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Growth and Distribution

SECOND EDITION

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For Betsy, Glyn, and Meredith
For Ruth and Gerry
For Silvia and Luca

Contents

Preface to the Second Edition

By the time this book is in print, two decades will have passed since the publication of the first edition. We have benefited enormously from the feedback we have received from the students, instructors, and researchers who have used this book. The current edition incorporates both the positive and the negative reactions to the first edition. In this regard we would like to thank Deepankar Basu, Scott Carter, Laura Carvalho, Heinz Kurz, Javier Lopez Bernardo, Ulrich Morawetz, Michalis Nikiforos, Engelbert Stockhammer, and Luca Zamparelli for their insightful comments and suggestions, and Adalmir Marquetti, who maintains the EPWT database we use in this book and has also given us helpful comments on this second edition revision. We are especially grateful to Mike Aronson, who edited the first edition, for his encouragement in getting the new edition off the ground and insightful comments on drafts. We have ruthlessly cut chapters that did not work out. Also, we have tried to build on our distinctive emphasis on the wage-profit schedule and the problem of closure that runs like a thread through the text. The original authors are pleased to have been joined in this effort by Daniele Tavani, who is part of a new generation of political economists influenced by the first edition.

Aside from improving the pedagogy and exposition, we have been motivated by the stunning changes in modern capitalism at the close of the twentieth century and opening of the twenty-first century. At the time the first edition was written, the major crisis of capitalism that occupied our attention had taken place in the 1970s. This was, by most accounts, a crisis of low or declining profitability. The focus of the book was then on explaining the slowdowns in economic growth that had occurred in the intervening years through this lens. One pattern that seemed to predominate was a persistent capital-using bias in technical change that we dubbed "Marx-biased technical change." We continue to believe that understanding biased technical change is one of the central tasks of a theory of economic growth. But over the last decades, the patterns of technical change in the advanced world have grown increasingly diverse, and in the developing world, as users of our text reported, we find instances of labor-using technical change that turn the Marx-biased pattern on its head.

While its outlines were only beginning to come into focus in the 1990s, it has become clear that a new variant of capitalism has emerged that is now generally referred to as neoliberal capitalism. Among its characteristic features, most sharply visible in the United States but global in scope, has been an unrelenting rise in income inequality taking the form of a decoupling between real wage growth and productivity growth that has shifted massive amounts of income from wages toward profits and toward the compensation of top corporate executives. This has been accompanied by an increased role for financial mechanisms and a hypertrophied financial system, where executive compensation has reached almost unbelievable levels. The Global Financial Crisis that began in 2008 can only be understood as a crisis of neoliberal capitalism, in a sense a crisis of high or rising profitability. It is not surprising that there has been a renewed interest among professional economists and the lay public in the growing polarization of income and wealth. We have attempted to engage with this new reality in the current edition by including new material on financial markets, corporate capitalism, and wealth distribution, and by revising our treatment of aggregate demand.

There have also been exciting theoretical developments in alternative macroeconomics that demanded our attention. In order to provide the necessary background, we have added material on induced technical changes and the Goodwin cycle that deepens the treatment of biased technical change and provides a view of distribution that is an alternative to the neoclassical theory. We also included a discussion of the structuralist approach that broadens the scope of the demand-constrained growth and distribution model. Because they integrate distribution, technical change, and capital accumulation, we hope that these additions contribute to an understanding of neoliberal capitalism and make the book more useful for instructors and economists in the post-Keynesian and Classical traditions.

Finally, a near-universal consensus that global warming presents an existential challenge to humanity has emerged in the last decades. The economic analysis of global warming we have added as the culminating chapter of the book builds on the existing chapters by applying the models of land- and resource-limited growth. Some understanding of the theory of growth and distribution is indispensable for any serious attack on the problem of global warming.

In order to facilitate the formation of a community around the approach of this book, as well as to make its use as current as possible, this edition has a companion website, www.growthdistribution.net. Our immediate goal is to provide regular updates to keep the content alive. But we also hope that the users of the book will contribute to the project by sharing their own teaching materials.

Preface to the First Edition

This book began as a set of notes for courses at Barnard College and Colgate University.

Inspiration for this work and a good deal of the substance of the models came from André Burgstaller, who gave us the privilege of reading the manuscript of his *Property and Prices* (Cambridge: Cambridge University Press, 1994), and with whom we have had extensive conversations on the topics covered here. In particular, Burgstaller's idea that equilibrium prices in a classical model can be viewed as the outcome of speculation in forward-looking asset markets is central to the point of view developed in Chapters 13 and 14.

Other important sources for our general approach are Stephen Marglin's *Growth, Distribution, and Prices* (Cambridge, MA: Harvard University Press, 1984), and John Broome's *The Microeconomics of Capitalism* (London: Academic Press, 1983).

We thank Adalmir Marquetti for preparing the Extended Penn World Tables dataset, which made an indispensable contribution to our work and to this book.

We would like to thank Milind Rao, Peter Hans Matthews, Sergio Parrinello, Christophre Georges, and our students at Colgate University and Barnard College of Columbia University for their help in rectifying errors in earlier drafts.

We retain the responsibility for all the things that are wrong.

Notation

A	Scale parameter in Cobb-Douglas production function
α	Capital coefficient in Cobb-Douglas production function
B	Effective labor productivity; Nominal value of government liabilities (ch. 16)
b	Social Security benefit (ch. 16)
β	Capitalist propensity to save out of wealth
β_w	Worker propensity to save out of wealth (ch. 17)
C, c	Consumption, social consumption per worker
C^w, C^c	Worker consumption, capitalist consumption
C^w, C^r	Consumption of active, retired workers (chs. 16, 17)
c^r, c^w	Consumption of household in retired and working periods (ch 16)
CD	CO2 concentration (ch. 18)
χ	Rate of capital-saving technical progress
D	Depreciation; Climate damage (ch. 18)
δ	Depreciation rate per unit of capital
E	Nominal primary fiscal surplus (ch. 16); Number of corporate stocks (ch. 15)
e	Employment rate (chs. 6, 7)
ϵ	Rate of CO2 dissipation (ch. 18)
η	Propensity to invest out of profit (ch. 12); Investment sensitivity to q-ratio (ch. 15)
f	Production function; Invention possibility frontier (ch. 7); Social security reserve fund (ch. 16)
g_{variable}	Growth rate of variable. Thus:
g_K	Growth rate of capital
g_P	Growth rate of stock prices (ch. 15)
g_X	Growth rate of output
g_x	Growth rate of labor productivity (ch. 2)
g_ρ	Growth rate of capital productivity (ch. 2)
g_W	Growth rate of workers wealth (ch. 17)
γ	Rate of labor-saving technical progress
I, i	Gross investment, gross investment per worker
i	Real interest rate (ch. 18)

J	Capitalist wealth
J_F	Corporate net worth
K, k	Capital stock, capital stock per worker (or capital intensity)
K^w, K^c	Workers' wealth, capitalist wealth (ch. 17)
λ	Shadow price, Lagrange multiplier
μ	Bargaining power of workers (ch. 6); Shadow price of CO2 (ch. 18)
N	Labor employed
N^S	Total labor force
n	Growth rate of labor force
ω	Viability threshold
P	Price of stock share (ch. 15)
p	Price of capital goods in terms of consumption (ch. 3)
p^{cd}	Price of CO2 (ch. 18)
p_u, p_q	Price of land, oil
ϕ	Workers' share of wealth
π	Profit share
Q, ΔQ	Oil reserves, oil depletion
q	Tobin's q
R	Profits; Total return factor (ch. 16)
r	Net profit rate
r_E	Equity yield (ch. 15)
ρ	Output-capital ratio, capital productivity
S	Saving
S^r, S^w	Saving of retired, active workers (chs. 16, 17)
S^c, S^f	Capitalist household saving, firm saving (ch. 15)
s	Saving as a proportion of output
s_F	Corporate retention (saving) rate
s^w	Saving per worker (ch. 16)
σ	Elasticity of substitution in production
t	Lump-sum tax (ch. 16)
U	Land
u	Utility function (ch. 5); Capacity utilization (ch. 12)
V	Dividends
v	Gross profit rate
v_k, v_u	Rental on capital, land
W, w	Real wage bill, real wage per worker
$\bar{w}$	Conventional real wage
X, x	Gross output, gross output per worker
Y, y	Net output, net output per worker
Z, z	Cash flow, cash flow per worker

GROWTH AND DISTRIBUTION

SECOND EDITION

1

Introduction

Economic growth is the hallmark of our historical epoch. It finances and directs the ongoing revolution in technology that continually transforms our social and personal lives. The political preeminence of nation states and the emergence of supra-national institutions have their roots in the process of economic growth. The unprecedented growth and aging of the world's population are to a large extent the result of economic growth, as are the relative decline of agriculture and the dominance of industrial and post-industrial production centered in cities. National political and military power and influence increasingly reflect relative economic performance. Economic practices have transformed social relations and ideological beliefs. The great challenges we perceive for the future, including the protection of our environmental heritage and the preservation of social justice in a world polarized between wealth and poverty, arise from the effects of economic growth.

In this book we present *theories* that economists have devised over the last 200 years to analyze and explain various aspects of economic growth, and the movement of economies through time more generally. As a background to these theories, we review in this introductory chapter some of the *social history* of economic growth.

1.1 Economic Growth in Historical Perspective

Human history shows a slow improvement in technology and productivity from the earliest periods we know anything about. This improvement seems to have occurred in distinct waves, punctuated by such rapid leaps as the adoption of settled agriculture, the emergence of cities, the establishment of long-distance sea trade, and so forth. The earth's human population grew very slowly, if at all, for the thousand years before 1500 C.E. Around the fifteenth century in Europe we see a noticeable acceleration of the pace of social and technological change, and in the rate of growth of population. This acceleration was marked by the enlargement of towns and cities, the spread of trade in goods and money, the growing importance of wealth invested in capitalist trade and production in towns in relation to traditional landed wealth, and a systematic focus on the improvement of technologies in production and transportation. By the sixteenth century the more advanced European societies had become recognizable forerunners of capitalist nation states. During this period people began to view trade and production as the central sources of national influence and power. The phenomenon of economic growth, with its problems and promises, had arrived.

Toward the end of the eighteenth century these developments underwent another sharp acceleration with the emergence, most notably in Britain, of *industrialization*. The scale of production increased dramatically and became concentrated in large towns and cities. A pattern emerged in which traditional farming, still based heavily on the needs of local subsistence, gave way to market-oriented agriculture, in the process displacing large numbers of the rural poor as common lands and forests were appropriated by large landowners and converted to the production of marketable commodities. The displaced agricultural poor moved to towns and cities, becoming both the wage-seeking labor force necessary to run rapidly expanding industries, and the mass of the urban poor. These economic developments precipitated huge migrations of people, not just from the countryside to cities, but from continent to continent. The growing economic and military power of the advanced nineteenth-century European nations led to their race to carve out colonies, empires, and spheres of influence all over the globe. In this way the phenomenon of economic growth sooner or later invaded every corner of the earth.

From its earliest stages the fostering, shaping, and taxing of economic growth was a preoccupation of the politically powerful. Economic growth confers immense political and military advantages on nations. *Political economy* arose as a discussion of the impact of national policies toward trade, labor markets, and taxation on economic growth.

Despite the evident fact that world economic growth is a unified, articulated, self-reinforcing phenomenon, political economy emphasizes national differences in policy and their impact on national economies. Thus the theories we will examine below take the national economy as their starting point, and treat each nation's economic growth as a separate experimental observation.

1.2 Quality and Quantity

We experience economic growth overwhelmingly as *qualitative* change. Economic growth has moved most of us from small rural communities where individuals had lifelong personal relationships and employed simple and undifferentiated techniques of production to large urban agglomerations where most interactions are mediated by the anonymity of the market and we specialize in tiny aspects of a bewilderingly complex technology in order to produce. Economic growth means qualitatively new products and services—railroads, airplanes, and automobiles; electrical and electronic appliances; radio, television, telephones, and computers; anesthesia, X-rays, and MRI scans.

But despite constant change in the commodities actually produced and the techniques through which people produce them, economic growth reproduces the same basic social relationships on an ever-increasing *quantitative scale*. Capitalist economic growth arises from the organization of production in particular firms or enterprises, which assemble human workers and the means of production they require to transform inputs available on the market into marketable outputs. Capitalist production rests on the quantitative increase in the money value of the product through the process of production, insofar as the marketed output is worth more than the inputs that were consumed to produce it. This *value added* appears as the wages of the workers who actually transform the inputs into outputs, and the profit, interest, and rent that constitute the incomes of the capitalist owners of factories and machines, money and natural resources including land. Economic growth is financed by the decisions of capitalists to reinvest some part of their incomes to allow production to take place on a larger scale.

The reinvestment of profits in the expansion of capitalist production, however, always involves a qualitative change in the technique of production and the actual commodities produced. The larger scale of production is carried out with somewhat different machines, in different locations, with differently trained and organized workers. On a larger scale, improvements and adaptations of the output are possible. In the process of economic growth the quantitative aspect of simple expansion of production through the reinvestment of profit incomes and the qualitative aspect of change in the products and the lives of the producers of the product are inextricably intertwined.

While the mathematically based theories of political economy emphasize the quantitative aspects of economic growth, it is important not to lose sight of the profound qualitative changes that ensue.

1.3 Human Relationships

The self-reinforcing cycle of capitalist economic growth cannot establish itself without deep changes in the ways people relate to each other. The constant expansion and restless change of capitalist production require a flexible labor force that can be redeployed, expanded, and contracted rapidly. Before the emergence of capitalism, these changes simply could not take place: workers were bound either to their employers as slaves or to the land they worked on as serfs. Capitalist economic growth rests on the *free worker*, who can accept or reject jobs, move from city to city and country to country in response to the incentives of wage differentials, and who takes the ultimate responsibility for her or his own survival and reproduction. The preoccupation of the free worker is to control the massive insecurity that wage labor brings with it. Thus in the capitalist world economy the great mass of the population becomes free labor that works for a wage. Because workers are free, and their wages are regulated only by the vagaries of competition, some prosper and some find themselves on the margin of existence.

On the other hand, the organization of work on a national and world scale requires the separately flexible deployment of factories, machines, and transportation facilities. This is the realm of *capital*, enormous concentrations of money available to finance production.

The owners and managers of capital have very different interests from those of workers. Wages and profit incomes divide the value added from production, so that capital and labor often find themselves on opposite sides of issues of social policy that affect the level of wages. Capital seeks a flexible and adaptable labor force, a goal that runs counter to the workers' desire for stability and security in their employment and conditions of life.

The political economic theory we survey in this book centers on the impact of the distribution of income between workers and capitalists on the quantitative aspects of economic growth, and the impact of growth on distribution.

1.4 Economic Theories of Growth

Adam Smith, whose *Inquiry into the Nature and Causes of the Wealth of Nations* (Smith 1937 [1776]) marks a key turning-point in the development of political economy, was primarily concerned with economic growth. In Smith's view the central aspects of economic growth were the *division of labor*, the separation of production processes into smaller tasks that can be assigned to specialists, and the *extent of the market*, the growth of population, income, and transportation and communication facilities that allow more output to be sold. The division of labor raises *labor productivity* as labor becomes more skilled in specialized tasks, and as machinery can be devised to take over the routine aspects of production. Smith sees technological progress as an aspect of the widening division of labor. The increasing division of labor and the widening extent of the market are mutually reinforcing tendencies in Smith's vision, since a wider market makes possible a more detailed division of labor, and a higher degree of division of labor increases productivity and incomes, encourages investment in transportation and the growth of population, and thus widens the market. The two phenomena are linked through a set of positive feedbacks into an unstable cycle of upward spiraling development. Smith thought that governments should try to foster this process by securing property, providing cheap legal services and national security, and otherwise staying out of private decisions about investment (the policy now known as *laissez-faire*). He argues that the cycle of growth is virtuous in that it benefits both workers and capitalists (a version of *trickle-down* economics): capitalists will be free to pursue maximum profitability of their investments, but the growth of capital will create a demand for labor and tend to pull up workers' wages as well. While population will grow along with capital in the process of growth, Smith thought that it would lag enough to assure a long period of higher wages. In Smith's version economic growth is spontaneous, or *endogenous*: it tends to take hold like the spread of a wildfire unless restrictive government policies repress it. We will study a simplified version of Smith's model in Chapter 6.

Thomas Malthus, whose *Essay on the Principle of Population* first appeared in 1798 (Malthus 1986), had a distinctly gloomier view than Smith. Malthus could see that capital accumulation is a self-reinforcing feedback system, but doubted that it could do anything in the long run for the well-being of workers. Malthus reasoned that an increase in the real wage would raise workers' standards of living, encourage them to marry earlier, and reduce infant mortality among their offspring, thus producing a surge in population. The growing population would in turn crowd the labor market, driving real wages back down to the point where infant mortality and later marriage would stabilize population growth. The real wage at this *demographic equilibrium* would constitute a *natural* wage level around which actual wages could only fluctuate temporarily.

David Ricardo in his *Principles of Political Economy and Taxation*, published in 1817 (Ricardo 1951), took up Malthus's ideas about population and the real wage and combined them with his own theory that rent arises from the limited supply of fertile land. In Ricardo's view, Smith's virtuous cycle was doomed to extinction as capital accumulation and population growth eventually used up all the fertile land, food prices rose, and profit rates declined to zero in what he called the *stationary state*. Ricardo's methods of analysis had an immense influence on later thinking about political economy. In particular, Ricardo emphasized the class divisions of early industrial capitalist society. Workers, with wages depressed to the minimum compatible with reproduction by Malthusian forces, had no surplus available to save. Landowners, the remnants of the feudal aristocracy, dissipated their incomes in the support of retainers and clients for political advantage and social status. Capitalists, on the other hand, forced by competition with each other to accumulate as much of their incomes as possible, were the engine of capital accumulation and growth. As profit rates fell as a result of rising rents and wages with population growth, however, Ricardo argued that the capitalist engine of growth would be choked off by a falling rate of profit. We work out Ricardo's reasoning in modern terms in Chapter 13.

Karl Marx published the first volume of his work *Capital* (Marx 1977) in 1867, after spending his youth in the development of a revolutionary philosophy of *historical materialism*. Marx, along with his close associate Friedrich Engels, saw the secret of human history in the ways in which particular classes controlled the *surplus product* of their societies. In a slave-based society, for example, slaveowners controlled the whole product of the slave producers and were able to use the surplus over the required maintenance of the slaves to perpetuate the system. Feudal lords bound serfs to work a certain proportion of each week on their own fields, thus providing themselves with a surplus product (the serfs providing for their own needs by cultivating their own land the rest of the week) that allowed them to maintain armies to fight each other and repress the serfs. Each form of society has its own level of development, and its own characteristic class structure, from the point of view of Marx's historical materialism, and a clear understanding of these human relations is the key to understanding the society and its history.

Marx saw in Ricardo's picture of industrial capitalism a perfect example of a class society. Because they owned the means of production (factories, land, and so forth), landowners and capitalists were in a position to appropriate the surplus labor time of workers in the form of monetary profits and rents, which Marx called *surplus value*. Marx, however, disagreed with Ricardo's view that diminishing returns to capital and labor because of limited land would eventually bring capital accumulation to a halt through rising rents and wages. Marx took a more Smithian view, arguing that the historical genius of capitalism is its technological progressivity, enforced by the pressure on each capitalist to find cost-reducing technical innovations to keep ahead of its competitors. Thus Marx thought that capitalism could always overcome diminishing returns to limited land resources by finding cheaper technologies. What would lead to a fall in the rate of profit, Marx argued, was that these cheaper technologies would use more and more capital per worker, thus driving down the rate of profit. In the end, according to Marx and Engels, the very success of capitalism in raising labor productivity would lead to its replacement by a class-free socialist organization of production in which scarcity would have been eliminated. Some elements of Marx's theory of technical progress underlie the discussion of patterns of economic growth in Chapter 8. Marx's theory of induced technical change is the inspiration for the models of Chapter 7.

Turning away from the explosive social and political issues that the Classical theory of growth seemed to lead to, marginalist economists focused their attention on the static *efficiency* of economic allocation, and the tendency for markets to equalize marginal costs and marginal benefits across society. The twentieth-century crises of the two World Wars and the Great Depression raised again the questions of the stability and long run tendencies of economic growth.

Roy Harrod (Harrod 1939) argued that the process of economic growth was inherently problematic for two reasons. First, the rate of growth necessary to absorb society's saving in investment projects (which Harrod called the *warranted rate of growth*) would only by accident equal the underlying rate of growth of population adjusted for the rate of increase of labor productivity (which Harrod called the *natural rate of growth*). This is the Harrod *existence problem*. Second, if the actual growth rate exceeded the warranted rate, chronic labor shortages, wage increases, and inflation would disrupt the growth process, but if the actual growth rate fell short of the warranted rate, the economy would slip into increasing unemployment, stagnation, and deflation. This is the Harrod *stability problem*. We look at a modernized extension of Harrod's model in Chapter 12.

Harrod's existence problem was addressed by Robert Solow's seminal *neoclassical growth model*. Solow argued that the possibility of substitution of capital for labor along the isoquant of an *aggregate production function* could adjust the warranted rate to any level of the natural rate of growth. We work through Solow's model in detail in Chapters 10 and 11.

While neoclassical economists generally accepted Solow's arguments and methods as settling the basic questions of the analysis of economic growth, economists working in the Keynesian, Marxian, and Ricardian traditions, led by Joan Robinson, strongly criticized the neoclassical model. The central point of controversy was Solow's assumption that there existed a well-behaved aggregate production function that could summarize the possibilities of substitution of capital for labor in the economy as a whole. The critics argued that capital was just the market valuation of a huge range of different *capital goods*: as the wage rate changes, the prices of all these goods can undergo any pattern of change, depending on the exact structure of their costs of production. In the end there is no guarantee, according to the critics, that a lower wage rate will lead to a lower value of capital per worker or more employment for a given stock of accumulated capital value, as the neoclassical production function analysis predicts. Since Solow and his supporter in this debate, Paul Samuelson, taught at M.I.T. in Cambridge, Massachusetts, and Joan Robinson and many of her supporters taught or were students at Cambridge University in England, this debate is known as the *Cambridge capital controversy*. While the neoclassicals conceded the theoretical possibility of the effects of changing wages on capital values pointed to by their critics, they argued that these possibilities were relatively unlikely in real economies and continued to assume that an aggregate production function would give a good approximation to the behavior of real economies.

The controversies of contemporary growth and capital theory create a dilemma for us in writing this book. Which basic approach should we use in setting forth and developing theories of growth? We have chosen to resolve this dilemma by presenting the basic framework of production and capital theory in Chapters 2 and 3 in terms of the *growth-distribution schedule*, a flexible starting point that is consistent with both neoclassical and nonneoclassical models, and that allows us to explain what is at issue in the capital controversy. Through most of this book we use production models with only a single produced good that can either serve as a consumption good or be accumulated as capital. Under that particular assumption, there can be no divergence between the conclusions of neoclassical and nonneoclassical models in the area of capital theory, and we focus attention on different theories of labor supply, saving, resource availability, demand generation, and technical change.

Our aim in presenting growth theory through the perspective of the growth-distribution schedule is to bring out the insights that both the Classical and neoclassical theories of economic growth have reached, and to introduce the reader to the fascinating range of economic issues and concepts that growth theory raises.

1.5 Using This Book

For instructors who are planning a course around this book (or readers planning to navigate it), we have some thoughts that may help. The core tools and concepts used throughout the book are laid out in Chapters 2–5, which provide a logical starting point. Chapter 5 explains an agent's intertemporal consumption and saving choice using the Lagrangian method. Some may find the level of mathematics here to be challenging, and choose to treat this chapter as optional reading without losing the ability to follow the rest of the book. Because we have used a logarithmic utility function, the solution to the consumption problem always has a simple, intuitive form: agents consume a constant fraction of their wealth. We stick to logarithmic utility and this transparent consumption function throughout the book so that readers can follow the argument without mastering the Lagrangian method.

Chapters 6, 10, and 12 present basic versions of the Classical, neoclassical, and Keynesian growth models with an emphasis on the alternative modeling choices discussed above. Readers should be able to discern how different visions of the growth process lead to contrasting emphases on key causal relationships in these three schools of thought, as well as form their own preliminary opinion about the relative merits of competing models.

The remaining chapters can be grouped into four main broad categories. First, in Chapters 7, 8, 9 and 11 we explore the role that technical change plays in the Classical and neoclassical growth models. Second, in Chapters 13, 14, and 15 we explore the distinction between capital and wealth (which also includes assets like land, natural resources, or financial instruments). Third, in Chapters 16 and 17 we introduce worker life-cycle saving and explore the distribution of wealth among active workers, retirees, and capitalist households. Fourth, we attack the problem of global warming in Chapter 18 using some of the insights about the economic role of scarce resources developed in Chapters 13 and 14. Chapter 18 explains the *social coordination problem* that underlies the phenomenon of global climate change, and the use of the Lagrangian method in this chapter only adds to a deeper appreciation of its economic logic.

We have for the most part avoided explicit discussion of the policy implications of growth theories, choosing to leave that to readers. There are several points where political economy questions rise close to the surface. In Chapters 7 and 9 on induced and endogenous technical change, a natural question is whether economic policies can be devised that encourage technological progress by favoring spending on R&D, by creating strong aggregate demand that allows for Smithian returns to scale, or by boosting wages to incentivize labor-saving changes in production techniques. Chapter 12 raises the possibility that under the right circumstances greater income equality can stimulate faster growth because workers tend to consume a higher fraction of their incomes than capitalists. Chapter 16 provides the basic tools for understanding the economics of the national debt and fiscal programs like social security that are perennial sources of controversy. Finally, Chapter 18 outlines the core argument for a carbon tax or similar policy designed to steer the accumulation process toward green technology and away from a global ecological catastrophe.

1.6 Suggested Readings

To explore the history of economic theory, a good point of departure is the survey provided by Foley (2006). Also see the masterly treatment of the history of thought by Dobb (1973) and the influential paper by Kaldor (1956), which is devoted specifically to growth and distribution theories.

Gram and Walsh (1980), a textbook exposition of the Classical versus the neoclassical approach, combines clear formal exposition with well-chosen textual passages from seminal works.

The early development of growth theory is surveyed at the professional level by Hahn and Matthews (1964); for an accessible textbook treatment, try Jones (1976). Many of the seminal contributions to early growth theory are contained in Stiglitz and Uzawa (1969). The recent contributions called New Endogenous Growth Theory are allied with the neoclassical approach in some ways, such as their devotion to the full employment assumption, but differ in their view of technical change. New Growth Theory and neoclassical theory are described in such advanced textbooks as Acemoglu (2009), Aghion and Howitt (1998), Barro and Sala-i-Martin (2011), and Romer (2012), as well as in the undergraduate texts by Jones and Vollrath (2013) and Aghion and Howitt (2009).

Three works that have deeply influenced the current text through their insightful comparative approach to the Keynesian, Classical, and neoclassical theories of growth and distribution are Harris (1978), Marglin (1984), and Taylor (2004).

2

Measuring Growth and Distribution

Economic growth is an increase in a country's *output* of goods and services. Output is equal to the number of workers employed in production, *labor*, multiplied by the output produced by each worker, *labor productivity*. Labor productivity depends on *technology*, which also determines the amounts of other inputs to production—previously produced raw materials, tools, equipment, and buildings, *capital goods*, and natural resources, *land*—required by each worker. The number of workers employed in production, given technology, is thus limited by the accumulated stock of capital and the available land.

A country's rate of economic growth ultimately depends on the growth of its productive population, its *accumulation* of stocks of capital goods, and on *technological change*. Our aim in this book is to examine each of these sources of economic growth in detail, and to explain how their interaction results in the patterns we observe in empirical data.

Before we discuss explanations of economic growth, we need to be able to measure and to account for an economy's outputs and inputs. In this chapter we present an accounting system that will be the foundation for a series of models that attempt to explain and analyze the various aspects of the economic growth process.

2.1 Measuring Output and Inputs

The *total production* of an economy in any year consists of all its newly produced goods and services. Much of the total production serves to replace goods and services used up in the process of production. *Gross production*, the difference between total production and the goods and services used up in production, is the collection of goods and services available for immediate use, *consumption*, and the accumulation of capital goods, *gross investment*. The *gross product* (GP)[1] is the value of gross production at current market prices, including consumption and gross investment. Gross investment, however, is offset by *depreciation*, the wear-and-tear and deterioration of existing long-lived capital goods. *Net product* (NP) is equal to GP less the value attributed to depreciation, and thus includes only *net investment*. Since depreciation is not measured by actual market transactions, the measurement of net product is subject to more uncertainty than the measurement of gross product.

The use of market prices to calculate gross product reduces the large number of actual goods and services that constitute gross production to a single number, which is a great simplification. Changes in gross product, however, can arise either because gross production has changed, or because market prices have changed (for example, through inflation). Economists measure price changes by constructing a *price index*, a weighted average of the actual prices observed during a year, divided by a similar weighted average of actual prices in some base year. Different systems of weights produce somewhat different price indexes. We estimate the *real output* of an economy by dividing its gross product by a price index, thus correcting for pure price changes. We will refer to the real gross product in an economy in a period simply as its *output*, and denote it by the mathematical symbol X.

A research team at the University of Pennsylvania, under the leadership of Robert Summers and Allen Heston, has undertaken the task of compiling a consistent set of measures of gross domestic product and price indexes based on purchasing power parity for most countries in the world starting in 1950 (or in later years in those countries that have no statistical sources for earlier years). This data set, often called the Penn World Tables (PWT), is available online. The PWT expresses the output of each country in each year in terms of 2011 international dollars, and thus corrects for differing price levels between countries and differing rates of inflation within coun-

[1]*Gross domestic product* (GDP) is the gross product produced in a given country in a year. Economists also refer to *gross world product* for the gross product of the entire world economy, *gross state product* for the product of a particular state, and so on.

tries. Adalmir Marquetti has supplemented the PWT tables by calculating capital stocks, including measures of the distribution of income, and adding information on population growth in the data set used in this book, which we will refer to as the *Extended Penn World Tables*, or EPWT. Unless we specify otherwise, we measure output in this book in terms of the Penn World Tables 2011 international dollar, and use the symbol $ as a shorthand for this unit. It is important to remember that this $ is a measure of real, inflation-corrected output.

In measuring the output of an economy it is important to keep track of the deterioration of the capital stock through wear-and-tear and the passage of time, *depreciation*, D. The net product, $Y = X - D$, measures the output of the society reduced by an estimate of depreciation.

We measure labor input as the number of employed workers (or in some cases, the number of hours of work), and denote it with the mathematical symbol N. In real economies workers vary in skills and ability, so that in principle it would be desirable to measure labor input as a weighted average of employed workers, with the weights representing the skill and ability levels of individual workers. In the theoretical parts of the book we could interpret the labor input as such a weighted average without changing the arguments. Since detailed data on the skill and ability levels of workers are not available for many countries, we simply abstract from differing skill and ability levels in presenting empirical measurements of labor input.

Capital goods in real economies represent a heterogeneous collection of stocks of raw materials and partly finished goods, plant, equipment, transportation facilities, and so forth. In principle, it would be desirable to measure capital input with a detailed list of all the different categories of capital goods. It is also possible to aggregate capital goods by measuring their value at market prices at the time of their construction, which is the procedure we use. Thus we calculate the capital input, denoted by the mathematical symbol K, as the sum of the real value of past gross investment, less the estimated sum of accumulated depreciation. Capital is measured in the same units as output, 2011 international dollars.

The measurement of capital inputs in this way has been the center of considerable theoretical debate among economists, particularly during the Cambridge capital controversy of the 1960s. The difficulty is that the same value aggregate of capital can represent completely different collections of actual capital goods, and that the same collection of actual capital goods can have a different aggregate value if the prices of individual capital goods change. In our theoretical models we assume that there is only one output, and that capital is accumulated output, thereby avoiding the problem of relative prices. The empirical measures of capital input we use, however, are subject to the limitations of the aggregate value method. As we will explain in more detail below, a further controversy over capital input arises in the context of the neoclassical production function, which assumes that the value of capital as such contributes to the level of real output. We do not agree with this position, since in our view the level of real output depends on technology, which in turn requires certain levels of capital goods, and, consequently, certain values of capital goods. The exploration of these different points of view is one of the main themes of the later chapters of this book.

In some theoretical models we consider land (conceived of broadly as natural resources and environmental quality) as an input to production. In the theoretical models we simply take the available quantity of land as the unit of land, so that the quantity of land is always 1. The measurement of natural resource and environmental inputs to production in real economies is an active but relatively underdeveloped area of economic research, so that we cannot present empirical data on land inputs.

In comparing different economies, or the same economy in different years, it is often useful to measure output and capital stock per employed worker. Output per employed worker, $x = X/N$, is a measure of average labor productivity, or, more simply, *labor productivity*: labor productivity has the units of output per worker per year, or \$/worker-year. Capital stock per worker, $k = K/N$, is a measure of *capital intensity*, and has the units \$/worker. $\rho = X/K = x/k$ (the Greek letter *rho*, pronounced "rō") is the *output-capital ratio*, which has the units \$/year/\$, or 1/years, a pure number like an interest rate. By analogy to x, average labor productivity, we often refer to ρ as the average productivity of capital or *capital productivity*. As we remarked above, we do not view capital as such as directly productive, since capital goods serve to enhance the productivity of workers, but this usage is so common and convenient that we have adopted it. The ratio of depreciation to the capital stock is $\delta = D/K$ (the Greek letter *delta*, pronounced "ˈdel-ta"). $y = Y/N$ is *net output per worker*. The ratio of the net output to the capital stock is $Y/K = (X - D)/K = \rho - \delta$.

These key ratios can be calculated on the basis of the data in the Extended Penn World Tables data set for many countries for recent years. Economic historians have made estimates of these variables for a few key countries over longer historical periods.

2.2 Time and Production

Because we are concerned with economic growth, *time* plays a key role in the analysis. We will measure variables in a sequence of discrete periods, $t = 0, 1, 2, \ldots$ Real-world economic time is much more complicated, with some processes (like stock and foreign exchange markets) moving extremely rapidly, even minute by minute, and other processes (the construction of large power plants or factories, the aging of the population) moving relatively much more slowly. However, the outcomes of all these processes are always measured statistically over fixed periods (each year, or quarter of a year, or month, or week, for example), and we can easily fit these real measurements into a period framework.

When we have actual economic data we will indicate its time by writing it explicitly as a subscript: X_{2005} will indicate real (that is, inflation-adjusted) gross domestic product (GDP) in 2005. In order to simplify mathematical expressions, we will assume that any variable without a subscript refers to the current year, and indicate the next year's variable with the subscript "+1". Thus X will be current year GDP (for whatever particular year we are analyzing) and X_{+1} next year's GDP.

In the analysis of economic growth the concept of *growth rates* plays an important role. We will write the change in a variable, for example, X, over one period as $\Delta X = X_{+1} - X$, and the one-period growth rate as $g_X = \Delta X / X$.[2] Economists generally refer to the growth rate of output, g_X, as the *growth rate* of the economy. In our models, the growth rate of the capital stock, g_K, plays a key role.

The *growth factor* for a variable is the ratio of the next period's value to the initial period's value. For one period, this is just the growth rate plus one. For example, the growth factor of output is $X_{+1}/X = 1 + g_X$.

[2]The actual average compound growth rate of a variable X between time 0 and time T is $g_X = (\ln X_T - \ln X_0)/T$, where ln is the natural logarithm. If $T = 1$, this becomes $g_X = \ln X_1 - \ln X_0 = \ln(X_1/X_0) = \ln(1 + (\Delta X/X)) \approx \Delta X/X$, since when ϵ is small, $\ln(1 + \epsilon) \approx \epsilon$. Thus if changes in variables are small relative to their levels, the definition of the growth rate in the text is close to the actual average compound rate of growth.

2.3 A Note on Units

In using this accounting system careful attention to the units involved is required to avoid confusion. X is the GDP in a period, usually a year, and is measured in units of output per year ($\$T^{-1}$). Here \$ stands for the units in which capital and output are measured, real dollars, and T stands for time, so that $\$T^{-1}$ means dollars per year. As can be seen in Table 2.1, US X_{2005} was about \$14.7 trillion/year, for example. The capital stock K is accumulated output (\$), and is measured in the same units as output. For example, private nonresidential US K_{2005} was about \$34 trillion.

Since $\rho = X/K = \$T^{-1}\$^{-1} = T^{-1}$, we can see that ρ must be measured in inverse time units, like an interest rate. The output-capital ratio, ρ, shows output as a proportion of the capital stock. ρ_{2005} = \$14.7 trillion/year/\$34 trillion/year = .43/year, or 43%/year.

Depreciation, D, is measured in the same units as output. US private non-residential D_{2005} was \$2.77 trillion/year. The depreciation rate, $\delta = D/K =$

Table 2.1 US, 2005 and 2014: Output Account

Variable	*Symbol*	*2005*	*2014*	*Units*
Output	X	14.68×10^{12}	16.60×10^{12}	\$/year
Consumption	C	10.67×10^{12}	13.15×10^{12}	\$/year
Gross investment	I	4.02×10^{12}	3.45×10^{12}	\$/year
Depreciation	D	2.772×10^{12}	3.143×10^{12}	\$/year
Net output	Y	11.911×10^{12}	13.455×10^{12}	\$/year
Capital	K	33.94×10^{12}	37.67×10^{12}	\$
Employment	N	143.99×10^{6}	148.46×10^{6}	workers
Labor productivity	x	101,974.82	111,799.27	\$/worker-year
Net labor productivity	y	82,724.48	90,633.34	\$/worker-year
Consumption per worker	c	74,086.75	88,564.31	\$/worker-year
Investment per worker	i	27,888.07	23,234.96	\$/worker-year
Capital-labor ratio	k	235,735.86	253,716.02	\$/worker
Capital productivity	ρ	43.26	44.06	%/year
Depreciation rate	δ	8.17	8.34	%/year

Source: Extended Penn World Tables 6.0. All values are at 2011 purchasing power parity.

$\$T^{-1}\$^{-1} = T^{-1}$, also has the dimension of inverse time, like an interest rate. US private nonresidential δ_{2005} = \$2.77 trillion/year/\$34 trillion/year = .08/year or 8%/year. δ measures the proportion of the value of the capital stock that disappears through deterioration each year.

N is the number of workers employed. US N_{2005} was 144 million. x is output per worker per period, ($\$N^{-1}T^{-1}$). US x_{2005} was \$14.7 trillion/year/144 million workers = \$102,000/worker-year.

It is very important to be sure that the time units are consistent in each problem; if you measure output per year and labor input per week, the result will be nonsense.

2.4 Technology in the Real World

The Extended Penn World Tables data set contains estimates of ρ, x, and k for many countries and many years. The EPWT reveals some broad patterns that are central to understanding the process of economic growth in the real world.

Figure 2.1 shows $\{\rho, x\}$ points for 49 countries[3] at different dates from the EPWT. Economic development tends to lower ρ as it raises x, as the figure indicates. Falling capital productivity arises because economic development leads to more capital-intensive methods of production. Thus workers become more productive, but the amount of capital they work with increases even more than their productivity, so that the productivity of capital actually tends to fall.

The same information can be plotted in $\{k, x\}$ terms. Figure 2.2 plots the same data in this form. The positive relation between k and x shows that the process of economic growth tends to increase the capital stock per worker at the same time that it increases output per worker. This strong correlation is

[3] Argentina, Australia, Bangladesh, Brazil, Burundi, Canada, Chile, China, Colombia, Ecuador, Egypt, Ethiopia, France, Germany, Ghana, Greece, Hong Kong, India, Indonesia, Ireland, Israel, Italy, Japan, Kenya, Malaysia, Mexico, Morocco, Netherlands, New Zealand, Nigeria, Pakistan, Peru, Philippines, Portugal, Republic of Korea, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Tanzania, Thailand, Turkey, Uganda, United Kingdom, United States, Venezuela, Viet Nam.

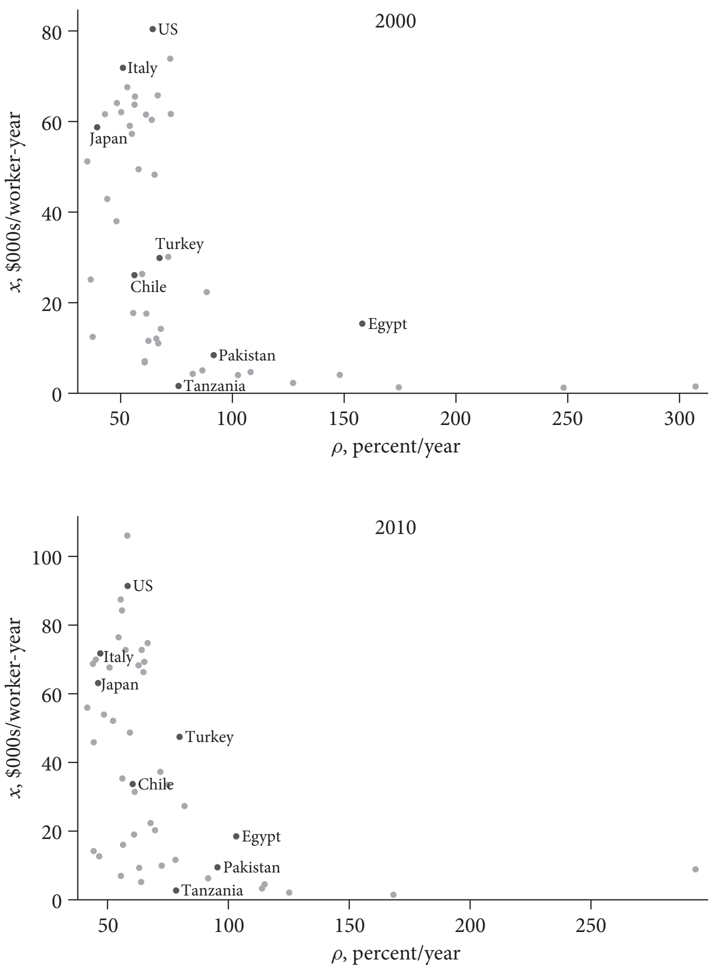

Figure 2.1 $\{\rho, x\}$ points for 49 countries at widely differing levels of economic development in 2000 and 2010, from the EPWT. There is a strong inverse correlation between ρ and x: as countries develop, ρ tends to fall (due to industrialization and the adoption of capital-intensive techniques of production) as x rises.

one reason some economists think that a stable *production function* links k and x.

One of the aims of different theories of economic growth and technical change is to account for these strongly marked (though not uniformly observed) patterns.

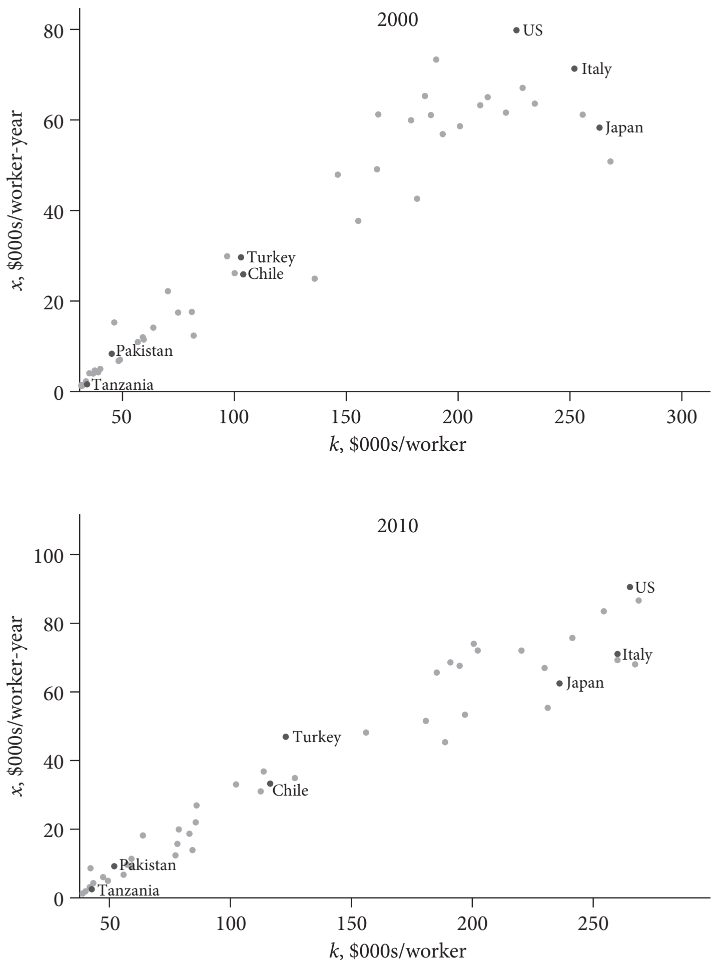

Figure 2.2 $\{k, x\}$ points for 49 countries at widely differing levels of economic development in 2000 and 2010, from the EPWT. There is a strong positive correlation between k and x: as countries develop, k tends to rise (due to industrialization and the adoption of capital-intensive techniques of production) as x rises.

2.5 The Uses of Output: Investment and Consumption

Output can be used either for *consumption*, C, or *gross investment*, I.

National income accounting gives us a system for measuring the output of an economy and its uses. The basic identity of national income accounting is that GDP = Consumption + Gross Investment + Government Expenditure

+ Net Exports. For the analysis of economic growth we want to divide the output of an economy into two categories: consumption, output that is used up in the period, and gross investment, output that is devoted to increasing the capital stock. A significant part of government expenditure in real economies takes the form of investment in productive facilities like roads, harbors, airports, and so forth. In our theoretical models, we interpret consumption and gross investment as including the corresponding parts of government expenditure. In presenting empirical data, when detailed breakdowns of government expenditure between consumption and investment are not available we have somewhat arbitrarily allocated both government expenditure and net exports to consumption, which may distort the resulting picture of gross investment. We will accordingly write the *output identity* as:

$$X \equiv C + I \tag{2.1}$$

We can divide through both sides of this equation by N to express output per worker, x, as the sum of consumption per worker, c (which may not all be consumed by workers), and gross investment per worker, i:

$$x \equiv c + i \tag{2.2}$$

Net output, Y, is the gross product less depreciation.

$$Y \equiv X - D = X - \delta K \tag{2.3}$$

We can also express this in per-worker terms:

$$y = x - \delta k \tag{2.4}$$

PROBLEM 2.1 Ricardia is a corn economy, where the capital completely depreciates each year. Suppose that 20 bushels of seed corn can be planted by one worker to yield 100 bushels of harvest at the end of the year. Find x, k, ρ, δ, and y for Ricardia. How many workers and how much seed corn would be needed to grow a million bushels of corn?

PROBLEM 2.2 In Industria \$50,000 worth of output requires one worker-year of labor working with \$150,000 worth of capital. If 1/15 = .0666 = 6.66% of the capital depreciates in each year, what x, k, ρ, δ, and y would you choose to represent the Industrian production system? How much labor and capital would be needed to produce \$8 trillion in output in this economy? What would its net output be?

2.6 The Social Consumption-Growth Rate Schedule

The change in the capital stock from one period to the next, the *accumulation of capital*, is a key aspect of economic growth. The next period's capital stock is equal to this period's capital stock less depreciation plus gross investment:

$$K_{+1} = K - \delta K + I = (1 - \delta)K + I \tag{2.5}$$

The growth rate of the capital stock, g_K, is equal to the increase in capital divided by the initial level of capital:

$$g_K = \frac{K_{+1}}{K} - 1 \tag{2.6}$$

By dividing (2.5) by K, we can express the relation between gross investment per worker and the growth rate of the capital stock:

$$g_K = \frac{K_{+1} - K}{K} = \frac{I - D}{K} = \frac{i}{k} - \delta \tag{2.7}$$

Every economy faces a trade-off between consuming output and investing it to provide for future consumption. This trade-off is the *production possibilities frontier* between consumption and investment. In real economies the production possibilities frontier may be concave, reflecting rising costs as resources are shifted from producing consumption to investment. We will approximate the production possibility frontier as a straight line with slope $= -1$ and intercepts equal to X, as illustrated in Figure 2.3.

In studying economic growth, it is convenient to express this trade-off directly in terms of consumption and the growth rate of the capital stock. To facilitate the comparison of different economies of different sizes, we measure consumption and gross investment per employed worker. Equation (2.7) allows us to construct this key relationship, the *social consumption-growth rate schedule*:

$$c = x - (g_K + \delta)k = y - g_K k \tag{2.8}$$

In words, social consumption per worker is the output left over after the replacement of depreciation and the increase in the stock of capital have been accounted for.

We can also write the social consumption-growth rate schedule in the form:

$$x = c + (g_K + \delta)k \tag{2.9}$$

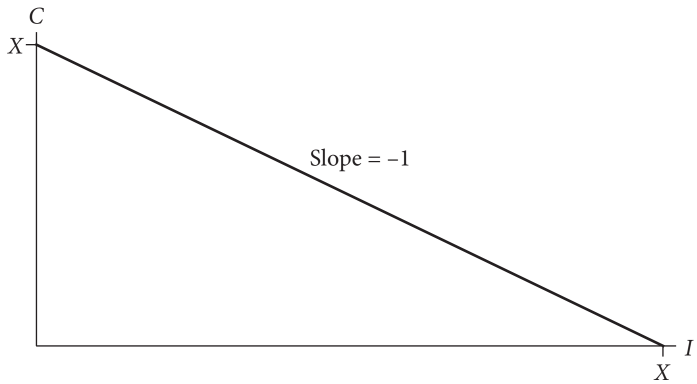

Figure 2.3 The production possibility frontier between consumption and investment is a straight line with slope equal to −1 and intercepts equal to output, if the economy can shift resources from consumption to gross investment without rising costs.

The social consumption-growth rate schedule is illustrated in Figure 2.4.

Sometimes it is convenient to express the social consumption-growth rate schedule in terms of the productivity of capital, ρ, rather than the capital intensity, k. In terms of ρ, x, and δ, the social consumption-growth rate schedule is:

$$c = x\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{2.10}$$

We can also solve the social consumption-growth rate schedule for $g_K + \delta$:

$$g_K + \delta = \frac{x - c}{k} = \left(1 - \frac{c}{x}\right)\rho \tag{2.11}$$

PROBLEM 2.3 Show the effect of an increase in labor productivity, holding the output-capital ratio and the depreciation rate constant, on the social consumption-growth rate schedule of an economy.

PROBLEM 2.4 Show the effect of an increase in the output-capital ratio, holding labor productivity and the depreciation rate constant, on the social consumption-growth rate schedule of an economy.

PROBLEM 2.5 Show the effect of an increase in the depreciation rate, holding labor productivity and the output-capital ratio constant, on the social consumption-growth rate schedule of an economy.

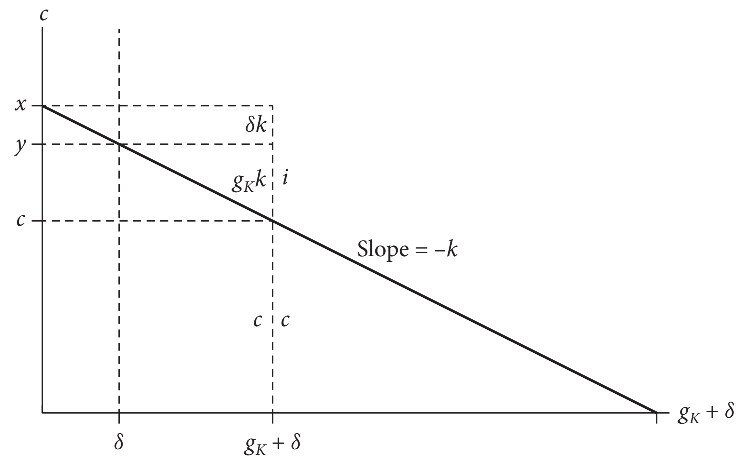

Figure 2.4 The *social consumption-growth rate schedule* expresses the trade-off between consumption and the growth of the capital stock. For a given technology the schedule is a straight line with slope equal to $-k$, the capital-labor ratio. If the economy consumed all its output, x, $g_K + \delta = 0$, and the capital stock would shrink at the rate of depreciation $-\delta$. If the economy invested all its output, consumption would be zero, $g_K + \delta = \rho$, and the capital stock would grow at the rate $\rho - \delta$. When the growth rate of the capital stock is zero, consumption is equal to the net product, y. At the actual $g_K + \delta$, output per worker is divided into consumption per worker, c, and gross investment per worker, i, which equals net investment per worker, $g_K k$, plus depreciation per worker, δk.

PROBLEM 2.6 Draw the social consumption-growth rate schedule for the US economy in 2005, using the data presented above.

PROBLEM 2.7 Draw the social consumption-growth rate schedule for Ricardia (see Problem 2.1). If the growth rate of the capital stock is 100% per year, how large is social consumption?

PROBLEM 2.8 Draw the social consumption-growth rate schedule for Industria (see Problem 2.2). If the growth rate of the capital stock is 10% per year, how large is social consumption?

2.7 The Distribution of Income: Wages and Profit

In capitalist economies capital is owned privately by profit-seeking capitalists, and workers work for a wage. The revenue from selling the output after the costs of intermediate inputs are deducted takes the form of wages and gross profit, including depreciation. Gross profit in turn is divided into depreciation and net profit, which is distributed in a variety of ways, as interest payments on debt, rents, royalties, taxes, and dividends. We will refer to gross profit simply as "profit."

Thus in a capitalist economy we can divide the value of output, X, into wages, W, and profit, Z, and profit in turn into net profit, R, and depreciation, D. This decomposition is the *income identity*. Profit, Z, the sum of depreciation and net profit, is also called *cash flow*.

$$X \equiv W + Z = W + R + D, \quad \text{or} \qquad (2.12)$$
$$Y \equiv X - D = W + R$$

The ratio of the total wage bill to employment, W/N, is the *average real wage*, w. We will often refer to the average real wage simply as the *wage*.

The ratio of profit to the capital stock, Z/K, is the *profit rate*, v. The ratio of net profit to the capital, R/K, is the *net profit rate*, r. The difference between the gross and the net profit rate is the depreciation rate: $v = r + \delta$.

2.8 The Real Wage-Profit Rate Schedule

In a capitalist economy there is a trade-off between wages and profit, given the value of output. Just as with the social consumption-growth rate trade-off, we can measure wages and profit per employed worker. This allows us to construct another key relationship, the *real wage-profit rate schedule*:

$$\frac{W}{N} = \frac{X}{N} - \frac{Z}{N} = \frac{X}{N} - \frac{D}{N} - \frac{R}{N}, \quad \text{or} \qquad (2.13)$$
$$w = x - vk = x - \delta k - rk = y - rk$$

In words, the wage can be regarded as the output left over after the capitalist has received her profit.

We can also write the real wage-profit rate relationship as:

$$x = w + vk \qquad (2.14)$$

The real wage-profit rate schedule is illustrated in Figure 2.5 and data for constructing an actual real wage-profit rate schedule are presented in Table 2.2.

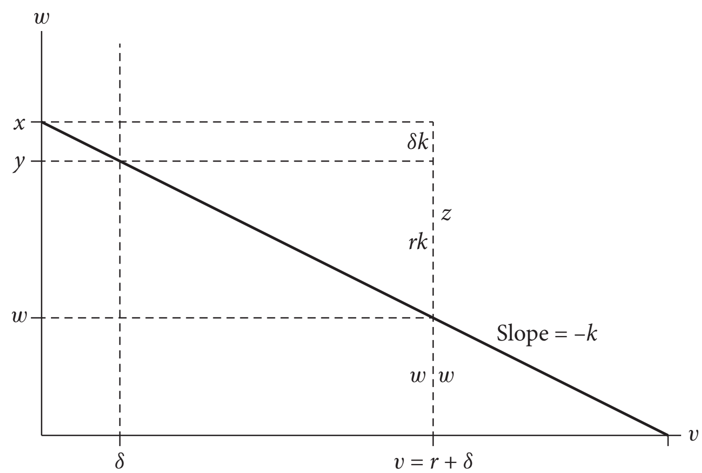

Figure 2.5 The *real wage-profit rate schedule* shows the relationship between real wages and the profit rate in a capitalist economy with given labor and capital productivity. For a given technology the schedule is a straight line with slope equal to $-k$, the capital-labor ratio. When real wages are equal to output per worker, x, the profit rate $v = 0$, and the net profit rate $r = -\delta$. When the real wage is zero, the profit rate $v = \rho$, and the net profit rate $r = \rho - \delta$. When the net profit rate is zero, the real wage is equal to the net product, y. At the actual profit rate output is divided into the components of income: the wage, w, and profit per worker, z, which equals net profit per worker, rk, plus depreciation per worker, δk.

Sometimes it is convenient to express the real wage-profit rate schedule in terms of the productivity of capital, ρ, rather than the capital intensity, k. In terms of ρ, x, and δ, the real wage-profit rate schedule is:

$$w = x\left(1 - \frac{v}{\rho}\right) \tag{2.15}$$

We can also solve the real wage-profit rate schedule for v:

$$v = \frac{x - w}{k} = \left(1 - \frac{w}{x}\right)\rho \tag{2.16}$$

PROBLEM 2.9 Draw the real wage-profit rate schedule for the US economy in 2005, using the data presented in Table 2.2.

Table 2.2 US, 2005 and 2014: Income Account

Variable	*Symbol*	*2005*	*2014*	*Units*
Output	X	14.683×10^{12}	16.598×10^{12}	\$/year
Wages	W	6.233×10^{12}	6.741×10^{12}	\$/year
(Gross) profit	Z	5.709×10^{12}	6.579×10^{12}	\$/year
Depreciation	D	2.772×10^{12}	3.143×10^{12}	\$/year
Net profit	R	2.937×10^{12}	3.437×10^{12}	\$/year
Net output	Y	11.911×10^{12}	13.455×10^{12}	\$/year
Capital	K	33.944×10^{12}	37.668×10^{12}	\$
Employment	N	143.99×10^{6}	148.46×10^{6}	workers
Labor productivity	x	101,974.82	111,799.269	\$/worker-year
Net labor productivity	y	82,724.48	90,633.34	\$/worker-year
Real wage	w	62,328.38	67,481.75	\$/worker-year
Profit per worker	z	39,646.40	44,318.53	\$/worker-year
Profit rate	v	16.82	17.47	%/year
Depreciation rate	δ	8.17	8.34	%/year
Net profit rate	r	8.65	9.12	%/year

Source: Extended Penn World Tables 6.0. All values are in 2011 purchasing parity.

PROBLEM 2.10 Draw the real wage-profit rate schedule for Ricardia (see Problem 2.1). If the real wage is 20 bushels of corn a year, what is the profit rate and the cash flow per worker?

PROBLEM 2.11 Draw the real wage-profit rate schedule for Industria (see Problem 2.2). If the real wage is \$10 per hour and workers work 2000 hours each year, what is the profit rate and the cash flow per worker?

2.9 Income Shares

The value of output, which accrues to workers and capitalists as income, is divided into the part going to workers as wages (the wage bill) and the part going to the owners of capital as profit. If we want to express these two parts as shares of income, we have only to divide them by output. The *profit share* is:

$$\pi \equiv \frac{X - W}{X} = \frac{x - w}{x} = \left(1 - \frac{w}{x}\right)$$

and the *wage share* is one minus the profit share, or:

$$1 - \pi \equiv \frac{W}{X} = \frac{w}{x}$$

It is sometimes helpful to use the profit or wage share instead of the wage in describing the distribution of the value of output in an economy. We can, for example, write the real wage-profit rate schedule in terms of π, using equation (2.16):

$$v = \left(1 - \frac{w}{x}\right)\rho = \pi\rho \qquad (2.17)$$

or

$$\pi = \frac{v}{\rho} \qquad (2.18)$$

Profit and wage shares can be calculated using national income accounts. Historical data for long periods, however, are available for only a few of the countries. Table 2.3 shows the profit share in the US, the UK, and Japan for selected years during the last century. These data show that the profit share has remained fairly stable during the twentieth century. It is usually around one-fourth to two-fifths of the GDP of these countries but because of differences in definition, we cannot make comparisons between countries in Table 2.3. The profit share gives some indication of decline since the nineteenth century. This decline was not spread out evenly, but seems to occur abruptly over a few decades, depending on the country.

On close inspection, time series data show that the profit share is not very stable. One source of instability occurs at frequencies of the business cycle, or every five years or so. During recessions, profit shares tend to decline, only to recover with the return of prosperity. The data in Table 2.3 try to correct for this cyclical variation (since we are concerned with long-term patterns) by choosing years near the peak of the business cycle, but some cyclical variation unavoidably remains.

The advanced capitalist countries now publish national income accounts, from which it is possible to calculate the profit shares over the last three decades. The Organization for Economic Cooperation and Development

Table 2.3 Profit Shares in the US, UK, and Japan for Selected Years from 1956–2016

US		UK		Japan	
Year	*Share*	*Year*	*Share*	*Year*	*Share*
1869	39.7	1856	40.9	1908	42.4
1880	51.9	1873	43.1	1917	50.2
1913	38.0	1913	38.8	1924	33.7
1924	40.4	1924	29.9	1938	40.0
1937	36.6	1937	32.1	1954	24.7
1951	39.7	1951	27.1	1964	25.16
1965	34.28	1964	29.68	1973	25.23
1973	32.50	1973	32.89	1990	29.42
1989	35.11	1990	33.20	1995	27.73
1995	35.79	1995	36.35	2000	30.31
2000	34.23	2000	33.40	2005	34.40
2005	37.37	2005	34.39	2010	36.42
2010	38.35	2010	32.42	2015	35.27
2015	38.35	2015	35.62	2016	35.27
2016	37.70	2016	35.02		

Sources: For the US, authors' calculations from Duménil and Lévy (1994, pp. 354–361); UK from Matthews et al. (1982, Table 6.8); Japan from Ohkawa and Rosovsky (1973, pp. 316–317); Extended Penn World Tables 5.0.

(OECD), as well as the European Commission, which publishes the Annual Macroeconomic database (AMECO), collect and compile this data in standardized form. Table 2.4 shows the average profit share over each of the last five decades for a group of six countries, adjusted for unpaid family members. (Differences in coverage make it hazardous to directly compare Tables 2.4 and 2.3.) The data reveal that the 1970s was generally a period of shrunken profit shares among these countries, compared to the 1960s. This historical event has been called the "profit squeeze" by some observers. In most cases, the profit share subsequently recovered during the 1980s and 1990s, and its upward trend persisted through the 2000s. It is remarkable that the profit share in the US and the UK actually increased on average after the 2008 crisis, while it decreased in the other countries included in the table.

Table 2.4 Profit Shares in Eight Countries in Selected Periods

Country	*1960–1973*	*1974–1979*	*1980–1989*	*1990–1999*	*2000–2006*	*2007–2010*	*2011–2016*
US	32.67	33.73	34.70	35.18	35.88	37.55	38.63
France	27.38	25.81	28. 32	34. 27	35.61	35.31	33.13
Germany	39.22*	39.89*	41.33*	43.49*	37.16	38.90	37.17
Netherlands	30.48	24.86	29.57	31.98	34.72	35.69	34.04
UK	30.40	28.91	32.67	34.84	33.28	33.07	34.73
Japan	25.66	19.52	24.53	28.71	33.19	35.79	34.92
China	35.23	35.23	35.23	35.58	40.96	44.95	43.98**
India	28.25	28.85	31.39	37.87	47.54	51.04	50.46**

* Authors' calculations from OECD (2016),
** 2011–2014.
Sources: AMECO adjusted wage share at current factor costs 2016; Penn World Tables 9.0.

It would not be strictly correct to say that the profit share is constant. Yet the profit share seems to remain near a value of one-third in the advanced capitalist countries over long periods of time.

2.10 The Growth-Distribution Schedule

If you experienced a sense of *déjà vu* in reading Section 2.8, it is not surprising, because the social consumption-growth rate schedule is exactly the same as the real wage-profit rate schedule. If you compare (2.8) to (2.13) you will see that the relations are exactly the same, except that w has been substituted for c, and v for $g_K + \delta$. The reason for this resemblance is that both the real wage-profit rate relation and the social consumption-growth rate relation depend only on k, x, and δ. The social consumption-growth rate relation represents the distribution of output between gross investment in future output and consumption. The real wage-profit rate schedule represents the distribution of the value of the output between wages and profit, including depreciation. The same technology underlies both relations. The combination of the two schedules is called the *growth-distribution schedule* for the economy, as illustrated in Figure 2.6.

Since the growth-distribution schedule describes both the distribution of output between consumption and gross investment and the distribution of

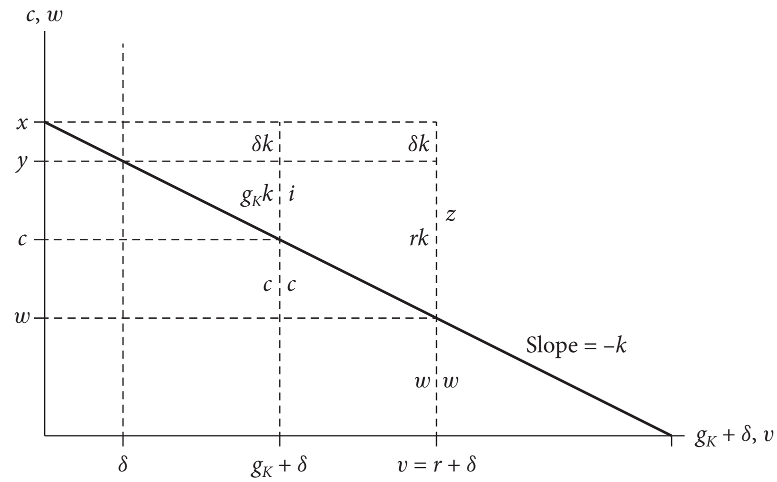

Figure 2.6 The *growth-distribution schedule* combines the social consumption-growth rate schedule and the real wage-profit rate schedule to give a complete view of the growth process in a capitalist economy. The growth rate need not equal the profit rate, because some part of the profits may be consumed. Social consumption per worker likewise exceeds the real wage because of the existence of capitalist consumption out of profits.

the value of output between wages and profit, it shows the aggregate national income and product accounts graphically. The key income and output identities in aggregate and per worker terms are:

$$X \equiv C + I = C + (g_K + \delta)K$$

$$x \equiv c + i = c + (g_K + \delta)k \tag{2.19}$$

$$Y \equiv X - D = C + (I - D) = C + g_K K$$

$$y \equiv x - \delta k = c + (i - \delta k) = c + g_K k \tag{2.20}$$

$$X \equiv W + Z = W + vK = W + R + D = W + rK + \delta K$$

$$x \equiv w + z = w + vk = w + rk + \delta k \tag{2.21}$$

$$Y \equiv X - D = W + R = W + rK$$

$$y \equiv x - \delta k = w + rk \tag{2.22}$$

The product accounts show that output is social consumption plus gross investment (2.19), and that net output is social consumption plus net investment (2.20). The income accounts show that the value of output is wages plus profit (2.21), and that the value of net output is wages plus net profit (2.22).

The growth-distribution schedule is a good starting point for the empirical analysis of growth in a real-world economy. The data you need to construct it are output per worker, x, the capital-labor ratio, k, or the productivity of capital, ρ, and the depreciation rate, δ, together with income and product account measures of consumption per worker, c, gross investment per worker, i, the wage, w, and the profit rate, v. These data are available for many countries and years in the Extended Penn World Tables. You can graph the growth-distribution schedule for one country for one year, or for the same country over several years to understand the chief factors responsible for growth, or for more than one country in a single year to compare their growth patterns.

PROBLEM 2.12 Graph the growth-distribution schedule for the US economy in 2005.

PROBLEM 2.13 Graph the growth-distribution schedule for Ricardia if the wage is 20 bu/worker-year and the growth rate of capital is 100% per year.

PROBLEM 2.14 Graph the growth-distribution schedule for Industria when the net profit rate is 13.33% per year and the growth rate of capital is 6.66% per year.

2.11 Changes in Labor and Capital Productivity

A very important aspect of economic growth is changes in the productivity parameters of the economy, x, ρ (or k), and δ over time. Increases in output per worker, x, are the main source of increases in wealth and standard of living. It is useful to classify patterns of change in these parameters so that these patterns can be compared to the experience of real-world economies.

Changes in labor and capital productivity can be described in terms of shifts of the growth-distribution schedule. The growth-distribution schedule is a straight line defined by two points, for example, the point $(0, x)$, which corresponds to the minimum rate of profit and the maximum level of the real wage, and the point $(\rho, 0)$, which corresponds to the maximal profit rate and zero real wage. Since changes in δ leave output and cash flow per worker unchanged, we will classify movements of the growth-distribution schedule, and hence changes in technique, by changes in x and ρ.

An increase in x holding ρ constant corresponds to a pure increase in labor productivity (more output per worker-year) with no change in capital productivity (since the output-capital ratio ρ is unchanged). This type of technical change is called *labor-saving* since it has the effect of increasing output per unit of labor input, x. The measured rate of change in labor productivity is g_x, the percentage increase in output per worker from one period to the next:

$$g_x \equiv \frac{x_{+1}}{x} - 1 \tag{2.23}$$

A rise in ρ holding x constant corresponds to an increase in capital productivity, since it raises the output-capital ratio. This type of technical change is called *capital-saving*. The measured rate of change in capital productivity is g_ρ, the percentage increase in output per unit of capital from one period to the next:

$$g_\rho \equiv \frac{\rho_{+1}}{\rho} - 1 \tag{2.24}$$

Figure 2.7 illustrates an arbitrary shift of the growth-distribution schedule and shows how the rates of labor-saving and capital-saving technical progress can be calculated.

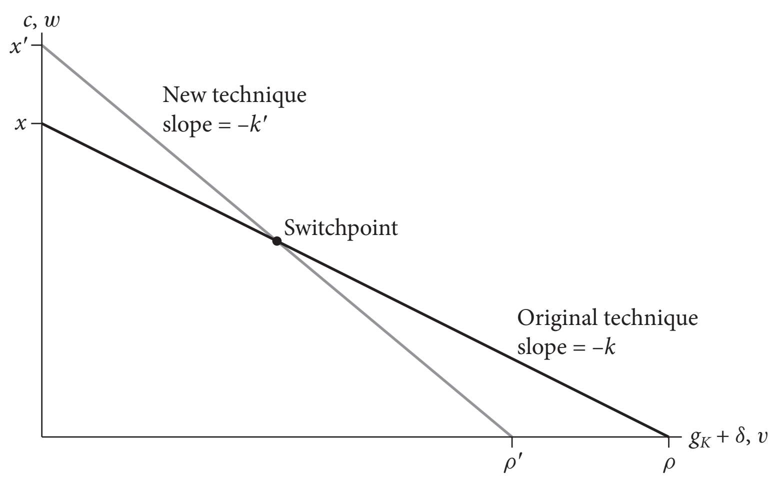

Figure 2.7 Technical change corresponds to a shift in the straight line defining the growth-distribution schedule. An upward shift in x corresponds to labor-saving technical progress, since output per worker increases. An outward shift in ρ corresponds to capital-saving technical progress, since output per unit of capital increases. The shift pictured here combines labor-saving technical progress with negative capital-saving (or capital-using) technical change.

Table 2.5 US Technical Change, 1988–1989, 2004–2005, and 2013–2014

Variable	*1988*	*1989*	*Growth Rate*
x	$76,754/worker-year	$79,130/worker-year	
g_x			3.1%/year
ρ	65.51%/year	65.75%/year	
g_ρ			0.37%/year
Variable	*2004*	*2005*	*Growth Rate*
x	$100,293.63/worker-year	$101,974.82/worker-year	
g_x			1.68%/year
ρ	43.44%/year	43.26%/year	
g_ρ			−0.41%/year
Variable	*2013*	*2014*	*Growth Rate*
x	$110,686.27/worker-year	$111,799.27/worker-year	
g_x			1.00%/year
ρ	43.32%/year	44.06%/year	
g_ρ			1.71%/year

Source: Extended Penn World Tables 6.0. All values are at 2011 purchasing power parity.

As Table 2.5 shows, the US economy experienced labor-saving technical change at the rate of 3.1% per year and capital-saving technical change at the rate of .37% per year between 1988 and 1989. Conversely, between 2004 and 2005 labor productivity grew by 1.68%, while capital productivity fell by .41 points.

PROBLEM 2.15 Graph on the same graph the new and old growth-distribution schedules for Ricardia if it experiences a 50% labor-saving and 0% capital-saving technical change.

PROBLEM 2.16 Graph on the same graph the new and old growth-distribution schedules for Industria if it experiences a 2% labor-saving and −2% capital-saving technical change.

PROBLEM 2.17 Graph on the same graph the growth-distribution schedule for the US in 2004 and 2005, using the data in Table 2.5.

2.12 Comparing Economies

As we have seen in analyzing the US economy in 1988 and 1989, 2004 and 2005, and 2013 and 2014, the growth-distribution schedule is a good way to visualize the changes in a single economy over time. It illustrates the type of technical change that is occurring, shows how the economy allocates its product between growth and consumption, and reveals the underlying distributional relations between real wages and profits.

The growth-distribution schedule is also a good way to compare the productivity and growth patterns of two different economies. If we plot the growth-distribution schedules of the two economies on the same graph with the same units of output per worker, the relative productivities of the two economies and their relative patterns of distribution and growth can be visualized clearly.

We can use the Extended Penn World Tables data to compare the US and Japanese economies in 2014, for example, as Table 2.6 and Figure 2.8 show.

PROBLEM 2.18 Use the data in Table 2.6 to graph growth-distribution schedules for the US and China in 2014.

Table 2.6 Comparison of US, Japan, and China, 2014

Variable	*US*	*Japan*	*China*
x	\$111,799/wkr-yr	\$63,394/wkr-yr	\$21,464/wkr-yr
k	\$253,716/wkr	\$179,082/wkr-yr	\$60,865/wkr-yr
ρ	44.07%/yr	38.75%/yr	35.26%/yr
δ	8.34%/yr	10.17%/yr	6.53%/yr
c	\$88,564/wkr-yr	\$54,282/wkr-yr	\$11,428/wkr-yr
i	\$23,325/wkr-yr	\$15,112/wkr-yr	\$10,035/wkr-yr
g_K	0.81%/yr	−1.73%/yr	9.95%/yr
w	\$67,482/wkr-yr	\$41,830/wkr-yr	\$12,175/wkr-yr
z	\$90,631/wkr-yr	\$51,175/wkr-yr	\$17,486/wkr-yr
v	17.47%/yr	15.39%/yr	15.26%/yr
r	9.12%/yr	5.21%/yr	8.72%/yr

Source: Extended Penn World Tables 6.0. All values are at 2011 purchasing power parity.

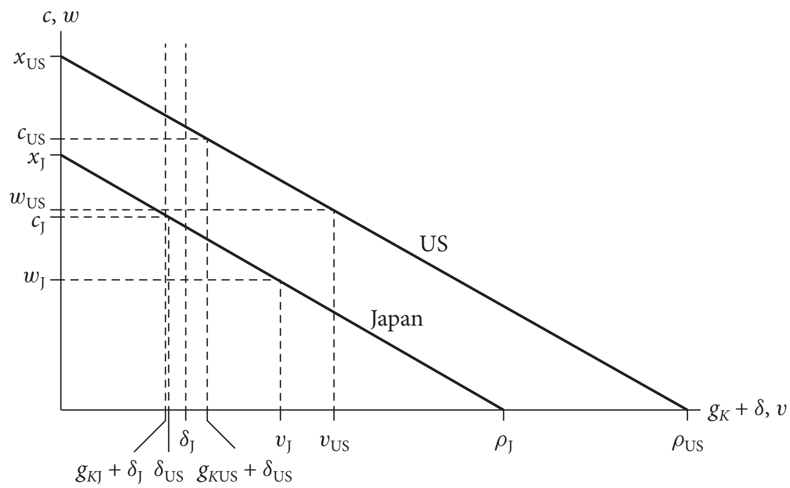

Figure 2.8 The growth-distribution schedules for the US and Japan in 2014 are drawn on the same scale. The US has higher productivity of both labor and capital, so its growth-distribution schedule lies above the Japanese growth-distribution schedule. The US real wage of \$67,482/wkr-yr gives a profit rate of 17.47%/yr, and the Japanese real wage of \$41,830/wkr-yr a profit rate of 15.39%/yr. US consumption per worker of \$88,564/wkr-yr leaves room for a growth rate of capital of 0.81%/yr, while Japanese consumption per worker of \$54,282/wkr-yr determines a negative growth of the Japanese capital stock at a rate of −1.73%/yr.

2.13 Global Economic Leadership

Modern economists have a distinct advantage over previous generations of growth theorists because much more data are now available. *Time series* data express the historical patterns of the main variables, while *cross sectional* data allow for comparisons between countries at a point in time. Often, these sorts of data are combined into *longitudinal* or *panel* data sets.

The late Angus Maddison assembled an important panel data set that includes six leading advanced capitalist countries (US, France, Germany, Netherlands, UK, and Japan) over nearly two centuries. Table 2.7 presents the relative levels of labor and capital productivity and relative capital intensity for these countries for selected years since 1820. The levels are measured as index numbers relative to the US, owing to its status as current world leader in labor productivity. Thus, productivity in the US is 100 by definition, while in 1992, for example, Japan's index shows that its labor productivity

Table 2.7 Catching up and Falling behind: Productivity Relative to the US for Six Countries in Selected Years from 1820–1992

	1820	*1870*	*1913*	*1929*	*1938*	*1950*	*1973*	*1992*
	Labor Productivity (US = 100)							
US	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
France	94.6[a]	59.3	55.7	55.1	61.9	45.5	75.3	101.8
Germany	86.4[a]	68.6	68.3	58.0	56.0	34.4	70.5	94.7
Netherlands	121.3[a]	101.4	78.3	84.0	72.3	51.3	80.6	99.0
UK	111.0	115.0	83.6	73.6	69.6	62.0	67.5	82.4
Japan	33.1	20.2	20.2	23.6	25.4	16.0	47.2	68.8
	Capital Intensity (US = 100)							
US	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
France						30.4	55.2	95.3
Germany					31.1	25.5	64.6	92.2
Netherlands						43.2	75.3	93.1
UK	80.1	60.6	21.3	21.1	17.5	20.5	42.0	61.6
Japan		5.0[b]	5.4	8.7	8.2	11.6	38.9	85.6
	Capital Productivity (US = 100)							
US	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
France						149.8	136.3	106.8
Germany					179.8	134.9	109.0	102.7
Netherlands						118.7	107.1	106.4
UK	138.6	189.9	392.9	349.0	397.5	302.3	160.6	133.7
Japan		428.9[b]	372.6	270.9	308.4	138.3	121.3	80.4

[a] Using GDP per person.
[b] 1890.
Sources: Maddison (1995a, Tables K-1, A3a, C16a, J-2, J-4, and D1a) and Maddison (1995b, pp. 148–164).

was 75.5 percent of the level in the US. Unfortunately the original Maddison dataset stops at 1992.

From 1820 to 1973 the global lead in labor productivity has changed hands three times. In 1820, the Netherlands was the world's most productive nation, but by 1870 the lead had passed to the UK. By 1913, the US had overtaken the UK, and maintained leadership through the post-WWII decades, up to the present day. This pattern is sometimes called "leapfrogging." We do not yet know if leapfrogging will continue.

Another clear possibility is that leapfrogging will make way for *convergence* in productivity levels. Between 1950 and 1992, the labor productivity lead of the US narrowed. This "catching-up" occurred at different periods for different nations. Most of this "catching-up" by other advanced nations occurred between 1950 and 1973.

On the other hand, during the late nineteenth and early twentieth centuries, the other countries were "falling behind" while the US surged ahead. We need to be careful not to confuse falling behind in relative terms, which the table illustrates, with falling behind in absolute terms. Since the US productivity level is growing over time, countries that experience less growth will fall behind in relative terms, even though they are growing in absolute terms. During the period spanning the two world wars, the US lead continued to grow, partly because of the devastations of war that were visited upon the other countries, and partly because of the dynamism of the US economy.

Convergence in labor productivity levels has been associated with convergence in the capital-labor ratio, or capital intensity. Both variables have converged "from below." By contrast, capital productivity seems to converge on the world leader "from above." Nations at low levels of labor productivity have high levels of capital productivity, which then falls in the course of economic development. In 1950, all five other countries had much higher output-capital ratios than the US. By 1973, all had lower capital productivity ratios than the US.

The challenge presented to modern growth theory is to explain and interpret the relative growth performance of the world's nations, as well as the absolute growth performance of each individual country. Some data on absolute growth performance of the same six countries appear in the next section.

2.14 Labor Productivity Growth in Real Economies

Labor productivity has grown more or less continuously in the six advanced capitalist countries whose history we have been following. Angus Maddison divided the last one hundred and seventy-five years of history into five sub-periods up to 1992. He periodized the phases of modern growth sensibly, based on his own judgment, and we have adopted his periodization in Table 2.8.

Table 2.8 Growth Rates of Selected Variables (%/year) for Six Countries in Selected Periods from 1820–2010

	1820–1870	*1870–1913*	*1913–1950*	*1950–1973*	*1973–1992*	*1993–2002*	*2003–2006*	*2007–2010*
				US				
Labor Productivity, g_x	1.10	1.88	2.48	2.74	1.11	2.10	1.37	1.11
Capital Intensity, g_k	2.30	3.44	1.65	2.10	1.84	2.32	2.41	0.63
Capital Productivity, g_ρ	−1.18	−1.51	0.81	0.63	−0.72	−0.20	−1.01	0.52
				France				
Labor Productivity, g_x		1.74	1.87	5.11	2.73	1.82	2.10	0.80
Capital Intensity, g_k				4.79	4.78	−0.51	3.50	3.75
Capital Productivity, g_ρ				0.22	−1.96	2.34	−1.35	−2.83
				Germany				
Labor Productivity, g_x		1.87	0.60	5.99	2.69	2.19	2.68	0.27
Capital Intensity, g_k				5.93	3.76	1.08	−0.43	−0.26
Capital Productivity, g_ρ				0.05	−1.04	1.11	3.13	0.51
				Netherlands				
Labor Productivity, g_x		1.27	1.31	4.78	2.21	2.69	3.60	0.65
Capital Intensity, g_k				4.59	3.14	0.96	1.74	2.60
Capital Productivity, g_ρ				0.18	−0.90	1.72	1.82	−1.91
				UK				
Labor Productivity, g_x	1.16	1.13	1.66	3.12	2.18	3.13	1.03	0.18
Capital Intensity, g_k	1.74	0.96	1.56	5.33	3.91	1.69	2.67	1.48
Capital Productivity, g_ρ	−0.55	0.16	0.10	−2.10	−1.67	1.42	−1.59	−1.27
				Japan				
Labor Productivity, g_x	0.09	1.89	1.85	7.69	3.13	1.64	0.84	0.10
Capital Intensity, g_k		3.03	3.75	7.63	6.16	2.66	−1.39	−1.76
Capital Productivity, g_ρ		−0.95	−1.85	0.06	−2.85	−0.97	2.26	1.91

Sources: Maddison (1995a, Table 2-6); Extended Penn World Tables 5.0.

Despite its continuity, the growth of labor productivity has not been steady. Instead, it has a stop-go quality. Maddison's last two phases in particular attracted much attention. In the period from 1950–1973, labor productivity grew at unprecedented rates. By contrast, since 1973, labor productivity growth slowed everywhere; economists often refer to this event simply as the *productivity slowdown*. In the US, labor productivity growth was lower in the 1973–1992 period than in any previous period. In other countries, labor productivity growth remained at levels that were respectable by historical standards, but lower than in the previous phase. And yet, these two decades saw the US economy losing ground relative to other advanced nations, especially Japan and Germany.

During the 1990s, labor productivity growth in the US proceeded again at a healthier growth rate of over 2 percent a year on average. The latter part of the decade was characterized by the surge of information technology and the internet in particular, and a strong wave of investment in "dot.com" companies. As it often happens, however, by 1999 the dot.com boom turned into bust. The US economy went into recession in 2001, and labor productivity growth slowed down again in the early 2000s. The Great Recession and the slow recovery that has been following are responsible for the low labor productivity growth in the 2007–2010 period.

Capital productivity has not behaved uniformly. The period 1973–1992 saw capital productivity decline in all the countries in the table. In the decades that followed, however, the picture has been less clear. In the US, capital productivity declined during the first two growth phases, but increased from 1913–1973 and in the latter period 2007–2010. Over the whole expanse of time, the output-capital ratio in the US declined, from 1.055 in 1820 to 0.5827 in 2010. This corroborates and qualifies our earlier observation that the output-capital ratio tends to fall in the course of economic development.

The patterns of growth in Table 2.8 could reflect patterns of technical change, or they could reflect technical choices from among the existing techniques. The first case represents a shift in the production function, while the second case represents movement along the production function. Obviously, some combination of the two movements is also possible.

If we interpret the patterns as technical changes, it is clear that the rate of labor-saving technical change has been persistently positive. On the other hand, the rate of capital-saving technical change has been positive in some periods and negative in others. Negative capital-saving technical change, or capital-using technical change, is economically possible when it occurs in combination with labor-saving technical change. A new technique that uses more capital may be more profitable if it saves enough labor. Chapter 8 is dedicated to this issue.

There is an important connection between Table 2.8 and Tables 2.3 and 2.4. For countries where the productivity of labor has grown persistently and the profit share has remained roughly constant, we can deduce that, over the very long run, the real wage must have grown at a rate equal or close to the growth rate of labor productivity. One important exception, at least according to the available data, is the United States. The rising trend in the profit share after the 1990s is apparent, and it persisted even during the Great Recession and its aftermath. The implication is that real wage growth has not kept up with labor productivity growth, and the distribution of income has shifted in favor of profits.

The pattern of rising labor productivity and declining capital productivity is common, but by no means universal in looking at real economies. Figure 2.9, for example, plots the (g_ρ, g_x) pairs for the same 49 countries plotted in Figure 2.1 for 2000–2010, from the EPWT data. While rising labor productivity appears to be coupled with falling capital productivity in 2010, the graph for 2000 shows the opposite.

2.15 Stylized Facts

In this chapter we have developed a system of accounting that allows us to present the empirical facts that are the foundation of the analysis of economic growth. Several patterns, or *stylized facts*, emerge strongly from the data. One challenge for theories of growth is to explain these common patterns.

As capitalist economies develop, there is a strong tendency for labor productivity to increase, while capital productivity stagnates or slowly declines. As a result capital per worker rises. As labor productivity increases, the real wage also rises, at roughly the same rate. As a result, the wage and profit shares in income, despite definite fluctuations, show no strong trend.

In the next chapters, we will develop the technical concepts that are the basis of the various theories of economic growth and technical change.

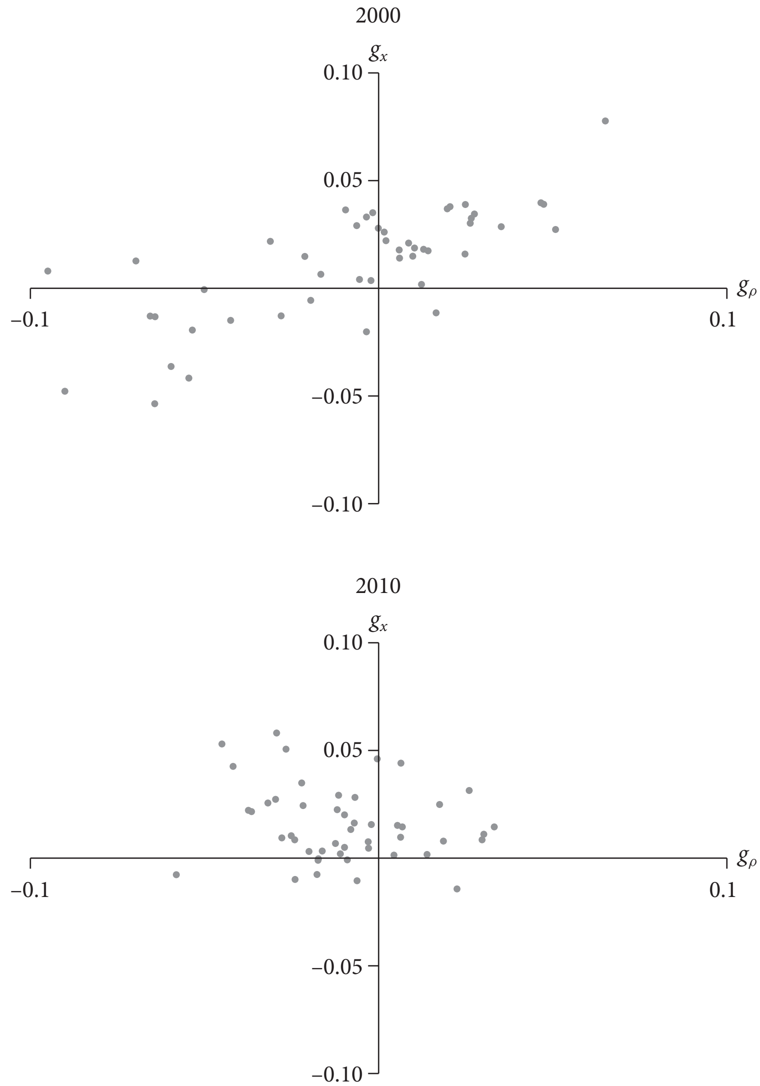

Figure 2.9 The (g_ρ, g_x) observed for 49 countries in 2000 and 2010.

2.16 Suggested Readings

The *Survey of Current Business*, published by the US Department of Commerce, regularly reports on developments in national income accounting in the US through clear, well-documented articles. The measurement of consumer prices in the US has been the subject of considerable debate, which is reviewed in a symposium published in the winter 1998 issue of the *Journal of Economic Perspectives*. Economists in the classical tradition have developed alternative interpretations of national income accounts, particularly by recognizing the distinction between productive and unproductive activities; see Wolff (1987) or Shaikh and Tonak (1994).

An important contribution to the measurement of profit and wage shares is Gollin (2002).

Documentation for the Penn World Tables can be found in Feenstra et al. (2015). Angus Maddison provides useful commentary along with his historical data sets in Maddison (1995a, 2001, 2003). A unique and readable work that combines comprehensive macroeconomic statistics with political economy and history, Armstrong et al. (1991) is a particularly good source on the Golden Age of Capital Accumulation and the profit squeeze of the 1970s. A widely cited source on catching up and falling behind is Abramovitz (1986), while Nelson and Wright (1992) focus on the particular characteristics associated with the rise to leadership of the US. An accessible and comprehensive treatment of productivity, the productivity slowdown, catching up, and US leadership is given in Baumol et al. (1989). The causes and consequences of the productivity slowdown are explored in a popularly written but careful book by Madrick (1995). The idea that the computer revolution will affect productivity growth with a time lag is attributed to David (1990). A pessimistic view of future productivity growth is presented in Gordon (2016).

Finally, the original list of “stylized facts” can be found in Kaldor (1965).

3

Models of Production

3.1 Accounting Frameworks and Explanatory Models

Description is an important step toward a complete understanding of the process of economic growth. But economists would like to go further than mere description, to explain and even to predict the consequences of historical developments and policies on the pattern of growth. In order to give explanations and make predictions, the economist needs a complete *model* of the growth process, in which the factors to be explained or predicted are *endogenous variables* determined within the model, and the factors explaining or predicting consequences are *exogenous parameters*. The model specifies enough relations among the endogenous variables and the exogenous parameters so that once we know the exogenous parameters we can calculate mathematically (or graphically) the corresponding values of the endogenous variables. *Explanation* in such a model consists of showing what change in the exogenous parameters would lead to an observed change in the endogenous variables. *Prediction* consists of calculating the effect on the endogenous variables of hypothetical changes in the exogenous variables. As a first step to developing complete models of economic growth, we need to develop a *model of production*, since so far we have only set up an accounting framework. The specification of such a model inevitably loses certain features of complex reality, so that we must consider each of the assumptions of the model carefully, to understand what real-world situations the resulting model can and cannot explain.

3.2 A Model of Production

In our model, as in the accounting framework, we will conceive of time as passing in discrete units (usually years), $t = 2016, 2017, \ldots$ Economic decisions, such as the decision to produce or to consume, and prices are fixed at the beginning of each period, and cannot change until the next period. The period framework forces us to conceive of all economic decisions proceeding synchronously on the same time scale, which is a drawback. More realistic modeling treatments of time, however, involve much more complicated mathematics, and we choose the period scheme as the simplest that can explicitly reflect the economic passage of time.

In the accounting framework X is GDP, the market value of all the disparate goods and services actually produced in an economy. In our model, in order to simplify as much as possible, we will assume that there is only one good produced, *output*, which we will also denote as X, and that this output can be accumulated as a single kind of capital K. (In the real world, of course, K is the value of capital goods of many different kinds.) Capital, K, and labor, N, together produce the output X. We will model the fact that production takes time by assuming that inputs must be employed at the beginning of the period, while the output becomes available at the end of the period.

A *technique of production* can be described by specifying how much capital is necessary at the beginning of a period to equip one unit of labor, how much output is produced at the end of the period, and how much of the capital stock deteriorates during the period. We will assume that techniques exhibit *constant returns to scale*, that is, that it is possible to produce exactly twice as much output with twice as much of both inputs.

A technique of production can be described by three numbers, (k, x, δ), where k is the capital stock per worker, x is the output per worker, and δ is the proportion of the value of the capital stock lost to depreciation over the period of production. In general δ will be larger than zero (some deterioration of the capital stock always takes place over the period of production) and smaller than or equal to 1 (some of the capital stock may survive to the next period). (If $\delta = 1$, the capital stock lasts only one period, like seed corn, and the corresponding model is often called a *corn model*.) It is also possible to describe the technique as (ρ, x, δ), where $\rho = x/k$ is the productivity of capital, or as (ρ, k, δ), since if we know any two of the parameters (k, x, ρ), we can derive the other one. We can describe a single technique of production schematically as:

1 labor and k capital at the beginning of the period
$\rightarrow$
x output and $(1-\delta)k$ capital at the end of the period

We can also describe a technique as a table of input-output coefficients:

outputs	output	x
	capital	$(1-\delta)k$
inputs	capital	k
	labor	1

The technique of production determines the relation between output and the input of capital and labor:

$$N = \frac{X}{x}$$
$$K = \frac{kX}{x} = \frac{X}{\rho}$$

The technique in use determines the productivity of labor and capital and the capital-labor ratio in the economy. Each technique of production corresponds to a particular growth-distribution schedule, like the one in Figure 2.6.

The *technology* of an economy is the collection of all the known usable techniques. We could represent the technology as a matrix, each column of which is a technique of production. At any given real wage, different available techniques will yield different profit rates.

We assume that the technology defined by the input-output coefficients is an exogenous parameter in each period. In real-world economic growth a crucial role is played by *technological change*, which appears in the model as a change in the collection of techniques from period to period. We will study models of technical change later in this book.

3.3 Agents and Distribution

To clarify the exact mechanisms through which capitalist production functions, we will distinguish three types of agents in our model. First are *workers*, who supply labor for a wage. Second are *capitalists*, who own the capital. Third are *entrepreneurs*, who on behalf of the capitalists hire workers, organize production, sell the output, and return the residual revenue as profit to the capitalist after paying the workers their wages. In real-world capitalist economies these functions are sometimes combined in various ways. Workers may own part of the capital through pension funds, for example, or as members of producer cooperatives. Capitalists may act as entrepreneurs, both owning capital and organizing production (and, indeed, this was a common pattern in the early days of industrial capitalism). But even if the same persons sometimes act out the three roles, our analysis will be clearer if we separate them carefully.

We will always assume that there is a large number of each type of agent, even though they are all alike, so that competition rules and each agent, worker, entrepreneur, or capitalist takes output prices and wages as given.

The entrepreneurs hire the workers for a wage measured in terms of output, w, paid at the end of the period, and organize them to produce. Entrepreneurs must choose a technique of production defined by coefficients k (or ρ), x, and δ from available technology determined by engineering and scientific knowledge and social and cultural practices that limit the possible techniques of production. For example, health and safety legislation might prevent entrepreneurs from using workers in ways that cause occupational diseases or preventable accidents.

Given the technique chosen, (ρ, x, δ), in order to produce X output in a period, the entrepreneur must hire $N = X/x$ workers for the period. The wage bill will be $W = wX/x$. The entrepreneur must also secure the services of capital equal to X/ρ from owners of capital. Competition will force the entrepreneurs to pay the residual revenue after the payment of wages, the *(gross) profit*, to the capitalists at the end of the period. Since the profit share $\pi = (1 - (w/x))$, the profit will be:

$$Z = X - W = \left(1 - \frac{w}{x}\right) X = \pi X$$

The *(gross) profit rate*, v, will be:

$$v = \frac{Z}{K} = \rho \left(1 - \frac{w}{x}\right) = \pi\rho \tag{3.1}$$

The profit rate is quite different from the *price* of a unit of capital, since capital may last several periods if $\delta < 1$. The profit rate v is what the entrepreneur pays the capitalist as the result of using a unit of capital for a single period, at the end of which the depreciated capital returns to the capitalist. The price of a unit of capital is always 1, since we are reckoning prices in terms of output, and one unit of output can be invested as one unit of capital.

We assume that the entrepreneurs pay themselves a wage for whatever actual work they do in production (which is accounted for in W), and that their motivation for undertaking the entrepreneurial activity is their pure joy in bossing other people around.

Now let us consider the situation of the capitalist, who owns the capital stock. She begins the period with capital K, and receives a gross profit from the entrepreneurs of vK. At the end of the period she gets back the depreciated capital $(1 - \delta)K$ along with the profit. Thus the *net profit* consists of the profit less depreciation, $R = vK - \delta K$, and the *net profit rate*, the ratio of profit to the initial capital, is:

$$r \equiv \frac{R}{K} = \frac{vK - \delta K}{K} = v - \delta = \pi\rho - \delta$$

3.4 Social Accounting Matrix

We can summarize the relationships between the agents in this economy by means of a social accounting matrix (SAM) that records all the transactions in a unified system of accounts. We have presented the national accounts for each sector individually. A SAM presents the equations for all the sectors in one comprehensive statement that makes the interrelationships more transparent. The SAM presented in Table 3.1 is designed around a few accounting conventions that follow the pioneering work of Lance Taylor in the application of consistent accounting foundations for macroeconomic models. For example, the sums of corresponding rows and columns should be equal.

The first row represents the expenditures on gross output or the uses to which output can be put. This is the familiar output identity. The first column represents the familiar national income identity showing that output is exhausted by the costs of its production, wages, and gross profits. The accounting convention that rows and columns add up implements the principle that value added can either be measured by adding up incomes or by adding up expenditures on final goods.

The next three rows show how the gross incomes (X) of workers, capitalists, and firms arise, using the symbols w, c, and f as superscripts and

Table 3.1 A SAM for the Capitalist Economy

		Expenditures				
	Output costs	w	c	f	I	*Sum*
Output uses		C^w	C^c		$\Delta K + \delta K$	X
Incomes						
w	W					X^w
c				vK		X^c
f	vK					X^f
Flow of funds						
c			S^c		$-(\Delta K + \delta K)$	0
Sum	X	X^w	X^c	X^f	0	

to identify the rows. By convention, gross profits are allocated to firms. The firms pay workers wages in the w-row and pay rents to the capitalists in the c-row.

The next three columns labeled w, c, and f show how each agent allocates her income between consumption spending (C) and gross saving (S), using superscripts to identify the agent. In the basic model of a capitalist economy, we will assume that workers consume all their income so worker saving is zero. As a result we have excluded it from this SAM, but in later chapters we will consider worker saving. We also assume that firms rent the services of capital goods from the capitalist agents and do not engage in any investment spending of their own. Firm saving is zero so it is not shown here either. In later chapters we will introduce firm saving in the SAM.

The second-to-last column records gross investment, which equals new capital formation plus replacement of depreciated capital. This column provides a kind of pivot to the flow of funds accounts because it reveals how investment is being financed or funded. The flow of funds row labeled c uses a sources-and-uses of funds approach to accounting. The accounting convention is that a source of funds receives a positive sign and a use of funds receives a negative sign.

The SAM shows that the flow of saving provides a source of funds that can be used to purchase new capital goods and replace worn-out capital goods. In the basic model of a capitalist economy, there is only one asset that can be accumulated, capital. In this simple model, saving and investment can be considered to be identical acts, but that will change as we introduce more realism to the model by providing other assets that can be accumulated besides capital. The coordination of saving and investment when these are not carried out by the same agents is one of the deepest questions in macroeconomics.

The SAM provides a comprehensive accounting of the important transactions involving flows of output, income, spending, and saving. To complete the picture, it is also useful to record the assets and liabilities of agents in each period. The *balance sheet* of an economic agent uses the accounting definition that net worth is equal to assets minus liabilities. Net worth is sometimes called net wealth, or just wealth, and we use the symbol J to refer to it in this book. The convention for recording assets and liabilities rearranges the definition of net worth by putting assets on the left-hand side. The balance sheet then looks like this (in practice, a vertical line is often used to separate the two sides rather than an equal sign):

$$\text{Assets} = \text{Liabilities} + \text{Net Worth}$$

In our simple model, there are no liabilities and the only asset is capital. Because the capitalist agents are the only agents holding any asset, we need only consider their balance sheet. It is simply

$$K = J$$

and we can use the terms capital and wealth interchangeably in the basic model. Later we will consider other forms of wealth such as land, natural resources, or financial assets, and we will introduce some liabilities as well.

The flows recorded in the SAM cumulate smoothly into changes in the balance sheet. A balance sheet reports stocks measured in real dollars, without the time dimension that is needed when measuring flows. The balance sheet is like a snapshot of the financial position of an agent taken on a specific date in time. Having both the SAM and the balance sheets gives us a complete accounting of how financial positions evolve from period to period.

The accounting convention that requires equality between the corresponding row and column sums enforces *stock-flow consistency*. Stock-flow consistency is sometimes described colorfully as the requirement that there are no "black holes" where flows simply disappear. In the basic model, these accounts appear almost trivially simple, but as we add complicating factors the value of having a unified system of accounts presented in a transparent way will become self-evident.

3.5 Choice of Technique and Production Functions

Each combination of the parameters k, x, and δ defines a single technique of production, one particular method of combining labor and capital to produce output, and therefore one growth-distribution schedule. Suppose there is another possible way to produce output, in which each worker is equipped with k' units of capital and produces x' units of output, assuming, for simplicity, that the depreciation rate is the same for both techniques, $\delta' = \delta$. The alternative technique has its own real wage-profit rate schedule. In Figure 3.1 we plot the real wage-profit rate schedules corresponding to both the original technique defined by k and x, and the alternative defined by k' and x'. From the entrepreneur's point of view the real wage-profit rate schedule shows how large a profit rate she can secure the capitalist at any wage. Entrepreneurs who secure a larger profit rate v will be the most popular with capitalists.

In a capitalist society entrepreneurs choose techniques of production to maximize profit. In the case illustrated, the alternative technique will pay a higher profit rate than the original when the wage is high, but a lower profit

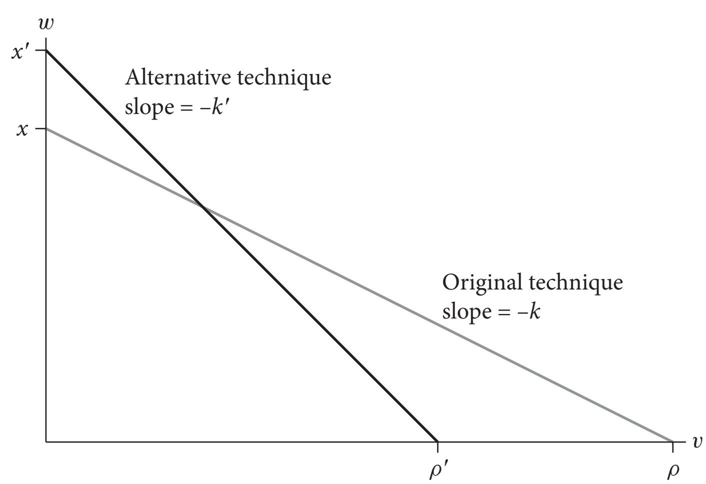

Figure 3.1 When there are two or more available techniques, each has its own real wage-profit rate schedule.

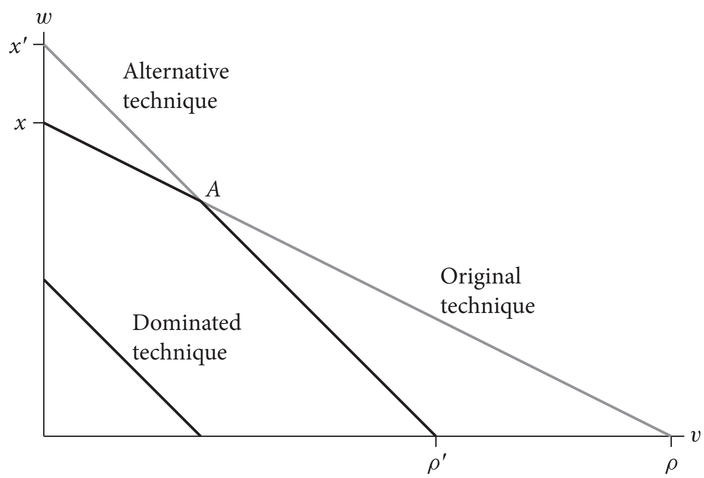

Figure 3.2 The relevant technological choices are those on the *efficiency frontier*, which is the northeast boundary of the real wage-profit rate schedules corresponding to the available techniques.

rate when the wage is low. An entrepreneur who has the option of using either technique of production would use the alternative technique at high wage rates and the original technique at low wage rates.

A technique is *dominated* by another technique when the real wage-profit rate schedule of the first lies entirely below and to the left of the corresponding schedule of the second. The third technique illustrated in Figure 3.2 is dominated by both the original and alternative techniques. It is not profit-maximizing for an entrepreneur to use a dominated technique at any wage rate. The *efficiency frontier* for a technology is the northeast boundary of the real wage-profit rate schedules corresponding to its undominated techniques. The efficiency frontier is shown as the gray line in Figure 3.2.

(The economic definition of efficiency is not the same as the engineering definition. Engineering efficiency measures the fraction of the available energy that is turned into useful work in a system. Economic efficiency means not wasting any resources from a social point of view.)

The point A in Figure 3.2 represents a real wage at which the two undominated techniques have the same profit rate, and is called the *switchpoint* between the two techniques. Entrepreneurs will select the original technique at wages below the switchpoint, and the alternative technique at wages above the switchpoint.

This same construction works with any number of alternative techniques, even an infinite continuum. Each technique corresponds to one real wage-profit rate schedule, and the northeast boundary of the real wage-profit rate schedules of the available techniques is the efficiency frontier for the economy. Profit-maximizing entrepreneurs will choose the technique on the efficiency frontier for any level of the wage.

Neoclassical economists often assume a *production function* that shows the output, X, that can be produced by arbitrary inputs of capital, K, and labor, N:

$$X = F(K, N) \tag{3.2}$$

If the production function has constant returns to scale, which means that it is possible to increase output by any given factor, say $1/N$, by increasing both inputs by the same factor, then it can be viewed as describing a technology, that is, a collection of techniques of production. A pair of numbers (k, x) is an available technique given the production function $F(K, N)$ if

$$x = \frac{X}{N} = F\left(\frac{K}{N}, \frac{N}{N}\right) = F(k, 1) \equiv f(k)$$

This means that k units of capital and 1 unit of labor can be combined to produce x units of output. The function $f(k) \equiv F(k, 1)$ is called the *intensive production function*. If the production function is a continuous, smooth function, the corresponding technology is an infinite continuum of techniques. The efficiency frontier for a smooth production function is also smooth, and looks like Figure 3.3.

When the efficiency frontier is a smooth curve arising from a technology described by a smooth production function, every point on the efficiency frontier is a switchpoint. A small rise in the real wage will change the profit-maximizing technique slightly to one that employs a bit more capital per worker.

As we have seen, profit-maximizing entrepreneurs will choose the technique that has the highest profit rate for any wage. If the production function is smooth, the profit-rate maximizing technique of production at a given wage will combine labor and capital in proportions such that the marginal product of labor is equal to the wage and the marginal product of capital is equal to the profit rate. Thus the equality of the marginal products to factor

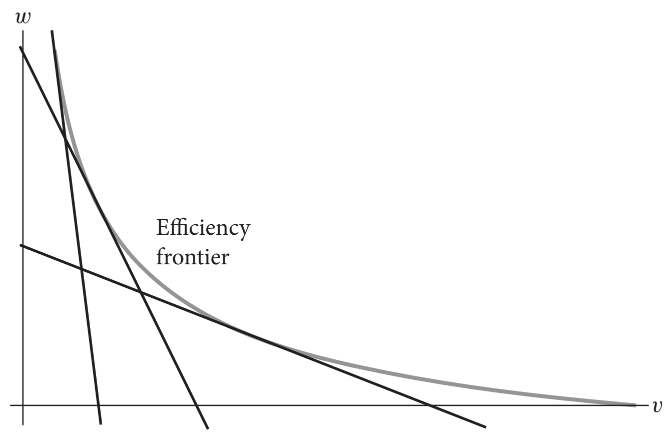

Figure 3.3 A smooth production function describes a technology with an infinite continuum of techniques. The real wage-profit rate schedules are each tangent to the efficiency frontier at one point. Only three of the individual real wage-profit rate schedules are drawn here. In fact, there is one tangent to every point on the efficiency frontier, which is the envelope of the real wage-profit rate schedules corresponding to the technology.

prices is just another way of describing the entrepreneur's choice of the most profitable technique of production.

To see this point, consider that profit is just output less wages:

$$Z = vK = X - wN = F(K, N) - wN$$

For a given amount of capital employed, the entrepreneur will want to choose the technique of production so as to maximize this profit. Holding K constant, if the entrepreneur can continuously vary the amount of labor working with the given amount of capital, the condition for maximization is:

$$\frac{dZ}{dN} = \frac{\partial F(K, N)}{\partial N} - w = 0$$

This implies that the entrepreneur must choose a technique at which:

$$w = \frac{\partial F(K, N)}{\partial N}$$

Another way to view this situation is that a profit-maximizing entrepreneur always chooses the technique of production at a switchpoint between a slightly more- and slightly less-capital intensive technique. The equality of

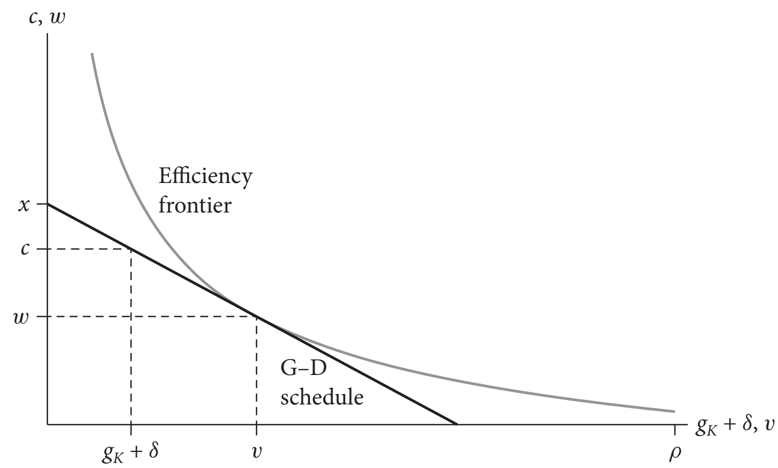

Figure 3.4 The wage determines the profit-maximizing technique, which establishes the growth-distribution schedule for an economy.

the marginal product of labor to the wage is just another way of defining this switchpoint.

The choice of the profit-maximizing technique at any real wage is the fundamental principle at work here, not the equalization of the marginal product of labor to the real wage. If there are only a finite number of techniques available, it may not be possible to determine a marginal product of labor, but entrepreneurs can still choose the available technique that has the highest profit rate given the real wage. The growth-distribution schedule for that technique will then determine the average productivity of labor, x, and the average productivity of capital, ρ. We would use this one particular growth-distribution schedule to analyze the relations between aggregate consumption and investment. The real wage determines, through profit maximization, the technique in use, and the growth-distribution schedule for that technique determines the social trade-off between gross investment and consumption.

Figure 3.4 summarizes the situation when there is a continuum of techniques represented by a smooth production function. Given the wage (or the profit rate), there is one profit rate-maximizing technique, corresponding to a point on the unit isoquant of the production function. All the entrepreneurs will adopt this technique, which will then determine the growth-distribution schedule for the economy.

3.6 Particular Production Functions

In this book, we will use several different production functions in examples and problems.

3.6.1 The Leontief production function

The first is called the *Leontief* or *fixed coefficients* production function. The fixed coefficients production function specifies that capital and labor can be combined in just one way to produce output, so that it corresponds to a single technique of production. The Leontief production function is written mathematically:

$$X = \min(\rho K, xN) \tag{3.3}$$

Dividing through by N, we can write the intensive fixed coefficients production function:

$$x = \min(\rho k, x) \tag{3.4}$$

The min(., .) function of two numbers always takes the value of the smaller of the numbers. Thus this production function says that the output X is limited by either the output of the capital employed or the output of the labor employed, whichever is smaller. In other words, for each x units of output the entrepreneur has to have at least ρk units of capital and 1 unit of labor. The fixed coefficients production function exactly describes one technique of production.

With a Leontief production function technology there is only one available technique, that is, only one possible way to combine labor and capital to produce output. If there is only one way to combine labor and capital, the *marginal products* of capital and labor are not well defined. Adding more labor without the corresponding necessary capital will give zero extra output while subtracting labor reduces output proportionately.

The unit isoquant for the Leontief production function has the shape of the letter "L," as Figure 3.5a illustrates. The corner occurs at the input point $(1/\rho, 1/x)$. The corresponding efficiency frontier is a single straight-line real wage-profit rate schedule with horizontal intercept ρ and vertical intercept x, as shown in Figure 3.5b. The intensive Leontief production function is a straight line from the origin to the point (k, x), and a horizontal line at the level x for higher k, as shown in Figure 3.5c. In the first part of the intensive

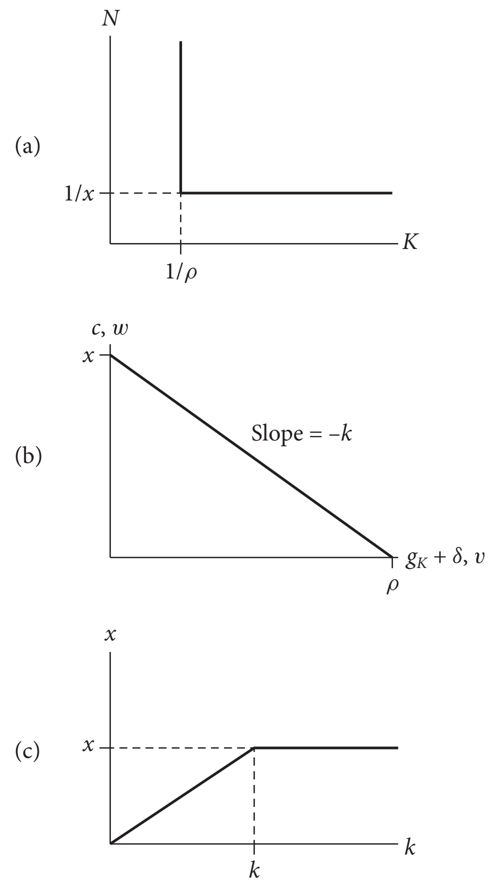

Figure 3.5 (a) The *Leontief production function* has an "L-shaped" isoquant, with a corner at the input proportions, $(1/\rho, 1/x)$. (b) The corresponding efficiency frontier is a single real wage-profit rate schedule with horizontal intercept ρ, and vertical intercept x. (c) The corresponding intensive production function consists of two lines, one from the origin to (k, x), the other a horizontal line at the level x.

Leontief production function output is constrained by the capital input, and is proportional to k, and in the second part output is constrained by the labor input, and is equal to x no matter how much more capital may be available.

3.6.2 The Cobb-Douglas production function

Another widely used production function is the *Cobb-Douglas* production function, which allows for smooth substitutability between capital and labor in production. It is written mathematically:

$$X = AK^{\alpha}N^{1-\alpha} \tag{3.5}$$

Here α (the Greek letter *alpha*, pronounced "'al-fa") is a parameter that lies between 0 and 1, and A is a scale factor used to make the units of measurement consistent. Using (3.5) we can see that a technique (k, x) is allowed by the Cobb-Douglas production function with parameter α if:

$$\begin{aligned} x &= Ak^{\alpha}(1)^{1-\alpha} \quad \text{or} \\ x &= Ak^{\alpha} \end{aligned} \tag{3.6}$$

With the Cobb-Douglas production function we can choose the capital required for one unit of labor, k, to be any number we wish, and then find the amount of output, x, the unit of labor can produce with that capital from equation (3.6). Notice that the Cobb-Douglas production function implies a very high degree of substitutability between capital and labor, since enough labor can always make up for any reduction of capital (and vice versa).

The unit isoquant for the Cobb-Douglas production function is the modified hyperbola asymptotic to the axes shown in Figure 3.6a. Each point such as A, B, or C on the isoquant corresponds to a particular technique of production, with its own (ρ, x), and to a particular real wage-profit rate schedule, as shown in Figure 3.6b. The efficiency frontier is the envelope of these growth-distribution functions. The intensive Cobb-Douglas production function is Ak^{α}.

With the Cobb-Douglas production function (or any production function with a smooth isoquant), it is possible to define the marginal product of labor or capital as the increase in output that could be achieved from a small increment in one factor of production, holding the other constant. In mathematical terms the marginal product of either factor is the partial derivative of the production function with respect to that factor. The marginal products of labor and capital with the Cobb-Douglas production function, for example, are:

$$MP_N = \frac{\partial X}{\partial N} = (1-\alpha)A\left(\frac{K}{N}\right)^{\alpha} = (1-\alpha)Ak^{\alpha}$$

$$MP_K = \frac{\partial X}{\partial K} = \alpha A\left(\frac{K}{N}\right)^{-(1-\alpha)} = \alpha Ak^{\alpha-1}$$

The technical rate of substitution (TRS) between capital and labor is defined as the ratio of the two marginal products. For the Cobb-Douglas

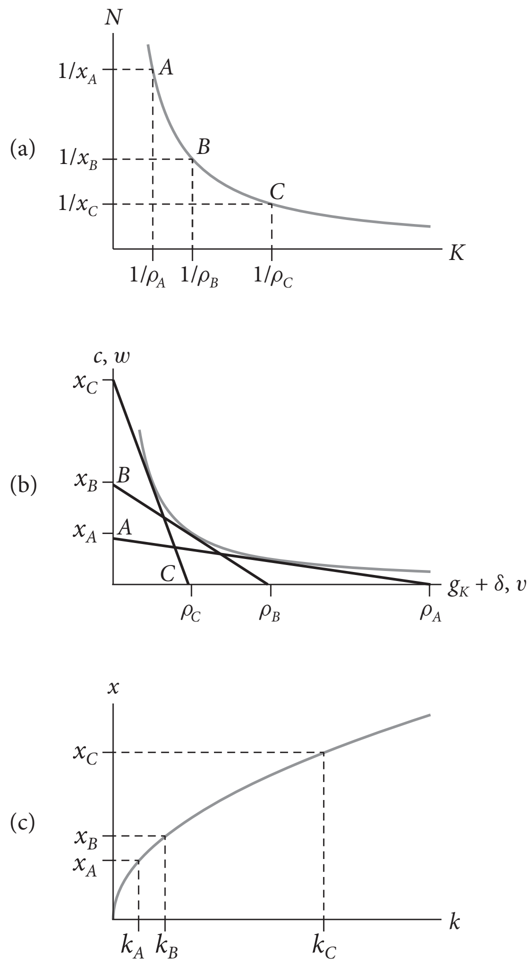

Figure 3.6 (a) The *Cobb-Douglas production function* has a smooth isoquant, representing a continuum of techniques, three of which are shown at points A, B, and C. (b) The efficiency frontier is the envelope of the growth-distribution schedules of the techniques on the isoquant. The growth-distribution schedules for points A, B, and C are shown. (c) The capital intensities and labor productivities for points A, B, and C are shown on the intensive production function.

production function, we have:

$$TRS = \frac{MP_N}{MP_K} = \frac{1-\alpha}{\alpha}k$$

With the Cobb-Douglas production function, the technique that maximizes the profit rate for a wage w must satisfy the condition:

$$w = (1-\alpha)Ak^{\alpha} = (1-\alpha)x$$

Once we know k, we can derive x and ρ. From the intensive Cobb-Douglas production, we see that:

$$x = Ak^{\alpha}$$

Dividing the Cobb-Douglas production function through by k, we see that:

$$\rho = \frac{x}{k} = Ak^{\alpha-1}$$

The profit rate will equal the marginal product of capital:

$$v = \alpha Ak^{\alpha-1} = \alpha\rho$$

The parameter α in the Cobb-Douglas production function is therefore the profit share, since:

$$\pi = \frac{vk}{x} = \frac{\alpha x}{x} = \alpha$$

Furthermore, we can see that w and v satisfy the growth-distribution schedule equation for this particular choice of k and x:

$$w + vk = (1-\alpha)x + \alpha x = x$$

PROBLEM 3.1 Draw the production isoquant (the combinations of capital and labor required to produce one unit of output), the real wage-profit rate schedule, and the intensive production function for the Leontief technology with $k = \$100{,}000$/wkr and $x = \$50{,}000$/wkr-yr. What is the marginal product of labor in the Leontief technology?

PROBLEM 3.2 Draw the production isoquant (the combinations of capital and labor required to produce one unit of output), the real wage-profit rate schedule, and the intensive production function for the Cobb-Douglas technology with $A = \$10$ and $\alpha = .25$. What is the marginal product of labor in the Cobb-Douglas technology?

PROBLEM 3.3 What technique of production will profit rate-maximizing entrepreneurs choose if they face a Cobb-Douglas production function and a given real wage, $\bar{w}$? What if they face a fixed coefficients production function and the same real wage?

PROBLEM 3.4 Show that the efficiency frontier expressing w as a function of v for the Cobb-Douglas production function has the same mathematical form as the unit isoquant expressing $1/x$ as a function of $1/\rho$.

3.6.3 The CES production function

A general form of production function, which encompasses both the Cobb-Douglas production function and the Leontief production function as special cases, is the *constant elasticity of substitution* (CES) production function:

$$X = A\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)N^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{3.7}$$

where $0 \leq \sigma \leq \infty$ is the *elasticity of substitution*.

The elasticity of substitution is defined as the ratio between the rate of change of the capital intensity, dk/k, and the rate of change of the technical rate of substitution between capital and labor, $dTRS/TRS$. It is often easier to consider the *logarithmic derivative* of a variable x because $d \ln x$ equals the (exponential) rate of change of the variable. Using the logarithmic derivative, the elasticity of substitution can be written more compactly as $d \ln k/d \ln TRS$.

The technical rate of substitution for this production function is actually quite simple:

$$TRS = \frac{\partial X/\partial N}{\partial X/\partial K} = \frac{1-\alpha}{\alpha}\left(\frac{K}{N}\right)^{\frac{1}{\sigma}} = \frac{1-\alpha}{\alpha}k^{1/\sigma}$$

Taking logs of both sides and differentiating we find that since $d \ln((1-\alpha)/\alpha) = 0$,

$$\frac{d \ln k}{dTRS} = \sigma$$

This confirms that the elasticity of substitution for this production function is indeed constant and equal to σ.

Consider now the following special cases:

1. $\sigma = 0$. In this case, the technical rate of substitution is not defined, just as in the Leontief production function. Capital and labor are complements.

2. $\sigma = 1$. In this case, the technical rate of substitution is equal to

$$\frac{1-\alpha}{\alpha}k$$

just as in the Cobb-Douglas production function.

3. $\sigma = \infty$. In this case, the technical rate of substitution is equal to $(1-\alpha)/\alpha$. A linear production function of the form $X = (1-\alpha)K + \alpha N$ would produce the same result. Hence, with the elasticity of substitution equal to infinity, the CES production function takes the form of a linear production function. Capital and labor are perfect substitutes.

The intensive production function for the CES technology is defined as

$$x = A\left[\alpha k^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\right]^{\frac{\sigma}{\sigma-1}}$$

From the intensive production function, it is not too hard to show that the profit share satisfies:

$$\pi = \alpha\left(\frac{k}{x}\right)^{\frac{\sigma-1}{\sigma}}$$

Income shares are not constant if the production function takes the CES form unless the elasticity of substitution happens to be exactly equal to one. From this standpoint, the Cobb-Douglas case appears quite special despite its popularity.

PROBLEM 3.5 Using the fact that profit-maximization with a CES production function implies $v = \partial x/\partial k$ and that $w = x - vk$, derive the expression for the profit share above.

PROBLEM 3.6 Show that if the elasticity of substitution σ is greater than one, the profit share is increasing in the capital/output ratio k/x.

Problem 3.6 engages with recent explanations of a fall in the wage share in advanced economies and its relation with the increase in the capital/output ratio (that is, $1/\rho$). The French economist Thomas Piketty has argued that, because the elasticity of substitution between a broad measure of wealth and labor is higher than one, an increase in the capital/output ratio will be accompanied by a fall in the wage share. As we will see in Section 10.6, however, this technological explanation can be contrasted with an emphasis on the social and institutional determination of the wage share.

3.7 Classifying Technical Change

A single technique of production is determined by its capital productivity, ρ, and labor productivity, x. A change in the technique can therefore be described in terms of the change in these two parameters. For example, a purely *labor-saving* technical change corresponds to a rise in x while ρ remains unchanged. As in Chapter 2, we measure the amount of labor-saving technical change by the growth rate of the productivity of labor, g_x:

$$1 + g_x = \frac{x_{+1}}{x}$$

The growth-distribution schedule corresponding to the technique of production rotates clockwise around its ρ-axis intercept when there is pure labor-saving technical change, as in Figure 3.7.

Similarly, a purely *capital-saving* technical change corresponds to a rise in ρ with x unchanged. We can measure the degree of purely capital-saving technical change by the growth rate g_ρ:

$$1 + g_\rho = \frac{\rho_{+1}}{\rho}$$

Purely capital-saving technical change rotates the growth-distribution schedule counter-clockwise around its w-intercept, as Figure 3.8 shows.

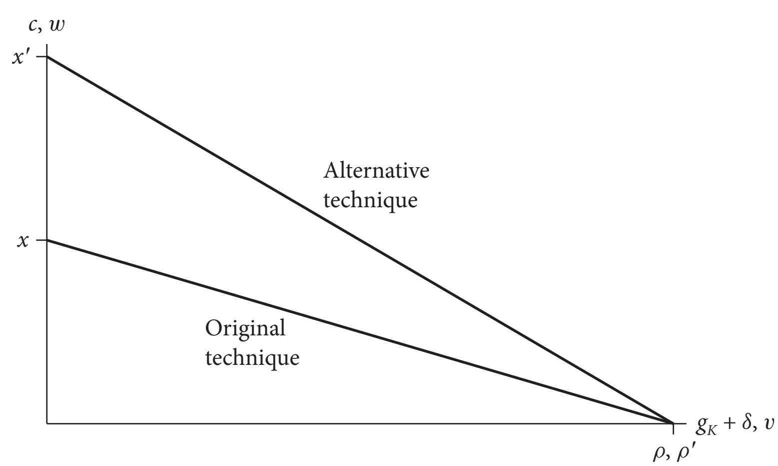

Figure 3.7 A purely labor-saving technical change corresponds to a rotation of the growth-distribution schedule around its ρ-intercept.

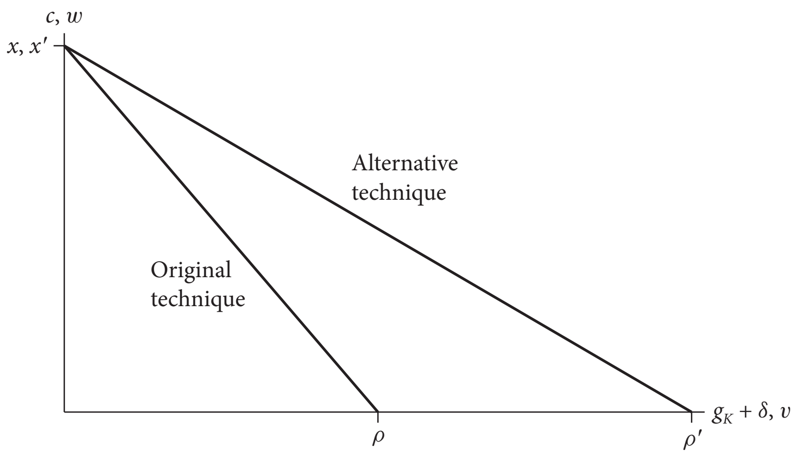

Figure 3.8 Purely capital-saving technical change rotates the growth-distribution schedule around its w-intercept.

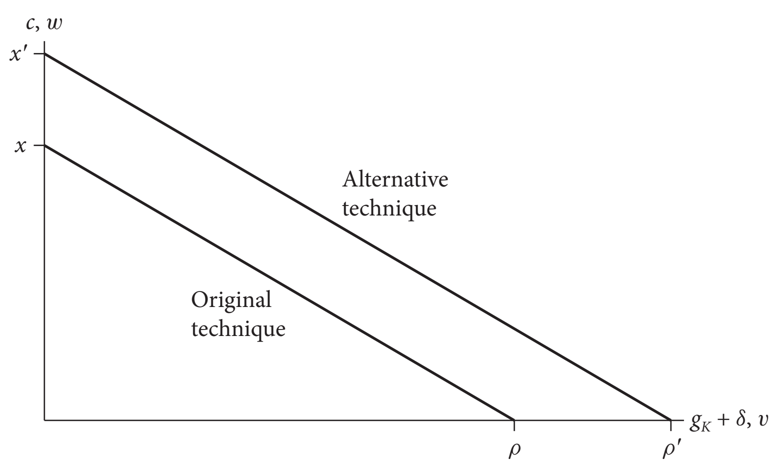

Figure 3.9 Factor-saving technical change shifts the growth-distribution schedule outward parallel to itself.

If technical change saves both capital and labor equally, the growth-distribution schedule moves outward parallel to itself, as illustrated in Figure 3.9. In this case $g_x = g_\rho$, so that both intercepts move by the same proportion. This form of technical change is called *factor-saving*. Factor-saving technical change can also be thought of as a rescaling of the output itself: the same labor and capital inputs produce more output.

A moment's thought shows that we can represent *any* pattern of technical change for a single technique either by a combination of purely labor-saving and purely capital-saving technical change, or by a combination of purely labor-saving and factor-saving technical change.

It is more complicated in general to describe technical change when the technology consists of a collection of many techniques, as in the case of a neoclassical production function. In principle, technical change might affect each of the techniques differently, leading to a wholly new technology. The situation is simpler if we assume that all the different techniques in the technology undergo the same pattern of technical change.

If all the techniques in a technology undergo the *same* degree of labor-saving technical change, the result is a *labor-augmenting* technical change, which is also called *Harrod-neutral*. Purely labor-augmenting technical change can be represented by multiplying the labor input in the production function by the factor $1 + \gamma$ (the Greek letter *gamma*, pronounced "ˈgam-ma"):

$$F'(K, N) = F(K, (1 + \gamma)N)$$

Another way to think of labor-augmenting, or Harrod-neutral technical change is as a rescaling of the measure of labor input: each worker after the technical change functions as if her efforts were magnified by a factor representing the size of the change. Economists often make this transformation and refer to *effective labor input*, which means multiplying the number of workers in any year by a factor representing the degree of labor-augmenting technical change that has taken place since the base year. Each actual worker after the Harrod-neutral technical change is the equivalent of more than one effective worker. If the wage rate increases by the same proportion as labor productivity, the profit rate will remain unchanged with purely labor-augmenting technical change, which is why Harrod called this type of technical change "neutral."

Capital-augmenting technical change, which is also called *Solow-neutral* technical change, is defined analogously to labor-augmenting technical change. In this case each unit of capital acts as if its productivity were multiplied by the factor $1 + \chi$ (the Greek letter *chi*, pronounced "kī").

Factor-augmenting technical change consists of equal amounts of labor- and capital-augmenting technical change, and can be represented by multiplying the whole production function by the factor $1 + \gamma = 1 + \chi$, under the assumption that the production function exhibits constant returns to scale:

$$F'(K, N) = (1 + \gamma)F(K, N) = F((1 + \gamma)K, (1 + \gamma)N)$$

Factor-augmenting technical change is often called *Hicks-neutral*. Factor-augmenting technical change can also be thought of as a rescaling of the output itself: the same labor and capital inputs produce more output, which is why Hicks regarded it as "neutral."

Notice that these various types of neutral technical change to the whole technology assume that *every* technique experiences the *same* degree and type of technical change. This need not be true in reality, since technical change might affect some techniques differently from others. In the case of a technology represented by a neoclassical production function, it might be the case that one part of the unit isoquant shifts in a different pattern from the rest, for example.

It is useful to distinguish the parameters γ and χ from the growth rates g_x and g_ρ. For the Leontief production function, which has only one technique, they will be the same, but for a more general neoclassical production function, x and ρ will change not only because of technical change, but also because of changes in the technique in use, so that g_x and g_ρ may not be equal to γ and χ.

3.8 Two-Sector Growth-Distribution Schedules

We have been working under the *one-sector* assumption that there is only one produced commodity in the economy, which can be used interchangeably as a consumption good or as investment to add to the capital stock. This assumption greatly simplifies the analysis of a model economy. The production possibilities frontier for a one-sector economy is a straight line. As long as a one-sector economy is not specialized to the production of consumption or investment goods, the price of capital goods in terms of consumption goods is fixed at unity, and as a result we do not need to analyze the effect of changes in exogenous variables on the consumption goods price of capital. The output per worker, x, cannot be affected by changes in the price of capital. Under these circumstances the social consumption-growth rate schedule and the real wage-profit rate schedule are represented by the same straight line, which coincides with the growth-distribution schedule whose intercepts are the output per worker and the output per unit of capital.

In an economy with two (or more) produced outputs, relative price changes do matter, and considerably complicate the analysis. Since real economies have many different commodities, this problem is important in principle. The Cambridge capital controversy centered on the issues raised in moving from the one-sector model to the analysis of economies with more than one sector.

We can see the range of issues raised by looking briefly at a two-sector economy. In this economy, each good can be used as an input in its own production and in the production of the other good. We will refer to the goods as good a and good b to identify the production coefficients.

To describe a technique of production we will need to specify the input intensity in each sector. In the a-sector each worker uses k_{ab} units of good b and k_{aa} units of good a to produce x_a units of good a. (For example, if good a is corn and good b is steel, the units might be bushels and tons.) In the b-sector, each worker uses k_{ba} and k_{bb} units of good a and good b respectively to produce x_b units of good b. Here x_a and x_b represent the labor productivity in each industry.

We will take the a-good to be the numeraire (so that its price is by definition one) and denote the price of the b-good in terms of units of the a-good by p. We will also measure the real wage rate, w, in terms of the numeraire. We can calculate the wage-profit rate frontier for a technique in the two sector model on the assumption it is in a *steady state*, so that the prices of the two goods as outputs are the same as their prices as inputs. We will need to find the price of the b-good, p, and the real wage rate, w, consistent with the same profit rate, v, in the two sectors. This requires solving the two equations:

$$x_a = v(k_{aa} + pk_{ab}) + w \tag{3.8}$$

$$px_b = v(k_{ba} + pk_{bb}) + w \tag{3.9}$$

Equations (3.8) and (3.9) express the requirement that the profit rate be the same in the two sectors. For any level of v, these equations can be solved for p and w. The solutions take the form of somewhat complicated polynomials that can be written in general terms as

$$p = p(v)$$
$$w = w(v)$$

We can simplify the two-sector model in order to reveal the economic properties of the price equation and real wage-profit rate schedule by assuming that the *a*-good is a pure consumption good. A pure consumption good is not used as an input into any production process. In this case, the *b*-good becomes a pure capital good used to produce itself and the consumption good. This amounts to setting $k_{aa} = k_{ba} = 0$.

With this simplification, the price and wage equations become considerably more transparent:

$$p = \frac{x_a}{v(k_{ab} - k_{bb}) + x_b}$$

$$w = x_a - pvk_{ab} = x_a \left(1 - \frac{vk_{ab}}{v(k_{ab} - k_{bb}) + x_b}\right)$$

These equations will not be linear unless $(k_{ab} - k_{bb}) = 0$. This term measures the relative capital intensity of production in the two sectors. When both sectors have the same capital intensity (so that this term is zero), then changes in distribution do not affect the price of the capital good relative to the consumption good. For example, if the wage rate declines and the profit rate increases, it will affect the costs of production symmetrically across the two sectors. The price of the capital good will reflect the relative productivity in the two sectors, $p = x_a/x_b$.

The term $(k_{ab} - k_{bb})$ is positive when the consumption good sector is more capital intensive than the investment good sector. The real wage-profit rate schedule for this case is shown in Figure 3.10. Here, as the rate of profit increases it affects the cost of producing capital goods by less than it affects the cost of producing consumption goods, so the relative price of the capital good falls. The overall *value* of capital (measured in units of the *a*-good) to labor will thus decline as the rate of profit increases. Mathematically, this accounts for the shape (inward curving) of the wage-profit schedule in Figure 3.10.

But the wage-profit curve can also curve outward if the capital good sector is more capital intensive than the investment good sector. This case is shown in Figure 3.11.

The economic intuition behind Figures 3.10 and 3.11 remains valid in the more complicated case in which both goods are used as inputs in production in at least one sector besides their own. In this case, capital goods are heterogeneous and the only way to measure the capital intensity of the economy is by using prices to calculate the value of capital per worker. When capital

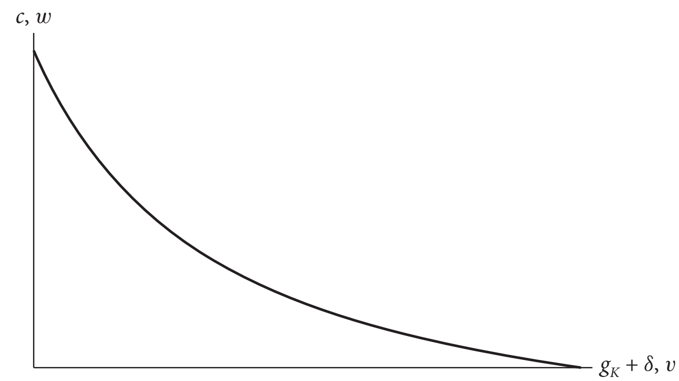

Figure 3.10 The real wage-profit rate relation for a single technique in a two-sector model need no longer be a straight line, because the price of capital changes with the profit rate.

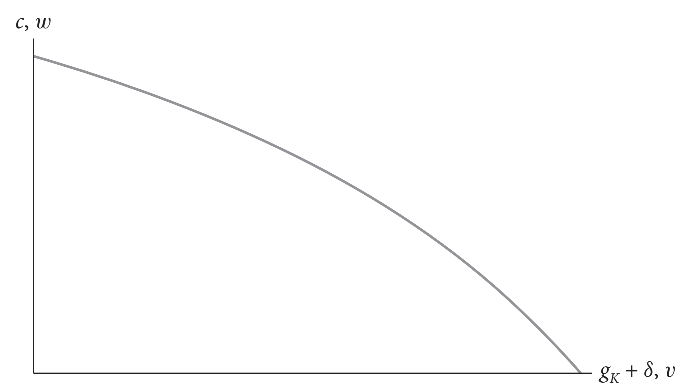

Figure 3.11 The real wage-profit rate relation for a single technique in a two-sector model can curve outward as well as inward, depending on whether k_{ab} is larger or smaller than k_{bb}.

goods are heterogeneous, they cannot be measured in some common physical unit—it makes no sense to add up the number of laptop computers and the number of blast furnaces. While the value of capital per worker is more difficult to define with heterogeneous capital goods, it is clear that a change in the distribution of income will affect the relative prices and therefore the value of capital per worker for the same reason it did in the simpler case with a pure consumption good and a pure investment good.

As in the one-sector model, the efficiency frontier is the envelope of the real wage-profit rate schedules for the available techniques. The effi-

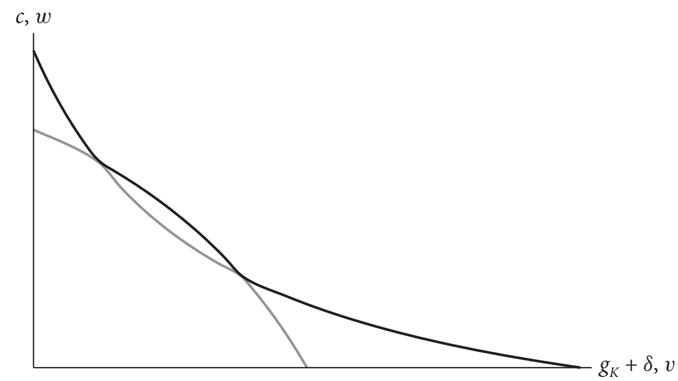

Figure 3.12 The efficiency frontier in a two-sector model is the envelope of the real wage-profit rate schedules for the available techniques. Unlike the one-sector model, the efficiency frontier in a two-sector model is not necessarily convex toward the origin.

ciency frontier for two techniques that generate paging nonlinear wage-profit curves like those in Figures 3.10–3.11 is graphed in Figure 3.12. (We assume that both goods are used as inputs in production in at least one sector besides their own.)

Several features of Figure 3.12 are of fundamental economic importance. The efficiency frontier is not necessarily convex toward the origin, so that it may have the same slope at several different points. In the one-sector model, each available technique of production contributes only one segment to the efficiency frontier (only a point, in the case of a technology defined by a smooth production function), but in the two-sector model (and in models with more than two sectors), it is possible for a technique to contribute two (or more) segments to the efficiency frontier. Since there are two switch-points in Figure 3.12, the same technique is the most profitable both at a low wage rate and at a high wage rate, a phenomenon known as *reswitching of techniques*. Thus in general there is no one-to-one correspondence between profit or wage rates and techniques of production in the two-sector model. In this case it is impossible to make sense of the idea of a marginal product of capital. These points are of fundamental importance to the theory of capital and the Cambridge capital controversy.

We can generalize the growth-distribution schedule method to economies with two (or more) sectors. But the growth-distribution schedule in these more complex economies will not represent the real wage-profit rate schedule for the technique in use. It does, however, give us some information about the real wage-profit rate schedule. The growth-distribution schedule intersects the real wage-profit rate schedule of the profit-maximizing technique at the (v, w) point and at the $(g_K + \delta, c)$ point. If the real wage-profit rate schedule is not very curved, the observed growth-distribution schedule approximates it.

3.9 Models of Production and Models of Growth

We have developed a simple model of production in which there is only one output, which can be used either for consumption or investment. A technique of production is defined by its capital intensity, k, its labor productivity, x, and its depreciation rate, δ, and corresponds to a single growth-distribution schedule. When the available technology consists of more than one technique, profit-maximizing entrepreneurs will choose the technique with the highest profit rate at the ruling real wage. The efficiency frontier is the envelope of the real wage-profit rate schedules for the technology. The point on the efficiency frontier at the ruling real wage determines the profit-maximizing technique, which in turn determines the social consumption-growth rate schedule for the economy. When the technology is described by a smooth production function, such as a Cobb-Douglas production function, the profit-maximizing technique has the marginal product of labor equal to the real wage. But there are technologies, such as the Leontief technology, in which the marginal product of capital is not well defined.

The basic model of production can be combined with models of labor supply and of saving to create a model of economic growth.

3.10 Suggested Readings

Further detail on the mathematical properties of the Cobb-Douglas and more general neoclassical production functions can be found in Allen (1968). Pasinetti (1977) provides a rich exposition of the fixed-coefficient model with many sectors. Piketty (2014) studies the effect of capital accumulation on distribution using the elasticity of substitution.

The principles behind the construction of social accounting matrices are laid out in Taylor (2004). The Integrated Macroeconomic Accounts of the US are published jointly by the Bureau of Economic Analysis and the Federal Reserve Board, and they combine the balance sheets and national income accounts of the major sectors in a stock-flow consistent system.

For conceptually critical views of the neoclassical production function, see Nelson and Winter (1982) and Robinson (1953). The latter began the Cambridge Capital Controversy, which is surveyed with considerable style by Harcourt (1972); also see Cohen and Harcourt (2003) for a retrospective. Two important summary statements of this debate are given by Samuelson (1966) and Garegnani (1970). A particularly clear exposition of the two-commodity model developed in the text can be found in Morishima (1966). Samuelson (1962) first tried to show (unsuccessfully, it turned out) that the neoclassical production function and efficiency frontier could be generalized in the context of a multi-commodity world. An exhaustive treatment of the theory of production, which includes coverage of the Cambridge Controversy and other topics in the history of economic thought, is Kurz and Salvadori (1995). Ochoa (1989) was an early effort to present evidence that the wage-profit curves found in real economies are well approximated by linear functions.

Finally, an important paper on unbalanced growth in a multi-sector economy by William Baumol (1967) points out that the output from a sector that resists automation, such as education, medical care, or concert performance, will grow increasingly expensive in relative terms, which is now sometimes called the "Baumol cost disease."

4

The Labor Market

4.1 Models of Economic Growth

A model of economic growth is a set of mathematical assumptions that allow us to predict the behavior of an imaginary economy. In any model certain factors are taken as given *exogenous parameters* of the process. The model does not try to explain why the exogenous parameters have the values they do, but simply accepts them as determined by processes outside the model's scope. The other variables in the system are taken to be *endogenous variables*. The model is supposed to determine, and therefore explain, the endogenous variables on the basis of the values taken by the exogenous parameters. A typical analysis, for example, asks mathematically what would happen to the endogenous variables if one of the exogenous parameters were to be changed. In the real world we almost always see all the exogenous parameters changing at once, but the analytical procedure holds all of them constant except one, in order to isolate the influences of that particular exogenous parameter. If we want to use the model to explain real historical events, we have to superimpose the effects of all the changes in the exogenous parameters that have occurred.

What we take as exogenous and endogenous depends on our point of view and the type of question we want to analyze. What we take as an exogenous parameter in one model we might regard as endogenous in another model that tries to explain what determines its evolution. Thus, identifying the exogenous and endogenous variables explicitly is very important when studying a model (or presenting a model to others).

In general a mathematical system consists of a certain number of relations between variables expressed as equations or inequalities. The number of endogenous variables that can be explained by a model is limited to the number of relations it contains. If we try to explain three endogenous variables with only two equations, for example, we will fail, because we can typically take any arbitrary value of one of the variables and solve the equations consistently for the other two. Thus a model can have any number of exogenous parameters, but the number of endogenous variables is limited to the number of relations specified in the model.

In modeling economic growth in a capitalist economy we will take as our endogenous variables the growth rate of the capital stock, the profit rate, the level of consumption, and the wage: g_K, v, c, and w. Among the exogenous parameters will be capital intensity, k, or the output-capital ratio, ρ, labor productivity, x, and the depreciation rate, δ. The growth-distribution schedule gives us two relations among the four endogenous variables, given the exogenous parameters:

$$w = x - vk$$
$$c = x - (g_K + \delta)k$$

A complete model of growth, however, must have two more relations expressed as equations in order to determine all four of these variables. For example, if we knew the real wage and social consumption per worker in an economy, the growth-distribution schedule would determine the profit rate and the growth rate of the capital stock. These additional relations are sometimes said to *close* the model.

Different schools of economic thought add different conditions to close the growth model. The doctrinal differences among the schools are reflected in these differences. One reason we have begun by explaining the model of production is that most of the models generated by the different major schools of thought are consistent with these core production relations. The models we will study all close the growth model by making some hypothesis about labor supply and demand and equilibrium in the labor market, which adds a third determining relation to the two expressed by the growth-distribution schedule, and some hypothesis about the behavior of households in allocating income between investment and consumption, which adds the fourth. Different hypotheses about the labor market and about household consumption patterns can lead to models that make quite different predictions about the patterns of growth.

We will begin in this chapter by discussing different models of the labor market.

4.2 Demand for Labor

The real wage can be regarded as the price that equates the supply of and demand for labor. In a one-sector production model with a single given technique of production, the demand for labor is given by the amount of capital available for it to work with and the coefficient k, which determines how many jobs each unit of capital supports, since we have the relations:

$$N^d = \frac{X}{x} = \frac{K}{k}$$

This means that in any period the demand for labor is a vertical line, as illustrated in Figure 4.1.

If, on the other hand, we assume that there is a spectrum of techniques of production defined by the smooth isoquant of a production function as in Figure 3.6a, the demand for labor will depend on the profit rate-maximizing technique as well as on the amount of capital accumulated. We could express this by writing $x(w)$ and $k(w)$ as functions of the wage:

$$N^d(w) = \frac{X}{x(w)} = \frac{K}{k(w)}$$

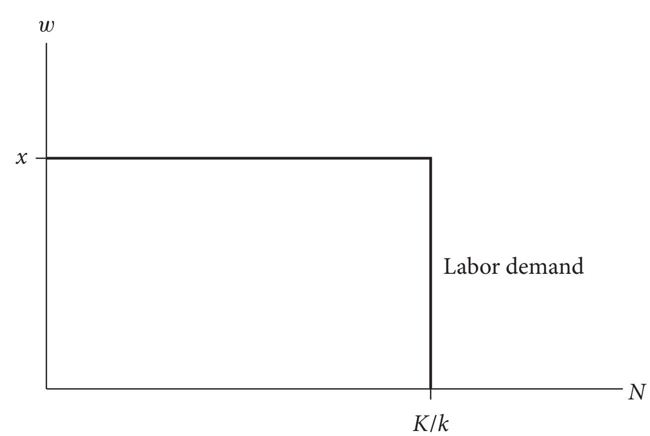

Figure 4.1 When there is only one available technique of production the demand for labor is limited by the amount of capital, which determines the level of output. At any real wage below the productivity of labor the demand for labor is equal to K/k.

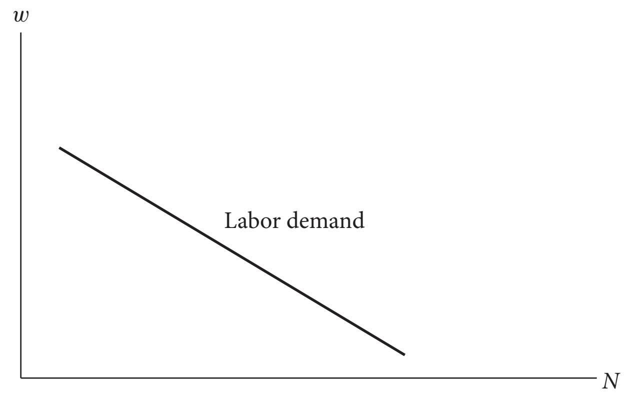

Figure 4.2 With a smooth spectrum of techniques the demand for labor given the capital stock will rise gradually as the wage falls, due to the shift to lower capital intensity techniques.

If the spectrum of techniques falls along a smooth isoquant, a fall in the wage will lower the profit rate-maximizing k, and the demand for labor will be a smoothly decreasing function of the wage, as in Figure 4.2.

With a spectrum of techniques, the elasticity of the demand for labor depends on the exact shape of the unit isoquant of the production function.

4.3 The Classical Conventional Wage Model

The Classical economists, Smith, Ricardo, Malthus, and their critic, Marx, viewed labor supply as growing or shrinking in response to the demand for labor at an exogenously given real wage. Ricardo, following Malthus, argued that the population and hence the supply of labor would rise if the real wage rose above a subsistence level, and that the population would fall if the real wage fell below this level. In the long run, at least, this theory implies that the supply curve of labor is horizontal at the subsistence wage.

The theory of a subsistence wage rests on Malthus's model of *demographic equilibrium*. Malthus argued that death and birth rates in any society would be stable functions of the standard of living, which he associated with the level of the real wage. A higher wage would lower the death rate, especially among infants, by allowing workers to consume a better standard of nutrition. A higher wage would also raise the birth rate, by encouraging earlier marriages.

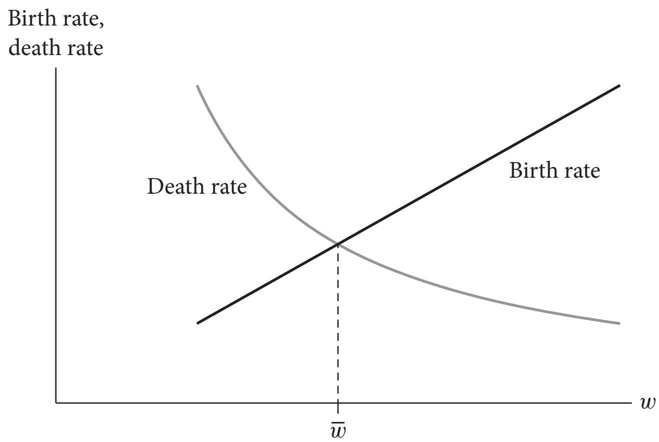

Figure 4.3 Malthus's theory of wages is based on the assumption that death rates decline and birth rates rise with increases in the wage. The intersection of the death and birth rate schedules is a demographic equilibrium that determines the subsistence level of the wage.

As illustrated in Figure 4.3, the intersection of the death and birth rate schedules is a demographic equilibrium at which population would remain constant. Malthus assumed that labor supply would be closely related to population, so that a constant population would also mean a constant supply of labor. The level of the real wage at which birth and death rates equalize can be viewed as *subsistence*, since it is just high enough to reproduce the population and labor force without change. If the wage were to rise above subsistence, the population would grow, and the increased supply of labor would tend to force the wage downward. If the wage were to fall below subsistence, high infant mortality would lead the population to shrink, and the resulting decline in the supply of labor would tend to force the wage upward. Over a period of time long enough to allow for these changes in population, the wage in this model will tend to remain close to the subsistence level.

Marx criticized Malthus's theory on two grounds. First, Marx argued that the schedules of birth and death rates were themselves the product of specific social relations. Malthus's theory, in Marx's view, applied to early nineteenth-century capitalism, which lacked any regulation of the exploitation of labor and had no social "safety net" to protect workers from extreme poverty, but might not hold for different social relations (such as those of a socialist society). History has borne out this criticism of Malthus, since the modifications of capitalism to provide for the protection and education of workers have coincided with dramatic changes in fertility, and falsified Malthus's projections.

Marx also argued against Malthus's assumption that labor supply is proportional to population. Marx pointed out that capitalist production always coexists with noncapitalist production such as domestic labor and subsistence agriculture, and draws part of its labor supply from these noncapitalist sectors through migration and the mobilization of female and child labor. Marx viewed these noncapitalist sectors as *reserve armies of labor*. Thus the capitalist labor supply might not vary proportionally with population because of offsetting changes in these labor reserves.

Marx agreed, however, with Malthus's conclusion that the supply of labor was horizontal at a given real wage because the movement of labor from the reserve armies would increase the labor supply if the real wage rose. The real investment costs (transport, relocation, training, and so on) involved in migrating from backward sectors to industrial employment establish a *value of labor-power*, which Marx viewed as determining the level of real wages. In Marx's view this was not a subsistence real wage in the sense of a biological minimum, but reflected social and historical factors affecting the cost of reproducing labor-power in different economies.

We will call these Classical and Marxian theories the *conventional wage model*. The supply of labor in the conventional wage model is horizontal at the exogenously given conventional wage, as shown in Figure 4.4.

Arthur Lewis explains the conventional wage model as reflecting a vision of economic development in which labor is drawn into a modern production sector from reserves that can support themselves by traditional production. In order to permit workers to move from the traditional sector to the modern sector, they must be supplied with the wherewithal to survive in that sector, because they cannot carry on their traditional production and work in the modern sector at the same time.

This vision of the process of labor supply is relevant to highly industrialized economies as well. In highly industrialized economies labor is supplied by migration from less developed countries or less developed regions within a given country, or by drawing workers from other pursuits (such as childcare and housework) into industrial production. The Classical model assumes that these reserves of labor are practically limitless and that the subsistence wage necessary to attract labor to the modern sector is given in each period.

If there is only one technique of production, the conventional wage model determines the wage, and the accumulated capital stock determines output and employment, as in Figure 4.5.

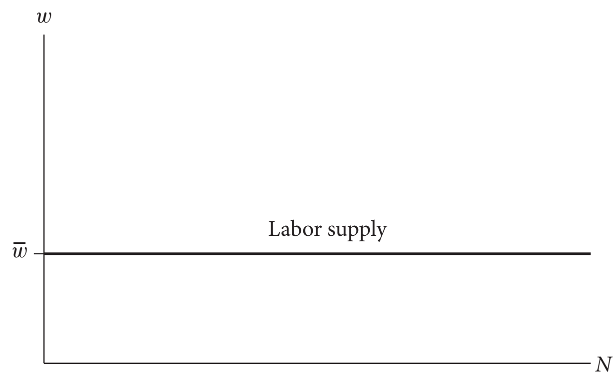

Figure 4.4 In the conventional wage model the supply of labor in each period is horizontal at an exogenously given real wage determined by the costs of reproducing labor-power.

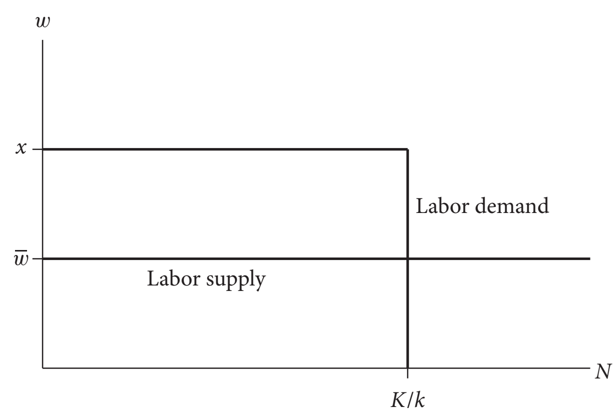

Figure 4.5 When there is only one technique, the conventional wage determines the wage, and the technique together with the accumulated capital determines the level of employment and output.

When there is a spectrum of techniques, the conventional wage determines the wage level, and also determines the profit rate-maximizing technique of production. The capital stock, together with the profit rate-maximizing technique of production, determines the level of employment and output, as shown in Figure 4.6.

The conventional wage model thus can add one further condition to the growth model, by determining the real wage as an exogenous parameter. The real wage-profit rate schedule then determines the profit rate and the technique of production, leaving social consumption per worker and the

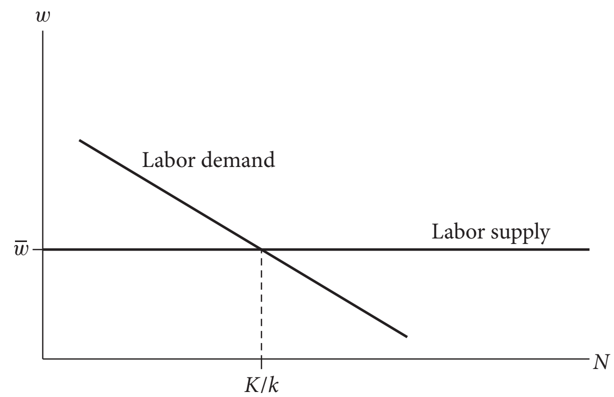

Figure 4.6 When there is a spectrum of techniques, the conventional wage determines the profit rate-maximizing technique, which, together with the accumulated capital stock, determines the level of output and employment.

growth rate of the capital stock still to be explained. The conventional wage assumption can be written algebraically as:

$$w = \bar{w} \tag{4.1}$$

Here $\bar{w}$ is the exogenously given conventional level of the real wage.

4.4 The Neoclassical Full Employment Model

At the opposite extreme from the conventional wage model is the assumption that the supply of labor in any period is exogenously given. As we will see in Section 6.5, the full employment closure can be embedded in a Classical model of growth and distribution. The focus of this section is on neoclassical models. They allow for a shift in the labor supply over time as the result of population growth, but view the rate of population growth, n, as an exogenous parameter. This approach views labor as an inelastically supplied input to production like land, though it allows for the exogenous increase in the quantity of labor supplied.

Neoclassical labor economists view the supply of labor in any period as depending on the real wage, due to the possible disutility of labor, with the supply of labor at any real wage determined by the population. In the context of economic growth theory the inclusion of the wage elasticity of the supply of labor in each period complicates the analysis without adding important new insights. Neoclassical growth models therefore typically abstract from labor supply responses to the real wage, and assume that households supply

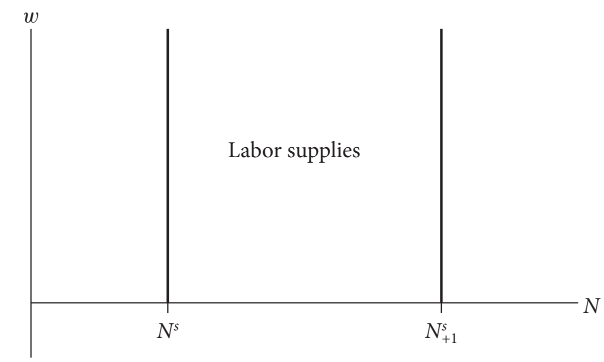

Figure 4.7 Neoclassical growth models assume an inelastically given supply of labor at any real wage. The shift from N^s to N^s_{+1} represents the exogenous growth of the labor force from one period to the next.

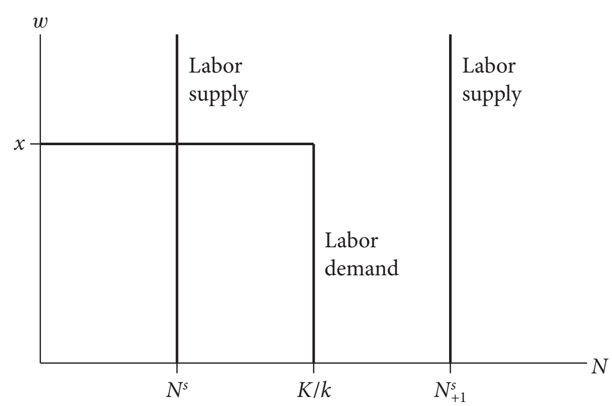

Figure 4.8 When there is only one technique of production, and a fixed supply of labor, either there will be a shortage of labor, which will drive the wage up to x and the profit rate down to zero, or there will be a surplus of labor, which will drive the wage down to zero and the profit rate up to ρ.

labor inelastically. In these models the supply of labor is a vertical line at the level given by the population, as illustrated in Figure 4.7.

When there is only one technique of production, the demand for labor is determined by the accumulated capital stock. If the supply of labor is smaller than this demand for labor, the real wage will rise to x, the productivity of labor, and the profit rate will fall to zero. If, on the other hand, the supply of labor is larger than the demand, the real wage will fall to zero, and the profit rate will rise to its maximum value, ρ, as shown in Figure 4.8.

Since neither of these outcomes is compatible with steady growth, the full employment assumption requires that the demand for labor and the supply of labor be matched in every period. Since the supply of labor is assumed to grow at the exogenously given rate n, this requires the demand for labor determined by the capital stock to grow at the same rate. If there is no change in the capital-labor ratio over time, this implies that the growth rate of the capital stock must also be equal to n. Thus the full employment assumption also adds one determining equation to the growth model, the requirement that the growth rate of the capital stock be equal to the exogenously given growth rate of the population. This assumption can be written

$$\begin{aligned} &\frac{N^d_{+1}}{N^d} = \frac{\frac{K_{+1}}{k_{+1}}}{\frac{K}{k}} = \frac{N^s_{+1}}{N^s} = 1 + n \\ &\text{or, if } k_{+1} = k, \\ &1 + g_K \equiv \frac{K_{+1}}{K} = 1 + n \end{aligned} \tag{4.2}$$

In the case where there is a spectrum of techniques and an exogenously fixed supply of labor, it is possible for a change in the technique in use to allow full employment. The way this would work is that if the wage were to fall in response to an excess supply of labor, entrepreneurs would shift to a less capital-intensive technique, thereby increasing the demand for labor. If this process could take place smoothly and rapidly in a single period of production, equilibrium could be reached in the labor market as in Figure 4.9.

The great difficulty in applying the neoclassical model of full employment through flexible wages and changes in technique to real economies is that it can take a long time in real economies for the wage and the technique of production to adjust. Thus there may be considerable periods of time when the labor market fails to reach full employment equilibrium. The neoclassical defense of the model is that it should work on average over time, and since the purpose of a growth model is to analyze the long run behavior of the economy, the assumption of full employment is a permissible abstraction from real frictions. The Classical economists might respond that it is precisely over longer time periods that it makes sense to consider the labor supply itself as endogenously adjusting to the wage.

The idea that full employment could be achieved in the short run through changes in the technique of production in response to changes in the wage

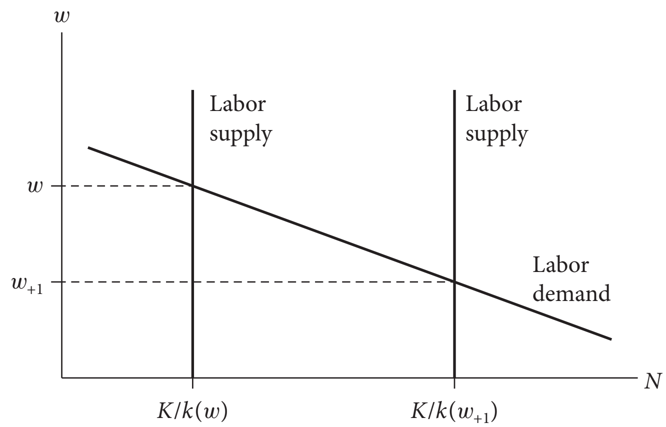

Figure 4.9 Wage flexibility can in theory achieve full employment by inducing entrepreneurs to change the technique of production. Here a rise in the supply of labor is accommodated by a move to a less capital-intensive technique mediated by a fall in the wage.

misleads some economists into thinking that the marginal product of labor determines the wage. As we have seen, however, the equality of the marginal product of labor (when it can be defined) to the wage is the result of profit maximization by entrepreneurs. A more accurate way to understand labor market equilibrium under the full employment assumption is that the wage determines the marginal product of labor (through the profit rate-maximizing decisions of entrepreneurs) and the supply of and demand for labor determine the wage so as to clear the labor market. Under these assumptions, the capital intensity of the technique in use has to change in each period to maintain full employment.

The full employment assumption can add one relation to an economic growth model: the wage rate is determined in each period so as to provide full employment of the exogenously given labor force. The growth rate and the level of consumption per worker still remain undetermined, even with the addition of the full employment assumption.

The full employment assumption can be written algebraically as:

$$\frac{K}{k(w)} = \bar{N} \tag{4.3}$$

Here $\bar{N}$ is the exogenously given labor supply.

4.5 Toward a Model of Economic Growth

A theory of the labor market adds one more determining relation to the growth-distribution schedule, but a full growth model still requires one more theoretical relation to determine the four endogenous variables, g_K, c, v, and w. As we have seen, the conventional wage model takes the real wage as given exogenously and thus determines the profit rate (and the profit rate-maximizing technique of production), but leaves the growth rate of the capital stock and social consumption per worker unexplained. The full employment model with no choice of technique forces the growth rate of the capital stock to be equal to the exogenously determined growth rate of the labor force, and thus through the social consumption-growth rate schedule also determines the level of social consumption per worker, but leaves the profit rate and the real wage unexplained. The Classical conventional wage model sees the labor market as determining distribution, but leaves growth to be determined elsewhere. The full employment model without choice of technique sees the labor market as determining growth, but leaves distribution to be determined elsewhere.

In the full employment model with choice of technique, the structure of the model is different in the short run and the long run. In the short run, the full employment condition determines the wage and the profit rate-maximizing technique, but leaves the growth rate and social consumption per worker unexplained. On a long run steady state growth path on which the wage is constant, however, the capital stock must grow at the same rate as the labor force. In this time frame the full employment assumption determines the growth rate of the capital stock and social consumption per worker, leaving the wage and the profit rate unexplained.

4.6 Growth in Real Economies

In the theoretical models, the exogenous parameters are assumed to be constant or growing in some orderly way, such as a constant rate of Harrod-neutral technical change. In real economies, this rarely happens. We turn once again to the six countries that we have been following through the phases of growth identified by Angus Maddison. Table 4.1 focuses on the growth of output, capital, and employment. The measure of employment in this table is total labor hours, which is the product of the number of workers and the annual hours worked by the average worker.

The volume of output and the capital stock have grown more or less continuously during the nearly two-century stretch of time. There are no

Table 4.1 Growth Rates of Output, Employment, and Capital (%/year) for Six Countries in Selected Periods from 1820–2010

	1820–1870	*1870–1913*	*1913–1950*	*1950–1973*	*1973–1992*	*1993–2002*	*2003–2006*	*2007–2010*
				US				
Output, X	4.22	3.94	2.84	3.92	2.39	2.77	1.93	0.92
Employment, N	3.09	2.02	0.35	1.15	1.27	1.29	1.43	−1.02
Capital Stock, K	5.46	5.53	2.01	3.27	3.13	3.64	3.87	−0.42
				France				
Output, X	1.27	1.63	1.15	5.02	2.26	2.41	0.99	1.48
Employment, N		−0.10	−0.75	0.01	−0.46	1.13	0.87	−0.09
Capital Stock, K				4.80	4.30	0.62	4.41	3.65
				Germany				
Output, X	2.00	2.81	1.06	5.99	2.30	2.32	1.92	1.54
Employment, N		0.92	0.45	0.00	−0.38	0.35	0.69	0.94
Capital Stock, K				5.93	3.37	1.43	0.26	0.68
				Netherlands				
Output, X	1.93	2.20	2.43	4.74	2.14	3.61	2.51	2.31
Employment, N		0.92	1.10	−0.04	−0.07	1.78	1.23	0.15
Capital Stock, K				4.55	3.07	2.76	2.99	2.75
				UK				
Output, X	2.04	1.90	1.19	2.96	1.59	3.16	1.81	0.25
Employment, N	0.86	0.76	−0.46	−0.15	−0.57	1.08	0.86	−0.02
Capital Stock, K	2.61	1.73	1.09	5.17	3.32	2.78	3.56	1.46
				Japan				
Output, X	0.31	2.34	2.24	9.25	3.76	1.65	0.88	−0.05
Employment, N	0.21	0.45	0.40	1.44	0.61	−0.47	0.37	−0.73
Capital Stock, K		3.49	4.17	9.18	6.81	2.18	−1.02	−2.48

Sources: Maddison (1995a, Table 2-6), Extended Penn World Tables 5.0.

negative entries, and only a few growth rates below 1 percent per year. If this sounds small, consider that a sum of money invested at 1 percent compound interest will double in value in only 70 years.

As was true with labor productivity, the growth of output and capital has an irregular, stop-go character. Again, the period right after World War II (1950–1973) contrasts vividly with the next period (1973–1992). Because of the high rate of growth in the former period, it is sometimes called the Golden Age of Capital Accumulation.

We need to be careful about the variations in the growth of employment because some of these variations are reflections of changes in the average number of hours worked per worker, and some are due to changes in the number of workers. The variations in the growth of employment can be interpreted through the different models in very different ways. Models that assume full employment of an exogenously given labor supply would treat these variations as exogenous changes in the supply of labor. The Classical conventional wage model would interpret these variations as changes in the demand for labor caused by variation in the rate of capital accumulation. In either case, we might need to superimpose some exogenous technical change in order to make sense out of the real data. In models that assume a spectrum of techniques, we might seek evidence of changes in the technique chosen, in response to changes in the real wage that clears the labor market.

4.7 Suggested Readings

The Classical doctrine on labor supply originated with Malthus (1986), Ricardo (1951), and Marx (1977), and an influential modern view is given by Lewis (1954). For a modern treatment of population in a Classical growth model, see Foley (2000). The distinction that Marx draws between labor and labor-power is elaborated in Marglin (1974) and Bowles (1985); this distinction plays a major role in the theory of economic development and capital-labor relations presented in Gordon et al. (1982). Goodwin (1967) studies the cyclical dynamics that can arise from variations in the Marxian reserve army of labor. The theoretical and applied literature on labor demand is surveyed from a neoclassical perspective in Hamermesh (1993). For a view of the evidence that is skeptical of the neoclassical approach on the grounds that it amounts to estimating an accounting identity, see Shaikh (1974) or Felipe and McCombie (2013). The Golden Age of Capital Accumulation is theoretically evaluated in Marglin and Schor (1990).

5

Models of Consumption and Saving

In order to close a model of economic growth, even given one of the labor market theories, we need to add a theory that explains how the society divides its income between consumption and investment.

The Classical economists, Smith, Malthus, and Ricardo, assumed that workers as a class consumed their whole wage. From one point of view this idea is a tautology, since if workers as a class saved, they would have positive wealth and would no longer be purely workers.

The assumption that workers as a class do little or no saving does not, of course, rule out the possibility that *individual* worker households might save. For example, workers might save in their youth and middle age in order to finance their retirement, as in the life-cycle theory of saving. Workers might save in order to meet certain contingencies, like unemployment, or the need to pay for their children's education. But this saving by some households will be offset by dissaving (spending out of accumulated saving) by other worker households. While some households are saving for retirement, others are spending their life saving on retirement consumption. While some households are saving to finance their children's education, others are spending out of saving to pay for that education. The Classical view amounts to the assumption that for workers as a class the saving of some households is matched by the dissaving of others.

Throughout much of this book, we will make the Classical assumption that workers spend all their wage income as a class, and contribute nothing to social saving. From the Classical point of view social saving is the function of the capitalists who already own wealth. We examine the complications that arise with worker saving in Chapter 17.

The model we will use to explain and predict capitalist consumption and saving, however, is quite neoclassical. Neoclassical economic theory views consumption decisions as the result of a trade-off between consuming in the present and saving in order to consume in the future, and we will rigorously adopt this point of view in modeling capitalist consumption and saving decisions. One great advantage of this modeling approach is that it forces us to make explicit the *intertemporal budget constraint* of the capitalists, which, in turn, is the key to understanding *portfolio decisions* when there is more than one asset in the economy, and the basic principles of *financial arbitrage*.

We will also use other models of consumption and saving when they are appropriate. In our discussion of the neoclassical growth model in Chapter 10, we will assume that all households, including worker households, save a fixed fraction of their income. In the *overlapping generations model* we analyze in Chapter 16, which focuses on the economic consequences of limited time horizons, workers are the sole source of saving.

The particular model of capitalist consumption and saving we will use adopts a particular intertemporal utility function, of the Cobb-Douglas form. The mathematical optimization model that arises from this utility function is particularly easy to solve, and the solution has a particularly simple form: the capitalist household consumes a constant fraction of its wealth at the end of each period, regardless of the net profit rates it anticipates in future periods. This simple behavior is traceable to the fact that in the Cobb-Douglas demand system the wealth and substitution effects of changes in future net profit rates exactly offset each other.

An important limitation of this model of intertemporal consumption is that it abstracts from *uncertainty*, which plays a major role in real economies. The analytical tools we develop could also be used to study consumption and saving under uncertainty, but uncertainty greatly complicates the mathematics involved. In our models the capitalist always knows current and future net profit rates with certainty.

The remainder of this chapter is devoted to working out the basic model of capitalist consumption and saving behavior in detail, as the basis for closing the model of economic growth.

5.1 A Two-Period Consumption-Saving Model

To begin with, let us start by working through an example used in many economic theory courses: the *two-period saving* model with a *Cobb-Douglas utility function*. A capitalist lives for two periods, 0 and 1. We measure her wealth, consumption, and saving in terms of real output. At the beginning of period 0 she has an endowment of wealth, K_0, which she can invest at a net profit rate r_0. The net profit rate represents the real return on her investment, and is thus comparable to an inflation-adjusted interest rate. At the end of the first period her wealth will have increased to $K_0 + r_0 K_0 = (1 + r_0) K_0$, which she can consume, C_0, or save, K_1, in order to consume at the end of the second period. In period 1 she again will invest at the net profit rate r_1, and at the end of the period will consume, C_1, her whole principal and return, $(1 + r_1) K_1$. We can write the capitalist's budget constraint in two parts:

$$
\begin{aligned}
&C_0 + K_1 \leq (1 + r_0) K_0 \\
&C_1 + K_2 \leq (1 + r_1) K_1 \leq (1 + r_1)((1 + r_0) K_0 - C_0) \qquad (5.1)
\end{aligned}
$$

For completeness we have included the possibility that the capitalist might save something at the end of period 1 (K_2), in order to provide for a still more distant future either for herself or her heirs. If the capitalist does not care about the future after period 1, she will set $K_2 = 0$, and consume all her wealth at the end of period 1. We have also included the possibility that she might throw some wealth away (neither consuming nor saving it), though if the capitalist gets positive marginal utility from consumption she will never do this, and the inequality signs will always be equalities.

The Cobb-Douglas utility function is defined for a parameter β (the Greek letter *beta*, pronounced "ˈbay-ta"), which is called the *utility discount factor* and lies between 0 and 1, as the weighted average of the logarithms of consumption in the two periods, with a weight of $1 - \beta$ on the first period and β on the second period:

$$
u(C_0, C_1) = \ln(C_0^{1-\beta} C_1^{\beta}) = (1 - \beta) \ln C_0 + \beta \ln C_1 \qquad (5.2)
$$

The natural logarithm function here plays the role of a utility function for each period. The utility a capitalist gains from consuming C in a period is $\ln C$, and the marginal utility of consumption when the capitalist is consuming C is the derivative of the natural logarithm, $1/C$, so that the higher is the capitalist's consumption, the lower is her marginal utility: the logarithmic utility function displays the property of *diminishing marginal utility*.

The form of equation (5.2) implies that the capitalist gets no utility from consumption after period 1.

To choose the pattern of consumption that maximizes utility subject to the budget constraint, the capitalist has to solve the mathematical programming problem:

$$\begin{aligned} &\max_{C_0, C_1 \geq 0} (1-\beta) \ln C_0 + \beta \ln C_1 \\ &\quad \text{subject to } C_0 + K_1 \leq (1+r_0)K_0 \\ &C_1 + K_2 \leq (1+r_1)K_1 \leq (1+r_1)((1+r_0)K_0 - C_0) \\ &\quad \text{given } \beta, K_0, r_0, r_1 \end{aligned} \tag{5.3}$$

The solution to this problem, which we will work out in detail below, is:

$$\begin{aligned} C_0 &= (1-\beta)(1+r_0)K_0 \\ K_1 &= \beta(1+r_0)K_0 \\ C_1 &= (1+r_1)K_1 = \beta(1+r_1)(1+r_0)K_0 \end{aligned} \tag{5.4}$$

The capitalist spends a fraction, $1-\beta$, of her wealth at the end of the first period on consumption, regardless of what the net profit rate will be in the second period. As we will see, this feature of the solution carries over no matter how long the time horizon may be.

5.1.1 Solving the two-period consumption problem

A convenient and economically insightful way to solve maximization problems of this type is through the *Lagrangian technique*. We define two new variables, λ_0 and λ_1 (the Greek letter *lambda*, pronounced " 'lam-da"), one for each constraint, called *shadow prices* or *Lagrange multipliers*, and view them as penalties for violating the constraints. Then we form the *Lagrangian function*, which is the utility of the capitalist less penalties for violating the constraints:

$$\begin{aligned} L(C_0, C_1, K_1, K_2; \lambda_0, \lambda_1) &\equiv (1-\beta) \ln C_0 + \beta \ln C_1 \\ &- \lambda_0(C_0 + K_1 - (1+r_0)K_0) - \lambda_1(C_1 + K_2 - (1+r_1)K_1) \end{aligned} \tag{5.5}$$

It is convenient to refer to the first part of the Lagrangian as the *utility function* and the rest of the Lagrangian as the *penalty function*. If we can choose C_0^* and $C_1^* \geq 0$ (and as a result K_1^* and K_2^*) and λ_0^* and $\lambda_1^* \geq 0$ so that C_0^*, C_1^*, K_1^*, and K_2^* *maximize* the Lagrangian holding λ_0^* and λ_1^* constant, and λ_0^* and λ_1^* *minimize* the Lagrangian holding C_0^*, C_1^*, K_1^*, and K_2^* constant, the resulting C_0^* and C_1^* will be the maximum for the original constrained problem. Such a combination $(C_0^*, C_1^*, K_1^*, K_2^*; \lambda_0^*, \lambda_1^*)$ is called a *saddle-point* of the Lagrangian function.

To see why a saddle-point of the Lagrangian must solve the original constrained maximization problem, notice first that C_0^*, C_1^*, K_1^*, and K_2^* must satisfy the constraints of the original problem. If they did not (if, for example, $C_0^* + K_1^* > (1 + r_0)K_0$), then it would always be possible to make the Lagrangian function smaller by taking λ_0 bigger, since λ_0 would be multiplied by a negative number in the Lagrangian expression. This would contradict the saddle-point property that λ_0^* and λ_1^* minimize the Lagrangian function holding C_0^*, C_1^*, K_1^*, and K_2^* constant. In fact, the penalty function must be zero at a saddle-point: either the constraint is exactly satisfied, so that the corresponding penalty term is zero, or the number multiplying the shadow price is negative, so that the Lagrangian function will be minimized only if the corresponding shadow price is zero.

Now suppose that there were some alternative C_0, C_1, K_1, and K_2 that satisfied the constraints and also gave a larger utility than C_0^* and C_1^*. At this alternative plan, the utility function would be larger than at the assumed saddle-point. Since this alternative plan satisfies the constraints, λ_0^* and λ_1^* must be multiplied by negative or zero numbers in the Lagrangian function, so that the penalty function is also either positive or zero. But the penalty function for the saddle-point was exactly zero, so the value of the Lagrangian at the alternative plan would be larger than at the saddle-point. This contradicts the saddle-point property that C_0^*, C_1^*, K_1^*, and K_2^* maximize the Lagrangian function holding λ_0^* and λ_1^* constant. This argument shows that there can be no alternative plan that satisfies the constraints and gives a larger utility. But this means that C_0^* and C_1^* are the maximum solution to the original problem.

To find the saddle-point of the Lagrangian, we find its critical points by setting its derivatives with respect to C_0, C_1, K_1, K_2, λ_0, and λ_1 equal to zero and solve the resulting set of *first-order conditions*. (This is not always possible, but does work for the Cobb-Douglas utility function.)

$$
\begin{aligned}
\frac{\partial L}{\partial C_0} &= \frac{1-\beta}{C_0} - \lambda_0 \le 0 \; (= 0 \text{ if } C_0 > 0) \\
\frac{\partial L}{\partial C_1} &= \frac{\beta}{C_1} - \lambda_1 \le 0 \; (= 0 \text{ if } C_1 > 0) \\
\frac{\partial L}{\partial K_1} &= -\lambda_0 + (1+r_1)\lambda_1 \le 0 \; (= 0 \text{ if } K_1 > 0) \\
\frac{\partial L}{\partial K_2} &= -\lambda_1 \le 0 \; (= 0 \text{ if } K_2 > 0) \\
\frac{\partial L}{\partial \lambda_0} &= -(C_0 + K_1 - (1+r_0)K_0) \ge 0 \; (= 0 \text{ if } \lambda_0 > 0) \\
\frac{\partial L}{\partial \lambda_1} &= -(C_1 + K_2 - (1+r_1)K_1) \ge 0 \; (= 0 \text{ if } \lambda_1 > 0)
\end{aligned}
\tag{5.6}
$$

The first-order condition for K_1, for example, says that the coefficient multiplying K_1 in the Lagrangian must be less than or equal to zero: if it were positive, we could increase the value of the Lagrangian without limit by choosing K_1 to be very large, and there would be no saddle-point. If K_1 is chosen positive, then this coefficient must be equal to zero, since if it were negative and K_1 were positive, we could increase the value of the Lagrangian by reducing K_1. This coefficient could be negative at the saddle-point only if K_1 were zero, since we cannot reduce K_1 below zero. Similar reasoning underlies the other first-order conditions.

The first two first-order conditions can be satisfied only if C_0 and $C_1 > 0$, which further implies that λ_0 and $\lambda_1 > 0$. The economic intuition behind this mathematical condition is that the marginal utility of the logarithmic utility function grows without bound as consumption becomes smaller and smaller, so that the capitalist will always consume something in each period. Since $\lambda_1 > 0$, we see that $K_2 = 0$ (which we figured out already above). Since the penalty function is zero at the saddle-point, we can see that:

$$\lambda_0 C_0 + \lambda_1 C_1 = K_1(-\lambda_0 + (1+r_1)\lambda_1) - \lambda_1 K_2 + \lambda_0(1+r_0)K_0.$$

But we can also see from the first-order conditions that

$$
\begin{aligned}
&\lambda_0 C_0 + \lambda_1 C_1 = 1 \\
&K_1(-\lambda_0 + (1+r_1)\lambda_1) = 0 \\
&\lambda_1 K_2 = 0
\end{aligned}
$$

Solving these equations, we get the *Cobb-Douglas demand system*:

$$\begin{aligned}
\lambda_0 &= \frac{1}{(1+r_0)K_0} \\
\lambda_1 &= \frac{1}{(1+r_1)(1+r_0)K_0} \\
K_1 &= \beta(1+r_0)K_0 \\
C_0 &= (1-\beta)(1+r_0)K_0 \\
C_1 &= (1+r_1)K_1 = \beta(1+r_1)(1+r_0)K_0
\end{aligned} \tag{5.7}$$

The Cobb-Douglas demand system has the peculiar feature that the wealth and substitution effects of a change in future net profit rates are equal and opposite in sign, so that the consumption in period zero does not depend on the net profit rate in the second period, r_1. This simplification is a great help in the types of growth models we will be studying. For example, the capitalist's saving, K_1, is just a constant fraction β of her wealth at the end of the period $(1+r_0)K_0$.

PROBLEM 5.1 Write down the capitalist choice problem for a capitalist facing three periods. Indicate clearly the utility function and the budget constraints, and explain your notation.

PROBLEM 5.2 Write down the Lagrangian function for the three-period capitalist choice problem, and find the first-order conditions characterizing its critical points. How many shadow prices will there be?

PROBLEM 5.3 Solve the first-order conditions for the three-period capitalist choice problem, and show that the resulting demand system is given by the equations:

$$\begin{aligned}
C_0 &= (1-\beta)(1+r_0)K_0 \\
K_1 &= \beta(1+r_0)K_0 \\
C_1 &= (1-\beta)(1+r_1)K_1 = (1-\beta)\beta(1+r_1)(1+r_0)K_0 \\
K_2 &= \beta(1+r_1)K_1 \\
C_2 &= (1+r_2)K_2
\end{aligned} \tag{5.8}$$

5.2 An Infinite-Horizon Model

Ricardo and the Classical economists argued that wealth-holders would take into account the interests of their descendants in making consumption and saving decisions. Thus they would act as if their planning horizon stretched infinitely far into the future, even though their individual lives would come to an end in a finite time. This dynastic hypothesis is called *Ricardian equivalence*.

It is, happily, possible to generalize the two-period Cobb-Douglas saving problem to a longer horizon.

To allow for an *infinite horizon*, we let $t = 0, 1, 2, \ldots$ without any ending point.

The capitalist begins period 0 with a stock of wealth K_0, just as in the two-period model. She can invest this at the net profit rate r_0, and at the end of the period will have $(1 + r_0)K_0$ to divide between consumption and saving for the next period, just as in the two-period model. In fact, the budget constraints for the infinite horizon model are just the same as in the two-period model, except that there is an infinite sequence of them:

$$\begin{aligned} C_0 + K_1 &\leq (1 + r_0)K_0 \\ C_1 + K_2 &\leq (1 + r_1)K_1 \\ &\vdots \\ C_t + K_{t+1} &\leq (1 + r_t)K_t \\ &\vdots \end{aligned} \tag{5.9}$$

The capitalist has to make a sequence of decisions of this kind in each period. As a result her consumption will be a series

$$\{C_0, C_1, C_2, \ldots, C_t, \ldots\} = \{C_t\}_{t=0}^{\infty}$$

extending from period zero to infinity.

We will assume that the typical capitalist ranks consumption paths $\{C_t\}_{t=0}^{\infty}$ by calculating the discounted logarithmic utility function:

$$\begin{aligned} u(\{C_t\}_{t=0}^{\infty}) &= (1 - \beta)\Sigma_{t=0}^{\infty}\beta^t \ln C_t \\ &= (1 - \beta) \ln C_0 + (1 - \beta)\beta \ln C_1 + (1 - \beta)\beta^2 \ln C_2 + \ldots \end{aligned} \tag{5.10}$$

This utility function is a generalization of the Cobb-Douglas utility function we used in analyzing the two-period saving problem. It is a weighted average of the logarithms of consumption in each period. (Remember that the geometric sequence can be summed: $\Sigma_{t=0}^{\infty}\beta^t = 1/(1 - \beta)$ so that $(1 - \beta)\Sigma_{t=0}^{\infty}\beta^t = 1$.) The effect of multiplying $\ln C_t$ by $(1 - \beta)\beta^t$ is to shrink down or *discount* the utility from consumption in period t. Utility farther in the future counts for less in the typical capitalist's calculations. We are assuming that the capitalist has *perfect foresight*, that is, that she correctly anticipates all future net profit rates. The capitalist thus has to solve a planning problem, which is to maximize her utility subject to the series of budget constraints in each period:

$$\begin{aligned}&\text{choose } \{C_t \geq 0,\ K_{t+1} \geq 0\}_{t=0}^{\infty}\\ &\quad \text{so as to maximize } (1-\beta)\Sigma_{t=0}^{\infty}\beta^t \ln C_t \\ &\quad \text{subject to } C_t + K_{t+1} \leq (1+r_t)K_t \quad t = 0, 1, \ldots\\ &\quad K_0,\ \{r_t\}_{t=0}^{\infty} \text{ given}\end{aligned} \tag{5.11}$$

The solution to this problem, worked out below, is, as we would expect from the two-period problem, that the capitalist consumes a fraction $1-\beta$ of her end-of-period wealth in every period:

$$C = (1-\beta)(1+r)K \tag{5.12}$$

This implies that she also saves a fraction β of her wealth:

$$K_{+1} = \beta(1+r)K \tag{5.13}$$

The growth of her wealth in each period depends only on the discount rate, β, and the net profit rate, r:

$$1 + g_K = \frac{K_{+1}}{K} = \beta(1+r) \tag{5.14}$$

In growth theory equation (5.14) is called the *Cambridge equation*.

5.2.1 Solving the infinite-horizon problem

We can solve the infinite-horizon problem by the Lagrangian technique exactly as in the two-period model. We now have an infinite sequence of shadow prices $\{\lambda_t\}_{t=0}^{\infty}$, one for each period's budget constraint. The Lagrangian function for the capitalist's planning problem is:

$$\begin{aligned}&L(\{C_t, K_{t+1}; \lambda_t\}_{t=0}^{\infty})\\ &\quad = (1-\beta)\Sigma_{t=0}^{\infty}\beta^t \ln C_t - \Sigma_{t=0}^{\infty}\lambda_t(C_t + K_{t+1} - (1+r_t)K_t)\\ &\quad = (1-\beta)\Sigma_{t=0}^{\infty}\beta^t \ln C_t\\ &\qquad - \Sigma_{t=0}^{\infty}\lambda_t C_t - \Sigma_{t=0}^{\infty}(\lambda_t - \lambda_{t+1}(1+r_{t+1}))K_{t+1} + \lambda_0(1+r_0)K_0\end{aligned}$$

In order to find a saddle-point for the Lagrangian, which is a set of values $\{C_t^*, K_{t+1}^*; \lambda_t^*\}_{t=0}^{\infty} \geq 0$ that have the property that $\{C_t^*, K_{t+1}^*\}_{t=0}^{\infty}$ maximizes

L taking $\{\lambda_t^*\}_{t=0}^{\infty}$ as given and that $\{\lambda_t^*\}_{t=0}^{\infty}$ minimizes L taking $\{C_t^*, K_{t+1}^*\}_{t=0}^{\infty}$ as given, we find the first-order conditions corresponding to each of the variables. If we can find a saddle-point, for the same reasons as in the two-period case, the values $\{C_t^*, K_{t+1}^*\}_{t=0}^{\infty}$ must be the solution to the original problem (5.11). These first-order conditions, which are necessary and sufficient to solve the capitalist's planning problem (5.11), are:

$$\frac{\partial L}{\partial C_t} = \frac{(1-\beta)\beta^t}{C_t} - \lambda_t \leq 0 \ (\ = 0 \text{ if } C_t > 0) \tag{5.15}$$

$$\frac{\partial L}{\partial K_{t+1}} = -\lambda_t + (1 + r_{t+1})\lambda_{t+1} \leq 0 \ (\ = 0 \text{ if } K_{t+1} > 0) \tag{5.16}$$

$$\frac{\partial L}{\partial \lambda_t} = -(C_t + K_{t+1} - (1 + r_t)K_t) \geq 0 \ (\ = 0 \text{ if } \lambda_t > 0) \tag{5.17}$$

These first-order conditions are always necessary for a saddle-point. In general, they will also be sufficient provided that the objective function is concave in its arguments, and the constraint set is convex. It is not hard to see that both these conditions are satisfied in our problem: the natural logarithm is a concave function, and each budget constraint $K_{+1} = (1 + r)K - C$ is linear, and therefore convex, in (K, K_{+1}). Finally, the first-order conditions have to be satisfied for all $t = 0, 1, \ldots, \infty$.

Equation (5.15) can be satisfied only if $\lambda_t > 0$ and $C_t > 0$. We can use the saddle-point conditions, as in the two-period example, to figure out the typical capitalist's consumption function. At the saddle-point the value of the penalty function must be zero. But then we have:

$$\begin{aligned} \Sigma_{t=0}^{\infty}\lambda_t C_t &= (1-\beta)\Sigma_{t=0}^{\infty}\beta^t = 1 \\ &= \Sigma_{t=0}^{\infty}K_{t+1}(-\lambda_t + (1 + r_{t+1})\lambda_{t+1}) + \lambda_0(1 + r_0)K_0 \end{aligned}$$

According to the first-order conditions:

$$\Sigma_{t=0}^{\infty}K_{t+1}(-\lambda_t + (1 + r_{t+1})\lambda_{t+1}) = 0$$

so we have, as in the two-period case,

$$\lambda_0 = \frac{1}{(1 + r_0)K_0}$$

Equation (5.15) for $t = 0$ implies that $C_0 = (1 - \beta)/\lambda_0$, so we have

$$C_0 = (1 - \beta)(1 + r_0)K_0$$

Since every period is actually just like the first, a similar argument shows that the first-order conditions lead to the consumption function:

$$C = (1 - \beta)(1 + r)K \tag{5.18}$$

As we have seen, this leads to the formulas for the growth of the capitalist's wealth:

$$K_{+1} = \beta(1 + r)K \tag{5.19}$$

$$1 + g_K = \frac{K_{+1}}{K} = \beta(1 + r) \tag{5.20}$$

PROBLEM 5.4 In the infinite-horizon Cobb-Douglas consumption model, prove (5.13), and express C_t in terms of K_{t+1}.

PROBLEM 5.5 Show that along the optimal consumption path in the infinite-horizon Cobb-Douglas consumption model the sum of realized consumption and the value of the capital at the shadow price, $\Sigma_{t=0}^{T}\lambda_t C_t + \lambda_T K_{T+1}$, remains constant over time and is equal to $\lambda_0(1 + r_0)K_0$.

5.3 The Constant Saving Rate Model

In the neoclassical growth model gross investment is often assumed to be a constant fraction, s, of output:

$$I = sX$$

In the Classical saving model we have developed here, gross investment depends on the stock of wealth of the capitalists, which in this model consists only of capital, K. Remembering that $\rho K = X$ and $r = v - \delta$, we see that in the Classical model:

$$I = K_{+1} - K + \delta K = (\beta(1 + r) - (1 - \delta))K = \frac{\beta v - (1 - \beta)(1 - \delta)}{\rho} X$$

Thus the Classical saving model also predicts that investment will be a constant proportion of output as long as the profit rate, v, the productivity of capital, ρ, and the depreciation rate, δ, do not change. Thus the two saving models differ only when the profit rate, productivity of capital, or depreciation rate are changing.

5.4 Saving Rates and Growth Rates

The *saving rate*, $s = I/X$, is often used by economists to indicate where a country lies on its social consumption-growth schedule. As its definition indicates, the saving rate is actually the proportion of gross investment in output. (The identification of the investment rate with the saving rate reflects the implicit assumption of *Say's Law* in these models.) In a closed economy saving and investment are identical, but open economies may export or import capital, so that saving may exceed or fall short of investment. For the purposes of studying economic growth in a country, the key factor is gross investment, so we measure the saving rate as the ratio of gross investment to output. Countries with high saving rates are devoting more effort to growth and less to consumption. From its definition, we can see that:

$$s = \frac{I}{X} = \frac{X - C}{X} = 1 - \frac{C}{X} = 1 - \frac{c}{x}$$

By rearranging this equation, we can see the relationship of the saving rate to the social consumption-growth schedule:

$$c = (1 - s)x$$

It is possible to use $1 - s$ in place of c to express social consumption in the social consumption-growth rate schedule:

$$1 - s = 1 - \frac{g_K + \delta}{\rho} \tag{5.21}$$

or:

$$g_K + \delta = s\rho \tag{5.22}$$

An important question in economic growth analysis is whether high saving rates lead to more rapid economic growth. In some theories higher saving rates accelerate economic growth only temporarily, and in others, higher saving rates have a permanent positive effect on economic growth.

Maddison's data set measures the saving rates for the six countries included in Table 2.8. We compare saving rates to growth rates of the capital stock for these countries by subperiod over the last century in Table 5.1. If you study each country over time, you can see that increases in the saving rate are usually associated with increases in the growth rate. It also seems true that the high-saving countries (such as Japan) grow more rapidly than the low-saving countries. There are some exceptions to these generalizations,

Table 5.1 Rates of Saving (I/X in %) and Capital Accumulation (I/K in % per year) for Six Countries for Selected Intervals, 1870–2014

	1870–1890	*1890–1913*	*1913–1938*	*1938–1950*	*1950–1973*	*1973–1981*	*1981–1987*	*1988–1998*	*1999–2006*	*2007–2010*	*2011–2014*
US											
I/X	16.3	15.9	14.2	13.1	18.0	18.0	17.7	17.40*	18.95*	15.72*	15.84*
I/K	6.79	4.17	2.10	1.75	3.22	3.31	3.01	2.56**	4.83**	0.53**	1.98**
France											
I/X	12.8	13.9	16.1		21.2	21.7	20.2	17.10*	18.00*	18.77*	18.73*
I/K					4.68	4.47	3.44	4.68*	4.79*	4.47*	4.20*
Germany											
I/X			12.9		23.2	20.6	20.2	20.72*	18.75*	17.67*	17.48*
I/K				−1.09	6.11	3.37	2.91		3.82*	3.46*	3.36*
Netherlands											
I/X			17.5		23.8	20.2	19.4	19.08*	17.93*	17.41*	15.44*
I/K					4.45	2.92	2.54		6.78*	6.09*	4.85*
UK											
I/X	8.4	8.5	7.8	6.5	16.3	17.7	16.5	18.08*	16.85*	13.96*	14.07*
I/K	1.66	1.75	1.29	0.65	5.04	3.08	2.81	4.4**	4.37**	8.8**	4.4**
Japan											
I/X	12.6	14.4	16.2	18.6	28.3	30.4	29.2	21.88*	18.89*	18.12*	17.70*
I/K		3.43	4.71	2.78	8.79	6.70	5.40	7.92**	4.01**	1.41**	1.54**

* Authors' calculations from OECD Economic Database;
** Authors' calculations from Penn World Tables 9.0.
Sources: Maddison (1995a, Table K-1), Maddison (1995b, pp. 148–164 and p. 172),

which must reflect differences in the parameters (such as x or ρ) of the social consumption-growth schedule.

The saving rate in the US has remained fairly constant over the last century. Perhaps because the US economy has been the main source of data for macroeconomists, they once believed that a good theory of consumption and saving should explain the "fact" that the saving rate is a constant that lacks any trend.

But the data from other countries support an idea associated with John Maynard Keynes: countries save a greater proportion of their income as they grow richer. For most countries, saving rates have increased over the course of this century. One major qualification is that during the 1980s, and for some countries during the period following the Global Crisis of 2008, saving rates declined, as can be seen by examining the last five columns in Table 5.1.

5.5 Suggested Readings

The dynamic model of optimal consumption originated with the English mathematician Frank Ramsey (1928). John Maynard Keynes's view of the class structure of saving, essentially a defense of capitalism based on its ability to deliver a high rate of capital accumulation, can be found in Keynes (1920). His view that the saving rate tends to rise with income is elaborated in Keynes (1936). The two-class assumption plays a major role in the work of Kaldor (1956) and Pasinetti (1974), and was an object of debate (Samuelson and Modigliani 1966, for example) during the Cambridge capital controversy. Stephen Marglin (1984) devotes considerable attention to the comparison between neoclassical and Classical theories of saving.

6

Classical Models of Economic Growth

A particular theory of the labor market, a particular theory of consumption and saving, and the growth-distribution schedule constitute a *growth model*. The *Classical growth models* we will analyze in this chapter combine the main insight of the infinite-horizon model of capitalist consumption, that is, that capitalist consumption is a constant fraction of the end-of-period wealth, with the assumption of either a conventional wage or full employment in the labor market.

6.1 The Classical Conventional Wage Model

As we have seen, a key idea in the Classical approach to growth theory developed by Smith and Ricardo, and used as a base for Marx's critique of the capitalist economy, is that labor-power is elastically supplied at a given conventional wage. The Classical model thus assumes that labor supply is a horizontal line at a given real wage $\bar{w}$. This determines one (w) of the four variables, v, w, g_K, and c:

$$w = \bar{w} \tag{6.1}$$

As we have seen in Chapter 5, in the Classical view social saving is the result of decisions of capitalists not to consume their wealth. We assume that there are many identical capitalists, all of whom begin with the same initial wealth K. If the number of capitalists stays the same over time, the decisions of the typical capitalist summarize what happens to the whole economy.

We have seen that in the one-sector model the typical capitalist receives the profit rate v on each unit of capital she rents to entrepreneurs. This is the residual profit per unit of capital after the entrepreneur has paid the wage bill, or, equivalently, the profit share in output times the output-capital ratio:

$$v = \frac{x - w}{k} = \left(1 - \frac{w}{x}\right)\rho = \pi\rho \tag{6.2}$$

At the end of the period the typical capitalist has to divide her wealth, which consists of the profit she has received on her capital and her depreciated capital, between her consumption, C^c, and accumulation of capital for the next period:

$$C^c + K_{+1} = (1 - \delta)K + vK \tag{6.3}$$

As we have seen in Chapter 5, assuming that the typical capitalist maximizes a discounted logarithmic utility function, she will choose to consume a constant fraction $(1 - \beta)$ of her end-of-period wealth:

$$C^c = (1 - \beta)(1 + r)K \tag{6.4}$$

and, as a consequence:

$$\begin{aligned} K_{+1} &= \beta(1 + r)K \\ 1 + g_K \equiv \frac{K_{+1}}{K} &= \beta(1 + r) \end{aligned} \tag{6.5}$$

This relation between the capital growth factor $1 + g_K$, the discount factor or saving propensity of capitalists, β, and the net profit rate $1 + r$ plays an important role in many modern models of growth, and is often called the *Cambridge equation*. It will hold whenever workers spend all their wages and capitalists save a fraction β of their end-of-period wealth.

We can also express the Cambridge equation as a relation between the $g_K + \delta$, and the gross profit rate, v:

$$g_K + \delta = \beta v - (1 - \beta)(1 - \delta) \tag{6.6}$$

The Classical theories of the labor market and of capitalist consumption give us two equations to add to the real wage-profit rate relation and the social consumption-growth rate relation to make a complete model that will determine all four endogenous variables: social consumption, c, the growth rate of capital, g_K, the wage, w, and the profit rate, v.

We can write the four relations that make up the Classical model in terms of the parameters x, δ, β, and $\bar{w}$, and either k or ρ, depending on which we use to describe the technology as in Table 6.1.

Table 6.1 The Classical Conventional Wage Model

Endogenous variables: w, v, c, g_K
Exogenous parameters: k, x, δ, β, $\bar{w}$

$$w = x - vk \tag{6.7}$$

$$c = x - (g_K + \delta)k \tag{6.8}$$

$$\delta + g_K = \beta v - (1-\beta)(1-\delta) \tag{6.9}$$

$$w = \bar{w} \tag{6.10}$$

Exogenous parameters: ρ, x, δ, β, $\bar{w}$

$$w = x\left(1 - \frac{v}{\rho}\right) \tag{6.11}$$

$$c = x\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{6.12}$$

$$g_K + \delta = \beta v - (1-\beta)(1-\delta) \tag{6.13}$$

$$w = \bar{w} \tag{6.14}$$

We can also find capitalist consumption per worker, c^c, from the fact that social consumption equals capitalist consumption plus workers' consumption, and workers' consumption is equal to the wage:

$$c = c^c + c^w = c^c + w$$

We can visualize the full determination of the Classical system as in Figure 6.1.

The Classical model has a straightforward explanatory structure. The conventional real wage determines the profit rate, given the real wage-profit rate relation determined by the production coefficients, and also workers' consumption, given the assumption that workers as a class consume the entire real wage. The profit rate then determines the growth rate through the Classical profit rate-growth rate relation (the Cambridge equation), and the growth rate in turn determines social consumption, divided into capitalists' consumption and workers' consumption.

The Classical conventional wage model can be applied to real economies. For example, we can use the data in Tables 2.1 and 2.2 to determine the appropriate parameters for the US economy in 2005. We can read k, x,

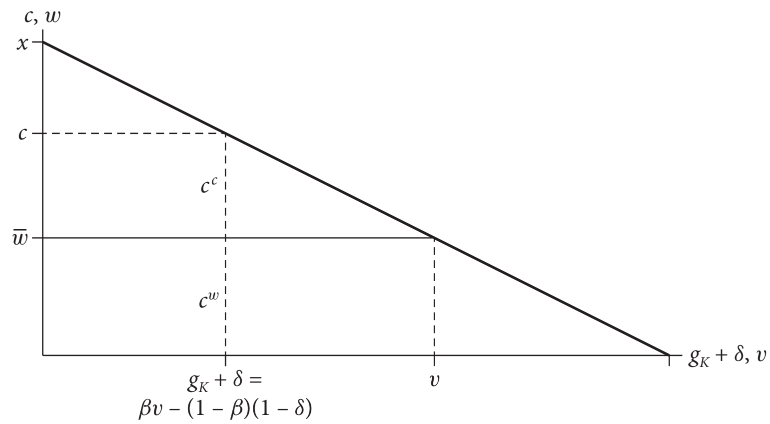

Figure 6.1 The addition of the Classical assumptions of a conventional real wage and capitalist consumption (which is a constant fraction of wealth), closes the growth model, determining g_K, w, r, and c, given the real wage-profit rate and social consumption-growth rate schedules.

δ, and $\bar{w}$ directly from these tables. To calculate β, we need first to find $g_K = ((i/k) - \delta) = (.118 - .0817)/yr = 0.0367\%/yr$. $\beta = (1 + g_K)/(1 + v - \delta) = 1.0367/1.0865 = .954$.

PROBLEM 6.1 If the real wage in Ricardia (see Problem 2.1) is 20 bu/worker-year and $\beta = .5$, find the growth rate of capital, social consumption per worker, and capitalist consumption per worker.

PROBLEM 6.2 If the real wage in Industria (see Problem 2.2) is \$10/hr., workers work 2000 hours per year, and $\beta = .97$, find the growth rate of capital, social consumption per worker, and capitalist consumption per worker.

6.2 Comparative Dynamics in the Conventional Wage Model

A model explains changes in the endogenous variables as the result of changes in the exogenous parameters. In order to carry out this kind of analysis, we need to figure out what the effect of a change in the various parameters of the model will be on the endogenous variables.

In the Classical model the endogenous variables are the real wage, w, the profit rate, v, social consumption, c (divided into workers' consumption $c^w = w$ and capitalists' consumption c^c, both measured per worker), and the growth rate of capital, g_K. The parameters are the capital-labor ratio, k, or capital productivity, ρ, output per worker, x, the depreciation rate, δ, the capitalists' utility discount factor, β, and the conventional wage, $\bar{w}$. A typical comparative dynamic exercise is to work out the effect of an increase in labor productivity, x, holding capital productivity, ρ, constant, on the endogenous variables, w, v, c, and $g_K + \delta$. In doing this type of comparative exercise, it is important to be very clear about which parameters are changing, and which are remaining constant. In this case, ρ, δ, β, and $\bar{w}$ remain the same: *only* x and $k = x/\rho$ change.

We can work this problem out either on the basis of the equations that define the equilibrium, or by looking at the graphical representation of the solution. The equilibrium conditions are equations (6.11) through (6.14).

Since we are assuming that $\bar{w}$ does not change when we change x, the wage will remain the same when x increases. Then we can see from equation (6.11) that the profit rate v will rise (since x increases and ρ stays constant). According to equation (6.13), the Cambridge equation, a rise in v will increase $g_K + \delta$. Since both x and $g_K + \delta$ increase in Equation (6.12), it is not immediately possible to conclude whether c rises or falls. We can, however, look back at the capitalist's consumption function, which tells us that:

$$c^c = (1 - \beta)(1 + r)\frac{x}{\rho}$$

We know that $r = v - \delta$ and x increase, while by assumption β and ρ remain constant, so that capitalist consumption per worker, c^c, must increase. Workers' consumption is just equal to the real wage, which is held constant by assumption, so that total social consumption per worker must rise. We summarize this experiment in Table 6.2.

The same conclusions could be reached by studying the graphical representation of the equilibrium. For variety, let us work out another comparative statics exercise graphically, for example, an increase in the capital-labor

Table 6.2 The Comparative Dynamics of the Classical Conventional Wage Model

Parameter changes					Effects				
ρ	k	x	β	$\bar{w}$	v	w	g_K	c	c^c
same	up	up	same	same	up	same	up	up	up
down	up	same	same	same	down	same	down	up	up
same	same	same	up	same					
same	same	same	same	up					

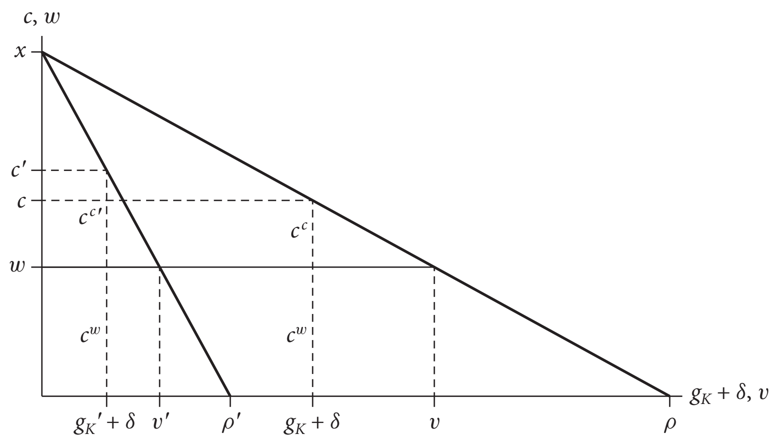

Figure 6.2 An increase in k, holding x, δ, β, and $\bar{w}$ constant (but lowering $\rho = x/k$), to k', makes the growth-distribution schedule steeper. Since the wage does not change, the profit rate must fall, which, given a constant β, leads to a fall in the rate of growth of capital, g_K. Since $c^c = (1-\beta)(1+r)k = (1-\beta)(1+v-\delta)k = (1-\beta)((1-\delta)k + vk)$, and $vk = x - w$ is constant, capitalist consumption per worker and social consumption per worker both rise.

ratio, k, holding constant x, δ, β, and $\bar{w}$. (Since $\rho = x/k$, this experiment also lowers the productivity of capital ρ.) As we can see in Figure 6.2, the growth-distribution schedule becomes steeper, rotating around the intercept $(0, x)$. With an unchanging real wage the profit rate and growth rate of capital must fall. It is also possible to conclude from the capitalist's consumption function that capitalist consumption per worker rises.

PROBLEM 6.3 Analyze the effect of an increase in β on the endogenous variables in the Classical conventional wage model.

PROBLEM 6.4 Analyze the effect of an increase in the conventional real wage $\bar{w}$ on the endogenous variables in the Classical conventional wage model.

6.3 Labor-Saving Technical Change in the Classical Model

The Classical conventional wage model can explain continuing economic growth, since with constant capital productivity the growth rate of output, g_X, will equal the growth rate of capital, g_K. But the conventional wage model cannot explain the increases in labor productivity and the wage observed in historical capitalist economies, since it assumes that x and $\bar{w}$ are constant over time.

The simplest way to modify the Classical model to accommodate increasing labor productivity is to add exogenous labor-saving technical change. If labor productivity steadily rises and the wage remains constant, however, the wage share, w/x, will fall steadily toward zero. In real capitalist economies the wage share, despite considerable fluctuation, does not tend to zero, but remains roughly constant. This suggests modifying the Classical model by assuming a conventional wage *share* rather than a conventional wage. These two modifications together yield a *Classical conventional wage share model*, which is a good first approximation to observed patterns of capitalist economic growth.

The assumption that labor productivity grows steadily translates into the algebraic formula

$$x = x_0(1+\gamma)^t$$

where x_0 is the labor productivity in some arbitrarily chosen base year, and γ is the exogenously given rate of growth of labor productivity. Since labor-saving technical change leaves capital productivity, ρ, constant, capital intensity,

$$k = x/\rho = (x_0/\rho)(1+\gamma)^t = k_0(1+\gamma)^t$$

where $k_0 \equiv x_0/\rho$ is the capital intensity in the base year, also grows steadily at the rate γ. The assumption that the wage share is given translates into

$$w = (1-\bar{\pi})x = (1-\bar{\pi})x_0(1+\gamma)^t = w_0(1+\gamma)^t$$

where $1-\bar{\pi}$ is the conventionally given wage share (and $\bar{\pi}$ is the corresponding profit share) and $w_0 \equiv (1-\bar{\pi})x_0$ is the wage in the base year, so that the wage will also be increasing steadily at the rate γ. Thus we could write the real wage-profit rate schedule as:

$$w = w_0(1+\gamma)^t = x - vk = x_0(1+\gamma)^t - vk_0(1+\gamma)^t$$

If we divide both sides of this equation through by $(1+\gamma)^t$, it becomes:

$$w_0 = x_0 - vk_0$$

which is just the same as the real wage-profit relation for the Classical conventional wage model. This observation suggests that it would be easier to

Table 6.3 The Classical Conventional Wage Share Model

Endogenous variables: $\tilde{w}, v, \tilde{c}, g_K$
Exogenous parameters: $\tilde{k}, \tilde{x}, \delta, \beta, \bar{\pi}$

$$\tilde{w} = \tilde{x} - v\tilde{k} \tag{6.15}$$

$$\tilde{c} = \tilde{x} - (g_K + \delta)\tilde{k} \tag{6.16}$$

$$\delta + g_K = \beta v - (1 - \beta)(1 - \delta) \tag{6.17}$$

$$\tilde{w} = (1 - \bar{\pi})\tilde{x} \tag{6.18}$$

Exogenous parameters: $\rho, \tilde{x}, \delta, \beta, \bar{\pi}$

$$\tilde{w} = \tilde{x}\left(1 - \frac{v}{\rho}\right) \tag{6.19}$$

$$\tilde{c} = \tilde{x}\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{6.20}$$

$$\delta + g_K = \beta v - (1 - \beta)(1 - \delta) \tag{6.21}$$

$$\tilde{w} = (1 - \bar{\pi})\tilde{x} \tag{6.22}$$

analyze the model in terms of a new set of variables, $\tilde{x} = x/(1+\gamma)^t$, $\tilde{k} = k/(1+\gamma)^t$, $\tilde{w} = w/(1+\gamma)^t$, and $\tilde{c} = c/(1+\gamma)^t$, which will remain constant over time. In these variables, the Classical conventional wage share model can be expressed algebraically as in Table 6.3.

If we compare these equations with equations (6.7)–(6.13), we see that the Classical conventional wage share model has exactly the same mathematical form as the Classical conventional wage model, with "˜" variables taking the places of the corresponding variables in the original model. Thus all the comparative dynamics results from the conventional wage model carry over to the conventional wage share model with the appropriate change in interpretation.

One way to think about the change in variables to the "˜" form is to recognize that labor-saving technical change effectively makes each employed worker in year t the productive equivalent of $(1+\gamma)^t$ workers in the base year. Thus dividing through output variables by $(1+\gamma)^t$ expresses them in terms of *effective workers*. In effective worker units, then, the Classical conventional wage share model is mathematically identical to the Classical conventional wage model expressed in terms of real workers.

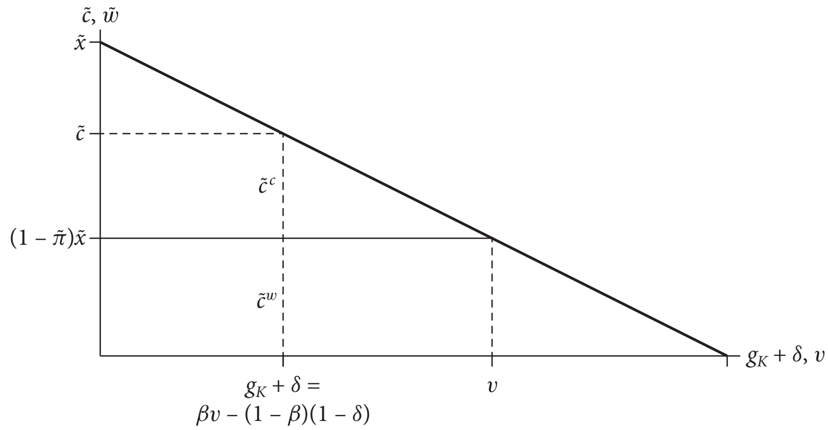

Figure 6.3 The Classical conventional wage share model with pure labor-saving technical change is mathematically identical to the Classical conventional wage model, substituting the effective labor units $\tilde{x}$, $\tilde{k}$, $\tilde{w}$, and $\tilde{c}$ for x, k, w, and c.

Figure 6.3, which, except for the relabeling of the vertical axis, is the same as Figure 6.1, shows the Classical conventional wage share model in effective labor units.

The convenience of working in effective labor units comes at the price of having to reinterpret the predictions of the model to apply them to real economies. For example, a Classical conventional wage share economy with labor-saving technical change at rate γ will have constant output per effective worker, $\tilde{x}$, but output per real worker will be growing at the steady rate γ. The same reinterpretation has to be applied to the constant wage per effective worker, $\tilde{w}$, and the constant social consumption per effective worker, $\tilde{c}$, since the wage per real worker and social consumption per real worker will be growing steadily at the rate γ. This is roughly the pattern observed over long periods of time in real capitalist economies, so the Classical conventional wage share model is at least a first approximation to a workable theory of economic growth.

We can take a further step in simplifying the mathematical form of the Classical conventional wage share model by shifting to using the profit share, $\pi = 1 - (w/x) = z/x$, to measure the division of the value of output between wages and profits, and the saving rate, $s = 1 - (c/x) = i/x$, to measure the division of output between consumption and investment. In these variables, the Classical conventional wage share model has the form of Table 6.4.

Table 6.4 The Classical Conventional Wage Share Model in Share Variables

Endogenous variables: π, v, s, g_K
Exogenous parameters: $\rho, \delta, \beta, \bar{\pi}$

$$v = \pi\rho \tag{6.23}$$

$$g_K + \delta = s\rho \tag{6.24}$$

$$g_K + \delta = \beta v - (1-\beta)(1-\delta) \tag{6.25}$$

$$\pi = \bar{\pi} \tag{6.26}$$

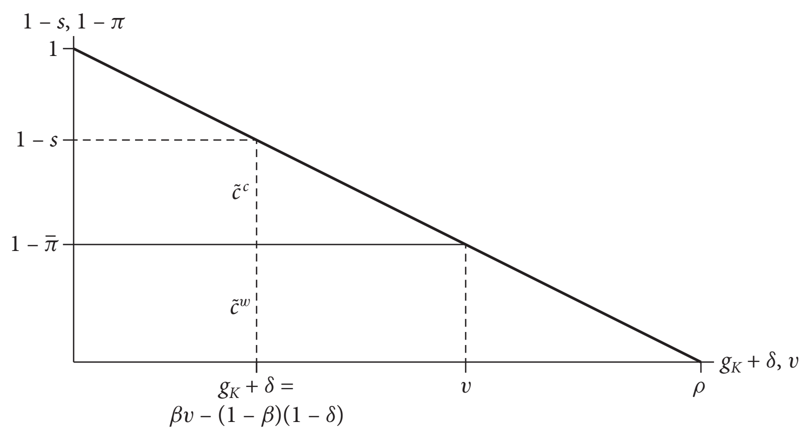

Figure 6.4 In terms of share variables, π and s, the growth-distribution schedule has vertical intercept 1, and horizontal intercept ρ. The exogenous wage share, $1-\pi$, determines the profit rate, v, and the growth rate of capital, $g_K + \delta = \beta v - (1-\beta)(1-\delta)$, determines the consumption rate $1-s$.

In this form, the model determines the basic ratios of the economic growth process. In order to calibrate it to a real economy, we need to know both the base year output per worker, x_0, and the growth rate of labor productivity, γ. Then in any year we can calculate $x = x_0(1+\gamma)^t$, $w = (1-\pi)x$, and $c = (1-s)x$ from the solution of the model.

Figure 6.4 presents the Classical conventional wage share model graphically in share variables.

If γ is zero, the Classical conventional wage share model reduces exactly to the conventional wage model. From this point on, we will analyze only the more general conventional wage share model with labor-saving technical change, since the conclusions can be translated into the conventional wage model simply by setting $\gamma = 0$.

Growth in the Classical conventional wage share model is endogenous, since it is determined by the saving behavior of capitalists. If the saving propensity of capitalists, β, rises, the growth rates of capital and output will rise as well, because capital will be accumulated more rapidly. The labor force and its growth rate are also endogenous, adapting to the accumulation of capital.

PROBLEM 6.5 Analyze the effect of an increase in β on the endogenous variables in the Classical conventional wage share model.

PROBLEM 6.6 Analyze the effect of an increase in the effective wage on the endogenous variables in the Classical conventional wage share model.

6.4 Choice of Technique in the Classical Model

We have been analyzing the Classical conventional wage share model on the assumption that the technology consists of a single technique, so that it could be described by a Leontief production function. What if the technology consists of a continuum of techniques described by a smooth production function? In this case the assumption of labor-saving technical change must be translated into the assumption of pure labor-augmenting (Harrod-neutral) technical change that affects all the techniques of production uniformly. In this case the production function in year t can be written as:

$$X = F(K, N) = F_0(K, (1+\gamma)^t N)$$

where $F_0(K, N)$ describes the technology in a base year. Dividing through by effective labor, $(1+\gamma)^t N$, we see that the intensive production function is constant in effective labor units:

$$\tilde{x} = F(\tilde{k}, 1) = F_0(\tilde{k}, 1)$$

Thus we can define the effective labor intensive production function:

$$\tilde{x} = f(\tilde{k}) \equiv F_0(\tilde{k}, 1)$$

A technique can be characterized in effective labor terms by its effective capital intensity, $\tilde{k}$, or, equivalently, by its capital productivity $\rho = \tilde{x}/\tilde{k} = x/k$, and its effective labor productivity $\tilde{x}$. Each technique of production

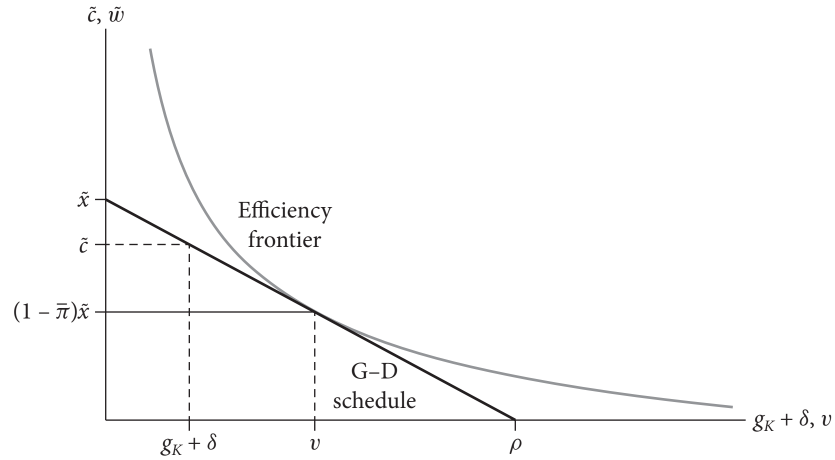

Figure 6.5 With pure labor-augmenting technical change the efficiency frontier defined in terms of the profit rate, v, and the effective wage, $\tilde{w}$, is constant over time. The effective wage, $\tilde{w}$, determines the profit-maximizing technique of production, and the effective growth-distribution schedule, including the effective labor productivity, $\tilde{x}$.

in effective labor terms corresponds to an effective real wage-profit rate schedule:

$$\tilde{w} = \tilde{x} - v\tilde{k}$$

Thus we can translate the whole growth-distribution schedule analysis, including the efficiency frontier, into effective labor terms by simply replacing the output, wage, and social consumption variables by their effective labor equivalents, as Figure 6.5, which is equivalent to Figure 3.4, illustrates.

For any effective wage, $\tilde{w}$, there will be a profit-maximizing technique, $(\tilde{k}(\tilde{w}), \tilde{x}(\tilde{w}))$, which determines the effective growth-distribution schedule for the economy, including the effective labor productivity $\tilde{x}$. Thus the Classical conventional wage model behaves in the same way with or without a choice of technique from a technology defined by a smooth production function. Only one profit-maximizing technique is ever used, and the others are irrelevant unless the effective wage changes.

For most production functions, it is possible to determine the profit-maximizing technique from the wage share as well as from the effective wage, because there will be a one-to-one correspondence between the wage (or profit) share and the effective wage. In the Cobb-Douglas case, however, the wage share is equal to $1 - \alpha$ for any effective wage, so that in this case it is necessary to specify the effective wage in order to close the model.

PROBLEM 6.7 Analyze the effect of an increase in β on the endogenous variables in the Classical conventional wage share model.

PROBLEM 6.8 Analyze the effect of an increase in the wage share on the choice of technique and endogenous variables in the Classical conventional wage share model.

6.5 A Classical Model of Growth with Full Employment

The alternative to the conventional wage share closure of the labor market is the *full employment* assumption that the wage is determined in each period so as to equate the demand for labor to a given supply of labor, which grows at an exogenously given rate, n, independent of the wage. We will also assume pure labor-saving technical change at the rate γ, so that k and x both grow steadily at the rate γ, and $\tilde{k}$ and $\tilde{x}$ are constant. Thus the effective labor supply grows at the rate $n + \gamma$, which is called the *natural rate of growth*. We will assume that the depreciation rate, δ, is unchanging.

The Classical full employment model, first analyzed by Luigi Pasinetti, has the same models of production and saving as the Classical conventional wage share model, but shares the full employment model of the labor market with the neoclassical growth model we will analyze in later chapters. Thus the growth-distribution schedule and the Classical growth rate–profit rate relation (Cambridge equation), equations (6.15)–(6.17), continue to hold in the Classical full employment model.

The labor market equation, however, is different. The full employment theory of growth assumes that the supply of labor grows exogenously independently of the wage. In this case we must drop the assumption of a given conventional wage share, and substitute instead the assumption that the wage adjusts to employ the given labor force. The supply of labor in each year follows the path:

$$N^s = N_0(1 + n)^t$$

As we have seen in Chapter 4, it might not be possible for the economy to achieve full employment in any particular year, if there is only one technique of production, because the accumulated capital may not offer the right number of jobs. If the number of jobs offered by the accumulated capital is smaller than the labor force, there will be unemployment of labor. If, on the other hand, the number of workers required to employ the accumulated capital is larger than the labor force, some of the capital stock will be unemployed.

If there is unemployment of labor, the wage in the period will fall to zero. If there is unemployment of capital, the real wage will rise to equal output per worker, x. Figure 4.8 illustrates the labor market equilibrium under these conditions.

If the economy manages to provide exactly the number of jobs necessary to reach full employment in one year, so that:

$$\frac{K}{k} = N^s$$

it will achieve full employment in the next year only if:

$$N^s_{+1} = (1+n)N^s = (1+n)\frac{K}{k} = \frac{K_{+1}}{k_{+1}} = \frac{(1+g_K)K}{(1+\gamma)k}$$

or, when n and γ are small, so that $n\gamma$ can be neglected:

$$1 + g_K = (1+n)(1+\gamma) \approx 1 + n + \gamma \tag{6.27}$$

The maintenance of full employment requires that the rate of growth of the capital stock equals the natural rate of growth, $n + \gamma$, the sum of the growth rates of the population and labor productivity, which supplies the fourth equation necessary to close the growth model.

The four equations that make up the Classical model with full employment are summarized in Table 6.5.

Figure 6.6 shows how the full employment model works graphically. In this figure output, wages, and consumption are measured per *effective* worker.

A Classical full employment economy reaches equilibrium through the impact of profits on the accumulation of capital. Suppose the wage were

Table 6.5 The Classical Full Employment Model

Endogenous variables: $\tilde{w}, v, \tilde{c}, g_K$
Exogenous parameters: $\tilde{k}, \tilde{x}, \delta, \beta, n, \gamma$

$$\tilde{w} = \tilde{x} - v\tilde{k} \tag{6.28}$$

$$\tilde{c} = \tilde{x} - (g_K + \delta)\tilde{k} \tag{6.29}$$

$$g_K + \delta = \beta v - (1-\beta)(1-\delta) \tag{6.30}$$

$$1 + g_K = (1+n)(1+\gamma) \tag{6.31}$$

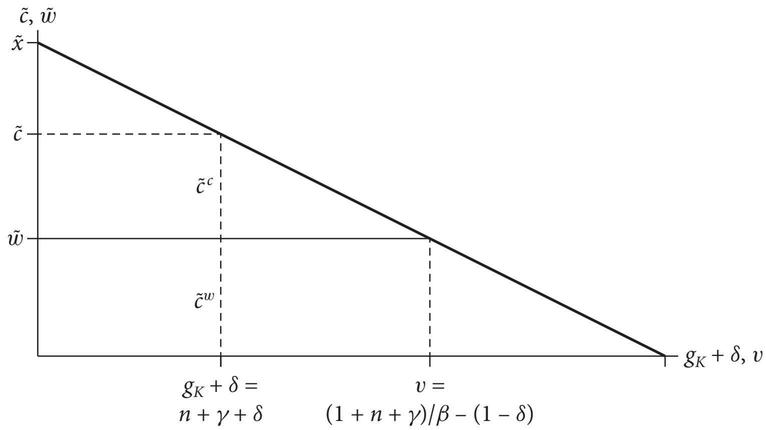

Figure 6.6 In the Classical full employment model the capital growth rate, g_K, is equal to the natural rate of growth $n + \gamma$. The Cambridge equation then determines the profit rate, and the growth-distribution schedule determines the wage and social consumption per effective worker.

lower than the equilibrium level. Then profits would be higher, and the capital stock would grow more rapidly than the labor force, creating an upward pressure on the wage. Similarly, if the wage were higher than the equilibrium level, capitalist saving out of profits would lead to a growth of capital that fell short of the growth of the labor force, and unemployment would emerge, pushing down the wage. Thus full employment in the labor market indirectly determines the profit rate and the real wage in this type of model.

The Classical mechanism for achieving full employment contrasts with the neoclassical mechanism that we studied in Section 4.4. In the neoclassical full employment model, changes in the wage create changes in the technique of production. If the wage were lower than the equilibrium level, firms would choose a technique that is too labor intensive, creating an excess demand for labor since more workers would be needed to operate the technique. This would generate upward pressure on the wage until firms chose the technique that just matched the existing supply of workers to the number of workers needed for that technique. Similarly, if wages were higher than the equilibrium level, the same mechanism would create pressure to reduce wages in order to employ more workers. These alternative interpretations of full employment are expressing deeper scientific differences about the nature of capitalist economies.

In the Classical full employment model the growth rate of the capital stock and of output are determined by the exogenously given natural rate of growth. The growth of the labor force, rather than being endogenously determined, as in the Classical conventional wage share model, poses an external limit to the capital accumulation process. Growth, rather than being endogenous, as in the Classical conventional wage share model, is exogenous, ultimately imposed on the economy by independent demographic behavior. Thus changes in capitalist saving behavior cannot influence the rate of growth of the Classical full employment model: such changes will be offset by changes in the wage, leaving the growth rate unaffected.

PROBLEM 6.9 Analyze the effect of an increase in the population growth rate, n, on the growth rate, profit rate, wage, social consumption, and capitalist consumption per worker in the Classical full employment model.

PROBLEM 6.10 Analyze the effect of an increase in the capitalist propensity to save, β, on the growth rate, profit rate, wage, social consumption, and capitalist consumption per worker in the Classical full employment model.

PROBLEM 6.11 Analyze the effect of a rise in the capital labor ratio, k, on the growth rate, profit rate, wage, social consumption, and capitalist consumption per worker in the Classical full employment model.

PROBLEM 6.12 Analyze the effect of a rise in the productivity of effective labor, $\tilde{x}$, on the growth rate, profit rate, wage, social consumption, and capitalist consumption per worker in the Classical full employment model.

6.6 Choice of Technique in the Classical Full Employment Model

Suppose now that the technology provides a choice of techniques in the Classical full employment model.

Since we continue to maintain the assumption that capitalists save a given fraction β of their end-of-period wealth, the Cambridge equation must hold, and if we assume full employment, we have the two relations:

$$(1+n)(1+\gamma) = 1 + g_K = \beta(1+r) = \beta(1+v-\delta)$$

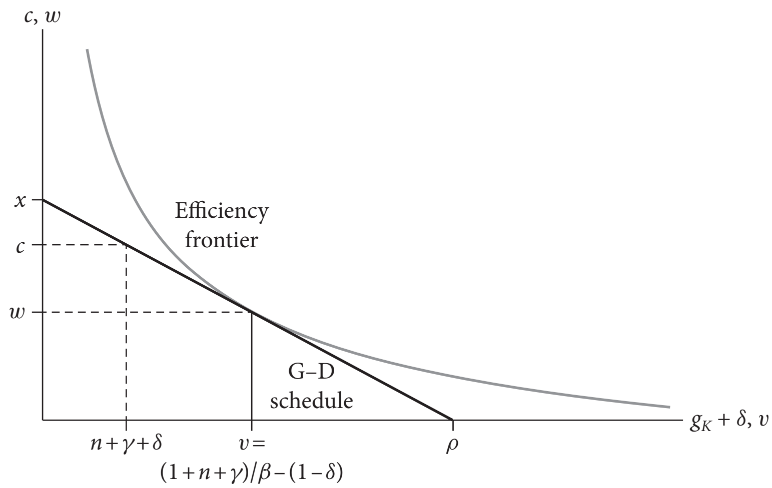

Figure 6.7 When there is a choice of technique in the full employment model, the natural rate of growth determines the rate of growth of capital, and hence the profit rate through the Cambridge equation. There will be one technique that maximizes real wages at this profit rate, and competition will force capitalists to choose it. Then the real wage is determined by the real wage-profit relation corresponding to the chosen technique.

Thus the rate of profit is still determined by the natural rate of growth and the capitalists' propensity to save. As we have seen, this rate of profit will correspond to a particular technique, which will be tangent to the efficiency frontier at the given rate of profit, as in Figure 6.7.

The fact that the chosen technique is on the efficiency frontier for the given rate of profit can also be expressed by saying that the marginal product of capital (if it exists) is equal to the profit rate at this point. But it is clear from the reasoning that it is the profit rate that determines the marginal product of capital in this model, not the other way around. Just as in the model with only one technique, changes in the wage will bring about an equilibrium, in this case through affecting both the choice of technique and the profit rate.

PROBLEM 6.13 If $n = .02$, $\gamma = 0$, $\delta = 0$, and $\beta = .9$, find the equilibrium real wage, profit rate, capitalist consumption, and technique in use in a Classical full employment economy with a Cobb-Douglas production function, where the techniques satisfy $\tilde{x} = A\tilde{k}^{\alpha}$, where $\alpha = .2$ and $A =$ (\$10,000/worker)$^{1-\alpha}$. What is the equilibrium marginal productivity of capital?

PROBLEM 6.14 What effect does an increase in the population growth rate have on the technique in use and the profit rate in a Classical full employment economy with choice of technique?

6.7 Growth and Cycles

One of the key aspects of the Classical vision, especially prominent in Marx's analysis, is that economic growth is far from being a harmonious process. Quite the opposite, growth is inherently unstable. Marx emphasized the role of *crises* in capitalist economies, arguing however that crises play a role in removing short run obstacles to continuing capital accumulation. He viewed the economy as cyclically alternating between expansions and recessions around its long run growth trend. Recessions and expansions are reflected in the state of the labor market: unemployment increases during recessions, and decreases during expansions.

In particular, Marx viewed the interaction between the size of the *reserve army* of labor and the distribution of income as the main indicator of the cyclical nature of economic growth. If the profit share increases, so will investment and capital accumulation, as we know from the Cambridge equation. Industrial production will require hiring more workers to keep pace with the increased size of the capital stock, and firms will seek to attract workers by offering higher real wages. The economy is expanding: the size of the reserve army is reduced, and the higher wages result in an increase in the labor share, at the expense of the share of profits.

As the profit share decreases, however, the pace of accumulation will also slow down. Progressively, firms will lay workers off, and the remaining workers will accept a reduction in the real wage in order to maintain being employed. The economy is now contracting: the size of the reserve army increases, and real wages are declining. Both these features are typically seen in recessionary times. However, lower real wages reduce the labor share and increase the profit share, thus providing a jump start to new accumulation. At this point, the cycle can repeat itself. Thus, the distributive conflict between capital and labor is never solved, because it is inherently intertwined with their mutual need for one another.

The late American economist Richard Goodwin worked out a model describing Marx's cyclical view of the growth process. The model outlines the evolution of the employment rate and the profit share, and can be seen as combining the Classical conventional wage share model with the Classical full employment model. Goodwin assumed a Leontief production function, zero depreciation, purely Harrod-neutral technical change, and a constant growth rate of the labor supply: $N^s_{+1} = (1+n)N^s$.

We define e, the *employment rate* at time t, as the ratio of labor demand N over labor supply N^s. Because the production function is Leontief, we know that, at time $t+1$,

$$e_{+1} = \frac{N_{+1}}{N^s_{+1}} = \frac{\rho K_{+1}}{x N^s_{+1}} = \frac{\rho(1+g_K)K}{(1+\gamma)x(1+n)N^s} = \frac{(1+g_K)}{(1+\gamma)(1+n)}e \quad (6.32)$$

For small values of labor productivity growth γ and labor supply growth n, their product γn can be neglected, and the denominator can be approximated by $1+\gamma+n$ as in equation (6.27).

From the Cambridge equation,

$$1 + g_K = \beta(1+r) = \beta(1+v-\delta) = \beta(1+\rho\pi)$$

(remember that depreciation is assumed to be zero). Once plugged into (6.32), the last equation establishes that employment next period increases with the current profit share. Regarding the latter, we have at time $t+1$:

$$\pi_{+1} = 1 - \frac{w_{+1}}{x_{+1}} = 1 - \frac{w_{+1}}{(1+\gamma)x}$$

Goodwin assumed that the real wage grows with employment, in line with the notion of a Phillips curve:

$$w_{+1} = [1 + h(e)]w \quad (6.33)$$

where the function $h(e)$ is defined as:

$$h(e) = -\xi + \mu e$$

which is a straight line with negative intercept $-\xi$ and positive slope μ. The intuition behind this equation is simple. The parameter μ captures the effect of an increase in the employment rate at time t on the real wage at time $t+1$. It can be seen as describing the *bargaining power* of workers in the labor market. The negative intercept tells us that for zero employment wages must decline.

After a little bit of algebra (see Problem 6.15), we can then write a dynamic equation describing the evolution of the profit share as follows:

$$\pi_{+1} = \pi + (1-\pi)\frac{1}{1+\gamma}[\gamma - h(e)] \tag{6.34}$$

which shows that, since the real wage grows following an increase in employment, the profit share will decrease next period.

PROBLEM 6.15 Using the fact that $\pi = 1 - w/x$, derive equation (6.34).

Equations (6.32) and (6.34) make up a *dynamical system.* In studying such a dynamical system, we are interested in (i) determining its *steady state*, that is calculating values for the employment rate and the profit share such that both are constant over time; and (ii) finding out whether the steady state is *stable* or unstable, meaning whether the profit share and employment rate move back to their steady state after a perturbation occurs.

At a steady state, employment is constant and therefore $e_{+1} = e = \bar{e}$. This gives

$$1 + g_K = 1 + \gamma + n$$

which is basically the full employment closure (6.31). On the other hand, using the Cambridge equation we can solve for the steady state value of the profit share as:

$$1 - \bar{\pi} = 1 - \frac{1+\gamma+n-\beta}{\rho\beta} \tag{6.35}$$

which is independent of employment, and only depends on exogenous parameters. Thus, we have the interesting feature that steady employment in the Goodwin model requires a constant wage share, just like in the conventional wage share model. In Marxian terms, the conventional wage share can be interpreted as the particular value of the labor share such that the reserve army of labor (which is just one minus the employment rate) is constant.

Similarly, setting $\pi_{+1} = \pi = \bar{\pi}$ in equation (6.32), we obtain the steady state value of the employment rate as:

$$\bar{e} = \frac{\xi + \gamma}{\mu} \tag{6.36}$$

Table 6.6 Steady State of the Goodwin Model

Endogenous variables: π, e, v, s, g_K
Exogenous parameters: $\rho, \beta, \xi, \mu, \gamma, n$

$$v = \pi\rho \tag{6.37}$$

$$g_K + \delta = s\rho \tag{6.38}$$

$$g_K + \delta = \beta v - (1-\beta)(1-\delta) \tag{6.39}$$

$$\pi = \bar{\pi} = \frac{1+\gamma+n-\beta}{\beta\rho} \tag{6.40}$$

$$e = \bar{e} = \frac{\xi+\gamma}{\mu} \tag{6.41}$$

independent of income distribution just like in the Classical full employment model. Table 6.6 summarizes the steady state of the Goodwin model.

PROBLEM 6.16 With $\gamma = .02 = n$, $\beta = .9$, $\mu = .73$, $\rho = .4$, calculate the steady state wage share and corresponding profit share, and the value of the intercept parameter ξ that ensures that the steady state employment rate is 94%.

PROBLEM 6.17 What is the effect of an increase in the capitalist propensity to save β on the steady state wage share of the Goodwin model? Can workers use a higher bargaining power μ in order to increase the steady state wage share in this model?

Is the steady state of the Goodwin model stable? That is, suppose that, starting from the steady state, a shock in the economy occurs so that the employment rate and the labor share are not in equilibrium anymore. Will they get back to their steady state values? It turns out that they will not: any perturbation to the steady state will be self-reinforcing, as opposed to self-correcting. In this respect, the model is able to reproduce the Marxian insight regarding the instability of the growth process. Yet it also turns out that the unstable behavior of income distribution and employment occurs within a bounded region. This is because neither employment nor income shares can grow above one or decrease below zero. The implication is that distribution and employment evolve in cyclical fashion, where economic

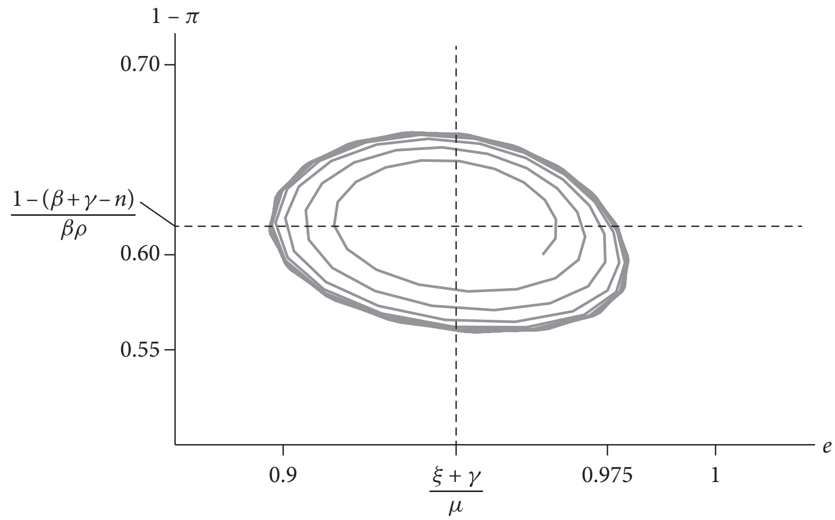

Figure 6.8 Given an initial condition, employment rate and the labor share move away from the steady state until they reach a limit cycle, that is, the thicker region in the graph. At this point, oscillations become perpetual, although bounded. This simulation plot is obtained using the parameters found in Problem 6.16.

expansions alternate with recessions, without ever reaching their long run steady state values. Thus, the Goodwin model also reproduces the never-ending conflict, and the symbiotic relationship between capital and labor, described by Marx.

Figure 6.8 shows cycles around the steady state of the Goodwin model. It can be noted that, given an initial condition, the employment rate and the wage share move away from the steady state in counterclockwise fashion. Mathematically, we say that the dynamics of the model displays a *limit cycle*: although the steady state is unstable, both the employment rate and the labor share are bounded above. Thus, their evolution is trapped within (that is, in the limit it tends to) a certain region, which will delimit the oscillations. Notice also that, once the limit cycle is reached, oscillations will never cease to happen.

PROBLEM 6.18 Suppose that $\gamma = .02 = n$, $\beta = .9$, $\mu = .73$, $\rho = .4$, $\xi = .6862$. Suppose further that the employment rate at time zero is 93%, while the profit share at time zero is 30%. Calculate $(e_{+1,}, \pi_{+1})$, (e_{+2}, π_{+2}), (e_{+3}, π_{+3}).

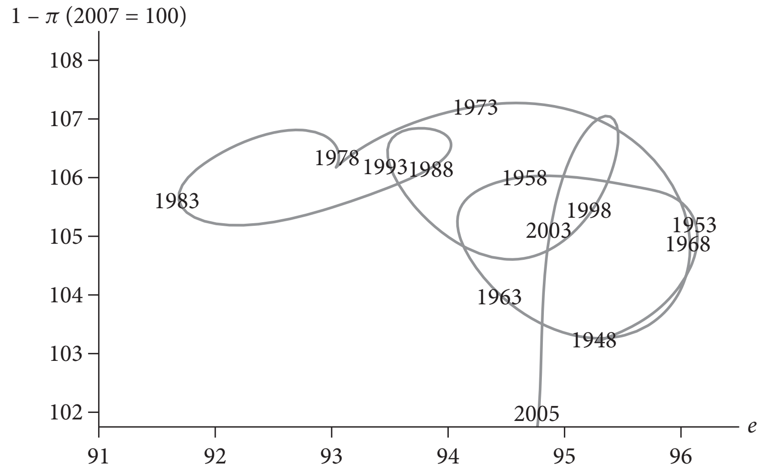

Figure 6.9 Cycles in the employment rate and the labor share in the nonfinancial business sector (2007=100) in the US, 1948–2005. Source: Federal Reserve Economic Database, quarterly data.

6.7.1 Employment-distribution cycles in the US

Figure 6.9 displays counterclockwise loops in the labor share of the nonfinancial business sector and the employment rate in the US for the period 1948–2005. The direction of motion is roughly in line with the dynamics implied by the Goodwin model, even though the cycles shift over time.

6.8 The Classical Approach to Growth

Smith, Malthus, Ricardo, and Marx, the main thinkers who developed the Classical approach to growth, saw class divisions between capitalists and workers as the central drama of capitalist economies. Capitalist accumulation, driven by competition, is the engine of growth. The consumption of workers poses limits to growth by restricting the proportion of output available for accumulation. Capital accumulation increases the demand for labor and induces population growth so that the population itself is endogenous to the economic growth process. The growth rates of capital and population are determined primarily by the class distribution of income. The Classical conventional wage share model developed in this chapter reflects the central preoccupations of the Classical political economists.

The Classical full employment model retains the class structure of the Classical vision, but tames it considerably by regarding population growth, rather than the class distribution of income, as the factor ultimately limiting growth. In this model, class distribution, rather than population growth, adjusts to equilibrate the labor market, and becomes endogenous.

The Goodwin model combines elements of both the conventional wage share model and the full employment model. Its emphasis is on the cyclical nature of the interaction between economic growth and the distribution of income.

The main empirical challenge to the Classical growth models is that rates of growth and rates of profit often tend to decline over time in real capitalist economies, while the Classical models in this Chapter predict constant growth and profit rates. To address this problem we have to look more closely at the process of technical change.

6.9 Suggested Readings

The Classical model of growth has led a lively if somewhat subterranean existence during this century in the writings of Marxian political economists such as Luxemburg (1951) and Sweezy (1949). For an introduction to Marxian political economy, consult Foley (1986). The great mathematician John von Neumann developed an essentially Classical model, and although his seminal contribution (von Neumann 1945) is difficult, an accessible exposition of it (and the Classical model in general) can be found in Gram and Walsh (1980). Richard Goodwin's pioneering growth cycle (Goodwin 1967), another important contribution, uses the mathematics of biological predator-prey systems to depict an economy with a growing labor force whose unemployment rate cycles around an average rate. For empirical evidence of the Goodwin cycle, see Mohun and Veneziani (2008) and Grasselli and Maheswari (2017). Luigi Pasinetti has been a major force in extending the Classical approach; see Pasinetti (1977); Pasinetti (1974) for examples.

7

Induced Technical Change, Growth, and Cycles

7.1 The Induced Invention Hypothesis

The Classical models of growth and distribution we studied in Chapter 6 assumed technical change to be exogenous and purely Harrod-neutral, with labor productivity growing but capital productivity remaining constant over time. The conventional wage share model was closed by an exogenously given wage share.

While these assumptions are consistent with the stylized facts of long run growth in advanced capitalist economies, it would be desirable to provide an economic explanation of the reason *why* technical change takes the Harrod-neutral form (that is, is biased toward labor), as well as why there are economic forces that push income shares toward being constant in the long run.

With regard to the latter issue, the Goodwin model provided a rationale for the constancy of the wage share in the long run through its formalization of the Marxian notion of a reserve army of labor. The wage share increases with employment, but decreases with labor productivity growth. In the long run, these two effects offset each other: as we have seen, the steady state of the Goodwin model involves a constant share of wages independent of labor market conditions, just like the conventional wage share model. According to the Goodwin model, the constancy of the wage share in the long run ensures that the size of the reserve army remains constant. However, since the Goodwin steady state is unstable, the economy endlessly cycles around its long run growth path, and the constancy of the wage share is actually never achieved. Yet a constant wage share, as well as a constant employment rate, emerges once we average out fluctuations occurring at the business cycle frequency.

It then remains to be explained why technical change is labor-biased. While it is possible that technical change takes a biased form more or less spontaneously, economists have long suspected that specific characteristics of capitalist society, such as the large share of wages in total costs, impart a labor-saving bias to technical change. Marx believed that by making labor more expensive, the nineteenth-century Factory Acts in Britain stimulated the discovery of labor-saving machinery. The theory of induced technical change provides a lens through which to look at the empirical phenomenon of capital-labor substitution. The basic idea is simple: as the share of one factor of production in a firm's costs increases, the firm will have an incentive to economize more on that factor of production.

A simple model that illustrates the basic idea of induced technical change begins by assuming that firms are presented with a menu of possible technical changes, which takes the form of a concave technical progress function (also known as the *invention possibility frontier*):

$$\gamma = f(\chi)$$

If, as seems reasonable, a greater saving on labor comes at the price of lesser saving on capital inputs (or even, at some point, an increase in capital inputs), this function will slope downward and have a negative derivative, written $f'(\chi) < 0$. If larger saving in labor requires proportionately larger sacrifices of capital saving, then the function will be concave and have a negative second derivative, written $f'' < 0$. We assume that the technical progress function has both these mathematical properties in Figure 7.1.

This simple technical progress function can actually encompass many different configurations of technical change. For instance, a purely Harrod-neutral path of technical change emerges at the point where the invention possibility frontier intersects the vertical axis in Figure 7.1. There, capital productivity growth is zero, and labor productivity growth is positive. On the other hand, it is not hard to imagine a path along which some amount of labor-saving requires an actual increase in capital requirements. This would be the case with labor-saving, capital-using (or Marx-biased) technical changes, which is the focus of Chapter 8. Finally, it should also be possible to imagine capital-saving (Solow-neutral) technical changes.

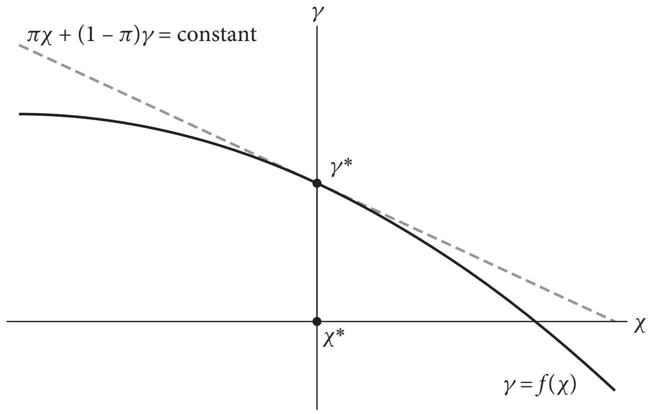

Figure 7.1 Given a technical progress function, $\gamma = f(\chi)$, represented by the heavy curve, profit-maximizing entrepreneurs will choose the pattern of technical change, (χ^*, γ^*), that leads to the largest reduction in costs, $\pi\chi + (1 - \pi)\gamma$. The profit-maximizing pattern occurs at the point where the technical progress function is tangent to the isocost-reducing locus, represented by the dashed line. In this case, the emerging pattern of technical change is Harrod-neutral.

According to the theory of induced technical change, the bias of technical change results from the response by capitalists to economic incentives. Each individual manager will seek to reduce the cost of producing one unit of output as fast as possible, taking as given the wage and rental rate of capital. This is just another way of saying that the manager seeks to maximize profits for the capitalists who own the firm. Costs consist of the wages paid to her workers, labor costs, and rents paid to the owners of the firm, capital costs. Since the firm operates in a competitive environment, it will in general start out with a profit rate equal to the average profit rate. As a result, the profit share represents the share of capital costs in total costs, and the wage share represents the share of labor costs in total costs. The rate of reduction of costs per unit of output produced is given by the expression $(1 - \pi)\gamma + \pi\chi$.

The planning problem for the entrepreneur-manager of the firm can thus be expressed as a one-period choice problem:

$$\begin{aligned}&\text{Choose } \chi \text{ to maximize } (1-\pi)\gamma + \pi\chi\\&\quad\text{subject to } \gamma = f(\chi)\\&\quad\text{where } f' < 0,\ f'' < 0\\&\quad\text{given } \pi\end{aligned}$$

Substituting the constraint into the objective function, we see that the entrepreneur wants to maximize:

$$(1-\pi)f(\chi)+\pi\chi$$

The first-order condition for a maximum is:

$$(1-\pi)f'(\chi)+\pi=0$$

The solution value for χ clearly must satisfy:

$$f'(\chi)=-\frac{\pi}{(1-\pi)} \tag{7.1}$$

If the technical progress function is well-behaved, then the inverse of $f'(\chi)$ will exist. Thus, we can solve the above equation for the growth rate of capital productivity χ as a function of income shares. We can thus write: $\chi=\chi(\pi)$, with $\chi'(\pi)>0$.

It is not hard to verify that, because the growth rate of capital productivity is increasing in the profit share, the growth rate of labor productivity will be increasing in the wage share. Using the technical progress function again, we can emphasize the dependence of γ on income shares as follows:

$$\gamma(\pi)=f(\chi(\pi))$$

Because of the properties of the invention possibility frontier, the growth rate of labor productivity will decrease with the profit share and increase with the wage share. Thus, this model delivers a relationship between income distribution and specific patterns of technical change. Remember that the wage share $1-\pi$ is defined as the ratio of real wage-to-labor productivity w/x. An increase in the share of wages in firms' costs means that wages are rising faster than labor productivity. According to the induced technical change hypothesis, if this is the case firms will have an incentive to increase the productivity of labor so as to maintain the labor share in check, or even reduce it. Vice versa, if the profit share were larger, that would create a greater incentive to save on capital and the solution value for χ would increase.

In solving their planning problem, entrepreneurs take the going income shares as given. At a solution to the problem, the ratio of income shares will be equal to the negative of the slope of the invention possibility frontier. One possible solution is represented in Figure 7.1, which indeed shows the first-order condition as the point where the tangent to the technical progress function has the slope $-\pi/(1-\pi)$. In this particular case, the solution occurs at a value of the profit share such that $f'(\chi)$ crosses the vertical axis: the corresponding value of χ will be zero, signifying purely Harrod-neutral technical change. The solution value for γ can then be read off the vertical axis.

PROBLEM 7.1 Suppose that the technical progress function takes a quadratic form: $\gamma = -\chi^2 - .52\chi + .0149$. Find the optimum rate of capital-saving (or -using) technical change when the profit share is 1/3. Find the optimum rate when the profit share is 1/4. Compare your answers.

PROBLEM 7.2 Using the same technical progress function above, find an expression for the growth rate of labor productivity γ as a function of the wage share $1 - \pi$.

PROBLEM 7.3 Using the data in Problem 7.1, find the rate of increase in the real wage under the conventional wage share assumption when the profit share is 1/3. Find the rate when the profit share is 1/4. Compare your answers.

7.2 Induced Technical Change in the Classical Full Employment Model

We can combine the above firm-level model of induced technical change with the Classical model with a full employment closure. In such a model, instantaneous changes in the profit share will ensure that the economy remains on a Harrod-neutral path of technical change.

We know that steady state growth requires the growth rate of capital productivity χ to be zero; therefore, using the technical progress function we find $\gamma = f(0)$. We can then use equation (7.1) to solve for the profit share as follows:

$$f'(0) = -\frac{\pi}{1-\pi}, \text{ which we can write as } \pi = -\frac{f'(0)}{1-f'(0)}$$

The above expression means that we evaluate the derivative of the invention possibility frontier $f(\chi)$ at the point where it intercepts the vertical axis, and then we use the first-order condition in the firm's planning problem in order to find the slope of $f(\chi)$ at that point.

PROBLEM 7.4 Using the invention possibility frontier function above, solve for the profit share and the wage share corresponding to a Harrod-neutral path of technical change.

Table 7.1 The Classical Full Employment Model with Induced Technical Change

Endogenous variables: $\tilde{w}, v, \tilde{c}, g_K, \gamma, \pi$

Exogenous parameters and functions: $x, \rho, \delta, \beta, n, f(\chi)$

$$\tilde{w} = \frac{x}{(1+\gamma)^t}\left(1 - \frac{v}{\rho}\right) \tag{7.2}$$

$$\tilde{c} = \frac{x}{(1+\gamma)^t}\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{7.3}$$

$$g_K + \delta = \beta v - (1-\beta)(1-\delta) \tag{7.4}$$

$$1 + g_K = (1+n)(1+\gamma) \tag{7.5}$$

$$\pi = -\frac{f'(0)}{1 - f'(0)} \tag{7.6}$$

$$\gamma = f(0) \tag{7.7}$$

The implication of this result is that both the profit share and the wage share are endogenous, and depend on the features of the invention possibility frontier alone. In contrast to the full employment model presented in Chapter 6, however, the bias of technical change here is endogenous: the profit share adjusts so as to maintain a constant growth rate of labor productivity through induced technical change. Given labor productivity growth, the full employment closure determines the growth rate. Given the growth rate, we can solve for social consumption per worker, and recover the profit rate from the Cambridge equation. The real wage will be determined from the real wage-profit relation. Table 7.1 illustrates the model.

PROBLEM 7.5 Consider the Classical full employment model with induced technical change, and suppose that the labor force is constant. Use the technical progress function in the previous problems. Characterize the steady state equilibrium with induced technical change by calculating the values of γ, χ, g_K, g_X, and π.

7.3 Growth Cycles with Induced Technical Change

The theory of induced technical change can be fruitfully incorporated into the Goodwin model of cyclical growth. From a Classical political economy point of view, the ability to change the technique in use in response to changes in income shares gives capitalist firms an additional “weapon” in the conflict over the distribution of income that is at the heart of the growth cycle.

In the Goodwin model, the assumption of exogenous Harrod-neutral technical change made the bargaining position of workers and capitalists perfectly symmetrical: higher profits translated into further accumulation of capital stock and therefore faster growth. Growth, in turn, puts pressure on the labor market, thus increasing the real wage and lowering the profit rate. We have seen that, as a result, the economy reached a limit cycle where conflict was never settled.

Conversely, with induced technical change firms do not have to passively accept the increase in the real wage that occurs as a byproduct of accumulation. When the wage share increases, capitalist firms can turn to more labor-saving techniques of production. This possibility was precluded under an exogenously given growth rate of labor productivity. In this sense, induced technical change breaks the symmetry in bargaining positions that produced the growth cycle. As a result, the conflict over income distribution will be settled in the long run, and the economy will actually converge to a steady state growth path involving constant income shares, a constant employment rate, and a Harrod-neutral profile of technical change. The convergence toward the steady state, however, will be cyclical.

We thus modify the Goodwin model of Chapter 6 by introducing the following dynamic equations describing the evolution of labor and capital productivity over time:

$$x_{+1} = (1 + \gamma(\pi))x \tag{7.8}$$

$$\rho_{+1} = (1 + \chi(\pi))\rho \tag{7.9}$$

The dynamic equation describing the evolution of the employment rate over time (refer back to Chapter 6 for the derivation) is

$$e_{+1} = \left(\frac{\chi(\pi) + \beta(1 + \pi\rho)}{1 + \gamma(\pi) + n} \right) e \tag{7.10}$$

Finally, the evolution of the profit share is

$$\pi_{+1} - \pi = \left(\frac{\gamma(\pi) - h(e)}{1 + \gamma(\pi)} \right)(1 - \pi) \tag{7.11}$$

The dynamical system is now formed by equations (7.9), (7.10), and (7.11). We proceed by first characterizing the steady state. Setting $\rho_{+1} = \rho$

we have that $\chi(\pi) = 0$, which gives the profit share as the solution to the inverse function

$$\bar{\pi} = \chi^{-1}(0)$$

In other words, the profit share adjusts in the long run so as to ensure a constant value for the productivity of capital stock, which is endogenous and to be determined just below. This is exactly the configuration represented in Figure 7.1 above. Given an initial condition on the profit share, the economy will move dynamically so as to ensure that, in the long run, the productivity of capital remains constant while labor productivity grows at a constant rate.

Once $\bar{\pi}$ is found, the long run growth rate of labor productivity will follow as $\gamma(\bar{\pi}) = \gamma(\chi^{-1}(0))$. Next, by setting $e_{+1} = e$ in equation (7.10) we can solve for the steady state value of capital productivity as

$$\bar{\rho} = \frac{1 + \gamma(\bar{\pi}) + n - \beta}{\beta\bar{\pi}}$$

Finally, a constant share of profits in equation (7.11), ruling out the uninteresting case $\bar{\pi} = 1$, requires the employment rate to solve the equation $h(e) = \gamma(\bar{\pi})$. For instance, if as we did in Chapter 6 we assume a simple linear Phillips curve $h(e) = -\xi + \mu e$, then the steady state employment rate is

$$\bar{e} = \frac{\xi + \gamma(\bar{\pi})}{\mu}$$

Thus, the steady state of the growth cycle model with induced technical change adds the long run values of the employment rate and of capital productivity as additional endogenous variables to the full employment version presented above. In this model, income shares do not adjust instantaneously, but *evolve* over time in order to ensure a Harrod-neutral profile of technical change. Once labor productivity growth is determined through the invention possibility frontier, the long run employment rate follows through the real wage Phillips curve $h(e)$, and the long run value of capital productivity is found given the saving propensity β, the long run profit share $\bar{\pi}$, and the growth rate of population n. As Figure 7.2 shows, the employment rate and the wage share converge cyclically to their long run values.

PROBLEM 7.6 Suppose that: (i) $h(e) = -.665 + .7e$; (ii) $\chi(\pi) = .02 - .03(1 - \pi)$; (iii) $\gamma(\pi) = .3(1 - \pi)$; (iv) $\beta = .9$. Calculate the steady state of the Goodwin model with induced technical change.

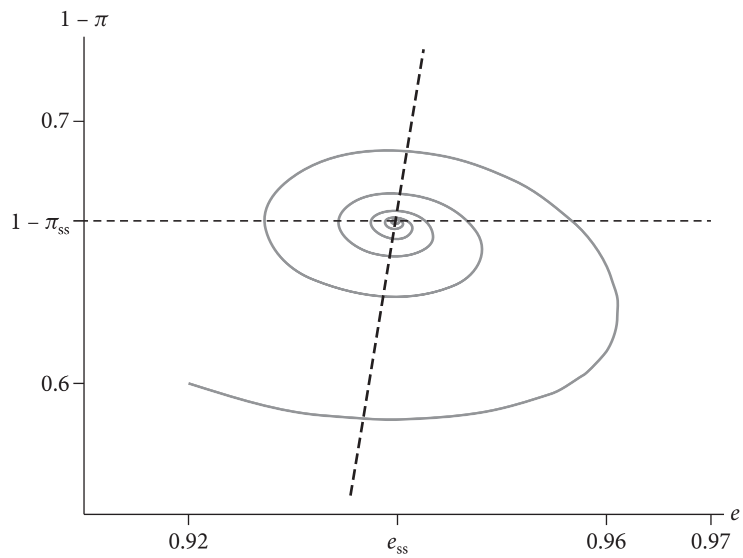

Figure 7.2 Because of induced technical change, the Goodwin oscillations in the wage share and the employment rate become smaller and smaller, until convergence to the steady state is eventually achieved.

7.4 Comparative Dynamics

The presence of induced technical change not only changes the dynamical properties of the growth cycle, as Figure 7.2 shows. It also affects the comparative dynamics of the steady state of the model.

First, in the basic Goodwin model (see Problem 6.17), the long run value of the wage share was directly related to the capitalists' propensity to save: an increase in β lowered the profit share, thus increasing the wage share. Here, instead, the long run value of income shares is entirely pinned down by the invention possibility frontier, and therefore it is invariant to changes in the capitalists' saving behavior.

Changes in the propensity to save, on the other hand, affect the steady state value of capital productivity. An increase in β determines higher capital accumulation, and this lowers the long run value of capital productivity $\bar{\rho}$. The mechanism is that an increase in the saving propensity would push the accumulation rate up in equation (7.4). However, since $\bar{\pi}$ is fixed (and therefore $\gamma(\bar{\pi})$ is, too), the employment closure (7.5) prevents the growth rate from increasing. Therefore, the profit rate v must fall in order to offset the increase in β. Recalling that $v = \rho\pi$, if the profit share is constant, the only way that a fall in the rate of profit can occur is through a decrease in capital productivity.

Once the invention possibility frontier has determined the distribution of income, the behavior of the steady state employment rate is very much similar to the basic Goodwin model: an increase in the Phillips curve parameter μ only lowers the steady state employment rate without changes in distribution (see again Problem 6.18).

7.5 Conclusions

The Classical growth cycle model with induced technical change provides a representation of the growth process that emphasizes the distributional conflict between capital and labor. Technological change emerges as a result of the struggle over income distribution, as capitalist firms are able to counter increases in the share of wages in costs through increases in labor productivity. The model describes the cyclical adjustments that occur before income shares settle into long run values that ensure a Harrod-neutral profile of technical change, and the employment rate reaches its long run value that maintains a constant size of the reserve army of labor.

One weakness of the theory of induced technical change is the absence of a counterpart to the technical progress function confronting firms in real economies. It seems more plausible to believe that the set of possible technical changes is stochastic or probabilistic. In this case, if we search for more labor-saving technical changes, the probability decreases of finding any that are not also more capital-using. However, it can be shown that even a probabilistic representation of technical change gives rise to something very similar to the invention possibility frontier described above. Thus, even though the interpretation of the emergence of a certain pattern of technical change is different, the mathematical structure of the firm's planning problem remains mostly unchanged.

A second, perhaps more critical weakness of the induced invention hypothesis regards the position of the invention possibility frontier on the (χ, γ) plane, which the firm takes as given in its planning problem. This is important, because the intercept of the technical progress function on the vertical axis determines the long run growth rate of labor productivity in the economy. In other words, firms can only choose a point along the given trade-off represented by the technical progress curve, but cannot, for example, divert resources from accumulation to research and development (R&D) in order to shift the curve up and attain an even higher growth rate in the long run. Thus, induced technical change can explain the bias of technical change in relation to income shares, but not the role of R&D investment in growth and income distribution. Addressing this issue is the focus of Chapter 9.

Even with these limitations, however, when introduced in the Classical growth model the theory of induced technical change provides an explanation of the phenomenon of capital-labor substitution that is alternative to the neoclassical production function. As such, the approach is not prone to the conceptual issues highlighted by the Cambridge capital controversy we discussed in Chapter 3.

7.6 Suggested Readings

The theory of induced technical change owes much to Kennedy (1964): Drandakis and Phelps (1966) provide a neoclassical analysis. For a model in which technical change is treated as a stochastic process, see Duménil and Lévy (1995). The presentation of the Goodwin model with induced technical change adapts the paper by Shah and Desai (1981). A clear exposition of the model presented in this chapter appears in Julius (2005), which elaborates on Foley (2003). Tavani (2012) presents a model where wage bargaining is explicit, while Tavani and Zamparelli (2015) study how R&D spending by capitalist firms affects the growth cycle. Van der Ploeg 1985 shows that substitution along a CES production function can also stabilize the growth cycle much like induced technical change. Finally, Acemoglu (2002) has applied the logic of directed technical change to endogenous growth models: a detailed account of this literature is provided in Acemoglu (2009).

8

Biased Technical Change in the Classical Model

The Classical model can be extended so that it matches more of the qualitative features of real economies. The historical record presented earlier in Table 2.8 shows that although labor productivity has improved persistently over the last two centuries, Harrod-neutral technical change has not always prevailed. In the US, for example, the periods from 1820–1913 and 1973–1992 were characterized by declining capital productivity. Sandwiched in between these periods was a span of rising capital productivity. Similarly, Japan witnessed two long periods of declining capital productivity from 1870–1950 and 1973–1992, with a period of near-neutrality sandwiched in between. Declining capital productivity, together with a roughly constant wage share, reduces the rate of profit, which in turn can slow down the accumulation of capital and the growth of output. We can understand these periods of declining profitability and slowing growth through the Classical model with Marx-biased technical change.

Marx-biased technical change is a mix of capital-using and labor-saving technical change. In this chapter we will assume that Marx-biased technical change occurs at constant rates, γ and χ. There are two ways in which this pattern can arise. First, technical change may tend to be inherently biased toward the mechanization and automation of the labor process as a reflection of the antagonistic social relations of production under capitalism. In this case, Marx-biased technical change arises exogenously during those historical periods when this tendency asserts itself.

Secondly, we can appeal to the theory of induced technical change developed in the previous chapter to explain Marx-biased technical change as an endogenous response to economic incentives. If the conventional wage

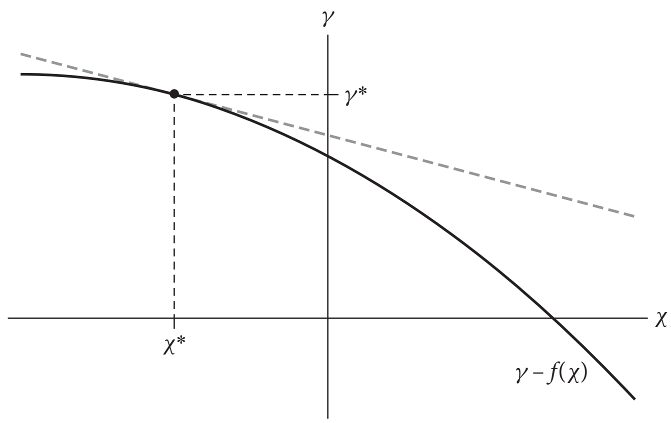

Figure 8.1 If the conventional wage share is large enough, entrepreneurs will choose a point on the technical progress function with capital-using, labor-saving technical change. This pattern is called Marx-biased technical change.

share is relatively high, it will induce firms to search aggressively for labor-saving innovations. Since these innovations are more likely to come at the expense of low or negative rates of capital-saving according to the theory of induced technical change, they can take a Marx-biased form. Figure 8.1 shows this configuration of parameters using the technical progress function developed in the previous chapter. We will consider both these explanations in this chapter.

8.1 The Classical Conventional Wage Share Model with Biased Technical Change

Marx-biased technical change can be included in the Classical conventional wage share model. The equations written out in Chapter 6 for purely labor-saving technical change remain valid for biased technical change, and we continue to maintain the conventional wage share assumption. The key difference is that with biased technical change, the economy never reaches a steady state because the net rate of profit changes over time, generating changes in the rates of capital accumulation and growth. We return to accounting in real workers rather than effective workers in order to emphasize the relation of the model to real-world growth-distribution schedules, and write the equations for the Classical conventional wage share model with biased technical change as in Table 8.1.

Table 8.1 The Classical Conventional Wage Share Model with Marx-Biased Technical Change

Endogenous variables: x, ρ, w, v, c, g_K
Exogenous parameters: $x_0, \rho_0, \delta, \beta, \bar{\pi}, \gamma, \chi$

$$x = x_0(1+\gamma)^t \tag{8.1}$$

$$\rho = \rho_0(1+\chi)^t \tag{8.2}$$

$$w = x\left(1 - \frac{v}{\rho}\right) \tag{8.3}$$

$$c = x\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{8.4}$$

$$\delta + g_K = \beta v - (1-\beta)(1-\delta) \tag{8.5}$$

$$w = (1-\bar{\pi})x \tag{8.6}$$

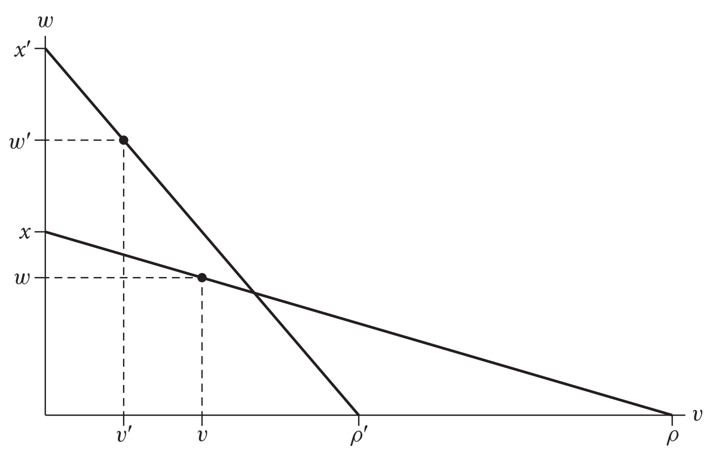

Figure 8.2 Marx-biased technical change raises labor productivity, x, lowers capital productivity, ρ, and hence rotates the growth-distribution schedule around a switchpoint in the positive quadrant. If the wage share, $1-\pi$, remains constant, the wage, w, rises proportionately to labor productivity, and the profit rate, v, must fall.

We can visualize the path of the economy under these assumptions in Figure 8.2, which shows the growth-distribution schedules for two adjacent time periods, t and $t+1$. The biased nature of technical change rotates the growth-distribution schedules around the switchpoint between the techniques. For clarity, Figure 8.2 omits social consumption and the growth rate of capital in order to focus attention on the profit rate and the wage rate. We can see that the profit rate has declined, and it is easy to show that this must be true, if the wage rises proportionately to the productivity of labor: since the profit rate is equal to $\pi\rho$, as capital productivity declines, the rate of profit will also decline in the absence of offsetting rises in the profit share.

We can work out the paths of social consumption and the rate of capital accumulation from the Classical conventional wage share model equations. Since the net rate of profit, r, declines, the rate of capital accumulation, $g_K = \beta(1 + r) - 1$, will also decline. The rate of growth of output, g_X, is equal to $g_K + \chi$, and with χ constant it will decline as well. The level of social consumption per worker increases, since workers' consumption, which is equal to the wage, has risen, and with some effort it can also be shown that capitalist consumption per worker has increased as well.

This pattern could not go on forever. Eventually, the rate of profit would fall so low that the economy would stop growing altogether. From the Cambridge equation, $1 + g_K = \beta(1 + r)$, we see that when the net rate of profit reaches $(1 - \beta)/\beta$, capital accumulation halts completely. If the profit rate fell farther, the rate of capital accumulation would become negative, and the capitalists would eat up the capital stock until it was exhausted. Real economies have never reached this point because the episodes of capital-using bias that underlie this pattern have not lasted indefinitely. As we have seen, periods of capital-using technical change in real capitalist economies eventually have given way to periods of stable or rising capital productivity.

PROBLEM 8.1 The Industrian economy (see Problem 2.2) now experiences biased technical change with $\gamma = 5\%$ per year and $\chi = -2\%$ per year. In the initial period, the wage is \$20,000 per worker-year, labor productivity is \$50,000 per worker-year, and capital productivity is $33\frac{1}{3}\%$ per year. What will be the rate of profit and the wage rate (per worker-year) in the next period? Compare the rate of profit in the next period to the rate of profit in the base year.

PROBLEM 8.2 Calculate the wage share and profit share in Industria. Show that the wage share did not change in the first year.

PROBLEM 8.3 Find the level of consumption per worker and the growth rate of capital in the Industrian economy for the base year and the first year, assuming $\beta = .97$.

PROBLEM 8.4 In how many years will the net rate of profit in Industria reach zero if there is no change in the rates of technical change? In how many years will the growth rate of capital reach zero?

8.2 Viability of Technical Change

Why would entrepreneurs introduce technologies that lower the rate of profit?

Individual entrepreneurs can choose their own technologies, but they cannot control the social forces that technological change sets in motion that result in increases in real wages. Each entrepreneur acts under competitive pressure to be the first to adopt a profit rate-increasing technology. Marx argues that this fact lies behind the technologically progressive character of capitalist production. As individual entrepreneurs race to adopt more profitable technologies, they raise labor productivity in the society as a whole, and set in motion forces that raise wages as well (in the conventional wage share model). The net result, if technical change takes a Marx-biased form, is a fall in the rate of profit. But capitalist entrepreneurs could avoid this outcome only through an agreement not to pursue their individual self-interests by pursuing profit-increasing technical changes. Such an agreement is impossible to enforce in advanced, highly competitive, capitalist economies.

Each individual entrepreneur decides whether to adopt a new technique based only on the private rate of profit that she anticipates. Techniques that raise the rate of profit at the current level of wages and prices are called *viable*. It generally takes some time before competitors catch up with an innovator, so that entrepreneurs are motivated to adopt viable techniques by the prospect of reaping temporary above-normal profits before other firms have time to catch up.

The fate of the average profit rate for the whole economy, however, depends on what happens to wages as labor productivity rises. Because the Classical tradition sees the conventional wage, rather than the supply of labor, as the exogenously given factor in the labor market, it can embrace the possibility that institutional and political factors contribute to determining wage levels. Trade unions and legislated changes in the minimum wage, for example, keep upward pressure on the wage during periods of rising labor productivity. The conventional wage share assumption implies that wages increase proportionately with labor productivity. Thus a rise in wages is a predictable indirect effect of the widespread adoption of techniques of production that raise labor productivity, but is not under the control of any single entrepreneur.

When all the entrepreneurs act on their perception that a new technique will increase their rate of profit, the increase in the productivity of labor creates the conditions for institutional factors to enforce a proportionate increase in the wage. At the end of the day the entrepreneurs will have to raise pay by the same amount as labor productivity. The average rate of profit declines because the entrepreneurs have fallen victim to the *fallacy of composition*: actions that appear to be advantageous to individual capitalists are not always advantageous when all capitalists take them.

On a deeper level, the decline in the rate of profit reflects a *social coordination problem*. If the entrepreneurs could somehow coordinate their actions, they would refrain from choosing techniques that lower the rate of profit. But each entrepreneur makes her decision independently, without taking into consideration the externality she imposes on the other firms by choosing to raise labor productivity and create the conditions for a general increase in the wage. The individual entrepreneurs are acting in a way that defeats their shared collective purpose. Social coordination problems are a pervasive feature of modern capitalist economies.

The expectation of increasing wages as a result of social patterns of technical change only increases the pressure on individual entrepreneurs to adopt labor-saving techniques in an effort to protect their rate of profit from erosion by higher wages. Each entrepreneur may understand quite well that the falling rate of profit is the result of the general adoption of labor-saving techniques, and still see it as in her own best interest to adopt precisely such techniques.

In judging the viability of a new technique, (ρ', x'), entrepreneurs focus on the rate of profit they would get if they adopted it while paying the existing wage, w. Let us call the private rate of profit expected by the typical entrepreneur $v^e = (1 - w/x')\rho'$. We can simplify this equation by substituting the equations that describe technical change:

$$x' = (1 + \gamma)x$$

$$\rho' = (1 + \chi)\rho$$

Remembering the definition of the wage share, $w = (1 - \pi)x$, the expected rate of profit can be written:

$$v^e = \frac{\rho(1 + \chi)(\gamma + \pi)}{1 + \gamma}$$

The entrepreneurs compare this expected rate of profit with the prevailing rate of profit, $v = \pi\rho$. The condition for a technical change to be considered viable by the entrepreneurs is that its expected rate of profit should exceed the prevailing rate of profit, or $v^e > v$. This *viability condition* can be expressed in terms of the profit share:

$$\pi < \frac{\gamma(1+\chi)}{\gamma - \chi} \tag{8.7}$$

The economic intuition behind this condition is that a technical change that saves on labor but requires more capital will be profitable if labor costs are a sufficiently large proportion of total costs. The viability condition plays an important role in implementing the Classical model empirically, and in distinguishing the Classical theory from the neoclassical theory.

Entrepreneurs who anticipate a rising wage will have that much more incentive to adopt viable new techniques, because a labor-saving technique that is viable at a given wage is also viable at any higher wage. Equation (8.7) reflects this fact, since a higher wage will correspond to a lower profit share, making the inequality even stronger.

PROBLEM 8.5 In Industria (see Problem 8.1) in the base year, calculate the private rate of profit that entrepreneurs perceive they would receive in the next year if they adopt the new technique. Would this technical change be considered viable?

PROBLEM 8.6 Show that the viability condition is met in Industria (see Problem 8.1).

PROBLEM 8.7 Show that if entrepreneurs expect the wage to increase at the same rate as labor productivity, γ, they will still adopt new labor-saving techniques that satisfy the viability condition. For simplicity, assume that δ is zero. (Hint: show that at the new wage, w', the rate of profit will be higher using the new technique, $\{\rho', x'\}$, than with the old technique, $\{\rho, x\}$, if the viability condition is satisfied.)

8.3 Biased Technical Change and the Fossil Production Function

The Classical model with Marx-biased technical change and a conventional wage share provides a way of understanding capital-labor substitution that is alternative to the neoclassical production function. A history of Marx-biased technical change leaves behind a trail of evidence that is hard to distinguish from movement along a preexisting production function. In fact, if the rates of capital-using ($\chi < 0$) and labor-saving ($\gamma > 0$) are constant, the historical path of labor and capital productivities left behind by technical change will exactly resemble a Cobb-Douglas production function.

To see this, consider an economy undergoing Marx-biased technical change with constant $\gamma > 0$ and $\chi < 0$. First, the measured rate of labor productivity growth will be:

$$g_x = \gamma$$

and the growth rate of the capital-labor ratio will be:

$$g_k = \frac{1+\gamma}{1+\chi} - 1 = \frac{\gamma - \chi}{1+\chi}$$

Dividing the latter equation into the former and rearranging, we obtain the following expression linking labor productivity growth to the rate of growth of the capital-labor ratio:

$$g_x = \frac{\gamma(1+\chi)}{\gamma - \chi} g_k = \omega g_k \tag{8.8}$$

where the coefficient $\omega \equiv \gamma(1+\chi)/(\gamma - \chi)$ (the Greek letter *omega*, pronounced "o′-may′-ga") is a positive fraction less than unity since $\gamma > 0$ and $\chi < 0$. Notice that ω appears in the viability condition as the profit share at which the new techniques will just be viable.

Now consider an economy moving along a Cobb-Douglas production function, $X = K^{\alpha} N^{1-\alpha}$, or $x = k^{\alpha}$. Then in successive periods:

$$\frac{x_{+1}}{x} = \left(\frac{k_{+1}}{k}\right)^{\alpha}$$

or, taking logarithms of both sides:

$$\ln(x_{+1}) - \ln(x) = \alpha(\ln(k_{+1}) - \ln(k))$$

The first difference of the natural logarithm of a dated variable is equal to its exponential compound growth rate, which in turn is very close to the growth rate formula that we are using here, as we saw in Chapter 2. In other words, the Cobb-Douglas economy will have a measured growth of labor productivity:

$$g_x \approx \ln(x_{+1}) - \ln(x) = \alpha(\ln(k_{+1}) - \ln(k)) = \alpha g_k \tag{8.9}$$

Comparing equations (8.8) and (8.9), we see that they are identical when we substitute ω for α. A history of Marx-biased technical change at a constant rate is indistinguishable from movement along a Cobb-Douglas production function, since the Cobb-Douglas growth path has the same mathematical form as the biased technical change growth path.

In the Classical conventional wage share economy with Marx-biased technical change, the historical path of labor and capital productivity creates a *fossil production function*. The history of past techniques appears to trace out a production function, but is in fact just the fossil record of past technology.

This similarity in form between the Classical fossil record and the neoclassical production function invites us to ask what the substantive difference between the Classical conventional wage model with biased technical change and the neoclassical model with a Cobb-Douglas production function actually is. At issue in the contest between the Classical and neoclassical theories are some of the deepest questions in political economy. The Classical theory regards capital as a social relationship between two classes: the owners of wealth (the actual capital goods) and the direct producers, workers. It regards profit as the form of the social surplus appropriated by capitalists through the capitalist property relations. The neoclassical theory, with its essentially harmonious vision of the economy, imputes a definite productive contribution to capital as well as to labor. It explains profit and wage income symmetrically, as the equilibrium of supply and demand in the capital and labor markets. The neoclassical theory attaches great significance to equality between the wage (profit rate) and the marginal product of labor (capital). Modern Classical economists criticize the neoclassical theorists for misrepresenting social reality by reifying capital and treating a social relationship as if it were a thing.

It would be useful to be able to distinguish empirically between hypotheses generated by these competing theories. As we saw in Chapter 3, neoclassical theory assumes a smooth production function like the Cobb-Douglas, and assumes that the economy is always operating at a switchpoint on the efficiency frontier of the production function. By contrast, in the Classical model the best practice technique will generally be chosen over a range of wage rates. The Classical theory, therefore, allows the economy to operate at a wage higher than the switchpoint; only in the limiting case is the wage at the switchpoint. In the economy represented in Figure 8.2, for example, the initial wage is greater than the wage at the switchpoint. Since, as we saw in Chapter 3, the assumption that the wage is equal to the marginal product of labor is just another way of saying that the economy is at a switchpoint, we can also say that in the Classical conventional wage share model the wage may be higher than the apparent marginal product of labor.

The distance between the actual wage and the wage at the switchpoint is given in terms of the profit share by $\omega - \pi$. When $\pi = \omega$, the viability condition is satisfied as an equality and the economy operates at a switchpoint. When $\pi < \omega$, the viability condition is satisfied as a strict inequality and the economy operates above the switchpoint. Therefore, we can use the viability condition to evaluate competing hypotheses generated by the neoclassical and Classical theories. An implication of neoclassical theory's insistence that the wage is equal to the marginal product of labor is that the viability condition will be satisfied as an equality, while the Classical theory allows it to be satisfied as a strict inequality. Note that finding the viability condition to be an equality does not falsify the Classical theory that apparent capital-labor substitution is a result of a historical pattern of technical change but finding it to be a strict inequality does falsify the neoclassical theory that the wage is equal to the marginal product of labor.

We have assembled statistics to evaluate the viability condition in Figure 8.3. Averages for the profit share and the growth rates of x and ρ (measuring γ and χ) have been calculated over the period 1965–2011 for twelve countries in which technical change took the Marx-biased form,[1] using the Extended Penn World Tables 5.0. These data have been used to calculate the viability condition, which is displayed visually in the figure. The 45-degree line divides the figure, with the viable region lying above the diagonal. The neoclassical theory predicts that the data points should lie along the diagonal (or at least close to it). The Classical theory allows for the possibility that the data points should lie above the diagonal, which they clearly do. In fact, the average value of ω is around .85 while the average profit share, π, is around .5. The neoclassical theory of distribution appears from this test to be off by a fairly large margin. We will see in Chapter 11 that neoclassical theorists have had to make auxiliary assumptions in order to explain these basic discrepancies between their predictions and real observations.

PROBLEM 8.8 In a classical model with biased technical change and a wage share $1 - \pi = 0.8$, $1 + \gamma = 1.02$/year, and $1 + \chi = 0.99$/year, find the

[1] US, Australia, Austria, Belgium, Canada, Denmark, Spain, France, UK, Italy, New Zealand, Portugal.

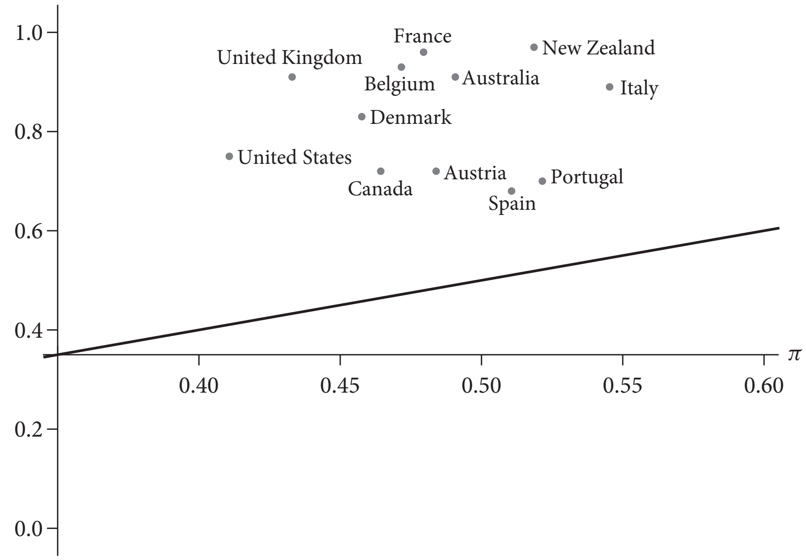

Figure 8.3 Average ω and π for countries exhibiting Marx-biased technical change over the period 1965–2011 plotted against each other reveal an overwhelming tendency for $\omega > \pi$, or in other words, for advanced capitalist economies to operate at a wage above the apparent marginal product of labor. Source: Authors' calculations from the Extended Penn World Tables 5.0.

relationship between the growth rates of x and k. If you were able to estimate this relationship without knowing how the data had been generated, would you accept or reject the hypothesis that there is a Cobb-Douglas production function with perfectly competitive markets?

PROBLEM 8.9 Use the data in Tables 2.4 and 2.8 to check whether technical change from 1973–1992 in the six countries satisfied the viability condition. (Use the value of the profit share during the 1980s.) Do the values you compute satisfy the predictions of the neoclassical theory?

8.4 The Classical Full Employment Model with Marx-Biased Technical Change

Marx-biased technical change can also occur in a Classical full employment model. The six equations defining the Classical full employment model with Marx-biased technical change are summarized in Table 8.2. Recall that biased technical change is the assumption that $\gamma > 0$ and $\chi < 0$. We will also

Table 8.2 The Classical Full Employment Model with Marx-Biased Technical Change

Endogenous variables: x, ρ, w, v, c, g_K
Exogenous parameters: $x_0, \rho_0, \delta, \beta, n, \gamma, \chi$

$$x = x_0(1+\gamma)^t \tag{8.10}$$

$$\rho = \rho_0(1+\chi)^t \tag{8.11}$$

$$w = x\left(1 - \frac{v}{\rho}\right) \tag{8.12}$$

$$c = x\left(1 - \frac{g_K + \delta}{\rho}\right) \tag{8.13}$$

$$\delta + g_K = \beta v - (1-\beta)(1-\delta) \tag{8.14}$$

$$1 + g_K = (1+n)(1+g_k) \tag{8.15}$$

assume that the initial profit share satisfies the viability condition, or $\pi_0 = 1 - w_0/x_0 < \omega$.

In developing this model, we need to specify that we are assuming that the economy undergoes a process of biased technical change that is independent of the distribution of income. In other words, we interpret technical change as exogenous in order to study its effect on a capitalist economy under full employment conditions. This treatment might be seen as a complement to the previous chapter, where we assumed that distribution regulates the bias of technical change under full employment according to the theory of induced technical change.

In this case, we must assume that the wage rate adjusts in each period so the demand for labor created by the capital stock exactly matches the supply of labor. As in the Classical full employment model of Chapters 6 and 7, we assume that the supply of labor grows at the constant rate n. Under this assumption we can determine the wage rate, wage share, and profit share that satisfy the full employment assumption.

The demand for labor in each period will depend on the amount of capital, K, and the technique chosen as represented by its capital intensity, k, or

$$N^d = K/k$$

The demand for labor grows by the factor N^d_{+1}/N^d or $(1+g_K)/(1+g_k)$. Substituting $g_k = (\gamma - \chi)/(1+\chi)$, we see that the demand for labor grows at the rate

$$\frac{g_K(1+\chi) - (\gamma - \chi)}{(1+\gamma)}$$

Imposing the assumption of full employment restricts the growth of labor demand to be equal to the rate of labor force growth, n. This lets us determine the rate of accumulation required for continuous full employment:

$$g_K = \frac{n(1+\gamma) + (\gamma - \chi)}{(1+\chi)} \tag{8.16}$$

We need to turn to the Cambridge equation to determine what profit rate and share will be needed to satisfy the full employment assumption. Substituting equation (8.16) into the Cambridge equation and rearranging, we arrive at

$$r = \frac{1}{\beta}\left(\frac{(1+n)(1+\gamma)}{(1+\chi)} - \beta\right)$$

This equation establishes the rate of profit needed to maintain full employment in the presence of Marx-biased technical change. Since we are assuming that the depreciation rate remains constant (a simplifying assumption), and since $v = r + \delta$, it is clear that the gross rate of profit, v, must also remain constant on a full employment growth path. But the gross rate of profit is the product $\pi\rho$, and capital productivity will be falling as the result of Marx-biased technical change. This means that the profit share must be rising (and the wage share must be falling) in order to preserve the rate of profit at its full employment level. We can work out the exact growth rate for the profit share by taking differences of the constant term $\pi\rho$ and dividing through by $\pi\rho$:

$$\frac{\Delta\pi}{\pi} + \frac{\Delta\rho}{\rho} + \frac{\Delta\pi\,\Delta\rho}{\pi\rho} = 0$$

Solving for the growth rate of the profit share, $g_\pi = \Delta\pi/\pi$, and assuming Marx-biased technical change ($\chi < 0$) gives us the full employment growth rate for the profit share:

$$g_\pi = \frac{-\chi}{1+\chi}$$

This is a remarkable result because it shows how even though workers are enjoying some benefits from biased technical change since their real wage is rising, capitalists must receive a greater and greater share of output in order to keep their rate of accumulation at the full employment level. Workers' real wages are rising more slowly than labor productivity or $g_w < \gamma$. Marx identified this pattern with a rising rate of surplus value.

A neoclassical economist consulting the marginal productivity theory of distribution would interpret this growth path using the aggregate production function. A real wage that is rising more slowly than labor productivity (so that the wage share is declining) is consistent with an elasticity of substitution greater than one, as we saw in Section 3.6.3. We discuss the elasticity of substitution in more detail in Chapter 10.

We have assumed that technical changes are viable or that $\pi < \omega$, but it is clear that as the profit share rises in order to maintain full employment, at some point it will equal the viability threshold parameter, ω. At this point, the wage will correspond to the wage at the switchpoint. If the profit share rises beyond this point, the new techniques will not be immediately viable. In order to model this growth regime, we would need to specify how unused techniques are processed by entrepreneurs. In Chapter 10, we study the neoclassical growth model, which assumes a smooth production function with many techniques that are not viable at the prevailing wage.

8.5 Reverse Marx-Biased Technical Change

The process of industrialization and growth has historically been accompanied by capital-using, labor-saving patterns of technical change. As we saw in Chapter 2, this was true almost universally for the currently advanced countries when they were developing. Yet in the last decades of the twentieth century some economies have exhibited signs of the polar opposite pattern—labor-using capital-saving technical change, which we will call *reverse Marx-biased technical change* or RMBTC.

A closer look at the EPWT 6.0 data for subperiods 1967–1985 and 1986–2014 (Figures 8.4–8.8) reveals different patterns of technical change for the world economy as a whole, for the advanced capitalist regions of the world, for the rapidly growing economies of South and East Asia, on the one hand, and for the "global South," Central and South America, Africa, the Middle East, and Central Asia, on the other.

The world economy as a whole exhibits mostly Marx-biased technical change in the earlier period, which drifts toward a pattern of Hicks-neutral technical change in which both capital and labor inputs become more productive. There is no evidence of RMBTC at the world level.

The data tell a somewhat different story for the various regions that make up the world economy.

The advanced or "post-industrial" regions of North America, Europe, and Oceania (primarily Australia) shown in Figure 8.6 exhibit familiar patterns of

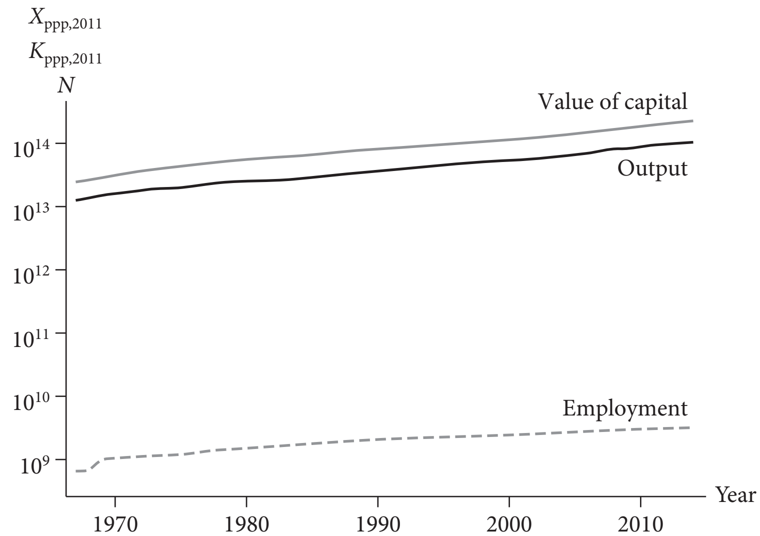

Figure 8.4 This plot shows the totals over all the economies in EPWT 6.0 of output, the value of capital (both measured in terms of 2011 purchasing power parity), and employment. On the log scale of the plot the main feature is steady growth of inputs and output.

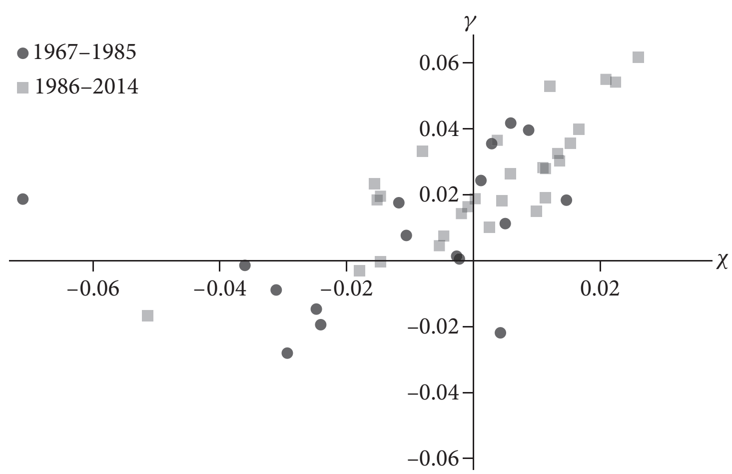

Figure 8.5 Rates of change of capital ($\chi = g_\rho$) and labor ($\gamma = g_x$) productivity for all the economies in EPWT 6.0, divided into subperiods 1967–1985 and 1986–2014. Marx-biased technical change observations appear in the northwest quadrant ($\chi < 0$, $\gamma > 0$), while RMBTC observations appear in the southeast quadrant ($\chi > 0$, $\gamma < 0$). The world economy as a whole exhibits Marx-biased technical change in the earlier period, shifting toward a Hicks-neutral pattern in the second period. There is no evidence of RMBTC.

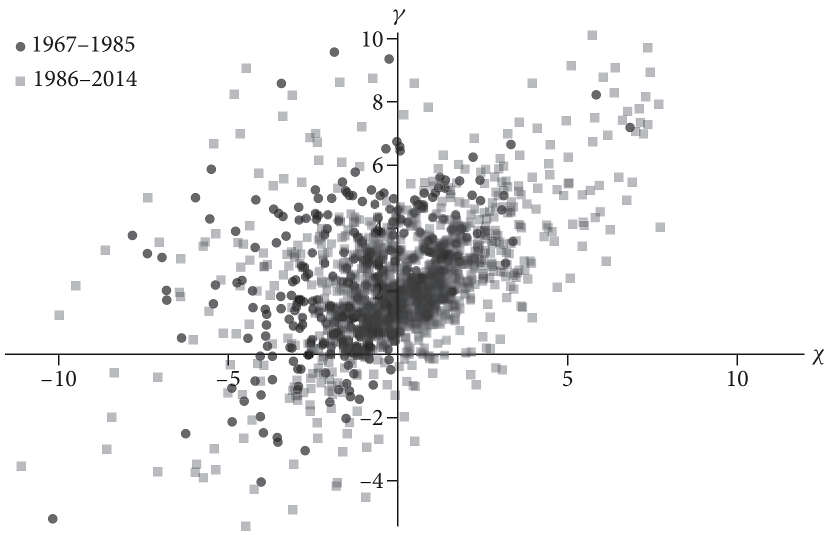

Figure 8.6 Observations of patterns of technical change from North American, European, and Oceanian economies plotted as in Figure 8.5. As for the world economy, Marx-biased technical change dominates the earlier period, and the pattern drifts toward Hicks-neutral patterns in the second period, and there is no sign of RMBTC patterns.

Marx-biased technical change in the earlier period, with some sign of Hicks-neutral technical change favoring both inputs in the second period. This pattern reflects the impact of globalization of production through falling transport costs and changing world political-economic institutions. The advanced countries outsourced much of their production chains to South and East Asia and as a result were able to scrap their oldest and least productive capital facilities. There is no sign of RMBTC in these economies.

The rapidly industrializing South and East Asian region (Figure 8.7) exhibits Marx-biased technical change patterns in the early period, also drifting toward Hicks-neutral patterns of increases in both labor- and capital-productivity in the later period of globalization. In these economies rapid growth encouraged the installation of more efficient technologies through heavy capital investment. There is no sign of RMBTC. These stylized facts were recently discovered by Luis Villanueva and Xiao Jiang.

The rest of the world, Central and South America, Africa, the Middle East, and Central Asia (Figure 8.8), shows a dramatically different pattern. These economies were attempting to follow the Marx-biased pattern of industrialization in the earlier period, but shifted to deindustrialization and RMBTC in many cases in the later period. This change reflects the limited success of

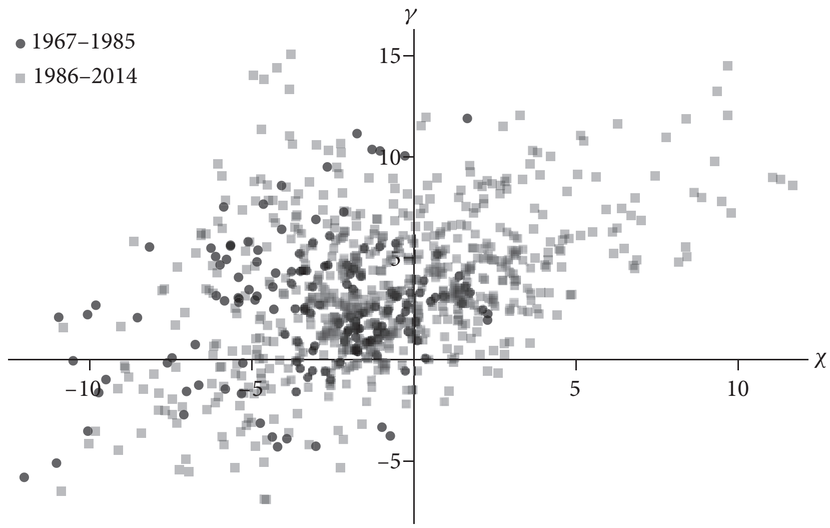

Figure 8.7 Observations of patterns of technical change from East and South Asian economies plotted as in Figure 8.5. These regions experienced rapid growth and industrialization particularly in the second period, 1986–2014. Marx-biased technical change dominates the earlier period, and the pattern drifts toward Hicks-neutral patterns in the second period. There is no sign of RMBTC.

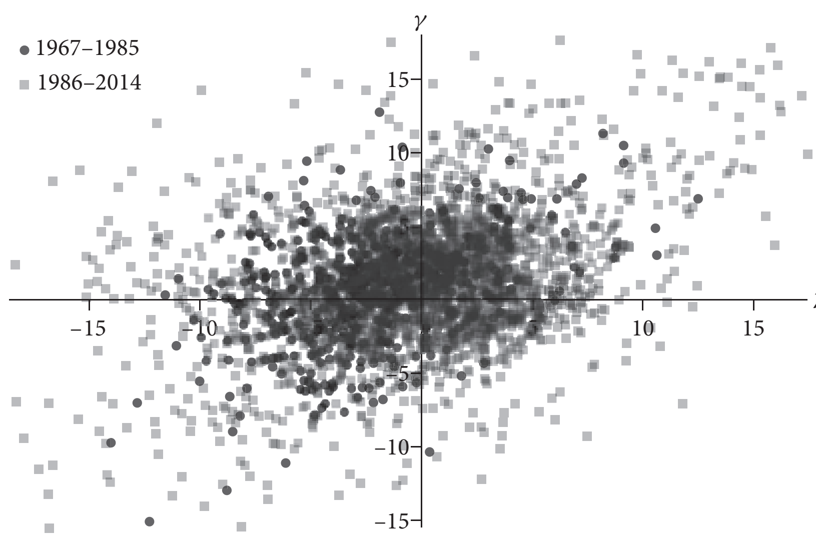

Figure 8.8 Observations of patterns of technical change from the "global South," Central and South American, African, Middle Eastern and Central Asian economies, plotted as in Figure 8.5. In these regions, Marx-biased technical change dominates the earlier period, and the pattern moves toward RMBTC patterns in the second period. These economies show signs of deindustrialization through the period of globalization.

these regions of the world in attracting industrial and manufacturing production from the advanced economies in the period of globalization. Many of these economies effectively specialized in the export of labor rather than goods and services in the globalizing second period.

The Classical theories of technical change might help shed some light on the economic mechanisms that would produce these patterns. According to the theory of induced technical change, a reduction in the wage share can in principle incentivize the generation of RMBTC. This would not require an outright decline in wages; a depression of the rate of growth of wages below the rate of labor productivity growth would suffice. In this case, the equilibrium shown in Figure 8.1 would occur somewhere on the technical progress function in the fourth quadrant where RMBTC can be found.

The other approach taken in this chapter regards the fossil production function as an essentially exogenous process of innovation. As new techniques are adopted, this approach invites us to imagine a backlog of old, discarded techniques that have accumulated in the institutional memory of society. These techniques are therefore available to return to viability on the condition that the real wage declines sufficiently. In this case, the substitutions toward more labor intensive techniques and sectors have a lot in common with the neoclassical theory of factor substitution since they involve entrepreneurs choosing from among a given set of known techniques. Both cases raise the question of what caused wages to stagnate or decline in the first place.

Villanueva and Jiang's research suggests that RMBTC describes a pattern of "deindustrialization" that has resulted from specific economic policies associated with the rise of neoliberal capitalism. Beginning in the 1980s, developing countries were pressured by international authorities to adopt a suite of policies that came to be known as the Washington Consensus, given that both the World Bank and International Monetary Fund have headquarters there. Prominent examples of these policies have been the privatization of state run enterprises as well as the liberalization of trade and finance.

There are many unanswered questions about the dynamics of globalization and its impact on technical change and the world division of labor. But these initial summaries suggest that the basic story of induced technical change, the incentive for capitalist producers to reduce labor costs, plays a decisive part throughout the years for which EPWT 6.0 provides data. The earlier period saw the continuation of traditional patterns of industrialization in the advanced countries. Globalization, however, offered an alternative path for reduction in labor costs, namely the shift of production from high-wage advanced economies to lower wage rapidly growing economies in South and East Asia. Even though these economies started from a lower level of wages, rapid growth began to pull wages and the standard of living up in those economies, so that on a world scale the overall incentives for wage cost reduction resulted in continuing Marx-biased patterns. The regional impact of these changes was, however, very uneven, and those parts of the world that did not benefit from the relocation of industrial and manufacturing production found themselves undergoing a process of deindustrialization and specialization in the export of labor.

Deindustrialization and reverse Marx-biased technical change represent an important part of the ongoing research program of the Classical approach to accumulation and technical change.

8.6 One Vision of Economic Growth

The Classical conventional wage share model with exogenous Marx-biased technical change represents one vision of the process of capitalist economic growth. This Classical vision sees world capitalist development as a historically unique event shaped by specific social factors. The fundamental driving force of capitalist development is the class division it induces in society. The tendency of class conflict to maintain a high wage share in income creates strong incentives for labor-saving innovations, which accounts for the technologically progressive character of capitalism. The resulting bias toward capital-using technical change, however, poses an obstacle to capitalist growth because it depresses rates of profit and growth. Periodic bursts of capital-saving innovation have so far sufficed to restore capital productivity and profitability. The future of capitalism in this vision is open and undetermined, hanging on many historical contingencies. Continuing rises in labor productivity and standards of living, for example, depend on the maintenance of a high wage share in the technologically leading economies. Within this framework the ultimate fate of capitalism is a genuine intellectual, moral, and historical problem.

The Classical vision, however, is not the only way economists have tried to put together the complex aspects of economic growth in a coherent gestalt. Neoclassical growth theory, to which we now turn, offers an alternative perspective from which many of the same empirical facts appear in a quite different light.

8.7 Suggested Readings

The first significant reference to the phenomenon of mechanization is probably David Ricardo's speculation (Ricardo 1951, Ch. 31, "On Machinery") that machinery could generate what today would be called technological unemployment. Karl Marx discussed the underlying causes and forms of capitalist technical change extensively in the first volume of *Capital* (Marx 1977). For a modern continuation of this investigation, see Lazonick (1990). Marx's law of the tendency for the rate of profit to fall has been the object of considerable controversy, which was reopened by Okishio (1961); for an overview, consult Foley (1986). The model in this chapter is basically an extension of Okishio's approach; for further elaboration of the model, see Michl (1999) or Michl (2008, chs 10, 11), and for a "putty-clay" version with embodied technical change, see Michl (2002). It has also been influenced by Duménil and Lévy (1994). For a more general derivation and rigorous econometric test of the viability threshold condition, see Basu (2010). The relationship between deindustrialization and reverse Marx-biased technical change is explored in Villanueva and Jiang (2018).

9

Endogenous Technical Change

9.1 Technical Change in a Capitalist Economy

Industrial capitalism is a powerful mechanism for the accumulation of wealth in the form of means of production. The concentration of social surplus production in the form of profits in the hands of private capitalists creates conditions for a massive increase in the quantity of factories, machines, and transportation facilities. This quantitative growth in the means of production leads to a qualitative change in the organization of production and in the productivity of labor. Competing capitalists seek out new methods of organizing production, new processes, and new products in an attempt to achieve an advantage over their rivals. As a byproduct of this competitive struggle, the productivity of labor rises. The steady increases in labor productivity in capitalist society are as important an influence on modern society as the accumulation of capital itself.

Attempts to make technical change endogenous in a model of economic growth generally fall into one of two broad categories. One approach treats technical change as a byproduct of ordinary economic activity (sometimes called an *externality*). The second approach regards technical change as the output of a distinct research and development (R&D) sector. This chapter will consider examples of both categories.

9.2 Learning by Doing

Intel co-founder Gordon Moore once predicted that the capacity of silicon chips would double every eighteen months. “Moore’s Law” has become an article of faith in the computer industry. In other industries, managers speak of the *learning curve* to describe improvements that result from experience. The late economist Kenneth Arrow called this *learning by doing*.

The learning by doing of greatest interest to growth theorists creates knowledge that spills over to other firms and workers in the economy. The size of the region over which such spillovers occur is open to interpretation. It could be an industry, a country, a region, or the global economy. Arrow argues that learning by doing is most important in the production of new capital goods. When the knowledge that is gained is accessible to other producers through spillover effects, it can lead to self-sustaining technical change. Because the capital stock represents the accumulation of past investment, the stock of knowledge will depend on the stock of capital.

To demonstrate how learning by doing can be formalized as an external economy of scale, and hence compatible with a competitive equilibrium, we will need to distinguish between firm-level and economy-level variables. We do this by using a subscript to describe variables for the i-th firm. An unsubscripted variable refers by default to its aggregate value. Each firm is assumed to operate with a Leontief production function, or

$$X_i = \min(AK_i, x(K)N_i)$$

We will assume that technical change is Harrod-neutral, so capital productivity is constant. We will explain below why we have replaced the usual symbol ρ with A. The level of technology depends on the size of the aggregate capital stock through the function $x(K)$, which models the learning by doing effect. We are assuming that learning effects are too small to matter at the firm level. We can add a little structure by letting $x(K)$ take the convenient form of a power function, as in

$$x = K^a$$

The power function signifies that a 1 percent increase in K generates an a percent change in x, with $a > 0$ to reflect labor-saving technical change.

In this model, firms take the technology as given, but as they collectively accumulate capital, they contribute to the discovery of new techniques. By aggregating (summing) over all the firms, we arrive at an aggregate production function that reveals the effects of these spillovers:

$$X = \min[AK, K^a N]$$

Ordinarily, firms will operate with no excess capital stock and hire no excess labor, which means that both the constraints in the min(., .) function will be satisfied as equalities. The labor constraint is in the Cobb-Douglas family of production functions, with increasing returns since $1 + a > 1$. This kind of scale effect, operating at an aggregate level, was one explanation put forward by Kaldor for Verdoorn's Law, which states that the rate of productivity growth tends to be positively and strongly correlated with the rate of growth of output. Estimates of the Verdoorn Law (as it is also known) suggest that each 1 percent increase in the growth rate of output generates around a 0.5 percent increase in the growth rate of labor productivity.

The capital constraint is in the form of a constant times capital. This production function is the basis for the "AK" family of growth models. We switched notation for capital productivity to make this connection clear. Obviously, the Classical model belongs to this family of models, even without the increasing returns to scale we are about to include in it.

To introduce learning by doing into our Classical model with a conventional wage share, let us continue to assume that the rate of capital accumulation depends on the rate of profit through the Cambridge equation. To economize on notation, assume that the rate of depreciation is zero in this section (we return to positive depreciation in the rest of the chapter). This makes the net rate of profit equal to the constant profit share times capital productivity, or $r = \pi A$. The rate of accumulation will thus be determined by

$$1 + g_K = \beta(1 + r) = \beta(1 + \pi A)$$

The rate of labor-saving technical change depends in a straightforward fashion on the rate of accumulation. From the definitions of γ and g_K we have

$$1 + \gamma = \frac{x_{t+1}}{x_t} = \frac{K^a{}_{t+1}}{K^a{}_t} = (1 + g_K)^a$$

This expression can be simplified further by using the mathematical fact that when a variable, z, is small in magnitude, $\ln(1 + z)$ is approximately equal to z. Taking logs of both sides, substituting from the Cambridge equation, and applying this handy fact gives us

$$\gamma \approx a((\beta - 1) + \beta\pi A)$$

Thus, the rate of technical change depends on the rate of capital accumulation, since knowledge grows as an unintended consequence of investment. An increase in the propensity to save out of wealth will cause an increase in the rate of technical change, which might be one explanation for the observed correlation between saving and growth in per capita income across countries. An increase in the profit share (or the productivity of capital) will cause an increase in the rate of technical change. This means that an increase in the real wage, insofar as it squeezes down the profit share, will cause the rate of technical change to decline, which is at odds with what we concluded in the model of induced technical change in Chapter 7.

PROBLEM 9.1 Derive an expression for the rate of growth of employment in the model with learning by doing. Must employment be increasing?

PROBLEM 9.2 Suppose that learning by doing tends to lead to more mechanized technologies, so that $A = K^b$, where $b < 0$. Derive the expression that describes the rate of capital-using technical change, χ.

9.3 R&D Investment in Technical Change

Another approach to understanding endogenous technical change focuses on the decisions of individual capitalists to invest in productivity increases through research and development spending. The resulting technological advances may then *spill over* to other producers as they are revealed in patents, publications, the products themselves, or the movement of technical workers from one firm to another.

To construct a model of endogenous technical change through R&D, assume that a typical capitalist starts each period with a stock of capital K, and that there is a socially available technology of production (x, k) that anyone can use. The rate of profit with this technology, assuming that the wage is w, is:

$$v = \frac{x - w}{k}$$

For simplicity we will assume that all technical change is Harrod-neutral, so that capital productivity $\rho = x/k$ and δ never change. But we will allow for changes in labor productivity, x. Remember also that $w/x = 1 - \pi$ is the share of the gross product going to wages.

Now suppose that the typical capitalist can use some of her capital to increase labor productivity above the socially given level x. Essentially the capitalist is in a position to buy technical progress. In order to make the model consistent with steady state growth, assume that the *fraction* of the total capital allocated to improving productivity determines the amount of technical progress in the period. We could think of the improvement in technical progress as the result of teaching workers better methods of production. The larger a capitalist's stock of capital, the more workers she will employ, and the more resources it will take to educate the workers. Call the proportion of her capital spent on technical improvement rd. Then the capitalist can achieve the technology $(x/g(rd), k/g(rd))$ by spending a proportion rd of her capital on technical innovation.

To make sense of this picture, we have to assume that the capitalist cannot raise more capital by borrowing. If she could, the trade-off between the resources she puts into technical change and the resources she uses for production would not exist. In real capitalist economies firms can borrow, but there are limits to how much a firm can borrow in relation to its own equity (which corresponds to its capital in the model we are studying). Thus the assumption that capitalists cannot borrow at all is not too inaccurate as a first approximation.

The function $g(.)$ expresses the productivity of resources in improving labor productivity. If the capitalist spends nothing on innovation, she will just use the average social technique (x, k); we reflect this by assuming that $g(0) = 1$. The more she spends on innovation, the higher will be her workers' productivity, $x/g(rd)$. Thus we assume that the derivative $g'(rd)$ is negative. Because we are assuming that capital productivity remains constant, this implies that capital intensity, $k = x/\rho$, will also rise to $k/g(rd)$. As a particular example, assume that $g(rd) = (1 - rd)^{\theta}$, where θ is a parameter that measures how productive resources devoted to innovation are in raising labor productivity. Figure 9.1 illustrates $g(rd)$.

A capitalist who invests a proportion rd of her capital in innovation will have $(1 - rd)$ left for production. Thus her profit rate after paying wages will be:

$$\frac{(x/g(rd)) - w}{k/g(rd)}(1 - rd) = \frac{x - g(rd)w}{k}(1 - rd)$$

Thus the effect of research and development from the point of view of the capitalist is the same as a reduction in the wage.

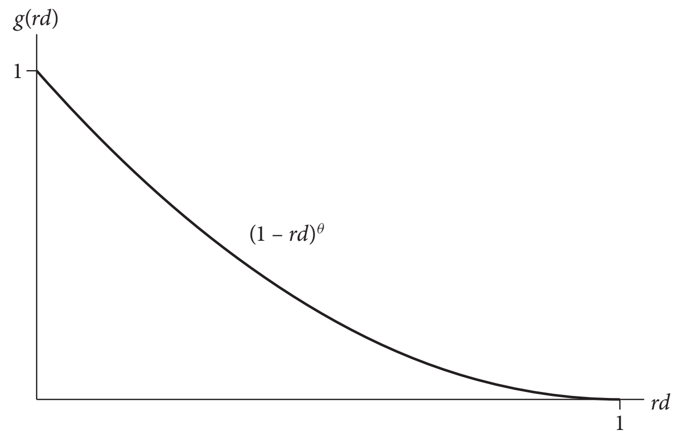

Figure 9.1 A capitalist who invests a proportion rd of her capital in innovation can increase her labor productivity by a factor $1/g(rd)$ of the standard level. This graph is drawn on the assumption that $\theta > 1$.

The capitalist's net profit rate when she spends rd on innovation is:

$$\begin{aligned} r(rd) &= \frac{x - wg(rd)}{k}(1 - rd) - \delta \\ &= \frac{x - w(1 - rd)^{\theta}}{k}(1 - rd) - \delta \end{aligned}$$

Her budget constraint, then, is:

$$\begin{aligned} K_{+1} + C &= (1 + r(rd))K \\ &= \left(\frac{x - wg(rd)}{k}(1 - rd) + (1 - \delta)\right) K \end{aligned} \tag{9.1}$$

We can write the typical capitalist's planning problem as:

choose $\{C_t \geq 0, 0 \leq rd_t \leq 1\}_{t=0}^{\infty}$

so as to maximize $(1 - \beta) \sum_{t=0}^{\infty} \beta^t \ln(C_t)$

subject to $K_{t+1} + C_t = (1 + r(rd_t))K_t t = 0, \ldots, \infty$

given K_0, $\{w/x\}_{t=0}^{\infty}$

As we already know, a capitalist who maximizes a Cobb-Douglas intertemporal utility function spends a fraction $1 - \beta$ of her wealth at the end of the period on consumption.

$$C = (1 - \beta)(1 + r(rd))K \tag{9.2}$$

What is new in this model is that the capitalist has to decide how much of her capital to devote to innovation in each period. Thus the capitalist's decision problem will be solved once we understand how she will choose rd_t. This also turns out to be the key to understanding the forces governing the growth of labor productivity.

9.4 How Much R&D?

The advantage in research and development spending from the point of view of the capitalist is that it raises her profit rate. The capitalist will choose the level of research and development spending that maximizes her profit rate. If she decides to invest in innovation at all, she should continue to invest until her net profit rate with respect to her R&D investment is maximized:

$$r'(rd) = \left(-\frac{w}{x}g'(rd)(1-rd) - \left(1 - \frac{w}{x}g(rd)\right)\right)\rho = 0$$
$$\text{or} \tag{9.3}$$
$$g(rd) - g'(rd)(1-rd_t) = \frac{x}{w}$$

This first-order condition expresses the trade-off the capitalist faces. She can determine her rate of profit

$$(1 + r(rd)) = \frac{x - wg(rd_t)}{k}(1 - rd) + (1 - \delta)$$

by choosing the level of innovative expenditure, rd. An increase in rd lowers her labor cost by raising labor productivity, which has a positive effect on profitability. But an increase in rd also leaves the capitalist with fewer resources to devote to production because of the $(1 - rd)$ term. If we graph the rate of profit as a function of rd, as in Figure 9.2, we can see that when $rd = 0$ the rate of profit is just the level available by taking the existing social technology, and when $rd = 1$, the rate of profit is zero. Somewhere in between is the maximal level, which is determined by equation (9.3).

For the particular $g(rd) = (1 - rd)^\theta$, we can see that the profit rate for any level of rd will be:

$$1 + r(rd) = \frac{x - w(1 - rd_t)^\theta}{k}(1 - rd) + (1 - \delta)$$

In this case the first-order condition is satisfied when:

$$1 - rd^* = \left(\frac{1}{(1+\theta)(1-\pi)}\right)^{\frac{1}{\theta}} \tag{9.4}$$

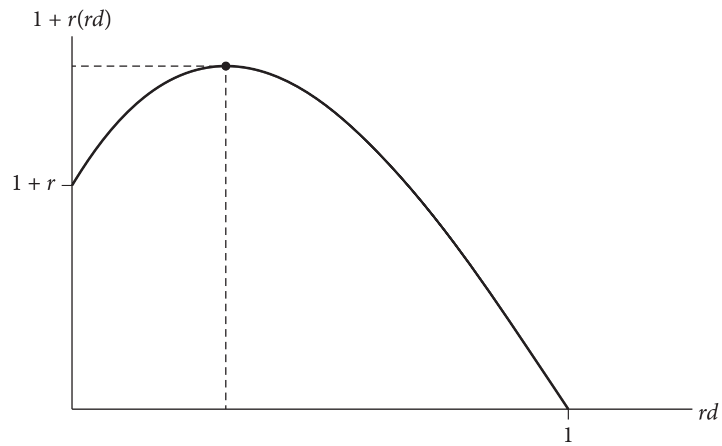

Figure 9.2 As the capitalist spends more on innovation her profit rate initially rises because of the effect of innovation on labor productivity, but eventually falls because she has so little capital left for actual production.

This expression makes sense only if $(1+\theta)(1-\pi) > 1$. If $(1+\theta)(1-\pi) \leq 1$, the capitalist is better off not investing in innovation at all, because the profit rate is actually declining in rd even when $rd = 0$. Thus if innovation is very expensive (because θ is small) or if the wage share, $1-\pi = w/x$, is small, there is no incentive for the capitalist to innovate.

If $(1+\theta)(1-\pi) > 1$, so that the capitalist does invest in innovation, the higher is the wage share in the gross product, the more resources the capitalist will find it profitable to devote to increasing the productivity of labor. This makes good sense, since if wage costs are only a small share of total costs, there isn't much point in trying to reduce them further, while if wage costs are a large share of total costs, an increase in labor productivity increases profits a lot.

We can also calculate the resulting labor requirement and wage share after the innovation since:

$$g(rd^*) = (1-rd^*)^\theta = \frac{1}{(1+\theta)(1-\pi)}$$

$$(1-\pi)g(rd^*) = \frac{1}{1+\theta} \tag{9.5}$$

As a result the profit rate for the capitalist after innovation will be:

$$r^* = \frac{\theta}{1+\theta}\left(\frac{1}{(1+\theta)(1-\pi)}\right)^{\frac{1}{\theta}}\rho - \delta$$

This expression shows the final effect of endogenous technical change on the capitalist's profit rate.

PROBLEM 9.3 In the model with $g(rd) = (1 - rd)^{\theta}$, if $w/x = .5$, and $\theta = 2$, find the proportion of capital a capitalist will devote to innovation, rd^*. What will the level of $g(rd^*)$ be? What will be her actual share of wages in costs? If $\rho = .333$/year and $\delta = .05$/year, calculate her gross rate of profit when $rd = 0$ and at the optimum level of innovation.

PROBLEM 9.4 Find the formula for the optimal rd^* if $g(rd) = 1 - \theta rd$.

9.5 Steady State Growth with No Persistent Effects of R&D

We now have a model that explains what proportion of resources capitalists would devote to innovation, or training, or whatever activities increase the productivity of the labor force. What happens to the economy over time depends on what we assume about the effects of this innovative expenditure on labor productivity and wages in succeeding periods.

The simplest hypothesis is that the effects of the innovative expenditure wear off completely each period, putting the productivity of the labor back at a given level x, and that the conventional wage, $\bar{w}$, is constant. Under this assumption $1 - \bar{\pi} = \bar{w}/x$ in every period, so the capitalists will devote the same proportion of their resources to innovation in each period. We have:

$$1 - rd^* = \left(\frac{1}{(1+\theta)(1-\bar{\pi})}\right)^{\frac{1}{\theta}} \quad \text{if } (1+\theta)(1-\bar{\pi}) > 1$$

$$rd^* = 0 \quad \text{if } (1+\theta)(1-\bar{\pi}) \leq 1$$

As a result of the innovative expenditure, the actual labor productivity will be higher in every period. If $(1+\theta)(1-\bar{\pi}) > 1$:

$$g(rd^*) = (1 - rd^*)^{\theta} = \frac{1}{(1+\theta)(1-\bar{\pi})}$$

$$(w/x)g(rd^*) = \frac{1}{1+\theta}$$

$$\frac{x}{g(rd^*)} = w(1+\theta) \geq x \tag{9.6}$$

Thus labor productivity $x/g(rd^*)$ will be higher as a result of the innovative expenditure. The higher is the wage, the more resources the capitalists will be induced to put into innovation, and the higher will be the labor productivity in the steady state.

The steady state net profit rate in this economy will be:

$$r^* = \rho(1-rd^*)\left(1-\frac{\bar{w}}{x}g(rd^*)\right) - \delta \geq 1+r = \rho\left(1-\frac{\bar{w}}{x}\right) - \delta$$

Since the Cambridge equation holds for this economy, we have:

$$1+g_K = \beta(1+r^*) \geq \beta(1+r)$$

Thus the steady state growth rate for this economy will also be higher than if the capitalists did not innovate.

PROBLEM 9.5 Find the steady state gross profit rate and gross growth rate for the economy described in Problem 9.3 under the assumption that there is no persistence to the improvement in labor productivity that results from innovative expenditure and that $\beta = .9$.

PROBLEM 9.6 Can an increase in the wage raise the profit rate and growth rate in an economy where improvements in labor productivity from innovative expenditures do not persist?

9.6 Steady State Growth with Persistent Effects of R&D

In the last section we saw that innovative expenditure can raise the steady state profit rate and growth rate of an economy, even if there are no persistent effects of innovation, so that the level of labor productivity is not increasing.

Innovative expenditure may have much more far-reaching effects, however. There may be *spillovers* from one capitalist's innovation to the average level of labor productivity in later periods. For example, if innovative expenditures raise the productivity of labor by training workers to be more efficient, in the next period there will be a larger pool of trained workers in the economy as a whole. In a competitive economy some of these workers will move to other firms. In this situation the effect of innovative expenditure in one period by all the capitalists is to raise the social level of labor productivity in the future.

In our model we assume that each individual capitalist takes the level of labor productivity in the system, x, as given in each period and beyond her control. She sees her innovative expenditure as helping her workers in one period improve over the social standard level. We also assume, however, that all the capitalists are exactly alike; whatever one of them does, they all will do, since they all face the same incentives. In this situation innovation is an *externality*: the innovative expenditures of one capitalist improve the productivity and profitability of the other capitalists in future periods, but because each capitalist makes her decision as to how much to spend on innovation taking future productivity as given, she does not take this external effect into account. In this circumstance the capitalists will spend too little on innovation, in the sense that they would all have higher utility if they agreed to increase innovative expenditure in each period, and thereby raised the whole path of labor productivities. When each makes this decision separately, she has no incentive to spend the increased amount.

In the case of persistent effects of innovation on labor productivity we have an additional equilibrium condition in the model, to take account of the effect of the average level of innovative expenditure on the future level of labor productivity. A simple (though somewhat extreme) assumption is to suppose that the level of social labor productivity (which will apply in the absence of any further innovative expenditure by the capitalists) in a period will equal the level of labor productivity actually achieved (including the effects of innovation) in the last period by the typical capitalist. Mathematically this amounts to the assumption:

$$x_{+1} = \frac{x}{g(rd^*)}$$

First let us assume that the wage is constant at $\bar{w}$. Then we know from (9.6) that after the first period

$$x_1 = \frac{x_0}{g(rd_0^*)} = (1+\theta)\bar{w}$$

In this case we have $(\bar{w}/x_1) = \frac{1}{1+\theta}$, and $(1+\theta)(1-\pi_1) = 1$, so that after period 0 it will not pay any capitalist to innovate. Thus in this economy with persistent effects of innovation and a constant real wage the labor productivity, x, will immediately rise to the level $(1+\theta)\bar{w}$, and at that point the wage share will be so small that there will be no further incentive for the typical capitalist to innovate. After period 0, $rd^* = 0$, and labor productivity will continue at its steady state level.

The steady state level of labor productivity will be the level where $rd = 0$, or where

$$1+\theta = \frac{x}{\bar{w}} \quad \text{or}$$
$$x = \bar{w}(1+\theta)$$

But, as we have seen, this steady state is not actually the best that the capitalists could achieve, since it would be to their collective advantage to lower the labor requirement even more, if they could make a group decision to do so.

PROBLEM 9.7 Find the steady state level of labor productivity for the economy described in Problems 9.3 and 9.5 when there is complete persistence of the productivity enhancing effect of innovative expenditure and the wage is $\bar{w} = \$10{,}000/\text{year}$.

9.7 Persistent Effects of R&D with a Conventional Wage Share

In real capitalist economies the wage tends to rise with labor productivity, so that the wage share does not decline as in the last example. The typical capitalist, of course, takes both the wage and the social level of labor productivity as parameters in making her decisions about innovative expenditure. What happens if market forces beyond the capitalist's control act to keep the labor share $1 - \pi$ constant at a given level, $1 - \bar{\pi}$, but innovative expenditure has persistent effects on future levels of labor productivity?

We know, since $\bar{\pi}$ is constant, that, if $(1+\theta)(1-\bar{\pi}) > 1$:

$$g(rd^*) = \frac{1}{(1+\theta)(1-\bar{\pi})}$$

Since improvements in labor productivity are persistent, if $(1+\theta)(1-\bar{\pi}) > 1$ we have:

$$x_{+1} = \frac{x}{g(rd^*)} = x(1+\theta)(1-\bar{\pi})$$

Thus the effect of a sufficiently high conventional wage share is to induce an indefinitely continuing rise in labor productivity. The higher is the wage share, the more resources the typical capitalist will put into innovation, and the faster labor productivity will grow. This interaction will give rise to Harrod-neutral technical progress at the rate:

$$1 + \gamma = \frac{x_{+1}}{x} = \frac{1}{g(rd^*)} = (1+\theta)(1-\bar{\pi})$$

What is happening here is that each capitalist, consulting her own incentives to innovate to get ahead of the market, invests in innovation that raises the labor productivity of the workers. This improvement spills over into higher labor productivity in the next period. At the same time, the wage is rising at the same rate as labor productivity, so that the wage share is constant. This means the typical capitalist has exactly the same incentive to spend on innovation in the next period. In this situation labor productivity improvement is a kind of unintended side-effect of the capitalists' pursuit of profit.

It is striking that the incentive for the capitalist to innovate rises with the wage share. A high wage share economy, on these hypotheses, tends to be an economy with rapidly rising labor productivity, other things being equal, as was the case in the model of induced technical change in Chapter 7. This view of the link between income distribution and technological change is deeply rooted in the Classical tradition.

It is also striking that the ability of a capitalist economy to sustain high rates of labor productivity growth depends both on the costs of innovation (represented by the parameter θ), and on the incentives to innovate (represented by the labor share, $1 - \bar{\pi}$). A capitalist economy whose labor share is low relative to the costs of innovation will settle into a stagnant regime where no capitalist puts resources into innovation and there is no productivity growth. This contrasts with the model of learning by doing, where a high profit share causes rapid accumulation and high productivity growth.

As Figure 9.3 shows, the decline in the US wage share that began in the early 2000s has been by and large accompanied by a decline in the growth rate of labor productivity. These developments appear to be in line with the Classical view presented both in the induced technical change model of chapter 7 and in this Chapter. Institutional forces might be behind a falling share of wages in national income; because of the reduced incentives to innovate, labor productivity growth declines as a result.

As with any abstract model, it is important to view the conclusions of this analysis with caution. Many assumptions are required to reach the conclusions, and some of them may not hold in any particular real economy.

PROBLEM 9.8 Find the growth rate of labor productivity for the economy described in Problem 9.3 under the assumption that labor productivity improvements are persistent and that the wage share is 60%.

PROBLEM 9.9 What rd would lead to the maximum rate of growth of labor productivity in this model? Would it be a good idea to follow this policy? Explain why or why not.

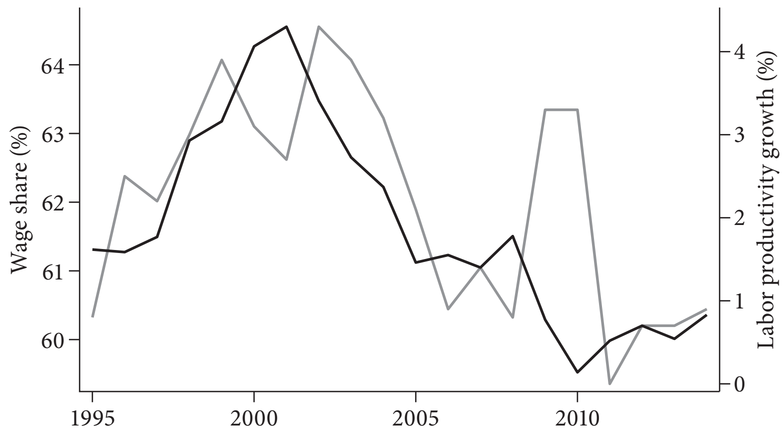

Figure 9.3 The wage share (black, left axis) and the growth rate of labor productivity (gray, right axis) are plotted in the same graph for the United States during the period 1995–2014. By and large, changes in the wage share are followed by changes in labor productivity growth the following year. Source: Bureau of Economic Analysis.

9.8 Suggested Readings

Learning by doing was elaborated by Arrow (1962). Nicholas Kaldor (1966); Kaldor (1967) rediscovered the argument of Allyn Young (1928) that growth creates opportunities for greater specialization among firms and industries, giving rise to the Verdoorn Law. The importance of the division of labor in the growth process of course traces all the way back to Smith (1937 [1776]). For recent contributions and empirical evidence, consult McCombie et al. (2002). For the connection between the Classical-Marxian tradition and Kaldor's work on technical change, see Ricoy (1987).

The AK-models originated with Paul Romer (1986), who, along with Robert Lucas (1988), is often credited with starting the New Endogenous Growth Theory. Aghion and Howitt (1992) revived the Schumpeterian approach to growth in this same tradition. An accessible summary of this approach is provided in Aghion and Howitt (2009).

Models that incorporate R&D also owe a debt to Romer, as in Romer (1987a) and Romer (1990). For some insight into the larger controversy between proponents of the New Growth Theory and devotees of the Solow–Swan approach, Romer (1994) and Grossman and Helpman (1994) provide a New Growth perspective, rounded out on the other side by Solow (1994) and Mankiw (1995). An influential study of the contribution of R&D is Coe and

Helpman (1995). Finally, don't overlook Kremer (1993), which contemplates technical change from the Ice Age onward.

The models in this chapter and Chapter 7 are further elaborated in Zamparelli (2015) and Tavani and Zamparelli (2015). For a comprehensive survey of the role of endogenous technical change in Classical and Keynesian growth models, see Tavani and Zamparelli (2017).

10

The Neoclassical Growth Model

10.1 The Solow–Swan Model

During the 1940s and 1950s, economists debated the Keynesian proposition that unemployment tends to persist indefinitely unless special actions are taken by the government. Roy Harrod, a follower of Keynes, and Evsey Domar, a student of socialist planned economies, argued that only by accident would a capitalist economy's *warranted* growth rate (at which planned saving would equal planned investment) equal its *natural* growth rate (the growth needed to create jobs to employ a growing population in the presence of labor-augmenting technical change). Robert Solow and T. W. Swan independently developed a neoclassical model of growth to show that full employment is compatible with steady state growth. The Solow–Swan model assumes, like the Classical model, that planned investment and planned saving are identical, so that it does not directly address the problem of the stability of the actual growth path. (We will study explicit models of the warranted rate of growth in Chapter 12.) The Solow–Swan model is now a standard theoretical explanation of why some countries grow faster than others, and it plays an important role in many policy discussions related to the long run significance of saving and investment.

The Solow–Swan growth model reaches closure by assuming full employment. This is achieved by the choice of the appropriate technique of production from a production function, guided by changes in the real wage. The Solow–Swan model also assumes that there is one representative type of household that saves and invests a constant fraction of its gross income.

10.2 The Intensive Production Function

Constant returns to scale in production means that a proportional increase in all inputs makes it possible to increase output in the same proportion. If we can produce one ton of corn using one bushel of seed corn and one worker, we ought to be able to replicate that result by hiring one more worker and buying one more bushel of seed, and producing two tons of corn. This replication argument makes constant returns an attractive assumption. There is strong evidence, however, that real production is subject to *increasing returns to scale*, because it is possible to adopt new techniques of production involving a more detailed division of labor at a larger scale.

As we saw in Chapter 3, constant returns to scale permits us to work with the intensive production function, $x = f(k)$, which in the Cobb-Douglas case is $x = Ak^{\alpha}$. The graph of this function, shown in Figure 10.1, has an inverted dish shape. Economically, the intensive production function exhibits a *diminishing marginal product of capital*: increasing the amount of capital per worker raises output per worker, but by progressively smaller increments. Mathematicians describe such functions as *concave*.

The neoclassical production function in general, and the Cobb-Douglas function in particular, can be viewed as the result of increasing the number of techniques available, until there is an infinite continuum of techniques. Therefore, each point on the Cobb-Douglas represents a single technique (ρ, x). One feasible technique has been highlighted by the thin lines in Figure 10.1. The slope of the thin line up to the capital intensity k represents the output-capital ratio, ρ, for that technique. With that technique, adding more capital per worker above k will yield no further output, so the thin line representing the technique becomes horizontal to the right of k.

While the Classical theory of technical change sees more capital intensive techniques as coming into being historically as the result of technical innovation, the neoclassical production function implies that a broad spectrum of techniques of every capital intensity have already been invented and are available in any historical period.

An important corollary of the assumption of diminishing marginal productivity of capital is that the productivity of capital, the output-capital ratio, ρ, will be a decreasing function of the capital-labor ratio, k. This point can be seen geometrically in Figure 10.1: the ray through the origin representing the productivity of capital declines in slope as k rises. In the Cobb-Douglas case, the relation between the productivity of capital and the capital intensity

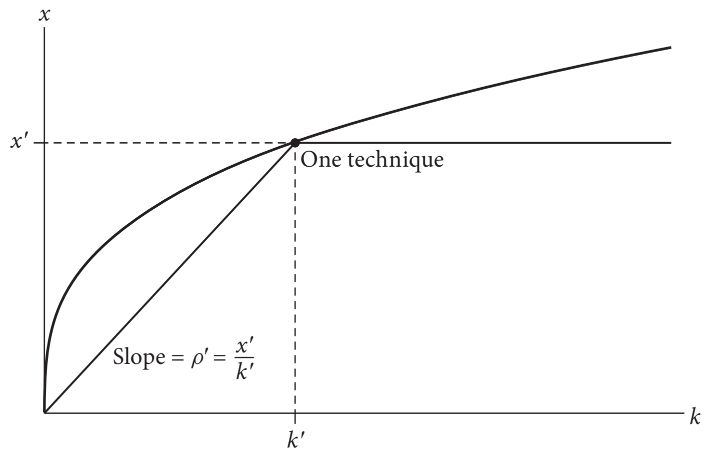

Figure 10.1 The intensive production function.

is described by the function $\rho = Ak^{\alpha-1}$, which is a decreasing function when $\alpha < 1$.

PROBLEM 10.1 Write the Leontief production function in intensive form.

PROBLEM 10.2 Find the value of x and ρ for the Cobb-Douglas production function $X = AK^{\alpha}N^{1-\alpha}$, when $k = \$14{,}000$/worker, $A = 1{,}000$, and $\alpha = .2$.

PROBLEM 10.3 Show that the Cobb-Douglas production function implies that $\rho = Ak^{\alpha-1}$.

10.3 Saving, Population, and Steady State Growth

The Solow–Swan growth model assumes that the economy saves a constant, exogenously given fraction of its income, and that the population and labor force grow at a constant, exogenously given rate. Note that the Solow–Swan model, in assuming that households save the same proportion of profit and wage income, abstracts from the distinction between workers and capitalists that is central to the Classical model. Furthermore, unlike later neoclassical growth theory, the Solow–Swan model does not base the saving equation on the household's utility maximizing problem as we did in Chapter 5. The

Solow–Swan model simply assumes that gross saving is a constant fraction of gross output:

$$S = sX$$

Here S represents the flow of gross saving and s is the fraction of gross income that is saved, the saving ratio, also called the *saving propensity*. Notice that saving in the Solow–Swan model is a constant fraction of the *flow of output*, rather than being a constant fraction of the *stock of wealth*, as in the model of Chapter 5.

The Solow–Swan model assumes, like the Classical model, that saving is identical to investment, which implies that the change in the capital stock per period is the excess of saving over depreciation:

$$K_{+1} - K = sX - \delta K$$

Dividing both sides of this equation by K, we obtain an equation for the rate of capital accumulation, g_K:

$$g_K = \frac{sX}{K} - \delta = s\rho - \delta \tag{10.1}$$

When we recall that the output-capital ratio, ρ, is an inverse function of the capital-labor ratio, we can see that this makes the rate of accumulation an inverse function of the capital-labor ratio too. In the Cobb-Douglas case, this function will be:

$$g_K = sAk^{\alpha-1} - \delta$$

which is shown in Figure 10.2.

The labor force is assumed to grow at a constant rate, n, which is an exogenous parameter of the model. The Solow–Swan model assumes that labor remains fully employed at all times. The mechanism that assures that any excess labor will be absorbed by the demand for labor in production is the constant adjustment of the wage, so that entrepreneurs' profit-maximizing choice of technique creates enough jobs to clear the labor market. If labor were to become unemployed, the wage would decline, leading entrepreneurs to choose more labor-intensive techniques, and create more jobs. The existence of techniques with arbitrarily high and low capital intensity, as is the case for the Cobb-Douglas production function, guarantees that there will

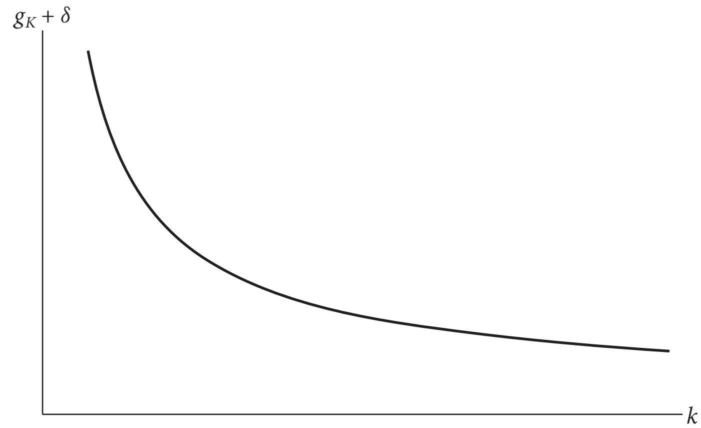

Figure 10.2 The rate of capital accumulation is a decreasing function of the capital-labor ratio under the assumptions of the Solow–Swan model.

always be a technique that will provide full employment, no matter how much or how little capital has been inherited from the past.

In order to predict the direction of the choices of technique in a dynamic setting, we need to know whether the supply of capital or labor is growing faster. Mathematically, this amounts to solving an equation for the growth rate of the capital-labor ratio, g_k. Since $k = K/N$ is a ratio, its growth rate (like the growth rate of any ratio) can be expressed as the difference between the growth rates of its numerator and denominator:

$$g_k = \frac{\frac{K_{+1}}{N_{+1}} - \frac{K}{N}}{\frac{K}{N}} = \frac{1 + g_K}{1 + n} - 1 \approx g_K - n$$

The approximation holds when g_K and n are small. When we substitute the expression for g_K from equation (10.1) into this equation, we arrive at the fundamental equation of the Solow–Swan growth model, (10.2):

$$g_k \approx (s\rho - \delta) - n \tag{10.2}$$

which, multiplying both sides by the capital intensity k and rearranging, can be written as

$$g_k k = \Delta k = sx - (\delta + n)k \tag{10.3}$$

Equation (10.2) tells us that when the rate of capital accumulation (which is the term bracketed on the right-hand side) exceeds the rate of population growth, the capital-labor ratio will be growing. As capital increases faster than labor, the wage is bid up so that more capital-intensive techniques become the most profitable. Otherwise, there would be an excess demand for labor (overemployment). This path of increasing capital intensity is known as *capital deepening*.

In the converse situation, with labor growing faster than capital, the wage would be falling. This would cause firms to switch to more labor-intensive techniques to soak up the excess supply of labor.

Equation (10.3) tells us the same story but from a different perspective. The capital-labor ratio will be increasing ($\Delta k > 0$) when saving per worker sx exceeds the amount of investment per worker required to maintain the current capital stock per worker. This, in turn, is equal to the sum of capital depreciation δk and nk, where the latter is the additional capital needed in order to compensate for the increase in the labor force. When $(n + \delta)k > sx$, the capital-labor ratio will decrease.

We can visualize these cases better by specializing the Solow–Swan model to the case of the Cobb-Douglas production function. In this case, the fundamental equation becomes:

$$g_k = (sAk^{\alpha-1} - \delta) - n$$

Figure 10.3 graphs equation (10.3). Saving per worker is drawn using a Cobb-Douglas production function, and thus is sAk^{α}. The investment per worker required to maintain the capital stock per worker is equal to $(n + \delta)k$. When saving per worker is equal to $(n + \delta)k$, the capital-labor ratio will remain constant.

At the intersection of these saving and required investment curves, $g_K = n$. Here the economy has reached its *steady state equilibrium*, k^*, where there is no change in the capital-labor ratio ($g_k = 0$). The capital accumulation occurring in this state is called *capital widening*. By setting $g_k = 0$ in equation (10.2), we can see that output per worker and the capital intensity in the steady state must be related by the equation:

$$k^* = \frac{s}{n + \delta} x^*$$

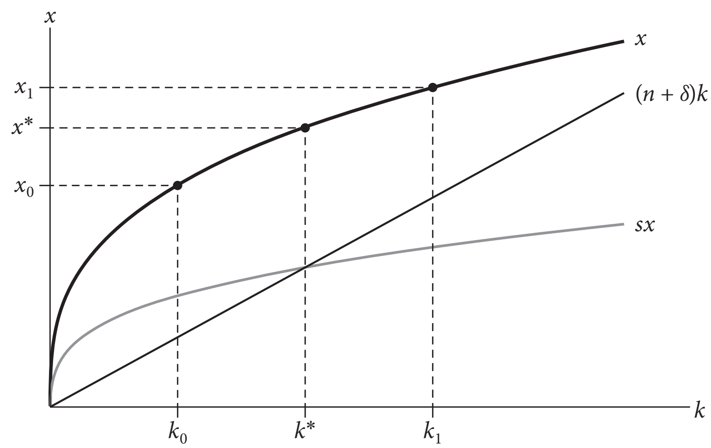

Figure 10.3 Steady state equilibrium in the Solow–Swan model.

The asterisk superscript identifies the steady state values of the capital-labor and output-labor ratios.

In the case of the Cobb-Douglas production function, we can derive an explicit or *closed-form* solution for the equilibrium capital-labor and output-labor ratios in terms of the parameters of the model:

$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$x^* = Ak^{*\alpha}$$

The upper curve in Figure 10.3 shows output per worker along the Cobb-Douglas production function. The lower curves show saving per worker and the investment per worker required to maintain the capital stock for different levels of k. Their intersection determines the steady state capital intensity, k^*. Corresponding to the steady state capital-labor ratio will be the equilibrium level of output per worker, x^*. Using Figure 10.3, we can work out many of the important characteristics of the Solow–Swan model.

The steady state at (k^*, x^*) is stable because if the economy starts out at a low level of capital per worker, such as k_0 in Figure 10.3, the fundamental equation (10.2) tells us that capital will be growing faster than the labor force. By the same token, starting with a high level of capital per worker at k_1 results in capital growing more slowly than the labor force. In the long run, the system converges on (k^*, x^*).

PROBLEM 10.4 Production in Solowia is described by a Cobb-Douglas production function with $A = 1000$, $\alpha = .2$. The saving rate is .15, the rate of depreciation, δ, is .1 per year, and the population growth rate, n, is .02 per year. What will the growth rates of capital and the capital-labor ratio be when the capital-labor ratio is $5,000 per worker?

PROBLEM 10.5 Find the steady state equilibrium values of the capital-labor ratio, productivity of labor, and productivity of capital for Solowia (see Problem 10.4).

10.4 The Solow–Swan Model and the Growth-Distribution Schedule

The Solow–Swan model can be analyzed by means of the growth-distribution diagram, as in Figures 10.4 and 10.5.

Recall that the efficiency frontier contains the same information as the intensive production function; each technique is represented by a growth-distribution schedule which contributes one point to the frontier. The profit-maximizing technique for any wage, w, is represented by the growth-distribution schedule tangent to the efficiency frontier at w, and the slope of the growth-distribution schedule is equal to the negative of the corresponding capital-labor ratio, k. In the Classical model, the wage is given exogenously and determines the technique in use and the capital intensity of production. In the Solow–Swan model, by contrast, the *capital intensity*, $\bar{k}$, is given exogenously in each period by the past growth of the population and the past accumulation of capital. If the efficiency frontier is concave toward the origin, as in Figure 10.4, there will be one tangent to the efficiency frontier whose slope is equal to $-\bar{k}$. This tangent is the growth-distribution schedule for the technique in use, and determines the wage and profit rate in the period. Consumption per worker is just $c = (1 - s)x$, and the growth rate of the capital stock is determined by the growth-distribution schedule.

Figure 10.4 also shows the growth of the labor force plus the depreciation rate, $n + \delta$. As the figure is drawn, the gross growth rate of the capital

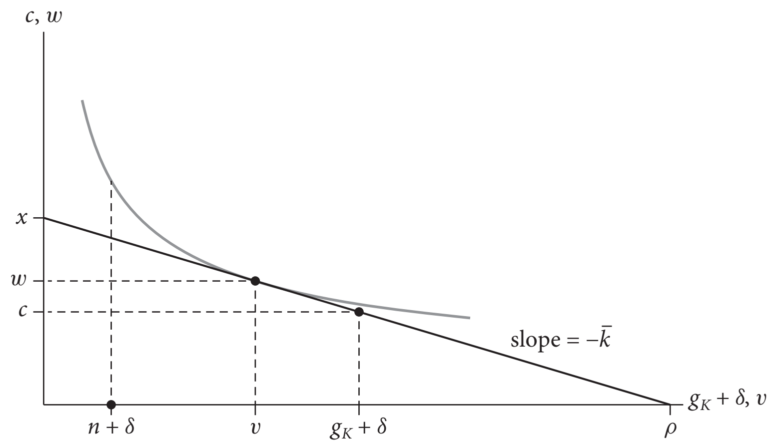

Figure 10.4 The Solow–Swan model takes the capital-labor ratio in each period, $\bar{k}$, as exogenously determined by past population growth and capital accumulation. The technique in use, the wage, and the profit rate are determined by the point on the efficiency schedule where the slope of the tangent is equal to $-\bar{k}$. The saving propensity, s, then determines consumption per worker and the growth rate of the capital stock.

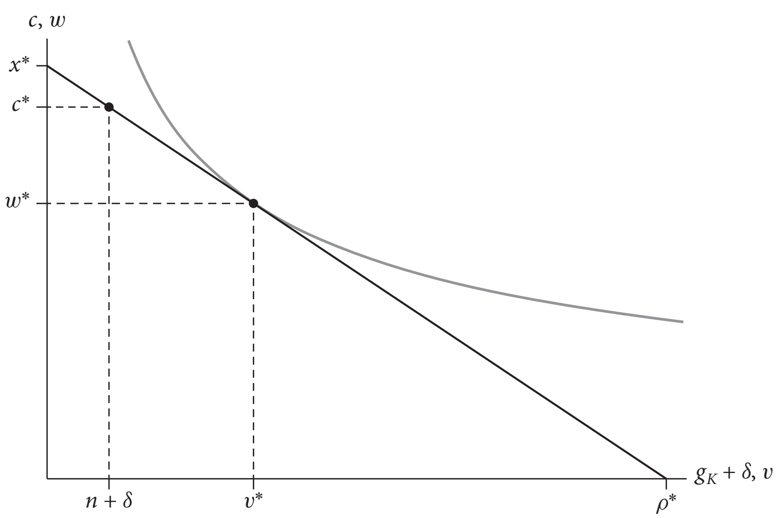

Figure 10.5 The Solow–Swan model reaches a steady state when the capital intensity rises to the point where saving finances just enough investment to offset depreciation and the growth of the labor force.

stock, $g_K + \delta$, exceeds $n + \delta$, so the capital intensity next period, k_{+1}, will be higher:

$$k_{+1} = k + \Delta k = (1 + g_K - n)k$$

Thus in the next period the economy will move to a point where the efficiency schedule is steeper, the profit rate lower, the real wage higher, and the growth rate of the capital stock lower. This process will continue until the economy reaches the *steady state capital intensity*, k^*. The steady state is represented in Figure 10.5, which shows the efficiency frontier and the growth-distribution schedule associated with the steady state technique. The slope of this schedule equals the steady state capital-labor ratio, or $-k^*$. The vertical intercept equals the steady state productivity of labor, x^*.

Once any technique has been chosen by entrepreneurs, its growth-distribution schedule determines the trade-off between social consumption per worker and growth. In the steady state, the growth rate of capital will be the exogenously given rate of growth of the labor force, n. The rate of growth of output is $g_X = g_\rho + g_K$. Since capital productivity remains constant at its steady state value, $g_\rho = 0$, and the rate of growth of output will also be equal to n.

10.5 The Complete Model

In the Classical conventional wage share model, each period is identical to the last except that all the aggregate variables—output, capital stock, and employment—have grown at the same rate. In the Solow–Swan model, however, outside of the steady state each period differs from the last because the capital intensity of production changes. Thus in the Classical model the analysis of equilibrium in a single period is the same as the analysis of the steady state, whereas in the Solow–Swan model it is necessary to consider equilibrium outside of the steady state separately from the analysis of the steady state.

We can summarize the equilibrium in an arbitrary period in the Solow–Swan model in Table 10.1.

The accumulated capital stock and population determine the given capital stock per worker, $\bar{k}$, for the period, which in turn determines output per worker, x. The profit rate is equal to the marginal product of capital corresponding to $\bar{k}$, and consumption per worker is determined by the sav-

Table 10.1 Short Run Equilibrium in the Solow–Swan Model

Endogenous variables: x, w, v, c, g_K, k_{+1}
Exogenous Parameters: $\bar{k}$, δ, s, n
$x = f(\bar{k})$
$v = f'(\bar{k})$
$c = (1 - s)x$
$w = x - v\bar{k}$
$g_K + \delta = \frac{sx}{\bar{k}}$
$k_{+1} - k = sf(\bar{k}) - (n + \delta)\bar{k}$

Table 10.2 Steady State in the Solow–Swan Model

Endogenous variables: k^*, x^*, w^*, v^*, c^*
Exogenous parameters: $f(.)$, δ, s, n
$sf(k^*) - (n + \delta)k^* = 0$
$x^* = f(k^*)$
$v^* = f'(k^*)$
$c^* = (1 - s)x^*$
$w^* = x^* - v^*k^*$
$g_K^* + \delta = n$

ing propensity and the output per worker. The growth-distribution schedule then determines the remaining variables, w and g_K.

With the Cobb-Douglas production function, we can solve these equations explicitly in terms of the parameters of the model in each period: $x = A\bar{k}^\alpha$, $v = \alpha A\bar{k}^{\alpha-1}$, $c = (1 - s)x$, $w = (1 - \alpha)x$, $g_K = sA\bar{k}^{\alpha-1} - \delta$, and $k_{+1} = sA\bar{k}^\alpha - (n + \delta)\bar{k}$.

The steady state capital intensity of the Solow–Swan model, k^*, is defined by the condition $g_k k = sf(k^*) - (n + \delta)k^* = 0$. The steady state conditions for a general production function are shown in Table 10.2.

Table 10.3 Steady State in the Solow–Swan Model with Cobb-Douglas Production Function

Endogenous variables: k^*, x^*, w^*, v^*, c^*
Exogenous parameters: A, α, δ, s, n

$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$x^* = Ak^{*\alpha}$$

$$\rho^* = Ak^{*\alpha-1}$$

$$w^* = (1-\alpha)x^*$$

$$v^* = \alpha\rho^*$$

$$c^* = (1-s)x^*$$

$$g_K{}^* + \delta = n$$

With the Cobb-Douglas production function, we can use these equations to solve for the steady state variables in terms of the exogenous parameters in Table 10.3.

10.6 Substitution and Distribution

During the convergence to a steady state, capital deepening will cause the wage to rise and the profit rate to fall. The effect a rising wage has on the distribution of income between wages and profits depends on the ease with which capital and labor can be substituted for one another. If it is easy to substitute capital for labor, entrepreneurs will shift to much more capital-intensive techniques in the face of a small increase in wages, and wages will become a smaller proportion of income. If it is very difficult to substitute capital for labor, large increases in the wage will be required to induce entrepreneurs to choose even slightly more capital-intensive techniques so that wages will become a larger proportion of income.

The ease of substitution between capital and labor implied by a particular production function at a particular capital intensity is measured by the elasticity of substitution between capital and labor, σ, that we first encountered in Section 3.6.3.

In the neoclassical model, the wage and profit rates will be equal to the marginal products of labor and capital. Using this fact we can write the elasticity of substitution in a form that emphasizes the relationship between substitution and distribution:

$$\sigma = \frac{\%\Delta(K/N)}{\%\Delta(w/v)}$$

To understand how the value of the elasticity of substitution affects the distribution of income, it is helpful to remember the following definitions of the wage share and the profit share of income:

$$1 - \pi = \frac{wN}{X}$$

$$\pi = \frac{vK}{X}$$

As the economy converges on its steady state capital intensity from below, the wage is rising and the profit rate is falling. This induces entrepreneurs to switch to more capital-intensive technologies. What happens to distribution depends on how much these substitutions affect labor and capital productivity.

A borderline case occurs with $\sigma = 1$ as in the Cobb-Douglas production function. If wages rise by, say, 1%, the productivity of labor will rise by exactly 1% as well in this case. The wage share will stay constant (since N/X has fallen by 1%), and clearly the profit share will also stay constant. As we saw in Chapter 3, the wage share will equal $1 - \alpha$ and the profit share will equal α with a Cobb-Douglas production function and perfect competition.

If the elasticity of substitution is less than one, a 1% increase in wages will induce a less than 1% increase in the productivity of labor through substitution effects. As a result, an increase in wages will lead to an increase in the wage share and a decrease in the profit share.

If the elasticity of substitution is greater than one, a 1% increase in wages will induce a greater than 1% increase in labor productivity. As a result, an increase in wages will actually lead to a decline in the wage share and a rise in the profit share.

In general the elasticity of substitution may change with the technique in use, but for the production functions we have used in this book, the elasticity of substitution is constant because they all belong to the family of CES production functions. Much empirical research on production functions makes a similar assumption.

As we have seen, a constant wage share has sometimes been a good first approximation to the behavior of real capitalist economies, and appears as a fundamental assumption in the Classical conventional wage share model. That model attributes the constant wage share to the behavior of labor supply: it assumes that economic, political, and social forces will tend to make the wage rise roughly at the same rate as labor productivity. The neoclassical model explains the constant wage share as a property of a specific production function that happens to describe substitution possibilities between labor and capital, the Cobb-Douglas function. This is undoubtedly one reason for the popularity of this production function among neoclassical economists.

But we have also seen that the wage and profit shares have changed dramatically in some historical periods such as the last three decades. Some economists have argued that the simultaneous appearance of a falling wage share and capital deepening in this period is a sign that the elasticity of substitution must be greater than one. On the other hand, most econometric studies of neoclassical production functions find that the estimated elasticity of substitution is close to one (indicating a Cobb-Douglas production function) or less than one. If this is true, the falling wage share must have some other cause. The Classical approach would emphasize changes in the social structures and institutions that determine the conventional wage share. Some examples of these changes might include the decline of trade unions and falling statutory minimum wages. The explanation of the distribution of income between wages and profits is a central point of divergence between the neoclassical and Classical approaches.

10.7 Comparative Dynamics

As in the Classical model, it is useful to compare one steady state of the Solow–Swan model with another when only one parameter of the model has changed. This can be done by using the diagrams in Section 10.3 and the steady state equations for the Solow–Swan model with a Cobb-Douglas production function in Section 10.5.

For example, consider the effect of an increase in the saving rate from s to s'. An increase in the saving rate will increase the saving per worker for every

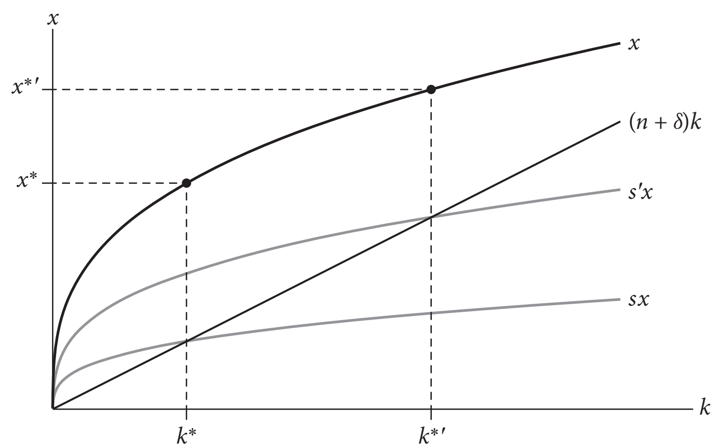

Figure 10.6 The effect of an increase in the saving rate on the steady state in the Solow–Swan model.

level of the capital-labor ratio, as shown in Figure 10.6. The new steady state capital intensity will therefore be higher. With more capital per worker, the economy will enjoy more output per worker. The same conclusion emerges from examination of Table 10.3.

The increase in the saving rate will not affect the growth rate, which in the long run returns to the exogenously given rate of population growth, n. This somewhat disconcerting result is characteristic of *exogenous growth models*, in which the rate of growth is fixed by the exogenous growth of some input to production, such as labor or land. The increase in saving will result in a temporary increase in the rate of growth, as the system converges on its new equilibrium.

An increase in the saving rate has offsetting effects on steady state consumption per worker. Increasing labor productivity from a higher capital intensity raises consumption per worker, but the higher saving rate tends to reduce it. There is one steady state at which consumption per worker is maximized in the Solow–Swan model. At lower rates of saving (and therefore capital intensity), a small increase in the saving rate tends to increase consumption per worker because the productivity effect dominates. At higher rates of saving and capital intensity, the saving effect dominates any increase in productivity, so small increases in the saving rate will decrease consumption per worker.

Edmund Phelps called the equilibrium capital stock per worker at which consumption per worker is maximized the *Golden Rule* capital stock. The net profit rate, $r = v - \delta$, will equal the population growth rate, n, at the Golden Rule equilibrium.

Increasing the saving rate will always bring about an increase in the real wage and decrease in the profit rate. This can be seen by inspection of the last two equations in Table 10.2. Another way to see this is to recognize that increasing the saving rate will push the economy up along its efficiency frontier, to a point with a higher capital intensity.

PROBLEM 10.6 In a Solow–Swan model with a Cobb-Douglas production function, where $A = 1{,}000$, $\alpha = .2$, $\delta = .1$, and $n = .02$/year, what is the capital intensity, labor productivity, and consumption/worker at the original and new steady state when the saving rate rises from $s = .15$ to $s' = .17$? Show these two steady states on the efficiency frontier.

PROBLEM 10.7 Find the Golden Rule values of $\hat{s}$, $\hat{k}$, $\hat{c}$, and $\hat{r}$ for the economy of Problem 10.5.

PROBLEM 10.8 Analyze the comparative dynamics of an increase in the population growth rate, n, using the equations and the diagram for the Solow–Swan growth model. What effect would this change have on k, x, c, g, r, and w?

PROBLEM 10.9 Prove that $r = n$ at the Golden Rule steady state.

10.8 Transitional Dynamics

If the economy starts out with less than the steady state level of capital per worker, it finds the capital stock growing more rapidly than the labor force, and the wage rising to clear the labor market. This would propel the economy along the efficiency frontier in the direction of the steady state position since higher wages would lead to more capital-intensive techniques. This is the process of capital deepening we have already seen, during which the economy is in transit between disequilibrium and its steady state. This process is the *transitional dynamics* of the Solow–Swan model.

The Solow–Swan model explains the growth of output per worker as the effect of the transitional dynamics of the economy while it converges on its steady state equilibrium. When an economy that saves a constant proportion of its income starts out with little capital per worker, it will have a high rate of capital accumulation because saving will be large relative to the investment required to offset depreciation and the growth of the labor force. As the capital-labor ratio increases, saving per unit of capital decreases owing to the operation of diminishing returns, while the investment per unit of capital required to maintain the capital per worker in the face of depreciation and labor force growth remains constant.

There is no guarantee, however, that diminishing returns will be strong enough to extinguish growth in the capital-labor ratio. With the Cobb-Douglas production function, the system will always converge to a steady state, because the marginal productivity of capital approaches zero as k approaches infinity. This assumption has already been incorporated into the figures above.

For other production functions, however, no such guarantee can be made. For example, if we merely add a linear term to the Cobb-Douglas function:

$$X = BK + AK^{\alpha}N^{1-\alpha}$$

then the marginal productivity of capital will approach the parameter B as k approaches infinity. If $sB - \delta > n$, diminishing returns will not be strong enough to shut down growth in the capital-labor ratio, which will continue asymptotically forever at the rate $sB - \delta - n$, with capital accumulating at the rate $g_K = sB - \delta$. Here we have an example of *endogenous growth*, where the long run growth rate is affected by changes in s. The last decade has seen a revival of interest in models of endogenous growth. The Classical conventional wage share model, for example, is an endogenous growth model.

Most extensions of the Solow–Swan model, however, assume that diminishing returns are strong enough to extinguish growth in capital per worker, so that a steady state exists. In this case, the rate of growth will converge on n, the exogenously given rate of population growth, regardless of the saving ratio s.

PROBLEM 10.10 Consider a Solow–Swan model with the production function $X = K + 1000K^{.2}N^{.8}$, $s = .15$, $\delta = .1$/year, and $n = .02$/year. Derive the equation for the rate of accumulation as a function of k, and graph it as in Figure 10.2. Add a line showing the growth rate of the labor force to your figure. Why won't this economy ever achieve a steady state?

10.9 Limitations of the Solow–Swan Model

Two limitations of the Solow–Swan model deserve mention, one pertaining to its internal consistency and the other to its ability to explain features of real economic growth.

First, economists working in the Classical tradition have put forward serious criticisms of the concept of the one-sector production function as a basis for the explanation of growth in real economies. These criticisms were the issues in *Cambridge Capital Controversy* debates in the 1960s and early 1970s. The Classical critics of the Solow–Swan model argue that it cannot be generalized rigorously to economies that have more than one produced output, where the efficiency frontier may not be concave to the origin. We have seen this possibility arise in Chapter 3. The difficulty is that when the efficiency frontier is not concave to the origin, there may be more than one point at which its slope is equal to any given value of capital per worker, pk. In this case the accumulated capital per worker is not sufficient information to determine the technique in use, or the wage and profit rate as the Solow–Swan approach requires. There may be several techniques, and several levels of the wage and profit rate that are consistent with a given value of capital per worker.

The Classical critics of the Solow–Swan model argue that this problem arises because *capital* cannot be defined independently of *capital goods*, and have pointed out numerous logical contradictions, paradoxes, and inconsistencies that arise from trying to reason purely in terms of the value of capital in economies where capital takes the form of many different commodities. These problems were discovered through Piero Sraffa's work studying the properties of the efficiency frontier in a model that allows capital to consist of a multiplicity of commodities.

Second, the Solow–Swan model makes several specific strong empirical predictions that appear inconsistent with the historical record of capitalist economic growth. It predicts that the wage and profit rate should be equal to the marginal products of labor and capital, yet we have already seen evidence that contradicts this prediction. In Chapter 8, we saw that the Cobb-Douglas production function that appears to fit the OECD economies has a parameter α that is significantly larger than the observed profit share. The Solow–Swan model predicts that the profit share should be equal to α. The Solow–Swan model also predicts that growth in labor productivity and the capital-labor ratio should eventually fade out under the operation of the law of diminishing returns. We have seen in Chapter 2, however, that there has been scarcely any sign in Maddison's data of productivity growth dying out over the last two centuries. In Chapter 11, we will see how the Solow–Swan model proposes to solve both these empirical problems with the auxiliary assumption of some form of exogenous neutral technical change.

10.10 Suggested Readings

The basic neoclassical growth model is called the Solow–Swan model in honor of its simultaneous discovery by Solow (1956) and Swan (1956). (The sociologist Robert K. Merton points out that such twin scientific discoveries are surprisingly common.) The Keynesian growth model to which Solow and Swan were responding is due to Roy Harrod (1942) and Evsey Domar (1946).

On the Golden Rule, see Phelps (1966). The Solow–Swan model raises the issue of whether modern economies might be saving too much (called dynamic inefficiency), which researchers such as Abel et al. (1989) have found not to be the case.

Finally, Piero Sraffa's difficult but rewarding little book (Sraffa 1960) is a milestone in the ongoing Classical critique of neoclassical economic theory.

11

Technical Change in the Neoclassical Model

11.1 Technical Change and the Production Function

The Solow–Swan model presented in Chapter 10 predicts that if the production function has strongly diminishing marginal productivity of capital, the growth of output per worker will ultimately cease altogether as the economy reaches a steady state. Since no advanced capitalist country shows signs of having reached this plateau, the Solow–Swan model needs to be extended to explain the growth in per capita output at or near the steady state. This can be done by assuming the presence of exogenous technical change.

Growth models with technical change will not converge on a steady state unless the technical change is Harrod-neutral, or purely labor-augmenting, with no effect on the productivity of capital associated with each technique. Hicks-neutral technical change, which presumes proportional capital- and labor-saving effects on each technique, plays an important role in empirical studies of productivity growth, but not in theoretical growth models because it is not consistent with the existence of a steady state.

Harrod-neutral technical change is illustrated in Figure 11.1, where one technique has been highlighted. The point representing the technique has been projected along the ray through the origin whose slope measures the productivity of capital, ρ, which Harrod-neutral change leaves unchanged. All the other techniques have been projected in the same proportion. Hicks-neutral technical change, which preserves the capital-labor ratio for each technique, would project each point on the intensive production function vertically. Both types of neutral technical change shift the whole efficiency

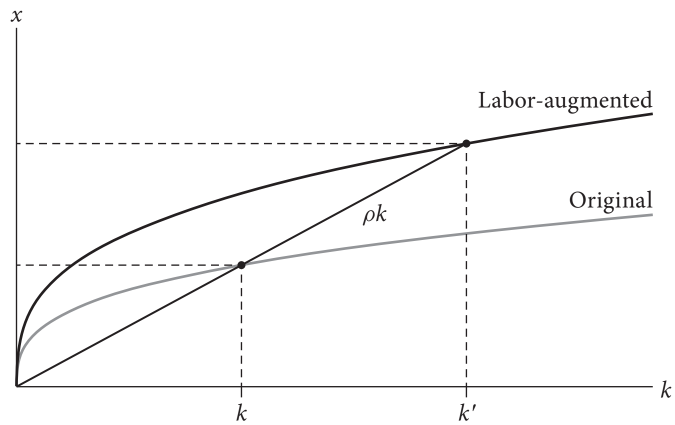

Figure 11.1 Harrod-neutral technical change with a smooth neoclassical production function.

frontier to the northeast, since the frontier is made up of the growth-distribution schedules of all the techniques.

Harrod-neutral technical change in the Cobb-Douglas production function is described by the equation:

$$X = AK^{\alpha}((1+\hat{\gamma})^{t}N)^{1-\alpha}$$

In the Cobb-Douglas case, Harrod-neutral technical change is also Hicks-neutral. We can see this since:

$$X = AK^{\alpha}((1+\hat{\gamma})^{t}N)^{1-\alpha} = \left((1+\hat{\gamma})^{1-\alpha}\right)^{t}(AK^{\alpha}N^{1-\alpha})$$

This equivalence between Hicks and Harrod neutrality holds only for the Cobb-Douglas family of production functions.

As in Chapter 6, we will find it convenient to translate the Solow–Swan model with Harrod-neutral technical change into effective labor units. We continue to use a tilde ("˜") over a variable when it is measured in effective labor terms. The mathematical form of the Solow–Swan model conveniently remains unchanged when we transform all the per-worker variables to effective labor units. As in Chapter 6, in order to recover variables such as x, w, and c in real labor units, we have to multiply the effective labor variables by $(1+\hat{\gamma})^{t}$. We can convert growth rates from effective to real worker terms by adding $\hat{\gamma}$, as long as the growth rates are small. For example, the growth rate of the real capital intensity, k, is given by $g_k \approx g_{\tilde{k}} + \hat{\gamma}$, and the growth rate of real labor productivity by $g_x \approx g_{\tilde{x}} + \hat{\gamma}$. The *effective labor input*, $\tilde{N}$, is the number of actual workers, N, multiplied by $(1 + \hat{\gamma})^t$. The effective workforce will therefore grow at a rate equal to $(\hat{\gamma} + n)$, which plays the role of the natural rate of growth, n, in the basic model.

The intensive Cobb-Douglas production function with Harrod-neutral technical change can be written as:

$$\tilde{x} = A\tilde{k}^{\alpha}$$
$$\rho(\tilde{k}) = A\tilde{k}^{\alpha-1}$$

The neoclassical model of neutral technical change differs conceptually from the Classical model with biased technical change. The neoclassical approach regards capital-labor substitution as a process of moving along a static or timeless production function, while the Classical approach regards it as a historical process of discovery of new techniques. The neoclassical approach treats technical change as global in the sense that it affects every technique, from the most to the least mechanized, in exactly the same way. The Classical approach regards technical change as a sequence of improvements, each slightly more capital intensive than the last. Classical technical changes are localized since they have no effect on old, less capital intensive fossil techniques. In Chapter 8, we saw that some predictions of the Classical and neoclassical models of capital-labor substitution could be tested against real economic data. When the neoclassical theory is augmented to incorporate neutral technical change, however, it becomes difficult to devise a simple empirical test of the two approaches.

PROBLEM 11.1 If the rate of Harrod-neutral technical change is 2% per year, what is the rate of growth of the capital-labor ratio if the ratio of capital per effective worker grows at 5% per year?

PROBLEM 11.2 Suppose the production function is Cobb-Douglas with $A = 1000$ and $\alpha = 0.2$. If technical change is Harrod-neutral at 2% per year and there is \$14,000 per worker of capital in the base year, find the value of output per effective worker and per worker after two years, assuming that the capital stock grows at the same rate as the labor force.

11.2 The Solow–Swan Model with Harrod-Neutral Technical Change

If we retrace the steps we took in developing the equations for the Solow growth model by using the definitions for $\tilde{x}$ and $\tilde{k}$, we can derive the main equations for the Solow–Swan model with Harrod-neutral technical change. For example, substituting the definition for the growth rate of $\tilde{k}$ into the fundamental equation of the Solow growth model gives us the fundamental equation of a Solow growth model with Harrod-neutral technical change:

$$g_{\tilde{k}} = (s\rho - \delta) - (n + \hat{\gamma})$$

This looks just like the fundamental equation in the original Solow–Swan model, but with $(n + \hat{\gamma})$ replacing n. The similarity between these two fundamental equations suggests that it would be easy to extend most of the apparatus developed earlier, provided we redefine the variables in effective labor terms. As before, the fundamental equation predicts that the economy will converge on a steady state equilibrium $(\tilde{x}^*, \tilde{k}^*)$, as long as the production function exhibits a sufficiently diminishing marginal product of capital. The equilibrium effective capital intensity, $\tilde{k}^*$, and the equilibrium effective labor productivity, $\tilde{x}^*$, will be related by:

$$\tilde{k}^* = \left(\frac{s}{n + \hat{\gamma} + \delta}\right) \tilde{x}^*$$

Specializing the Solow–Swan model to the Cobb-Douglas family of production functions lets us derive a closed-form solution for $\tilde{k}^*$ and $\tilde{x}^*$. Using the Cobb-Douglas function in effective labor terms, we find:

$$\tilde{k}^* = \left(\frac{sA}{n + \hat{\gamma} + \delta}\right)^{\frac{1}{1-\alpha}}$$

$$\tilde{x}^* = A(\tilde{k}^*)^{\alpha}$$

This equilibrium is depicted in Figure 11.2, which shows that we can continue to use the same diagrammatic apparatus developed for the Solow–Swan model, provided we do our accounting in effective labor terms.

We know that in the steady state, output per effective worker, $\tilde{x}$, will be stationary. Output per worker, however, will be growing at the rate of technical change, $\hat{\gamma}$, since $g_x = g_{\tilde{x}} + \hat{\gamma}$. Similar reasoning shows that capital intensity also grows at the rate $\hat{\gamma}$. Output and capital expand at the natural rate of growth, $n + \hat{\gamma}$.

PROBLEM 11.3 Draw the growth-distribution schedules over two periods, t and $t + 1$, for the technique that has been selected in the steady state equilibrium of the Solow model with neutral technical change. Identify the

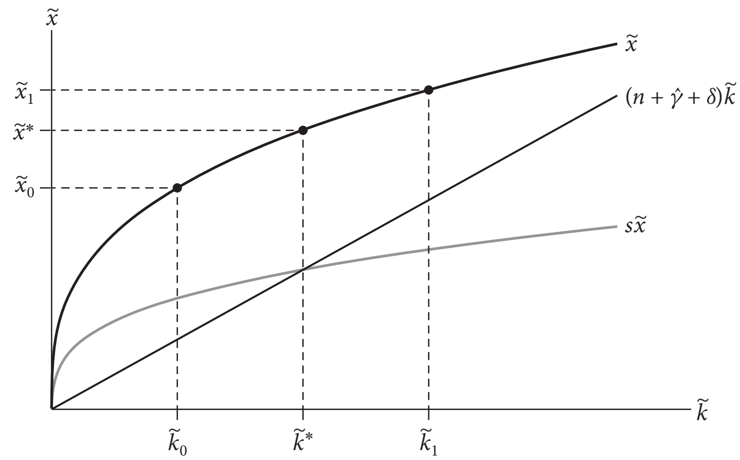

Figure 11.2 The Solow–Swan model with Harrod-neutral technical progress.

wage rate and profit rate, and growth rate and consumption per worker-year on the growth-distribution schedule in each year.

PROBLEM 11.4 Let the Cobb-Douglas production function have $A = 1{,}000$ and $\alpha = .2$. Find the steady state values of $\tilde{x}$, $\tilde{k}$, and ρ when the saving propensity is 15%, depreciation $\delta = 10\%$/year, the rate of population growth $n = 1\%$/year, and the rate of Harrod-neutral technical change $\hat{\gamma} = 2\%$/year.

PROBLEM 11.5 In the economy described in Problem 11.4, what would be the growth rate of capital in the steady state? the capital-labor ratio?

PROBLEM 11.6 If the economy described in Problem 11.4 began in its steady state in the base year, and remained there, what would be the value of output/worker after ten years?

11.3 Growth Accounting

From the perspective of the neoclassical growth model, improvements in living standards come about for two reasons: technical change and increased capital per worker. Since economic policies to raise the level of national saving operate through the latter channel, there is a need to develop an accounting system that separates these two sources of growth. In general (no matter whether technical change is biased or neutral), we can write the neoclassical production function as $X = F(K, N; T)$, where technical change is represented as it occurs over time by the variable T. If we use the symbol F_K to represent the marginal product of capital, F_N the marginal product of labor, and F_T the change in output associated with a unit change in time owing to improved technology, differencing this equation (and assuming $\Delta T = 1$) gives us:

$$\Delta X = F_K \Delta K + F_N \Delta N + F_T$$

Dividing both sides by X and manipulating leads to:

$$\frac{\Delta X}{X} = \left(\frac{F_K K}{X}\right)\frac{\Delta K}{K} + \left(\frac{F_N N}{X}\right)\frac{\Delta N}{N} + \frac{F_T}{X}$$

or

$$g_X = \left(\frac{F_K K}{X}\right) g_K + \left(\frac{F_N N}{X}\right) g_N + \frac{F_T}{X} \tag{11.1}$$

The neoclassical theory assumes that the wage is equal to the marginal productivity of labor, and the profit rate to the marginal productivity of capital. Under this assumption, we can substitute the wage and profit shares for the terms in brackets on the right hand side of this equation. Since the wage and profit rate are observable from macroeconomic data, while marginal products are not directly observable, this assumption makes it operationally possible to decompose the growth of output into a part due to the growth of resources (i.e., capital and labor) and a part due purely to technical change. This decomposition, which is only possible under the neoclassical assumption of equality of factor prices and marginal products, can be written:

$$g_X = \pi g_K + (1 - \pi) g_N + \frac{F_T}{X}$$

The first two terms on the right-hand side of this expression represent the part of output growth due to input growth, and the last term represents the part of output growth due to technical change.

This method of allocating the sources of growth is often called *Solow decomposition*. The part of growth due to the increased availability of capital and human resources is easily understood. But the remainder, sometimes called the *Solow residual*, has been dubbed a "measure of our ignorance" because it is not clear how it is generated by economic activity. The Solow–Swan model attributes the part of output growth represented by the Solow residual to exogenous shifts in the production function, but the model has nothing to say about the origins of these shifts in economic reality.

Another concept of the residual associates it with *total factor productivity*, sometimes called *multifactor productivity*. This is distinguished from *labor productivity* (X/N), because total factor productivity attempts to measure the output produced by a combination of *both* capital and labor inputs. Suppose that technical change takes the form of capital- and labor-augmenting technical change at rates $\hat{\chi}$ and $\hat{\gamma}$ respectively applied to an unchanging, constant returns to scale production function. Letting $\tilde{K}_t = (1+\hat{\chi})^t K_t$ and $\tilde{N}_t = (1+\hat{\gamma})^t N_t$ represent the effective capital and labor inputs, the production function can be written:

$$X_t = F(\tilde{K}, \tilde{N}) = F((1+\hat{\chi})^t K, (1+\hat{\gamma})^t N)$$

Applying equation (11.1) to this production function, we see that:

$$g_X = \left(\frac{F_{\tilde{K}}\tilde{K}}{X}\right) g_{\tilde{K}} + \left(\frac{F_{\tilde{N}}\tilde{N}}{X}\right) g_{\tilde{N}}$$

If we adopt the neoclassical assumption that the wage is equal to the marginal product of effective labor, and the profit rate equal to the marginal product of effective capital, we can express this decomposition in terms of the profit and wage shares:

$$g_X = \pi g_{\tilde{K}} + (1-\pi) g_{\tilde{N}}$$

By definition, $g_{\tilde{K}} = g_K + \hat{\chi}$ and $g_{\tilde{N}} = g_N + \hat{\gamma}$, so that:

$$g_X = \pi g_K + (1-\pi) g_N + \pi\hat{\chi} + (1-\pi)\hat{\gamma}$$

In this decomposition the technical change term F_T/X appears as a weighted average of the rates of capital-augmenting and labor-augmenting technical change, $\pi\hat{\chi} + (1-\pi)\hat{\gamma}$. But if we attempt to use this equation to measure technical change with macroeconomic data, the best we can do is calculate the weighted average of capital- and labor-augmenting technical change:

$$\pi\hat{\chi} + (1-\pi)\hat{\gamma} = g_X - (\pi g_K + (1-\pi) g_N) \qquad (11.2)$$

Table 11.1 Solow Decomposition of Four East Asian Economies, 1966–1990 (%/year)

	Country			
	Hong Kong	*Singapore*	*South Korea*	*Taiwan*
g_X	7.3	8.7	10.3	8.9
g_K	8.0	11.5	13.7	12.3
g_N	3.2	5.7	6.4	4.9
$\pi g_K + (1-\pi)g_N$	5.0	8.5	8.6	6.8
$\hat{\gamma}$	2.3	0.2	1.7	2.1
Memo item:				
$(1-\pi)$ (%)	62.8	50.9	70.3	74.3

Source: Young (1995).

If we assume that technical change is Hicks-neutral, however, $\hat{\chi} = \hat{\gamma}$, so that $\pi\hat{\chi} + (1-\pi)\hat{\gamma} = \hat{\gamma}$, and equation (11.2) becomes:

$$\hat{\gamma} = g_X - (\pi g_K + (1-\pi)g_N) \tag{11.3}$$

Under the assumption of Hicks-neutral technical change, the total factor productivity approach becomes operational. Perhaps for this reason most studies of total factor productivity rely on the assumption of Hicks neutrality. If technical change is biased, on the other hand, it is not possible to measure total factor productivity unambiguously, because the residual determines only the weighted average of capital- and labor-augmenting rates of technical change.

Since the early 1980s the Bureau of Labor Statistics in the US has compiled official statistics on multifactor productivity based on the approach of equation (11.3).

This analytical framework has been used by Alwyn Young to investigate the growth of the celebrated "Four Dragons of East Asia," Hong Kong, Singapore, South Korea, and Taiwan. Was their phenomenal growth from 1966 to 1990 due to an increase in the efficiency with which they used their resources (i.e., total factor productivity) or was it due to an increase in the resources themselves? Table 11.1 displays the Solow decomposition of data for these economies. Their GDP growth rates are extraordinary, ranging from 7.3 to 10.3% per year. But they also experienced very high rates of capital accumulation, from 8.0 to 13.7% per year, and growth in their labor forces. Thus,

Table 11.2 Growth Decomposition of US Productivity (%/year)

	1987–1995	*1995–2007*	*Change*	*2007–2016*	*Change*
g_x	1.6	2.7	+1.2	1.2	−1.5
g_k	2.0	3.3	+1.3	1.4	−1.9
πg_k	0.6	1.1	+0.5	0.5	−0.6
$\hat{\gamma}$	0.9	1.6	+0.7	0.4	−1.2

Notes: Multifactor productivity growth is calculated according to the formula in the text, and differs slightly from the Bureau of Labor Statistics measure because the BLS also corrects for the skill composition of the labor force.

Sources: Bureau of Labor Statistics (2017) and authors' calculations.

the Solow residual measuring total factor productivity growth is not nearly as large, which can be seen in the penultimate row. In fact, Young argues, the performance of these economies was not very different from the rest of the world when it is measured by total factor productivity.

We can also derive the following equation, which decomposes the growth rate of labor productivity into a part attributable to increasing capital intensity, and a residual:

$$g_x = \pi g_k + \hat{\gamma} \tag{11.4}$$

This equation lets economists estimate the relative importance of technical change and capital deepening during selected historical periods. Table 11.2 shows data assembled by the Bureau of Labor Statistics for the nonfarm private sector of the United States over the last three decades. Labor productivity increased dramatically during the 1990s and early 2000s but then declined just as dramatically after the Great Recession of 2008. How much of this pattern was the result of capital accumulation, which also increased during the 1990s and then declined sharply? The rest of the table shows that about two-thirds of the increase in labor productivity and over three-fourths of the slowdown after the Great Recession can be accounted for by total factor productivity growth. Capital deepening played only a supporting role.

Many economists attribute the spurt of total factor productivity growth during the 1990s to the rapid pace of innovation in information technology as it diffused through the economy in this period. Since technical change is not measured directly in growth accounting exercises but only as a residual, there is room for debate about competing hypotheses. It is not unusual for most of the historical variation in labor productivity growth to be accounted for by the residual in practical applications of the Solow decomposition.

PROBLEM 11.7 Use the data in Table 11.1 to determine what proportion of labor productivity growth in each country was caused by capital deepening.

PROBLEM 11.8 Derive the formula for the Solow decomposition of labor productivity growth assuming that technical change is Harrod-neutral.

11.4 Classical and Neoclassical Interpretations of the Residual

The Classical model dispenses with the need for a Solow residual to interpret macroeconomic data by attributing all the growth in labor productivity to technical change. In other words, the Classical model assumes that its technical change parameters, χ and γ, are identical to the measured increases in capital and labor productivity: $\chi = g_\rho$ and $\gamma = g_x$. In contrast, the neoclassical Hicks-neutral growth accounting scheme measures its technical change parameters, $\hat{\gamma}$ and $\hat{\chi}$, as $\hat{\gamma} = \hat{\chi} = g_x - \pi g_k$. From the Classical perspective, the Solow decomposition appears to be a device for explaining the discrepancy between the viability coefficient and the actual value of the profit share that we explored in Chapter 8.

We can see the relation between the Classical and neoclassical approaches by writing out the mathematical expression for total factor productivity growth as it would be measured by a neoclassical growth accountant in terms of the Classical parameters. Substituting for $g_x = \gamma$ and $g_k = (\gamma - \chi)/(1 + \chi)$ in equation (11.4), we have:

$$\hat{\gamma} = \gamma - \pi \frac{\gamma - \chi}{1 + \chi}$$

Multiplying through by $(1 + \chi)/(\gamma - \chi)$, we arrive at an expression connecting the viability condition to the Solow residual.

$$\gamma \frac{1 + \chi}{\gamma - \chi} = \omega = \pi + \frac{1 + \chi}{\gamma - \chi}\hat{\gamma} \qquad (11.5)$$

We can see from equation (11.5) that the Classical viability condition will *always* be an inequality, $(\omega > \pi)$, when there is a positive rate of total factor productivity growth, $\hat{\gamma}$, as measured by a neoclassical accountant. Conversely, when $(\omega > \pi)$, a neoclassical growth accountant will see an increase in total factor productivity. The neoclassical economist will thus be disinclined to take data showing $\omega > \pi$ as evidence that the wage is greater than the marginal product of labor. Instead, she will argue that some of the productivity growth used to calculate ω has been misclassified, and that when the viability condition is recalculated using the corrected values it will turn out to be satisfied as an equality. The Classical response is that the method for correcting the data—translating them into effective labor terms using the measured rate of technical change—is tautological since the accounting framework used to measure technical change rests on the assumption that the marginal productivity theory is true. Consequently, the Classical economist regards the Solow residual as an accounting device to explain the gap between the viability condition and actual profit shares.

11.5 Comparative Dynamics in the Solow–Swan Model

In analyzing the steady state of the Solow–Swan model with exogenous Harrod-neutral technical change, the endogenous variables are $\tilde{k}^*$, $\tilde{x}^*$, v^*, $\tilde{w}^*$, g_K^*, and $\tilde{c}^*$. The exogenous variables are the production function, $f(.)$, n, $\hat{\gamma}$, δ, and s. Comparative steady state analysis studies the question: what effect does a change in one of the exogenous variables have on the steady state endogenous variables? It does this by comparing the steady states before and after the change, either by working out the mathematical solution or by interpreting the relevant diagram. The equations for the steady state of the Solow–Swan model with Harrod-neutral technical change and a Cobb-Douglas production function are:

$$\tilde{k}^* = \left(\frac{sA}{n + \hat{\gamma} + \delta}\right)^{\frac{1}{1-\alpha}}$$

$$\tilde{x}^* = A(\tilde{k}^*)^{\alpha}$$

$$\rho^* = A(\tilde{k}^*)^{\alpha-1}$$

$$\tilde{w}^* = (1-\alpha)\tilde{x}^*$$

$$v^* = \alpha\rho^*$$

$$\tilde{c}^* = (1-s)\tilde{x}^*$$

$${g_K}^* + \delta = n + \hat{\gamma}$$

We saw earlier that a leading candidate to explain the decline in labor productivity growth in the US (and other countries) after 1970 is a decline

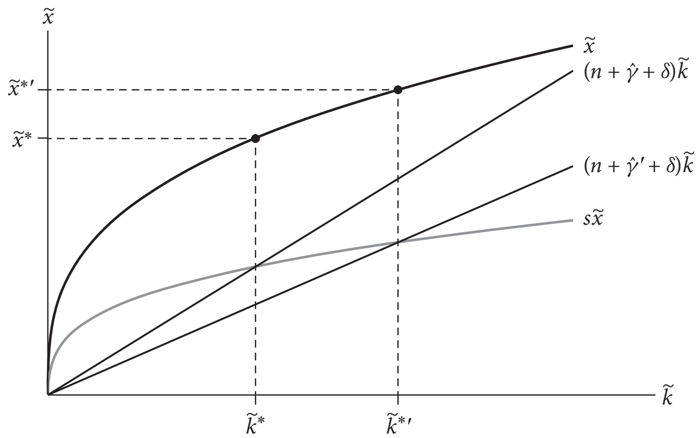

Figure 11.3 Productivity slowdown in the Solow–Swan model.

in the rate of labor-augmenting technical change. It is instructive to analyze the effects of a decline in the rate of technical change, $\hat{\gamma}$, on the steady state growth path of the Solow–Swan model.

In Figure 11.3 we have represented the old and new steady states associated with the old and new rates of technical change, $\hat{\gamma}$ and $\hat{\gamma}'$. Remember that the axes for this diagram are capital and output per effective worker. A reduced rate of Harrod-neutral technical change rotates the line representing required investment per effective worker downward. Thus, a lower $\hat{\gamma}$ will raise the steady state capital-effective labor ratio.

This increase in capital per effective worker will reduce the marginal product of capital and the profit rate, in conformity with the principle of diminishing marginal productivity. This change can also be visualized by means of the efficiency frontier for the economy. Figure 11.4 represents the efficiency frontier in effective labor terms.

We can see that the economy will be pushed up (to the northwest) along its efficiency frontier, and the effective wage will rise. This leads to the paradoxical situation that the wage received by each actual worker began to grow more slowly after 1970 (since it grows at the rate $\hat{\gamma}$), while the wage per effective worker must have increased. Similarly, output per worker grew more slowly after 1970, though output per effective worker increased. These conclusions underline the need to distinguish between comparative dynamics effects on the endogenous variables measured in effective and actual labor units.

Policy makers in the US have used the Solow–Swan model to evaluate proposals to reduce the government's budget deficit. Our model does not

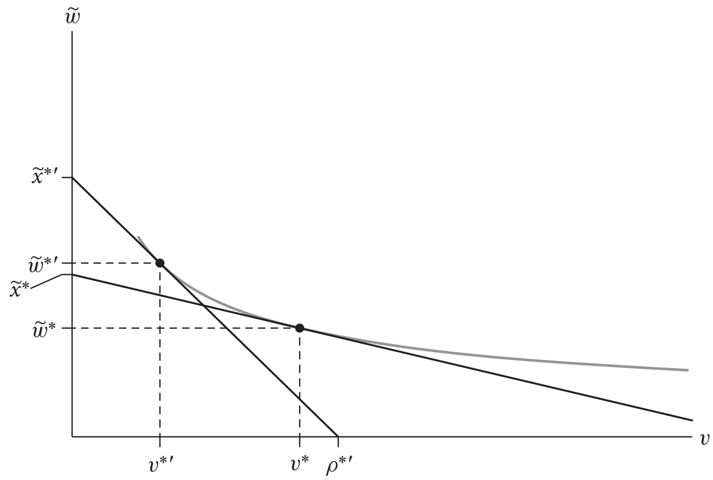

Figure 11.4 The steady state effective worker wage rises with a productivity slowdown.

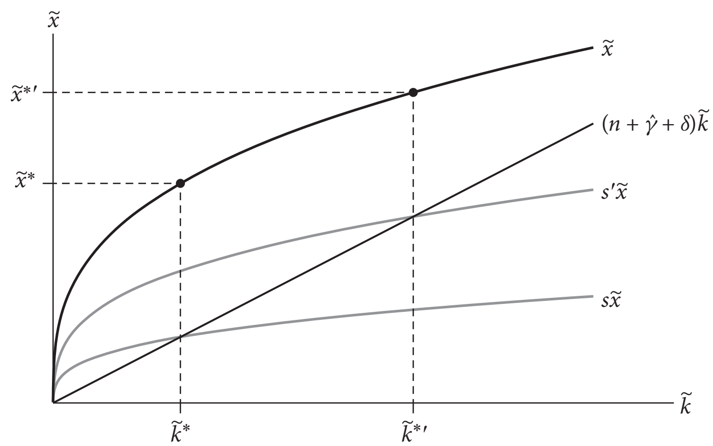

Figure 11.5 Effects of an increase in the saving rate on the steady state of the Solow–Swan model.

contain an explicit government sector, but one possible assumption is that an increase in government saving (i.e., reduction in the budget deficit) would raise the national saving rate. Analytically, an increase in the parameter s will shift up the saving function, as shown in Figure 11.5. The new steady state will therefore have more capital per effective worker and more capital per real worker. This increase in capital per worker will imply a higher level of output per worker. If the economy starts out below its Golden Rule capital-labor ratio, the increased productivity of labor will also permit more consumption per effective worker.

As in Chapter 10, an increase in the saving rate has no effect on the long run growth rate of the economy. During the transition to the new equilibrium, of course, there will be a temporary increase in capital accumulation and in output growth. However, at the new steady state, the growth rate of output will settle down to its natural rate, $(n + \hat{\gamma})$, and the growth rate of output per worker and capital per worker will return to the rate of technical progress, $\hat{\gamma}$.

During a period when the US government's fiscal deficit was a major concern among policy makers, the Council of Economic Advisors in its 1994 Economic Report of the President calibrated the Solow–Swan model to fit the US economy in an effort to estimate the long run benefits of reducing the budget deficit. They used a Cobb-Douglas production function with $\alpha = 1/3$: as we saw above, this is approximately the value of the profit share in the US. They assumed that $n + \hat{\gamma} = 2.5\%$/year, $\delta = 9\%$/year, and that the deficit-reduction package then being contemplated would raise the national saving rate from 13 to 14% of GDP. With these parameters, they calculate that it will take about 50 years to reach a new steady state. At the new steady state they predict that the capital stock (per effective worker) will increase by nearly 12% and that this increase will drive down the rate of profit by about 2 percentage points. It will also raise wages and productivity by about 3.75%, and consumption by more than 2.5%. Estimates derived from implementations of the Solow–Swan model play an important role in policy debates.

PROBLEM 11.9 Analyze the effects of an increase in the rate of population growth on the steady state in the Solow–Swan model with Harrod-neutral technical change. Explain your results in terms of both the figure and the equations representing the model.

PROBLEM 11.10 Use the figures given in the text to check the calculations of the Council of Economic Advisors. Assume that the scale parameter in the production function $A = 750$, the rate of population growth $n = 1.5\%$/year, and the rate of Harrod-neutral technical change $\hat{\gamma} = 1\%$/year. Calculate the old and new values of $\tilde{k}^*$, $\tilde{x}^*$, and $\tilde{c}^*$.

PROBLEM 11.11 Assuming it takes 50 years to reach the new equilibrium, by how much will the increase in national saving considered by the Council of Economic Advisors succeed in raising consumption/worker (*not* /effective worker)?

PROBLEM 11.12 Calculate the profit rate before and after the increase in national saving using the same Council of Economic Advisors assumptions.

11.6 Transitional Dynamics in the Solow–Swan Model

The fact that the Council of Economic Advisors predicts that it would take fifty years to reach a new steady state after a major change in the national saving rate underlines the importance of transitional dynamics in the Solow growth model. An economy that is not in its steady state can undergo a lengthy process of adjustment during the transition to a new steady state. Is it possible that the enormous disparities in levels of development of economies we observe in the world could be the result of such long processes of adjustment? If this were true, economic growth would gradually equalize levels of development through a process of *convergence*.

A tendency for countries to converge on the same level of labor productivity is called *absolute convergence*. Absolute convergence implies that economies that are farther behind the leader will have faster labor productivity growth. We have seen in Chapter 8 that there is evidence for absolute convergence among already advanced economies, but not for all economies. Large differences in the growth rates of output and productivity among some economies persist over time. If these economies are assumed to be on their individual steady state growth paths, then differences in rates of labor productivity growth can be explained within the Solow–Swan model only by differences in the exogenous rate of technical change. But the model itself does not explain the exogenous rate of technical change, so the persistence of differences in rates of labor productivity growth among economies is a challenge to the neoclassical model. One way the neoclassical model can be defended is to assume that most countries are undergoing a process of adjustment toward their long run steady states, and to attribute differences in rates of labor productivity growth to capital deepening.

In this context, the Solow–Swan model makes three predictions about economic growth. First, it predicts that among economies that share access to the same technology, have similar population growth and similar saving behavior, there will be a tendency for productivity to converge over long enough periods of time. This would explain the prevalence of absolute convergence among the advanced economies.

Second, the neoclassical model predicts that among economies that share access to the same technology but have different population growth rates and saving behaviors, there will still be a tendency for economies that are far behind to grow faster, but only after controlling for these differences in population growth and saving. This tendency is called *conditional convergence*. Economies tend to grow faster the farther they are from their *own* steady state. There is now widespread agreement that conditional convergence prevails at the global level, although its meaning remains open to alternative interpretations. The conditional convergence interpretation makes the Solow–Swan model consistent with the failure of absolute convergence to prevail globally. Thus, economic data can be regarded as qualitatively consistent with the predictions of the Solow–Swan model with respect to absolute and conditional convergence.

Third, the Solow–Swan model makes specific predictions about the *speed* of convergence, which can be checked against the empirical experience of actual economies. Most recent studies suggest that convergence among real capitalist economies is slower than the Solow–Swan model predicts it should be. In this respect, the predictions of the Solow–Swan model, at least in its basic form, are quantitatively inconsistent with economic data.

When the Solow–Swan economy reaches its steady state, output and capital per worker grow at the rate of technical change, $\hat{\gamma}$. But when the economy lies below its steady state level of capital intensity, the capital stock will grow faster than the labor force. The fundamental equation of the Solow–Swan model describes this rate of accumulation. With a Cobb-Douglas production function, the fundamental equation (in effective labor terms) is:

$$g_{\tilde{k}} = sA\tilde{k}^{\alpha-1} - (n + \hat{\gamma} + \delta)$$

This equation shows that the growth rate of capital per effective worker will be higher the less capital there is per effective worker (since α is less than 1). This higher rate of capital accumulation owes its existence to diminishing marginal productivity of capital, which makes the level of output per effective worker higher when there is less capital per worker.

If two economies share similar values of the parameters s, n, $\hat{\gamma}$, and δ, it follows that the poorer country (with the lower level of capital per worker) will grow faster, illustrating how absolute convergence works. This same equation shows that a poor country may not grow faster than a rich country if the poor country has a low enough saving rate or a high enough rate of population growth, illustrating how conditional convergence works.

If we had data on output and capital per worker for a large number of economies, we might be able to estimate the fundamental equation. In its

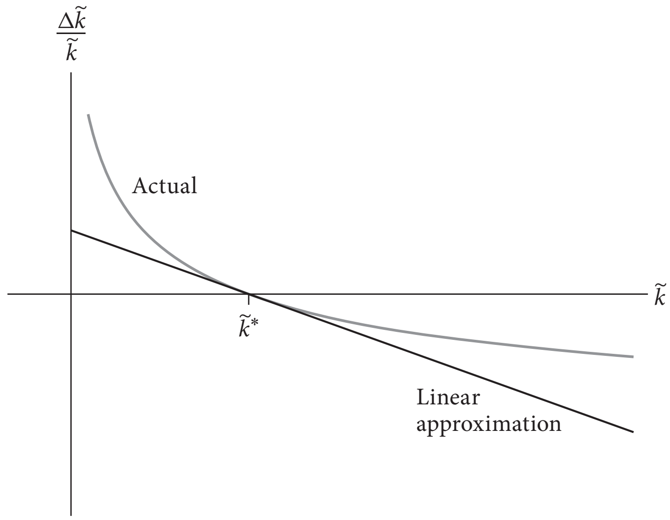

Figure 11.6 The linear approximation to the fundamental equation near the steady state.

present nonlinear form, however, it is a difficult equation to estimate, so economists use a linear approximation that is regarded as reasonably accurate if economies are close enough to their steady state positions. The approximation is based on the mathematical technique of taking a first-order Taylor expansion of the fundamental equation around the steady state value of $\tilde{k}$ as illustrated in Figure 11.6. This figure shows the actual shape of the fundamental equation, and the linear approximation that provides a good fit for points near the steady state position.

In most applications, economists find it easier to work with a linear approximation of a version of the fundamental equation that is expressed in terms of the logarithm of output per effective worker. The details of the derivation are left to the Appendix. The logarithmic output convergence equation for the Solow growth model is:

$$\Delta \ln(\tilde{x}) = \phi(\ln(\tilde{x}^*) - \ln(\tilde{x}))$$

where

$$\phi = (n + \hat{\gamma} + \delta)(1 - \alpha)$$

There is now a large econometric literature estimating this convergence equation, and the value of ϕ is generally estimated to be near .02, which is too low to be consistent with observed profit shares of 1/4 to 1/3. For example, if we take the commonly used values $n = .02$, $\hat{\gamma} = .02$, and $\delta = .03$, we will find that $\phi = .02$ implies that $\alpha = .71$. Since the marginal productivity theory of distribution predicts that the profit share should be equal to α, there is a conflict between the neoclassical predicted and actual profit shares.

Some neoclassical economists, notably Gregory Mankiw, David Romer, and David Weil, explain this conflict by arguing that the relevant concept of capital in the Solow–Swan model needs to be broadened to include human capital as well as physical capital. They interpret the share of profits implied by the convergence coefficient to include both profits as normally understood and the returns to education. These returns show up in the higher wages that more skilled workers are able to earn. This would imply that about half the wages of workers are really returns to human capital. Under this hypothesis only about 1/3 of the national income takes the form of labor income per se, while 1/3 represents the return to human capital, and 1/3 represents the return to physical capital. The Cobb-Douglas production function consistent with this hypothesis would add human capital, H, to the other two inputs, taking the form $X = AK^{1/3}N^{1/3}H^{1/3}$.

This effort to rescue the Solow–Swan model remains controversial. There is little dispute that education, particularly primary and secondary schooling, plays an important role in the growth process. The neoclassical interpretation is that the intellectual skills produced by education are a form of capital, so that increases in the amount of skill directly produce increases in output. The alternative interpretation coming from the technology gap literature is that intellectually skilled workers facilitate the transfer of technology, which speeds up the catching-up process. This suggests that a high level (rather than a high growth rate) of intellectual skills is associated with increases in output. If the alternative interpretation is correct, the conflict between the predicted and actual profit share may not be so easily resolved.

PROBLEM 11.13 Find the approximate rate of growth of labor productivity for an economy whose current level of labor productivity is 3/4 of its steady state value under the assumptions that the rate of Harrod-neutral technical change is 1%/year, the depreciation rate is 4%/year, population grows 2%/year, and the gross profit share is 1/3.

PROBLEM 11.14 If population growth is 2%/year, Harrod-neutral technical change is 1%/year, and the depreciation rate is 4%/year, find the implicit gross profit share for a convergence coefficient $\phi = .02$.

PROBLEM 11.15 If the gross profit share were 1/3, what should be the value of the convergence coefficient in Problem 11.14?

11.7 Suggested Readings

The Solow decomposition appears in Solow (1957). Opinion about the size of the residual ranges from Denison (1967) to Jorgenson (1995), with the latter arguing that more careful measurement reduces the residual considerably. A critical review of the whole growth accounting approach is given by Nelson (1973; 1981). For the New Growth Theory approach, see Romer (1987b).

For an early example of the study of absolute convergence among advanced countries, see Baumol (1986). The methodology of estimating convergence speeds owes much to Robert Barro (cross-country equations are often called Barro Equations); see Barro and Sala-i-Martin (1992). A modern defense of the extended Solow–Swan model appears in Mankiw (1995) and the article mentioned in the text, Mankiw et al. (1992). There is now a large literature on growth regressions. Benhabib and Spiegel (1994) find that intellectual skills do not affect growth significantly when treated as a form of capital (contradicting Mankiw, Romer, and Weil) but that they do affect growth through the transfer of technology discussed in the text. For some overviews on the empirical methodologies, see Quah (1993) or Durlauf et al. (2005), as well as Barro and Sala-i-Martin (2011). An influential critique of the Mankiw, Romer, Weil result is Klenow and Rodriguez (1997); also see Caselli (2005). Generally, this work supports a larger role for international differences in technology. Bernanke and Gürkaynak (2001) find that growth is significantly related to the saving rate, calling into question the assumption of exogenous technical change.

For two different perspectives on intellectually skilled labor, consult Becker (1964) for the traditional treatment of human capital as an ordinary input and Nelson and Phelps (1966) for the idea that intellectually skilled labor chiefly facilitates the transfer of technology.

The source cited in this chapter for applying the Solow–Swan model to the East Asian economies is Young (1995).

Appendix: Deriving the Convergence Equation

First, using the fact that $g_k = \Delta k/k$, rearrange the fundamental equation as follows:

$$\Delta\tilde{k} = sf(\tilde{k}) - (n + \hat{\gamma} + \delta)\tilde{k}$$

Next, take a first-order Taylor expansion of the right hand side of this equation, around the steady state value of $\tilde{k}$:

$$\Delta\tilde{k} \approx sf'(\tilde{k}^*) - (n + \hat{\gamma} + \delta)(\tilde{k} - \tilde{k}^*)$$

At the steady state $s = (n + \hat{\gamma} + \delta)(\tilde{k}/\tilde{x})$, so that we can write:

$$\Delta\tilde{k} \approx \left(\frac{f'(\tilde{k}^*)\tilde{k}^*}{\tilde{x}^*} - 1\right)(n + \hat{\gamma} + \delta)(\tilde{k} - \tilde{k}^*)$$

According to the neoclassical theory of distribution, the profit rate is equal to the marginal product of effective capital, $v = f'(\tilde{k}^*)$, and $f'(\tilde{k}^*)\tilde{k}^*/\tilde{x}^*$ is equal to the profit share, π. Now define $\phi \equiv (1 - \pi)(n + \hat{\gamma} + \delta)$ and collect terms to get:

$$\frac{\Delta\tilde{k}}{\tilde{k}} = \phi\left(\frac{\tilde{k}^*}{\tilde{k}} - 1\right)$$

Using the fact that $(\frac{\tilde{k}^*}{\tilde{k}} - 1) \approx \ln(\frac{\tilde{k}^*}{\tilde{k}})$ when $\tilde{k}$ is close to $\tilde{k}^*$, we can write this equation in the form of a convergence equation for effective capital intensity, $\tilde{k}$:

$$\frac{\Delta\tilde{k}}{\tilde{k}} \approx \phi \ln\left(\frac{\tilde{k}^*}{\tilde{k}}\right)$$

If we assume a Cobb-Douglas production function, for simplicity:

$$\frac{\Delta\tilde{x}}{\tilde{x}} = \alpha\frac{\Delta\tilde{k}}{\tilde{k}}$$

$$\ln\left(\frac{\tilde{x}^*}{\tilde{x}}\right) = \alpha \ln\left(\frac{\tilde{k}^*}{\tilde{k}}\right)$$

Substituting these expressions into the effective capital intensity convergence equation gives us a convergence equation in terms of effective labor productivity:

$$\frac{\Delta\tilde{x}}{\tilde{x}} = \phi \ln\left(\frac{\tilde{x}^*}{\tilde{x}}\right) = \phi(\ln(\tilde{x}^*) - \ln(\tilde{x}))$$

(It can be shown that this convergence equation holds for any constant returns to scale production function.)

12

Demand-Constrained Economic Growth

12.1 The Global Crisis

On September 15, 2008, the US investment bank Lehman Brothers filed for protection under Chapter 11 of the US bankruptcy code. This event marks the beginning of a crisis that spread throughout the entire world economy and whose consequences were still unfolding nearly a decade later.

From the financial sector, the crisis spread to the real economy rather quickly. Because most of the largest banks found themselves in stressful financial positions, the US credit market came to a halt. It became harder and harder for firms to finance their investment projects, and a wave of layoffs followed. The US economy experienced the worst economic downturn since the Great Depression of the 1930s. By the beginning of 2010, the US unemployment rate was over 10 percent. Because of its pervasiveness and severity, this event is commonly referred to as the Great Recession. The crisis quickly spread to the rest of the world through global financial markets and through international trade. While the actual recession only lasted for about a year and a half in the US, the recovery has been particularly slow.

The Great Recession raises the theoretical issue of the role played by aggregate demand in the growth process. This chapter introduces a class of growth models in which growth is demand-constrained so that aggregate demand plays a central role in economic growth. It also addresses the relationship between a demand-constrained growth model and models that suppress the distinction between aggregate demand and aggregate supply, such as the Classical and neoclassical growth models.

12.2 Measuring Demand Shocks

An important concept in the measurement of economic activity is *potential GDP*, which represents the level of output when capital and labor resources are fully utilized. In a recession or depression, substantial unemployment and underemployment of machines, workers, and other resources express themselves in an *output gap* between potential and actual GDP.

Economic data showing the behavior of potential and actual GDP raise some challenging questions about the constraints on economic growth. We have seen that in the neoclassical growth model and in the full employment version of the Classical growth model, the ultimate constraint on growth is the effective labor force, that is, the labor force taking into account the role of technical progress. Capital accumulation must adapt to this constraint in the long run. Economists call models with this property *exogenous growth models* because the factors governing long run growth are taken to be exogenous to the economic relationships within the model.

Because recessions are usually short-lived, economists guided by exogenous growth theories consider the output gaps created by recessions to be temporary deviations from a long-term growth path. These gaps are often the subject of specialized short run macroeconomic models that abstract from growth. This conventional intellectual division of labor makes sense as a research framework and it is profoundly influential in the formation of macroeconomic policy. But the Great Recession and its aftermath raise serious questions about its limitations.

The conventional approach assumes that the long run growth path is governed by supply factors that are independent of the demand shocks that cause recessions. But large demand shocks such as the Great Recession can potentially affect supply factors. For example, an extended period of unemployment may discourage workers from participating in the labor force, convince immigrant workers to return home, or result in a deterioration of workers' skills, so that when the economy recovers, it achieves an equilibrium level of employment and output that is lower than it would have been in the absence of the demand shock. The economy does not return to a preexisting path of potential GDP because the shock itself alters potential GDP. Since the deviations from potential GDP alter the path the economy ultimately takes, economists refer to this behavior as *path dependence*. Under path dependence, temporary shocks have permanent effects.

We have also seen that in the conventional wage share version of the Classical growth model the rate of capital accumulation is free to vary because

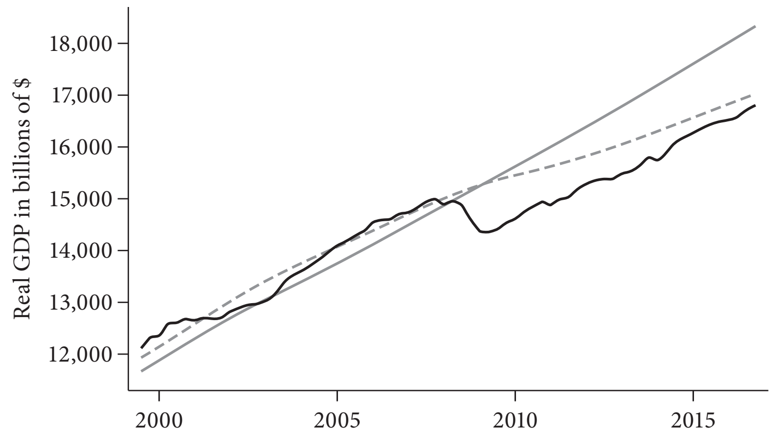

Figure 12.1 United States Real GDP (black) *versus* the CBO potential GDP estimate in 2008 (gray), and the CBO potential GDP estimate in 2017 (dashed) are plotted for the period 1999–2016. While the US economy has been growing since 2010, it is still operating below its potential. The downward revision in the potential GDP estimate by the CBO underlines the depth of the 2008 crisis. Sources: Federal Reserve Economic Database and CBO.

the rate of growth of the labor force can adapt. Models with this property are called *endogenous growth models* because the rate of capital accumulation is a variable determined within the model itself. The demand-constrained growth model we develop in this chapter is also an endogenous growth model. Endogenous growth models exhibit path dependence with respect to capital and employment because these variables do not return to a preexisting path after a temporary shock.

In order to make fully informed judgments, it makes sense to be alert to the possibility that growth can be exogenous, endogenous, or path dependent.

To illustrate the issues involved, Figure 12.1 shows actual GDP (in constant dollars) of the US economy and estimates of the potential GDP produced by the Congressional Budget Office (CBO). The figure shows estimates of potential GDP made just before and well after the Great Recession.

The CBO methodology accepts the conventional view that output gaps are temporary deviations from a supply-determined growth path and that the growth path is free of path dependence. It defines potential GDP as the level of output that stabilizes the inflation process, so that it corresponds to full employment equilibrium in the labor market. In fact, the CBO explicitly uses the Solow growth model in its methodology.

The persistence of the output gap in Figure 12.1 calls into question the assumption that output gaps are temporary deviations that express themselves in V-shaped recessions. The long recovery evident in the figure suggests an L-shaped recession that would be consistent with path dependence. The methodology of the CBO guarantees that as long as the inflation process stabilizes, their estimate of potential GDP will converge eventually with actual GDP.

Moreover, in Figure 12.1 it is clear that the growth rate of potential GDP fell quite dramatically after the Great Recession. (Over such a short interval of time, the slope of the graph approximately reflects the growth rate.) It is not clear why the trends of population growth and technical change would change so abruptly in 2008, and in fact the CBO's own forecasts of potential GDP made before the Great Recession did not project this slowdown. An alternative explanation might be that growth is endogenous, and that the underlying rate of capital accumulation declined in the aftermath of the Great Recession, bringing down the growth of the effective labor force with it. The methodology used by the CBO does not permit us to determine whether the slower growth evident in the figure reflects an exogenous change in supply conditions or an endogenous reduction in capital accumulation since it treats any change in the trend of the labor force and technology as an exogenous factor by assumption.

These issues assume practical importance in policy discussions about decisions that affect economic growth. Under the conventional or mainstream approach, the projected future path of potential GDP represents a policy barrier that cannot be crossed without causing inflation or other problems. But if growth is endogenous, it makes more sense to visualize the projected path of potential GDP as a cone-shaped range of possibilities rather than a linear and predetermined future.

12.3 Saving, Investment, and Output

Entrepreneurs play a limited and passive role in the Classical and neoclassical models: their only functions are to translate capitalist saving decisions into investment, and to choose the profit-maximizing technique of production. Keynesian economic theory, however, insists on the importance of the distinction between decisions to save, taken by the capitalist wealth holder, and decisions to invest, taken by entrepreneurs. *Investment* in this context refers to decisions to purchase new capital goods, while *saving* refers to decisions to refrain from consuming a portion of income. Entrepreneurs' decisions to invest can play a pivotal role in determining the actual growth path of an economy. The Classical and neoclassical models implicitly abstract from this distinction by assuming that all savings are automatically invested in real capital.

In real capitalist economies, the relation between saving and investment is more complicated. Savers generally accumulate financial assets such as money, stock certificates issued by firms, or bank deposits. Assets can provide potential financing for real investment, but are not directly purchases of real capital goods. To take the simplest example, households in a monetary economy can save by hoarding money, without any corresponding increase in the purchase of capital goods. In real economies, the decisions of savers and entrepreneurs' decisions to invest are linked by complex financial mechanisms.

Keynes argued that, at least in the short run, it is the level of output that adjusts in order to equalize planned investment and saving. Changes in the short run level of output determine changes in the degree of *capacity utilization* of the economy. The "Keynesian cross" diagram found in most macroeconomics textbooks describes the multiplier process through which levels of output change to generate the saving required to match any level of planned investment.

Keynesian models of economic growth build on these insights. Like the Classical models, Keynesian growth models do not view growth as constrained by the availability of labor. When a Keynesian economy is operating at less than full capacity utilization, the existing capital stock cannot constrain output, either. In the Keynesian tradition the willingness of entrepreneurs to invest is the key constraint on output and the growth of capital. Thus, growth is constrained by the demand for investment, which in turn is one of the determinants of aggregate demand: for this reason, we will refer to this model as a *demand-constrained* growth model.

Demand-constrained growth models lead to two characteristic conclusions that appear paradoxical from a Classical or neoclassical perspective. The *paradox of thrift* shows that an increase in the propensity to save, holding the willingness to invest constant, results in slower growth of capital at lower levels of capacity utilization, because it reduces the demand for consumption goods. The paradox of thrift is at odds with the Classical conventional wage share model, in which an increase in the capitalists' propensity to save raises the rate of growth of capital. The *paradox of costs* shows that an increase in the wage, holding the willingness to invest constant, will increase capacity utilization (and perhaps the growth of capital), because it increases the demand for wage goods (consumption goods for workers). The paradox of costs is also at odds with the Classical conventional wage share model, which predicts that a rise in the wage, by lowering the profit rate, will reduce the growth rate through the Cambridge equation.

12.4 A Model of Demand-Constrained Growth

We will develop a model of demand-constrained growth in the Keynesian tradition, keeping as close as possible to the familiar elements of the Classical conventional wage share model.

First, we continue to assume that capitalists save a constant fraction, β, of their end-of-period wealth. The Cambridge equation therefore continues to describe the accumulation of wealth in the economy. Since capitalists own the firms operated by entrepreneurs, they receive the profits that are distributed, in the form of monetary dividends or interest payments. We will use the superscript s to identify the growth rate of the capitalist's financial wealth:

$$1 + g_K^s = \beta(1 + r) = \beta(1 + v - \delta), \quad \text{or} \tag{12.1}$$
$$g_K^s + \delta = \beta v - (1 - \beta)(1 - \delta)$$

The hallmark of a Keynesian model is the introduction of independent entrepreneurial decisions to invest. An investment equation proposed by Joan Robinson relates entrepreneurs' target rate of growth of capital to the expected rate of profit. The central idea is that if entrepreneurs expect a higher rate of profit, their *animal spirits* will be excited and they will be willing to take more gambles on investment projects whose uncertain returns lie far in the future.

This theory can be represented mathematically by an equation relating the target rate of growth of capital, g_K^i, to the actual rate of profit, v. We must be careful about the interpretation of the rate of profit in this equation. Robinson argued that the actual rate of profit would provide entrepreneurs with a forecast about the future only if it persisted at a stable level for some time. Thus, the Robinsonian investment equation is not meant to be true instantaneously, but only after the economy has been in a stable position for some time, so that the actual rate of profit accurately reflects the expected rate of profit. We write Robinson's investment equation $g_K^i + \delta$ proportional to v:

$$g_K^i + \delta = \eta v \tag{12.2}$$

In this equation the parameter η (the Greek letter *eta*, pronounced "ay′-ta") represents the propensity to invest out of profits, the animal spirits of the entrepreneurs. If you have difficulty visualizing exactly how entrepreneurs formulate the investment decisions described by (12.2), you can take some comfort from the fact that Keynes himself regarded this decision as inherently resistant to economic theorizing or modeling.

In equilibrium, the actual rate of growth of the capital stock must be consistent both with entrepreneurs' investment plans and capitalists' saving plans:

$$g_K^i = g_K^s (= g_K) \tag{12.3}$$

When we add equations such as (12.2) and (12.3) to the Classical conventional wage share model, we face the problem of *overdetermination*: since we have added two equations and only one additional endogenous variable (g_K^i, since g_K^s takes the place of g_K), we have too many equations for the number of endogenous variables. In order to avoid this dilemma, we have to add another endogenous variable as well.

There is a natural resolution to this problem, which is to recognize that a Keynesian economy operates with some *excess capacity*, and add the *rate of capacity utilization*, u, to the list of endogenous variables. The rate of capacity utilization is a positive fraction between 0 and 1, indicating how much of the economy's productive potential is being realized.

The impact of changes in capacity utilization on the growth-distribution schedule depends on how entrepreneurs adjust inputs of labor and capital to fluctuations in demand, and such adjustment depends on economic conditions.

In general, labor productivity falls less than capital productivity during recessions. Figure 12.2 shows the growth-distribution schedules for two deep recessions experienced by the US economy. The left panel compares 1979, when capacity utilization was high, to recession year 1982, when capacity utilization was low. The right panel compares the growth-distribution schedule for 2006, when capacity utilization was high, to recession year 2009. In both cases, capital productivity decreased more than labor productivity.

In the comparisons presented in Figure 12.2, the growth-distribution schedule rotates clockwise. Keynesian economists have looked at this issue by assuming that only measured capital productivity is affected by the utilization rate u. Thus, labor productivity remains at x, but the *actual* productivity

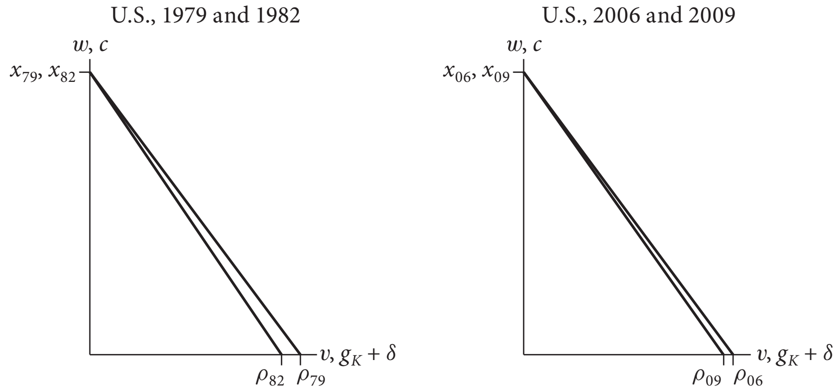

Figure 12.2 When the US economy experienced a serious recession in the early 1980s, and in 2008–2009, capacity utilization declined. The US growth-distribution schedules for 1979 and 1982 on the one hand, and for 2006 and 2009 on the other, illustrate the tendency for ρ to decline more than x.

of capital is equal to $u\rho$. As a result, the capital intensity, and therefore the slope of the growth-distribution schedule $k = x/(u\rho)$, will be affected by the utilization rate u.

As in the Classical conventional wage share model, we assume that labor will be elastically supplied at a conventional wage share. Because money and a well-developed financial system lie in the background of the Keynesian model, we should think of workers being paid in money and purchasing wage goods at the prevailing price level. Prices must adjust to changes in money wages to keep the wage share in income constant. Since labor productivity is unaffected by capacity utilization, the conventional wage share equation is:

$$w = (1 - \bar{\pi})x \tag{12.4}$$

On the other hand, when the rate of capacity utilization is less than 1, the real wage-profit rate equation and the social consumption-growth rate equation depend on the actual productivity of capital, $u\rho$ (as opposed to potential capital productivity ρ), and on labor productivity x. The slope of the growth-distribution schedule is, once again, $-k = x/(u\rho)$.

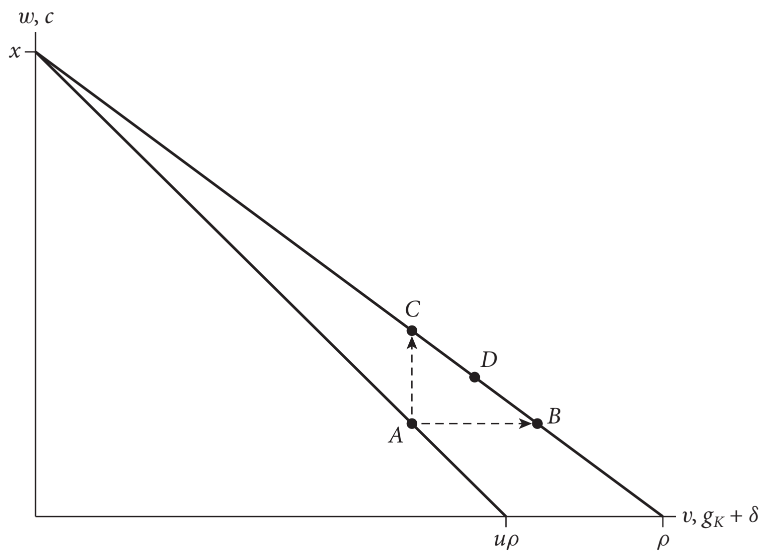

Figure 12.3 When capacity utilization is lower than 1, the growth-distribution schedule rotates inward, so that actual capital productivity $u\rho$ falls while labor productivity x stays constant. The capital intensity k increases, and the growth-distribution schedule becomes steeper.

$$w = x - vk = x\left(1 - \frac{v}{u\rho}\right) \tag{12.5}$$

$$c = x - (g_K + \delta)k = x\left(1 - \frac{g_K + \delta}{u\rho}\right) \tag{12.6}$$

The growth-distribution schedule for the demand-constrained Keynesian economy is illustrated in Figure 12.3 for a given rate of utilization, u. This needs to be distinguished from the *full capacity growth-distribution schedule* that applies only when $u = 1$. As long as utilization is below full capacity, the actual growth-distribution schedule lies inside the full capacity growth-distribution schedule. Changes in capacity utilization in this model are capital-using, like Solow-neutral technical change. Most of our attention will focus on the real wage and profit rate, so let us focus on equation (12.5), which describes the *actual real wage-profit rate schedule*.

Along the full capacity real wage-profit rate schedule, an increase in the profit rate is always associated with a decrease in the wage. But when the economy is below full capacity, an increase in the rate of capacity utilization rotates the actual growth-distribution schedule outward, and therefore it creates the possibility (which did not exist before) of an increase in the profit rate with no change in the wage, or of an increase in the real wage with no change in the profit rate. It is even possible for the wage and profit rate to increase simultaneously when the rate of utilization rises. These possibilities are represented respectively by the movement from point *A* to point *B*, from point *A* to point *C*, and from point *A* to point *D* in Figure 12.3.

The actual profit rate is the product of the rate of capacity utilization, the profit share, and potential capital productivity according to equation (12.5).

$$v = \pi \rho u \tag{12.7}$$

If we substitute this into the investment equation (12.2), we find:

$$g_K^i + \delta = \eta u \pi \rho \tag{12.8}$$

Thus, the demand-constrained model assumes that entrepreneurs forecast the rate of profit on the basis of the prevailing rate of capacity utilization and wage share.

PROBLEM 12.1 Kaldoria is an economy similar to Industria (see Problem 2.2), with $x = \$50{,}000$/wkr-yr, $\rho = 1/3 = .33 = 33.33\%$, and $\delta = 1/15 = .0666 = 6.66\%$. The wage share in Kaldoria is 60%. Find the capital-labor ratio, actual capital productivity, and the profit rate when $u = 100\%$ and when $u = 85\%$.

PROBLEM 12.2 Graph on the same diagram the full capacity growth and distribution schedule for Kaldoria and the actual growth and distribution schedule at 85% utilization. Identify the points on both schedules when the wage share is 60%.

12.5 Equilibrium in the Demand-Constrained Model

The six equations of the demand-constrained model exactly determine the six endogenous variables, u, v, w, g_K^s, g_K^i, and c. The whole system is shown in Table 12.1. We will focus first on the subsystem of equations (12.1), (12.2), and (12.3). These equations can be solved for the equilibrium rate of profit, v:

$$v = \frac{(1-\beta)(1-\delta)}{\beta - \eta} \tag{12.9}$$

Table 12.1 The Demand-Constrained Model

Endogenous variables: u, v, w, g_K^i, g_K^s, c
Exogenous parameters: k, x, δ, β, $\bar{\pi}$, η

$$w = x\left(1 - \frac{v}{u\rho}\right) \tag{12.5}$$

$$c = x\left(1 - \frac{g_K + \delta}{u\rho}\right) \tag{12.6}$$

$$g_K^s + \delta = \beta v - (1-\beta)(1-\delta) = \beta(1 + v - \delta) - (1-\delta) \tag{12.1}$$

$$g_K^i + \delta = \eta v \tag{12.2}$$

$$g_K^s = g_K^i (= g_K) \tag{12.3}$$

$$w = (1 - \bar{\pi})x \tag{12.4}$$

In order to avoid negative profit rates, we must assume that $\beta > \eta$. The economic interpretation of this condition is that the propensity to save out of profits must be greater than the propensity to invest. Once we have calculated the equilibrium profit rate, it is straightforward to find the equilibrium values of the other endogenous variables.

From equations (12.7) and (12.9) we can see that the equilibrium level of capacity utilization is just:

$$u = \frac{(1-\beta)(1-\delta)}{\bar{\pi}\rho(\beta - \eta)} \tag{12.10}$$

On the other hand, the equilibrium real wage is simply:

$$w = (1 - \bar{\pi})\, x \tag{12.11}$$

Next, from equation (12.2), we see that the equilibrium rate of growth of capital is:

$$g_K = \eta v \tag{12.12}$$

Finally, we can calculate the equilibrium level of social consumption per worker, c:

$$c = x\left(1 - \frac{(g_K + \delta)}{u\rho}\right) = x\left(1 - \eta\pi - \frac{\delta}{u\rho}\right) \tag{12.13}$$

When the Cambridge equation and the Robinson investment function are plotted together, as in Figure 12.4, we see that, provided that $\beta > \eta$, there is a

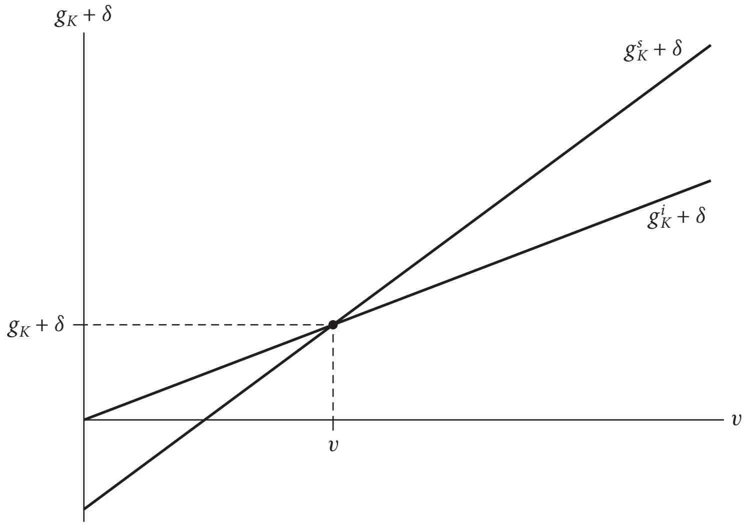

Figure 12.4 For each rate of profit the Cambridge equation determines the growth rate of capital consistent with capitalists' saving plans, and the Robinson investment function determines the growth rate of capital consistent with entrepreneurs' investment plans. Since the Cambridge equation has a lower intercept and higher slope than the Robinson investment function, under the assumption that $\beta > \eta$, there is a unique equilibrium level of the profit rate, v, and the gross growth rate of capital, $g_K + \delta$. The equilibrium capacity utilization rate is then $u = v/\pi\rho$.

unique equilibrium level of the profit rate, and hence of capacity utilization. At profit rates below the equilibrium level, capitalists save too little to finance entrepreneurs' investment plans: the resulting excess demand will raise the capacity utilization rate and hence the profit rate. At profit rates above the equilibrium level, capitalist saving exceeds entrepreneurial investment, creating excess supply that will drive down the capacity utilization and profit rates. Thus, this unique equilibrium is also stable when $\beta > \eta$. This explains why $\beta > \eta$ is referred to in the literature as the *Keynesian stability condition*.

Violation of the Keynesian stability condition can shed some light on Roy Harrod's stability problem.[1] With $\eta > \beta$, a profit rate below the equilibrium level would create an excess supply of output because capitalist saving would exceed entrepreneurial investment. As a result of low utilization, firms would

[1]The formal model would need to be modified to accommodate this change so that there is a meaningful equilibrium with positive growth and profit rates. For example, we could add an intercept term to the investment equation.

cut back on production and utilization would decline further. This would further reduce investment spending and exacerbate the excess supply of output. Harrod emphasized that in this situation individual firms respond to excess capacity by sharply reducing their investment, hoping to eliminate the excess by bringing the growth of their capital stocks below the growth of sales. But when all firms do this together, it results in such a drop in demand that utilization goes down even further. In Harrod's view, any increase in the warranted rate of growth caused by an increase in the saving rate would disrupt the (unstable) equilibrium and send the economy into a downward spiral of stagnation of output, employment, and investment. Modern followers of Harrod have proposed that some other mechanisms might be expected to contain the instability that results from such a strong response of investment to excess capacity.

We will proceed on the assumption that the Keynesian stability condition is satisfied.

PROBLEM 12.3 Entrepreneurs in Kaldoria (see Problem 12.1) have a Robinsonian investment function with $\eta = .7$. What is their desired gross rate of capital accumulation, $g^i_K + \delta$, when the rate of utilization, u, is .9?

PROBLEM 12.4 Capitalist households in Kaldoria (see Problem 12.1) have a propensity to save out of wealth $\beta = .97$. What is their desired gross rate of wealth accumulation, $g^s_K + \delta$, when the rate of utilization, u, is .9?

PROBLEM 12.5 If the entrepreneurs had expected the rate of utilization to be .7, and invested on the basis of the corresponding profit rate, what utilization rate would the Kaldorian economy achieve? Would the entrepreneurs find that they had chosen the right amount of investment? How would they respond?

PROBLEM 12.6 Calculate the equilibrium rate of utilization, gross rate of growth of capital, and rate of profit in Kaldoria.

PROBLEM 12.7 Graph the saving and investment equations for Kaldoria, and identify the equilibrium. Where on your graph does the economy lie in Problem 12.3? Discuss the dynamics in this position.

12.6 Comparative Dynamics in the Demand-Constrained Model

The demand-constrained model gives rise to three comparative dynamic results which are characteristic of Keynesian models, but appear paradoxical

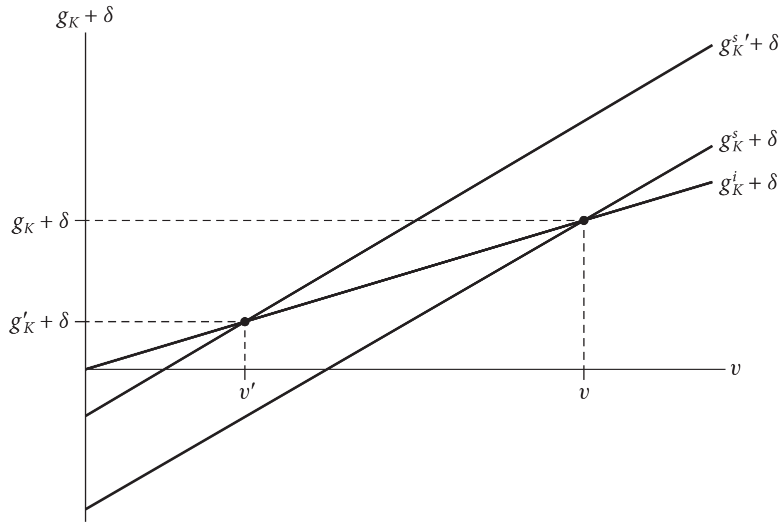

Figure 12.5 The paradox of thrift. When β rises, the savings schedule shifts and rotates upward, cutting the unchanged investment schedule at a lower rate of profit, v, and hence lower rates of capacity utilization and gross capital growth.

from the point of view of the Classical or neoclassical tradition. First, there is a *paradox of thrift* in the demand-constrained model. When the saving rate, β, increases, holding the investment propensity, η, constant, it shifts and rotates the savings schedule, equation (12.1), upward. The new equilibrium occurs at lower rates of profit, utilization, and gross capital growth, as Figure 12.5 shows.

This paradox of thrift contrasts sharply with both the Classical theory, in which increases in saving increase growth, and the neoclassical theory, where an increase in the saving rate increases growth only in the short run. In the Keynesian model, in which investment demand can be held constant, less capitalist consumption leads to insufficient demand to maintain the original rate of utilization. The reduced rate of profit that results will induce less growth. Perhaps nothing illustrates the nature of a demand-constrained economic system better than the paradox of thrift. It is easy to see why economists who view growth as demand-constrained rarely join campaigns to raise national saving rates.

A second distinctive feature of Keynesian models is the *paradox of costs*. An increase in the conventional wage share, holding constant the investment propensity, has a positive effect on the rate of capacity utilization and level of output, despite the fact that each individual capitalist perceives it as increasing her costs. An increase in the conventional wage share lowers the profit share, π, but leaves the equilibrium rates of profit, $v = (1 - \beta)(1 - \delta)/(\beta - \eta)$, and gross growth of capital, $g_K + \delta = \eta v$, unchanged in Figure 12.4. But the equilibrium rate of capacity utilization, $u = v/\pi\rho$, rises with a fall in π.

When the wage share increases, the redistribution of income from capitalists to workers reduces the rate of growth of financial wealth, since workers consume all their income. Thus, a wage share increase reduces overall saving and raises effective demand at each rate of capacity utilization. As a result, the rate of capacity utilization rises to generate the saving needed to finance investment. In its emphasis on the adjustment of capacity utilization to accommodate an unchanging propensity to invest, the paradox of costs bears a noticeable family resemblance to the paradox of thrift.

The third characteristic comparative dynamics result in the demand-constrained model arises from considering the impact of a rise in η, the propensity to invest. Keynes called the outcome of this experiment the *Widow's Cruse*. If entrepreneurs decide to spend more on investment, they will find that once they begin spending, the profits to finance the expenditure miraculously appear, just as in the Biblical tale of the widow whose jar (cruse) of oil would miraculously refill whenever it was drawn down. For example, a surge in animal spirits, represented by an upward shift in η, will rotate the investment schedule, equation (12.2), upward, increasing the rates of profit, gross growth of capital, and capacity utilization to provide entrepreneurs with enough capitalist saving to finance their projects, as Figure 12.6 shows. The equilibrium rate of profit, $v = (1 - \beta)(1 - \delta)/(\beta - \eta)$, rises with an increase in η, and the rate of capacity utilization, $u = v/\pi\rho$, and the gross growth rate of capital, $g_K + \delta = \eta v$, follow suit. The ability of entrepreneurs to invest profits before they have earned them in the Keynesian analysis relies critically on the existence of a financial system in the background that can advance funds to the entrepreneurs.

PROBLEM 12.8 Suppose the saving propensity in Kaldoria increased from .97 to .98. What are the new equilibrium rates of profit, gross growth of capital, and capacity utilization? Would an increased rate of saving benefit the Kaldorian economy?

PROBLEM 12.9 Suppose the saving propensity in Kaldoria remained at .97, but the wage share increased to 65%. What are the new equilibrium rates

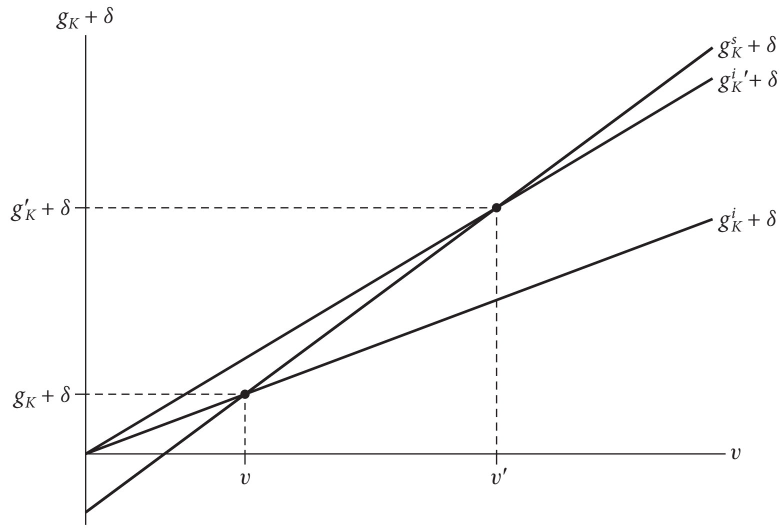

Figure 12.6 The Widow's Cruse. An increase in the entrepreneurial propensity to invest, η, rotates the investment schedule upward to intersect the unchanged saving schedule at a higher rate of profit, v, and gross growth rate of capital, $g_K + \delta$. The higher profits and saving are made possible by an increase in capacity utilization, $u = v/\pi\rho$.

of profit, gross growth of capital, and capacity utilization? Would an increased wage share benefit the Kaldorian economy?

PROBLEM 12.10 Calculate social consumption per worker before and after the increase in the wage share in Problem 12.9. How has this change been divided between capitalists' consumption and workers' consumption?

PROBLEM 12.11 Graph the real wage-profit rate and social consumption-growth rate schedules for Kaldoria before and after the wage share increases from 60% to 65%. Find the equilibrium wage-profit rate and social consumption-growth rate points on your graph before and after the change (four points in all).

12.7 Profit-Led or Wage-Led Growth?

The Robinsonian investment function makes the entrepreneurs' target gross rate of investment, $g_K^i + \delta$, proportional to the profit rate, v. But the profit rate is itself the product of the profit share, π, the capacity utilization rate, u, and the productivity of capital, ρ. One way to generalize Robinson's idea, proposed by Stephen Marglin and Amit Bhaduri, is to allow each of these factors to have an independent influence on investment plans. The simplest way to do this is to make planned gross investment a linear function of the three components of the profit rate:

$$g_K^i + \delta = \eta_u u + \eta_\pi \pi + \eta_\rho \rho \tag{12.14}$$

Here $\eta_u > 0$ represents the impact of capacity utilization on investment plans, and analogously for $\eta_\pi > 0$ and $\eta_\rho > 0$.

This generalization adds some intriguing insights to the demand-constrained model. The equilibrium of the generalized demand-constrained model requires saving and investment plans to be consistent, which implies a level of capacity utilization:

$$u = \frac{\eta_\pi \pi + \eta_\rho \rho + (1-\beta)(1-\delta)}{\beta\pi\rho - \eta_u} \tag{12.15}$$

To keep the rate of capacity utilization positive, we must assume that $\beta\pi\rho > \eta_u$. This condition is analogous to the condition $\beta > \eta$ in the basic model, and implies that saving rises faster than investment as capacity utilization increases, which ensures the stability of the equilibrium.

As in the simpler version of the demand-constrained model, the profit rate is proportional to the rate of capacity utilization:

$$v = \pi\rho u \tag{12.16}$$

The gross rate of growth of capital can be calculated from the Cambridge equation:

$$g_K + \delta = \beta v - (1-\beta)(1-\delta) = \beta(1 + v - \delta) - (1-\delta) \tag{12.17}$$

The paradox of thrift continues to hold in the generalized model: an increase in the capitalist saving propensity increases the denominator and reduces the numerator of equation (12.15), and thus lowers the rates of capacity utilization and profit. An increase in β has offsetting effects on the gross growth rate of capital, since β rises and v falls in equation (12.17). In the Appendix to the chapter we show, however, that the v effect predominates, and the gross growth rate of capital falls with an increase in β, as in the simple model.

The generalized demand-constrained model, however, can respond to an increase in the wage share in more complex ways than the paradox of costs of the basic model. If the value of $\eta_\pi = 0$, for example, an increase in the wage share lowers the profit share, which lowers the denominator of equation (12.15), and thus raises the equilibrium level of capacity utilization. A rise in the wage share has offsetting effects on the profit rate, $v = \pi\rho u$, since u rises but π falls. It is possible to show that when $\eta_\pi = 0$ the rise in u prevails, and the profit rate rises. (See the Appendix.) In this case the gross growth rate of capital, $g_K + \delta$, also rises with a rise in the wage share, and the paradox of costs continues to hold. This case is called *wage-led growth*, since an increase in the wage share raises the rates of capacity utilization and growth. Wage-led growth occurs because the increase in workers' consumption demand has a positive feedback effect on investment, through raising the rate of utilization. Since $\eta_\pi = 0$, there is no dampening effect through changes in profitability from the wage share increase at all.

On the other hand, if $\eta_u = 0$, then $\eta_\pi \pi > \eta_u u$, and the profit rate will rise with a rise in the profit share, leading to an increase in the gross growth rate of capital. This case is sometimes called *profit-led growth*. Profit-led growth occurs because an increase in the profit share can have offsetting effects on demand: a higher profit share reduces consumption demand by redistributing income away from workers, but increases investment demand, through raising profitability. For profit-led growth to occur, the increase in investment demand must dominate the reduction in consumption demand.

As these two extremes suggest, and the Appendix shows, whether growth is wage-led or profit-led depends critically on the relative value of the parameters in the investment and saving equations. Real economies may alternate between periods in which one or the other regime prevails.

PROBLEM 12.12 Suppose the entrepreneurs in Kaldoria are behaving according to the following investment function: $g^i + \delta = .25\pi$, and the wage share is $.6 = 60\%$. Use the equations in the Appendix to this chapter to find the equilibrium rates of capacity utilization, profit, and gross growth of capital. What happens to the endogenous variables when the wage share rises to $.65 = 65\%$? Explain what has happened.

PROBLEM 12.13 Suppose the entrepreneurs in Kaldoria shift to behaving according to the following investment function: $g^i + \delta = .1u$. Find the equilibrium rates of capacity utilization, profit, and gross growth of capital when the wage share is $.6 = 60\%$, and when the wage share is $.5 = 50\%$. Explain what has happened.

12.8 Long Run or Short Run?

Even among economists who accept Keynes's vision, opinions are divided about whether the conclusions we have reached with the demand-constrained model are valid in the long run. The terms *long run* and *short run* can take on different meanings, depending on context. The key idea is that the short run equilibrating processes operate much faster than the long run processes so that it is analytically and practically useful to separate the two concepts of equilibrium.

It is important to be aware that the speeds of adjustment in question are relative rather than absolute. A short run equilibrium does not refer to any specific short period of time such as a quarter or a year. The experience of the US economy in the aftermath of the Great Recession chronicled in Figure 12.1 illustrates that it can take many years for a capitalist economy to adjust fully after a major shock.

There are some aspects of the demand-constrained model which suggest that it might apply only when the economy is out of equilibrium along some dimension, even though it has adjusted fully in other dimensions. In particular, the model assumes that entrepreneurs continue to invest in the face of excess capacity, which suggests that they have not fully adjusted to long run equilibrium. Excess capacity would be defined as a rate of utilization below what is regarded as desired or normal utilization of the capital stock, which we might take to be $u = 1$. It seems paradoxical that entrepreneurs would continue to add to capacity when they are not even using their existing capital stock fully. The demand-constrained model can be seen as a long run model only if some convincing resolution of this paradox is offered.

One resolution is that the normal or desired rate of capacity utilization cannot be reduced to a single well-defined value. Instead, normal utilization might consist of a range of values. If this is true, then the demand-constrained model does remain valid as a long run model as long as the economy remains within this range, which might in practice be quite broad.

In an alternative resolution, firms form expectations about the normal or desired rate of capacity utilization by looking back at the rates that have prevailed in the past, and thus come to accept excess capacity as a normal condition (i.e., a "norm"). In this case, the normal rate of utilization adjusts over time until it is equal to the actual rate. For example, a period of high utilization will lead entrepreneurs and firms to revise upward their sense of normal utilization. Models with this mechanism for distinguishing the short run from the long run often include a similar adjustment process for the expected growth of demand, which can also be a determinant of investment spending.

Under either of these treatments of capacity utilization, the demand-constrained model continues to make distinctive predictions about the growth process. In particular, the paradox of thrift, whereby an increased saving rate tends to reduce utilization and growth, continues to apply even in the long run. However, as we have seen, the paradox of cost does not necessarily apply since growth in the demand-constrained model can be wage-led or profit-led depending on the parameter values.

Many economists, on the other hand, view the demand-constrained model as a short run model. The relation between short and long run equilibrium in demand-constrained models remains controversial.

One interpretation sees the investment function as the expression of financial constraints on entrepreneurs' plans to invest. (In this view the demand-constrained model is actually a *finance constrained* model of growth.) Profits relieve entrepreneurs from the need to go to financial markets or banks to raise funds to finance investment. The profit rate thus could influence investment plans through changing entrepreneurs' ability to finance expenditures from their retained earnings. In this interpretation, entrepreneurs anticipate that the economy will gravitate toward full capacity utilization, and make investment decisions based on their forecast of the full capacity utilization rate of profit, in spite of the presence of excess capacity in the present.

If this interpretation is to be complete, it must explain how the economy gets to the long run position, where utilization is at its normal capacity level. In our presentation of the demand-constrained model, this adjustment to long run equilibrium could take place through shifts over time in the coefficients of the investment demand equation or through shifts in the capitalist saving equation.

Keynes saw shifts in the investment function as representing changes in financial constraints on investment spending. Any forces tending to push the economy to a long run equilibrium level of capacity utilization must, in this perspective, operate through the financial system that lies in the background of the demand-constrained model.

One familiar mechanism of financial adjustment is the *Keynes effect*, which is used in many intermediate macroeconomics textbooks to explain the existence of an "aggregate demand curve." The Keynes effect occurs when the price level falls during periods of excess capacity, but the nominal money supply remains constant. In this case, the real money supply will increase, which, according to the Keynesian theory of liquidity preference, reduces the interest rate on loans to entrepreneurs to finance investment. More generally, we get a Keynes effect when the rate of growth of prices (i.e., the inflation rate) drops below the rate of growth of the nominal money supply. The Keynes effect can lead to an upward shift in the investment function, through increases in η. But most economists (including Keynes) regard the Keynes effect as too weak to self-stabilize modern capitalist economies.

A more convincing argument is that active demand management by the central bank is required to stabilize the economy. A central bank, faced with widespread excess capacity, might inject liquidity into the banking system, making loans more readily available to firms. Central banks monitor the inflation rate closely, and inflation tends to respond to variations in capacity utilization. Most central banks in advanced economies have explicit or implicit inflation targets so that when excess capacity emerges and inflation drops below the target, they ease up the key interest rates they control in order to reflate the economy. Again, this relaxation of financial constraints on investment could shift the investment function upward. If these shifts take place more slowly than the saving and investment decisions we have been examining, the demand-constrained model would be valid in the short run, but the economy might gravitate toward a long run equilibrium of normal capacity utilization.

Yet another possible mechanism might involve capitalist consumption and saving. The same tendency for central banks to inject liquidity during periods of excess capacity will tend to boost the prices of financial assets such as stocks and bonds that are disproportionately held by capitalist households. If capitalist consumption spending depends on capitalist wealth, the rise in asset prices will make the households feel richer, consume more, and save at a lower rate. The saving function will shift downward, which will create (through the paradox of thrift) an expansion in demand. We examine a model with financial assets and wealth effects in Chapter 15.

These equilibrating mechanisms work equally well (perhaps better) in a situation of overutilization of capacity, when $u > 1$. (For this to occur, we must treat u as a fraction of a desired rate of capacity utilization, which leaves some spare capacity to cover surges in demand, rather than as a strict technological limit.) When a surge in demand occurs, it will push the level of utilization closer to the technological limit. Inflationary forces in such an overheated economy would tend to increase the financial constraints on

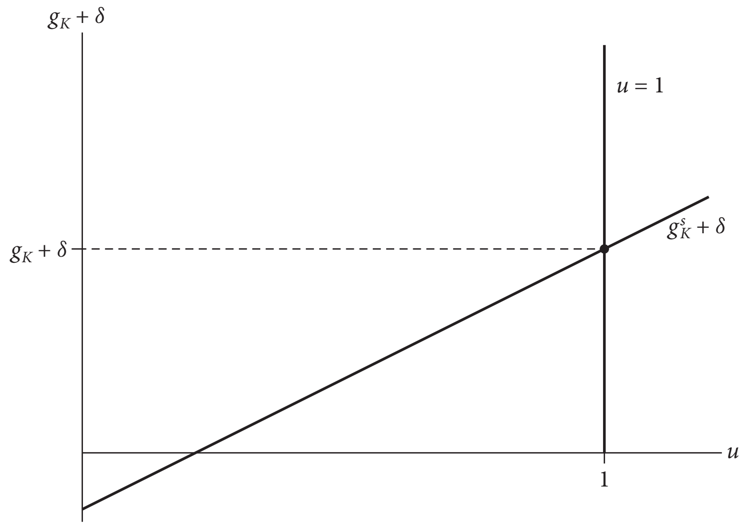

Figure 12.7 In long run equilibrium investment demand adjusts to saving, and the Cambridge equation again determines the rate of growth of capital.

investment since monetary tightening is a dependable response from modern central banks when inflation breaks above the target rate.

In the long run, the economy would be attracted to the full utilization of capacity at $u = 1$, as illustrated by Figure 12.7. In this figure, we are assuming that the equilibrating mechanism operates through the investment function. In this situation, the investment function becomes superfluous, replaced by the condition that $u = 1$. In the long run, then, the demand-constrained model transforms itself into the Classical model with a conventional wage share. Economists sometimes say that the economy is Keynesian in the short run but Classical in the long run.

This way of compartmentalizing macroeconomics into short and long runs comes naturally to a wide range of modern economists, even though they may disagree about which models are right in each instance. Such an approach offers the advantages of an intellectual division of labor by having specialized models to deal with short run and long run issues separately.

The debate about capacity utilization and capital accumulation remains unresolved. In this text, we have taken the view that the demand-constrained model provides a valuable addition to the macroeconomic toolkit. For some truly long range issues we think the Classical models are more appropriate, at least as a starting point for attacking the problems at hand. But the insights of the demand-constrained model, such as the Keynesian problem of coordinating saving and investment decisions of disparate agents, cannot be easily dismissed, particularly in the historical shadow of the Great Recession, which was a long-lasting aggregate demand failure of epic proportions.

12.9 The Distributive Curve

The demand-constrained growth model can be seen as embedding the distributional insights of the Classical conventional wage share closure into a model where the Keynesian notion of effective demand is the main factor driving equilibrium output and growth. It is reasonable to assume, however, that at least in the short-to-medium run the real wage can vary with the rate of utilization.

Such considerations are at the heart of the so-called *structuralist* models that focus on the social and institutional causes that determine the interaction between effective demand, growth, and income distribution, and on the effect of redistributive policies in favor of workers. On the one hand, these models look at whether the rate of utilization increases or decreases with the profit share, that is, whether aggregate demand is wage-led or profit-led. On the other hand, it is also important to evaluate the response of income shares to changes in aggregate demand. Suppose that the profit share depends on utilization through the *distributive curve*:

$$\pi = \pi_0 + \pi(u) \tag{12.18}$$

The sign of the derivative $\partial\pi/\partial u$ determines whether the profit share decreases or increases with the utilization rate. For instance, a negative $\partial\pi/\partial u$ means that the profit share decreases and the wage share increases as the economy approaches full capacity. This case is often referred to as *profit squeeze*, because when the economy is expanding toward full capacity the labor market becomes tighter and real wages increase at the expense of profitability.

In our model we have assumed a linearized investment function for simplicity. In this case, as is shown in the appendix, the level of utilization will always be negatively related to the profit share. This pattern represents *wage-led aggregate demand*. With a more general investment function, however, it is possible for the effect of the profit share on investment to be strong enough to generate *profit-led aggregate demand*. Empirical work by Lance Taylor with several coauthors has pointed out that, in post–World War II

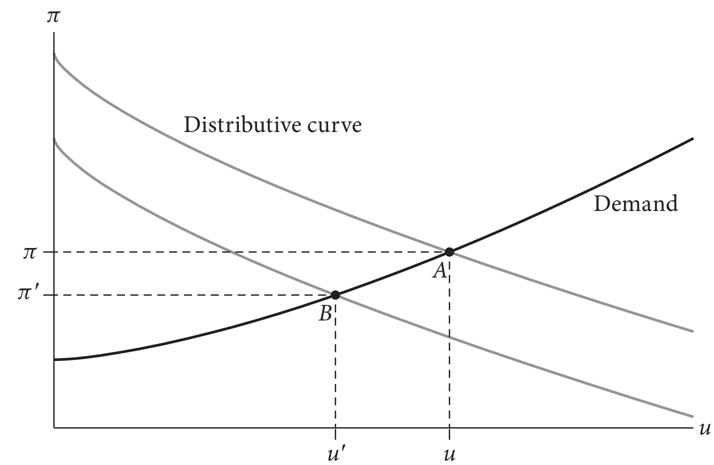

Figure 12.8 A redistribution in favor of workers reduces the profit share. Because aggregate demand and growth are profit-led, utilization and the growth rate decrease.

United States, aggregate demand appears to be profit-led while the distributive curve is characterized by a profit squeeze. This scenario is represented in Figure 12.8, where effective demand is upward sloping in the profit share, while the distributive curve is downward sloping.

The intercept parameter π_0 in the distributive curve captures the effect of redistributive policies. For instance, a higher tax on profits or an increase in the minimum wage would reduce the value of π_0. In the profit-led/profit-squeeze scenario, the effect of such policy would be to lower the equilibrium profit share; since demand is profit-led, this will lower the equilibrium rate of utilization and growth rate. The movement from point *A* to point *B* in Figure 12.8 illustrates this situation.

Suppose instead that aggregate demand is wage-led, and that the distributive curve is upward sloping as depicted in Figure 12.9. An upward-sloping distributive curve is sometimes referred to as exhibiting *wage squeeze*, or Kaldorian behavior, because as economic activity (that is, u) increases, distribution must shift in favor of profits in order to maintain the current level of investment. Consider as before the effect of a redistributive policy that reduces the value of π_0. Because utilization and growth are wage-led, utilization and growth will increase. This case is illustrated by the movement from point *A* to point *B* in Figure 12.9.

Different configurations of demand and distribution are also possible: the main conclusion of the structuralist growth model is that the ultimate effect

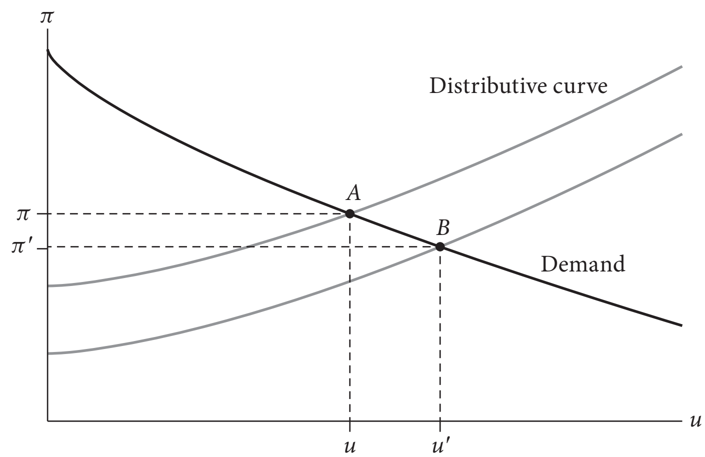

Figure 12.9 A redistribution in favor of workers reduces the profit share. Because aggregate demand and growth are wage-led, utilization and the growth rate increase.

of economic policies aimed at improving the workers' distributional position depends on the structural characteristics of the economy, as captured by the actual slopes of the demand curve and the distributive curve. These characteristics are not set in stone, but can change over time: the challenge for policy makers is to empirically evaluate whether redistributive policies (which might be desirable per se) will foster economic activity and growth as in the wage-led/forced-saving case, or will result in an economic slowdown as in the profit-led/profit-squeeze case.

PROBLEM 12.14 Consider an expansionary policy, be that fiscal or monetary, that shifts the aggregate demand schedule so that utilization increases at any given level of the profit share. Illustrate the effects of this policy on the equilibrium utilization rate and profit share in the wage-led/wage-squeeze scenario and in the profit-led/profit-squeeze scenario.

12.10 The Keynesian Contribution to Growth Theory

The demand-constrained model introduces fundamentally new considerations into the modeling of capitalist economic growth. Both Classical and neoclassical traditions see saving as the engine of capital accumulation and assume that saving decisions always lead to a corresponding decision to invest. In these models *Say's Law* holds, and there can be no discrepancy between aggregate demand and supply (though Say's Law does allow for disequilibrium between demand and supply in particular markets). The introduction of an independent investment demand function together with the rate of capacity utilization as an endogenous variable breaks the identity between saving and investment, and generates a class of Keynesian models in which Say's Law does not hold. As we have seen, the results of comparative dynamics experiments in demand-constrained models are quite different from the parallel results in Classical and neoclassical models. Parametric changes, such as a rise in the saving propensity, or in the profit share, that raise the rate of growth of capital in Classical and neoclassical models can lower the rate of growth of capital in demand-constrained models when the investment demand schedule is unchanged. These differences in comparative dynamics lead to different evaluations of policies toward growth, as well. In the Classical and neoclassical perspective redistribution of income from profits to wages may be viewed as desirable in itself, but comes at a price in terms of slower capital accumulation. In the Keynesian models these trade-offs are less painful or even nonexistent: redistribution can move the economy to a higher rate of capacity utilization and thus create a larger income to be divided between profits and wages, and a larger output to be used as consumption and investment.

The great policy debates in macroeconomics and growth economics of the last half of the twentieth century have concerned the limits of applicability of the Keynesian models. Many economists agree that Keynesian effects are important in the short run, but question whether Keynesian analysis can safely be used to guide long run economic policy toward economic growth. These debates should presumably be settled by looking at the empirical evidence as to how strong the tendencies moving capitalist economies toward full capacity utilization actually are. But econometric techniques for answering this question are themselves in dispute, and macroeconomic evidence is limited, so the policy dilemma remains unresolved.

12.11 Suggested Readings

Roy Harrod (1939) is probably most responsible for the birth of Keynesian growth theory, while Nicholas Kaldor (1956) and Luigi Pasinetti (1974) developed Keynesian/Classical models of full employment growth. These economists in turn were influenced by Michal Kalecki (1971), who discovered the theory of the multiplier simultaneously with Keynes (1936).

The model in the text (which does not presume full employment) owes much to Joan Robinson (1964) and of course Amit Bhaduri and Stephen

Marglin (Bhaduri and Marglin 1990). See Blecker (2002) for a particularly clear treatment of wage-led versus profit-led regimes. The Bhaduri-Marglin paper spawned a large literature; e.g., the October 2016 and subsequent three issues of the *Review of Keynesian Economics* are devoted to this paper's legacy. The survey papers there by Setterfield (2016) and Blecker (2016) cover the theoretical and empirical controversies over distribution-led demand. Taylor (2010) and Kiefer and Rada (2015) apply the distributive curve and provide some evidence for profit-led Goodwin-style cycles.

For other contributions in this broad tradition, see Taylor (1983); Dutt (1990), and Palley (1996). Skott (1989) advocates a neo-Harrodian approach. For the view that normal utilization is a range or band of values, see Dutt (2010). For a survey of the debates over capacity utilization in the long and short run, see Lavoie (2014) and Nikiforos (2015).

The original reference to the Widow's Cruse can be found in Keynes (1930).

Appendix: The Marglin-Bhaduri Model

We can solve the investment and saving demand equations for the equilibrium level of capacity utilization by equating planned saving and investment.

$$g_K^i + \delta = \eta_u u + \eta_\pi \pi + \eta_\rho \rho$$
$$g_K^s + \delta = \beta(1 + v - \delta) - (1 - \delta)$$

Using $v = \pi\rho u$, we get the equilibrium equation:

$$\beta\pi\rho u - (1 - \beta)(1 - \delta) = \eta_u u + \eta_\pi \pi + \eta_\rho \rho$$

which can be solved for equilibrium u:

$$u = \frac{\eta_\pi \pi + \eta_\rho \rho + (1 - \beta)(1 - \delta)}{\beta\pi\rho - \eta_u}$$

Differentiating the equilibrium conditions with respect to β, we see that:

$$\frac{du}{d\beta} = -\frac{1 + v - \delta}{\beta\pi\rho - \eta_u} < 0$$
$$\frac{dv}{d\beta} = \pi\rho\frac{du}{d\beta} < 0$$
$$\frac{d(g_K + \delta)}{d\beta} = -\frac{\eta_u(1 + v - \delta)}{\beta\pi\rho - \eta_u} < 0$$

Thus the paradox of thrift results continue to hold in the generalized demand-constrained model.

Differentiating the equilibrium conditions with respect to π, however, yields:

$$\frac{du}{d\pi} = \frac{\eta_\pi - \beta\rho u}{\beta\pi\rho - \eta_u}$$

Thus the sign of $du/d\pi$ is the same as the sign of $\eta_\pi - \beta\rho u$, and will always be negative, as we can see from the equilibrium condition above. Similarly:

$$\frac{dv}{d\pi} = (\eta_\pi \pi - \eta_u u)\frac{\rho}{\beta\pi\rho - \eta_u}$$

Thus the sign of $dv/d\pi$ is the same as the sign of $\eta_\pi \pi - \eta_u u$, and will be positive when $\eta_\pi \pi > \eta_u u$.

$$\frac{d(g_K + \delta)}{d\pi} = \beta\frac{dv}{d\pi}$$

If $\eta_\pi = 0$, we see that:

$$\frac{dv}{d\pi} = -\eta_u u \frac{\rho}{\beta\pi\rho - \eta_u} < 0$$

If $\eta_u = 0$, on the other hand:

$$\frac{dv}{d\pi} = \frac{\eta_\pi}{\beta} > 0$$

Thus the impact of an increase in profit share on growth depends on the coefficients of the investment function, leading to the distinction between wage-led and profit-led growth regimes.

13

Land-Limited Growth

13.1 Non-Reproducible Resources

In the Classical conventional wage model where labor-power is supplied elastically at a given wage, all inputs are reproducible. The economy can produce capital by itself, and, practically speaking, it can reproduce labor-power as well by paying the wage. In this type of model there are no resource limitations to growth. The growth rate of the economy is determined entirely by productivity and the propensity of capitalists to accumulate.

When labor-power is inelastically supplied and grows at an exogenously given rate, as in the Solow–Swan model, the forces determining the steady state growth rate change sharply. The long run growth rate of the economy has to adjust to the given natural growth rate of the labor force. Input prices must vary to make this adjustment.

In this chapter we will study a Ricardian economy where there is a fixed and limited amount of land that is necessary for production. We will suppose that property rights in land exist, creating both a *rental market* for the productive use of land in each period and a *land market* through which land can be bought and sold.

Capitalists' asset portfolios now include both capital and land, and their portfolio choices determine a price for land. Since capitalists can invest either in capital or in land, the returns from owning land and their expectations about the path of the future price of land play a central role in the economy. The introduction of this second asset raises issues of asset pricing that are fundamental to the modern theory of finance as well.

13.2 Ricardo's Stationary State

David Ricardo, a successful London stockbroker whose implacably logical analysis of economic growth and distribution was a major influence in the development of political economy, analyzed the growth of an economy with limited land in his *Principles of Political Economy and Taxation*.

Ricardo works with a *corn model* of production very similar to the Classical model we presented earlier. In this model he abstracts from the real diversity of commodities and assumes that the only produced good is *corn* (the comprehensive English term for all food grains), which we will take as the numéraire. Production of corn requires workers, whose wages must be paid during the production period between the planting of the corn and its harvesting. These advanced wages, together with the seed corn, constitute the capital required to carry on production. Ricardo follows Malthus in assuming that the wage (in terms of corn) is fixed at a level where the birth and death rates of the population are close to equal, as in Chapter 4. In our terms Ricardo's corn model is a conventional wage model.

In Ricardo's world there are three classes: workers, capitalists, and landowners. The capitalists rent land from the landowners and perform the entrepreneurial function of organizing production, as well as the capitalist function of owning and accumulating capital. The landowners own the land and rent it to the capitalists.

The central economic idea of Ricardo's model is that different plots of land have different natural fertilities. Conceptually we divide up all the land in the economy into *plots* that require the same *dose* of capital and labor to cultivate (though they may not all have the same actual area). Each plot of land has a certain *yield*, the average harvest that can be expected when the standard dose of labor and capital of standard quality are applied to it. Ricardo imagines that we can rank all the land in the economy according to its fertility, starting from the most fertile, and proceeding to the least fertile. If we graph the plots of land along the horizontal axis and the yield of each plot of land on the vertical axis, as in Figure 13.1, we can visualize the *diminishing returns* that are at the heart of Ricardo's thinking as a *marginal product schedule* of capital and labor applied together in fixed proportions. Since each plot of land requires the same amount of capital and labor, the distance along the horizontal axis measures the labor force (which Ricardo takes as proportional to population) and capital employed. The yield of the least fertile land in cultivation (the *extensive margin* in Ricardo's language) is

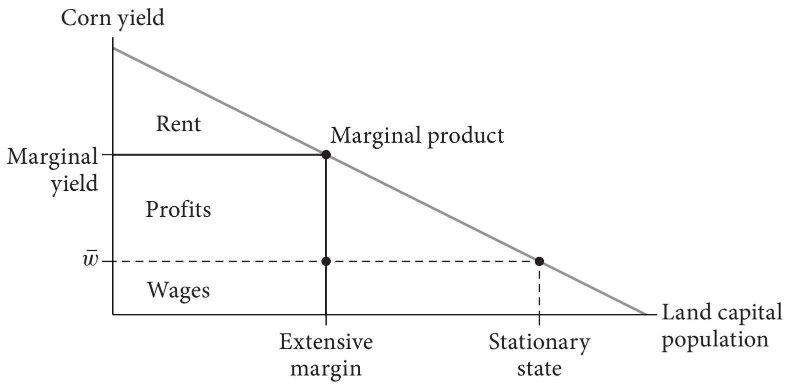

Figure 13.1 In Ricardo's model, plots of land are graphed along the horizontal axis in order of diminishing fertility, and the yield of each plot constitutes the marginal product schedule for capital and labor. The extensive margin is the least fertile land in cultivation. The Malthusian conventional wage divides the yield of the marginal land between wages and profit, and determines the profit rate for the whole economy. Rents on more fertile land are equal to the excess of the output on that plot over the output on the marginal land. The stationary state is the point where the yield on the marginal land is just equal to the conventional wage: when capital accumulates and population grows to this point, the profit rate falls to zero and accumulation stops.

the marginal product of capital and labor together, since the removal of one dose of capital and labor would remove the marginal land from cultivation and reduce the total output by its yield. The area under the marginal product schedule up to the extensive margin is the total corn output of the economy.

On this graph we can draw a horizontal line at the height of the real wage, determined so as to bring about a demographic equilibrium of births and deaths. This is the familiar conventional wage supply schedule of labor. The amount of capital accumulated in the past by capitalists can offer employment for a certain number of workers, and thus determines the population, as well as the amount of land in cultivation and the extensive margin, in Ricardo's framework. The area under the marginal product schedule above the real wage is the *surplus product* of the economy, since it represents the excess of production over what is necessary to keep the current population of workers alive.

This surplus is divided between landowners and capitalists as they bargain over the rents on the various plots of land. Ricardo argued that the owner of the marginal land in cultivation could charge only a nominal rent on her land, since other land almost as good is available and earning no rent at all. He also argued that competition among the capitalists would force the rent on any cultivated plot of land to the point where the capitalist's profit rate on that land after paying rent would just be equal to the profit rate on the marginal land that paid practically no rent. Thus in Figure 13.1 the horizontal line at the level of the yield of the marginal land divides the surplus into rent and profit. Profit is the rectangle between the wage line and the yield on the extensive margin, and rent is the area above the yield on the marginal land under the marginal product schedule.

Ricardo assumed, extrapolating the behavior he thought he observed in British society, that workers and landowners would spend all their income on consumption (workers on wages for subsistence, and landowners on staffing great houses with armies of liveried servants), and that capitalists would save all or most of their profit income, and accumulate it to expand production. As long as profits are positive, capital will be expanding, creating more jobs, supporting a larger population, and pushing the extensive margin into less fertile land. As this accumulation occurs, the wage remains constant, but the profit rate falls.

In the end, Ricardo predicts that capital accumulation and growth will stop when the extensive margin is pushed out to the point where the yield on the marginal land is just equal to the conventional wage. At this point the profit rate and total profits are zero, so there is no more capital accumulation, and all the surplus takes the form of rent. Ricardo called this situation the *stationary state*. At the stationary state most of a large population lives at the edge of subsistence, pressing on the limited resources of the earth, while a small wealthy class of landowners appropriate the social surplus product.

Ricardo's analysis of the stationary state has strong echoes in contemporary anxieties about resource depletion and environmental degradation as a result of economic growth. The diminishing returns that Ricardo modeled in terms of land could be seen as arising from the exhaustion of nonrenewable resources and the destruction of the environment, and the stationary state as the unhappy fate awaiting an overcrowded humanity on a finite planet.

13.3 Production with Land

Let us consider the one-sector model of production, but with the added assumption that land is required as an input. In this chapter we will assume that the technology of production is unchanging over time. We will use the letters U and u for land and land per worker (since L is easy to confuse with labor). The technique of production then becomes:

$$1 \text{ labor} + k \text{ capital} + u \text{ land} \rightarrow x \text{ output} + (1-\delta)k \text{ capital} + u \text{ land}$$

In other words, in this model one worker equipped with k units of capital and u units of land can produce x units of output at the end of a year. Capital depreciates at the rate δ, but land does not depreciate at all. We are free to measure land in any units (acres or hectares, for example), and the coefficient u will change proportionately. In order to simplify the equations of the model, we will take as our unit of land the amount of land required per unit of capital, so that $u = k$. Thus we can measure land and capital directly on the same scale.

In any period there is a fixed amount of capital, K_t, inherited from the past, and a fixed amount of land U. Unlike Ricardo's model, all the land here is assumed to be of the same fertility. The quantity of capital will change from period to period because the economy can produce capital and capitalists can accumulate it. The quantity of land, on the other hand, can never change.

The idea that there is a fixed resource limitation of some kind (like land in this model) is very strongly rooted in human attitudes toward economic growth, but is perhaps not very well confirmed by human experience. First of all, all resources require some development. Agricultural land must be cleared, drained, and plowed. Mineral resources must be discovered and developed (through the construction of mines, wells, and transportation facilities). Second, the process of technical change frequently renders resources obsolete before they are exhausted. The iron deposits in the Eastern United States that were the basis of pre–Civil War industrial development have become economically irrelevant because of the emergence of larger-scale iron mines in the West and in other countries. These deposits still exist, but it is unlikely that they will play any important role in economic production. This way of thinking suggests that we might best regard all resources as potentially producible, though some may be producible only at a very high cost. If this point of view is accurate, the model of land we are studying will be misleading.

As before, there are entrepreneurs who actually organize production by renting land and capital from capitalists and hiring labor. We suppose that labor is available at the conventional wage $\bar{w}$. We denote the profit rate on capital by v_{kt}, which is the amount of output the entrepreneur has to pay for the use of capital for one period.

The use of land as an input in production leads to the emergence of *land rent*. If entrepreneurs want to use all the land available in production, capitalists will be in a position as landowners to bargain for a rent on land, which we will call v_{ut}. The payer of rent gets to use the land in production for one period. The dimensions of v_{ut} are \$/unit of land/year. Since our land unit is the amount of land required to employ one dollar of capital, we could also express v_{ut} as \$/\$/year, or %/year, like the profit rate. If the profit rate is 10%/year and the rent on land 5%/year, the entrepreneur has to pay 15%/year to rent capital and land from capitalists.

The profit an entrepreneur makes on each worker employed will be output per worker less the wage and rent on both capital and land. Remembering that we are measuring land in units so that $u = k$, the entrepreneurs' profit is:

$$x - w - v_{kt}k - v_{ut}k$$

The entrepreneur must make zero profit, for the same reasons as in the one-input production model. If we reinterpret the profit share, π, to include rent on both land and capital, we can write this condition as:

$$v_{kt} + v_{ut} = \frac{x - w}{k} = \pi\rho \qquad (13.1)$$

13.4 The Capitalist's Decision Problem with Land

We will attack the analysis of this economy with the same methods we developed in Chapter 5. We will assume that there are a large number of identical capitalist wealth holders, each of whom begins owning the same share of the total capital and land in the economy. We continue to assume that these capitalists maximize the discounted sum of the logarithm of their consumption of output.

The introduction of land into the picture adds a new dimension to the typical capitalist's decision. In the model of Chapter 5, the typical capitalist had to choose between holding capital and consuming at the end of each period. Now the typical capitalist has an additional choice: she has to choose how much of her wealth to invest in land and how much in capital.

Even though the amount of land in the whole economy is fixed, each individual capitalist could in principle own more or less land. Thus the *asset price of land* in terms of output (or capital), which we will call p_{ut}, must adjust in each period to make the capitalists willing to hold the existing stocks of capital and land. The asset price of land is quite different from rent. The renter gets to use the land only for one period, while the purchaser of the land itself gets to keep the land until she wants to sell it, and collect rent on the land in all the periods she owns it.

The typical capitalist starts each period holding some land, U_t, and some capital, K_t.

At the beginning of period $t + 1$, then, the typical capitalist's source of funds will be the value of her depreciated capital together with the capital rent she has received, plus the value of her land together with the land rent she has received:

$$
\begin{aligned}
&v_{kt}K_t + (1-\delta)K_t + (p_{ut+1} + v_{ut})U_t \\
&\quad = K_t + (v_{kt} - \delta)K_t + (p_{ut+1} + v_{ut})U_t \\
&\quad = (1 + v_{kt} - \delta)K_t + (p_{ut+1} + v_{ut})U_t \qquad (13.2)
\end{aligned}
$$

These funds must be divided between consumption, C_t, and holdings of capital and land in the next period, K_{+1} and $p_{ut+1}U_{t+1}$. The typical capitalist's budget constraint with land is thus:

$$K_{t+1} + p_{ut+1}U_{t+1} + C_t \leq (1 + v_{kt} - \delta)K_t + (p_{ut+1} + v_{ut})U_t \qquad (13.3)$$

This constraint defines the typical capitalist's utility maximization problem, which is summarized in Table 13.1.

Table 13.1 Capitalist's Utility Maximization with Land

$$\text{choose } \{C_t, K_{t+1}, U_{t+1}\}_{t=0}^{\infty} \geq 0 \qquad (13.4)$$

$$\text{so as to maximize } (1-\beta)\sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to

$$K_{t+1} + p_{ut+1}U_{t+1} + C_t \leq (1 + v_{kt} - \delta)K_t + (p_{ut+1} + v_{ut})U_t \qquad (13.3)$$

$$K_0, U_0, \{v_{kt}, p_{ut}, v_{ut}\}_{t=0}^{\infty} \text{ given} \qquad (13.5)$$

13.5 The Arbitrage Principle

The new element in the capitalist's utility maximization problem is the decision as to how much of her wealth to invest in land and how much in capital in each period. We assume that the capitalist knows the paths of the price of land, the rental rate on land, and the profit rate with certainty, which greatly simplifies this problem. In the real world, the portfolio decision as to how to apportion wealth between competing assets, like equities and bonds, is highly dependent on the relative risk the wealth holder perceives in each choice. In our model, however, the issue of risk is absent. As a result, the model's typical capitalist will choose between holding land and capital purely on the basis of which has the higher rate of return.

A capitalist who chooses to hold a unit of land during period t at the price p_{ut} will have $v_{ut} + p_{ut+1}$ at the end of the period, since she will collect the rent on the land and still have the land to sell. She could, alternatively, have invested the money in capital instead and had $(1 + v_{kt} - \delta) p_{ut}$ at the end of the period. These two returns must be equal if the capitalist is to be willing to hold both land and capital in her portfolio. This is the *arbitrage principle*, which plays a central role in modern financial theory. Rational wealth holders will hold assets with equal risk only if their anticipated rates of return are equal. In our model the two assets are capital and land. They have the same (zero) risk, so capitalists will hold both only if they have the same rate of return. Furthermore, the capitalist is indifferent as to how much of her wealth she holds in capital and land as long as the rates of return on the two assets are identical. The mathematical expression of the arbitrage principle is:

$$1 + r_t \equiv 1 + v_{kt} - \delta = \frac{p_{ut+1} + v_{ut}}{p_{ut}} = 1 + g_{p_u t} + \frac{v_{ut}}{p_{ut}} \qquad (13.6)$$

The arbitrage principle tells us immediately that the rate of return to capital and land in each period must be equal, establishing a single rate of return, r_t, that applies to both assets. The rate of return to capital is the rental to capital, v_{kt}, less the rate of depreciation, δ, and the rate of return to land is the ratio of the rental to land to the land price, v_{ut}/p_{ut}, plus the capital gain or loss on land due to the change in its price, $g_{p_u t}$.

The arbitrage principle reduces the capitalist's utility maximization problem with land to the same form as the utility maximization problem with one asset, capital, that we have solved in Chapter 5. To see this, define the capitalist's total wealth in each period, $J_t = K_t + p_{ut} U_t$. The arbitrage principle assures us that she will get the same rate of return, r_t, whether she holds land or capital. Thus we can write the budget constraint as:

$$J_{t+1} + C_t \leq (1 + r_t) J_t \tag{13.7}$$

But this is exactly the budget constraint in Chapter 5, with wealth, J_t, substituted for capital, K_t. This makes sense, because in the earlier model the only form of wealth was capital. We already know the solution to this problem: the typical capitalist consumes a fraction $1 - \beta$ of her wealth at the end of the period:

$$C_t = (1 - \beta)(1 + r_t) J_t = (1 - \beta)(1 + r_t)(K_t + p_{ut} U_t) \tag{13.8}$$

The Cambridge equation now applies to the growth of total wealth:

$$J_{t+1} = \beta(1 + r_t) J_t \tag{13.9}$$

But the growth of capital itself is governed by the rule:

$$\begin{aligned} K_{t+1} &= J_{t+1} - p_{ut+1} U_{t+1} = \beta(1 + r_t) J_t - p_{ut+1} U_{t+1} \\ &= \beta(1 + r_t)(K_t + p_{ut} U_t) - p_{ut+1} U_{t+1} \end{aligned} \tag{13.10}$$

The implications of capitalists' utility maximization in the model with land boil down to the arbitrage principle of equation (13.6) and the consumption function of equation (13.8).

PROBLEM 13.1 Write down the Lagrangian function for the capitalist's utility maximization problem with land, and find the first-order conditions describing the saddle point. Use these conditions to derive the arbitrage principle and the consumption function.

13.6 Equilibrium Conditions

The analysis of Section 13.5 tells us how the typical capitalist will behave if she is confronted with a given path of prices, rents, and profit rates $\{p_{ut+1}, v_{ut}, 1 + r_t\}_{t=0}^{\infty}$.

But the prices, rents, and profit rates must be chosen so that the markets for capital, land rental, and land owning clear in each period.

First consider the market for land as an asset. We have allowed the typical capitalist to make a free choice as to how much land she will own in each period. In equilibrium, however, she has to wind up owning her share of the actual amount of land in the economy U. So land market clearing requires:

$$U_t = U \quad t = 0, 1, \ldots, \infty \tag{13.11}$$

But the rental land market has to clear as well. Entrepreneurs cannot plan to rent more land for production than exists. Furthermore, the rent on land will depend on whether entrepreneurs want to rent all the land or not. If there is so little capital in the economy that the entrepreneurs cannot use all the existing land, the land rent must be zero. If the land rent is positive, it must be the case that all the land is used. This turns out to be a key aspect of the growth path of this economy. Since we measure land in the same units as capital, rent will be zero if $K_t < U$, and can be positive only when $K_t = U$:

$$\begin{aligned} &K_t \leq U \quad (= \text{ if } v_{ut} > 0) \quad \text{or} \\ &v_{ut} = 0 \quad \text{if } K_t < U \end{aligned} \tag{13.12}$$

Thus we have two possible *regimes* in this economy. In the *abundant land regime* there isn't enough capital to cultivate all the land, so some land will remain uncultivated, and land rent will be zero. As far as production goes, the abundant land economy is exactly like the Classical conventional wage model of Chapter 6.

But if capital grows to the level $K^* = U$, the economy enters the *scarce land regime*. In this case the level of production is determined by the amount of land, not by the amount of capital. General equilibrium in both regimes is summarized in Table 13.2.

Table 13.2 Equilibrium in the Land Model

Endogenous variables: v_{kt}, v_{ut}, r_t, p_{ut+1}, J_{t+1}, K_{t+1}
Exogenous parameters: ρ, δ, β, $\bar{\pi}$, U, K_t, J_t, p_{ut}

$$v_{kt} + v_{ut} = \bar{\pi}\rho \tag{13.13}$$

$$v_{ut} = 0 \text{ if } K_t < U \tag{13.14}$$

$$r_t = v_{kt} - \delta \tag{13.15}$$

$$p_{ut+1} + v_{ut} = (1 + r_t)p_{ut} \tag{13.16}$$

$$J_{t+1} = \beta(1 + r_t)J_t \tag{13.17}$$

$$K_{t+1} = J_{t+1} - p_{ut+1}U \tag{13.18}$$

From equation (13.18) we see that the capital gains from land can soak up some of capitalist saving, and thereby reduce investment in capital. This can be an important factor in the development of capitalist economies where a large proportion of wealth is in the form of land.

13.7 The Abundant Land Regime

We can work out the pattern of growth in the abundant land regime from the general equilibrium conditions. The mathematical details are summarized in Table 13.3.

In the abundant land regime $K_t < U$, and the rent on land $v_{ut} = 0$. The only way that land can compete with capital for a place in portfolios, as equation (13.16) shows, is for the price of land to be rising at the net profit rate. Thus in the abundant land regime:

$$p_{ut+1} = (1 + r_t) p_{ut} \tag{13.19}$$

Notice that the expectation of this price appreciation can justify a positive price for land, *even though the rental on land is zero*.

Now consider what is happening to the capital stock, by looking at equation (13.18). In the abundant land regime equation (13.19) holds, so the growth path of the capital stock follows the path:

$$\begin{aligned} K_{t+1} &= \beta(1 + r_t)K_t + \beta(1 + r_t)p_{ut}U - p_{ut+1}U \\ &= \beta(1 + r_t)K_t - (1 - \beta)(1 + r_t)p_{ut}U \end{aligned}$$

If the price of land is low, there will be enough saving to allow the capital stock to grow.

In the abundant land regime both the price of land and the capital stock will rise, but as the price of land increases, capitalists will feel richer and

Table 13.3 Equilibrium in the Abundant Land Regime

$$v_{ut} = 0$$

$$r_t = v_{kt} - \delta = \bar{\pi}\rho - \delta$$

$$p_{ut+1} = (1 + r_t) p_{ut}$$

$$K_{t+1} = (1 + r_t)(\beta K_t - (1 - \beta) p_{ut} U)$$

richer and will consume a larger part of their resources, so that the growth of the capital stock will tend to slow down over time.

13.8 The Scarce Land Regime

Eventually the capital stock will grow to the point where $K_t = U$, and the economy will shift to the *scarce land regime*, as summarized in Table 13.4.

In the scarce land regime output is limited by the availability of land, and there is no point in accumulating capital, since without more land extra capital will be worthless in production. Thus in the scarce land regime we know that:

$$K_{t+1} = K_t = U = K^*$$

We also know that the net profit rate will be:

$$r_t = v_{kt} - \delta = \bar{\pi}\rho - v_{ut} - \delta$$

From equation (13.18) we can see that:

$$\begin{aligned} K_{t+1} &= \beta(1+r_t)K_t + \beta(1+r_t)p_{ut}U - p_{ut+1}U \\ &= K_t = K^* = U \end{aligned}$$

Substituting for $1 + r_t$, and using equation (13.16), we have:

$$\begin{aligned} K_{t+1} = K^* &= \beta(1+\bar{\pi}\rho - \delta - v_{ut})K^* + \beta(p_{ut+1} + v_{ut})U - p_{ut+1}U \\ &= \beta(1+\bar{\pi}\rho - \delta)K^* + v_{ut}(U - K^*) - (1-\beta)p_{ut+1}U \end{aligned}$$

Since $K^* = U$, the rents disappear from this expression, leaving:

$$K^* = \beta(1+\bar{\pi}\rho - \delta)K^* - (1-\beta)p_{ut+1}U \qquad (13.20)$$

In the scarce land regime everything besides p_{ut+1} in equation (13.20) is unchanging, so the price of land must be unchanging as well, at some level p_u^*. Since the capital stock and price of land do not change from period to period in the scarce land regime, the wealth of the capitalists must not change either, so that:

$$J_{t+1} = J_t = J^* = \beta(1+r^*)J^*$$

The requirement that wealth be constant in the scarce land regime thus implies that the profit factor $1 + r^*$ is equal to the inverse of the capitalist

Table 13.4 Equilibrium in the Scarce Land Regime

$$K_{t+1} = K_t = K^* = U$$

$$1 + r_t = 1 + r^* = \frac{1}{\beta}$$

$$v_{ut} = v_u^* = \bar{\pi}\rho - \delta - r^*$$

$$p_{ut} = p_{ut+1} = p_u^* = \frac{v_u^*}{r^*}$$

saving propensity or utility discount factor, β:

$$1 + r^* = \frac{1}{\beta}$$

Land rent, from equation (13.13), must satisfy:

$$v_u^* = \bar{\pi}\rho - \delta - r^*$$

From the land speculation condition, equation (13.16), we see that in the scarce land regime the price of land must be the present discounted value of future rents:

$$p_u^* = \frac{v_u^*}{r^*} \tag{13.21}$$

The scarce land regime is very much like Ricardo's stationary state. The price of land is so high that capitalists consume all of their net income, and there is no growth. The profit rate after rent has fallen from its high level when land is abundant. Since the same capitalists both own land and capital, in this model there is saving and consumption out of both rents and profits, in contrast to Ricardo's assumption that all rents were consumed and all profits accumulated. Thus in the scarce land regime the net profit rate can be positive, rather than falling to zero, as Ricardo predicted for the stationary state.

13.9 From the Abundant to the Scarce Land Regime

How does the economy that grows rapidly in the abundant land regime link up with the stationary economy of the scarce land regime? The key is the initial price of land. As we have seen, for any initial price of land we can predict the paths of the price of land and the capital stock in the abundant land regime. On this path the price of land is going to rise to its stationary state level p_u^*, in some period. If the capital stock in that same period has grown to its stationary state level, K^*, the two regimes will fit together and the expectations of the capitalists will be exactly fulfilled. In the same period that the price of land rises high enough to stop the growth of capital, the capital stock will be just large enough to raise the rent on land above zero. The expectations of rising land prices will turn out to have been correct, and when land prices stop rising, rents will become positive to make land competitive with capital in capitalists' portfolios. This is a *perfect foresight* equilibrium growth path. The price of land in the initial period is actually determined by the requirement that the two regimes fit together in this way.

If the price of land were too high in the first period, the capital stock would stop growing before it reached K^* level, and rents would never become positive. The price of land and wealth would have to continue rising indefinitely, and eventually on such a path the capitalists would eat up all the capital in consumption. If the price of land were too low in the first period, the capital stock would reach K^* while the price of land was still below p_u^*. Thus the capital stock would continue to grow, leading to unemployment of capital. Only when the market in the first period prices land as an asset at exactly the correct level will it be possible for the growth path to fulfill the expectations.

EXAMPLE 13.1 *Let x = \$50,000/worker/year, δ = 1/year, k = \$12,500/worker, $\bar{w}$ = \$20,000/worker/year, and $\beta = .5$. $\rho = x/k = 4/year$, and $\bar{\pi} = (1 - (w/x)) = .6$. Suppose one hectare of land can employ \$1,000 of capital, and there are 1 million hectares of land available. The unit of land that can employ \$1 of capital is 1/1000 hectare, so there are 1 billion units of land. Find the scarce land regime equilibrium, and the growth path leading to it starting two periods before.*

Answer: In the scarce land regime we have

$$K^* = U = \$1 \text{ billion}$$

$$1 + r^* = \frac{1}{\beta} = 2/\text{year}, \text{ so}$$

$$r^* = 1/\text{year} = 100\%/\text{year}.$$

$$v_u^* = \bar{\pi}\rho - (r^* + \delta)$$

$$= (2.4 - 2)/\text{year} = \$.40/\text{unit of land/year} = \$400/\text{hectare/year}$$

$$p_u^* = \frac{v^*}{r^*} = \frac{\$.40}{1} = \$.40/\text{unit of land} = \$400/\text{hectare}$$

Thus in the scarce land regime the capital stock is worth \$1 billion and the land is worth \$.4 billion.

Suppose we take one step backward from the stationary state. In the abundant land regime we have

$$\begin{aligned}
v_{ut} &= 0\\
r_t &= \bar{\pi}\rho - \delta = (.6)(4) - 1/\text{year} = 1.4/\text{year} = 140\%/\text{year}\\
(1 + r_t)p_{u-1} &= p_u^*, \text{ so}\\
p_{u-1} &= \$400/(2.4) = \$166.67/\text{hectare}\\
v_{u-1} &= 0\\
K^* &= (1 + r_t)(\beta K_{-1} - (1 - \beta)p_{u-1}U\\
\$1 \text{ billion} &= (2.4/\text{year})(.5K_{-1} - .5(\$166.67(1 \text{ million hectares}))
\end{aligned}$$

and

$$K_{-1} = \$1 \text{ billion}$$

In the period in which the capital stock reaches its maximum level, the land rent is still zero and the price of land continues to rise one more period before reaching the scarce land regime level.

If we take one more step backward, we have:

$$\begin{aligned}
(1 + r_t)p_{u-2} &= pu_{u-1}, \text{ so}\\
p_{u-2} &= \$166.67/(2.4) = \$69.44/\text{hectare}\\
v_{u-1} &= 0\\
K_{-1} &= (1 + r_t)(\beta K_{-2} - (1 - \beta)p_{u-2}U)\\
\$1 \text{ billion} &= (2.4/\text{year})(.5K_{-2} - .5(\$69.44(1 \text{ million hectares})))\\
K_{-2} &= \$902.78 \text{ million}
\end{aligned}$$

PROBLEM 13.2 Suppose in Ricardia (see Problem 2.1) the production of 100 bushels of corn requires 1 acre of land, together with 20 bushels of seed corn and 1 worker-year. If there are 10,000 acres of land available, what is the maximum amount of seed corn capital that could be employed, and the maximum amount of corn output? If the wage rate is 20 bu/worker-year, what are the gross and net profit rates in the abundant land regime?

PROBLEM 13.3 Find the land price, land rent, and gross and net profit rates in Ricardia (see Problem 13.2) when the wage is 20 bu/worker-year, and $\beta = 4/5$.

PROBLEM 13.4 Make a spreadsheet program to calculate the growth path for Ricardia starting from the scarce land regime and working backward 20 years, calculating the asset price of land and the capital stock in each year.

13.10 Lessons of the Land-Limited Model

The model of growth with an absolutely limited resource like land underlines several fundamental insights of economic analysis.

A long-lived asset can have a positive price even when it yields no current return, like land in the abundant land regime, where the price of land is positive and rising even though the rent on land is zero. The price of land in this model is determined by forward looking *speculative* forces. Capitalists will pay a positive price for land because they believe, correctly, that the rent on land will eventually become positive. Even before the rent becomes positive, landowners are rewarded at the average rate of return by the rising price of land.

Speculative pricing of assets is central to the operation of equity markets in capitalist economies. Equity claims on companies that pay no dividends and even have no earnings from which they might pay dividends can command a positive and rising price in speculative stock markets because asset-holders believe that the company may eventually become profitable. Even if the eventual profitability of a company is quite uncertain, and there is a significant probability in the minds of investors that it will never become profitable, its equity can still command a positive price because investors believe there is some probability that it will earn profits in the future. This effect, which the land model explains in a highly simplified setting, is the source of the *wealth-creating* powers of speculative asset markets. Hopes and dreams can be turned into hard cash, as long as enough speculators are convinced of the possibility that they will come to pass.

In the abundant land regime, the capital gains on land come to absorb a larger and larger part of the saving of capitalists, until at the moment land becomes scarce, the wealth represented by land is so large that capitalists stop saving altogether. This effect also occurs in real economies where wealth in land is very great. Wealth holders may believe themselves to be so rich that they stop saving, and make no funds available for investment in capital. The resulting economic stagnation has been a problem in some developing countries.

As Ricardo divined, a capitalist economy facing an absolute land constraint eventually reaches a stationary state where accumulation of capital ceases. In the limited land model the profit rate in the stationary state remains positive, in contrast to Ricardo's differential rent model. But the profit rate falls to the point where capitalists choose to consume all their net income, leaving nothing left over to finance new investment.

Ricardo disliked the idea of the stationary state, since he thought the accumulation of capital was the chief source of social change and improvement. He saw two factors that might, at least temporarily in his view, delay the stationary state. The first was international trade, and Ricardo's analysis became the bible of British free-trade advocates in the middle years of the nineteenth century. The effect of free trade, in Ricardo's eyes, was to incorporate all (or at least more) of the whole world's land in the economy, and thus to moderate the effects of diminishing returns. The extensive margin, instead of being confined to the narrow and rocky islands of Britain, could migrate to the fertile and empty prairies of North America, the pampas of Argentina, or the savannahs of east Africa.

The other factor Ricardo saw as delaying the stationary state was technical change, particularly land-augmenting technical change that would raise the marginal product schedule for capital and labor, and move the stationary state further away from the current extensive margin.

But Ricardo, like contemporary theorists of limited economic growth, viewed both free trade and technical change as only temporary stopgaps delaying, but not preventing, the arrival of the stationary state. He believed that the law of diminishing returns would sooner or later assert itself.

The world economy has seen dramatic increases in the scale and scope of foreign trade since Ricardo's time, and equally dramatic technical change in agricultural and other resource-intensive production. The stationary state seems no nearer today than it did when Ricardo wrote. But perhaps it equally seems no further off. The warnings of contemporary theorists of limited growth, who see human society threatened by the exhaustion of natural resources and the deterioration of the environment, are a reminder of the depth of Ricardo's vision.

13.11 Suggested Readings

The seminal work on the theory of rent is generally acknowledged to be Ricardo (1951, Ch. 2, "On Rent"). The modern formalization of the Ricardian treatment by Pasinetti (1974) is highly recommended. For an in-depth discussion of land-rent and modern developments of the theory in the Classical tradition, see Kurz and Salvadori (1995, Ch. 10). For treatment of land (and natural resources in general) in the basic Solow–Swan model, consult Meade (1961). The model in this chapter forms the basis for an analysis of the economics of antebellum US slavery in Clegg and Foley (forthcoming).

14

Exhaustible Resources

14.1 Growth with an Exhaustible Resource

Land can neither be created nor used up in the model in Chapter 13. It is either abundant and has no immediate effect on production, or scarce, in which case it is an absolute limit on production.

Another important aspect of resource limitation is that some resources are *exhaustible*, in the sense that they get used up in production, and cannot be renewed. For example, certain mineral and oil resources exist as a quantity of ore or oil in deposits under the ground. As they are mined or pumped out they disappear. Once used, they cannot be replaced. We can use the same modeling approach as we used in the case of land to understand the fundamental economics of growth with exhaustible resources. This analysis is called the *Hotelling model* after Harold Hotelling, the economist who first solved this problem.

Just as it might make more sense to view land as ultimately producible than as absolutely fixed, it might make more sense to view mineral and oil resources as renewable rather than as nonrenewable. First of all, new reserves of ores and oil can always be found by exploration and prospecting. In reality we do not know for sure exactly how large the ultimate reserves are; in practice it is possible at a cost to find new reserves. Second, the exploitation of mineral and oil reserves depends on the mining and drilling technology available. At any time there are known reserves that are too costly to exploit with existing technology. If society is willing to pay higher costs, more of these reserves become available. Furthermore, the technology is always changing, thus lowering the costs of exploiting known reserves. Oil companies now routinely drill wells that would have been impossibly deep fifty years ago. Some shallow reserves still have oil in them, but are not being pumped because the quantities are too small to justify the fixed cost required. Finally, technological innovation constantly turns up new alternatives to existing resources. The development of solar technology may make oil reserves economically irrelevant before they are physically exhausted.

Nonetheless, in this chapter we will assume that there is an exhaustible resource and an unchanging technology. We will investigate the economic forces that govern the pricing and utilization of this type of resource.

14.2 Production with an Exhaustible Resource

In order to introduce an exhaustible resource into the growth model, we begin with production. Suppose that oil is a source of energy that lowers costs of production. We will use the symbols Q and q for oil. The production technique (now assuming that there are no land limitations) is:

$$1 \text{ labor} + k \text{ capital} + x \text{ oil} \rightarrow x \text{ output} + (1-\delta)k \text{ capital} - x \text{ oil}$$

Here we are again taking advantage of our freedom to choose the units in which we measure oil. This model of production assumes that to produce a unit of output you need to burn up one unit of oil. Thus at the end of the period, there are three results of the productive process: the x units of new output, the depreciation of capital, and the depletion of x units of oil. We are using the amount of oil required to produce one unit of output as our unit of measurement for oil.

Since the amount of oil available is finite, we have to have some theory of what happens when the oil runs out. If the only known production technology required oil, then we would have to assume that production would stop altogether when the oil reserves were exhausted. A more realistic modeling assumption is that there is another method of production (for example, one that depends on solar energy) that does not require oil:

$$1 \text{ labor} + k \text{ capital} \rightarrow x' \text{ output} + (1-\delta)k \text{ capital}$$

For simplicity we assume that the alternative solar technology has the same capital intensity, k, and depreciation rate, δ, as the oil technology. If oil is

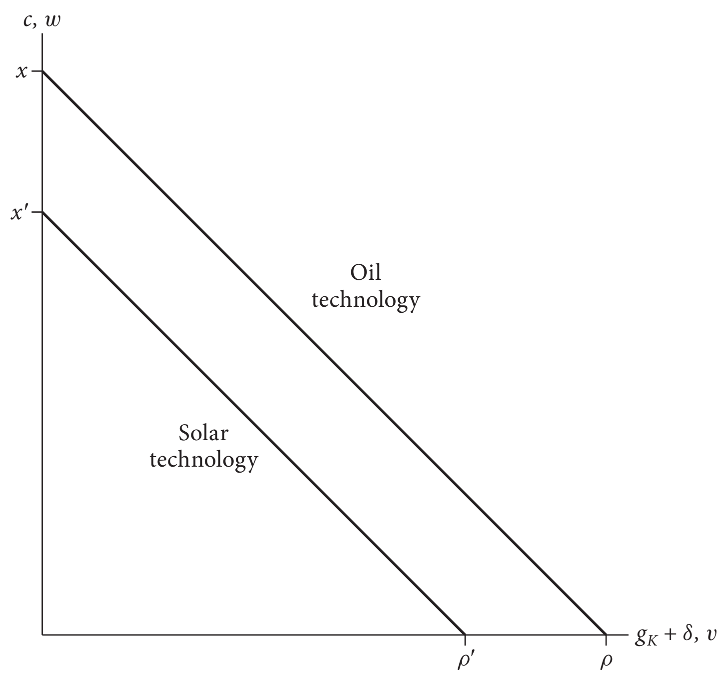

Figure 14.1 The real wage-profit rate relations for the oil and solar technologies. The oil technology dominates the solar technology, since it is more profitable at every real wage. The slopes are the same, since we assume the two technologies have the same capital-labor ratios: the use of oil saves equal proportions of labor and capital.

an economically relevant resource, it must increase the productivity of other resources, such as labor and capital, so we assume that labor productivity is higher with the oil technology: $x > x'$. Under the assumption that the capital intensity is the same for the two technologies, the oil technology also raises the productivity of capital: $\rho' = x'/k > x/k = \rho$.

Figure 14.1 shows the real wage-profit rate schedules for the two techniques. The assumption that capital intensity, k, is the same in the two technologies implies that oil saves labor and capital in equal proportions. In other words, the use of oil is the equivalent of Hicks-neutral technical progress. For a given wage, w, the wage share for the oil technology is $1 - \pi = w/x$, while the wage share for the solar technology is $1 - \pi' = w/x'$.

Since oil is a productive, scarce, and wasting resource, it will command a price in the market. If p_{qt} is the price of oil at the beginning of period t, which is the end of period $t-1$, and entrepreneurs pay for their oil, like wages, at the end of the period after they have sold their output, the profit an entrepreneur using oil technology will make on each worker employed is:

$$v_{qt}k = x - w - p_{qt+1}x = (1 - p_{qt+1})x - w$$

The profit per worker using solar technology is:

$$v_s k = x' - w$$

The corresponding profit rates for the two technologies are:

$$v_{qt} = (\pi - p_{qt+1})\rho$$
$$v_s = \pi' \rho'$$

The actual profit rate in any period will depend on whether the oil technology or the solar technology is the more profitable:

$$v_t = r_t + \delta = \max(v_{qt}, v_s) \tag{14.1}$$

There is a price of oil, p_q^*, at which the two technologies have the same profit rate for every real wage. If we set $w = 0$, we have $\pi' = \pi = 1$, so that:

$$\rho(1 - p_q^*) = \rho' \text{ or}$$
$$p_q^* = \frac{\rho - \rho'}{\rho}$$

The price of oil at which the two technologies have the same profit rates is equal to the proportion of capital (and labor) saved by oil, $(\rho - \rho')/\rho$. The price of oil cannot rise above p_q^*, because if it did the oil technology would have a lower profit rate than solar and no one would use it, so p_q^* is also the *maximum* price of oil. The more capital and labor oil saves, the higher will be its maximal price.

PROBLEM 14.1 Consider an economy with an oil technology where $x =$ \$50,000/worker/year, $k =$ \$100,000/worker, $\delta = 0$/year, and solar technology is 50% less productive, with the same rate of depreciation. Find the price of oil at which solar technology would just compete with oil.

PROBLEM 14.2 For the economy described in Problem 14.1, suppose that the capitalist $\beta = .95$ and that the wage is \$10,000/worker/year. Find the profit rate and the growth rate of the capital stock using solar technology.

14.3 Saving and Portfolio Choice

The typical capitalist now has to choose how much oil reserves, Q_t, to hold as an asset in each period, and also how much oil to pump out of the reserves, ΔQ_t. If she pumps oil at the beginning of the period, she can sell it to entrepreneurs and receive p_{qt+1} at the end of the period, but in this case she will also have a smaller amount of oil reserves left at the end of the period.

The typical capitalist's budget constraint with oil in each period, given the net profit rate r_t, is thus:

$$K_{t+1} + C_t + p_{qt+1}Q_{t+1} \leq (1 + r_t)K_t + p_{qt+1}(Q_t - \Delta Q_t) + p_{qt+1}\Delta Q_t \tag{14.2}$$

We can also write the budget constraint as:

$$K_{t+1} + C_t + p_{qt+1}Q_{t+1} \leq (1 + r_t)K_t + p_{qt+1}Q_t \tag{14.3}$$

The capitalist utility maximization problem is shown in Table 14.1. A capitalist who chooses to hold a unit of oil reserves during period t at the price p_{qt} will have p_{qt+1} at the end of the period, due to the change in the price of oil over the period. She could, alternatively, invest the money in capital instead and receive $(1 + r_t)p_{qt}$ at the end of the period. These two returns must be equal if the capitalist is to be willing to hold both oil reserves and capital in her portfolio. As Hotelling pointed out, the *arbitrage principle* of Chapter 13 applies to reserves of exhaustible resources like oil. Owners of reserves under conditions of competition must believe that the price of the reserves is rising at the same rate as the net profit rate on capital, since rational wealth holders will hold assets with equal risk only if their anticipated rates of return are equal. In this model the two assets are capital

Table 14.1 Capitalist's Utility Maximization Problem with Oil

choose $\{C_t, K_{t+1}, \Delta Q_t, Q_{t+1} \geq 0\}_{t=0}^{\infty}$ so as to maximize

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to

$$K_{t+1} + p_{qt+1}Q_{t+1} + C_t \leq (1 + r_t)K_t + p_{qt+1}Q_t$$

$$\{p_{qt}, 1 + r_t\}_{t=0}^{\infty}, K_0, Q_0 \text{ given}$$

and oil reserves. They have the same (zero) risk, so capitalists will hold both only if they have the same rate of return. Furthermore, the capitalist is indifferent as to how much of her wealth she holds in capital and oil reserves as long as the rates of return on the two assets are identical. The mathematical expression of the arbitrage principle for the oil model is:

$$(1 + r_t)p_{qt} = p_{qt+1} \tag{14.4}$$

This insight greatly simplifies the typical capitalist's budget constraint. Writing $J_t = K_t + p_{qt}Q_t$ for the capitalist's wealth at the beginning of period t, we can express the budget constraint as:

$$J_{t+1} + C_t \leq (1 + r_t)J_t$$

This is just the same as the budget constraint in Chapter 5, so we know that the solution to the utility maximization problem will be:

$$C_t = (1 - \beta)(1 + r_t)J_t, \text{ and}$$
$$J_{t+1} = (1 + r_t)J_t - C_t = \beta(1 + r_t)J_t$$

Thus wealth grows at the rate $\beta(1 + r_t)$.

PROBLEM 14.3 Write down the Lagrangian function for the capitalist's utility maximization problem with oil, and find the first-order conditions describing the saddle-point. Use these conditions to derive the arbitrage principle and the consumption function.

14.4 The Growth Path

The final piece of the puzzle is provided by recognizing that the amount of oil pumped, ΔQ_t, must be equal to X_t since each unit of output requires one unit of oil. Output, X_t, as in all Classical models where labor-power is elastically supplied, is determined by the amount of capital accumulated: $X_t = \rho K_t$. So we have:

$$\Delta Q_t = X_t = \rho K_t$$

But this allows us to trace the depletion of the oil reserves, since we know that:

$$Q_{t+1} = Q_t - \Delta Q_t = Q_t - \rho K_t$$

Table 14.2 Equilibrium in the Oil Model

$r_t = (\pi - p_{qt+1})\rho - \delta$	(14.5)
$p_{qt+1} = (1 + r_t)p_{qt}$	(14.6)
$J_{t+1} = \beta(1 + r_t)J_t$	(14.7)
$Q_{t+1} = Q_t - \rho(J_t - p_{qt}Q_t)$	(14.8)

We can put all these relations together as in Table 14.2 to see the laws governing the changes in market equilibrium prices and quantities in the oil model during periods while some oil reserves still remain. It turns out to be easier to express the quantities in terms of capitalist wealth $J = K + p_q Q$ and the remaining oil reserve, Q, rather than in terms of the capital stock K and Q, but if we know J, p_q, and Q in any period, we can easily find $K = J - p_q Q$.

Now we have a complete picture of the process of growth with the exhaustible resource. The price of oil rises steadily to provide capital gains on the oil reserves equal to the net profit rate, r_t. The net profit rate itself, r_t, declines in each period as oil becomes more expensive. Wealth grows through the saving of capitalists. Some part of this increase in wealth goes to increase the capital stock, thereby raising output and using up more oil.

There comes a time when the oil runs out. In this period the price of oil must rise to p_q^*, at which the solar technology is just competitive with the oil technology.

As in Chapter 13, the two regimes have to be fitted together by the correct speculative pricing of oil in the initial period. The price of oil in the initial period must be set by speculation in such a way that it reaches p_q^* in exactly the period that the oil reserves will be exhausted. Clearly this depends on the size of the oil reserves in the initial period in relation to the capital stock. If the oil price is too high, the resulting large oil wealth will induce the capitalists to consume at a high rate, and output and the capital stock will not grow fast enough to use up the oil reserves by the time the price of oil rises to its maximum level. If the oil price is set too low, on the other hand, the high rate of capitalist saving will exhaust oil reserves before the solar technology becomes competitive. Forward-looking speculation attempts to find the initial price of oil that induces just the rate of growth of output and capital stock compatible with using up the oil reserves at the equilibrium rate.

The larger are initial reserves, the longer it will take to exhaust them, and the lower the initial equilibrium price of oil will be. The more efficient the alternative solar technology, the lower is the maximal price of oil (since it measures the efficiency advantage of oil over solar), and the lower will be the initial equilibrium price. Thus speculators in the oil market have to consider the size of known reserves, the likely rate of economic growth and demand for oil, and the rate at which alternative technologies are developing to form the equilibrium price.

EXAMPLE 14.1 *Let the oil technology have $x =$ \$100,000/worker/year, $\delta =$ 1/year, $k =$\$12,500/worker, and suppose that the alternative solar technology is half as productive, so that $x' =$ \$50,000/worker/year. $\rho = x/k = 8$/year and $\rho' = x'/k = 4$/year. The conventional wage, $\bar{w} =$\$20,000/worker/year. Thus $\pi = 1 - (w/x) = .8$ and $\pi' = 1 - (w/x') = .6$. Find the maximum price of oil, and the price of oil and the net profit rate in the period before oil reserves are exhausted.*

Answer: The maximum price of oil is:

$$p_q^* = \frac{\rho - \rho'}{\rho} = .5$$

In each period before the oil reserves are exhausted we have

$$\begin{aligned} p_{qt+1} &= (1 + r_t) p_{qt} = (v_{qt} + 1 - \delta) p_{qt} \\ &= ((\pi - p_{qt+1})\rho + 1 - \delta) p_{qt} \end{aligned}$$

This implies that

$$p_{qt} = \frac{p_{qt+1}}{\pi\rho + 1 - \delta - \rho p_{qt+1}}$$

Suppose we take one step backward from the period in which the oil runs out.

The profit rate in the period before the oil runs out will be

$$v_{q-1} = (\pi - p_q^*)\rho = (.8 - .5)(8) = 2.4$$

Thus we have:

$$p_{q-1} = \frac{p_q^*}{\pi\rho + 1 - \delta - \rho p_q^*}$$
$$= \frac{.5}{(.8)(8) - (8)(.5)} = .5/2.4 = .21$$

or 21% of output.

PROBLEM 14.4 Explain what effect the following would have on oil prices, using the exhaustible resources model as a basis: (a) a discovery that would allow wells four times as deep as at present to be drilled at the same cost; (b) a drastic cheapening of solar cells; (c) an increase in the capitalist propensity to save.

PROBLEM 14.5 Consider the economy described in Problem 14.2. Suppose that the economy has just exhausted its oil reserve. Work backward one period and find the price of oil and the profit rate in the period just before the oil reserve was exhausted.

PROBLEM 14.6 Make a spreadsheet program to calculate the growth path for an oil economy starting from the period in which oil reserves are exhausted and working backward 20 years, calculating the price of oil in each year.

14.5 Exhaustible Resources in the Real World

This model gives us some fundamental insights into the way a market capitalist economy will value reserves of exhaustible resources. The general outlines of the solution look plausible: profit rates and growth rates decline as the reserve is depleted, and the price of the exhaustible resource gradually rises until it makes the next best technology competitive.

In the real world, however, the prices of exhaustible resources do not always rise, and, in fact, sometimes fall dramatically and over a long period. These observations could mean that some assumption of the model is wrong. For example, the exhaustible resources in question might not be priced competitively in some periods. But a fall in the price of an exhaustible resource could also occur if new information about the size of reserves, or the costs of alternative technology, or the growth rate of the economy arrives. New information of this kind requires the owners of oil reserves to reprice them, taking the information into account. Anticipated slower economic growth, or more rapid improvement in alternative technologies, or the discovery of new reserves can drive the price of the exhaustible resource down.

Thus the chief aspect of the real world the model leaves out is uncertainty about future technological developments, economic growth, and resource discoveries. We have assumed that the initial oil reserve Q_0 is known, and that the solar costs, which determine p_q^*, are also known and unchanging. In the real world new information constantly changes the best estimates of the reserves and of the costs of competing technologies. This type of information is particularly important in the pricing of a speculative asset like oil reserves. To explain this rigorously would require a model where capitalists took account of the uncertainty of the relevant future developments.

14.6 Suggested Readings

The seminal work on the theory of optimal use of an exhaustible resource is Hotelling (1931). For additional discussion of the model of oil and solar power developed in the text, see Kurz and Salvadori (1995, ch. 12), where an overview of the history of thought on the subject can also be found. The model in this chapter is explored further in Michl and Foley (2007).

15

Corporate Capitalism

In the Classical conventional wage model the capitalist agents make decisions about the accumulation of capital, which they then rent to the entrepreneurs who organize the production process. The growth rate of the economy is determined by the profitability of capital when it is fully utilized and by the propensity of the capitalists to accumulate. In the demand-constrained growth model of Chapter 12, the capitalist agents do not directly invest in capital but instead save by accumulating financial assets. It is the entrepreneurs that make the investment decisions, and these are coordinated with the saving decisions through changes in capacity utilization. But we concluded that there are plausible mechanisms that would push utilization toward full utilization in the long run.

In this chapter we will study a model of corporate capitalism that preserves the separation between saving and investment decisions but focuses on a long run in which the product market has achieved full capacity utilization. As in the land-limited growth model in Chapter 13, there is an asset market. We will study the role this market—the stock market—plays in coordinating the saving decisions of the capitalists with the investment decisions of corporate managers.

In modern capitalist economies, a substantial fraction of economic activity is organized around corporate business enterprises that issue stock certificates, usually simply called *stocks* or *shares*. Another alternative term for stocks—*equities*—derives from the fact that stocks are certificates of ownership: they entitle the stockholders to any residual profits (distributed in the form of dividends) left over after the firm has paid for inputs and retained some earnings to finance investment in new capital goods. Corporations also operate under *limited liability*, which means the stockholders are not legally responsible for the debts of the corporation.

We will change terminology in discussing corporate capitalism because the owners of the firm effectively hire professional managers to make the business decisions, including the decision about how much to invest and how to finance the investment using a combination of retained earnings and new stock issuance. We will dispense with references to entrepreneurs in this chapter accordingly.

Corporations also have the option of borrowing through the bond market or through a bank loan to obtain external financing for investment. It makes some sense to focus initially on the simplest form of corporate capitalism that abstracts from these funding options. In this chapter, we will assume that corporations rely entirely on stock issuance for external finance.

15.1 Accounting in the Corporate Capitalist Economy

We will assume that labor-power is supplied elastically at the conventional wage. Workers are assumed to consume all their wage income so we won't need to specify their balance sheets. The balance sheets of the capitalist households and the firms are given by two equations, written in the standard order with assets on the left-hand side and financial liabilities and net worth on the right. The balance sheets give us snapshots of the financial positions at the beginning of the period:

$$PE = J$$
$$K = PE + J_F$$

Here E (for equities) is the number of stock shares and P is the price of a share. Firms carry the value of their outstanding shares as a liability because this in a sense represents what they owe to their stockholders, but shares are not an enforceable liability like debt that can create solvency issues for the firm. Capitalist net worth at the beginning of the period is J and firms are also assumed in our accounting convention to have beginning-of-period net worth, J_F. The valuation ratio, q (since it is similar to Tobin's Q), is the ratio of wealth to capital or $q = (PE)/K = J/K$, making $J_F = (1 - q)K$. With this accounting convention, firms will have negative net worth when the *q-ratio* exceeds unity but this does not mean that they are in financial distress.[1]

Corporate managers decide on the growth of capital (investment) and make financial decisions about external financing. For simplicity, we assume that capital does not depreciate, making the net and gross rates of profit identical and equal to r and the growth rate of capital $g_K = I/K$. Managers retain a fraction of profits, s_F, called the *retention rate*, and distribute $(1 - s_F)$ as dividends to the households. Retained earnings represent corporate saving. Managers issue new stocks to finance the remaining investment. We will make extensive use of delta notation to describe changes over time in a stock variable in this chapter. For example, new stock issues are written $\Delta E = E_{+1} - E$. The managers' financial plan is to combine retained earnings and stock issuance to finance investment:

$$I = s_F(rK) + P\Delta E \tag{15.1}$$

In practice, corporate managers finance most or all of their investment spending out of retained earnings. One possible explanation for this aversion to external finance is that information about corporate performance is imperfect and financial markets interpret stock issuance as a sign of weakness (i.e., insufficient profitability). Low or falling stock prices can present a problem for managers; for example, their compensation may be tied to the price of their company's stock. As a result, managers prefer to tap their internal funds first before turning to external sources. In most advanced capitalist countries, a substantial proportion of national saving comes from corporate retained earnings. Later we will adopt the assumption that managers finance investment out of retained earnings when we develop the corporate capitalist model in detail, both because it simplifies the exposition and because it contributes realism.

[1]A system of interlinked balance sheets must respect the fact that one agent's asset will have its counterpart in another agent's liability, with the exception of pure outside assets like capital goods that are a liability to no one. An alternative accounting convention would deny that the value of shares is a true liability for the firm. In this case, the net worth of the firm would have to be equal to the value of stocks held by households, PE. Consistent accounting practice would then require that the firm book its capital goods as assets using the valuation ratio so that $qK = J_F = PE = J$.

Table 15.1 A SAM for the Corporate Capitalist Economy

	Output Costs	Expenditures				*Changes in Claims*	*Sum*
		w	c	f	I		
Output Uses		C^w	C^c		ΔK		Y
Incomes							
w	W						Y^w
c				$(1-s_F)rK$			Y^c
f	rK						Y^f
Flows of Funds							
c			S^c			$-P\Delta E$	0
f				S^f	$-\Delta K$	$P\Delta E$	0
Sum	Y	Y^w	Y^c	Y^f	0	0	

The social accounting matrix (SAM) for the corporate capitalist economy is shown in Table 15.1. Since we have assumed for simplicity that capital does not depreciate, the SAM records net income and saving. The SAM for the corporate economy highlights several salient features of the corporate form of organization. First, capitalist agents do not receive rental payments from entrepreneurs as they did in the Classical model. Instead they receive dividend payments from the corporations. As a result, they experience the profitability of capital indirectly as a return on their stock holdings that we will call the *equity yield*. Financial analysts often call this the *required rate of return on equity* since it will be the return expected by stock market participants.

Second, managers make the decisions about the accumulation of capital rather than capitalist agents as in the Classical model. To finance the firm's investment spending, they have the option of issuing stock, which must be purchased by capitalist agents, or using retained profits. Thus, the social saving in the economy that must be coordinated with the investment spending consists of the sum of firm saving and capitalist household saving.

Together these distinctive features illustrate a substantive fact about the corporate form of organization. Presumably, the corporate form arises because it offers advantages beyond limited liability that are not available to individual capitalists. For example, corporations can pool the capital of a large number of capitalists and undertake large-scale investment projects that exceed the resources of each individual. The direct ownership of capital goods by capitalist agents that we assumed in the Classical model has been replaced by stock ownership.

Capitalist and corporate saving are defined in the SAM using the national income accounting definition that saving equals value-added minus consumption. We will call this definition "SNA saving" in reference to the System of National Accounts that has been codified by the United Nations and is used as a standard internationally.

Shares are traded in a stock market, opening up the possibility that households will experience capital gains when stock prices rise. An alternative definition of saving includes capital gains as part of income. This more inclusive definition of saving is sometimes called *comprehensive* saving. Comprehensive saving is the change in net worth of each sector or the sum of its SNA saving from the SAM and capital gains from rising stock prices. With this adjustment the flow of funds from the SAM cumulates smoothly into changes in balance sheets.

This cumulation operates through the *revaluation accounts*, which can be written as two equations that result from differencing the balance sheets and solving for the change in net worth (i.e., comprehensive saving). For example, households benefit from capital gains on their beginning-of-period equities and from gains on stocks bought during the period at the prevailing price, P. To see this mathematically, we take the first difference of their assets and group the terms:

$$P_{+1}E_{+1} - PE = (P + \Delta P)(E + \Delta E) - PE = P\Delta E + \Delta P E_{+1}$$

On the far right hand side, the first term represents capitalist SNA saving while the second term represents total capital gains earned during the period since at the end of the period the households can sell their stocks at the new price, P_{+1}, including any stocks newly issued during the period.

Using the fact that with no depreciation $I = \Delta K$, we can expand the firms' balance sheets in the same way and write out the revaluation accounts:

$$\Delta J = P\Delta E + E\Delta P + \Delta P\Delta E = S^c + \Delta P E_{+1}$$

$$\Delta J_F = I - P\Delta E - E\Delta P - \Delta P\Delta E = S^f - \Delta P E_{+1}$$

Notice that capital gains and stock transactions are offset between firms and their owners so that the familiar national accounting identity between saving and investment holds for both definitions of saving, or

$$\Delta J + \Delta J_F = S^c + S^f = I$$

Capital gains are a source of some potential confusion. For the individual capitalist, the fact that others are willing to purchase her stocks for a higher price creates the opportunity to realize a capital gain in order to finance consumption and constitutes an increase in private wealth. But at a social level, capital gains represent purely fictitious income or wealth since they cannot increase the aggregate wealth of the capitalist households and the firms they own which in our model is the actual capital stock, $K = J + J_F$. This equation is the consolidated national balance sheet.

The United Nations' System of National Accounts (SNA) establishes standards for complete accounting at the aggregate level. In the US, the Federal Reserve Board and Bureau of Economic Analysis now publish the Integrated Macroeconomic Accounts (IMA) that follow SNA principles and put many of the conventions we use in this chapter into practice. The Federal Reserve Board includes the IMA as part of the Financial Accounts of the US (formerly known as the Flow of Funds Accounts).

PROBLEM 15.1 Construct a SAM for the corporate capitalist economy in which workers save (S^w), accumulate equities, and own a fraction (call it ϕ) of the total financial wealth. Assume that they receive this share of the dividends paid out by corporations and that they purchase this share of newly issued stocks.

PROBLEM 15.2 Download the Integrated Macroeconomic Accounts (IMA) from the website of the US Federal Reserve Board or the Bureau of Economic Analysis. Construct the balance sheets for households and firms for the most recent year. Use the nonfinancial corporate sector to represent firms. Create the categories "other assets" and "other liabilities" to record the values of all items besides capital and equity (net worth).

PROBLEM 15.3 Using the IMA, calculate the q-ratio for nonfinancial corporate business as the ratio of total assets to total liabilities for the available years and make a chart. Comment on any patterns you can see in your figure. How does the IMA account for corporate equity and net worth—is it the same as or different from our textbook treatment?

15.2 Stocks and the Capitalist Decision Problem

We will assume that a large number of capitalist households begin each period owning the same proportion of the corporate shares outstanding and that they maximize the discounted sum of the logarithm of their future consumption. The price of stocks is determined in the asset market and we will have more to say about its determination below. The typical capitalist receives a dividend, $V = (1 - s_F)rK/E$, on each share she owns, or VE in total. She can also sell her shares at the end of the period. These sources of funds must be sufficient to finance her consumption during the period and her holdings of stock in the next period. Her budget constraint is thus:

$$P_{+1}E_{+1} + C \leq VE + P_{+1}E$$

A capitalist who holds a share of stock at price P during the period will receive its dividend, V, and will be able to sell the share for its price at the end of the period. Her equity yield will be this total return divided by the price of the stock, or

$$1 + r_E = \frac{V + P_{+1}}{P}$$

The equity yield is the sum of the *dividend yield* (V/P) and the capital gain or loss from the percentage change in stock prices ($g_P = \Delta P/P$), or

$$r_E = \frac{V}{P} + g_P \qquad (15.2)$$

What is the relationship between the equity yield that capitalist agents experience and the underlying rate of profit on capital? To get a clear picture of these two returns, we will study the economy in a steady state position where the q-ratio remains constant and where the capital stock is growing at the constant rate g_K. A steady state refers to a state of affairs where all the important variables grow at the same rate so that the important ratios between them stay constant. In this case, from the constancy of the q-ratio, $q = PE/K$, we can express the growth of stock prices on a steady state path simply by the difference between the growth of capital and the growth of shares, g_E:

$$g_P = \frac{g_K - g_E}{1 + g_E} \approx g_K - g_E$$

Under the assumption that investment is financed fully out of retained earnings (as in most of the instances below), corporations will stop issuing stock and this will be a precise equality. In other cases, we will assume that because g_E is a small number, the approximation will be close enough to be treated as an equality.

Substituting this into the definition of the equity yield reveals that the relationship between it and the rate of profit on a steady state growth path depends critically on the q-ratio:

$$1 + r_E = \frac{1}{q}\left(r - g_K + q(1 + g_K)\right) \tag{15.3}$$

The q-ratio and the equity yield are inversely related. An increase in the q-ratio implies a lower equity yield and a decrease in the q-ratio implies a higher equity yield, given the rate of profit and the growth rate.

If we rearrange equation (15.3) we can see that one interpretation is that the q-ratio measures the profitability of capital relative to equity:

$$q = \frac{r - g_K}{r_E - g_K}$$

A q-ratio greater than unity implies that for any given growth rate, the equity yield will be lower than the rate of profit. A q-ratio less than unity implies that the equity yield will exceed the rate of profit. Only when $q = 1$ will the two returns be the same. We will see that this interpretation helps us understand the investment response of managers to the q-ratio.

We can use the equity yield to rewrite the capitalist budget constraint in a familiar and transparent form. Using the fact that $(1 + r_E)PE = VE + P_{+1}E$ and $J = PE$, we have:

$$J_{+1} + C \leq (1 + r_E)J$$

Just as in Chapter 5 we know that the solution to the capitalist's maximization problem with a logarithmic utility function will be to consume a constant fraction of her end-of-period wealth so that

$$C = (1 - \beta)(1 + r_E)J \tag{15.4}$$

With this consumption equation, a version of the Cambridge equation specialized for the corporate capitalist economy describes the capitalist's accumulation of wealth held in the form of stocks:

$$(1 + g_J) = \beta(1 + r_E) \tag{15.5}$$

An important feature of corporate capitalism is that the owners of corporations do not experience the rate of profit on capital directly, but instead respond to the return on their holdings of stocks. While they consume a constant fraction of their wealth, their wealth depends critically on the asset market valuation of the capital stock. We see this when we substitute the steady state equity yield from equation (15.3) and $J = qK$ into the definition of end-of-period wealth:

$$(1 + r_E)J = (r - g_K + q(1 + g_K))K$$

This expression tells us that stock prices create wealth effects on capitalist consumption. A stock price boom that raises the q-ratio will be experienced as an increase in wealth by the capitalist households. From equation (15.4) we see that they will increase their consumption and reduce saving. From equation (15.5) we see that their desired rate of wealth accumulation will be lower since from equation (15.3) it is clear that a higher q-ratio implies a lower equity yield. Conversely, a decline in the q-ratio will have a negative wealth effect, reducing consumption and raising saving; the capitalists' desired rate of wealth accumulation will be correspondingly higher. It is significant that only when $q = 1$ will the capitalists' wealth be identical to the capital stock and the equity yield equal to the rate of profit.

15.3 Investment-Saving Equilibrium

The possibility that the equity yield can deviate persistently from the rate of profit reflects the lack of arbitrage between financial assets and real capital goods. Presumably, if capitalist households could invest directly in capital goods (which could be rented out to entrenpreneurs as in the Classical growth model), they would never hold stocks when the rate of profit exceeds the equity yield. Publicly traded corporations would vanish as a species. On the other hand, capitalist households would prefer stocks to capital goods when the equity yield exceeds the rate of profit. Privately held companies would be floated off on the stock market at a financial gain to their owners. Investment-saving equilibrium ($g_K = g_J$) would require that $q = 1$ so that $r_E = r$. Yet in real economies most firms adopt the corporate form of organization and the q-ratio is far from stable. The source and extent of the barriers to arbitrage that lie behind these stylized facts about corporate capitalism remain a bit of a mystery to economists. We proceed on the assumption that barriers to arbitrage are absolute.

How are the saving decisions of capitalist households and investment decisions of managers coordinated under this assumption? In the demand-constrained model of growth in Chapter 12 this problem was resolved by changes in utilization that brought the saving of capitalists into line with the investment plans of managers or entrepreneurs. Here we are assuming that utilization remains at its normal level. We will show how the price of assets can adjust to coordinate saving and investment.

Any misalignment between the saving of the capitalist households and the investment spending of the managers would express itself as an imbalance in the asset market. There are two polar cases or regimes to consider, as well as a spectrum of intermediate cases that combine elements of the two regimes. In each of the polar cases, a different mechanism coordinates household saving decisions and managerial investment decisions.

First, it is possible that investment will adjust. If the managers are fully sensitive to the q-ratio in formulating investment plans, managers would respond vigorously to any deviation between the q-ratio and its equilibrium value of $q = 1$. The asset market will set stock prices to satisfy this value in a perfect foresight equilibrium. In this case, the wishes of the capitalist households to accumulate wealth are translated so smoothly into the path of capital accumulation that it is as if they were making the investment decisions themselves. We will call this regime *rentier capitalism*, since the preferences of wealthy households rule the roost (*rentier*, a word of French origin pronounced "ron-teeyay," refers to an agent who lives off financial wealth).

Second, it is possible that consumption will adjust. If the managers are fully insensitive to the stock market valuation of the capital stock and they formulate investment plans without paying attention to the q-ratio, a change in stock prices will make the capitalist agents feel wealthier or poorer, as we saw in Section 15.2. Because they consume a constant fraction of their wealth, changes in perceived wealth will stimulate the capitalist households to change their consumption and saving plans. In a perfect foresight equilibrium, the price of stocks and the q-ratio adjust to eliminate any imbalance between saving and investment plans. We will call this regime *managerial capitalism*.

Finally, there is a spectrum of intermediate cases in which both mechanisms can be expected to operate simultaneously. We will call this the *hybrid capitalist regime*.

In all these cases, investment-saving equilibrium implies that the growth of wealth given by the Cambridge equation must be equal to the growth of capital chosen by corporate managers. By substituting from equation (15.3) for r_E in the Cambridge equation, specialized to the case where $g_J = g_K$ to match saving and investment, we arrive at the *IS equation* for the corporate capitalist economy:

$$q = \frac{\beta(r - g_K)}{(1-\beta)(1+g_K)} \tag{15.6}$$

IS is mnemonic for the investment-saving equilibrium that we assumed as a condition for this equation's derivation. It will describe the steady state equilibrium in all three regimes that we need to consider. Once again it is significant that when $q = 1$ this expression reduces to the Cambridge equation that we have seen before in the Classical growth model: $1 + g_K = \beta(1 + r)$. We will see that this will be the form the IS equation takes in the regime of rentier capitalism.

15.4 The Corporate Capitalist Model

The IS equation is valid for any given level of investment but it does not determine the level of investment. In order to close the corporate capitalist model, we need to specify an investment equation.

One important theory of investment suggests that managers compare the value of an investment in capital goods with the valuation of the investment in the stock market. The q-ratio makes precisely this comparison. As John Maynard Keynes first proposed, if the q-ratio is less than unity, managers will find that it is cheaper to add to the capital stock of their firms by buying titles to already-existing capital goods—i.e., by buying stocks that confer ownership of other firms. They will be reluctant to spend money on newly produced capital goods, and investment will be low. On the other hand, if the q-ratio is greater than unity, every \$1 raised by selling new stocks can be used to purchase new capital goods worth less than \$1, providing a stimulus to investment spending. An alternative interpretation is that if the q-ratio exceeds unity, that signifies that the rate of profit is greater than the required rate of return in the asset market so that expanding the capital stock would serve the interest of shareholders.

James Tobin formulated a neoclassical version of this theory that focuses on the marginal q-ratio or the ratio of the marginal benefits of an investment to its marginal costs. Our q-ratio is an average q-ratio.

A straightforward way to close the basic model of corporate capitalism is to use a modified form of the q-theory of investment. We can put Keynes's original ideas into practice by writing out an investment equation in simple

Table 15.2 The Corporate Capitalist Model

Endogenous variables: $w, r, c, g_K, r_E, q, g_J$
Exogenous parameters: $k, x, \beta, \bar{w}, \bar{g}\eta$

$$w = x - rk \tag{15.7}$$

$$c = x - g_K k \tag{15.8}$$

$$w = \bar{w} \tag{15.9}$$

$$g_K = \bar{g} + \eta(q - 1) \tag{15.10}$$

$$(1 + g_J) = \beta(1 + r_E) \tag{15.11}$$

$$(1 + r_E) = \frac{1}{q}(r - g_K + q(1 + g_K)) \tag{15.12}$$

$$g_K = g_J \tag{15.13}$$

linear form that we will refer to as the GQ-equation below:

$$g_K = \bar{g} + \eta(q - 1) \tag{15.14}$$

The intercept term, $\bar{g}$, captures the effect of animal spirits that we discuss further below. The slope term, η, is the *q-sensitivity of investment*. It measures how responsive managers are to the relative profitability of investment projects, which we have seen is one possible interpretation of the q-ratio. Since the q-ratio measures profitability relative to the return expectations of capitalist households, this parameter can be seen as a measure of how responsive the managers are to the wishes of the capitalist households. At one extreme, as $\eta \to \infty$ the q-ratio will not diverge from unity and we are in the rentier capitalist regime. When $q = 1$, the model of corporate capitalism collapses into the basic Classical growth model that makes no distinction between owners and managers. At the other extreme, if $\eta = 0$, the managers are fully autonomous and we are in the managerial capitalist regime where investment reflects animal spirits.

In between these extremes lies a spectrum of hybrid regimes that probably are closer to real capitalist economies. We will focus on studying the properties of the equilibrium steady states in the polar regimes in order to build insights and intuition that will make the hybrid regime more transparent.

We can write out the seven relations that make up the corporate capitalist model in terms of the parameters $k, x, \beta, \bar{w}, \bar{g}$, and η as in Table 15.2 (recall that we have assumed $\delta = 0$ for simplicity). This model uses the same conventional wage closure used in the basic Classical model. But it also distinguishes between investment, equation (15.10), and saving, equation (15.11). As we saw in Chapter 12, this requires that we add additional variables (in this case r_E and q) and equations to avoid the problem of overdetermination. Solving equations (15.11)–(15.13) gives the IS equation (15.6) that describes investment-saving equilibrium for any given level of investment. The investment equation, which we label as GQ in figures below, then determines the specific level of investment and growth in the steady state equilibrium.

15.5 Stock Prices and the Asset Market

Our default assumption about the financial plan is that corporate managers are reluctant to rely on external funding for investment spending. Because they are able to finance all their investment spending through retained earnings, they are relieved of the need to issue new stocks so that $\Delta E = 0$. We will assume that at some time in the past, the managers did in fact issue some stock, perhaps through an initial public offering or IPO, and take the quantity of stock, E, to be constant.

We can see from the SAM that this implies that SNA saving is zero since there are no new stocks to absorb household saving. Capitalist households' desire to increase their wealth is satisfied entirely by capital gains. We can work out from the financial plan, equation (15.1), that for a given rate of capital accumulation, g_K, the retention rate will be the constant $s_F = g_K/r$. Managers will distribute any profits not needed to finance investment to stockholders as dividends. Capitalists receive some of their equity returns in the form of capital gains rather than as a pure dividend yield. A constant q-ratio in a steady state with no stock issuance implies that the rate of capital gains is equal to the rate of accumulation. We will find that these steady state relationships are useful in developing the model of corporate capitalism with internal financing of investment:

$$\Delta E = 0 \qquad S^c = 0 \qquad s_F = g_K/r \qquad g_P = g_K$$

Because they are in fixed supply, stocks are like land in the land-limited growth model in Chapter 13. They will be priced in an asset market by forward-looking agents with perfect foresight. Stock prices, like land prices in the scarce land regime, will be the present value of the future stream of dividends on a share of stock. We will attack the problem of asset pricing by first making the assumption that the whole system has achieved a steady state equilibrium before turning in the next section to the equilibrating mechanisms.

Since arbitrage with capital goods is ruled out, capitalist households will use the equilibrium equity yield established in the asset market as the discount factor for valuing stocks. Dividends per share of stock must be growing at the same rate as the capital stock, g_K, because $V = (1 - s_F)rK/E$ and E is constant by assumption. To remain consistent with our definition of the equity yield, which excludes the reinvestment of dividends, we assume that dividends on traded stocks are transferred at the end of the period so they are only available in the next period. The present value of the future stream of dividends on a share of stock evaluated in period t is

$$P_t = \sum_{T=0}^{\infty} \frac{V_t(1+g_K)^T}{(1+r_E)^{T+1}}$$

When this equation is solved for P_t, it takes the form of the Gordon Growth Model well-known to financial analysts and first proposed by Myron Gordon in the 1950s.

$$P_t = \frac{V_t}{r_E - g_K} \tag{15.15}$$

It is reassuring that when we solve the Gordon equation for the equity yield, we recover our original definition of it in equation (15.2) with the rate of capital gains equal to the rate of growth of the capital stock, $g_P = g_K$.

We have derived the Gordon equation on the assumption that the whole system has achieved a steady state equilibrium and that forward-looking agents are able to predict the growth of capital and dividends. In order to see how the system achieved a steady state equilibrium with coordination between the saving plans of households and the investment plans of managers, we need to examine specific regimes in detail.

15.6 The Rentier Capitalist Regime

By focusing on polar extremes we can gain insight into the operation of asset markets and their role in generating investment-saving equilibrium. The defining feature of rentier capitalism is that managers are fully sensitive to the signals that capitalist households (i.e., the rentiers) send through the asset market. In the most extreme version of this regime where $\eta \to \infty$, managers respond instantly to stock prices and the q-ratio cannot deviate from unity for any significant time. In a sense, the managers are performing precisely the arbitrage that we have assumed to be unavailable to the households.

It is clear from equation (15.3) that the equilibrium condition that $q = 1$ in the rentier economy implies that the equity yield and the rate of profit will be identical, $r_E = r$. The IS schedule, equation (15.6), in the rentier economy simplifies to the Cambridge equation

$$(1 + g_K) = \beta(1 + r)$$

familiar from the Classical growth model.

Like the asset market in the land-constrained growth model of Chapter 13, the stock market in this model of corporate capitalism operates with perfect foresight. Capitalist households recognize that the sensitivity of managers to the q-ratio requires that it remain at the equilibrium level $q = 1$. We can see how this works by studying the model in some initial period with an existing volume of stocks issued in the past.

If the price of stocks were too large, so that $q > 1$, the managers would plan on accumulating capital more rapidly than households plan on accumulating wealth, causing persistent excess demand for the output being produced, upward pressure on prices, and a tendency for utilization to rise above normal levels. If the price of stocks were too small, households would plan on increasing their wealth faster than managers are accumulating capital, causing persistent overproduction of goods, downward pressure on prices, and a tendency for utilization to collapse. Only when the market prices stocks at exactly the correct level (so that $P = K/E$ and $q = 1$) will the plans of capitalist households and managers be consistent with each other. Forward-looking capitalist households perform the function of ensuring that stock prices continuously achieve the equilibrium level that coordinates their own saving plans with the investment plans of the managers.

We can also see that by substituting $V = (1 - s_F)rK/E$ and $s_F = g_K/r$ into the Gordon Growth Model, equation (15.15), we arrive at the same pricing equation, $P = K/E$. Stock prices will be rising continuously at the same rate as capital. The equity yield will comprise a dividend yield equal to $r - g_K$ and a capital gain equal to g_K, which sum, as we have already seen, to r.

An increase in the capitalist propensity to save, β, would have no immediate effect on the price of stocks. However, because it would raise the equilibrium rate of growth through the Cambridge equation, it would raise the rate of growth of stock prices for all subsequent periods. This increased growth in stock prices will actually induce managers to raise the rate of accumulation to maintain balanced growth continuously.

Asset markets are unlikely to be so well-behaved in practice. We have abstracted from forecasting errors created by uncertainty and random shocks that are characteristic of real economies, as well as from purely speculative behavior that is prevalent in real financial markets.

PROBLEM 15.4 Consider the Industrian economy (see Problem 2.2) with a corporate capitalist structure, a conventional wage of \$30,000/worker-year, and a depreciation rate of zero. The capitalist households save 90% of their end-of-period wealth. The economy has initial capital of \$100,000, with 25,000 shares of stock previously issued. Find the equilibrium values for the rentier capitalist regime in Industria when the corporations finance all investment using retained earnings. Calculate the rate of growth of capital, retention rate, and dividend per share of stock. Find the price of a share of stock using the q-ratio. Is it consistent with the Gordon Growth Model?

PROBLEM 15.5 Calculate the breakdown of the equity yield in Problem 15.4 into the dividend yield and the rate of capital gains.

15.7 The Managerial Capitalist Regime

At the other extreme, managers are completely impervious to the signals emanating from the asset market and $\eta = 0$. In this case, they have chosen to expand the capital stock at a constant and given rate, $\bar{g}$. This idea can also be traced back to Keynes, who observed that many investment decisions reflect the animal spirits of managers rather than any cold calculation of the costs and benefits of investment. If investment depends on animal spirits, $q = 1$ is no longer in general an equilibrium condition.

Investment-saving equilibrium in the managerial capitalist regime implies a steady state valuation ratio, q^*, which we can derive by substituting the animal spirits growth rate into the IS equation:

$$q^* = \frac{\beta(r - \bar{g})}{(1-\beta)(1+\bar{g})}$$

The equity yield associated with this q-ratio satisfies

$$q^* = \frac{r - \bar{g}}{r_E{}^* - \bar{g}}$$

In the managerial capitalist regime, changes in the q-ratio have no effect on investment, but as we have seen in Section 15.2, they will create wealth effects for the capitalists' consumption plan and desired rate of wealth accumulation. Investment-saving equilibrium requires that the wealth accumulation plan of the capitalist agents must agree with the investment plan chosen by the managers. If the q-ratio does not bring about an appropriate rate of wealth accumulation, that would express itself in an excess supply or demand for the shares in existence. A perfect foresight equilibrium in the asset market rules out these imbalances.

With SNA saving set to zero, households' desire to accumulate wealth must be entirely satisfied by capital gains on their existing stock holdings. If the price of stocks were too high in the first period, it would make capitalist households feel wealthy and consume too much. They would be dissaving by unloading their stocks. Planned investment spending would exceed saving and there would be a chronic excess demand for goods. If the price of stocks were too low, the households would feel impoverished and consume too little. They would be saving and competing with each other to buy up the existing stocks. Saving would exceed planned investment spending, causing chronic overproduction of goods. Only when the asset market sets the price of stocks at exactly the correct level will the expectations of both managers and households be fulfilled, with households' desire to accumulate wealth satisfied fully by the capital gains they enjoy. As in the rentier regime, forward-looking capitalist households perform the function of ensuring that stock prices continuously achieve the equilibrium level that coordinates their own saving plans with the investment plans of the managers.

This same reasoning applies in each subsequent period. On a perfect foresight equilibrium path, the market will price stocks correctly in the initial period and the price of stocks will then grow at the rate $g_P = \bar{g}$ in order to preserve the coordination of household saving and managerial investment decisions over time.

We can also see that by substituting $r_E = r_E{}^*$, $V = (1 - s_F)rK/E$, and $s_F = \bar{g}/r$ into the Gordon Growth Model, equation (15.15), we arrive at a pricing equation

$$P_t = \frac{r - \bar{g}}{r_E{}^* - \bar{g}}(K_t/E)$$

that simplifies to $q = q^*$. In a steady state stock prices will be rising at the same rate as the capital stock, and the equity yield will comprise a dividend yield, $(r - \bar{g})/q$, and capital gains, $\bar{g}$, that sum to r_E^*.

An increase in the capitalist propensity to save, β, will have an immediate effect on the stock price. From the IS equation, we can see that a higher propensity to save implies a higher q-ratio and lower equity yield, r_E. The equity yield is the discount factor used to value stock prices, so stock prices will immediately rise to the level predicted by the new higher equilibrium q-ratio. Stock prices will subsequently increase as before to generate capital gains at the rate $g_P = \bar{g}$. In this example, the price of stock is a *jump variable* since it is capable of making abrupt, discrete changes.

PROBLEM 15.6 Consider the Industrian economy (see Problem 2.2) with a corporate capitalist structure, a conventional wage of $30,000/worker-year, and a depreciation rate of zero. The capitalist households save 90% of their end-of-period wealth. The economy has initial capital of $100,000 with 25,000 shares of stock previously issued. Find the equilibrium values for the managerial regime in Industria when the corporations finance all investment using retained earnings and the managers plan to increase the capital stock by 1% per period. Calculate the q-ratio, the retention rate, the dividend per share, and the price of a share of stock. Explain why the q-ratio is not equal to unity.

PROBLEM 15.7 Calculate the equity yield in Problem 15.6 by showing how it breaks down into a dividend yield and capital gains. Verify that the stock price conforms to the Gordon Growth Model.

15.8 The Hybrid Capitalist Regime

The two polar cases are useful for building insights about the role of the stock market in coordinating saving and investment, but they each make extreme assumptions unlikely to be satisfied under realistic conditions. In between these extremes lies a hybrid regime that combines elements of the polar cases. In the hybrid regime, changes in the q-ratio determined in the asset market serve to coordinate saving and investment through both mechanisms we studied in Sections 15.6 and 15.7: equilibrating changes in investment by corporate managers and equilibrating changes in consumption by capitalist households.

Putting together the IS equation (15.6) showing steady state equilibria for given levels of investment (growth) with the q-theory investment equation (15.14) that describes how investment decisions respond to the valuation ratio produces a complete model of the corporate capitalist economy.

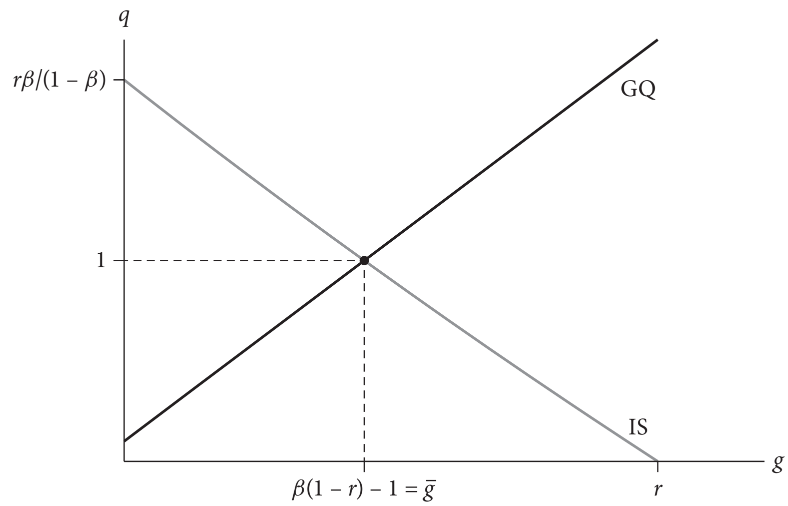

Figure 15.1 The IS-GQ model of corporate capitalism. The parameters have been chosen so that $q^* = 1$. The nonlinearity of the IS schedule is imperceptible even though the figure uses realistic parameter values.

Figure 15.1 provides a helpful way to visualize the IS-GQ model of corporate capitalism. It has been constructed so that the equilibrium where the IS and GQ schedules intersect happens to be at $q^* = 1$ because that makes the equilibrium growth rate $g^* = \beta(1 + r) - 1$ equal to the animal spirits term, $\bar{g}$. As a result, all the key landmarks on the diagram are readily identifiable.

From this benchmark equilibrium with $q = 1$ we can conduct a series of experiments with parameter changes using the comparative equilibrium method in which we examine one parameter at a time. The effects on the steady state equilibrium, (g^*, q^*), can be easily derived using Figure 15.1 and the underlying equations themselves. We will set aside the extreme values for η discussed in the previous section, and return to those below.

First consider an increase in the profit rate, which itself could reflect either an increase in the profit share, π, or in capital productivity, ρ. (Recall that we have assumed that the depreciation rate is 0 so $r = \pi\rho$.) Let us restrict ourselves to a pure redistribution from wages to profits. We see from equation (15.6) that this will shift the IS curve outward, and raise both g^* and q^*. The increased growth rate reflects the importance of profitability in regulating investment spending, which operates through the increased q-ratio. We have here an instance of profit-led growth. But the higher q-ratio also makes capitalist households feel wealthier enough to increase their consumption. Some of the increase in profitability is dissipated in a stock market boom that drives up capitalist consumption out of end-of-period wealth. Which one of these responses dominates depends on the parameters of the model, in particular on the magnitude of the q-sensitivity of investment.

To determine the effects on consumption per worker, we use the consumption-growth schedule to determine that the increase in growth requires social consumption per worker to fall. We know that worker consumption has fallen because the real wage has declined. Capitalist consumption per worker is a fraction, $(1 - \beta)$, of end-of-period wealth per worker, $(1 + r_E)qk$. By substituting from equation (15.3) we can determine that end-of-period wealth per worker is $(q + r + g_K(q - 1))k$. Since each term in parentheses increases, capitalist consumption per worker must increase, which makes sense because the redistribution has increased the capitalist share of national income.

To take a second example (which is not so obvious), an increase in the q-sensitivity of investment rotates the GQ function in a clockwise direction. The effect depends on whether the initial valuation ratio is greater or less than unity. (In the equilibrium shown, where q^* is already unity, an increase in η merely rotates the GQ schedule around the existing equilibrium point.) Since we know that as $\eta \rightarrow \infty$, $q \rightarrow 1$, it is clear that if we started at $q^* > 1$, this would reduce the valuation ratio, and vice versa. One way of seeing this intuitively is to recognize that in the extreme case with $\eta \rightarrow \infty$, the valuation ratio approaches unity, so in either case q is being pushed toward that outcome. The effects on social consumption per worker and capitalist consumption per worker can be worked out easily using the consumption-growth schedule and the fact that with worker consumption constant, any change in social consumption will be the result of a change in capitalist consumption since $c = c^w + c^c$.

The results of these experiments are summarized in Table 15.3, with the remaining comparative dynamic experiments left for the problems below.

PROBLEM 15.8 Analyze the effect of an increase in β on the endogenous variables in the corporate capitalist economy.

PROBLEM 15.9 Analyze the effect of an increase in the animal spirits of managers on the endogenous variables in the corporate capitalist economy.

PROBLEM 15.10 Use the following linear approximation for the IS curve: $q = \beta(r - g)/(1 + \beta)$. Solve the IS-GQ system algebraically for q^* and g^*.

Table 15.3 Comparative Dynamics of the Corporate Capitalist Economy

	Parameter Changes				
	r up	*β up*	*ḡ up*	*η* up	
Effects				$q < 1$	$q > 1$
g^*	up			down	up
q^*	up			up	down
c	down			up	down
c^c	up			up	down

(The linear approximation, which is quite close, eliminates the need to use the quadratic formula to solve the IS-GQ model.) Use your answer to approximate a hybrid capitalist regime equilibrium in Industria assuming $\bar{g} = 0.01$ and $\eta = .1$ (see Problem 15.6 for the other parameters).

15.9 Corporate Saving and the Equity Yield

A remarkable feature of the model of corporate capitalism is that the financial policy of the managers—their choice of a retention rate—has no effect on any major outcome. The decision about how to finance a given investment plan affects how the equity yield is distributed over its components, the dividend yield and capital gains, but it has no effect on the equity yield itself.

The key to understanding this point is to see that the equity yield and the q-ratio are determined by the IS equation and the investment equation independently of the financial policy of the managers. (In Table 15.2 the retention rate is not among the parameters.) Given an equilibrium equity yield, r_E^*, it follows that there is a one-to-one trade-off between the dividend yield and capital gains:

$$g_P = r_E^* - \frac{V}{P}$$

The choice of a retention rate, s_F, only affects where on this trade-off schedule the asset market will lie.

There is one qualification worth remarking on, however. When managers finance all their investment spending out of retained earnings, the retention rate will be $s_F = g_K/r$. If they increase the retention rate beyond this point, we can see from the SAM that stock issuance, $P\Delta E$, must become negative. Negative stock issuance occurs when corporations buy back stock that has been previously issued. Corporate buybacks are a well-established feature of modern asset markets. It is clear, however, that because there is a finite amount of previously issued stock in existence, a corporate capitalist economy cannot operate with $s_F > g_K/r$ indefinitely.

In Sections 15.6 and 15.7, we worked with examples in which dividends per share grew at the same rate as the capital stock. But the Gordon equation requires only that the retention rate remains constant so that dividends per share expand at some constant rate, $(1 + g_K)/(1 + g_E)$. The general discounting equation with a constant retention rate is

$$P_t = \sum_{T=0}^{\infty} \frac{V_t \left(\frac{1+g_K}{1+g_E}\right)^T}{(1 + r_E)^{T+1}}$$

Since a constant q-ratio implies that $1 + g_P = (1 + g_K)/(1 + g_E)$, this leads to the general form of the Gordon equation:

$$P_t = \frac{V_t}{r_E - g_P}$$

If we solve the Gordon equation for the equity yield, we arrive back at equation (15.2), our original expression for r_E.

The idea that corporate financial policies have no real effects beyond shuffling around the composition of the equity yield is controversial. It is clearly true at the level of abstraction of the model of corporate capitalism we have developed here, but we may find that including more concrete determinations that are important in real economies changes this conclusion. For example, corporate managers may experience financial or credit constraints that affect their investment behavior so that pressure from the asset markets to distribute profits to shareholders has implications for their investment plans. Much research into the growing significance of financial mechanisms in modern capitalism (often called *financialization*) emphasizes this kind of effect. In practice, corporations seem to observe a "pecking order," preferring to fund investment by drawing first on retained earnings, then turning to borrowing in bond markets and from banks, and finally relying on equity sales.

PROBLEM 15.11 Find the steady state equilibrium in the rentier capitalist regime in Industria (see Problem 15.4) when the corporate retention rate is $s_F = .1$. Calculate the rate of capital gains and the dividend yield. Verify that the price of a share satisfies the Gordon Growth Model. Hint: use the financial plan and the fact that $g = g_P + g_E$ to determine g_P.

PROBLEM 15.12 Find the steady state equilibrium in the managerial capitalist regime in Industria (see Problem 15.6) when the corporate retention rate is $s_F =.05$. Calculate the rate of capital gains and the dividend yield. Verify that the price of a share satisfies the Gordon Growth Model. Use the hint from Problem 15.11.

15.10 Ownership and Control

The IS-GQ model for a corporate capitalist economy is a transparent tool for interpreting the role of the stock market in the structure of capital accumulation. We have gained important insights by pushing the model to the two extremes: $\eta \to \infty$ corresponding to the rentier capitalist regime and $\eta = 0$ corresponding to the managerial capitalist regime. These cases differ by the degree of separation of ownership and control. Readers should have no trouble following the remarks below if they recognize that the former case would be represented on Figure 15.1 by a horizontal GQ curve and the latter case would be represented by a vertical GQ curve.

In the rentier capitalist regime, the managers have effectively internalized the preferences of the capitalist households by virtue of the careful attention they pay to the required rate of return established in financial markets. We can see this dramatically by returning to one of our earlier thought experiments. Suppose the capitalists' propensity to accumulate wealth increased. This would be represented by an outward shift in the IS curve in Figure 15.1. With a horizontal GQ curve the q-ratio remains at unity and the growth rate increases according to the Cambridge equation, $(1 + g_K) = \beta(1 + r)$, exactly as in the basic Classical growth model with a conventional wage share. The managers' accommodation of the preferences of the capitalist households would translate all the available saving into capital accumulation. The q-ratio remaining at unity means there would be no attenuating influence from the wealth effect on capitalist consumption.

But in the managerial capitalist regime with investment driven entirely by the animal spirits of managers there is an absolute separation of ownership and control. In this case, the q-ratio serves the purpose of incentivizing the capitalist households to consume out of their end-of-period wealth at the rate consistent with the growth plans of the managers. We can see this dramatically by repeating the same thought experiment, an increase in the capitalists' propensity to accumulate. In this case, the vertical GQ schedule forces all the increase in the capitalist desire to accumulate into a stock market boom. Through a wealth effect, the rise in the q-ratio then incentivizes the capitalist households to maintain their consumption out of end-of-period wealth, thus keeping the rate of growth of their wealth constant and equal to the rate of growth of capital dictated by the animal spirits of managers. This is an extreme example of the role of the stock market in coordinating saving decisions of capitalist households and the investment decisions of managers.

Real corporate capitalist economies surely lie somewhere in between these polar extremes. The IS-GQ model suggests that the Classical models of growth are good approximations to modern economies that lie close to the idealization of rentier capitalism. The intermediate cases and the extreme case of managerial capitalism support the Keynesian insistence that coordination between saving and investment is a central scientific question in understanding modern capitalism. The IS-GQ model lends support to a synthesis of the Keynesian and Classical growth theories.

15.11 An Application

The model of a corporate capitalist economy can be used to interpret the behavior of the US economy over the last fifty years. The political and economic system that emerged after the early 1980s is often called *neoliberal capitalism* or just *neoliberalism* to distinguish it from the variant of capitalism that preceded it, often called managerial capitalism because of the relative autonomy from financial markets exercised by managers.[2] Under neoliberalism, the distribution of income has grown more unequal for a variety of economic, social, and political reasons, including changes in corporate governance that have put more financial pressure on managers to suppress real wages. In our model, this would correspond to an increase in the conventional profit share.

[2]This use of the term managerial capitalism should be distinguished from the specific meaning we have assigned in this chapter to the managerial capitalist regime.

Table 15.4 Selected Data for the Nonfinancial Corporate Sector and for the Overall US Economy, 1980–2010

Variable	*1960–1985*	*1985–2010*
π (%)	36.52	37.38
r (%/year)	16.95	18.48
g_K (%/year)	3.74	2.35
q	0.74	0.92
C/Y	0.90	0.96

Sources: From Financial Accounts of the US and US National Income and Product Accounts, Fixed Nonresidential Assets (Bureau of Economic Analysis).

Our comparative dynamic analysis suggests that an increased profit share will increase the steady state values of growth and the q-ratio, with the relative size of the effects depending on the shape of the investment equation.

Table 15.4 shows that the redistribution from wages to gross profits over this interval raised the gross rate of profit in the nonfinancial corporate sector of the US economy that accounts for around 90 percent of the corporate economy. (The Integrated Macroeconomic Accounts do not report data on the financial corporate sector separately.) The national income accounts from which the data are taken probably understate the extent of the redistribution because they classify all corporate executive compensation as wages. As Thomas Piketty has pointed out, the composition of the incomes of the top households shifted dramatically away from property income toward compensation, reflecting the massive increases in managerial pay in the neoliberal era. Many economists believe that at least some proportion of executive compensation is really a disguised form of profit-type income.

The increase in profitability has the predicted effect on the valuation ratio, but it does not have the predicted effect on accumulation or social consumption. If we want to interpret the world through the model of corporate capitalism, something else must have changed to explain these results. The most obvious candidate is the investment equation. Another striking difference between neoliberalism and the preceding variant of capitalism has been the rise of the importance of financial markets and financial performance as a constraint on managerial and household behavior (financialization). Many writers believe that these changes have disincentivized investment, which we can model as a decline in animal spirits in the investment equation. This would help explain why the rise in profitability did not result in an increase in accumulation, but instead seems to have been dissipated in an increase in capitalist consumption. Table 15.4 measures social consumption as the share of consumption spending in net output. In practice, it is not likely that all the increase in social consumption resulted from increased capitalist consumption. There is substantial evidence that noncapitalist (worker and other) households have also reduced their saving rates (which our model assumes equal zero) in the neoliberal era, perhaps in response to financialization, housing price bubbles, and the impulse to "keep up with the Joneses" fostered by rising inequality.

15.12 Suggested Readings

The importance of the separation between ownership and control for the corporate capitalist economy was noticed in the nineteenth century by Veblen (1904) but the modern literature on this phenomenon was initiated in the 1930s by Berle and Means (1968). Kaldor (1966) constructs a growth model with a corporate institutional structure similar to the model in this chapter. He was responding to Luigi Pasinetti's celebrated Cambridge Theorem (see Chapter 17) and derived an equation he dubbed the "neo-Pasinetti Theorem" which attributes considerable importance to the corporate retention rate. Subsequent work in this tradition includes Moore (1975), Skott (1989), and Moss (1978), who provides a clear exposition of the difference between the Pasinetti and neo-Pasinetti Theorems. The q-theory of investment shows up in Keynes (1936). Tobin (1969) formalized a neoclassical version of the q-theory that remains the foundation for the neoclassical theory of investment. Crotty (1990) provides a critique of the q-theory that draws heavily on the importance of the separation of ownership and control. The Gordon Growth Model was outlined in Gordon (1959). The literature on financialization in the neoliberal era is now quite large; Hein and van Treeck (2010) provide a survey and Lazonick and O'Sullivan (2000) is an influential contribution. The rise of the consumption rate of middle-income households in the US is studied by Cynamon and Fazzari (2014), while Duménil and Lévy (2011) argue that the neoliberal model of capitalism depends to an unwholesome extent on rising capitalist consumption. The rise of income and wealth inequality is treated extensively in Piketty (2014).

16

Government Debt and Social Security: The Overlapping Generations Model

16.1 Government Finance and Accumulation

In this chapter we will study the impact of government finance in the form of social security programs and deficit spending on the accumulation of capital. Social security benefits and government debt are an asset to private households, but do not necessarily correspond to any real investment on the part of the government. The key question is whether the existence of these government-created assets can reduce private saving and capital formation.

Government taxes and transfers can have effects on the allocation of resources if the taxes and transfers are linked to economic decision variables like saving or profit. This is because these taxes affect the rates of return perceived by decision-makers, and will influence their decisions to save and invest by changing these rates of return. In this chapter, however, we are interested in whether government programs can divert private saving from the financing of real investment. In order to focus our attention on this particular impact of government fiscal policy, we will consider only programs financed by *lump-sum* taxes and transfers, which do not depend on agents' wealth or income, and thus do not change their economic incentives at the margin.

The effects of a social security system or a deficit spending policy of the government on household saving plans depend critically on whether we assume that each generation takes into account the welfare of future generations in making its spending plans. As Robert Barro has pointed out, if the welfare of future generations enters into the utility function of the current generation, then there will be no macroeconomic effects of deficits or social security plans. The assumption that the current generation takes the future generation's welfare into account in making its spending plans is called *Ricardian equivalence*. We have been using this assumption in all the models where saving decisions are made by a representative capitalist who maximizes utility over an infinite horizon.

It is not hard to see intuitively why the assumption of Ricardian equivalence implies that deficit spending by the government will have no real effects. Under these assumptions the typical household of the current generation can enforce whatever level of next-generation consumption seems optimal by changing its bequest to the next generation and thus undo any effects of deficit spending or social security on social saving. In the next section we work through this problem rigorously by examining the budget constraints of the government and the typical capitalist household under the assumption of Ricardian equivalence.

In considering the importance of Ricardian equivalence in the real world, remember that from the economic point of view a bequest does not have to be an inheritance at the time of death of a member of the current generation. Ricardian equivalence holds as well if the current generation invests in the education of their children (since this investment is an intergenerational transfer, just like a bequest), or, indeed, if the children support their parents in retirement (which is like a negative bequest). If households are rational and forward-looking, the government social security policies and deficits will have an impact on social saving only if each generation acts selfishly.

16.2 Government and Private Budget Constraints

The difference between government revenues and outlays is the *fiscal surplus*. If outlays exceed revenues, the fiscal surplus is negative and is often referred to as a *fiscal deficit*. (It is crucial not to confuse the fiscal surplus and deficit with the *balance of payments surplus or deficit* of a country. The balance of payments surplus or deficit reflects the transactions of all sectors of an economy, private as well as public, with the rest of the world. The fiscal surplus reflects the transactions of the public sector with the private sector.) Revenues and outlays include interest payments received and made by governments. The difference between government revenues and outlays excluding interest payments is called the *primary fiscal surplus*, and measures the degree to which current noninterest revenues are financing current noninterest expenditures.

When governments spend more than their tax revenue, they must finance the resulting primary fiscal deficit by borrowing. In our models, where prices and profit rates are known with certainty, the government will have to pay the same real rate of interest as capitalists can get by investing their money in capital. In this chapter we will assume that the price level is constant, so that real and monetary quantities are the same. We will also assume that the only asset or liability held by the government is its own debt, B. The growth of the government debt under these assumptions will depend on the primary fiscal surplus, E, and its interest payments on the accumulated debt, rB:

$$B_{t+1} = B_t + r_t B_t - E_t = (1 + r_t)B_t - E_t \tag{16.1}$$

From this series we can see that:

$$
\begin{aligned}
B_1 &= (1 + r_0)B_0 - E_0 \\
B_2 &= (1 + r_1)B_1 - E_1 = (1 + r_1)(1 + r_0)B_0 - (E_1 + (1 + r_1)E_0) \\
&\vdots \\
B_T &= (1 + r_{T-1})(1 + r_{T-2}) \ldots (1 + r_0)B_0 \\
&\quad - (E_{T-1} + (1 + r_{T-1})E_{T-2} + (1 + r_{T-1})(1 + r_{T-2})E_{T-3} \\
&\quad \cdots + (1 + r_{T-1})(1 + r_{T-2}) \ldots (1 + r_1)E_0)
\end{aligned}
$$

The economic meaning of this way of looking at the government budget constraint is that the government effectively has to pay an opportunity cost for running a primary fiscal deficit ($-E$) equal to all the future interest it would save if it financed the expenditures out of current taxes. If we define the *total return factor over the horizon* T, $R_T = (1 + r_{T-1})(1 + r_{T-2}) \ldots (1 + r_0)$, we can divide through by R_T and write this equation as:

$$B_0 = \frac{B_T}{R_T} + \Sigma_{t=0}^{T-1} \frac{E_t}{R_{t+1}}$$

The value of the government debt in the current period is equal to the present discounted value of the primary fiscal surpluses over the horizon T plus the present discounted value of the debt at time T.

The government budget constraint depends on what we assume happens to B_T/R_T as $T \to \infty$. If we allow $\lim_{T\to\infty}(B_T/R_T) > 0$, we are assuming that the government can escape the intertemporal budget constraint by indefinitely paying the interest on its debt by new borrowing. Economists call such a path a *Ponzi game*, after a Boston financier who had temporary success with this creative financing method in the 1920s. The *conventional government budget constraint* requires that $\lim_{T\to\infty}(B_T/R_T) = 0$. Under the conventional budget constraint, taking the limit as $T \to \infty$:

$$B_0 = \Sigma_{t=0}^{\infty} \frac{E_t}{R_t}$$

The conventional government budget constraint implies that the value of the government debt in the current period is equal to the present discounted value of the primary fiscal surpluses over the whole future.

Lump-sum government tax and transfer programs that respect the conventional government budget constraint will have no macroeconomic effects in the Classical model where a representative capitalist makes consumption decisions over an infinite horizon. The reason is that the typical capitalist will take into account all the future tax payments and benefits involved in the government's programs, and adjust her own consumption accordingly. Since the government must abide by its budget constraint, the capitalist's consumption and saving decisions cannot be altered by anything the government does. This is the essence of Ricardian equivalence.

To see this point, return to the model of capitalist consumption where the capitalist earns a certain sequence of rates of return $\{r_t\}_{t=0}^{\infty}$ on her wealth in each period, J_t (which may consist of capital or a mixture of capital and other assets like land and government bonds). In each period the capitalist's budget constraint can be written:

$$J_{t+1} = (1 + r_t)J_t - C_t$$

This constraint is exactly the same as the government budget constraint, (16.1), with the capitalist's wealth, J_t, taking the place of the government debt, B_t, and the capitalist's consumption, C_t, taking the place of the primary fiscal deficit, $-E_t$. Thus we can draw the same conclusion:

$$J_0 = \Sigma_{t=0}^{\infty} \frac{C_t}{R_t} \tag{16.2}$$

Economically this means that we can summarize the capitalist's budget constraint as the requirement that the present discounted value of the capitalist's consumption over the infinite future must be equal to her initial wealth.

Now, suppose that the government, starting from a position where $B_0 = 0$, introduces a system of taxes and transfers that imply a series of primary fiscal surpluses (or deficits) $\{E_t\}_{t=0}^{\infty}$ that satisfy the conventional government budget constraint. Suppose for simplicity that the government invests any surpluses in real investment. The typical capitalist household's budget constraint in period t will now have to include these taxes and transfers:

$$J_{t+1} = (1 + r_t)J_t - C_t - E_t$$

So the capitalist can choose any consumption path that satisfies:

$$\Sigma_{t=0}^{\infty} \frac{C_t + E_t}{R_t} = J_0 \tag{16.3}$$

But if the government respects the conventional government budget constraint:

$$\Sigma_{t=0}^{\infty} \frac{E_t}{R_t} = 0$$

then (16.3) represents exactly the same constraints as (16.2), so the government tax and transfer policy has no effect whatsoever on the capitalist's consumption path.

In the types of models we are using, when the government runs a surplus and invests the resources in real capital, this government investment will just take the place of the reduction in saving of the capitalist households as they maintain their consumption plan in the face of higher taxes. (If the government provides consumption services, the capitalist households will take that into account and reduce their consumption accordingly, leaving the path of investment unchanged.) Similar reasoning applies to the periods in which the government runs a deficit: capitalist households will exactly offset the deficit to maintain the overall consumption and investment path unchanged.

16.3 Saving and Consumption with Selfish Households

In order to analyze real macroeconomic effects of social security programs and deficit spending, we need a model in which households make saving decisions over a limited horizon, so that Ricardian equivalence does not hold. One influential model of this kind is the *overlapping generations* model, in which each generation lives a finite number (usually two) periods, and makes its saving and consumption decisions without regard to the future. In these models workers rather than capitalists save in order to finance their retirement consumption. We will look at a Classical version of the overlapping generations model, in which the growth rate of the population varies in order to keep the wage (or the wage share) constant. In this setting government finance decisions can affect the growth rate of the economy. Neoclassical economists have analyzed the overlapping generations model under the assumption of full employment of an exogenously growing labor force, so that the growth rate is determined in the labor market. Under this assumption government fiscal policy cannot have an impact on the growth rate itself, but can have impacts on saving and consumption decisions, wage and profit rates, and the average welfare of the agents in the society.

The overlapping generations model sees the source of social saving as worker households looking toward eventual retirement. The prospect of a period of life in which the household will not be able to earn money and still must live is a powerful motive for saving. This view of saving was developed by Franco Modigliani, and is often called the *life-cycle theory of saving*, because the motive for saving is to allow a steady stream of consumption over the whole life cycle, despite the fact that earnings from work are concentrated at one stage of the life cycle.

This approach differs from the model of capitalist consumption because households in the life-cycle theory plan for finite lifetimes, and therefore consume their whole wealth in retirement. The capitalist household, by contrast, considers the welfare of its whole posterity. Ricardian equivalence, which holds in the capitalist consumption model, does not hold in the life-cycle model.

In order to keep the model simple, we will make some other key assumptions: that households can borrow or lend freely at a single market rate of interest; that no one tries to cheat the system by dying in debt; and that all the funds lent by savers are borrowed by firms for investment, so that the rate of interest is equal to the rate of profit. We will also explain the model assuming that there is no inflation or deflation of money prices, so that all the transactions take place and are measured in terms of real output.

It is possible to analyze the overlapping generations model in two-dimensional diagrams if we assume that households live two periods, so that the only decision they have to make is how to divide their total lifetime income between consumption in their youth and in their retirement.

To begin with, consider a single household that lives two periods. Suppose that it is willing to supply one unit of labor-power to the market in its first (working) period at any positive wage, and will supply no labor-power at any wage in its second (retirement) period. Assume as well that the household leaves no bequests, so that it consumes all of its wealth and income in the retirement period. If we call the household's consumption when it is working c^w, its saving s^w, and its consumption in retirement c^r, we have the following budget constraints, writing $r = v - \delta$ for the net profit rate:

$$c^w + s^w = w$$
$$c^r = (1 + r_{+1})s^w$$

The households will receive the net profit rate on their saving in the second period of their lives, when they are retired. These two constraints can be combined into a single household budget constraint showing the consumption levels in the working and retirement periods the household can achieve:

$$c^w + \frac{c^r}{1 + r_{+1}} = w \tag{16.4}$$

The neoclassical tradition explains household saving on the assumption that households have given *preferences* over different patterns of lifetime consumption. We can represent these patterns as *indifference curves* between consumption in the working period and consumption in the retirement period. These indifference curves reflect such factors as the household's *time preference*, that is, its relative valuation of consumption in the present and consumption in the future, and the different consumption possibilities and demands on the household in the working and retirement periods.

Given these indifference curves, the household will choose the point on its budget constraint that reaches the highest indifference curve. If the indifference curves are smooth and concave to the origin, this implies that the household will choose to consume at a point where the budget constraint is tangent to the indifference curve through that point.

This theory allows for a very wide range of responses of households to changes in wages and interest rates, depending on the relative size of wealth and substitution effects. A rise in the interest rate makes future consumption cheaper in terms of present consumption. A change in the price of future consumption affects present consumption (and saving) in the same ways that the change in the price of one good can affect the demand for another good in the general model of consumer demand. In particular, when interest rates increase, saving may either increase or decrease, depending on the exact shape of the indifference curves, which determines whether the substitution or wealth effect of an increase in interest rates predominates.

To make our analysis simpler, we will assume that the indifference curves of households arise from Cobb-Douglas utility functions:

$$U(c^w, c^r) = (1-\beta)\ln(c^w) + \beta\ln(c^r)$$

Then we know that the household will spend a fraction $1-\beta$ of its lifetime wealth on current consumption, and save a fraction β of its lifetime wealth. The consumption and saving functions are:

$$\begin{aligned} c^w(r,w) &= (1-\beta)w \\ s^w(r,w) &= \beta w \\ c^r(r,w) &= (1+r_{+1})\beta w \end{aligned} \tag{16.5}$$

The Cobb-Douglas assumption implies that current consumption and saving will both increase in a constant proportion to the wage rate and working period income. As we have seen in previous models, the substitution and wealth effects arising from a change in the interest rate exactly offset each other in the Cobb-Douglas case.

16.4 Accounting in the Overlapping Generations Model

We can summarize the overlapping generations model by constructing its social accounting matrix (SAM), which is presented in Table 16.1. Active worker households are signified using w while retired worker households are signified using r.

The first column shows that output costs comprise wages that are assigned as income to workers and gross profits that are assigned as income to firms, f. The next two columns show the disposition of incomes between consumption and gross saving by active workers and retirees.

Firms are assumed to rent the capital from retired workers, so the entries in the column labeled f show how these payments are distributed. Firms do no spending of their own out of their income, so these payments exhaust firm income. Firm saving will be zero and is not shown on the SAM.

The penultimate column shows that gross investment spending is carried out by active worker households. The saving of active workers has to buy the undepreciated capital stock, $(1-\delta)K$, from the retired generation and finance gross investment, $\Delta K + \delta K$. The net result of these transactions, $K + \Delta K = K_{+1}$, is that active worker saving finances the capital stock for the next period as shown in the SAM. Retired worker households finance

Table 16.1 A SAM for the Overlapping Generations Model

	Output	Expenditures				
	Costs	*w*	*r*	*f*	*I*	*Sum*
Output Uses		C^w	C^r		$\Delta K + \delta K$	X
Incomes						
w	W					X^w
r				vK		X^r
f	vK					X^f
Flows of Funds						
w		S^w			$-K_{+1}$	0
r			S^r		$(1-\delta)K$	0
Sum	X	X^w	X^r	X^f	0	

their consumption by selling off the undepreciated capital they accumulated when they were active. (Recall the convention in the SAM that a positive sign represents a source of funds.)

The bottom rows show the flows of funds. Saving by active workers provides the source of funding for additions to capital stocks and replacement of depreciated capital. Retired workers are dissaving since their rental income, vK, does not cover their consumption, $C^r = (1+r)K$. As in the basic model, saving and investment are aligned by construction since the only available use of funds is investment.

16.5 A Classical Overlapping Generations Growth Model

The overlapping generations saving model can be combined with the Classical conventional wage closure of the labor market to construct a model of economic growth.

In this Classical overlapping generations model all saving comes from workers who are looking forward to retirement. The capital stock is owned by retired workers, who save nothing at all because they do not care (by assumption) about future generations. Life-cycle saving theory thus explains social saving on the basis of the preferences of households as represented by their indifference curves, and on the demographics of the society, as represented by the ratio of retired to active workers. All saving comes from wages, in contrast to the capitalist consumption model, where all saving comes from accumulated capitalist wealth.

The technology is a Leontief system described by the parameters k, x, and δ, with no technical change. (It would be straightforward to incorporate pure labor-augmenting technical change.) Thus the demand for labor in each period depends on the amount of capital that exists in that period. The growth-distribution relations continue to hold:

$$w = x - vk \tag{16.6}$$

$$c = x - (g_K + \delta)k \tag{16.7}$$

Now let us make the Classical assumption that the wage is fixed at $\bar{w}$ because the labor force will grow if the wage is above $\bar{w}$ or decline if the wage is below $\bar{w}$, and the labor market clears at full employment in each period. Then the number of young, working households in period $t+1$ will be determined by the saving of working households in period t, since the number of jobs in period $t+1$ depends on the capital stock at the beginning of the period. Thus the growth rate of the population, n, on an equilibrium perfect-foresight path under these assumptions must be equal to the growth rate of the capital stock, g_K. The growth rate of output, g_X, will then also be equal to g_K.

The capital stock of the next generation must be financed entirely by the saving of the current working generation, since the current retired generation consumes all of its wealth and income. Thus the saving of the current generation, $s^w = \beta w$, has to buy back the undepreciated capital stock, $(1-\delta)k$, from the retired generation, and finance gross investment, $(g_K + \delta)k$. Thus we have the overlapping generations *growth-wage relation*:

$$(1-\delta)k + (g_K + \delta)k = (1 + g_K)k = \beta w = s^w \tag{16.8}$$

This savings-investment relation takes the place of the Cambridge equation to determine the growth rate of the capital stock in the Classical overlapping generations model. We can also write this as a relation between the wage, w, and the gross growth rate of the capital stock, $g_K + \delta$:

$$w = \frac{(1-\delta + (g_K + \delta))k}{\beta} = \frac{(1 + g_K)k}{\beta} \tag{16.9}$$

The model is closed by assuming a conventional wage:

$$w = \bar{w} \tag{16.10}$$

In the Classical conventional wage overlapping generations model, the wage, w, is equal to the conventional wage, $\bar{w}$, and the rate of profit, v, is determined from the wage through the real wage-profit rate relation. The wage also determines the growth rates of the capital stock, labor force, and output through equation (16.8). Social consumption, c, then follows from the growth-distribution schedule.

The social consumption per worker, c, is divided between the consumption of the current generation of workers, c^w, and the consumption of the retired generation, c^r. Since the labor force grew at the rate g_{K-1} in the last period, there are $1 + g_{K-1}$ active workers for every retired worker, and since each active worker supplies one unit of labor-power to the economy, the social consumption per active worker will be:

$$c = c^w + \frac{c^r_{-1}}{1 + g_{K-1}} \tag{16.11}$$

EXAMPLE 16.1 *Find the Classical overlapping generations equilibrium for Ricardia (see Problem 2.1) when the wage is 50 bushels of corn/worker-year and workers' households save 50% of the wage.*

Answer: In Ricardia: $x = 100$ bushels/worker-year; $\rho = 5$/year; $k = 20$ bushels/worker; and $\delta = 1$/year. Here we have $\bar{w} = 50$ bushels/worker-year, so the profit rate $v = (x - w)/k = 2.5$/year. The gross growth rate of capital $g_K + \delta = \beta w/k - (1 - \delta) = (.5)(50/20) = 1.25$/year, and the growth rate $g_K = .25$/year. Social consumption per worker is $c = x - (g_K + \delta)k = 100 - (1.25)(20) = 75$ bushels/worker-year. Worker consumption is $c^w = (1 - \beta)w = (.5)(50) = 25$ bushels/worker-year. Retired households consume $c^r = (1 - \delta + v)\beta w = (2.5)(.5)(50) = 62.5$ bushels/worker-year. We see that $c = c^w + (c^r/(1 + g_K))$ holds, since $c^w = 25$ bushels/worker-year and $c^r/(1 + g_K) = 62.5/1.25 = 50$ bushels/worker-year.

The retired generation consumes its saved principal and return:

$$c^r_{-1} = (1 - \delta + v)s^w_{-1} = (1 - \delta + v)(1 + g_{K-1})k \tag{16.12}$$

We can also find social saving per worker, which is just the difference between output and social consumption per worker:

$$x - c = w - c^w + vk - \frac{c^r_{-1}}{1 + g_{K-1}} = s^w - (1 - \delta)k \tag{16.13}$$

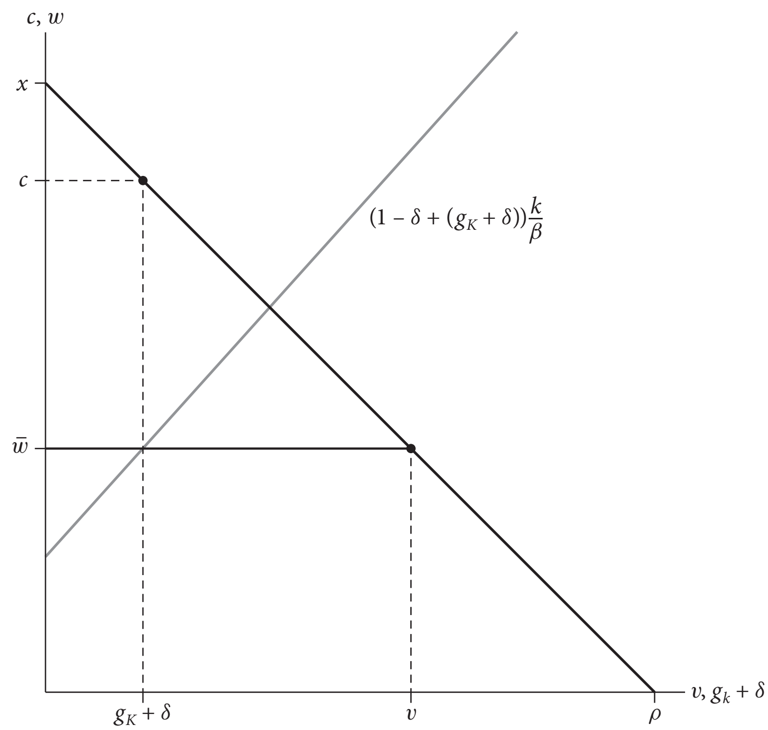

Figure 16.1 In the Classical overlapping generations model the conventional wage, $\bar{w}$, determines the profit rate, v, through the growth-distribution schedule, and the gross growth rate of capital through the growth-wage relation shown in gray, $w = (1 - \delta + (g_K + \delta))k/\beta$. Social consumption per worker, c, is then determined by the growth-distribution schedule. Workers' consumption is $c^w = (1 - \beta)w$, and the consumption of the retired generation is $c^r = (1 + g_{K-1})(1 - \delta + v)k$.

Social saving per worker, $x - c$, differs from the saving per working household, s^w, because retired households dissave the value of the undepreciated capital stock, $(1 - \delta)k$.

Figure 16.1 illustrates the Classical overlapping generations model.

PROBLEM 16.1 Find the Classical overlapping generations equilibrium for Industria (see Problem 2.2) when the wage is \$30,000/worker-year and workers' households save 80% of the wage.

PROBLEM 16.2 In the Classical overlapping generations model, what is the effect of an increase in the conventional real wage $\bar{w}$ on w, v, g_K, c, c^w, and c^r?

PROBLEM 16.3 In the Classical overlapping generations model, what is the effect of an increase in the saving propensity β on w, v, g_K, c, c^w, and c^r?

16.6 A Neoclassical Overlapping Generations Growth Model

The overlapping generations model can also be closed by assuming that the growth rate of the labor force, $\bar{n}$, is given exogenously, and that the wage, w, adjusts to assure the clearing of the labor market. As in the Classical version of the model, the rate of growth of the capital stock must equal the rate of growth of the labor force, $g_K = \bar{n}$, in order to assure full employment of the available labor.

The neoclassical overlapping generations model has the same growth-distribution and saving relations as the Classical overlapping generations model:

$$w = x - vk$$
$$c = x - (g_K + \delta)k$$
$$(1 + g_K)k = \beta w$$

The model is closed, however, by assuming a given growth rate of the labor force:

$$g_K = \bar{n} \tag{16.14}$$

The wage in the neoclassical overlapping generations model is determined by the requirement that the saving of the working generation finance enough investment to employ the next working generation completely:

$$\beta w = (1 + \bar{n})k \tag{16.15}$$

Figure 16.2 illustrates the neoclassical overlapping generations model. The growth rate of capital is determined by the exogenously given growth rate of the labor force, $g_K = \bar{n}$, which determines social consumption, c. The wage is determined by the requirement that the saving of workers equal the whole capital stock necessary to employ the next generation, $w = (1 + \bar{n})(k/\beta)$.

There may be no equilibrium profit rate if β is small and $\bar{n}$ is large, because there may be no wage high enough to induce workers to save enough to employ the entire next generation of workers. We can read c^w from the graph as the distance from the efficiency schedule to the βw line. The consumption of the typical retired household, c^r, is equal to $(1 - \delta + v)(1 + \bar{n})k$. Retired households consume the value of the undepreciated capital stock plus the profit, $(1 - \delta + v)k$, while working households consume a part of the real

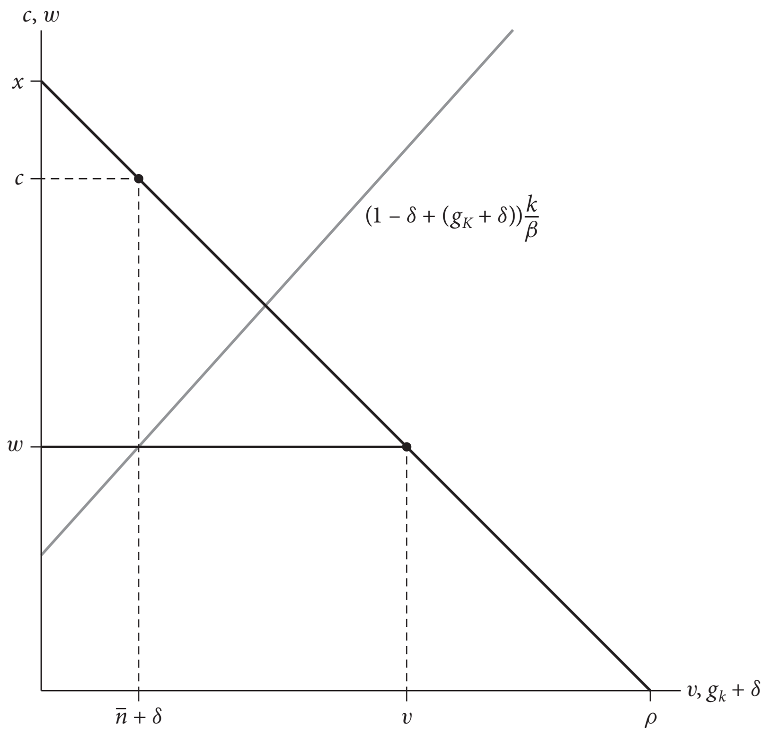

Figure 16.2 In the neoclassical overlapping generations model the growth rate of the labor force determines the growth rate of the capital stock directly, and the wage through the growth-wage relation, $w = (1 + \bar{n})(k/\beta)$. The growth rate of the capital stock determines social consumption per worker, c, and the wage determines the profit rate through the growth-distribution schedule.

wage and save enough of the rest to buy the undepreciated capital stock from the retired workers and provide new capital to replace depreciation and create jobs for the next generation.

EXAMPLE 16.2 *Find the neoclassical overlapping generations equilibrium for Ricardia (see Problem 2.1) when the growth rate of the labor force is .1/year and workers' households save 50% of the wage.*

Answer: In Ricardia: $x = 100$ bushels/worker-year; $k = 20$ bushels/worker; $\rho = 5$/year; and $\delta = 1$/year. Here we have $\bar{n} = g_K = .1$/year, so the wage $w = (1 + \bar{n})(k/\beta) = 44$ bushels/worker-year, and the profit rate $v = (x - w)/k = 2.8$/year. The gross growth rate of capital $g_K + \delta = \beta w/k - (1 - \delta) = (.5)(44/20) = 1.1$/year, and the growth rate $g_K = .1$/year, which will just maintain full employment in the face of the growth of the labor force. Social consumption per worker is $c = x - (g_K + \delta)k = 100 - (1.1)(20) = 78$ bushels/worker-year. Worker consumption is $c^w = (1 - \beta)w = (.5)(44) = 22$ bushels/worker-year. Retired households consume $c^r = (1 - \delta + v)\beta w = (2.8)(.5)(44) = 61.6$ bushels/worker-year. We see that $c = c^w + (c^r/(1 + g_K))$ holds, since $c^w = 22$ bushels/worker-year and $c^r/(1 + g_K) = 61.6/1.1 = 56$ bushels/worker-year.

PROBLEM 16.4 Find the Classical overlapping generations equilibrium for Industria (see Problem 2.2) when the growth rate of the labor force is zero and workers' households save 80% of the wage.

PROBLEM 16.5 Analyze the equilibrium of the overlapping generations model when there are several techniques available.

PROBLEM 16.6 What is the effect of a rise in β on the equilibrium growth path of the overlapping generations model, in terms of v, w, g_K, c, c^w, and c^r?

PROBLEM 16.7 What is the effect of an increase in the growth rate of the population on the equilibrium of the overlapping generations model, in terms of v, w, g_K, c, c^w, and c^r?

16.7 Pareto-Efficiency in the Overlapping Generations Model

An important idea in the history of economic thought is the claim, put forward vividly by Adam Smith, that free competition leads to a socially desirable use of resources. In the twentieth century economic theorists have worked to develop logical concepts to analyze this claim more precisely.

A key concept in this discussion is the notion of a *Pareto-efficient* allocation of resources. We imagine that, instead of having a market economy with private ownership and exchange, an economic dictator has the power to decide what will be produced, how it will be produced, and who will get the output. The dictator has control of exactly the same resources as exist in the market economy, and faces exactly the same production possibilities. The households in the market economy are imagined to have exactly the same preferences under the dictator as they do when there is exchange. An *allocation of resources* is a plan that specifies what will be produced, and what techniques of production will be used, and how the output will be distributed between the consumption of various generations and investment. An allocation is called *feasible* if it would actually be possible to carry it out with the existing resources and technology of the economy. The dictator is assumed to be so powerful that she can order any feasible allocation of resources that she wishes.

Now consider some particular feasible allocation, the *test allocation*. The test allocation could come about in any arbitrary way, but we are particularly interested in test allocations that are equilibrium paths of the overlapping generations model. To analyze the Pareto-efficiency of the test allocation, we consider whether there is another feasible allocation, the *alternative allocation*, that gives every household a consumption plan that it likes at least as well as its consumption under the test allocation, and gives at least one household a consumption plan that it prefers to the test allocation. This is the same thing as asking whether the dictator could rearrange production plans and distribution in such a way as to give at least one household something it likes better without forcing any household to accept something it likes worse than the test allocation. If an alternative allocation exists that does leave every household at least as well off and makes at least one household better off, the alternative allocation is said to be *Pareto-superior* to the test allocation, and the test allocation is *not* Pareto-efficient. If, on the other hand, there is *no* Pareto-superior alternative allocation, the test allocation is *Pareto-efficient*.

In order to prove that a test allocation is not Pareto-efficient, all we have to do is to construct one Pareto-superior alternative allocation. To prove that a test allocation is Pareto-efficient, on the other hand, is logically much more difficult, because it requires us to show that *no* alternative feasible allocation is Pareto-superior.

It is important to see why we use the term *Pareto-efficient* rather than calling Pareto-efficient allocations *optimal*. Optimal means *best*: an optimal allocation is the best allocation under some method of ranking allocations. In particular, we can refer to an optimal allocation only if we have a method of ranking every pair of allocations including cases in which some households are better off and some worse off. But the concept of Pareto-superiority does not allow us to compare *any* two allocations, and in particular cannot rank two allocations in which some households are better off and some worse off. If we have two allocations, and the first makes some households better off and some worse off, and the second makes some other households better off, and some worse off, neither is Pareto-superior to the other. Thus it makes no sense to say that one allocation was best, or optimal, using the logic of Pareto-superiority. The fact that no allocation is Pareto-superior to a Pareto-efficient allocation does *not* imply that a Pareto-efficient allocation is Pareto-superior to every other allocation.

Some economists have been tempted to think that Pareto-efficiency is at least a part of full optimality and argue that an optimal allocation chosen according to any reasonable method of ranking must be Pareto-efficient. But this is not true. The reason is that the Pareto method of comparison of allocation completely ignores the *relative levels* of consumption of different households. An allocation in which one household consumes almost everything and the rest consume almost nothing can be Pareto-efficient because any change that would improve the lot of low-consuming households would have to make the high-consuming household worse off, for example. If the method of ranking we use to decide which allocation is the best allocation includes some consideration of the *distribution* of consumption among the households, it might turn out under certain circumstances that the overall best, or optimal, allocation of resources was not Pareto-efficient.

This is a difficult point for some people to follow. They reason as follows: take the allocation you called the best allocation but that is not Pareto-efficient. Then there is by definition an alternative allocation that makes some households better off without making any households worse off. Surely that alternative allocation is better than the test allocation, so that the test allocation could not be the best after all. The flaw in this argument is that it might not be possible in reality to reach the alternative allocation. For example, the alternative allocation might be achievable in a real-world market economy only by using taxes that are unconstitutional in the country in question, or only by using private information that the government cannot collect. The only way to settle the question of whether an optimal allocation according to some ranking is Pareto-efficient is to specify exactly what the ranking criterion is, and what the institutional setting is within which allocations are going to be determined. Only with this information can we determine the optimal allocation in particular circumstances.

There is a famous economic argument, often called the *First Welfare Theorem*, that says that if an allocation arises as a market clearing equilibrium in an economy where all agents have full information about the qualities of commodities and the technology, where there are no external effects of one agent's economic activities on other agents, that is, effects that cannot be bought and sold for a price on a market, and where there is vigorous competition, so that each agent takes the market price as given, then that allocation will be Pareto-efficient. This theorem can be proved in an economy with a finite number of commodities by showing that if a test competitive allocation were not Pareto-efficient, the alternative allocation would be more profitable for some producer, or provide a higher level of satisfaction at the same income for some household than the test allocation, so that the test allocation could not in fact be an equilibrium allocation.

It is a striking fact that this theorem does not hold in the overlapping generations economy. It is possible to have a competitive equilibrium in the overlapping generations model that is not Pareto-efficient.

To see how this comes about, let us take for our test allocation a steady state equilibrium of the overlapping generations model with a net profit rate $r = v - \delta$. Imagine that we are the dictator, and that we will try to make the retirees in the first period better off without making any of the later generations worse off.

First of all, in each period we have to assign enough output as investment so that the next generation will be fully employed. This requires us to set aside $(\delta + n)k$ units of output for every employed household, because, as we have seen, that will provide the next, larger, generation with just enough capital for all of them to work. As equation (16.7) shows, this means that the total consumption per active worker in each period on the alternative path $(c = x - (\delta + n)k)$ will have to be the same as on the equilibrium path. The only freedom we have is to rearrange that consumption between the working generation and the retired generation.

If we want to make the first retired generation better off, we have to give them more consumption. But to do this, the first working generation will have to consume less. Is there any way to make them better off, despite the fact that they are consuming less in their working period? The only way would be to give them enough more consumption in their retirement so that they liked the alternative situation just as well.

Suppose for definiteness that we took a very small amount of consumption, Δc_1^w units of output, from each of the first generation of workers and gave it to the retirees in the first period. Clearly the first period retirees are better off, because they are consuming more. How much more must we give the first generation of workers when they are retired to keep them just as well off as under the original stationary equilibrium allocation? We know from the theory of saving that the marginal rate of substitution between working consumption and retirement consumption for every generation is $(1+r)$, that is, that each household would view getting $(1+r)\Delta c_1^w$ more units of consumption in its retirement as compensation for losing Δc_1^w units of consumption in its working period, as long as Δc_1^w is very small. In order to make the first generation of workers as well off in the alternative allocation, we have to have:

$$\Delta c_1^r = -(1+r)\Delta c_1^w$$

Here Δc_1^r is the increase in household retirement consumption of the first generation in the second period (when the first generation of workers retire), and Δc_1^w is the decrease in consumption of the first generation in the first period, when they are working. But if we give the first generation more consumption in the second period, we have to take away consumption from the *second* generation while it is working. How much? We know that there are $(1+n)$ households in the second generation for each household in the first generation, so:

$$\Delta c_2^w = -\frac{\Delta c_1^r}{1+n} = \frac{1+r}{1+n}\Delta c_1^w$$

Now we are in exactly the same position with regard to the second generation as we were previously with respect to the first. The first generation of retirees is definitely better off, because they are consuming more. The first generation of workers is no worse off, because we have given them enough extra consumption in their retirement to compensate them for the loss when they were working. Now we have to compensate the second generation of workers in their retirement, by taking some away from the third generation of workers. We can see that following this plan will require:

$$\Delta c_t^w = \left(\frac{1+r}{1+n}\right)^{t-1} \Delta c_1^w$$

Is this plan going to work? If $1+r > 1+n$, it will not work, because we will have to take larger and larger amounts from each generation of workers to keep the last generation as well off as at the stationary equilibrium. But if $1+r < 1+n$ it will work, because the amount we have to take from each succeeding generation of workers will be getting smaller and smaller, and eventually will practically vanish. Thus if $1+r < 1+n$, that is, if the profit rate is smaller than the growth rate of the labor force, the competitive equilibrium is not Pareto-efficient. This example shows that the First Welfare Theorem does not hold in the overlapping generations model, despite the fact that all the assumptions of the theorem are satisfied: the agents have full information, there are no externalities, and the households and firms take market prices as given.

Notice that in proving that the stationary equilibrium with $r < n$ is not Pareto-efficient, we did not compare it to an alternative *stationary* allocation: the alternative we constructed was not stationary because we allowed different generations to have different consumption plans (even though they were all required to save the same amount).

PROBLEM 16.8 Does the argument given above prove that if $r > n$ the stationary overlapping generations equilibrium is Pareto-efficient? Is the alternative path we constructed to show that the $r < n$ stationary equilibrium is not Pareto-efficient itself Pareto-efficient?

PROBLEM 16.9 If you were a dictator in an overlapping generations economy, and you had to choose a stationary path for the economy, which one would you choose to maximize the utility of the representative household? (Hint: how much consumption do you have to allocate between workers and retirees in each period, after you have allowed for enough capital to permit steady growth to continue?) Is this path the one the market will choose?

16.8 Analyzing Social Security and Budget Deficits

Under the assumptions of the overlapping generations model, we can give some definite answers to questions often raised about the economic effects of social security programs and of deficit spending. A government runs a deficit when it spends more than it takes in currently in taxes and has to borrow to cover the difference. One important criticism of deficit spending as a policy is that it might impoverish future generations. A model like the overlapping generations model is a natural setting in which to examine this question.

Social security systems tax active workers and make benefit payments to retired workers. Within a model that distinguishes different generations, we can trace through the effects of social security systems on saving, wages, profit rates, growth rates, and patterns of life-cycle consumption.

In the Classical overlapping generations model, social security systems and budget deficits can alter household saving decisions and change the growth rate of the capital stock and of population. Since the conventional wage is exogenously given, however, the wage and profit rate will not be affected by social security or budget deficits.

Since the neoclassical overlapping generations model is a full employment growth model, the growth rate in the model is determined by the exogenously given growth of the labor force and the rate of labor-augmenting technical progress. As a result, social security programs and government deficits can, by assumption, have no impact on the growth rate itself under the assumptions of the model, though they can have an impact on saving, investment, the wage, and the profit rate.

In the analysis that follows, it is important to keep in mind several limiting assumptions. First, the models we are studying do not distinguish different households in the same generation. In the real economy, social security taxes and benefits differ according to the wages earned by a particular household; income tax burdens also depend on the income level of the household. The models we examine because of this feature cannot say anything about the distributional or insurance effects of the policies within generations.They are limited to examining the effects of the policies between generations.

Second, both social security and income taxes have many economic effects. In the real world, for example, workers might react to high tax rates by cutting down the amount of hours they work, or by retiring earlier. In the model we study we assume that each household supplies exactly one unit of labor-power regardless of the after-tax wage in its first period of life, so that we assume away at the very beginning incentive effects of this kind. These are lump-sum taxes and benefits that have no effects on marginal incentives to work or consume. They do, however, have important wealth and income effects, which the model does reflect.

Third, because our model has no explicit treatment of money, our analysis will be carried out completely in real terms after correcting for inflation. Thus these models cannot tell us anything about the impact of government deficits or social security on inflation. We measure taxes, benefits, and government spending in terms of real output, and the interest rates we work with are real interest rates.

Finally, we will limit our discussions to comparisons of steady state growth paths. The changes we see when we alter some parameter of the system, like the social security tax level, correspond to the differences between two economies, each of which has always had a constant social security tax at the two different levels. Thus we must be cautious in drawing conclusions from this analysis about what would happen in a real economy as it adjusted to a new level of social security benefits and taxes.

16.9 Social Security in the Overlapping Generations Model

We can model a social security system by assuming that the government taxes each working household an amount t and pays each retired household a benefit b, both measured in terms of real output. (The tax and benefit levels might in principle be different for different generations, to reflect changes in social security policies.) Then the budget constraint for the household facing the net profit rate $r = v - \delta$ is:

$$c^w + s^w = w - t$$

$$c^r = b + (1 + r_{+1})s^w, \quad \text{or}$$

$$c^w + \frac{c^r}{1 + r_{+1}} = w - \left(t - \frac{b}{1 + r_{+1}}\right)$$

Because the taxes and benefits are lump-sum, the slope of the household's budget constraint is still $-(1 + r_{+1})$. The effect of the social security system is to reduce the household's lifetime wealth by the difference between its tax payment and discounted benefit $\left(t - \frac{b}{1+r_{+1}}\right)$.

If the typical household maximizes a Cobb-Douglas utility function of consumption in the working and retired periods, the typical household demand functions are:

$$c^w = (1 - \beta)\left(w - \left(t - \frac{b}{1 + r_{+1}}\right)\right)$$

$$s^w = w - t - c^w = \beta w - t + (1 - \beta)\left(t - \frac{b}{1 + r_{+1}}\right)$$

$$c^r = (1 + r)s^w + b$$

As the social security system collects taxes and pays out benefits, it may accumulate a *reserve fund*, representing the excess of taxes over benefits. This reserve fund may become negative if benefits exceed taxes. We assume that the reserve fund is invested (or the deficit is financed by borrowing) at the net profit rate r. We will write f for the size of the reserve fund per worker. The reserve fund per worker will be depleted in each generation by the payment of benefits and the growth of the labor force, and replenished by the interest collected on it and taxes. Thus we have:

$$f = \frac{(1+r)f_{-1} - b_{-1}}{1+g_{K-1}} + t \tag{16.16}$$

Since the reserve fund is invested, it represents an additional source of finance for the capital stock. Thus the saving-investment condition must be modified to include the reserve fund:

$$(1+g_K)k = s^w + f = \beta w - t + (1-\beta)\left(t - \frac{b}{1+r_{+1}}\right) + f \tag{16.17}$$

The growth-distribution equations continue to hold when there is a social security system, providing two more determining conditions to the model, which can be closed either with the Classical assumption of a conventional wage, or the neoclassical assumption of an exogenous rate of growth of the labor force.

The impact of a social security system on the growth path of an overlapping generations economy depends on how much of the taxes are actually accumulated in a reserve fund.

16.9.1 Fully funded social security

In a *fully funded* social security system the government invests the taxes of each generation at the market rate of return by buying bonds or equity investments in enterprises. Thus at any moment in a funded social security system the government has a reserve equal to the taxes paid in that it has not yet paid out in benefits. The relation between the tax and benefit for a funded system is:

$$b = (1+r_{+1})t \tag{16.18}$$

The reserve of a fully funded system is $f = t$ (measured per working household). Thus the aggregate reserve of the fully funded system grows at the same rate as the labor force.

The existence of a fully funded social security system makes no difference whatsoever to the allocation of resources in the economy. Households will consume the same amount when they are working, and the same amount during retirement, regardless of the size of the social security system. To see this formally, notice that when $b = (1 + r_{+1})t$, the discounted lifetime income of the typical household in the budget constraint is exactly the same as when $b = t = 0$. Therefore the decision the household makes about working consumption is exactly the same.

We can see this point mathematically from equation (16.17), since:

$$(1 + g_K)k = \beta w + (f - t) + (1 - \beta)\left(t - \frac{b}{1 + r_{+1}}\right) = \beta w \qquad (16.19)$$

when $f = t$ and $b = (1 + r_{+1})t$.

In a fully funded social security system households save and invest collectively through the social security system on exactly the same terms they could save and invest individually. Some of their saving passes through the social security fund as taxes, but rational households will adjust their private saving exactly enough to compensate.

Most real-world social security systems, however, are *partially funded*. They hold reserves equal to only a fraction of the benefits they owe to retirees. In order to understand clearly the impact of partially funded systems on economic growth and distribution, let us look at the extreme case of an *unfunded* system.

16.9.2 Unfunded social security

In an *unfunded social security system* the government uses the taxes on current working households to pay benefits to the current generation of retired households. Thus it has no reserve fund at all. Each generation's contributions to the system are already consumed by the time that generation retires. The relation between the benefit level and the tax level for an unfunded social security system is:

$$b_{-1} = (1 + g_{K-1})t \qquad (16.20)$$

The benefit of each retired household, b_{-1}, is equal to the tax on each working household, t, adjusted by the difference in the number of working and retired households, $1 + g_{K-1}$. If $g_{K-1} > 0$ there are more working households in each period than retired households, so that a given tax on working households can support a proportionately larger benefit for each retired household.

The existence of an unfunded social security system does make a real difference to saving decisions, and hence to growth rates in the Classical overlapping generations model, and to the wage and profit rate in the neoclassical overlapping generations model. The budget constraint for the typical household under an unfunded social security system, taking account of the benefit-tax relationship (16.20), and assuming that the benefit level is constant over time, so that $b_{-1} = b$, is:

$$\begin{aligned} c^w + \frac{c^r}{1+r_{+1}} &= w - \left(t - \frac{b}{1+r_{+1}}\right) \\ &= w - \left(\frac{b_{-1}}{1+g_{K-1}} - \frac{b}{1+r_{+1}}\right) \\ &= w - b\left(\frac{1}{1+g_{K-1}} - \frac{1}{1+r_{+1}}\right) \\ &= w - b\left(\frac{r_{+1} - g_{K-1}}{(1+r_{+1})(1+g_{K-1})}\right) \end{aligned}$$

When $r_{+1} > g_{K-1}$, the unfunded system reduces the lifetime wealth of the typical household, while when $r_{+1} < g_{K-1}$, the unfunded system increases the wealth of the typical household. According to the Cobb-Douglas demand system, the working household will consume a fraction $1 - \beta$ of its lifetime resources:

$$\begin{aligned} c^w &= (1-\beta)\left(w - b\left(\frac{r_{+1} - g_{K-1}}{(1+r_{+1})(1+g_{K-1})}\right)\right) \\ &= (1-\beta)\left(w - b\left(\frac{1}{1+g_{K-1}} - \frac{1}{1+r_{+1}}\right)\right) \end{aligned}$$

We can use this consumption function to see exactly what effects a change in the size of an unfunded system (corresponding to a change in the benefit of a typical working household, b) will have on the growth path of the economy. Saving per worker will be $w - c^w - t$, so with an unfunded social security system, and assuming that the benefit is constant, so that $b_{-1} = b$, we have:

$$s^w = w - c^w - t = \beta w - b\left(\frac{\beta}{1+g_{K-1}} + \frac{1-\beta}{1+r_{+1}}\right)$$

Workers in an economy with an unfunded social security system save less than workers in an economy with no social security system for every wage rate. The effect of the social security system is to shift the growth-wage relation upward:

$$w = \frac{(1-\delta+(g_K+\delta))k}{\beta} + b\left(\frac{1}{1+g_{K-1}} + \frac{1-\beta}{\beta(1+r_{+1})}\right) \quad (16.21)$$

In the Classical overlapping generations model the conventional wage, $\bar{w}$, determines the net profit rate, $r = v - \delta$, in all periods, so that $r_{+1} = r = (x - \bar{w})/k - \delta$. In this model the growth-wage relation relates the wage to the current period growth rate of capital, taking the last period growth rate of capital, g_{K-1}, as given by the history of the economy. In the neoclassical overlapping generations model, the growth rate of the labor force, $\bar{n}$, determines the growth rate of the capital stock in all periods, so that $g_{K-1} = g_K = \bar{n}$. In this model the growth-wage relation relates the current period wage to the growth rate of the capital stock, taking the next period's net profit rate, r_{+1} (and wage), as given by expectations.

We can see through Figure 16.3 that with an unfunded social security system the growth rate will be lower for any conventional wage, $\bar{w}$, and last period growth rate, g_{K-1}, in the Classical overlapping generations model than with no social security system or a fully funded social security system. Exactly parallel reasoning shows that in the neoclassical overlapping generations model with an unfunded social security system the wage will be higher (and the profit rate lower) for any labor force growth rate, $\bar{n}$, and expected next period net profit rate, r_{+1}, than with no social security system or a fully funded social security system.

To find the steady state of the economy in the Classical overlapping generations model, we have to substitute the steady state growth rate, g_K^*, on both sides of the growth-wage relation, remembering that the net rate of profit, r, is constant over time:

$$\bar{w} = \frac{(1+g_K^*)k}{\beta} + b\left(\frac{1}{1+g_K^*} + \frac{1-\beta}{\beta(1+r)}\right) \quad (16.22)$$

This is a quadratic equation in the steady state growth factor, $1 + g_K^*$.

We can see that when $b = 0$, corresponding to an economy with no social security system, the solution of (16.22) is just the steady state growth rate for the Classical overlapping generations model without social security:

$$1 + g_K^* = \frac{\beta\bar{w}}{k}$$

We could use equation (16.22) to analyze the effect of a larger or smaller unfunded social security system on the growth rate, and hence on the divi-

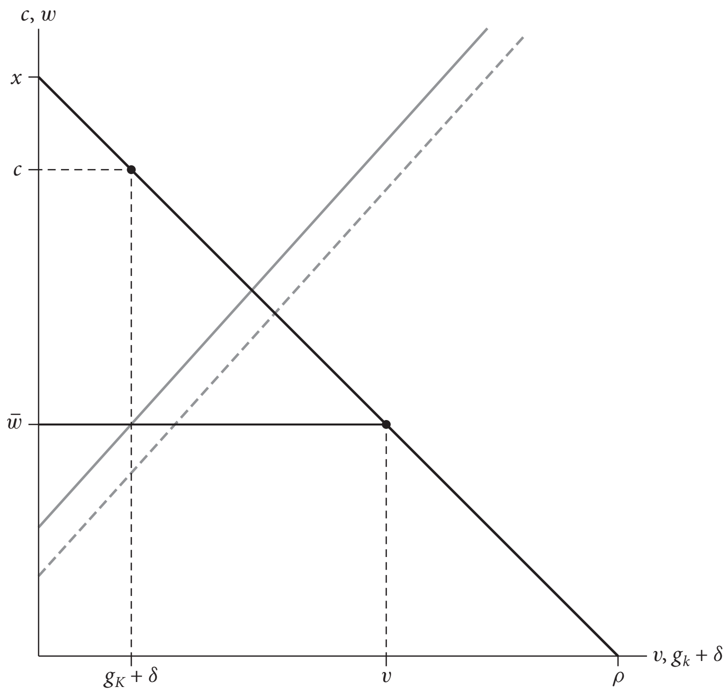

Figure 16.3 Workers in an economy with an unfunded social security system will save less at every wage. As a result the growth-wage relation shifts upward (from the dashed to the undashed gray line above). In the Classical overlapping generations model, the growth rate is lower for any conventional wage and last period growth rate. In the neoclassical overlapping generations model, the wage is higher for any rate of growth of the labor force and anticipated next-period profit rate.

sion of social consumption between working and retired households. Since a change in b will change the equation, it is clear that an unfunded social security system has real effects on the economy that are reflected in changes in the steady state growth rate.

PROBLEM 16.10 Consider an overlapping generations economy where the typical household has Cobb-Douglas utility with $\beta = .2$, the wage $\bar{w} =$ \$750,000/period, x = \$900,000/period, $\delta = 1$, so that capital depreciates completely each period, and k = \$100,000 /worker. Find the steady state equilibrium profit rate, growth rate, and the pattern of consumption of the typical household in the absence of any social security system.

PROBLEM 16.11 For the economy in Problem 16.10, find the steady state equilibrium growth rate, and profit rate with a funded social security system where the benefit for a typical working household is \$5,000/period. Find the steady state equilibrium growth rate, and profit rate for an unfunded social security system with the same social security benefit. What will the tax on working households be?

EXAMPLE 16.3 *Find the steady state equilibrium of the economy described in Example 16.1 with an unfunded social security system paying a benefit, b, of 5 bushels of corn/worker/year.*

Answer: In Ricardia: $x = 100$ bushels/worker-year; $k = 20$ bushels/worker; $\rho = 5$/year; and $\delta = 1$/year. Here we have $\bar{w} = 50$ bushels/worker-year, so the profit rate $\delta + r = 1 + r = v = (x - w)/k = 2.5$/year. Simplifying equation (16.22) we get

$$(1 + g_K^*)^2 + \left(\frac{\beta}{k}\right)\left(b\frac{1-\beta}{\beta(1+r)} - \bar{w}\right)(1 + g_K^*) + \frac{\beta b}{k} = 0$$

or

$$(1 + g_K^*)^2 - 1.2(1 + g_K^*) + .125 = 0$$

The larger root is

$$(1 + g_K^*) = (\delta + g_K^*) = 1.173$$

so the growth rate $g_K = .173$/year. Social consumption per worker is

$$\begin{aligned} c &= x - (g_K + \delta)k \\ &= 100 - (1.173)(20) \\ &= 76.5 \text{ bushels/worker-year} \end{aligned}$$

Worker consumption is

$$\begin{aligned} c^w &= (1-\beta)w - b\left(\frac{r - g_K^*}{(1+r)(1+g_K^*)}\right) \\ &= (.5)(50) - 5(1.5 - .173)/(2.5)(1.173) \\ &= 22.74 \text{ bushels/worker-year} \end{aligned}$$

Retired households consume

$$\begin{aligned} c^r &= (1 + g_K^*)(c - c^w) \\ &= (1.173)(76.5 - 22.74) \\ &= 63.1 \text{ bushels/worker-year} \end{aligned}$$

PROBLEM 16.12 Consider a neoclassical overlapping generations economy where the typical household has Cobb-Douglas utility with $\beta = .2$, the population grows at $\bar{n} = .5$/period, $x = \$900{,}000$/period, $\delta = 1$, so that capital depreciates completely each period, and $k = \$100{,}000$/worker. Find the steady state equilibrium real wage and profit rate, and the pattern of consumption of the typical household in the absence of any social security system.

PROBLEM 16.13 For the economy in Problem 16.12, find the steady state equilibrium wage and profit rate with a funded social security system where the benefit to a typical working household is \$5,000/period. Find the steady state equilibrium wage and profit rate for an unfunded social security system with the same social security benefit level. What will the tax be?

16.10 Government Debt in the Overlapping Generations Model

We can analyze the problem of government debt in the overlapping generations model with the same methods we used to analyze the social security problem. Suppose that the government continues to pay a retirement benefit, b, to each retired household, but that instead of financing it with a tax on workers' wages, as in the social security model, it finances it entirely through borrowing. With this policy the government will have a negative reserve fund, f, which will grow over time. In this case the government has two types of expenditures: the benefit, b, per retired household, and the interest on the outstanding government debt, $B = -f$.

With the Cobb-Douglas utility function, the demand of the workers for consumption, assuming $t = 0$, will be, using the results derived in the last section:

$$c^w = (1-\beta)\left(w + \frac{b}{1+r_{+1}}\right)$$

$$s^w = w - c^w = \beta w - (1-\beta)\left(\frac{b}{1+r_{+1}}\right)$$

$$c^r = (1+r)s^w + b$$

As we saw in equation (16.16), the reserve fund debt will grow as a result of continued borrowing to finance the benefit:

$$f = \frac{(1+r)f_{-1} - b}{1+g_{K-1}} \tag{16.23}$$

The reserve fund deficit has to be financed by the saving of working households (who would hold it in the form of government bonds). Thus the saving-investment condition becomes:

$$(1 + g_K)k = s^w + f = \beta w - (1 - \beta)\left(\frac{b}{1 + r_{+1}}\right) + f \qquad (16.24)$$

As a result the growth-wage relation in any period is:

$$w = \frac{1 - \delta + (g_K + \delta)k}{\beta} + b\left(\frac{1 - \beta}{\beta(1 + r_{+1})}\right) - f$$

Since $f < 0$ under the deficit-financed policy, the effect of the policy is to raise the growth-wage relation even more than the unfunded social security system. In the Classical overlapping generations model, the deficit-financed social security system lowers the growth rate for any conventional wage and size of the government debt compared to either a fully funded or an unfunded social security system.

Can a Classical overlapping generations economy with a deficit-financed social security system reach a steady state? Writing f^* for the steady state debt per worker, this would require, according to equation (16.23):

$$f^* = \frac{(1 + r)f^* - b}{1 + g_K^*}$$

or:

$$f^* = -\frac{b}{g_K^* - r}$$

Since the social security benefit b is positive, and the steady state reserve fund f^* must be negative, a steady state is possible only if $g_K^* > r$. In this case the labor force will be growing fast enough to offset the constant growth in the debt from new borrowing and interest charges. If $g_K^* > r$, the government can violate the conventional government budget constraint and consistently run a Ponzi-game fiscal policy. Because the interest rate is lower than the rate of growth, the government debt will not explode in relation to GDP, even though the government is financing the interest payments on its accumulated debt entirely out of new borrowing.

PROBLEM 16.14 Show that the equilibrium profit rate is the same in the debt model when the debt is zero as in the social security model with no social security tax or benefit.

16.11 The Lessons of the Overlapping Generations Model

If households integrate their private consumption and saving decisions with the fiscal policy of the government, as they do in Ricardian equivalence models, government deficits and accumulated debt will have no effect on saving, investment, or the capital stock. Furthermore, even if households have foreshortened time horizons, as in the overlapping generations models, if the government itself matches its outstanding liabilities against real assets, as it does in operating a fully funded social security system, government finance will have no impact on saving and investment.

But in a world where households do not fully discount the future impact of government spending and taxation, there is room for viable government financial policies that households regard as real wealth, but result in no real accumulation of capital. Unfunded social security systems are an example. Households regard the guaranteed benefit in their retirement years as a real increment to their lifetime wealth, but the government does not invest contributions in real capital. As a result an unfunded social security system can influence social saving and investment, and the growth rate of the economy. In a similar fashion, government debt can appear to be real wealth to households that account for the impact of government taxation and spending over a limited time horizon, and as a result can displace capital from household portfolios.

In the real world, however, it is difficult to judge the degree to which households correctly anticipate the full, long run impact of government financial policies, and try to optimize the inheritance of future generations. Since we cannot independently observe the budget constraints or utility functions of households, it is difficult to construct independent tests of the hypothesis of Ricardian equivalence. If households do attempt to account correctly for the long run effects of government financial policy, then changes in policy, whatever effects they may have on the distribution of income, will not make any difference to social saving, investment, and capital accumulation.

16.12 Suggested Readings

The overlapping generations model was introduced by Samuelson (1958) and Diamond (1965). A modern exposition in this tradition is de la Croix and Michel (2002). For a critical discussion of the neoclassical version of this model, including the specific criticism that its stability requires a high elasticity of substitution, see Marglin (1984). Credit for reviving the Ricardian idea that government debt is neutral goes to Barro (1974). The overlapping generations model is the inspiration for "generational accounting," an alternative method of treating government in national income accounts; see Kotlikoff (1992). Michl and Foley (2004) elaborate the Classical model of social security.

17

Two-Class Models of Wealth Accumulation

In the Classical conventional wage share and full employment models of growth, we assumed that workers' saving is negligible in order to focus on the accumulation of wealth by capitalist households. In modern capitalist economies, workers' saving, for example in the form of pension contributions, accounts for a substantial amount of national saving. This raises the question of whether the Classical models will behave any differently if worker saving is included.

In the Classical and neoclassical overlapping generations models of growth we abstracted from capitalist households altogether in order to study the life-cycle saving of workers in some detail. But this raises the question of whether these models will behave differently if capitalist saving is included.

In this chapter, we combine capitalist households that save for bequest purposes with worker households that save for their retirement to address these questions. We will construct a pair of two-class models of wealth accumulation, one with a conventional wage closure and another with a full employment closure. We will see that Classical models of growth that abstract from worker saving will have the same rate of growth and the same rate of profit in the long run as more realistic two-class models that include worker saving. In this sense, the Classical models of growth provide a good first approximation to the more realistic two-class models. This is the implication of the remarkable *Cambridge Theorem* discovered by Luigi Pasinetti (and sometimes called the *Pasinetti Theorem* or the *Pasinetti Paradox*).

In addition, if both workers and capitalists are saving and accumulating wealth, the distribution of wealth (which is distinct from the distribution of income that we have been studying) emerges as an important analytical variable. Through the two-class models in this chapter, we will be able to study the economic forces that shape the distribution of wealth across social classes in a transparent and tractable analytical framework.

17.1 Worker and Capitalist Saving

Since both classes are now assumed to own capital, we need to distinguish between the wealth of worker households and the wealth of capitalist households. Because households are assumed to accumulate ownership rights over capital goods directly, capital and wealth will be equivalents in this chapter and we will use these terms interchangeably. The total capital stock is simply the sum of these two subcategories, using superscripts to identify them, or

$$K = K^w + K^c$$

The distribution of wealth will be measured by the workers' share of wealth, or

$$\phi = \frac{K^w}{K}$$

As in Chapter 16, the worker households are assumed to maximize the discounted logarithm of their lifetime consumption. Since we now have two kinds of agents saving actively, we will label the workers' discount factor β_w to distinguish it from the capitalist discount factor, β. We know that under these assumptions the worker households will save a constant fraction, β_w, of their lifetime wealth. In the absence of taxes or subsidies, lifetime wealth is equivalent to their wage, w. Workers live for two periods, an active working period and an inactive period of retirement when consumption is funded out of the wealth they accumulated during the active period.

With each worker saving $w\beta_w$ during her active (working) years, total worker saving will be individual saving multiplied by the number of active workers. Since each worker operates k units of capital, the number of active workers depends strictly on the amount of capital being used, or $N = K/k$. We can write the equation for total workers' capital accumulation as

$$K^w_{+1} = \left(\frac{w\beta_w}{k}\right) K \tag{17.1}$$

Worker saving is generated by the wage income of workers employed by the total capital stock, which includes both worker-owned and capitalist-owned wealth. If we were in a one-class overlapping generations setting, as in Chapter 16, equation (17.1) would determine the growth factor, K_{+1}/K, since workers would own all the capital. We can treat the factor $w\beta_w/k$ in this equation as the hypothetical growth factor that would prevail in the one-class setting. We will call this the *workers' incipient growth factor*. This growth factor plays a role in how we interpret the distribution of wealth in the two-class model.

The capitalist agents also maximize the discounted logarithm of their future consumption, but over an infinite horizon that indicates that they are taking into account the utility of their descendants who receive bequests of wealth. The capitalist households save a constant fraction, β, for bequest purposes. Their wealth evolves according to the familiar Cambridge equation,

$$K^c_{+1} = \beta(1+r)K^c \tag{17.2}$$

We will construct our two-class models of wealth accumulation around these two equations. As in previous chapters, we will start with an endogenous growth closure with a conventional real wage before moving on to consider an exogenous closure with the full employment of a constantly growing workforce. The models we develop will thus be directly comparable to the Classical models in Chapter 6 and to the overlapping generations models in Chapter 16.

17.2 Accounting in the Two-Class Models

We can summarize the two-class model by constructing its social accounting matrix (SAM), which is presented in Table 17.1. Active worker households are signified using w, while retired worker households are signified using r.

The first column shows that output costs comprise wages that are assigned as income to workers and gross profits that are assigned as income to firms, f. The next three columns show the disposition of incomes between consumption and gross saving by active workers, retirees, and capitalist households.

Firms are assumed to rent the capital from retired workers and from capitalists, so the entries in the column labeled f show that these payments are distributed in proportion to the ownership shares. Firms do no spending of their own out of their income, so these payments exhaust firm income. Firm saving will be zero and is not shown on the SAM.

The penultimate column shows that gross investment spending is carried out by active worker households and capitalist households. The saving of

Table 17.1 A SAM for the Two-Class Model

	Output Costs	Expenditures					
		w	r	c	f	I	*Sum*
Output Uses		C^w	C^r	C^c		$\Delta K + \delta K$	X
Incomes							
w	W						X^w
r					vK^w		X^r
c					vK^c		X^c
f	vK						X^f
Flows of Funds							
w		S^w				$-K^w_{+1}$	0
r			S^r			$(1-\delta)K^w$	0
c				S^c		$-K^c_{+1} + (1-\delta)K^c$	0
Sum	X	X^w	X^r	X^c	X^f	0	

active workers has to buy the undepreciated capital stock, $(1-\delta)K^w$, from the retired generation and finance gross investment, $\Delta K^w + \delta K^w$. The net result of these transactions shown in the SAM is that the saving of the active generation of workers finances the wealth of workers in the next period. Retired worker households finance their consumption by selling off the undepreciated capital they accumulated when they were active. (Recall the convention in the SAM that a positive sign represents a source of funds.) For capitalist households where we have not imposed any generational structure, all these transactions are summarized in one entry in the penultimate column.

The bottom four rows show the flows of funds. Saving by active workers and by capitalist households provides the source of funding for additions to capital stocks and replacement of depreciated capital. Retired workers are dissaving since their rental income, vK^w, does not cover their consumption, $C^r = (1+r)K^w$. As in the basic model, saving and investment are aligned by construction since the only available use of funds is capital.

PROBLEM 17.1 Construct a SAM for a two-class model in which retirees do not consume all their remaining wealth (perhaps because they can not predict their life span with great accuracy). In this case, the retirees save and leave a small bequest to the next generation of active workers (their grandchildren), so that active workers have some initial wealth. Label this bequeathed wealth K^b.

17.3 Accumulation with a Conventional Wage

As in the Classical overlapping generations model, we are assuming that the wage is fixed at a conventional level, $\bar{w}$, and that labor is supplied elastically at that wage. The labor force will grow if the wage exceeds this level or decline if it falls below this level. Because the labor market clears at full employment in each period and workers are forward-looking, the number of young, active households in period $t+1$ will be determined by the total saving of the capitalist and worker households, K_{+1}, which determines the number of jobs at the beginning of the period, $t+1$.

With a given technology and a conventional real wage, $w=\bar{w}$, the rate of profit will be determined by the wage-profit schedule, $w=y-rk$. This makes the rate of accumulation of capitalist wealth a constant according to equation (17.2). We use the symbol g to denote the growth rate of capitalist wealth because, as we will see, in the steady state the growth rate of total wealth will typically be governed by the behavior of the capitalist households. It is convenient to focus on the growth factor for capitalist wealth, $(1+g)$, which obeys the growth version of the Cambridge equation:

$$(1+g)=\beta(1+r)$$

The capital stock grows as a result of both capitalist saving and worker saving,

$$K_{+1}=(1+g)K^c+\left(\frac{w\beta_w}{k}\right)(K^c+K^w)$$

but since capitalist wealth grows at a constant rate, capitalist saving, $(1+g)K^c$, can be eliminated from both sides of this equation. We can then focus on equation (17.1) to derive an expression for the evolution of worker wealth. If we assume the system has evolved from some initial conditions given by wealth holdings in period 0, K_0^w and K_0^c, then capitalist wealth at any period, t, will be given by

$$K^c=K_0^c(1+g)^t$$

By making substitutions and rearranging, we see that worker wealth will obey a first-order (meaning there is one time lag) difference equation:

$$K^w_{+1} - \left(\frac{w\beta_w}{k}\right) K^w - \left(\frac{w\beta_w}{k}\right) K^c_0(1+g)^t = 0 \tag{17.3}$$

Here we are confronted with a race between capitalist wealth, growing by the factor $(1+g)$, and worker wealth, governed by this difference equation. Will the system achieve a steady state in which both forms of wealth grow at the same rate and the distribution of wealth stabilizes?

Let us initially assume that it will, and that the steady state rate of growth will be the rate of growth of capitalist wealth, $g^* = g$. (We will demonstrate that the system will in fact achieve this steady state below.) From the worker saving function, equation (17.1), we can see that with worker wealth also growing at the steady state growth rate we have

$$(1+g)K^w = \left(\frac{w\beta_w}{k}\right) K$$

Dividing both sides by K reveals the steady state wealth distribution (indicated by an asterisk) to be

$$\phi^* = \frac{w\beta_w/k}{1+g} \tag{17.4}$$

This expression conveys the important insight that the distribution of wealth reflects the contest between the workers' incipient growth factor (the numerator) and the capitalist growth factor (the denominator). It is immediately evident that a steady state outcome with workers owning a fraction of the capital stock ($\phi^* < 1$) depends on the inequality $(1+g) > w\beta_w/k$. In this case, with the capitalist growth factor winning the contest, we will have a two-class model of wealth accumulation.

If this inequality is violated, however, the workers' wealth will grow faster than capitalist wealth. Over time, the capitalist agents will become increasingly insignificant in the operation of the system, and the economy will asymptotically approach a one-class model of accumulation such as the model presented in Chapter 16. It is clear from equation (17.4) that there will be a critical value for β_w beyond which the system escapes the two-class regime and enters the one-class regime. The Classical overlapping generations model could arise if this condition is met.

Since we are primarily interested in the two-class regime, we will proceed on the assumption that the workers' saving propensity has not breached this limit.

PROBLEM 17.2 Find the equation (expressed in terms of the parameters of the model) for the critical value of β_w that separates the one- and two-class regimes in the model with a conventional wage. What factors make the two-class regime more likely?

17.3.1 The Cambridge Theorem in the conventional wage model

We can confirm these results more rigorously by solving equation (17.3) for workers' wealth, for example by using the method of undetermined coefficients (see the mathematics texts in Suggested Readings for more details on this method for solving a first-order difference equation) to arrive at the general solution given initial conditions on wealth:

$$K^w = (K_0^w - \phi^* K_0)\left(\frac{w\beta_w}{k}\right)^t + \left(\frac{\phi*}{1-\phi^*}\right) K_0^c(1+g)^t \qquad (17.5)$$

Here we have expressed the solution in terms of the steady state distribution of wealth in order to highlight the economic intuition and reveal an important insight. If the workers' initial wealth happens to coincide with the steady state distribution of wealth, the first term in parentheses on the right-hand side will be zero and workers' wealth will obviously grow at the steady state rate, $g^* = g$. In other words, the steady state growth rate is fully determined by the Cambridge equation.

We have arrived at a version of the Cambridge Theorem discovered by Luigi Pasinetti, which states that the relation between the rate of profit and the rate of growth in a two-class model of accumulation depends only on the saving behavior of the capitalist agents. In this case, given the distribution of income and the capitalist saving propensity, β, the growth rate is fully determined through the Cambridge equation and worker saving behavior has no effect on it in the long run.

The mechanism underlying the irrelevance of worker saving is that the wealth distribution adjusts so that the saving generated by active workers achieves exactly the same growth of worker-owned capital that would have prevailed counterfactually if capitalist agents had complete control over the accumulation process.

To gain a fuller appreciation of how exactly worker saving and the wealth distribution co-evolve, we need to demonstrate that the economy will achieve a steady state even if it starts with $K_0^w \neq \phi^* K$. Consider what happens when the initial worker-owned capital fails to correspond to the amount dictated by the steady state wealth distribution so that the first term in parentheses on the right-hand side of equation (17.5) is nonzero. Let the workers' wealth growth factor be represented by g_W so that by definition

$$K_{+1}^w = K^w(1 + g_W)$$

By substituting this expression into the workers' saving function, equation (17.1), and using the definition of the wealth distribution we can see that the growth of workers' wealth depends critically on the actual distribution of wealth through

$$(1 + g_W) = \frac{w\beta_w/k}{\phi}$$

From the fact that the steady state growth rate will be g, we can see immediately that when $\phi < \phi^*$ so that $K_0^w < \phi^* K_0$, workers' wealth will grow faster than capitalist wealth, or $g_W > g$. Conversely, starting from a position where $\phi > \phi^*$ so that $K_0^w > \phi^* K_0$, workers' wealth will grow more slowly than capitalist wealth. In either case, this suggests that over time the actual distribution of wealth will be gravitating toward the steady state distribution of wealth. It is through this mechanism that the growth rate of workers' wealth ultimately comes into alignment with the growth rate of capitalist wealth determined by the Cambridge equation, assuming that the system is stable and does converge on a steady state equilibrium.

We can demonstrate that the system will in fact be stable and that its convergence will be "monotonic"—meaning that there are no oscillations or reversals caused by overshooting the steady state. Using equations (17.1), (17.2), and (17.4) we can derive the equation that describes the motion of the wealth distribution. The wealth distribution obeys a nonlinear first-order difference equation:

$$\phi_{+1} = \frac{\phi^*}{\phi^* + (1 - \phi)}$$

An expression like this is often called the *equation of motion* of a dynamic system. As long as we are in a two-class regime so that $\phi^* < 1$, this particular form for the equation of motion will always converge on the steady state monotonically. (See the mathematics texts in Suggested Readings for further details.)

PROBLEM 17.3 Find the equilibrium for Industria (see Problem 2.2) assuming a two-class model with a conventional wage of $30,000/worker-year, capitalist households' $\beta = .97$, and worker households' $\beta_w = .80$. Compare your answer to the answer to Problem 16.1 for the Classical overlapping generations model with no capitalist households.

PROBLEM 17.4 Set up a spreadsheet for the dynamics of the two-class model with a conventional wage and compute it over twenty-five periods. Include an equation for workers' wealth and a separate equation for capitalists' wealth. Choose initial values for workers' and capitalists' wealth stocks and use parameter values from the previous problem. Verify that the model is stable, that it converges on the distribution of wealth you expected, and that it confirms the Cambridge Theorem after a change in the workers' saving propensity, β^w.

17.3.2 Comparative dynamics with a conventional wage

Establishing that the two-class model of wealth accumulation does converge on a steady state clears the way for a comparison of different equilibrium paths that share all parameter values except for one that we have chosen to single out for study. To facilitate this comparative dynamic equilibrium analysis we use equation (17.4) to describe the workers' share of wealth required for any given steady state growth rate, g^*. This schedule is labeled "workers" in Figure 17.1. And we will invoke the Cambridge Theorem and use the fact that g^* is determined by the capitalist saving function, equation (17.2), independently from the distribution of wealth. This schedule is labeled "capitalists" in Figure 17.1. Together, these two equations determine the steady state values, g^* and ϕ^*.

We can use this framework to analyze any change in parameters. First, let us consider an increase in the worker's propensity to save, β_w. From the Cambridge equation describing capitalist saving, we can see that this change will not affect the steady state growth rate. This is the implication of Pasinetti's Cambridge Theorem. From equation (17.4) describing the steady state wealth distribution, however, we can see that this will shift the schedule labelled "workers" in Figure 17.1 to the right (or out). The new steady state share of workers' wealth will therefore increase as a result of higher worker

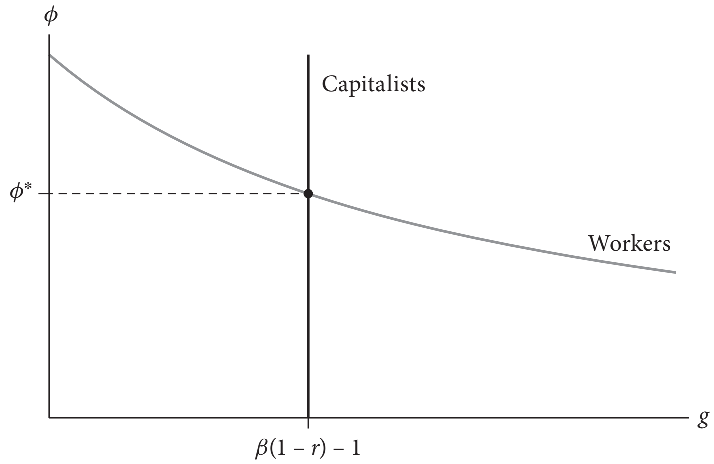

Figure 17.1 In the conventional wage two-class model, the steady state workers' share of wealth declines as the growth rate increases. The Cambridge Theorem dictates that the growth rate is determined by the capitalists' saving propensity and the rate of profit, independently of worker saving and the wealth distribution.

saving. This makes good sense intuitively, and fits well with our interpretation of the wealth distribution as the outcome of a contest between the workers' incipient growth factor (which we have just raised) and the capitalist growth factor.

While the increase in workers' saving has no effect on the steady state growth rate, it would be misleading to say it had no growth effects at all. In this case, during the transition to the new steady state, the growth rate will be higher than g^* until the system has had time to adjust fully and the workers' wealth share has risen to its new equilibrium value. The initial effect of the parameter change is to put the model in a position where $\phi < \phi^*$, which as we have already seen will raise the growth rate temporarily above its steady state value. The increase in workers' saving will have *level effects* in the long run because the economy will ultimately be operating at a higher level of activity (output, capital, and employment) despite having the same growth rate as before.

Second, consider an increase in the capitalist propensity to save, β. In this case, the workers' wealth schedule in Figure 17.1 will not be affected at all, but the Cambridge equation, labeled "capitalists" on the diagram, will shift outward. Workers will thus wind up with the lower share of wealth needed to accommodate the faster rate of accumulation dictated by capitalist behavior.

Table 17.2 Comparative Dynamics of the Two-Class Model with Conventional Wage

Parameter Changes						Effects	
β_w	β	w	x	k	ρ	ϕ^*	g^*
up	same	same	same	same	same	up	same
same	up	same	same	same	same	down	up
same	same	up	same	same	same		
same	same	same	up	up	same		
same	same	same	up	same	up		
same	same	same	same	down	up		

Again, this fits well with our interpretation of the wealth distribution as the outcome of a contest between workers' incipient growth factor and the capitalist growth factor.

Table 17.2 summarizes the results of these two comparative dynamic exercises. It leaves the remaining comparative dynamic exercises unfinished so that students can tackle them on their own.

PROBLEM 17.5 Analyze the effect of an increase in the conventional wage, $\bar{w}$ in the two-class model. Comment on the effect this has on the distribution of wealth.

PROBLEM 17.6 Analyze the effect of a Harrod-neutral technical change that increases output per worker, x, but keeps the output-capital ratio, ρ, constant in the two-class model with a conventional wage.

PROBLEM 17.7 Analyze the effect of a Hicks-neutral technical change that increases output per worker but keeps the capital-labor ratio, k, constant.

PROBLEM 17.8 Analyze the effect of a Solow-neutral technical change that increases the output-capital ratio but keeps output per worker constant.

17.4 Accumulation in the Full Employment Model

An alternative closure for the two-class model of wealth accumulation assumes that the labor force grows at the natural rate, n, and that the distribution of income adjusts to maintain continuous labor market clearing at full employment. This is the assumption we used in the neoclassical overlapping generations model. In each period, retired workers and capitalists start with capital stocks inherited from the past and the wage rate and rate of profit must adjust so that the saving and investment decisions of active workers and capitalists result in just the right amount of accumulation to keep the growing workforce fully employed. Mathematically, we must satisfy the following equation in each period:

$$K^c_{+1} + K^w_{+1} = K_{+1} = (1+n)K$$

By substituting from the worker and capitalist saving functions, equations (17.1) and (17.2), and using the fact that the capitalist wealth share is $(1-\phi)$, we can arrive at a more detailed expression for the wage and profit rates required to maintain full employment in the next period:

$$\beta(1+r)(1-\phi) + (w\beta_w/k) = (1+n) \tag{17.6}$$

To solve for the two unknowns, w and r, we will need one more equation, the wage-profit schedule:

$$w = y - rk \tag{17.7}$$

In each period, these two equations uniquely determine the distribution of income needed for continuous full employment growth. They describe the distribution of income needed for full employment in the next period along a perfect-foresight growth path. Since the capital stocks at the beginning of the period are already determined, the distribution of wealth, ϕ, will also be fully determined. Using the fact that $y/k = \rho - \delta$ to simplify terms, we find that the equilibrium wage in each period must satisfy

$$w = \frac{k\ (1+n-\beta(1+\rho-\delta)(1-\phi))}{\beta_w - \beta(1-\phi)} \tag{17.8}$$

The equilibrium rate of profit can then be found by substituting this expression into the wage-profit schedule, equation (17.7). Thus, the path of the wage rate and profit rate will be determined by the path of the wealth distribution.

To derive the equation of motion describing the path of the wealth distribution, we can substitute equation (17.8) back into equation (17.1). Using the fact that $g = n$, we arrive at the following dynamic equation:

$$\phi_{+1} = \frac{\beta_w}{1+n}\left(\frac{1+n-\beta(1+\rho-\delta)(1-\phi)}{\beta_w - \beta(1-\phi)}\right) \tag{17.9}$$

The equation of motion in this case is a first-order nonlinear difference equation. However, unlike the difference equation in Section 17.3, it does generate oscillations or reversals in the distribution of wealth and by implication, in the distribution of income.[1] Rather than converging monotonically on the equilibrium distribution of wealth, this system is destined to approach its steady state by alternating between overshooting and undershooting.

Moreover, these oscillations will not always be stabilizing in the sense that they become smaller over time and lead toward the steady state value for the wealth distribution. For our purposes, it is important to check that the dynamics of this equation can lead toward a steady state, at least potentially. It can be shown (see the Appendix to this chapter) that there will indeed be parameter values that meet this criterion, and we will restrict our attention to this subset of possibilities.

17.4.1 The Cambridge Theorem in the full employment model

Confining ourselves to stable two-class models of accumulation with full employment, we can turn our attention to the steady state distribution of income and wealth. Away from a steady state, worker and capitalist wealth will typically be growing at rates that do not correspond to the natural rate of growth but in the steady state they will both grow at the natural rate of growth. From the Cambridge equation, this lets us pin down the steady state rate of profit, $r^* = (1+n)/\beta - 1$, and from the wage-profit schedule we can easily find the steady state wage, w^*.

It is significant that we were able to derive the steady state rate of profit using the Cambridge equation without any reference to the workers' saving rate. This result is the Cambridge Theorem in the full employment model setting. It was in this setting that Pasinetti originally discovered that in a two-class model the steady state rate of profit in the long run is completely determined by the capitalist saving propensity and the natural rate of growth, without any reference to worker saving at all.

Once again, adjustments in the distribution of wealth provide the mechanism that underpins the Cambridge Theorem. Workers as a group will wind

[1]Giancarlo Gandolfo suggests that it would be more precise to describe these dynamics as "improper oscillations" or "alternations" to distinguish them from the smoother trigonometric (sine) oscillations that arise in higher-order difference equations.

up saving so that their wealth grows at the same rate that would prevail if the capitalists were making all the saving decisions.

As in the conventional wage model, the steady state distribution of wealth can be interpreted as the outcome of a contest between worker saving and capitalist saving. If we substitute the steady state wage into the workers' saving function, equation (17.1), specialized to the natural rate of growth, we can see this clearly:

$$\phi^* = \frac{w^* \beta_w / k}{1+n}$$

The numerator of this equation represents the workers' incipient growth factor at the equilibrium wage and the denominator represents the steady state growth factor for capitalist wealth in the two-class regime. By substituting the values we derived for r^* and w^* into this equation, we can eliminate w^*. Using the fact that $y/k = \rho - \delta$ to simplify terms we arrive at a more definitive expression for the steady state distribution of wealth:

$$\phi^* = \frac{(1+\rho-\delta)\beta_w}{1+n} - \frac{\beta_w}{\beta}$$

It is reassuring that if we solve equation (17.9), the equation of motion, for a fixed point where $\phi_{+1} = \phi$ we arrive at the same expression for ϕ^* as one of two possible solutions.

The other possible solution corresponds to the one-class model in which $\phi^* = 1$. Just as in the model with a conventional wage, there will be a critical value for the workers' saving propensity that separates the two-class and one-class models of accumulation. When the workers' saving propensity exceeds this value, their saving will eclipse the wealth accumulation of capitalists and the model will asymptotically approach a one-class model. One interpretation of the neoclassical overlapping generations model is that it results when this critical value has been exceeded, and workers are capable on their own of generating enough saving to maintain full employment growth at the natural rate.

Since we are interested in studying the two-class model, we proceed on the assumption that this critical value for the workers' saving propensity has not been breached.

It is clear that in comparison to the conventional wage version the full employment version of the two-class model requires more parameter restrictions in order to get it to behave properly. When it does behave, it creates oscillations that some economists find implausible. We can probably learn more from this model by concentrating on its steady states than by studying its dynamic behavior.

PROBLEM 17.9 Consider an economy where the worker households save 10% of their wage, capitalist households save 90% of their end-of-period wealth, the population grows at the rate $n = .5$/period, $x = \$900{,}000$/period, $\delta = 1$ so that capital depreciates completely each period, and $k = \$100{,}000$/worker. Find the steady state equilibrium real wage, profit rate, and distribution of wealth.

PROBLEM 17.10 Set up a spreadsheet for the dynamics of the two-class model with full employment and compute it over twenty-five periods. Include an equation for the workers' wealth share and a separate equation for the wage. Choose an initial value for the workers' wealth share of 0.3, and use parameter values from the previous problem. Verify that the model is potentially stable, that it converges on the distribution of wealth you expected, and that it confirms the Cambridge Theorem after a (small!) change in the workers' saving propensity, β^w.

17.4.2 Comparative dynamics in the full employment model

In order to study the comparative dynamics of the full employment model, it is useful to concentrate on the relationship between the steady state distribution of wealth and the steady state profit rate. From the workers' saving function, equation (17.1), specialized to the steady state, we have already seen that $\phi^* = (w^* \beta_w)/k(1+n)$. Using the fact that $w = y - rk$ and $y/k = \rho - \delta$, we find that

$$\phi^* = \frac{\beta_w}{1+n}(\rho - \delta - r)$$

In Figure 17.2, this schedule is labeled "workers" to indicate that it represents the locus of steady state wealth distributions and income distributions consistent with worker saving behavior. Which of these possible wealth distributions prevails in an equilibrium depends on the distribution of income as measured by the rate of profit.

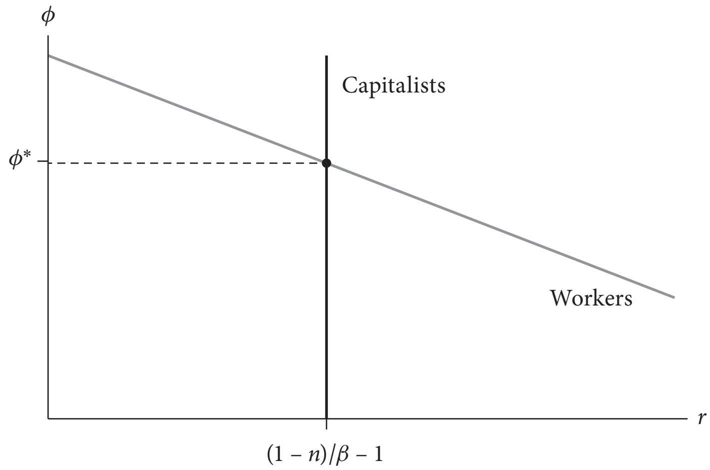

Figure 17.2 In the full employment version of the two-class model, the steady state workers' share of wealth declines with the equilibrium rate of profit. The Cambridge Theorem dictates that the rate of profit is determined by the capitalists' saving propensity and the natural rate of growth, independently of worker saving and the wealth distribution.

The other schedule in Figure 17.2, labeled "capitalists," is the Cambridge equation specialized for full employment growth in a steady state so that $r^* = (1 + n)/\beta - 1$. This schedule pins down the equilibrium rate of profit. The fact that there is no relationship between the workers' wealth share and the profit rate is an implication of the Cambridge Theorem. With these two schedules, we can analyze any parameter change much as we did in the conventional wage model in the previous section.

First, consider an increase in the workers' saving propensity, β_w. This has no effect on the Cambridge equation as we have seen. But it will shift the workers' schedule in Figure 17.2 to the right, resulting in an increase in the workers' share of wealth.

This makes intuitive sense but to get a deeper understanding of the model, we can see how the transitional dynamics of the two-class model bring about this outcome. Initially, the increase in β_w creates an incipient excess demand for workers (because the increased worker saving boosts capital accumulation from which the demand for workers derives). In order to eliminate this incipient excess demand, wages must rise and profits must fall so that the capitalist households are forced to save and invest at a temporarily lower rate.

Table 17.3 Comparative Dynamics of the Two-Class Model with Full Employment

Parameter Changes						Effects	
β_w	β	n	x	k	ρ	ϕ^*	g^*
up	same	same	same	same	same	up	same
same	up	same	same	same	same	up	down
same	same	up	same	same	same		
same	same	same	up	up	same		
same	same	same	up	same	up		
same	same	same	same	down	up		

The temporary increase in wages allows workers to accumulate wealth at a faster rate than the capitalists and as a result, their share of wealth rises even though the wage and profit rate eventually return to their original values.

Second, consider an increase in the capitalists' saving propensity, β. This has no effect on the schedule labeled "workers" in Figure 17.2, but it will shift the Cambridge equation labeled "capitalists" to the left. With capitalists saving at a higher rate, the system achieves steady state growth at the natural rate with a lower rate of profit and a correspondingly higher wage. The higher wage enables workers to save more, and their wealth share rises. It is interesting that we found the opposite effect on the wealth share in the two-class model with a conventional wage when β increased.

Table 17.3 summarizes the results of these two comparative dynamic exercises. It leaves the remaining comparative dynamic exercises unfinished so that students can tackle them in the problems.

PROBLEM 17.11 Analyze the effect of an increase in the natural rate of growth n in the two-class model with full employment. Comment on the effect this has on the distribution of wealth.

PROBLEM 17.12 Analyze the effect of a Harrod-neutral technical change that increases output per worker, x, but keeps the output-capital ratio, ρ, constant in the two-class model with full employment.

PROBLEM 17.13 Analyze the effect of a Hicks-neutral technical change that increases output per worker but keeps the capital-labor ratio, k, constant.

PROBLEM 17.14 Analyze the effect of a Solow-neutral technical change that increases the output-capital ratio but keeps output per worker constant.

17.5 Wealth Distribution in the US

Partly in response to the work of Thomas Piketty and a group of empirically oriented researchers who study inequality, interest in the wealth and income distribution has grown in recent years. These economists have reported substantial increases in inequality in the income distribution over the last decades of the twentieth century and first decades of the twenty-first century.

Data on the distribution of wealth are not as readily available as data on the distribution of income. There are three methods that economists have used to estimate the extent of inequality in the wealth distribution, which is often measured by the share of wealth owned by the wealthiest 1% of households or individuals. The first method uses income tax records to estimate wealth by capitalizing streams of property income. This requires imputing values for the discount rates used to perform the capitalization, usually by referring to observed rates of return on different kinds of financial asset.

The second method uses estate tax records and treats decedents as a sample from the living population. The third method surveys households directly. In the US, the Federal Reserve Board conducts the Survey of Consumer Finances (SCF) that asks respondents to provide information on wealth, income, and spending. The SCF includes a separate high-income sample to compensate for the tendency of wealthy households to underreport their assets.

Each of these methods offers distinct challenges, advantages, and disadvantages, and there is no consensus about which method is most accurate. Consequently, it is worthwhile looking at all three. Table 17.4 presents some data from three different sources, each using one of the methods. The years have been selected to maximize the coverage of each series, and to include years when two or more of the sources report data. The SCF has been conducted at three-year intervals, starting in 1989, but precursor surveys do provide some coverage for prior years.

The income tax and estate tax methods agree broadly that wealth inequality increased in the 1920s, but the income tax data show a much larger increase. These two methods agree that wealth inequality diminished sub-

Table 17.4 Estimates of Top 1% Wealth Share in Percentages US Economy, Selected Years

	Method		
Year	*Income Tax*	*Estate Tax*	*Survey*
1913	43.4	—	—
1916	42.3	38.1	—
1927	49.5	39.2	—
1939	41.9	26.0	—
1950	30.5	22.8	—
1962	29.6	24.4	31.8
1969	27.9	22.9	31.1
1983	24.7	21.1	33.8
1989	27.8	22.0	30.1
1998	32.3	21.7	33.9
2004	33.5	18.1	33.4
2012	41.8	—	—
2013	—	—	35.8

Sources: Online data appendix from Kopczuk (2015). For original sources, consult Kopczuk (2015).

stantially during the Great Depression, and remained relatively low in the post–World War II period, up until around 1980.

After that there is again disagreement between these two series, as the income tax method shows a sharp increase in wealth inequality that becomes particularly noticeable after 1990. The estate tax method shows no dramatic trend over this time interval. Similarly, the SCF method shows only modest increases in wealth inequality with little change over the decades of the 1980s and 1990s, and some increase in the 2000s.

From the standpoint of the conventional wage two-class model, the rise in inequality in the distribution of wealth at the beginning of the twenty-first century makes sense. A rise in the profit share associated with a decline in the conventional wage would unambiguously reduce the workers' share of wealth in the conventional wage model. However, this model also predicts that the growth rate should increase, which has not occurred. The survey data and estate tax data presented in Table 17.4 are also puzzling because they suggest only a modest rise in wealth inequality in comparison to the quite substantial rise in income inequality. Interpreting either of these puzzles through the conventional wage two-class model would require us to ask if other changes have occurred at the same time. For example, if the capitalist propensity to save also declined, that would help explain these patterns.

17.6 Conclusions

We have seen that the interaction between worker and capitalist saving changes the behavior of models of growth. The Classical conventional wage model will have the same long run rate of growth if we include worker saving because the distribution of wealth adjusts so that worker saving behaves as if it were being generated by the Cambridge equation. However, worker saving does affect the growth path. A higher worker saving propensity, for example, will result in a higher level of capital because it has temporary effects on the growth rate even though it does not affect the long run rate of growth. It will also raise the workers' share of wealth.

The Classical full employment model will have the same long run rate of profit if we include worker saving because the distributions of income and wealth adjust so that worker and capitalist wealth both grow at the natural rate of labor force expansion. However, worker saving does affect the distribution of wealth. Higher worker saving will result in workers' wealth growing temporarily faster than capitalist wealth so that the long run share of wealth owned by workers will increase.

These results are implications of Pasinetti's Cambridge Theorem that states that in a two-class model, the relationship between growth and profitability is mediated by the capitalist saving function, independently of workers' saving behavior.

Including capitalist saving in the Classical overlapping generations model increases its long run rate of growth, which represents a qualitative change in the nature of the model. Including capitalist saving in the neoclassical overlapping generations model also transforms the model fundamentally since it will have a different rate of profit and wage rate, now dictated by the Cambridge Theorem. The exceptions occur when the workers' saving propensity is so high that their wealth grows faster than the capitalists' wealth and the economy drifts toward a one-class model over a long time span.

The two-class model gives us a transparent framework through which we can interpret recent developments in the distribution of wealth. There is substantial evidence that in many advanced capitalist countries, particularly the US and UK, the distributions of income and wealth in the twenty-first century have grown more unequal than they were in the middle of the twentieth century.

17.7 Suggested Readings

The models in this chapter are explored in more depth and in connection with fiscal policies in Michl (2008) and Michl and Foley (2004). The Cambridge Theorem was presented in 1962 and is explained in Pasinetti (1974). Samuelson and Modigliani (1966) showed that there is also a possible one-class solution to the models in this chapter; they called this the "anti-Pasinetti Theorem." See Fazi and Salvadori (1985) for a careful discussion of the conditions under which Pasinetti's Cambridge Theorem holds.

Two mathematics texts that are helpful for solving the type of difference equations used in this chapter are Gandolfo (1997) and Elaydi (2005).

The work of Piketty (2014) on income and wealth inequality has been very influential; also see Pressman (2016) and the symposium on Piketty's work in the *Journal of Economic Perspectives*, Winter 2015. For discussion from a Classical viewpoint, see Michl (2016). Wolff (2017) explores the distribution and accumulation of wealth in the US in meticulous detail.

There has been considerable debate about the relative importance of bequests versus life-cycle saving, with Modigliani (1988) taking the position that life-cycle saving dominates while Kotlikoff (1988) takes the opposite position. Further discussion can be found in Kessler and Masson (1988). For empirical evidence on the class structure of saving, Wolff (1981) suggests a three-class model, with the wealthy primarily engaged in bequest saving, a middle class engaged in life-cycle saving, and a working class that does no saving. Yakovenko (2012) uses methods drawn from statistical mechanics to characterize empirically the class structure of the wealth and income distributions.

Appendix: Stability in the Full Employment Model

Local stability (in the neighborhood of the steady state) requires that the absolute value of the derivative of the equation of motion be less than unity, or $|d\phi_{+1}/d\phi| < 1$. (For more details, consult the mathematics textbooks listed in Suggested Readings.) Taking the derivative of the equation of motion and substituting for ϕ^* results in this implication of the stability inequality:

$$\beta_w < \frac{\beta(1+n)}{\beta(1+\rho-\delta)+(1+n)}$$

This condition on permissible workers' saving propensities is even more restrictive than the condition for a two-class regime with $\phi^* < 1$, which is that $\beta_w < (1+n)/(1+\rho-\delta)$.

18

Global Warming

18.1 Global Warming and Economic Growth

Historically industrial production, which is the foundation of the prosperity of the richer economies of the world, has depended on the application of larger and larger quantities of energy in producing goods. This increase in energy has gone hand in hand with the use of machinery and other nonlabor inputs to production, which have been the main focus of our economic analysis of growth in previous chapters, represented by the value of the capital stock.

Historically the energy applied in industrial production has been overwhelmingly derived from burning fossil fuels, such as coal, petroleum, and natural gas. These fuels are the remains of plants that lived and died millions of years ago. We have discussed the issue of the depletion of fossil fuel reserves in Chapter 14. Renewable sources of energy such as hydroelectric generation, wind and wave power, and geothermal energy recovery have contributed relatively little energy to production until very recently. Nuclear power generation also remains a relatively small fraction of energy generation.

Burning fossil fuels releases energy through the fundamental chemical reaction of oxidation in which the carbon in the fuels combines with atmospheric oxygen to produce carbon dioxide (CO_2). Carbon dioxide is an odorless, tasteless gas that is not harmful to human beings at low concentrations. But, as scientists who study the earth's environment as a whole realized as much as a hundred years ago, a relatively small increase in the carbon dioxide content of the earth's atmosphere disproportionately increases the heat-trapping capacity of the atmosphere (often called the "greenhouse effect").

The release of carbon dioxide into the atmosphere tends to raise global temperatures, although global warming is affected by many other non–human-controlled factors, such as the amount of ash discharged into the atmosphere by volcanic eruptions. Other gases, such as methane and hydro- and chlorofluorocarbons, also contribute to global warming through the greenhouse effect, but carbon dioxide emissions are the most important of the human-controlled factors that promote global warming, and in this chapter we will confine our discussion to the analysis of carbon dioxide emissions and economic growth.

CO_2 stays in the atmosphere for a long time. The half-life of CO_2 in the earth's atmosphere, the time period required for one-half of a given amount of atmospheric CO_2 to break down or dissipate, is on the order of 300 years.

Geoscientists measure the concentration of CO_2 in the atmosphere as *parts per million* or *ppm*, the fraction of atmospheric gas represented by CO_2. Preindustrial levels of CO_2 concentration were around 280 ppm. Even though early industrial production emitted a large quantity of CO_2 relative to output, the scale of industrial production (together with the burning of fossil fuels for residential heating and to power transport networks) has grown large enough to affect atmospheric CO_2 only in the last century. Measured concentrations of atmospheric CO_2 have climbed in proportion, and approached 400 ppm in 2016.

Global warming matters to economic well-being because higher global temperatures will damage economic production through sea-level rise, changes in agricultural productivity, increased incidents of flood and drought, and increased severity of storms. Climate damage retards economic production and accumulation. A social planner who could control all aspects of economic life would take these effects of climate damage on production into account in deciding what energy technologies to invest in and how much of various goods to produce.

The phenomena of global warming and climate damage have important implications for economic growth based on the burning of fossil fuels, beyond the depletion of fossil fuel reserves. If human beings want to stabilize or reduce the climate damage due to rising atmospheric concentration of CO_2, we have to reduce fossil fuel emissions essentially to zero, find an economically viable technology to take CO_2 out of the atmosphere, or discover other technologies to offset global warming (often called *geoengineering*), such as releasing sunlight-reflecting aerosols into the atmosphere, or sequestering CO_2 in mines or beneath the sea to keep it from entering the atmosphere.

The problem of global warming combines several of the issues we have discussed in earlier chapters. From one point of view, the limited capacity of the earth's atmosphere to absorb CO_2 emitted by production is parallel to the limitation land imposes on production that we discussed in Chapter 13. Because the carrying capacity of the atmosphere is reduced with the emission of CO_2, there is also a parallel to the exhaustion of resources that we analyzed in Chapter 14. In this chapter we will use the analytical methods we have developed to examine these issues to understand the problems of greenhouse gas emissions, climate damage, and economic growth and distribution.

The case of CO_2, however, highlights another important economic issue, the *pricing* of emissions. Until very recently human economies have had no market in which to price emissions. As a result economic incentives for individual households, firms, or nations to control greenhouse gas emissions have been limited to the direct net costs of fossil fuel use.

The control of greenhouse gas emissions is a *public good* because the impact of emissions on welfare depends on total emissions, and essentially affects everyone. (Not necessarily uniformly, because there are some regions of the world where climate damage from global warming is small, or there may even be economic benefits to global warming.) There is some incentive for one economy (or even a very large international corporation), for example, to control its emissions because even a small reduction in emissions will have some benefits for that economy. But these individual incentives, as is generally the case with public goods, are insufficient to enforce an economically efficient outcome, because each individual (or national or regional economy) will ignore the benefits its reduction in emissions confers on other individuals (or regional or national economies). An important aspect of the economics of global warming is the *economic externality* greenhouse gas emissions impose on other economies. In analyzing the economics of global warming we will highlight the importance of incentives, through prices or direct controls, in controlling emissions.

We can use the same modeling approach as we used in the case of land and exhaustible resources to understand the fundamental economics of growth with greenhouse gas emissions and climate damage.

The atmosphere's capacity to absorb CO_2 is in a very long run (over several centuries) a renewable resource, because atmospheric CO_2 appears to dissipate spontaneously at a very low rate. Furthermore, as in the analysis of exhaustible resources, a key concept in understanding the economics of global warming is that there are two technologies available: traditional fossil-fuel-using technologies, and renewable energy technologies (which we will call "solar"). (For simplicity we will abstract from intermediate cases like nuclear energy.)

In this chapter we will assume that the atmosphere's capacity to absorb CO_2 is a very slowly renewable resource and that there are two alternative unchanging technologies, one based on fossil fuel burning, and the other on non-emitting (clean) renewable energy sources. Two scenarios are important. In one, we assume a *business-as-usual* (*BAU*) world in which the emission of CO_2 through burning fossil fuels has a zero price. In the other, we assume that through carbon taxes, direct controls, or tradable emissions permits there is an effective price on emissions that corresponds to their long run social marginal cost. In order to develop the second scenario, we will have to investigate the economic forces that govern the efficient pricing and utilization of this type of externality.

18.2 Production with Greenhouse Gas Emissions

In order to introduce greenhouse gas emissions into the growth model, we begin with production. The basic model is very similar to the model of production with an exhaustible resource we presented in Chapter 14. There are two alternative production technologies, one based on burning fossil fuels that emits greenhouse gases that increase the concentration of carbon dioxide in the atmosphere ("FF"), the other a renewable "solar" technology that does not emit greenhouse gases ("SOL"). We will use the symbols CD and cd for the atmospheric concentration of CO_2 and CO_2 emissions. Climate scientists argue that it would be impossible to burn all the earth's fossil fuel reserves without catastrophic climate damage, which leads us in this chapter to assume that the reserves of fossil fuels are effectively unlimited. We assume that climate damage takes the form of destruction of means of production, and is expressed by increasing the depreciation rate of capital by $D(CD)$, where $D(CD)$ is a damage function that represents the proportion of existing capital lost to climate damage in a production period when CO_2 concentration is CD. The fossil-fuel production technique is:

$$\begin{array}{c} 1\,\text{labor} + k^{FF}\,\text{capital} \rightarrow \\ x\,\text{output} + (1 - \delta - D(CD))k^{FF}\,\text{capital} + x\,CO_2 \end{array}$$

Here we again take advantage of our freedom to choose the units in which we measure CO_2 concentration in assuming that to produce a unit of output you need to emit one unit of CO_2. Thus at the end of the period, there are three results of the productive process: the x units of new output, taking account of climate damage; the capital depreciated by the factor $1 - \delta - D(CD)$; and the emission of x units of CO_2 into the atmosphere. Again, we are using the amount of CO_2 emitted to produce one unit of output as our unit of measurement for CO_2.

The other method of production, which does not emit CO_2 (for example, one that depends on solar energy), is:

$$\boxed{1 \text{ labor} + k^{SOL} \text{ capital} \rightarrow x \text{ output} + (1 - \delta - D(CD))k^{SOL} \text{ capital}}$$

For simplicity we assume that the alternative solar technology has the same potential labor productivity, x, and depreciation rate, $\delta + D(CD)$, as the fossil-fuel technology, but a higher capital intensity, $k^{SOL} > k^{FF}$. This implies that the capital productivity of the fossil-fuel technology is higher than the capital productivity of the solar technology, $\rho^{FF} > \rho^{SOL}$. If fossil fuels are an economically relevant resource, they must increase the productivity of other resources, such as labor and capital, which is why we assume that capital productivity is higher with the fossil-fuel technology.

Climate damage depends on the actual atmospheric concentration of CO_2, which depends not just on the current technology in use, but on the history of technologies in the past. Given the atmospheric concentration of CO_2, climate damage increases depreciation of the two types of capital to the same degree.

Figure 18.1 shows the real wage-profit rate schedules for the two techniques. The assumption that labor productivity, x, and the depreciation rate, $\delta + D(CD)$, are the same in the two technologies implies that the use of fossil fuels is the equivalent of Solow-neutral technical progress. For a given wage, w, the wage share for the fossil technology is $1 - \pi = w/x$, the same as the wage share for the solar technology. We also assume that climate damage takes the same proportion of the capital stock of the two technologies.

The profitability of the two techniques of production depends on the price the producer has to pay for emissions, which may be zero. If p^{cd} is the price of emitting a unit of CO_2 at the beginning of period t, which is the end of period $t - 1$, and entrepreneurs pay for their emissions, like wages, at the end of the period after they have sold their output, the profit an entrepreneur

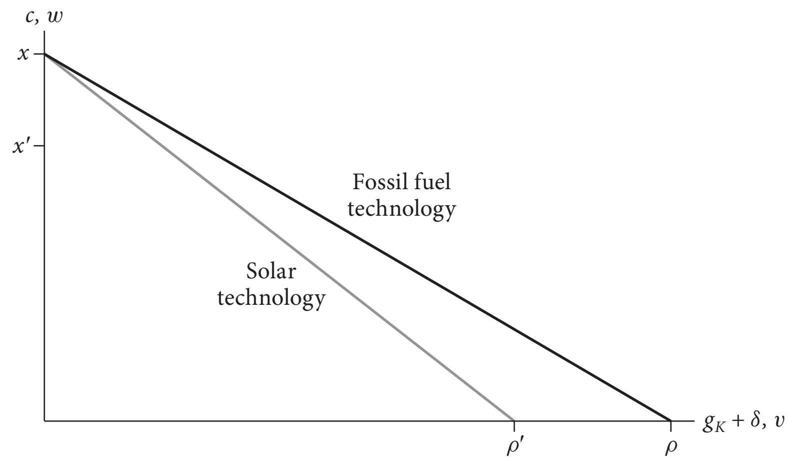

Figure 18.1 The real wage-profit rate relations for the fossil-fuel (black) and solar (gray) technologies. The fossil-fuel technology dominates the solar technology, since it is more profitable at every real wage. The slope of the fossil fuel real wage-profit rate relation is higher than the slope of the solar real wage-profit rate relation, while they have the same vertical intercept when the price of emissions is zero.

using fossil-fuel technology will make on each worker employed is:

$$v^{FF}k^{FF} = x - w - p^{cd}x = (\pi - p^{cd})x$$

The profit per worker using solar technology is:

$$v^{SOL}k^{SOL} = \pi x$$

The corresponding gross profit rates for the two technologies are:

$$v^{FF}(p^{cd}) = (\pi - p^{cd})\rho^{FF}$$
$$v^{SOL} = \pi\rho^{SOL}$$

This way of looking at profits and profitability shows that climate damage is a cost to the entrepreneur parallel to wages and any possible emission charge.

The actual profit rate in any period will depend on whether the fossil-fuel technology or the solar technology is the more profitable, and therefore in use:

$$v = r + \delta = \max(v^{FF}(p_{cd}(t)), v^{SOL}) \quad (18.1)$$

For every real wage, w, and the corresponding potential profit share $\pi(w) = 1 - \frac{w}{x}$, there is a price of CO_2, $p^*_{cd}(w)$, at which the two technologies have the same profit rate:

$$(\pi - p^*_{cd}(w))\rho^{FF} = \pi\rho^{SOL} \quad \text{or} \quad p^*_{cd}(w) = \pi(w)\frac{\rho^{FF} - \rho^{SOL}}{\rho^{FF}}$$

The price of CO_2 at which the two technologies have the same profit rate is equal to the proportion of capital costs saved by fossil fuel, $\pi(w)(\rho^{FF} - \rho^{SOL})/\rho^{FF}$. There is no point in imposing a price of CO_2 above $p^*_{cd}(w)$, because at a higher price for emissions the fossil-fuel technology would have a lower profit rate than solar and no one would use it. The more capital fossil fuel saves, the higher will be the price of CO_2 at which the two technologies have the same profit rate. Since climate damage increases depreciation of the two types of capital by the same amount, it does not affect the relative profitability of the two technologies.

PROBLEM 18.1 Consider an economy with a fossil-fuel technology where $x =$ \$60,000/worker/year, $w =$ \$20,000/worker/year, $k^{FF} =$ \$200,000/worker, $\delta = 0$/year, and $k^{SOL} =$ \$300,000/worker, so that solar technology is 50% less capital-productive, with the same rate of depreciation. Find the price of CO_2 at which solar technology would just compete with fossil fuels.

PROBLEM 18.2 For the economy described in Problem 18.1, suppose that the capitalist's $\beta = .95$ and that the wage is \$20,000/worker/year. Find the profit rate and the growth rate of the capital stock using solar technology.

18.3 Saving and Portfolio Choice

The typical capitalist's consumption decision follows the same logic as in an economy that does not suffer climate damage from greenhouse gas emissions, given her holding of capital and the net profit rates of the two technologies.

In the real world, it is costly to transform capital built for one technology to another, and in the extreme case the only way to do this is to allow the capital embodying the first technology to depreciate and replace it with capital embodying the second technology. To keep our model as simple as possible, however, we will assume that the capitalist can decide in each period how much of her capital to assign to fossil-fuel technology, and how much to assign to solar technology. This is equivalent to assuming that the typical capitalist can costlessly shift capital from one technology to the other.

(We will also assume that the time period is a decade, so that this assumption is not quite so unrealistic as it would be if the time period were a year.)

The typical capitalist's net profit rates for the two types of capital are the gross profit rates less depreciation, which includes climate damage:

$$r^{FF}(p^{cd}, CD) = v^{FF}(p^{cd}) - \delta - D(CD) = (\pi - p^{cd})\rho^{FF} - \delta - D(CD)$$
$$r^{SOL}(p^{cd}, CD) = v^{SOL} - \delta - D(CD) = \pi\rho^{SOL} - \delta - D(CD)$$

The typical capitalist's budget constraint in each period, assuming that her holding of capital is K and net profit rates on the two technologies are r^{FF}, r^{SOL}, is:

$$K_{+1} + C \leq (1 + r^{FF})K^{FF} + (1 + r^{SOL})K^{SOL} \quad (18.2)$$

We assume that the typical capitalist correctly forecasts the path of the atmospheric level of CO_2 concentration and the resulting climate damage. In the real world there are substantial uncertainties about both of these variables, but we abstract from these uncertainties to highlight the social coordination problem posed by the fact that the use of the more profitable fossil-fuel technology leads to higher climate damage through greenhouse gas emissions.

The capitalist utility maximization problem under these assumptions is summarized in Table 18.1.

The assumption that the capitalist can shift capital costlessly between the two technologies is represented by the first constraint.

We can solve this constrained maximization problem as in Chapter 5, so we know that the solution to the utility maximization problem will be:

$$C = (1 - \beta)(1 + r)K \quad \text{and} \quad K_{+1} = (1 + r)K - C = \beta(1 + r)K$$

Table 18.1 The Capitalist's Utility Maximization Problem

$$\text{choose } \{C_t, K_t, K_t^{FF}, K_t^{SOL} \geq 0\}_{t=0}^{\infty}$$

$$\text{so as to maximize } (1 - \beta)\sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

$$\text{subject to} \quad (18.3)$$

$$K_{t+1}^{FF} + K_{t+1}^{SOL} \leq K_t$$

$$K_{t+1} + C_t \leq (1 + r_t^{FF})K_t^{FF} + (1 + r_t^{SOL})K_t^{SOL}$$

$$\{1 + r_t^{FF}\}_{t=0}^{\infty}, \{1 + r_t^{SOL}\}_{t=0}^{\infty}, K_0, \{CD_t\}_{t=0}^{\infty} \text{ given}$$

Thus wealth grows at the rate $\beta(1+r)$. The current rate of profit depends on which technology the typical capitalist adopts and on the level of climate damage resulting from past greenhouse gas emissions.

18.4 The Growth Path with Fossil-Fuel Technology

Let us first study the growth path of the economy under the business-as-usual (BAU) assumption that the economy imposes no price on CO_2 emissions and as a result entrepreneurs operate exclusively with the fossil-fuel technology, so that $K^{FF} = K$.

The growth path depends on the fact that with fossil-fuel technology the increase in atmospheric concentration of CO_2, ΔCD, is equal to the amount of CO_2 emitted less the dissipation of CO_2 in the atmosphere, ϵCD, where ϵ is the (quite small) rate of CO_2 dissipation. The emission of CO_2 in turn is equal to the output (all from the fossil-fuel sector), $X = \rho^{FF} K$, since the production of each unit of output with fossil-fuel technology emits one unit of CO_2. So we have:

$$\Delta CD = X - \epsilon CD = \rho^{FF} K - \epsilon CD$$

But this allows us to trace the concentration of CO_2, since we know that:

$$CD_{+1} = CD + \Delta CD = (1-\epsilon)CD + \rho^{FF} K$$

We can put all these relations together to see the laws governing the changes in the capital stock, output, CO_2 concentration, and climate damage summarized in Table 18.2.

Now we have a complete picture of the process of growth with the climate damage. Starting from a low CO_2 concentration that implies negligible climate damage, output will create CO_2 emissions above the dissipation rate, and CO_2 concentration will increase. If the profit rate is high enough to allow capital accumulation, the economy and output will continue to grow.

Table 18.2 BAU Growth Path in the Global Warming Model

$$r = \pi\rho^{FF} - \delta - D(CD) \tag{18.4}$$

$$CD_{+1} = CD + \Delta CD = (1-\epsilon)CD + \rho^{FF} K \tag{18.5}$$

$$K_{+1} = \beta(1+r)K \tag{18.6}$$

Table 18.3 BAU Growth Path in the Global Warming Model

$$r^{SS} = \pi\rho^{FF} - \delta - D(CD^{SS}) \tag{18.7}$$

$$K^{SS} = \frac{\epsilon}{\rho^{FF}} CD^{SS} \tag{18.8}$$

$$1 = \beta(1 + r^{SS}) \tag{18.9}$$

Eventually, if the climate damage function reflects increasing marginal climate damage, CO_2 concentration will inflict significant climate damage by destroying capital. The net profit rate itself, r, will then decline in each period as climate damage becomes more severe. Eventually the profit rate will fall to the point where $\beta(1 + r) < 1$ and capital and output will start to decline. This process of economic decline will come to a halt around a stationary state when the CO_2 concentration stabilizes, which, according to equation (18.5), requires $X^{SS} = \rho^{FF} K^{SS} = \epsilon CD^{SS}$. The stationary state level of CO_2 concentration has to stabilize the capital stock, so we have $1 + r^{SS} = 1 + \pi\rho^{FF} - \delta - D(CD^{SS}) = 1/\beta$, which determines CD^{SS} given the climate damage function. The equations determining the stationary state are summarized in Table 18.3.

Because the dissipation of atmospheric CO_2 is so slow, it may be the case that the BAU path actually oscillates over very long time periods and reaches the stationary state only after thousands of years.

Capitalist saving determines how low the stationary state profit rate has to be driven in order to stabilize the capital stock and output. The impact of climate damage on the profit rate determines how much CO_2 concentration is necessary to create enough climate damage to lower the profit rate to the stationary state level. The dissipation rate of CO_2 sets the limit on how much output and how big a capital stock is compatible with the stationary state.

One important insight from this model is that the BAU path will push CO_2 concentration and climate damage as high as necessary to shut down economic growth and limit economic production to whatever level results in emissions just equal to the dissipation of CO_2. The reason for this is that climate damage in this model does not differentiate between solar and fossil-fuel technologies, so that climate damage itself produces no change in the incentives for entrepreneurs to use the more profitable fossil-fuel technology.

The BAU stationary state enforced by climate damage can have very low output and incomes.

The smaller the capitalist economy is in relation to the carrying capacity of the earth's atmosphere, the longer it will take for production and emissions to reach the point where climate damage becomes significant, and the economy is inevitably driven into the stationary state.

The system of dynamic equations describing the BAU path does not yield an analytical solution that would allow us to describe the whole path from its initial conditions and the model's parameters alone. In order to study the dynamic behavior of the economy on the BAU path, we resort to numerical simulations using *Mathematica*, a powerful computational environment. We will use this simulation approach to characterize other possible growth paths as well.

Figure 18.2 shows the evolution of consumption, the capital stock, the atmospheric concentration of CO_2, and climate damage on the BAU path. The central features of this path are the climate catastrophe that occurs as atmospheric CO_2 concentration rises and climate damage erodes the productivity of resources, and the resulting stationary state at a low level of production and consumption and a high level of atmospheric CO_2 concentration and climate damage.

EXAMPLE 18.1 *Let the fossil-fuel technology have $x = \$100{,}000$/worker/year, $\delta = .1$/year, $k = \$125{,}000$/worker, $\rho^{FF} = x/k = .8$/year. The conventional wage $\bar{w} = \$20{,}000$/worker/year. Capitalist saving is $\beta = 1/3$. Thus $\pi = 1 - (w/x) = .8$. Find the BAU stationary state profit rate and the stationary state level of climate damage.*

Answer: The stationary state profit rate is determined by equation (18.9): $r^{SS} = \frac{1}{3}(1 + r^{SS})\ r^{SS} = \frac{1}{2}$. The stationary state level of climate damage $D^{SS} = D(CD^{SS})$ is determined by equation (18.7): $r^{SS} = \frac{1}{2} = \pi\rho^{FF} - \delta - D^{SS} = (.8)(.8) - .1 - D^{SS}\ D^{SS} = .04$. The CO_2 concentration has to increase until 4% of existing capital is lost to climate damage each year.

PROBLEM 18.3 Find the stationary state concentration of CO_2 if the damage function is $D(CD) = CD^2$.

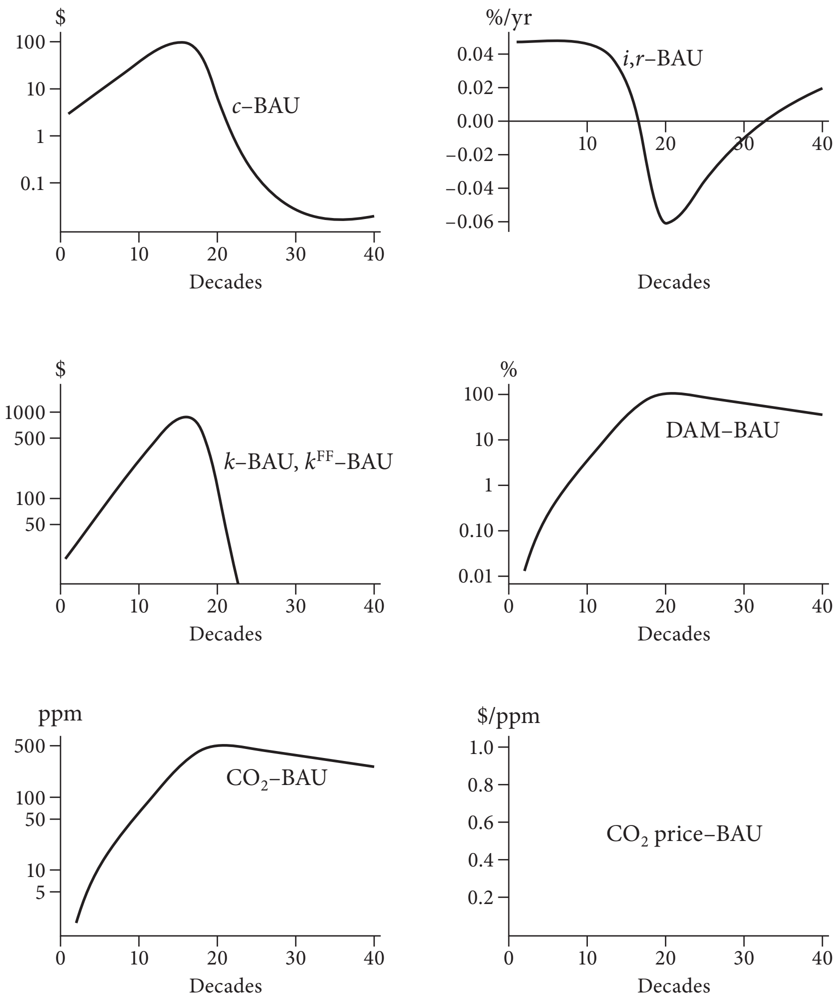

Figure 18.2 The model economy's growth path under the BAU assumption. Uncontrolled production with the fossil-fuel technology results in an initial period of rapid growth of production and consumption and large emissions of CO_2, leading to a climate catastrophe when climate damage rises to the point where production and consumption fall. The decline is halted only at the stationary state where production and emissions are small enough to balance the natural dissipation of CO_2.

18.5 The Growth Path with Solar Technology

As a contrast, let us study the *solar* (*SOL*) growth path of the economy where the economy summarized in Table 18.4 imposes a high price on CO_2 emissions and as a result entrepreneurs operate exclusively with the solar technology.

The SOL growth path results in no further human-controlled emissions of CO_2. The atmospheric concentration of CO_2 will decline through dissipation until it reaches equilibrium with non–human-controlled emissions of CO_2. In any case CO_2 concentrations are decoupled from economic production.

This scenario is the basic Classical growth model with the solar technology.

As in the Classical growth model, the solar economy grows at a steady rate $g^{SOL} = \beta(1 + r^{SOL})$, where the profit rate is determined by the distribution and production parameters in equation (18.10), which assume CO_2 concentration is low enough to avoid climate damage.

There is a sharp contrast with the BAU path on which the economy continues to use the fossil-fuel technology because it offers a cost advantage to individual entrepreneurs. The BAU path leads to a low-level stationary state where growth is suppressed completely and output and income are forced to low levels by uncontrolled climate damage. The SOL path leads to steady state growth at a rate determined by capitalist saving and distribution. (Sooner or later, however, it seems inevitable that some other limits to growth will come into play even if global warming is averted.)

EXAMPLE 18.2 *Let the fossil-fuel technology have $x = \$100{,}000$/worker/year, $\delta = .1$/year, $k^{FF} = \$125{,}000$/worker, $\rho^{FF} = \frac{x}{k^{FF}} = .8$/yr, and suppose that the alternative solar technology is half as profitable, so that $\rho^{SOL} = .4$/yr, and $k^{SOL} = \frac{x}{\rho^{SOL}} = \$250{,}000$/worker/year. The conventional wage $\bar{w} = \$20{,}000$/worker/year. Thus $\pi = 1 - (w/x) = .8$. Capitalist saving is $\beta = 1/3$. Find the SOL steady state profit rate and growth rate.*

Answer: The steady state profit rate is determined by equation (18.10): $r = \pi\rho^{SOL} - \delta = (.8)(.4) - .1 = .22$. The steady state growth rate is determined by equation (18.11): $g = \beta(1 + r) = \frac{1}{3}(.22) = .073$.

Table 18.4 SOL Growth Path

$$r = \pi\rho^{SOL} - \delta \tag{18.10}$$

$$K_{+1} = \beta(1 + r)K \tag{18.11}$$

18.6 Coordinated Growth with Global Warming

We have seen that there is a dramatic difference between a BAU growth path that sticks indefinitely with fossil-fuel technology and the SOL growth path where a carbon tax or other pricing mechanism, such as tradable emission permits, makes fossil-fuel technology unprofitable.

If a typical capitalist had to choose one or the other of these paths, she would choose the SOL path, because it provides indefinite growth of output and capitalist consumption, whereas the BAU path dooms both output and capitalist consumption to stagnation at low levels.

The global warming scenario, however, is an example of an economic *social coordination problem*. The typical capitalist seeking the highest profit rate will not invest capital with an entrepreneur who chooses the solar technology if the price of carbon emissions is zero. This is true for any level of climate damage that has the same effect on solar and fossil-fuel depreciation and profitability. Climate damage itself creates no profit rate signal to induce individual entrepreneurs to shift to solar technology. An individual entrepreneur who tried to buck the trend by using solar technology would find no capitalist willing to invest at the lower profit rate. As in Prisoners' Dilemma models in game theory, the rational capitalists and entrepreneurs lock into a stable equilibrium of producing with fossil-fuel technology that results in a very unfavorable long run growth path and pattern of capitalist consumption.

If one single capitalist controlled the whole world economy (somewhat like a socialist world government), what path of technology, investment, production, and consumption would she choose, supposing that she had to pay a conventional subsistence wage that she could not control for labor input? Such a single capitalist would *internalize* the global warming externality.

This world capitalist would have to decide in each period how much of her capital to use with fossil-fuel technology, K_t^{FF}, and how much to use with solar technology, K_t^{SOL}. The world capitalist's utility maximization problem is summarized in Table 18.5.

The first four constraints are the same as those faced by any typical capitalist, given a level of climate damage beyond her control. The last two constraints apply to the economy as a whole. $K_{t+1} - (1 - \delta - D(CD_t))K_t$ is gross investment, which cannot be negative for the economy as a whole. While it is possible for any individual capitalist to finance a high level of

Table 18.5 World Capitalist's Utility Maximization Problem with Global Warming

$$\text{choose } \{C_t, K_t, K_t^{FF}, K_t^{SOL}, K_{t+1}, CD_{t+1} \geq 0\}_{t=0}^{\infty}$$

so as to maximize

$$(1-\beta)\sum_{t=0}^{\infty} \beta^t \ln(C_t)$$

subject to

$$K_{t+1} + C_t \leq (1 + r_t^{FF})K_t^{FF} + (1 + r_t^{SOL})K_t^{SOL}$$

$$K_t^{FF} + K_t^{SOL} \leq K_t \tag{18.12}$$

$$r_t^{FF} = \pi\rho^{FF} - \delta - D(CD_t)$$

$$r_t^{SOL} = \pi\rho^{SOL} - \delta - D(CD_t)$$

$$0 \leq K_{t+1} - (1 - \delta - D(CD_t))K_t$$

$$CD_{t+1} = (1 - \epsilon)CD_t + \rho^{FF}K_t^{FF}$$

$$CD_0, K_0 \text{ given}$$

consumption by selling capital to other capitalists, it is not possible for the economy as a whole to convert existing capital directly into consumption, a constraint the world capitalist has to respect in her planning. The typical individual capitalist understands perfectly well that it is her (and other capitalists') emissions that increase atmospheric CO_2 and raise climate damage to her capital; she regards her own production as a relatively small part of this social problem and therefore assumes changing her own production or technology will have negligible impact on climate damage. This is true even if she correctly anticipates the climate damage implied by the emissions of the whole economy.

The world capitalist has to trade off the productivity advantage of the fossil-fuel technology against the climate damage it creates. As opposed to an individual typical capitalist, the world capitalist has the advantage of not only correctly *foreseeing*, but also *controlling* all the relevant economic variables in making these decisions. In effect she chooses her own level of emissions under the assumption that all the other capitalists will do the same.

One way to approach the mathematical solution of the world capitalist's global warming problem is with the *Lagrange multiplier method*. This mathematical method is based on the idea that the decision-maker should charge herself for violating the constraints she faces. For each constraint (in each period) the coordinating capitalist establishes a corresponding Lagrange multiplier, or *shadow price*, and subtracts the shadow price times the amount by which her plan violates the constraint from the utility the plan promises.

For the coordinating capitalist, the utility function is the discounted sum of the logarithm of consumption over the whole future:

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t \ln(C_t)$$

There are two constraints in each period. The first constraint is the production constraint:

$$K_{t+1} + C_t \leq \pi(\rho^{FF}K_t^{FF} + \rho^{SOL}K_t^{SOL}) + (1 - \delta - D(CD_t))K_t$$

We assign a Lagrange multiplier or shadow price $\beta^t\lambda_t$ to each period's constraint. The value of $\beta^t\lambda_t$ represents how much discounted utility the coordinating capitalist would gain if the constraint were relaxed slightly, for example, if the economy somehow had a windfall gain of output in that period. Because β^t is the factor by which utility in period t is discounted, λ_t represents the marginal undiscounted utility of consumption in period t. The real interest rate determined by the growth path of consumption in each period is equal to the rate of change of the shadow price on output: $i_t = \frac{\beta\lambda_{t+1}}{\lambda_t} - 1$.

The second constraint is accumulation of CO_2 in the atmosphere due to production with fossil-fuel technology:

$$CD_{t+1} \geq (1-\epsilon)CD_t + \rho^{FF}K_t^{FF}$$

The coordinating capitalist cannot plan on magically reducing the CO_2 concentration below the level implied by her production plan insofar as she plans to use the fossil-fuel technology for production at all. We assign a Lagrange multiplier $\beta^t\lambda_t\mu_t$ to this constraint. This shadow price represents how much utility the coordinating capitalist would gain if the constraint were relaxed slightly, for example, if some of the CO_2 in the atmosphere were somehow removed. Since $\beta^t \lambda_t$ represents the gain in utility for the coordinating capitalist of an increase in output (measured in trillions of \$), μ_t represents the amount of current output a unit of CO_2 concentration is equivalent to, or the shadow price of CO_2.

We can write the Lagrangian for the coordinating capitalist's maximization problem using these shadow prices:

$$
\begin{aligned}
&L\big(\{K_t^{FF}, K_t^{SOL}, C_t, CD_t, \lambda_t, \mu_t, \kappa_t\}_{t=0}^{\infty}\big) \\
&\quad = (1-\beta)\sum_{t=0}^{\infty}\beta^t\Big\{\ln(C_t) - \lambda_t\Big[\Big(K_{t+1}^{FF} + K_{t+1}^{SOL} + C_t \\
&\quad - \Big(\pi\big(\rho^{FF}K_t^{FF} + \rho^{SOL}K_t^{SOL}\big) + \big(1-\delta-D(CD_t)\big)\big(K_t^{FF} + K_t^{SOL}\big)\Big)\Big) \\
&\quad + \mu_t\Big(CD_{t+1} - \big((1-\epsilon)CD_t + \rho^{FF}K_t^{FF}\big)\Big) \\
&\quad + \kappa_t\Big(K_{t+1}^{FF} + K_{t+1}^{SOL} - \big(1-\delta-D(CD_t)\big)\big(K_t^{FF} + K_t^{SOL}\big)\Big)\Big]\Big\} \qquad (18.13)
\end{aligned}
$$

18.6.1 Solving for the coordinated growth path

If the coordinating capitalist can assign the correct shadow prices to the constraints, then she can find the optimal production and growth path by maximizing the Lagrangian with respect to the variables she controls, $\{K_t^{FF}, K_t^{SOL}, C_t, CD_t, \lambda_t, \mu_t, \kappa_t\}_{t=0}^{\infty}$. For example, if she thinks of increasing consumption in some period t, the impact on the Lagrangian is:

$$
\frac{\partial L}{\partial C_t} = (1-\beta)\beta^t\left(\frac{1}{C_t} - \lambda_t\right)
$$

If this effect is positive, the coordinating capitalist will want to raise C_t, which would increase her utility. But on the optimal growth path any increase in C_t must violate the production constraint. What we mean by the correct shadow price λ_t is that it is just large enough as a penalty to make the net change in the Lagrangian from a small increase (or decrease) in C_t equal to zero. With this shadow price the coordinating capitalist maximizing the Lagrangian has no incentive either to raise or lower C_t. The correct shadow price λ_t is the marginal utility of consumption.

Similar reasoning applies to all of the decision variables, giving rise to the *first-order conditions*, which we can write, eliminating nonnegative constant factors:

$$\frac{\partial L}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0 \tag{18.14}$$

$$\frac{\partial L}{\partial K_t^{FF}} = -\lambda_{t-1}(1 - \kappa_{t-1}) + \beta\lambda_t\big((\pi - \mu_t)\rho^{FF} - \delta - D(CD_t) + \kappa_t\big) = 0 \tag{18.15}$$

$$\frac{\partial L}{\partial K_t^{SOL}} = -\lambda_{t-1}(1 - \kappa_{t-1}) + \beta\lambda_t\big(\pi\rho^{SOL} - \delta - D(CD_t) + \kappa_t\big) = 0 \tag{18.16}$$

If the coordinating capitalist has the correct λ_t, representing the opportunity cost of output in period t and μ_t, representing the opportunity cost of higher atmospheric CO_2 concentration in period t, these first-order conditions represent basic economic logic. The first instructs the coordinating capitalist to choose a level of consumption in period t such that the marginal utility of consumption is equal to the cost of output. The second instructs her to choose a level of fossil-fuel capital, K^{FF}, that balances the cost of output (which is the same as the cost of capital) in period $t-1$ with the contribution of fossil fuel capital to output weighted by the shadow value of output, and taking account of the cost of emissions of fossil-fuel production through the term $\mu_t\rho^{FF}$, where the shadow price κ prevents negative gross investment. The third instructs her to choose a level of solar capital, K^{SOL}, that balances the cost of output (which is the same as the cost of capital) in period $t-1$ with the contribution of solar capital to output weighted by the shadow value of output, and taking account of the fact that solar capital makes no contribution to CO_2 concentration or climate damage, where the shadow price κ prevents negative gross investment.

It turns out that the first-order conditions related to the shadow prices also represent economically meaningful concepts, namely the production and CO_2 accumulation constraints:

$$\frac{\partial L}{\partial \lambda_t} = K_{t+1}^{FF} + K_{t+1}^{SOL} + C_t - \pi\big(\rho^{FF}K_t^{FF} + \rho^{SOL}K_t^{SOL}\big) - \big(1 - \delta - D(CD_t)\big)\big(K_t^{FF} + K_t^{SOL}\big) = 0 \tag{18.17}$$

$$\frac{\partial L}{\partial \mu_t} = CD_{t+1} - (1-\epsilon)CD_t - \rho^{FF}K_t^{FF} = 0 \tag{18.18}$$

If the coordinating capitalist (or the economist) can solve all of the first-order conditions simultaneously, they define a possible solution for the original maximization problem (Table 18.5).[1]

18.7 Optimal and BAU Growth Paths

Figure 18.3 shows the growth path that results from solving the coordinating capitalist's maximization problem using the Lagrangian technique.

The optimal growth path begins with a period of use of fossil-fuel technology, which results in rapid growth of consumption and output, but also raises CO_2 concentrations due to emissions of CO_2. After a certain point, the productivity advantage of the fossil-fuel technology is offset by its future contribution to climate damage as measured by the shadow price of CO_2, μ_t. At this point the coordinating capitalist switches almost entirely over to solar technology, using just enough fossil-fuel technology so that emissions are balanced by the natural dissipation of CO_2 from the atmosphere. The growth rate of consumption and output fall when this transition occurs, and the CO_2 concentration and climate damage stabilize, as does the shadow price of CO_2.

Figure 18.4 compares the optimal and BAU growth paths.

The BAU path is the result of solving the first-order conditions of the Lagrangian, but setting the $\mu_t \rho^{FF}$ term in the condition on K^{FF} equal to zero. The resulting equations represent the situation of a typical capitalist who correctly foresees the accumulation of CO_2 and climate damage, but does not alter her production plan to control them, on the grounds that as a single small contributor to emissions, she cannot make any difference.

The two paths begin with the same period of rapid growth based on the productive fossil-fuel technology. They diverge sharply because on the BAU path the typical capitalist is compelled to maximize her profit as an individual by sticking with fossil-fuel technology, even though she knows that the social consequences will be disastrous. There is no effective signal to the typical capitalist to induce her to make the socially desirable shift to solar technology. In contrast to the optimal coordinated path chosen by the coordinating capitalist who controls all capitalist production decisions, not just her own, the BAU path implies a stabilization of emissions entirely through

[1]When the objective function in a constrained maximization problem is concave in the decision variables, and the constraints represent a convex set in the space of the decision variable, the Lagrangian first-order conditions are both necessary and sufficient to characterize the solution. These conditions hold for the coordinated capitalist's problem.

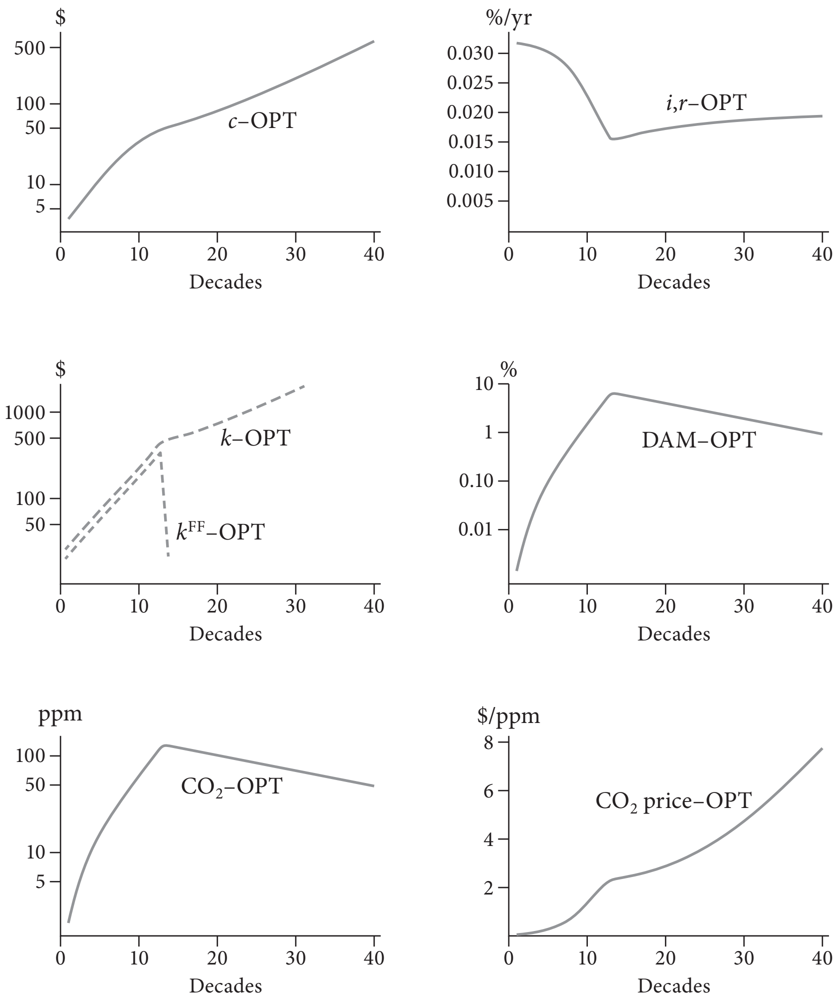

Figure 18.3 The model economy's optimal growth path. The coordinating capitalist will use the fossil fuel technology only up to the point where its further contribution to climate damage through CO_2 emissions offsets its productivity advantage. At this point (around 170 years in this simulation, which starts with CO_2 concentration at the preindustrial level of 280 ppm) the coordinating capitalist shifts over almost completely to solar technology, limiting the amount of fossil-fuel technology to a level that just offsets the natural dissipation of CO_2. As a result CO_2 concentration and climate damage remain limited.

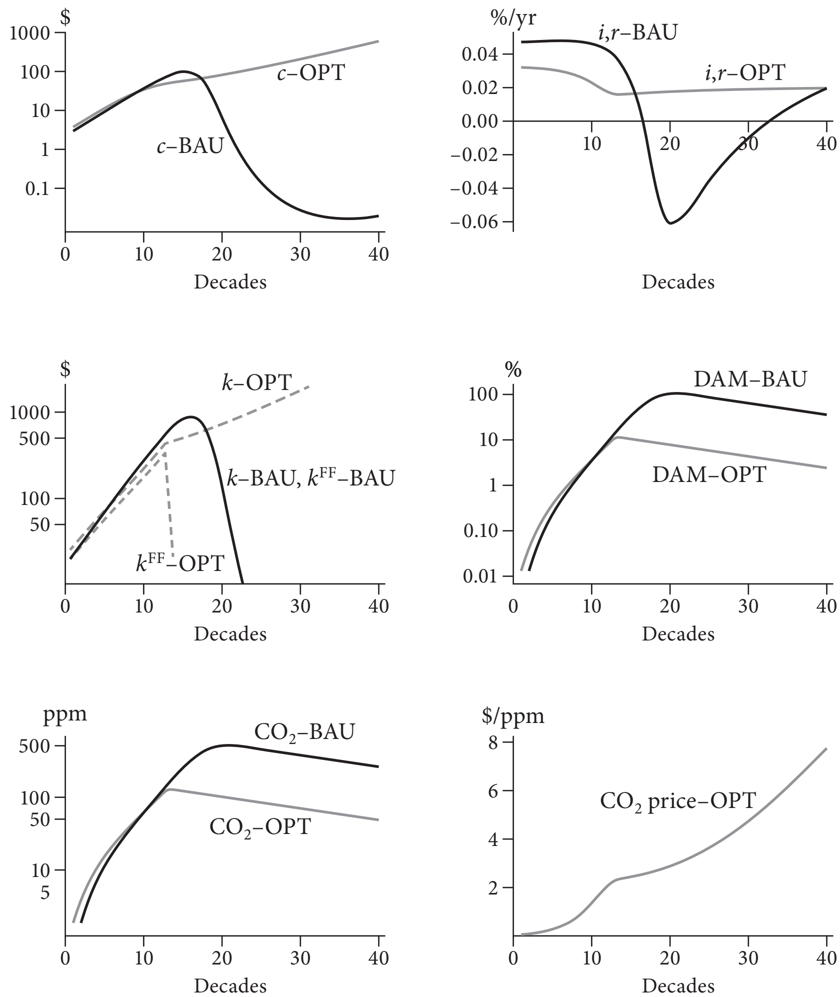

Figure 18.4 The BAU and optimal paths are plotted together for comparison. The optimal path shifts to solar technology and grows at a lower rate than when using the fossil-fuel technology, but thereby stabilizes CO_2 concentrations and controls climate damage. On the BAU path individual capitalists are compelled by profit-maximization to stick with the fossil-fuel technology, dooming the economy to very high climate damage. Eventually climate damage after the climate catastrophe shuts down production and consumption to levels that stabilize the CO_2 concentration, but at much lower consumption levels than on the optimal path.

climate damage, which also greatly diminishes consumption. We do not compute the implicit shadow price of CO_2 on the BAU path, because it has no impact on capitalists' behavior.

18.8 Centralized and Decentralized Economic Control

The problem of climate damage highlights a key issue with economic systems that depend on decentralized decision-making guided by market prices to control resource allocation. If the price incentives that decentralized decision makers see fail to represent social marginal costs correctly, the decentralized outcome will fall short of the first-best efficient outcome. In the climate damage scenario we have been examining, the centralized decision-maker we have called the "coordinating capitalist" internalizes the externality represented by the emission of greenhouse gases. In one way or another the coordinating capitalist sees the overall advantage of switching from fossil fuel to solar technology when the social cost of emissions as measured by the shadow price on CO_2 outweighs the higher private productivity of fossil fuels. In a decentralized economy the only way to avoid the climate catastrophe under the assumptions of this scenario is to make individual decision-makers respond to the social costs of their actions. In the case of climate damage, this might be accomplished by imposing a carbon tax calibrated to the shadow price of CO_2, or by capping emissions through regulation, perhaps allowing trade in emissions licenses.

The costs imposed by incorrect price signals may in some cases be so small that they can safely be ignored, but the example of climate damage shows that they may also be enormous. We may hope that the externality itself will go away, for example, that due to a large enough improvement in the productivity of non-emitting technologies, fossil-fuel technology becomes privately unprofitable. (In the case of fossil fuel burning, this would require a very large improvement in solar and other noncarbon technologies, since the market prices of fossil fuels are much higher than their actual production costs due to rents.) If solar technology is more productive than fossil-fuel technology, no one will burn fossil fuels anyway and the externality will not exist.

Our analysis of growth with climate damage also underlines another important feature of these social coordination problems. The BAU paths we studied are not the result of incorrect evaluation of private profitability on the part of entrepreneurs and capitalists, nor of their inability to predict the actual path of CO_2 concentrations and climate damage. If anything the model assumes better private allocations decisions than real-world conditions are likely to produce. The climate catastrophe in this model is due to the lack of feedback from the long-term costs of emissions through climate damage to the perceived individual interests of decentralized decision-makers (in this case the typical capitalist). This feedback is represented mathematically by the shadow price on CO_2 in the Lagrangian problem. When we remove that term from the first-order condition that evaluates the relative desirability of fossil-fuel and solar technologies, the growth path changes qualitatively with the emergence of the climate catastrophe.

18.9 Suggested Readings

The analysis of global warming as an economic externality is developed in Foley (2009), and in more detail in Rezai et al. (2011). More general economic perspectives on this and related environmental problems are discussed in Georgescu-Roegen (1999) and Daly (2008). For the approach that emphasizes intergenerational equity issues, see Nordhaus (2008).

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