FUNDAMENTAL CONSTANTS

Evaluating Measurement Uncertainty

Boris M. Menin

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Cambridge Scholars Publishing

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This book first published 2019

Cambridge Scholars Publishing

Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN (10): 1-5275-2318-7
ISBN (13): 978-1-5275-2318-0

Any true is only one facet of the truth
—Anonymous

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PREFACE

The book is an attempt to apply the methods of information theory, similarity theory, modeling theory and experimental design theory to assess the *a priori* model mismatch before the actual experiment or computer calculations.

It contains rich experimental material, confirming the attractiveness of the information-oriented method for experimental and theoretical physics, including measurement of fundamental physical constants.

The focus is on the organic link between the original mathematical terms of information theory, similarity theory, and the theory of planning of experiments. So, the information-oriented approach of modeling physical phenomena is perceived as a system of ideas that have a clear physical meaning.

The book is based on experimental and theoretical investigations carried out by the author over 35 years, as well as development experience and extensive research activities in modeling measurements of the fundamental physical constants.

The introduced method is very simple and easy to digest, so appropriate technical skills are easily acquired. But even the experience of its formal use cannot teach the relevance and reasonable use of it without a stencil or even without direct mistakes occurring. To apply the information method, you must first understand its physical content.

The book is supplemented by a rich bibliography with internet addresses.

It may be useful for scientists, engineers working in the enterprises and organizations of the corresponding profile, and students of universities and colleges. Comments and suggestions about the content of the book should be sent to the following email: meninbm@gmail.com.

ABOUT THE AUTHOR

Boris M. Menin was Director of the Laboratory of Ice Generators and Plate Freezers in St. Petersburg (then Leningrad) from 1977 to 1989, at which time he emigrated from the Soviet Union to Israel. He has since managed at Crytec Ltd. (Beer-Sheba, Israel) on the development, production and marketing of pumpable ice generators, cold energy storage systems, and also high accuracy instrumentation for heat and mass processes among other matters. Dr. Menin is credited with developing an entirely new branch of modeling: an information-oriented approach, by which the lowest achievable absolute and relative uncertainties of measured quantity can be calculated before the realization of experiments or a model's computerization. An earlier book by the same author called *Information approach for modeling physical phenomena and technological processes* (published by Lambert Academic Publishing, Germany, 2017) presents a theoretical explanation and grounding of application of information and similarity theories for the calculation of the threshold discrepancy between a model and researched phenomena. This current book focuses on the methods for experimental data processing of fundamental constants measurements and considers this aspect of physics in greater depth.

Introduction

The illiterate of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and relearn
—Alvin Toffler

"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." This worldview statement was by Lord Kelvin in 1900, but it was shattered only five years later when Einstein published his paper on special relativity.

In the 21st century, it can be safely asserted that absolutely all modern achievements in the field of science are based on the successes of the theory of measurements, on the basis of which the practical recommendations useful in physics, engineering, biology, sociology, etc. are extracted. In addition, this is because the application of the principles of the theory of measurements in determining the fundamental constants allows us to verify the consistency and correctness of the basic physical theories. Complementing the above, quantitative predictions of the basic physical theories depend on the numerical values of the constants involved in these theories: each new sign can lead to the discovery of a previously unknown inconsistency or, conversely, can eliminate the existing inconsistency in our description of the physical world. At the same time, scientists came to a clear understanding of the limitations of our efforts to achieve very high measurement accuracy.

The very act of the measurement process already presupposes the existence of the physical-mathematical model describing the phenomenon under investigation. Measurement theory focuses on the process of measuring the experimental determination of the values by using special hardware called measuring instruments [1]. This theory only covers the aspects of data analysis and measurement procedures of the quantity observed or after formulating a mathematical model. Thus, the problem that there is uncertainty *before* experimental or computer simulation and caused by the limited number of quantities recorded in the mathematical model is generally ignored in the measurement theory.

The proposed information approach—to assess the model's noncompliance with the physical phenomenon under study—has introduced an additional measurement accuracy limit that is more stringent than the Heisenberg

Uncertainty Principle. And it turns out that the "fuzziness" of the observed object, strangely enough, depends on the personal philosophical prejudices of scientists, which are based on their experience, acquired knowledge and intuition. In other words, when modeling a physical phenomenon, one group of scientists can choose quantities that will differ fundamentally from the set of quantities that are taken into account by another group of scientists. The fact is that the same data can serve as the basis for radically opposite theories. This situation assumes an equally probable accounting of quantities by a conscious observer when choosing a model. A possible, though controversial, example of such an assertion is the consideration of an electron in the form of a particle or wave, for the description of which various physical models and mathematical equations are used. Indeed, it is not at all obvious that we can describe physical phenomena with the help of one single picture or one single representation in our mind.

This book aims to introduce a fundamentally new method for the characterization of the model uncertainty (threshold discrepancy) that is associated with only a finite number of the registered quantities. Of course, in addition to this uncertainty, the total measurement uncertainty includes *a posteriori* uncertainties related to the internal structure of the model and its subsequent computerization: inaccurate input data, inaccurate physical assumptions, the limited accuracy of the solution of integral-differential equations, etc.

The novel analysis introduced is intended to help physicists and designers to clarify the limits of the achievable accuracy of measurements and to determine the most simple and reliable way to select a model with the optimal number of recorded quantities calculated according to the minimum achievable value of the model uncertainty.

The book contains five chapters. Chapter 1 gives base elements of similarity theory, information theory, theory of planning of experiments, and group theory. It includes a classification of measurement inaccuracy and postulates the theory of measurements. The basic definitions and explanations introduced are needed for further development of the main principles of the information-oriented method.

Chapter 2 contains the analysis of publications related to usage of the concepts of "information quantity" and "entropy" for real applications in physics and engineering, calculating information quantity inherent in the physical-mathematical model, and the formulation of a system of base dimensional quantities (SBQ), from which a modeler chooses a number of quantities in order to describe the researched process. Such a system must meet a certain set of axioms that form an Abelian group. This in turn allows the author to employ the approach for the calculation of the total number of dimensionless criteria in the existing International System of Units (SI). Mathematically, the exact expression for the calculation of the comparative uncertainty of the developed model with a limited number of quantities obtained by counting the amount of information contained in the model is formulated.

Chapter 3 is devoted to applications of the information-oriented approach, including its most attractive application which is the measurement of fundamental physical constants. The data and calculations of the accuracy of the Avogadro number, Boltzmann constant, Planck constant, and gravitational constant are presented. In addition, the puzzle of the Maxwell demon and the amount of information related to ordinary matter are analyzed from the point of view of the information approach.

Chapter 4 is expanded to discuss using comparative uncertainty instead of relative uncertainty in order to compare the measurement results of the main quantity of the recognized phenomenon, including the fundamental physical constants, and to verify their true-target value. Moreover, drawbacks and advantages of the introduced method are carefully analyzed.

Chapter 5 focuses on emphasizing that the information-oriented approach is a living topic. This is extremely important because successfully demonstrating its use has many consequences in the measurement of fundamental physical constants, quantum mechanics and cosmology.

CHAPTER ONE

BASIC KNOWLEDGE ABOUT APPLIED THEORIES

Don't let your ears hear what your eyes didn't see,
and don't let your mouth say what your heart doesn't feel
—Anonymous

1.1. The measurement theory basics

To begin with, the first task of the scientist studying a phenomenon is usually to determine the conditions under which the phenomenon can be repeatedly observed in other laboratories and can be verified and confirmed. For an accurate knowledge of the physical variable, you need to measure it. And for its measurement, a certain device is always required (this presupposes the existence of a physics-mathematical model already formulated) that somehow influences this value, as a result of which it becomes known with some degree of accuracy. In tum, the amount of information obtained by measurement can be calculated by reducing the uncertainty resulting from the measurement. In other words, uncertainty about a particular situation is the total amount of potential information in this situation [2].

For all the instructions below, it is important to indicate the difference between the error and uncertainty. The error is in how much the measurement corresponds to the true value. This error is rarely what interests us. In science, we usually do not know the "true" meaning. Rather, we are interested in the uncertainty of measurements. This is what we need to quantify in any measurement. Uncertainty is the interval around the measurement, in which measurements will be repeated. Uncertainty describes the distance from the measurement result within which the true value is likely to lie.

The introduction of measurable quantities and the creation of their units are the basis of the measurements. However, any measurement is always performed on a specific object, and the general definition of the measured quantity must be formulated taking into account the properties of the object and the purpose of the measurement. Essentially, the true value of the measured quantity is introduced and determined in this way. Unfortunately, this important preparatory stage of measurements is usually not formulated [1].

The idealization necessary for constructing the model generates an inevitable discrepancy between the parameters of the model and the real property of the object. We will call this nonconformity a threshold discrepancy. The uncertainty caused by the threshold discrepancy between the model and the object should be less than the total measurement uncertainty. If, however, this component of the error exceeds the limit of the permissible measurement uncertainty, it is impossible to perform the measurement with the required accuracy. This result shows that the model is inadequate. To continue the experiment, if this is permissible for the measurement target, the model must be redefined. If an object is a natural object, the threshold discrepancy means that the model is not applicable and needs to be reviewed. The preceding logic reduces to the following postulates of measurement theory [3]:

- There is a true value of the measured quantity;
- In every measurement there is one true value;
- The true value of the measured quantity is constant;
- True value cannot be found due to the existence of an inevitable discrepancy between the parameters of the model and the real property of the object, called the threshold discrepancy.

In addition, there are other inevitable limitations to the approximation of the true value of the measured quantity. For example, the accuracy of measuring devices is inevitably limited. For this reason, we can formulate the following statement: the result of any measurement always contains an error. Thus, the accuracy of the measurement is always limited, and in particular, it is limited by the correspondence between the model and the phenomenon. We add that the achievable measurement accuracy is determined by *a priori* information about the measurement object.

The accepted model can be considered as corresponding to the studied physical phenomenon, if the differences between the obtained estimates of the mathematical expectation of the process are much smaller than the permissible measurement error. If, however, these differences are close to or exceed the error, then the model must be redefined, which is most easily done by increasing the observation interval.

It is interesting to note that the definitions of some quantities seem at first sight sufficient for high accuracy of measurements (if the errors of the measuring device are ignored). Examples of these are the parameters of stationary random processes, the parameters of distributions of random variables, and the mean value of a quantity. One would think that to achieve the required accuracy in these cases it is sufficient to increase the number of observations during the measurement. However, in reality, the accuracy of measurements is always limited, and in particular it is limited by the correspondence between the model and the phenomenon, i.e., the threshold discrepancy.

When the true value cannot be determined, measurement is impossible. For example, in the last few years, much has been written about the measurement of random variables. However, these values, as such, are not of true value, and for this reason they cannot be measured [1]. It is important to emphasize that *the present study refers only to variables for which a true value may exist*.

1.2. The similarity theory basics

Usually in textbooks on the theory of similarity, we first introduce the necessary concepts, including "quantity", "likeness", "dimensionality", "homogeneity" and others. Then, Buckingham's theorem is derived and many examples of the application of this theory are given in mechanics, heat transfer, hydraulics, etc. In contrast to this scheme, the author strives to focus only on those points that are directly related to the formulation of the presented approach. This, in turn, requires the reader to undertake some preliminary preparation and possess knowledge of the fundamental aspects of the theory.

The similarity theory is suitable for several reasons. When studying phenomena occurring in the world around us, it is advisable to consider not individual quantities but their combinations or complexes, which have a certain physical meaning. The methods of the similarity theory, based on the analysis of integral-differential equations and boundary conditions, additionally determine the possibility of identifying these complexes. Furthermore, the transition from dimensional physical quantities to dimensionless quantities reduces the number of counted values. The specified value of the dimensionless complex can be obtained using various combinations of dimensional quantities included in the complex. This means that when we consider problems with new quantities, we consider not an isolated case but a series of different events, united by some common properties. It is important to note that the universality of the similarity transformation is determined by invariant relationships that characterize the structure of all laws of nature, including the laws of relativistic nuclear physics. Moreover, dimensional analysis from the point of view of a mathematical apparatus has a group structure, and the transformation coefficients (similarity complexes) are invariants of groups. The concept of a group is a mathematical representation of the concept of symmetry, which is one of the most fundamental concepts of modern physics [4].

At the same time, it should be noted that the similarity theory does not answer the question of the number of possible combinations of dimensional characteristics included in the description of the dimensionless physical process and the form of these combinations. In addition, it is unclear what criteria for several interacting parts of an object are suitable for describing the physical process and how necessary they are for a given uncertainty in determining the selected base quantity [5].

It is obvious that all the physical dimensional quantities appearing in the mathematical model cannot make an infinite interval of changes in the real world. These values lie in certain intervals, the boundaries of which can be selected based on experience and intuition of the modeler, and an analysis of published scientific, technical, regulatory and technological literature.

The reasons for choosing the allowable intervals for the remaining physical characteristics included in the developed system of equations can be explained. The rules for the transition from differential equations to expressions in the final form are described in detail in [6]. In any case, for any physical phenomena and processes, as well as for any models describing a material object, it is necessary to choose the interval of expected changes in the main observable or measured quantity (criterion).

Bridgman [7] showed that for *all* physical dimensional quantities, a monomial formula satisfies the principle of absolute significance of relative magnitude *only* if it has the power-law form:

$$q \supset C_1^{\tau_1} \cdot C_2^{\tau_2} \cdot \ldots \cdot C_H^{\tau_h}, \tag{1}$$

where q is the dimensional quantity, C_1, C_2, … C_H, are numerical values of base quantities, and exponents τ_1, τ_2, … τ_h, are real numbers whose values distinguish one type of derived quantity from another. *All monomial derived quantities have this power-law form*; no other form represents a physical quantity.

If you know the range of variation of the exponents τ_1, τ_2, … τ_h of the base quantities C_1, C_2, … C_H and these exponents take, for example, an integer value, then it is possible to calculate the total number of possible combinations contained in a finite set that includes all dimensional quantities. This statement will be discussed in Chapter 2.2, and necessary calculations with respect to the SI system will be carried out.

The logical continuation of (1) is the question of the possible number of dimensionless complexes that can be built on the basis of the selected base quantities. This question is answered by π-theorem that was proved by Buckingham [8].

Buckingham's π-theorem states that when the total relationship between dimensional physical quantities is expressed in dimensionless form, the number of independent quantities that appear in it decreases from the initial n to n-k, where k is the maximum number of initial n that are independent of dimension. The dimensional analysis reduces the number of values that must be specified to describe the event. This often leads to a huge simplification. At the same time, the π-theorem simply indicates to us the number of dimensionless quantities that affect the value of a particular dimensionless recognized value. It does not tell us about the form of dimensionless quantities. The form should be opened by experiments or theoretically solved problems.

There is no point in adding or subtracting quantities that have different units. You cannot add length to the mass. The point is that all the terms of the equation must have the same dimensions. This is called the dimensional homogeneity requirement.

A homogeneous equation is one in which each independent additive term has the same dimensions. There are functions that are homogeneous in their structure. The homogeneity of these functions does not depend on any additional assumptions on the properties of the transformations. Such functions are properly called unconditionally homogeneous. Only the degree complexes possess the property of absolute homogeneity.

All other operations related to the theory of similarity and dimensional analysis, and containing a choice of argument complexes and construction of parametric criteria, are based on considerations that are not within the scope of this study. Therefore, we have finished the discussion of the similarity theory. Only the above definitions will be used to formulate the proposed approach.

1.3. The information theory basics

The definition of information comes from statistical considerations. In this case, we define information as a result of a choice that always has a positive value. In our approach, we do not consider information as a result, which can be used to make a different choice. In this case, *the human evaluation of information is completely ignored.*

Random events—in our case, the choice of quantities in the model at the desire of the researcher—can be described using the concept of "probability". The probability theory allows us to find (calculate) the probability of one random event or a complex experience, combining a number of independent or unrelated events. If the event is accidental, it means there is a lack of full confidence in its implementation, which in turn creates uncertainty in the results of experiments related to the event. Of course, the degree of uncertainty is different for different situations.

Consider a system that represents P different events, when a particular quantity will be equally probable. When we impose restrictions on the quantities that reduce freedom of choice, these conditions exclude some of the pre-existing features. The new number of events P' with the restrictions should clearly be smaller than the original P. It should be noted that any limitation, additional requirement or condition imposed on the possible freedom of choice leads to a reduction in information. Therefore, we need to get the new value of information $\boldsymbol{Z}' < \boldsymbol{Z}$:

- without limitations: P equally probable outcomes, $Z = K \ln P$;
- with limitations: P' equally probable outcomes at $P' < P$, and

$$\boldsymbol{Z}' = K \cdot \ln P' < \boldsymbol{Z}, \tag{2}$$

$$\Delta \boldsymbol{Z} = \boldsymbol{Z} - \boldsymbol{Z}' = K \cdot \ln P - K \cdot \ln P' = K \cdot \ln(P/P'), \tag{3}$$

where ΔZ is a change of information during the experiment, K is a constant, and ln is the natural logarithm.

One can prove [9] that in this case, information Z has a maximum when all events P are equal. The use of the logarithm in (2) is justified by the fact that we wish that the information is additive. For the first time, a logarithmic measure of information is suggested by Hartley [10].

In information theory, the information is usually regarded as a dimensionless quantity, and, therefore, the constant K is an abstract number, which depends on the choice of unit system. The most convenient system is based on binary units, which gives us:

$$K = 1/\ln 2 = \log_2 e \tag{4}$$

Another system of units can be introduced, if we compare the information with the thermodynamic entropy and measure both values in the same units. As it is known, the entropy has the dimension of energy divided by the temperature. For the entropy there is the Boltzmann formula that is very similar to (2) and contains the factor:

$$k_b = 1.38 \cdot 10^{-23} m^2 \mathrm{kg}/(s^2 K). \qquad (5)$$

This constant (k_b) is known as the Boltzmann constant. When we are interested in physical problems, such a choice of units allows us to compare the information with entropy itself. It should be noted that the ratio of units in equations (4) and (5) is equal to:

$$k_b/K = k_b \cdot \ln 2 \approx 10^{-23}. \qquad (6)$$

This numerical magnitude plays an important role in all applications of information theory [9].

At the same time, entropy is directly related to the "surprise" of the occurrence of the event. From this, it follows its information content: if the event is more predictable, it is less informative. This means that its entropy is lower. It remains an open question about the relationship between the properties of information, entropy properties and the properties of its various estimates. But we are just dealing with the estimates in most cases. All this lends itself to the study of the information content of different indexes of entropy regarding the controlled changes of properties and processes, i.e., in essence, their usefulness to specific applications [11].

Our definition of information is very useful and practical. It corresponds exactly to the task of the scientist, who must retrieve all the information contained in the physical- mathematical model, regardless of the limits to the achievable accuracy of the measuring instruments used for observation of the object. According to the suggested approach, the human evaluation of information is completely ignored. In other words, the set of 100 musical notes played by chimpanzees will have exactly the same amount of information as that of the 100 notes played by Mozart in his Piano Concerto No.21 (Andante movement).

The following explanations are specifically intended for a possible application of information theory to the modeling of physical phenomena and experiments.

Let us start with a simple example. We see the position of the point x on the segment of length S (range of observation) with uncertainty Δx. We introduce the definitions:

absolute uncertainty is Δx, (7)

relative uncertainty is $r_x = \Delta x / x$, (8) comparative uncertainty is $\varepsilon_x = \Delta x / S$. (9)

The accuracy of the experiment ω can be defined as the value inverse to ε_x:

$$\omega = 1/\,\varepsilon_x = S/\Delta x\,. \quad (10)$$

This definition satisfies the condition that greater accuracy corresponds to lower comparative uncertainty. The absolute and relative uncertainties are familiar to physicists, but not comparative uncertainty because it is seldom mentioned. But the comparative uncertainty value is of great importance in the application of information theory to physics and engineering sciences [9].

If all the events are equiprobable, the amount of information obtained by observing the object ΔZ, according to (2) and (3), is equal to:

$$\Delta Z = k_b \cdot \ln(S/\Delta x) = -k_b \cdot \ln\varepsilon_x = k_b \cdot \ln\omega\,. \quad (11)$$

If the range of observation S is not defined, the information obtained during the observation/measurement cannot be determined, and the entropic price becomes infinitely large [9].

In turn, the efficiency Q of experimental observation, on the assumption that some perturbation is added into the system under study, may be defined as the ratio of the obtained information ΔZ to a value equal to the increase in entropy ΔH accompanying observation:

$$Q = \Delta Z / \Delta H \quad (12)$$

It follows from all the above that the modeling is an information process in which information about the state and behavior of the observed object is obtained by the developed model. This information is the main subject of interest of modeling theory. During the modeling process, the information increases, while the information entropy decreases due to increased knowledge about the object [12]. The extent of knowledge W of the observed object may be expressed in the form:

$$W = 1 - H/H_{max}, \quad (13)$$

where H is the information entropy of the object and H_{max} is its maximum value where the amount of knowledge can become A (0, 1). The impossibility of reaching the boundary values A=0 and A=1 is contained within the modeling theorems. These boundaries express ideal states.

It follows from the above, *a priori* and *a posteriori* information of the object must be known. The amount of the model information Z can be determined from the difference between initial H_1 and residual H_2 entropy:

$$Z = H_1 - H_2. \tag{14}$$

We intend to use all the above for defining a model's uncertainty considered and analyzed from an information measure-based perspective. In this case, entropy is used as a measure of uncertainty, and depends only on the amount and the probability distribution of quantities taken into account by the conscious observer for the development of a model.

1.4. Basics of the theory of modeling the phenomena

The key problem for modeling is one of cognition of physical reality, which is viewed through the prism of a set of physical laws that objectively describe the real world. In this regard, one of the main tasks of modeling is the development of theoretical and methodological aspects and procedures for achieving accurate knowledge of objects and processes in the surrounding world, related to the improvement of measurement accuracy. As a concentrated and most universal form of purposeful experience, modeling makes it possible to verify the reliability of the most general and abstract models of the real world, realizing the principle of observability. Modeling is a method of studying objects of cognition (actually existing) in their models; the construction and study of models of objects and phenomena (physical, chemical, biological, social) to determine or improve their characteristics, rationalize the methods of their construction, management, etc.

The model is a concrete image of the object under study, in which real or perceived properties, structures, etc. are displayed. Therefore, increasing the accuracy of measurements is given particular importance in modeling. In turn, the purpose of measurement is the formation of a certain objective image of reality in the form of a symbolic symbol, namely a number. At the same time, "potential measurement accuracy" does not receive enough attention. The task of this book is to fill, if possible, this gap. In its turn, the purpose of measurement is the formation of some objective image of reality in the form of a representative symbol, namely a number. At the same time, "the potential accuracy of measurements" has been given insufficient attention. The task of this book is to fill, if possible, this gap. We will understand by "the ultimate accuracy of measurements" the accuracy with which a physical quantity can be measured at a given stage in the development of science and technology, i.e., the highest accuracy achieved at the present time. "Potential accuracy of measurements" is understood as the maximum achievable accuracy, which has not yet been realized at the present stage of development of science and technology.

Modeling can be defined as a translation of the physical behavior of phenomena components and collections of components into a mathematical representation [13]. This representation must include descriptions of the individual components, as well as descriptions of how the components interact.

Mathematical modeling of various physical phenomena and technological processes is a challenge for the 2010s and beyond. The study of any physical phenomena or processes begins with the creation of the simplest experimental facts. They can formulate laws governing the analyzed material object, and write them in the form of certain mathematical relationships. The amount of prior knowledge, the purpose of analysis, and the expected completeness and accuracy of the necessary decisions determine the level schematic of the test process.

A model is a physical, mathematical or otherwise logical representation of the real system, entity, phenomenon or process. Simulation is a method for implementing a model over time. The real system, in existence or proposed, is regarded as fundamentally a source of data.

In general, every model of the object does not contain the wording of the causal relationships between the elements of the object in the form of ready-made analytical expressions. In some cases, we have to be satisfied with such bonds (qualitative and quantitative) which characterize the material object only in the most general terms, and express a much smaller amount of knowledge about the internal structure of the test process. In all cases, the model is a user-selectable abstraction in the first place because it was built for an intuitively designated object, and also because of the incomplete or inaccurate knowledge (conscious simplification) of the laws of nature. From the point of view of developers, if the difference between the results of theoretical calculations and the data obtained in the course of experiments is less than the measurement uncertainty achieved, the chosen physical and mathematical model is considered acceptable.

However, comprehensive testing of the model is impossible [14]. Exhaustive checking is realized only upon receipt of all results from a model sweep for all possible variants of the input data. In practice, model validation aims to increase confidence in the accuracy of the model. Estimations produced by the model can be made with different levels of detail, but there is no generally accepted or standard procedure which would establish the minimum quantitative requirements for the design of model testing [15].

Over last two decades, many studies have been conducted to identify which method will demonstrate the most accurate agreement between observation and prediction. Unfortunately, the confirmation is only inherently partial. Complete confirmation is logically precluded by the incomplete access to the material object. At the same time, the general strategies of matching models and a recognized object that have been particularly popular from both a theoretical and applied perspective are *verification* and *validation* (V&V) techniques [16].

In [17] the following definition is proposed: *verification* is the process of determining that a computational model accurately represents the basic mathematical model and its solution; *validation* is the process of determining to what degree a model is an accurate representation of the real world from the perspective of the intended use of the model.

Given the above definition, we can say that the quality validation may be useful in certain scenarios, especially when identifying possible causes of errors in the model. However, at the moment, the validation is not able to provide a quantitative measure of the agreement between the experimental and computer data. This makes it difficult to use in determining at what point the accuracy requirements are met [16]. We refer the reader to [18, 19] for a more detailed discussion of the existing developments in V&V.

However, some scholars suggest that the V&V of numerical models of natural systems is impossible [20]. The authors argue that the models can never fully simulate reality in all conditions and, therefore, cannot be confirmed.

So, the causes of numerous attempts to direct the use of the experimental results are the limited applicability of different applied methods (analytical and numerical), and the difficulties with the use of computers and the methods of computational mathematics because of the lack of qualified researchers. Decisions resulting from the correlation of experimental data in the form of graphs, nomograms and criteria equations allow us to judge the quality and, to a certain extent, the proportion of the observed parameters of the process. Nevertheless, the experimental method cannot explain why the process is in the direction of what is observed in practice, nor accurately substantiate the list of selected process parameters.

Experience in dealing with the problems associated with various applications has shown that a preliminary analysis of a mathematical model using the theory of similarity (the definition of a set of physical criteria, each of which controls a specific behavior of a physical phenomenon) and the subsequent application of numerical methods to implement them on a computer allows us to obtain detailed information that cannot be obtained by analytical methods. However, analytical methods, by contrast to numerical methods, allow the creation of more visual solutions with which the influence of selected factors on the result of the decision can easily be analyzed. In addition, in practice it is considered a good result if it is obtained with an accuracy of up to 10% or even more [21]. Thus, research to consider various processes is basically a synthesis based on analytical and numerical methods.

The modern idea of combining the analytical and numerical methods is in the computational experiment [22]. This experiment consists of several stages. The first step is to compile equations of the problem, expressing in quantitative form a general idea of the physical mechanism of the process. They are based on the analysis of the process as a particular application of the fundamental principles of physics. In most cases they are in the form of differential (integral, integral-differential) equations.

Since the studied process is quite complicated and it cannot be investigated on the basis of only one physical law, there is a need to consider various aspects of the model and also different physical laws. Therefore, the overall process is usually determined by the system of equations.

In addition to the basic equations, there are written boundary conditions: a set of constant parameters characterizing the geometric and physical properties of the system that are essential for the process as well as conditions for uniqueness.

After the mathematical model is made, it is necessary to determine the correctness of its formulation (the existence of a solution, its uniqueness, whether it continuously depends on the boundary conditions). However, in practice, for many applications it is impossible to rigorously prove theorems of existence and uniqueness. So, there are some “illegal” mathematical techniques used that do not have a precise mathematical justification [23].

In the second stage of the computational experiment, the selection of the computation algorithm is realized. In a broad sense, the algorithm refers to the exact prescription that specifies the computational process, starting from an arbitrary initial datum and aiming to obtain results which are completely defined by this initial data [24]. In a narrow sense, computational algorithms are the sequence of arithmetic and logical operations, by which the mathematical problem is solved [22].

A computational algorithm focused on the use of modern computers must meet the following requirements: 1) provide a solution of the original problem with a given accuracy after a finite number of actions; 2) implement the decisions of the problem by taking the least possible computer time; 3) ensure the absence of an emergency stop of computers during the calculations; and 4) be sustainable (in the calculation process, rounding errors should not be accumulated). For more detailed information about this phase of the computational experiment, see [25].

In the third stage, the computer programming of a computational algorithm is organized. A huge amount of work is devoted to this issue. Given the specificity of this study, the greatest work of interest can be found in [26].

The fourth stage involves performing calculations on a computer, and the fifth involves the analysis of the numerical results and the subsequent refinement of the mathematical model.

From the standpoint of saving computer time and the practical value of the information obtained, the organization and planning of the last two stages of the computational experiment are important. So, just before the start of the computational experiment, the question of the scope and methods of processing (convolution) output data should be carefully considered. Obviously, in the study of any process, the experimenter has to accommodate a large number of quantities, and accordingly, the solving of the multi-criteria problem.

It should be noted that to find hidden relationships between quantities in the case of the multi-quantity model is very difficult. So, it is valuable to use the methods of the theory of similarity, which are in accordance with modem ideas and can be called a theory of generalized quantities [6]. Application of this theory is advisable for several reasons mentioned in Chapter 1.1. At the moment, the similarity theory does not answer the question about the number of possible combinations of dimensional characteristics included in the description of the dimensionless physical process, and the form of these combinations. In addition, it is not clear what criteria, for many interacting quantities, are suitable for the description of the physical process and how much they are required for a given uncertainty in the determination of the chosen main quantity [5].

So, realized in the form of a computer program, the mathematical model is a kind of computational experimental unit [27] that has several advantages over the conventional technology experimental construction:

- universality, because for the study of a new version of the computing installation it is only necessary to introduce new background information, whereas the technologically realized experiment will need a lot of raw materials and sometimes reinstalling, reconstruction and even full-scale installation of the new design;
- the possibility to obtain complete information about the effect of process parameters on the temperature field of the interacting bodies.

However, the array of information provided by the computing unit has a very large volume, making it difficult to process it.

At the same time, implementation of the full-scale experiment at the test conditions of the process equipment would be fraught with even greater difficulties. In order to be able to compare the numerical calculations and the experimental data, it is necessary to hold at least the same number of experiments, with the options as calculated by the computer. To make the experimental data statistically significant, it is needed to organize three to five replications in each experiment. This will lead to a further increase in labor costs and an increase in the duration of the experiments, which, in turn, affects the accuracy of the experimental data.

On the other hand, it is obvious that with random, haphazard use of any sorting options, usage of the fastest computer does not provide optimal solutions. It needs a deliberate and planned recognition of these options. However, not all parameters equally affect the researched process. So, the reduction of the number of quantities to a minimum on the basis of their relative influence and the selection of essential process quantities is the most important goal in the correct formulation of the problem. For this reason, the active principles of the theory of experimental design [28] are most valuable.

There are various methods for global sensitivity analysis of an output data model. Numerous statistical and probabilistic tools (regression, smoothing, tests, statistical training, Monte Carlo, random balance, etc.) are aimed at determining the input quantities which most affect the selected target quantity of the model. This value may be, for example, the variance of the output quantity. Three types of methods are distinguished: screening (coarse sorting being the most influential among a large number of inputs), the measure of importance (quantitative sensitivity indices) and in-depth study of the behavior of the model (measuring the effects of inputs on their variation range) [29]. As an example of the organization and usage of phases of the sensitivity analysis, we will discuss the method of random balance here.

In the method of random balance, linear effects and pairwise interactions are eliminated. But, at the same time, there is an additional constraint: it is assumed that the number of significant effects is significantly less than the total number of effects taken into consideration.

The application of random balance in the study of any process has, in principle, two features. The solution to any practical problem will be of great value when the independent quantities are used as generalized criteria, rather than individual factors of the physical dimension. The rationale for this approach is justified in [30]. In this case, the monitoring process is less sensitive to variations of similarity criteria, rather than to the combined effect of variations of the parameters in the similarity criteria. Application of the theory of similarity to solve problems by using the theory of experiment planning is due to the desire to reduce the number of independent quantities, and, therefore, the number of experiments, and dramatically reduce the amount of computational work.

Another feature is the fact that all methods of the theory of experimental design, including the method of random balance, are used in a full-scale natural experiment. At the same time, the methods of Monte Carlo and random balance can be used to identify significant factors in the framework of the developed mathematical model [31]. Such an approach from the standpoint of mathematics does not currently have theoretical studies. However, from an engineering point of view, by the condition of the availability of positive experimental evaluation, which is, of course, only a partial justification, the suggested approach to the engineering processes seems possible.

In the method of random balance [32] there are supersaturated plans used in which the number of trials (experiments) μ is less than the number of the effects, but it is greater than the number of significant effects q ($\mu >$ q). This method is used to determine the most significant factors that characterize the object under study [33].

The application of the method of random balance is based on two assumptions: 1) if, for the development of the experimental plan, one uses random sampling of the rows of full factorial experiment, then the probability of separating the dominant effects will be great enough because of the small number of these effects; and 2) factors do not affect the response of the system, i.e., they can be ranked (ranking is exponential) in descending order of influence on ξ, and most of them can be attributed to background noise. Compliance with the condition μ-q ≥ 0 gives a possibility of the quantity measure of the chosen effects by regression analysis [33].

The abundance of the most detailed information obtained from numerical studies on the basis of the developed finite-difference algorithm is not always necessary, or rather, almost never required, to produce correct and effective design solutions. In each case, it is necessary to clearly understand for what purpose each version of a computer-aided calculation should be performed. Detailed information is sometimes harmful as it is not subject to consolidation and synthesis. Therefore, in the development of design solutions, it is a very important skill to carry out the required level of aggregation of information and at the right scale (i.e., its consolidation in order to reduce the amount of information considered).

From this perspective, the use of the random balance will highlight the significant factors, the number of which is sufficient for the desired number of design studies in the shortest possible period of time.

Here, it is appropriate to make a few observations on the further procedure of processing the results of the numerical experiment. The random balance is the first step in the experiment. Subsequently, there is "movement in the area and a description of the area of the optimum" performed [34].

Time of termination of the shifting of effects in the random balance method is assessed by using Fisher's criterion *F* [35]:

$$F = S^2\{y_u\}/S^2\{y\} \tag{15}$$

where $S^2\{y_u\}$ is the dispersion of the results of experience and $S^2\{y\}$ is the variance calculated based on the results of several parallel experiments in the center of the experiment ('reproducibility variance').

Sieving effects cease if the value *F* calculated by the formula (15) is less than the table value for the selected level of significance, i.e., it found that the remaining variation of points is not different from the scattering results related to the experimental uncertainty.

In view of the fact that the computational experiment is conducted on models of different physical nature [36], the notion of "reproducibility variance" appears problematic. It should be noted that, despite the positive results of applying the methods of the experiment planning in the creation of mathematical models of complex physical objects, there is a problem in reducing the time and cost for the required number of experiments in the subsequent stages of finding the optimum area.

The complexity of solving this problem is determined primarily by the impossibility of the *a priori* definition of the order of the mathematical model due to the complexity of the physical nature of processes [37], as well as the fact that for every recognized object there is its own region of the optimum. In addition, because of the essential nonlinearity of main functions of different physical processes, and the high accuracy requirements to describe the area of the optimum, it is required to build mathematical models of higher orders. This in turn makes it difficult and sometimes impractical to use the obtained results in practice.

Considering all of the above, it appears to the author that the only solution regarding a decision on the termination of screening experiments and the selection of the desired value of Fisher's factor is the calculation of the minimum absolute and comparative uncertainties of the model, which depends only on the number of selected quantities. The absolute uncertainty can be used to calculate the exact Fisher's criteria, since it is the smallest achievable uncertainty of the developed model. In addition, it does not depend on an experimental measurement uncertainty. The purpose of this work is to formulate a method for calculating the comparative uncertainty inherent in any physical-mathematical model.

1.5. Basics of group theory

This section contains the minimum required explanation about concepts and definitions of group theory needed to understand the logical chain of reasoning in the formulation of the information approach for calculating the minimum achievable comparative uncertainty.

Definition 1. The binary operation is specified on the set, if there is the defined law that puts in line any two elements of the set and a unique element of the same set.

Definition 2. The set Y with a binary algebraic operation defined on it is called a group if:

- this operation is associative, i.e., $(ab)c = a(bc)$ for all elements a, b, c of Y;
- Y contains a single element e: $ae = ea = a$ for every element a from Y;
- for each element a of Y, in Y there exists an inverse element a^{-1}: $a^{-1}\,a = a\,a^{-1} = e$.

Definition 3. A group is called Abelian (commutative) if all elements of the group commute with each other, i.e., there is a commutative law performed $ab = ba$ for any elements a, b of the group Y ($a, b \in$ Y). Set P $\subset$ Y is called a subgroup of Y, if it is closed with respect to a multiplication operation which is done on Y.

Examples of Abelian groups include the set of rational numbers, real numbers and complex numbers, considered with respect to the operation of addition. Non-Abelian groups are groups of substitutions of more than two elements, or matrix groups with respect to multiplication.

Depending on the number of elements of Y (more precisely, on its power), there are distinguished groups: finite, infinite discrete, continuous and mixed continuous. The number of elements of a finite group is called its order. Elements of an infinite discrete group can be enumerated using natural series of numbers, or any countable set of symbols. Elements of the continuous group are defined by a finite number of constantly changing parameters. The group is called compact if its parameters are located in a limited range of values. In mixed groups, some parameters have a discrete (for example, finite) set of values.

All the even numbers form a group with respect to the addition operation. A set of integer numbers that are multiples of a given number n is also a group. The set of odd numbers will not be a group for the addition operation, since this operation takes us beyond the given set. All the non-zero positive rational numbers also form a group with respect to multiplication. The numbers 1 and -1 when multiplied constitute a finite group.

Definition 4. Order of element is the smallest positive integer n such that $a^n = e$. It is represented by $|a|$.

Definition 5. Order of the group Y is the number of its elements.

Order of the group Y is represented by $|Y|$. If the set of elements is infinite, it can be said that Y has infinite order and written as $|Y| = \infty$.

The groups are divided into two broad classes by the number of elements: the finite, in which a plurality of elements is finite, and the infinite with an infinite number of elements. Examples of finite groups are permutations of a finite number of elements and the number of roots of 1 with the multiplication operation. If the set of elements of finite order is a subgroup, then this is called the periodic part of the group. Groups having a periodic part are also highlighted in a special class of groups.

In the study of the group theory there is highlighted the class of Abelian groups, i.e., groups in which all elements commute with one another. The theory of these groups is already quite well developed [38]. Abelian groups include rational numbers, integer numbers, complex numbers for any operation, all of the groups with one generating element, and quasi-cyclic groups.

Let the different types of quantities be denoted by A, B, C. Then, the following relations must be realized [39]:

a. From A and B, a new type of value is obtained as: $C = A \cdot B$ (multiplicative relationship);
b. There are unnamed numbers, denoted by $(\mathbf{I}) = (A^0)$, which when multiplied by A do not change the dimensions of this type of quantity. $A \cdot (\mathbf{I}) = A$ (single item);

c. A quantity must exist which corresponds to the inverse of the quantity A, which we denote A^{-1} such that $A^{-1} \cdot A = (I)$;
d. The relation between the different types of quantities obeys the laws of associativity and commutativity:
   Associativity: $A \cdot (B \cdot C) = (A \cdot B) \cdot C$,
   Commutativity: $A \cdot B = (B \cdot A)$;
e. For all $A \neq (1)$ and $m \in N$; $m \neq 0$, the expression $A^m \neq 1$ is the case;
f. The complete set consisting of an infinite number of types of quantity has a finite generating system.

This means that there are a finite number of elements $C_1, C_2, \ldots C_\Omega$, through which any type of quantity q can be represented as:

$$q \supset C_1^{\tau_1} \cdot C_2^{\tau_2} \cdot \ldots \cdot C_\Omega^{\tau_h}, \qquad (16)$$

where the symbol $\supset$ means "corresponds to dimension"; and τ_i means integer coefficients, $i \in [1, \Omega]$, $\tau_i \in \lambda$, where λ is the set of integers.

The uniqueness of such a representation is not expected in advance. Axioms "a-f" form a complete system of axioms of an Abelian group [40]. By taking into account the basic equations of the theories of electricity, magnetism, gravity and thermodynamics, they remain unchanged.

Now, we use the theorem that holds for an Abelian group: among Ω elements of the generating system $C_1, C_2, \ldots C_\Omega$ there is a subset $h \leq \Omega$ of elements $B_1, B_2, \ldots B_h$, with the property that each element can be *uniquely* represented in the form:

$$q \supset B_1^{\beta_1} \cdot B_2^{\beta_2} \cdot \ldots \cdot B_h^{\beta_h}, \qquad (17)$$

where β_k are integers, $k \in [1, h]$, $h \leq \Omega$; elements $B_1 \cdot B_2 \cdot \ldots B_h$ are called the basis of the group; and B_k are the basic types of quantities. $\prod_1^k B_k^{\beta k}$ is the product of the dimensions of the main types of quantities B_k.

For the above-stated conditions, the following statement holds: the group, which satisfies axioms a-f, has at least one basis $B_1 \cdot B_2 \cdot \ldots B_h$. In the case $h > 2$, there are infinitely many valid bases.

How to determine the number of elements of a basis? In order to answer this question, we will apply the above-mentioned theorem for the SI in Chapter 2.2. In this case, attention must be paid to the following irrefutable situation. One should be aware that condition (16) is a very strong constraint. It is well known that not every physical system can be represented as an Abelian group. Presentation of experimental results as a formula, in which the main parameter is represented in the form of the correlation function of the one-parameter selected functions, has many limitations. However, in this study, condition (16) can be successfully applied to the dummy system, in terms of being absent in nature, which is based on SI. In this system, the derived quantities are always presented as the product of the base quantities in different powers.

1.6. Summary

In this chapter, we presented an overview of the fundamental principles that will be used in this book. At the same time, we defined mathematical modeling of phenomena, provided motivation for its use in science and technology, and outlined a fundamental approach to mathematical modeling. We also outlined some important tools that will be used in more detail later: the fundamentals of similarity theory, information theory, experiment planning theory, and group theory.

CHAPTER TWO

INFORMATION MEASURE OF THE MODEL'S DISCREPANCY

The most important thing you cannot see with your eyes
—Antoine de Saint-Exupéry

2.1. Analysis of publications

The human desire to learn about the macrocosm, to understand the laws of the invisible microcosm, to enhance the quality of everyday life, to protect against natural disasters and prevent an ecological catastrophe on our planet stimulates researchers and designers in a bold and ambitious search, and generates the desire of scientists and engineers to create energy-efficient appliances and equipment. This equipment is compact, characterized by a high degree of computerization and robotics, and can implement complex algorithms.

All of the above causes systematic research of processes and phenomena by methods of physical and mathematical modeling. In addition, the demand increases for a clear understanding of the results obtained using these methods.

In recent years, new tools and methods have been developed to detect the proximity between the researched material object and the designed physical-mathematical model, to evaluate modeling errors, as well as to quantify the uncertainties inherent in the numerical calculations, and for choosing the appropriate and adequate model [41, 42, 43, 44].

Specific examples of the selection of the expedient physical-mathematical model to describe the studied material object are presented in the following papers.

The systematic approach is used for qualitative analysis of the measurement procedure [45]. This procedure is considered as a system containing different elements interacting with each other, including the material object, and mathematical models describing it. The traditional analysis of the accuracy of the measurements is supplemented by a study of qualitative characteristics such as reliability and complexity of the measurement procedure. Mathematical modeling of the measurement procedure is developed and studied. The qualitative characteristics of the mathematical model are also investigated, including the adequacy of the number of quantities used.

Authors of [46] developed a method for estimating the systematic error of a model and proposed its introduction into a physical experiment for the case of correlated measurements of unequal accuracy. They obtained algorithms for calculating the confidence limits of the systematic error of the mathematical model and also demonstrated their efficiency.

In [47], methods for measuring uncertainty in the form of the different mathematical models were demonstrated. The authors discussed and analyzed a class of models in engineering and sciences, taking into account the relationship between input and output quantities for a system. These models are built on the basis of knowing the underlying physical laws such as material mechanics, and utilizing constraints such as boundary conditions.

In [48], the authors stated that the criterion for choosing a method to estimate the values of a measure is not clearly addressed in the Guide to the Expression of Uncertainty in Measurement (GUM). This statement is true if repeated measurements are performed. The two methods recommended in the GUM to estimate the values of a measure are compared. Thus, a certain criterion is formulated for selecting the preferable method based on the calculation of contributions to the acquisition uncertainty.

In research [49], three criteria (robustness, fidelity and prediction-looseness) were used to assess the credibility of mathematical or numerical models. It is shown that these criteria are mutually antagonistic. The recommended main strategy is to explore the trade-offs between robustness and uncertainty, fidelity and data, and tightness of predictions.

Thus, there is no shortage of methods and techniques to identify the matching of the physical-mathematical model and the studied natural phenomena or processes. However, given the theme of the study, we are interested in focusing on the works connecting information theory and measurement theory.

One of the first innovative works must be considered in [9]. In this book, Brillouin related the concept of entropy with the uncertainty of the physical experiment results in order to determine the accuracy of the experiment. For a more detailed study of the accuracy achieved in the experiment, an additional metric was proposed. It is called the comparative uncertainty and it is the ratio of the absolute uncertainty of measurement of the quantity to the magnitude of its interval of changes. It has been explained in detail that without any knowledge about this interval, any experimental research loses its physical meaning.

Despite numerous scientific publications that the author is aware of related to the possibility of using the concept of "amount of information" and "entropy" in conducting field experiments and computer modeling, examples of the practical use of information theory with concrete numerical calculations in physics and engineering are few. In the context of this book, a number of articles should be noted.

The first is [50] in which Akaike Information Criterion (AIC) is proposed. It is a metric of the relative quality of a statistical model for a chosen set of data. If one has a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. AIC is founded on the concept of entropy in information theory: it offers a relative estimate of the information lost when a given model is used to represent the process that generates the data. AIC can be conceived of as a theoretical tool for empirical modeling. When we wish to determine calculated values to represent theoretical data of an experiment, a researcher should usually choose the model with the smallest AIC. Unfortunately, AIC does not determine the quality of a model in an absolute sense. If all the candidate models fit poorly, AIC will not give any indication of this. Although AIC can be used for concrete practical cases, its application is quite different to the approach proposed here.

In [51] an upper limit has been calculated, called the Bekenstein bound, of the quantity of information contained within a given framed object which represents the maximum amount of information required to perfectly describe a given physical system. It was implied that the quantity of information of a physical system must be finite if the space of the object and its energy are finite. In informational terms, this bound is given by:

$$\Upsilon \leq (2 \cdot \pi \cdot R \cdot E)/(\hbar \cdot c \cdot \ln 2), \tag{18}$$

where Υ is the information expressed in the number of bits contained in the quantum states of the chosen object sphere; the ln2 factor comes from defining the information as the natural logarithm of the number of quantum states; R is the radius of an object sphere that can enclose the given system; E is the total mass-energy including any rest masses; $\hbar$ is the reduced Planck constant; and c is the speed of light. The results are purely theoretical in nature, although it is possible, judging by the numerous references to this article, that one may find applications of the proposed formula in medicine or biology.

A study of quantum gates has been developed in [52]. The author considered these gates as physical devices which are characterized by the existence of random uncertainty. Reliability of quantum gates was investigated from the perspective of information complexity. In turn, the complexity of the gate operation was determined by the difference between the entropies of the quantities characterizing the initial and final states. The study stated that the gate operation may be associated with unlimited entropy, implying the impossibility of realization of the quantum gates function under certain conditions. The relevance of this study comes from its conceptual approach of the use of quantities as a specific metric for calculation of information quantity changing between input and output of the apparatus model.

The information-theory-based principles have been investigated in relation to uncertainty of mathematical models of water-based systems [53]. In this research, the mismatch between physically-based models and observations has been minimized by the use of intelligent data-driven models and methods of information theory. The real successes were achieved in developing forecast models for the Rhine and Meuse rivers in the Netherlands. In addition to the possibility of forecasting the uncertainties and accuracy of model predictions, the application of information theory principles indicates that, alongside appropriate analysis techniques, patterns in model uncertainties can be used as indicators to make further improvements to physically-based computational models. At the same time, there have been no attempts to apply these methodologies to results of other physical or engineering tasks.

The design information entropy was introduced as a state that reflects both complexity and refinement in [54]. The author argued that it can be useful as some measure of design efficacy and design quality. The method has been applied to the conceptual design of an unmanned aircraft, going through concept generation, concept selection and parameter optimization. For the purposes of this study, it is important to note that introducing the design information entropy as a state can be used as a quantitative description for various aspects in the design process, both with regards to structural information of architecture and connectivity, as well as for parameter values, both discrete and continuous.

In [55] there has been a systematic review conducted of major applications of information theory to physical systems, its methods in various subfields of physics, and examples of how specific disciplines adapt this tool. In the context of the proposed approach for practical purposes in experimental and theoretical physics and engineering, the physics of computation, acoustics, climate physics and chemistry have been mentioned.

However, no surveys, reviews or research studies were found with respect to applying information theory for calculating an uncertainty of models of the phenomenon or technological process.

The approach that uses the tools of estimation theory to fuse together information from multi-fidelity analysis, resulting in a Bayesian-based approach to mitigating risk in complex design has been proposed [56]. Maximum entropy characterizations of model discrepancies have been used to represent epistemic uncertainties due to modeling limitations and model assumptions. The revolutionary methodology has been applied to multidisciplinary design optimization and demonstrated on a wing-sizing problem for a high altitude, long endurance aircraft. Uncertainties have been examined that have been explicitly maintained and propagated through the design and synthesis process, resulting in quantified uncertainties on the output estimates of quantities of interest. However, the proposed approach focuses on the optimization of the predefined and computer-ready simulation model.

For these reasons there are only a handful of different methods and techniques used to identify the matching of physical-mathematical models and studied physical phenomena or technological processes by the uncertainty formulated with usage of the concepts of "information quantity" and "entropy". All the above-mentioned methodologies are focused on identifying *a posteriori* uncertainty caused by the ineradicable gap between a model and a physical system. At the same time, according to our data, in modem literature there does not exist any physical or mathematical relationship which could formulate the interaction between the level of detailed descriptions of the material object (the number of recorded quantities) and the lowest achievable total experimental uncertainty of the main parameter.

Thus, it is advisable to choose the appropriate/acceptable level of detail of the object (a finite number of registered quantities) and formulate the requirements for the accuracy of input data and the uncertainty of the specific target function (similarity criteria), which describes the "likelihood" and characterizes the behavior of the observed object.

2.2. System of base quantities

De facto, the physical-mathematical model formulation is based on two guidelines:

1. ***Observation is framed by a System of Base Quantities*** (SBQ). Absolutely all physicists and engineers try to describe the observed phenomena with the help of concepts inculcated by everyday experience, acquired knowledge and, not infrequently, intuition. At the same time, despite 90-year efforts, it has not been possible until now to combine classical determinism with the probabilistic laws of quantum mechanics. The only characteristic that unites all modem physics so far is that scientists use SBQ, such as the SI or CGS (centimeter–gram–second system of units), to realize their ideas. It means that the harmonic construction of modern science is based on a simple consensus that any physical laws of micro- and macro-physics are described by quite certain dimensional quantities: base and derived quantities. Taking quantity as the fundamental aspect means that it can be assigned as a standard of measurement, which is independent of the standard that is chosen for the other fundamental quantity. The base quantities are selected arbitrarily, while the derived quantities are chosen to satisfy discovered physical laws or relevant definitions.

The concept of SBQ is taken from our everyday experience and is valid only for the momentary perception of the observed phenomena. It would be surprising if it would be possible some day to exclude from the physical theory concepts that are the very foundation of our daily life. True, the history of science reveals the amazing fruitfulness of human thought and one should not lose hope. However, until we have succeeded in spreading our ideas in this direction, we should try with greater or lesser difficulty to squeeze the observed phenomena into the framework of the concept of SBQ. Although we will always be troubled by the feeling that we are trying to put a huge human foot into a small diamond shoe that does not suit her.

SBQ, in its essence, is some new element in scientific knowledge, completely alien to classical concepts. It exists only because of the consensus of the researchers, although SBQ is absent in nature. By default, the use of the dimensional quantities contained in the SBQ to describe the micro- and macro-cosmos implies a certain framework that limits our knowledge.

So, the quantities are selected within a pre-agreed SBQ that is a set of dimensional quantities, which are base and can generate derived quantities. These quantities are necessary and sufficient to describe the known laws of nature, as in the quantitative physical content [40]. This means that any scientific knowledge and, without exception, all formulated physical laws are discovered due to information contained in the SBQ. This is a unique chamel (generalizing carrier of information [57]) through which information is transmitted to the observer or the observer extracts information about the object from the SBQ. The SBQ includes a finite number of physical dimensional quantities, which have the potential to characterize the world's physical properties and, in particular, observed phenomenon qualitatively and quantitatively. So, an observation of a material object and its modeling are framed by the SBQ. We model only what we can imagine or observe, and the mere presence of a selected SBQ, such as the lens, sets a specific limit on the measurement of the observed object.

Each quantity carries a certain amount of information about the object under study. Since the number of elements in the SBQ is finite, the total amount of information contained in the SBQ is finite. Thus, we conclude that there exists the specific limit of knowledge of the surrounding reality. This limit is not due to any existing physical laws, but to the presence of collective human consciousness. The mere fact of the measurement process presupposes the existence of a physical-mathematical model of the researched object that has already been formulated, including equations and boundary conditions. In this case, it is already possible to compile a list of the registered dimensional quantities and calculate their number in advance. Most importantly, it is also possible to calculate the entropy change between the initial state corresponding to the maximum number of variables in the SBQ.

2. ***The number of quantities taken into account in the physical-mathematical model is limited.***

The SBQ includes the base and derived quantities used for descriptions of different classes of phenomena (*CoP*). In other words, the additional limits of the description of the studied material object are defined by the choice of *CoP* and the number of derived quantities taken into account in the mathematical model [58]. For example, in mechanics SI uses the basis {***L***– length, ***M***– mass, ***T***– time}, i.e., $CoP_{SI} \equiv LMT$. Basic accounts of electromagnetism here add the magnitude of electric current I. Thermodynamics requires the inclusion of thermodynamic temperature $\boldsymbol{\Theta}$. For photometry it needs to add ***J***– luminous intensity. The final base quantity of SI is an amount of substance F.

If the SBQ and *CoP* are not given, then the definition of "information about researched object" loses its force. Without the SBQ, the modeling of the phenomenon is impossible. You can never get something out of nothing, not even by watching [9]. It is possible to interpret the SBQ as a basis of all accessible knowledge that humans are able to have about their environment at the present time.

At the same time, the uncertainty of a mathematical model with a finite number of quantities cannot be achieved as low as desired. It is explained by the fact that this uncertainty relates to the validity of each natural or computer-based experiment, and should be a part of the theory of measurements. When this theory is used as a physical model, it becomes the object of applying both the above restrictions. In physics, this leads to an assumption of the possibility of the existence of certain uncertainties (limited accuracy) before the mathematical model is applied.

There are fundamental objective (e.g., thermodynamic) limits for accuracy during the experimental study. This, in turn, determines the existence of *a priori* source of inaccurate knowledge on all material objects, the information about which is received and processed by the observer.

Fundamental limits on the maximum precision with which we can determine the physical quantities are created by Heisenberg's Uncertainty Principle [59]. However, Planck's constant is extremely small, so the uncertainty in the macro-measurements is devoid of practical meaning. Uncertainties of position and momentum, which follow from it, lie far beyond the achievable accuracy of the experiments.

In turn, the establishment of a specific SBQ (e.g., SI units) means that we are talking about trying to restrict the set of possible quantities by a smaller number of basic quantities and the corresponding units. Then, all other required quantities can be found or determined based on these base quantities, which must meet certain criteria [40] that were introduced in Chapter 1.4. The entire information above can be represented as follows:

1. There are $\xi = 7$ base quantities: $\boldsymbol{L}$ is length, $\boldsymbol{M}$ is mass, $\boldsymbol{T}$ is time, $\boldsymbol{I}$ is electric current, $\boldsymbol{\Theta}$ is thermodynamic temperature, $\boldsymbol{J}$ is luminous intensity, $\boldsymbol{F}$ is the amount of a substance [40].

2. The dimension of any derived quantity $\boldsymbol{q}$ can only be expressed as a unique combination of dimensions of the main base quantities to different powers (1) [40]:

$$\boldsymbol{q} \ni \boldsymbol{L}^{l} \cdot \boldsymbol{M}^{m} \cdot \boldsymbol{T}^{t} \cdot \boldsymbol{I}^{i} \cdot \boldsymbol{\Theta}^{\Theta} \cdot \boldsymbol{J}^{j} \cdot \boldsymbol{F}^{f} \qquad (19)$$

In this case, one should note that condition (19) is a very strong constraint. It is well known that not every physical system can be represented as an Abelian group. The presentation of experimental results as a formula, where the main quantity is represented in the form of the correlation function of the one-quantity selected functions, has many limitations. However, in this study, condition (1) can be successfully applied to a system that is not in nature; for example, SI. In this system, the derived quantities are always represented as the product of the base quantities to different degrees.

3. $l, m \ldots f$ are exponents of the quantities and the range of each has a maximum and minimum value; according to [60], integers are the following:

$$-3 \le l \le +3, -1 \le m \le +1, -4 \le t \le +4, -2 \le i \le +2$$
$$-4 \le \theta \le +4, -1 \le j \le +1, -1 \le f \le +1. \qquad (20)$$

4. The exponents of quantities can only take integer values [60], so the number of choices of dimensions for each quantity $e_l, \ldots e_f$, according to (20), is the following:

$$e_l = 7; e_m = 3; e_t = 9; e_i = 5; e_\theta = 9; e_j = 3; e_f = 3 \qquad (21)$$

where, for example, L^{-3} is used in a formula of density, and Θ^4 in the Stefan-Boltzmann law.

5. The total number of dimension options of physical quantities equals $\Psi^\circ = \prod_l^f e_i - 1$

$$\Psi^\circ = \left(e_l \cdot e_m \cdot e_t \cdot e_i \cdot e_\theta \cdot e_j \cdot e_f - 1\right) = (7 \cdot 3 \cdot 9 \cdot 5 \cdot 9 \cdot 3 \cdot 3 - 1)$$
$$= 76{,}544, \qquad (22)$$

where "-1" corresponds to the case where all exponents of the base quantities in formula (19) are treated to zero dimension.

6. The value Ψ° includes both required and inverse quantities (for example, L^1 is the length, L^{-1} is the running length). The object can be judged knowing only one of its symmetrical parts, while others structurally duplicating this part may be regarded as information empty. Therefore, the number of options of dimensions may be halved. This means that the total number of dimension options of physical quantities without inverse quantities equals $\Psi = \Psi^\circ/2 = 38{,}272$.

7. For further discussion we use the methods of the theory of similarity, which is expedient for several reasons mentioned in Chapter 1.2. It is important to note that the universality of similarity transformations is defined by the invariant relationships that characterize the structure of all

the laws of nature, including the laws of relativistic nuclear physics. Moreover, dimensional analysis from the point of view of the mathematical apparatus has a group structure, and conversion factors (the similarity complexes) are invariants of the groups. The concept of the group (Chapter 1.5) is a mathematical representation of the concept of symmetry, which is one of the most fundamental concepts of modem physics [4].

According to π-theorem [61] (Chapter 1.2), the number μ_{SI} of possible dimensionless criteria with $\xi = 7$ base dimensional quantities for SI will be:

$$\mu_{SI} = \Psi - \xi = 38{,}265. \quad (23)$$

Applying the theory of similarity is motivated by the desire to generalize obtained results in the future for different areas of physical applications. The numerical value of μ_{SI} can only increase with the deepening of knowledge about the material world. It should be mentioned that the set of dimensionless quantities μ_{SI} is a fictitious system, since it does not exist in physical reality. However, this observation is true for proper SI too. At the same time, the object which exists in actuality may be expressed by this set.

The relationships (19)–(23) are obtained on the basis of the principles of the theory of groups (Chapter 1.5) as set forth in [40]. The presented results provide a possible use of information theory to different physical and engineering areas with a view to formulating precise mathematical relationships to assess the minimum comparative uncertainty (see Chapter 2.3) of the model that describes the studied physical phenomenon or process.

Thus, at the information processing stage of the material object modeling, it is appropriate to consider tasks allowing the following: improvement in the reliability and accuracy of the results of physical and mathematical modeling; reduction in the amount and duration of natural and computer simulations; mathematical formulation of “life-activity” of the material object in the consolidated criteria form; and dissemination of the obtained results on similar material objects.

All the above can be attributed to the basic task involved in the problem of improving research efficiency and accelerating its practical implementation.

2.3. Amount of information inherent in the model

The validity of a mathematical model structure is confirmed, to a researcher, by the small differences between theoretical calculations and the experimental data. In doing so, a question is overlooked: to what extent does the chosen model correctly describe the relevant natural phenomenon or process.

At the beginning of the 20th century it was thought that the mathematical models of physical phenomena, of course, must be correct. Therefore, any incorrect model cannot simulate any meaningful physical phenomena. This idea was changed many years later during the 1940s with the work of Tikhonov, who provided the approach to stabilize the incorrect problem. This approach is known as the inverse problem and is now being applied to many important areas of science. In the inverse problem, we use data from physical phenomena to detect certain characteristics, or the reasons for the phenomena themselves.

In [62] it has been shown that by setting *a priori* the total value of uncertainties of an experiment and the formulated model, one can determine the necessary number of measurements of the chosen quantity and the validity of the selected model. The specified approach at the decision of inverse mathematical tasks is based on the legitimacy of a condition [63]:

$$\boldsymbol{\rho_D}(\boldsymbol{B}y, v) \leq \boldsymbol{\Delta}, \tag{24}$$

where $\boldsymbol{y}$ is the set of characteristics of the investigated process; $\boldsymbol{v}$ is an experimental field of measurement; D denotes the set of possible theoretical fields of measurements $\boldsymbol{d}$; $\mathbf{B}$ is the law connecting the characteristic of investigated object $\boldsymbol{y}$ with $\boldsymbol{d}$; $\boldsymbol{\rho_D}$ $(\boldsymbol{d}_1, \boldsymbol{d}_2)$ is a measure of affinity ("distance") between two fields; and Δ is an absolute uncertainty of definition of a field $\boldsymbol{d}$.

Condition (24) means that the field calculated under the characteristic $\boldsymbol{y}$ is separated from $\boldsymbol{v}$ by a distance which is less than or equal to Δ. In what follows, we denote Δ_{pmm} as the uncertainty in determining the dimensionless theoretical field $\boldsymbol{u}$, "embedded" in a physical-mathematical model and caused only by its dimension that is the property of the model to reflect a certain number of characteristics of researched phenomena, as well as its external and internal connections. What is the possible structure of Δ_{pmm}? To answer this question, we turn to [64], in which attempts to find a more general measure of information than the Shannon concept have been reviewed. In addition, the need for such an alternative measure has been demonstrated based on a historical review of the problems concerned with the conceptualization of information. The author has proven that an alternative measure can be presented in the context of a modified definition of information applicable outside of the conduit metaphor of Shannon's approach. Several features superior to those of entropy have been shown. For instance, unlike entropy it can be easily and consistently extended to continuous probability distributions, and unlike differential entropy this extension is always positive and invariant with respect to linear transformations of the coordinates. The author has proven a theorem which is interpreted as an assertion that the total information amount can be separated into information identifying the element of the partition, plus the average information identifying an element within subsets of the partition. Taking into account this conclusion, we can represent Δ_{pmm} as the sum of two terms, in which the first term of an alternative measure of information defines Δ_{pmm}' and the second term dictates the choice of Δ_{pmm}'':

$$\Delta_{\mathrm{pmm}} = \Delta_{\mathrm{pmm}}' + \Delta_{\mathrm{pmm}}'', \tag{25}$$

where Δ_{pmm}' is the uncertainty due to *CoP*, which is associated with the reduction in the number of recorded base quantities compared with the SBQ; and Δ_{pmm}'' is the uncertainty due to the choice of the number of recorded influencing quantities within the framework of the set of *CoP*.

The equation (25) is an expression of the fact that during modeling of any phenomenon or technological process and equipment there is a gap between the researched object and its theoretical representation in physical-mathematical form due to the conscious observer choosing only *CoP* and a number of quantities based on their own knowledge, experience and intuition. The reality of the environment is the obvious *a priori* condition for the modeling of the investigated material object. The "enclosure" of the process or phenomenon being investigated by the boundary conditions leads to the fact that the unknown relationships between the contents of the object and the environment are "broken". In this context it is obvious that an overall uncertainty of the model including inaccurate input data, physical assumptions, the approximate solution of the integral-differential equations, etc., will be larger than Δ_{pmm}. Thus, Δ_{pmm} is only one component of a possible mismatch of the real object and its modeling results. In turn, Δ_{pmm}'' cannot be defined without declaration of the chosen *CoP* (Δ_{pmm}'). So, according to its nature, Δ_{pmm} will be equal to the sum of two terms. When comparing different models (according to a value of Δ_{pmm}) describing the same object, preference should be given to the model for which $\Delta_{\mathrm{pmm}}/\Delta_{\mathrm{exp}}$ is closer to 1. The term Δ_{exp} is the estimated uncertainty in the determination of the generalized objective function (similarity criterion) during an experiment or computer simulation. It will always be larger than Δ_{pmm}. Many different models may describe fundamentally the same object, where two models are considered to be essentially the same if they are indistinguishable from a value of Δ_{pmm}.

We formulate an approach for the introduction of a measure of the information quantity about an object in the SBQ and the definition of a sequence of actions (algorithm) allowing a measurement of this quantity. A certain complexity of the observed material object is offered as a measure of the complexity of the object model. Each observer can decide only the category of the model. Any claim can be made only with respect to the model. Of course, the notion of "complexity" also requires definition and there is a possibility of arbitrariness. However, the process of cognition of a real object as a physical system, in general, is infinite. Thus, the model of the system is a formal structure built according to certain rules, and the design certainly is predictable. In this case, a material object (a certain totality) can be represented in two different ways. By merely listing its elements when the researcher supposes that a set of values is finite, or by specifying a system of rules (algorithm), based on which you can perform such an enumeration. This means a totality is thus accounted for. From a practical point of view, the most natural assertion is that the measure of complexity of the totality is the number of elements contained therein. So, one of the simplest ways is to find the magnitude calculated according to the number of elements included in this description. This value is an information quantity measure contained in the description of a physical system. In order to calculate an information quantity, we choose $\mathbf{X}_1$, $\mathbf{X}_2$, ... $\mathbf{X}_n$ ($\mathbf{n} \in \mathbf{N}$) base quantities. Then for a derived quantity, base quantities enter into the formula of dimension with exponents $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$, ... $\boldsymbol{\tau}_n \in \mathbf{P}$, where $\mathbf{P}$ is the set of rational numbers [40]. If the set of values $\mathbf{E}_{\tau n}$, which can accept $\boldsymbol{\tau}_n$ in various variants of formulas of dimension for derived quantities, has the top and bottom bounds, then $\mathbf{E}_{\tau n}$ is finite [70]. Consideration of a case $\boldsymbol{\tau}_n \in \mathbf{R}$, $\boldsymbol{\tau}_n \in \mathbf{E}_{\tau n}$, $\mathbf{E}_{\tau n} \in \mathbf{R}$, where $\mathbf{R}$ is a totality of real numbers, is invalid because it is possible $\boldsymbol{\tau}_n \in \mathbf{R} \backslash \mathbf{P}$, where $\boldsymbol{\tau}_n$ represents an irrational number, does not have physical meaning. The number of elements in $\mathbf{E}_{\tau n}$ will make e_n. The variant dimension number of physical quantities describing the internal structure of a material object reaches $\boldsymbol{\Psi}^{\circ} = \Pi e_n - 1$, where "-1" corresponds to the occasion when all exponents of base quantities in the formula are treated to zero dimension.

As the information quantity of an object is connected to its symmetry [65], the number $\boldsymbol{\Psi}^{\circ}$ can be reduced by a factor of $\boldsymbol{\omega}$ (the quantity of equivalent parts in the researched material object): $\boldsymbol{\Psi} = \boldsymbol{\Psi}^{\circ}/\omega$. Obviously, the equivalent parts of a symmetrical object $\{\mathbf{E}_{\tau n}\}$ have identical structure, where $\{\mathbf{E}_{\tau n}\}$ is the totality including elements of $\mathbf{E}_{\tau n}$ totalities. Consequently, the object can be judged, knowing only one of its symmetrical parts, while others structurally duplicating this part may be regarded as information empty. Knowing $\boldsymbol{\Psi}$ and using π-theorem [61], as mentioned in Chapter 1.2, the number of dimensionless criteria $\boldsymbol{\mu}_{SI}$ is equal to the number of dimensional physical parameters $\boldsymbol{\Psi}$, net amount of ξ base quantities, i.e., $\boldsymbol{\mu}_{SI} = \boldsymbol{\Psi} - \xi$ (23).

For further discussion we will use an analogy with a theory of signals transmission. Imagine that you want to convert the analog signal to digital by three operations: time sampling, quantization and coding in amplitude. To convert the analog signal to digital, the analog-to-digital converter (ADC) is used. At regular intervals the ADC measures the amplitude of the analog signal and receives the instantaneous values or readouts of the signal, and then it converts readouts in binary words/codes. ADC operation parameters are determined by the theorem of Shannon-Kotelnikov, on which a continuous signal with limited spectrum is completely characterized by a discrete number of values (readouts). Since any continuous medium may be written in a discrete form, it is a viable probabilistic approach for the measurement of information in the case of continuous environments. It is designed by Wiener, who proceeded from the fact that in cybernetic systems the elementary form of information is the memorization of the choice of one of two equally likely possibilities, which he called the solution.

For the purposes of our research, and with some practical intuition thrown in, assume that the recognized material object has a huge number of properties (criteria, quantities) that characterize its content and interaction with the environment. Then, we assume that each dimensionless complex represents the original readout (reading [9, 66]), through which some information on the dimensionless researched field $\boldsymbol{u}$ (observed object) can be obtained by the observer. In other words, the researcher observing a physical phenomenon, analyzing the process or designing the device, selects—according to his experience, knowledge and intuition—certain characteristics of the object. With this selecting of the object, connections of the actual object with the environment enveloping it are destroyed. In addition, the modeler takes into account the relatively smaller number of quantities than the current reality due to constraints of time, and technical and financial resources. Therefore, the "image" of the object being studied is shown in the model with a certain uncertainty, which depends primarily on the number of quantities taken into account. In addition, the object can be addressed by different groups of researchers, who use different approaches for solving specific problems and, accordingly, different groups of quantities, which differ from each other in quality and quantity. Thus, for any physical or technical problem, the occurrence of a particular quantity in the model can be considered as a random process.

It is supposed that the accounting of readouts (criteria, quantities) is equiprobable. We want to emphasize again that the use of the concept of "readout" in examining some object at the stage of model development is due to the expediency of the vector (positional) ways of representing information of the observed phenomena. When there are large numbers of components (a large-dimensional vector space) it is possible to distinguish only two states of the vector component: for example, the presence or absence of a signal; in our case, the appearance or lack of a readout quantity [67]. It should be noted that the approval of the equiprobable occurrence of a readout is justified by the purpose of the research: finding the absolute uncertainty Δ_{pmm} stipulated by the level of detail of the researched object. Indeed, any other distribution of readouts yields less information [68–70], which leads to a larger uncertainty of the model in comparison with an uncertainty calculated at the uniform distribution of readouts.

Then, let there be a situation wherein all quantities $\boldsymbol{\mu}_{\mathrm{SI}}$ of SI can be taken into account, provided the choice of these quantities is considered, *a priori*, equally probable. In this case, $\boldsymbol{\mu}_{\mathrm{SI}}$ corresponds to a certain value of entropy and may be calculated by the following formula [9]:

$$\boldsymbol{H} = \boldsymbol{k}_b \cdot \ln \boldsymbol{\mu}_{\mathrm{SI}}, \tag{26}$$

where **H** is entropy of SI including $\boldsymbol{\mu}_{\mathrm{SI}}$, equally probable accounted quantities, and $\boldsymbol{k}_{\mathrm{b}}$ is the Boltzmann's constant.

When a researcher chooses the influencing factors (the conscious limitation of the number of quantities that describe an object, in comparison with the total number $\boldsymbol{\mu}_{\mathrm{SI}}$), entropy of the mathematical model changes *a priori*. The entropy change is generally measured as follows [9]:

$$\boldsymbol{\Delta H} = \boldsymbol{H}_{\mathrm{pr}} - \boldsymbol{H}_{\mathrm{ps}}, \tag{27}$$

where **ΔH** is the entropy difference between two cases, and pr is "*a priori*" and ps is "*a posteriori*".

"The efficiency **Q** of the experimental observation method can be defined as the ratio of the information obtained to the entropy change accompanying the observation." [9] During a thought experiment, no distortion is brought into the real system, that is why **Q**=1. Then, one can write it according to (27):

$$\boldsymbol{\Delta A} = \boldsymbol{Q} \cdot \boldsymbol{\Delta H} = \boldsymbol{H}_{\mathrm{pr}} - \boldsymbol{H}_{\mathrm{ps}} \tag{28}$$

where $\Delta \mathbf{A}$ is the *a priori* information quantity pertaining to the observed object.

Using equations (26) to (28) and imposing symbols where z' is the number of physical quantities in the selected *CoP* (see Chapter 2.2) and β' is the number of base quantities in the selected *CoP* leads to the following equation:

$$\Delta \mathrm{A}' = \boldsymbol{Q} \cdot \left(\boldsymbol{H}_{\mathrm{pr}} - \boldsymbol{H}_{\mathrm{ps}}\right) = 1 \cdot [\boldsymbol{k}_b \cdot \ln \boldsymbol{\mu}_{\mathrm{SI}} - \boldsymbol{k}_b \cdot \ln(\boldsymbol{z}' - \boldsymbol{\beta}')] = \tag{29}$$

$$= \boldsymbol{k}_b \cdot \ln[\boldsymbol{\mu}_{\mathrm{SI}}/(\boldsymbol{z}' - \boldsymbol{\beta}')],$$

where $\Delta \mathbf{A}'$ is the *a priori* amount of information pertaining to the observed object due to the choice of the *CoP*.

The value $\Delta \mathbf{A}'$ is linked to the *a priori* absolute uncertainty of the model, caused only by the choice of the *CoP*, Δ_{pmm}' and S, the dimensionless interval of observation of the main researched dimensionless quantity $\boldsymbol{u}$, through the following dependence [9]:

$$\Delta_{\mathrm{pmm}}' = \boldsymbol{S} \cdot \exp\left(-\Delta \mathrm{A}'/\boldsymbol{k}_b\right). \tag{30}$$

Substitution of (29) into (30) gives the following dependence:

$$\Delta_{\mathrm{pmm}}' = \boldsymbol{S} \cdot (\boldsymbol{z}' - \boldsymbol{\beta}')/\mu_{\mathrm{SI}}. \tag{31}$$

Following the same reasoning, it can be shown that the *a priori* absolute uncertainty of a model of the observed object, caused by the number of recorded dimensionless criteria chosen in the model, Δ_{pmm}'' takes the following form:

$$\Delta_{\mathrm{pmm}}'' = \boldsymbol{S} \cdot (\boldsymbol{z}'' - \boldsymbol{\beta}'')/(\boldsymbol{z}' - \boldsymbol{\beta}'), \tag{32}$$

where z'' is the number of physical dimensional quantities recorded in a mathematical model; β'' is the number of base physical dimensional quantities recorded in a model; and Δ_{pmm}'' cannot be defined without declaring the chosen *CoP* (Δ_{pmm}').

All the above derivations can be summarized in the form of the μ_{SI}-hypothesis: *In model formulation, let the system of base quantities with a total number of dimensional physical* quantities *be denoted by* Ψ, *where* ξ *of which are chosen and are independent of dimension. In the framework of the phenomena class (*z' *is the total number of dimensional* quantities *and* β' *is the number of base quantities), there is a dimensionless main quantity* $\boldsymbol{u}$

that is raised to a given range of values $\boldsymbol{S}$*. Then, the absolute uncertainty* Δ_{pmm} *that contains* $\boldsymbol{u}$ *(for a given number of physical dimensional* quantities, z'' *is recorded in a model where* $\boldsymbol{\beta}''$ *of which are the number of chosen base quantities) can be determined from the relationship*:

$$\Delta_{\mathrm{pmm}} = \boldsymbol{S} \cdot [(z' - \boldsymbol{\beta}')/\boldsymbol{\mu}_{\mathrm{SI}} - (z'' - \boldsymbol{\beta}'')/(z' - \boldsymbol{\beta}')] \;, \qquad (33)$$

where $\varepsilon = \Delta_{\mathrm{pmm}}/\boldsymbol{S}$ is the comparative uncertainty [9].

Using equation (33), one can find the recommended uncertainty value with the theoretical analysis of the physical phenomena. Moreover, equation (33) can also inform a limit on the advisability of obtaining an increase of the measurement accuracy in conducting pilot studies or computer simulation. It is not a purely mathematical abstraction. Equation (33) has physical meaning. The $\boldsymbol{\mu}_{\mathrm{SI}}$-hypothesis lays down that, in nature, there is a fundamental limit to the accuracy of measuring any process, which cannot be surpassed by any improvement of instruments, measurement methods or the model's computerization. The value of this limit is much higher and stronger than what the Heisenberg uncertainty relation provides.

At its core, Δ_{pmm} is an *a priori* conceptual uncertainty that is inherent to any physical-mathematical model and is independent of the measurement process. The uncertainty determined by the proposed principle is not the result of measurement; it represents an intrinsic property of the model, and it is caused only by the number of selected quantities and the chosen *CoP*. Therefore, the overall uncertainty model including additional uncertainties associated with the structure of the model and its subsequent computerization will be much greater than Δ_{pmm}.

In fact, equation (33) can be regarded as the *conformity principle* (uncertainty relation) for the process of model development. No model can produce results that contradict the relation (33). That is, any change in the level of the detailed description of the observed object (z''-$\boldsymbol{\beta}''$; z'-$\boldsymbol{\beta}'$) causes a change in the minimum comparative uncertainty value $\Delta_{\mathrm{pmm}}/\boldsymbol{S}$ of the model of a specific *CoP* and in the achieved accuracy of each main quantity, characterizing the internal structure of the object. In other words, the *conformity principle* fundamentally establishes the accuracy limit (for a given class of phenomena) by simultaneously defining a pair of quantities, observed by a conscious researcher, particularly the absolute uncertainty in the measurement of the investigated quantity and the interval of its changes.

Thus, it follows that the fuzziness (inaccurate representation) of the object in the eyes of the researcher depends both on the chosen class of phenomena and on the number of quantities taken into account by the conscious observer; the latter directly depends on the knowledge, life experience and intuition of the researcher. Objectively, these factors, already stated above, render it possible to consider the choice of a quantity as a random process, with an equally probable account of a particular quantity.

It is to be noted that the relative and comparative uncertainties of the dimensional quantity U and the dimensionless quantity $\boldsymbol{u}$ are equal:

$$(\boldsymbol{\Delta U}/\boldsymbol{S}*) = (\boldsymbol{\Delta U}/\boldsymbol{a})/(\boldsymbol{S}*/\boldsymbol{a}) = (\boldsymbol{\Delta u}/\boldsymbol{S})$$
$$(\boldsymbol{r}/\boldsymbol{R}) = (\boldsymbol{\Delta U}/\boldsymbol{U})/(\boldsymbol{\Delta u}/\boldsymbol{u}) = (\boldsymbol{\Delta U}/\boldsymbol{U}) \cdot (\boldsymbol{a}/\boldsymbol{\Delta U}) \cdot (\boldsymbol{U}/\boldsymbol{a}) = 1 \quad (34)$$

where $\boldsymbol{S}$ and $\boldsymbol{\Delta u}$ are the dimensionless quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensionless quantity $\boldsymbol{u}$); $\boldsymbol{S^*}$ and ΔU are the dimensional quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensional quantity U); $\boldsymbol{a}$ is the dimensional scale parameter with the same dimension as that of U and $\boldsymbol{S^*}$; $\boldsymbol{r}$ is the relative uncertainty of the dimensional quantity U; and R is the relative uncertainty of the dimensionless quantity $\boldsymbol{u}$.

Equating the derivative of $\Delta_{\mathrm{pmm}}/\boldsymbol{S}$ (33) with respect to $\boldsymbol{z'}$-$\boldsymbol{\beta'}$ to zero, gives the following condition for achieving the minimum comparative uncertainty for a particular *CoP*:

$$(\boldsymbol{z'} - \boldsymbol{\beta'})^2/\boldsymbol{\mu}_{\mathrm{SI}} = (\boldsymbol{z''} - \boldsymbol{\beta''}). \quad (35)$$

By using (35), one can find the values for the lowest achievable comparative uncertainties for different CoP_{SI}; moreover, the values of the comparative uncertainties and the numbers of the chosen variables are *different* for each CoP_{SI}:

1. For mechanics processes ($CoP_{\mathrm{SI}} \equiv \boldsymbol{LMT}$), taking into account the aforementioned explanations and (19), the lowest comparative uncertainty ε_{LMT} can be reached at the following conditions:

$$(\boldsymbol{z'} - \boldsymbol{\beta'}) = (\boldsymbol{e}_l \cdot \boldsymbol{e}_m \cdot \boldsymbol{e}_t - 1)/2 - 3 = 91, \quad (36)$$

$$\boldsymbol{z''} - \boldsymbol{\beta''} = (\boldsymbol{z'} - \boldsymbol{\beta'})^2/\boldsymbol{\mu}_{\mathrm{SI}} = 0.2164 < 1 \quad (37)$$

where "-1" corresponds to the case when all the base quantity exponents are zero in formula (2); dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 3 corresponds to the three base quantities $\boldsymbol{L}$, $\boldsymbol{M}$, T.

According to (33) ε_{LMT} equals:

$$\varepsilon_{LMT} = (\Delta u/S)_{LMT} = 91/38{,}265 + 0.2164/91 = 0.0048. \quad (38)$$

In other words, according to (37), even one dimensionless main quantity does not allow one to reach the lowest comparative uncertainty. Therefore, in the frame of the suggested approach, the original comparative uncertainty cannot be realized using any mechanistic model ($CoP_{SI} \equiv \boldsymbol{LMT}$). Moreover, the greater the number of mechanical parameters, the greater the embedded uncertainty. In other words, the Cavendish method, for example, in the frame of the suggested approach is not recommended for measurements of the Newtonian gravitational constant.

Such statements appear to be highly controversial, and one might even say very unprofessional, not credible and far from current reality. However, as we shall see below, the proposed approach allows the obvious conclusions to be made consistent with practice.

2. For electromagnetism processes ($CoP_{SI} \equiv \boldsymbol{LMTI}$), taking into account (19), the lowest comparative uncertainty can be reached at the following conditions:

$$(z' - \beta') = \frac{e_l \cdot e_m \cdot e_t \cdot e_i - 1}{2} - 4 = \frac{7 \cdot 3 \cdot 9 \cdot 5 - 1}{2} - 4 = 468, \quad (39)$$

$$z'' - \beta'' = (z' - \beta')^2 / \mu_{SI} = 5.723873, \quad (40)$$

where "-1" corresponds to the case when all the base quantity exponents are zero in formula (19); dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 4 corresponds to the four base quantities $\boldsymbol{L}$, $\boldsymbol{M}$, T, $\boldsymbol{I}$.

Then, one can calculate the minimum achievable comparative uncertainty ε_{LMTI}:

$$\varepsilon_{LMTI} = (\Delta u/S)_{LMTI} = 468/38{,}265 + 5.723873/468 = 0.0245. \quad (41)$$

3. For combined heat and electromagnetism processes ($CoP_{SI} \equiv \boldsymbol{LMT\theta I}$), taking into account (19), the lowest comparative uncertainty $\varepsilon_{LMT\theta I}$ can be reached at the following conditions:

$$(z' - \beta') = (e_l \cdot e_m \cdot e_t \cdot e_\theta \cdot e_i - 1)/2 - 5 = 4{,}247, \quad (42)$$

$$z''\text{-}\beta'' = (z'\text{-}\beta')^2/\mu_{SI} = 4{,}247^2/38{,}265 \approx 471, \quad (43)$$

where "-1" corresponds to the case when all the base quantity exponents are zero in formula (19); dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 5 corresponds to the five base quantities $\boldsymbol{L}, \boldsymbol{M}, T, \boldsymbol{\Theta}, I$.

Then, one can calculate the minimum achievable comparative uncertainty $\varepsilon_{LMT\theta I}$:

$$\varepsilon_{LMT\theta I} = (\boldsymbol{\Delta u})_{\boldsymbol{LMT\theta I}} = 4{,}247/38{,}265 + 471/4{,}247 = 0.0222. \quad (44)$$

4. For heat processes ($CoP_{SI} \equiv \boldsymbol{LMT\theta}$), taking into account (19), the lowest comparative uncertainty $\varepsilon_{LMT\theta}$ can be reached at the following conditions:

$$(\boldsymbol{z'} - \boldsymbol{\beta'}) = (\boldsymbol{e}_l \cdot \boldsymbol{e}_m \cdot \boldsymbol{e}_t \cdot \boldsymbol{e}_\theta - 1)/2 - 4 = 846, \quad (45)$$

$$z''\text{-}\beta'' = (z'\text{-}\beta')^2/\mu_{SI} = 846^2/38{,}265 \approx 19, \quad (46)$$

where "-1" corresponds to the case when all the base quantity exponents are zero in formula (19); dividing by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length, L^{-1} is the run length; and 4 corresponds to the four base quantities $\boldsymbol{L}, \boldsymbol{M}, T, \boldsymbol{\Theta}$.

Then, one can calculate the minimum achievable comparative uncertainty $\varepsilon_{LMT\theta}$:

$$\varepsilon_{LMT\theta} = (\boldsymbol{\Delta u})_{\boldsymbol{LMT\theta}} = 846/38{,}265 + 19/846 = 0.0442. \quad (47)$$

Below is a summary table of comparative uncertainties and the optimal number of dimensionless criteria considered in the model for each class of phenomenon:

Table I. Comparative uncertainties and optimal number of dimensionless criteria.

CoP_{SI}	Comparative uncertainty	Optimal number of criteria
LMT	0.0048	0.2 < 1
LMTF	0.0146	≅ 2
LMTI	0.0245	≅ 6
LMTθ	0.0442	≅ 19

LMTIF	0.0738	≅ 52
LMTθF	0.1331	≅ 169
LMTθI	0.2220	≅ 471
LMTθFI	0.6665	≅ 4,249

The reader must keep in mind that the optimal number of criteria chosen in a model corresponds to the comparative uncertainty inherent in a specific *CoP*. Because the values of comparative uncertainties and the required number of the chosen quantities are completely independent and different for each class of a phenomena, the attained approach can become an arbitrary metric for comparing different models that describe the same recognized object. Let us now try to apply the aforementioned method for the analysis of the accuracy of the fundamental constants measurement and the determination of their minimum absolute, relative and comparative uncertainties.

2.4. Summary

In this chapter we have analyzed the existing literature related to applying information theory to different cases of physics in a perspective to calculate the uncertainty of the developed model of the recognized object. In addition, we described an important aspect of the system of base quantities (SBQ) and class of phenomena (*CoP*) by the usage of which there can be calculated a total number of dimensionless criteria inherent to the SI. Taking into account the achieved results and instruments of information theory, we have calculated the comparative uncertainty of any model describing a physical phenomenon. The construction of this uncertainty allows verification of the required number of dimensionless criteria of the different classes of phenomena.

We close by noting that while our use of the information approach will be limited, we will extensively use the comparative uncertainty for the below-mentioned applications. This will help us to evaluate the importance of different effects to bolster our physical understanding, as well as organize our numerical calculation, data collection, and design experiments.

This treatise is an attempt to show the inextricable link between the information approach and the concept of the comparative uncertainty with the developed physical-mathematical model by tracing it back to its physical base. We will clarify the terms used in the assessment of the comparative uncertainty of the model, explain why and how it works, and utilize the calculations to prove its usefulness, as well as discuss some of the challenges and issues that typically arise in its application. The procedure is the same for all applications, a specific case of which will be presented in a subsequent section.

Finally, one needs to compare the features of the application of the theory of measurements and the measure of the similarity of the model to the phenomenon under study on the basis of calculating the amount of information contained in the model. The trait that unites measurement theory and the information-oriented approach is that they study in physical theory only what can be observed directly, excluding such things as unobservable quantities.

One can understand the fundamental difference that exists between measurement theory and the information-based approach. To do this, we note that the measured variables—the simultaneous knowledge of which is necessary in the theory of measurements in order to strictly predict the measured value—are those whose number is precisely calculated by the information approach in the process of model formulation. Areas of their application are delineated by a threshold discrepancy. The uncertainty caused by the threshold discrepancy between the model and the object must be less than the permissible uncertainty in the measurement. If the predetermined measurement uncertainty exceeds this limit, then the main quantity cannot be measured with the required accuracy. This result shows that the model is inadequate. To implement the experiment, the model must be redefined.

Summarizing this idea, the information-oriented approach raises the following basic postulate, which can be called the *principle of limitation*: **the value of any physical quantity can be found only with a minimum absolute uncertainty, depending on the chosen class of the phenomenon and the number of quantities considered in the model**.

For classical physics, quantum mechanics and technical applications, this postulate is not trivial. Any theorist or experimenter, based on his experience, knowledge and intuition, determines the design of the test bench or theoretical model, thereby limiting (decreasing) the number of quantities reflecting the observed phenomenon, compared with the total number of quantities contained in the SBQ. Thus, this intangible disturbance of the system is primordial, although much smaller in comparison with the quantities considered by the theory of measurements and including inaccuracy of the initial data, boundary conditions, differential and integral equations with their subsequent computerization, etc. Therefore, any model significantly distorts the phenomenon under investigation.

Is this information approach, although very beautiful and very clear, somewhat arbitrary? Why are its concepts so contradictory to the usual notions of the scientific community (by this, it is meant the equally probable accounting of quantities by a conscious observer when choosing a model)? It turns out that equally probable interpretation is the only possible approach for today. This means that today it alone allows us to explain within the information-based approach reasonable boundaries of expedient accuracy before carrying out any theoretical or experimental research. None of the attempts made in any other direction have led to success: absolutely all the methods now developed are aimed at reducing the *a posteriori* uncertainties associated with the optimization of an already formulated model. So, we can say that the above fundamental postulate is justified by the fact that it is possible to build on its basis a theory consistent with all the experimental facts. Unlike the traditional theory of measurements, the new information-oriented approach provides a theoretically substantiated and reasonable estimate of the minimum achievable measurement absolute uncertainty of any developed model.

CHAPTER THREE

APPLICATIONS OF THE COMPARATIVE UNCERTAINTY METRIC FOR THE MEASUREMENTS OF THE FUNDAMENTAL PHYSICAL CONSTANTS

Your bait of falsehood takes this carp of truth
—W. Shakespeare, "Hamlet"

The physical laws express in mathematical form the quantitative relationships between different physical quantities. They are set on the basis of generalization of obtained experimental data, and reflect the objective laws existing in nature. So, fundamentally important is that all physical laws are an approximation to reality, since the construction of the theories is formulated by certain models of phenomena and processes. Outside these models, the laws do not work or work poorly. Therefore, the laws have certain limits of applicability. In other words, physical laws give good predictions in a specific area of experimental conditions, and the corresponding theory explains them. A more accurate or more correct theory has a wider range of applications. Scientists believe that physical laws, at least, enable us to predict results to an arbitrary accuracy. For example, classical mechanics, based on Newton's three laws and the law of universal gravitation, is valid only for the motion of bodies with velocities much lower than the speed of light. If the velocities of the bodies are comparable to the speed of light, predictions of classical mechanics are wrong. Special relativity has successfully coped with these problems. In fact, all physical theories are limited. The correspondence principle requires that a new theory with a broader area of applicability was limited to the old theory within the limits of its applicability. Turning to the theory of new concepts creates important preconditions for further development.

Among the various explanations for the admissibility of the possible limits of applicability of physical laws, the following reasons are the most used. The first is the assumption that there is the limited detailing of phenomena, for which the Heisenberg inequality gives a quantitative expression. The second reason is that the restrictions are determined by the real nature of the macroscopic instrument or measuring system. Most devices are presented, finally, as a solid. In principle, it could be argued that any device has an educational effect only within its field of reality. Thus, the research results should be expressed in terms of the macroscopic. In other words, concepts and images can be identified and are associated only with the ordinary macroscopic views. The final argument is the point of view of the principle of the electromagnetic nature of all modem means of measurement and their role in determining the boundaries of experimental and measurement capabilities, and harmonization of the data with the theoretical postulates. Thus, there are possible explanations, but any quantitative approaches to numerically estimate the difference between a model (formulated physical law) and existing reality have not been proposed to date.

Since Newton's law of gravity in all the basic equations of the physical-mathematical theories together with the quantities, there are isolated values of physical quantities called the fundamental physical constants. These constants are parameters in the equations that describe physical phenomena and have the units that are necessary for dimensional consistency. For example, in Newton's theory it is a gravitational constant G, in special relativity it is the speed of light in a vacuum c, in general relativity they are c and G, in the quantum (non-relativistic) mechanics it is the Planck constant h, and in quantum electrodynamics they are c and h.

Regarding the fundamental physical constants, it should be noted that their values are the accuracy of our knowledge of the fundamental properties of matter. On the one hand, very often the verification of the physical theories is determined by the accuracy of a measured physical constant. On the other hand, the firmly established experimental data are put into the foundations of the new physical theories.

In the study of physical constants it is noteworthy that they are measured with very high accuracy, which is steadily growing and is itself a testament to the development and perfection of techniques of physical experimentation. Precision research on the measurement and specification of the values of physical constants and meticulous work on harmonization of data obtained by different methods and different groups of researchers are both currently being carried out. However, there is an urgent need to further improve the accuracy of measurement of fundamental physical constants. This is explained by the desire to improve the axiomatic basis of the SI.

The CODATA (Committee on Data of the International Science Council) recommended values and units for the constants [71] are based on the conventions of the current SI, and any modifications of those conventions will have consequences for the units. One consequence of this is that mathematics provides no information on how to incorporate units into the analysis of physical phenomena. One role of the SI is to provide a systematic framework for including units in equations that describe physical phenomena.

The desire to reduce the value of uncertainty in the measurement of fundamental physical constants is due to several reasons. Firstly, achieving the accurate quantitative description of the physical universe depends on the numerical values of the constants that appear in the theories. Secondly, the overall consistency and validity of the basic theories of physics may be proved by the careful examination of the numerical values of these constants as determined from different experiments in different fields of physics.

To assess the accuracy achieved in the measurements of fundamental physical constants, the concept of relative uncertainty is used. It should be mentioned that this method for identifying the measurement accuracy does not indicate the direction in which one can find the true value of a fundamental physical constant. In addition, it involves an element of subjective judgment [72]. For this reason, we offer the information-oriented novel method of assessing the credibility of the obtained measurements results.

In 2018, it was expected that there would be a redefinition of the SI based on specified values of certain fundamental constants [73], but it has not occurred yet. This constitutes a dramatic change with one of the consequences being that there will no longer be a clear distinction between base units and derived units [74]. In view of this change, it is timely to revise the units in the SI and their definitions. One of the goals is to ensure that all such units are consistent; that is, they consist of a coherent system of units. In the current SI, various quantities are designated as dimensionless. This means that they are considered as not having a unit or have what is called a coherent derived unit "one". In some cases, this designation leads to ambiguous results for these quantities. In this book, we will look at units in SI that are considered dimensionless and other units that are not currently included in the SI which can be added to bring it into closer correspondence with widespread scientific use.

One of the seminal works exclusive to the problem of interpretation of measurement accuracy, as well as methods to improve the uncertainty assessment in the measurements of fundamental constants, is [72]. The authors noted that precise estimates of the fundamental constants of physics are subject to uncertainty from various sources. Reliable estimates of uncertainty are required (a) to compare the accuracy of different measurements of the same quantity, (b) to evaluate the accuracy of other quantities derived from them, (c) to help in defining and revising models, and most importantly, (d) to assess compliance of the physical theory with current best measurements. In order to prove their conclusions, the authors used the Birge ratio [75] that assesses the compatibility of a set of measurements by comparing the variability among experiments to the reported uncertainties of the light velocity, the fine structure constant and the gravitational constant.

Two quite different kinds of fundamental physical constants uncertainty must be considered: first, the relevant quantities and functional relationships between them are known, but the values of key coefficients are not known; second, when the developer is not sure what all the relevant quantities are, or what the functional relationships are among them. Often, uncertainty about model form is more important than uncertainty about values of coefficients [76].

Developers often have difficulty evaluating or even estimating the model discrepancy from the fundamental physical constants under realistic conditions. Many of the model structures do not quantify uncertainty resulting from factors such as developer knowledge, intuition, experience and environmental properties. In addition, without at least some quantification, qualitative descriptions of uncertainty convey little useful information.

Actually, the act of fundamental physical constant measurement itself already implies an existence of the formulated physical-mathematical model describing the phenomenon under investigation. At the same time, most of the research focuses on data analysis and a calculation of the fundamental constant uncertainty value *after* formulating the mathematical model. But the unavoidable uncertainty existing before the start of the experiment or computer simulation and caused only by the finite number of quantities recorded in the mathematical model of the fundamental physical constants is generally ignored. Of course, in addition to this uncertainty, the overall uncertainty of the fundamental physical constant measurement includes the *a posteriori* uncertainties related to the internal structure of the model, its subsequent computerization and the testing equipment characteristics: inaccurate input data, inaccurate physical assumptions, the limited accuracy of the solution of integral-differential equations, etc. Detailed definitions of many different sources of uncertainty are given in [1].

Here, we investigate the information cost of measurements in the modeling. Starting with the frame set in Chapter 2.3, we introduce the metric called the comparative uncertainty of the measurement, which is implemented in a real experiment. Thanks to the introduction of this quantitative tool, we obtain the lower limit of the achieved absolute and relative uncertainties associated with the act of observation, which is characteristic of and inherent in measurement. The flexibility of our experimental setup allows us to calculate the amount of information retrieved from the system. This method also allows us to determine how much the developed model (before carrying out the experiment or computer calculations) can extract information in order to achieve the lowest threshold inconsistency in comparison with the object under study.

The true and precise values of the most fundamental physical constants are not known at the moment. Therefore, the CODATA task group on fundamental constants (TGFC) periodically reviews and declares recommended values of the fundamental physical constants and their measurement relative uncertainty. It should be mentioned that the concept of relative uncertainty was used when considering the accuracy of the achieved results (absolute value and absolute uncertainty of the separate quantities and criteria) during the measurement process in different applications. However, this method for identifying the measurement accuracy does not indicate the direction of deviation from the true value of the main quantity. In addition, it involves an element of subjective judgment [74]. That is why, for the purposes of this book, along with a relative uncertainty, this study recommends a comparative uncertainty for analyzing published results.

If the range of observation S is not defined, the information obtained during the observation/measurement cannot be determined, and the entropic price becomes infinitely large [9].

In the framework of the information-oriented approach it seems that the theoretical limit of the absolute and relative uncertainties depends on the empirical value, i.e., possible interval of placing (the observed range of variations) S of the measured physical constant. In other words, the results will be completely different if a larger interval of changes is considered in the measured fundamental physical constant. However, if S is not declared, the information obtained in the measurement cannot be determined. Any specific measurement requires a certain (finite) *a priori* information about the components of the measurement and interval of observation of the measured quantity. These requirements are so universal that it acts as a postulate of metrology [77]. This, the observed range of variations, depends on the knowledge of the developer prior to undertaking the study. "If nothing is known about the system studied, then S is determined by the limits of the measuring devices used." [9] That is why, taking into account the Brillouin suggestions, there are two options of applying the conformity principle to analyze the measurement data of the fundamental physical constants.

The first, this principle dictates, is analyzing the data of the magnitude of the achievable relative uncertainty at the moment, taking into account the *latest* results of measurements. The extended range of changes in the quantity under study S indicates an imperfection of the measuring devices, which leads to a large value of the relative uncertainty. The development of measuring technology, an increase in the accuracy of measuring instruments and the improvement in existing and newly created measurement methods together lead to an increase in knowledge of the object under study and, consequently, the magnitude of the achievable relative uncertainty decreases. However, this process is not infinite and is limited by the conformity principle. The reader should bear in mind that this conformity principle is not a shortcoming of the measurement equipment or engineering device, but of the way the human brain works. When predicting behavior of any physical process, physicists are in fact predicting the perceivable output of instrumentation. It is true that, according to the μ-hypothesis, observation is not a measurement, but a process that creates a unique physical world with respect to each particular observer. Thus, in this case, the range of observation (possible interval of placing) of the fundamental physical constant S is chosen as the difference between the maximum and minimum values of the physical constant measured by different scientific groups during a certain period of recent years. Only in the presence of the results of various experiments one can speak about the possible appearance of a measured value in a certain range. Thus, using the smallest attainable comparative uncertainty inherent in the selected class of phenomena during measurement of the fundamental constant, it is possible to calculate the recommended minimum relative uncertainty that is compared with the relative uncertainty of each published study. In what follows, this method is denoted as *IARU* and includes the following steps:

1. From the published data of each experiment, the value z, relative uncertainty r_z and standard uncertainty u_z (possible interval of placing) of the fundamental physical constant are chosen;
2. The experimental absolute uncertainty Δ_z is calculated by multiplying the fundamental physical constant value z and its relative uncertainty r_z attained during the experiment,
   $\Delta_z = z \cdot r_z$;
3. The maximum z_{max} and minimum z_{min} values of the measured physical constant are selected from the list of measured values z_i of

the fundamental physical constant mentioned in different studies;

4. As a possible interval for placing the observed fundamental constant S_z, the difference between the maximum and minimum values is calculated, $S_z = z_{max} - z_{min}$;
5. The selected comparative uncertainty ε_T (Table I) inherent in the model describing the measurement of the fundamental constant is multiplied by the possible interval of placement of the observed fundamental constant S_z in order to obtain the absolute experimental uncertainty value Δ_{IARU} in accordance with the *IARU*, $\Delta_{IARU} = \varepsilon_T \cdot S_z$;
6. To calculate the relative uncertainty r_{IARU} in accordance with the *IARU*, this absolute uncertainty Δ_{IARU} is divided by the arithmetic mean of the selected maximum and minimum values, $r_{IARU} = \Delta_{IARU} / ((z_{max} + z_{min})/2)$;
7. The relative uncertainty obtained r_{IARU} is compared with the experimental relative uncertainties r_i achieved in various studies;
8. According to *IARU*, a comparative experimental uncertainty of each study ε_{IARUi} is calculated by dividing the experimental absolute uncertainty of each study Δ_z by the difference between the maximum and minimum values of the measured fundamental constant S_z, $\varepsilon_{IARUi} = \Delta_z / S_z$. These calculated comparative uncertainties are also compared with the selected comparative uncertainty ε_T (Table I).

In the second option of applying the conformity principle to analyze the measurement data of the fundamental physical constants, *S* is determined by the limits of the measuring devices used [9]. This means that as the observation interval in which the expected true value of the measured fundamental physical constant is located, a *standard uncertainty* is selected when measuring the physical constant in each particular experiment. Compared with various fields of technology, experimental physics is better for the fact that in all the research the experimenters introduce the output data of the measurement with uncertainty bars. At the same time, it should be remembered that the standard uncertainty of a particular measurement is subjective, because the conscious observer may not take into account various uncertainties. The experimenters calculate the standard uncertainty, taking into account all measured uncertainties that they have observed. Then, one calculates the ratio between the absolute uncertainty reached in an experiment and the standard uncertainty, acting as a possible interval for allocating a fundamental physical constant. So, in the framework of the information approach, the comparative uncertainties achieved in the studies are calculated, which in turn are compared with the theoretically achievable comparative uncertainty inherent in the chosen class of phenomena.

Standard uncertainty can also be calculated for quantities that are not normally distributed. Transformation of different types of uncertainty sources into standard uncertainty is very important. In what follows, this method is denoted as *IACU* and includes the following steps:

1. From the published data of each experiment, the value z, relative uncertainty r_z and standard uncertainty u_z (possible interval of placing) of the fundamental physical constant are chosen;
2. The experimental absolute uncertainty Δ_z is calculated by multiplying the fundamental physical constant value z and its relative uncertainty r_z attained during the experiment, $\Delta_z = z \cdot r_z$;
3. The achieved experimental comparative uncertainty of each published research ε_{IACUi} is calculated by dividing the experimental absolute uncertainty Δ_z by the standard uncertainty u_z, $\varepsilon_{IACUi} = \Delta_z / u_z$;
4. The experimental calculated comparative uncertainty ε_{IACUi} is compared with the selected comparative uncertainty ε_T (Table I) inherent in the model, which describes the measurement of the fundamental constant.

We will apply *IARU* and *IACU* for analyzing data of the measurement of the different fundamental physical constants.

3.1. Boltzmann constant

The analysis of the Boltzmann constant k_b plays an increasingly important role in our physics today to ensure the correct contribution to the next CODATA value and to the new definition of the Kelvin. This task is more difficult and crucial when its true-target value is not known. This is the case for any methodology intending to look at the problem from another possible view and which, maybe, has different constraints and needs special discussion.

A detailed analysis of the measurements of Boltzmann's constant taken since 1973 is available in [78, 79]. The more recent of these measurements, taken during 2011–2018 [78–87], were analyzed for this study. The data are summarized in Table II. The noted scientific articles in most cases belong to $CoP_{SI} \equiv$ ***LMTΘF*** [80–84, 86], and some to $CoP_{SI} \equiv$ ***LMTΘI*** [85, 87]. Although the authors of the research studies cited in these papers mentioned all the possible sources of uncertainty, the values of absolute and relative uncertainties can still differ by more than two times. And a similar situation exists in the spread of the values of comparative uncertainty.

Table II. Boltzmann's constant determinations and relative and comparative uncertainties achieved.

Year	CoP	Boltzmann's constant	Achieved relative uncertainty	Absolute uncertainty	k_b possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
		$k\cdot 10^{23}$	$r_k\cdot 10^{6}$	$\Delta_k\cdot 10^{29}$	$u_k\cdot 10^{29}$	$\varepsilon_k' = \Delta_k/u_k$	$\varepsilon_k'' = \Delta_k/S_k$	
		$m^2\,kg/(s^2\,K)$		$m^2\,kg/(s^2\,K)$	$m^2\,kg/(s^2\,K)$	*IACU*	*IARU*	
2011	***LMTΘF***	**1.38065170**	12.0	1.66	3.7	4.4778	4.7337	[80]
2015	***LMTΘF***	1.38064871	2.0	2.76	2.7	1.0227	0.7889	[81]
2015	***LMTΘF***	1.38065080	1.1	1.51	2.9	0.5237	0.4339	[82]
2017	***LMTΘF***	1.38064879	6.0	8.28	1.6	0.5177	0.2367	[83]
2017	***LMTΘF***	1.38064861	0.7	0.97	1.8	0.5370	0.2761	[84]
2017	***LMTΘI***	**1.38064820**	1.9	2.62	5.3	0.4949	0.7495	[85]
2017	***LMTΘF***	1.38064843	2.0	2.76	5.5	0.5020	0.7889	[86]
2017	***LMTΘI***	1.38064974	2.7	3.73	6.5	0.5735	1.0651	[87]
2018		1.38064904	3.7	0.51	1.0	0.5108	0.1460	[78]
2018		1.38064900	3.7	0.51	1.0	0.5108	0.1460	[79]

* Data are introduced in [78, 88]

Following the method *IARU*, one can argue about the order of the desired value of the relative uncertainty of $CoP_{SI} \equiv \boldsymbol{LMT\Theta F}$, which is usually used for obtaining measurements of Boltzmann's constant. An estimated observation interval of k is chosen as the difference in its values obtained from the experimental results of two projects: $k_{max} = 1.3806517 \cdot 10^{-23}$ m^2·kg·s^{-2}·K^{-1} [80] and $k_{min} = 1.3806482 \cdot 10^{-23}$ m^2·kg·s^{-2}·K^{-1} [85]. In this case, the possible observed range S_k of k placing is equal to:

$$S_k = k_{max} - k_{min} = 3.5 \cdot 10^{-29} \mathrm{m}^2 \cdot \mathrm{kg}/(\mathrm{s}^2 \cdot K). \qquad (48)$$

For this purpose, taking into account data of Table I, one can arrive at the lowest comparative uncertainty $\varepsilon_{LMT\theta F}$ using the following conditions:

$$(\boldsymbol{z}' - \boldsymbol{\beta}')_{\boldsymbol{LMT\theta F}} = \left(e_l \cdot e_m \cdot e_t \cdot e_\theta \cdot e_f - 1\right)/2 - 5 = 2{,}546, \qquad (49)$$

$$(\boldsymbol{z}'' - \boldsymbol{\beta}'')_{\boldsymbol{LMT\theta F}} = (\boldsymbol{z}' - \boldsymbol{\beta}')^2/\boldsymbol{\mu}_{SI} = 2{,}546^2/38{,}265 \approx 169, \qquad (50)$$

where "-1" corresponds to the case where all the base quantities exponents are zero in formula (1); 5 corresponds to the five base quantities ***L***, ***M***, *T*, $\boldsymbol{\Theta}$ and *F*; and division by 2 indicates that there are direct and inverse quantities, e.g., L^1 is the length and L^{-1} is the run length. The object can be judged based on the knowledge of only one of its symmetrical parts, while the other parts that structurally duplicate this one may be regarded as information empty. Therefore, the number of options of dimensions may be reduced by 2 times.

According to (49) and (50):

$$\varepsilon_{LMT\theta F} = (\Delta u/S)_{LMT\theta F} = 0.1331. \qquad (51)$$

Then, the lowest possible absolute uncertainty for $CoP_{SI} \equiv \boldsymbol{LMT\Theta F}$ is given by the following:

$$\boldsymbol{\Delta_{LMT\theta F}} = \boldsymbol{\varepsilon_{LMT\theta F}} \cdot \boldsymbol{S_k} = 4.66 \cdot 10^{-30} \mathrm{m}^2 \cdot \mathrm{kg}/(\mathrm{s}^2 \cdot K). \qquad (52)$$

In this case, the lowest possible relative uncertainty $(r_{min})_{LMT\theta F}$ for $CoP_{SI} \equiv \boldsymbol{LMT\Theta F}$ is the following:

$$r_{LMT\theta F} = \Delta_{LMT\theta F}/\left((k_{max} + k_{min})/2\right) = 3.4 \cdot 10^{-7}. \qquad (53)$$

This value agrees well with the recommendation of $3.7 \cdot 10^{-7}$ cited in [78, 79].

Guided by the *IACU* method, one can calculate the achieved comparative uncertainty in each experiment (Table II). There is not *significant difference* between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMT\theta F} = 0.1331$ and the experimental magnitudes achieved during measuring *k*; for example, 0.1460 [79]. The difference may be explained by the fact that experimenters take into account a very contrasting number of quantities in comparison to the recommendations (see Table I). This means that further future improvements of test benches can be recommended. That is why the information approach can be used as an additional tool for the new definition of the Kelvin and for revising the SI.

3.2. Planck's constant

Planck's constant ***h*** is of great importance in modem physics. This is explained by the following reasons [89]:

a. It defines the quanta (minimum amount) for the energy of light and therefore also the energies of electrons in atoms. The existence of a smallest unit of light energy is one of the foundations of quantum mechanics.
b. It is a factor in the Uncertainty Principle, discovered by Werner Heisenberg in 1927;
c. Planck's constant has enabled the construction of transistors, integrated circuits, and chips that have revolutionized our lives.
d. For over a century, the weight of a kilogram has been determined by a physical object, but that could change in 2019 under a new proposal that would base it on Planck's constant.

Therefore, a huge amount of research has been dedicated to the Planck constant measurement [90]. The most summarized data published in scientific journals in recent years about the magnitude of the standard uncertainty of the Planck constant and the Boltzmann constant measurements are presented in [88].

The measurements taken during 2011–2018 were analyzed for this study. The data are summarized in Table III [78, 91–102]. At the time of writing, it has been demonstrated that two methods have the capability of realizing the kilogram according to its future definition with relative standard uncertainties of a few parts in 10^8: the Kibble balance ($CoP_{SI} \equiv$

LMTI) and the x-ray crystal density (XRCD) method ($CoP_{SI} \equiv LMT\Theta F$). From Table III it follows that the values of absolute and relative uncertainties differ by more than two times, despite the fact that the authors of the presented studies calculated all possible—from their point of view—sources of uncertainty. The values of absolute and relative uncertainties may still differ by more than twice. A similar situation exists in the range of values of comparative uncertainty.

Following the *IARU* method, one can argue about the order of the desired value of the relative uncertainty $(r_{min})_{LMTI}$. For this purpose, we take into account the following data: $(\varepsilon_{min})_{LMTI}$ = 0.0245 (Table I); and an estimated observation interval of h is chosen as the difference in its values obtained from the experimental results of two projects: h_{max} = $6.626070406 \cdot 10^{-34}$ m^2·kg·s^{-2} [98] and h_{min} = $6.626069216 \cdot 10^{-34}$ m^2·kg·s^{-2} [99]. In this case, the possible observed range S_h of h placement is equal to:

$$S_h = h_{\max} - h_{\min} = 1.1898 \cdot 10^{-40} \mathrm{m}^2 \cdot \mathrm{kg} \cdot s^{-2}. \qquad (54)$$

Then, the lowest possible absolute uncertainty for $CoP_{SI} \equiv LMTI$ equals:

$$\Delta_{LMTI} = \varepsilon_{LMTI} \cdot S_h = 2.9103 \cdot 10^{-42}\ \mathrm{m}^2 \cdot \mathrm{kg} \cdot s^{-2}. \qquad (55)$$

In this case, taking into account (55), the lowest relative uncertainty $(r_{min})_{LMTI}$ for $CoP_{SI} \equiv LMTI$ is the following:

$$r_{LMTI} = \Delta_{LMTI} / ((h_{\max} + h_{\min}) / 2) = 4.4 \cdot 10^{-9}. \qquad (56)$$

This value corresponds to the recommendation mentioned in [78] of $1.0 \cdot 10^{-8}$ and should be satisfactory to the existing mass standards community. Of course, the choice of the values of Δ_{LMTI} and $(h_{max} - h_{min})/2$ is controversial because of their apparent subjectivity. With all of this, the capability for prediction of the Planck constant value by usage of the comparative uncertainty allows the improvement of our fundamental comprehension of complex phenomena, as well as the application of this comprehension to the solution of specific problems.

Table III. Planck's constant determinations and relative and comparative uncertainties achieved.

Year	*CoP*	Planck's constant, $h\cdot 10^{34}$	Achieved relative uncertainty	Absolute uncertainty	k_b possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
				$\Delta_h\cdot 10^{41}$	$u_h\cdot 10^{41}$	$\varepsilon_h' = \Delta_h/u_h$	$\varepsilon_h'' = \Delta_h/S_h$	
			$r_h\cdot 10^8$	$m^2\ kg/s^2$	$m^2\ kg/s^2$	*IACU*	*IARU*	
2011	***LMTθF***	6.626070082	3.0	1.9878	4.0	0.4970	0.1671	[91]
2011	***LMTθF***	6.626069942	3.0	1.9878	4.1	0.4848	0.1671	[92]
2014	***LMTI***	6.626070341	1.4	0.9542	2.4	0.3976	0.0802	[93]
2015	***LMTθF***	6.626070221	2.0	1.3252	2.6	0.5097	0.1114	[94]
2015	***LMTI***	6.626069364	5.7	3.7769	7.7	0.4905	0.3174	[95]
2016	***LMTI***	6.626069832	3.4	2.2529	4.4	0.5120	0.1894	[96]
2017	***LMTθF***	6.626070134	9.1	6.0297	1.2	0.5025	0.0507	[97]
2017	***LMTθF***	**6.626070406**	1.2	7.9513	1.6	0.4970	0.0668	[98]
2017	***LMTI***	**6.626069216**	2.4	1.5903	3.1	0.5130	0.1337	[99]
2017	***LMTI***	6.626069935	1.3	8.6139	1.8	0.4786	0.0724	[100]
2017	***LMTθF***	6.626070132	2.4	1.5903	3.2	0.4970	0.1337	[101]
2017	***LMTI***	6.626070404	5.7	3.7769	7.7	0.4905	0.3174	[102]
2018	***LMTI***	6.626070151	1.0	6.6261	1.4	0.4733	0.0557	[78]

* Data are introduced in [103]

It is obvious that such findings will cause a negative reaction on the part of the scientific community. At the same time, an additional view of the existing problem will, most likely, help to understand the existing situation and identify concrete ways for its solution. Reducing the comparative uncertainty of the Planck constant measurement obtained from different experiments to the value of 0.0245 will serve as a convincing argument for professionals involved in the evolution of the SI.

It is seen from the data given in Table III that there was a dramatic improvement in the accuracy of the measurement of Planck's constant during the last decade. It is authorized as true when based on a calculation of the relative uncertainty. Judging the data by the comparative uncertainty following *IACU*, one can see that the measurement accuracy has significantly changed too. There is not *significant difference* between the comparative uncertainty calculated according to the information-oriented approach ε_{LMTI} = 0.0245 and the experimental magnitudes achieved during measuring *h*; for example, 0.0557 [78]. At the same time, it must be mentioned that the exactness of Planck's constant, as with other fundamental physical constants, most likely cannot be infinite. Therefore, the development of a larger number of designs and improvement of the various experimental facilities for the measurement of Planck's constant is an absolute must [104]. The requirements of accuracy and methodological diversity of the prerequisites for the redefinition of a unit of mass and for its realization in terms of a fundamental constant nature must be continued.

The results of the definitions of the Planck constant, obtained with various measurements from recent years, are very consistent. This issue may become more significant for future comparisons when the uncertainty of the implementation experiments will become less. Current research and development, as well as improvements in measurement methods, will probably reduce some components of uncertainty in the future and, therefore, will steadily increase the accuracy of measurement of the Planck constant.

3.3. Avogadro constant

The Avogadro constant N_A is the physical constant that connects microscopic and macroscopic quantities, and is indispensable especially in the field of chemistry. In addition, the Avogadro constant is closely related to the fundamental physical constants, namely the electron relative atomic mass, fine structure constant, Rydberg constant, and Planck constant.

During the period 2011 to 2017, several scientific publications were analyzed, based on the available relative and comparative uncertainty values

[92, 94, 97, 100–102, 105–108], and the results are summarized in Table IV. In order to apply a stated approach (*IARU*), an estimated observation interval of the Avogadro constant is chosen as the difference in its values obtained from the experimental results of two projects: $N_{Amin} = 6.0221405235 \cdot 10^{23}$ mol^{-1} [102] and $N_{Amax} = 6.0221414834 \cdot 10^{23}$ mol^{-1} [107].

Then, the dimensional possible observed range S_N of N_A variations is given by the following:

$$\boldsymbol{S}_N = N_{\mathrm{Amax}} - N_{\mathrm{Amin}} = 9.6 \cdot 10^{16} (\mathrm{mol}^{-1}). \tag{57}$$

The choice by the author of (N_{Amax} - N_{Amin}) seems subjective and arbitrary. However, we need to emphasize that only in the presence of the results of various experiments, one can speak about the possible occurrence of a measured quantity in a certain range.

The true and precise value of the Avogadro constant is not known at the moment. Therefore, the CODATA task group on fundamental constants (TGFC) periodically reviews and declares its recommended value of the Avogadro constant and its relative uncertainty.

Applying the present approach, one can argue about the order of the desired value for the relative uncertainty $(r_{min})_{\boldsymbol{LMTF}}$ for $CoP_{SI} \equiv \boldsymbol{LMTF}$. For this purpose, the following values are considered: $(\varepsilon_{min})_{\boldsymbol{LMTF}} = 0.0146$ (Table I) and $S_N = 6.9 \cdot 10^{17}$ mol^{-1} (57). Then, the lowest possible absolute uncertainty is given by the following:

$$(\Delta_{\mathrm{min}})_{\boldsymbol{LMTF}} = (\varepsilon_{\mathrm{min}})_{\boldsymbol{LMTF}} \cdot \boldsymbol{S}_N = 2.348 \cdot 10^{15} (\mathrm{mol}^{-1}). \tag{58}$$

In this case, the lowest possible relative uncertainty $(r_{min})_{LMTF}$ is as follows:

$$(r_{\mathrm{min}})_{\boldsymbol{LMTF}} = (\Delta_{\mathrm{min}})_{\boldsymbol{LMTF}} / \big((N_{\mathrm{Amax}} + N_{\mathrm{Amin}})/2\big) = 3.9 \cdot 10^{-9}. \tag{59}$$

This value (59) agrees well with the recommendation mentioned in [97] of $9.1 \cdot 10^{-9}$, and can be particularly relevant in the run-up to the adoption of new definitions for SI units.

Table IV. Avogadro constant determinations and relative and comparative uncertainties achieved.

Year	*CoP*	Avogadro's constant	Achieved relative uncertainty	Absolute uncertainty	N_A possible interval of placement,	Reached comparative uncertainty,	Calculated comparative uncertainty,	Ref.
		$N_A \cdot 10^{-23}$	$r_N \cdot 10^{8}$	$\Delta_N \cdot 10^{-15}$	$u_N \cdot 10^{-16}$	$\varepsilon_N' = \Delta_N / u_N$	$\varepsilon_N'' = \Delta_N / S_N$	
		m^2 kg/(s^2 K)		m^2 kg/(s^2K)	m^2 kg/(s^2K)	*IACU*	*IARU*	
2011	*LMTF*	6.0221407818	3.0	18	3.7	0.4883	1.0000	[105]
2011	*LMTF*	6.0221409918	3.0	18	3.6	0.5018	1.0000	[92]
2015	*LMTF*	6.0221407612	2.4	14	2.4	0.6022	0.8000	[94]
2016		6.0221408577	1.2	7.2	1.5	0.4818	0.4000	[106]
2016	*LMTI*	**6.0221414834**	5.6	34	5.4	0.6245	1.8667	[107]
2017	*LMTF*	6.0221408415	2.4	14	2.9	0.4984	0.8000	[101]
2017	*LMTF*	6.0221405267	1.2	7.2	1.4	0.5162	0.4000	[108]
2017	*LMTI*	6.0221409538	1.3	7.8	1.7	0.4605	0.4333	[100]
2017	*LMTI*	**6.0221405235**	5.7	34	6.9	0.4975	1.9000	[102]
2017	*LMTI*	6.0221407726	0.9	5.5	1.1	0.4982	0.3033	[97]

It seems that the theoretical limit of the absolute and relative uncertainties depends on the empirical value, i.e., on the observed range of variations in S. In other words, the results will be completely different if a larger interval of changes of the Avogadro constant is considered. For example, in the case $N_{Pmax} = 7.15 \cdot 10^{23}$ mol^{-1} [109], Perrin's experiments belong to $CoP_{SI} \equiv \boldsymbol{LMT\theta}$. Then, taking into account that $N_{amin} = 6.0221405235 \cdot 10^{23}$ mol^{-1} [102], $(\varepsilon_{min})_{LMT\theta} = 0.0446$ (Table I), the lowest possible absolute uncertainty $(\Delta_{min})_{PLMT\theta}$ and relative uncertainty $(r_{min})_{PLMT\theta}$ would be equal to the following:

$$\boldsymbol{S}_{PN} = N_{\mathrm{Pmax}} - N_{\mathrm{Amin}} = 1.1278 \cdot 10^{23}\mathrm{mol}^{-1}. \tag{60}$$

$$(\boldsymbol{\Delta}_{\mathrm{min}})_{\boldsymbol{PLMT\theta}} = (\boldsymbol{\varepsilon}_{\mathrm{min}})_{LMT\theta} \cdot S_{PN} = 5.03 \cdot 10^{21}\ \mathrm{mol}^{-1}. \tag{61}$$

$$(r_{\mathrm{min}})_{\boldsymbol{PLMT\theta}} = (\Delta_{\mathrm{min}})_{\boldsymbol{PLMT\theta}} / \big((N_{\mathrm{Pmax}} + N_{\mathrm{Amin}})/2\big) = 7.6 \cdot 10^{-3}. \tag{62}$$

Thus, within the framework of the proposed information approach, and with 100-year-old imperfect measurement equipment, the achievable relative uncertainty is $7.6 \cdot 10^{-3}$ (62), which is much higher than $3.9 \cdot 10^{-9}$ (59) that can be achieved by the accuracy of modem measuring instruments and the knowledge about the true-target magnitude of the Avogadro constant.

It is seen from the data given in Table IV that there was an impressive improvement in the accuracy of measurement of the Avogadro constant during the last decade. It is authorized as true when based on a calculation of the relative uncertainty (*IARU*). Judging the data by the comparative uncertainty following *IACU*, one can see that the measurement accuracy has significantly changed too. Unfortunately, there is *significant difference* between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMTF} = 0.0146$ and the experimental magnitudes achieved during measurement of N_A; for example, 0.3033 [97]. The difference may be explained by the fact that experimenters take into account a very contrasting number of quantities in comparison to the recommendations (see Table I). This means that further future improvements of test benches can be recommended.

With all of this, the ability to predict the relative uncertainty of the Avogadro constant by using the comparative uncertainty allows the improvement of the fundamental comprehension of complex phenomena, as well as the application of this comprehension to solving specific problems.

3.4. Gravitational constant

The importance of high precision of the gravitational constant G not only stems from pure metrological interest; it has a key role in different theories including gravitation, cosmology, particle physics and astrophysics. At the same time, the current spread of values of G considered as a fundamental constant of nature, at present, is very poor compared with other physical fundamental constants. The measurement of G is one of the most difficult of all experiments. The constant G remains the basic physical constant with the highest measurement relative uncertainty. At present, active efforts are being made to disclose the sources of large discrepancies in the latest measurements. There is a hope that the search for a more accurate G will not stop, because science has not yet developed a solution to the riddle of why the measurements of G do not converge. Changes to G are generally believed to be a result of measurement discrepancies because it is very difficult to measure, in part due to the fact that gravity is much weaker than the other fundamental forces [110].

When measuring G, it is desirable to identify and assess all relevant quantities chosen by the conscious observer, based on his knowledge, experience and intuition. There can be pitfalls, such as objective and subjective uncertainties of the physical-mathematical model and the methods of calculation associated with it. Many inferences and assumptions can be justified on the basis of experience (and sometimes uncertainties can be estimated), but the degree to which our assumptions correspond to the study of G is never established. The present analysis of data of G variations is associated with both the latest observations and the impending reform in fundamental metrology: the introduction of new definitions of basic SI units.

The measurements taken during 2000–2014 were analyzed for this study. The data are summarized in Table V [111–121].

Table V. Gravitational constant determinations and relative and comparative uncertainties achieved.

Year	Gravitational constant	Achieved relative uncertainty	Absolute uncertainty	G possible interval of placing*	Reached comparative uncertainty	Calculated comparative uncertainty	Ref.
	$G \cdot 10^{11}$	$r_G \cdot 10^5$	$\Delta_G \cdot 10^{16}$	$u_G \cdot 10^{15}$	$\varepsilon_G' = \Delta_G / u_G$	$\varepsilon_G'' = \Delta_G / S_G$	
	$m^3\ kg^{-1}\ s^{-2}$		$m^3\ kg^{-1}\ s^{-2}$	$m^3\ kg^{-1}\ s^{-2}$	*IACU*	*IARU*	
2000	6.6742559	1.4	0.934396	1.8	0.5191	0.0254	[111]
2001	**6.6755927**	4.0	2.670237	8.1	0.3297	0.0727	[112]
2002	6.6742298	15	10.01134	20	0.5006	0.2726	[113]
2003	6.6738727	4.0	2.669549	5.4	0.4944	0.0727	[114]
2005	6.6723900	13	8.674107	18	0.4819	0.2362	[115]
2006	6.6742512	1.9	1.268108	2.4	0.5284	0.0343	[116]
2010	6.6723414	2.1	1.401192	2.8	0.5004	0.0382	[117]
2010	6.6734900	2.6	1.735107	3.6	0.4820	0.0472	[118]
2014	6.6755420	2.4	1.602130	8.3	0.1930	0.0436	[119]
2014	**6.6719199**	15	10.00788	20	0.5004	0.2725	[120]
2014	6.6743513	1.9	1.268127	2.6	0.4877	0.0345	[121]

* Data are introduced in [122]

To apply a stated approach (*IARU*), an estimated observation interval of G, one can choose the difference of its value reached by the experimental results of two projects: $G_{min} = 6.6719199 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} [120] and $G_{max} = 6.6755927 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2} [112]. Then, the possible observed range S_G of G variations equals:

$$S_G = \mathrm{G_{max}} - \mathrm{G_{min}} = 3.6728 \cdot 10^{-14}(m^3 \cdot \mathrm{kg}^{-1} \cdot s^{-2}). \qquad (63)$$

The analyzed publications fall into two classes of phenomena: $CoP_{SI} \equiv$ ***LMT*** and $CoP_{SI} \equiv$ ***LMTI*** for which the comparative uncertainties, respectively, equal 0.0048 and 0.0245 (Table I). It should be mentioned that within the proposed approach, to achieve the equal comparative uncertainties of mathematical models describing the same material object but with different *CoP*, a distinctive number of dimensionless complexes used in a mathematical model or during field experiments is required. For further discussion, we will use 0.0245 as a weaker restriction corresponding to the information-oriented approach recommendations.

Applying the present approach (*IARU*), one can argue about the order of the desired value of the relative uncertainty r_{LMTI}. For this purpose, we take into account the following quantities: $\varepsilon_{LMTI} = 0.0245$ (Table I), and $S_G = 3.6728 \cdot 10^{-14}$ (63). Then, the lowest possible absolute uncertainty for $CoP_{SI} \equiv$ ***LMTI*** equals:

$$\boldsymbol{\Delta_{LMTI}} = \varepsilon_{\boldsymbol{LMTI}} \cdot \boldsymbol{S_G} = 8.984034 \cdot 10^{-16}(m^3 \cdot \mathrm{kg}^{-1} \cdot s^{-2}). \qquad (64)$$

In this case, the lowest possible relative uncertainty r_{LMTI} for $CoP_{SI} \equiv$ ***LMTI*** is as follows:

$$\boldsymbol{r_{LMTI}} = \boldsymbol{\Delta_{LMTI}}/((\mathrm{G_{max}} + \mathrm{G_{min}})/2) \approx 1.4 \cdot 10^{-5}(m^3 \cdot \mathrm{kg}^{-1} \cdot s^{-2}). \qquad (65)$$

This value is in good agreement with the recommendation mentioned in [111] of $1.4 \cdot 10^{-5}$ and very close to the $1.9 \cdot 10^{-5}$ in [121], and could be particularly relevant in the run-up to the adoption of new definitions of SI units. Experimental application of the information approach is not difficult at the present time. Besides, it is expected that this work will stimulate discussion of the precise measurement of G and its application to gravity.

It is seen from the data given in Table V that the assertions presented in [123, 124] are fully confirmed. The fact is that there was not a dramatic improvement in the accuracy of the measurement of the gravitational constant during the last 15 years. This is true when based on the calculation of the relative uncertainty. In addition, judging the data by the comparative uncertainty according to the proposed approach, one can see that the measurement accuracy has not significantly changed either. Perhaps this situation has arisen as a result of unaccounted systematic errors in these experiments [123, 124]. It is authorized as true too when based on a calculation of the relative uncertainty (*IARU*). Judging the data by the comparative uncertainty following *IACU*, one can see that the measurement accuracy has not significantly changed either. At the same time, there is not *significant difference* between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMTI} = 0.0245$ and the experimental magnitudes achieved during measurement of G; for example, 0.0345 [121]. The difference may be explained by the fact that experimenters take into account a very contrasting number of quantities in comparison to the recommendations (see Table I). This means that further future improvements of test benches can be recommended.

However, it must be mentioned that the exactness of the gravitational constant, as with other fundamental physical constants, most likely cannot be infinite, and, in principle, must be calculable. Therefore, the development of a larger number of designs and an improvement of the various experimental facilities for the measurement of the gravitational constant by using schemes combining a torsion balance and electromagnetic equipment (electrostatic servo control) [124] is absolutely necessary in order to obtain closer results to the minimum comparative uncertainty ε_{LMTI}.

One can continue to use the information method to analyze the results of measurements of fundamental physical constants. At the same time, the four above-presented examples are quite sufficient for the reader to become acquainted with the recommended procedure for analyzing experimental data using the proposed method.

One needs to note the fundamental difference between the described method and the CODATA technique in determining the relative uncertainty of one fundamental physical constant or another. For using CODATA technique, tables of values that allow direct use of relative uncertainty are constructed, using modem, advanced statistical methods and powerful computers. This, in turn, allows for checking the consistency of the input data and the output set of values. However, at every stage of data processing, one needs to use his own intuition, knowledge and experience (one's personal philosophical leanings [125]). In the framework of the presented approach, a theoretical and informational grounding and justification are carried out for calculating the relative uncertainty. Detailed data description and processing do not require considerable time. Thus, the μ-hypothesis is an exact mathematical—and thus scientific—concept.

3.5. Fundamental minimum resolutions of energy, length and information

In the age of the internet, the Big Bang theory and the colonization of Mars, the pervasive computerization, concepts and methods of information theory are widely used in a variety of areas of human activity: physics, chemistry, biology, physiology, technology, etc. Of course, information theory plays a fundamental role in the modeling of various processes and phenomena. This is because modeling is an information process, in which information about the state and behavior of the observed object is obtained from the developed model. During the modeling process, information is increased, and information entropy is reduced due to increased knowledge about the object [126].

The harmonic construction of modern science is based on separate blocks which are still not joined together: quantum electrodynamics, cosmology, biology, thermodynamics, chemistry, computer science, information theory. At the same time, there have been numerous attempts to create a picture of the objective reality covering the whole body of knowledge. However, they do not lead to proper results. One of the main obstacles standing in the way is, perhaps, that researchers and scientists consider continuous space-time but there are still unresolved problems associated with the processes of observation and measurement. Although discrete and continuous features coexist in any natural phenomenon, depending on the scales of observation [126], one can suppose a deeper level of reality which exhibits some kind of discrete elementary structure.

A review paper considering many possibilities of something similar to a discrete quantum length scale is [127]. In [128], the authors demonstrated that in non-perturbative quantum descriptions, the existence of a minimum uncertainty in physical time is generally avoidable when gravitational effects are taken into account. Minimum time and length uncertainty in rainbow gravity have been reported in [128–130]. Unfortunately, the absolute minimum values of quantities under consideration are not introduced in the majority of studies.

In the 1980s, a brilliantly elegant formula was developed, and the upper limit of the amount of information (called the Bekenstein boundary) was calculated [51]. It is contained in a body of limited volume and has the maximum amount of information needed to fully describe this physical system. This means that the volume of information of a physical system must be finite if the space of the object and its energy are finite. In informational terms, this bound is given by:

$$\Upsilon \leq (2 \cdot \pi \cdot \mathrm{R} \cdot \mathrm{E})/(\hbar \cdot c \cdot \ln 2), \tag{66}$$

where Υ is the information expressed in the number of bits contained in the quantum states of the chosen object sphere; the ln2 factor comes from defining the information as the natural logarithm of the number of quantum states; R is the radius of an object sphere that can enclose the given system; E is the total mass-energy including any rest masses; $\hbar$ is the reduced Planck constant; and *c* is the speed of light.

After almost 35 years of the use of Bekenstein theory, this study proposes an information-oriented method, according to which the information quantity inherent in the model can be calculated and it dictates the required number of quantities that should be taken into account.

The idea is to combine the Bekenstein formula and the information-oriented method with the help of a theoretically grounded approach for numerically calculating the lowest possible energy, length and number of information resolutions without going into theoretical debates and ineffective discussions. Note that one of the constructive ways to answer this question is to promote the hypothesis; this work is an attempt to do so. We do this through the use of a universal metric called the comparative uncertainty.

Hints of graininess stem from attempts to unify the general theory of relativity, Einstein's theory of gravity, with quantum mechanics, which describes the work of three other forces: electromagnetism, and strong and weak nuclear interactions. The result is a single structure, sometimes called quantum gravity, which explains all the particles and forces of the universe.

Bekenstein proved in [51] that a bound of a given finite region of space with a finite amount of energy contains the maximum finite amount of information required to perfectly describe a given physical system. In informational terms, this bound is given by (66) or:

$$S \leq (2 \cdot \pi \cdot \boldsymbol{\kappa}_b \cdot \mathrm{R} \cdot \mathrm{E})/(\hbar \cdot c), \tag{67}$$

where S is the entropy, and $\boldsymbol{k}_b$ is the Boltzmann constant.

The results are purely theoretical in nature, although it is possible to find application of the proposed formula in medicine or biology. Indeed, the act of the Bekenstein modeling process already implies an existence of the formulated physical-mathematical model describing the sphere under investigation. In this model, the quantities are taken into account from the SI.

This point of view is related to a more general one: any physical theory is based on the representation of the system and concerns only this representation, whereas reality always remains outside and never completely captured. Therefore, the mathematical analysis of any physical model should be extended by an additional pragmatic status for subsequent statements. Since the model is only an approximation of the real system, only plausible assumptions should be put forward. Therefore, there should be some tolerance for implausible ideas: a property that is not observed can, in some cases, be considered true. This underlines the gap between mathematics and physical research and is important for moving beyond simple model analysis and the transition to real-world understanding.

It can be shown (Chapter 2.3) that an amount of information quantity $\Delta\mathrm{A}_e$ about the observed modeled sphere is calculated according to the following:

$$\boldsymbol{\Delta}\mathbf{A}' = \boldsymbol{Q}\cdot\left(\boldsymbol{H}'_{\mathrm{pr}} - \boldsymbol{H}'_{\mathrm{ps}}\right) = \boldsymbol{k}_b \cdot \ln\,[\boldsymbol{\mu}_{\mathrm{SI}}/(\boldsymbol{z}' - \boldsymbol{\beta}')], \qquad (68)$$

$$\boldsymbol{\Delta}\mathbf{A}'' = \boldsymbol{Q}\cdot\left(\boldsymbol{H}''_{\mathrm{pr}} - \boldsymbol{H}''_{\mathrm{ps}}\right) = \boldsymbol{k}_b \cdot \ln\,[(\boldsymbol{z}' - \boldsymbol{\beta}')]/(\boldsymbol{z}'' - \boldsymbol{\beta}'')], \qquad (69)$$

$$\boldsymbol{\Delta}\mathbf{A}_e = \boldsymbol{\Delta}\mathbf{A}' + \boldsymbol{\Delta}\mathbf{A}'' = \boldsymbol{k}_b \cdot \ln\,[\boldsymbol{\mu}_{\mathrm{SI}}/(\boldsymbol{z}'' - \boldsymbol{\beta}'')], \qquad (70)$$

where $\Delta\mathrm{A}_e$ is measured in units of entropy; z'' is the number of the dimensional physical quantities recorded in the mathematical model; β'' is the number of the dimensional base physical quantities recorded in a model; and μ_{SI} is the number of possible dimensionless complexes (criteria) with $\xi = 7$ base dimensional quantities of SI and equals (Chapter 2.2):

$$\boldsymbol{\mu}_{\mathrm{SI}} = (e_l \cdot e_m \cdot e_t \cdot e_i \cdot e_\theta \cdot e_j \cdot e_f - 1)/2 - 7 = 38{,}265, \qquad (71)$$

where "-1" corresponds to the occasion when all exponents of base quantities in the formula (19) are treated as having zero dimensions; dividing by 2 means that there are both required and inverse quantities (for example, the length L^1 and the running length L^{-1}), or in other words, the object can be judged knowing only one of its symmetrical parts, while others structurally duplicating this part may be regarded as information empty; and 7 corresponds to seven base quantities L, M, T, Θ, I, J, F.

In order to transform $\Delta\mathrm{A}_e$ to bits $\Delta\mathrm{A}_b$, one should divide it by the following abstract number $\kappa\ln 2 = 9.569926\cdot 10^{-24}$ kg·m^2·s^{-2}·K^{-1} [9, 131]. Then:

$$\boldsymbol{\Delta A}_b = \ln\,[\boldsymbol{\mu}_{\mathrm{SI}}/(\boldsymbol{z}'' - \boldsymbol{\beta}'')]/\ln 2. \qquad (72)$$

Let us speculate how we can apply it for analyzing the elementary structure of the existing universe.

3.5.1. Dose of energy

In the case of the Bekenstein bound, the information quantity contained in a sphere ϒ equals the information quantity ΔA_b obtained by modeling process:

$$\Upsilon = \Delta A_b, \tag{73}$$

or, taking into account (18), (72) and (73):

$$(2\pi \cdot R \cdot E)/(\hbar \cdot c \cdot \ln 2) = \ln[\mu_{SI}/(z''-\beta'')]/\ln 2. \tag{74}$$

So:

$$(RE)_{min} = \hbar \cdot c \cdot \ln[\mu_{SI}/(z''-\beta'')]/(2\pi) = 5.031726 \cdot 10^{-27} \cdot \ln[\mu_{SI}/(z''-\beta'')] \tag{75}$$

According to analysis of recorded quantities dimensions, the Bekenstein model is classified by $CoP_{SI} \equiv LMT\Theta$, for which μ_{SI} = 38,265 (71) and $(z'-\beta')$ = 846 (Table I). Then, we get:

$$(z''-\beta'') = (z'-\beta')^2/\mu_{SI} = 846^2/38{,}265 \approx 18.704194. \tag{76}$$

Taking into account (76), the achievable value of $(R \cdot E)_{min}$ equals:

$$(RE)_{min} = \hbar \cdot c \cdot \ln[\mu_{SI}/(z''-\beta'')]/(2\pi) = 3.835958 \cdot 10^{-26} \tag{77}$$

$(R \cdot E)_{min}$ can be applied to verify the lowest energy uncertainty E_{min} indicating that the universe itself cannot distinguish energy levels lower than a special limit [132].

The age of the universe T_{univ} is about 13.7 ± 0.13 billion years or $4.308595 \cdot 10^{17}$ s [133]. Therefore, a radius of the universe is given by:

$$R_{univ} = T_{univ} \cdot c = 1.291684 \cdot 10^{26} (m). \tag{78}$$

So, the minimum energy resolution E_{min} is the following:

$$E_{min} = 3.835958 \cdot 10^{-26} / 1.291684 \cdot 10^{-26} \approx 3 \cdot 10^{-52} \ (m^2 \cdot kg \cdot s^{-2}). \tag{79}$$

E_{min} is difficult to imagine and its lower value was introduced in [132] as 10^{-50} J. At the same time, the value obtained in (79) is of the same order as $\sim 10^{-45}$ ergs = 10^{-52} m^2 kg s^{-2} provided in [134]. E_{min} can be used, along with μ_{SI} and combining the thought experiment with field studies, for measurement of the uncertainty values of fundamental physical constants [135].

3.5.2. "Graininess" of space

Until recently, scientists believed that the diameter of the grain of space or the minimum possible length in nature is nothing more than Planck's length ($\sim 1.6 \cdot 10^{-35}$ meters). There are numerous concepts, approaches, methodologies and formulas proposed for identifying the boundary, or transition zone, where space-time becomes granular R_{min} or, in other words, a resolution limit of length in any experiment [136].

In connection to this, attention should be paid to the undeservedly forgotten fact that European scientists reported the results of the most outstanding attempt to detect the quantization of space [137]. To carry out their calculations, a group of physicists from France, Italy and Spain used data from the European space telescope Integral, namely its capture of the gamma-flash GRB 041219A, which occurred in 2004. According to calculations, the grain of space, if it exists, must influence the polarization of transmitted rays. And the influence is more noticeable, the more intense the radiation and the greater distance it had to go through. GRB 041219A was included in 1% of the brightest gamma outbreaks among all that scientists caught and the distance to the source was at least 300 million light-years. It was a very fortunate case, allowing the checking of existing performances. It must be added that the degree of influence of the quantization of space on transmitted light depends also on the dimensions of the grain itself, so the parameters of a distant flash could indicate this value or at least its order.

Scientists have already made attempts to find the grain of space, decoding the light of distant gamma-flares. The current observation was ten thousand times more accurate than all the previous experiments. The analysis showed that if the granularity of space exists at all, then it should be at a level of 10^{-48} meters or less.

Following ideas introduced in Chapter 2, we have supposed that any of our measurements has a certain intrinsic limited length about small-scale physics and we shall calculate it as follows. Hooft [138] introduced S_{HS} which is the holographic entropy bound expressed in terms of the entropy:

$$S_{HS} \leq \pi \cdot c^3 \cdot R^2/(\hbar\, G), \tag{80}$$

or:

$$\Upsilon \leq \Upsilon_{HS} \leq \pi \cdot c^3 \cdot R^2/\,(\hbar \cdot G \cdot \kappa_b \cdot \ln 2), \tag{81}$$

where Υ_{HS} is the information quantity expressed in bits and corresponding to S_{HS}; c is the light speed, c = 299,792,458 $m^1 s^{-1}$; $\hbar = h/(2\pi)$ is the reduced Planck constant, $\hbar = 1.054572 \cdot 10^{-34} m^2\, kg\, s^{-1}$; G is the gravitational constant, G = $6.67408 \cdot 10^{-11}\, m^3\, kg^{-1}\, s^{-2}$; R is the radius of an object sphere expressed in meters; $\kappa_b \cdot \ln 2 = 9.569926 \cdot 10^{-24}$; and $\pi = 3.141592$.

Equating ΔA_b (72) to (81) and using the known values of physical quantities, we get:

$$\pi \cdot c^3 \cdot R^2/(\hbar \cdot G \cdot \kappa_b \cdot \ln 2) = \ln[\mu_{SI}/(z''-\beta'')]/\,ln\,2, \tag{82}$$

$$1.256712 \cdot 10^{93} \cdot R^2 = \ln\,[\mu_{SI}/(z''-\beta'')]/\ln 2, \tag{83}$$

$$R_{\min} = 3.388203 \cdot 10^{-47} \cdot \{\ln[\mu_{SI}/(z''-\beta'')]\}^{1/2}. \tag{84}$$

Taking into account (71), (76) and (84), the minimum achievable value of the length discretization or the universal, global standard of length equals:

$$R_{\min} = 9.3 \cdot 10^{-47}(m). \tag{85}$$

This $R_{min} = 9 \cdot 10^{-47}$ m is in excellent agreement with the result of [137]. It could be suggested that this metric of space is only a purely mathematical concept that measures a “degree of distinguishability”. In addition, maybe, the minimal length scale is not necessarily the Planck length. The scale of distance, just like the duration of time, turns out to be a property not of the world but of the models we employ to describe it [139]. With the help of these calculations, it is possible to identify a boundary or a transition zone, where space-time becomes granular and physically non-local.

3.5.3. “Grain” of information

Taking into account (79) and (85), let us calculate a possible achievable minimum amount of information Υ_q; in other words, an information quantum bit, or “qubit” [140] which can be viewed as the basic building block of quantum information systems [141]:

$$\Upsilon_q \leq (2 \cdot \pi \cdot R_{\min} \cdot E_{\min})/(\hbar \cdot c \cdot \ln 2) = 0.79411 \cdot 10^{-71}(\text{bit}). \quad (86)$$

On the question of "Does information exist by itself?" a completely reasonable answer would be "Yes and no". "Yes", since we cannot deny the availability of information and its storage, transfer, processing, etc., which we encounter in our daily lives. We know that information is of great importance and can significantly affect the course of events. Information exists independently of people's consciousness [125]. On the other hand, the answer "No" also has a rational grain. Is it possible to "touch" this notorious information? Most likely, information exists objectively but not materially in itself.

If information can be stored in the position of the smallest particle, the activation energy for its motion will be sufficiently lowest [142]. If the information, like some substance, is granular beyond a certain scale, it means that there is a "base scale", a fundamental unit that cannot be broken down into anything less. This hypothesis so far contradicts the generally accepted opinion in the scientific community.

In [143] it was noted that information is a quantity which is both discrete and continuous, where time and other physical phenomena might be reconceived as simultaneously discrete and continuous with an information theoretic formulation. Perhaps equation (86) will spur researchers to further understanding of the concept of information. In addition, this value may have use in the definition of qubits for quantum computation.

3.5.4. Information embedded in a photon

The radius of the particle is determined by the region in which it can produce some effect. According to [144], the radius of a single photon in the energy region of 2.1 GeV equals $2.8 \cdot 10^{-15}$ meters. In this case, taking into account (66), the amount of information contained in one photon is:

$$\Upsilon \leq (2 \cdot \pi \cdot R \cdot E)/(\hbar \cdot c \cdot \ln 2) = 270(bit). \quad (87)$$

In fact, the author does not offer anything so concrete. First, these are all assumptions. Secondly, too much that is doubtful and untrustworthy has been written. However, if you are still reading and you like this unorthodox application of information theory, then all of the above can stimulate your imagination.

3.5.5. Maxwell's demon

Over the past 20 years, both information in the form of a certain substance and methods of information theory have been the subject of special attention by scientists, engineers and philosophers. A great number of studies have been devoted not only to clarifying the internal content of the concept of information, but also to the application of this unique substance in all fields of human activity: physics, chemistry, biology, psychology, business, etc. The number of theories offered is uncountable. Impressive practical results were obtained using information theory in the field of quantum mechanics, telecommunications, medicine, marketing and the development of non-lethal weapons. At the same time, in theoretical and experimental physics, the number of research papers (with a specific quantitative result) using information theory is catastrophically small; they can be counted on the fingers. The author, being a convinced practitioner, has taken the courage to present, without going into endless and unconvincing theoretical discussions, ordinary calculations (in the sense that known and generally accepted formulas are used) to quantify the amount of information on several examples.

What do the measurements of the fundamental constants and Maxwell's demon have in common? In fact, only a little. Adapting the μ_{SI}-hypothesis, which was used in recent years to test the achievable relative uncertainty in measuring fundamental constants, we are developing a way to better understand specific problems that are close to the Maxwell problem.

In one of his versions, the standard Maxwell's demon is a very small intellectual being endowed with free will, and a fairly subtle tactile and perceptive organization to enable him to observe and influence individual molecules of matter. In Maxwell's thought experiment, two gas chambers, maintained at equal temperatures, are separated by an adiabatic wall with a small hole and a gate that the demon opens and closes. Observing the speed of individual molecules, the demon selectively opens and closes the gate to quickly separate slow molecules, creating a clear temperature difference between the two chambers. Thus, as the collisions with the shutter are elastic, and moving the shutter is frictionless, no work is performed by the demon. The temperature difference that develops could be exploited by a conventional heat engine to extract work, in violation of the second law of thermodynamics.

Various researchers suggested different ways by which a demon could select particles in a reversible manner. In 1929, Leó Szilárd [145] argued that the demon must consume energy in the act of measuring the particle speeds and that this consumption will lead to a net increase in the system's entropy. In fact, Szilárd formulated an equivalence between energy and information, and calculated that $k_b \cdot \theta \cdot \ln 2$ is both the minimum amount of work needed to store one bit of binary information and the maximum that is liberated when this bit is erased, where θ is the temperature of the storage medium. Through the latest publications [146, 147–149] one must remember [142], in which it was finally clarified that the demon's role does not contradict the second law of thermodynamics, implying that we can, in principle, convert information to free energy. Toyabe et al. [150] showed that since the energy transformed from the information is compensated by the cost of the demon's energy for manipulating information, the second law of thermodynamics is not violated when a general system involving both a particle and a demon is considered. In the proposed research system, the demon consists of macroscopic devices, such as computers. The microscopic device receives energy due to the energy consumption of the macroscopic device. In other words, using information as an energy transfer medium, this transformation of information into energy can be used to transfer energy to nanomachines, even if they cannot be directly controlled. In [151] it was declared that Maxwell's demon can be converted into free energy by one bit of information obtained by measurement. The authors implemented an electronic Maxwell's demon based on a one-electron unit operating as a Szilard engine, where $k_b \cdot \theta \cdot \ln 2$ of heat is extracted from the reservoir at a temperature θ by one bit of generated information. The information was encoded in the form of an additional electron in the box. The authors provided, to their knowledge, the first demonstration of extracting nearly $k_b \cdot \theta \cdot \ln 2$ of work for one bit of information.

After 150 years, a satisfactory additional solution of this paradox can be given [5]. In order to prevent the violation of the second law of thermodynamics, one must assume that the demon is a conscious observer with knowledge, experience and intuition. Then, before performing any actions, in order to know the velocity of every molecule in the box, he must compose a mental model of the experiment, with no disturbances being brought into the box. In turn, for the development of the model the demon will take advantage of the already well-known SI. When modeling particle movement, the demon may choose quantities—for example, velocity, mass, angle of motion of the particle with respect to the shutter, and temperature—that may substantially differ from those chosen by another demon, as happened, for example, during the study of electrons that behave like particles or waves. That is why SI can be characterized by equally probable accounting of any quantity chosen by the demon. In this case, the total number of possible dimensionless criteria μ_{SI} of SI with the seven base quantities L, M, T, I, Θ, J and F could be calculated (23):

$\mu_{SI} = 38{,}265,$

Then, μ_{SI} corresponds to a certain value of entropy and may be calculated by the following formula [5]:

$$\boldsymbol{H} = \boldsymbol{k}_b \cdot \ln\mu_{SI}, \tag{88}$$

where **H** is entropy of SI including μ_{SI}, equally probable accounted quantities, and k_b is the Boltzmann constant.

When a demon chooses the influencing factors (the conscious limitation of the number of quantities that describe an object, in comparison with the total number μ_{SI}), entropy of the mathematical model changes *a priori*. The entropy change is generally measured as follows:

$$\Delta\boldsymbol{H} = \boldsymbol{H}_{pr} - \boldsymbol{H}_{ps}, \tag{89}$$

where ΔH is the entropy difference between two cases, and *pr* is "*a priori*" and *ps* is "*a posteriori*".

"The efficiency **Q** of the experimental observation method can be defined as the ratio of the information obtained to the entropy change accompanying the observation." [9] During a demon's thought experiment, no distortion is brought into the real system, that is why **Q** = 1. Then, one can write it according to [9]:

$$\Delta\mathbf{A} = \mathbf{Q} \cdot \Delta\boldsymbol{H} = \boldsymbol{H}_{pr} - \boldsymbol{H}_{ps}, \tag{90}$$

where Δ**A** is the *a priori* information quantity pertaining to the observed object.

Using equations (88)–(90) and imposing symbols where z' is the number of physical dimensional quantities in the selected *CoP* and β' is the number of base quantities in the selected *CoP* leads to the following equation:

$$\Delta\boldsymbol{A}' = \mathbf{Q} \cdot \left(\boldsymbol{H}'_{pr} - \boldsymbol{H}'_{ps}\right) = \boldsymbol{k}_b \cdot \ln[\mu_{SI}/(\boldsymbol{z}' - \boldsymbol{\beta}')] \tag{91}$$

where Δ**A'** is the *a priori* amount of information pertaining to the observed object due to the choice of the *CoP*.

Following the same reasoning, one can calculate the *a priori* amount of information Δ**A"**, caused by the number of recorded dimensionless criteria chosen in the model. Δ**A"** takes the following form:

$$\Delta\mathbf{A}'' = \boldsymbol{k}_b \cdot \ln[(\boldsymbol{z}' - \boldsymbol{\beta}')/(\boldsymbol{z}'' - \boldsymbol{\beta}'')], \tag{92}$$

where Δ**A**" cannot be defined without declaring the chosen *CoP* (Δ**A**'); z" is the number of physical dimensional quantities recorded in a mathematical model; and β" is the number of base quantities recorded in a model of the box.

A minimal amount of information $\Delta\mathbf{A}_E$ about the observed modeled box is calculated according to the following:

$$\Delta\mathbf{A}_{\mathrm{E}} = \Delta\mathbf{A}' + \Delta\mathbf{A}'' = \kappa_b \cdot \ln[\mu_{\mathrm{SI}}/(z'' - \beta'')], \qquad (93)$$

where $\Delta\mathbf{A}_E$ is measured in units of entropy [152]; z" is the number of physical dimensional quantities recorded in the mathematical model; and β" is the number of base dimensional quantities recorded in the model.

In order to transform $\Delta\mathbf{A}_E$ to bits $\Delta\mathbf{A}_b$, one should divide it by the following abstract number $k_b \cdot \ln 2 = 9.569926 \cdot 10^{-24}$ kg·m^2·s^{-2}·K^{-1} [9, 153]. Then:

$$\Delta\mathbf{A}_b = \ln\,[\mu_{\mathrm{SI}}/(z'' - \beta'')]/\ln 2(\mathrm{bits}). \qquad (94)$$

Taking into account that μ_{SI} = 38,265 and suppose z" - β" = 1 (one can choose a larger number of dimensionless criteria, but this does not affect the course of further reasoning and conclusions), one can calculate the minimum boundary of the motion blur of a particle in the eyes of a demon:

$$\Delta\mathbf{A}_b = \ln\,[38{,}265/1]/0.6931472 \approx 11(\mathrm{bits}). \qquad (95)$$

In this case, the mathematical theory of information does not cover all the wealth of information content, because it is distracted from the semantic content side of the message. From the point of view of the information-based approach, a phrase of 100 words taken from the newspaper, Shakespeare, or Einstein's theory has about the same amount of information.

Thus, equation (95) contains a very strong hint that the demon is not able to clearly distinguish the exact state of a large number of particles. There are no glasses that could correct the sight of the demon. This closes the possibility of developing a device that could distinguish between fluctuations in individual particle velocities. Hence, it is clear that any material physical device, in comparison with a mental thought experiment (conscious, without a demon's material shell), will require much more information and energy for the release of any gate movement.

Let us apply $\Delta\mathbf{A}_b$ (95) corresponding to the insurmountable threshold mismatch ("cloud" of blurring) between the vision of the demon and the actual situation in the box with the particles, i.e., amount of information inherent in a particle. As an example, consider that the radius of the particle is determined by the region in which it can produce some effect. According to [144], the radius of a single photon r_p in the energy region of $E_p = 2.1$ GeV equals $2.8 \cdot 10^{-15}$ meters. In this case, the amount of information contained in one photon is [154]:

$$\Upsilon_{bp} \leq \left(2 \cdot \pi \cdot r_p \cdot E_p\right)/(\hbar \cdot c \cdot \ln 2) = 270(bit), \qquad (96)$$

where Υ_{bp} is the information amount expressed in bits and corresponding to the photon's sphere; r_p is the radius of a photon expressed in meters, $2.8 \cdot 10^{-15}$ m [144]; c is the light speed, c = 299,792,458 m/s; ħ is the reduced Planck constant, $\hbar = 1.054572 \cdot 10^{-34} m^2 \cdot kg \cdot s^{-1}$; ln2 = 0.693147; and π = 3.141593.

Thus, the minimum boundary of the motion blur of the particle in the eyes of the demon (in bits) is much less than the information contained in the photon (270 bit » 11 bit). However, this fact does not in any way allow us to state that the demon, after preliminary modeling, will be able to carry in one direction particles moving at high speed, and in the other direction particles having a low speed, thereby violating the second law of thermodynamics. On the contrary, the demon will need information through a measuring device that is comparable in magnitude to the information inherent in the particle. This, in turn, will require the performance of work, which will lead to an increase in entropy in the total volume of the box.

The proposed approach provides only a hint of how much information a demon and the observed particle have before starting any action with a system of "box-demon" or about "uncertainty" in the mind of someone about to receive a message [155].

3.5.6. Universe energy associated with information

In connection with the foregoing, there is an amazing possibility (and for the readers, a very controversial one) of applying the results obtained in analyzing the status of the miniature Maxwell's demon to the problems associated with clarifying the energy of the observed universe.

Experiments and theories developed in theoretical physics over the past decades have demonstrated the significant role of information, the amount of which physicists usually identify with entropy but which can be more general when used to explain the emerging complexity of the universe. One of the most attractive features of the Bekenstein formula [51] is that it allows us to compose an idea of the possible connection between energy and information contained in the universe.

For this purpose, let us recall [51], in which it was proved that the amount of information in a physical system must be finite if the space of an object and its energy are finite. In informational terms, this bound is given by:

$$\Upsilon_b \leq (2 \cdot \pi \cdot R \cdot E) \,/\, (\hbar \cdot c \cdot \ln 2), \tag{97}$$

where Υ_b is the information expressed in the number of bits contained in the quantum states of the chosen object sphere; the ln2 factor (approximately 0.693149) comes from defining the information as the natural logarithm of the number of quantum states; R is the radius of an object sphere that can enclose the given system; E is the total mass-energy, including rest masses; $\hbar$ is the reduced Planck constant; and c is the speed of light.

Further, Landauer's principle [142] asserts that there is a minimum possible amount of energy required to erase one bit of information, known as the *Landauer limit*:

$$\boldsymbol{k}_b \cdot \theta \cdot \ln 2, \tag{98}$$

where $\boldsymbol{\theta}$ is the temperature in Kelvins of the environment.

It is important to note that the equivalent bit energy depends on the temperature of the described system. The average temperature of the universe today is approximately $\boldsymbol{\theta}$ =2.73K [156], based on measurements of cosmic microwave background radiation.

Therefore, with imagination and an essential assumption, in order to transform Υ_b to terms of the ordinary energy Υ_E, one should multiple it by $\boldsymbol{k}_b \cdot \boldsymbol{\theta} \cdot \ln 2$:

$$\Upsilon_E = \Upsilon_b \cdot \boldsymbol{k}_b \cdot \boldsymbol{\theta} \cdot \ln 2 \leq ((2 \cdot \pi \cdot R \cdot E) \,/\, (\hbar \cdot c \cdot \ln 2)) \cdot \boldsymbol{k}_b \cdot \boldsymbol{\theta} \cdot \ln 2, \tag{99}$$

or:

$$\Upsilon_E \,/\, E \leq (2 \cdot \pi \cdot R \cdot \boldsymbol{k}_b \cdot \theta) \,/\, (\hbar \cdot c). \tag{100}$$

Using the dimensional analysis, we verify the achieved dimension of equation (100):

$$\dim \mathrm{R} \ni \mathrm{m};\ \dim \boldsymbol{k}_b \ni \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}};\ \dim \boldsymbol{\theta} \ni \mathrm{K};\ \dim \hbar \ni \mathrm{m^2 \cdot kg \cdot s^{-1}};$$

$$(101)$$

$$\dim c \ni \mathrm{m \cdot s^{-1}};\ \dim (\Upsilon_E \,/\, E) \ni \mathrm{m \cdot kg \cdot m^2 \cdot s^{-2} \cdot K^{-1} \cdot K / (m^2 \cdot kg \cdot s^{-1} \cdot m \cdot s^{-1})} = 1$$

So, at least from the point of view of the dimensional analysis, there is not a fatal mistake.

Further, the age of the universe T_{univ} is about 13.7 ± 0.13 billion years or 4.308595×10^{17} s [157]. Then, taking into account c = 299,792,458 m/s, a radius of the universe is:

$$R_{univ} = T_{univ} \cdot c = 1.291684 \cdot 10^{26} \text{ (m)}. \tag{102}$$

It should be noted that there is no known boundary, that is, R_{univ} is an approximate value. When people talk about the size of the observable universe, this means the estimated distance to the most distant objects that we can see. This does not mean that there is nothing further, it simply means that we do not see it.

In this case, one can get the numerical relationship between energy corresponding to the amount of information and the material energy contained in a universe sphere:

$$\Upsilon_E / E \leq (2 \cdot \pi \cdot R \cdot \kappa_b \cdot \theta) / (\hbar \cdot c) = 9.6 \cdot 10^{29} \approx 10^{30} \tag{103}$$

Thus, we have shown that the energy represented by the information makes a significant contribution to the total energy of the universe. Of course, the value in (103) is a rough estimate. It is interesting to note that 10^{30} is much less than 10^{122}. According to the holographic principle, the latter huge number represents an upper bound on the information content of the universe [158]. Since information energy can make a significant contribution to the dark energy and dark matter of the universe, scientists need to study it more closely. Maybe this value (10^{30}) can also be a signal of some kind of new interaction between matter and information.

Therefore, more is unknown than known. Besides this, it is a complete but important secret. The rest—everything on the Earth, everything that has ever been observed with all our instruments, all normal energy—is a meager part of the universe. Think about it: perhaps it is not "normal" at all, since it is such a small part of the universe. But what kind of information is this? Perhaps information itself is a fundamental component of the physical universe. Is it "ontological"; the real substance from which space, time and matter emerge? Or is it "epistemic"; something that only represents our state of knowledge about reality? Ultimately, information can be a key element in the constitution of physical reality. The explicit relationship between entropy and information, using Shannon's concept of objective quantitative information, was formalized in [159], and this can be regarded as irrefutable confirmation of information as a physical entity.

Such a dramatic gap of 10^{30} between the amounts of energy associated with ordinary matter and the energy due to information can be associated with initially accepted assumptions: the universe is not a sphere; the average temperature of the universe can be much lower than the observed temperature; and for the giant distance scale, the Landauer's limit is not satisfied.

The presented results (95), (96) and (103) are simply routine calculations by formulas known in the scientific literature. At the same time, only experts on quantum electrodynamics or the theory of gravity can "separate the wheat from the chaff". However, if the Bekenstein formula and Landauer's limit have a physical explanation, perhaps the result of (103) can be used to study the universe.

Additional explanation of how information acquires energy comes from the quantum theory of matter. In this theory, "empty space" is actually full of temporal ("virtual") particles that are constantly being formed, possessing certain information, and then disappear. But when we tried to calculate how much energy this information gives to the empty space, the answer turned out to be erroneous—by a considerable margin: the number is 10^{30} times too large. It is difficult to get such an answer. So, the mystery exists.

Another explanation of the significant magnitude of the energy corresponding to the information contained in the universe is that it is a new kind of field energy that fills the whole space. But if the information itself is the answer, we still do not know what it is, what it interacts with or how it exists in the universe. Thus, the mystery continues.

More speculatively, a last possibility is that Einstein's theory of gravity requires clarification. Einstein's formula does not cause any reasonable doubt. But now, in order to qualify the result (103), we need to clearly state the need to improve this formula by adding a component due to the information and that at the moment it is only possible to directly measure the component of conventional energy. This fact would provide a way to decide if the solution of the amount of information is a possible and admissible part of the new gravity theory or not. Thus, there are many questions and no answers. That is why things are still not so bad as to expect improvement.

As an alternative to dark energy and dark matter, the energy due to information contained in the universe can serve as a "cementing" component or a "hard disk". The huge difference between the two types of energy (103) makes it possible to assert that the universe is isotropic—the same in all directions—and homogeneous, without the regions of the cosmos, which have special, peculiar characteristics. Equation (103) cannot be an illusion caused by mathematics. Does this mean that our universe consists of information, and the associated energy is responsible for the inhibition of space and time and the accelerating expansion that we observe? It is difficult for the matter-of-fact physicist to agree with this point of view. Maybe there are better ideas. It is tempting to look for links and analogies, even if they are at first considered unsuitable for discussion. Perhaps in the future these two problems will not be as fragmented as they might seem. Formulating a problem that at first glance seems completely extravagant can sometimes, with further reflection, acquire real significance and become very meaningful for the further development of science.

3.5.7. Summary

Continuing the discussion of issues involving information theory that began in Chapter 2, we have focused here on very important applications of the comparative uncertainty concept. We have shown how its definition affects and influences our understanding of different phenomena and processes in the micro-world. In fact, we have shown that the constructive and creative potential of analyzing any material object by the comparative uncertainty is enormous, and it is encouraging further research on the issue of deep knowledge of natural phenomena and processes.

If the measure of the beauty of the theory is the ratio of the number of things that it explains to how many assumptions it makes for their explanation, then the information-oriented approach seems very promising. The μ_{SI}-hypothesis does refer to a real place of the surrounding world. It might be applicable to experimental verification. In general, it is available when the researcher has all the information about the uncertainty interval of the main quantity. Moreover, the μ_{SI}-hypothesis provides new functionalities useful for micro- and macro-physics including engineering, astronomy and quantum electrodynamics. The comparative uncertainty can be a peculiar metric for assessing the measurement accuracy of physical laws and fundamental physical constants.

Obviously, the coordination of a probabilistic subatomic world with a macroscopic everyday world is one of the greatest unsolved problems in physics. The use of the μ_{SI}-hypothesis opens up the possibility of combining these two worlds: from Maxwell's demon to cosmology and astrophysics.

The μ_{SI}-hypothesis allows us to obtain the entropy cost associated with the acquisition of the demon information. Any demon, no matter how smart it is, must perform measurements. Certainly, when creating a model for the separation of particles, it is necessary to consider in detail the constitution of a rational being. The possession of information can indeed be regarded as a decrease in entropy. However, in the case of mental modeling, obtaining information does not require the dissipation of heat and there is no threat to the generalized form of the second law of thermodynamics.

Mental modeling requires us to say something about the demon itself as a physical being. A demon can perform modeling without dissipation. This fully corresponds to the position of Brillouin. He characterized the information as "connected" if it was embodied in states of the physical device, but he bluntly stated that information contained only in the mind is "free" and not "connected".

Now, the connection between entropy and information becomes more understandable. When the demon leaves the system, he can be viewed as an agent that has information about the system. Uncertainty in the description of the system can be considered as a lack of knowledge by the demon about the exact state of the system. If the demon has more information, the system's entropy is smaller. However, once the demon can obtain information without dissipation, the system's entropy decreases and the only compensation appears to be an increase in the uncertainty of the state of the demon itself.

CHAPTER FOUR

DISCUSSION

It's easier to fool people than to convince them that they have been fooled
—Mark Twain

Although this approach is considered to be very promising, attractive and versatile, some limitations have kept scientists away. The main problems are as follows:

- The information-based approach requires the equally probable appearance of quantities chosen by a conscious observer. It ignores factors such as developer knowledge, intuition, experience and environmental properties;
- The approach requires the knowledge or declaration of the observed interval of the main observed or researched quantity. In fact, the standard uncertainty can be used as the value of this parameter;
- The method does not give any recommendations on the selection of specific physical quantities, but only places a limit on their number.

Nevertheless, the approach yields the universal metric by which the model discrepancy can be calculated. A more effective solution to finding the minimum uncertainty can be reached using the principles of information and similarity theories. Qualitative and quantitative conclusions drawn from the obtained relations are consistent with practice. They are as follows:

Based on the information and similarity theories, a theoretical lowest value of the mathematical model absolute uncertainty of the measured fundamental physical constant can be derived. A numerical evaluation of this relationship requires the knowledge of the uncertainty interval of the main researched quantity and the required number of quantities to be taken into account. In order to estimate the discrepancy between the chosen model measurement and the observed material object, a universal metric called comparative uncertainty has been developed further. Our analytical result for $\varepsilon = \Delta_{pmm}/S$ is a surprisingly simple relationship.

The information-oriented approach—in particular, *IARU*—makes it possible to calculate with high accuracy the relative uncertainty, which is in good agreement with the recommendations of CODATA. The principal difference of this method, in comparison with the existing statistical and expert methodology of CODATA (actually, all statistical methods are unreliable—some more and some less [160]), is the fact that the information method is theoretically justified.

Significant differences in the values of the comparative uncertainties achieved in the experiments and calculated in accordance with the *IACU* can be explained as follows. The very concept of comparative uncertainty, within the framework of the information approach, assumes an equally probable account of various quantities, regardless of their specific choice by scientists when formulating a model for measuring a particular fundamental constant. Based on their experience, intuition and knowledge, the researchers build a model containing a small number of quantities, which, in their opinion, reflects the fundamental essence of the process under investigation. In this case, many phenomena, perhaps not significant or just secondary, which are characterized by specific quantities are not taken into account.

For example, when measuring a value of the Planck constant by the LNE Kibble balance ($Co\boldsymbol{P}_{\mathrm{SI}} \equiv \boldsymbol{LMTI}$), located inside and shielded, temperature (base quantity is $\boldsymbol{\theta}$) and humidity are controlled, the air density (base quantity is F) is calculated [102]. Thus, the possible influence of temperature and the use of other types of gas, for example, inert gas, are neglected by developers. In this case, we get a paradoxical situation. On one side, different groups of scientists dealing with the problem of measuring a certain fundamental constant and using the same method of measurement "learn" from each other and improve the test bench to reduce uncertainties known to them. This is clearly seen using the *IARU* method: when measuring $\boldsymbol{h}$, $\boldsymbol{k}$, and $\mathrm{N_A}$, all the comparative uncertainties are very consistent, especially for measurements made in recent years. However, on the other side, ignoring a large number of secondary factors, which are neglected by experimenters, leads to a significant variance in the comparative uncertainties calculated by the *IACU* method.

Although the goal of our work is to obtain a primary restriction on the measurement of fundamental physical constants, we can also ask whether it is possible to reach this limit in a physically well-formulated model. Since our estimation is given by optimization in comparison with the achieved comparative uncertainty and the observation interval, it is clear that in the practical case, the limit cannot be reached. This is due to the fact that there is an unavoidable uncertainty of the model. It implies the initial preferences of the researcher, based on his intuition, knowledge and experience, in the process of his formulation. The magnitude of this uncertainty is an indicator of how likely it is that your personal philosophical inclinations will affect the outcome of this process. When a person mentally builds a model, at each stage of its construction there is some probability that the model will not match this phenomenon with a high degree of accuracy.

The proposed methodology is an initial attempt to use a comparative uncertainty instead of relative uncertainty in order to compare the measurements results of fundamental physical constants and to verify their true values. A direct way to obtain reliable results has always been open, namely to assume that a fundamental physical constant value lies within a chosen interval. Of course, the choice of a value of the variation of any fundamental physical constant and its interval is controversial because of its apparent subjectivity. With these methods, our capacity to predict the fundamental physical constants values by usage of the comparative uncertainty allows for an improvement of our basic comprehension of a complex phenomenon, as well as allowing us to apply this understanding to the solution of specific problems. It may be the case that such findings will induce a negative reaction on the part of the scientific community and some readers who consider the above examples as a game of numbers. In his defense, the author notes that eminent scientists such as Arnold Sommerfeld, Wolfgang Pauli and others have followed a similar approach in order to approximate values for the fundamental physical constants. The calculated results are routine calculations from formulas known in the scientific literature. At the same time, an additional perspective of the existing problem will, most likely, help us to understand the situation and identify concrete ways for its solution. Reducing the value of the comparative uncertainty of fundamental physical constants to the lowest achievable value will serve as a convincing argument for professionals involved in perfecting the SI.

The strength and special value of the suggested approach is that, in revealing features of the distribution of quantities in the model and the pattern of the numerical calculation of comparative uncertainty of measured fundamental physical constants, it not only allows the results to be understood, but can also predict the future. In other words, can the proposed method augment the study of the fundamental physical constants? The analysis of scientific data, in our opinion, can give this question quite a clear answer.

Opportunities for rigorous analysis and in-depth knowledge are on the side of CODATA scientists who conduct extensive research on the measurement of fundamental physical constants. The obtained data undergo a comprehensive statistical and expert testing and serve as a benchmark for conducting any research in various fields of science and technology. Why do the real results of the information method allow you to go the other way? The most important reason is that the analysis is centered on evidence built on a theoretically grounded approach, rather than on a biased statistical expert who is motivated by personal beliefs or preferences. In addition, the facts necessary for scientific analysis simply appeared only in the last decade. And the theoretically grounded approach, based on the theory of information, was unexpectedly introduced only in 2015.

There are three non-empirical arguments that give rise to confidence in the information-oriented method among its supporters. Recognizing that non-empirical confirmation is a part of science, one can discuss the pros and cons of these arguments in a certain context.

First, there seems to be only one version, based on the theory of information and capable of consistently achieving a unification of the estimation of the accuracy of the model of the observed physical phenomenon; moreover, no other theory capable of formulating a criterion for calculating the initial (before the experiment or computer simulation) uncertainty of the model was found, despite enormous efforts. This argument of "no alternatives" increases the confidence of theorists that there are no other alternative principles at all. This makes a more likely situation in which the μ-hypothesis is the correct approach.

Secondly, the μ-hypothesis has grown from the theory of information—accepted, empirically confirmed theory, applicable to all, without exception, known fundamental and applied sciences in a unified mathematical form. The theory of information had no alternatives in the years of its formation and it will expand the scope of its application in the future. This "meta-inductive" argument reinforces the "no alternatives" argument, showing that it worked earlier in similar contexts, opposing the fact that physicists simply do not have enough imagination to find alternatives that exist.

The third non-empirical argument is that the information-oriented approach unexpectedly set up interrelations for other theoretical problems, in addition to the unification problem for which it was intended. For example, the μ-hypothesis makes it possible to calculate the achievable relative uncertainty and, as a result of an unexpected discovery that has caused a surge in research over the past 5 years, is mathematically applicable for high-precision measurements of fundamental physical constants.

Like any other method, the proposed hypothesis has contradictory provisions and assumptions that are difficult to be perceived by the reader. Moreover, we have to be very careful with the results. At the same time, the universe in which we live is a unique object, and therefore it is not clear whether it is an accident or natural [161]. The approach does not give any recommendations on the selection of specific physical quantities, but only limits their number; the information-based method requires the equiprobable appearance of quantities chosen by the modeler; it fully ignores developer knowledge, intuition and experience; and the approach requires the knowledge or declaration of changes in the range of the fundamental physical constants values.

For obvious practical results, this method gives a generic metric of the influence of a number of chosen quantities on the model's discrepancy. Such integral characteristics are of a physical nature, but most scientists do not understand this opinion. To determine these characteristics we need to calculate the total number of dimensionless criteria in SI, and to declare a specific interval of the fundamental physical constants changes. Moreover, this metric has an inherent duality, as follows. On the one hand, it is obvious that the choice of the class of phenomena and number of chosen quantities is entirely determined by the researcher. On the other hand, before the beginning of the experiment, and regardless of the particular implementation of the experimental setup "in hardware", against the will of the researcher, the magnitude of the lowest achievable comparative uncertainty is already known, provided that the changes in the interval of the fundamental physical constants are defined.

An information-oriented approach leads us to the following conclusions. If the mathematics and physics that describe the surrounding reality are effective human creations, then we must take into account the relationship between human consciousness and reality. In addition, the ultimate limits of theoretical, computational, experimental and observational methods, even using the best computers and the most complex experiments such as the Large Hadron Collider, are limited to the μ-hypothesis applicable to any human activity. Undoubtedly, the current unprecedented scientific and technological progress will continue. However, since a limit for this progress exists, the speed of discoveries will slow down. This remark is especially important for artificial intelligence, which seeks to create a truly super-intelligent machine.

CHAPTER FIVE

FINAL REMARKS

Sense of humor and self-criticism should always be present in a presentation of one's own theory, which differs from the generally accepted opinion
—Boris Menin

We have used information and similarity theories to formulate general principles and derived effects, which are amenable to rigorous experimental verification of measurements of fundamental physical constants.

The main value of the results of the proposed method in the field of modeling is that they give a universal criterion by which to compare various models describing the observed phenomenon, in terms of their ability to achieve the lowest comparative uncertainty. Models may differ materially by class of phenomenon, number of quantities, magnitude of the observed changes interval of the main quantity, and qualitative composition of chosen quantities. Under these conditions, the information metric (comparative uncertainty) allows evaluating the extent to which the various models fit together, and what is their proximity to the object under study. This requires, on the basis of the selected class of phenomena, to calculate their information-modeling indicators: the number of quantities, the changes interval of the main quantity, and the comparative uncertainty. The calculated comparative uncertainty and the minimum achievable relative uncertainty corresponding to the selected class of phenomena are great measures by which one can judge the discrepancy between the model and a real system.

A measure of evaluation of the achievable accuracy of measurement of different physical constants is suggested, and we formulated the method of calculating the comparative uncertainty realized during the experiment. Various applications of the information-oriented method are presented. At the same time, the following must be noted. The μ_{SI}-hypothesis made it possible to establish the fact that scientists may approach, but never reach, the comparative uncertainty corresponding to the chosen class of phenomena. Regardless of the implementation of super-power computers, brilliant modem data processing methods and unique test benches, comparative uncertainty—even with the required number of dimensionless criteria—will be unattainable. In addition, the μ_{SI}-hypothesis made it possible to judge the appropriate limit of the accuracy of measurements in each individual case.

The present analysis of published studies on the measurement of fundamental physical constants allows us to hope that our approach will be used to compare the accuracy achieved in various experimental settings and by applying methods that differ from each other.

●f course, the proposed method does not claim to be the completely universal way and does not give an answer to the question of the selection of specific physical parameters for the best representation of the surrounding world. However, within the above concept, we can firmly assert that the future theory, the uniting gravity theory and quantum electrodynamics carmot be based on the use of only three base quantities: the meter, second and kilogram. This is because for such a class of phenomena it is impossible to reach the minimum comparative uncertainty.

The proposed information approach has its own implications. Any physical process, from quantum mechanics to palpitation, can be viewed by the observer only through the idiosyncratic "lens". Its material is a combination of not only mathematical equations, but also of the researcher's desire, *intuition*, experience and knowledge. These, in turn, are framed by SB●, which is chosen with the consensus of the researchers. Thus, a sort of aberration—a distortion of reality—creeps into modeling, prior to the formulation of any physical, or even mathematical, statement. The degree of distortion of the image in comparison with the actual process depends essentially on the chosen class of phenomena and the number of the "quantities created by observation". [162]

The accuracy of a model of any physical phenomena can no longer be considered limited by the boundaries, determined by the Heisenberg uncertainty relation. "Potential accuracy of the measurement" [77] is limited by the initially known unrecoverable comparative uncertainty determined by the μ-hypothesis and depending on the class of phenomena and the number of quantities chosen by the strong-willed researcher. This is where equation (33) can be considered a kind of compromise solution between future possibilities, limitations in improving measuring devices, diversity in mathematical calculation methods, and the increasing power of computers.

Under the unrecoverable uncertainty of the model, we mean the initial preferences of the researcher, based on his intuition, knowledge and experience, in the process of its formulation. The magnitude of this uncertainty is an indicator of how likely it is that one's personal philosophical leanings will affect the outcome of this process. Therefore, modeling, like any information process, looks like any similar process in nature: noisy, with random fluctuations and, in our case, an equiprobable choice of quantities that depends on the observer. When a person mentally builds a model, at each stage of construction there is some probability that it will not match the phenomenon to a high degree of accuracy.

The quality of the scientific hypothesis should be judged not only by its correspondence to empirical data, but also by its predictions. In this study, information theory was used to give a theoretical explanation and grounding of the experimental results which determine the precision of different fundamental constants. A focus on "the real" is what allows the information measure approach to explore new avenues in the different physical theories and technologies. The approach proposed here can answer one fundamental question—How are we seeing?—because it is based on the fundamental subject, namely the *System of Base Quantities*. The information approach allows for crafting of a meaningful picture of future results, because it is based on the realities of the present. In this sense, when applying the results of precision research to the limitations that constrain modern physics, it is necessary to clearly understand the research framework and the way the original data can be modified [161]. This can be considered as an additional reason for speedy implementation of the μ_{SI}-hypothesis, the concept of SBQ and, in general, the information approach for analyzing existing experimental data on the measurement of fundamental physical constants. The experimental physics segment is expected to be the most rewarding application for the information method, thanks to a greater demand for high accuracy measurements. The proposed information approach allows for calculating the absolute minimum uncertainty of the measurement of the investigated quantity of the phenomenon, using formula (33). Calculation of the recommended relative uncertainty is a useful consequence of the formulated μ-hypothesis and is presented for application in calculation of relative measurement uncertainty of different physical constants.

The main purpose of most measurement models is to make predictions in verifying the true-target magnitude of the researched quantity. The quantity that needs to be predicted is generally not experimentally observable before the prediction, since otherwise no prediction would be needed. Assessing the credibility of such extrapolative predictions is challenging. In terms of validation, in CODATA's approach, the model outputs for observed quantities are constructed using modern, advanced statistical methods and powerful computers to determine if they are consistent. By itself, this consistency only ensures that the model can predict the measured physical constants under the conditions of the observations

[163]. This limitation dramatically reduces the utility of the CODATA effort for decision making because it implies nothing about predictions for scenarios outside of the range of observations. The μ-hypothesis proposes and explores a predictive assessment process of the relative uncertainty that supports extrapolative predictions for models of measurement of the fundamental physical constants.

The findings of this study are applicable to all the models in physics and engineering, including climate, heat- and mass-transfer, and theoretical and experimental physics systems in which there is always a trade-off between the model's complexity and the accuracy required. On the other side, the proposed method is not claimed to be universally applicable, because it does not answer the question about the selection of specific physical quantities for the best representation of the surrounding world. The information-oriented approach for estimating the model's uncertainty does not involve any spatio-temporal or causal relationship between the quantities involved; instead, it considers only the differences between their numbers. However, it can be firmly asserted that the findings presented here reveal, contrary to what is generally believed, that the precision of physics and engineering devices is fundamentally bounded by certain constraints and cannot be improved to an arbitrarily high degree of accuracy. The outcome of this study, which seems to be too good to be true, indeed turns out to be a real breakthrough.

It is now possible to design optimal models, which use the required number of dimensional quantities that corresponds to the selected SBQ, chosen according to the experimental physics considerations.

The theory of measurements and its concepts remain the correct science today in the 21st century, and will remain faithful forever (paraphrase of Prof. L.B. Okun [164]). The use of the μ_{SI}-hypothesis only limits the scope of the measurement theory for uncertainties exceeding the uncertainty in the physical-mathematical model due to its finiteness. The key idea is that although the basic principles of measurement remain valid, they need to be applied discretely, depending on the stage of model's computerization.

Though the data in and explanations to Tables II–V appear to confirm the predictive power of the μ-hypothesis, the author is skeptical of considering them as "confirmation". In fact, the μ-hypothesis is considered a Black Swan [165] among the existing theories related to checking the discrepancy between a model and the observed object, because none of the existing methods for validating and verifying the constructed model take into account the smallest absolute uncertainty of the model's measured quantity, caused by the choice of the class of the phenomena and the number of quantities created by observation.

"Our knowledge of the world begins not with matter but with perceptions." [125] According to the μ-hypothesis, there are no physical quantities independent of the observer. Instead, all physical quantities refer to the observer. This is motivated by the fact that, according to the information approach, different observers can take different account of the same sequence of events. Therefore, each observer assumes to "dwell" in his own physical world, as determined by the context of his own observations.

An information-oriented approach leads us to the following conclusions. If the mathematics and physics that describe the surrounding reality are effective human creations, then we must take into account the relationship between human consciousness and reality. In addition, the ultimate limits of theoretical, computational, experimental and observational methods, even using the best computers and the most complex experiments such as the Large Hadron Collider, are limited by the μ-hypothesis applicable to any human activity. Undoubtedly, the existing unprecedented scientific and technological progress will continue. However, since a limit for this progress exists, the speed of discoveries will slow down. This remark is especially important for artificial intelligence, which seeks to create a truly super-intelligent machine.

Finally, because the values of comparative uncertainties and the required number of chosen quantities are completely independent and different for each class of phenomena, the attained approach can now, in principle, become an arbitrary metric for comparing different models that describe the same recognized object. In this way, the information measure approach will radically alter the present understanding of the modeling process. In conclusion, it must be said that, fortunately or unfortunately, one sees everything in the world around him through a haze of doubts and errors, excepting love and friendship. If you did not know about the μ-hypothesis, you would not come to this conclusion.

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