean that it is less reliable. One the contrary, if this person is a much higher personality such as being omniscient, then the reliability is much greater. As an imperfect analogy, think of Einstein as a support of his Theory of Relativity (Special and General). Both theories are solely his great invention and discovery. Does concentration just on him mean a loss of reliability? Of course not.

10.44 (To 10.14:) Hume's view that there cannot be miracles depends on his definition of miracles: that miracles contradict the laws of nature. As has been elaborated in section 10.3.2.2 (2), the definition of miracles (of religion) in the Christian tradition speaks of events for which the capacity of creatures is not sufficient. However, such events, as has been shown there, are compatible with laws of nature. Independently of that, Hume had no clear understanding of laws of nature. On the one hand, he is right to point out that past events do not have a logical (logically necessary) connection with an occurring future event. On the other hand, he wrongly concludes from this that there is no causal or natural connection at all except a psychological one: our customary expectation or habit, for example, that the sun will rise again. He had no understanding that with the help of Newton's laws plus past events one can correctly predict such events as the sunrise or planetary motion or many other future events. Therefore, his argument against miracles with the help of laws of nature is quite useless.

11 Whether There is a Voluntary Component in Scientific and in Religious Belief

11.1 Arguments Contra

11.11 Activities of the intellect are not activities of the will. Scientific belief is an activity of the intellect. Therefore it is not an activity of the will. Further: Only activities of the will have a voluntary component. Therefore: Scientific belief does not have a voluntary component.

11.12 If scientific belief would be justified by voluntary reasons as components of the belief then it would not be justified by objective reasons. However, scientific belief means belief in scientific hypotheses, laws, and theories which is justified by objective reasons. Therefore: Scientific belief is not justified by voluntary reasons as components of the belief.

11.13 Fermat's belief in his "Last Theorem", i. e. that $x^n + y^n = z^n$ has no solutions for $n > 2$ was scientific true belief. Moreover, his justification was not voluntary because he said he found a simple proof for it; even if this proof could have been hardly complete—if we compare it to the arduous and long correct proof of Wiles (1994)—his proof-idea of descending series might be a correct possibility. Therefore it seems that scientific beliefs in which mathematical proofs are involved do not have voluntary components.

11.14 If scientific belief in the outcome of scientific observations or experiments is based on universal theoretical conjectures or scientific laws then such scientific belief does not contain a voluntary component. As the experiment of Millikan concerning the charge of the electron shows his (and Ehrenfest's) scientific belief was based on a theoretical conjecture (recall section 2.3.3.2). Moreover, scientific belief concerning both idea and outcome of scientific observations or experiments seem always to be based on theoretical conjectures or scientific laws. As Popper says: "We do not 'have' an observation but we 'make' an observation. An observation is always preceded by … a question, or a problem—in short, by something theoretical. After all, we can put every question in the form of a hypothesis or conjecture…"[1] Therefore: Scientific belief seems not to contain a voluntary component.

1 Popper (1972) pp. 342f.

https://doi.org/10.1515/9783110585797-011

11.15 Since *what is believed* is something which is true or false (and not good or bad) in both scientific and religious belief, belief is an activity of the intellect and not of the will. Moreover, it seems that only activities of the will have a voluntary component. Therefore: Religious belief does not seem to have a voluntary component.

11.2 Arguments Pro

11.21 According to definition 10 D 1 (ch. 10), belief is trust in what we hope. Since hope is the desire of some good which is not yet present, hope contains a component of will. As the following quotations of a letter of Planck and a letter of Heisenberg to Einstein show, hope is also included in scientific belief:

> I believe and hope that a strict mechanical significance can be found for the second law along this path, but the problem is obviously extremely difficult and requires time.[2]

> Sie werden wohl recht darin haben, daß unsere Formulierung der Quantenmechanik mehr der Bohr-Kramers-Slaterschen Auffassung angepasst ist, aber die ist ja auch eine Seite der Strahlungsphänomene. Die andere ist Ihre Lichtquantentheorie, und wir haben doch die Hoffnung, daß die Gültigkeit des Energie-und Impulssatzes in unserer Quantenmechanik auch den Anschluß an Ihre Theorie einmal ermöglicht.[3]

Therefore: Since hope is included in scientific belief, it holds that there is a voluntary component in scientific belief.

11.22 As Salamucha says, an important role in an act of faith is played by the will.

> In an act of faith, reason is not forced to accept a given truth by its obviousness, nor by the power of purely rational arguments. A truth which requires faith remains as if outside the scope of reason; here the factor which decides whether a given truth is accepted and acknowledged is always an act of will.[4]

Therefore: If Salamucha is right there is a voluntary component in faith (=religious belief).

---

2 Planck in a letter to his friend Leo Graetz. Cited in Kuhn (1978) p. 27.

3 Letter to Einstein on Nov. 30 1925. Cited in Pais (1982) ch. 25.

4 Salamucha (2003) p. 279.

11.3 Proposed Answer

Every action of knowing or believing involves an assent of the intellect to what is known or to what is believed. We may distinguish four cases: (1) The intellect assents because it is forced by true immediate objective understanding (K 1) or by true immediate intrasubjective understanding (K 2). These are cases of immediate knowledge. (2) The intellect assents because it is forced after consideration, investigation, and checking either by proof (K 3) or by direct or indirect observation and experiment (K 4). These are cases of provable knowledge. (3) The intellect assents because it is convinced by corroboration (K 5), by justified true belief (K 6) or by possessing epistemic entropy and epistemic information (K 7). These are cases of justified knowledge (recall chs. 4 and 5). (4) The intellect assents—without being forced sufficiently after consideration, investigation and checking—partially because of a decision of the will. This happens in every religious belief but also in scientific belief. The distinction between cases (1)–(3) on the one hand and (4) on the other is well described by Thomas Aquinas:

> Now the intellect assents to a thing in two ways. First, through being moved to assent by its very object, which is known either by itself (as in the case of first principles, which are held by the habit of understanding), or through something else already known (as in the case of conclusions which are held by the habit of science). Secondly, the intellect assents to something, not through being sufficiently moved to this assent by its proper object, but through an act of choice, whereby it turns voluntarily to one side rather than to the other: and if this be accompanied by doubt and fear of the opposite side, there will be opinion, while, if there be certainty and no fear of the other side, there will be faith.[5]

The subsequent sections of this chapter will be concerned with case (4), i. e. with the voluntary component of religious and scientific belief.

11.3.1 There is a voluntary component in every religious belief

This is so in a fivefold sense: (1) Religious belief is connected with the desire of happiness and reward after this life (cf. section 10.323 above). (2) Religious belief includes considerations and investigations of what is believed, and this is subject to free choice. (3) Religious belief includes searching for reasons for what is believed, and this is subject to free choice. (4) Religious belief means accepting a higher authority in an epistemic and in a deontic sense. (5) Religious belief contains a voluntary component directly referring to the assent of the intellect.

5 Thomas Aquinas (STh) II–II, 1,4.

(1) As has been elaborated on in section 10.323 religious belief is connected with the desire of happiness. This is so at least in the Christian tradition. The connection was given by the postulates 10 P 3–10 P 5 and the theorem 10 T 1: If a person *a* believes religiously that *p* (let '*p*' represent a proposition of the Christian creed), then—under the condition that *a* wants to achieve eternal happiness and believes that if *a* does not believe religiously that *p*, *a* will not achieve happiness—*a* wills that *a* believes that *p*. Since the condition above between the dashes is satisfied for religious belief (at least under normal conditions), the conclusion is satisfied too. Thus, this conclusion, that *a* is willing to believe that *p*, is *one sense* in which religious belief has a voluntary component. As Swinburne points out this aspect of religious belief affects also respective actions:

> He will thus seek not his own fame, but long-term and deep well-being for himself and others. Seeking these things, he may believe that they are only to be had if there is a God who provides such well-being in this world and in the world to come. Hence he may act on the assumption that there is a God—for unless there is, that which is most worthwhile cannot be had.[6]

(2) Consideration and Investigation: There is certainly religious belief which is immature and not reflected upon (as in children for example). However, we do not deal with these forms here; we presuppose mature believers with critical self-reflection. In this case, religious belief includes interest in what is believed and consequently repeated consideration and permanent further investigation of what is believed. This can be done in different ways like discussions with friends and colleagues, reading religious texts (including the Bible or the Koran), etc. However, all such activity, depending also in the strength of interest, is open to free choice and needs the will to realize it. This is a *second sense* in which (mature) religious belief has a voluntary component.

(3) Searching for reasons: Only in the first four kinds of knowledge K 1–K 4 are the reasons sufficient for the assent of the intellect. In the case of K 5–K 7 searching for further reason is necessary. Similarly in religious belief, the reasons for belief are never sufficient in the sense that it develops from belief into knowledge of the kind K 1 or K 2; or also into K 3 or K 4 like the belief in a mathematical conjecture develops into knowledge (K 3) after the proof has been given or the belief in an empirical prediction develops into knowledge (K 4) after a confirming experiment has been carried out. An exception are saints who could have such a development after a vision. Thus, mature religious belief includes searching for further supporting reasons. These reasons can be very different as is manifest

6 Swinburne (1981) p. 115.

from the elaboration of ch. 10 above. Similarly, as with further considerations and investigations, the activity of searching for further supporting reasons is an action of free choice which needs the will for realization. This is a *third sense* in which (mature) religious belief has a voluntary component.

(4) Higher epistemic and deontic authority: Every rational believer prefers to believe that what an expert says (w. r. t. that domain) compared to that what a layman says (w. r. t. that domain). This also holds for the religious believer. For every religious believer, the optimal expert is the God of the respective religion. Therefore every religious believer prefers to believe what God says compared to that what men say. However, since "what God says" is not directly accessible but only via the mediation of man (i. e. through the writings of prophets and evangelists in the Judaic-Christian tradition) the strength of belief has to be relativized to the strength of trust to these mediators. To *prefer* involves an act of the will. Therefore religious belief in what God says involves a voluntary component. We may elaborate somewhat further the belief in experts by saying it is a belief in an *epistemic authority* (an expression introduced by Joseph Bochenski). *Epistemic authority* can be defined in the following way:

11 D 1 x is an *epistemic authority* for y within the domain D iff one of the following conditions (i) or (ii) are satisfied:
- (i) every proposition p communicated by x has a higher degree of subjective probability for y at time t_2 than p had at time t_1 ($t_1 < t_2$) for y when p was not yet communicated to y by x.
- (ii) every proposition p communicated by x has a higher degree of reliability for y at time t_2 than p had at time t_1 ($t_1 < t_2$) for y when p was not yet communicated to y by x.[7]

Condition (i) holds for singular propositions or very restricted hypotheses, condition (ii) holds for universal hypotheses and laws since the application of the usual concept of probability to universal hypotheses and laws leads to difficulties. Example: The domain of an internist are internal diseases. Thus an internist is an epistemic authority for patients with internal diseases.

Since God is understood as omniscient[8], God is an epistemic authority for men w. r. t. every domain. However, he has not communicated (revealed) propositions

7 x, y are persons, D is a domain of discourse or investigation. Bochenski's definition contains only condition (i). Cf. Bochenski (1965) section 51.6.

8 Cf. Weingartner (2008)

of every domain; he left many things to man's discovery since he has endowed man with reason. He has communicated (revealed) especially those things which serve man's salvation and concerning which man could hardly find the truths because they surpass his natural reason. Therefore God's epistemic authority is of special importance in the domain of the revealed context.

If x is an epistemic authority for y w. r. t. domain D then y will usually also accept advice from x w. r. t. domain D. Thus if the internist x is an epistemic authority for the patient y who has a stomach disease then y will accept therapeutic advice given by x. Similarly concerning religious belief: If God is an epistemic authority for the religious believer y, then y will accept advice and commandments given by God. In this case, we say that the internist or God is also a *deontic authority* for y. A definition of deontic authority is as follows:

11 D 2 x is a *deontic authority* for y within domain D iff every norm n representing an advice or commandment, communicated by x has a higher degree of obligatory force for y at time t_2 than n had at time t_1 ($t_1 < t_2$) when n was not yet communicated to y by x.

That this advice and these commandments should be satisfied by man's activity is an essential feature of Judaism, Christianity, and Islam (cf. section 10.312 above). To satisfy them and to act accordingly is still a matter of free choice even if the belief in the deontic authority is quite strong. However, already the belief in the deontic authority means willingly accepting a stronger obligatory force. Moreover *doing* what one believes is correct or in accordance with the commandments of the deontic authority needs still more force of the will. Thus, religious belief in a deontic authority contains a component of the will.

Observe, however, that a deontic authority does not imply an epistemic authority; i. e. x may be a deontic authority for y w. r. t. domain D but x may not be (at the same time) an epistemic authority for y w. r. t. D. For example, a director of a research laboratory (as Max Perutz was for Francis Crick and James Watson) may be a deontic authority for the research fellows in the laboratory w. r. t. a certain domain D concerning the research in the laboratory. However, from this, it does not follow that the director is also an epistemic authority w. r. t. D for these researchers. In fact, Perutz investigated the structure of haemoglobin (for which he got the Noble-Prize together with Kendrew in 1962) whereas Crick and Watson investigated the structure of the DNA (for which they received the Noble-Prize together with Wilkins in the same year). Moreover, Perutz was a deontic authority to Crick and Watson distinguished by a lot of freedom and encouragement for them.

A school-teacher should be both a deontic and an epistemic authority for his pupils. If he is only a deontic authority (which he has received partially by his teaching certificate and by his supervisory school authority) but lacking epistemic authority, then he is a bad teacher.

In conclusion to section (4), we may say that both the religious belief in an epistemic authority and that in a deontic authority involves a voluntary component. And this is a *fourth sense* in which (mature) religious belief has a voluntary component.

(5) Direct voluntary component in religious belief: In the four senses in which a voluntary component is involved in religious belief—discussed so far—the voluntary component is only indirectly or conditionally concerned with the assent of the intellect in the action of believing: In (1) what is willed or devised is ultimate happiness and conditionally religious believing. In (2) what is willed by free choice are considerations and investigations of what is believed and in (3) what is willed by free choice is searching for reasons. In (4) a higher authority is accepted with voluntary choice. However, there also seems to be a voluntary component directly referring to the assent of the intellect in an action of religious believing. This direct voluntary component may not be present at the beginning of a religious belief; and it may have been motivated and supported by the four other voluntary components. However, it seems to be necessary that also a direct voluntary component is involved in religious belief. As Aquinas says: "The intellect of the believer is determined to one object, not by reason, but by the will, wherefore assent is taken here for an act of the intellect as determined to one object by the will."[9]

11.3.2 Is There a Voluntary Component in Scientific Belief?

(1) If we compare scientific belief with religious belief w. r. t. the five senses of the voluntary component then it turns out that scientific belief certainly does not have all five of the voluntary components. A voluntary component which is connected with the desire of ultimate happiness does not exist in scientific belief, at least not in the first sense in which it exists in religious belief. There might exist another kind of happiness in the sense that the *scientific activity* is something which makes a scientist happy. In religious belief such happiness lies in the future and believing in the Creed is understood as a necessary condition for achieving it. Thus the first voluntary component of religious belief does not occur in an analogous way in scientific belief. The other four senses in which a voluntary component exists in religious belief, however, have an analogy in scientific belief.

9 Thomas Aquinas (STh) II–II, 2, 1, ad 3.

(2) Consideration and Investigation: Belief in scientific hypotheses needs permanent consideration and investigation concerning their corroboration. Moreover, it includes critical investigation concerning hidden preconditions and problematic assumptions. It needs reading related scientific texts and studying methods for finding severe tests in order to examine the hypotheses. All that can be guided and motivated by strong interest in what is believed scientifically. That interest in what is believed is very important and is also revealed by biographies of great scientists:

> The most fascinating subject at the time that I was a student was Maxwell's Theory.[10]

> My own interest in those years was less concerned with the detailed consequences of Planck's results, however important these might be. My major question was: What general conclusions can be drawn from the radiation-formula concerning the structure of radiation and even more generally concerning the electromagnetic foundation of physics?[11]

From this consideration, it is plain that there are two different kinds of volitive components in scientific belief. One is connected with *interest*, i. e. an evaluation of the content of the scientific belief. The second is connected with the scientific activity of consideration and investigation. Although the second is more a matter of free choice than the first, both can be realized only with the involvement of the will.

(3) Searching for reasons: Many scientific beliefs are not directly testable as in simple cases of K4. However, they are far from being a stray conjecture, there is a vast searching for supporting reasons in the respective discipline. Examples: The hypothesis "there have been dinosaurs" is not directly testable. However, there is world-wide searching for the respective fossils such as bones, teeth, eggs, and footprints. The hypothesis "there exists the Higgs boson", a particle of the scalar isopin field energetically required by the Glashow-Salam-Weinberg theory unleash a long series of investigations and experiments in elementary particle physics until the particle was discovered in CERN in 2014.[12]

Needless to say that such searching for reasons and experiments which uncover fossils and elementary particles require a lot of free will decisions of scientists and also politicians to give the money for the research projects.

(4) Higher epistemic and deontic authority: As it was already pointed out, every rational believer prefers to believe what an expert (of some domain *D*) says compared to what a layman says w. r. t. *D*. This certainly holds for scientific belief.

10 Einstein (1949) p. 33.

11 Ibid. p. 47

12 Wolschin (2013)

However here, in contradistinction to religious belief, the epistemic and deontic authority is not supernatural (even if mediated through man) but human.

Great scientists are certainly *epistemic authorities* for scientists working in the same domain. Although, to become an epistemic authority usually takes some time: it is rarely at the beginning when a scientist announces his new ideas but sometimes much later. An exception was the discovery of the DNA-structure by Watson and Crick published in *Nature* (1953): both immediately became famous epistemic authorities. For Newton to become an epistemic authority also took longer although it began in his lifetime and lasted for almost 300 years until Planck's discoveries of discontinuous dependencies. With Einstein it began after his publications of his "annus mirabilis" 1905, but his epistemic authority was still controversial. The break-through to the world-wide accepted scientific authority was when the expeditions of the Royal Society to South Africa and Brazil confirmed in 1919 one of the most important predictions of the General Theory of Relativity (published in *Annalen der Physik* of 1916): the deviation of light rays by big masses.[13]

Boltzmann became a famous epistemic authority only after his death. Moreover, the unjustified attacks of many physicists at that time against Boltzmann's atomic theory were serious reasons for his depression.[14]

Some famous scientists are very critical, if not skeptical, against epistemic authorities; and they have a good point of two such authorities male claims which are mutually incompatible. In this sense, Wolfgang Pauli wrote to Niels Bohr in 1924:

> Sie sehen: selbst wenn es für mich psychologisch möglich wäre, mir meine wissenschaftlichen Meinungen auf Grund irgendeiner Art von Autoritätsglauben zu bilden (was jedoch, wie Sie wissen, nicht der Fall ist), so wäre dies doch (in diesem Fall wenigstens) logisch unmöglich da die Meinungen zweier Autoritäten einander hier so sehr widersprechen.[15]

Great scientists are also a *deontic authority* for scientists working in the same or a similar field but sometimes even for any scientists or educated people in general.

One important domain in which great scientists are deontic authorities for other scientists is that of advice and rules for scientific methodology. Examples: "Furthermore, among theories of equally 'simple' foundation that one is to be taken as superior which most sharply delimits the qualities of systems (i. e. contains the most definite claims)."[16] This is an advice that one should higher estimate

13 Pais (1982) ch. 16.

14 Cf. Broda (1986) p. 32ff.

15 Pauli (1979). The two authorities meant are Einstein and Bohr concerning the Bohr-Kramers-Slater Theory of the atoms which was refuted soon afterwards.

16 Einstein (1949) p. 23

that theory which is more informative, i. e. which has more epistemic information (recall ch. 5) compared to another theory which has less epistemic information.

The following advice is a warning against philosophical prejudices in science:

> The antipathy of these scholars [Ostwald and Mach] towards atomic theory can indubitably be traced back to their positivistic philosophical attitude. This is an interesting example of the fact that even scholars of audacious spirit and fine instinct can be obstructed in the interpretation of facts by philosophical prejudices.[17]

Another domain in which great scientists are deontic authorities for other scientists is that of an inspiring stimulants for doing further investigation and research. Examples: Erwin Schrödinger's book "What is life?" became an important stimulus for theoretical biology to understand life on the basis of the second law of thermodynamics, i. e. the law of entropy.[18] An interesting hint or stimulus which has been taken up by theoretical physics about 100 years later[19] is Boltzmann's doubt about the background-assumption of the continuity of time. The hint to look at the respective passage (quoted below) was given by Erwin Schrödinger to Engelbert Broda when he wrote his book about Boltzmann:

> For my taste there still lies an obscurity in the differential coefficient w. r. t. time … Except for a few cases … in order to construct a mapping to numbers time will be always thought to be divided into a finite number of time-slices before proceeding to the limit. Maybe our formulas are only the very approximate expression for average-values which could be constructed from more refined elements that are not differentiable. However there is no indication for that in experience so far.[20]

(5) Direct voluntary component in scientific belief: Analogously to what has been discussed in 11.31 (5), there seems to be a direct voluntary component also in scientific belief. That means that believing scientifically in a hypothesis implies—in many cases even if not in all cases—willing to believe in it.

Even before Einstein had found the right value for the deviation of light (because in his first calculation he did not incorporate the curvature of space) he wrote in March 1914 to Besso about his system of General Relativity: "Ich zwei-

17 Einstein (1949) p. 49.

18 Schrödinger (1944)

19 Investigations about discrete spacetime began with P. Alexandrow (1937) but were continued only much later with causal set-theory and spacetime quantisation: Kronheimer, Penrose (1967), Sorkin (1991), Markopoulou (1998), Meessen (1989, 2000).

20 Boltzmann (1904) p. 12, 26. Cf. Broda (1986) p. 48.

fle nicht mehr an der Richtigkeit des ganzen Systems, mag die Beobachtung der Sonnenfinsternis gelingen oder nicht."[21]

Not only the belief but also the disbelief can be supported voluntarily. When Pauli wrote to Bohr concerning the Bohr-Kramers-Slater theory, he stressed that he could not believe in it although there is no proof for either side: "Logisch beweisen kann man da nichts, und auch die vorliegenden Erfahrungsergebnisse reichen nicht aus, um für oder gegen Ihre Auffassung zu entscheiden."[22]

11.4 Answers to the Objections

11.41 (To 11.11:) As has been elaborated on in the proposed answer, every activity of the intellect such as knowing or believing involves an assent of the intellect. In the case of knowledge the intellect, assents because it is forced by immediate understanding (K1, K2) or after consideration (K3, K4) or by sufficient justification or corroboration (K5–K7). In the case of scientific belief, however, this is not the case in a sufficient way, such that a support of the will can be necessary for the assent of the intellect. Therefore: If the activity of the intellect is scientific belief then it may contain a voluntary component in support of the assent of the intellect. As has been elaborated on in 11.32, there are four different possibilities for such a volitive component, and it is very likely that in most scientific beliefs more than one of them are present.

11.42 (To 11.12:) Objective justification for scientific belief of hypotheses, laws, theories needs serious consideration, investigation and penetrating research for reasons. However, these scientific activities are subject to free choice and need the force of the will to be executed. Therefore, objective justification for scientific belief needs the support of the will and consequently the assent of the intellect when scientifically believing contains a voluntary component.

11.43 (To 11.13:) We may concede that in mathematical proofs there is no acceptance of an epistemic or deontic authority. We may further concede that there need not be a direct voluntary component in the sense of willing to believe in the proof or in its correctness. However, there certainly is a voluntary component in the choice of serious consideration w. r. t. the idea and method of the proof and the penetrating control of its steps.

21 Besso (1972) Letter from March 1914

22 Pauli (1979) Letter to Bohr of 1924.

11.44 (To 11.14:) The first premise of 11.41 is not correct. The statement describing the outcome of a scientific observation or experiment let's call it A, may be more or less dependent on the theoretical conjecture (hypothesis) or law; i. e. "based on…" may have different meanings. In the strongest sense, A is derived from a law together with initial conditions. In this case, A is a scientific prediction. Take as an example that a particular body possessing a certain mass attracts gravitationally a particular another body. This follows from Newton's law of gravitation which predicts gravitational attraction between any bodies. Newton believed this prediction, although it could not be tested within his lifetime, only 111 years later by Cavendish with his torsion balance. Nothing hinders however that there can be a voluntary component in Newton's belief of such a prediction. Independently there might also be a voluntary component in Newton's belief of his law of gravitation. We assume that the logical derivation of the prediction from the law plus initial condition is not a matter of belief but of knowledge by proof (K3). In this knowledge of a logically necessary connection between premises and conclusion, there need not be any voluntary component. That is, we might assume that the following schema of derivation is valid: $[aGp \wedge aK_3(p \rightarrow q)] \rightarrow aGq$

If the type of belief G contains a voluntary component then it is contained in both the premise aGp and the conclusion aGq. However, we can also consider the possibility that a scientific belief in laws is a different type of belief than a scientific belief in a statement which describes a singular event or the outcome of a particular observation or experiment. In this case, we have to distinguish two types of belief G_1 and G_2. And then G_2 could contain a voluntary component even if G_1 does not, and consequently the above schema of derivation would not be valid.

Summing up, we may say: Since the first premise of the argument in 11.14 is not generally true, the conclusion is not proved. Thus scientific belief may contain a voluntary component.

11.45 (To 11.15:) That, what is (scientifically or religiously) believed is something which is true or false, does not exclude a support of the will for the assent of the intellect. Therefore, there might be a voluntary component in religious belief as it has been elaborated on in a fivefold sense in section 11.3.1.

12 Whether Religious Belief Can Be a Kind of Knowledge

12.1 Arguments Pro

12.11 Every scientific approach concerning some domain D contains some kind of knowledge concerning that domain D. And consequently a scientific approach concerning religious belief contains some kind of knowledge concerning religious belief. A theology (of some religion) is a scientific approach concerning religious belief (of that religion).

Therefore: If theology is a science and consequently a scientific approach concerning religious belief then theology contains some kind of knowledge concerning religious belief.

As Thomas Aquinas says, theology—in his terminology the "Sacred Doctrine" (meant as the theology of the Christian Religion)—is a science: "I answer that, Sacred Doctrine is a science."[1] Therefore: According to Aquinas, theology contains some kind of knowledge concerning religious belief.

12.12 Religious belief can be a kind of knowledge:

> Things which are demonstrated are known; since a demonstration is a syllogism that produces knowledge. Now certain matters of faith have been demonstrated by the philosophers, such as the existence and unity of God, and so forth. Therefore things that are of faith can be known.[2]

12.13 According to Plantinga's approach in his "Warranted Christian Belief" faith is a kind of knowledge: "Faith is not to be contrasted with knowledge: faith (at least in paradigmatic instances) is knowledge, knowledge of a certain special kind."[3] Therefore, if Plantinga is right, religious belief can be a kind of knowledge.

12.14 If there is *knowledge* of God in the proper sense of that word then we need not speak about "religious belief". As Torrance says there is knowledge of God: "Knowledge of God is *knowledge* in the proper sense of that word. Here we are using that term *knowledge* formally in the same way in which we use it in

---

1 Thomas Aquinas (STh) I, 1, 2.

2 Thomas Aquinas (STh) II, II, 1, 5 objection 3.

3 Plantinga (2000) p. 256

https://doi.org/10.1515/9783110585797-012

every branch of true knowledge or scientia."[4] Therefore: If Torrence is right, what usually is called religious belief is knowledge in the proper sense.

12.15 If there are valid logical proofs of the existence of God then religious belief can be a kind of knowledge. There have been valid logical proofs of the existence of God in the tradition; moreover there exist such proofs which are written in precise, logical terms and of which the validity has been checked with rigorous methods, like Gödel's Ontological Proof.[5] Therefore, religious belief can be a kind of knowledge.

12.2 Arguments Contra

12.21 There are propositions which belong to religious belief that have to be believed equally by all. According to Thomas Aquinas, these propositions cannot be the object of science:

> … that which is proposed to be believed equally by all, is equally unknown by all as an object of science: such are the things which are of faith simply. Consequently, faith and science are not about the same things.[6]

Therefore: Religious belief (faith) cannot be a kind of knowledge.

12.22 All a posteriori arguments for the existence of God start from evident facts of experience. And there is no doubt about the truth of such facts and even of the truth that our universe exists. However, to conclude by an argument from such facts that God or the Creator of the universe exists cannot be valid according to Swinburne:

> It seems to me equally evident that no argument from any of such starting points to the existence of God is deductively valid. For if an argument from, for example, the existence of a complex physical universe to the existence of God were deductively valid, then it would be incoherent to assert that a complex physical universe exists and that God does not. There would be a hidden contradiction buried in such co-assertions… Now notoriously, attempts to derive obviously incoherent propositions from such co-assertions have failed through the commission of some elementary logical error.[7]

4 Torrance (1969) p. 12.

5 Gödel (1970) pp. 403–404. Further examples of such proofs are given in Szatkowski (2012), Swietorzecka (2015), Weingartner (2010).

6 Thomas Aquinas (STh) II–II, 1, 5.

7 Swinburne (1979) p. 119.

Therefore: If Swinburne is right, a posteriori arguments cannot turn a belief in the existence of God into knowledge since such arguments are deductively invalid.

12.3 Proposed Answer

To answer the question of ch. 12, we have to distinguish two different domains concerning religious belief. The first Domain 1 is that in which some kind of knowledge is possible. Concerning the second Domain 2 only belief is possible assuming normal human conditions. Independently of this division there can be a kind of knowledge concerning both domains in the sense that logic can be applied to both domains. This concerns conceptual clarification, interpretation of sentences as propositions or norms, logical relations among sentences such as logical consequences of belief-sentences, etc.:

(1) To reach knowledge concerning Domain 1 is more complicated: First, we have to observe that Domain 1 can be divided further into a domain of propositions stating the existence of some creator or cause for the world (universe), the spirituality of the human soul, the freedom of man's will, etc. And into a domain of norms regulating man's life as the "Ten Commandments". Secondly, it seems plain that w. r. t. the above propositions either no knowledge in the sense of K 1-K 4 is possible or it is insufficient, assuming normal human conditions. However, knowledge in the sense of K 5 (explanation and corroboration), K 6 (justified true belief) and K 7 (possessing epistemic entropy and epistemic information) may be possible. Thirdly, w. r. t. norms and their validity regulating man's life that are prescribed by religions—like the Ten Commandments—also knowledge in the sense of K 5, K 6 and K 7 may be possible.

(2) The second Domain 2 contains states of affairs that would not have come to man's mind if they would not have been revealed in scriptures. This domain can also be further subdivided:

(2a) Into those states of affairs which have been reported in scriptures in such a way that their historical content can—at least partially—be confirmed by historical sources outside the Bible. Concerning their historical content these states of affairs are accessible by knowledge in the sense of corroboration and explanation (K 5) and also in the sense of justified true belief (K 6) with respect to historical research. For example: David (governing ca. 1000–965 B. C.) and his regime and governmental power. Salomon (governing 965–926 B. C.) and his government. The birth of Jesus, his life with his disciples, his death and resurrection.

(2b) Into those states of affairs which have been revealed in scriptures but did not occur as historical events. For example: That Jesus was sent by God (although there are historical facts as his kind of life and his miracles which support that), that he came to save humankind, that there will be some continuation of life after death with reward or punishment, that Christ is present in the Eucharistic Sacrament, that God is *one* in nature and *three* in persons. These are things humankind would be ignorant of if God would not have made himself known through gracious disclosure as Polkinghorne says.[8] These states of affairs cannot be known by any kind of knowledge (K 1–K 7), assuming normal human conditions. The expression "assuming normal human conditions" means that we have not taken under consideration cases as saints, prophets or the apostles. Saints could have visions concerning (1a) and even (2 b) such that knowledge in the sense of K 4 or K 2 or K 6 is possible. Prophets could get orders to transmit norms for life to people such that they could have a very strong kind of K 6 concerning these norms. The apostles who saw Christ and his doing miracles could have knowledge of (1a) in the sense of K 4 and K 6. We can add persons who have been miraculously cured in places like Lourdes or Fatima, and those who watched such a process. All these cases are exceptional and do not satisfy normal human conditions.

Concerning Domain 1 of religious belief, it has been said already that we cannot have sufficient knowledge in the sense of K 1–K 4 in any domain of religious belief, assuming normal human conditions. An exception is the application of K 3 concerning interrelations and logical consequences of belief-statements. Such an application is possible in both domains 1 and 2. However, the belief-statement as a premise cannot be proved by K 3. Knowledge in the sense of K 5–K 7 is possible concerning Domain 1. We shall consider in detail one important case in Domain 1: The existence of a creator.

There are other important cases that have a great impact on religious belief: The spirituality of the soul and the freedom of man's will. However, these are accessible to scientific research at least to a great extent and are not cases of religious belief although they are important preliminaries to religious belief.

12.3.1 The Existence of a Creator

For the possibility of having a kind of knowledge of the existence of a Creator (or first cause), two main approaches have been proposed. One is by some kind of demonstration, the other is by "religious experience".[9] To begin with the latter, we

8 Polkinghorne (1996) p. 33.

9 For an account of "religious experience" see Hick (1993) ch. 2 and Weingartner (1994) pp. 161–182.

may distinguish two sorts. The first is neither unique nor specific nor restricted to one person. It is something like admiring the miracles of nature (recall section 10.3.2.2 above) and assuming and worshiping a creator for it. The second is a unique and specific experience restricted to one person, similar but perhaps weaker than in the case of saints and prophets or in the case of the apostles. In this case, knowledge (that there is a creator) is possible in the sense of K 4 and K 2 besides K 6. However, as has been said already these are exceptional cases at least in the sense that they are not accessible for everyone. In the first case mainly knowledge in the sense of K 6, K 5 and K 7 is possible. Here, K 6 and K 5 can be used also in a very rudimentary way and also by uneducated people.

The approach by some kind of demonstration for the existence of a creator or a first cause has a long tradition and goes back at least to the Greeks. Already Anaxagoras gave reasons for a spiritual cause of the world. Aristotle gave a demonstration for a first unmoved mover in book 8 of his physics. This tradition splits into two different ways. One way is from the creation or creatures to a creator or in another terminology from the effect (world) or from the effects (things and events of the world) to the cause of the world or its things and events. This way is usually called a posteriori. The other way is from a supposed definition of God or description of God[10] by his properties to his existence. This way is usually called a priori.[11] A more demanding form of the a posteriori way was first given by Aristotle (ch. 8 of his physics) and one of the a priori way was given first by Anselm of Canterbury (in his proslogion).

12.3.2 The A Posteriori Way

At the beginning of article 2 of questio 2[12], Thomas Aquinas distinguishes two types of demonstrations. (1) Those where we start from known or already proved premises in order to derive a conclusion which he calls “through the cause” and a

10 In a priori arguments the expression ‘God’ is used whereas in a posteriori arguments ‘first mover’, ‘first cause’, ‘necessary being’, ‘most perfect being’ etc. are used. The latter expressions are more modest since their equation with God in the sense of a special—say Christian—religion would need an extra justification. Aquinas is aware of this since he finishes his arguments in the five ways with a nominal (not real) definition saying this (the first cause) all c a l l God (in Latin: Quam omnes Deum nominat).

11 Today such arguments are called “Ontological Proofs” which is a very misleading name introduced by Kant for Anselm’s argument. Arguments from the world to its creator are at least as “ontological” as those which are mostly conceptual. Observe however that Anselm also starts with an empirical premise, cf. Weingartner (2012).

12 Thomas Aquinas (STh) I, 2, 2.

priori. (2) Those where we start from known or observed facts interpreting them as conclusions and search for premises as explanations which he calls "through the effect" and a posteriori. The second is the a posteriori way which he also uses in his Five Ways.

The a posteriori way starts with empirical premises. They describe contingent and observable events of this world as effects of some causes which might be also unobservable. Alternatively, they state the existence of the whole world as an effect of some unobservable first cause. There is a certain analogy in science to such an approach.

12.3.2.1 Analogy in Science

In 1795 Laplace postulated that a celestial body of sufficient size—because of the influence of gravitation on light—could not send light rays and would, therefore, be non-observable to us; i. e. he postulated the existence of black holes.[13] The star HDE 226868 having a mass of 30 times and a diameter of 25 times of that of the sun is revolved by an unobservable satellite having half of its mass and a diameter of only 50 km. That such an unobservable satellite must exist is sure, but about its structure and its properties, we have only consistent hypotheses which suggest that it must be a black hole. What can be measured and observed in the case of the big star HDE 226868 is that the big star is disturbed in its spherical shape by forming a top from which matter is sucked away in the direction of the (unobservable) black hole. It has to be noted that the kind of unobservability of the black hole is of a principle kind. Since to be observable light is needed and a black hole does not send out light and arriving light is swallowed up.

There are other examples where the kind of unobservability is only of a temporal or technical kind: the existence of a further planet (Neptune) was hypothesized as the cause for disturbing the orbit of Uranus and was found near the place predicted. Similarly with the Higgs-Boson. The similarities and differences are as follows:

13 The respective article of Laplace was published in an unknown journal. For his conjecture Laplace used Newton's law of gravitation and Newton's corpuscle-theory of light, where the tiny corpuscles are sensitive to gravitational attraction. An English translation of Laplace's article in included in Hawking and Ellis (1973), Appendix.

Similarities:
(1) Both demonstrations are from the existence of the effect to the existence of a cause
(2) The effects are observable and scientifically describable as the effects of some existent cause.
(3) The object which is the existing cause for the observable effects is unobservable.
(4) The kind of unobservability is not of a temporal or technical kind, but of a principle nature.
(5) The effects are part of this world (universe) or the whole world (universe).

Differences:
(1) The unobservable object which is the actually existing cause for the observable effects
    (a) belongs to this world (universe) or is an *immanent cause*.
    (b) does not belong to this world (universe) or is a *transcendent* cause.
(2) The observable and scientifically describable effects of the unobservable cause
    (a) are only part of this world (universe).
    (b) are both parts of this world (universe) and the world (universe) as a whole.

12.3.2.2 The A Posteriori Way as an *Explanation*

Keeping this analogy in mind, we shall show that the types of knowledge K 5, K 6, and K 7 can be used for the a posteriori way.

We can achieve knowledge as K 5 of a hypothesis by corroboration when their consequences turn out to be true by severe tests and by explanation when statements about empirical facts follow from this hypothesis (usually in connection with initial and boundary conditions). Thus the hypothesis "there exists a black hole as a revolving satellite of the star HDE 226868" can be known in the sense of K 5 by corroboration and explanation. In order to fulfill the explanatory function, the unobservable black hole has to have some very specific properties (for example a huge mass compressed to a small size). In order to fulfill the corroborating function, observations of the behavior of HDE 226868 have to be interpretable as consequences of the hypothesis together with some initial and boundary conditions.

In an analogous way, the hypothesis "God as the creator of our universe exists" can be known in the sense of K 5 by explanation and corroboration. In order to fulfil the explanatory function God has to have some very specific properties; for example to be very powerful, best *omnipotent*, to be able to create such a huge and complicated universe and to know so many difficult things, best being *omniscient*, in order to create this sophisticated universe; further to be an extremely powerful spirit to be able to create the human spiritual soul, best being an infinite spirit and therefore not-embodied since bodies are finite. In order to fulfil the corroborating function descriptions of the universe and its parts have to be interpretable as consequences of the action of an almighty and omniscient creator on the one hand; and since man with the help of science discovers always new admirable things of this universe, these corroborate and confirm the existence of such a creator on the other.

Here a further analogy can be observed between scientific and religious hypotheses. Scientific hypotheses and theories can be ordered w. r. t. depth. For example, Electromagnetic Optics is deeper than Ray Optics; or Quantum Mechanics is deeper than Classic Kinematics; or General Relativity is deeper than Newton's theory of Gravitation; or Nonequilibrium Statistical Mechanic is deeper than Classical Thermodynamics; or Synthetic Evolution Theory is deeper than Darwin's Theory Evolution. These differences may be comprised by the following definition:

12 D 1 H 1 is *deeper* than H 2 iff the following conditions (i)–(iii) are satisfied:
- (i) H 1 includes higher-level constructs or higher-level unobservables,
- (ii) these constructs and unobservables occur in hypotheses which explain facts referred to by H 2,
- (iii) H 1 logically explains H 2, i. e. H 2 follows logically from H 1.[14]

In a similar way, religious beliefs or religious hypotheses belonging to Domain 1 (cf. 12.3 above) can be ordered w. r. t. depth: "There exists an omnipotent, omniscient, perfect, benevolent creator of the universe, being a person[15] who is called God" (H 1), is deeper than: "there is a first mover" or "there is a first cause" or "there is a necessary being" (H 2).

"Every human person has a spiritual, non-material immortal soul (self) unified with its body (including brain), causally interacting with its body, bearer of psychic actions (like perceiving, imagining, asserting, feeling, willing… etc.) which are intentional and non-spatial" (H 1), is deeper than: "Every human person has psychic actions" (H 2).

Also here the first-religious hypothesis H 1 is deeper than the second H 2 by definition 12 D 1: For example *omnipotent* is a higher-level construct and a higher-level unobservable than *first mover*. Alternatively, *spiritual*, *non-material*, *immortal*

14 Cf. the definition in Bunge (1967) I, p. 508.

15 These properties can be understood in the sense as Swinburne (1979) describes them roughly on page 8; except I do not agree that God is in time since time belongs to our universe, i. e. his creation and not to him. For this problem and a detailed account of omniscience see Weingartner (2008).

soul (as the bearer of psychic actions) is a higher-level construct and a higher-level unobservable than *psychic action*.

Further, these higher-level constructs and unobservables may occur in religious hypotheses which explain facts referred to by H 2, for example, that the world is a moving and changing being or that we humans have psychic actions. Thus H 1 explains H 2 and if more accurately formulated H 2 may logically follow from H 1.

12.3.2.3 Summarizing the Gist of the A Posteriori Demonstration

(1) It is a demonstration which uses as premises two kinds of empirical facts, represented by propositions as premises:

(a) observable empirical facts about events and the states of this world (universe) and empirical facts about its laws[16]

(b) the empirical fact that the world (universe)as a whole exists.

(2) It is a demonstration which uses the methodological principle of scientific explanation: to search for causes as explaining observable, empirical facts as effects. This principle of scientific explanation is as old as science itself. It has been known and used from Antiquity (Greek Science) over the Middle Ages to the Enlightenment and today's science.

(3) It is a demonstration which has the structure of a scientific explanation. However, in some scientific explanations the effects are not yet observable, but they are described by predictions as events to happen in the future. In this case the causes are observable events which have been observed in the past. Such an explanation which connects past with future by laws of nature (dynamical laws), corresponds to the first type of demonstration distinguished by Thomas Aquinas. In other scientific explanations, the observable events are the effects which have been observed in the past up to the present. In this case, the causes may be observable events or objects, or even unobservable ones, though described to some extent (not completely) with the help of the laws. This latter type is the one that corresponds to the second type of demonstration described by Thomas Aquinas. In this type of scientific explanation, the conclusion describes known empirical facts as the effects of some causes. These causes are twofold (a) and (b_1) or (b_2):

16 Such facts about the laws of this world (universe) are for example that they are invariance or symmetry principles of different kinds or that the laws do not change in time (of this world). For more details see Mittelstaedt/Weingartner (2005), chs. 5 and 6.

(a) A general law which connects effects with causes. Such laws may differ from domain to domain, but a minimal common feature of their explanatory function is this implication: if the effect exists, then some suitable cause must pre-exist; i. e. the causality relation expressed by the law is such that the causes are necessary conditions. Recall the example of star HDE226868.

(b_1) If the effect is an observable event or thing of this world (for example that matter is sucked away or that a new individual comes into existence) or, more generally, an event in a later section of the past light cone (say between t_1 and t_0 (present) where $t_1 < t_0$), then the proper or efficient cause (*causa efficiens*) is an earlier event or thing (observable or not yet observable or never observable) or, more generally an event in earlier sections ($t_2 < t_1$) of the past light cone. In this case, both events or things, as effects and as causes, are events or things of this world (universe).

(b_2) If the effect is the existence of the whole world (universe), the proper or efficient cause cannot be of this world (universe), it must be "outside" or "transcendent" of this world (universe). Point (b_2) is well expressed by passages of Wittgenstein's *Tractatus*:

> The sense of the world must lie outside the world. In the world everything is as it is and happens as it does happen… For all happening and being-so is accidental. What makes it non-accidental cannot lie *in* the world, for otherwise this would again be accidental. It must lie outside the world.[17]
>
> So people stop short at natural laws as at something unassailable, as did the ancients at God and Fate. And they both are right and wrong. But the ancients were clearer, in so far as they recognized one clear terminus, whereas the modern system makes it appear as though everything were explained.[18]

12.3.3 Logical Proofs of A Posteriori Ways

The famous a posteriori ways proposed in the history of philosophy started with empirical contingent premises. As law-like principles, they used metaphysical principles and definitions as the second premises. From both, they derived the conclusion. As an example, we take the Second Way of Thomas Aquinas:

Empirical contingent premises:

(1) Something causes efficiently something else.

17 Wittgenstein (1960, TLP) 6.41.

18 Ibid. 6.372.

(2) There is no case in which a thing is the efficient cause of itself, for then it would be prior to itself which is impossible.[19]
(3) To take away the cause is to take away the effect.[20]

Definition of the first cause:
(4) The first cause is uncaused and is the cause of the intermediate cause (be it several or only one) which causes the ultimate effect.

Metaphysical principle:
(5) In efficient causes, it is not possible to go on to infinity.
(6) Empirical justification: If it is possible to go on to infinity there will be no first cause and neither an ultimate effect nor an intermediate cause. However, this is empirically impossible.

Conclusion:
(7) Therefore there exists a first cause.
(8) To this everyone gives the name of God.[21]

There are also contemporary a posteriori ways. For example Meixner's Cosmo-Ontological Argument.[22]

In the mentioned references it is shown that these arguments are logically valid.[23] Thus the question remains about the truth of the premises. Are some or all premises evident? According to Aquinas, "a proposition is evident because the predicate is included in the essence of the subject, as 'Man is an animal', for animal is contained in the essence of man."[24]

---

19 Thomas Aquinas presupposes here (STh) I, 2, 3 that every causal propagation needs time, an empirical fact which we know today from experience and from the Theory of Relativity. With this fact, he proves the irreflexivity of the efficient cause.
20 This unambiguously interprets *cause* as *necessary condition*. Efficient cause should therefore not be interpreted as *sufficient* condition. This premise is not only empirical but also conceptual.
21 For a formulation in First Order Predicate Logic (PL1) and a formalized proof see Weingartner (2010) p. 62 f. There the other four ways of Aquinas are treated in PL1 too. For other proofs of Aquina's Five Ways see Bochenski (2000) and Niezmanski (2003).
Bochenski thinks that the first and the fourth way of Aquinas are not correct proofs because of false (physical and other) empirical premises. This is however not correct as is shown in Weingartner (2010), p. 57f and 89f.
22 Meixner (2012)
23 In contradistinction to what Swinburne claims too generally and wrongly (cf. 12.22 and 12.46).
24 Thomas Aquinas STh I, 2,1. For an exposition of this definition with the help of modern logic cf. Weingartner (2010, p. 9ff.)

Looking at the premises of Aquinas' second way we may say that premise (2) is self-evident and so is (3) even if (3) selects a special type of causes, i. e. cause as necessary condition. (1) is an acceptable empirical premise known by knowledge K 4 (observation and experiment). It has experimental evidence. (4) is a definition, though not just a convenient abbreviation. Premise (5) stops an infinite regress of causes. This premise is not self-evident. Moreover, it also seems not self-evident in the sense of Aquinas. However, it can be supported quite well even by today's science. Also premise (6) can be supported in this way, which is as follows:

(i) Thomas Aquinas presupposes that the universe is finite; i. e. that the universe as God's creation is finite, and that also the number of creatures is finite. One of his reasons among others is that a body, a material thing or some whole consisting of material substances, cannot be spatially infinite. Since the material parts of the whole (universe) are all finite in extension and finite in number. This assumption is in good accordance with the Theory of General Relativity. And since this theory is scientifically very well confirmed, the assumption that the universe is spatially finite is much more probable than its opposite. More specifically: all the Friedmann solutions of the Field Equations of General Relativity are finite solutions in the sense that they describe a finite universe U such that: U is spatially closed and finite and the time-like curves of U are not closed, but also finite.

(ii) Thomas Aquinas also assumes that the universe has a finite age. Still, he quite critically points out that a beginning of the universe, a finite time ago, cannot be proved in a strong scientific way; that means it cannot be proved with the help of laws (of nature) because universal laws abstract from *hic* (*here*, space) and *nunc* (*now*, time). In modern terms: from laws of nature we cannot derive a singularity. Moreover, this second assumption is confirmed quite well experimentally by the cosmic background radiation discovered in 1965 by Penzias and Wilson. And it is confirmed theoretically by the so-called Singularity Theorems of Penrose and Hawkins (cf. 10.3.1). Therefore this assumption of a universe with a finite age, i. e. with a beginning, is also more probable than the opposite one; the latter is defended by some mathematical models of cosmology which have, however, no experimental confirmation so far (cf. 10.3.1).

(iii) Concerning the causal relation, Thomas Aquinas defended that it is irreflexive, that within the universe the cause is always earlier than the effect, that it has to be interpreted as a necessary condition and that it is (as a necessary condition) transitive. These defended properties of the causal relation are also very well confirmed today. For example, the causal relation which connects events from the past light cone with those of the future light cone is—according to the Theory of Special Relativity—irreflexive and transitive; moreover, any causal propagation needs time (its speed limit is the velocity of light in vacuum) such that the cause must be earlier than the effect. It also holds that the events in the past light cone are necessary conditions for the events in the future light cone; i. e. if those in the past light cone would not occur (exist), those in the future light cone could not occur (exist) either. This shows that there is a well confirmed scientific model of contemporary physics (i. e. that of the Theory of Special Relativity) for the kind of causality Thomas Aquinas uses in the Second Way, and also in the First Way.[25]

(iv) The part of Thomas Aquinas' Second Way which stops the regress is now the following: If it holds for two-place relation R
   (a) it is true between at least two objects x and y: Rxy
   (b) it is irreflexive: $\forall x \neg Rxx$
   (c) it is transitive: $\forall xyz(Rxy \wedge Ryz \rightarrow Rxz)$
   (d) its domain is finite

   then there is something z to which no thing u stands in this relation R : $(\exists z \neg \exists u)Ruz$. This is a principle which is satisfied in every finite domain. Instantiating R by the causal relation C (Cxy) and interpreting Cxy as x is a cause (as a necessary condition) of y (for short: $CNxy$) the principle reads thus: If there is an object x and an object y such that x causes (as a necessary condition) y ($CNxy$) and if CN is irreflexive and transitive and defined on a finite domain, then there is some object z which is not caused by any other object u, i. e $\exists z \forall u \neg CNuz$. In other words the conclusion is: there is an entity which is uncaused.

   If the real objects and the real events, states and processes based on these objects are finite, then an infinite causal process cannot exist as a real process. Since every causal process is such that state (or event) S_1 is a cause for state (or event) S_2 or more completely—in case of the application of the Theory of Special Relativity—the states of the past light cone are causes for the states in the future light cone. However, all these states as causes and as effects are finite in number.

The reasons elaborated in (i)–(iv) above have often been neglected in discussions of the "infinite regress" concerning proofs of the existence if God. There are however some notable exceptions like Salamucha (2003), pp. 113f., Kenny (1969), Bochenski (2000) and very detailed Nieznanski (1980). Many discussions concerning the

25 Thomas Aquinas did not claim universality for this type of causal relation which he uses in the Second and also in the First Way (here w. r. t. the application to celestial bodies where one moves the other). He also accepts other types of causes as his commentary to the four causes of Aristotle shows. Cf. Thomas Aquinas (CMA), p. 120–133.

"infinite regress" are rather superficial and so short that one does not have the impression that the underlying reasons are understood.[26] The above reasons show however that Thomas Aquinas was very well justified in rejecting an infinite regress of causes.

12.3.4 The A Priori Way

The tradition of the a priori way, if it is understood in a more broad sense, is as old as the a posteriori way. It starts from a supposed definition or from a description of God by his properties, more or less independent from the world as a creation of God. Platon (Pol, 380 df.) characterizes God as most perfect and demonstrates that consequently he must be immutable. Augustine defines God as the one than which nothing greater exists (De lib. arb. II, 6, 14, ML 32). Whereas the first demanding demonstration of an a posteriori way was already given by Aristotle in book 8 of his physics, the first demanding demonstration of an a priori way was only given about 1400 years later by Anselm of Canterbury.

(1) Anselm's Argument (Proslogion ch. 2): Contrary to a widespread opinion Anselm's argument is not purely a priori or conceptual. It starts with the empirical premise that "we believe that You are *something than which no greater can be conceived*"[27] and continues that even the fool, i. e. the non-believer understands the word QM and so what both, believers and non-believers, understand exists in their mind. All these are empirical premises.[28] Only then begins the second part of Anselm's argument to show by reductio ad absurdum that QM cannot only exist in the mind but must also exist in reality.

---

26 A recent example can be found on the pages 100–101 of R. Dawkins' (2006). Also, Dawkin's concern with "the larger problem of who designed the designer" (i. e. with an infinite regress of designers, p. 188) shows that he did not consider such reasons at all. How "the main conclusion" of his book, that God almost certainly does not exist (p. 189) follows from his "central argument" of his book (summarized p. 188f.) is—at least from a logical point of view (i. e. where the premises should be sufficient to derive the conclusion by logically valid deduction rules)—a complete *non-sequitur*. Dawkins makes fun of the arguments of Thomas Aquinas (p. 102), but these arguments have a logical structure and can be brought into logically (deductively) valid arguments (as has been shown by Salamucha, Bochenski, Essler, Nieznanski and Weingartner (2010). In contradistinction Dawkins' "central argument" and many others in the book are formulated in a rather journalistic and polemical style such that it will be hard to find some logical structure at all. For a detailed criticism cf. Anglberger et al. (2010) p. 181–197.
27 In Latin: id quo maius cogitare non potest (QM).
28 For a detailed examination of this point and a defense that incorporating an empirical premise of that sort into the demonstration suggests to apply intuitionistic logic, see Weingartner (2012).

One problem in Anselm's argument is that 'QM' is used in two meanings: first *without* existential import such that QM as it is understood (by believers and non-believers) exists in their minds and as such is sufficient for QM. Second, *with* existential import when it is assumed that a QM existing in the mind but not, in reality, is not a QM.

This point has been criticized by several philosophers in the tradition, notably by Thomas Aquinas[29] and Franz Brentano. Brentano expresses the point in a more general way concerning an ambiguity of the copula is; and this is also underlying Anselm's argument:

(1) Whatever is an infinitely perfect being exists necessarily.
(2) God is an infinitely perfect being.
(3) Therefore: God exists necessarily.

The second premise contains the ambiguity: If the 'is' has existential import, the argument is a petitio principii. If the 'is' concerns just the meaning of the word 'God' then (2) says: By *God* we understand an infinitely perfect being. However, in this case, the consequence is only: By *God* we understand a being which exists necessarily.[30]

Where the ambiguity is hidden also depends on the type of reconstruction of the argument. In an accurate reconstruction with the help of the theory of description of Principia Mathematica, Morscher has shown that the purported contradiction which is used by Anselm for the reductio ad absurdum, is not a real contradiction.[31]

There is still great interest for Anselm's argument in contemporary philosophy especially in the domain of analytic philosophy.[32]

(2) Gödel's Argument: The most famous a priori argument of contemporary philosophy is that of Kurt Gödel, called by him "Ontologischer Beweis" dated from February 10, 1970.[33] It has been shown that under several reconstructions Gödel's "Ontologischer Beweis" (GOB) is a logically valid argument.[34] What remains to be considered are the premises (axioms and definitions). Gödel's main idea about God is that God possesses all positive properties. This idea reminds us immediately of Leibniz and Kant.

29 (STh) I, 2, 1 ad 2.
30 Brentano (1929), p. 44.
31 Morscher (1991)
32 Cf. Szatkowski (2012)
33 Gödel (1970)
34 Anderson (1990), Czermak (2002), Fitting (2002), Benzmüller (2013), (2014)

According to Leibniz, God has metaphysical perfection where perfection $=_{df}$ the magnitude of positive reality without limits.[35]. According to Kant, God is the sum total of all possible predicates (omnitudo realitatis)[36], where *all possible predicates* are selected by the *Principle of Complete Determination*: of all pairs of opposite predicates one must belong to a thing.[37] Whereas Leibniz definitely stresses the *positive* reality, Kant seems to think that possessing all possible predicates means some positive maximum.

The axioms of GOB:
Ax 1 The conjunction of any two positive properties is positive.
Ax 2 For any property ϕ: if ϕ is positive—P(ϕ)— then the opposite (complement) of ϕ is not positive.[38]
Ax 2a If ϕ is positive then necessarily so.
Ax 2b If ϕ is negative then necessarily so.
Ax 3 Necessary existence is positive.
Ax 4 Any property which is necessarily implied by a positive property, is also positive.

These axioms seem uncontroversial and intuitively plausible. The harder problem are Gödel's definitions.

The definitions of GOB: GOB uses 3 definitions of which the first two are most important.

D 1 $Gx \leftrightarrow \forall\phi(P(\phi) \rightarrow \phi x)$

This definition is too liberal since contingent properties could be instantiated for ϕ and this is against Gödel's intentions: "Positive means positive in the moral, aesthetic sense (*independently* of the *accidental* structure of the world). Only then the ax. are true."[39]

This idea seems to be best represented by Czermak's T-restriction: Only those properties are meant for which it holds: they belong possibly to a subject iff they belong necessarily to that subject. Such properties are not accidental:

35 Monadology 45 and GP 7, p. 195
36 KRV, B604
37 KRV, B600
38 This is already the weakened axiom 2, corrected by Gödel. Cf. Gödel (1970), p. 435. The original one (ϕ or its complement is positive) is too strong and leads to difficulties as Anderson showed.
39 Gödel (1970), p. 403.

$T\phi \leftrightarrow \forall x(\Diamond\phi x \leftrightarrow \Box\phi x)$

D 1 $Gx \leftrightarrow \forall\phi[T\phi \rightarrow (P(\phi) \rightarrow \phi x)]$ (Czermak)[40]
D 2 $\phi Essx \leftrightarrow \phi x \wedge \forall\psi(\psi x \rightarrow \Box\forall y(\phi y \rightarrow \psi y))$

This definition is also too liberal. Instantiating for x Socrates, for ϕ to be a man (rational animal) and for ψ to have brown eyes makes the definiendum true, and the definiens false and this makes the definition false. Anderson's restriction is helpful to meet this counterexample.[41]

D 2 $\phi\, Ess\, x \leftrightarrow \forall\psi(\Box\psi x \leftrightarrow \forall y(\phi y \rightarrow \psi y))$ (Anderson)

However, the following counterexample makes Anderson's definition false: Instantiating for ϕ morally good makes $\phi\, Ess\, x$ true and instantiating for ψ being almighty makes the definiens false since almightiness does not follow from being morally good.

D 3 $Ex \leftrightarrow \forall\phi(\phi\, Ess\, x \rightarrow \Box\exists y\phi y)$[42]

From the axioms Ax 1–Ax 4 and the definitions D 1, D 2, D 3 one can derive the main theorem $\Box\exists xGx$—necessarily God exists—on the basis of the modal system S5 with the help of its axiom $\Diamond\Box A \rightarrow \Box A$. However, one can also circumvent this rather problematic axiom of S5 by using the much weaker modal system T plus the Barcan-Formula of First Order and de-re modalities.[43] Since the definitions of *essence* make special difficulties, Czermak has given a version without using *essence*[44]: As preliminary theses, he uses the theorems of Classical Propositional Logic with the usual rules of MP and universal generalization and a restricted necessitation. The underlying modal system is S 4. The proper axioms are:
1. Instantiation of Comprehension for property ϕ.
2. D 1 (Czermak) necessitated (see above).
3. Definition of $T\phi$ necessitated (see above).
4. Ax 4 necessitated.
5. To be God is positive.
6. $P\phi \rightarrow T\phi$

40 Czermak (2010), p. 253, D 13
41 Anderson (1990)
42 Ex… x exists necessarily
43 Czermak (2010), p. 254
44 Czermak (2015), p. 61ff.

In addition, one of the following two principles are used: (i) Leibniz's Principle "If it is possible that God exists then it is necessary that God exists", or (ii) Special Barcan Formula "If it is possible that God exists then there exists someone who is possibly God".

From these preliminary theses and the proper axioms and one of the above principles, one can derive the main-theorem of Gödel's Ontological Proof: Necessarily God exists. This version of Gödel's Ontological Argument seems to be the most simple, valid argument which also uses as few assumptions as possible.

12.3.5 Summarizing the A Posteriori and the A Priori Way

The gist from describing both ways, the a posteriori way and the a priori way, can be summarized as follows:

1. There are logically valid proofs with the conclusion: there exists a first cause, there exists a necessary being, there exists God, etc.
2. These conclusions do not claim that the *first cause*, the *necessary being*, this *God* is identical with the God of the Bible, i. e. the God of Judaism or the God of Christianity, or the God of the Koran. Most of what can be said here is: "this is what all *call* God", or "this everyone understands to be God" as Thomas Aquinas modestly formulates at the end of each of his *Five Ways*.[45]
3. Both types of proofs go beyond logic in the sense that they do not exclusively contain principles of logic as their premises. This is clear concerning the a posteriori way, but it has sometimes been claimed wrongly for the a priori way (cf. 12.3.4, 1.).
4. Proofs of the a posteriori way do not only contain empirical premises—besides logical principles—but also very general principles which are partially empirical, partially metaphysical (for example: stopping an infinite regress of causes) and principles which are partially empirical and partially conceptual (for example: the irreflexivity of the causal relation).
5. Proofs of the a priori way do not either contain exclusively logical principles as their premises. They contain a definition of God (Anselm, Gödel), other definitions (for example of essence) and axioms which are metaphysical theses (for example: necessary existence is positive). Furthermore, they may even contain empirical premises like in the beginning of Anselm's Argument (Proslogion, ch. 2) where he says that everyone, even the fool understands by God an *id quo maius cogitare non potest* and what he understands is in his mind.

45 Cf. Weingartner (2010), p. 51 and Siegwart (1991).

6. A further important point is the logical system in which the proof is carried out. In the case of the a posteriori way it is usually First Order Predicate Logic and therefore not burdened with special problems. For a priori ways, stronger logical tools are necessary in most cases. For example, the modal system S 5, or at least the Barcan-Formula, further an abstraction—operator (λ), i. e. some degree of comprehension, Second Order Logic or at least many-sorted First Order Logic. It is a difficult question how to justify such tools.[46]

12.4 Answer to the Arguments

12.41 (To 12.11:) It is correct as said in the argument that if theology is a scientific approach concerning religious belief, then theology contains some kind of knowledge concerning religious belief. To clarify that point, we have first to give a demarcation between theology and other approaches which are concerned with similar questions concerning things that transcend the world. Such approaches are mainly philosophy and "Weltanschauung" or world-outlook. We agree here with Thomas Aquinas that as long as the argumentation uses only "profane" premises we are in the domain of philosophy or "Weltanschauung". However, as soon the argumentation uses in addition premises from the Bible (or more generally from religious scriptures or texts) we are in the domain of theology. This criterion is not yet sharp enough; what is meant are not historical facts reported in the Bible (or religious texts) since as historical facts they belong first to the domain of history, although they may be interpreted also as a religious context related to the object of religion (God) and man's salvation. If the argumentation contains (in addition to profane premises) premises of specific theological kind as those listed in section 12.3(2b) then it belongs to the domain of theology. Having this demarcation in mind, the kind of knowledge concerning religious belief in theology can be of three sorts:

(1) There can be knowledge in the sense of K 3 if principles of logic are used in the argumentation, and logical relations among statements of religious belief are described.

(2) Concerning the reports of the apostles and evangelists in the New Testament that have been critically examined by historical and exegetical investigation, we have to distinguish two possibilities: Belief in reported historical facts can be

46 Cf. the discussion in Czermak (2015), p. 67ff.

justified true belief in the sense of K 6, i. e. knowledge of this kind. On the other hand belief in states of affairs of the sort, 12.3(2b) can never be knowledge (in this life) although it might be justified belief for the believer and it may even, *in fact*, be justified true belief. In the latter case, there is a similarity between the cases in science where we have justified true belief but not (yet) knowledge by proof or experimental confirmation (recall 4.12 and 4.13) and cases of religious belief but not knowledge in this life which will turn into knowledge in the later life. To mention another analogy: What some scientists believe, but do not know, others, i. e. their colleagues, might know since they have scientifically investigated and proved these facts. Similarly, what religious believers in this world believe but do not know, others, i. e. the blessed, might know.[47]

In his first *questio* of his Summa Theologica Aquinas describes what kind of scientific approach theology (*sacra doctrina* in his terminology) is. His view of science is the view of Aristotle's Posterior Analytics, i. e. science is, in its optimal form, an axiomatic system depending on first true principles (axioms).[48] Parts of the axioms and parts of their consequences are revealed in the scriptures for the salvation of man. On the other hand, the truths of that science are known in the sense of an ordered scientific system by the blessed (and by God). However, they are neither known by religious believers nor by theologians although some of them are partially accessible by hard work for theologians. Thus theology is like a "secondary science" that constructs its scientific system by taking its principles from a higher science, i. e. the science (or scientific system) known by the blessed:

> … thus the science of perspective proceeds from principles established by geometry, and music from principles established by arithmetic. So it is that sacred doctrine is a science, because it proceeds from principles established by the light of a higher science, namely the science of God and of the blessed.[49]

The idea of Thomas Aquinas, that there can be a science in the sense of an axiomatic system built up from axioms of which we have no knowledge is also held by Leibniz with respect to the domains of physics, ethics, and jurisprudence: in these domains man can build up scientific systems *more geometrico* (i. e. axiomatic systems) which are neither complete (that is we cannot deduce all true sentences of the

47 Cf. Thomas Aquinas STh I, 1, 2. Weingartner (1992)
48 Cf. Bochenski (1991). Although Aquinas accepts also principles which hold for "most cases" in the application, for example in ethics, i. e. in modern terms *statistical laws*. Cf. Aristotle Met 1026b31, Thomas Aquinas STh I–II, 94, 5 and Weingartner (1997).
49 Thomas Aquinas STh I, 1, 2. For an interpretation cf. Weingartner (1992).

respective domain in finitely many steps) nor can we know all the general principles (axioms).[50]

Moreover, this view comes very close to present science. The very general laws of nature (as the first principles or axioms) that we use in science are *not known* in the strong sense of knowledge satisfying the principle $Kp \rightarrow p$, but they can be known in the sense of K 5 as good approximations to the true laws which can be corroborated (not verified) by severely testing their consequences.

(3) There is a third sense in which theology can be viewed as a scientific approach that contains some kind of knowledge. In any science, we can distinguish the universal statements representing the hypothesis, laws, and theories from the singular statements representing particular states of affairs restricted by a certain space-time point. We speak of a scientific approach based on scientific methodology if there is explanation and corroboration among these statements: The singular statements representing particular states of affairs are *explained* by the universal ones since they can be derived from them with the help of additional singular assumptions. On the other hand, the universal statements can be *corroborated* or *criticized* by testing their particular consequences. This structure of explanation, corroboration, and criticism is also present in theology—at least in theology as it is investigated and thought at universities.

Thus in the moral domain particular obligations can be explained and deduced from universal principles like the Ten Commandments. Moreover, the evidence of particular obligations in the application to special cases corroborate the universal principles on the one hand or criticize and restrict these principles by introducing conditions when applied to specific subdomains on the other. In this case of the moral domain, the universal principles (the Ten Commandments) occur already in the religious text which is understood to be revealed.

In other domains of the scriptures, only particular statements occur dispersed in the text. Thus particular statements of God's mercy are dispersed in the scriptures (PS 107,1; 136; Lk 1, 50; 9, 56; 15, 11ff; Joh 3, 17; 12, 47; 2 Cor 1,3.; 2 Petr 3, 9) and illustrated with examples (lost son, lost sheep etc.). A theological theory about God's mercy has to be first *compatible* with these cases on record (assuming that they have been already critically investigated by exegesis) and then has to provide an explanation for them in general terms. On the other hand, if one or more of the biblical records were incompatible with the theological theory then this theory

50 The reason for Leibniz is that in these domains contingent proposition have to be used. Logic, mathematics, and metaphysics can be built up *more geometrico* as complete systems according to him. Today we know that this is only possible for a part of logic (First Order Logic) and neither for mathematics nor metaphysics.

is criticized and restricted or even refuted. In a similar way theological theories about the resurrection, life after death, the Eucharist, and original sin, etc. are used as explanations for dispersed biblical records and can be corroborated and criticized *relative* to these records. Like scientific hypotheses, laws and theories can explain observational data and experimental results and can be corroborated or criticized or even refuted by them, in a similar way theological theories can explain dispersed biblical records and may be corroborated, criticized or refuted by them. The basis (in Popper's sense represented by basic statements) from which scientific hypotheses and theories are corroborated or criticized are to a great extent direct or indirect observation or experiment that are scientifically interpreted in the light of other presupposed and well corroborated laws and theories. Concerning this basis we can speak of *knowledge* in the sense of K 4. The basis from which theological theories are corroborated or criticized are to a great extent biblical records that are exegetically interpreted in the light of linguistic proficiency, of historical and archaeological investigations (including chemical C^{14} methods) and of other biblical records. Concerning this basis, we can speak of *justified religious belief*.[51]

12.42 (To 12.12:) Concerning the quotation "Now certain matters of faith have been demonstrated by the philosophers…" we may distinguish two possibilities. If the demonstration concerns logical relations among propositions of faith, for example, that one logically follows from another then such demonstrations are knowledge in the sense of proof (K 3). If the demonstration concerns the existence of God, then the question of the impact of kinds of knowledge has been elaborated in detail in sections 12.3.1–12.3.4 and was summarized in section 12.35. Concerning the demonstration of the unity of God, Aquinas uses empirical, conceptual, metaphysical and theological premises. Therefore the answer concerning the impact of (different kinds of) knowledge is similar to that given concerning the existence of God (of a creator) in sections 12.3.1–12.3.5.

12.43 (To 12.13:) As it has been elaborated in 12.3 we have to distinguish different domains concerning religious belief. Concerning domain (1a) i. e. questions of the existence of a creator, of the spirituality of the human soul or the freedom of man's will we cannot have knowledge in the sense of K 1–K 4. However, knowledge in the sense of explanation and corroboration (K 5), in the sense of justified true belief (K 6) and in the sense of possessing epistemic entropy and information (K 7) is

51 This view of theology as a scientific approach has been proposed by Bochenski (1965) section 20 and by Weingartner (1978) section 6.44.

possible. In this sense, Plantinga's claim "faith is knowledge … of a certain special kind" can be accepted. There is, however, the question whether Plantinga's "faith" as *warranted Christian belief* has this meaning. Since if Plantinga has in mind his *basic beliefs* which are not conclusions of arguments then the possibility to have knowledge in the sense of K 5, K 6, K 7 for domain (1a) is not Plantinga's view because K 5, K 6, K 7 are argumentative: "In the model, the beliefs constituting faith are typically taken as basic; that is, they are not accepted by way of argument from other propositions or on the evidential basis of other propositions."[52]

The only types of knowledge in our list of seven types which are immediate and basic and not argumentative are K 1 K 2 and K 4 (1), i. e. direct observation. Moreover (1a) cannot be the knowledge of one's own existence. The only possibility is that by some inner experience similar to introspection someone could receive some insight concerning religious states of affairs belonging to domain (1a). This is—as we have said—possible but not under normal human conditions (recall 12.3).

Therefore, concerning domain (1a) of religious belief, there can be knowledge at most in the sense of K 5, K 6, and K 7. Observe however that concerning K 5, K 6, and K 7 the principle $Kp \rightarrow p$ is not generally presupposed (recall ch. 6); this principle is only guaranteed for K 1–K 4. This does not rule out however that concerning domain (1a) there can be "basic" religious belief that is very strong, is immediate, evident, is not a part of an argument, etc. The main point is nevertheless that it is *belief* and *not knowledge*.

Concerning domain (2b)[53], i. e. things humankind would be ignorant of if God would not have revealed them, like that Jesus was sent by God, that he came to save humankind, that there will be some continuation of life after death with reward or punishment, etc., there cannot be any kind of knowledge K 1–K 7 in this life, under normal conditions. Concerning (2b), we can have religious belief or faith but not knowledge in this life.

Again, that does not rule out that there can be "basic" religious belief concerning domain (2b) that is very strong evident, immediate, without being part of an argument, intuitive, etc. However, it is *belief* and it cannot be knowledge or develop into knowledge in this life—assuming normal conditions. Although what is believed in this life can develop into knowledge in the life to come (after death).

12.44 (To 12.14:) Most of what has been said in the answer 12.43 to 12.13 (Plantinga) applies also to 12.14 (Torrance). In our view it is *impossible* that religious belief

52 Plantinga (2000), p. 250

53 We need not deal with logical relations between belief-statements, and (2a), historical questions since they do not belong to genuine religious belief.

is knowledge in the same way as scientific knowledge. Although we agree that there are similarities and analogies, especially between *scientific* and *religious belief* (recall chs. 12.3.1–12.3.5). And there are also analogies in the structure of scientific and methodological approach in science and in theology as has been elaborated in 12.41 above. But this does not mean that one should neglect the differences. Although every science contains conjectures and scientific beliefs besides knowledge,their hypotheses, laws and theories can be severely tested by observation and experiment (otherwise they would remain blind conjectures). On the other hand statements of religious belief and theological hypotheses and theories cannot be tested in this way; they can be *tested relative* to a *basis of beliefs* that concern biblical records, even if they have been carefully and critically examined by exegesis.

12.45 (To 12.15:) The answer to this claim has been elaborated in detail in sections 12.3.1–12.3.5 and need not be repeated here.

12.46 (To 12.22:) The claim of Swinburne in the quotation of 12.22 is not correct. There are deductively valid arguments which prove the existence of an entity (first cause, necessary being, etc.) that all call God. To find generally accepted premises is a more complicated problem in both a posteriori and a priori ways. However, the a posteriori way to find a cause or explanation for the world seems to be the most natural and convincing argumentation for the existence of God.

13 A Theory of Knowledge, Belief and Assumption

In this chapter, an epistemic deductive system for concepts of knowledge, belief, and assumption is proposed. This is done in the following way:

First (13.1), properties and desiderata of the three concepts knowledge—belief—assumption are discussed. As to the three concepts, these of knowledge are subdivided into one strong concept of knowledge and two weaker ones, those of belief are subdivided into two, a stronger and a weaker one. Moreover it is stressed that knowledge as described here (in the system KBA) is *human* knowledge, not an ideal knowledge and not just logical knowledge or provability. Further desiderata concern *assumption* and iterations of epistemic operations.

Second (13.2), the main points of the terminology and the semantics of KBA are described.

Third (13.3), the definition of the system (theory) KBA is given, and in the fourth part (13.4) the theorems of the theory are presented.

13.1 Knowledge—Belief—Assumption

All the three are essential for everyday life and for science: Without *belief* learning cannot start. However, learning starts in babies who have trust and belief beginning at birth. Without belief in experts and colleagues, today's science could not exist. Some immediate basic *knowledge* is necessary to start any intellectual investigation. "All teaching and all intellectual learning come about from already existing knowledge."[1] A great deal of experience in everyday life and in science starts from *assumptions* and continues to test them.

13.1.1 Desiderata for Knowledge, Belief, and Assumption

According to ch. 4 we can distinguish seven types of knowledge. For the system KBA we select two types:

(1) One strong type of knowledge (denoted by K) which may represent types K 1–K 4 since it satisfies the principle K[2]:

1 Aristotle (APost) 71a

2 To avoid the danger of misunderstanding, we abbreviate the epistemic operators by dropping the variables for persons 'a', 'b' and write 'Kp' instead of 'aKp', 'bKp'.

https://doi.org/10.1515/9783110585797-013

K $\quad Kp \rightarrow p$

That such a strong concept of knowledge is defensible has been shown already by describing the types K 1 (simple logical or mathematical principles) and K 2 (simple empirical principles like *fallor ergo sum*) in chs. 1 and 4, and K 3 (knowledge by proof) and K 4 (knowledge by verification) in ch. 4. It has also been defended by several contemporary philosophers as for example by Chisholm, Hintikka, and Lehrer.[3] A simple defense which is frequently used is this: To say that the person a knows that p (is the case) *and* p is false is contrary to a widespread usage of the concept of knowledge; therefore—according to this widespread usage—the principle K above must hold.

(2) A weaker type of knowledge denoted by K^- which does not satisfy principle K but satisfies, together with the strong type, several other principles and consistency conditions. That such a weaker type of knowledge is needed has been defended in section 4.3.7, 3 and 6.3.2. It concerns types of knowledge K 5 (knowledge by corroboration), K 6 (justified true belief) and K 7 (knowledge as epistemic entropy and information).

A further weaker type of knowledge K' is needed in connection with the involvement of two persons a and b. K' satisfies principle K but is weaker with respect to a when b is involved (see below):

(3) *Human* Knowledge: Logical Omniscience and deductive infallibility are not human properties. By *logical omniscience* (LO) one understands that all propositions which are logically true (say theorems of First Order Predicate Logic with Identity) are known. By *deductive infallibility* (DI) one understands that all consequences which follow logically from known premises are known too. Both properties are not human properties as has been pointed out in detail in section 6.3.1. LO and DI fit much better to the concept "provable" than to the concept "knowledge" or "known". Therefore the system KBA does not include LO or DI. It includes however for both concepts of knowledge the consistency conditions 6C1–6C5 and the principles MPK and DK$\rightarrow$ of section 6.3.1.

(4) Distribution over $\rightarrow$ and $\wedge$: As mentioned above KBA includes distribution over $\rightarrow$, i. e. principle DK$\rightarrow$ and also over $\wedge$ (principle DK$\wedge$) in one direction, as theorems. However, it does not contain principle FK$\wedge$ for the reasons given in section 6.3.1.

3 Chisholm (1963), (1966); Hintikka (1962); Lehrer (1974).

(5) Ignorance: It is a theorem of KBA that ignorance exists and consequently that there are unknown truths.

(6) Knowing that and knowing whether, if two persons are involved, can be handled in KBA with suitable definitions and distinctions as discussed shortly in section 6.3.3.

(7) The system KBA contains two different concepts of belief, a stronger one (denoted by G) that is nowledge-exclusive and a weaker one (denoted by B) that is knowledge-inclusive. The need for both concepts and the fact that the stronger one, G-belief, is the one used in scientific and religious belief has been elaborated already in sections 8.3.1 and 8.3.2. G-belief is however not knowledge-exclusive with the weaker type of knowledge K^- for which the principle K (see (1) above) does not hold. Thus $Gp \wedge K^- p$ is compatible.

(8) Assumption: If an epistemic system includes a concept of assumption, it seems to be a desideratum that the kind of assumption used in scientific discourse can be described in a suitable way. Since assumptions used in science are frequently used in *indirect proofs*, it is desirable that their properties satisfy such situations. That means that it must not be inconsistent to make the assumption that p but at the same time to assume—just for the sake of the indirect proof: non-p. However, this is not the same as to assume the contradiction $p \wedge \neg p$. In other words, $Ap \wedge A\neg p$ ('Ap' for 'Assumption that p') must not be inconsistent whereas $A(p \wedge \neg p)$ is inconsistent. The system KBA includes these as theorems.

(9) As has been pointed out already in section 6.3.1 none of the epistemic operations aK, aK^-, aK', bK, bK', aG, aB, aA is extensional in the sense that inside its scope logical equivalence transformation or logical closure would in general be possible salva validitate. This is so although the system KBA defines these epistemic operations by finite matrices (truth-tables).

(10) Iterations and mixed epistemic operations: There are different kinds of iterations and mixed operations. It seems useless to deal with all the possible ones. The selection adopted here is guided by two principles: First not to allow more than two operations (of the same or of a different category) in an atomic wff. The reason is that the epistemic operations in an epistemic system are thought to give a precise representation of epistemic discourse. However, although in ordinary and scientific epistemic discourse more than two operations occur sometimes, they are used relatively seldom, at least compared to two operations. In addition they are sometimes hard to interpret unambiguously. Thus "a knows that he believes that

he knows" has hardly a good sense whereas "*a* knows that he does not believe this" makes good and clear sense. The same is true if I say "*a* knows that *b* assumes that *p*". Therefore, we omit expressions which contain more than two operators. The second restrictive principle is to find out "important" iterations and to omit the others when presenting theorems. Since knowing, believing, and assuming are all not unconscious actions those iterations which are repetitions—i. e. "*a* believes that he believes" and "*a* assumes that he assumes" with the exceptions of the K-K-thesis "*a* knows that *a* knows"—do not seem to be so important. On the other hand, it makes a difference whether I say "*a* does not know *p*" or "*a* knows that he does not know *p*" because "*a* does not know" is open to both sides: knowing that one does not know and ignorance. The same holds for the forms "*a* knows that he does not believe" (or: "that he does not assume").

(11) Contingency of epistemic assertions: Atomic epistemic assertions represented by Ep, see 13.2.1 (3) below, are understood to be *contingent human states of affairs*. Therefore the specific ontological status of a state of affairs represented by p (i. e. being logically or factually true or false) does not determine the ontological status of the epistemic assertion which will still be contingent.

13.2 Terminology and Semantics

13.2.1 Well-Formed Formulas (wffs)

(1) The usual wffs of two-valued propositional logic are constructed out of atomic formulas with the connectives $\neg$, $\rightarrow$, $\vee$, $\wedge$, $\leftrightarrow$ (negation, implication, disjunction, conjunction, equivalence). $p, q, r, s, \ldots$ are used as propositional variables.

(2) The wffs of (1) are extended by the following epistemic operators to produce epistemic wffs: If p is atomic then aKp, bKp, $aK'p$, $bK'p$, $aK^{-}p$, aGb, aBp, aAp are atomic epistemic wffs. Compound epistemic wffs may be build up of them with the above connectives (see (3)).

(3) If Ep and Eq are wffs (where E stands for the operators aK, bK, aK', bK', aK^{-}, aG, aB, aA) then so are $\neg Ep$, $\neg E\neg p$, $E(p \rightarrow q)$, $E(p \vee q)$, $E(p \wedge q)$, $E(p \leftrightarrow q)$.

(4) Atomic epistemic wffs can have at most 2 iterated epistemic operators.

(5) If E_1 and E_2 are different epistemic operators of (2) above (aK, bK, aK', bK', aK^{-}, aG, aB, aA) then E_1E_2p, $E_1\neg E_2p$ are iterated epistemic wffs.

(6) Compound iterated epistemic wffs can be build up of them with the above connectives.

(7) Definitions are expressed by equivalences.

(8) Theorems are understood as holding for universal propositional quantifiers. For example, $aKp \rightarrow aBp$ is understood as $(\forall p)(aKp \rightarrow aBp)$. The universal propositional quantifier is therefore not used (except in some cases to avoid misunderstanding). However, there are some important theorems which hold only with an existential quantifier. For example: $(\exists p)(p \wedge \neg aKp)$, i. e. there are unknown truths. Therefore the existential propositional quantifier is used whenever needed.[4]
Suggested translation of some examples: The formulas aKp, $aK^{-}p$, $bK^{O}\neg p$, aGp, aBp, aAp, $aK'\neg aKp$, $bK'\neg aGp$, $aK'aBp$, $aK'\neg aA\neg p$ are to be read as "*a* knows that *p*" (in the strong sense), "*a* knows that *p*" (in the weak sense), "*b* knows whether *p*", "*a* *G*-believes that *p*" or "*a* strongly believes that *p*", "*a* *B*-believes that *p*" or "*a* weakly believes that *p*", "*a* assumes that *p*", "*a* knows that he does not know that *p*", "*b* knows that *a* does not strongly believe that *p*", "*a* knows that *a* weakly believes that *p*", "*a* knows that *a* does not assume that not-*p*". It is assumed that *a* and *b* are different persons, i. e. that $a \neq b$.

13.2.2 Semantics of KBA

KBA has a many-valued semantics with finite matrices (truth tables). Every atomic wff has 10 truth-values, 5 for true and 5 for false. Compound wffs with 2 variables have 100 truth-values (10^n truth-values for n variables). Therefore the system KBA is decidable and consistent. Because of the decidability also the invalid formulas can be proved to be invalid. Therefore the sign '$\vdash$' is used subsequently to stand for 'provable valid' and the sign '$\nvdash$' for 'provable invalid'.

The question whether one can find a finite number of axioms for the system KBA has only partially been investigated. Based on these considerations a reasonable conjecture seems to be this: There is an axiom system either for KBA or for a system KBA' which is similar to KBA in that it meets the following two conditions:

(1) It is at least so strong as to give all the theorems of KBA where only the single epistemic operators K, G, and B are involved.
(2) It is at least so weak as not to allow the principles of deductive infallibility or logical omniscience.

The classical 2-valued propositional calculus is included in KBA. This can be seen easily by selecting the values 1 and 0 in the matrices of the atomic wffs p and $\neg p$

4 Notice that extending two-valued propositional logic by propositional quantifiers is only a *conservative extension*. The same is true for ten values (5 for true, 5 for false) as used here. For a proof see Kreisel (1981).

and by selecting the 4 values in the 4 corners of the matrices of the compound wffs $p \rightarrow q$, $p \vee q$, $p \wedge q$. If we drop all other values, KBA reduces to two-valued propositional logic.

Moreover, the system KBA is built up as an extension of two-valued propositional logic. This can be seen from the structure of the matrices (see 13.3 below): The values for true are $T = 1, 2, 3, 4, 5$; those for false are $F = 6, 7, 8, 9, 0$. The atomic wff p has values 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. its negation $\neg p$ has its mirror image 0, 9, 8, 7, 6, 5, 4, 3, 2, 1. The matrix for $\rightarrow$ contains F-values in the right upper corner, T-values in the other three corners. The matrix for $\vee$ contains F-values in the right lower corner, T-values in the other three corners. The matrix for $\wedge$ contains T-values in the left upper corner, F-values in the other three corners.

13.3 Definition of the System KBA

13.3.1 The System KBA

The system KBA is the set of all formulas which are satisfied by the matrix $M =< T, F, \neg, \rightarrow, aK, aK^{-}, bK, aK^{'}, bK^{'}, aG, aB, aA >$ where $T = 1, 2, 3, 4, 5$, $F = 6, 7, 8, 9, 0$ and the operations $\neg, \rightarrow, aK, aK^{-}, bK, aK^{'}, bK^{'}, aG, aB, aA >$ are defined by the following matrices:

p	***¬p***	***p → q***	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**0**
1	0	1	1	2	4	4	5	7	7	8	7	0
2	9	2	1	2	4	4	5	7	7	8	9	7
3	8	3	1	2	3	4	5	8	7	8	8	8
4	7	4	1	2	4	4	5	7	7	7	7	7
5	6	5	1	2	4	4	5	6	7	8	7	7
6	5	6	5	5	5	4	5	5	5	5	5	5
7	4	7	5	5	4	4	4	4	4	4	4	4
8	3	8	1	5	3	4	5	4	4	3	4	4
9	2	9	5	2	5	5	5	2	2	2	2	2
0	1	0	1	5	1	5	5	1	1	1	1	1

aKp	$aK^{-}p$	bKp	$aK'p$	$bK'p$	aGp	aBp	aAp
0	0	1	0	1	1	1	1
2	2	9	2	9	9	2	2
8	8	3	3	3	8	8	8
7	7	7	4	4	7	7	7
5	5	5	5	5	6	5	5
6	6	6	6	6	6	6	5
7	7	7	7	7	7	7	7
8	3	8	8	8	3	3	3
9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0

The other connectives satisfy the following equivalences:
$p \vee q \leftrightarrow \neg p \rightarrow q$
$p \wedge q \leftrightarrow \neg(\neg p \vee \neg q)$
$p \leftrightarrow q \leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Equivalence means only sameness of T-values and sameness of F-values but not sameness of particular values inside the 5 T-values or inside the 5 F-values. Therefore, we define explicitly the matrices for $\wedge$ and $\vee$.

$p \wedge q$	**1 2 3 4 5 6 7 8 9 0**	$p \vee q$	**1 2 3 4 5 6 7 8 9 0**
1	1 4 4 4 4 6 7 7 9 0	1	1 5 1 5 5 1 1 1 1 2
2	4 2 3 4 4 6 7 7 9 0	2	5 2 5 5 5 2 2 2 2 2
3	4 3 3 4 3 6 7 9 9 0	3	1 5 3 4 5 4 4 3 4 4
4	4 4 4 4 4 6 7 7 9 0	4	5 5 4 4 4 4 1 4 4 4
5	4 4 3 4 5 7 7 7 9 0	5	5 5 5 4 5 5 5 5 5 5
6	6 6 6 6 7 6 6 6 6 6	6	1 2 4 4 5 6 7 8 7 7
7	7 7 7 7 7 6 7 7 6 6	7	1 2 4 1 5 7 7 7 7 7
8	7 7 9 7 7 6 7 8 6 6	8	1 2 3 4 5 8 7 8 8 8
9	9 9 9 9 9 6 6 6 9 6	9	1 2 4 4 5 7 7 8 9 7
0	0 0 0 0 0 6 6 6 6 0	0	2 2 4 4 5 7 7 8 7 0

13.3.2 Further Epistemic Operations

The following operations can be constructed from the basic epistemic operations defined in 13.3.1 above. They are listed here for more transparency.

$aK\neg p$	$\neg aKp$	$\neg aK\neg p$	$aK^O p$	$bK^O p$	$aK'bKp$	$aK'\neg aKp$	$aK'\neg aGp$	$aK'aBp$
0	1	1	0	1	0	0	0	0
9	9	2	2	9	9	9	2	2
8	3	3	8	4	3	3	3	8
7	4	4	7	7	7	4	4	7
6	6	5	5	5	5	6	5	5
5	5	6	5	5	6	5	5	6
7	4	4	7	7	7	4	4	7
8	3	3	8	4	8	3	8	3
2	2	9	2	9	9	2	2	9
0	1	1	0	1	0	0	0	0

13.3.3 Size of Matrices

The size of matrices is determined by the following two rules:

R 1: If the number of different atomic formulas p, q, r is n then the number of values of the matrix of the compound formulas is 10^n. Thus p has 10 values, q has 10 values, $p \to q$, $p \vee q$, $p \wedge q$, $p \leftrightarrow q$ have 100 values. On the other hand formulas like $aKp \vee aK\neg p$, $aKp \to \neg aGp$, $aKp \to aK'\neg bK\neg p$ have only 10 values since only p (or $\neg p$) is involved.

R 2: Formulas which are defined (D 3, . . . , D 13, see 13.4 below) and have (round) parenthesis in their definiendum are the only exceptions of rule R 1: they have matrices of 100 values, i. e. the parts of the compound definiens (being a conjunction or a disjunction) are treated like p and q.

13.4 Theorems and Non-Theorems of KBA

13.4.1 General Theorems

Let $E^\star$ stand for the operations aK, aK^-, bK, aK', bK', aG, aB (i. e. without aA, cf. 13.2.2 (3)). Then the following are general theorems:

$\vdash$ 1 $E^\star p \to \neg E^\star \neg p$
$\vdash$ 2 $E^\star \neg p \to \neg E^\star p$
$\vdash$ 3 $E^\star (p \wedge q) \to (E^\star p \wedge E^\star q)$
$\vdash$ 4 $E^\star (p \to q) \to (E^\star p \to E^\star q)$
$\vdash$ 5 $[E^\star p \wedge E^\star (p \to q)] \to E^\star q$

$\vdash 6$ If p is true then $\vdash A(p \to q) \to (Ap \to Aq)$.
$\vdash 7$ If p is true then $\vdash [Ap \wedge A(p \to q)] \to Aq$.

Ignorance: A simple form of ignorance concerning states of affairs represented by p may be defined as $\neg aKp \wedge \neg aK\neg p$ (cf. section 6.3.1 above). A very general type of ignorance can be defined thus:

D 1 $aIGp \leftrightarrow (\neg Ep \wedge \neg E\neg p)$, (where E is like E^{*} above plus aA)

Then there is ignorance of that sort concerning some states of affairs and concerning some facts (represented by true propositions):

$\vdash 8$ $(\exists p)(\neg Ep \wedge \neg E\neg p)$
$\vdash 9$ $(\exists p)(p \wedge \neg\exists p \wedge \neg E\neg p)$
$\vdash 10$ $(\exists p)(p \wedge \neg\exists p)$

Theorem 10 says that there are truths that are neither known nor believed nor assumed.

Important non-theorems are as follows:

$\nvdash 1$ $(p \to q) \to E(p \to q)$
$\nvdash 2$ $\vdash (p \to q) \vdash E(p \to q)$
$\nvdash 3$ $[Ep \wedge (p \to q)] \to Eq$
$\nvdash 4$ $[Ep \wedge \vdash (p \to q)] \to Eq$
$\nvdash 5$ $(Ep \vee Eq) \to E(p \vee q)$
$\nvdash 6$ $(\forall p)$: if p is logically true then Ep, i. e. logical omniscience does not hold for any of the epistemic operations.

Non-theorems 1–4 state that deductive infallibility does not hold for any of the epistemic operations.

$\vdash 11$ For every of the epistemic operations represented by E there is some p (i. e. some value true of the matrix of p) such that it holds: $Ep \wedge p$. Semantically this means:Every pair of matrices, consisting of the matrix of p conjoined with one of the matrices of the 8 epistemic operations aK, aK^{-}, aK', bK, bK', aG, aB, aA (see section 13.3.1) has at least one (or other) common truth-value true.
$\vdash 12$ $\neg E(p \wedge \neg p)$, i. e. an explicit contradiction is neither known nor believed nor assumed (cf. 6 C 1 of 6.3.1).

$\vdash$ 12.1 $\neg(E^{*}p \wedge E^{*}\neg p)$

Whereas theorem 12 holds for all epistemic operations, theorem 12.1 holds for all operations of knowledge and belief but not for assumption (cf. 6 C 2 of section 6.3.1 and theorem 43 and non-theorem 10 below).

13.4.2 Theorems and Non-Theorems for Knowing

$\vdash$ 13	$aKp \rightarrow p$	$\vdash$ 13.1	$\neg p \rightarrow \neg aKp$
$\vdash$ 14	$bKp \rightarrow p$	$\vdash$ 14.1	$\neg p \rightarrow \neg bKp$
$\vdash$ 15	$aKp \rightarrow \neg bK\neg p$	$\vdash$ 15.1	$bK\neg p \rightarrow \neg aKp$
$\vdash$ 16	$bKp \rightarrow \neg aK\neg p$	$\vdash$ 16.1	$aK\neg p \rightarrow \neg bKp$
$\vdash$ 17	$aKp \rightarrow aKaKp$	$\vdash$ 17.1	$bKp \rightarrow bKbKp$
$\vdash$ 18	$aK'p \rightarrow aK'aK'p$	$\vdash$ 18.1	$bK'p \rightarrow bK'bK'p$

Theorem 17 is called the K-K-Thesis: Someone who knows that p also knows that he(she) knows that p. This does not hold for the weaker kind K^{-} of knowledge.

$\nvdash$ 7 $aK^{-}p \rightarrow aK^{-}aK^{-}p$

It does not hold for belief or assumption either:

$\nvdash$ 8	$aGp \rightarrow aGaGp$	$\nvdash$ 8.1	$aBp \rightarrow aBaBp$
$\nvdash$ 9	$aAp \rightarrow aAaAp$		

D 2 $aK^{O}p \leftrightarrow (aKp \vee aK\neg p)$
D 2.1 $bK^{O}p \leftrightarrow (bKp \vee bK\neg p)$

D 2 (D 2.1) are the definitions of “a knows whether p is the case” and of “b knows whether p (is the case)”. They are in accordance with everyday language use and with scientific discourse: we say that the person a knows whether p is the case iff either a knows that p is the case or a a knows that not-p is the case.

$\vdash$ 19	$aKp \rightarrow aK^{O}p$	$\vdash$ 19.1	$\neg aK^{O}p \rightarrow \neg aKp$
$\vdash$ 20	$aK\neg p \rightarrow aK^{O}p$	$\vdash$ 20.1	$\neg aK^{O}p \rightarrow \neg aK\neg p$
$\vdash$ 21	$bKp \rightarrow bK^{O}p$	$\vdash$ 21.1	$\neg bK^{O}p \rightarrow \neg bKp$
$\vdash$ 22	$bK\neg p \rightarrow bK^{O}p$	$\vdash$ 22.1	$\neg bK^{O}p \rightarrow \neg bK\neg p$

Theorems 19–22.1 hold analogously for aK^{-}, aK', bK' if aK^{-O}, aK'^{O}, bK'^{O} are defined accordingly.

$\vdash 23 \quad aK^O p \rightarrow aK(p \vee \neg p)$

Example for a proof: We take theorem 13: $aKp \rightarrow p$. For the proof, we have to look at the matrix of aKp of p and of $p \rightarrow q$ in section 13.3.1. We have to relate each of the 10 values of aKp to those of p in the same order: 0 to 1, 2 to 2 etc. Since these values have to be related by an implication we have to apply the matrix of $p \rightarrow q$. We see then $(0 \rightarrow 1) = 1, (2 \rightarrow 2) = 2, (8 \rightarrow 3) = 3 \ldots (0 \rightarrow 0) = 1$. The produced values from such an application are the following 10 values: 1, 2, 3, 4, 5, 5, 4, 3, 2, 1. Since all these values are values for true theorem 13 is proved.

If one of the resulting values would be > 5 the proposition in question is contingent and not a theorem. If all values are > 5 the proposition is logically false and its negation would be a theorem.

13.4.3 Theorems and non-theorems for knowing, believing and assuming

$\vdash 24$	$aKp \rightarrow aK^- p$	$\vdash 25.1$	$\neg aK^- p \rightarrow \neg aKp$
$\vdash 25$	$aK^- p \rightarrow aBp$	$\vdash 25.1$	$\neg aBp \rightarrow \neg aK^- p$
$\vdash 26$	$aKp \rightarrow aBp$	$\vdash 26.1$	$\neg aBp \rightarrow \neg aKp$
$\vdash 27$	$aBp \rightarrow aAp$	$\vdash 27.1$	$\neg aAp \rightarrow \neg aBp$
$\vdash 28$	$aKp \rightarrow aAp$	$\vdash 28.1$	$\neg aAp \rightarrow \neg aKp$
$\vdash 29$	$aKp \rightarrow aK^- p \rightarrow aBp \rightarrow aAp$		
$\vdash 30$	$aKp \rightarrow aK' p$	$\vdash 30.1$	$bKp \rightarrow bK' p$
$\vdash 31$	$aKp \rightarrow \neg aGp$	$\vdash 31.1$	$aGp \rightarrow \neg aKp$
$\vdash 32$	$aKp \rightarrow \neg aG\neg p$	$\vdash 32.1$	$aGp \rightarrow \neg aK\neg p$
$\vdash 33$	$aK\neg p \rightarrow \neg aGp$	$\vdash 33.1$	$aG\neg p \rightarrow \neg aKp$
$\vdash 34$	$aK\neg p \rightarrow \neg aG\neg p$	$\vdash 34.1$	$aG\neg p \rightarrow \neg aK\neg p$
$\vdash 35$	$aK^O p \rightarrow (\neg aGp \wedge \neg aG\neg p)$	$\vdash 35.1$	$(aGp \vee aG\neg p \rightarrow \neg aK^O p)$
$\vdash 36$	$aGp \rightarrow aBp$	$\vdash 36.1$	$\neg aBp \rightarrow \neg aGp$
$\vdash 37$	$aGp \rightarrow aAp$	$\vdash 37.1$	$\neg aAp \rightarrow \neg aGp$
$\vdash 38$	$aGp \rightarrow (aBp \wedge \neg aKp)$		
$\vdash 39$	$(aKp \vee aGp) \rightarrow aBp$		
$\vdash 40$	$aKp \rightarrow (aBp \wedge \neg aGp)$		
$\vdash 41$	$aKp \rightarrow (\neg aGp \wedge \neg aG\neg p)$		
$\vdash 42$	$aGp \rightarrow (\neg aKp \wedge \neg aK\neg p)$		
$\vdash 43$	$\neg aA(p \wedge \neg p)$	i e. $aA(p \wedge \neg p)$ is inconsistent.	
$\nvdash 10$	$\neg(aAp \wedge aA\neg p)$	i. e. $(aAp \wedge aA\neg p)$ is *not* inconsistent.	

In an indirect proof, we use non-theorem $\nvdash$ 10 and not assuming a contradiction (cf. $\vdash$ 43).

$\nvdash$ 11	$\neg(aGp \wedge bKp)$	i. e. $(aGp \wedge bKp)$ is *not* inconsistent.
$\nvdash$ 12	$\neg(aGp \wedge aK^{-}p)$	i. e. $(aGp \wedge aK^{-}p)$ is *not* inconsistent.

This means: strong belief is compatible with weak knowledge although it is incompatible with strong knowledge (see theorems 31–35) which satisfies theorem 13.

$\vdash$ 44	$\neg(aGp \wedge aKp)$	i. e. $(aGp \wedge aKp)$ is inconsistent.
$\vdash$ 45	$[aKp \wedge (p \rightarrow (q \wedge \neg q))] \rightarrow aK(q \wedge \neg q)$	i. e. $[aKp \wedge (p \rightarrow (q \wedge \neg q))]$ is inconsistent (cf. $\vdash$ 12).
$\vdash$ 46	$[bKp \wedge (p \rightarrow (q \wedge \neg q))] \rightarrow bK(q \wedge \neg q)$	(cf. $\vdash$ 12)
$\nvdash$ 13	$[aK^{-}p \wedge (p \rightarrow (q \wedge \neg q))] \rightarrow aK^{-}(q \wedge \neg q)$	i. e. $[aK^{-}p \wedge (p \rightarrow (q \wedge \neg q))]$ is *not* inconsistent though $aK^{-}(p \wedge \neg p)$ is (cf. $\vdash$ 12).
$\nvdash$ 14	$[aGp \wedge (p \rightarrow (q \wedge \neg q))] \rightarrow aG(q \wedge \neg q)$	i. e. $[aGp \wedge (p \rightarrow (q \wedge \neg q))]$ is *not* inconsistent though $aG(q \wedge \neg q)$ is (cf. $\vdash$ 12).

Non-theorems 13 and 14 may be used for an interpretation of Frege's strong belief (G) or his not strong knowledge (K^{-}) in his 5th axiom (the comprehension axiom) of the *Grundgesetze*.[5]

5 Frege says in his "Vorwort" (preface) that a controversion could arise only about this principle (axiom V) though he believes in it as a principle of logic.

> Ein Streit kann hierbei, soviel ich sehe, nur um mein Grundgesetz der Werthverläufe (V) entbrennen, das von den Logikern vielleicht noch nicht eigens ausgesprochen ist, obwohl man danach denkt, z. B. wenn man von Begriffumfängen redet. Ich halte es für rein logisch. (Frege 1893, Vol I, p. VII)

In his "Nachwort" (postscript) he says:

> Einem wissenschaftlichen Schriftsteller kann kaum etwas Unerwünschteres begegnen, als dass ihm nach Vollendung einer Arbeit eine der Grundlagen seines Baues erschüttert wird. In diese Lage wurde ich durch einen Brief des Herrn Bertrand Russell versetzt, als der Druck dieses Bandes sich seinem Ende näherte. Es handelt sich um mein Grundgesetz (V). Ich habe mir nie verhehlt, dass es nicht so einleuchtend ist, wie die andern, und wie es eigetnlich von einem logischen Gesetze verlangt werden muss. (Frege 1903, Vol. II, p. 253)

As Russell showed him, a contradiction followed from this axiom, although Frege did not know this and did not believe this. Therefore we should not say that his strong belief (as an action of believing) or his not-strong knowledge (as an action of knowing) is (was) inconsistent although *what* he believed (weakly knew) implied an inconsistency. In other words, from the consistency of aGp or of $aK^{-}p$ the consistency of its logical consequences does not follow, and consequently, from the inconsistency of some of its consequences it does not follow that aGp or $aK^{-}p$ are inconsistent.

$\nvdash$ 15 $[aBp \wedge (p \rightarrow (q \wedge \neg q))] \rightarrow aB(q \wedge \neg q)$ — i. e. $[aBp \wedge (p \rightarrow (q \wedge \neg q))]$ is *not* inconsistent though $aB(q \wedge \neg q)$ is (cf. $\vdash$ 12).

$\nvdash$ 16 $[aAp \wedge (p \rightarrow (q \wedge \neg q))] \rightarrow aA(q \wedge \neg q)$ — i. e. $[aAp \wedge (p \rightarrow (q \wedge \neg q))]$ is *not* inconsistent though $aA(q \wedge \neg q)$ is (cf. $\vdash$ 43).

The results are the same if in theorems 45 and 46 and in non-theorems 13, 14, 15 and 16 '$p \rightarrow (q \wedge \neg q)$' is replaced by '$\vdash [p \rightarrow (q \wedge \neg q)]$', i. e. by '$p$ logically implies $q \wedge \neg q$'.

13.4.4 Theorems for iterated and combined epistemic operations

The operations aK', bK' are weaker notions than aK and bK. They are needed for getting enough differentiation concerning iterations and combinations of epistemic operations. Such differentiations will become clear from the next section 13.4.5.

In this section, some general theorems for building iterations and combinations with aK' and bK' will be given. Let E, $E^{\star}$ be as in 13.2.1 and 13.4.1.

$\vdash$ 47	$aK'Ep \rightarrow Ep$	$\vdash$ 47.1	$aK'\neg Ep \rightarrow \neg Ep$
$\vdash$ 48	$bK'Ep \rightarrow Ep$	$\vdash$ 48.1	$bK'\neg Ep \rightarrow \neg Ep$
$\vdash$ 49	$aKp \rightarrow aK'\neg E^{\star}\neg p$		
$\vdash$ 50	$aK\neg p \rightarrow aK'\neg E^{\star}p$		
$\vdash$ 51	$bKp \rightarrow bK'\neg bK\neg p$	$\vdash$ 51.1	$bKp \rightarrow bK'\neg aK\neg p$
$\vdash$ 52	$bK\neg p \rightarrow bK'\neg bKp$	$\vdash$ 52.1	$bK\neg p \rightarrow bK'\neg aKp$
$\vdash$ 53	$aKp \rightarrow aK'\neg aGp$	$\vdash$ 53.1	$aK\neg p \rightarrow aK'\neg aG\neg p$
$\vdash$ 54	$aK^{O}p \rightarrow (aK'\neg aGp \wedge aK'\neg aG\neg p)$		
$\vdash$ 55	$bK'aKp \rightarrow (bK'\neg aGp \wedge bK'\neg aG\neg p)$	$\vdash$ 55.1	$bK'aK\neg p \rightarrow (bK'\neg aGp \wedge bK'\neg aG\neg p)$

$\vdash 56 \quad (E_1^\star p \to E_2^\star p) \to (aK' E_1^\star p \to aK' E_2^\star p)$

Theorem 56 is very general, it holds for all epistemic operations represented by $E^\star$. For transparency, we shall give two instantiations as examples:

(1) $(aGp \to \neg aG\neg p) \to (aK' aGp \to aK' \neg aG\neg p)$
Since $aGp \to \neg aG\neg p$ follows from theorem 1, $aK' aGp \to aK' \neg aG\neg p$ is also a theorem.

(2) $(\neg aBp \to \neg aK^- p) \to (aK' \neg aBp \to aK' \neg aK^- p)$
Since $\neg aBp \to \neg aK^- p$ is theorem 25.1, $aK' \neg aBp \to aK' \neg aK^- p$ is also a theorem.

13.4.5 Theorems and non-theorems for knowing that and whether someone else knows

D 3 $\quad aK(bKp) \leftrightarrow (aKp \wedge aK' bKp)$
D 4 $\quad aK(bK\neg p) \leftrightarrow (aK\neg p \wedge aK' bK\neg p)$

$aK(bKp)$ and $aK(bK\neg p)$ are the strong propositions "*a* knows that *b* knows that *p*" and "*a* knows that *b* knows that not-*p*" from which (cf. theorems 57 and 58) both aKp and bKp are derivable. On the other hand $aK' bKp$ and $aK' bK\neg p$ are the respective weak versions from which—according to theorem 47—bKp and $bK\neg p$ are derivable, although aKp or $aK\neg p$ are not derivable (cf. non-theorem 17).

The stronger versions defined in D 3 and D 4 are expressed in everyday language and also in scientific discourse in such a way as to make sure that also the first person knows that *p* is the case, i. e. as something like "*a* knows that *b* also knows that *p*" or something similar.

The weaker versions on the other hand (which also occur in the definitions of the stronger as constituents) need no definition since they have a matrix which is obtained by applying the matrix of the operation aK' to the matrix of bKp (or of $bK\neg p$) (cf. 13.2.1 (5) and 13.3.3). A case for the weaker version is for example if a child (or in general if a person *a* whose knowledge is on a lower level as compared to another person *b*) says that he knows that his father (*b*) knows that something is the case. Although it should then follow (from a normal interpretation of the statement) that the father knows that *p* but one does not assume (and does not want) in general the consequence that the child also knows that *p*. The child (or the student w. r. t. the professor) may have a much weaker understanding of the case such that one wouldn't like to use 'know' here although we accept that the child (student) knows that his (her) father (professor) knows that *p*. For such a kind of "weaker knowledge" the system presented provides the weaker notions aK' and bK'.

However, observe that they are weaker in *one* respect, namely in respect to the state of affairs p but they need not be and are in fact not (as theorems 47 and 48 show) weaker with respect to what they are applied to, i. e. to the *knowing* of p of the other person.

$\vdash$ 57	$aK(bKp) \to aKp$	$\vdash$ 57.1	$aK(bK\neg p) \to aK\neg p$
$\vdash$ 58	$aK(bKp) \to bKp$	$\vdash$ 58.1	$aK(bK\neg p) \to bK\neg p$
$\nvdash$ 17	$aK'bKp \to aKp$	$\nvdash$ 17.1	$aK'bK\neg p \to aK\neg p$
$\vdash$ 59	$aK(bKp) \to aK(\neg bK\neg p)$	$\vdash$ 59.1	$aK(bK\neg p) \to aK'\neg bKp$ (cf. D 6)

D 5 $\quad aK(\neg bK\neg p) \leftrightarrow (aKp \wedge aK'\neg bK\neg p)$

D 6 $\quad aK(\neg bKp) \leftrightarrow (aKp \wedge aK'\neg bKp)$

Similar examples as those above also apply to D 5 and D 6.

D 5: The statement "the professor of mathematics a knows that his colleague b does not now (did not prove) the opposite (the negation) of p" can have two meanings, a stronger one—$aK(\neg bK\neg p)$—and a weaker one. According to the stronger one the professor a knows that p; according to the weaker one—$aK'\neg bK\neg p$—a just knows that b does not know (hasn't proved) the opposite ($\neg p$).

D 6: Similarly the statement "a knows that b does not know that p" can have a stronger and a weaker meaning: The stronger sense—$aK(\neg bKp)$—may be expressed by saying "a knows that b does not yet know that p" from which aKp follows. The weaker sense—$aK'\neg bKp$—says that a just knows that b does not know that p from which only $\neg bKp$ follows.

D 7 $\quad bK(aK^{-}p) \leftrightarrow (bKp \wedge bK'aK^{-}p)$

D 7 is analogous to D 3 for the weaker concept aK^{-} of knowledge. This leads to a similar distinction between a stronger version—$bK(aK^{-}p)$—and a weaker version—$bK'aK^{-}p$—for $aK^{-}p$ as above for $aK(bKp)$:

$\vdash$ 60	$bK(aK^{-}p) \to bKp$	$\vdash$ 60.1	$bK(aK^{-}p) \to aK^{-}p)$
$\vdash$ 61	$bK'aK^{-}p \to aK^{-}p$		
$\nvdash$ 18	$bK'aK^{-}p \to bKp$		

For further combinations we might give a semantic definition for $bK^{-}p$: 1937567390 and then define $aK^{-}(bK^{-}p) \leftrightarrow (aK^{-}p \wedge aK'bK^{-}p)$ which includes a further distinction of a stronger and weaker version concerning the weaker concepts aK^{-} and bK^{-}. Other combinations can be received by combining bK with aG or aB: $bK(aGp)$, $aG(bKp)$, $bK(aBp)$, $aB(bKp)$.

D 8 $aK(bK^{O}p) \leftrightarrow [aK(bKp) \vee aK(bK\neg p)]$ (cf. D 2, D 3, D 4)
D 9 $aK(bK^{OO}p) \leftrightarrow [aK(bK^{O}p) \vee (bK^{O}p \wedge \neg aK^{O}p)]$ (cf. D 8, D 2)

$aK(bK^{O}p)$ is the strong proposition "*a* knows that *b* knows whether *p* is the case" whereas $aK(bK^{OO}p)$ is the weak proposition which is usually expressed by the same sentence in everyday language. The difference between both is important: $aK^{O}p$ is derivable only from $aK(bK^{O}p)$ (cf. theorem 63) but not from $aK(bK^{OO}p)$. On the other hand, $bK^{O}p$ is derivable from both (cf. theorems 64 and 66).

An illustration for the difference is the following example: If one says: The professor of mathematics (*a*) knows that his student (*b*) knows whether a certain sentence *p* is a theorem of mathematics then one supposes usually that the professor of mathematics will also know whether *p* is a theorem (or not). Thus in this case $aK(bK^{O}p)$ is applied. On the other hand if one says: The student (*a*) knows that his professor (*b*) knows whether *p* is a theorem of mathematics then one doesn't presuppose that the student will also know whether *p* is a theorem; i. e. one applies $aK(bK^{OO}p)$ which is weaker since $aK^{O}p$ *is not derivable from it*.

⊢ 62	$aK(bKp) \rightarrow aK(bK^{O}p)$	⊢ 62.1	$aK(bK\neg p) \rightarrow aK(bK^{O}p)$
⊢ 63	$aK(bK^{O}p) \rightarrow aK^{O}p$		(cf. D 2, D 3, D 4, D 8)
⊢ 64	$aK(bK^{O}p) \rightarrow bK^{O}p$		(cf. D 2.1, D 3, D 4, D 8)
⊢ 65	$aK(bK^{O}p) \rightarrow aK(bK^{OO}p)$		(see D 9)
⊢ 66	$aK(bK^{OO}p \rightarrow bK^{O}p$		
⊬ 19	$aK(bK^{OO}p) \rightarrow aK^{O}p$	⊬ 19.1	$aK(bK^{OO}p) \rightarrow \neg aK^{O}p$

D 10 $aK^{O}(bKp) \leftrightarrow [aK(bKp) \vee aK(\neg bKp)]$ (cf. D 3 and D 6)

D 10 is the definition of "*a* knows whether *b* knows that *p*" which is of course different from "*a* knows that *b* knows whether *p*" (D 8 and D 9). The sense in which $aK^{O}(bKp)$ is defined or best expressed by saying "*a* knows whether *b* already knows that *p*". This is a phrase which is very often used in everyday language and in scientific discourse. Its meaning presupposes clearly that *a* knows that *p* (and thus also that *a* knows whether *p*). That means that both aKp and $aK^{O}p$ must be derivable from it (cf. theorems 67 and 68). However, it is left open whether *b* knows that *p* or whether he does not yet know it. Thus neither bKp, nor $bK^{O}p$, nor $\neg bKp$ must be derivable from it (cf. non-theorems 20–22).

D 11 $aK^{O}(bK\neg p) \leftrightarrow [aK(bK\neg p) \vee aK(\neg bK\neg p)]$ (cf. D 4 and D 5)

D 11 is a disjunction of the definientia of D 4 and D 5. From looking at these two definientia it will be clear that there is not a full analogy to definition 10 and that the following are non-theorems, although 68.1 and 69.1 are theorems:

$\nvdash$ 23 $\quad aK^O(bK\neg p) \rightarrow aK\neg p$
$\nvdash$ 24 $\quad aK^O(bK\neg p) \rightarrow bK\neg p$
$\nvdash$ 25 $\quad aK^O(bK\neg p) \rightarrow bK^O p$
$\nvdash$ 26 $\quad aK^O(bK\neg p) \rightarrow \neg bK\neg p$

D 12 $\quad aK^O(bK^O p) \leftrightarrow [aK(bKp) \vee aK(\neg bKp) \vee aK(bK\neg p)]$ (cf. D 3, D 4, D 6)

D 12 is the definition of "*a* knows whether *b* knows whether *p*". Although such phrases are not used so frequently as those which are defined in D 2, D 6, D 7, and D 8, there are a lot of examples in everyday language use and in scientific discourse.

The meaning seems best expressed by saying "*a* knows whether *b* already knows whether *p*". An example would be: The physician knows whether the parents of the patient already now whether the operation was successful or not. In this case, we assume that the physician knows whether the result was good or not. Thus $aK^O p$ must be derivable from $aK^O(bK^O p)$ (cf. theorem 71).

On the other hand, we do not assume that the parents in fact already know whether the operation had a good result or not. Thus $bK^O p$ must not be derivable from $aK^O(bK^O p)$ (cf. non-theorem 27). Moreover, one would agree to the following: if the physician *knows that* the parents already *know whether* the operation had a good result then it holds also that the physician *knows whether* the parents already *know whether*... i. e. $aK(bK^O p)$ implies $aK^O(bK^O p)$ (cf. theorem 72). Further, if the physician *knows whether* the parents already *know that* the operation had a good result then again the physician *knows whether* the parents already *know whether*..., i. e. $aK^O(bK^O p)$ must also be derivable from $aK^O(bKp)$ (cf. theorem 73).

$\vdash$ 71 $\quad aK^O(bK^O p) \rightarrow aK^O p$
$\nvdash$ 27 $\quad aK^O(bK^O p) \rightarrow bK^O p$
$\vdash$ 72 $\quad aK(bK^O p) \rightarrow aK^O(bK^O p)$
$\vdash$ 73 $\quad aK^O(bKp) \rightarrow aK^O(bK^O p)$

There is even a weaker version of "*a* knows whether *b* knows whether *p* is the case". Take the following example: "Do you know whether there was a Brentano-translation into English before 1930?"—"I don't, Rod (Chisholm) might know! But he isn't here. So you might ask Keith (Lehrer) who knows Rod and his work on

Brentano very well: He (Keith) will know whether Rod knows whether there was such a translation." Then we expect (under normal conditions) Keith to answer in one of the following ways: "Yes, I know that Rod will certainly know whether p", or "I am sure (I know) that Rod does not know whether p" (i. e. that he neither knows that there was a translation nor that there was none).

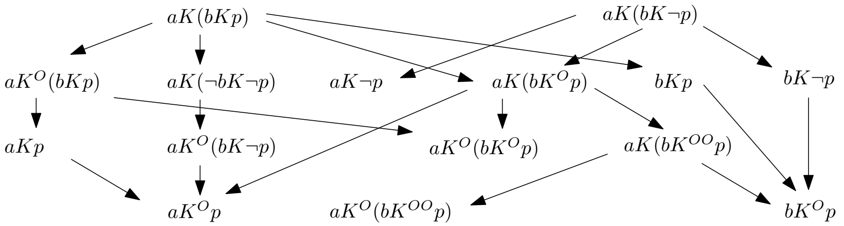
In the first case not $aK^O(bK^O p)$ will be the correct translation since we do nor assume here that Keith knows whether there was such a translation, i. e. $aK^O p$ must not be derivable from Keith's first answer. However, in Keith's first answer it is claimed that Rod knows whether p, i. e. $bK^O p$ must be derivable from Keith's first answer and that means that $aK(bK^{OO}p)$ is the correct translation of Keith's first answer.

In Keith's second alternative answer, he says that he knows that it is not the case that Rod knows that p nor that Rod knows that not-p. Thus from the disjunction of both alternatives neither $aK^O p$ nor $bK^O p$ will follow (cf. non-theorems 28, 29). The disjunction of both answers will be then the correct translation of this weak sense of "a knows whether b knows whether p". This is expressed in D 13:

D 13 $\quad aK^O(bK^{OO}p) \leftrightarrow [aK(bK^{OO}p) \vee [aK(\neg bKp) \wedge aK(\neg bK\neg p)]]$

$\vdash$ 74 $\quad aK(bK^{OO}) \rightarrow aK^O(bK^{OO}p)$
$\nvdash$ 28 $\quad aK^O(bK^{OO}p) \rightarrow aK^O p$
$\nvdash$ 29 $\quad aK^O(bK^{OO}p) \rightarrow bK^O p$

An overview of important deductive relations of the notions "a knows that b knows that p" and "a knows that b knows that not-p" is as follows:



Bibliography

[Alexandrow et al. 1937] Alexandrow, P. (1937). “Diskrete Räume”, *Mathematickeskii Sbornik* 2, pp. 501–519.

[Anderson et al. (1986)] Anderson, M./Page, D. C/de la Chapelle, A. (1986). “Chromosome Y-Specific DNA is Transferred to the Short Art of X Chromosome in Human XX Males”, *Science* 233, pp. 786–788.

[Anderson (1990)] Anderson, C. A. (1990). “Some Emendations of Gödel’s Ontological Proof”, *Faith and Philosophy* 7, pp. 291–303.

[Anglberger (2010)] Anglberger, A. J. J. et al. (2010). Richard Dawkins Hauptargument wissenschaftstheoretisch betrachtet. In: A. J. J. Anglberger/ P. Weingartner: *Neuer Atheismus wissenschaftlich betrachtet*. Frankfurt a. M.: Ontos, pp. 181–197.

[Aristotle (Cat)] Aristotle (Cat). Categories. In: J. Barnes (ed.): *The Complete Works of Aristotle* Vol. I, Princeton, 1985.

[Aristotle (APost)] Aristotle (APost). Posterior Analytics. In: J. Barnes (ed.): *The Complete Works of Aristotle* Vol. I, Princeton, 1985.

[Aristotle (Met)] Aristotle (Met). Metaphysics. In: J. Barnes (ed.): *The Complete Works of Aristotle* Vol. II, Princeton, 1985.

[Audi (1995)] Audi, R. (1995). *The Cambridge Dictionary of Philosophy*. Cambridge: Cambridge Univ. Press.

[Augustine (Ord)] Augustine (Ord). *De Ordine*. Opera Omnia, ML 32, pp. 977–1020.

[Augustine (Trin)] Augustine (Trin). *De Trinitate*. Opera Omnia, ML 42, pp. 819–1098.

[Augustine (Civ)] Augustine (Civ). *De Civitate Deii*. Opera Omnia, ML 41.

[Balakier (1993)] Balakier, H. et al. (eds.) (1993). “Experimentally Induced Pathogenetic Activation of Human Ocytes”, *Human Reproduction* 8, pp. 740–743.

[Barrow/Tipler (1986)] Barrow, J. D./Tipler, F. J. (1986). *The Anthropic Cosmological Principle*. Oxford: Oxford Univ. Press.

[Batens (2000)] Batens, D. et al. (eds.) (2000). *Frontiers of Paraconsistent Logics*. Baldock, UK: Research Studies Press.

[Behmann (1931)] Behmann, H. (1931). Zu den Widersprüchen der Logik und Mengenlehre. Jahresbericht der deutschen Mathematiker Vereinigung 40, pp. 37–48.

[Benzmüller/Woltzenlogel (2013)] Benzmüller, C./Woltzenlogel, P. B. (2013). “Formalization, Mechnization and Automation of Gödel’s Proof of God’s Existence”, arXiv: 1308.4526[cs.lo].

[Benzmüller/Woltzenlogel (2014)] Benzmüller, C./Woltzenlogel, P. B. (2014). Automating Gödel’s Ontological Proof of God’s Existence with Higher-Order Automated Theorem Provers. In: T. Schaub et al. (eds.): *21st European Conference on Artificial Intelligence. Proceedings*. Amsterdam–Berlin: OS Press, pp. 93–98.

[Bernays (1958)] Bernays, P. (1958). *Axiomatic Set Theory*. Amsterdam: North-Holland Publ. Comp.

[Bernays (1976)] Bernays, P. (1976). *Abhandlungen zur Philosophie der Mathematik*. Darmstadt: Wissenschaftliche Buchgesellschaft.

[Besso (1972)] Besso, M. (1972). P. Speziali (ed.): *Albert Einstein—Michele Besso Correspondence 1903–1955*. Paris: Hermann.

[Bochenski (1961)] Bochenski, J. M. (1961). *History of Formal Logic*. Indiana: Notre Dame Press.

https://doi.org/10.1515/9783110585797-007

[Bochenski (1965)] Bochenski, J. M. (1965). *The Logic of Religion*. New York: New York Univ. Press.

[Bochenski (1974)] Bochenski, J. M. (1974). An Analysis of Authority. In: F.J. Addmann (ed.):*Boston College Studies in Philosophy*. Vol. 3. The Hague: Martinus Nijhoff, pp. 56–85.

[Bochenski (1991)] Bochenski, J. M. (1991). Faith and Science. In: G. Schurz, G. Dorn (eds.):*Advances in Scientific Philosophy. Essays in Honour of Paul Weingartner*. Amsterdam: Rodopi, pp. 531–550.

[Bochenski (2000)] Bochenski, J. M. (2000). The Five Ways. In: A. G. de la Sienra (ed.): *The Rationality of Theism*. Amsterdam: Rodopi, pp. 61–92.

[Boltzmann (1904)] Boltzmann, L. (1904). *Vorlesungen über die Prinzipien der Mechanik*. 2 Vols. Leipzig: J. A. Barth.

[Boltzmann (1968)] Boltzmann, L. (1968). *Wissenschaftliche Abhandlungen*. 3 Vols. New York: Chelsea Publ. Co.

[Bolzano (1929)] Bolzano, B. (1929). *Wissenschaftslehre*. 4 Vols. Ed. by A. Höfler, W. Schultz. Leipzig: Felix Meiner.

[Bonola (1955)] Bonola, R. (1955). *Non-Euclidean Geometry*. New York: Dover.

[Brentano (1929)] Brentano, F. (1929). *Vom Dasein Gottes*. Ed. by A. Kastil. Reprint 1968. Hamburg: Felix Meiner.

[Brentano (1963)] Brentano, F. (1963). *Geschichte der Philosophie*. Bern: Francke Verlag.

[Broda (1986)] Broda, E. (1986). *Ludwig Boltzmann. Mensch, Physiker, Philosoph*. Vienna: Denticke.

[Broglie (1947)] Broglie, L. de (1947). *Physique et Metaphysique*. Paris: Albin-Michel.

[Bulloff (1969)] Bulloff, J. et al. (1969). *Foundations of Mathematics*. Amsterdam: North Holland.

[Bunge (1967)] Bunge, M. (1967). *Scientific Research*. Vol. I. Heidelberg: Springer.

[Bunge (1983)] Bunge, M. (1983). *Exploring the World. Treatise on Basic Philosophy*. Vol. V. Dordrecht: Reidel.

[Buttelmann/Carpenter/Tomasello (2009)] Buttelmann, D./Carpenter, M./Tomasello, M. (2009). "Eighteen-month-old infants show false-belief understanding in an active helping paradigm", *Cognition* 112, pp. 357–342.

[Carnap (1932a)] Carnap, R. (1932a). "Die physikalische Sprache als Universalsprache der Wissenschaft", *Erkenntnis* 2, pp. 432–465.

[Carnap (1932b)] Carnap, R. (1932b). "Psychologie in physikalischer Sprache", *Erkenntnis* 3, pp. 107–142.

[Carnap (1932c)] Carnap, R. (1932c). "Über Protokollsätze", *Erkenntnis* 3, pp. 215–228.

[Carnap (1950)] Carnap, R. (1950). *Logical Foundations of Probability*. Chicago: Univ. of Chicago Press.

[Carnap (1971/1980)] Carnap, R./Jeffrey, C. (1971/1980). *Studies in Inductive Logic and Probability*. Vols. I and II. Berkeley: Univ. of California Press.

[Chapelle (1981)] Chapelle, A. (1981). "The Etiology of Maleness in XX Men", *Human Genetics* 58, pp. 105–116.

[Chisholm (1963)] Chisholm, R. (1963). "The Logic of Knowing", *Journal of Philosophy* 60, pp. 373–395.

[Chisholm (1966)] Chisholm, R. (1966). *Theory of Knowledge*. New York: Prentice Hall.

[Cohen (1963)] Cohen, P. J. (1963). "The Independence of the Continuum Hypothesis", *I. Proceedings of the National Academy of Sciences of the United States of America* 50, pp. 1143–1148.

[Cohen (1964)] Cohen, P. J. (1964). "The Independence of the Continuum Hypothesis", *II. Proceedings of the National Academy of Sciences of the United States of America* 51, pp. 105–110.

[Cohen (1966)] Cohen, P. J. (1966). *Set Theory and the Continuum Hypothesis*. New York: W. A. Benjamin. Repr. 2008. Mineola, NY: Dover.

[Coplestone (1947)] Coplestone, F. (1947). *A History of Philosophy*. Vol. I, London: Burns Oates and Washbourne.

[Czermak (2002)] Czermak, J. (2002). Abriss des ontologischen Arguments. In: Buldt et al. (eds.): *Kurt Gödel. Wahrheit und Beweisbarkeit*. Vol. 2. Wien: öbv&hpt, pp. 208–224.

[Czermak (2010)] Czermak, J. (2010). Überlegungen zum Thema: Gott und Logik. In: A. J. J. Anglberger, P. Weingartner (eds.): *Neuer Atheismus wissenschaftlich betrachtet*. Frankfurt: Ontos, pp. 235–260.

[Czermak (2015)] Czermak, J. (2015). From Ontological Proof to Theological Theories. In: K.Swietorzecka (ed.): *Gödel's Ontological Argument. History, Modifications and Controversies*. Warszawa: Semper, pp. 47–83.

[Dalla Chiara et al. (2001)] Dalla Chiara et al.: "Quantum Logics", arXiv:quant-ph/0101028v2.

[Dawkins (1996)] Dawkins, R. (1996). *Climbing Mount Improbable*. London: Vicking.

[Dawkins (2006)] Dawkins, R. (2006). *The God Delusion*. New York: Bantam Books.

[Denton (1998)] Denton, M. J. (1998). *Nature's Destiny. How the Laws of Biology Reveal Purpose in the Universe*. New York: The Free Press.

[Descartes (RD)] Descartes, R.: Rules for the Direction of the Mind. In: C. Adam, P. Tannery (eds.): *OEuvres de Descartes*. 13 Vols. Paris 1897–1913. Volume 10.

[Descartes (AT)] Descartes, R.: *OEuvres de Descartes*. Edited by C. Adam, P. Tannery. 13 Vols. Paris 1897–1913.

[Diels (1960)] Diels, H. (1960). *Die Fragmente der Vorsokratiker*. Griechisch und Deutsch. 10^{th} edition by W. Kranz, 3 Vols. Berlin.

[Dyson (1972)] Dyson, F. J. (1972). The fundamental constants and their time variation. In: A. Salam, E. P. Wigner (eds.): *Aspects of Quantum Theory*. Cambridge: Cambridge Univ. Press, pp. 213–236.

[Einstein (1905)] Einstein, A. (1905). "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt", *Annalen der Physik* 322/6, pp. 132–148.

[Einstein (1944)] Einstein, A. (1944). Remarks on Bertrand Russell's Theory of Knowledge. In: P. A. Schilpp (ed.): *The Philosophy of Bertrand Russell*. New York: Harper Torchbooks, pp. 277–291.

[Einstein (1949)] Einstein, A. (1949). P. A. Schilpp (ed.): *Albert Einstein. Philosopher-Scientist*. New York: Harper Torchbooks.

[Faber (2001)] Faber, S. M. (2001). The Big Bang as Scientific Fact. In: J. B. Miller (ed.): *Cosmic Questions. Annals of the New York Academy of Sciences*. Vol. 950. New York: New York Academy of Sciences.

[Fitch (1963)] Fitch, F. B. (1963). "A logical analysis of some value concepts", *Journal of Symbolic Logic* 28, pp. 135–142.

[Fitting (2002)] Fitting, M. (2002). Ontological Arguments. In: M. Fitting (ed.): *Types, Tableaus and Gödel's God*. Dordrecht: Kluwer, pp. 133–172.

[Fraenkel (1973)] Fraenkel, A./Bar-Hillel, Y./Levy, A. (1973). *Foundations of Set Theory*. Amsterdam: North-Holland Publ. Comp.

[Freeman (1959)] Freeman, K. (1959). *The Presocratic Philosophers. A Companion to Diels, Fragmente der Vorsokratiker*. Oxford: Blackwell.

[Frege (1884)] Frege, G. (1884). *Die Grundlagen der Arithmetik. Eine logisch-mathematische Untersuchung über den Begriff der Zahl*. Darmstadt: Wissenschaftliche Buchgesellschaft, 1961.

[Frege (1892)] Frege, G. (1892). "Über Sinn und Bedeutung", Zeitschrift für Philosophie und Philosophische Kritik, NF 100, pp. 25–50. Engl. Transl. in: P. Geach, M. Black (eds.): *Translation from the Philosophical Writings of Gottlob Frege*. Oxford, 1952.

[Frege (1893/1903)] Frege, G. (1893/1903). *Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet.* Vol. I (1893), Vol. II (1903). Reprinted: Darmstadt: Wissenschaftliche Buchgesellschaft, 1962.

[Frege (1967)] Frege, G. (1967). *The Basic Laws of Arithmetic.* Transl. and ed. by M. Furth. Los Angeles: Univ. of California Press.

[Fries (1828)] Fries, J. F. (1828). *Neue oder anthropologische Kritik der Vernunft.* Heidelberg: Christian Friedrich Winter.

[Gallagher (2015)] Gallagher, S. (2015). "The Problem with 3-Year-Olds", *Journal of Consciousness Studies* 22, pp. 160–182.

[Gettier (1963)] Gettier, E. (1963). "Is Justified True Belief Knowledge?", *Analysis* 23/6, pp. 121–123.

[Gödel (1931)] Gödel, K. (1931). "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme", *Monatshefte für Mathematik und Physik* 38, pp. 173–198.

[Gödel (1940)] Gödel, K. (1940). "The Consistency of the Axiom of Choice and the Generalized Continuum Hypothesis with the Axiom of Set Theory", *Annals of Mathematics Studies* 3, Princeton.

[Gödel (1949)] Gödel, K. (1949). "An example of a new type of cosmological solution of Eintein's field equations of gravitation", *Reviews of Modern Physics* 21, pp. 444–450. Reprinted in: S. Feferman et al. (eds.): *Kurt Gödel. Collected Works* Vol. II. Oxford: Oxford Univ. Press, 1990.

[Gödel (1952)] Gödel, K. (1952). "Rotating universes in general relativity theory", *Proceedings of the International Congress of Mathematicians* 1950. American Math. Society Vol. I, pp. 175–181. Reprinted in: S. Feferman et al. (eds.): *Kurt Gödel. Collected Works* Vol. II. Oxford: Oxford Univ. Press, 1990.

[Gödel (1970)] Gödel, K. (1970). Ontological Proof. In: S. Feferman et al. (eds.): *Collected Works* Vol. III. Oxford: Oxford Univ. Press, 1995, pp. 403–404.

[Gödel (MaxPhil)] Gödel, K. (MaxPhil). Philosophical Remarks of Gödel's Nachlass. Unpublished.

[Haken (1988)] Haken, H. (1988). *Information and Self-Organisation. A Macroscopic Approach to Complex Systems*. Berlin: Springer.

[Hamilton (2005)] Hamilton, A. (2005). Proprioception as Basic Knowledge of the Body. In: R. v. Woudenberg et al.: *Basic Belief and Basic Knowledge: Papers in Epistemology*. Frankfurt a. M.: Ontos, pp. 269–292.

[Hartle/Hawking (1983)] Hartle, J. B./Hawking, S. W. (1983). "Wave Function of the Universe", *Physical Review* D 28, pp. 2960–2975.

[Hawking/Penrose (1970)] Hawking, S. W./Penrose, R. (1970). "The Singularities of Gravitational Collapse and Cosmology", *Proceedings of the Royal Society* A314/1519, pp. 529–548.

[Hawking/Ellis (1973)] Hawking, S. W./Ellis, G. F. R. (1973). *The Large Scale Structure of Space Time*. Cambridge: Cambridge Univ. Press.

[Hawking/Israel (1979)] Hawking, S. W./Israel, W. (1979) (eds.). *General Relativity. An Einstein Centenary Survey*. Cambridge: Cambridge Univ. Press.

[Hawking (1988)] Hawking, S. W. (1988). *A Brief History of Time*. London: Bantam.

[Heller (2004)] Heller, M. (2004). Can the Universe Explain Itself? In: W. Lüffler, P. Weingartner (eds.): *Knowledge and Belief. Proceedings of the 26th International Wittgenstein Symposium*. Vienna: öbv-hpt.

[Heller (2010)] Heller, M. (2010). *The Sense of Life and the Sense of the Universe*. Krakow: Copernicus Center Press.

[Helmholtz (1868)] Helmholtz, H. (1868). "Über die Tatsachen, welche der Geometrie zugrunde liegen", *Nachrichten der Königlichen Gesellschaft der Wissenschaften und der Georg August Universität* 9, pp. 193–221.

[Hick (1993)] Hick, J. (1993). *Disputed Questions in Theology and the Philosophy of Religion*. London: Macmillan.

[Hick (1994)] Hick, J. (1994). Religious Experience. In: P. Weingartner (ed.): *Scientific and Religious Belief*. Dordrecht: Springer, pp. 161–173.

[Hilbert/Barnays (1939)] Hilbert, D./Barnays, P. (1939). *Grundlagen der Mathematik*. Vol. 2. Berlin: Springer.

[Hintikka (1962)] Hintikka, J. (1962). *Knowledge and Belief*. Ithaca, New York: Cornell Univ. Press.

[Hintikka (1964)] Hintikka, J. (1964). "The Once and Future Sea Fight: Aristotle's Discussions of Future Contingents in De Interpretatione 9", *Philosophical Review* 73, pp. 461–492.

[Hintikka (1966a)] Hintikka, J./Hilpinen, R. (1966a). Knowledge Acceptance and Inductive Logic. In: J. Hintikka, P. Suppes (eds.): *Aspects of Inductive Logic*. Amsterdam: North-Holland Publ. Comp.

[Hintikka (1966b)] Hintikka, J. (1966b). A Two-Dimensional Continuum of Inductive Methods. In: J. Hintikka, P. Suppes (eds.): *Aspects of Inductive Logic*. Amsterdam: North-Holland Publ. Comp.

[Hintikka (1968)] Hintikka, J. (1968). Induction by Enumeration and Induction by Elimination. In: I. Lakatos (ed.): *The Problem of Inductive Logic*. Amsterdam: North-Holland Publ. Comp., pp. 191–216.

[Hintikka (1974)] Hintikka, J. (1974). *Knowledge and the Known*. Dordrecht: Reidel.

[Hintikka (1991)] Hintikka, J. (1991). *Defining Truth, the whole Truth and Nothing But the Truth*. Reports from the Department of Philosophy, University of Helsinki.

[Hintikka (1996)] Hintikka, J. (1996). *The Principle of Mathematics Revisited*. Cambridge: Cambridge Univ. Press.

[Hintikka (2013)] Hintikka, J. (2013). *Knowledge is Justifiable Information*. Mimeographed Typoscript J. Hintikka 1110.1210.120810.

[Hoyle/Wickramasinghe (1981)] Hoyle, F/Wickramasinghe C. (1981). *Evolution aus dem Weltraum*. Berlin: Ullstein.

[Husserl (1989)] Husserl, E. (1989). *Ideas Pertaining to a Pare Phenomenology and to a Phenomenological Philosophy*. Second Book, transl. by R. Rojcewicz and A. Schuwer. Dordrecht: Kluwer.

[Iserloh (1962)] Iserloh, E. (1962). *Luthers Thesenanschlag. Tatsache oder Legende?* Wiesbaden: Steiner.

[Jammer (1966)] Jammer, M. (1966). *The Conceptual Development of Quantum Mechanics*. New York: McGraw-Hill.

[Junker/Scherer (1998)] Junker, R./Scherer, S. (1998). *Evolution. Ein kritisches Lehrbuch*. Giessen: Weyel.

[Kant (1783)] Kant, I. (1783). *Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wird auftreten können*. Riga: Hartknoch. Reprinted in London: McMillan, 1929.

[Kant (1787)] Kant, I. (1787). *Kritik der reinen Vernunft*. Riga: Hartknoch. Reprinted in London: McMillan, 1929.

[Kemp (2005)] Kemp, T. S. (2005). *The Origin and Evoution of Mammals*. Oxford: Oxford Univ. Press.

[Kenny (1969)] Kenny, A. (1969). *The Five Ways. St. Thomas Aquinas' Proofs of God's Existence*. London: Routledge.

[Ketland (1999)] Ketland, J. (1999). "Deflationism and Tarski's Paradise", *Mind* 108/429, pp. 69–94.

[Kreisel (1967)] Kreisel, G. (1967). Informal Rigour and Completeness Proofs. In: L. Lakatos: *Problems in the Philosophy of Mathematics*. Amsterdam: North Holland Publ. Comp., pp. 138–186.

[Kreisel (1981)] Kreisel, G. (1981). Aussagenquantoren: Ein Beitrag der mathematischen Logik zur Erkenntnistheorie. In: K. Weinke: *Logik, Ethik und Sprache. Festschrift für Rudolf Freundlich*. Vienna–Munich: Oldenburg, pp. 266–287.

[Kripke (1975)] Kripke, S. (1975). "Outline of a Theory of Truth", *Journal of Philosophy* 72, pp. 690–716.

[Kronheimer/Penrose (1967)] Kronheimer, E. H./Penrose, R. (1967). "On the structure of causal spaces", *Proceedings of the Cambridge Philosophical Society* 63, pp. 481–501.

[Kuhn (1978)] Kuhn, Th. (1978). *Black-Body Theory and the Quantum Theory 1894–1912*. Oxford: Oxford Univ. Press.

[Kuipers (1982)] Kuipers, T. A. (1982). "Approaching Descriptive and Theoretical Truth", *Erkenntnis* 18, pp. 343–178.

[Küppers (1990)] Küppers, B. O. (1990). *Information and the Origin of Life*. Cambridge, Mass.: MIT Press.

[Kutschera (2008)] Kutschera, U. (2008). *Evolutionsbiologie*. Stuttgart: Eugen Ulmer.

[Lambert (1983)] Lambert, K. (1983). *Meinong and the Principle of Independence*. Cambridge: Cambridge Univ. Press.

[Lebowitz (1994)] Lebowitz, J. L. (1994). Time's Arrow and Boltzmann's Entropy. In: J. J. Halliwell et al. (eds.): *Physical Origins of Time Asymmetry*. Cambridge: Cambridge Univ. Press.

[Lehrer (1974)] Lehrer, K. (1974). *Knowledge*. Oxford: Oxford Univ. Press.

[Leibniz (GP)] Leibniz, G. W. (1965). *Die philosophischen Schriften von G. W. Leibniz*. Edited by C. I. Gerhardt (ed.), 7 Vols. Berlin 1875–90. Hildesheim: Georg Olms.

[Leibniz (NE)] Leibniz, G. W. (1978): *Nouveaux essais sur l'intendament humaine*. In: C. I. Gerhardt (ed.): *Die philosophischen Schriften von G. W. Leibniz*, 7 Vols. Berlin 1875–90. Hildesheim: Georg Olms.

[Leibniz (OF)] Leibniz, G. W. (1903) *Opuscules et fragments inédits de Leibniz*. Ed. by L. Couturat. Hildesheim: Georg Olms.

[Lighthill (1986)] Lighthill, J. (1986). "The Recently Recognized Failure of Predictability in Newtonian Dynamics", *Proceedings of the Royal Society London* A 407, pp. 35–50.

[Löb (1955)] Löb, M. H. (1955). "Solution of a Problem of Leon Henkin", *Journal of Symbolic Logic* 20/2, pp. 115–118.

[Lorentz (1895)] Lorentz, H. A. (1895). *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern*. Leiden: E. J. Brill.

[Łukasiewicz (1957)] Łukasiewicz, J. (1957). *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*. Oxford: Oxford Univ. Press.

[Mahner/Bunge (2000)] Mahner, M./Bunge, M. (2000). *Philosophische Grundlagen der Biologie*. Berlin: Springer.

[Mainzer (2004a)] Mainzer, K. (2004). Geometrie. In: Mittelstrass et al. (eds.): *Enzyklopädie Philosophie und Wissenschaftstheorie*. Vol. I. Stuttgart: Metzlersche Verlagsbuchhandlung, pp. 737–744.

[Mainzer (2004b)] Mainzer, K. (2004). Nicht-Euklidische Geometrie. In: Mittelstrass et al. (eds.): *Enzyklopädie Philosophie und Wissenschaftstheorie*. Vol. II. Stuttgart: Metzlersche Verlagsbuchhandlung, pp. 1006–1007.

[Markopoulou (1998)] Markopoulou, F. (1998). "The internal description of a causal set: What the universe looks like from the inside", *Commun. Math. Phys.* 211, pp. 559–583.

[Matijasevic (1970)] Matijasevic, Yu. V. (1970). "Enumerable sets are diophantine", *Dokladý Akademia Nauk SSSR* 191, pp. 279–282. Revised Engl. transl. in: *Soviet Math. Dokladý* 11 (1970), pp. 354–358.

[Maynard-Smith/Szathmary (1999)] Maynard-Smith, J./Szathmary, E. (1999). *The Major Transition in Evolution*. Oxford: Oxford Univ. Press.

[Mayr (1942)] Mayr, E. (1942). *Systematics and the Origin of Species*. New York: Columbia Univ. Press.

[Meessen (1989)] Meessen, A. (1989). Is it Logically Possible to Generalize Physics through Space-Time Quantization? In: P. Weingartner, G. Schurz: *Philosophy of the Natural Sciences. Proceedings of the 13th Int. Wittgenstein Symposium 1988*. Vienna: Pichler-Tempsky.

[Meessen (2000)] Meessen, A. (2000). "Spacetime Quantisation. Elementary Particles and Cosmology", *Foundation of Physics* 29, pp. 281–316.

[Meschkowski (1978)] Meschkowski, H. (1978). *Problemgeschichte der neueren Mathematik*. Mannheim: B. I. Wissenschaftsverlag.

[Meixner (2012)] Meixner, U. (2012). A Cosmo-Ontological Argument for the Existence of a First Cause—perhaps God. In: M. Szatkowski: *Ontological Proofs Today*. Frankfurt a. M.: Ontos, pp. 193–202.

[Meyer et al. (1990)] Meyer, A. et al. (1990). "Monophyletic Origin of Lake Viktoria Cichlid Fishes Suggested by Mitochondrial DNA Sequences", *Nature* 347/6293, pp. 550–553.

[ML] Migne, J. P. (1887). Patrologiae. Series Latina.

[Milgrom] Milgrom, A. (1983). "A Modification of Newtonian Dynamics", *Astrophysical Journal* 270, pp.365–389.

[Miller (1974)] Miller, D. (1974). "Popper's Qualitative Theory of Verisimilitude", *British Journal for the Philosophy of Science* 25, pp. 166–177.

[Miller (1978)] Miller, D. (1978). "The Distance Between Constituents", *Synthese* 38, pp. 197–212.

[Mittelstaedt/Weingartner (2005)] Mittelstaedt, P./Weingartner, P. (2005). *Laws of Nature*. Heidelberg: Springer.

[Mittelstaedt (2008)] Mittelstaedt, P. (2008). Concepts of Time in Physics and Cosmology. In: E. Agazzi (ed.): *Time in the Different Scientific Approaches*. Genova: Tilgher, pp. 113–141.

[Moffitt et al. (2014)] Moffitt, T. E. et al. (2014). "Ein besseres Leben dank früher Selbstbeherrschung", *Spektrum der Wissenschaft*, 14.11.2014.

[Monod (1972)] Monod, J. L. (1972). *Chance and Necessity*. New York: Knopf.

[Morscher (1991)] Morscher, E. (1991). Was sind und was sollen die Gottesbeweise? Bemerkungen zu Anselms Gottesbeweis(en). In: F. Ricken (ed.): *Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstheorie*. 2nd edition. Stuttgart: Kohlhammer, pp. 60–84.

[Morscher (2016)] Morscher, E. (2016). *Bernard Bolzanos Lehre von Grund und Folge. Eine axiomatische Rekonstruktion*. Beiträge zur Bolzano-Forschung 27. St. Augustin: Academia.

[Myhill (1950)] Myhill, J. (1950). “A System which can Define its own Truth”, *Fundamenta Mathematica* 37, pp. 190–192.

[Nagel/Newman (1959)] Nagel, E./Newman, J. R. (1959). *Gödel’s Proof*. London: Routledge.

[Neurath (1931)] Neurath, O. (1931). “Soziologie im Physikalismus”, *Erkenntnis* 2, p. 403.

[Newton (Princ)] Newton, I. (1934). *Mathematical Principles of Natural Philosophy*. Ed. by F. Cajori. Berkeley: Univ. of California Press.

[Nieznanski (1980)] Nieznanski, E. (1980). “Formalized Attempts to Set Formal Logical Bases to State First Elements of Relations Considered in the Thomistic Theodicy”, *Miscellanea Logica* 1, pp. 7–194.

[Nieznanski (2003)] Nieznanski, E. (2003). Logical Analysis of Thomism. In: J. Salamucha: *Knowledge and Faith*, ed. by K. Swietorecka, J. J. Jadacki. Amsterdam: Rodopi.

[Niiniluoto (1987)] Niiniluoto, I. (1987). *Truthlikeness*. Dordrecht: Reidel.

[Oderberg (2007)] Oderberg, D. S. (2007). *Real Essentialism*. London: Routledge.

[Onishi/Baillargeon (2005)] Onishi, K. H./Baillargeon, R. (2005). “Do 15-month-old infants understand false belief?”, *Science* 308, pp. 255–258.

[Pais (1982)] Pais, A. (1982). *Subtle is the Lord. The Science and the Life of Albert Einstein*. Oxford: Oxford Univ. Press.

[Parkinson (1966)] Parkinson, G. H. R. (1966). *Leibniz. Logical Papers*. A Selection translated and edited with an introduction by G. H. R. Parkinson. Oxford: Clarendon Press.

[Pascal (EGA)] Pascal, B. (1963). De l’esprit géometrique et de l’art de persuader. In: H. Gouhier, L. Lafua (eds.): *Pascal. OEuvres complètes*, Paris: Editions du Seuil.

[Pauli (1979)] Pauli, W. (1979). *Wolfgang Pauli. Scientific Correspondence with Bohr, Einstein, Heisenberg*. Ed. by A. Hermann et al. Vol. I: 1919–1929. New York: Springer.

[Page (1985)] Page, D. et al. (1985). “Chromosome-Specific DNA in Related Human XX Males”, *Nature* 315, pp. 224–226.

[Peano (1889)] Peano, G. (1889). *Arithmetics principia, novo methodo exposita*. Torino: Fratres Bocca.

[Peirce (1960)] Peirce, C. S. (1960). Collected Papers of Charles Sanders Peirce. Ed by Ch. Hartshorne, P. Weiss. Cambridge, Mass.: Harvard Univ. Press.

[Penrose (1965)] Penrose (1965). “Gravitational Collapse and Space-Time-Singularities”, *Physical Review Letters* 14/3, pp. 57–59.

[Penrose (1979)] Penrose (1979). Singularities and Time Asymmetry. In: S. W. Hawking, W. Israel: *General Relativity. An Einstein Centenary Survey*. Cambridge: Cambridge Univ. Press, pp. 581–638.

[Penzias/Wilson (1965)] Penzias, A. A./Wilson, R. W. (1965). “A Measurement of Excess Antenna Temperature at 4080 Mc/s”, *Astrophysical Journal* 142, pp. 419–421.

[Perelman (2002)] Perelman, G. (2002). “The entropy formula for the Ricci flow and its geometric applications”, arXiv:math/0211259.

[Petit (1987)] Petit, C. (1987). “In Abnormal Terminal X-Y Interchange Accounts for Most but not all Cases of Human XX-Maleness”, *Cell* 49, pp. 595–602.

[Plantinga (1967)] Plantinga, A. (1967). *God and other Minds*. Ithaca, New York: Cornell Univ. Press.

[Plantinga (2000)] Plantinga, A. (2000). *Warranted Christian Belief*. Oxford: Oxford Univ. Press.

[Plato (The)] Plato (1958). Theaitetos. In: W. F. Otto et al. (eds.): *Plato. Sämtliche Werke*. Hamburg: Rowohlt.

[Poincaré (1898)] Poincaré, H. (1898). "La mesure du temps", *Revue de Métaphysique et Morale* 6/1, pp. 1-13.

[Polanyi (1958)] Polanyi, M. (1958). *Personal Knowledge*. Chicago: Univ. of Chicago Press.

[Polkinghorne (1996)] Polkinghorne, J. (1996). *The Faith of a Physicist*. Princeton: Fortress Press.

[Popper (1959)] Popper, K. R. (1959). *The Logic of Scientific Discovery*. London: Hutchinson. (German edition: 1935).

[Popper (1963)] Popper, K. R. (1963). *Conjectures and Refutations. The Growth of Scientific Knowledge*. London: Routledge and Kegan Paul.

[Popper (1968)] Popper, K. R. (1968). Epistemology without a knowing subject. In: B. van Rootselaar, J. F. Staal (eds.): *Logic, Methodology and Philosophy of Science III*. Amsterdam: North-Holland Publ. Comp., pp. 333–373.

[Popper (1972)] Popper, K. R. (1972). *Objective Knowledge*. Oxford: Oxford Univ. Press.

[Popper (1983)] Popper, K. R. (1983). *Realism and the Aim of Science*. London: Hutchinson.

[Prigogine (1981)] Prigogine, I. (1981). *From Being to Becoming. Time and Complexity in Physical Sciences*. San Francisco: Freeman.

[Prigogine (1985)] Prigogine, I. (1985). *Vom Sein zum Werden. Zeit und Komplexität in den Naturwissenschaften*. München: Piper.

[Quine (1951)] Quine, W. v. O. (1951). *Mathematical Logic*. Cambridge, Mass.: Harvard Univ. Press.

[Rescher (1968)] Rescher, N. (1968). *Topics in Philosophical Logic*. Dordrecht: Reidel.

[Rescher (1969)] Rescher, N. (1969). *Many-valued Logic*. New York: McGraw-Hill.

[Rescher (2005)] Rescher, N. (2005). *Epistemic Logic*. Pittsburgh: Pittsburgh Univ. Press.

[Routley (1981)] Routley, R. (1981). Necessary Limits to Knowledge: Unknowable Truths. In: E. Morscher et al. (eds.): *Philosophie als Wissenschaft. Essays in Scientific Philosophy*. Bad Reichenhall: Comes Verlag, pp. 93–115.

[Rubio-Fernandez/Geurts (2013)] Rubio-Fernandez, P./Geurts, B. (2013). "How to pass the false-belief task before your fourth birthday", *Psychological Science* 24, pp. 27–33.

[Ruelle (1991)] Ruelle (1991). *Chance and Chaos*. Princeton: Princeton Univ. Press.

[Russell (1940)] Russell (1940). *An Inquiry into Meaning and Truth*. London: Ullen and Unwin.

[Saari (2015a)] Saari, D. G. (2015a). "N-Body Solutions and Computing Galactic Masses", *Astronomical Journal* 149/174, pp. 1–6.

[Saari (2015b)] Saari, D. G. (2015b). "Mathematics and the "Dark Matter" Puzzle", *American Mathematical Monthly* 122, pp. 407–423.

[Salam/Wigner (1972)] Salam, A./Wigner, E. P. (1972). *Aspects of Quantum Theory*. Cambridge: Cambridge Univ. Press.

[Salamucha (2003)] Salamucha, J. (2003). *Knowledge and Faith*. Ed. by K. Świetorzecka, J. J. Jadacki. Amsterdam: Rodopi.

[Salzburger/Meyer (2004)] Salzburger, W./Meyer, A. (2004). "he Species Flocks of East African Cichlid Fishes: Recent Advances in Molecular Phylogenetics and Population Genetics", *Naturwissenschaften* 91, pp. 277–290.

[Scholz (2009)] Scholz, O. (2009). “Experts: What They Are and How We Recognize Them—A Discussion of Alvin Goldman’s Views”, *Grazer Philosophische Studien* 79, pp. 187–205.

[Schrödinger (1944)] Schrödinger, E. (1944). *What is Life?* Cambridge: Cambridge Univ. Press.

[Schurz/Weingartner (1987)] Schurz, G./Weingartner, P. (1987). Verisimilitude Defined by Relevant Consequence-Elements. A new Reconstruction of Popper’s Original Idea. In: Th. Kuiper (ed.): *What is Closer-to-the-Truth?* Amsterdam: Rodopi, pp. 47–77.

[Schurz (1991)] Schurz, G. (1991). “Relevant Deduction”, *Erkenntnis* 35, pp. 391–437.

[Schurz (1997)] Schurz, G. (1997). *The Is-Ought Problem. An Investigation in Philosophical Logic*. Dordrecht: Kluwer.

[Schurz/Weingartner (2010)] Schurz, G./Weingartner, P. (2010). “Zwart and Franssen’s Impossibility Theorem Holds for Possible-World-Accounts to Verisimilitude”, *Synthese* 172/3, pp. 415–436.

[Schurz (2011)] Schurz, G. (2011). “Verisimilitude and Belief Revision. With a Focus on the Relevant Element Account”, *Erkenntnis* 55, pp. 203–221.

[Schütte (1960)] Schütte, K. (1960). *Beweistheorie*. Berlin: Springer. Engl. transl.: Proof Theory. Berlin: Springer, 1977.

[Sextus Empiricus (OPr)] Sextus Empiricus (1933/1961). *Outlines of Pyrrhonism*. 4 Vols. Cambridge, Mass.: Harvard Univ. Press.

[Sextus Empiricus (AMt)] Sextus Empiricus (1914). *Adversus Mathematicos* Vols VII–XI of Adversus Dogmaticos. Leipzig.

[Shannon/Weaver (1949)] Shannon, C. E./Weaver, W. (1949). *The Mathematical Theory of Communication*. Urbano, Illinois: Univ. of Illinois Press.

[Siegwart (1991)] Siegwart, G. (1991). “Et hoc dici mus deums”. Eine definitionstheoretische Betrachtung zu STHIq2a3. In: F. Ricken: *Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstheorie*. 2nd edition. Stuttgart: Kohlhammer, pp. 85–108.

[Sorkin (1991)] Sorkin, R. D. (1991). Spacetime and Causal Sets. In: J. C. D’Olivo et al. (eds.): *Relativity and Gravitation: Classical and Quantum*. Singapur: World Scientific.

[Stachel (2001)] Stachel (2001). *Einsteins Annus Mirabilis*. Hamburg: Rowohlt.

[Stegmüller (1958)] Stegmüller, W. (1958). “Der Phänomenalismus und seine Schwierigkeiten”, *Archiv für Philosophie* 8, pp. 36–100.

[Šwietorzecka (2015)] Šwietorzecka, K. (ed.) (2015). *Gödel’s Ontological Argument. History, Modifications and Controversies*. Warszawa: Semper.

[Swinburne (1979)] Swinburne, R. (1979). *The Existence of God*. Oxford: Oxford Univ. Press.

[Swinburne (1981)] Swinburne, R. (1981). *Faith and Reason*. Oxford: Oxford Univ. Press.

[Szatkowski (2012)] Szatkowski, M. (2012). *Ontological Proofs Today*. Frankfurt a. M.: Ontos.

[Tarski (1935)] Tarski, A. (1935). “Der Wahrheitsbegriff in den formalisierten Sprachen”, *Studia Philosophica*, Vol. I, Leopoli. Engl. transl. in: A. Tarski: *Logic, Semantics, Mathematics*. Oxford: Oxford Univ. Press, pp. 152–268.

[Tarski (1936)] Tarski, A. (1936). “Über den Begriff der logischen Folgerung”, Actes du Congress Internationale de Philosophie Scientifique 7, Paris 1936, pp. 1–11. Engl. transl. in: A. Tarski: *Logic, Semantics, Mathematics*. Oxford: Oxford Univ. Press.

[Tarski (1956)] Tarski (1956). *Logic, Semantics, Mathematics*. Oxford: Oxford Univ. Press.

[Thomas Aquinas (CMA)] Thomas Aquinas (1961). *Commentary on the Metaphysics of Aristotle*. 2 volumes. Transl. by J. P. Rowen. Chicago: Henry Regnery Company.

[Thomas Aquinas (SCG)] Thomas Aquinas (1974–1996). *Summa Contra Gentiles*. 4 Vols. Latin/German. Darmstadt: Wissenschaftliche Buchgesellschaft.

[Thomas Aquinas (STh)] Thomas Aquinas (1948). *Summa Theologica*. Transl. by Fathers of the English Dominican Province. Christian Classics. Maryland: Westminster. Reprinted: 1981.

[Thomas Aquinas (Ver)] Thomas Aquinas (1953). *De Veritate*. The Disputed Questions on Truth. Vol. I–III. Transl. by J. V. McGlynn. Chicago: Henry Regnery Company.

[Tichy (1974)] Tichy, P. (1974). "On Popper's Definition of Verisimilitude", *British Journal for the Philosophy of Science* 25, pp. 155–165.

[Tichy (1978)] Tichy, P. (1978). "Verisimilitude Revisited", *Synthese* 38, pp. 175–196.

[Tipler (2007)] Tipler, F. J. (2007). *The Physics of Christianity*. New York: Doubleday.

[Torrance (1969)] Torrance, T. F. (1969). *Theological Science*. Oxford: Oxford Univ. Press.

[Turing (1936)] Turing, A. (1936). "On computable numbers, with an application to the Entscheidungsproblem", *Proceedings of the London Mathematical Society*, 2/42, pp. 230–265.

[Weinberg (1977)] Weinberg, S. (1977). *The First Three Minutes. A Modern View of the Origin of the Universe*. London: Fontana.

[Weinberg (1987)] Weinberg, S. (1987). Towards the Final Laws of Physics. In: R. P. Feynman, S. Weinberg: *Elementary Particles and the Laws of Physics. The 1986 Dirac Memorial Lectures*. Cambridge: Cambridge Univ. Press.

[Weingartner (1964)] Weingartner, P. (1964). Vier Fragen zum Wahrheitsbegriff. In: *Salzburger Jahrbuch für Philosophie* 8, pp. 31–74. Engl. transl. in: Weingartner (2000): *Basic Questions on Truth*. Dordrecht: Kluwer, chs. 1–4.

[Weingartner (1968)] Weingartner, P. (1968). "Modal Logics with Two Kinds of Necessity and Possibility", *Notre Dame Journal of Formal Logic* 9, pp. 97–159.

[Weingartner (1978)] Weingartner, P. (1978). *Wissenschaftstheorie I. Einführung in die Hauptprobleme*. 2nd edition. Stuttgart–Bad Cannstatt: Frommann-Holzboog.

[Weingartner (1981)] Weingartner, P. (1981). "A System of Rational Belief, Knowledge and Assumption", *Grazer Philosophische Studien* 12/13, pp. 143–165.

[Weingartner (1982)] Weingartner, P. (1982). "Conditions of Rationality for the Concepts Belief, Knowledge and Assumption", *Dialectica* 36, pp. 243–263.

[Weingartner (1983a)] Weingartner, P. (1983a). The Ideal of the Mathematization of all Sciences and of 'More Geometrico' in Descartes and Leibniz. In: W. R. Shea (ed.): *Nature Mathematized*. Dordrecht: Reidel, pp. 151–195.

[Weingartner (1983b)] Weingartner, P. (1983b). Definition of Value-Judgement. In: E. Agazzi (ed.): *La Responsibilité de la Science*. Epistemologia 6, pp. 79–86.

[Weingartner (1984)] Weingartner, P. (1984). On the Introduction of Teleological Arguments into Scientific Discourse. Archives de l'Institute Internationale des Sciences Théoretiques 26, Bruxelles, pp. 196–218.

[Weingartner/Schmetterer (1987)] Weingartner, P./Schmetterer, L. (1987). *Gödel Remembered*. Napoli: Bibliopolis.

[Weingartner (1991)] Weingartner, P. (1991). A Note on Aristotle's Theory of Definition and Scientific Explanation. In: W. Spohn (ed.): *Existence and Eplanation*. Dordrecht: Reidel.

[Weingartner (1992)] Weingartner, P. (1992). Hat in der Heiligen Schrift ein und dasselbe Wort einen mehrfachen Sinn? Kommentar zur Summa Theologica I, 1, 10. In: G. Putz, H. Dachs, F. Horner, F. Reisinger (eds.): *Politik und Christliche Verantwortung. Festschrift für F. M. Schmölz.*. Innsbruck: Tyrolia, pp. 94–111.

[Weingartner (1994)] Weingartner, P. (1994). Scientific and Religious Belief. *Philosophical Studies Series* 59. Dordrecht: Kluwer.

[Weingartner (1996a)] Weingartner, P. (1996a). A note on Gettier's Problem. In: G. Heinzmann (ed.): *La Science e l'hypothèse. Proceedings of the 1994 Conference of the Academie Internationale de Philosophie des Sciences. Philosophia Scientia*, pp. 221–231.

[Weingartner (1996b)] Weingartner, P. (1996b). Under What Transformations are Laws Invariant. In: P. Weingartner, G. Schurz (eds.): *Law and Prediction in the Light of Chaos Research. Lecture Notes in Physics*. Vol 473. Heidelberg: Springer, pp. 47–88.

[Weingartner (1997)] Weingartner, P. (1997). Kann sich das natürliche Gesetz ändern? Kommentar zu Thomas von Aquins S.Th. I–II, 94.5. In: W. Löffler, E. Runggaldier (eds.): *Dialog und System. Otto Muck zum 65. Geburtstag. Conceptus Studien* 12, pp. 197–230.

[Weingartner (2000a)] Weingartner, P. (2000a). *Basic Questions on Truth*. Dordrecht: Kluwer.

[Weingartner (2000b)] Weingartner, P. (2000b). Reasons for Filtering Classical Logic. In: D. Batens et al.: *Frontiers for Filtering Classical Logic*. Dordrecht: Kluwer, pp. 315–327.

[Weingartner (2001)] Weingartner, P. (2001). Applications of Logic outside Logic and Mathematics. Do such applications force us to deviate from Classical Logic? In: W. Stelzner, M. Stöckler (eds.): *Zwischen traditioneller und moderner Logik*. Paderborn: Mentis, pp. 53–64.

[Weingartner (2008)] Weingartner, P. (2008). *Omniscience. From a Logical Point of View*. Hengenstamm-Frankfurt: Ontos.

[Weingartner (2009)] Weingartner, P. (2009). "Matrix Based Logic for Application in Physics", *The Review of Symbolic Logic* 2, pp. 132–163.

[Weingartner (2010a)] Weingartner, P. (2010a). "Basic Logic for Application in Physics and its Intuitionistic Alternative", *Foundations of Physics* 40, pp. 1578–1596.

[Weingartner (2010b)] Weingartner, P. (2010b). "An Alternative Propositional Calculus for Application to Empirical Sciences", *Studia Logica* 95, pp. 233–257.

[Weingartner (2010c)] Weingartner, P. (2010c). *God's Existence. Can it be Proven? A logical Commentary on the Five Ways of Thomas Aquinas*. Frankfurt a. M.: Ontos.

[Weingartner (2012)] Weingartner, P. (2012). The Premises of Anselm's Argument. In: M. Szatkowski: *Ontological Proofs Today*. Frankfurt a. M.: Ontos, pp. 423–441.

[Weingartner (2015a)] Weingartner, P. (2015a). A 6-Valued Calculus which Avoids the Paradoxes of Deontic Logic. In: J. Y. Beziau, D. Krause (eds.): *Essays in Honor of Patrick Suppes' 90th Birthday*. College Publication, Tribute Series, pp. 217–228.

[Weingartner (2015b)] Weingartner, P. (2015b). *Nature's Teleological Order and God's Providence. Are They Compatible with Chance, Free Will, and Evil?*. Berlin: De Gruyter.

[Weingartner (2016)] Weingartner, P. (2016). "The Need for Pluralism of Causality", *Logic and Logical Philosophy* 25, pp. 461–498.

[Whitehead/Russell (1927)] Whitehead, A. N./Russell, B. (1927). *Principia Mathematica*. Cambridge: Cambridge Univ. Press.

[Wigner (1967)] Wigner, E. P. (1967). Symmetries and Reflections. In: W. J. Moore, M. Scriven: *Scientific Essays of Eugene P. Wigner*. Bloomington, Indiana: Indiana Univ. Press, pp. 15–27.

[Wiles (1995)] Wiles, A. (1995). "Modular Elliptic Curves Fermat's Last Theorem", *Annals of Mathematics* 141/3, pp. 443–551.

[Williamson (1990)] Williamson, T. (1990). "Two incomplete anti-realist modal epistemic logics", *Journal of Symbolic Logic* 55, pp. 297–314.

[Williamson (2000)] Williamson, T. (2000). *Knowledge and its Limits*. Oxford: Oxford Univ. Press.

[Wimmer/Perner (1983)] Wimmer, H./Perner, J. (1983). "Beliefs about beliefs: Representation and constraining function of wrong beliefs in young children's understanding of deception", *Cognition* 13, pp. 103–128.

[Winston et al. (1991)] Winston et al. (1991). "Pathenogenetic Activation and Development of Fresh and Aged Human Ocytes", *Fertility and Sterility* 53, pp. 266–279.

[Wittgenstein (1956)] Wittgenstein, L. (1956). *Bemerkungen über die Grundlagen der Mathematik. Remarks on the Foundations of Mathematics*. Ed. by G. H. von Wright, R. Rhees, G. E. M. Anscombe. Oxford: Blackwell.

[Wittgenstein (1960)] Wittgenstein, L. (1960). *Tractatus Logico Philosophicus*. Schriften 1. Frankfurt a. M.: Suhrkamp.

[Wittgenstein (1969)] Wittgenstein, L. (1969). *On Certainty*. Ed. by G. E. M. Anscombe, G. H. von Wright. Oxford: Blackwell.

[Wolschin (2013)] Wolschin, G. (2013). "Krönender Abschluss des Standardmodells. Peter W. Higgs und Francis Englert erhalten den Physiknobelpreis 2013", *Spektrum der Wissenschaft* 12, pp. 19–23.

[Wolters (1987)] Wolters, G. (1987). *Mach I, Mach II, Einstein und die Relativitätstheorie. Eine Fälschung und ihre Folgen*. Berlin: De Gruyter.

[Woudenberg (2005)] Woudenberg, R. v. (2005). *Basic Belief and Basic Knowledge: Papers in Epistemology*. Frankfurt: Ontos Verlag.

[Zermelo (1908)] Zermelo, E. (1908). "Untersuchungen über die Grundlagen der Mengenlehre I", *Mathematische Annalen* 65, pp. 261–281. Engl. transl. in: J. v. Heijenoort (ed.): *From Frege to Gödel—a source book in mathematical logic 1879–1931.* Cambridge, Mass.: Harvard Univ. Press.

[Zwart/Franssen (2007)] Zwart, S. D./Franssen, M. (2007). "An Impossibility Theorem for Verisimilitude", *Synthese* 158, pp. 75–92.

Index

https://doi.org/10.1515/9783110585797-015