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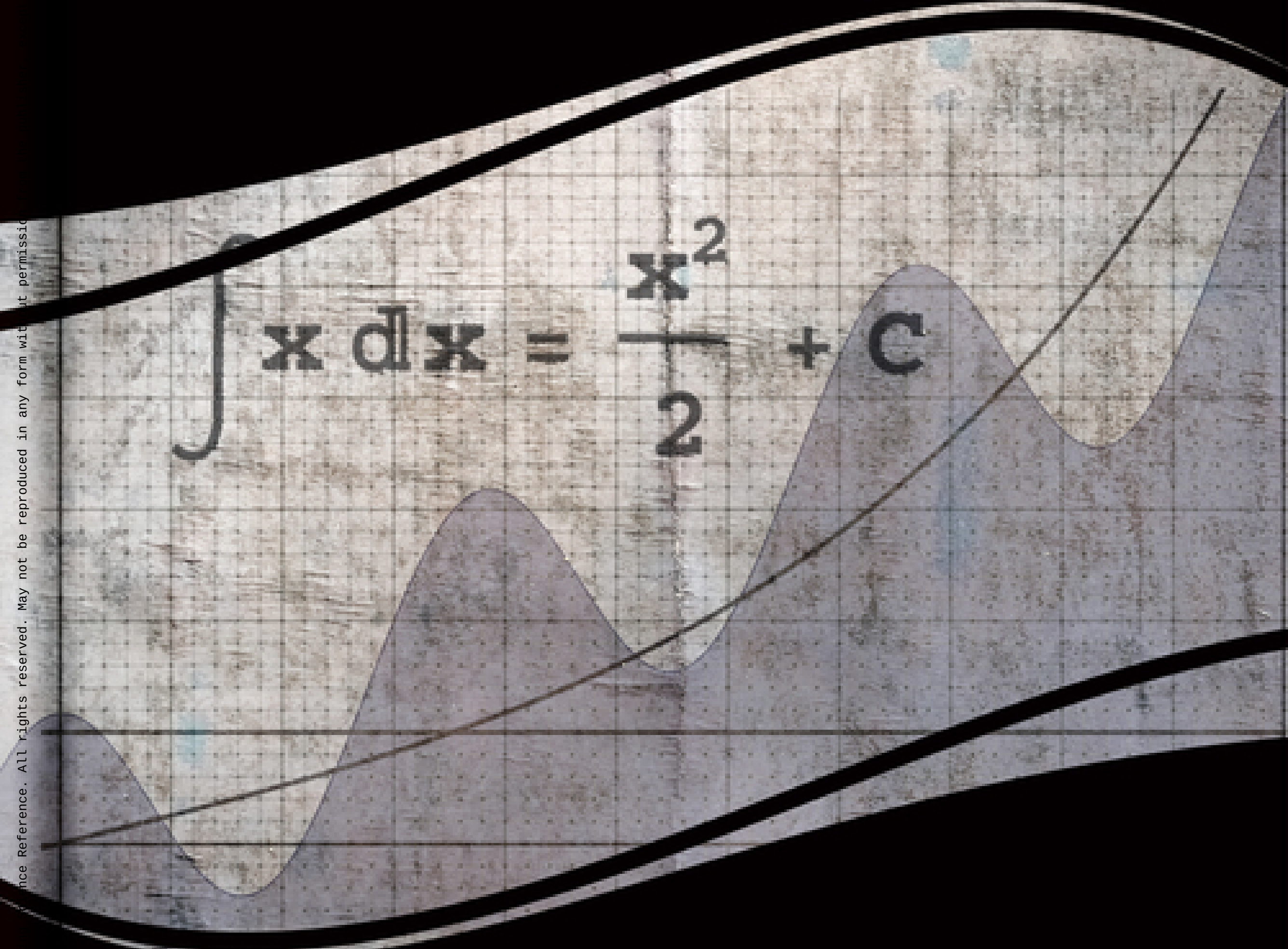
Neutrosophic Graph Theory and Algorithms



Florentin Smarandache and Said Broumi

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Neutrosophic Graph Theory and Algorithms

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A cycle passing through all the vertices exactly once in a graph is a Hamiltonian cycle (HC). In the field of network system, HC plays a vital role as it covers all the vertices in the system. If uncertainty exists on the vertices and edges, then that can be solved by considering fuzzy Hamiltonian cycle. Further, if indeterminacy also exist, then that issue can be dealt efficiently by having neutrosophic Hamiltonian cycle. In computer science applications, objects may not be a crisp one as it has uncertainty and indeterminacy in nature. Hence, new algorithms have been designed to find interval neutrosophic Hamiltonian cycle using adjacency matrix and the minimum degree of a vertex. This chapter also applied the proposed concept in a network system.

The aim of this chapter is to impart the importance of domination in various real-life situations when indeterminacy occurs. Domination in graph theory plays an important role in modeling and optimization of computer and telecommunication networks, transportation networks, ad hoc networks and scheduling problems, molecular physics, etc. Also, there are many applications of domination in fuzzy and intuitionistic fuzzy sets for solving problems in vague situations. Domination in neutrosophic graph is introduced in this chapter for handling the situations in case of indeterminacy. Dominating set, minimal dominating set, independent dominating set, and domination number in neutrosophic graph are determined. Some definitions, characterization of independent dominating sets, and theorems of neutrosophic graph are also developed in this chapter.

neutrosophic sets of value as well as a combination of neutrosophic and crisp value for analyzing the every real-life uncertain situation.

This chapter gives some essential scopes to study some plithogenic algebraic structures. Here the notion of plithogenic subgroup has been introduced and explored. It has been shown that subgroups defined earlier in the crisp, fuzzy, intuitionistic fuzzy, as well as neutrosophic environments, can also be represented as plithogenic fuzzy subgroups. Furthermore, introducing function in plithogenic setting, some homomorphic characteristics of plithogenic fuzzy subgroup have been studied. Also, the notions of plithogenic intuitionistic fuzzy subgroup, plithogenic neutrosophic subgroup have been introduced and their homomorphic characteristics have been analyzed.

In this chapter, the concept of cylindrical single-valued neutrosophic number whenever two of the membership functions, which serve a crucial role for uncertainty conventional problem, are dependent to each other is developed. It also introduces a new score and accuracy function for this special cylindrical single valued neutrosophic number, which are useful for crispification. Further, a minimal spanning tree execution technique is proposed when the numbers are in cylindrical single-valued neutrosophic nature. This noble idea will help researchers to solve daily problems in the vagueness arena.

Neutrosophic soft sets are an important tool to deal with the uncertainty-based real and scientific problems. In this chapter, the idea of neutrosophic soft (NS) digraph has been developed. These digraphs are mainly the graphical representation of neutrosophic soft sets. A graphical study of various set theoretic operations such as union, intersection, complement, cross product, etc. are shown here. Also, some properties of NS digraphs along with theoretical concepts are shown here. In the last part of the chapter, a decision-making problem has been solved with the help of NS digraphs. Also, an algorithm is provided to solve the decision-making problems using NS digraph. Finally, a comparative study with proposed future work along this direction has been provided.

Foreword

Neutrosophic set theory is a generalization of crisp, fuzzy, Pythagorean fuzzy, and intuitionistic fuzzy sets. And refined neutrosophic set, when the neutrosophic components T, I, F are split/refined into subcomponents respectively (T_1, T_2, …; I_1, I_2, …; F_1, F_2, …) is an extension of picture fuzzy set and neutrosophic set.

Neutrosophic Set deals with indeterminacy in the given data sets, beyond the truth-membership and false-nonmembership values. It was started by Prof. F. Smarandache in 1998 to represent win, loss, and tight (neutral/indeterminacy) of any game independently. Previously to that, the fuzzy set, which was introduced by LA. Zadeh, represents membership and non-membership values of any attributes via a single membership-values. The intuitionistic fuzzy sets tried to represent the uncertainty in attributes based on its membership-values and non-membership-values in the given interval [0, 1]. The neutrosophic set theory gave an alternative and a generalized way to represent the uncertainty in given attributes based on its truth, falsity and indeterminacy membership-values independently in $[0, 1]^3$.

The neutrosophic set and its applications haves received significant attention of researchers among the world in various fields. The most prominent are data science, mathematical algebra, graph theory, networking, expert systems, pattern recognition, operations research, robotics, medicine, as well as cognitive science.

Recently, its theory was first introduced in field of applied lattice theory for knowledge discovery and processing tasks by Prem Kumar Singh. He has also introduced interval-valued neutrosophic concept lattice, bipolar neutrosophic concept lattice, complex neutrosophic concept lattice and n-valued neutrosophic concept lattice at different granulation. There are several researchers who started working in this field around the world and completed their theses. The three-ways procedure on a space X is actually a neutrosophication of the space X, i.e. to split the space X into three regions: of acceptance (T), of rejection (F), and of indeterminacy (neutrality) (I).

Recently most of the peer reviewed journals special issues came on this topic which approves its trends in scientific communities.

Most of the real-life problem data contains uncertainty, indeterminacy and incompleteness to deal with them this set theory is considered as one of the prominent. This book will represent various applications and algebra of neutrosophic set. The objectives of this book is to present the development of several mathematical approach for neutrosophic set, and its recent extension i.e. called as plithogenic set. The authors and editors of this book are well-known researchers in field of neutrosophic set theory and given many interesting concept in this fields. Now, he has made another scientific contribution to the neutrosophic set theory and its application in various fields with publication of this book.

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Preface

Graph Theory is a significant area of Discrete Mathematics. Graphs are practical to explain discrete structures, since they use a basic functional structure that can model discrete objects. Starting with Konigsberg's Bridge Problem, this adventure continues to be a very important tool in many areas. We can say that Graph Theory is a haunt for almost every workspace that can be represented by feature networks. When a little research is done on the web for Graph Theory, which has already turned into a field in itself, it can be witnessed its many applications ranging from biology to engineering, and from medical to geographical systems. Graphs consist of dots and lines connecting these dots. The dots of a graph (the graph doesn't refer to the picture one gets when one plots a function on the coordinate plane) are called vertices and the lines are called edges. Briefly, Graph Theory is the study of graphs. Graph theory is the domain of study of many different disciplines in the literature. It is used in different fields from sociology to mathematics, business to computer science. For example, in discrete mathematics under computer science, or operational research under industrial engineering, or as a field of study of mathematics. Graph theory is basically based on the principle that a problem is modeled with edges and nodes and that this model is represented as a graph. Some of the features defined in graph theory help to solve this model and hence solve the real problem. So, for graph theory to work with, first of all, the problem should be modeled as a graph from the real world, this model is solved and then applied. Graph theoretic approach provides to analyze and comprehend systems as a whole by identifying systems and subsystems up to their component levels. The approach and its matrix model are formed by matrices, digraphs and permanent representations. Digraph representation is useful for modelling and visual analysis and its transformation to matrix representation is operable in analyzing the digraph model mathematically, or for computer processing.

The Graph Theory mentioned so far is the crisp one. Real life is full of uncertain systems and models. The first model that springs to mind when it comes to models under uncertainty is of course Fuzzy. When modeling real-life problems, sometimes vertices and edges (or both) may need ambiguous structures. This need laid the groundwork for Fuzzy Graph Theory field. Kauffman (1973) introduced the notion of a fuzzy graph. The main contributions to Fuzzy Graph Theory and the development of fuzzy graph theory come from the papers of Rosenfeld et al. (1975) and Yeh and Bang (1975). Yeh and Bang gave different connectivity parameter definitions and results, and discussed their applications. Rosenfeld's paper presented basic structural and connectivity concepts. Rosenfeld obtained the fundamental fuzzy notions of varied graph-theoretic properties such as paths, cycles, trees, connectedness, and bridges. Fuzzy graph theory is finding more and more applications. Cluster analysis, decision analysis, data base theory, pattern classification, social sciences, portfolio management, neural networks, and many other areas (Mordeson and Nair, 2000) have been found applications. Graphs obtained from the Intuitionistic Fuzzy Sets where uncertainty is evaluated one step further are found when Fuzzy Sets and their graph models are inadequate.

Intuitionistic fuzzy graph and its concepts are introduced by Karunambigai and Parvathi (2006). These concepts had been applied to find the shortest path in networks using Dynamic Programming Problem approach and further, some important operations on intuitionistic fuzzy graphs were defined and their properties were studied by Karunambigai, Parvathi and Atanassov (2009). Akram and his collaborations studied some properties like operations, cycles, trees, hypergraphs and decision support systems on Intuitionistic Fuzzy Graphs (Akram and Dayyaz, 2012; Akram, Ashraf and Swrwar, 2015; Akram and Dudek, 2013; Akram and Al-Shehrie, 2014; Akram and Akmal, 2017; Akram and Akmal, 2016).

It is observed that these areas, where their studies and applications are produced in a wide frame, are inadequate in modeling networks and different types of systems with certain limits when dealing with Indeterminacy. Neutrosophic Set Theory was proposed by Florentin Smarandache to overcome these limits.

Neutrosophic set is a generalization of fuzzy set and of intuitionistic fuzzy set. Neutrosophic models give more precision, flexibility, and compatibility to the system as compared to Intuitionistic Fuzzy models. The key distinction between the neutrosophic set and other types of sets is the introduction of the degree of indeterminacy / neutrality (I) as independent component in the neutrosophic set. In the neutrosophic set, the degree of membership-truth (T), the degree of indeterminacy (I), and the degree of non-membership-falsehood (F) are independent, therefore their sum (as single valued numbers) can be up to 3. Neutrosophic set has been used in solving problems that involve indeterminacy, uncertainty, impreciseness, vagueness, inconsistent, incompleteness etc. In the past years, the field of neutrosophic set, logic, measure, probability and statistics, pre-calculus and calculus etc. and their applications in multiple fields have been extended to various fields. For more information, see the University of New Mexico's website on: http://fs. unm.edu/neutrosophy.htm.

Operational Research, II. It is a relatively new discipline originating from World War II and very popular around the world. India is one of the first countries in the world to use Operational Research. Operational Research has been successfully used not only in army / military operations, but also in business, government and industry. Nowadays, Operational Research is used in almost all areas. It is difficult to offer an exact definition for Operational Research since its borders and content are not stable. The tools for Operational Research are provided by the subject exam, psychology, mathematics, economics, engineering, statistics etc. Operational Research tools / techniques contain: linear programming, nonlinear programming, dynamic programming, integer programming, Markov operation, queuing theory, etc. Operational Research has many applications. Similarly, there is a number of constraints that basically relate to time, money, and the problem of model building. Day-by-day Operational Research is gaining popularity and respect because it increases the efficiency of managers to decide. Almost all business areas are investigating operations to make better decisions. Operational Research is necessary for a particular purpose in a situation where there are certain constraints to simply ensure that the most appropriate solution is found - using a set of methods. An organization, in the coordination and execution of operations related to complex real-world problems, aims to achieve effective results using the science of statistics. When Operational Research was put forward, scientists realized that most business problems they faced could be mapped to a number of lower level generic families of problems. For this reason, they devoted much of their time and bandwidth to studying these issues, giving them specific names and offering solutions for them. Generic Operations Research problems are sufficient to be described in mathematical notation (especially Graph Theoretic). However, Operations Research practitioners usually express these problems using higher-level modelling languages. A few main examples: The Set Covering Problem, The Travelling Salesman Problem (TSP) The Shortest Path Problem (SPP), The Graph Coloring Problem as an example of the higher-level modelling language, scheduling problems are often described using terms such as precedence constraints, resources and activities. Addition to this, decision analysis and simulation, statistics, optimization, randomness, queuing, game and graph theory, often used by Operations research practitioners. There too many details such as basics of graphs, connectivity in graphs: paths, cycles, diameter, connected components, cuts, etc.; adjacency matrix algebra and transitive closure, pseudocode for algorithms and algorithmic, search algorithms: breadth first and depth first using queues and recursive function; trees: minimum cost spanning tree problem, Kruskal and Prim algorithms, shortest path problem: Bellman-Ford and Dijkstra algorithms; generalized Dijkstra algorithm: max-flow min-cut problem and applications; other graph problems such as matching, coloring, and clustering; analyzing large networks (Markov chains): transition matrix, states classification, marginal distribution, stationary probabilities; solving and simulate Markov chains; linear programming: definition, graphical solution in the case of two decision variables, overview of the simplex method; modeling real-world problems using linear programming and solving when using Graph Theory in Operations Research. Thus, the enrichment of neutrosophic models, systems, and environments with Neutrosophic Graph Theory can help many discoveries and solutions in industry and engineering to work on a much richer and broader range of problems in the field of operations research. Neutrosophic Graph Theory can serve as a life preserver where the problems and systems of Fuzzy and Intuitionistic Fuzzy Graphs are insufficient or incomplete. With Neutrosophic Graph Theory, today's social networks can be examined in more detail. In social networks, people, animals, cars, houses, diseases can be the main objects with many different features. When evaluating people only as objects, gigantic data emerges depending on the skill of the study. For each person, some data cannot be measured with certainty within the scope of work. In particular, when many features such as sentiment, taste, reaction, hatred and pleasure are measured with precise judgments, inadequate or incorrect data are used in order to obtain accurate results. Being the focus of advertising, human beings carry out numerous researches on social media and tools every day. These research utilize many tools that can be applied to graph basis such as optimization, network research and decision making. In this direction, Neutrosophic Graph Theory can contribute under uncertain environments in the field of work of both human network and industrial fields, especially in Operations Research. On the other hand, Fuzzy and Intuitionistic Fuzzy digraph theories have numerous applications in modern sciences and technology, especially in the fields of operations research, neural networks, artificial intelligence, and decision making. Intuitionistic fuzzy models give more precision, flexibility, and compatibility to the system as compared to the fuzzy models.

The objective of this volume is to point out the importance of neutrosophic graph theory with its problem solving operations called algorithms. As the world is full of indeterminacy, the neutrosophic theory establishes its place into modern research. This book contains 12 chapters covering various aspects of neutrosophic graph theory and algorithms from theoretical to application problems.

The concept of neutrosophic set is a more functional tool for handling indeterminate, inconsistent and uncertain information that exist in real life compared to fuzzy sets or to intuitionistic fuzzy sets.

If the relation between the elements of the vertex is indeterminate, fuzzy graph and its extension fails. In Chapter 1 by Said Broumi, Shio Gai Quek, Ganeshsree Selvachandran, Florentin Smarandache, Assia Bakali and Mohamed Talea study a kind of network where the edge weights are characterized by single-valued triangular neutrosophic numbers were introduced. First, rigorous definitions of nodes, edges, paths, and cycles of such a network were proposed, which must be defined in algebraic terms. Then, characterization on the length of paths in such a network were presented. This follows by the presentation an algorithm for finding the shortest path length between two given nodes on the network. The proposed algorithm gives the shortest path length from source node to destination node based on a ranking method which takes both the length of edges and the number of nodes into account. Finally, a numerical example based on a real life scenario is also presented to illustrate the efficiency and usefulness of the proposed approach. An attempt to combine the two concepts of neutrosophic garphs and soft expert set theory is given in Chapter 2 by Vakkas and Memet Şahin introduce a new graph structure called neutrosophic soft expert graph, establishing a link between graphs and neutrosophic soft expert sets. They discussed different operations defined on neutrosophic soft expert graphs using examples to make the concept easier to understand. They introduced the concepts of neutrosophic soft expert graph, strong neutrosophic soft expert graph, their union and intersection, and explained them with examples. They have wide applications in the fields of modern sciences and technology, especially in research areas of computer science, including database theory, data mining, neural networks, expert systems, cluster analysis, control theory, and image capturing. The concept of strong neutrosophic soft expert graphs and the complement of strong neutrosophic soft graphs are also discussed. Neutrosophic soft expert graphs are pictorial representation in which each vertex and each edge is are elements of neutrosophic soft sets.

There is a proficiency to define neutrosophic graph, single-valued neutrosophic graph, and interval valued neutrosophic graphs.

It is possible to apply neutrosophic graphs in network systems, especially to solve neutrosophic shortest path problems. In Chapter 3, Malayalan Lathamahesw ari,Deivanayagampillai Nagarajan,Said Broumi, Florentin Smarandache and Jacob Kavikumar introduce an extended version of Floyd's algorithm to solve the shortest path problem under neutrosophic environment. Since the data collected for the real world problems may be interval data and expressing the measure of indeterminacy interms of interval instead of crisp numbers provides improved result of fuzzy and neutrosophic mathematics, here the shortest path has been found for the network by considering the edge weights are interval neutrosophic numbers, trinangular interval neutrosophic numbers and trapezoidal interval neutrosphic numbers. Hence this algorithm will be very useful to find the shortest path of the given network for interval data. Also Also comparative analysis has been done with the existing methods.

Also in Chapter 4, Malayalan Lathamaheswari, Deivanayagampillai Nagarajan, Said Broumi, Florentin Smarandache and Jacob Kavikumar introduce the methodology to find the Hamiltonian cycle for the network under interval neutrosophic environment. New algorithms have been introduced to find interval Hamiltonian path and interval Hamiltonian cycle using adjacency matrix and minimum vertex degree Here, the network in which the cities are connected by the flight routes is considered and interval neutrosophic Hamiltonian path and interval neutrosophic Hamiltonian cycle have been found. Also comparative analysis has been done with the existing methods.

Also, in Chapter 5 by Mullai Murugappan the concept of dominations in neutrosophic graph is established. The various types of dominations in neutrosophic graph such as domination number, dominating set, minimal dominating set, independent neutrosophic dominating set and isolated vertex are also derived with suitable examples. Some theorems and propositions on dominations in neutrosophic graphs are developed. Also the characterization theorem of independent neutrosophic dominating set and applications of domination in neutrosophic graphs are examined as well.

Contuining the studies related to neutrosophic shortest path problem. Chapter 6 by Ranjan Kumar, Arindam Dey, Said Broumi and Florentin Smarandache presents mathematical model of SPP in neutrosophic environment is approached as a neutrosophic shortest path problem (NSPP). The utility of neutrosophic set as arc lengths and its real life applications are described. Further, different operators are applied to handle multi-criteria decision making problems.

In Chapter 7 by Jayanta Pratihar, Ranjan Kumar, Arindam Dey and Said Broumi solve neutrosophic transportation problems using the extended version of linear programming principle, extension of north west principle, extension of Vogel's approximation method (VAM) principle and extended principle of MODI method.

Chapter 8 by Sudipta Gayen, Florentin Smarandache, Sripati Jha, Manoranjan Singh, Said Broumi and 1Ranjan Kumar explores the advancement of neutrosophic theory and plithogenic theory, which can be helpful for scientists and researchers in mathematics, computer science and other disciplines. It provides a detailed introduction and it explains the fundamental ideas of plithogenic subgroup.

The prerequisites are some knowledge of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic theory. Also, some understandings in fuzzy, intuitionistic fuzzy and neutrosophic algebraic structures are required.

In this chapter the notion of plithogenic subgroup is introduced and explored with proper examples. It has been shown that subgroups defined earlier in the crisp, fuzzy, intuitionistic fuzzy, as well as neutrosophic environments, can also be represented as plithogenic fuzzy subgroups. Furthermore, some homomorphic characteristics of plithogenic fuzzy subgroup are studied. Also, the notions of plithogenic intuitionistic fuzzy subgroup, plithogenic neutrosophic subgroup are introduced and their homomorphic characteristics are studied.

In Chapter 9 by Avishek Chakraborty able to define a new form of neutrosophic number, namely cylindrical single –valued neutrosophic number. This new concept is related with the neutrosophic number of type-2, whenever any two membership functions are dependent to each other. Demonstration of score and accuracy function has been developed which plays an important role in crispification. Utilizing this crispification idea solved a minimum spanning tree problem in cylindrical single-valued environment. Finally, comparison is done with the other established method of minimal spanning tree problem and this noble idea will help researchers to solve different kinds of daily life problems in neutrosophic arena.

Chapter 10 by İrfan Deli and Emel Kırmızı Öztürk is about two centroid points called 1. and 2. centroid point for single valued neutrosophic numbers (SVN-numbers) are given. Then, a new single valued neutrosophic multiple attribute decision making method based on concept of 1. and 2. centroid point is developed. Moreover, an application is proposed in fuzzy graph because centroid points of normalized SNTN-numbers or SNTrN-numbers are fuzzy values. Furthermore, in Chapter 11 by Jacob Kavikumar, Deivanayagampillai Nagarajan, Said Broumi, Malayalan Lathamaheswari and Gan Jian Yong introduce and define some notions neutrosophic finite automata such as distinguishable, rational states, semi-inverses and inverses. Interestingly, every state in finite automata is said to be rational when its inputs are ultimately periodic sequence that yields an ultimately periodic sequence of outputs. Based on that they show that any given state is rational when its corresponding sequence of states is distinguishable. Furthermore, they have proved that the semi-inverses of two neutrosophic finite automata are indistinguishable. They proved that any neutrosophic finite automata and its inverse are distinguished, and then their reverse relation also distinguished.

Finally, the Chapter 12 by Kalyan and Pinaki presents the concept of neutrosophic soft digraph and its properties are developed. An algorithm is provided to solve decision making problems.

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Chapter 1
Finding the Shortest Path With Neutrosophic Theory:
A Tool for Network Optimization

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ABSTRACT

In this chapter, the authors study a kind of network where the edge weights are characterized by single-valued triangular neutrosophic numbers. First, rigorous definitions of nodes, edges, paths, and cycles of such a network were proposed, which are then defined in algebraic terms. Then, characterization on the length of paths in such a network were presented. This is followed by the presentation of an algorithm for finding the shortest path length between two given nodes on the network. The proposed algorithm gives the shortest path length from source node to destination node based on a ranking method that takes both the length of edges and the number of nodes into account. Finally, a numerical example based on a real-life scenario is also presented to illustrate the efficiency and usefulness of the proposed approach.

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1. INTRODUCTION

In 1995, the concept of the neutrosophic sets and neutrosophic logic were introduced by (Smarandache, 2005;2006) in order to efficiently handle the indeterminate and inconsistent information which exists in the real world. Unlike fuzzy sets which associate to each member of a fuzzy set a degree of membership T and intuitionistic fuzzy sets which associate a degree of membership T and a degree of non-membership F, where $T,F\in[0,1]$, NSs are characterized by a truth-membership function $T(x)$, indeterminacy-membership function $I(x)$ and a falsity- membership function $F(x)$, each of which belongs to the non-standard unit interval of $]^{-}0, 1^{+}[$. Although, IFSs do have the ability to consider some indeterminacy or hesitation margin denoted by π, and can be computed via $\pi=1-T-F$. NSs have the ability of handling uncertainty in a better way since in the case of NSs, the indeterminacy is handled independently from the truth and falsity aspects of the information. NSs are in fact a generalization of the theory of fuzzy sets (Zadeh,1965), intuitionistic fuzzy sets (Atanassov,1986), interval-valued fuzzy sets(Turksen) and interval-valued intuitionistic fuzzy sets(Atanassov and Gargov,1989). However, neutrosophic theory is difficult to be directly applied in real life problems in the areas of engineering, science and technology.To easily use it in science and engineering areas, (Wang et al.,2010) proposed the concept of a single-valued neutrosophic set. The SVNS model is also characterized by the truth, indeterminacy and falsity membership functions, but all of these membership functions lie in [0, 1], instead of the non-standard unit interval of $]^{-}0, 1^{+}[$. Presently research on NS theory and its applications are progressing very rapidly, and some of the prominent works can be found in (http://fs.gallup.unm.edu/NSS., Liu et al. ;2016 ;2016a ; 2016b ; 2016c ; 2017 ; 2017, ŞAHİN and Liu ;2016).Recently, (Subas et al., 2015) presented the concept of triangular and trapezoidal neutrosophic numbers and applied it to multiple-attribute decision making (MADM) problems. (Biswas et al., 2014) then presented a special case of trapezoidal neutrosophic numbers which included triangular fuzzy numbers in neutrosophic sets and applied it to MADM problems by introducing a cosine similarity measure. (Deli and Subas, 2016) proposed the concept of single-valued triangular neutrosophic numbers (SVTrNNs) as a generalization of the intuitionistic triangular fuzzy numbers, and subsequently proposed a methodology for solving MADM problems using SVTrNNs.

The shortest path problem which concentrates on finding a shortest path from a source node to other nodes, is a fundamental network optimization problem that has appeared in many domains including road networks application, transportation, routing in communication channels, and scheduling problems in various fields. The main objective of the shortest path problem is to find a path with minimum length between the starting node and the terminal node which exist in a given network. The edge (arc) length (weight) of the network may represent any real life quantities such as cost, time and weight. In the conventional shortest path problem, the distance between the edges and different nodes of a network are assumed to be certain. In literature, many algorithms have been developed that considers the weights of the edges on a network, where the weights may be in the form of fuzzy numbers, intuitionistic fuzzy numbers, type-2 fuzzy numbers, and vague numbers (Porchelvi and Sudha, 2013; Jayagowri and Ramani, 2014;Anuuya and Sathya, 2013; Kumar and Kaur, 2011; Majumdaer and Pal,2013; Kumar and Kaur, 2011a)

In more recent times, (Broumi et al., 2016; 2016a; 2016b; 2016c; 2016d; 2016e; 2016f) presented the concept of neutrosophic graphs, interval-valued neutrosophic graphs and bipolar single-valued neutrosophic graphs, and proceeded to study some of their related properties. (Smarandache, 2015;2015a) proposed another variant of neutrosophic graphs based on literal indeterminacy. Presently, only a few papers dealing with the shortest path problem in neutrosophic environments have been developed. One of the pioneering works in this area is due to (Broumi et al.,2017), in which the authors proposed an algorithm for solving neutrosophic shortest path problems based on score function. The same authors (Broumi et al.,2016g),also proposed another algorithm for solving shortest path problems in a bipolar neutrosophic environment. For more information on the application of neutrsophic theory we refer the readers (Jeyanthi and Radhika, 2018; Ranjan et al. 2019; Singh PK,2019; Broumi et al.,2019;Broumi et al., 2018; Singh PK,2017; Singh PK; 2018; Singh PK; 2018a, Singh PK; 2018b Singh PK; 2019a). In (Broumi et al.,2019), the authors proposed a shortest path algorithm in a network with its edge lengths denoted as interval-valued neutrosophic numbers. However, up till now, SVTrNNs have not been applied to shortest path problems. The main objective of this paper is to propose an approach for solving shortest path problem in a network where the edge weights are represented by SVTrNNs.

The remainder of this paper is organized as follows. In Section 2, we review some of important concepts related to NSs, SVNSs and SVTrNSs. In Section 3, we propose some modified operations for SVTrNNs. In Section 4, we propose an algorithm for finding the shortest path and shortest distance between SVTrN graphs. In Section 5, our proposed algorithm is applied to a hypothetical example. Concluding remarks are presented in Section 6, followed by the list of references.

2. PRELIMINARIES

In this section, we present an overview of the concepts related to NSs, SVNNs and SVTrNSs that are relevant to this study.

Definition 2.1 (Smarandache, 2005). Let *X* be a space of points (objects) with generic elements in *X,* denoted by *x*. Let *A* be defined as:

$$A = \left\{\left(x, T_A\left(x\right), I_A\left(x\right), F_A\left(x\right)\right) : x \in X\right\}, \tag{1}$$

where *T*, *I*, and *F* are functions from *X* to]−0,1+[$= \mathbb{R} - \left[0,1\right]$. Then:

1. *A* is called a neutrosophic set (NS) on *X*.

T_A, I_A and F_A are called the truth-membership function, the indeterminacy-membership function, and the falsity-membership function of *A*, respectively.

Since it is difficult to apply NSs to practical problems, (Wang et al., 2010) introduced the concept of single-valued neutrosophic sets (SVNSs) which is an instance of a NS and can be used more conveniently in real-life scientific and engineering applications.

Definition 2.2 (Wang et al.,2010). Let *X* be a space of points (objects) with generic elements in *X* denoted by *x*. Let:

$$A = \left\{\left(x, T_A\left(x\right), I_A\left(x\right), F_A\left(x\right)\right) : x \in X\right\},$$

where T_A, I_A and F_A are functions from *X* to [0,1]. Then:

1. *A* is called a *single-valued neutrosophic set* (SVNS) on *X*.
2. T_A, I_A and F_A are called the *truth-membership function*, the *indeterminacy-membership function*, and the *falsity-membership function* of *A*, respectively.

The restriction on the sum of the values of $T_A(x)$, $I_A(x)$, $F_A(x)$ is

$$0 \le T_A\left(x\right) + I_A\left(x\right) + F_A\left(x\right) \le 3,$$

which holds for all $x \in X$.

Definition 2.3 (Deli and Subas, 2016). A *single valued triangular neutrosophic number* (abbr. SVTrN-number) $A=(a,b,c); t,l,f$ is a kind of SVNS on the real number set R, whose truth membership, indeterminacy-membership, and a falsity-membership are characterized as follows:

$$T(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}t & a<x<b\\ t & x=b\\ \dfrac{(c-x)}{(c-b)}t & b<x<c\\ 0 & otherwise\end{cases}. \tag{3}$$

$$I(x)=\begin{cases}1+\dfrac{(x-a)}{(b-a)}(l-1) & a<x<b\\ l & x=b\\ 1+\dfrac{(c-x)}{(c-b)}(l-1) & b<x<c\\ 1 & otherwise\end{cases}. \tag{4}$$

$$F(x)=\begin{cases}1+\dfrac{(x-a)}{(b-a)}(f-1) & a<x<b\\ f & x=b\\ 1+\dfrac{(c-x)}{(c-b)}(f-1) & b<x<c\\ 1 & otherwise\end{cases}. \tag{5}$$

where $a, b, c \in \mathbb{R}$ with $a \leq b \leq c$, and $t,l,f \in [0,1]$.

In (Deli and Subas, 2016) proposed a way of "adding" two SVTrN-numbers, as shown in the following definition.

Definition 2.4 (Deli and Subas, 2016). Let $A_1=(a_1,b_1,c_1)$; t_1,l_1,f_1 and $A_2=(a_2,b_2,c_2)$; t_2,l_2,f_2 be two SVTrN-numbers. Then the operator ⊕ is defined on A_1 and A_2 as follows.

$$\mathrm{A}_1 \oplus \mathrm{A}_2 = \left(a_1 + a_2, b_1 + b_2, c_1 + c_2\right); \min(t_1, t_2), \max(l_1, l_2), \max(f_1, f_2) \tag{6}$$

Remark: Note that the operator min is a T-norm; and the operator max is the S-norm (i.e. the T-conorm) correspond to min.

With the method of adding two SVTrN-numbers established, we then need a way to describe the "size" of a SVTrN-number. We will also need a method of comparing two SVTrN-numbers to determine which one is "greater" or "smaller", based on their sizes as described. These were likewise proposed by Deli and Subas with their concept of *score function*, *accuracy function,* as well as a method to rank SVTrN-numbers, as shown in the following two definitions.

Definition 2.5 (Deli and Subas, 2016). Let A-(a,b,c); t,l,f be a SVTrN-number. Then, the *score function* and the *accuracy function* of A, denoted respectively as $\sigma(A)$ and $\alpha(A)$, are defined as follows:

$$\sigma\left(A\right) = \left(\frac{1}{12}\right)\left(a + 2b + c\right)\left(2 + t - l - f\right) \tag{9}$$

$$\alpha\left(A\right) = \left(\frac{1}{12}\right)\left(a + 2b + c\right)\left(2 + t + l - f\right) \tag{10}$$

Definition 2.6 (Deli and Subas, 2016). Let $A_1=(a_1,b_1,c_1)$; t_1,l_1,f_1 and $A_2=(a_2,b_2,c_2)$; t_2,l_2,f_2 be two SVTrN-numbers. Then the ranking of A_1 and A_2 by score function and accuracy function are defined as follows:

1. If $\sigma(A_1)>\sigma(A_2)$, then $A_1 \succ A_2$.
2. If $\sigma(A_1)<\sigma(A_2)$, then $A_1 \prec A_2$.
3. If $\sigma(A_1)=\sigma(A_2)$, and if

a. α(A_1)>α(A_2), then $A_1 \succ A_2$.
b. α(A_1)<α(A_2), then $A_1 \prec A_2$.
c. α(A_1)=α(A_2), then $A_1 \sim A_2$ (i.e. the same rank)

Remark 1: If it is either $A_1 \prec A_2$ or $A_1 \sim A_2$, then we shall denote $A_1 \preceq A_2$. Likewise, if it is either $A_1 \succ A_2$ or $A_1 \sim A_2$, then we shall denote $A_1 \succeq A_2$.

Remark 2: By convention, if $(a_1,b_1,c_1, t_1,l_1,f_1)=(a_2,b_2,c_2, t_2,l_2,f_2)$, then $A_1=A_2$ (which clearly implies $A_1 \sim A_2$).

3. OUR CHOICE OF ADDING TWO SVTrN-NUMBERS

In our paper, we faithfully adopt Definition 2.5 and 2.6 as proposed by (Deli and Subas, 2016). However, a slight modification has been made on the operator ⊕ in Definition 2.4 by choosing a different T-norm (and hence the corresponding S-norm will also differ). This is required for our real life scenario which we shall discuss in the later sections.

We now give the following definition.

Definition 3.1 Let $A_1=(a_1,b_1,c_1)$; t_1,l_1,f_1 and $A_2=(a_2,b_2,c_2)$; t_2,l_2,f_2 be two SVTrN-numbers. Then the operator + is defined on A_1 and A_2 as follows.

$$A_1 + A_2 = \left(a_1 + a_2, b_1 + b_2, c_1 + c_2\right); t_1 t_2, l_1 + l_2 - l_1 l_2, f_1 + f_2 - f_1 f_2 \tag{11}$$

Remark 1: $l_1+l_2-l_1l_2=1-(1-l_1)(1-l_2)$.

4. NETWORK TERMINOLOGY

Before we come the shortest path problem, we must rigorously define all terminology involved, such as “node”, “edge”, “path”, “cycle” etc. Those definitions, which may seem fundamental, are indispensable as we will be dealing with our shortest path problem in an algebraic manner.

The set of all SVTrN-number shall be denoted as $\mathfrak{T}$ for the following section of this paper.

Definition 4.1 Let *V* be a non empty set. Let E ⊆ *V*×*V*. Let $d : E \to \mathfrak{T}$. Then *G*=*V,E,d* is said to be an *SVTrN-network.*

Remark 1: In analogy to classical graph theory, each *v*∈*V* is said to be a *node* (or *vertex*) of *G*, and each (*u,v*)∈*E* is said to be *directed edge* of *G* with *d*(*u,v*) being its length. Furthermore, *u* is said to be a *predecessor node* of *v*, and *v* is said to be a *succesor node* of *u*, whenever (*u,v*)∈*E*.

Remark 2: $d\left(u,v\right) = \left(a_{u,v}, b_{u,v}, c_{u,v}\right); t_{u,v}, l_{u,v}, f_{u,v} \in \mathfrak{T}$ for all (*u,v*)∈*E*.

Definition 4.3a Let *G*=*V,E,d* be an SVTrN-network. Let *P* be an ordered set $\left(\kappa_1, \kappa_2, \ldots, \kappa_{\lambda-1}, \kappa_\lambda\right)$ for some λ≥1, with $\kappa_1, \kappa_2, \ldots, \kappa_{\lambda-1}, \kappa_\lambda$ being λ distinct nodes in *V*, and $\left(\kappa_\varrho, \kappa_{\varrho+1}\right) \in \mathrm{E}$ whenever 1≤ϱ≤ λ -1. Then *P* is said to be a *path* in *G*.

Remark 1: The set of all paths in *G* shall be denoted as $\mathfrak{P}_G$. So we shall denote $P \in \mathfrak{P}_G$.

Remark 2: If λ=1, then the entire path consists of a single node $\left(\kappa_1\right)$. Moreover, if λ=2, then the entire path consists of a single directed edge $\left(\kappa_1, \kappa_2\right)$.

Remark 3: Let *v*∈*V*. Recall that $P = \left(\kappa_1, \kappa_2, \ldots, \kappa_{\lambda-1}, \kappa_\lambda\right)$. The ordered set obtained by appending *P* with the node *v*, $\left(\kappa_1, \kappa_2, \ldots, \kappa_{\lambda-1}, \kappa_\lambda, v\right)$, shall be denoted as apnd(*P,v*).

Definition 4.3c Let *G*=*V,E,d* be an SVTrN-network. Let *u,v*∈*V*. Let *P*=(*u,…,v*) $\in \mathfrak{P}_G$. Then:

1. *u* is said to be the *source node* of *P*, for which we may denote *P*=*P*[*u*→].
2. *v* is said to be the *destination node* of *P*, for which we may denote *P*=*P*[→*v*].

   P is said to be a path *from u to v*, for which we may denote *P*=*P*[*u*→*v*].

Remark 1: There can be multiple paths from *u* to *v,* for example: *P*[*u*→*v*]=(*u,w,v*), *Q*[*u*→*v*]=(*u,x,v*), and with *u,v,w,x* being 4 *distinct* nodes in *V* .

Recall that, in classical graph theory, if a path consists of a single node, it is said to have the length zero. In order for the concept of zero length being carried onto paths within SVTrN-network. We must have an element in $\mathfrak{T}$ that behaves like zero in numbers.

Denote $\tilde{0} = \left(0,0,0\right); 1,0,0 \in \mathfrak{T}$, it follows directly from Definition 3.1 that $A + \tilde{0} = \tilde{0} + A = A$. This property clearly resembles the role of 0 in real numbers. Moreover, from definition 2.5, $\sigma\left(\tilde{0}\right) = \alpha\left(\tilde{0}\right) = 0$.

We now defined the following

Definition 4.4 Let G be an SVTrN-network. Let $P = \left(\kappa_1, \kappa_2, \ldots, \kappa_{\lambda-1}, \kappa_\lambda\right) \in \mathfrak{P}_G$. Then:

1. λ is said to be the *number of nodes along P*, and is denoted as $N(P)$.
2. The *length* of P, denoted as $D(P)$, is defined to be $\sum_{r=1}^{\lambda-1} d\left(\kappa_r, \kappa_{r+1}\right)$ if $N(P)>1$, and $\tilde{0}$ if $N(P)=1$.

Remark 1: Hence, $N : \mathfrak{P}_G \to \mathbb{N}$, and $D : \mathfrak{P}_G \to \mathfrak{T}$

Remark 2: As mentioned, the way of adding SVTrN-numbers shall follow Definition 3.1 instead of Definition 2.4 of this paper.

Remark 3: It is clear that $N(P) \leq |V|$.

Definition 4.5 Let $G=V,E,d$ be an SVTrN-network. Let $u,v \in V$. Let P, $Q \in \mathfrak{P}_G$. Then:

1. P is said to be *shorter than Q* if $D\left(P\right) \prec D\left(Q\right)$.
2. P is said to be *longer than Q* if $D\left(P\right) \succ D\left(Q\right)$.
3. P is said to be *the same length as Q* if $D(P) \sim D(Q)$.

Definition 4.6 Let $G=V,E,d$ be an SVTrN-network. Let $P=P[u \to v] \in \mathfrak{P}_G$. Then P is said to be *a shortest path* from u to v if either one of the conditions holds for all $Q = Q\left[u \to v\right] \in \mathfrak{P}_G$:

1. $D\left(P\right) \prec D\left(Q\right)$.
2. $D(P) \sim D(Q)$, but $N(P) \leq N(Q)$. (i.e. length prioritize over number of nodes)

Remark 1: Let $\mathfrak{h} \subseteq \mathfrak{P}_G$, the set of shortest path(s) among $\mathfrak{h}$ shall be denoted as $\mathfrak{S}\left(\mathfrak{h}\right)$. Note that, despite being a set itself, it is clear that all paths in $\mathfrak{S}\left(\mathfrak{h}\right)$ have *equal length* and contain *equal number of nodes*. Therefore we shall apply the notation D and N from section ii of Definition 4.4, denoting the length of those path in $\mathfrak{S}\left(\mathfrak{h}\right)$ by $D\left(\mathfrak{S}\left(\mathfrak{h}\right)\right)$, and the number of nodes by $N\left(\mathfrak{S}\left(\mathfrak{h}\right)\right)$.

5. SVTrN PATH PROBLEM

5.1 Preliminaries

In this section, we are specifically given two nodes s,t of an SVTrN-network $G=V,E,d$. We want to find *the shortest path(s)* from s to t in G in accordance with Definitions 4.3, 4.4 and 4.5. Our work is motivated by Kumar in [15]. Let us be reminded that, by part iii of Definition 4.5, it is theoretically possible to have multiple shortest paths from s to t.

In order to describe our algorithm, we shall precisely define certain sets related to G, which shall be referred throughout the following passages of this paper.

We shall also denote the power set of $\mathfrak{P}_G$ by $\wp\left(\mathfrak{P}_G\right)$.

Definition 5.1.1:

$$\mathfrak{f}_k = \left\{P \in \mathfrak{P}_G : P = P\left[s \to\right]; N\left(P\right) = k; \text{moreover, if } P \text{ contains } t, \text{then } P = P\left[\to t\right]\right\}. \tag{12}$$

In other words, $\mathfrak{f}_k$ consists of those paths from s, that contain k nodes, and does not go beyond t, this is because we do not have to go beyond a path who reached t, our destination. Remark 1: Here we define $\mathfrak{f}_k$ only as a set to be referred to in the following passages. In other words, there is no need to find $\mathfrak{f}_k$ explicitly for any k.

Remark 2: It is clear that $\mathfrak{f}_1 = \left\{\left(s\right)\right\}$.

Remark 3: $\mathfrak{f}_i$ and $\mathfrak{f}_j$ are mutually exclusive for all $i \neq j$.

Definition 5.1.2:

$$S_k = \left\{u \in V : \text{there exists } R_0 \in \mathfrak{f}_h \text{ for some } h \leq k \text{ where } R_0 = R_0\left[\to u\right]\right\}. \tag{13}$$

In other words, S_k consists of all of those nodes we have stepped onto/pass, upon considering all paths from s containing up to k nodes.

$$\mathcal{L}_k : S_k \to \wp\left(\mathfrak{P}_G\right), \text{ where } \mathcal{L}_k\left(v\right) = \mathfrak{S}\left(\bigcup_{1 \leq h \leq k} \left\{R \in \mathfrak{f}_h : R = R\left[\to v\right]\right\}\right). \tag{14}$$

In other words, $\mathcal{L}_k\left(v\right)$ yields the shortest path(s) from s to v among those with at most k nodes, which is what we have considered thus far.

Remark: It follows directly from definition that

$$\mathcal{L}_k\left(v\right) = \mathfrak{S}\left(\left\{R \in \mathfrak{f}_{h_v} : R = R\left[\to v\right]\right\}\right)$$

for some $h_v \leq k$.

$$T_k = \left\{u \in S_k - \left\{t\right\} : N\left(\mathcal{L}_k\left(u\right)\right) = k\right\}. \tag{15}$$

In other words, T_k consists of all of those nodes that we will be searching on their successor nodes. As mentioned, we do not have to search for successor nodes if a path has reached *t*, which is our destination.

$$Y_k = \left\{v \in \mathrm{V} : \text{there exists } u = T_k \text{, with } \left(u, v\right) \in E \text{, and } v \text{ is not a node in some } P \in \mathcal{L}_k\left(u\right)\right\} \tag{16}$$

In other words, Y_k consists of all of those successor nodes of the elements in T_k.

Remark: $Y_k \cap S_k$ need not be empty.

Thus, given any

$$\mathfrak{Y} \subseteq \{P \in \mathfrak{P}_G : P = P\left[\rightarrow u\right]\}$$

and $(u,v) \in E$, we shall denote $\operatorname{apnd}\left(\mathfrak{Y}, v\right)$ to mean

$$\left\{\operatorname{apnd}\left(P, v\right) : P \in \mathfrak{Y}, v \text{ is not a node in } P\right\} \subseteq \mathfrak{P}_G.$$

We now discuss some lemmas.

Lemma 5.1.3 *There exists* k_0 *and* k_1 , *with* $1 \leq k_0 \leq k_1$, *such that*

a) $t \in S_k$ *for all* $k \geq k_0$ (This is because we will eventually reach *t* by a path passing through k_0 nodes)
b) $\mathrm{Y}_{k_1} = \varnothing$ *for all* $k \geq k_1$ (This is because we will eventually exhaust all the nodes in *V*)

Since is *V* finite, a) and b) of the lemma must follows. The proof for this lemma is therefore ommitted.

Lemma 5.1.4:

$$Y_k = \left\{ v \in V : \text{there exists } u = T_k, \text{such that apnd}\left(\mathcal{L}_k\left(u\right), v\right) \subseteq \mathfrak{f}_{k+1} \right\}$$

Proof: Let $u=T_k$. Then $u \neq t$ and $N\left(\mathcal{L}_k\left(u\right)\right) = k$. Note that $\mathcal{L}_k\left(u\right) \subseteq \mathfrak{P}_G$ with s being its source node.

[⟹] Suppose $(u,v) \in E$ and v is not a node in some $P \in \mathcal{L}_k\left(u\right)$.

It follows that

$$\text{apnd}\left(\mathcal{L}_k\left(u\right), v\right) \subseteq \mathfrak{P}_G$$

with s being its source node as well. Moreover, as $N\left(\mathcal{L}_k\left(u\right)\right) = k$, so

$$N\left(\text{apnd}\left(\mathcal{L}_k\left(u\right), v\right)\right) = k + 1.$$

On the other hand, as $u \neq t$, so t is not the destination node $\mathcal{L}_k\left(u\right)$. This implies that $\mathcal{L}_k\left(u\right)$ does not contain the node t. As a result, if $\text{apnd}\left(\mathcal{L}_k\left(u\right), v\right)$ contains t, then $v=t$ is the only way.

$\text{apnd}\left(\mathcal{L}_k\left(u\right), v\right) \subseteq \mathfrak{f}_{k+1}$ now follows.

[⟹] Suppose

$$\text{apnd}\left(\mathcal{L}_k\left(u\right), v\right) \subseteq \mathfrak{f}_{k+1} \subseteq \mathfrak{P}_G.$$

Then implies $(u,v) \in \text{E}$. We also have some $P \in \mathcal{L}_k\left(u\right)$ where v is not a node in P.

5.2 The Iterative Formulae

Our algorithm will therefore consist of explicitly determining S_k, $\mathcal{L}_k$, T_k, Y_k in the order as follows,

S_1, $\mathcal{L}_1$, T_1, Y_1, S_2, $\mathcal{L}_2$, T_2, Y_2, S_3, $\mathcal{L}_3$, T_3, Y_3, S_4, ……

By Lemma 5.1.2, we will stop upon reaching a k_1 with $Y_{k_1} = \varnothing$, for which $\mathcal{L}_{k_1}(t)$ will be the shortest path(s) yielded. Note that $T_k = \varnothing$ implies $Y_k = \varnothing$ for all k, the converse however is not nessecarily true.

5.2.1 Initial Condition

$S_1 = \{s\}$ (because $\mathfrak{f}_1 = \{(s)\}$)

Remark: Though not the initial condition, it follows directly that

a) $\mathcal{L}_1 : S_1 \to \wp(\mathfrak{P}_G)$, where $\mathcal{L}_1(s) = \{(s)\}$.

b) $T_1 = \{s\}$.

From Y_1 onwards, however, will differ from graph to graph.

5.2.2 The Iterative Formulae of $S_k, \mathcal{L}_k, T_k, Y_k$

Since T_k is related only to S_k and $\mathcal{L}_k$ by equation (4); and Y_k is related only to $\mathcal{L}_k$ and T_k by equation (5), we only have to establish two iterative formulae that espress S_{k+1} and $\mathcal{L}_{k+1}$ in terms of $S_k, \mathcal{L}_k, T_k, Y_k$ and E. This establishment is legitimate, because $S_k, \mathcal{L}_k, T_k, Y_k$ are found in prior of S_{k+1} and hence $\mathcal{L}_{k+1}$, and E is given in G.

Lemma 5.2.2.1 $S_{k+1} = S_k \cup \mathrm{Y}_k$.

Proof: It is clear to us that $S_k \subseteq S_{k+1}$.

1. Let $v \in Y_k$. By Lemma 5.1.4, there exists $u = T_k$ such that

$$\operatorname{apnd}\left(\mathcal{L}_k\left(u\right), v\right) \subseteq \mathfrak{f}_{k+1}.$$

Note that $\operatorname{apnd}\left(\mathcal{L}_k\left(u\right), v\right)$ is itself a path to *v*. We thus have $v \in S_{k+1}$ as well. $\Upsilon_k \subseteq S_{k+1}$ now follows.

$S_k \cup \Upsilon_k \subseteq S_{k+1}$ is thus proven.

2. Now let $w \in S_{k+1}$.

Suppose $w \notin S_k$, then $w \in S_{k+1} - S_k$, which implies that there exists $R_0\left[\rightarrow w\right] \in \mathfrak{f}_{k+1}$ but no such $R\left[\rightarrow w\right] \in \mathfrak{f}_h$ for all $h \leq k$.

Let

$$\mathfrak{h} = \left\{R \in \mathfrak{f}_{k+1} : R = R\left[\rightarrow w\right]\right\}.$$

Clearly, for each $R_0 \in \mathfrak{h}$, there must exist a unique predecessor node $c_{R_0} \neq t$ for *w*.

Let $c_1, c_2, \ldots, c_n$ be *all* the distinct predecessor nodes of *w* found in some $R \in \mathfrak{h}$. Then $\mathfrak{h}$ can be partitioned into $\mathfrak{h}_1, \mathfrak{h}_2, \ldots, \mathfrak{h}_n$ where

$$\begin{aligned}
\mathfrak{h}_i &= \left\{R \in \mathfrak{f}_{k+1} : R = \left(s, \ldots, c_i, w\right)\right\} \\
&= \left\{\operatorname{apnd}\left(P, w\right) \in \mathfrak{f}_{k+1} : P = \left(s, \ldots, c_i\right)\right\} \\
&= \left\{\operatorname{apnd}\left(P, w\right) : P \in \mathfrak{f}_k, P\left[\rightarrow c_1\right], c_i \neq t, \left(c_i, w\right) \in E\right\} \\
&= \left\{\operatorname{apnd}\left(P, w\right) : P \in \mathfrak{f}_k, P\left[\rightarrow c_1\right]\right\}
\end{aligned}$$

(because c_i is known to be a distinct predecessor nodes of *w*), and c_i cannot possibly be *t*, for all *i*.

Consider $i=1$. We have

$$\begin{aligned}\mathfrak{S}\left(\mathfrak{h}_1\right) &= \mathfrak{S}\left(\left\{\text{apnd}\left(P,w\right) : P \in \mathfrak{f}_k, P\left[\to c_1\right]\right\}\right) \\ &= \text{apnd}\left(\mathfrak{S}\left(\left\{P \in \mathfrak{f}_k : P = P\left[\to c_1\right]\right\}\right), w\right)\end{aligned}.$$

Suppose there exists $P_0 = P_0\left[\to c_1\right] \in \mathfrak{f}_h$ for some $h<k$, then

$$\text{apnd}\left(P\left[\to c_1\right], w\right) \in \mathfrak{f}_{h+1}$$

which implies that $w \in S_k$, a contradiction.

We now have proved that

$$\left\{P \in \mathfrak{f}_k : P = P\left[\to c_1\right]\right\} = \bigcup_{1 \le h \le k} \left\{P \in \mathfrak{f}_h : P = P\left[\to c_1\right]\right\}.$$

As a result,

$$\begin{aligned}\mathfrak{S}\left(\mathfrak{h}_1\right) &= \text{apnd}\left(\mathfrak{S}\left(\left\{P \in \mathfrak{f}_k : P = P\left[\to c_1\right]\right\}\right), w\right) \\ &= \text{apnd}\left(\mathfrak{S}\left(\bigcup_{1 \le h \le k} \left\{P \in \mathfrak{f}_h : P = P\left[\to c_1\right]\right\}\right), w\right). \\ &= \text{apnd}\left(\mathcal{L}_k\left(c_1\right), w\right)\end{aligned}$$

Moreover, as

$$\bigcup_{1 \le h \le k} \left\{P \in \mathfrak{f}_h : P = P\left[\to c_1\right]\right\} \supseteq \mathfrak{S}\left(\bigcup_{1 \le h \le k} \left\{P \in \mathfrak{f}_h : P = P\left[\to c_1\right]\right\}\right) = \mathcal{L}_k\left(c_1\right),$$

all paths in $\mathcal{L}_k\left(c_1\right)$, must have exactly *k* nodes, $\mathrm{N}\left(\mathcal{L}_k\left(c_1\right)\right) = k$ follows.

As $c_1 \in S_k$-$\{t\}$ with $N\left(\mathcal{L}_k\left(c_1\right)\right) = k$, we now have $c_1 \in T_k$.

On the other hand,

$$\text{apnd}\left(\mathcal{L}_k\left(c_1\right), w\right) = \mathfrak{S}\left(\mathfrak{h}_1\right) \subseteq \mathfrak{h}_1 \subseteq \mathfrak{h} \subseteq \mathfrak{f}_{k+1}.$$

$w \in Y_k$ is thus proven.

Lemma 5.2.2.2:

$$\mathcal{L}_{k+1}(v) = \begin{cases} \mathfrak{S}\left(\bigcup_{u \in T_k, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u), v\right)\right), & \text{if } v \in \mathrm{Y}_k - S_k \\ \mathfrak{S}\left(\left(\bigcup_{u \in T_k, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u), v\right)\right) \cup \mathcal{L}_k(v)\right), & \text{if } v \in S_k \cap \mathrm{Y}_k \quad \text{for all} \\ \mathcal{L}_k(v), & \text{if } v \in S_k - \mathrm{Y}_k \end{cases}$$

$v \in S_k \cup \mathrm{Y}_k$

Proof:

Suppose $N\left(\mathcal{L}_{k+1}(v)\right) \leq k$, then $v \in S_k$ is certain (though $v \in Y_k$ *may* occur). Therefore $\mathcal{L}_{k+1}(v) = \mathcal{L}_k(v)$.

Suppose $N\left(\mathcal{L}_{k+1}(v)\right) = k+1$, then $v \in Y_k$ is certain (though $v \in S_k$ *may* occur) . As

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\left\{R \in \mathfrak{f}_{h_v} : R = R[\rightarrow v]\right\}\right)$$

for some $h_v \leq k+1$, we conclude that

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\left\{R \in \mathfrak{f}_{k+1} : R = R[\rightarrow v]\right\}\right).$$

Let $R_0 = R_0[\rightarrow v] \in \mathfrak{f}_{k+1}$. Then there must exists a node $u_0 \in S_k$-$\{t\}$ with $(u_0,v) \in \mathrm{E}$, such that

$$R_0 \in \left\{\operatorname{apnd}\left(P[\rightarrow u_0], v\right) : P \in \mathfrak{f}_k\right\}$$

which is actually a subset of $\left\{R \in \mathfrak{f}_{k+1} : R = R[\rightarrow v]\right\}$ itself. We now have

$$\left\{R \in \mathfrak{f}_{k+1} : R = R\left[\to v\right]\right\} = \bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \left\{\operatorname{apnd}\left(P\left[\to u\right], v\right) : P \in \mathfrak{f}_k\right\}.$$

As a result,

$$\begin{aligned}\mathcal{L}_{k+1}\left(v\right) &= \mathfrak{S}\left(\left\{R \in \mathfrak{f}_{k+1} : R = R\left[\to v\right]\right\}\right)\\ &= \mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \left\{\operatorname{apnd}\left(P\left[\to u\right], v\right) : P \in \mathfrak{f}_k\right\}\right)\\ &= \mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \mathfrak{S}\left(\left\{\operatorname{apnd}\left(P\left[\to u\right], v\right) : P \in \mathfrak{f}_k\right\}\right)\right)\\ &= \mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathfrak{S}\left(\left\{P \in \mathfrak{f}_k : P = P\left[\to u\right]\right\}\right), v\right)\right)\end{aligned}$$

Now, for each $u \in S_k - \{t\}$ with $(u,v) \in \mathrm{E}$, we first find the shortest among all paths from s to such u with at most k nodes. This will yield $\mathcal{L}_k\left(u\right)$ for all such u. As $(u,v) \in \mathrm{E}$, $\operatorname{apnd}\left(\mathcal{L}_k\left(u\right), v\right)$ can be defined as well, for each $\in S_k - \{t\}$ with $(u,v) \in \mathrm{E}$. We now arrived at a union set:

$$\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k\left(u\right), v\right)$$

We then find the shortest path(s) among that union set as well, which is

$$\mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k\left(u\right), v\right)\right)$$

Note that, $\{u \in S_k - \{t\} : (u,v) \in \mathrm{E}\}$ is indeed the set of *all* predecessor nodes of v found among *all* paths in $\left\{R \in \mathfrak{f}_{k+1} : R = R\left[\to v\right]\right\}$. As a result:

$$\mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k\left(u\right), v\right)\right) = \mathfrak{S}\left(\bigcup_{1 \le h \le k+1} \left\{R \in \mathfrak{f}_h : R = R\left[\to v\right]\right\}\right) = \mathcal{L}_{k+1}\left(v\right).$$

Recall that

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathfrak{S}\left(\left\{ P \in \mathfrak{f}_k : P = P[\to u] \right\} \right), v \right) \right).$$

It follows that

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u) \cap \mathfrak{S}\left(\left\{ P \in \mathfrak{f}_k : P = P[\to u] \right\} \right), v \right) \right)$$

Note that

$$\mathcal{L}_k(u) \cap \mathfrak{S}\left(\left\{ P \in \mathfrak{f}_k : P = P[\to u] \right\} \right) = \mathcal{L}_k(u) \cap \mathfrak{f}_k$$

which is empty whenever $N\left(\mathcal{L}_k(u)\right) < k$. As a result,

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\bigcup_{u \in S_k - \{t\}, N\left(\mathcal{L}_k(u)\right) = k, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u), v \right) \right) = \mathfrak{S}\left(\bigcup_{u \in T_k, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u), v \right) \right).$$

Hence:

If $v \in \Upsilon_k - S_k$, then $N\left(\mathcal{L}_{k+1}(v)\right) = k+1$, so

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\bigcup_{u \in T_k, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u), v \right) \right)$$

follows.

If $v \in S_k \cap \Upsilon_k$, then it is either $N\left(\mathcal{L}_{k+1}(v)\right) \leq k$ or $N\left(\mathcal{L}_{k+1}(v)\right) = k+1$. So

$$\mathcal{L}_{k+1}(v) = \mathfrak{S}\left(\left(\bigcup_{u \in T_k, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k(u), v \right) \right) \cup \mathcal{L}_k(v) \right),$$

as we take whichever shorter paths from the 2 sets

$$\mathfrak{S}\left(\bigcup_{u\in T_k,(u,v)\in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_k\left(u\right),v\right)\right) \text{and } \mathcal{L}_k\left(v\right).$$

If $v\in S_k$-Y_k, then $N\left(\mathcal{L}_{k+1}\left(v\right)\right)\leq k$, so $\mathcal{L}_{k+1}\left(v\right)=\mathcal{L}_k\left(v\right)$.

Remark 1: $\mathcal{L}_{k+1}\left(v\right)$ will yield the shortest of all the newly found paths from *s* to the point *v* (each contains exactly *k*+1 nodes), if *v* is a new point.

Remark 2: $\mathcal{L}_{k+1}\left(v\right)$ will compare the existing path(s) in $\mathcal{L}_k\left(v\right)$ (each contains at most *k* nodes), with the newly found paths from *s* to the point *v* (each contains exactly *k*+1 nodes), and take the shortest among them.

Remark 3: $\mathcal{L}_{k+1}\left(v\right)$ will retain the pre-existing path $\mathcal{L}_k\left(v\right)$, if no new paths (which contains exactly *k*+1 nodes) from *s* to the point *v* are found.

5.2.3 The Flow Chart of Our Algorithm

We have thus established our algorithm to find the shortest path(s) as shown in Figure 1.

6. ILLUSTRATIVE EXAMPLE

6.1 The Scenario

Let *s*, *a*, *b*, *c*, *d*, *t* be 6 town areas, and with the network of traffic shown in Figure 2.

A crew from a cement company in *s* is carrying trucks of concrete to a construction site in *t*. The crew consist of one experienced supervisor and many fellow workers.

And Table 1 shows the crew's comment and agreements about each of the path through the roads.

In this scenario, the time spent *within* a town area is negligible compared to the time driving along a path. However, driving lots of heavy trucks through a town area will possibly increase the traffic burden in that town area. *Hence, if there are two paths of the same length, the construction crew will take the path that passes through fewer town areas.*

Figure 1. The flow chart of our algorithm

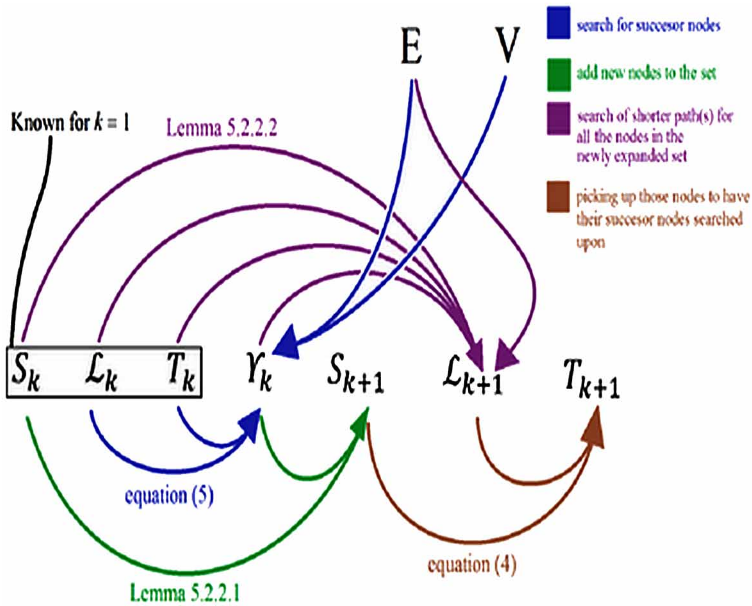

Figure 2. The network of traffic

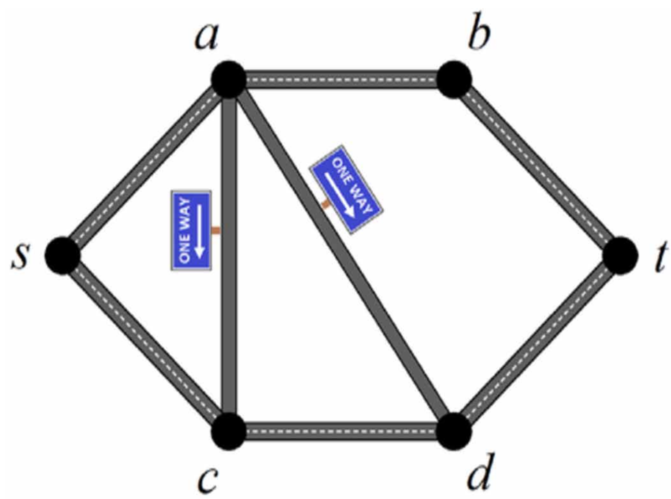

Table 1. Crew's comment and agreements about each of the path through the roads

Path	Supervisor's Experience on the Time to Travel	Workers' Response on the Supervisor's Claim
from *s* to *a*, or vice versa	At least 2 hours Most likely 5 hours At most 7 hours	20% "agree"* 50% "wonder"* 40% "disagree"*
from *s* to *c*, or vice versa	At least 1 hour Most likely 2 hours At most 3 hours	40% "agree" 60% "wonder" 70% "disagree"
from *a* to *c*,	At least 3 hours Most likely 7 hours At most 8 hours	10% "agree" 40% "wonder" 60% "disagree"
from *a* to *b*, or vice versa	At least 2 hours Most likely 4 hours At most 8 hours	50% "agree" 30% "wonder" 10% "disagree"
from *a* to *d*,	At least 3 hours Most likely 4 hours At most 5 hours	30% "agree" 40% "wonder" 70% "disagree"
from *c* to *d*, or vice versa	At least 1 hours Most likely 5 hours At most 7 hours	70% "agree" 60% "wonder" 80% "disagree"
from *b* to *t*, or vice versa	At least 7 hours Most likely 8 hours At most 9 hours	30% "agree" 20% "wonder" 60% "disagree"
from *d* to *t*, or vice versa	At least 2 hours Most likely 4 hours At most 5 hours	60% "agree" 50% "wonder" 30% "disagree"

*Note:

"agree": The workers find evidence that the expected length of travel is indeed *as claimed* by the supervisor.

"wonder": The workers find evidence that the expected length can *either be as claimed or shorter*, depending on many other conditions.

"disagree": The workers find evidence that the expected length of travel is *way shorter* than supervisor's opinion.

Furthermore, the percentages of workers "agree", "wonder", or "disagree", are *independent* of the order of traveling through the roads.

Without loss of generality, consider the path (*s,a,b*), such independence will implies that, a worker will "agree" on the time taken through (*s,a,b*) only if he "agree" on *both* (*s,a*) and (*a,b*) . On the other hand, if a worker "wonder" or/and "disagree", on *any* one of (*s,a*) or (*a,b*), then it suffices for him to "wonder" or/and "disagree" on the entire (*s,a,b*). This is the indeed reason of using Definition 3.1 instead of Definition 2.4 from [11] .

The crew now wants to find the best path to go from *s* to *t*.

6.2 Modeling With SVTrN-Network

We now model the scenario with the following SVTrN-network.
G=V,E,d where

0) $V=\{s,a,b,c,d,t\}$.

$E=\{(s,a), (s,c), (a,b), (c,d), (b,t), (d,t), (a,c), (a,d), (a,s), (c,s), (b,a), (d,c), (t,b), (t,d)\}$.

$d : E \to \mathfrak{T}$,

where

$d(s,a) = d(a,s) = (2,5,7)$; 0.2,0.5,0.4,

$d(c,d)=d(d,c)=(1,5,7)$; 0.7,0.6,0.8,

$d(s,c)=d(c,s)=(1,2,3)$; 0.4,0.6,0.7,

$d(b,t)=d(t,b)=(7,8,9)$; 0.3,0.2,0.6,

$d(a,c)=(3,7,8)$; 0.1,0.4,0.6,

$d(d,t)=d(t,d)=(2,4,5)$; 0.6,0.5,0.3 .

$d(a,b)=d(b,a)$-(2,4,8); 0.5,0.3,0.1,

$d(a,d)=(3,4,5)$; 0.3,0.4,0.7,

6.3 Calculation With Our Algorithm

We now determine the shortest path(s) with our iterative method

1a) $S_1=\{s\}$.
1b) $\mathcal{L}_1 : S_1 \to \wp\left(\mathfrak{P}_G\right)$, where $\mathcal{L}_1\left(s\right) = \left\{\left(s\right)\right\}$.

Note that $N(\mathcal{L}_1\left(s\right)) = 1$ and $D(\mathcal{L}_1\left(s\right)) = \left(0,0,0\right);1,0,0$

1c) $T_1=\{s\}$.

1d)

$$\begin{aligned}Y_1 &= \left\{v \in V : \text{there exists } u = T_1, \text{with} \left(u,v\right) \in \mathrm{E}, \text{and } v \text{ is not a node in some } P \in \mathcal{L}_1\left(u\right)\right\} \\ &= \left\{v \in V : \left(s,v\right) \in \mathrm{E}, \text{and } v \text{ is not a node in} \left(s\right)\right\} = \left\{a,c\right\}\end{aligned}$$

2a) $S_2 = S_1 \cup \mathrm{Y}_1 = \left\{s,a,c\right\}$.

2b) $\mathcal{L}_2 : S_2 \to \wp\left(\mathfrak{P}_G\right)$, where

$$\mathcal{L}_2\left(v\right) = \begin{cases} \mathfrak{S}\left(\bigcup\limits_{u \in T_1, \left(u,v\right) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_1\left(u\right), v\right)\right), & \text{if } v \in Y_1 - S_1 \\ \mathfrak{S}\left(\left(\bigcup\limits_{u \in T_1, \left(u,v\right) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_1\left(u\right), v\right)\right) \cup \mathcal{L}_1\left(v\right)\right) & \text{if } v \in S_1 \cap Y_1 \\ \mathcal{L}_1\left(v\right), & \text{if } v \in S_1 - Y_1 \end{cases}$$

$$= \begin{cases} S\left(\operatorname{apnd}\left(\left\{\left(s\right)\right\}, v\right)\right), & \mathit{if}\ v \in \left\{a,c\right\} \\ \left\{\left(s\right)\right\}, & \mathit{if}\ v \in \left\{s\right\} \end{cases}$$

Hence, $\mathcal{L}_2\left(a\right) = \left\{\left(s,a\right)\right\}$, $\mathcal{L}_2\left(c\right) = \left\{\left(s,c\right)\right\}$, $\mathcal{L}_2\left(s\right) = \left\{\left(s\right)\right\}$.

2c)

$$T_2 = \left\{u \in S_2 - \left\{t\right\} : N\left(\mathcal{L}_2\left(u\right)\right) = 2\right\} = \left\{u \in \left\{s,a,c\right\} : N\left(\mathcal{L}_2\left(u\right)\right) = 2\right\} = \left\{a,c\right\}$$

2d)

$$\begin{aligned}Y_2 &= \left\{v \in V : \text{there exists } u = T_2, \text{such that} \left(u,v\right) \in \mathrm{E}, \text{and } v \text{ is not a node in some } P \in \mathcal{L}_2\left(u\right)\right\} \\ &= \left\{v \in V : \left(a,v\right) \in \mathrm{E}, \text{and } v \text{ is not a node in} \left(s,a\right)\right\} \\ &\quad \cup \left\{v \in V : \left(c,v\right) \in \mathrm{E}, \text{and } v \text{ is not a node in} \left(s,c\right)\right\} \\ &= \left\{b,c,d\right\} \cup \left\{d\right\} = \left\{b,c,d\right\}\end{aligned}$$

3a) $S_3 = S_2 \cup \mathrm{Y}_2 = \left\{s,a,c,b,d\right\}$

3b) $\mathcal{L}_3 : S_3 \to \wp\left(\mathfrak{P}_G\right)$, where

$$\mathcal{L}_3(v) = \begin{cases} \mathfrak{S}\left(\bigcup_{u \in T_2, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_2(u), v\right)\right), & \text{if } v \in \Upsilon_2 - S_2 \\ \mathfrak{S}\left(\left(\bigcup_{u \in T_2, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_2(u), v\right)\right) \cup \mathcal{L}_2(v)\right) & \text{if } v \in S_2 \cap \Upsilon_2 \\ \mathcal{L}_2(v), & \text{if } v \in S_2 - \Upsilon_2 \end{cases}$$

$$= \begin{cases} \mathfrak{S}\left(\bigcup_{u \in \{a,c\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_2(u), v\right)\right), & \text{if } v \in \{b, d\} \\ \mathfrak{S}\left(\left(\bigcup_{u \in \{a,c\}, (u,v) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_2(u), v\right)\right) \cup \mathcal{L}_2(v)\right) & \text{if } v \in \{c\} \\ \mathcal{L}_2(v), & \text{if } v \in \{s, a\} \end{cases}$$

Hence,

$$\mathcal{L}_3(a) = \mathcal{L}_2(a) = \left\{(s,a)\right\},$$

$$\mathcal{L}_3(s) = \mathcal{L}_2(s) = \left\{(s)\right\}.$$

$$\mathcal{L}_3(b) = \mathfrak{S}\left(\bigcup_{u \in \{a,c\}, (u,b) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_2(u), b\right)\right) = \operatorname{apnd}\left(\mathcal{L}_2(a), b\right) = \operatorname{apnd}\left(\left\{(s,a)\right\}, b\right) = \left\{(s,a,b)\right\}$$

$$\begin{aligned} \mathcal{L}_3(d) &= \mathfrak{S}\left(\bigcup_{u \in \{a,c\}, (u,d) \in \mathrm{E}} \operatorname{apnd}\left(\mathcal{L}_2(u), d\right)\right) = \mathfrak{S}\left(\operatorname{apnd}\left(\mathcal{L}_2(a), d\right) \cup \operatorname{apnd}\left(\mathcal{L}_2(c), d\right)\right) \\ &= \mathfrak{S}\left(\operatorname{apnd}\left(\left\{(s,a)\right\}, d\right) \cup \operatorname{apnd}\left(\left\{(s,c)\right\}, d\right)\right) = \mathfrak{S}\left(\left\{(s,a,d), (s,c,d)\right\}\right) \end{aligned}.$$

D((*s,a,d*))=(5,9,12); 0.06.0.70.0.82,

D((*s,c,d*))=(2,7,10); 0.28,0.84,0.94,

with

σ((5,9,12)); 0.06,0.70,0.82 = 1.575,

σ((2,7,10); 0.28,0.84,0.94 = 1.0833333333.

1.575 > 1.0833333333.

Therefore, $\mathcal{L}_3\left(d\right)=\left\{\left(s,c,d\right)\right\}$.

$$\mathcal{L}_3\left(c\right)=\mathfrak{S}\left(\left(\bigcup_{u\in\{a,c\},(u,c)\in\mathrm{E}}\operatorname{apnd}\left(\mathcal{L}_2\left(u\right),c\right)\right)\cup\mathcal{L}_2\left(c\right)\right)=\mathfrak{S}\left(\operatorname{apnd}\left(\mathcal{L}_2\left(a\right),c\right)\cup\mathcal{L}_2\left(c\right)\right)$$
$$=\mathfrak{S}\left(\operatorname{apnd}\left(\left\{\left(s,a\right)\right\},c\right)\cup\left\{\left(s,c\right)\right\}\right)=\mathfrak{S}\left(\left\{\left(s,a,c\right),\left(s,c\right)\right\}\right)$$

D((*s,a,c*)) = (5,12,15)); 0.02,0.70,0.76,

D((*s,c*)) = ((1,2,3)); 0.4,0.6,0.7,

with

σ((5,12,15)); 0.02,0.70,0.76 = 2.0533333333,

σ((1,2,3)); 0.4,0.6,0.7 = 0.7333333333 .

2.0533333333 > 0.7333333333.

Therefore, $\mathcal{L}_3\left(c\right)=\left\{\left(s,c\right)\right\}$.

3c)

$$T_3=\left\{u\in S_3-\left\{t\right\}:N\left(\mathcal{L}_3\left(u\right)\right)=3\right\}=\left\{u\in\left\{s,a,c,b,d\right\}:N\left(\mathcal{L}_3\left(u\right)\right)=3\right\}=\left\{b,d\right\}$$

$$Y_3 = \begin{Bmatrix} v \in V : \text{there exists } u = T_3, \text{such that } (u,v) \in \mathrm{E}, \\ \text{and } v \text{ is not a node in some } P \in \mathcal{L}_3(u) \end{Bmatrix}$$

3d) $= \{v \in V : (b,v) \in \mathrm{E}, \text{and } v \text{ is not a node in } (s,a,b)\}$

$\cup \{v \in V : (d,v) \in \mathrm{E}, \text{and } v \text{ is not a node in } (s,c,d)\}$

$= \{t\} \cup \{t\} = \{t\}$

4a) $S_4 = S_3 \cup Y_3 = \{s,a,c,b,d,t\}$

4b) $\mathcal{L}_4 : S_4 \to \wp(\mathfrak{P}_G)$, where

$$\mathcal{L}_4(v) = \begin{cases} \mathfrak{S}\left(\bigcup_{u \in T_3, (u,v) \in \mathrm{E}} \operatorname{apnd}(\mathcal{L}_3(u), v) \right), & \text{if } v \in Y_3 - S_3 \\ \mathfrak{S}\left(\left(\bigcup_{u \in T_3, (u,v) \in \mathrm{E}} \operatorname{apnd}(\mathcal{L}_3(u), v) \right) \cup \mathcal{L}_3(v) \right) & \text{if } v \in S_3 \cap Y_3 \\ \mathcal{L}_3(v), & \text{if } v \in S_3 - Y_3 \end{cases}$$

$$= \begin{cases} S\left(\bigcup_{u \in \{b,d\}, (u,v) \in \mathrm{E}} \operatorname{apnd}(\mathcal{L}_3(u), v) \right), if\ v \in \{t\} \\ \mathcal{L}_3(v), if\ v \in \{s,a,c,b,d\} \end{cases}$$

Hence

$$\mathcal{L}_4(t) = \mathfrak{S}\left(\bigcup_{u \in \{b,d\}, (u,t) \in \mathrm{E}} \operatorname{apnd}(\mathcal{L}_3(u), t) \right) = \mathfrak{S}\left(\operatorname{apnd}(\mathcal{L}_3(b), t) \cup \operatorname{apnd}(\mathcal{L}_3(d), t)\right)$$

$$= \mathfrak{S}\left(\operatorname{apnd}(\{(s,a,b)\}, t) \cup \operatorname{apnd}(\{(s,c,d)\}, t)\right) = \mathfrak{S}\left(\{(s,a,b,t), (s,c,d,t)\}\right)$$

$D((s,a,b,t))$ = (11,17,24); 0.03,0.72,0.784,

$D((s,c,d,t))$ = (4,11,15); 0.168,0.90,0.958, with

σ((11,17,24)); 0.03,0.72,0.784 = 3.0245,

σ((4,11,15)); 0.168,0.90,0.958 = 0.9908333333 .

Therefore, $\mathcal{L}_4\left(t\right)=\left\{\left(s,c,d,t\right)\right\}$.

$\mathcal{L}_4\left(s\right)=\mathcal{L}_3\left(s\right)=\left\{\left(s\right)\right\}$,

$\mathcal{L}_4\left(a\right)=\mathcal{L}_3\left(a\right)=\left\{\left(s,a\right)\right\}$,

$\mathcal{L}_4\left(c\right)=\mathcal{L}_3\left(c\right)=\left\{\left(s,c\right)\right\}$,

$\mathcal{L}_4\left(b\right)=\mathcal{L}_3\left(b\right)=\left\{\left(s,a,b\right)\right\}$,

$\mathcal{L}_4\left(d\right)=\mathcal{L}_3\left(d\right)=\left\{\left(s,c,d\right)\right\}$,

4c)

$$T_4=\left\{u\in S_4-\left\{t\right\}\;:N\left(\mathcal{L}_4\left(u\right)\right)=4\right\}=\left\{u\in\left\{s,a,c,b,d\right\}\;:N\left(\mathcal{L}_4\left(u\right)\right)=4\right\}=\varnothing$$

4c) $Y_4=\varnothing$, because $T_4=\varnothing$.

So there are no more nodes to be considered. We have thus reached the end of our algorithm.

Therefore, P_0=(*s,c,d,t*) is the one and only best path, with $D(P_0)$=(4,11,15); 0.168,0.92,0.958, and $N(P_0)$=4.

CONCLUSION

In this chaptre, an algorithm has been developed for solving the shortest path problem on a network where the edges weight are characterized by a neutrosophic numbers called single valued triangular neutrosophic numbers. To show the performance of our proposed methodology in the shortest path problem, a hypothetical example was presented. Our future research works would involve developing more efficient and computationally inexpensive algorithms for solving shortest path problems in a SVTrN environment, as well as introducing an algorithm for solving shortest path problems in single-valued trapezoidal neutrosophic environment.

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Chapter 2
Decision-Making Method Based on Neutrosophic Soft Expert Graphs

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ABSTRACT

This chapter defines the concept of neutrosophic soft expert graph. The authors have established a link between graphs and neutrosophic soft expert sets. Basic operations of neutrosophic soft expert graphs such as union, intersection, and complement are defined here. The concept of neutrosophic soft expert soft graph is also discussed in this chapter. The new concept is called neutrosophic soft expert graph-based multi-criteria decision-making method (NSEGMCDM, for short). Finally, an illustrative example is given, and a comparison analysis is conducted between the proposed approach and other existing methods to verify the feasibility and effectiveness of the developed approach.

INTRODUCTION

The concept of fuzzy set theory was introduced by (Zadeh 1965) to solve difficulties in dealing with uncertainties. Since then the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real life problems involving ambiguous and uncertain environment. Atanassov(Atanassov 1986) introduced the

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concept of intuitionistic fuzzy sets as an extension of Zadeh's fuzzy set.The theory of neutrosophic set is introduced by Smarandache (Smarandache 1998, 2005) which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. The theory is generalization of classical sets and fuzzy sets and is applied in decision making problems, control theory, medicines, topology and in many more real life problems.Molodtsov (Molodtsov 1999) introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Molodtsov's soft sets give us new technique for dealing with uncertainty from the viewpoint of parameters.Maji *et al.* [(Maji, Roy & Biswas 2003), (Maji 2013)] proposed fuzzy soft sets and neutrosophic soft sets. Broumi and Smarandache(Broumi and Smarandache 2013)proposed intuitionistic neutrosophic soft set and its application in decision making problem. Alkhazaleh and Salleh [(Alkhazaleh & Salleh2011), (Alkhazaleh & Salleh2014)] defined the concept of soft expert set, which were later extended to vague soft expert set theory (Hassan and Alhazaymeh 2013), generalized vague soft expert set (Alhazaymeh & Hassan 2014) and multi Q-fuzzy soft expert set (Adam & Hassan 2016). Şahin et al.(Sahin, Alkhazaleh & Ulucay 2015) introduced neutrosophic soft expert sets, while Al-Quran and Hassan (Al-Quran & Hassan 2016) extended it further to neutrosophic vague soft expert set.

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. The graph is a widely used tool for solving combinatorial problems in different areas, such as geometry, algebra, number theory, topology, optimization and computer science. When the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs, intuitionistic fuzzy graphs and their extensions [(Basha & Kartheek 2015), (Deepa, Praba & Chandrasekaran 2015),(Deepa, Praba & Chandrasekaran 2016), (Gani & Ahamed 2003), (Gani & Begum 2010), (Mohinta & Samanta 2015), (Muthuraj & Sasireka 2016), (Mathew & Sunitha 2009), (Mathew & Sunitha 2010), (Rashmanlou et al.2016), (Shahzadi & Akram 2016)].Smarandache defined four main categories of neutrosophic graphs. Two of them, called I-edge neutrosophic graph and I-vertex neutrosophic graph, are based on literal indeterminacy (I); these concepts are deeply studied and gained popularity among the researchers due to applications via real world problems [(Kandasamy & Smarandache 2004), (Kandasamy & Smarandache 2013),(Kandasamy & Smarandache 2015)].More related works can be seen in [(Akram & Shahzadi 2017), (Broumi and Smarandache 2016), (Broumi, Bakali, Talea, & Smarandache 2016a), (Broumi, Bakali, Talea, & Smarandache 2016b), (Broumi, Talea, Bakali, & Smarandache 2016c), (Broumi, Talea, Smarandache & Bakali, 2016d), (Broumi, Talea, Bakali, & Smarandache 2016e), (Broumi, Talea, Bakali, & Smarandache 2016f), (Broumi, Talea, Bakali, & Smarandache 2016g), (Mathew J.K & Mathew. S 2016), (Shah 2016), (Shah & Hussain 2016), (Sahoo & Pal 2015), (Singh 2017), (Ulucay, Sahin & Olgun 2018), (Ulucay et al. 2018), (Bakbak, Ulucay & Sahin 2019), (Sahin et al. 2017), (Sahin, Ulucay & Acıoglu 2018), (Sahin, Ulucay & Broumi 2018)].

We have discussed different operations defined on neutrosophic soft expert graphs using examples to make the concept easier. The concept of strong neutrosophic soft expert graphs and the complement of strong neutrosophic soft graphs are also discussed. Neutrosophic soft expert graphs are pictorial representation in which each vertex and each edge is an element of neutrosophic soft sets. This paper has been arranged as the following;

In section 2, some basic concepts about graphs and neutrosophic soft sets are presented which will be employed in later sections. In section 3, concept of neutrosophic soft expert graphs is given and some of their fundamental properties have been studied. In section 4, the concept of strong neutrosophic soft expert graphs and its complement is studied. In section 5, we present an application of neutrosophic soft expert graphs in decision making and then an illustrative example is given. In section 6, a comparison analysis is conducted between the proposed approach and other existing methods, in order to verify its feasibility and effectiveness. Finally, the conclusions are drawn in section 7.

1. BACKGROUND

In this section, we have given some definitions about graphs and neutrosophic soft sets. These will be helpful in later sections.

Definition 1.1. (Smarandache 2005) Let U be a universe of discourse, with a generic element in U denoted by u, then a neutrosophic (NS) set A is an object having the form

$$A = \{<u: T_A(u), I_A(u), F_A(u) >, u \in U\}$$

where the functions T, I, F: $U \rightarrow\ ]^{-}0, 1^{+}[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $u \in U$ to the set A with the condition.

$$^{-}0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3^{+}$$

Definition 1.2. [(Maji 2013)] Let U be an initial universe set and E be a set of parameters. Consider AE. Let $NS(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the neutrosophic soft set over U, where F is a mapping given by

$F: A \rightarrow NS(U)$.

Definition 1.3. (Sahin, Alkhazaleh and Ulucay 2015)A pair (F,A) is called a neutrosophic soft expert set over U, where F is mapping given by

$F: A \rightarrow P(U)$

where $P(U)$ denotes the power neutrosophic set of U.

Definition 1.4. [(Gani & Begum 2003)] A fuzzy graph is pair of functions $G=(\sigma,\mu)$ where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ. i.e. $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u,v \in V$ where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

Definition 1.5.[(Gani & Begum 2003)] The fuzzy subgraph $H=(\tau,\rho)$ is called a fuzzy subgraph of $G=(\sigma,\mu)$, if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for all $u,v \in V$.

Definition 1.6.[(Gani and Begum 2010)] An intuitionistic fuzzy graph is of the form $G=(V,E)$ where

1. $V=\{v_1,v_2,\ldots,v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and

$$0 \leq \mu_1\left(v_i\right) + \gamma_1\left(v_i\right) \leq 1$$

for every $v_i \in V$, $(i=1,2,\ldots,n)$

2. $E \subseteq V \times V$ where $\mu_2: V \times V \rightarrow [0,1]$ and $\gamma_2: V \times V \rightarrow [0,1]$ are such that

$$\mu_2\left(v_i, v_j\right) \leq min\left[\mu_1\left(v_i\right), \mu_1\left(v_j\right)\right]$$

and

$$\gamma_2\left(v_i, v_j\right) \geq max\left[\gamma_1\left(v_i\right), \gamma_1\left(v_j\right)\right]$$

and

$$0 \leq \mu_2\left(v_i, v_j\right) + \gamma_2\left(v_i, v_j\right) \leq 1$$

for every $(v_i, v_j) \in E$, $(i,j=1,2,\ldots,n)$.

Definition 1.7. [(Broumi et al. 2016e)] Let $G^*=(V,E)$ be a simple graph and A be the set of parameters. Let $N(V)$ be the set of all neutrosophic sets in V. By a neutrosophic soft graph NSG, we mean a 4-tuple $G=(G^*,A,f,g)$ where f: $A \to N(V)$, g: $A \to N(V \times V)$ defined as

$$f\left(e\right) = f_e = \left\{x, T_{f_e}\left(x\right), I_{f_e}\left(x\right), F_{f_e}\left(x\right) : x \in V\right\}$$

and

$$g\left(e\right) = g_e = \left\{\left(x,y\right), T_{f_e}\left(x,y\right), I_{f_e}\left(x,y\right), F_{f_e}\left(x,y\right) : \left(x,y\right) \in V \times V\right\}$$

are neutrosophic sets over V and $V \times V$ respectively, such that

$$T_{g_e}\left(x,y\right) \leq min\left\{T_{f_e}\left(x\right), T_{f_e}\left(y\right)\right\},$$

$$I_{g_e}\left(x,y\right) \leq min\left\{I_{f_e}\left(x\right), I_{f_e}\left(y\right)\right\},$$

$$F_{g_e}\left(x,y\right) \geq max\left\{F_{f_e}\left(x\right), F_{f_e}\left(y\right)\right\}.$$

For all $(x,y) \in V \times V$ and $e \in A$. We can also denote a NSG by

$$G = \left(G^*, A, f, g\right) = \left\{N\left(e\right) : e \in A\right\}$$

which is a parameterized family of graphs $N(e)$ we call them Neutrosophic graphs.

2. NEUTROSOPHIC SOFT EXPERT GRAPH

In this section, we introduce the definition of a neutrosophic soft expert graph and give basic properties of this concept.

Let V be a universe, Y a set of parameters, X a set of experts (agents), and

$$O = \begin{Bmatrix} 1 = agree, \\ 0 = disagree \end{Bmatrix}$$

a set of opinions. Let $Z=YXO$ and AZ.

2.1 Definition Let $G^*=(V,E)$ be a simple graph and A be the set of parameters. Let $N(V)$ be the set of all neutrosophic sets in V. By a neutrosophic soft expert graph NSEG, we mean a 4-tuple $G=(G^*,A,f,g)$ where $f: A \to N(V)$, $g: A \to N(V \times V)$ defined as

$$f(\alpha) = f_\alpha = \left\{x, \mu_{f_\alpha}(x), \vartheta_{f_\alpha}(x), w_{f_\alpha}(x) : x \in V\right\}$$

and

$$g(\alpha) = g_\alpha = \left\{(x,y), \mu_{f_\alpha}(x,y), \vartheta_{f_\alpha}(x,y), w_{f_\alpha}(x,y) : (x,y) \in V \times V\right\}$$

are neutrosophic sets over V and $V \times V$ respectively, such that

$$\mu_{g_\alpha}(x,y) \leq min\left\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\right\},$$

$$\vartheta_{g_\alpha}(x,y) \leq min\left\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\right\},$$

$$w_{g_\alpha}(x,y) \geq max\left\{w_{f_\alpha}(x), w_{f_\alpha}(y)\right\}.$$

For all $(x,y)\in V\times V$ and $\alpha\in A$. We can also denote a NSEG by

$$G=\left(G^{*},A,f,g\right)=\left\{N\left(\alpha\right):\alpha\in A\right\}$$

which is a parameterized family of graphs $N(\alpha)$ we call them Neutrosophic graphs.

2.2 Example Suppose that $G^{*}=(V,E)$ be a simple graph with

$$V=\left\{x_1,x_2,x_3\right\},Y=\left\{e_1,e_2,e_3\right\}$$

be a set of parameters and $X=\{p,q\}$ be a set of experts. A NSEG is given in Table 1 and

$$\mu_{g_\alpha}\left(x_i,x_j\right)=0,\vartheta_{g_\alpha}\left(x_i,x_j\right)=0$$

and $w_{g_\alpha}\left(x_i,x_j\right)=1,$ for all

$$\left(x_i,x_j\right)\in V\times V\setminus\left\{\left(x_1,x_2\right),\left(x_2,x_3\right),\left(x_3,x_1\right)\right\}$$

and for all $\alpha\in A$

2.3 Definition A neutrosophic soft expert graph $G=(G^{*},A^{1},f^{1},g^{1})$ is called a neutrosophic soft expert subgraph of $G=(G^{*},A,f,g)$ if

1. $A^{1}\subseteq A$
2. $f_{\alpha}^{1}\subseteq f$, that is,

$$\mu_{f^1{}_\alpha}\left(x\right)\le\mu_{f_\alpha}\left(x\right),\vartheta_{f^1{}_\alpha}\left(x\right)\le\vartheta_{f_\alpha}\left(x\right),w_{f^1{}_\alpha}\left(x\right)\le w_{f_\alpha}\left(x\right).$$

3. $g_{\alpha}^{1}\subseteq g$, that is,

$$\mu_{f^1{}_\alpha}\left(x,y\right)\le\mu_{f_\alpha}\left(x,y\right),\vartheta_{f^1{}_\alpha}\left(x,y\right)\le\vartheta_{f_\alpha}\left(x,y\right),w_{f^1{}_\alpha}\left(x,y\right)\le w_{f_\alpha}\left(x,y\right).$$

Table 1. Neutrosophic soft expert graph

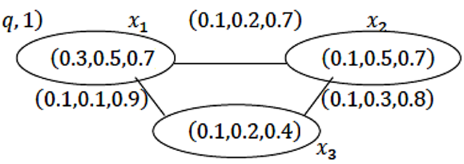
f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1, q, 1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_2, p, 1)$	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
$(e_2, q, 1)$	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
$(e_3, p, 1)$	(0.1,0.1,0.5)	(0.1,0.2,0.4)	(0.1,0.2,0.6)
$(e_3, q, 1)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_2, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, q, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
$(e_3, p, 0)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_3, q, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_3, p, 1)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_3, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, p, 0)$	(0.1,0.2,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, q, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)

Table 2. Neutrosophic soft expert subgraph

f^1	x_1	x_2	x_3
$(e_1,p,1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1,q,1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_1,p,0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1,q,0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
g^1	(x_1,x_2)	(x_2,x_3)	(x_1,x_3)
$(e_1,p,1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1,q,1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_1,p,0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)
$(e_1,q,0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)

Figure 1.

$N(e_1,q,1)$ Corresponding to $(e_1,q,1)$



for all $\alpha \in A^1$.

2.4 Example Suppose that $G^*(V,E)$ be a simple graph with $V=\{x_1,x_2,x_3\}$, $Y=\{e_1\}$ be a set of parameters and $X=\{p\}$ be a set of experts. A neutrosophic soft expert subgraph of example 2.2 is given in Table 2 and

$$\mu_{g_\alpha}\left(x_i,x_j\right)=0, \vartheta_{g_\alpha}\left(x_i,x_j\right)=0$$

and $w_{g_\alpha}\left(x_i,x_j\right)=1,$ for all

$$\left(x_i,x_j\right)\in V\times V\setminus\left\{\left(x_1,x_2\right),\left(x_2,x_3\right),\left(x_3,x_1\right)\right\}$$

Figure 2.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

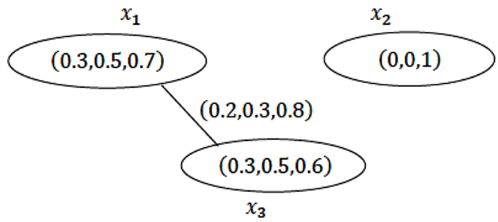


Figure 3.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

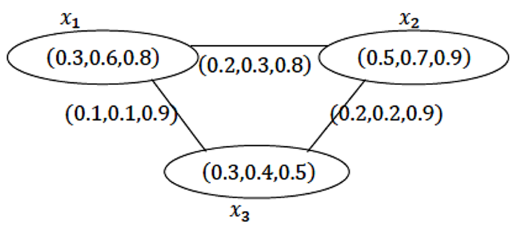


Figure 4.

$N(e_1, q, 0)$ Corresponding to $(e_1, q, 0)$

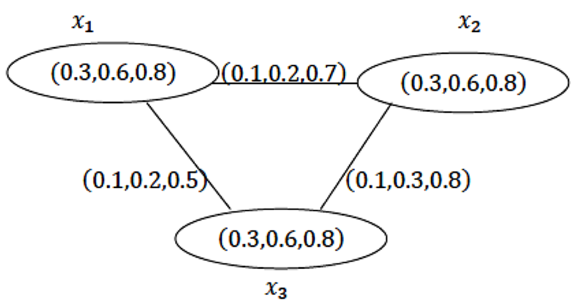


Table 3. The agree-neutrosophic soft expert graph

f_1	x_1	x_2	x_3
$(e_1,p,1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1,q,1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_2,p,1)$	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
$(e_2,q,1)$	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
$(e_3,p,1)$	(0.1,0.1,0.5)	(0.1,0.2,0.4)	(0.1,0.2,0.6)
$(e_3,q,1)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
g_1	(x_1,x_2)	(x_2,x_3)	(x_1,x_3)
$(e_1,p,1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1,q,1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2,p,1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2,q,1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_3,p,1)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_3,q,1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)

and for all $\alpha \in A$.

2.5 Definition A neutrosophic soft expert subgraph $G=(G^*,A^1,f^1,g^1)$ is said to be spanning neutrosophic soft expert subgraph of $G=(G^*,A,f,g)$ if $f_\alpha^1(x)=f(x)$, for all $x \in V$, $\alpha \in A^1$.

2.6 Definition An agree-neutrosophic soft expert graph $G_1=(G^*,A,f_1,g_1)$ over $G^*=(V,E)$ is a neutrosophic soft expert subgraph of $G=(G^*,A,f,g)$ defined as follow

$$G_1=\left(G^*,A,f_1,g_1\right)=\left\{f_1\left(\alpha\right),g_1\left(\alpha\right):\alpha \in E\times X\times\left\{1\right\}\right\}.$$

2.7 Example Consider Example 2.2. Then the agree-neutrosophic soft expert graph $G_1=(G^*,A,f_1,g_1)$ over $G^*=(V,E)$.

2.8 Definition An disagree-neutrosophic soft expert graph $G_0=(G^*,A,f_0,g_0)$ over $G^*=(V,E)$ is a neutrosophic soft expert subgraph of $G=(G^*,A,f,g)$ defined as follow

Table 4. The disagree-neutrosophic soft expert graph

f_2	x_1	x_2	x_3
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_2, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, q, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
$(e_3, p, 0)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_3, q, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
g_2	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, p, 0)$	(0.1,0.2,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_3, q, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)

$$G_0 = \left(G^*, A, f_0, g_0\right) = \left\{f_0\left(\alpha\right), g_0\left(\alpha\right) : \alpha \in E \times X \times \left\{0\right\}\right\}.$$

2.9 Example Consider Example 2.2. Then the disagree-neutrosophic soft expert graph $G_0=(G^*,A,f_0,g_0)$ over $G^*=(V,E)$.

2.10 Definition The union of two neutrosophic soft expert graphs $G^1=(G^*,A^1,f^1,g^1)$ and $G^2=(G^*,A^2,f^2,g^2)$ is denoted by $G=(G^*,A,f,g)$ with $A = A^1 \cup A^2$ where the truth-membership, indeterminacy-membership and falsity-membership of union are as follows

$$\mu_{f_\alpha}(x) = \begin{cases} \mu_{f_\alpha^1}(x) = \begin{cases} if\ e \in A^1 - A^2\ and\ x \in V^1 - V^2\ or \\ if\ e \in A^1 - A^2\ and\ x \in V^1 \cap V^2\ or \\ if\ e \in A^1 \cap A^2\ and\ x \in V^1 - V^2. \end{cases} \\ \mu_{f_\alpha^2}(x) = \begin{cases} if\ e \in A^2 - A^1\ and\ x \in V^2 - V^1\ or \\ if\ e \in A^2 - A^1\ and\ x \in V^1 \cap V^2\ or \\ if\ e \in A^2 \cap A^1\ and\ x \in V^2 - V^1. \end{cases} \\ \max\left\{\mu_{f_\alpha^1}(x), \mu_{f_\alpha^2}(x)\right\}\left\{if\ e \in A^1 \cap A^2\ and\ x \in V^1 \cap V^2\right\} \\ 0, otherwise \end{cases}$$

$$\vartheta_{f_\alpha}(x) = \begin{cases} \vartheta_{f_\alpha^1}(x) = \begin{cases} if\ e \in A^1 - A^2\ and\ x \in V^1 - V^2\ or \\ if\ e \in A^1 - A^2\ and\ x \in V^1 \cap V^2\ or \\ if\ e \in A^1 \cap A^2\ and\ x \in V^1 - V^2. \end{cases} \\ \vartheta_{f_\alpha^2}(x) = \begin{cases} if\ e \in A^2 - A^1\ and\ x \in V^2 - V^1\ or \\ if\ e \in A^2 - A^1\ and\ x \in V^1 \cap V^2\ or \\ if\ e \in A^2 \cap A^1\ and\ x \in V^2 - V^1. \end{cases} \\ \max\left\{\vartheta_{f_\alpha^1}(x), \vartheta_{f_\alpha^2}(x)\right\}\left\{if\ e \in A^1 \cap A^2\ and\ x \in V^1 \cap V^2\right\} \\ 0, otherwise \end{cases}$$

$$w_{f_\alpha}(x) = \begin{cases} w_{f_\alpha^1}(x) = \begin{cases} if\ e \in A^1 - A^2\ and\ x \in V^1 - V^2\ or \\ if\ e \in A^1 - A^2\ and\ x \in V^1 \cap V^2\ or \\ if\ e \in A^1 \cap A^2\ and\ x \in V^1 - V^2. \end{cases} \\ w_{f_\alpha^2}(x) = \begin{cases} if\ e \in A^2 - A^1\ and\ x \in V^2 - V^1\ or \\ if\ e \in A^2 - A^1\ and\ x \in V^1 \cap V^2\ or \\ if\ e \in A^2 \cap A^1\ and\ x \in V^2 - V^1. \end{cases} \\ \max\left\{w_{f_\alpha^1}(x), w_{f_\alpha^2}(x)\right\}\left\{if\ e \in A^1 \cap A^2\ and\ x \in V^1 \cap V^2\right\} \\ 0, otherwise \end{cases}$$

$$\mu_{g_\alpha}\left(x,y\right)\begin{cases}\mu_{g_\alpha^{1}}\left(x,y\right)=\begin{cases}if\; e\in A^1-A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)-\left(V^2\times V^2\right) or\\ if\; e\in A^1-A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)\cap\left(V^2\times V^2\right) or\\ if\; e\in A^1\cap A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)-\left(V^2\times V^2\right).\end{cases}\\ \mu_{g_\alpha^{2}}\left(x,y\right)=\begin{cases}if\; e\in A^2-A^1\; and\left(x,y\right)\in\left(V^2\times V^2\right)-\left(V^1\times V^1\right) or\\ if\; e\in A^2-A^1\; and\left(x,y\right)\in\left(V^2\times V^2\right)\cap\left(V^1\times V^1\right) or\\ if\; e\in A^2\cap A^1\; and\left(x,y\right)\in\left(V^2\times V^2\right)-\left(V^1\times V^1\right).\end{cases}\\ \max\left\{\mu_{g_\alpha^{1}}\left(x,y\right),\mu_{g_\alpha^{2}}\left(x,y\right)\right\}\left\{if\; e\in A^1\cap A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)\cap\left(V^2\times V^2\right)\right\}\\ 0, otherwise\end{cases}$$

$$\vartheta_{g_\alpha}\left(x,y\right)\begin{cases}\vartheta_{g_\alpha^{1}}\left(x,y\right)=\begin{cases}if\; e\in A^1-A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)-\left(V^2\times V^2\right) or\\ if\; e\in A^1-A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)\cap\left(V^2\times V^2\right) or\\ if\; e\in A^1\cap A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)-\left(V^2\times V^2\right).\end{cases}\\ \vartheta_{g_\alpha^{2}}\left(x,y\right)=\begin{cases}if\; e\in A^2-A^1\; and\left(x,y\right)\in\left(V^2\times V^2\right)-\left(V^1\times V^1\right) or\\ if\; e\in A^2-A^1\; and\left(x,y\right)\in\left(V^2\times V^2\right)\cap\left(V^1\times V^1\right) or\\ if\; e\in A^2\cap A^1\; and\left(x,y\right)\in\left(V^2\times V^2\right)-\left(V^1\times V^1\right).\end{cases}\\ \max\left\{\vartheta_{g_\alpha^{1}}\left(x,y\right),\vartheta_{g_\alpha^{2}}\left(x,y\right)\right\}\left\{if\; e\in A^1\cap A^2\; and\left(x,y\right)\in\left(V^1\times V^1\right)\cap\left(V^2\times V^2\right)\right\}\\ 0, otherwise\end{cases}$$

Table 5. Neutrosophic soft expert graphs

f_1	x_1	x_3	x_5
$(e_1, p, 1)$	(0.5,0.6,0.7)	(0,0,1)	(0.3,0.4,0.6)
$(e_1, p, 0)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
g_1	(x_1, x_3)	(x_3, x_5)	(x_1, x_5)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.1,0.3,0.8)
$(e_1, p, 0)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)

$$w_{g_\alpha}(x,y)\begin{cases} w_{g_\alpha^{1}}(x,y)=\begin{cases} if\ e\in A^1-A^2\ and\left(x,y\right)\in\left(V^1\times V^1\right)-\left(V^2\times V^2\right)or\\ if\ e\in A^1-A^2\ and\left(x,y\right)\in\left(V^1\times V^1\right)\cap\left(V^2\times V^2\right)or\\ if\ e\in A^1\cap A^2\ and\left(x,y\right)\in\left(V^1\times V^1\right)-\left(V^2\times V^2\right).\end{cases}\\ w_{g_\alpha^{2}}(x,y)=\begin{cases} if\ e\in A^2-A^1\ and\left(x,y\right)\in\left(V^2\times V^2\right)-\left(V^1\times V^1\right)or\\ if\ e\in A^2-A^1\ and\left(x,y\right)\in\left(V^2\times V^2\right)\cap\left(V^1\times V^1\right)or\\ if\ e\in A^2\cap A^1\ and\left(x,y\right)\in\left(V^2\times V^2\right)-\left(V^1\times V^1\right).\end{cases}\\ \max\left\{w_{g_\alpha^{1}}(x,y),w_{g_\alpha^{2}}(x,y)\right\}\left\{if\ e\in A^1\cap A^2\ and\left(x,y\right)\in\left(V^1\times V^1\right)\cap\left(V^2\times V^2\right)\right\}\\ 0,otherwise\end{cases}$$

Figure 5.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

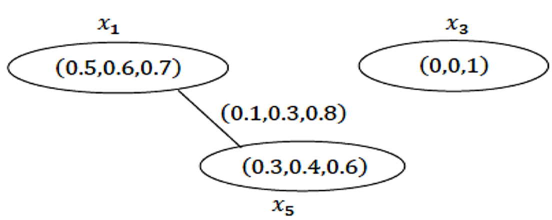


Figure 6.

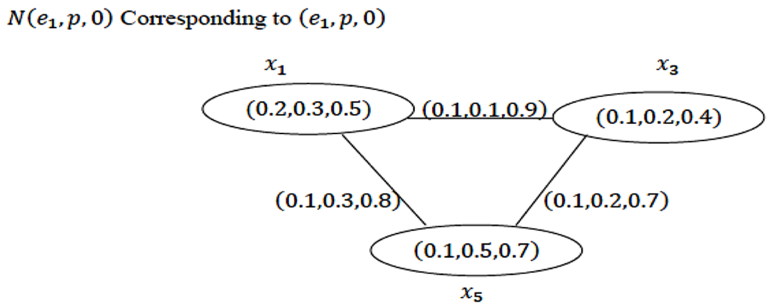


Table 6. Neutrosophic soft expert graphs

f_2	x_2	x_4	x_5
$(e_1,p,1)$	(0,0,1)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_2,p,1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_1,p,0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2,p,0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
g_2	**(x_2,x_4)**	**(x_4,x_5)**	**(x_2,x_5)**
$(e_1,p,1)$	(0,0,1)	(0.2,0.2,0.9)	(0,0,1)
$(e_2,p,1)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_1,p,0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2,p,0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

2.11 Example Suppose that $G^{1*}=(V^1,E^1)$ be a simple graph with

$$V = \{x_1, x_3, x_5\}, Y = \{e_1\}$$

be a set of parameters and $X=\{p\}$ be a set of experts. Let $G^{2*}=(V^2,E^2)$ be a simple graph with

$$V = \{x_2, x_4, x_5\}, Y = \{e_1, e_2\}$$

be a set of parameters and $X=\{p\}$ be a set of experts. ANSEG is given in Table 5 and

Figure 7.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

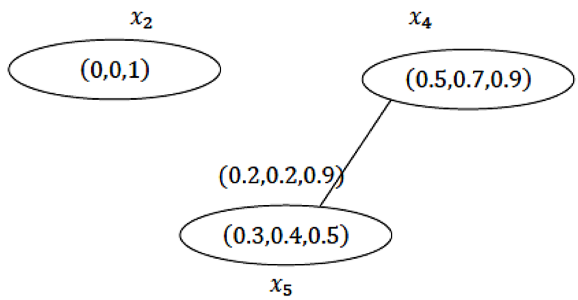
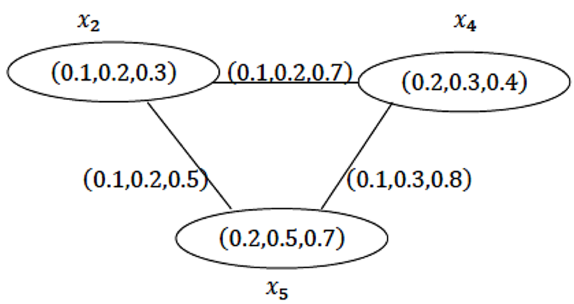


Figure 8.

$N(e_2, p, 1)$ Corresponding to $(e_2, p, 1)$



$$\mu_{g_\alpha}\left(x_i, x_j\right) = 0, \vartheta_{g_\alpha}\left(x_i, x_j\right) = 0$$

Figure 9.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

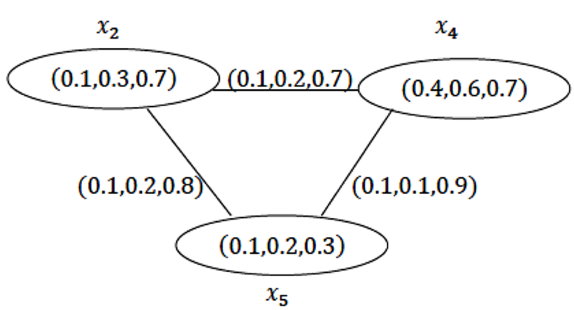


Figure 10.

$N(e_2, p, 0)$ Corresponding to $(e_2, p, 0)$

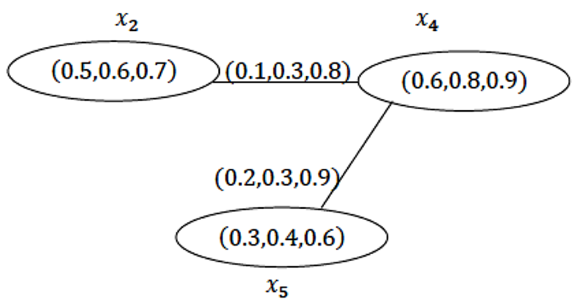


Table 7a. Neutrosophic soft expert graphs

f	x_1	x_2	x_3	x_4	x_5
(e_1,p,1)	(0,0,1)	(0.5,0.7,0.9)	(0.3,0.4,0.5)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
(e_2,p,1)	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
(e_1,p,0)	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
(e_2,p,0)	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)	(0.6,0.8,0.9)	(0.3,0.4,0.6)

and $w_{g_\alpha}\left(x_i, x_j\right) = 1,$ for all

$$\left(x_i, x_j\right) \in V^1 \times V^1 \setminus \left\{\left(x_1, x_2\right), \left(x_2, x_3\right), \left(x_1, x_3\right)\right\}$$

and for all $\alpha \in A^1$.

A NSEG $G^2=(G^*, A^2, f^2, g^2)$ is given in Table 6 and

$$\mu_{g_\alpha}\left(x_i, x_j\right) = 0, \vartheta_{g_\alpha}\left(x_i, x_j\right) = 0$$

Table 7b. Neutrosophic soft expert graphs.

g	(x_1,x_3)	(x_3,x_5)	(x_1,x_5)	(x_2,x_4)	(x_4,x_5)	(x_2,x=5)
(e_1,p,1)	(0,0,1)	(0.2,0.2,0.9)	(0,0,1)	(0,0,1)	(0.2,0.2,0.9)	(0,0,1)
(e_2,p,1)	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
(e_1,p,0)	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
(e_2,p,0)	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

Figure 11.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

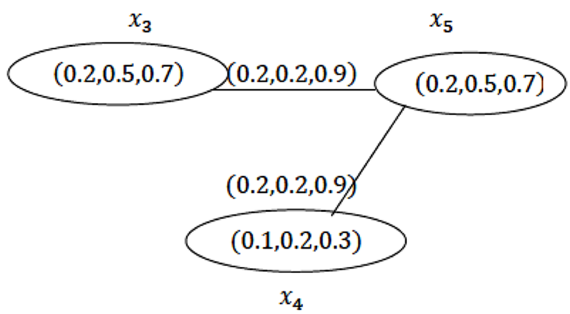


Figure 12.

$N(e_2, p, 1)$ Corresponding to $(e_2, p, 1)$

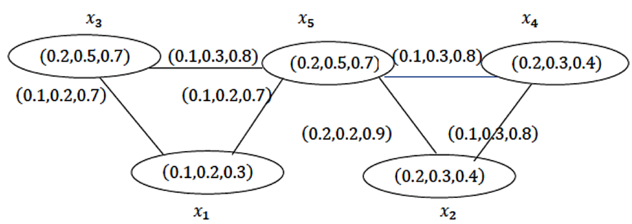


Figure 13.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

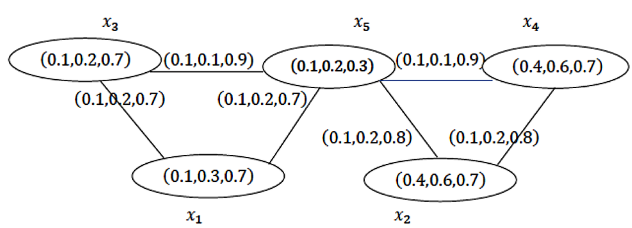
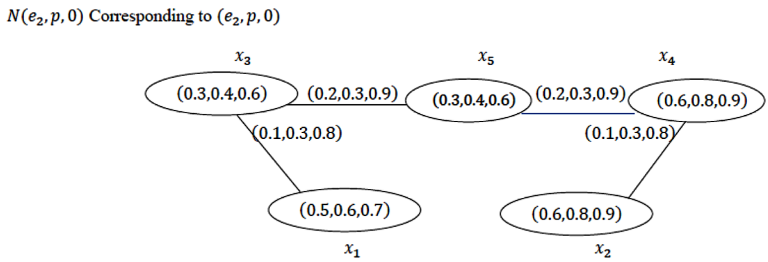


Figure 14.



and $w_{g_\alpha}\left(x_i, x_j\right) = 1,$ for all

$$\left(x_i, x_j\right) \in V^2 \times V^2 \setminus \left\{\left(x_1, x_2\right), \left(x_2, x_3\right), \left(x_1, x_3\right)\right\}$$

and for all$\alpha \in A^2$.

2.12 Proposition The union $G=(G^*,A,f,g)$ of two neutrosophic soft expert graph $G^2=(G^*,A^2,f^2,g^2)$ and $G^1=(G^*,A^1,f^1,g^1)$ is a neutrosophic soft expert graph.

Proof i.

$$\text{if } e \in A^1 - A^2 \text{and} \left(x, y\right) \in \left(V^1 \times V^1\right) - \left(V^2 \times V^2\right),$$

then

$$\begin{aligned}\mu_{g_\alpha}\left(x, y\right) = \mu_{g_\alpha^1}\left(x, y\right) &\le min\left\{\mu_{f_\alpha^1}\left(x\right), \mu_{f_\alpha^1}\left(y\right)\right\} \\ &= min\left\{\mu_{f_\alpha^1}\left(x\right), \mu_{f_\alpha^1}\left(y\right)\right\}\end{aligned}$$

So

$$\mu_{g_\alpha}\left(x, y\right) \le min\left\{\mu_{f_\alpha}\left(x\right), \mu_{f_\alpha}\left(y\right)\right\}$$

Also

$$= min\left\{\vartheta_{f_{\alpha}^{1}}\left(x\right), \vartheta_{f_{\alpha}^{1}}\left(y\right)\right\}$$

So

$$\vartheta_{g_{\alpha}}\left(x,y\right) \leq min\left\{\vartheta_{f_{\alpha}}\left(x\right), \vartheta_{f_{\alpha}}\left(y\right)\right\}$$

Now

$$w_{g_{\alpha}}\left(x,y\right) = w_{g_{\alpha}^{1}}\left(x,y\right) \geq max\left\{w_{f_{\alpha}^{1}}\left(x\right), w_{f_{\alpha}^{1}}\left(y\right)\right\}$$
$$= max\left\{w_{f_{\alpha}^{1}}\left(x\right), w_{f_{\alpha}^{1}}\left(y\right)\right\}$$

So

$$w_{g_{\alpha}}\left(x,y\right) \geq max\left\{w_{f_{\alpha}}\left(x\right), w_{f_{\alpha}}\left(y\right)\right\}$$

Similarly if

$$\left\{e \in A^{1} - A^{2} and \left(x,y\right) \in \left(V^{1} \times V^{1}\right) \cap \left(V^{2} \times V^{2}\right)\right\},$$

or

$$\text{if} \left\{e \in A^{1} \cap A^{2} \text{and} \left(x,y\right) \in \left(V^{1} \times V^{1}\right) - \left(V^{2} \times V^{2}\right)\right\},$$

we can show the same as done above.

Proof ii.

$$\text{if}\, e \in A^{1} \cap A^{2} \text{and} \left(x,y\right) \in \left(V^{1} \times V^{1}\right) \cap \left(V^{2} \times V^{2}\right),$$

then

$$\mu_{g_\alpha}(x,y) = max\left\{\mu_{f_\alpha^1}(x), \mu_{f_\alpha^1}(y)\right\}$$
$$\leq max\left\{min\left\{\mu_{f_\alpha^1}(x), \mu_{f_\alpha^1}(y)\right\}, min\left\{\mu_{f_\alpha^2}(x), \mu_{f_\alpha^2}(y)\right\}\right\}$$
$$\leq min\left\{max\left\{\mu_{f_\alpha^1}(x), \mu_{f_\alpha^2}(x)\right\}, max\left\{\mu_{f_\alpha^1}(y), \mu_{f_\alpha^2}(y)\right\}\right\}$$
$$= min\left\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\right\}$$

Also

$$\leq max\left\{min\left\{\vartheta_{f_\alpha^1}(x), \vartheta_{f_\alpha^1}(y)\right\}, min\left\{\vartheta_{f_\alpha^2}(x), \vartheta_{f_\alpha^2}(y)\right\}\right\}$$
$$\leq min\left\{max\left\{\vartheta_{f_\alpha^1}(x), \vartheta_{f_\alpha^2}(x)\right\}, max\left\{\vartheta_{f_\alpha^1}(y), \vartheta_{f_\alpha^2}(y)\right\}\right\}$$
$$= min\left\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\right\}$$

Now

$$\geq min\left\{max\left\{w_{f_\alpha^1}(x), w_{f_\alpha^1}(y)\right\}, max\left\{w_{f_\alpha^2}(x), w_{f_\alpha^2}(y)\right\}\right\}$$
$$\geq max\left\{min\left\{w_{f_\alpha^1}(x), w_{f_\alpha^2}(x)\right\}, min\left\{w_{f_\alpha^1}(y), w_{f_\alpha^2}(y)\right\}\right\}$$
$$= max\left\{w_{f_\alpha}(x), w_{f_\alpha}(y)\right\}$$

Hence the union $\mathrm{G} = G^1 \cup G^2$ is a neutrosophic soft expert graph.

2.13 Definition The inter section of two neutrosophic soft expert graphs $G^1=(G^{1*},A^1,f^1,g^1)$ and $G^2=(G^{2*},A^2,f^2,g^2)$ is denoted by $G=(G^*,A,f,g$ with

$$A = A^1 \cap A^2, V = V^1 \cap V^2$$

and the truth-membership, indeterminacy-membership and falsity-membership of inter section are as follows

Table 8. Neutrosophic soft expert graphs

f_1	x_2	x_3	x_5
$(e_1, p, 1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_1, p, 0)$	(0.1,0.3,0.7)	(0.2,0.4,0.4)	(0.4,0.6,0.7)
g_1	(x_2, x_3)	(x_3, x_5)	(x_2, x_5)
$(e_1, p, 1)$	(0.1,0,2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.8)
$(e_1, p, 0)$	(0.1,0.2,0.8)	(0.2,0.3,0.9)	(0,0,1)

$$\mu_{f_\alpha} = \begin{cases} \mu_{f_\alpha^1}(x) \, if \, e \in A^1 - A^2 \\ \mu_{f_\alpha^2}(x) \, if \, e \in A^2 - A^1 \\ min\left\{\mu_{f_\alpha^1}(x), \mu_{f_\alpha^2}(x)\right\} if \, e \in A^1 \cap A^2 \end{cases}$$

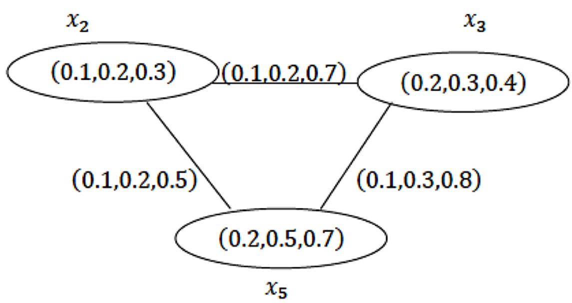
$$\vartheta_{f_\alpha} = \begin{cases} \vartheta_{f_\alpha^1}(x) \, if \, e \in A^1 - A^2 \\ \vartheta_{f_\alpha^2}(x) \, if \, e \in A^2 - A^1 \\ min\left\{\vartheta_{f_\alpha^1}(x), \vartheta_{f_\alpha^2}(x)\right\} if \, e \in A^1 \cap A^2 \end{cases}$$

$$w_{f_\alpha} = \begin{cases} w_{f_\alpha^1}(x) \, if \, e \in A^1 - A^2 \\ w_{f_\alpha^2}(x) \, if \, e \in A^2 - A^1 \\ max\left\{w_{f_\alpha^1}(x), w_{f_\alpha^2}(x)\right\} if \, e \in A^1 \cap A^2 \end{cases}$$

$$\mu_{g_\alpha} = \begin{cases} \mu_{g_\alpha^1}(x,y) \, if \, e \in A^1 - A^2 \\ \mu_{g_\alpha^2}(x,y) \, if \, e \in A^2 - A^1 \\ min\left\{\mu_{g_\alpha^1}(x,y), \mu_{g_\alpha^2}(x,y)\right\} if \, e \in A^1 \cap A^2 \end{cases}$$

Figure 15.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$



$$\vartheta_{g_\alpha} = \begin{cases} \vartheta_{g_\alpha^1}\left(x,y\right) if\, e \in A^1 - A^2 \\ \vartheta_{g_\alpha^2}\left(x,y\right) if\, e \in A^2 - A^1 \\ min\left\{\vartheta_{g_\alpha^1}\left(x,y\right), \vartheta_{g_\alpha^2}\left(x,y\right)\right\} if\, e \in A^1 \cap A^2 \end{cases}$$

$$w_{g_\alpha} = \begin{cases} w_{g_\alpha^1}\left(x,y\right) if\, e \in A^1 - A^2 \\ w_{g_\alpha^2}\left(x,y\right) if\, e \in A^2 - A^1 \\ max\left\{w_{g_\alpha^1}\left(x,y\right), w_{g_\alpha^2}\left(x,y\right)\right\} if\, e \in A^1 \cap A^2 \end{cases}$$

2.14 Example Suppose that $G^{1*}=(V^1,E^1)$ be a simple graph with

Figure 16.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

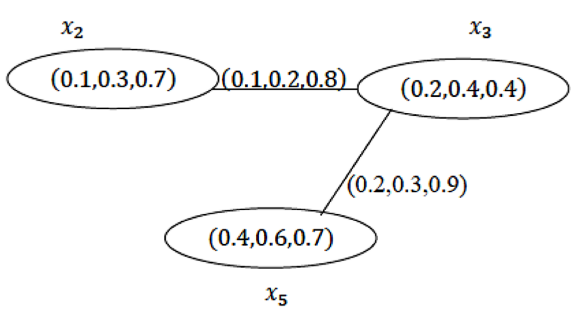


Table 9. Neutrosophic soft expert graphs

f_2	x_1	x_3	x_5
$(e_1, p, 1)$	(0.1,0.2,0.4)	(0.2,0.3,0.4)	(0.4,0.6,0.7)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
g_2	(x_1, x_3)	(x_3, x_5)	(x_1, x_5)
$(e_1, p, 1)$	(0.1,0,2,0.8)	(0.2,0.3,0.9)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.9)	(0.2,0.2,0.9)	(0.2,0.3,0.8)

Figure 17.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

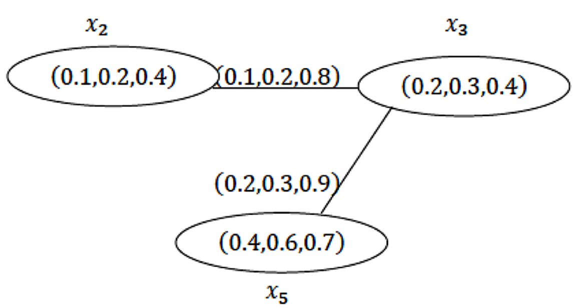


Figure 18.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

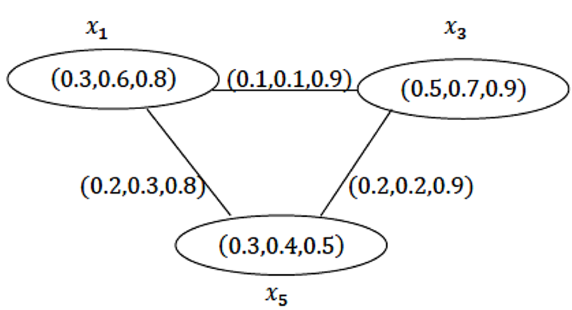
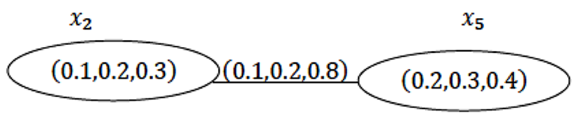


Table 10. The intersection of two neutrosophic soft expert graphs

f	x_2	x_5	g	(x_2, x_5)
$(e_1, p, 1)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	$(e_1, p, 1)$	(0.1,0.2,0.8)
$(e_1, p, 0)$	(0.1,0.3,0.7)	(0.2,0.4,0.4)	$(e_1, p, 0)$	(0,0,1)

Figure 19.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$



$$V^1 = \{x_2, x_3, x_5\}, Y = \{e_1\}$$

be a set of parameters and $X=\{p\}$ be a set of experts. A NSEG is given in Table 8 and

$$\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$$

and $w_{g_\alpha}(x_i, x_j) = 1,$ for all

$$(x_i, x_j) \in V^1 \times V^1 \setminus \{(x_2, x_3), (x_3, x_5), (x_2, x_5)\}$$

and for all $\alpha \in A^1$.

Let $G^{2*}=(V^2,E^2)$ be a simple graph with

$$V^2 = \{x_1, x_3, x_5\}, Y = \{e_1\}$$

Figure 20.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$

be a set of parameters and $X=\{p\}$ be a set of experts. A NSEG is given in Table 9 and

$$\mu_{g_\alpha}\left(x_i,x_j\right)=0,\vartheta_{g_\alpha}\left(x_i,x_j\right)=0$$

and $w_{g_\alpha}\left(x_i,x_j\right)=1,$ for all

$$\left(x_i,x_j\right)\in V^2\times V^2\setminus\left\{\left(x_1,x_3\right),\left(x_3,x_5\right),\left(x_1,x_5\right)\right\}$$

and for all $\alpha\in A^2$

Let

$$V=V^1\cap V^2=\left\{x_3,x_5\right\},A=A^1\cap A^2=\left\{\left(e_1,p,1\right),\left(e_1,p,0\right)\right\}.$$

The intersection of two neutrosophic soft expert graphs $G^1=(G^{1*},A^1,f^1,g^1)$ and $G^2=(G^{2*},A^2,f^2,g^2)$ is given in Table 10.

2.15 Proposition The intersection $G=(G^*,A,f,g)$ of two neutrosophic soft expert graph $G^1=(G^*,A^1,f^1,g^1)$ and $G^2=(G^*,A^2,f^2,g^2)$ is a neutrosophic soft expert graph where $A=A^1\cap A^2,V=V^1\cap V^2$.

Proof i. if $e\in A^1-A^2$, then

$$\begin{aligned}\mu_{g_\alpha}\left(x,y\right)&=\mu_{g_\alpha^1}\left(x,y\right)\leq min\left\{\mu_{f_\alpha^1}\left(x\right),\mu_{f_\alpha^1}\left(y\right)\right\}\\&=min\left\{\mu_{f_\alpha}\left(x\right),\mu_{f_\alpha}\left(y\right)\right\}\end{aligned}$$

So

$$\mu_{g_\alpha}\left(x,y\right)\leq min\left\{\mu_{f_\alpha}\left(x\right),\mu_{f_\alpha}\left(y\right)\right\}$$

Also

$$=min\left\{\vartheta_{f_\alpha}\left(x\right),\vartheta_{f_\alpha}\left(y\right)\right\}$$

So

$$\vartheta_{g_\alpha}\left(x,y\right) \le min\left\{\vartheta_{f_\alpha}\left(x\right), \vartheta_{f_\alpha}\left(y\right)\right\}$$

now

$$\begin{aligned} w_{g_\alpha}\left(x,y\right) &= w_{g_\alpha{}^1}\left(x,y\right) \ge max\left\{w_{f_\alpha{}^1}\left(x\right), w_{f_\alpha{}^1}\left(y\right)\right\} \\ &= max\left\{w_{f_\alpha{}^1}\left(x\right), w_{f_\alpha{}^1}\left(y\right)\right\} \end{aligned}$$

so

$$w_{g_\alpha}\left(x,y\right) \ge max\left\{w_{f_\alpha}\left(x\right), w_{f_\alpha}\left(y\right)\right\}.$$

Similarly if $e \in A^1 - A^2$ we can show the same as done above.
ii. if $e \in A^1 \cap A^2$, then

$$\begin{aligned} \mu_{g_\alpha}\left(x,y\right) &= max\left\{\mu_{f_\alpha{}^1}\left(x\right), \mu_{f_\alpha{}^1}\left(y\right)\right\} \\ &\le min\left\{min\left\{\mu_{f_\alpha{}^1}\left(x\right), \mu_{f_\alpha{}^1}\left(y\right)\right\}, min\left\{\mu_{f_\alpha{}^2}\left(x\right), \mu_{f_\alpha{}^2}\left(y\right)\right\}\right\} \\ &\le min\left\{min\left\{\mu_{f_\alpha{}^1}\left(x\right), \mu_{f_\alpha{}^2}\left(x\right)\right\}, min\left\{\mu_{f_\alpha{}^1}\left(y\right), \mu_{f_\alpha{}^2}\left(y\right)\right\}\right\} \\ &= min\left\{\mu_{f_\alpha}\left(x\right), \mu_{f_\alpha}\left(y\right)\right\} \end{aligned}$$

Also

$$\begin{aligned} &\le min\left\{min\left\{\vartheta_{f_\alpha{}^1}\left(x\right), \vartheta_{f_\alpha{}^1}\left(y\right)\right\}, min\left\{\vartheta_{f_\alpha{}^2}\left(x\right), \vartheta_{f_\alpha{}^2}\left(y\right)\right\}\right\} \\ &\le min\left\{min\left\{\vartheta_{f_\alpha{}^1}\left(x\right), \vartheta_{f_\alpha{}^2}\left(x\right)\right\}, min\left\{\vartheta_{f_\alpha{}^1}\left(y\right), \vartheta_{f_\alpha{}^2}\left(y\right)\right\}\right\} \\ &= min\left\{\vartheta_{f_\alpha}\left(x\right), \vartheta_{f_\alpha}\left(y\right)\right\} \end{aligned}$$

Now

Table 11. Neutrosophic soft expert graphs

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.6)	(0.2,0.4,0.6)	(0.4,0.5,0.9)
$(e_1, p, 0)$	(0.2,0.4,0.5)	(0.1,0.2,0.6)	(0.1,0.5,0.7)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0.1,0,3,0.7)	(0.2,0.4,0.9)	(0.2,0.4,0.9)
$(e_1, p, 0)$	(0.1,0.2,0.8)	(0.1,0.2,0.9)	(0.1,0.4,0.8)

$$\geq max\left\{max\left\{w_{f_\alpha^1}\left(x\right), w_{f_\alpha^1}\left(y\right)\right\}, max\left\{w_{f_\alpha^2}\left(x\right), w_{f_\alpha^2}\left(y\right)\right\}\right\}$$
$$\geq max\left\{max\left\{w_{f_\alpha^1}\left(x\right), w_{f_\alpha^2}\left(x\right)\right\}, max\left\{w_{f_\alpha^1}\left(y\right), w_{f_\alpha^2}\left(y\right)\right\}\right\}$$
$$= max\left\{w_{f_\alpha}\left(x\right), w_{f_\alpha}\left(y\right)\right\}$$

Hence the intersection $G = G^1 \cap G^2$ is a neutrosophic soft expert graph.

3. STRONG NEUTROSOPHIC SOFT EXPERT GRAPH

3.1 Definition A neutrosophic soft expert graph $G=(G^*,A,f,g)$ is called strong if

$$g_\alpha\left(x, y\right) = f_\alpha\left(x\right) \cap f_\alpha\left(y\right),$$

Figure 21.

$N(e_1, p, 1)$ Corresponding to $(e_1, p, 1)$

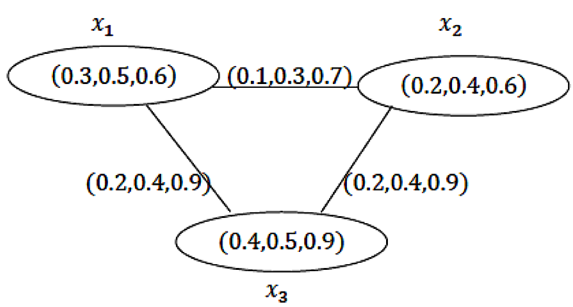
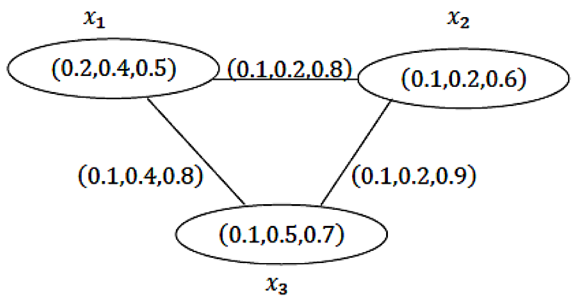


Figure 22.

$N(e_1, p, 0)$ Corresponding to $(e_1, p, 0)$



for all $x,y \in V$, $\alpha \in A$. That is if

$$\mu_{g_\alpha}(x,y) = min\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\},$$

$$\vartheta_{g_\alpha}(x,y) = min\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\},$$

$$w_{g_\alpha}(x,y) = max\{w_{f_\alpha}(x), w_{f_\alpha}(y)\},$$

for all $(x,y) \in E$.

3.2 Example Suppose that $G^*=(V,E)$ be a simple graph with

$$V = \{x_1, x_2, x_3\}, Y = \{e_1\}$$

be a set of parameters and $X=\{p\}$ be a set of experts. A NSEG is given in Table 11 and

$$\mu_{g_\alpha}(x_i, x_j) = 0, \vartheta_{g_\alpha}(x_i, x_j) = 0$$

and $w_{g_\alpha}(x_i, x_j) = 1,$ for all

Figure 23.

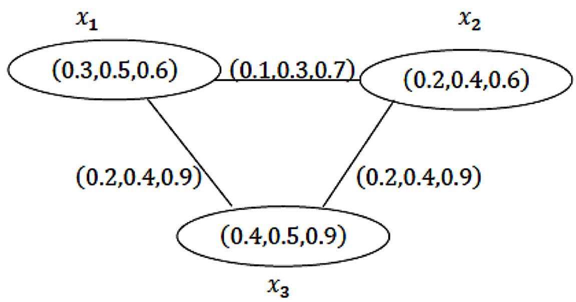
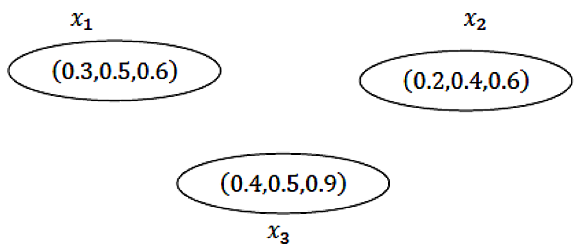


Figure 24.



$$\left(x_i, x_j\right) \in V \times V \setminus \left\{\left(x_1, x_2\right), \left(x_2, x_3\right), \left(x_1, x_3\right)\right\}$$

and for all $\alpha \in A$.

3.3 Definition Let $G=(G^*,A,f,g)$ be a strong neutrosophic soft expert graph that is

Figure 25.

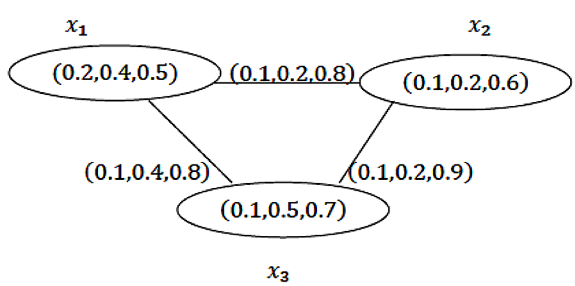
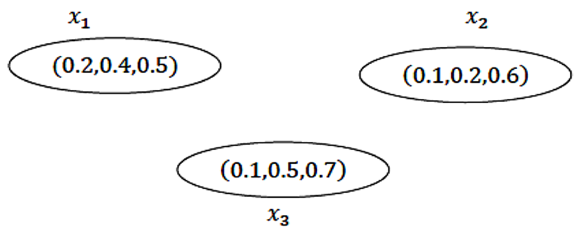


Figure 26.



$$g_\alpha(x,y) = f_\alpha(x) \cap f_\alpha(y),$$

for all $x,y \in V$, $\alpha \in A$. The complement $\overline{G} = \left(\overline{G}^*, \overline{A}, \overline{f}, \overline{g}\right)$ of strong neutrosophic soft expert graph $G=(G^*,A,f,g$ is neutrosophic soft expert expert graph where

1. $\overline{A} = A$
2. $\mu_{f_\alpha}(x) = \overline{\mu_{f_\alpha}}(x), \vartheta_{f_\alpha}(x) = \overline{\vartheta_{f_\alpha}}(x),\ w_{f_\alpha}(x) = \overline{w_{f_\alpha}}(x)$ for all $x \in V$
3. 3. $\overline{\mu_{f_\alpha}}(x) = \begin{cases} min\left\{\mu_{f_\alpha}(x), \mu_{f_\alpha}(y)\right\} \text{if} \mu_{g_\alpha}(x,y) = 0 \\ 0, \qquad \text{otherwise} \end{cases}$

$$\overline{\vartheta_{f_\alpha}}(x) = \begin{cases} min\left\{\vartheta_{f_\alpha}(x), \vartheta_{f_\alpha}(y)\right\} & \text{if} \quad \vartheta_{g_\alpha}(x,y) = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\overline{w_{f_\alpha}}(x) = \begin{cases} min\left\{w_{f_\alpha}(x), w_{f_\alpha}(y)\right\} & \text{if} \quad w_{g_\alpha}(x,y) = 0 \\ 0, & \text{otherwise} \end{cases}$$

3.4 Example For the strong neutrosophic soft graph in previous example, the complements are given below for $(e_1,p,1)$ and $e_1,p0)$

Corresponding to$(e_1,p,1)$ the complement of $N(e_1,p,1)$ is shown in Figure 23. Corresponding to $(e_1,p,1)$ is shown in Figure 24.

Corresponding to$(e_1,p,1)$ the complement of $N(e_1,p,0)$ is shown in Figure 25. Corresponding to $(e_1,p,0)$ is shown in Figure 26.

Table 12. Neutrosophic soft expert graphs

f	x_1	x_2	x_3
$(e_1, p, 1)$	(0.3,0.5,0.7)	(0,0,1)	(0.3,0.5,0.6)
$(e_1, q, 1)$	(0.2,0.3,0.5)	(0.1,0.2,0.4)	(0.1,0.5,0.7)
$(e_2, p, 1)$	(0.3,0.4,0.5)	(0.1,0.3,0.4)	(0.1,0.3,0.6)
$(e_2, q, 1)$	(0.3,0.2,0.5)	(0.3,0.2,0.6)	(0,0,1)
$(e_1, p, 0)$	(0.3,0.6,0.8)	(0.5,0.7,0.9)	(0.3,0.4,0.5)
$(e_1, q, 0)$	(0.1,0.2,0.3)	(0.2,0.3,0.4)	(0.2,0.5,0.7)
$(e_2, p, 0)$	(0.1,0.3,0.7)	(0.4,0.6,0.7)	(0.1,0.2,0.3)
$(e_2, q, 0)$	(0.5,0.6,0.7)	(0.6,0.8,0.9)	(0.3,0.4,0.6)
g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

4. APPLICATIONS OF NEUTROSOPHIC SOFT EXPERT GRAPH

In what follows, let us consider an illustrative example adopted from Adam et al.(Adam and Hassan 2016)and Shahzadi et al.(Shahzadi and Akram 2016).

4.1 Application in Decision-Making Problem

Assume that a hospital wants to fill a position to be chosen by an expert committee. Suppose that $G^*=(V,E)$ be a simple graph with

$$V = \{x_1, x_2, x_3\}, Y = \{e_1, e_2\}$$

Table 13. Neutrosophic soft expert graphs

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	(0,0,1)	(0,0,1)	(0.2,0.3,0.8)
$(e_1, q, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, p, 1)$	(0.1,0.1,0.9)	(0.1,0.2,0.7)	(0.1,0.3,0.8)
$(e_2, q, 1)$	(0.2,0.2,0.7)	(0,0,1)	(0,0,1)
$(e_1, p, 0)$	(0.1,0.1,0.6)	(0,0,1)	(0.1,0.2,0.6)
$(e_1, q, 0)$	(0.1,0.2,0.7)	(0.1,0.3,0.8)	(0.1,0.2,0.5)
$(e_2, p, 0)$	(0.1,0.2,0.7)	(0.1,0.1,0.9)	(0.1,0.2,0.8)
$(e_2, q, 0)$	(0.1,0.3,0.8)	(0.2,0.3,0.9)	(0,0,1)

Table 14. Tabular presentation of the agree-NSEG

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	0,333	0,333	0,433
$(e_1, q, 1)$	0,366	0,333	0,4
$(e_2, p, 1)$	0,366	0,333	0,4
$(e_2, q, 1)$	0,366	0,333	0,333

be a set of parameters computer knowledge and language fluency respectively. Let $X=\{p,q\}$ be a set of two expert committee members. A NSEG is given in Table 12 and

$$\mu_{g_\alpha}\left(x_i, x_j\right) = 0, \vartheta_{g_\alpha}\left(x_i, x_j\right) = 0$$

and $w_{g_\alpha}\left(x_i, x_j\right) = 1,$ for all

Table 15. Tabular presentation of the disagree-NSEG

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	0,266	0,333	0,3
$(e_1, q, 0)$	0,333	0,4	0,266
$(e_2, p, 0)$	0,333	0,366	0,333
$(e_2, q, 0)$	0,4	0,466	0,333

Table 16. Sum of agree-NSEG

	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 1)$	0,333	0,333	0,433
$(e_1, q, 1)$	0,366	0,333	0,4
$(e_2, p, 1)$	0,366	0,333	0,4
$(e_2, q, 1)$	0,366	0,333	0,333
$C_j = \sum_i x_{ij}$	1,431	1,332	1,566

Table 17. Sum of disagree-NSEG

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
$(e_1, p, 0)$	0,266	0,333	0,3
$(e_1, q, 0)$	0,333	0,4	0,266
$(e_2, p, 0)$	0,333	0,366	0,333
$(e_2, q, 0)$	0,4	0,466	0,333
$K_j = \sum_i x_{ij}$	1,332	1,565	1,232

$$\left(x_i, x_j\right) \in V \times V \setminus \left\{\left(x_1, x_2\right), \left(x_2, x_3\right), \left(x_3, x_1\right)\right\}$$

and for all $\alpha \in A$ After a serious deliberation the committee constructs the neutrosophic soft expert graph shown in Table 12.

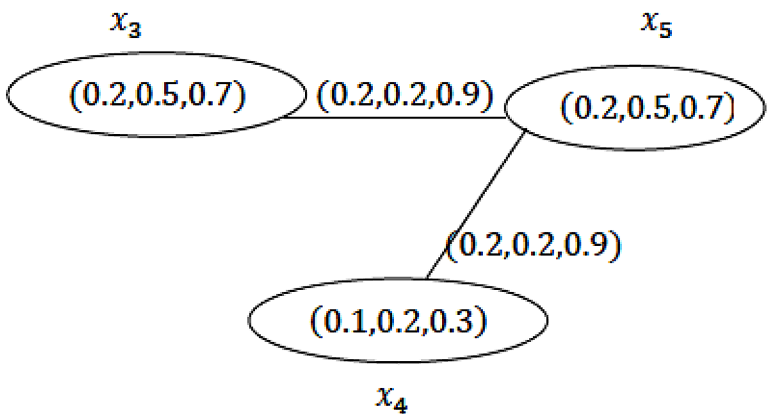
The following algorithm may be followed by the hospital to fill the position.

1. Input the NSEG.
2. Find the mean of each neutrosophic soft expert edges according to the relationship among criteria for each alternative.

Table 18. S_j=C_j-K_j

j	X	C_j	K_j	S_j
1	x_1	1,431	1,332	0,099
2	x_2	1,332	1,565	−0,233
3	x_3	1,566	1,232	0,334

Figure 27.



3. Find an agree-NSEG and a disagree-NSEG.
4. Find $C_j = \sum_i x_{ij}$ for agree-NSEG.
5. Find $K_j = \sum_i x_{ij}$ for disagree-NSEG.
6. Find S_j=C_j-K_j.
7. Find *r*, for which s_r=maxs_j, where, s_r is the optimal choice object. If r has more than one value, then any one of them could be chosen by the hospital using its option.

1. Neutrosophic soft expert edges according to the relationship among criteria for each alternative (Table 13).
2. Table 14 presents the agree-NSEG by using the mean of each NSEG.
3. Table 15 presents the disagree-NSEG respectively by using the mean of each NSEG.
4. $C_j = \sum_i x_{ij}$ for agree-NSEG (Table 16).
5. $K_j = \sum_i x_{ij}$ for disagree-NSEG (Table 17).
6. From Tables 16 and 17 we are able to compute the values of S_j=C_j-K_j as in Table 18.
7. Since max S_j=0,334, hence the committee will choose candidate x_3 with a masters degree for the job.

Figure 28.

NSEG $N(e_2, p, 1)$ w.r.t print communication

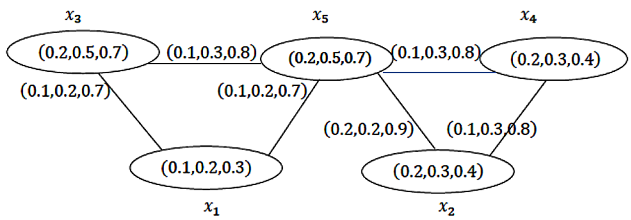
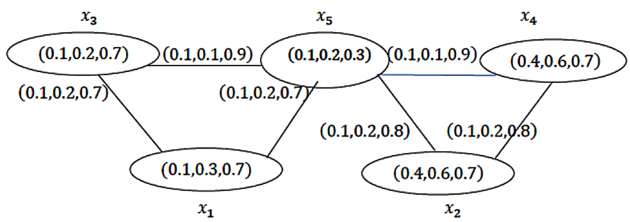


Figure 29.

NSEG$N(e_1, p, 0)$ w.r.t electronic communication

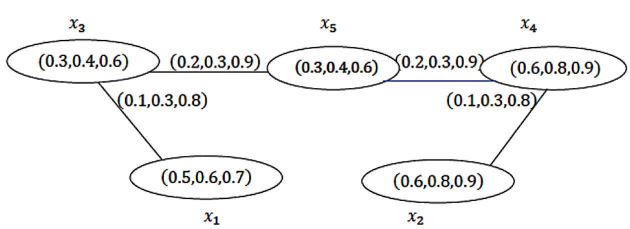


4.2 Application in Communication Network

A communication network model is used in an organization to manage, regulate information flows through proper channels. These networks form a pattern of person-to-person relationship by which information flows in an organization. In an organization, information is communicated through proper channels. We use graph to represent the communication networks. We consider a company in which company members share a common purpose to achieve specific goals. We can find the most useful channel for a company employee by considering a set of attributes or channels

Figure 30.

NSEG$N(e_2, p, 0)$w.r.t printcommunication



$$M = \begin{Bmatrix} e_1 = \text{electronic}, \\ e_2 = \text{print} \end{Bmatrix}.$$

Consider the graph G$*$ with vertex set V = {x_1=managing director(M.D), x_2=marketing manager(M.M), x_3=operation manager, x_4=accountant, x_5=sale staff} as shown in Table 7. The vertices represent company employees and edges represent any kind of communication relationship between them, if there is no edge between any two employees it means that there is no communication between them (Figs. 27, 28, 29, 30).

An NSEG

$$G = \left\{ N\left(e_1, p, 1\right), N\left(e_2, p, 1\right), N\left(e_1, p, 0\right), N\left(e_2, p, 0\right) \right\}$$

of G^* corresponding to the attributes electronic and print is represented in Table 7.

NSEG $N(e_1,p,1)$ w.r.t electronic communication. In the view of above NSEGs

$$N\left(e_1, p, 1\right), N\left(e_2, p, 1\right), N\left(e_1, p, 0\right) \text{and} N\left(e_2, p, 0\right),$$

we can see that the precise evaluation for each employee on each attributes is unknown while the lower and the upper limits for best communication device are given.

The neutrosophic soft expert graph, as a concept generalized of neutrosophic graph, fuzzy graph and intuitionistic fuzzy graph, provides additional capability to deal with uncertainty, inconsistent, incomplete and imprecise information by including a truth-membership, an indeterminacy-membership and a falsity membership with expert. Therefore, it plays a significant role in the network systems. Neutrosophic soft expert graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Neutrosophic models are becoming useful because of their aim of reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems.

6. COMPARISON ANALYSIS

In order to verify the feasibility and effectiveness of the proposed decision-making approach, a comparison analysis with interval valued neutrosophic decision method, used by Broumi et al. (Broumi et al .16f), Akram et al.'s method (Akram and Shahzadi

Table 19. Comparison of fuzzy soft set and its extensivese theory

Methods	Fuzzy soft	intuitionistic fuzzy soft	İnterval- Valued neutrosophic	Neutrosophic Soft	Neutrosophic soft expert
Methods		Shahzadi et al.'smethod (Shahzadi and Akram 2016)	Broumi et al.'smethod (Broumi et al .16f)	Akram et al.'s method (Akram and Shahzadi 2017)	Proposed Method
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Co-domain	Single-value in in [0,1]	Two-value in [0,1]	Unipolarintervalin [0,1]	$[0,1]^3$	$[0,1]^3$
Parameter	Yes	Yes	No	Yes	Yes
Uncertainty	Yes	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes	Yes
Falsity	No	Yes	Yes	Yes	Yes
Indeterminacy	No	No	Yes	Yes	Yes
Expert	No	No	No	No	Yes
Edge	Yes	Yes	Yes	Yes	Yes
Vertex	No	No	No	Yes	Yes
Ranking	-	$x_3 > x_2 > x_1$	$x_3 > x_1 > x_2$	$x_3 > x_2 > x_1$	$x_3 > x_1 > x_2$

2017) and Shahzadi et al.'s method (Shahzadi and Akram 2016)are given, based on the same illustrative example.

Clearly, the ranking order results are consistent with the result obtained in (Broumi et al .16f); however, the best alternative is the same as x_3, because the ranking principle is different, these four methods produced the same best alternatives.

Neutrosophic soft set is a generalization of the notion of fuzzy soft sets and intuitionistic fuzzy soft sets. Fuzzy soft graph theory is soft computing models in combination to study vagueness and uncertainty in graphs. Neutrosophic soft graphs are pictorial representation in which each vertex and each edge is an element of neutrosophic soft sets. Neutrosophic soft expert models give more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy and/ or intuitionistic fuzzy models. Neutrosophic soft expert models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems.

So, we think the proposed method developed in this paper is more suitable to handle this application example.

FUTURE RESEARCH DIRECTIONS

Using this concept we can extend our work in (1) Interval-valued neutrosophic soft expert graphs; (2) Bipolar neutrosophic soft expert regular graphs.

CONCLUSION

In this paper, we have introduced the concept of neutrosophic soft expert graph, strong neutrosophic soft expert graph, union and intersection of them has been explained with example which has wider application in the field of modern sciences and technology, especially in research areas of computer science including database theory, datamining, neural networks, expert systems, cluster analysis, control theory, and image capturing.

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Chapter 3
Application of Floyd's Algorithm in Interval Valued Neutrosophic Setting

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ABSTRACT

An algorithm with complete and incremental access is called a Floyd algorithm (FA). It determines shortest path for all the pairs in the network. Though there are many algorithms have been designed for shortest path problems (SPPs), due to the completeness of Floyd's algorithm, it has been improved by considering interval valued neutrosophic numbers as the edge weights to solve neutrosophic SPP (NSPP). Further, the problem is extended to triangular and trapezoidal neutrosophic environments. Also, comparative analysis has been done with the existing method.

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1. INTRODUCTION

Finding a shortest path is a common problem in the field of graph theory and optimization. To get the optimized path, there are many algorithms are available and the edge weights can be considered according to the environment taken namely crisp, fuzzy, intuitionistic fuzzy and neutrosophic and their extensions. SPP under fuzzy environment was examined by (Dubois and Prade, 1980). The length of the path is nothing but the sum of the arc lengths in the network considered. In traditional graph theory, the weight of all the edges is considered as a crisp numbers. SPP gets more attention in recent years among the researchers (Tajdin et al.2009). For example, in vehicle navigation, the main task of the system is to contribute the best routes and hence route selection is an essential task in all the communication and transportation system (Wei, 2010).

The most crucial combinatorial optimization problem is, finding the shortest path of the directed graph and its primary format unable to represent the situations where the value of the disconnected function should be found not only by the preference of the every single arc. While getting uncertainty in the set of vertices and edge then fuzzy graph can be adopted for SPP but if there is indeterminacy exist between the relation of nodes and vertices then neutrosophic will be the appropriate concept to deal the real life problems.

The technique of using fuzzy numbers (FNs) can be adopted for the environment with uncertainty. Using defuzzification function FN can be transformed into crisp number and it is widely used in an optimization techniques. SPP is not restricted to the geometric distance. Even though it is fixed, the travelling time between the cities may be represented as a fuzzy variable. Since the weight of the arcs is uncertain in almost all the communication and transportation networks, crisp graphs fail to work in an optimized way.

Since indeterminacy is also treated seriously, neutrosophic sets can be adopted to handle with uncertainty in a better way. The model of the neutrosophic set is an important mechanism to deal with real scientific and engineering as it is able to handle not only uncertain but also inconsistent and indeterminate information. Route maintenance or supply with uncertainty is playing a primary role in intelligent transport systems. Since most of the real world problems have uncertainty in nature t is necessary to use the concept of fuzzy and neutrosophic to solve SPP. Since fuzzy handles only grade membership, the concept of intuitionistic fuzzy was introduced by considering non-membership grade. Though these two cases handle uncertainty in a proper manner, both cannot deal with indeterminacy of the information. Hence neutrosophic concept is getting more attention in recent years. Always there is a problem of giving an exact number for any preference values for the decision makers or experts. This problem can be sorted out using interval based data.

Floyd's algorithm is used to find the shortest paths in a weighted graph by executing the algorithm to find the lengths between all the pairs of vertices. Though there is no possibilities of taking return details of the path themselves, reconstruction of the paths by considering simple modifications is possible in the algorithm even if one may want to save the existing path between all the pair of vertices. This algorithm analyzes all the possible paths between each pair of vertices. Though Jeyanthi and Radhika, 2018 used Floyd algorithm under fuzzy environment it could not deal with indeterminacy. Hence in this paper, interval valued neutrosophic (IVN) environment has been considered with triangular and trapezoidal cases.

2. REVIEW OF LITERATURE

(Kleeand Larman, 1979) explained the use of FA for getting shortest restricted path. (Shier, 1981) Done a study of computational method for FA. (Boulmakoul, 2004) proposed a generalized algorithm to find SP on semi rings and FSPP. (Bede, 2006) introduced operational laws of FNs and applied in Geology. (Rossi et al., 2006) proposed a prevailing structure and tractable algorithm for the fuzzy case. (Ortega et al.; 2006) examined the combination of small words and FA. (Natsheh et al., 2007) proposed Active Queue Management (AQM) method for traffic control in wireless Ad-Hoc under for fuzzy environment. (Whaley, 2008)Gave a perspective on Floyd's paper on destruction. (Akther et al., 2009) introduced an approach for the fuzzy arithmetic operations. (Mahdavia et al., 2009) introduced programming technique to find the shortest chains for fuzzy network.

(Gao et al.2009) proposed product operation on FNs. (Yadav and Biswas, 2009) Presented FSSP. (Tajdin et al., 2010) introduced a new method for SPP in mixed network. (Wei, 2010)Proposed FA for SPP. (Dahari andYang, 2011) reviewed about routing algorithm for auto- guided vehicles. (Oliveira and Fernandez, 2013) introduced a redirection algorithm under fuzzy setting for content delivery network. (Meenakshi and Kaliraja, 2012) determined SP for the networks under interval valued fuzzy. (Verma and Shukla, 2013) proposed a greedy algorithm for FSSP with the support of Quasi-Gaussian fuzzy weights. (Shukla, 2013) proposed fuzzy FA for SPP under uncertain setting. (Anusuya and Kavitha, 2013) recommended a genetic algorithm for SPP.

(Mamta and Ranga, 2014) introduced a system for location tracking for SPP to object using artificial intelligence and FL. (Qin and Yan, 2014) applied fuzzy AHP for distributing logistics. (Liang, 2015) proposed an improved algorithm for skeleton reconfiguration after temporary collapse of the power system. (Ergun et al. 2016) examined the evaluating performance of few routing algorithm using game theory technique. (Broumi et al., 2016) Applied Dijikstra algorithm for NSPP. (Broumi et al., 2016 a) applied Dijikstra algorithm for IVNSPP. (Broumi et al., 2017) proposed a novel concept of matrix algorithm for MST in undirected IVNG. (Broumi et al., 2017a) proposed computational methods to find MST using IVBNNs (Interval Valued Bipolar NNs). (Broumi et al., 2017b) applied TpNNs in MST problem. (Mullai et al. 2017) solved SPP using MST algorithm by considering BNNs.

(Broumiet al., 2018) solved SPP using IVNNs. (Broumiet al., 2019) solved spanning tree problem using neutrosophic arc weighs. (Broumi et al., 2019) solved MST problem using SVTpNNs. (Broumi et al., 2018) applied bipolar NNs in MST problem. (Dey et al., 2018) proposed a novel algorithm to solve MST with NGs, the undirected one. (Pandian and Priya, 2017) reviewed about path problem which is optimized. (Kumar et al., 2017) solved SPP using T2TrF arc length. (Broumi et al., 2017c) applied trapezoidal neutrosophic information for SPP. (Hameed, et al., 2018) proposed fast algorithm for SPP using matrices.

(Jeyanthi and Radhika, 2018) applied FA for NSPP. (Broumi et al., 2018a) proposed SVN clustering algorithm using Tsallis Entropy Maximization. (Li et al., 2018) explained about the habitude of A* algorithm for SPP. (Kumar et al., 2018) solved NSPP using single valued triangular and trapezoidal NNs. (Nagarajan et al. 2019) proposed Blockchain single and interval valued neutrosophic graphs. (Nagarajan et al. 2019) introduced Dombi interval valued neutrosophic graph. (Nagarajan et al. 2019) proposed new aggregation operators under interval fuzzy and interval neutrosophic environments and applied them in traffic control management. (Broumi et al. 2019) analyzed the SPP under various environments namely fuzzy, intuitionistic fuzzy and neutrosophic. (Broumi et al. 2019) solved SPP under triangular and trapezoidal interval valued neutrosophic environments

(Nabeeh et al. 2019) introduced integrated-TOPSIS method and applied them in decision making problem. (Basset et al. 2019) used TOPSIS methodology in supplier selection under type-2 neutrosophic environment. (Basset and Mohammed 2019) introduced a novel and efficient design on neutrosophic sets to aid patients with cancer. (Basset et al. 2019) introduced an integrated neutrosophic ANP and VIKOR method to obtain sustainable supplier selection. (Basset et al. 2019) proposed a framework for a decision making problem using the approach of neutrosophic TOPSIS for the selection of medical device.

This literature survey gives the motivation of the present work as there is no work has been contributed to find the shortest route using interval valued neutrosophic environment using Floyd's algorithm.

3. PRELIMINARIES

Definition 3.1: Let $t=\left[t_T,t_I,t_M,t_E\right]$ be the trapezoidal fuzzy number and $t_T \le t_I \le t_M \le t_E$ then the Centre of Gravity (COG) of *t* is

$$COG\left(t\right)=\begin{cases} t & if \qquad t_T=t_I=t_M=t_E \\ \dfrac{1}{3}\left[t_T+t_I+t_M+t_E-\dfrac{t_Et_M-t_It_T}{t_E+t_M-t_I-t_T}\right], & otherwise \end{cases} \tag{1}$$

Definition 3.2: Let

$$t=\left\langle\left[t_T,t_I,t_M,t_E\right],\left(T_t,I_t,F_t\right)\right\rangle$$

be a TpNN then the score, accuracy and certainty functions are as follows

$$\mathbb{S}\left(t\right)=COG\left(t\right)\times\frac{\left(2+T_t-I_t-F_t\right)}{3} \tag{2}$$

$$a\left(t\right)=COG\left(t\right)\times\left(T_t-F_t\right) \tag{3}$$

$$C\left(t\right)=COG\left(t\right)\times\left(T_t\right) \tag{4}$$

Definition 3.3 Let

$$r_N=\left\langle\left[r_T,r_I,r_P\right],\left(T_r,I_r,F_r\right)\right\rangle$$

be a TrNN then the score and accuracy function are,

$$\mathbb{S}\left(r\right)=\frac{1}{12}\left[r_T+2\cdot r_I+r_P\right]\times\left[2+T_r-I_r-F_r\right] \tag{5}$$

$$a(r) = \frac{1}{12}\left[r_T + 2 \cdot r_I + r_P\right] \times \left[2 + T_r - I_r + F_r\right] \tag{6}$$

Definition 3.4 Let

$$T = \left\langle \left[T^L, T^U\right], \left[I^L, I^U\right], \left[F^L, F^U\right] \right\rangle$$

be the interval valued neutrosophic number (IVNN) then, the ranking function and centre of gravity and the score function by considering both are defined by

Ranking function is,

$$R(IVNN) = \frac{1}{2}\left[\left(T^L + T^U\right) - \left(I^L I^U\right) + \left(I^U - 1\right)^2 + F^U\right] \tag{7}$$

Centre of gravity $= cog = \frac{1}{3}\left[t_L + 2t_M + t_R - \left(t_M\right)\right]$ (8)

Final score function is,

$$\mathbb{S} = cog \times R(IVNN) \tag{9}$$

Definition 3.5 For triangular interval valued neutrosophic numbers (TrIVNNs), the Centre of gravity is defined by

$$cog = \frac{1}{3}\left[t_L + 2t_M + t_R - \left(t_M\right)\right] \tag{10}$$

And the score function is defined by

Then the final score function is,

$$\mathbb{S} = cog \times R(IVNN) \tag{11}$$

Definition 3.6 For trapezoidal interval valued neutrosophic numbers centre of gravity is defined by

$$cog = \frac{1}{3}\left[t_T + t_I + t_M + t_E - \left(\frac{t_E t_M - t_I t_T}{t_E + t_M - t_I - t_T}\right)\right] \tag{12}$$

and with the ranking formula for IVNN, the score function is defined by

$$\mathbb{S} = cog \times R\left(IVNN\right) \tag{13}$$

4. PROPOSED METHODOLOGY USING FLOYD'S ALGORITHM (FA) UNDER INTERVAL VALUED NEUTROSOPHIC ENVIRONMENT

FA resolves shortest route between all the nodes in the network and hence it is more inference than Dijkstra algorithm. In the case of FA majority of the routes are undirected and there is a possibility of having directed routs also in the same network. In this paper, edge weights have been considered as interval valued neutrosophic numbers and triangular and trapezoidal neutrosophic cases as well.

Consider a network as a square matrix with number of columns and rows are equal to *p*. The distance between the nodes is represented by $\mathbb{C}_{uv}$. The entry $\left(u,v\right)$ of the matrix is,

1. *u* is directly connected to *v* and have finite distance
2. Otherwise, the distance is infinite.

Consider three nodes *u, v,* and *w*. We need to find the shortest distance from the node *u* to the node *w*. Here *u* is having direction connection to *w* by $\mathbb{C}_{uv}$, but during calculation it is found that, the distance is less to reach *w* from *u* travelling through *v* if $\mathbb{C}_{uv} + \mathbb{C}_{vu} < \mathbb{C}_{uw}$, where the replacement of the direct route from $u \to w$ is optimal with the indirect route $u \to v \to w$. Regular triple operation is used in the network.

Methodology:

Define the initial matrix $\mathbb{C}_0$ and sequence matrix $\mathbb{S}_0$ is given below. No loops are considered and hence the diagonal elements are blocked and is marked with (--) symbol.

Set $r = 1$

Define the matrix as shown in Table 1a and 1b.

Table 1a.

$\mathbb{C}_0$	1	2	3	…….	v	…….	p
1	--	$\mathbb{C}_{12}$	$\mathbb{C}_{13}$	…….	$\mathbb{C}_{1v}$	…….	$\mathbb{C}_{1p}$
2	$\mathbb{C}_{21}$	--	$\mathbb{C}_{23}$	…….	$\mathbb{C}_{2v}$	…….	$\mathbb{C}_{2p}$
3	$\mathbb{C}_{31}$	$\mathbb{C}_{32}$	--	…….	$\mathbb{C}_{3v}$	……	$\mathbb{C}_{3p}$
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
u	$\mathbb{C}_{u1}$	$\mathbb{C}_{u2}$	$\mathbb{C}_{u3}$	…….	--	…….	$\mathbb{C}_{up}$
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
p	$\mathbb{C}_{p1}$	$\mathbb{C}_{p2}$	$\mathbb{C}_{p3}$	…….	$\mathbb{C}_{pv}$	…….	--

Table 1b.

$\mathbb{S}_0$	1	2	3	…….	v	…….	p
1	--	2	3	…….	v	…….	p
2	1	--	3	…….	v	…….	p
3	1	2	--	…….	v	……	p
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
u	1	2	3	…….	--	…….	p
- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
p	1	2	3	…….	v	…….	--

Computation of Step *r*:

Define *r*th row and column as the pivot and apply triple operation to all the elements $\mathbb{C}_{uv}$ in $\mathbb{C}_{r-1}, \forall u, v$. If $\mathbb{C}_{ur} + \mathbb{C}_{r} < \mathbb{C}_{uv}$, for different *u,v,w* is satisfied then

1. Create $\mathbb{C}_{r}$ by replacing $\mathbb{C}_{uv}$ in $\mathbb{C}_{r-1}$ with $\mathbb{C}_{ur} + \mathbb{C}_{rv}$.
2. Create $\mathbb{S}_{r}$ by replacing $\mathbb{S}_{uv}$ in $\mathbb{S}_{r-1}$ with *r*.
3. Set $r = r + 1$. If $r = p$ then stop the process, else repeat step *r*.

After *p* steps, the shortest route between the nodes *u* and *v* from the matrices $\mathbb{C}_{p}$ and $\mathbb{S}_{p}$ using the following rules.

4. From $\mathbb{C}_{p}$, $\mathbb{C}_{uv}$ represents the weights of the shortest route between *u* and *v* as a crisp number.
5. From $\mathbb{S}_{p}$, find the transitional node $r = \mathbb{S}_{uv}$ which produce the route $u \rightarrow r \rightarrow v$.

If $\mathbb{S}_{ur} = r$ and $\mathbb{S}_{rv} = v$ stop (all transitional nodes of shortest route is found), otherwise repeat the process between the nodes *u* and *r*, and between *r* and *v*.

4.1 Finding Shortest Path for the Given Network Under Neutrosophic Environment

For this example, the above mentioned Floyd's algorithm, the score functions in Eqn. (9), Eqn. (11) and Eqn. (13) have been used for IVNN, TrIVNN and TpIVNN respectively. For this problem, the shortest route needs to be found between the nodes. The edge (3,5) is directional and hence traffic is not permitted form node 5 to 3 whereas the other edges permit the traffic on both the directions. Therefore finding the shortest distance from node 1 to node 5 is the problem of this example.

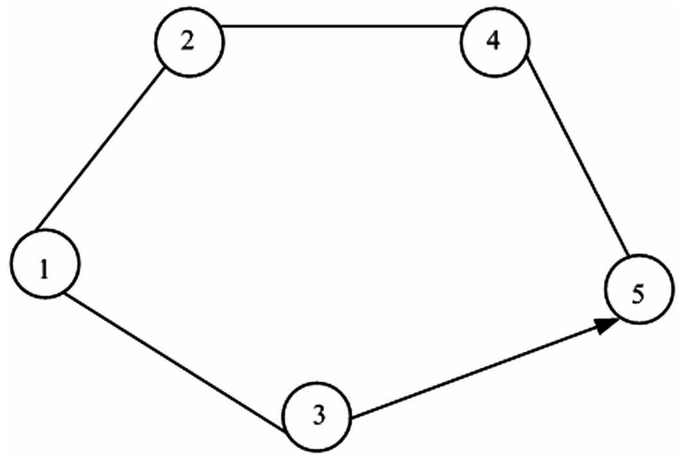
Figure 1 is the given network. For this network, finding the shortest path is the aim of this paper. For this purpose we analyzed three cases namely considering the edge weights as IVNN, TrIVNN and TpIVNN.

4.1.1 By considering the edge weights as IVNN

Consider the following edges and their edge weights.

$1 \rightarrow 2 : \left\langle [0.1, 0.2], [0.2, 0.3], [0.4, 0.5] \right\rangle,$

Figure 1. Network with interval valued neutrosophic weights



$$1 \to 3 : \left\langle [0.2, 0.4], [0.3, 0.5], [0.1, 0.2] \right\rangle$$

$$2 \to 4 : \left\langle [0.2, 0.3], [0.2, 0.5], [0.4, 0.5] \right\rangle,$$

$$4 \to 5 : \left\langle [0.3, 0.6], [0.1, 0.2], [0.1, 0.4] \right\rangle$$

$$3 \to 4 : \left\langle [0.2, 0.3], [0.3, 0.4], [0.1, 0.5] \right\rangle,$$

$$3 \to 5 : \left\langle [0.4, 0.6], [0.2, 0.4], [0.1, 0.3] \right\rangle$$

Using Eqn.(9), the score values are,

$$\mathbb{S}(1 \to 2) = 0.615,$$

$$\mathbb{S}(1 \to 3) = 0.45,$$

$$\mathbb{S}(2 \to 4) = 0.575,$$

Table 2. Initial matrix for the first step with IVNN as edge weight

$\mathbb{C}_0$	1	2	3	4	5
1	--	0.615	0.45	∞	∞
2	0.615	--	∞	0.575	∞
3	0.45	∞	--	0.68	0.79
4	∞	0.575	0.68	--	0.96
5	∞	∞	∞	0.96	--

$\mathbb{S}(4 \to 5) = 0.96$,

$\mathbb{S}(3 \to 4) = 0.68$,

$\mathbb{S}(3 \to 5) = 0.79$

Step 1.
Step 2. Fix $r = 1$

$\mathbb{C}_{23} = \mathbb{C}_{21} + \mathbb{C}_{13} = 0.615 + 0.45 = 1.065$

$\mathbb{C}_{32} = \mathbb{C}_{31} + \mathbb{C}_{12} = 0.45 + 0.615 = 1.065$, set $\mathbb{S}_{23} = \mathbb{S}_{32} = 1$

Step 3. Fix $r = 2$

Table 3. Sequence matrix for the first step with IVNN as edge weight

$\mathbb{S}_0$	1	2	3	4	5
1	--	2	3	4	5
2	1	--	3	4	5
3	1	2	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Table 4. Initial matrix for the second step with IVNN as edge weight

$\mathbb{C}_1$	1	2	3	4	5
1	--	0.615	0.45	∞	∞
2	0.615	--	**1.065**	0.575	∞
3	0.45	**1.065**	--	0.68	0.79
4	∞	0.575	0.68	--	0.96
5	∞	∞	∞	0.96	--

Table 5. Sequence matrix for the second step with IVNN as edge weight

$\mathbb{S}_1$	1	2	3	4	5
1	--	2	3	4	5
2	1	--	**1**	4	5
3	1	**1**	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

$\mathbb{C}_{14} = \mathbb{C}_{12} + \mathbb{C}_{24} = 0.615 + 0.575 = 1.19$

$\mathbb{C}_{41} = \mathbb{C}_{42} + \mathbb{C}_{21} = 0.575 + 0.615 = 1.19$, set $\mathbb{S}_{14} = \mathbb{S}_{41} = 2$

Step 4. Fix $r = 3$

Table 6. Initial Matrix for the third step with IVNN as edge weight

$\mathbb{C}_2$	1	2	3	4	5
1	--	0.615	0.45	**1.19**	∞
2	0.615	--	1.065	0.575	∞
3	0.45	1.065	--	0.68	0.79
4	**1.19**	0.575	0.68	--	0.96
5	∞	∞	∞	0.96	--

Table 7. Sequence matrix for the third step IVNN as edge weight

$\mathbb{S}_2$	1	2	3	4	5
1	--	2	3	**2**	5
2	1	--	1	4	5
3	1	1	--	4	5
4	**2**	2	3	--	5
5	1	2	3	4	--

Table 8. Initial matrix for the fourth step

$\mathbb{C}_3$	1	2	3	4	5
1	--	0.615	0.45	1.19	**1.24**
2	0.615	--	1.065	0.575	**1.855**
3	0.45	1.065	--	0.68	0.79
4	1.19	0.575	0.68	--	0.96
5	∞	∞	∞	0.96	--

$$\mathbb{C}_{15} = \mathbb{C}_{13} + \mathbb{C}_{35} = 0.45 + 0.79 = 1.24$$

$\mathbb{C}_{25} = \mathbb{C}_{23} + \mathbb{C}_{35} = 1.065 + 0.79 = 1.855$, set $\mathbb{S}_{15} = \mathbb{S}_{25} = 3$

Step 5. Fix $r = 4$

Table 9. Sequence matrix for the fourth step

$\mathbb{S}_3$	1	2	3	4	5
1	--	2	3	2	**3**
2	1	--	1	4	**3**
3	1	1	--	4	5
4	2	2	3	--	5
5	1	2	3	4	--

Table 10. Initial matrix for the fourth step

$\mathbb{C}_4$	1	2	3	4	5
1	--	0.615	0.45	1.19	**2.15**
2	0.615	--	**1.255**	0.575	**1.535**
3	0.45	**1.255**	--	0.68	**1.64**
4	1.19	0.575	0.68	--	0.96
5	**2.15**	**1.535**	**1.64**	0.96	--

$$\mathbb{C}_{15} = \mathbb{C}_{14} + \mathbb{C}_{45} = 1.19 + 0.96 = 2.15\,,$$

$$\mathbb{C}_{23} = \mathbb{C}_{24} + \mathbb{C}_{43} = 0.575 + 0.68 = 1.255$$

$$\mathbb{C}_{25} = \mathbb{C}_{24} + \mathbb{C}_{45} = 0.575 + 0.96 = 1.535\,,$$

$$\mathbb{C}_{32} = \mathbb{C}_{34} + \mathbb{C}_{42} = 0.68 + 0.575 = 1.255$$

$$\mathbb{C}_{51} = \mathbb{C}_{54} + \mathbb{C}_{41} = 0.96 + 1.19 = 2.15\,,$$

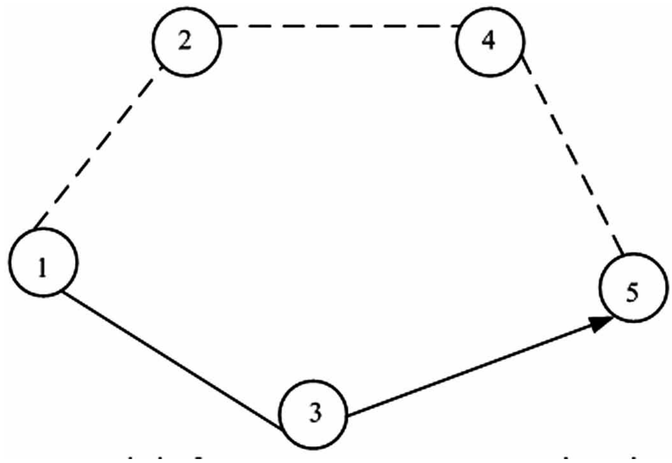
$$\mathbb{C}_{52} = \mathbb{C}_{54} + \mathbb{C}_{42} = 0.96 + 0.575 = 1.535$$

$$\mathbb{C}_{53} = \mathbb{C}_{54} + \mathbb{C}_{43} = 0.96 + 0.68 = 1.64\,,$$

Table 11. Sequence matrix for the fourth step

$\mathbb{S}_4$	1	2	3	4	5
1	--	2	3	**2**	**4**
2	1	--	**4**	4	**4**
3	1	**4**	--	4	**4**
4	**2**	2	3	--	5
5	**4**	**4**	**4**	4	--

Figure 2. Network with Shortest Path using Interval Valued Neutrosophic Weights



$\mathbb{C}_{35} = \mathbb{C}_{34} + \mathbb{C}_{45} = 0.68 + 0.96 = 1.64$

No more improvements are possible and hence $\mathbb{C}_4 = \mathbb{C}_5$ and $\mathbb{S}_4 = \mathbb{S}_5$.
And hence the shortest route is, $1 \to 2 \to 4 \to 5$ (Figure 2) and its value is **2.15.**
Therefore the process ends.

4.1.2 By Considering the Edge Eights as TrIVNN

Consider Figure 1 with triangular interval valued neutrosophic numbers as the edge weights and are defined by

$1 \to 2 : \left\langle \left(1,2,3\right);\left[0.1,0.2\right],\left[0.2,0.3\right],\left[0.4,0.5\right] \right\rangle,$

$1 \to 3 : \left\langle \left(2,5,7\right);\left[0.2,0.4\right],\left[0.3,0.5\right],\left[0.1,0.2\right] \right\rangle$

$2 \to 4 : \left\langle \left(3,7,8\right);\left[0.2,0.3\right],\left[0.2,0.5\right],\left[0.4,0.5\right] \right\rangle,$

$4 \to 5 : \left\langle \left(3,4,5\right);\left[0.3,0.6\right],\left[0.1,0.2\right],\left[0.1,0.4\right] \right\rangle$

$3 \to 4 : \left\langle \left(2,4,8\right);\left[0.2,0.3\right],\left[0.3,0.4\right],\left[0.1,0.5\right] \right\rangle,$

Table 12. Initial matrix for the first step with TrIVNN as edge weight

$\mathbb{C}_0$	1	2	3	4	5
1	--	1.230	2.102	∞	∞
2	1.230	--	∞	3.45	∞
3	2.102	∞	--	3.176	6.32
4	∞	3.45	3.176	--	3.84
5	∞	∞	∞	3.84	--

$$3 \to 5 : \left\langle \left(7,8,9\right);\left[0.4,0.6\right],\left[0.2,0.4\right],\left[0.1,0.3\right]\right\rangle$$

Using Eqn. (11), the score values are

$$\mathbb{S}\left(1 \to 2\right) = 2 \times 0.615 = 1.23,$$

$$\mathbb{S}\left(1 \to 3\right) = 4.67 \times 0.45 = 2.102,$$

$$\mathbb{S}\left(2 \to 4\right) = 6 \times 0.575 = 3.45,$$

$$\mathbb{S}\left(4 \to 5\right) = 4 \times 0.96 = 3.84,$$

$$\mathbb{S}\left(3 \to 4\right) = 4.67 \times 0.68 = 3.176,$$

$$\mathbb{S}\left(3 \to 5\right) = 8 \times 0.79 = 6.32$$

Step 1.
Step 2. Fix $r = 1$

$$\mathbb{C}_{23} = \mathbb{C}_{21} + \mathbb{C}_{13} = 1.23 + 2.102 = 3.332$$

$\mathbb{C}_{32} = \mathbb{C}_{31} + \mathbb{C}_{12} = 2.102 + 1.23 = 3.332$, set $\mathbb{S}_{23} = \mathbb{S}_{32} = 1$

Table 13. Sequence matrix for the first step with TrIVNN as edge weight

$\mathbb{S}_0$	1	2	3	4	5
1	--	2	3	4	5
2	1	--	3	4	5
3	1	2	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Table 14. Initial matrix for the second step with TrIVNN as edge weight

$\mathbb{C}_1$	1	2	3	4	5
1	--	1.230	2.102	∞	∞
2	1.230	--	**3.332**	3.45	∞
3	2.102	**3.332**	--	3.176	6.32
4	∞	3.45	3.176	--	3.84
5	∞	∞	∞	3.84	--

Step 3. Fix $r = 2$

$$\mathbb{C}_{14} = \mathbb{C}_{12} + \mathbb{C}_{24} = 1.23 + 3.45 = 4.68$$

$$\mathbb{C}_{41} = \mathbb{C}_{42} + \mathbb{C}_{21} = 3.45 + 1.23 = 4.68$$, set $\mathbb{S}_{14} = \mathbb{S}_{41} = 2$

Table 15. Sequential matrix for the first step with TrIVNN as edge weight

$\mathbb{S}_1$	1	2	3	4	5
1	--	2	3	4	5
2	1	--	**1**	4	5
3	1	**1**	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Table 16. Initial matrix for the third step with TrIVNN as edge weight

$\mathbb{C}_2$	1	2	3	4	5
1	--	1.230	2.102	**4.68**	∞
2	1.230	--	3.332	3.45	∞
3	2.102	3.332	--	3.176	6.32
4	**4.68**	3.45	3.176	--	3.84
5	∞	∞	∞	3.84	--

Table 17. Sequence matrix for the third step with TrIVNN as edge weight

$\mathbb{S}_2$	1	2	3	4	5
1	--	2	3	**2**	5
2	1	--	1	4	5
3	1	1	--	4	5
4	**2**	2	3	--	5
5	1	2	3	4	--

Step 4. Fix $r = 3$

$$\mathbb{C}_{15} = \mathbb{C}_{13} + \mathbb{C}_{35} = 2.102 + 6.32 = 8.422$$

$$\mathbb{C}_{25} = \mathbb{C}_{23} + \mathbb{C}_{35} = 3.332 + 6.32 = 9.652 \text{, set } \mathbb{S}_{15} = \mathbb{S}_{25} = 3$$

Table 18. Initial matrix for the fourth step with TrIVNN as edge weight

$\mathbb{C}_3$	1	2	3	4	5
1	--	1.230	2.102	4.68	**8.422**
2	1.230	--	3.332	3.45	**9.652**
3	2.102	3.332	--	3.176	6.32
4	4.68	3.45	3.176	--	3.84
5	∞	∞	∞	3.84	--

Table 19. Sequence matrix for the fourth step with TrIVNN as edge weight

$\mathbb{S}_3$	1	2	3	4	5
1	--	2	3	**2**	**3**
2	1	--	1	4	**3**
3	1	1	--	4	5
4	**2**	2	3	--	5
5	1	2	3	4	--

Step 5. Fix $r = 4$

$$\mathbb{C}_{15} = \mathbb{C}_{14} + \mathbb{C}_{45} = 8.52\,,$$

$$\mathbb{C}_{23} = \mathbb{C}_{24} + \mathbb{C}_{43} = 6.626$$

$$\mathbb{C}_{25} = \mathbb{C}_{24} + \mathbb{C}_{45} = 7.29\,,$$

$$\mathbb{C}_{32} = \mathbb{C}_{34} + \mathbb{C}_{42} = 6.626$$

$$\mathbb{C}_{51} = \mathbb{C}_{54} + \mathbb{C}_{41} = 8.52\,,$$

$$\mathbb{C}_{52} = \mathbb{C}_{54} + \mathbb{C}_{42} = 7.29$$

$$\mathbb{C}_{53} = \mathbb{C}_{54} + \mathbb{C}_{43} = 7.016\,,$$

$$\mathbb{C}_{35} = \mathbb{C}_{34} + \mathbb{C}_{45} = 7.016$$

No more improvements are possible and hence $\mathbb{C}_4 = \mathbb{C}_5$ and $\mathbb{S}_4 = \mathbb{S}_5$

And hence the shortest route is, $1 \to 2 \to 4 \to 5$ (Figure 2) with TrIVNN as the edge weight and its value is **8.52.**

Therefore the process ends.

Table 20. Initial matrix for the fifth step with TrIVNN as edge weight

$\mathbb{C}_4$	1	2	3	4	5
1	--	1.230	2.102	4.68	**8.52**
2	1.230	--	**6.626**	3.45	**7.29**
3	2.102	**6.626**	--	3.176	**7.016**
4	4.68	3.45	3.176	--	3.84
5	**8.52**	**7.29**	**7.016**	3.84	--

4.1.3 By Considering the Edge Weights as TpIVNN

Consider Figure 1 with trapezoidal interval valued neutrosophic numbers as the edge weights as follows.

$$1 \to 2 : \left\langle \left(1,2,3,4\right);\left[0.1,0.2\right],\left[0.2,0.3\right],\left[0.4,0.5\right]\right\rangle ,$$

$$1 \to 3 : \left\langle \left(2,5,7,8\right);\left[0.2,0.4\right],\left[0.3,0.5\right],\left[0.1,0.2\right]\right\rangle$$

$$2 \to 4 : \left\langle \left(3,7,8,9\right);\left[0.2,0.3\right],\left[0.2,0.5\right],\left[0.4,0.5\right]\right\rangle ,$$

$$4 \to 5 : \left\langle \left(3,4,5,10\right);\left[0.3,0.6\right],\left[0.1,0.2\right],\left[0.1,0.4\right]\right\rangle$$

$$3 \to 4 : \left\langle \left(2,4,8,9\right);\left[0.2,0.3\right],\left[0.3,0.4\right],\left[0.1,0.5\right]\right\rangle ,$$

Table 21. Sequence matrix for the fifth step with TrIVNN as edge weight

$\mathbb{S}_4$	1	2	3	4	5
1	--	2	3	**2**	**4**
2	1	--	**4**	4	**4**
3	1	**4**	--	4	**4**
4	**2**	2	3	--	5
5	**4**	**4**	**4**	4	--

Table 22. Initial matrix for the first step with TpIVNN as edge weight

$\mathbb{C}_0$	1	2	3	4	5
1	--	1.538	2.439	∞	∞
2	1.538	--	∞	3.778	∞
3	2.439	∞	--	3.876	6.715
4	∞	3.778	3.876	--	5.52
5	∞	∞	∞	5.52	--

$$3 \to 5 : \left\langle \left(7,8,9,10\right);\left[0.4,0.6\right],\left[0.2,0.4\right],\left[0.1,0.3\right]\right\rangle$$

Using Eqn. (13), the score values are

$$\mathbb{S}\left(1 \to 2\right) = 2.5 \times 0.615 = 1.538\,,$$

$$\mathbb{S}\left(1 \to 3\right) = 5.42 \times 0.45 = 2.439\,,$$

$$\mathbb{S}\left(2 \to 4\right) = 6.57 \times 0.575 = 3.778$$

$$\mathbb{S}\left(4 \to 5\right) = 5.75 \times 0.96 = 5.520\,,$$

$$\mathbb{S}\left(3 \to 4\right) = 5.7 \times 0.68 = 3.876\,,$$

Table 23. Sequence matrix for the first step with TpIVNN as edge weight

$\mathbb{S}_0$	1	2	3	4	5
1	--	2	3	4	5
2	1	--	3	4	5
3	1	2	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Table 24. Initial matrix for the second step with TpIVNN as edge weight

$\mathbb{C}_1$	1	2	3	4	5
1	--	1.538	2.439	∞	∞
2	1.538	--	**3.977**	3.778	∞
3	2.439	**3.977**	--	3.876	6.715
4	∞	3.778	3.876	--	5.52
5	∞	∞	∞	5.52	--

$$\mathbb{S}\left(3 \to 5\right) = 8.5 \times 0.79 = 6.715$$

Step 1.
Step 2. Fix $r = 1$

$$\mathbb{C}_{23} = \mathbb{C}_{21} + \mathbb{C}_{13} = 1.538 + 2.439 = 3.977$$

$\mathbb{C}_{32} = \mathbb{C}_{31} + \mathbb{C}_{12} = 2.439 + 1.538 = 3.977$, set $\mathbb{S}_{23} = \mathbb{S}_{32} = 1$

Step 3. Fix $r = 2$

$$\mathbb{C}_{14} = \mathbb{C}_{12} + \mathbb{C}_{24} = 1.538 + 3.778 = 5.316$$

$\mathbb{C}_{41} = \mathbb{C}_{42} + \mathbb{C}_{21} = 3.778 + 1.538 = 5.316$, set $\mathbb{S}_{14} = \mathbb{S}_{41} = 2$

Step 4. Fix $r = 3$

Table 25. Sequence matrix for the second step with TpIVNN as edge weight

$\mathbb{S}_1$	1	2	3	4	5
1	--	2	3	4	5
2	1	--	**1**	4	5
3	1	**1**	--	4	5
4	1	2	3	--	5
5	1	2	3	4	--

Table 26. Initial matrix for the third step with TpIVNN as edge weight

$\mathbb{C}_2$	1	2	3	4	5
1	--	1.538	2.439	**5.316**	∞
2	1.538	--	3.977	3.778	∞
3	2.439	3.977	--	3.876	6.715
4	**5.316**	3.778	3.876	--	5.52
5	∞	∞	∞	5.52	--

Table 27. Sequence matrix for the third step with TpIVNN as edge weight

$\mathbb{S}_2$	1	2	3	4	5
1	--	2	3	**2**	5
2	1	--	1	4	5
3	1	1	--	4	5
4	**2**	2	3	--	5
5	1	2	3	4	--

$\mathbb{C}_{15} = \mathbb{C}_{13} + \mathbb{C}_{35} = 2.439 + 6.715 = 9.154$

$\mathbb{C}_{25} = \mathbb{C}_{23} + \mathbb{C}_{35} = 3.977 + 6.715 = 10.692$, set $\mathbb{S}_{15} = \mathbb{S}_{25} = 3$

Step 5. Fix $r = 4$

Table 28. Initial matrix for the third step with TpIVNN as edge weight

$\mathbb{C}_3$	1	2	3	4	5
1	--	1.54	2.44	5.32	**9.15**
2	1.54	--	3.98	3.78	**10.69**
3	2.44	3.98	--	3.88	6.72
4	5.32	3.78	3.88	--	5.52
5	∞	∞	∞	5.52	--

Table 29. Sequence matrix for the third step with TpIVNN as edge weight

$\mathbb{S}_3$	1	2	3	4	5
1	--	2	3	2	**3**
2	1	--	1	4	**3**
3	1	1	--	4	5
4	**2**	2	3	--	5
5	1	2	3	4	--

$$\mathbb{C}_{15} = \mathbb{C}_{14} + \mathbb{C}_{45} = 10.836,$$

$$\mathbb{C}_{23} = \mathbb{C}_{24} + \mathbb{C}_{43} = 7.654$$

$$\mathbb{C}_{25} = \mathbb{C}_{24} + \mathbb{C}_{45} = 9.298,$$

$$\mathbb{C}_{32} = \mathbb{C}_{34} + \mathbb{C}_{42} = 7.654$$

$$\mathbb{C}_{51} = \mathbb{C}_{54} + \mathbb{C}_{41} = 10.836,$$

$$\mathbb{C}_{52} = \mathbb{C}_{54} + \mathbb{C}_{42} = 9.298$$

$$\mathbb{C}_{53} = \mathbb{C}_{54} + \mathbb{C}_{43} = 9.396,$$

Table 30. Initial matrix for the third step with TpIVNN as edge weight

$\mathbb{C}_4$	1	2	3	4	5
1	--	1.54	2.44	5.32	**10.84**
2	1.54	--	**7.65**	3.78	**9.30**
3	2.44	**7.65**	--	3.88	**9.40**
4	5.32	3.78	3.88	--	5.52
5	**10.84**	**9.30**	**9.40**	5.52	--

Table 31. Sequence matrix for the third step with TpIVNN as edge weight

$\mathbb{S}_4$	1	2	3	4	5
1	--	2	3	**2**	**4**
2	1	--	**4**	4	**4**
3	1	**4**	--	4	**4**
4	**2**	2	3	--	5
5	**4**	**4**	**4**	4	--

$$\mathbb{C}_{35} = \mathbb{C}_{34} + \mathbb{C}_{45} = 9.396$$

No more improvements are possible and hence $\mathbb{C}_4 = \mathbb{C}_5$ and $\mathbb{S}_4 = \mathbb{S}_5$

The shortest route is, 1→2→4→5 (Figure 2) with TpIVNN as the edge weight and its value is **10.84.** Therefore the process ends.

Table 32.

Method	Shortest Path	Value
(Jeyanthi and Radhika,2018) (using SVNN)	1→2→4→5	1.7
Our proposed method (using IVNN)	1→2→4→5	**2.15**
Our proposed method (using TrIVNN)	1→2→4→5	**8.52**
Our proposed method (using TpIVNN)	1→2→4→5	**10.84**

4.2 Comparative Analysis

The strength of our proposed method has been showed by comparing with the existing method as shown in Table 32.

CONCLUSION

As Floyd's algorithm covers all the paths, it is an efficient algorithm than other existing algorithms. Neutrosophic shortest path problem has been solved by considering the edge weight as IVNN, TrIVNN and TpIVNN. Also comparative analysis has been done with the existing method. Our proposed method can be applied in network and communication system as well. In future, it can be extended to soft neutrosophic environment.

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Chapter 4
New Algorithms for Hamiltonian Cycle Under Interval Neutrosophic Environment

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ABSTRACT

A cycle passing through all the vertices exactly once in a graph is a Hamiltonian cycle (HC). In the field of network system, HC plays a vital role as it covers all the vertices in the system. If uncertainty exists on the vertices and edges, then that can be solved by considering fuzzy Hamiltonian cycle. Further, if indeterminacy also exist, then that issue can be dealt efficiently by having neutrosophic Hamiltonian cycle. In computer science applications, objects may not be a crisp one as it has uncertainty and indeterminacy in nature. Hence, new algorithms have been designed to find interval neutrosophic Hamiltonian cycle using adjacency matrix and the minimum degree of a vertex. This chapter also applied the proposed concept in a network system.

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INTRODUCTION

The study of graphs is called graph theory where the realtion between the objects designed by the mathematical structures. A graph is made up of vertices and they are connected by the edges. Especially while solving shortest path problems, some of the concepts needed to be takec care namely walk (vertices and edges may be repaeated, open or closed) trail (vertices may be repeated but edges cannot be repeated, open), circuit (vertices may be repeated but edges cannot be repeated, closed), Path (both vertices and edges cannot be repeated, open) and cycle both vertices and edges cannot be repeated, closed). A cycle which passing through all the vertices in a network is a Hamilton cycle and the path is a Hamilton path (Hamilton, 1932).

A graph with HC is called a Hamiltonian graph. Every HC has Hamiltonian path (HP) and the cntraray is not inevitably true. The studies on HC and HP have been inspired by real life applications and the complexity issues (Rahman and Kaykobad, 2005). Identifying the existence of HC is a NP-complete problem and hence it's a slow process for huge graphs (Csehi and Toth, 2011). When there is impreciseness on the vertices and edges of a graph then the crisp graph fails and at this junction fuzzy graph can be applied to deal the uncertainty. The notion of connectivity performing an essential role in theory and as well as applications of fuzzy graphs (FGs). There are many applications in current technology namely information theory, intelligent systems, medical analysis, neural network and cluster analysis (Nirmala and Vijaya, 2012).

(Broumi, et al., 2016) Graph theory is a branch of combinatorics and applied mathematics and has been applied in various fields' namely number theory, topology, algebra, geometry, computer science and optimization. If there is indeterminacy exist in the relation between the vertices then FG fails. For this aspiration, Smarandache described neutrosophic graphs (NGs). Then single valued neutrosophic graphs (SVNGs) and interval valued neutrosophic graphs (IVNGs) have been introduced as a generalization of fuzzy and neutrosophic graphs. Also described degree, neighborhood degree of a vertex for SVNG. Smarandache, 2016 introduced IVNGs and described about adjacency vertices and matrices of IVNGs and degree of the vertices. Fuzzy set (Zadeh, 1965) is a concept introduced for handling uncertainty of the real world problems. It generalization called intuitionistic fuzzy set (Atanassov, 1986) deal membership as well as non membership of the elements of the set.

Fuzzy graph is capable of solving network problems when uncertainty exists on the vertices and edges (Rosenfeld, 1975). But indeterminacy can be handled by both fuzzy and intuitionistic fuzzy set and hence neutrosophic set (Smarandache, 2002) to deal indeterminacy of the data, vertices and edges. (Dogrusoz and Krishnamoorthy, 2015) Determining or identifying the Hamiltonian cycle in network or a problem is called Hamiltonian cycle problem (HCP). It has many applications if various fields especially in computer vision in formatting the object characterized by group of points. The relations in the fuzzy graphs are symmetric (Vanadhi et al., 2015).

If the number of vertices and edges are growing then identifying the Hamiltonian cycle will become a tedious process and hence simple algorithms are needed to be introduced. Adjacent matrices are useful to identify edge between the vertices easily and quickly. Network topologies can be designed using the notion of graphs namely traveling salesman problem, resources networking and designing of database. This causes the new algorithms and new theorems.

A graph is diagrammatic representation of the links (edges) between the objects (vertices) of the set. Adjacent vertices are connected by an edge. In a route map cities are the vertices and the roads connecting the cities are edges. Graph theory provides consolidated direction for specific problems and algorithms called graph algorithms can give the best solution (Nagoorgani and Latha, 2016).

Some of the problems may contain uncertainty on the vertices and edges in nature, and then fuzzy graph can be adopted to deal with uncertainty. For example, travelling time of the vehicle cannot be determined on the roads accurately. If the vertices and edges are in large number then graph analysis requires computer assistance. Also graphs need to be expressed through matrices especially adjacent matrices. These matrices are the square matrices to represent finite graph and tells about the adjacency of the vertices. Some algorithms have been proposed to solve shortest path problems by considering the edge weights by neutrosophic numbers and single and interval valued neutrosophic environments (Broumi et al., 2016).

Interval valued neutrosophic relation can be represented by interval valued neutrosophic graph. In computer network model, a computer is the vertex and every edge between two computers is represented by a telephone line operated in both directions. The intervals represent strength of the relation in computer network model under interval valued neutrosophic environment. The strength of the edges is less than the strength of the vertices (Akram and Nazir, 2017). Any network problem can be solved when the weights are known properly but most of the real world problems are uncertain in nature. Hence fuzzy sets, intuitionistic fuzzy sets and then neutrosophic sets have been applied to deal with uncertainty, nonmembership and indeterminacy of the objects of the sets (Mathew, et al., 2018). Hamiltonian cycle problem (HCP) is one of the widely studied problems in computer science and graph theory to determine whether Hamiltonian cycle exist in the graph (Mahasinge, 2019).

RELATED WORK

(Bjorklund and Husfeldt, 1975) introduced the parity of HCs. (Heinrich and Wallis, 1978) introduced HCs in sure graphs. (Dinh, 1992) presented a remark on HCs. (Reay and Zamfirescu 2000) proposed HC in T-graphs. (Kawarabayashi 2001) surveyed on HCs. (Rahman and Kaykobad 2005) introduced two theorems stating sufficient conditions for a graph to possess HCs and HPs and discussed the implication of the theorem. (Cuckler, 2007) introduced HCs in regular tournaments. (Fo and Lo, 2009) introduced multicolored parallelisms of HCs. (Csehi and Toth, 2011) extended the capability of HamiltonianQ which test the biconnectivity of a given graph. (Faudree and Gould, 2012) studied about distributing vertices on HCs.

(Ekstein, 2011) identified HCs in the square of a graph. (Nirmala and Vijaya, 2012) applied HCs on K 2n+1 fuzzy graphs. (Jenrol et al., 2013) introduced HCs by enforcing on vertices. (Maity and Upadhyay, 2014) introduced HCs in Polyherdral maps. (Dhavaseelan et al., 2015) introduced certain types of neutrosophic graphs. (Dogrusoz and Krishnamoorthy, 2015) studied about HCs in triangle graphs. (Sellappan et al., 2015) evaluated risk priority number in model future nodes and analyzed the effects using factor analysis.

(Broumi et al., 2016) applied interval valued neutrosophic graph in decision making problem. (Gani and Latha, 2016) introduced algorithms to find the HC in a fuzzy network using adjacency matrix and minimum vertex degree.

(Broumi et al., 2016) applied Dijkstra algorithm to find the shortest path of the network under interval valued neutrosophic environments. (Smarandache, 2016) introduced interval valued neutrosophic graphs. (Broumi et al., 2016) proposed single valued neutrosophic graphs.

(Broumi et al., 2016) proposed operations on IVNGs. (Vijaya and Kannan, 2017) determined the number of HCs in cubic fuzzy graph with n vertices.

(Sahin, 2017) introduced an approach to NG theory with applications. (Waligora, 2017) applied Hamilton theory of graph in new technologies. (Akram and Nazir, 2017) introduced a modified definition of IVNGs. (Kureethara, 2017) introduced a necessary and sufficient conditions for a complete multipartite graph to have a HC.

(Takaoka, 2018) discussed about the complexity of HC reconfiguration. (Pantic et al., 2018) enumerated HCs on a thick grid cylinder. (Mathew et al., 2018) introduced fundamentals of fuzzy graph theory. (Nagarajan et al., 2018) have done edge detection using type-2 fuzzy. (Lathamaheswari et al., 2018) reviewed the application of type-2 fuzzy in biomedicine. (Nagarajan et al., 2018) proposed image extraction methodology using type-2 fuzzy. (Nagarajan et al., 2019) proposed interval valued neutrosophic graphs using Dombi triangular norms.

(Cheng et al., 2019) introduced rainbow HCs in strongly edge-colored graphs. (Mahasinghe, 2019) solved HC problem using a quantum computer. (Nagarajan et al., 2019) analyzed traffic flow management using interval type-2 fuzzy sets and interval neutrosophic sets. (Broumi et al., 2019) solved shortest path problem using interval valued triangular and trapezoidal neutrosophic environments. (Lathamaheswari et al., 2018) reviewed the use of type-2 fuzzy controller in control system. (Nagarajan et al., 2019) introduced Blockchain technology using single and interval valued neutrosophic graphs. (Broumi et al., 2019) solved shortest path problem using triangular and trapezoidal neutrosophic environments. (Broumi et al., 2019) overviewed shortest path problem under various set environments. (Nenadov et al. 2018) introduced powers of HCs in random graphs and tight HCs in random Hypergraphs. (Rao, 2018) examined Eulerian, Hamiltonian and complete algebraic graphs.

(Li, 2019) introduced energy conditions for Hamiltonian and traceable graphs. (Liu et al. 2019) proposed spectral results on Hamiltonian problems. (Nikfar, 2019) proposed a novel concept of domination in neutrosophic graphs in his name. (Khan et al. 2019) introduced Laplacian energy of a graph under complex neutrosophic environment. (Zeps, 2019) analyzed Hamiltonian nodes and edges in uniquely Hamiltonian graphs. (Nhu, 2019) observed the Hamiltonian graph in an easy problem. (Asratian et al. 2018) introduced a new method in Hamiltonian graph theory called localization. (Samanta and Sarkar, 2018) examined generalized fuzzy Euler graphs and Hamiltonian graphs.

Preliminaries

Definition Graph (Broumi et al., 2016): A mathematical system $G=(V,E)$ is called a graph, where $V=V(G)$, a vertex set and $E=E(G)$ is an edge set. In this paper, undirected graph has been considered and hence every edge is considered as an unordered pair of different vertices.

Definition Fuzzy Graph (Broumi et al., 2016): Let $\mathbb{V}$ be a non-empty finite set, λ be a fuzzy subsets on $\mathbb{V}$ and δ be a fuzzy subsets on $\mathbb{V}\times\mathbb{V}$. The pair $\mathbb{G}=(\lambda,\delta)$ is a fuzzy graph over the set $\mathbb{V}$ if

$$\delta(x,y)\le \min\{\lambda(x),\lambda(y)\}$$

for all $(x,y)\in\mathbb{V}\times\mathbb{V}$ where λ is a fuzzy vertex and δ is a fuzzy edge. Where: A mapping $\lambda:\mathbb{V}\to[0,1]$ is called a fuzzy subset of $\mathbb{V}$, where $\mathbb{V}$ is the non-empty set. A mapping $\delta:\mathbb{V}\times\mathbb{V}\to[0,1]$ is a fuzzy relation on λ of $\mathbb{V}$. If

$$\delta(x,y) \le \min\{\lambda(x),\lambda(y)\}.$$

If

$$\delta(x,y) = \min\{\lambda(x),\lambda(y)\}$$

then $\mathbb{G}$ is a strong fuzzy graph.

Definition Interval Neutrosophic Set (Smarandache, 2016): Let U be the non-empty set. An interval neutrosophic set *B* is defined as follows.

$$B = \{x, \langle T(x), I(x), F(x)\rangle | x \in B\},$$

where the intervals

$$T(x) = [T^L(x), T^U(x)] \subseteq [0,1],$$

$$I(x) = [I^L(x), I^U(x)] \subseteq [0,1],$$

$$F(x) = [F^L(x), F^U(x)] \subseteq [0,1]$$

for $x \in U$ are the grades of the truth membership, indeterminacy membership and false membership respectively.

Definition Interval Neutrosophic Numbers (INNs) (Smarandache, 2016): Let $X = \{x_1, x_2, ..., x_n\}$ be an INS, where

$$x_j = \langle [T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U] \rangle$$

for $j = 1,2,3,...,n$ is a collection of INNs and

$$T_j^L, T_j^U, I_j^L, I_j^U, F_j^L, F_j^U \in [0,1].$$

Definition Interval Valued Neutrosophic Graph (IVNG) (Broumi et al., 2016): A pair $\mathbb{G}_{\mathbb{N}} = (P,Q)$ is IVNG, where

$$P = \left\langle \left[T_P^L, T_P^U\right], \left[I_P^L, I_P^U\right], \left[F_P^L, F_P^U\right] \right\rangle,$$

an IVN is set on $\mathbb{V}$ and

$$Q = \left\langle \left[T_Q^L, T_Q^U\right], \left[I_Q^L, I_Q^U\right], \left[F_Q^L, F_Q^U\right] \right\rangle$$

is an IVN edge set on $\mathbb{E}$ satisfying the following conditions:

1. Degree of truth membership, indeterminacy membership and falsity membership of the element $x_i \in \mathbb{V}$ are defined by

$$T_P^L : \mathbb{V} \to \left[0,1\right],\ T_P^U : \mathbb{V} \to \left[0,1\right], I_P^L : \mathbb{V} \to \left[0,1\right],\ I_P^L : \mathbb{V} \to \left[0,1\right],\ I_P^U : \mathbb{V} \to \left[0,1\right]$$

and

$$F_P^L : \mathbb{V} \to \left[0,1\right], F_P^U : \mathbb{V} \to \left[0,1\right]$$

respectively and

$$0 \le T_P\left(x_i\right) + I_P\left(x_i\right) + F_P\left(x_i\right) \le 3, \forall x_i \in \mathbb{V}, i = 1,2,3,\ldots,n$$

2. Degree of truth membership, indeterminacy membership and falsity membership of the edge $\left(x_i, y_j\right) \in \mathbb{E}$ are denoted by

$T_Q^L : \mathbb{V} \times \mathbb{V} \to \left[0,1\right]$, $T_Q^U : \mathbb{V} \times \mathbb{V} \to \left[0,1\right]$ $I_Q^L : \mathbb{V} \times \mathbb{V} \to \left[0,1\right]$, $I_Q^U : \mathbb{V} \times \mathbb{V} \to \left[0,1\right]$

and

$$F_Q^L : \mathbb{V} \times \mathbb{V} \to \left[0,1\right],\ F_Q^U : \mathbb{V} \times \mathbb{V} \to \left[0,1\right]$$

respectively and are defined by

$$T_Q^L\left(\left\{x_i, y_j\right\}\right) \leq \min\left[T_P^L\left(x_i\right), T_P^L\left(y_j\right)\right]$$

$$T_Q^U\left(\left\{x_i, y_j\right\}\right) \leq \min\left[T_P^U\left(x_i\right), T_P^U\left(y_j\right)\right]$$

$$I_Q^L\left(\left\{x_i, y_j\right\}\right) \geq \max\left[T_P^L\left(x_i\right), T_P^L\left(y_j\right)\right]$$

$$I_Q^U\left(\left\{x_i, y_j\right\}\right) \geq \max\left[I_P^U\left(x_i\right), I_P^U\left(y_j\right)\right]$$

$$F_Q^L\left(\left\{x_i, y_j\right\}\right) \geq \max\left[F_P^L\left(x_i\right), F_P^L\left(y_j\right)\right]$$

$$F_Q^U\left(\left\{x_i, y_j\right\}\right) \geq \max\left[F_P^U\left(x_i\right), F_P^U\left(y_j\right)\right]$$

where

$$0 \leq T_Q\left(\left\{x_i, y_j\right\}\right) + I_Q\left(\left\{x_i, y_j\right\}\right) + F_Q\left(\left\{x_i, y_j\right\}\right) \leq 3, \forall \left\{x_i, y_j\right\} \in \mathbb{E}\left(i, j = 1, 2, ..., n\right)$$

Score Function of INNs (Nagarajan et al., 2019): For ranking INNs, the following score function can be applied.

$$SF\left(\overline{F}\right) = \frac{1}{2}\left[\left(T_F^L + T_F^U\right) - \left(I_F^L I_F^U\right) + \left(I_F^U - 1\right)^2 + F_F^U\right]$$

Hamiltonian Cycle (HC) and Hamiltonian Path (HP) (Gani and Latha, 2016): A cycle passing through all the vertices without any repetition in a graph or network is called HC and the corresponding path is called a HP. Also a graph containing HC is called a Hamiltonian graph.

Figure 1. Adjacency Matrix of a ING of order n

$$A(\mathbb{G}) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & \cdot & \cdot & \cdot & v_n \end{array} \\ \left[\begin{array}{cccccccc} 0 & a_{12} & a_{13} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} & 0 & a_{23} & \cdot & \cdot & \cdot & a_{2n} \\ a_{31} & a_{32} & 0 & \cdot & \cdot & \cdot & a_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & 0 & \cdot & \cdot & \cdot & a_{nn} \end{array} \right] \end{array}$$

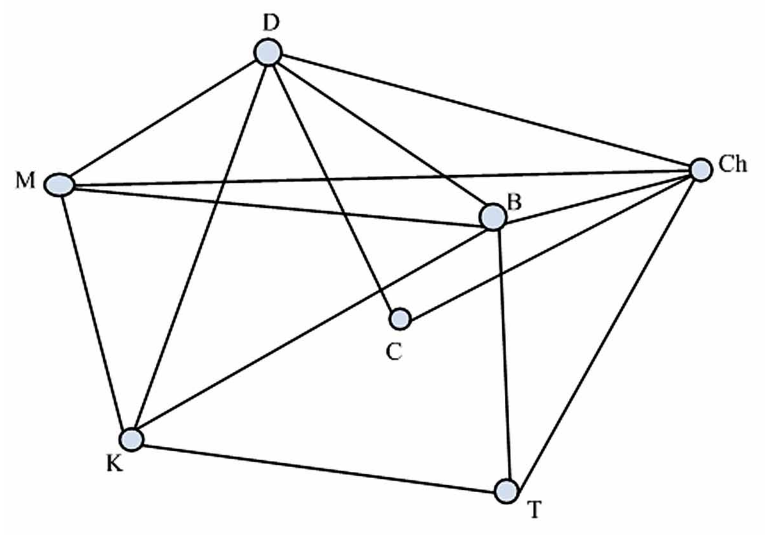
PROPOSED ALGORITHM TO FIND INHC IN INTERVAL NEUTROSOPHIC GRAPH

In this section, new algorithms have been proposed using adjacency matrix (AM) refer Fig.1 and vertex with minimum degree (VMD) to find interval neutrosophic Hamiltonian cycle (INHC) in an interval neutrosophic graph (ING).

Methodology to Find INHP and INHC In ING Using Adjacency Matrix

Let $\mathbb{G}_{\mathbb{N}} = \left(P, Q\right)$ be an ING of order n and $A(\mathbb{G})$ be its adjacency matrix. Choose the non-zero minimum entry from $A(\mathbb{G})$ say $a_{ij} = Q\left(v_i, v_j\right)$, the edge weight between the vertices v_i and v_j which provides the initial path that starts with the vertex v_i and travels to v_j. Now in the row of v_j, choose an appropriate non-zero entry and continue the process until getting HP which will extend to find the HC if exist.

Figure 2. Interval neutrosophic graph of chosen cities



Step 1: Using score function of INNs find the score values of all the edges in the ING.
Step 2: Form the AM using score values of the edges (Fig.1) of the ING.
Step 3: Find the degree of all the vertices using the definition of degree of INNs.
Step 4: Choose the VMD and start the process of finding INHP and INHC (if repetitions are there then choose any one). If NMD does not allow INHP and INHC then identify the next higher VMD and proceed.
Step 5: Repeat step 4, until getting NHP and NHC.

Experimental Section

Here model of the network formulation of the problem has been adopted from Gani and Latha, 2016, where map of the flight routes between the cities Mumbai (M), Delhi (D), Chennai (Ch), Coimbatore (C), Bangalore (B), Kochi (K) and Trivandrum (T) has been considered. In the present study, the edge weights are considered as an interval neutrosophic numbers which are adapted from Broumi et al. 2016.

Step 1: In Fig. 2, the vertices are the cities and edge weights have been considered as interval neutrosophic numbers as shown in Table 1.

Table 1. Membership values of the Vertices (Cities)

Vertices	Weights
M	$\langle [0.124, 0.126], [0.011, 0.021], [0.013, 0.014] \rangle$
D	$\langle [0.134, 0.141], [0.012, 0.031], [0.041, 0.043] \rangle$
K	$\langle [0.236, 0.271], [0.101, 0.104], [0.012, 0.023] \rangle$
C	$\langle [0.314, 0.320], [0.014, 0.016], [0.021, 0.024] \rangle$
B	$\langle [0.261, 0.271], [0.013, 0.017], [0.018, 0.021] \rangle$
Ch	$\langle [0.121, 0.124], [0.014, 0.016], [0.011, 0.014] \rangle$
T	$\langle [0.121, 0.125], [0.013, 0.014], [0.010, 0.013] \rangle$

Table 2. Airline routes from Mumbai to other cities

Cities	Weights	Score Value
M-M	**0**	**0**
M-D	$\langle [0.122, 0.125], [0.013, 0.032], [0.042, 0.043] \rangle$	0.613
M-K	$\langle [0.123, 0.124], [0.102, 0.105], [0.014, 0.024] \rangle$	0.531
M-C	**0**	**0**
M-B	$\langle [0.122, 0.125], [0.015, 0.018], [0.019, 0.022] \rangle$	0.616
M-Ch	$\langle [0.120, 0.123], [0.015, 0.022], [0.015, 0.016] \rangle$	0.607
M-T	**0**	**0**

Table 3. Airline routes from Delhi to other cities

Cities	Weights	Score Value
D-M	$\langle [0.122, 0.125], [0.013, 0.032], [0.042, 0.043] \rangle$	0.613
D-D	**0**	**0**
D-K	$\langle [0.132, 0.139], [0.103, 0.105], [0.043, 0.045] \rangle$	0.553
D-C	$\langle [0.133, 0.140], [0.015, 0.032], [0.043, 0.044] \rangle$	0.627
D-B	$\langle [0.132, 0.135], [0.015, 0.032], [0.042, 0.044] \rangle$	0.624
D-Ch	$\langle [0.119, 0.122], [0.016, 0.032], [0.041, 0.044] \rangle$	0.611
D-T	**0**	**0**

Table 4. Airline routes from Kochi to other cities

Cities	Weights	Score Value
K-M	$\langle [0.123, 0.124], [0.102, 0.105], [0.014, 0.024] \rangle$	0.531
K-D	$\langle [0.132, 0.139], [0.103, 0.105], [0.043, 0.045] \rangle$	0.553
K-K	**0**	**0**
K-C	**0**	**0**
K-B	$\langle [0.234, 0.237], [0.103, 0.105], [0.019, 0.024] \rangle$	0.643
K-Ch	**0**	**0**
K-T	$\langle [0.120, 0.124], [0.102, 0.106], [0.014, 0.024] \rangle$	0.528

Table 5. Airline routes from Coimbatore to other cities

Cities	Weights	Score Value
C-M	**0**	**0**
C-D	$\langle [0.133, 0.140], [0.015, 0.032], [0.043, 0.045] \rangle$	0.627
C-K	**0**	**0**
C-C	**0**	**0**
C-B	**0**	**0**
C-Ch	$\langle [0.121, 0.123], [0.016, 0.018], [0.023, 0.025] \rangle$	0.616
C-T	**0**	**0**

Table 6. Airline routes from Bangalore to other cities

Cities	Weights	Score Value
B-M	$\langle [0.122, 0.125], [0.015, 0.018], [0.019, 0.022] \rangle$	0.616
B-D	$\langle [0.132, 0.135], [0.015, 0.032], [0.042, 0.044] \rangle$	0.624
B-K	$\langle [0.234, 0.237], [0.103, 0.105], [0.019, 0.024] \rangle$	0.643
B-C	**0**	**0**
B-B	**0**	**0**
B-Ch	$\langle [0.119, 0.122], [0.016, 0.019], [0.021, 0.024] \rangle$	0.613
B-T	$\langle [0.120, 0.124], [0.016, 0.017], [0.019, 0.022] \rangle$	0.616

Table 7. Airline routes from Chennai to other cities

Cities	Weights	Score Value
Ch-M	$\langle[0.120, 0.123],[0.015, 0.022],[0.015, 0.016]\rangle$	0.607
Ch-D	$\langle[0.119, 0.122],[0.016, 0.032],[0.041, 0.044]\rangle$	0.611
Ch-K	**0**	**0**
Ch-C	$\langle[0.121, 0.123],[0.016, 0.018],[0.023, 0.025]\rangle$	0.616
Ch-B	$\langle[0.119, 0.122],[0.016, 0.019],[0.021, 0.024]\rangle$	0.613
Ch-Ch	**0**	**0**
Ch-T	$\langle[0.120, 0.123],[0.016, 0.017],[0.012, 0.015]\rangle$	0.607

Table 8. Airline routes from Trivandrum to other cities

Cities	Weights	Score Value
T-M	**0**	**0**
T-D	**0**	**0**
T-K	$\langle[0.120, 0.124],[0.102, 0.106],[0.014, 0.024]\rangle$	0.528
T-C	**0**	**0**
T-B	$\langle[0.120, 0.124],[0.016, 0.017],[0.019, 0.022]\rangle$	0.616
T-Ch	$\langle[0.120, 0.123],[0.016, 0.017],[0.012, 0.015]\rangle$	0.607
T-T	**0**	**0**

Table 9. Degree of the vertices and their score values

Vertices	Degree	Score Value
M	$\langle[0.487, 0.497],[0.145, 0.177],[0.090, 0.105]\rangle$	0.870
D	$\langle[0.638, 0.661],[0.162, 0.233],[0.211, 0.220]\rangle$	1.035
K	$\langle[0.609, 0.624],[0.410, 0.421],[0.090, 0.117]\rangle$	0.756
C	$\langle[0.254, 0.263],[0.031, 0.050],[0.066, 0.070]\rangle$	0.744
B	$\langle[0.727, 0.743],[0.165, 0.191],[0.120, 0.136]\rangle$	1.114
Ch	$\langle[0.599, 0.613],[0.079, 0.108],[0.112, 0.124]\rangle$	1.062
T	$\langle[0.360, 0.371],[0.134, 0.14],[0.045, 0.061]\rangle$	0.757

Edges (Flight Route)

See Tables 2-8.

Implementation of the Proposed Algorithm

Step 2: AM of the ING is

$$\begin{array}{c} \\ M \\ D \\ K \\ C \\ B \\ Ch \\ T \end{array}\begin{array}{c} \begin{array}{ccccccc} M & D & K & C & B & Ch & T \end{array} \\ \begin{bmatrix} 0 & 0.613 & 0.531 & 0 & 0.616 & 0.607 & 0 \\ 0.613 & 0 & 0.553 & 0.627 & 0.624 & 0.611 & 0 \\ 0.531 & 0.553 & 0 & 0 & 0.643 & 0 & 0.528 \\ 0 & 0.627 & 0 & 0 & 0 & 0.616 & 0 \\ 0.616 & 0.624 & 0.643 & 0 & 0 & 0.613 & 0.616 \\ 0.607 & 0.611 & 0 & 0.616 & 0.613 & 0 & 0.607 \\ 0 & 0 & 0.528 & 0 & 0.617 & 0.607 & 0 \end{bmatrix} \end{array}$$

Step 3: Degree of the vertices and their score values
Step 4: Choose minimum non zero entry (MNE) in AM.

$a_{37} = 0.528 \Rightarrow$ Row K reaches T⇒K-T

From T, the MNE is 0.67 which exist in the column Ch i.e., from T it goes to Ch (K-T-Ch).

Repeating step 4, we get, K-T-Ch-D-K (Rejected as it is a interval neutrosophic circuit)

Iteration 2: Start with other MNE $a_{73} = 0.528 \Rightarrow$ Row T reaches K ⇒T-K.

From K, MNE=0.531⇒K goes to M⇒K-M
From M, MNE=0.607⇒M goes to Ch⇒M-Ch
From Ch, MNE=0.611⇒Ch goes to D⇒Ch-D
From D, MNE=0.613⇒D goes to M⇒D-M (Rejected as it is not a INHP)

Iteration 3: Start with next higher MNE, $a_{13} = 0.531 \Rightarrow$ M-K

From K, MNE=0.528⇒K goes to T⇒K-T
From T, MNE=0.607⇒T goes to Ch⇒T-Ch
From Ch, MNE=0.611⇒Ch goes to D⇒Ch-D
From D, MNE=0.624⇒D goes to B⇒D-B
But B does not connect with C (Rejected as it is not formed INHP and INHC)

Iteration 4:

Start with next higher MNE, $a_{31} = 0.531 \Rightarrow$ K-M
From M, MNE=0.607⇒M goes to Ch⇒M-Ch
From Ch, MNE=0.616⇒Ch goes to C⇒Ch-C
From C, MNE=0.627⇒C goes to D⇒C-D
From D, MNE=0.624⇒D goes to B⇒D-B
From B, MNE=0.616⇒B goes to T⇒B-T
From T, MNE=0.528⇒T goes to K⇒T-K
Hence K-M-Ch-C-D-B-T is aINHP and the INHC is K-M-Ch-C-D-B-T-K with length = 4.149.

Algorithm to Find INHP and INHC in an ING Using Minimum Vertex Degree

This algorithm start the process of finding INHP and INHC with the vertex with minimum degree (MD) and one has to iterate the process until getting the desired result.

Step 1: Calculate the degree of all the vertices in the IVG.
Step 2: start the process with the vertex having MD (if repetitions exist choose anyone)
Step 3: Choose the vertex having next higher MD to the MD chosen.
Step 4: Identify the adjacent vertices of the vertex chosen with MD.
Step 5: Reach to the unvisited adjacent vertex of the MD. (If the adjacent vertex has more than one same identified MD, then select anyone).
Step 6: Repeat Step 4 and Step 5 until INHP and INHC is obtained.

Implementation:

Step 1: Here the score value of the vertices has been considered as the degree of the vertices.
Step 2: Choose the vertex C as it has the MD ($d\left(C\right) = 0.744$)
Step 3: The adjacent vertices (AVs) of C are D and Ch.
Step 4: From Step 1, $d\left(D\right) = 1.035$ and $d\left(Ch\right) = 1.062$, D has MD. Hence the path travels from C to D (C-D).
Iteration 1: The AVs of D: M, K, C, B and CH. Among these, K has a next higher minimum. Hence D-K and the path is C-D-K.
Itertaion 2: The AVs of K: M, D, B and T. Here T has a MD. Hence K-T and the path is C-D-K-T.
Iteration 3: The AVs of T: K, B and Ch. Here Ch has a MD. Hence T-Ch and obtained path is C-D-K-T-Ch.
Itertaion 4: The AVs of Ch: M, D, C, B and T. Here, M has a MD and Ch-M.

Hence, C-D-K-T-Ch-M.

Iteration 5: The AVs of M: D, K, B and Ch. Since B is the unvisited vertex, consider the degree of B as the MD. Hence, C-D-K-T-Ch-M-B.
Itertaion 6: The AVs of B: M, D, K, Ch and T. At this stage there is no unvisited vertex and INHP and INHC do not exist.

Therefore start with the next higher minimum degree i.e., $d\left(K\right) = 0.756$

AVs of K: M, D, B, T and T has a MD⇒K-T

AVs of T: K, B, Ch and Ch has a MD⇒T-Ch, hence K-T-Ch.

AVs of Ch: M, D, C, B, T. C has a MD⇒Ch-C, hence K-T-Ch-C.

AVs of C: D, Ch. D has a MD (Unvisited vertex)⇒C-D, hence K-T-Ch-C-D.

AVs of D: M, K, C, B, Ch. M has a MD (Unvisited vertex)⇒D-M, hence K-T-Ch-C-D-M.

AVs of M: D, K, B, Ch. B has a MD (Unvisited vertex)⇒D-M, hence K-T-Ch-C-D-M-B **(INHP)**

AVs of B: M, D, K, Ch, T. B is connected with K⇒K-T-Ch-C-D-M-B-K**(INHC).**

By considering next higher minimum of the degree of the vertex more NHPs and NHCs can be obtained (if exist).

COMPARATIVE ANALYSIS

A new algorithm has been proposed to find fuzzy Hamiltonian cycle for a given fuzzy network using adjacency matrix and minimum degree in Gani and Latha 2016, where indeterminacy of the edges cannot be handled. But using our proposed algorithm, the issue of handling indeterminacy is possible and for interval data under neutrosophic environment. Hence the novelty and efficiency of this proposed algorithm.

FUTURE RESEARCH DIRECTIONS

In future, this work shall be extended to complex neutrosophic environment.

CONCLUSION

Identifying the HP and HC is the tedious process when the number of vertices and edges are growing. That too for the case of interval based data, process of identifying

HP and HC takes more time. In this paper two algorithms have been proposed using adjacency matrix and MD of the vertex under interval neutrosophic environment. These algorithms may provide a simple way of testing the availability of the INHPs and INHCs in an interval neutrosophic graph.

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KEY TERMS AND DEFINITIONS

Adjacency Matrix: Is a connection matrix and represents the relation between the vertices whether connected or not.

Cycle in a Network: At least three vertices are connected in a closed chain.

Degree of a Vertex: The number of vertices incident to the vertex is called degree of a vertex.

Fuzzy Hamiltonian Cycle: In a fuzzy graph G, if a fuzzy cycle covers all the vertices exactly once then G is called Fuzzy.

Hamiltonian Cycle: Hamiltonian cycle is a cycle which visits each and every node exactly once.

Hamiltonian Path: A path which covers all the vertices exactly once is called a Hamiltonian path.

Interval Neutrosophic Hamiltonian Cycle: In interval neutrosophic graph G, if the interval neutrosophic cycle covers all the vertices exactly once then G is called interval neutrosophic Hamiltonian cycle.

Neutrosophic Hamiltonian Cycle: In a neutrosophic graph G, if a neutrosophic cycle covers all the vertices exactly once then G is called neutrosophic Hamiltonian cycle.

Neutrosophic Hamiltonian Path: A neutrosophic path which covers all the vertices exactly once is called Neutrosophic Hamiltonian path.

Path: A path in a graph is a finite or infinite sequence of distinct vertices.

Chapter 5
Dominations in Neutrosophic Graphs

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ABSTRACT

The aim of this chapter is to impart the importance of domination in various real-life situations when indeterminacy occurs. Domination in graph theory plays an important role in modeling and optimization of computer and telecommunication networks, transportation networks, ad hoc networks and scheduling problems, molecular physics, etc. Also, there are many applications of domination in fuzzy and intuitionistic fuzzy sets for solving problems in vague situations. Domination in neutrosophic graph is introduced in this chapter for handling the situations in case of indeterminacy. Dominating set, minimal dominating set, independent dominating set, and domination number in neutrosophic graph are determined. Some definitions, characterization of independent dominating sets, and theorems of neutrosophic graph are also developed in this chapter.

1. INTRODUCTION

Graph theory plays a vital role in applied mathematics. Many real-world situations can be described by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. For example, the points could represent people, with lines joining pairs of friends; or the points might be communication centres, with lines representing communication links. A mathematical abstraction of situations of this type gives rise to the concept of a graph (J. A. Bondy & U. S.

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R. Murty.,(1976)). Definition of fuzzy graph was proposed by Katmann by using the fuzzy relations introduced in (Zadeh, 1965). In (Rosenfield, 2010) another definition with fuzzy vertex and fuzzy edges and several concepts in graph theory such as paths and cycles etc., were introduced. Mathematical study of domination in graphs began around 1960. The terms “dominating set” and “domination number” were introduced in the book on graph theory by (Oystein Ore,1962). The problems described above were studied in more detail in (Yaglom and Yaglom, 1964). Solutions to some of these problems for rooks, knights, kings, and bishops were resulted by their study. In (Cockayne and Hedetniemi, 1977), the notation $\gamma(G)$ was first used for the domination number of a graph G. Also, the edge domination and its applications are presented in (Arumugam and Velammal, 1998 & Chang, 1998). In (Somasundaram and Somasundaram, 1998) domination in fuzzy set was investigated. The concept of independent domination in fuzzy graph is discussed in (Jayalakshmi et al.2014; Karunambigai et al., 2015). The concept of strong (weak) domination and total domination were developed by applying the domination in fuzzy graph in (A. Nagoor Gani et al, 2010). In (Atanassov, 1986) the concept of intuitionistic fuzzy set as a generalization of fuzzy sets was introduced. Definition of intuitionistic fuzzy graphs, intuitionistic hypergraph, intuitionistic digraphs and its applications, completeness, tolerance, degree structures and various operations in intuitionistic fuzzy graph were discussed in (Nagoor Gani et al, 2010). In Parvathi and Thamizhendhi(2010), dominating set, independent dominating set, domination number and total domination in intuitionistic fuzzy graph were introduced. Different types of domination in intuitionistic graph were investigated by many researchers in (Karunambigai et al., 2010, Nagoor Gani and Latha, 2012, Nagoorgani and Prasanna Devi, 2013, R. Parvathi and Karunambigai, 2006; Sahoo and Pal, 2016 & 2017). Neutrosophic set proposed by Smarandache (2006) is a powerful tool for dealing problems involving imprecise, inconsistant data and indeterminacy in the real world. It is the generalization of fuzzy sets and intuitionistic fuzzy sets. The results obtained from any application by using neutrosophic graphs will be better than by fuzzy graphs and intuitionistic fuzzy graphs, since indeterminacy membership function is used in neutrosophic graphs additionally. Fuzzy graph and intuitionistic fuzzy graph approaches are failed in some applications when indeterminacy occurs. So Smarandache (2011 & 2015) defined four main categories of neutrosophic graphs. In this chapter, certain types of domination in neutrosophic graph are developed and some theorems are explored.

2. PRELIMINARIES

In this section, some basic notions relating to neutrosophic sets, single valued neutrosophic graph, fuzzy graph and intuitionistic fuzzy graph connection with the present work are presented.

Definition 2.1 (Nagoor Gani et al, 2013): Let V be a non empty set. A fuzzy graph is a pair of functions G = (σ,μ), where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ, i.e., σ: V → [0, 1] and μ: V × V → [0, 1] such that μ(u, v) ≤σ(u) ∧σ(v), for all u,v in V.

Definition 2.2 (Nagoor Gani et al, 2013): A fuzzy graph G is said to be complete if μ(u, v) = σ(u) ∧σ(v), for all u, v in V.

Definition 2.3 (Nagoor Gani et al, 2013): Let G = (σ,μ)be a fuzzy graph on V. Let S of V and the fuzzy cardinality of S is defined to be $\Sigma_{v\in S}\sigma(v)$.

Definition 2.4 (Nagoor Gani et al, 2013): The domination number of a fuzzy graph G is the minimum cardinality taken over all dominating sets in G and is denoted by γ(G) or simply γ.

Definition 2.5 (Nagoor Gani et al, 2010): An intuitionistic fuzzy graph is of the form G = (V,E) where,

1. $V = \{v_1, v_2,\ldots,v_{n\}}$ such that μ_1: V → [0, 1] and γ_1: V → [0,1] denote the degree of membership and non-membership of the element $v_1 \in V$, respectively, and

$$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$$

for every $v_1 \in V$, (i = 1,2,..n),

2. E = V × V where μ_2: V × V→ [0, 1] and γ_2: V×V→ [0, 1] are such that

$$\mu_2(v_i, v_j) \le \min[\ \mu_1(v_i), \mu_1(v_j)]$$

and

$$\gamma_2(v_i, v_j) \le \max[\ \gamma_1(v_i),\gamma_1(v_j)]$$

and

$$0 \le \mu_2(v_i, v_j) + \mu_2(v_i, v_j) \le 1,$$

for every $(v_i; v_j) \in E$, $(i, j = 1,2,...,n)$

Definition 2.6 (Nagoor Gani et al, 2010). An intuitionistic fuzzy graph G = (V,E) is said to be complete intuitionistic fuzzy graph if

$$\mu_2(v_i, v_j) = \min\{\mu_1(v_i), \mu_1 v_j))$$

and

$$\upsilon_2(v_{i,} v_j) = \max\{\upsilon_1(v_i), \upsilon_1(v_j)\}$$

for every $(v_{i,} v_j) \in V$.

Definition 2.7 (Nagoor Gani et al, 2010) The complement of an intuitionistic fuzzy graph G = (V, E) is an intuitionistic fuzzy graph G = (V, E), where

1. V = V
2. $\mu_1(v_i) = \mu_1(v_i)$, and $\upsilon_1(v_i) = \{\upsilon_1(v_i)$,
3. $\mu_2(v_{\,i}, v_j) = \min\{\mu_1(v_i), \mu_1(v_j)\} - \mu_2(v_i, v_j)$ and
4. $\upsilon_2(v_{i,} v_j) = \max\{\upsilon_1(v_i), \upsilon_1(v_j)\} - \upsilon_2(v_{i,} v_j)$ for all i,j = 1,2,3,…,n.

Definition 2.8 (Nagoor Gani et al, 2010) Let G = (V,E) be an intuitionistic fuzzy graph. Let u,v∈ V, we say that u dominates v in G if there exist a strong arc between them. A subset D ⊆ V is said to be dominating set in G if for every v∈ V-D, there exist u dominates v.

Definition 2.9 (Jayalakshmi and Harinarayanan, 2014) A dominating set D of intuitionistic fuzzy graph G is said to be minimal dominating set of G if no proper subset of D is a dominating set of G. Minimum cardinality among all minimal dominating set of G is called the intuitionistic fuzzy domination number, and is denoted by $\gamma_{if}(G)$.

Definition 2.10 (Karunambigai et al., 2010) Let G = (V, E) be an intuitionistic fuzzy graph. A subset S of V is called dominating set in G if for every v∈V- S, there exists u∈ S such that u dominates v.

Definition 2.11 (Karunambigai et al., 2010) A dominating set S of an intuitionistic fuzzy graph G is said to be minimal dominating set of G if no proper subset of S is a dominating set of G.

Definition 2.12 (Broumi et al., 2016). Let X be a space of points (objects) with generic elements in X denoted by X, then the neutrosophic sets A (NS A) is an object having the form

$$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$$

where the functions T, I, F: X → $]0^-,1^+[$ define respectively the truth membership function, an indeterminacy membership function, and a falsity membership function of the element x ∈ X to the set A with the condition

$0^- \leq T_A(x)+I_A(x) + F_A(x) \leq 3^+$

The function $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$.

Definition 2.13 (Broumi et al., 2016). Let X be a space of points (objects) with generic elements in X denoted by X. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(X)$, an indeterminacy membership function $I_A(X)$ and a falsity membership function $F_A(X)$. For each point x in X $T_A(x)$; $I_A(x)$ and $F_A(x) \in [0,1]$. A SVNS A can be written as

$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$

Definition 2.14 (Broumi et al., 2016). Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set x. If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X, then $A = (T_A, I_A, F_A)$ is called a single valued neutrosophic relation on $B = (T_B, I_B, F_B)$ if

$T_B(x; y) \leq \min(T_A(x), T_A(y))$

$I_B(x; y) \geq \max(I_A(x), I_A(y))$

and

$F_B(x; y) \geq \max(F_A(x), F_A(y))$

for all x,y in X.

A single valued neutrosophic relation A on X is called symmetric if

$T_A(x, y) = T_A(y, x)$,

$I_A(x,y) = I_A(y, x)$,

$F_A(x, y) = F_A(y, x)$,

and

$T_B(x, y) = T_B(y, x)$,

$I_B(x, y) = I_B(y, x)$,

and

$F_B(x, y) = F_B(y; x)$

for all x,y in X.

Definition 2.15 (Broumi et al., 2016). A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G = (A,B) where,

1. The functions $T_A: V \to [0, 1]$, $I_A: V \to [0, 1]$ and $F_A: V \to [0, 1]$ denote the degree of truth membership, degree of indeterminancy membership, and degree of falsity membership of the element $v_i \in V$, respectively and

$0 \leq T_A(v_i)+I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in V$ (i =1,2,...,n)

2. The functions $T_B: E \subseteq V \times V \to [0, 1]$, $I_B: E \subseteq V \times V \to [0, 1]$ and $F_B: E \subseteq V \times V \to [0, 1]$ are defined by

$T_B(\{v_{i,} v_j\}) \leq \min(T_A(v_i), T_A(v_j))$,

$I_B(\{v_i; v_j\}) \geq \max(I_A(v_i), I_A(v_j))$

and

$I_B(\{v_i; v_j\}) \geq \max(F_A(v_i), F_A(v_j))$

denote the degree of truth membership, degree of indeterminacy membership, and degree of falsity membership of the edge $(v_i; v_j) \in E$ respectively, where

$0 \leq T_B(\{v_i; v_j\}) + I_B(\{v_i; v_j\}) + I_B(\{v_i; v_j\}) \leq 3$

for all $\{v_i; v_j\} \in E$, (i,j =1,2,...,n)

Table 1a.

	V_1	V_2	V_3	V_4	V_5
T_A	0.4	0.5	0.1	0.2	0.6
I_A	0.2	0.3	0.4	0.4	0.2
F_A	0.3	0.2	0.3	0.1	0.2

3. DOMINATIONS IN NEUTROSOPHIC GRAPH

Here we proceed on to define the notion of dominating set, minimal dominating set, domination number, independent domination and total domination in neutrosophic graphs. Also, the characterization of independent dominating set and some theorems in dominations in neutrosophic graph are developed. Throughout this chapter, G is considered as single valued neutrosophic graph.

Definition 3.1 Let G =(A,B) be a single valued neutrosophic graph on the vertex set V and let x,y ∈ V. x dominates y in G if

$T_B(x,y) = \min\{T_A(x); T_A(y)\}$,

$I_B(x,y) = \min\{I_A(x); I_A(y)\}$

and

$F_B(x,y) = \min\{F_A(x); F_A(y)\}$.

A subset D^N of V is called a dominating set in G if for every vertex $v \notin D^N$ there exists $u \in D^N$ such that u dominates v.

For example, consider a single valued neutrosophic graph G =(V, E) such that

$V= \{ V_1, V_2, V_3, V_4, V_5 \}$,

$E=\{ V_1 V_2, V_2 V_3, V_2 V_4, V_4 V_5, V_5 V_1\}$.

Let A be a single valued neutrosophic subset of V and let B a single valued neutrosophic subset of E denoted as shown in Tables 1a and 1b.

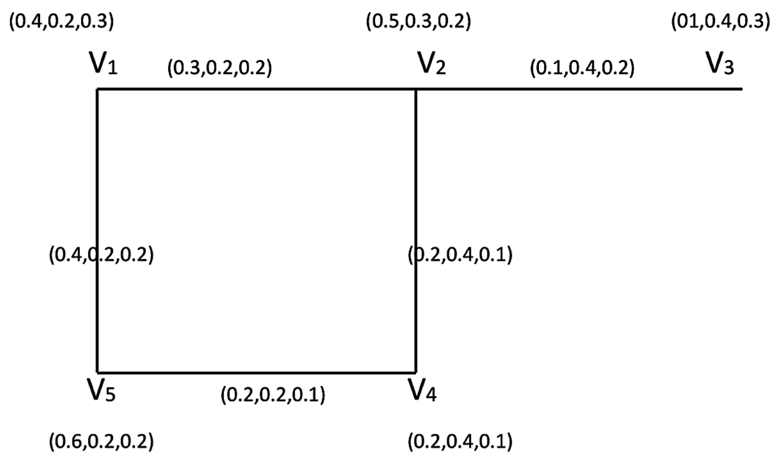
Definition 3.2 The minimum cardinality of a dominating set in a neutrosophic graph G is called the domination number of G and is denoted by $\gamma^N(G)$ (or) γ^N.

For example in figure 1, the minimum cardinality of the graph is 2. Hence $\gamma^N(G) = 2$.

Table 1b.

	$V_1 V_2$	$V_2 V_3$	$V_2 V_4$	$V_4 V_5$	$V_5 V_1$
T_B	0.3	0.1	0.2	0.2	0.4
I_B	0.2	0.4	0.4	0.2	0.2
F_B	0.2	0.2	0.1	0.1	0.2

Figure 1. G: Single valued neutrosophic graph



Note 3.3 Let G = (V,E) be a neutrosophic graph. Then

1. For any $x,y \in V$, if x dominates y in G then y dominates x in G and hence domination is a symmetric relation on V.\
2. For any x in V, N(x) is the set of all vertices which are dominated by x.
3. If $T_B(x,y) < \min\{T_B(x),T_B(y)\}$ (or)

$I_B(x,y) < \max\{I_B(x), I_B(y)\}$

and

$F_B(x,y) < \max\{F_B(x), F_B(y)\}$,

for all $x,y \in V$

Hence, the only dominating set in G is V.

Example 3.4 Let $G = (V,E)$ be a neutrosophic graph and $K^N_{(TA;IA;FA)}$ is a complete neutrosophic graph for all $v \in V$. Then,

1. $\gamma^N [K^N_{(TA,IA,FA)}] = \min \{T_A(x) + I_A(x) + F_A(x)\}$, since $\{v\}$ is a dominating set

$x \in V$

of $K^N_{(TA;IA;FA)}$.

2. $\gamma^N (G) = P$ iff

$T_B(x,y) < \min\{T_A(x), T_A(y)\}$ (or)

$I_B(x,y) < \min\{I_B(x), I_B(y)\}$

and

$F_B(x,y) < \min\{F_B(x), F_B(y)\}$,

for all $x,y \in V$,

3. $\gamma^N[K^N_{(TA1,TA2)(IB1, IB2)(FB1, FB2)}] = \min \{T_A(x) + I_A(x) + F_A(x)\}+$

$x \in V_1$

$\min \{T_A(x)+ I_A(x) + F_A(x)\}$,

$x \in V_2$

for all $V_1,V_2 \in V$.

Theorem 3.5 Let G be any neutrosophic graph. Then $\gamma^N {}_+ \gamma^N \leq 2p$, where γ^N is the domination number of G and equality holds if

$0 < T_A(xy) < \min\{T_A(x), T_A(y)\}$,

$0 < I_A(xy) < \max\{I_A(x), I_A(y)\}$

and

$0 < F_A(xy) < \max\{F_A(x), F_A(y)\}$,

for all $x,y \in V$.

Proof: The inequality is trivial. Also, γ^N=P iff

$T_A(xy) < \min\}T_A(x), T_A(y\}$,

$I_A(xy) < \max\{I_A(x), I_A(y)\}$

and

$F_A(xy) < \max\{F_A(x), F_A(y)\}$,

for all $x,y\in V$, and

γ^N =P iff

$\min\{T_A(x), T_A(y)\} - T_A(xy) < \min\{T_A(x), T_A(y)\}$,

$\max\{I_A(x), I_A(y)\} - I_A(xy) < \max\{I_A(x), I_A(y)\}$

and

$\max\{F_A(x), F_A(y)\} - F_A(xy) < \max\{F_A(x), F_A(y)\}$,

for all $x,y\in V$.

This is equivalent to $T_A(xy) > 0$, $I_A(xy) > 0$ and $F_A(xy) > 0$.

Therefore,

$\gamma^N{}_+ \gamma^N \leq 2p$ iff

$0 < T_A(xy) < \min\{T_A(x), T_A(y)\}$,

$0 < I_A(xy) < \max\{I_A(x), I_A(y)\}$

and

$0 < F_A(xy) < \max\{F_A(x), F_A(y)\}$,

for all $x,y\in V$.

Definition 3.6 Let G be a neutrosophic graph. A dominating set D^N of G is said to be a minimal dominating set if no proper subset of D^N is a dominating set of G.

Theorem 3.7 Let G be a neutrosophic graph. A dominating set D^N of G is a minimal dominating set iff for each $d\in D^N$, one of the following two conditions holds.

1. $N(d) \cap D^N = \phi$
2. $\exists\ a\in V\text{-}D^N$ such that $N(a)\cap D^N = \phi$

Proof: Let D^N be a minimal dominating set and $d\in D^N$. Then $D_d^N = D^N-\{d\}$ is not a dominating set.

Hence, there exists $x\in V\text{-}D_d^N$ such that x is not dominated by any element of D_d^N.

If x =d, then we get (i) and if $x\neq d$, then we get (ii).

The converse is trivially true.

Definition 3.8 A vertex x of a neutrosophic graph G(A,B) is said to be an isolated vertex if

$T_B(x,y) < \min\{T_A(x), T_A(y)\}$

$I_B(x,y) < \max\{I_A(x), I_A(y)\}$

and

$F_B(x,y) < \max\{F_A(x), F_A(y)\}$,

for all $y\in V\text{-}\{x\}$,

(ie) $N(x) = \phi$.

Thus an isolated vertex does not dominate any other vertex in G.

For example in Figure 2, V_3 is isolated vertex of G.

Theorem 3.9 Let G be a neutrosophic graph and it has no isolated vertices.

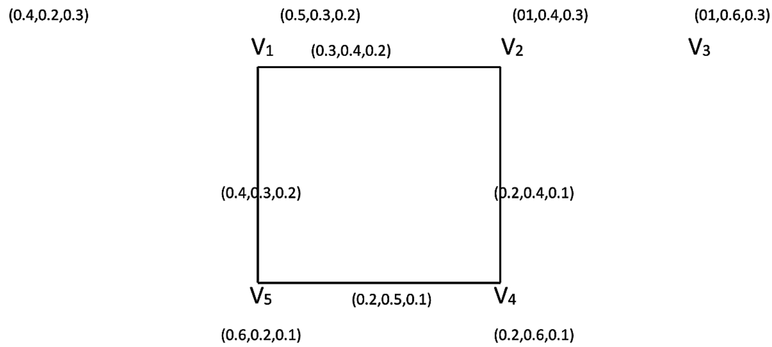
Let D^N be the minimal dominating set of G. Then $V\text{-}D^N$ is a dominating set of G.

(OR)

Let G be a neutrosophic graph and D^N be the minimal dominating set of G. If G has no isolated vertices then $V\text{-}D^N$ is a dominating set of G.

Proof: Let d be any vertex in D^N. Since G has no isolated vertex, there exists a vertex $c\in N(d)$. Then by theorem 3.7, $c \in V\text{-}D^N$.

Figure 2.



Hence every element of D^N is dominated by some element of V-D^N.

Corollary 3.10 For any neutrosophic graph without isolated vertex, $\gamma^N \leq p/2$.

Proof: Any neutrosophic graph G without isolated vertex has two disjoint dominating sets. Hence $\gamma^N \leq p/2$ in G.

Definition 3.11 A set of vertices D^N of a neutrosophic graph G(A,B) is said to be independent if

$T_B(xy) < \min\{T_A(x), T_A(y)\}$,

$I_B(xy) < \max\{I_A(x), I_A(y)\}$

and

$F_B(xy) < \max\{F_A(x), F_A(y)\}$,

for all x,y$\in D^N$

Theorem 3.12 (Characterization of independent neutrosophic dominating sets) If D^N is the independent neutrosophic dominating set of a neutrosophic graph G, then D^N is both minimal neutrosophic dominating set and a maximal independent set. Conversely any maximal independent set D^N is an independent neutrosophic dominating set of G.

Proof: Let G be a neutrosophic graph. If D^N is an independent neutrosophic dominating set of G, then $D_d^N = D^N - \{d\}$ is not a dominating set for every d$\in D^N$.

Also $D^N \cup \{x\}$ is not independent for every $x \notin D^N$.

Therefore, D^N is a minimal dominating set and maximal independent set of G.

Conversely, assume that D^N is a maximal independent set of G.

Then, $D^N \cup \{x\}$ is not independent for every $x \in$ V-D^N.

Hence, x is dominated by some element of D^N.

Thus, D^N is an independent dominating set of G.

Definition 3.13 Let G be a neutrosophic graph without isolated vertices. A subset D^N of V is said to be a total dominating set if every vertex in V is dominated by a vertex in D^N.

Definition 3.14 The minimum neutrosophic cardinality of a total dominating set is called total domination number of a neutrosophic graph G and is denoted by γ_1^N.

Theorem 3.15 For any neutrosophic graph G, γ_1^N =P iff every vertex of G has a unique neighbour.

Proof: Let G be any neutrosophic graph. If every vertex of G has a unique neighbour, then V is the only total dominating set of G so that γ_1^N =P.

Conversely, suppose γ_1^N =P.

If there exists a vertex v with two neighbours x and y, then V-{x} is a total dominating set of G. So $\gamma_1^N < P$ which is a contradiction to our assumption.

Hence, every vertex of G has a unique neighbour.

Corollary 3.16 Let G be a neutrosophic graph. If γ_1^N =P, then the number of vertices in G is even.

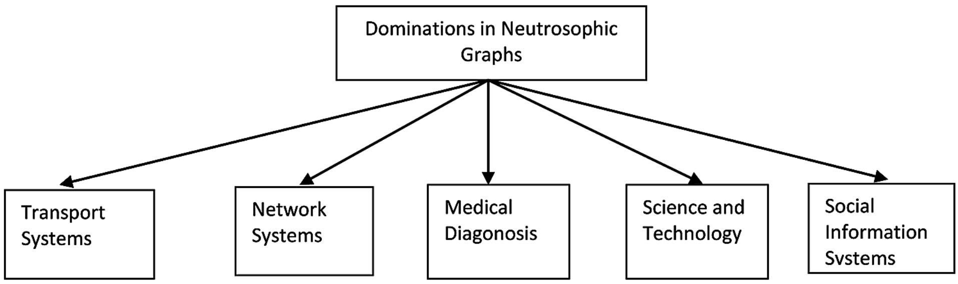
4. APPLICATIONS OF DOMINATION IN NEUTROSOPHIC GRAPHS

Domination in neutrosophic graphs has applications to several fields. Neutrosophic domination plays a vital role in many real life situations at the time of indeterminacy occurs. It gives better solutions to the problems in real life than other existing methods in unexpected and uncertain situations.

For eg, neutrosophic domination arises in facility location problems, where the number of facilities (e.g., hospital, bus stand, fire station) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility whenever indeterminacy occurs.

A social network is a structure made of individuals (or groups of individuals), which are connected by one or more specific types of interdependency. An important problem in the theory of social networks is the selection of sets of target individuals. When we are not able to identify the connection between particular types of interdependency and the choice of set of target individuals, neutrosophic dominating set is applied to easily identify the problems in such situations. Also it can be used to model social networks and to study the dynamics of relations among numerous individuals in different domains in indeterminacy and uncertain environment.

Figure 3.



The block diagram for some of its applications is as shown in Figure 3.

CONCLUSION

The main aim of this chapter is to present the importance of domination in neutrosophic graph ideas in various areas of science and technology for researchers that they can use the theoretical concept of domination in neutrosophic graphs for their research in various applications. Neutrosophic set is the generalization of fuzzy set and intuitionistic fuzzy sets. Neutrosophic models in real world applications are exible and compatibility than fuzzy and intuitionistic fuzzy models. Since neutrosophic graphs deals with imprecise, inconsistent and indeterminacy problems in the real world, the results obtained by using domination in neutrosophic graphs in various problems will give better results than by applying domination in fuzzy and intuitionistic fuzzy graphs. In this chapter, dominating set, minimal dominating set, domination number, independent domination and total domination in neutrosophic graph are defined. Also, the characterization of independent dominating set and some theorems in neutrosophic graph are developed. In future, the concept of domination in neutrosophic graph will be extented and applied to many fields like transportation, communication networks, biological and social information systems, linguistics, medical diagnosis, chemistry and physics etc.

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Chapter 6
A Study of Neutrosophic Shortest Path Problem

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ABSTRACT

Shortest path problem (SPP) is an important and well-known combinatorial optimization problem in graph theory. Uncertainty exists almost in every real-life application of SPP. The neutrosophic set is one of the popular tools to represent and handle uncertainty in information due to imprecise, incomplete, inconsistent, and indeterminate circumstances. This chapter introduces a mathematical model of SPP in neutrosophic environment. This problem is called as neutrosophic shortest path problem (NSPP). The utility of neutrosophic set as arc lengths and its real-life applications are described in this chapter. Further, the chapter also includes the different operators to handle multi-criteria decision-making problem. This chapter describes three different approaches for solving the neutrosophic shortest path problem. Finally, the numerical examples are illustrated to understand the above discussed algorithms.

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Table 1. Significance influences of neutrosophic set in different area of mathematics

Author and Year	Contribution in Different Area of Mathematics
Kandasamy & Smarandache (2003)	Introduced neutrosophic number.
Wang et al. (2005)	Introduced of interval neutrosophic set/logic.
Smarandache (2005)	Introduced the neutrosophic tri-polar set and neutrosophic multipolar set.
Smarandache (2009)	Introduced of n-norm and n-conorm.
Smarandache (2013a)	Development of neutrosophic measure and neutrosophic probability.
Smarandache (2013b)	Extended or refined the neutrosophic components.
Smarandache (2014a)	Introduced the law of included multiple middle.
Smarandache (2014b)	Suggested some of the development of neutrosophic statistics.
Smarandache (2014c)	Introduced neutrosophic crisp set and topology.
Smarandache (2015a)	Introduced neutrosophic pre-calculus and neutrosophic calculus.
Smarandache (2015b)	Extended or refined neutrosophic numbers.
Smarandache (2016)	Introduced degree of dependence and degree of independence between the neutrosophic components t, i, f
Smarandache (2017a)	In biology, Author introduced the theory of neutrosophic evolution: degrees of evolution, indeterminacy or neutrality, and involution.
Salama & Smarandache (2017b)	Introduced plithogeny (as generalization of dialectics and neutrosophy), and Plithogenic set/logic/probability/statistics.
Smarandache (2018)	Introduced neutrosophic psychology.

1. INTRODUCTION

Neutrosophic sets (NS) theory is proposed by Smarandache [(1999); (2005)], and this is generalised from the intuitionistic fuzzy set (Atanassov K. T., 1986) and fuzzy set (Zadeh, 1965). NS deals with indeterminate, uncertain and incongruous data where the indeterminacy is calculated explicitly. Moreover, indeterminacy, falsity and truth membership are fully independent. It overcomes some restrictions of the current methods in portraying uncertain decision. Samarandche's contribution towards neutrosophic theory is summarized in Table 1.

Moreover, Some extensions of NSs are also available such as interval NS [(Ye, 2014); (Liu & Shi, 2015)], bipolar NS [(Uluçay, Deli, & Şahin, 2018); (Wang, Zhang, & Wang, 2018)], single-valued NS [Wang et al. (Wang, Smarandache, Zhang, & Sunderraman, 2005); (Liu, 2016); (Liang, Wang, & Zhang, 2017)] multi-valued NS [(Peng, Wang, & Yang, 2017); (Ji, Zhang, & Wang, 2018)], neutrosophic linguistic set [(Ye, 2015); (Tian, Wang, Wang, & Zhang, 2017)], Rough neutrosophic sets [

(Broumi, Smarandache, & Dhar, 2014), (Mondal, Pramanik, & Smarandache, 2016), and so on. In the next section, SPP under different environment been discussed.

The Chapter is systemized as follows: In Segment 2, literature survey of shortest path problem. In Segment 3, the scientific problem and basic idea of solution methodology of SPP is given. In Segment 4, contains three different techniques are proposed for taking care of a similar issue along with numerical precedent is given to represent the proposed arrangement system. Finally, the conclusion of the section is provided.

2. LITERATURE SURVEY

In graph theory, the SPP is the problem of finding a path between two vertices in a graph where the total of the weight of its specified constituent edges is decreased. Recently, several researchers (Nie & Wu, 2009; Polychronopoulos & Tsitsiklis, 1996; Anusuya & Sathyaa, 2013) implement these shortest path methods in numerous atmospheres such as crisp, type 1 and type 2 fuzzy numbers, etc. Here, the application of shortest path problem in one of the available atmospheres (i.e. crisp atmosphere.) is discussed.

2.1. Crisp Shortest Path Problem

Network improvement is a well-known applied field among the foremost well studied area of research. Several sensible issues arising in industrial and commercial situations are developed such as network issues in communication networks, transportation network, neural networks, flow distribution networks, and so on. The first aim of network models rising from varied applications domains is to optimize the performance with relation to predefined objectives. Some effective algorithmic approaches were introduced by Bellman (1958) algorithm helps us find the shortest path from a vertex to all other vertices of a weighted graph. It is similar to Dijkstra's (1959) algorithm but it can work with graphs in which edges can have negative weights, Johnson's (1977) algorithm is a way to find the shortest paths between all pairs of vertices in a sparse, edge-weighted, directed graph. Floyd (1962) algorithm for finding shortest paths in a weighted graph with positive or negative edge weights (but with no negative cycles), in between 1950 and 1970. We have a tendency to refer these algorithms as classical algorithms. In classical algorithms for SPP, the costs of the arcs are replaced as real numbers. Most of the applications handling the networks need the computation of the most effective or shortest path from one node to a different node, and that they are referred to as SPP of a network. The objectives or criteria employed in checking out the shortest or best path in such networks is also one or a lot of, such as decrease of price, time, replication of knowledge and, risk. It also maximizes the comfort, stability, dependableness, quality etc. When just one objective (criteria) is taken into account within the SPP network, it is referred as single objective shortest path problem. A multiple objective (criteria) shortest path problem of a network consists of more than one criteria of either maximization or minimization of optimum value.

In the accepted approach to shortest path, the principal categories of shortest path problems are one pair problem, single source problem, single destination problem and all pair shortest path problem and so on.

2.1.1. *Mathematical Formulation for Crisp SPP*

In this section, the notation and existing linear model in crisp SPPs are studied.

Notations

Ω : Starting node
$\mho$: Final destination node
$\sum_{k=1}^{s} x_{mk}$: The total flow out of node *s*.
$\sum_{k=1}^{s} x_{mk}$: The total flow into node*s*.
RK_{mk} : The shortest distance from an m^{th} node to k^{th} node

According to Bazaraa and Sherali (2010), the crisp SPP model is as follows:

$$Min = \sum_{m=1}^{s} \sum_{k=1}^{s} RK_{mk} \cdot x_{mk} \tag{1}$$

Subject to:

$$\sum_{m=1}^{s} x_{mk} - \sum_{k=1}^{s} x_{km} = \underset{\sim}{\tilde{\kappa}}_{m} \tag{2}$$

for all $x_{mk} \in \Re$ and non-negative where $m, k = 1, 2, \ldots\ldots, s$ and:

$$\underset{\sim m}{\tilde{\kappa}} = \begin{cases} 1 \;\; if \quad m = \Omega, \\ 0 \;\; if \quad m = \Omega + 1, \Omega + 2, \ldots\ldots\ldots , \mho - 1 \\ -1 \;\; if \quad m = \mho. \end{cases} \tag{3}$$

2.2. Different Author's Contribution in SPP Under Uncertain and Inconsistent Data

Ahuja et al. (1990) recommended a new model for redistributive heap as a rapid algorithm to find SPP. Yang et al. (1992) suggested a graph-theoretic approach of rectilinear paths on bends and lengths. Ibarra & Zheng (1993) showed that the SPP for permutation graphs can be obtained by order of the logarithmic of n. Eilam-Tzoreff (1998) proposed a method for the disjoint SPP. Zhang & Lin (2003) compute the reverse SPP. Arnautović et al. (2013) suggested ant bee colony method for finding the SPP. Grigoryan & Harutyunyan (2015) offered SPP in the Knodel graph. Rostami et al. (2015) proposed quadratic SPP. Dragan & Leitert (2017) solved SPP on minimal peculiarity. since, crisp environment is not able to handle uncertainty so few techniques have been discussed which are able to handle the SPP under uncertainty are given in Table 2.

There are many researchers who have introduced some significant operators for decision-making in engineering technicalities under neutrosophic environment. Ye (2015) invented a new operator for trapezoidal neutrosophic set, Deli (2018) proposed innovative operators on single valued trapezoidal neutrosophic numbers, Islam & Ray (2018) introduced Neutrosophic Cubic Mean Operators and Entropy, Khan et al. (2018) attained Interval Neutrosophic Dombi Power Bonferroni Mean Operators, Fahmi et al. (2018) proposed Triangular Cubic Hesitant Fuzzy Einstein Hybrid Weighted Averaging Operator, Khan et al. (2019) suggested Neutrosophic Cubic Einstein Geometric Aggregation Operators and many others (Ye, 2014; Peng, Wang, Wu, Wang, & Chen, 2015; Liu & Shi, 2015; Liu & Wang, 2016; Liu & Shi, 2017). These operators are used in handling different real life problems. But this chapter mainly focuses on SPP under neutrosophic environment. Some significant influences of NSSPP have been charted in Table 3.

2.3. Motivation of the Work

One undertaking from the above discussions is that SPP underneath uncertain setting with non-consistent information is a terribly new and fruitful analysis field. Additionally the higher literature review reveals that there varied gaps within the study of SPP. As such, the subsequent gaps are studied.

Table 2. Different author's contribution for solving FSPP using different approaches under uncertainty

Author and Year	Different Approaches to Solve SPP Under Uncertainty
Li et al. (1996)	Artificial neural network
Jensen & Shen (2005)	Ant colony optimization
Mahadavi et al. (2009)	Dynamic programming approach
Dou et al. (2012)	Vague similarity measure
Zhang et al. (2013)	FPA algorithm
Zhang et al. (2014)	Modified biological inspired algorithm
Dey et al. (2016)	Dijsktra's algorithm
Ebrahimnejad et al. (2016)	Artificial bee colony algorithm
Mohiuddin et al. (2016)	PSO algorithm
Kumar et al. (2017)	Ranking approach to solve Type 2 SPP
Dey et al. (2018)	Genetic algorithm
Lu and Gazara (2019)	By branch and price and cut technique.
Kumar et al. (2020)	Linear programming approach

Table 3. Significance influences of different authors towards neutrosophic SPP

Author and Year	Significance Influences
Broumi et al. (2016a)	Neutrosophic setting along with triangular fuzzy for processing SPP
Broumi et al. (2016b)	Dijkstra principle for interval based data based problems.
Broumi et al. (2016c)	Single valued trapezoidal NS numbers for dijkstra principle
Broumi et al. (2016d)	Single valued NS Graphs in SPP
Broumi et al. (2017a)	Invented decision-making problem for maximization of deviation method with partial weight under the neutrosophic environment
Broumi et al. (2017b)	Neutrosophic setting along with trapezoidal fuzzy for processing SPP.
Broumi et al. (2017c)	Bipolar neutrosophic environment.
Kumar et al. (2018)	A new algorithm based on score function for finding the neutrosophic shortest path problems.
Broumi et al. (2018a)	Interval-valued NS setting environment for processing SPP.
Broumi et al. (2019)	Interval valued trapezoidal and triangular neutrosophic environment using improved score and center of gravity function for finding SPP.
Kumar et al. (2019a)	Multi-objective programming approach to solve SPP
Kumar et al. (2019b)	Linear programming approach is utilized to find SP under Gaussian neutrosophic environment.

- With crisp range you will be able to either predict the journey occurs or not happens.
- There are still very few investigation on problems related to SPP arising in different real world application.
- Although the majority of research work in the field has focused on crisp problems, there are still many open questions, further research on the optimization methods of SPP under different uncertain environment is limited to date.
- The seeking of solution of uncertain SPP is still a significant task that needs to focus on the development of the methods & on the modification (gap) of the current methods.

Therefore this motivates us to consider effective uncertain analysis to solve SPP.

2.4. Contribution of the Work

In veiw of above gaps, the aim in this research is to provide few evolutionary methods which are generally desribe through uncertainty theory. Some of the theoritical analysis like invalidity analysis of current methods have been taken into consideration. As such, objective related to the present work are included. Following are the objectives planned and accomplished during this analysis work.

- To study numerical solution and its application of SPP which are generally described through optimization theory.
- To provide few evolutionary methods for obtaining SP under uncertainty environments like neutrosophic environment.
- To predict the crisp shortest path length/time/cost underneath completely different uncertainty situations.

3. DESCRIPTION OF THE WORK

3.1. Research Problem

The problem of finding the shortest path (SP) from a specified source node to a destination node is a fundamental and well known combinatorial optimization problem in graph theory. The only certain thing in today's information intense age is uncertainty. Uncertainty exists almost in every real life application of SPP. The uncertainty theory is one of the popular tools to represent and handle uncertainty in information due to imprecise, incomplete, inconsistent, and indeterminate. Moreover the literature review reveals that there are varied gaps within the study of shortest path problem. The seeking of solution of uncertain SPP is still a significant task that needs to focus on the development of the methods & on the modification (gap) of the current methods. Therefore, the framework of this research work is based on shortest path problem i.e., to consider effective uncertain analysis to solve SPP.

3.2. Solution Methodologies

Solution methodologies for finding shortest path problem for linear programming method are mainly based on the choice of suitable environments. The two basic ideas are mentioned below which are discussed in following section:

1. Basic idea of FSPP using optimization theory.
2. Basic idea of neutrosophic SPP using optimization theory.

3.2.1. Basic Idea of Fuzzy Shortest Path Using Optimization Theory

In real life circumstances, unexpected events may occur with the goal that the edge weight in the system may change marginally. Subsequently many times, the load must be evaluated inside a specific interim. Hence in most situations, the weight can only be estimated within a certain interval. Consequently it is very difficult for a decision maker to give a single precise number to represent each edge weight more realistically and naturally achieved through the use of a fuzzy number. Therefore, the fuzzy methodology appears to be substantially more characteristic for taking care of the solving the shortest path problem.

The fuzzy shortest path problem (FSPP) was first introduced by Dubois and Prade (1980) in 1980. The analysis of fuzzy counter elements of the SPPs seems to become a preferred task in recent years. Within the non-fuzzy problems formulation, the decision maker is not forced into a certain formulation. There is a lot of inexactly quantified physical information in real life scenarios such as vertex restrictions, edge capacities, edge weights or arc lengths. The arc weight of a SPP used to represent the travelling price, distance, time or alternative variable within the real life situations. In application of SPP, the edge weights within the path of a graph have some parameters that is terribly exhausting to search out precisely, such as capacities, distance, cost, demands, traffic frequencies, etc. for instance, the geographical distance between two cities is also recognized precisely, however, the traveling price or traveling time could amendment because of weather, accident and traffic flow. So, the edge weights are non-deterministic in above situations and it is not possible to use the classical formula to search out optimum value of the SPP in such unsure surroundings. Several researchers thought that uncertainties are because of non-deterministic befit randomness and that they projected the idea of likelihood SPP (Nie & Wu, 2-009; Conde, 2017; Yang & Zhou, 2017) and random SPP (Bertsekas & Tsitsiklis, 1991; Polychronopoulos & Tsitsiklis, 1996; Chen, Lam, Sumalee, Li, & Tam, 2014; Randour, Rask-in, & Sankur, 2015). Fuzziness is applied as another to randomness for dealing uncertainties of SPP. The SPP with fuzzy arc lengths (FAL), outlined as FSPP, represents the type-1 fuzzy (T1-F). However, if the arc lengths of a graph amends under some specific condition like travelling time or arc lengths are collected from quite one supply that change often, it becomes very hard to represent those lengths by using T1-F range.For instance, it is generally impossible to give the mathematical description of traffic frequency of a road in different time. These types of information are collected from a set of person using some questionnaires which consists of uncertain words. The classical fuzzy set is unable to handle these types of uncertainties as their membership values are completely crisp. These sorts of uncertainty are sculptured by type-2 fuzziness. In type 2 fuzzy (T2-F) sets have membership values that also are fuzzy. Few researchers have work on SPP with T2-F set as arc lengths. Anusuya and Sathyaa (2013), have introduced a method for SPP between supply and destination vertices on a fuzzy graph. They needed to allot T2-F number to every edge as its weight. All doable ways from source vertex (SV) to destination vertex (DV) were computed and additionally their lengths of the trail were calculated in their proposed method. A path length is delineating by T2-F number and its corresponding rank is additionally determined. The SP is that path that has the lowest rank. Anusuya and Sathya (2013; 2014) have thought of a separate T2-F numbers to represent weight of every fringe of the graph and determined the lengths of the path of all ways from SV to DV supported as in (Anusuya & Sathyaa, 2013). The researchers have used a ranking technique to work out the minimum between two ways, i.e., separate T2-F numbers. The trail with highest similarity degree is that the SP between SV and destination vertices. Dey et al (2016) thought of interval T2-F set as edge weights of a fuzzy network for SPP. They changed the Dijkstra's formula for resolution the SPP with interval T2-F set as edge weights of the network. The researchers have additionally represented a generalized recursive approach for resolution this SPP, supported by path algebra. Dey et al. (2018), have introduced a genetic formula to work out the SP between a selected SV and one DV in an exceedingly fuzzy network, wherever the edge weights square measure delineate by interval T2-F set. The membership degree in fuzzy set or T2-F set is unable to model the ambiguous scenario of the SPP. To unravel this drawback, Atanassov (1986) has introduced the thought of intuitionistic fuzzy (INF) set that is represented by membership and non - membership degree and it may also deal the impreciseness information. Recently, several researchers (Karunambigai, Rangasamy, Atanassov, & Palaniappan, 2007; Kumar, Bajaj, & Gandotra, 2015; Motameni & Ebrahimnejad, 2018) have used INF set as arc length of SPP. However, T1-F set, INF set and T2-F set does not have any capability to grasp all potentialities of indeterminate or inconsistent info of SPP. Thus to handle such things Smarandache (1999) introduced neutrosophic principle.

3.2.2. *Mathematical Formulation of Fuzzy Shortest Path Problem*

If the parameter RK_{mk} is replaced into fuzzy cost parameters, *i.e.* RK_{mk}^{F}, then the model is as follows:

$$Min = \sum_{m=1}^{s}\sum_{k=1}^{s} RK_{mk}^{F}.\ x_{mk}$$

Subject to constraints **(2.2-2.3)**

3.2.3. Basic Idea of Neutrosophic Shortest Path Using Optimization Theory

Moving further, the edge weight is represented as neutrosophic number in the following study. In 1998, Smarandache (1999; 2005) has introduced the idea of NS set which can deal with vague, indeterminate and inconsistent information of the problem that may exist in the real word scenarios. It is an extension of crisp set, T1-F set and INF set. A NS set is described by three membership degrees of truth, indeterminate and falsity. This three independent membership degrees are within the non-standard unit interval] 0, 1+[. However, the membership degree of fuzzy set lies between the interval [0,1]. Recently, the NS set is used for modeling many engineering applications because it can deal with incomplete information as well as the inconsistent and indeterminate information are available in these references [(Broumi, et al., 2017; 2017; Deli I., 2019a; 2019); (Deli & Şubaş, 2017; 2017a); (Kumar R., Edalatpanah, Jha, Gayen, & Singh, 2019) (Kumar R., Edalatpanah, Jha, & Singh, 2019d) (Gayen, Jha, & Singh, 2019) (Gayen, Jha, Singh, & Kumar, 2019) (Broumi & Smarandache, 2015; Broumi, Smarandache, & Dhar, 2014); (Ye, 2015; Ye, 2014; Ye & Fu, 2016)] etc. Disaster like flood, earthquakes, landslide, tornadoes, windstorms, fires, tsunamis, chemical accidents are some extreme events inflicting from environmental issue of the world. These kinds of disaster will cause nice damage to the society. It is necessary to evacuate an oversized scale crowds from the danger space to a secure space once disaster happens. The fast rescue will facilitate to avoid wasting several lives of individual and property. Therefore, it is the foremost vital issue of disaster management that the way to utilize quickest speed to evacuate of an oversized populations from the danger space to a secure space when the disaster within the emergency evacuation, it's necessary to produce the SP from a specified path.

The idea is therefore proposed to seek out the shortest/fastest evacuation ways and rescue ways between the source and destination nodes. This idea is applied to find the quickest time within the evacuation method. The most issue of evacuation method throughout the natural disaster is that the majority of the bridges and roads square measure destructed in time of natural disaster like flood, earthquakes and alternative event. The shortest route and distance may be to verify for quickest evacuation supported the knowledge of the safe space and danger space of the remaining (after disaster) bridges and roads. However, uncertainties may exist during these situations. The knowledge is mostly collected from the opinion of a gaggle of specialist. For instance, experts say that "The transportation time of delivering the emergency medical things to the area would be five hours." Numerous kinds of uncertainties might exist with this statement which is as follows:

- The choice maker cannot finish up right away that the transportation time is precisely five hours. This sort of statement might have a point of neutral membership, that is independent from the degree of truth and false membership of the statement. Neutral membership worth may be accustomed handle the uncertainty in human perception of any statement.
- Experts might imagine that the chance of honesties of the statement is 0.7, the chance of untruth is capable 0.8 and therefore the risk of uncertainty of the statement is 0.5. Here, the addition of three membership degrees is bigger than one. Fuzzy set and INF sets can't handle these kinds of uncertainties. The thought of NS set will handle this sort of uncertainty.

3.2.4. *Mathematical Formulation of Neutrosophic Shortest Path Problem*

If parameter RK_{mk} is replaced into neutrosophic cost parameters, *i.e.* RK_{mk}^{N}, then the model is as follows:

$$Min = \sum_{m=1}^{s}\sum_{k=1}^{s} RK_{mk}^{N}.\ x_{mk}$$

Subject to constraints **(2.2-2.3)**

3.3. Preliminaries

Definition 3.3.1: Wang et al. (2005): Let X be a space point or objects, with a genetic element in X denoted by *x*. A single-valu-ed NS, W in X is characterised by three independent parts, namely truth-MF T_W^N, indeterminacy-MF I_W^N and falsity -MF F_W^N, such that

$$T_W^N : X \to [0,1], I_W^N : X \to [0,1], and\, F_W^N : X \to [0,1].$$

Now, W is denoted as

$$W = \left\{ < x, \left(T_W^N(x), I_W^N(x), F_W^N(x) \right) > \mid x \in X \right\},$$

satisfying

$$0 \le T_W^N(x) + I_W^N(x) + F_W^N(x) \le 3.$$

Definition 3.3.2: Chakraborty et al. (2018): Let

$$\hat{s}^n = \left\langle \left(\widehat{s_{ij,a}}, \widehat{s_{ij,b}}, \widehat{s_{ij,c}} \right), \left(s_{ij,a}, s_{ij,b}, s_{ij,c} \right) \left(\underline{s_{ij,a}}, \underline{s_{ij,b}}, \underline{s_{ij,c}} \right) \right\rangle$$

is a special NS on the real number set R, whose truth-MF $\Delta\eta_{\hat{s}}(x)$, indeterminacy-MF $\nabla\sigma_{\hat{s}}(x)$, and falsity-MF $\mho\mu_{\hat{s}}(x)$ are given as follows:

$$\eta_{\hat{s}}(x) = \begin{cases} \dfrac{\left(x - \widehat{s_{ij,a}} \right)}{\left(\widehat{s_{ij,b}} - \widehat{s_{ij,a}} \right)} & \widehat{s_{ij,a}} \le x < \widehat{s_{ij,b}}, \\ 1 & x = \widehat{s_{ij,b}}, \\ \dfrac{\left(\widehat{s_{ij,c}} - x \right)}{\left(\widehat{s_{ij,c}} - \widehat{s_{ij,b}} \right)} & \widehat{s_{ij,b}} < x \le \widehat{s_{ij,c}}, \\ 0 & otherwise. \end{cases}$$

$$\sigma_{\hat{s}}(x)=\begin{cases}\dfrac{\left(s_{ij,b}-x\right)}{\left(s_{ij,b}-s_{ij,a}\right)} & s_{ij,a}\le x<s_{ij,b},\\ 0 & x=s_{ij,b},\\ \dfrac{\left(x-s_{ij,b}\right)}{\left(s_{ij,c}-s_{ij,b}\right)} & s_{ij,b}<x\le s_{ij,c},\\ 1 & otherwise.\end{cases}$$

$$\mu_{\hat{s}}(x)=\begin{cases}\dfrac{\left(\underline{s_{ij,b}}-x\right)}{\left(\underline{s_{ij,b}}-\underline{s_{ij,a}}\right)} & \underline{s_{ij,a}}\le x<\underline{s_{ij,b}},\\ 0 & x=\underline{s_{ij,b}},\\ \dfrac{\left(x-\underline{s_{ij,b}}\right)}{\left(\underline{s_{ij,c}}-\underline{s_{ij,b}}\right)} & \underline{s_{ij,b}}<x\le \underline{s_{ij,c}},\\ 1 & otherwise.\end{cases}$$

Where

$$0\le \eta_{\hat{s}}(x)+\sigma_{\hat{s}}(x)+\mu_{\hat{s}}(x)\le 3, \qquad x\in \hat{s}^{n}$$

The parametric form is defined as follows

$$\left(\hat{s}^{n}\right)_{\delta,\nu,\eta}=\left[\eta_{\hat{s}^{N1}}\left(\delta\right),\eta_{\hat{s}^{N2}}\left(\delta\right);\sigma_{\hat{s}^{N1}}\left(\nu\right),\sigma_{\hat{s}^{N2}}\left(\nu\right);\mu_{\hat{s}^{N1}}\left(\lambda\right),\mu_{\hat{s}^{N2}}\left(\lambda\right)\right],$$

Where $\delta,\nu,\lambda\in(0,1)$

$$\eta_{\hat{s}^{N1}}\left(\delta\right)=\widehat{s_{ij,a}}+\delta\left(\widehat{s_{ij,b}}-\widehat{s_{ij,a}}\right)$$
$$\eta_{\hat{s}^{N2}}\left(\delta\right)=\widehat{s_{ij,c}}-\delta\left(\widehat{s_{ij,c}}-\widehat{s_{ij,b}}\right)$$
$$\sigma_{\hat{s}^{N1}}\left(\nu\right)=s_{ij,b}-\nu\left(s_{ij,b}-s_{ij,a}\right)$$
$$\sigma_{\hat{s}^{N2}}\left(\nu\right)=s_{ij,b}+\nu\left(s_{ij,c}-s_{ij,b}\right)$$
$$\mu_{\hat{s}^{N1}}\left(\lambda\right)=\underline{s_{ij,b}}-\lambda\left(\underline{s_{ij,b}}-\underline{s_{ij,a}}\right)$$
$$\mu_{\hat{s}^{N2}}\left(\lambda\right)=\underline{s_{ij,b}}+\lambda\left(\underline{s_{ij,b}}-\underline{s_{ij,c}}\right)$$

Definition 3.3.3: (Abdel-Basset, Gunasekaran, Mohamed, & Smarandache, 2018; Mohamed, Abdel-Basset, Zaied, & Smarandache, 2017)**:** Arithmetic operation: Let

$$\hat{s}^{n}=\left\langle\left(\widehat{s_{ij,a}},\widehat{s_{ij,b}},\widehat{s_{ij,c}}\right),\left(s_{ij,a},s_{ij,b},s_{ij,c}\right),\left(\underline{s_{ij,a}},\underline{s_{ij,b}},\underline{s_{ij,c}}\right)\right\rangle$$

and

$$\hat{u}^{n}=\left\langle\left(\widehat{u_{ij,a}},\widehat{u_{ij,b}},\widehat{u_{ij,c}}\right),\left(u_{ij,a},u_{ij,b},u_{ij,c}\right),\left(\underline{u_{ij,a}},\underline{u_{ij,b}},\underline{u_{ij,c}}\right)\right\rangle$$

be two arbitrary SVTNNs, and $\rho\geq 0$; then:

$$\hat{s}^{n}\oplus\hat{u}^{n}=\left\langle\left(\widehat{s_{ij,a}}+\widehat{u_{ij,a}},\widehat{s_{ij,b}}+\widehat{u_{ij,b}},\widehat{s_{ij,c}}+\widehat{u_{ij,c}}\right),\left(s_{ij,a}+u_{ij,a},s_{ij,b}+u_{ij,b},s_{ij,c}+u_{ij,c}\right),\left(\underline{s_{ij,a}}+\underline{u_{ij,a}},\underline{u_{ij,b}}+\underline{u_{ij,b}},\underline{u_{ij,c}}+\underline{u_{ij,c}}\right)\right\rangle$$

$$\hat{s}^{n}\otimes\hat{u}^{n}=\left\langle\left(\widehat{s_{ij,a}}\cdot\widehat{u_{ij,a}},\widehat{s_{ij,b}}\cdot\widehat{u_{ij,b}},\widehat{s_{ij,c}}\cdot\widehat{u_{ij,c}}\right),\left(s_{ij,a}\cdot u_{ij,a},s_{ij,b}\cdot u_{ij,b},s_{ij,c}\cdot u_{ij,c}\right),\left(\underline{s_{ij,a}}\cdot\underline{u_{ij,a}},\underline{u_{ij,b}}\cdot\underline{u_{ij,b}},\underline{u_{ij,c}}\cdot\underline{u_{ij,c}}\right)\right\rangle$$

$$\rho\hat{s}^{n}=\left\langle\left(\widehat{\rho s_{ij,a}},\widehat{\rho s_{ij,b}},\widehat{\rho s_{ij,c}}\right),\left(\rho s_{ij,a},\rho s_{ij,b},\rho s_{ij,c}\right),\left(\rho\underline{s_{ij,a}},\rho\underline{s_{ij,b}},\rho\underline{s_{ij,c}}\right)\right\rangle\ \ if\ \ \rho\geq 0$$

Definition 3.3.4: Chakraborty et al. (2018): Let

$$\hat{s}^n = \left\langle \left(\widehat{s_{ij,a}}, \widehat{s_{ij,b}}, \widehat{s_{ij,c}}\right), \left(s_{ij,a}, s_{ij,b}, s_{ij,c}\right), \left(\underline{s_{ij,a}}, \underline{s_{ij,b}}, \underline{s_{ij,c}}\right) \right\rangle$$

then the value of De-neutrosophication of neutrosophic number is as follows:

$$s\left(\hat{s}^n\right) = \frac{1}{12}\left[\left(\widehat{s_{ij,a}} + 2\cdot\widehat{s_{ij,b}} + \widehat{s_{ij,c}}\right) + \left(s_{ij,a} + 2\cdot s_{ij,b} + s_{ij,c}\right) + \left(\underline{s_{ij,a}} + 2\cdot\underline{s_{ij,b}} + \underline{s_{ij,c}}\right)\right]$$

4. PROPOSED SOME OF THE METHODS TO SOLVE NEUTROSOPHIC SPP

The present aims to investigate the predicted numerical solution of the SPP under uncertain, inconsistent data. Therefore three new method is proposed in this chapter to handle the problems mentioned in chapter title.

- Method 1: Extension of Dijsktra's principle
- Method 2: Extension of Multiple labelling principle.
- Method 3: Extension of Bellmond ford algorithm

4.1. Extension of Dijsktra's Principle

Algorithm for Extension of Dijsktra's principle is discussed in Table 4.

Numerical Illustration: See Figure 1.

Example 4.1 *Consider a network [*Figure 4.1 (Kumar, Edaltpanah, Jha, Broumi, & Dey, 2018; Broumi, Nagarajan, Bakali, Talea, Smarandache, & Lathamaheswari, 2019)*], with six nodes and eight edges, where node 1 is the SV and node 6 is the DV. The TrNS cost is given in* Table 5 (2018).

Solution: Calculating the SP using proposed algorithm 1 of TrNS path problem is given as follows.

Here $r = 6$, since the network consists of 6 vertices.

Let, $l_1 = \left\langle [0,0,0],[0,0,0],[0,0,0] \right\rangle$ and classify the SN

$n_1 = \left[\left\langle [0,0,0],[0,0,0],[0,0,0] \right\rangle, -\right]$.

To find the value of $l_m, m = 2,3,4,5,6$ (Box 1).

Table 4. Algorithm 1 for extension of modified Dijsktra's principle

Step 1:	Obtain the source vertex (SV) arc length $l_1 = \langle [0,0,0],[0,0,0],[0,0,0] \rangle$ and classify SV, vertex 1 by $\left[l_1 = \langle [0,0,0],[0,0,0],[0,0,0] \rangle, - \right]$
Step 2:	Obtain the minimum distance of n_1 with its corresponding vertex using $l_i = \min\{l_i \oplus l_{im}\}, m = 2,3,...,t$
Step 3:	If there is a minimum in the vertex and equating to the singular measure of *i* (i.e. $i = k$), then classify that node m as $[l_m, k]$
Step 4:	If the least value occurs in the vertex corresponding to more values from *i* then it can be determined that there are more triangular neutrosophic (TrNS) paths between SV (*i*) and DV (*m*) and select any value of *i*.
Step 5:	Classify the destination vertex (DV) (node *t*) by $[l_t, 1]$. Then the TrNS distance among SV l_t .
Step 6:	Find the triangular neutrosophic shortest path TrNSSP between primary and terminal vertex according to $[l_t, 1]$ and check the label of n_1 and is represented by $[l_d, a]$. Classify vertex *a* and so on. Repeat the process till get n_1.
Step 7:	By connecting all the vertices acquired by repeating the process in step 4, TrNSSP can be found **Note:** If $\mathbb{S}(N_i) < \mathbb{S}(N_p)$ then the TrNS number is the minimum of N_p , where $N_i, i = 1,2,...,t$ is the set of TrNS number and $\mathbb{S}$ is the de-neutrosophication function.
Step 8:	End

4.2. Extension of Multiple Labelling Principle

Extension of Multiple Labeling Principle aims in obtaining the SP from the source node *r* to the destinati -on *m* of a directed acyclic graph having neutrosophic arc length. The steps of the algorithm 2 are shown in Table 7.

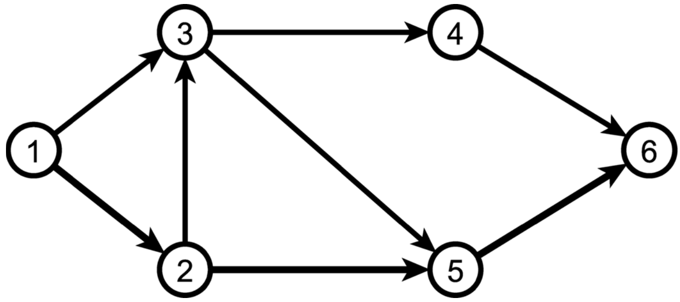
Consider the *Example 4.1*, and the solution approach is shown below:

Step 1: is shown in Table 8.

Execute Step 2 of algorithm 2 shown in Table 9.

After execute Step 3-4 of algorithm 2 to get the final result in Table 10.

Figure 1. A network consist 6 nodes and 8 edges
(2018; 2019)



Finally, the NSSP is 1→2→5→6 with the NSPL is < (2, 12, 24), (2.5, 6.5, 16), (12.5, 20.75, 40)>. And the crisp shortest path length is 14.625.

4.3. Extension of Bellmond Ford Algorithm

In this section, the algorithmic approach i.e., Extension of bellman dynamic programming is provided. Where n is the DV and 1 as a SV. a_{ij} denotes the neutrosophic distance between vertex *i* and *j*.

Algorithm 3: Extension of bellman dynamic programming

Consider a graph $G = (V, E)$ from SV '1' and the DV 'n' which is acyclic and they are organized by topological ordering $(E_{ij}; i < j)$. The Extension of Bellman programming is defined as follow:

$$k(i) = \begin{cases} 0 & , \quad i = 1 \\ \min\limits_{i<j}\left[k(i) + a_{ij}\right] & , \quad otherwise \end{cases}$$

Table 5. The conditions of Example 4.1 (2018)

T	H	TrNS Cost	T	H	TrNS Cost
1	2	<(1,3,4); (1,1,5);(1,2,6)>	3	4	<(1,2,3);(0.5,1.5,2.5);(1.2,2.7,3.5)>
1	3	<(1,5,8);(1.5,3,6.5);(4,7,9)>	3	5	<(1,4,7);(1,3,5);(3.5,6,7.5)>
2	3	<(2,4,6);(1.5,2.5,3.5);(3,5,7)>	4	6	<(10,15,20);(14,16,22); (12,15,19)>
2	5	<(0,4,11);(0,1,4.5);(7.5,11.75,24)>	5	6	<(1,5,9);(1.5,4.5,6.5);(4,7,10)>

*TrNS: Triangular neutrosophic

Box 1.

Iteration 1:	Since n_2 has only n_1 as the predecessor, let $i=1, m=2$ in step 2. To find l_2: $l_2 = \min\left\{l_1 \oplus l_{12}\right\}$ $= \min\left\{\left\langle[0,0,0],[0,0,0],[0,0,0]\right\rangle \oplus \left\langle[1,3,4],[1,1,5],[1,2,6]\right\rangle\right\}$ $= \min\left\{\left\langle[1,3,4],[1,1,5],[1,2,6]\right\rangle\right\}$ Since, minimum occurs for $i=1$, classify the node $n_2 = \left[\left\langle[1,3,4],[1,1,5],[1,2,6]\right\rangle,1\right]$ Since, minimum occurs for $i=1$, classify the node $n_2 = \left[\left\langle[1,3,4],[1,1,5],[1,2,6]\right\rangle,1\right]$
Iteration 2:	Since n_3 has two predecessors n_1 and n_2, let $i=1,2\ \&\ m=3$ in step 2. To find l_3: $l_3 = \min\left\{l_1 \oplus l_{13}, l_2 \oplus l_{23}\right\}$ $= \min\left\{\begin{array}{l}\left\langle[0,0,0],[0,0,0],[0,0,0]\right\rangle \oplus <[1,5,8];[1.5,3,6.5];[4,7,9]>,\\ \left\langle[1,3,4],[1,1,5],[1,2,6]\right\rangle \oplus <(2,4,6);(1.5,2.5,3.5);(3,5,7)>\end{array}\right\}$ $= \min\left\{<[1,5,8];[1.5,3,6.5];[4,7,9]>, \left\langle[3,7,10],[2.5,3.5,8.5],[4,7,13]\right\rangle\right\}$ $=<[1,5,8];[1.5,3,6.5];[4,7,9]>$ Since the de-neutrosophic function values are, $\mathbb{S}\left(<[1,5,8];[1.5,3,6.5];[4,7,9]>\right)$ $= \frac{1}{12}\left[\left(1+2\bullet 5+8+1.5+2\bullet 3+6.5+4+2\bullet 7+9\right)\right] = 5$ $\mathbb{S}\left(\left\langle[3,7,10],[2.5,3.5,8.5],[4,7,13]\right\rangle\right)$ $= \frac{1}{12}\left[\left(3+2\bullet 7+10+2.5+2\bullet 3.5+685+4+2\bullet 7+13\right)\right] = 6.3333$ and the minimum occurs for $i=1$, then classify the node $n_3 = \left[<[1,5,8];[1.5,3,6.5];[4,7,9]>,1\right]$.

Box continued on following page

Box 1. continued

<table>
<tr><td>Iteration 3:</td><td>Since n_4 has one predecessors n_3, let $i = 3 \;\&\; m = 4$ in step 2.
To find the value of l_4:
$l_4 = \min\left\{l_3 \oplus l_{34}\right\}$
$= \min\left\{< [1,5,8];[1.5,3,6.5];[4,7,9] > \oplus < [1,2,3];[0.5,1.5,2.5];[1.2,2.7,3.5] >\right\}$
$= \min\left\{< [2,7,11];[2,4.5,9];[5.2,9.7,12.5] >\right\}$
$= < [2,7,11];[2,4.5,9];[5.2,9.7,12.5] >$
Since, minimum occurs for $i = 3$, hence classify the node
$n_4 = \left[< [2,7,11];[2,4.5,9];[5.2,9.7,12.5] >, 3\right]$.</td></tr>
<tr><td>Iteration 4:</td><td>Since n_5 has two predecessors n_2 and n_3, let $i = 2,3 \;\&\; m = 5$ in step 2.
To find the value of l_5:
$l_5 = \min\left\{l_2 \oplus l_{25}, l_3 \oplus l_{35}\right\}$
$= \min\left\{\begin{matrix} < [1,3,4];[1,1,5];[1,2,6] > \oplus \langle < [0,4,11];[0,1,4.5];[7.5,11.75,24] > \rangle, \\ < [1,5,8];[1.5,3,6.5];[4,7,9] > \oplus < [1,4,7];[1,3,5];[3.5,6,7.5] > \end{matrix}\right\}$
$= \min\left\{< [1,7,15];[1,2,9.5];[8.5,13,30] >, < [2,9,15];[2.5,6,11.5];[7.5,13,16.5] >\right\}$
$= < [1,7,15];[1,2,9.5];[8.5,13,30] >$
Since the de-neutrosophication function values are,
$\mathbb{S}\left(< [1,7,15];[1,2,9.5];[8.5,13,30] >\right) = 9.083$
$\mathbb{S}\left(< [2,9,15];[2.5,6,11.5];[7.5,13,16.5] >\right) = 9.25$
and the minimum occurs for $i = 2$, hence classify the node
$n_5 = \left[< [1,7,15];[1,2,9.5];[8.5,13,30] >, 2\right]$</td></tr>
</table>

Box continued on following page

Table 6. Execute Method 1, to get

Vertex (*m*)	l_i	**TrNSP Between m^th and Vertex 1**
2	<[1,3,4],[1,1,5],[1,2,6]>	1→2
3	<[1,5,8],[1.5,3,6.5],[4,7,9]>	1→3
4	<[2,7,11],[2,4.5,9],[5.2,9.7,12.5]>	1→3→4
5	<[1,7,15],[1,2,9.5],[8.5,13.75,30]>	1→2→5
6	<[2,12,24],[25,65,16],[12.5,20.75,40]>	1→2→5→6

Box 1. Continued

Iteration 5:	Since n_6 has two predecessors n_4 and n_5, let $i = 4,5 \ \& \ m = 6$ in step 2. To find the value of l_6: $l_6 = \min\left\{l_4 \oplus l_{46}, l_5 \oplus l_{56}\right\}$ $= \min\left\{\begin{matrix} <[2,7,11];[2,4.5,9];[5.2,9.7,12.5]> \oplus <[10,15,20];[14,16,22];[12,15,19]>, \\ <[1,7,15];[1,2,9.5];[8.5,13,30]> \oplus <[1,5,9];[1.5,4.5,6.5];[4,7,10]> \end{matrix}\right\}$ $= \min\left\{\begin{matrix} <[12,22,31];[16,20.5,31];[17.2,24.7,31.5]>, \quad <[2,12,24];[2.5,6.5,16] \\ ;[12.5,20,40]> \end{matrix}\right\}$ $=<[2,12,24];[2.5,6.5,16];[12.5,20,40]>$ and the minimum occurs for $i = 5$ hence classify $n_6 = [<[2,12,24];[2.5,6.5,16];[12.5,20,40]>,5]$. Since n_6 is the DN of the given network, NSP between $n_1$and n_6 is $=<[2,12,24];[2.5,6.5,16];[12.5,20,40]>$
Iteration 6:	Now, TrNSP from $n_1$and n_6 is obtained as follows. Since, $n_6 = [<[2,12,24];[2.5,6.5,16];[12.5,20,40]>,5]$ a person is coming from $5 \to 6$ $n_5 = [<[1,7,15];[1,2,9.5];[8.5,13,30]>,2]$ a person is coming from 2→5 $n_2 = [\langle[1,3,4],[1,1,5],[1,2,6]\rangle,1]$ a person is coming from 1→2 By joining all the acquired nodes, TrNSSP from n_1 and n_6 is obtained. Hence TrNSSP of the given network is 1→2→5→6 The neutrosophic shortest path length is $<[2,12,24];[2.5,6.5,16];[12.5,20,40]>$ With predicted crisp shortest path length is 14.625
End	

$k(i)$ is the neutrosophic length of SP node i from the SV 1 (Box 2).

Solution approach is shown in Table 11.

Table 7. Algorithm 2: Extension of multiple labeling principles

Step 1:	Let *p* is the total number of routes from *r* to *m*. Compute the De-neutrosophication of each neutrosophic arc distance under the specified network using the Definition 3.3.4.
Step 2:	Find all possible routes H_i, and also find the neutrosophic distance of corresponding H_i, where $i = 1, 2, 3, \ldots\ldots, p$,for *p* possible number of neutrosophic routes. Now, each of *p* routes can be considered as an arc from *r* to *m*.
Step 3:	Calculate the summation of the De-neutrosophication value of each neutrosophic distance corresponding to the path H_i, and set that, $F(\psi_i)$ where $i = 1, 2, 3, \ldots\ldots, p$.
Step 4:	By sorting the de-neutrosophic value obtained in Step 3 in ascending order, find the lowest rank i.e. NSP of the given network.
Step 5:	End

Table 8. De-neutrosophication of each arc length

Neutrosophic Arc Distance	De-Neutrosophication of Each Neutrosophic Arc Distance
1-2	2.5
1-3	5
2-3	3.833
2-5	6.7083
3-4	2.0083
3-5	4.25
4-6	15.75
5-6	5.4167

Table 9. Path length of corresponding neutrosophic arc distance

Possible Route	$F(\psi_i)$
H_1: 1→2→5→6	<[2,12,24],[2.5,6.5,16],[12.5,20.75,40]>
H_2: 1→3→5→6	<[3,14,24],[4,10.5,18],[11.5,20,26.5]>
H_3: 1→2→3→5→6	<[5,16,26],[5,11,20],[11.5,20,30.5]>
H_4: 1→3→4→6	<[12,22,31],[16,20.5,31],[17.2,24.7,31.5]>
H_5: 1→2→3→4→6	<[14,24,33],[17,21,33],[17.2,24.7,35.5]>

Table 10. Final solution

Possible Route	$F(\psi_i)$	Sorting
H_1: 1→2→5→6	14.625	1
H_2: 1→3→5→6	14.666	2
H_3: 1→2→3→5→6	16	3
H_4: 1→3→4→6	22.7583	4
H_5: 1→2→3→4→6	24.09167	5

Table 11. Using the algorithm 3, the NSSP obtained as follows

Step 1:	$k(1) = 0$
Step 2:	$k(2) = min_{i<2}\{k(1) + a_{12}\} = a_{12}^{\ *} = 2.5$
Step 3:	$k(3) = min_{i<3}\{k(i) + a_{i3}\} = min\{k(1) + a_{13}, k(2) + a_{23}\}$ $= \{0 + 5, 2.5 + 3.833\} = \{5, 6.333\} = 5$
Step 4:	$k(4) = min_{i<4}\{k(i) + a_{i4}\} = min\{k(3) + a_{34}\}$ $= \{5 + 2.0083\} = 7.0083$
Step 5:	$k(5) = min_{i<5}\{k(i) + a_{i5}\} = min\{k(2) + a_{25}, k(3) + a_{35}\}$ $= \{2.5 + 6.7083, 5 + 4.25\} = \{9.2083, 9.25\} = 9.2083$
Step 6:	$k(6) = min_{i<6}\{k(i) + a_{i6}\} = min\{k(4) + a_{46}, k(5) + a_{56}\}$ $= \{7.0083 + 15.75, 9.2083 + 5.4167\} = \{22.7583, 14.625\} = 14.625$ thus, $k(6) = k(5) + a_{56} = k(2) + a_{25} + a_{56} = k(1) + a_{12} + a_{25} + a_{56} = a_{12} + a_{25} + a_{56}$
Step 7:	Therefore, the shortest route 1→2→5→6 is known as the NSSP, and the crisp shortest path is 14.625.
Step 8:	End

Box 2.

Step 1:	*nrank[s] ← 0*
Step 2:	*ndist[s] ← Empty* neutrosophic number.
Step 3:	Add *s* into *Q*
Step 4:	**For** every node *i* (excluding the *s*) in the neutrosophic graph *G*
Step 5:	*rank[i] ← ∞*
Step 6:	*Add i*into Q
Step 7:	**End For**
Step 8:	*u←s*
Step 9:	**While**(*Q* is not empty)
Step 10:	eliminate the vertex *u* from *Q*
Step 11:	**For** each adjacent vertex *v* of vertex *u*
Step 12:	relaxed←*False*
Step 13:	*temp_ndist[v] ← ndist[u]⨁ edge_weight(u,v) ⨁ represents the addition of* neutrosophic
Step 14:	temp_nrank[*v*] ←rank_of_neutrosophic(*temp_ndist[v]*)
Step 15:	**If** temp_nrank[*v*]<nrank[v] **then**
Step 16:	*ndist[v] ←temp_ndist[v]*
Step 17:	*nrank[v] ← temp_nrank[v]*
Step 18:	*prev[v] ←u*
Step 19:	**End If**
Step 20:	**End For**
Step 21:	**If** relaxed equals False **then**
Step 22:	exit the loop
Step 23:	**End If**
Step 24:	*u* ←Node in *Q* with minimum rank value
Step 25:	**End While**
Step 26:	**For** each arc***(u,v)*** in neutrosophic graph *G* do
Step 27:	**If** *nrank[v]>rank_of_neutrosophic(ndist[u]⨁ edge_weight(u,v))*
Step 28:	return false
Step 29:	**End If**
Step 30:	**End For**
Step 31:	The neutrosophic number *ndist[u]* is a neutrosophic number and its represents the SP from SV *s* and DV *u*.
End	

CONCLUSION

This chapter includes three different methods to solve the neutrosophic shortest path length. Method 1 extended the single valued triangular neutrosophic modified dijkstra's algorithm, method 2 is the extended version of multi labeling method and method 3 is the extended the single valued neutrosophic bellmond ford algorithm. A numerical example has been presented to illustrate the efficiency of the proposed method. These algorithms are not only suggested the NSP but also able to predict the neutrosophic path length and crisp path length. In the future, these proposed methods can be applied to real-world problems in the field of MCFP, job scheduling, transportation, and so on.

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Chapter 7
Transportation Problem in Neutrosophic Environment

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ABSTRACT

The transportation problem (TP) is popular in operation research due to its versatile applications in real life. Uncertainty exists in most of the real-life problems, which cause it laborious to find the cost (supply/demand) exactly. The fuzzy set is the well-known field for handling the uncertainty but has some limitations. For that reason, in this chapter introduces another set of values called neutrosophic set. It is a generalization of crisp sets, fuzzy set, and intuitionistic fuzzy set, which is handle the uncertain, unpredictable, and insufficient information in real-life problem. Here consider some neutrosophic sets of values for supply, demand, and cell cost. In this chapter, extension of linear programming principle, extension of north west principle, extension of Vogel's approximation method (VAM) principle, and extended principle of MODI method are used for solving the TP with neutrosophic environment called neutrosophic transportation problem (NTP), and these methods are compared using neutrosophic sets of value as well as a combination of neutrosophic and crisp value for analyzing the every real-life uncertain situation.

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1. INTRODUCTION

Uncertainties exist almost in real life problems, which cannot be solved by classical mathematics. Fuzzy set is one of the most popular and well known methods for handling the uncertainty, which is generalization of crisp set. Prof. Zadeh (1965) first introduced the fuzzy set in 1965, which is representing the membership values, but many real life scenarios result and decision are not enough to the level of accuracy. Fuzzy set and it extended version is used in shortest path problem [Dey, Pal, & Pal, 2016; Kumar et al., 2017; 2018; 2019; 2019a; 2019b; 2020; Dey et al., 2018; Broumi et al., 2019c], Spanning tree problem (Dey et al., 2018; Broumi et al., 2018), graph problem (Dey et al., 2015; Dey et al., 2018; Broumi et al., 2017; 2017a) and so on.

Later, Atanassov (1986; 1999) introduced intuitionistic fuzzy set to recover the limitation of accuracy to solve the problem with imprecision information and characterized by its membership and non-membership values (Atanassov 1999). Hence both fuzzy set and intuitionistic fuzzy set are used for handling the real life uncertainty but it have a limitation for solve the problem with indeterminate or inconsistence information.

To overcome this type of limitation Smarandache (1998) introduced neutrosophic set which is extension of classical set, fuzzy set and intuitionistic fuzzy sets. Neutrosophic set is basically used to represent the truth-membership degree, falsity-membership degree and indeterminacy-membership degree of an object and which is considered as an appropriate approach for representing the indeterminacy and inconsistent information. Wang et al. (2010) have introduced the idea of single valued neutrosophic set and it is more effective for solving the real life problem.

Many researchers have worked on fuzzy transportation problem. Some researchers (Guo et al., 2015; Yu et al., 2015; kumar et al. 2019d) have introduced some algorithmic approaches to find the transportation cost in uncertain environment. In 1941 Hitchcock (1941) has introduced the classical transportation problem. In real life, cost, supply and demand of a transportation problem are uncertain due to several reasons.

In 1978 Zimmerman (1978) has introduced linear programming model in fuzzy environment. It is used to solve TP with fuzzy. Chanas et al. (1984) have proposed fuzzy linear programming model with fuzzy supply and demand but transportation cost is a crisp value. Dinagar & Palanivel (2009) have described the TP with fuzzy supply, demand and Transportation cost. Kaur and Kumar (2012) have introduced an algorithm approach for solving the TP with trapezoidal fuzzy number.

Zadeh (1975) has introduced type 2 Fuzzy set (T2FS) which is extension of type 1 fuzzy set (T1FS). After 10 years, Zadeh has proposed T2FS whose membership function is a mapping from U to [0, 1]. The linguistic description of information does not represent using T1FSs due to human perception in membership degree. T2FS recover this limitation with higher degree. Mendel et al. (2001; 2002) enhanced the number of degree of fuzzy set. Liu (Liu, 2004) has introduced the idea of rough variable to handle the uncertainty. Jiménez & Verdegay (1999) has solved the fuzzy solid TP by an evolutionary algorithm based parametric approach. Yang & Liu, 2007 have proposed fuzzy fixed charge solid TP and algorithm. Ojha et al. (2009) have introduced solid transportation problem with entropy based for general fuzzy costs and time with fuzzy equality. Kundu et al. (2013) have introduced a solid transportation models with crisp and rough costs. Again Kundu et al. (2014) have utilized the multi-objective for transportation problems with budget constraint in uncertain environment. Sometimes fuzzy membership function was not suitable for the real life transportation problem. So, Atanassov (1986; 1999) has introduced other type of concept of fuzzy set called intuitionistic fuzzy set, which is degree of both membership and non membership of each element in the set. Now a day's many researchers (Gani & Abbas, 2012; Hussain & Kumar, 2012; Ebrahimnejad, 2016) used this type of institutional fuzzy set for solving the transportation problem which is more effective to handling various type of decision making. But very important thing is that this type of fuzzy set can only deal the incomplete information but cannot manage the indeterminate and inconsistent information for recovering this type of limitation considers neutrosophic set of values.

1.1 Motivation of This Work

Since the real life problems are in generally uncertain and always challenging. Decision makers/experts often face difficulty in taking proper decisions in the presence of uncertainty. But decisions can't be deferred indefinitely due to its impact in the process.

In the field of uncertainty, the real life problems are full of ambiguous and incomplete information, use of neutrosophic sets are becoming more significant, due to its specific potentiality to deal with uncertainty. In reality, human thinking and reasoning also generally involve uncertain information creating from inherently inexact human nature. This motivated us to consider some innovative methodologies for decision making in uncertain environment.

- In general mathematics TP solved by only crisp value.
- Some researcher solves this TP in fuzzy environment and uses others various type of methodology.
- Each and every case has some limitation for handling the real life uncertain situation.

Hence, in this chapter introduce neutrosophic set of values with various type of methodology with various combinations of set of values for proper handling the TP in real life situation.

1.2 Objectives of This Work

The only certain thing in today's information intense age is uncertainty. In several real life problems, the accessible data are not always exact due to various reasons such as insufficient information, lack of evidence, imperfect statistical analysis, etc. The imprecision of available data results in loss of information which in turn produces uncertainty in its description. This loss of information may also occur due to simplification of complex problems. Normally, the uncertainty in decision making problems may appear in the decision parameters which define the problem. Moreover, the decision situation in which the problem occurs also may be uncertain. In order to deal with the real life uncertain problems, a number of theories have been developed such as fuzzy set, vague set, rough set, soft set, etc. These theories and their extensions have been effectively applied by many researchers in the context of uncertain environment. All methodologies to deal with such incorrectness, uncertainty, partial truth, and approximation to achieve feasibility, robustness and least solution cost.

In veiw of above gaps, actual aim in this research is to provide few evolutionary methods which are generally describe through uncertainty theory. Some of the theoritical analysis like invalidity analysis of current methods have been taken into consideration. As such, objective related to the present work are included.

Following are unit the most objectives planned and accomplished during this analysis work.

- To study numerical solution and its application of TP which are generally described through optimization theory.
- To provide few evolutionary methods for obtaining TP under uncertainty environments like neutrosophic environment.

- To predict the cell cost supply and demand value completely different uncertainty situation.
- To analyze the results obtained by this suggested methods with current methods.

2. DESCRIPTION OF THE WORK

2.1 Research Problem

Vagueness exists in most of the real world problem. Uncertainty is inseparable from computation. Always measurement with instruments provides unavoidable errors. So handling of Uncertainty with real life world is very essential for human beings. Most of the cases in general mathematical model use crisp set for computing and reasoning. This is the measure limitation of Human beings for facing real world situation. Every problem related three types of characteristics uncertainty, complexity and credibility. Among from three characteristics uncertainty has major role for maintain the efficiency label of a problem. Uncertainty of a problem depends upon to two different parameter randomness and vagueness or incomplete information. Randomness basically indicates the probabilistic uncertainties, which are properly handled by the mathematical methods probability and statistics. But this type of method cannot handle the vagueness or incomplete information. In real life use some word say small, large, attractive, cloudy etc. for communication with natural language but these words cannot provide exact value.

In the TP from a specified source node to a destination node is a well known combinatorial optimization problem. The only certain thing in today's information intense age is uncertainty. Uncertainty exists in most of the real life application of TP. The uncertainty theory is very popular tools to represent and handle uncertainty in information due to imprecise, incomplete, inconsistent, and indeterminate. Moreover the literature review reveals that there varied gaps within the study of TP. There are many gaps when compared to existing methods available and very few studies on problems related to TP arising in different real world applications. The seeking of solution of uncertain TP is still a significant task that needs to focus on the development of the methods & on the modification (gap) of the current methods. Therefore, the framework of this research work is based on TP i.e., to consider effective uncertain analysis to solve transportation problem.

2.2 Solution Methodologies

Solution methodologies for finding TP for linear programming method are mainly based on the choice of suitable environments. The two basic ideas are mentioned below which are discussed in following section:

1. Basic idea of crisp transportation linear programming method.
2. Basic idea of neutrosophic TP with linear programming method.

2.3 Mathematical Formulation for Crisp Transportation Problem

In this section, the notation and existing linear model in crisp TP s are studied.

Notations

c_{ij} : Cost of transportation
a_i : Supply in quantity
b_i : Demand in quantity
x_{ij} : Transported quantity

Here m numbers of sources and n destination Let a_i is the neutrosophic number of supply units available at source *i* (*i* = 1,2,3,...,s) and let b_i is the neutrosophic number of demand units required at destination *j* (*j* = 1,2,3,...,d).

Mathematical model is given bellow

$$Min\, Z = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}.x_{ij} \tag{3.1}$$

subject to:

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,.....,s \\ &\sum_{i=1}^{m} x_{ij} = b_i, j = 1,2,.....,d \\ &x_{ij} \in \Re\, and\, are\, non\text{-}negative. \end{aligned} \tag{3.2}$$

2.4 Mathematical Formulation of NTP

Type 1 NTP: If parameter c_{ij} is replaced into neutrosophic cost parameters, *i.e.* c_{ij}^N, then the model is as follows:

$$Min\, Z = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}^N . x_{ij} \tag{3.3}$$

Subject to constraints (2.2)

Type 2 NTP: If parameter a_{ij}, b_{ij}, c_{ij} is replaced into neutrosophic parameters, *i.e.* $a_{ij}^N, b_{ij}^N, c_{ij}^N$, then the model is as follows:

$$Min\, Z = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij} . x_{ij} \tag{3.4}$$

Subject to constraints

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_{ij}^N, i = 1,2,.....,s \\ &\sum_{i=1}^{m} x_{ij} = b_{ij}^N, j = 1,2,.....,d \\ &x_{ij} \in \Re\, and\, are\, non\text{-}negative. \end{aligned} \tag{3.5}$$

Type 3 NTP: If parameter a_{ij}, b_{ij} is replaced into neutrosophic parameters, *i.e.* a_{ij}^N, b_{ij}^N, then the model is as follows:

$$Min\, Z = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}^N . x_{ij} \tag{3.6}$$

Subject to constraints

$$\sum_{j=1}^{n} x_{ij} = a_{ij}^{N}, i = 1,2,\ldots\ldots, s$$
$$\sum_{i=1}^{m} x_{ij} = b_{ij}^{N}, j = 1,2,\ldots\ldots, d \tag{3.7}$$
$$x_{ij} \in \Re \; and\; are\; non\text{-}negative.$$

3. PROPOSE SOME METHODS TO SOLVE TP UNDER NEUTROSOPHIC ENVIRONMENT

The present aims to investigate the predicted numerical solution of the TP under un-certain, inconsistent data. Therefore three new method is proposed in this chapter to handle the problems mentioned in chapter title.

- Method 1: based on extension of north west principle
- Method 2: based on extension of VAM principle
- Method 3: based on extension of Modi principle
- Method 4: based on extension linear programing principle

3.1 Prelimaniries

Definition 3.1.1: Wang et al (2005): Let X be a space point or objects, with a genetic element in X denoted by *x*. A single-valued NS, V in X is characterised by three independent parts, namely truth-MF T_V, indeterminacy-MF I_V and falsity -MF F_V, such that $T_V : X \to [0,1], I_V : X \to [0,1],\; and\; F_V : X \to [0,1]$. Now, V is denoted as $V = \left\{ < x, \left(T_V(x), I_V(x), F_V(x)\right) > \;\; | \, x \in X \right\}$, satisfying $0 \le T_V(x) + I_V(x) + F_V(x) \le 3.$

Definition 3.1.2: Chakraborty et al (2018): Let $\hat{r}^N = \left\langle \left(\overparen{r_{ij,l}}, \overparen{r_{ij,m}}, \overparen{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ is a special NS on the real number set R, whose truth-MF $\Delta_{\tilde{r}}(x)$, indeterminacy-MF $\nabla_{\tilde{r}}(x)$, and falsity-MF $\mho_{\tilde{r}}(x)$ are given as follows:

$$\Delta_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(x-\overset{\frown}{r}_{ij,l}\right)}{\left(\overset{\frown}{r}_{ij,m}-\overset{\frown}{r}_{ij,l}\right)} & \overset{\frown}{r}_{ij,l}\leq x<\overset{\frown}{r}_{ij,m},\\ 1 & x=\overset{\frown}{r}_{ij,m},\\ \dfrac{\left(\overset{\frown}{r}_{ij,k}-x\right)}{\left(\overset{\frown}{r}_{ij,k}-\overset{\frown}{r}_{ij,m}\right)} & \overset{\frown}{r}_{ij,m}<x\leq\overset{\frown}{r}_{ij,k},\\ 0 & otherwise.\end{cases} \tag{1}$$

$$\nabla_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(r_{ij,m}-x\right)}{\left(r_{ij,m}-r_{ij,l}\right)} & r_{ij,l}\leq x<r_{ij,m},\\ 0 & x=r_{ij,m},\\ \dfrac{\left(x-r_{ij,m}\right)}{\left(r_{ij,k}-r_{ij,m}\right)} & r_{ij,m}<x\leq r_{ij,k},\\ 1 & otherwise.\end{cases} \tag{2}$$

$$\mho_{\tilde{r}}(x)=\begin{cases}\dfrac{\left(\underline{r_{ij,m}}-x\right)}{\left(\underline{r_{ij,m}}-\underline{r_{ij,l}}\right)} & \underline{r_{ij,l}}\leq x<\underline{r_{ij,m}},\\ 0 & x=\underline{r_{ij,m}},\\ \dfrac{\left(x-\underline{r_{ij,m}}\right)}{\left(\underline{r_{ij,k}}-\underline{r_{ij,m}}\right)} & \underline{r_{ij,m}}<x\leq\underline{r_{ij,k}},\\ 1 & otherwise.\end{cases} \tag{3}$$

Where

$$0\leq\Delta_{\tilde{r}}(x)+\nabla_{\tilde{r}}(x)+\mho_{\tilde{r}}(x)\leq 3, x\in\hat{r}^{N}$$

The parametric form is defined as follows

$$\left(\hat{r}^N\right)_{\delta,\nu,\eta} = \left[\Delta_{\hat{r}^{N1}}\left(\delta\right), \Delta_{\hat{r}^{N2}}\left(\delta\right); \nabla_{\hat{r}^{N1}}\left(\nu\right), \nabla_{\hat{r}^{N2}}\left(\nu\right); \mho_{\hat{r}^{N1}}\left(\eta\right), \mho_{\hat{r}^{N2}}\left(\eta\right)\right],$$

Where $\delta, \nu, \eta \in (0,1)$

$$\Delta_{\hat{r}^{N1}}\left(\delta\right) = \overparen{r_{ij,l}} + \delta\left(\overparen{r_{ij,m}} - \overparen{r_{ij,l}},\right)$$
$$\Delta_{\hat{r}^{N2}}\left(\delta\right) = \overparen{r_{ij,k}} - \delta\left(,\overparen{r_{ij,k}} - \overparen{r_{ij,m}}\right)$$
$$\nabla_{\hat{r}^{N1}}\left(\nu\right) = r_{ij,m} - \nu\left(r_{ij,m} - r_{ij,l}\right)$$
$$\nabla_{\hat{r}^{N2}}\left(\nu\right) = r_{ij,m} + \nu\left(r_{ij,k} - r_{ij,m}\right)$$
$$\mho_{\hat{r}^{N1}}\left(\eta\right) = \underline{r_{ij,m}} - \eta\left(\underline{r_{ij,m}} - \underline{r_{ij,l}}\right)$$
$$\mho_{\hat{r}^{N2}}\left(\eta\right) = \underline{r_{ij,m}} + \eta\left(\underline{r_{ij,m}} - \underline{r_{ij,k}}\right)$$

Definition 3.1.3:(Abdel-Basset, Gunasekaran, Mohamed, & Smarandache, 2018; Mohamed, Abdel-Basset, Zaied, & Smarandache, 2017)**:** Arithmetic operation: Let $\hat{r}^N = \left\langle\left(\overparen{r_{ij,l}}, \overparen{r_{ij,m}}, \overparen{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right)\right\rangle$ and $\hat{s}^N = \left\langle\left(\overparen{s_{ij,l}}, \overparen{s_{ij,m}}, \overparen{s_{ij,k}}\right), \left(s_{ij,l}, s_{ij,m}, s_{ij,k}\right), \left(\underline{s_{ij,l}}, \underline{s_{ij,m}}, \underline{s_{ij,k}}\right)\right\rangle$ be two arbitrary SVTNNs, and $\theta \geq 0$; then:

$$\hat{r}^N \oplus \hat{s}^N = \left\langle\left(\overparen{r_{ij,l}} + \overparen{s_{ij,l}}, \overparen{r_{ij,m}} + \overparen{s_{ij,m}}, \overparen{r_{ij,k}} + \overparen{s_{ij,k}}\right), \left(r_{ij,l} + s_{ij,l}, r_{ij,m} + s_{ij,m}, r_{ij,k} + s_{ij,k}\right), \left(\underline{r_{ij,l}} + \underline{s_{ij,l}}, \underline{r_{ij,m}} + \underline{s_{ij,m}}, \underline{r_{ij,k}} + \underline{s_{ij,k}}\right)\right\rangle$$

$$\hat{r}^N \otimes \hat{s}^N = \left\langle\left(\overparen{r_{ij,l}} \cdot \overparen{s_{ij,l}}, \overparen{r_{ij,m}} \cdot \overparen{s_{ij,m}}, \overparen{r_{ij,k}} \cdot \overparen{s_{ij,k}}\right), \left(r_{ij,l} \cdot s_{ij,l}, r_{ij,m} \cdot s_{ij,m}, r_{ij,k} \cdot s_{ij,k}\right), \left(\underline{r_{ij,l}} \cdot \underline{s_{ij,l}}, \underline{r_{ij,m}} \cdot \underline{s_{ij,m}}, \underline{r_{ij,k}} \cdot \underline{s_{ij,k}}\right)\right\rangle$$

$$\theta\hat{r}^N = \left\langle\left(\theta\overparen{r_{ij,l}}, \theta\overparen{r_{ij,m}}, \theta\overparen{r_{ij,k}}\right), \left(\theta r_{ij,l}, \theta r_{ij,m}, \theta r_{ij,k}\right), \left(\theta\underline{r_{ij,l}}, \theta\underline{r_{ij,m}}, \theta\underline{r_{ij,k}}\right)\right\rangle \;\; if \;\; (\theta > 0)$$

Definition 3.1.4: Chakraborty et al(2018): Let $\hat{r}^N = \left\langle \left(\overset{\frown}{r_{ij,l}}, \overset{\frown}{r_{ij,m}}, \overset{\frown}{r_{ij,k}}\right), \left(r_{ij,l}, r_{ij,m}, r_{ij,k}\right), \left(\underline{r_{ij,l}}, \underline{r_{ij,m}}, \underline{r_{ij,k}}\right) \right\rangle$ then the value of De-neutrosophication of neutrosophic number is as follows:

$$s\left(\tilde{r}\right) = \frac{1}{12}\left[\left(\overset{\frown}{r_{ij,l}} + 2\cdot\overset{\frown}{r_{ij,m}} + \overset{\frown}{r_{ij,k}}\right) + \left(r_{ij,l} + 2\cdot r_{ij,m} + r_{ij,k}\right) + \left(\underline{r_{ij,l}} + 2\cdot\underline{r_{ij,m}} + \underline{r_{ij,k}}\right)\right]$$

3.2 Method 1: Based on Extension of North West Principle

It is a simple and efficient method for finding an initial solution of a TP. This method does not take any estimate the cost of transportation on any route of transportation.

This method uses for compute a basic feasible solution (BFS) of a **TP**. These basic variables are selected from the North West corner (i.e., top left corner). In most cases, researches use crisp sets, fuzzy set and intuitionistic fuzzy set these are unable to handle the uncertainty due to inexactness of human perception. Neutrosophic set is an extension of these sets which able to handle this uncertainty. Here introduce an algorithm to solve the TP based on **North West corner** method in neutrosophic environment.

Algorithm 1. Based on extension of north west principle

Step 1:	Select the top left-hand corner neutrosophic cell of the NTP and allocate the least amount between available supply and demand.
Step 2:	Accommodate the supply and demand numbers with the respective rows and columns.
Step 3:	Move horizontally, if demand is satisfied with first cell.
Step 4:	Move down, if supply for the first row is exhausted.
Step 5:	The next allocation will be made, if supply equals to demand in the cell.
Step 6:	Process will be continuing until all supply and demand values are exhausted.
Step 7:	End

3.3 Method 2: Based on Extension of VAM Principle

Vogel's Approximation method more efficiency than the North West corner method use for finding BFS of a TP. For that type reason in this chapter introduce this extension VAM method using neutrosophic set of values.

Algorithm 2. Based on extension of VAM principle

Step 1:	Consider the cell cost, supply and demand value as a neutrosophic set of values or combination of neutrosophic set of values and crisp values.
Step 2:	First, find out the penalty cost, the difference between least cost and next lest cost in each row and column.
Step 3:	Select the maximum penalty value from all row difference and column difference and select corresponding row or column as found in step 2.
Step 4:	Identify the cell for allotment which has lowest cost in that row or column as by step 3.
Step 5:	Allocate the least value in this allocated cell by comparing corresponding supply and demand as found in step 4.
Step 6:	Which one is least supply or demand, this row or column will be eliminated respectively.
Step 7:	Same procedure step 2 to step 6 will be applicable in rest of the unallocated cell, until all supply and demand will be adjusted.

3.4 Method 3: Optimality Test Using Extended Principle of MODI Method

This method mainly use for optimum solution of a TP. After finding the BFS of a problem can be test the optimality, whereas (s + d - 1) is exactly equal to number of non-negative unoccupied cell. Where s is the number of row and d is the number of column. These allocated cell allocation must be in independent position. This method is use for better solution of a problem.

Algorithm 3. Based on extension of MODI principle

Step 1:	Find out the sources S_i and destination O_j for each row and column respectively satisfying $u_i + v_j = c_{ij}$ for each occupied cell.
Step 2:	Select 0 along with any row or column which contain maximum number of allocation and calculate u_i and v_j.
Step 3:	The value of each unoccupied cell with the expression $x_{ij} = c_{ij} - (u_i + v_j)$. Case 1: If all $x_{ij} > 0$ then the solution is optimum and a unique solution is exist. Case 2: all $x_{ij} >= 0$ then the required solution is optimum but an alternative solution is exists. Case 3: If all $x_{ij} < 0$ then the solution does not optimum solution. For that type cases considering the next step, for better transportation cost.
Step 4:	If Case 3 is arise then select the most negative value of x_{ij} and form a closed path with occupied cell. Allocate the sign + and - alternately and then find the minimum allocation from the cell which is negative sign. This allocation added to the allocation which holding positive sign and subtracted from the allocation which having negative sign.
Step 5**:**	The above step yield a best solution by creation one or more occupied cell as empty and one empty cell as occupied. The new set of BFS allocations repeat from the step 1 until an optimum BFS is achieved.
Step 6:	End

3.5 Method 4: Based on Extension of Linear Programming Principle

Algorithm 4. Extension of Linear Programming Principle

Step 1:	Write the given transportation problem into linear programming model using equation.
Step 2:	Using Definition of De-neutrification principle write the standard linear model such as $Min\,Z = \sum_{i=1}^{m}\sum_{j=1}^{m}\left(c_{ij}^{N}\right)^{D}.x_{ij}$ (4.1) Subject to constraints $\sum_{j=1}^{n} x_{ij} = \left(a_{ij}^{N}\right)^{D}, i = 1, 2, ..., m$ $\sum_{i=1}^{m} x_{ij} = \left(b_{ij}^{N}\right)^{D}, j = 1, 2, ..., n$ (4.2) $x_{ij} \in \Re\, and\, are\, non\text{-}negative.$ Where, $\left(c_{ij}^{N}\right)^{D} \approx de\text{-}neutrification\, of\, cost,$ $\left(a_{ij}^{N}\right)^{D} \approx de\text{-}neutrification\, of\, supply,$ $\left(b_{ij}^{N}\right)^{D} \approx de\text{-}neutrification\, of\, demand.$
Step 3:	Using arithmetic operation (definition 4.1.3) on equation 4.1-4.2 and get a crisp standard model.
Step 4:	Solve the above model by using standard linear programming technique, then compute objective values and the optimum value of x_{ij}
Step 5:	**N**ow put all x_{ij} in equation 4.1 and compute the optimum cost.
Step 6:	End.

See Algorithm 4.

4. NUMERICAL ILLUSTRATION

Here use some examples for demonstrating extended north west corner method, extended VAM, extended MODI method and extended linear programming method to solve NTP with neutrosophic set of values, combination of neutrosophic and crisp values.

Example 4.1: Consider Type 1 Neutrosophic Transportation Problem

Example 4.1: In this problem, cost of transportation been considered as neutrosophic cost and supply and demand quantity in crisp numbers which is shown in Table 1.

Table 1. Transportation table with cell cost neutrosophic set of values and supply, demand crisp value

	O_1	O_2	O_3	**Source**
S_1	(1,4,7;1,3,5;3.5,6,7.5)	(0.5,2.5,4.5;1,2,3;1.5,3.5,5.5)	(1,3,5;0.5,1.5,3.5;2,4,6)	15
S_2	(1,2,3;0.5,1.5,2.5;1.5,2.5,3.5)	(1,1.5,4;0.5,1,2.5;1.25,3,4.25)	(1.5,2.5,3.5;1,1.5,3;2,3,4)	25
S_3	(2,4,6;1.5,2.5,4.5;3,5,7)	(1,5,8;1.5,4.5,7.5;4,6.5,9)	(1,5,8;1.5,3,6.5;4,7,9)	20
Demand	18	20	22	

4.1.1 Solution Approach of Extension of North West Method

After de-neutrisofication on the above mentioned problem given in Table 1, then execute the below steps

Step 1: Select the upper left-hand corner cell c_{11} from Table 2 and allocate 15, which is mini mum between corresponding supply and demand.

Table 2.

		O_1	O_2	O_3	**Source**
S_1	15	4.25	2.67	2.92	15
S_2		2.00	1.71	2.75	25
S_3		3.92	5.25	5.08	20
Demand		18	20	22	

Step 2: Remove the row S_1 and again select the upper left-hand corner cell c_{21}. Allocate 3, which is mini mum between available supply and demand.

Table 3.

		$\mathbf{O_1}$	$\mathbf{O_2}$	$\mathbf{O_3}$	**Source**
S_2	3	2.00	1.71	2.75	25
S_3		3.92	5.25	5.08	20
Demand		3	20	22	

Step 3: Eliminate the column O_1 and again select the upper left-hand corner cell c_{22}. Allocate 20 which is minimum between corresponding supply and demand.

Table 4.

		O_2	O_3	**Source**
S_2	20	1.71	2.75	22
S_3		5.25	5.08	20
Demand		20	22	

Step 4: Eliminate the column O_2 and again select the upper left-hand corner cell c_{23}. Allocate 2 which are minimum between available supply and demand. Rest 20 allocates the cell c_{33} and all supply and demand values are adjusted.

Table 5.

		O_3	**Source**
S_2	2	2.75	2
S_3	20	5.08	20
Demand		22	

Step 5: See Table 6.

Table 6. It is required solution table for example 4.1, type 1 neutrosophic transportation problem based on extension of north west principle.

		O_1		O_2		O_3	**Source**
S_1	15	4.25		2.67		2.92	15
S_2	3	2.00	20	1.71	2	2.75	25
S_3		3.92		5.25	20	5.08	20
Demand		18		20		22	

Here number of positive allocations is equal to (s + d – 1) = 5 where s is the number of source, d is number of destination.

Therefore it has BFS and required solution in crisp value

$$= 4.25 \times 15 + 2.00 \times 3 + 1.71 \times 20 + 2.75 \times 2 + 5.08 \times 20 = 211.05$$

Required solution in neutrosophic value

$$= 15\times(1,4,7;1,3,5;3.5,6,7.5) + 3\times(1,2,3;0.5,1.5,2.5;1.5,2.5,3.5)$$

$$+ 20\times(1,1.5,4;0.5,1,2.5;1.25,3,4.25) + 2\times(1.5,2.5,3.5;1,1.5,3;2,3,4)$$

$$+ 20\times(1,5,8;1.5,3,6.5;4,7,9)$$

$$= (61,201,361;\ 58.5,132.5,268.5;\ 166,303.5,396)$$

END

4.1.2 Solution Approach of Extension of VAM

Step 1: For finding the penalty cost, calculate the difference between least cost and next lest cost in each row and column. Select the maximum penalty 1.92 in column O_1 and allocate 18 in minimum cell cost field c_{21}

Table 7.

		O_1	O_2	O_3	**Source**	**Column Difference**
S_1		4.25	2.67	2.92	15	0.25
S_2	18	2.00	1.71	2.75	25	0.29
S_3		3.92	5.25	5.08	20	1.16
Demand		18	20	22		
Row Difference		1.92	0.96	0.17		

Step 2: Eliminate column O_1 and continue same as before select cell c_{21} with allocation 7

Table 8.

		O_2	O_3	**Source**	**Column Difference**
S_1		2.67	2.92	15	0.25
S_2	7	1.71	2.75	7	1.04
S_3		5.25	5.08	20	0.17
Demand		13	22		
Row Difference		0.96	0.17		

Step 3: Eliminate column S_2 and using same methodology allocate the values 13, 2 and 20 in the cell c_{12}, c_{13} and c_{33} respectively with allocation 13, 2 and 20. Here all supply and demand are exhausted.

Table 9.

		O_2		O_3	Source	Column Difference
S_1	13	2.67	2	2.92	15	0.25
S_3		5.25	20	5.08	20	0.17
Demand		13		22		
Row Difference		2.58		2.16		

Step 4: It is required solution which is shown in below Table 10 for example 4.1, type 1 NTP based on extension of VAM principle

Table 10.

		O_1		O_2		O_3	Source
S_1		4.25	13	2.67	2	2.92	15
S_2	18	2.00	7	1.71		2.75	25
S_3		3.92		5.25	20	5.08	20
Demand		18		20		22	

Here number of positive allocations is equal to (s + d – 1) = 5 where s is the number of source, d is number of destination.

Therefore it has BFS and

Required solution in crisp value

$$= 2.67 \times 13 + 2.92 \times 2 + 2.00 \times 18 + 1.71 \times 7 + 5.08 \times 20 = 190.12$$

Required solution in neutrosophic value

$$= 13\times(0.5,2.5,4.5;1,2,3;1.5,3.5,5.5) + 2\times(1,3,5;0.5,1.5,3.5;2,4,6)$$

$$+ 18\times(1,2,3;0.5,1.5,2.5;1.5,2.5,3.5) + 7\times(1,1.5,4;0.5,1,2.5;1.25,3,4.25)$$

$$+ 20 \times (1,5,8;1.5,3,6.5;4,7,9)$$

$$= (53.5, 185,310.5; 56.5, 123,238.5; 139.25, 259.5, 356.25)$$

END

4.1.3 Solution Approach of Extension of MODI Optimality Method

Step 1: Here consider Table 11 for MODI method after completing Vogel's Approximation Method for optimality test. First calculate all u_i and v_j with the expression $u_i + v_j = c_{ij}$ and u_1=0. Since x_{31} is negative, does not satisfy the condition and go next step.

Table 11.

		O_1		O_2		O_3	**Source**	u_i
S_1		4.25	13	2.67	2	2.92	15	$u_1 = 0$
S_2	18	2.00	7	1.71		2.75	25	$u_2 = -0.96$
S_3		3.92		5.25	20	5.08	20	$u_3 = 2.16$
Demand		18		20		22		
v_j		$v_1 = 2.96$		$v_2 = 2.67$		$v_3 = 2.92$		

$$x_{11} = 4.25 - (0 + 2.96) = 1.29;$$

$$x_{23} = 2.75 - (2.96 - 0.96) = 0.75;$$

$$x_{31} = 3.92 - (2.16 + 2.96) = -1.2;$$

$$x_{32} = 5.25 - (2.16 + 2.67) = 0.42;$$

Step 2: after reallocation the allocated cell, compute the value of all u_i and v_j with the expression $u_i + v_j = c_{ij}$ and u_2= 0, but x_{23} is negative. So, again reallocate the allocated cell for next step.

Table 12.

		O_1		O_2		O_3	**Source**	u_i
S_1		4.25		2.67	15	2.92	15	$u_1 = -0.24$
S_2	5	2.00	20	1.71		2.75	25	$u_2 = 0$
S_3	13	3.92		5.25	7	5.08	20	$u_3 = 1.92$
Demand		18		20		22		
v_j		$v_1 = 2$		$v_2 = 1.71$		$v_3 = 3.16$		

$$x_{11} = 4.25 - \left(2 \ - \ 2.04\right) = 4.29;$$

$$x_{12} = 2.67 - \left(1.71 \ - \ 0.24\right) = 1.2;$$

$$x_{23} == 2.75 - \left(3.16 \ + \ 0\right) = -0.41;$$

$$x_{32} = \ 5.25 - \left(1.71 \ + \ 1.92\right) = 1.62;$$

Step 3: Compute the value of all u_i and v_j with the expression $u_i + v_j = c_{ij}$ and $v_3 = 0$. Here all unoccupied x_{ij} are non negative. So, satisfy the optimum condition.

Table 13.

		O_1		O_2		O_3	**Source**	u_i
S_1		4.25		2.67	15	2.92	15	$u_1 = 2.92$
S_2		2.00	20	1.71	5	2.75	25	$u_2 = 2.75$
S_3	18	3.92		5.25	2	5.08	20	$u_3 = 5.08$
Demand		18		20		22		
v_j		$v_1 = -1.16$		$v_2 = -1.04$		$v_3 = 0$		

$$x_{11} = 4.25 - \left(2.92 \ - \ 1.16\right) = 2.49;$$

$$x_{12} = 2.67 - \left(2.92 \ - \ 1.04\right) = 0.79;$$

$$x_{21} = 2.00 - \left(2.75 \ - \ 1.16\right) = 0.41;$$

$$x_{32} = 5.25 - \left(5.08 \ - \ 1.04\right) = \ 1.21;$$

Since, all $x_{ij} > 0$ hence, obviously get an optimum value with a unique solution.
Required optimum solution in crisp value

$= 2.92 \times 15 + 1.71 \times 20 + 2.75 \times 5 + 3.92 \times 18 + 5.08 \times 2$

$= 172.47$

Required optimum solution in neutrosophic value

= 15×(1,3,5;0.5,1.5,3.5;2,4,6) + 20×(1,1.5,4;0.5,1,2.5;1.25,3,4.25)

+ 5×(1.5,2.5,3.5;1,1.5,3;2,3,4) + 18×(2,4,6;1.5,2.5,4.5;3,5,7)

+ 2×(1, 5, 8; 1.5, 3, 6.5; 4, 7, 9)

= (80.5, 169.5, 296.5; 52.5, 101,211.5; 127,239,339)

Step 4: End

Table 14. Execute the Example 4.1 using the proposed method 4

Step 1:	$Min\ Z = \left(1,4,7;1,3,5;3.5,6,7.5\right) \otimes x_{11} + \left(0.5,2.5,4.5;1,2,3;1.5,3.5,5.5\right) \otimes x_{12} + \left(1,3,5;0.5,1.5,3.5;2,4,6\right) \otimes x_{13} + \left(1,2,3;0.5,1.5,2.5;1.5,2.5,3.5\right) \otimes x_{21} + \left(1,1.5,4;0.5,1,2.5;1.25,3,4.25\right) \otimes x_{22} + \left(1.5,2.5,3.5;1,1.5,3;2,3,4\right) \otimes x_{23} + \left(2,4,6;1.5,2.5,4.5;3,5,7\right) \otimes x_{31} + \left(1,5,8;1.5,4.5,7.5;4,6.5,9\right) \otimes x_{32} + \left(1,5,8;1.5,3,6.5;4,7,9\right) \otimes x_{33}$ subject to: $x_{21} + x_{22} + x_{23} = 25;$ $x_{12} + x_{22} + x_{32} = 20;$ $x_{11} + x_{12} + x_{13} = 15;$ $x_{13} + x_{23} + x_{33} = 22;$ $x_{11} + x_{21} + x_{31} = 18;$ $x_{31} + x_{32} + x_{33} = 20;$ $x_{ij} \in \Re\ and\ are\ non\text{-}negative.$
Step 2-4:	Hence obtained,
Step 5:	**N**ow put all x_{ij} as per the above mentioned algorithm 3 and get optimum cost.
Step 6:	End.

Table 15. Represent the comparative computing values of Example 4.1 with proposed all the four Methods.

Method	IBFS/ Optimum Solution	Values of All x_{ij}	Neutrosophic Set's of Solution
Method 1:	211.05	$x_{11} = 15;\ x_{21} = 3;$ $x_{22} = 20;\ x_{23} = 2;$ $x_{33} = 20;$	(61,201,361; 58.5,132.5,268.5; 166,303.5,396)
Method 2:	190.12	$x_{12} = 13;\ x_{13} = 2;$ $x_{21} = 18;\ x_{22} = 7;,$ $x_{33} = 20;$	(53.5,185,310.5; 56.5,123,238.5; 139.25, 259.5, 356.25)
Method 3:	172.47	$x_{13} = 15;\ x_{22} = 20;$ $x_{23} = 5;\ x_{31} = 18;$ $x_{33} = 2$	(80.5,169.5,296.5; 52.5,101,211.5; 127,239,339)
Method 4:	172.47	$x_{13} = 15;\ x_{22} = 20;$ $x_{23} = 5;\ x_{31} = 18;$ $x_{33} = 2$	(80.5,169.5,296.5; 52.5,101,211.5; 127,239,339)

4.1.4 Solution Approach of Neutrosophic Programming Method

Depending upon the computations shown in Tables 14 and 15, it is clear that -

- Modified VAM is better than the Modified North west corner method.
- Modified MODI method provides better result than Modified Vogel's approximation method.

Example 4.2: Consider Type 2 Neutrosophic Transportation Problem

Example 4.2: In this problem, cost of transportation has been considered as crisp cost (Table 16) and supply and demand values are in neutrosophic numbers which is shown in Table 17

Table 16. Transportation cell cost

	O_1	O_2	O_3
S_1	4	2	2
S_2	2	1	2
S_3	3	5	5

Table 17. Transportation supply and demand quantities are in neutrosophic numbers

Supply	Demand
(10,15,20; 14,16,22; 12,15,19)	(13,18,23;17,19,25;15,18,22)
(20,25,30; 24,26,32; 22,25,29)	(15,20,25;19,21,27;17,20,24)
(15,20,25;19,21,27; 17,20,24)	(17,22,27;21,23,29;19,22,26)

Table 18, represent the comparative computing value of Example 4.2 with all four proposed Methods.

Table 18. Observation of Example 4.2

Method	IBFS/ Optimum Solution	Values of All x_{ij}
Method 1:	197.5	$x_{11} = 15.75; x_{21} = 3; x_{22} = 20.75; x_{23} = 2; x_{33} = 20.75;$
Method 2:	128.5	$x_{31} = 15.75; x_{22} = 20.75; x_{23} = 5; x_{31} = 18.75; x_{33} = 2;$
Method 3:	128.5	$x_{31} = 15.75; x_{22} = 20.75; x_{23} = 5; x_{31} = 18.75; x_{33} = 2;$
Method 4:	128.5	$x_{31} = 15.75; x_{22} = 20.75; x_{23} = 5; x_{31} = 18.75; x_{33} = 2;$

Example 4.3: Consider Type 3 Neutrosophic Transportation Problem

Example 4.3: In this problem, cost of transportation has been considered as neutrosophic cost (Table 19) and demand supply, values are also in neutrosophic numbers which is shown in Table 20

Table 19. Cost of transportation between source to destination.

	O_1	O_2	O_3
S_1	(1,4,7;1,3,5;3.5,6,7.5)	(0.5,2.5,4.5;1,2,3;1.5,3.5,5.5)	(1,3,5;0.5,1.5,3.5;2,4,6)
S_2	(1,2,3;0.5,1.5,2.5;1.5,2.5,3.5)	(1,1.5,4;0.5,1,2.5;1.25,3,4.25)	(1.5,2.5,3.5;1,1.5,3;2,3,4)
S_3	(2,4,6;1.5,2.5,4.5;3,5,7)	(1,5,8;1.5,4.5,7.5;4,6.5,9)	(1,5,8;1.5,3,6.5;4,7,9)

Table 20. Supply and demand quantities

Supply	Demand
(10,15,20; 14,16,22; 12,15,19)	(13,18,23;17,19,25;15,18,22)
(20,25,30; 24,26,32; 22,25,29)	(15,20,25;19,21,27;17,20,24)
(15,20,25;19,21,27; 17,20,24)	(17,22,27;21,23,29;19,22,26)

Table 21, represent the comparative computing value of Example 4.3

Table 21.

Method	IBFS/ Optimum Solution	Values of All x_{ij}	Neutrosophic Set's of Solution
Method 1	219.33	$x_{11} = 15.75;\ x_{21} = 3;$ $x_{22} = 20.75;\ x_{23} = 2$ $x_{33} = 20.75$	(63.25, 208.875, 375.25; 60.75, 137.75, 279; 172.5625, 315.5, 411.5625)
Method 2	197.43	$x_{12} = 13.75;\ x_{13} = 2;$ $x_{21} = 18.75;\ x_{22} = 7;$ $x_{33} = 20.75$	(55.375, 192.125, 322.125; 58.75, 127.875, 247.5; 144.5, 269.25, 369.75)
Method 3	178.88	$x_{13} = 15.75;$ $x_{22} = 20.75;\ x_{23} = 5;$ $x_{31} = 18.75;\ x_{33} = 2$	(83.5, 175.875, 307.75; 54.375, 104.75, 219.375; 131.6875, 248, 351.9375)
Method 4	178.88	$x_{13} = 15.75;$ $x_{22} = 20.75;\ x_{23} = 5;,$ $x_{31} = 18.75;\ x_{33} = 2;$	(83.5, 175.875, 307.75; 54.375, 104.75, 219.375; 131.6875, 248, 351.9375)

CONCLUSION

In this chapter, several types of algorithmic approach are used to solve the transportation problems in neutrosophic environment using. Some numerical examples are presented to demonstrate the propose approaches. In the first section of this chapter, a modified north west corner method is presented to find the BFS of a transportation problem. Next part of this chapter, an extension of VAM algorithm is introduced to solve this problem. The VAM algorithm is more efficient than north west corner method for finding initial BFS. Finally extended principle of MODI method is used for optimality test and minimization of transportation cost. Lastly it's clear that neutrosophic set is usable for real life transportation problem for handling the uncertainty in any situation. In our daily life, it is more efficiency for facing the real life transportation problem than the other any type of set of values and methodology.

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Chapter 8
Introduction to Plithogenic Subgroup

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ABSTRACT

This chapter gives some essential scopes to study some plithogenic algebraic structures. Here the notion of plithogenic subgroup has been introduced and explored. It has been shown that subgroups defined earlier in the crisp, fuzzy, intuitionistic fuzzy, as well as neutrosophic environments, can also be represented as plithogenic fuzzy subgroups. Furthermore, introducing function in plithogenic setting, some homomorphic characteristics of plithogenic fuzzy subgroup have been studied. Also, the notions of plithogenic intuitionistic fuzzy subgroup, plithogenic neutrosophic subgroup have been introduced and their homomorphic characteristics have been analyzed.

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PREFACE

This chapter is written for the advancement of Neutrosophic theory and Plithogenic theory, which can be helpful for scientists and researchers in mathematics, computer science and also other disciplines. It provides a detailed introduction and fundamental ideas of plithogenic subgroup. The prerequisites are some knowledge of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic set theories. Also, some understandings in fuzzy, intuitionistic fuzzy and neutrosophic algebraic structures are required. In this chapter, the notion of plithogenic subgroup is introduced and explored with proper examples. It has been shown that subgroups defined earlier in the crisp, fuzzy, intuitionistic fuzzy, as well as neutrosophic environments, can also be represented as plithogenic fuzzy subgroups. Furthermore, some homomorphic characteristics of plithogenic fuzzy subgroup are studied. Also, the notions of plithogenic intuitionistic fuzzy subgroup, plithogenic neutrosophic subgroup are introduced and their homomorphic characteristics are studied.

1. INTRODUCTION

Crisp set (CS) theory has certain drawbacks. It is quite insufficient in case of handling real-life problems. Fuzzy set (FS) theory (Zadeh, 1965) is more reliable in tackling such scenarios. Since the very beginning of FS theory, many researchers have carried out that perception in various realistic problems. But eventually some other set theories have emerged like intuitionistic fuzzy set (IFS) (Atanassov, 1986), neutrosophic set (NS) (Smarandache, 2005), Pythagorean FS (Yager, 2013), Plithogenic set (PS) (Smarandache, 2017), etc., which are capable of handling uncertainty better than FSs. As a result, these set theories are preferred by most of the researchers to solve different real-life problems in which uncertainty plays a crucial role. Actually, NS is a generalization of IFS, which is further a generalization of FS. Smarandache's contributions towards the development of NS theory are remarkable. For instance, he has contributed in developing neutrosophic measure and probability (Smarandache, 2013), calculus (Smarandache & Khalid, 2015), psychology (Smarandache, 2018), etc. Also, NS theory has a vast area of applications. Furthermore, Smarandache has introduced the notion of PS (Smarandache, 2018) theory which is a generalization of CS, FS, IFS and NS theories. He has further generalized PS and developed the notions of refined PS, plithogenic multiset, plithogenic bipolar set, plithogenic tripolar set, plithogenic multipolar set, plithogenic complex set, etc. Furthermore, he has developed plithogenic logic, probability and statistics (Smarandache, 2000; Smarandache, 2017) and shown that all those notions are generalizations of crisp logic, probability and statistics. Presently, PS theory is extensively applied in various

Table 1. Significance and influences of NS and PS in various fields

Author and Year	Contributions in Various Fields
(Vlachos & Sergiadis, 2007)	Implemented NS in pattern recognition problem.
(Guo & Cheng, 2009)	Implemented neutrosophic approach to image segmentation.
(Smarandache, 2013)	Mentioned some applications of neutrosophic logic in physics.
(Majumdar, 2015)	Implemented NS in decision-making problem.
(Kumar et. al., 2015)	Implemented neutrosophic cognitive maps in medical diagnosis.
(Deli et. al., 2015)	Applied neutrosophic refined sets in medical diagnosis.
(Broumi et. al., 2016)	Solved neutrosophic shortest path problem.
(Smarandache, 2017)	Introduced plithogenic set, logic, probability and statistics.
(Kumar et. al., 2018)	Solved neutrosophic shortest path problem.
(Smarandache, 2018)	Introduced aggregation plithogenic operator in physical fields.
(Smarandache, 2018)	Introduced physical plithogenic set.
(Smarandache, 2018)	Extended soft set to hypersoft set and introduced plithogenic hypersoft set.
(Kumar et. al., 2019)	Solved neutrosophic shortest path problem.
(Kumar et. al., 2019)	Solved neutrosophic Transportation problem.

decision-making problems as well as in other applied fields. The following Table 1 consists of some important contributions in NS and PS theory.

Again, plithogenic number (PN) has been introduced and some essential operations on PN, like, the summation of PNs, multiplication of PNs, power of a PN, etc., have been developed by Samandache. Furthermore, he has introduced some important measure functions like, dice similarity plithogenic number measure (based on (Ye, 2014)), cosine similarity plithogenic number measure, Hamming plithogenic number distance, Euclidean plithogenic number distance, Jaccard similarity plithogenic number measure (Smarandache, 2017), etc. Also, in plithogenic probability theory the concepts of plithogenic fuzzy probability, plithogenic intuitionistic fuzzy probability, and plithogenic neutrosophic probability have been proposed which are essential tools to handle various probabilistic problems.

This Chapter has been arranged as following: In Segment 2, literature surveys of the fuzzy subgroup (FSG), intuitionistic fuzzy subgroup (IFSG), neutrosophic subgroup (NSG) are given. In Segment 3, some preliminary notions like PS, the preeminence of PS over other set theories, homomorphic characteristics of FSG, IFSG, and NSG are discussed. In Segment 4, different aspects of plithogenic subgroup (PSG), like, plithogenic fuzzy subgroup (PFSG), plithogenic intuitionistic fuzzy subgroup (PIFSG), plithogenic neutrosophic subgroup (PNSG) are introduced and also, the effects of homomorphism on those notions are mentioned. Finally, in segment 5 the conclusion is given and some scopes of future researches are mentioned.

2. LITERATURE SURVEY

To tackle drawbacks and insufficiency of CS theory FS was introduced. But that too has certain limitations and hence IFS and further NS were introduced. Presently, Pythagorean FS (Yager, 2013) and PS (Smarandache, 2018) are more popular for their uncertainty handling capability. As a result, these set theories are preferred by most of the researchers to solve different realistic problems in which uncertainty is involved.

In this segment, we have discussed FS, IFS, NS and also some other essential notions like FSG, IFSG, NSG, level set, level subgroup, T-norm, T-conorm, etc. All these notions play vital roles in developing plithogenic subgroup (PSG).

Definition 2.1 (Zadeh, 1965) A FS σ of a CS U is a function from U to [0,1] i.e., $\sigma : U \to [0,1]$.

Definition 2.2 (Atanassov, 1986) A IFS $\boldsymbol{\gamma}$ of a CS U is denoted as

$$\gamma = \{(m, t_\gamma(m), f_\gamma(m)) : m \in U\},$$

where both t_γ and f_γ are FSs of U, known as the respective degree of membership and non-membership of any element $m \in U$. Here for every $m \in U$, t_γ and f_γ satisfy the condition

$$0 \leq t_\gamma(m) + f_\gamma(m) \leq 1.$$

Definition 2.3 (Smarandache, 1999) A NS η of a CS U is denoted as

$$\eta = \{(m, t_\eta(m), i_\eta(m), f_\eta(m)) : m \in U\},$$

where

$$t_\eta, i_\eta, f_\eta : U \to]^{-}0, 1^{+}[$$

are the respective degree of truth, indeterminacy and falsity of any element $m \in U$. Here for every $m \in U$ t_η, i_η and f_η satisfy the condition

$$^{-}0 \leq t_\eta(m) + i_\eta(m) + f_\eta(m) \leq 3^{+}.$$

Definition 2.4 (Zadeh, 1965) Let α be a FS of *U*. Then $\forall\, t \in [0,1]$ the set

$$\alpha_t = \{x \in U : \alpha(x) \geq t\}$$

is called a level subset (*t*-level subset) of α.

Definition 2.5 (Gupta & Qi, 1991) A function $T:[0,1] \to [0,1]$ is termed as T-norm iff $\forall\, m,u,t \in [0,1]$ subsequent conditions are fulfilled:

(i) $T(m,1) = m$

(ii) $T(m,u) = T(u,m)$

(iii) $T(m,u) \leq T(t,u)$ *if* $m \leq t$

(iv) $T(m,T(u,t)) = T(T(m,u),t)$

Definition 2.6 (Gupta & Qi, 1991) A function $T^*:[0,1] \to [0,1]$ is termed as T-conorm iff $\forall\, m,u,t \in [0,1]$ subsequent conditions are fulfilled:

(i) $T^*(m,0) = m$

(ii) $T^*(m,u) = T^*(u,m)$

(iii) $T^*(m,u) \leq T^*(t,u)$ *if* $m \leq t$

(iv) $T^*(m,T^*(u,t)) = T^*(T^*(m,u),t)$

In the next subsection, the notions of FSG, IFSG, and NSG are discussed and also, some of their basic fundamental properties are given.

2.1. Fuzzy Subgroup, Intuitionistic Fuzzy Subgroup and Neutrosophic Subgroup

Definition 2.7 (Rosenfeld, 1971) A FS α of a group *P* is termed as a FSG of *P* iff $\forall\, m,u \in P,$ the subsequent conditions are fulfilled:

(i) $\alpha(mu) \geq min\{\alpha(m),\alpha(u)\}$

(ii) $\alpha(m^{-1}) \geq \alpha(m)$.

Here $\alpha(m^{-1}) = \alpha(m)$ and $\alpha(m) \leq \alpha(e),$ where *e* represents the neutral element of *P*. Also, in the above definition if only condition (i) is satisfied by α then we call it a fuzzy subgroupoid.

Theorem 2.1 (Rosenfeld, 1971) α is a FSG of *U* iff

$\forall\, m,u \in U\ \ \alpha(mu^{-1}) \geq min\{\alpha(m),\alpha(u)\}$.

Definition 2.8 (Das, 1981) Let α be a FSG of a group *P*. Then $\forall\, t \in [0,1]$ and $\alpha(e) \geq t$ the subgroups α_t are called level subgroups of α.

Definition 2.9 (Biswas, 1989) An IFS

$$\gamma = \{(m, t_\gamma(m), f_\gamma(m)) : m \in U\}$$

of a crisp set *U* is called an IFSG of *U* iff $\forall m,u \in U$

(i) $t_\gamma(mu^{-1}) \geq \min\{t_\gamma(m), t_\gamma(u)\}$

(ii) $f_\gamma(mu^{-1}) \leq \max\{f_\gamma(m), f_\gamma(u)\}$

The collection of all IFSG will be denoted as IFSG(*U*).

Example 2.1 Let $U = \{1,-1,i,-i\}$ and δ be a NS of *U* such that

$$\gamma = \{(1, 0.6, 0.4), (-1, 0.7, 0.3), (i, 0.8, 0.2), (-i, 0.8, 0.2)\}.$$

Notice that $\gamma \in$ IFSG (*U*).

Definition 2.10 (Çetkin & Aygün, 2015) Let *U* be a group and δ be a NS of *U*. δ is called a NSG of *U* iff the subsequent conditions are fulfilled:

(i) $\delta(m \cdot u) \geq \min\{\delta(m), \delta(u)\}$, i.e.

$$t_\delta(m \cdot u) \geq \min\{t_\delta(m), t_\delta(u)\},$$

$$i_\delta(m \cdot u) \geq \min\{i_\delta(m), i_\delta(u)\}$$

and

$$f_\delta(m \cdot u) \leq \max\{f_\delta(m), f_\delta(u)\}$$

(ii) $\delta(m^{-1}) \geq \delta(m)$ i.e.

$$t_\delta(m^{-1}) \geq t_\delta(u),$$

$$i_\delta(m^{-1}) \geq i_\delta(u)$$

and

$$f_\delta(m^{-1}) \leq f_\delta(u)$$

The collection of all NSG will be denoted as NSG(*U*). Here notice that t_δ and i_δ are following Definition 2.7 i.e. both of them are actually FSGs of *U*.

Example 2.2 (Çetkin & Aygün, 2015) Let $U = \{1, -1, i, -i\}$ and δ be a NS of *U* such that

$$\delta = \{(1, 0.6, 0.5, 0.4), (-1, 0.7, 0.4, 0.3), (i, 0.8, 0.4, 0.2), (-i, 0.8, 0.4, 0.2)\}.$$

Notice that δ∈NSG(*U*).

Theorem 2.2 (Çetkin & Aygün, 2015) Let *U* be a group and δ be a NS of *U*. Then δ∈NSG(*U*) iff

$$\forall m, u \in U \ \delta(m \cdot u^{-1}) \geq \min\{\delta(m), \delta(u)\} .$$

Theorem 2.3 (Çetkin & Aygün, 2015) δ∈NSG (*U*) iff $\forall p \in [0,1]$ the *p*-level sets $(t_\delta)_p$, $(i_\delta)_p$ and *p*-lower-level set $(\overline{f_\delta})_p$ are CSGs of *U*.

Definition 2.11 (Çetkin & Aygün, 2015) Let *U* be a group and δ be a NS of *U*. δ is called a neutrosophic normal subgroup (NNSG) of *U* iff

$$\forall m, u \in U \ \delta(m \cdot u \cdot m^{-1}) \leq \delta(u)$$

i.e.

$$t_\delta(m \cdot u \cdot m^{-1}) \leq t_\delta(u) ,$$

$$i_\delta(m \cdot u \cdot m^{-1}) \leq i_\delta(u)$$

and

$$f_\delta(m \cdot u \cdot m^{-1}) \geq f_\delta(u) .$$

The collection of all NNSG of *U* will be denoted as NNSG (*U*). Some more references that can be helpful to various authors are (Kandasamy & Smarandache, 2004; Gayen et. al., 2019; Kumar et. al., 2019; Gayen et. al, 2019; Broumi et. al., 2014; Kumar et. al., 2020 Kumar et. al., 2019b; Kumar et. al., 2017.. In the Table 2, some sources have been mentioned which have some major contributions in the fields of FSG, IFSG, and NSG.

2.2. Motivation of the Work

From the above discussions, it is clear that the studies of FSG, IFSG, as well as NSG, have generated many fruitful research fields. Some researchers have studied their normal versions, homomorphic characteristics and different other algebraic structures. Also, some authors have implemented the soft set theory in these notions

Table 2. Significance and influences of some authors in FSG, IFSG, and NSG

Author and Year	Different Contributions in FSG, IFSG, and NSG
(Rosenfeld, 1971)	Introduced FSG.
(Das, 1981)	Introduced level subgroup.
(Anthony & Sherwood, 1979)	Introduced FSG using general T-norm.
(Foster, 1979)	Introduced product of FSGs.
(Anthony & Sherwood, 1982)	Introduced subgroup generated and function generated FSG.
(Sherwood, 1983)	Studied product of FSGs.
(Mukherjee & Bhattacharya, 1984)	Introduced fuzzy normal subgroups and cosets.
(Biswas, 1989)	Introduced IFSG.
(Eroğlu, 1989)	Studied homomorphic image of FSG.
(Kim & Kim, 1996)	Studied fuzzy symmetric groups.
(Ray, 1999)	Studied some properties on the product of FSGs.
(Hur et. al., 2004)	Introduced Intuitionistic fuzzy normal subgroups and intuitionistic fuzzy cosets.
(Yuan et. al., 2010)	Introduced (α,β)-IFSG.
(Sharma, 2011)	Studied homomorphism of IFSG.
(Çetkin & Aygün, 2015)	Introduced NSG and neutrosophic normal subgroup and studied some fundamental properties by introducing homomorphism in them.

and studied their fundamental properties. Presently, the PS theory has grabbed a lot of attention due to its uncertainty handling nature. Again, this set theory is more general than CS, FS, IFS, as well as NS theories. So, the notions of PSGs i.e. PFSG, PIFSG and PNSG can become effective research fields. Furthermore, whether PFSG, PIFSG, and PNSG will act as generalizations of the crisp subgroup (CSG), FSG, IFSG, and NSG or not that is needed to be discussed. Again, their normal forms, homomorphic properties and also some other essential algebraic structures are needed to be studied. In this chapter, the subsequent research gaps are discussed:

- Still, the notion of PFSG is undefined.
- Whether CSG, FSG, IFSG, NSG can be represented by PFSG or not, that is needed to be analyzed.
- Also, some other essential PSGs like PIFSG, PNSG are needed to be introduced and studied.
- Furthermore, homomorphic characteristics PFSG, PIFSG, PNSG are still unexplored.

Therefore, this motivates us to introduce these notions of PFSG, PIFSG, and PNSG and analyze their algebraic properties.

2.3. Contribution of the Work

On the basis of the above gaps, the purpose of this article is to provide some essential definitions, examples, theories, propositions, etc. in the fields of PSG i.e. PFSG, PIFSG, and PNSG. Also, effectiveness and excellence of PSG in comparison with CSG, FSG, IFSG, and NSG will be mentioned. Furthermore, some important analysis like homomorphic characteristics of these notions will be discussed. The following are some purposes that are planned and executed during this research work.

- To define PFSG and study its algebraic properties.
- To check whether PFSG is a generalization of CSG, FSG, IFSG, and NSG or not.
- To define PIFSG and study its algebraic properties.
- To define PNSG and study its algebraic properties.
- To study some homomorphic characteristics of PFSG, PIFSG, PNSG.

3. DESCRIPTION OF THE WORK

3.1. Research Problem

So far, many researchers have studied different algebraic structures and fundamental properties of FSG, IFSG, and NSG. We know that homomorphic functions preserve algebraic structures. Therefore, to study those essential algebraic properties one need to study the effects of homomorphism on them. Several researchers have already introduced and studied homomorphism in the environments of FSG, IFSG and NSG. Also, some researchers have introduced the normal forms of FSG, IFSG, NSG and studied their homomorphic properties. Till now the concept of PFSG is undefined and unexplored. Again, depending upon the different degree of appurtenance and degree of dissimilarity functions some other PSG can be introduced, like PIFSG, PNSG, etc. In addition, PS is a generalized version of CS, FS, IFS as well as NS and hence PSGs i.e. PFSG, PIFSG, and PNSG have the potentials to become a generalized version of FSG, IFSG, and NSG. Smarandache has showed that with only one set theory i.e. PS theory all the other set theories can be developed. Similarly, with only PSG the notions of FSG, IFSG, and NSG can be developed.

Again, not only these notions are needed to be defined but also some essential analysis of homomorphic images, pre-images, etc. are needed to be analyzed. However, before introducing homomorphism in PSG, PIFSG and PNSG one first need to understand the behavior of any mapping in plithogenic environment. In this chapter, these essential notions of PSGs have been introduced and analyzed with proper examples. In the following preliminary subsection, some essential notions have been discussed, which were introduced earlier.

3.1.1. Preliminaries

The term plithogenic means pertaining to genesis or evolution or creation. A PS is a set in which its elements are characterized by one or more attributes and each attribute consists of some values. In PS a relation between an element and any attribute's value is denoted as $d(m,u)$, which is known as the degree of appurtenance function. Also, the relation between any two attribute's values is denoted as $c(u,u)$, which is known as the degree of contradiction or dissimilarity function. The following is a formal definition of a PS:

Definition 3.1 (Smarandache, 2018) Let *U* be a universal set and $P \subseteq U$. A PS is denoted as

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F}),$$

where ψ be an attribute or appurtenance, V_ψ is the corresponding range of attribute's value,

$$p_{d_F} : P \times V_\psi \to [0,1]^s$$

is the degree of appurtenance function (DAF) and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]^t$$

is the corresponding degree of contradiction function (DCF). Here $s, t \in \{1,2,3\}$.

Note that, in the above definition for $s = 1$ and $t = 1$ p_{d_F} will become a fuzzy DAF (FDAF) and p_{c_F} will become a fuzzy DCF (FDCF). In general, for simplicity, we consider only FDAF and FDCF. In the case of FDCF p_{c_F} satisfies the following axioms:

$$\forall (u_i, u_j) \in V_\psi \times V_\psi \ p_{c_F}(u_i, u_i) = 0$$

and

$$p_{c_F}(u_i, u_j) = p_{c_F}(u_j, u_i).$$

Again, to increase more accuracy one may wish to take $s = 2$ or $s = 3$ with $t = 1$. In that case, we will have

$$p_{d_{IF}} : P \times V_\psi \to [0,1]^2$$

(intuitionistic fuzzy DAF (IFDAF)) and

$$p_{d_N} : P \times V_\psi \to [0,1]^3$$

(neutrosophic DAF (NDAF)) along with p_{c_F} as FDCF. Again, to generalize further and increase the level of accuracy and complexity one may wish to take $t = 2$ or $t = 3$ i.e.

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]^2$$

(intuitionistic fuzzy DCF (IFDCF)) or

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]^3$$

(neutrosophic DCF (NDCF)).

CS, FS, IFS and NS are characterized by a single attribute which has one value for CS (membership (M)), two values for FS (M, nonmembership (NM)) and three values for NS (M, indeterminacy (I), NM) and hence using PS one can easily denote any CS, FS, IFS, and NS. So, PS is a generalization of these sets.

The following are some preliminary homomorphic characteristics of FSG, IFSG, and NSG. These fundamental characteristics will help one to understand the effects of homomorphism in PFSG, PIFSG, and PNSG.

Definition 3.2 (Anthony & Sherwood, 1979) A FS α of *U* is said to have supremum property if for any $\alpha' \subseteq \alpha \; \exists m_0 \in \alpha'$ such that $\alpha(m_0) = \sup_{m \in \alpha'} \alpha(m)$.

Theorem 3.1 (Anthony & Sherwood, 1979) Let α be a fuzzy subgroupoid of *P* on the basis of a continuous t-norm *T* and *l* be a homomorphism on *P*, then the image (supremum image) of α is a fuzzy subgroupoid on $l(P)$ with respect to *T*.

Theorem 3.2 (Rosenfeld, 1971) Homomorphic image or pre-image of any FSG having supremum property is a FSG.

Theorem 3.3 (Sharma, 2011) Let *g* be a homomorphism of a group U_1 into another group U_2 then preimage of an IFSG γ of U_2 i.e. $g^{-1}(\gamma)$ is an IFSG of U_1.

Theorem 3.4 (Sharma, 2011) Let *g* be a surjective homomorphism of a group U_1 to another group U_2, then the image of an IFSG γ of U_1 i.e. $g(\gamma)$ is an IFSG of U_2.

Theorem 3.5 (Çetkin & Aygün, 2015) Homomorphic image or pre-image of any NSG is a NSG.

Theorem 3.6 (Çetkin & Aygün, 2015) Let δ∈NNSG(*U*) and *l* be a homomorphism on *U*. Then the homomorphic pre-image of δ i.e. $l^{-1}(\delta) \in$ NNSG(*U*).

Theorem 3.7 (Çetkin & Aygün, 2015) Let δ∈NNSG(*U*) and *l* be a surjective homomorphism on *U*. Then the homomorphic image of δ i.e. $l(\delta) \in$ NNSG(*U*).

Since PS is a generalization of CS, FS, IFS and NS one may guess superiority of PSG over CSG, FSG, IFSG, and NSG. In the next section, we have introduced different types of PSGs and studied their homomorphic characteristics.

4. PROPOSED NOTIONS OF PLITHOGENIC SUBGROUPS

4.1. Plithogenic Fuzzy Subgroup

Definition 4.1 Let

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F})$$

be a PS of a group *U*. Where ψ is an attribute, V_ψ is a range of all attribute's values,

$$p_{d_F} : P \times V_\psi \to [0,1]$$

is the corresponding FDAF and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

is the corresponding FDCF. Then P_s is called a PFSG of *U* iff p_{d_F} is a fuzzy subgroup i.e. in other words iff

$$\forall (m_1, u_1), (m_2, u_2) \in P \times V_\psi$$

the subsequent conditions are fulfilled:

(i) $p_{d_F}((m_1, u_1) \cdot (m_2, u_2)) \geq \min\{p_{d_F}(m_1, u_1), p_{d_F}(m_2, u_2)\}$ and

(ii) $p_{d_F}((m_1, u_1)^{-1}) \geq p_{d_F}(m_1, u_1)$

A set of all PFSG of a group *U* is denoted as PFSG(*U*).

Example 4.1 Let

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F})$$

be a PS of a group *U,* where $P = \{m, u, mu, e\}$ be the Klein's four group, ψ be an attribute, $V_\psi = \{a, e'\}$ be a group consisting of two attribute values (here $a^2 = e'$ and e' is the neutral element). Also, let

$$p_{d_F} : P \times V_\psi \to [0,1]$$

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

are respectively corresponding FDAF and FDCF defined in Table 3 and Table 4.

Then $P_s \in$ PFSG(*U*).

Table 3. FDAF

p_{d_F}	*m*	*u*	*mu*	*e*
a	0.2	0.5	0.3	0.2
e'	0.3	0.2	0.2	0.2

Note that in Definition 4.1 both p_{d_F} and p_{c_F} are FSs but only on p_{d_F} the conditions for FSG has been assigned because p_{c_F} will become a FSG only if

$\forall (u_i, u_j) \in V_\psi \; p_{c_F}(u_i, u_j) = 0$.

Example 4.2 Let

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F})$$

bet a PS of a group *U*, where $P = \{1, -1, i, -i\}$ be a cyclic group, ψ be an attribute, $V_\psi = \{m, u, mu, e\}$ be the Klein four-group. Also, let

$$p_{d_F} : P \times V_\psi \to [0,1]$$

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

are respectively corresponding FDAF and FDCF given in Table 5 and Table 6.

Theorem 4.1 $P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F}) \in$ PFSG(*U*) iff

Table 4. FDCF

p_{c_F}	*a*	e'
a	0	0.5
e'	0.5	0

Table 5. FDAF

$\alpha(p)_{d_{IF}}$	*m*	*u*	*mu*	*e*
1	0.4	0.4	0.6	0.2
-1	0.4	0.4	0.7	0.2
i	0.4	0.4	0.8	0.2
-*i*	0.4	0.4	0.8	0.2

$$p_{d_F}((m_1,u_1)\cdot(m_2,u_2)^{-1}) \geq \min\{p_{d_F}(m_1,u_1), p_{d_F}(m_2,u_2)\}$$

Proof: Using Theorem 2.1 this can be easily proved.

Using PS one can easily handle indeterminate, uncertain and incongruous data. As a result, it has become more general than CS, FS, IFS as well as NS. Hence, it is quite evident that PFSG can have the potentials to become more general than CSG, FSG, IFSG, and NSG. In the next section, the preeminence aspects of PFSG have been discussed with proper justifications.

4.1.1. PFSG as a Generalization of Other Subgroups

Proposition 4.1 Any CSG is a PFSG (given that the corresponding attribute value set is a union of two singleton crisp groups).

Proof: Let *C* be a CSG of a group *U*. So, $C \subseteq U$ and hence *C* is a PS of *U*. Here, one may consider corresponding $\psi =$ "appurtenance", $V_\psi = \{\mathrm{M}, \mathrm{NM}\}$ with cardinality 2,

$$p_{d_F} : C \times V_\psi \rightarrow [0,1]$$

Table 6. FDCF

c_{d_F}	*m*	*u*	*mu*	*e*
m	0	0.2	0.3	0.5
u	0.2	0	0.9	0.8
mu	0.3	0.9	0	0.6
e	0.5	0.8	0.6	0

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1].$$

Here

$$p_{c_F}(\mathrm{M},\mathrm{M}) = 0,$$

$$p_{c_F}(\mathrm{NM},\mathrm{NM}) = 0$$

and

$$p_{c_F}(\mathrm{M},\mathrm{NM}) = 1.$$

Also, V_ψ can be considered as

$$V_\psi = \{\mathrm{M}\} \cup \{\mathrm{NM}\},$$

where $\{\mathrm{M}\}$ and $\{\mathrm{NM}\}$ are singleton CSGs. Also, $p_{d_F}(m,\mathrm{M}) = 1$ and $p_{d_F}(m,\mathrm{NM}) = 0.$

Now,

$$\forall\, m_1, m_2 \in C m_1 m_2^{-1} \in C$$

i.e. $p_{d_F}(m_1,\mathrm{M}) = 1$ and $p_{d_F}(m_2,\mathrm{M}) = 1$ imply that

$$p_{d_F}(m_1 m_2^{-1},\mathrm{M}) = 1.$$

Where from it can be easily proved that

$$p_{d_F}(m_1,\mathrm{M}) \cdot (m_2,\mathrm{M})^{-1} = 1 \geq \min\{1,1\} = \min\{p_{d_F}(m_1,\mathrm{M}), p_{d_F}(m_2,\mathrm{M})\}.$$

Similarly, $\forall\, m_1, m_2 \notin C$ it can be proved that

$$p_{d_F}(m_1, \mathrm{NM})\cdot(m_2, \mathrm{NM})^{-1} = 0 \geq \min\{0,0\} = \min\{p_{d_F}(m_1, \mathrm{NM}), p_{d_F}(m_2, \mathrm{NM})\}.$$

Hence, $C \in \mathrm{PFSG}(U)$.

Proposition 4.2 Any FSG is a PFSG (given that the corresponding attribute value set is a singleton CSG).

Proof: Let $\alpha \in \mathrm{FSG}(U)$. So, $\alpha \in \mathrm{FS}(U)$ and hence $\alpha \in \mathrm{PS}(U)$. Here one may consider corresponding $\psi =$ "appurtenance", $V_\psi = \{\mathrm{M}\}$ with cardinality 1,

$$p_{d_F} : C_\alpha \times V_\psi \to [0,1]\,(C_\alpha = \{m \in U : (m, \alpha(m)) \in \alpha\})$$

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

with p_{c_F} (M, M) = 0. Note that here

$$p_{d_F}(m, \mathrm{M}) = \alpha(m) \in [0,1].$$

Then by Theorem 2.1, $\forall\, m_1, m_2 \in C_\alpha$,

$$\alpha(m_1 m_2^{-1}) \geq min\{\alpha(m_1), \alpha(m_2)\}$$

$$\Rightarrow p_{d_F}(m_1 m_2^{-1}, \mathrm{M}) \geq min\{p_{d_F}(m_1, \mathrm{M}), p_{d_F}(m_2, \mathrm{M})\}$$

$$\Rightarrow p_{d_F}((m_1, \mathrm{M})\cdot(m_2^{-1}, \mathrm{M})) \geq min\{p_{d_F}(m_1, \mathrm{M}), p_{d_F}(m_2, \mathrm{M})\}$$

$$\Rightarrow p_{d_F}((m_1, \mathrm{M})\cdot(m_2, \mathrm{M})^{-1}) \geq min\{p_{d_F}(m_1, \mathrm{M}), p_{d_F}(m_2, \mathrm{M})\}$$

Hence, from it can be concluded that, $\alpha \in \mathrm{PFSG}(U)$.

Proposition 4.3 Any IFSG is a PFSG (given that the corresponding attribute value set is a union of two singleton CSG).

Proof: Let

$\gamma = (m, t_\gamma(m), f_\gamma(m)) \in$ IFSG(U).

So, γ∈IFS(U)and hence γ∈PS(U). Here one may consider $\psi =$ "appurtenance", $V_\psi = \{\mathrm{M}, \mathrm{NM}\}$ with cardinality 2,

$$p_{d_F} : C_\gamma \times V_\psi \to [0,1]$$

$$(C_\gamma = \{m \in U : (m, t_\gamma(m), f_\gamma(m)) \in \gamma\})$$

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

(here $p_{c_F}(\mathrm{M},\mathrm{M}) = 0$ $p_{c_F}(\mathrm{NM},\mathrm{NM}) = 0$ and $p_{c_F}(\mathrm{M},\mathrm{NM}) = 1.$)

Here, V_ψ can be considered as $V_\psi = \{\mathrm{M}\} \cup \{\mathrm{NM}\}$, where $\{\mathrm{M}\}$ and $\{\mathrm{NM}\}$ are singleton CSGs and

$$p_{d_F}(m,\mathrm{M}) + p_{d_F}(m,\mathrm{NM}) \le 1.$$

Note that,

$$p_{d_F}(m,\mathrm{M}) = t_\gamma(m) \in [0,1]$$

and

$$p_{d_F}(m,\mathrm{NM}) = f_\gamma(m) \in [0,1].$$

Then by Theorem 2.1, $\forall\, m_1, m_2 \in C_\gamma$,

$$t_\gamma(m_1 m_2^{-1}) \ge min\{t_\gamma(m_1), t_\gamma(m_2)\}$$

$$\Rightarrow p_{d_F}(m_1 m_2^{-1}, \mathrm{M}) \ge min\{p_{d_F}(m_1,\mathrm{M}), p_{d_F}(m_2,\mathrm{M})\}$$

$$\Rightarrow p_{d_F}((m_1,\mathrm{M})\cdot(m_2^{-1},\mathrm{M}))\geq min\{p_{d_F}(m_1,\mathrm{M}),p_{d_F}(m_2,\mathrm{M})\}$$

$$\Rightarrow p_{d_F}((m_1,\mathrm{M})\cdot(m_2,\mathrm{M})^{-1})\geq min\{p_{d_F}(m_1,\mathrm{M}),p_{d_F}(m_2,\mathrm{M})\}$$

Again,

$$f_\gamma(m_1m_2^{-1})\leq\max\{f_\gamma(m_1),f_\gamma(m_2)\}$$

$$\Rightarrow p_{d_F}(m_1m_2^{-1},\mathrm{NM})\leq\max\{p_{d_F}(m_1,\mathrm{NM}),p_{d_F}(m_2,\mathrm{NM})\}$$

$$\Rightarrow p_{d_F}((m_1,\mathrm{NM})\cdot(m_2^{-1},\mathrm{NM}))\leq\max\{p_{d_F}(m_1,\mathrm{NM}),p_{d_F}(m_2,\mathrm{NM})\}$$

$$\Rightarrow p_{d_F}((m_1,\mathrm{NM})\cdot(m_2,\mathrm{NM})^{-1})\leq\max\{p_{d_F}(m_1,\mathrm{NM}),p_{d_F}(m_2,\mathrm{NM})\}$$

Hence, from and γ∈PFSG(*U*).

Proposition 4.4 Any NSG is a PFSG (given that the corresponding attribute value set is a union of three singleton CSGs).

Proof: Using Proposition 4.2 and Proposition 4.3 this can be easily proved.

To understand structures of different algebraic objects, one needs to study functions that preserve those algebraic structures i.e. one must study the effects of homomorphism on those algebraic entities. In the next section image and preimage of any PS under a function has been defined. Also, the homomorphic image, preimage of any PSG have been introduced and efficiently discussed.

4.1.2. Homomorphism on PFSG

Let ψ be an attribute and V_ψ be the corresponding range of attribute's values and

$$P_s=(P,\psi,V_\psi,p_{d_F},p_{c_F})$$

be a PS of a group *U*. Also, let *f* be a function defined on $U\cup V_\psi$. Then the image of P_s is denoted as

$$P'_s=(P',\psi,V'_\psi,p'_{d_F},p'_{c_F}),$$

where $p'_{d_F} : P' \times V'_\psi \to [0,1]$ is defined as

$$p'_{d_F}(m_2, u_2) = \sup_{\substack{m_1 \in f^{-1}(m_2) \\ u_1 \in f^{-1}(u_2)}} p_{d_F}(m_1, u_1)$$

and $p'_{c_F} : V'_\psi \times V'_\psi \to [0,1]$ is defined as

$$p'_{c_F}(u_3, u_4) = \sup_{\substack{u_1 \in f^{-1}(u_3) \\ u_2 \in f^{-1}(u_4)}} p_{c_F}(u_1, u_2).$$

Also, if

$$P'_s = (P', \psi, V'_\psi, p'_{d_F}, p'_{c_F})$$

is a PS of $f(U)$ then the preimage of P'_s will be denoted as

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F}),$$

where $p_{d_F} : P \times V_\psi \to [0,1]$ is defined as

$$p_{d_F}(m_1, u_1) = p'_{d_F}(f(m_1), f(u_1)),\ \forall (m_1, u_1) \in P \times V_\psi$$

and $p_{c_F} : V_\psi \times V_\psi \to [0,1]$ is defined as

$$p_{c_F}(u_1, u_2) = p'_{c_F}(f(u_1), f(u_2)),\ \forall (u_1, u_2) \in V_\psi \times V_\psi.$$

Theorem 4.2 Homomorphic preimage of a PFSG is a PFSG.

Proof: Let ψ be an attribute and V'_ψ be the corresponding range of attribute's values and

$$P'_s = (P', \psi, V'_\psi,\ p'_{d_F}, p'_{c_F})$$

be a PFSG of a group $f(U)$, where f is a homomorphism on $U \cup V_\psi$. Hence,

$$p'_{d_F} : P' \times V'_\psi \to [0,1]$$

is a FSG and

$$p'_{c_F} : V'_\psi \times V'_\psi \to [0,1]$$

is a FS.

Then preimage of P'_s is denoted as

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F}),$$

where

$p_{d_F} : P \times V_\psi \to [0,1]$ is defined as

$$p_{d_F}(m_1, u_1) = p'_{d_F}(f(m_1), f(u_1)) \ \forall (m_1, u_1) \in P \times V_\psi$$

and $p_{c_F} : V_\psi \times V_\psi \to [0,1]$ is defined as

$$p_{c_F}(u_1, u_2) = p'_{c_F}(f(u_1), f(u_2)) \ \forall (u_1, u_2) \in V_\psi \times V_\psi.$$

Let

$$(m_1, u_1), (m'_1, u'_1) \in P \times V_\psi,$$

then

$$p_{d_F}((m_1, u_1) \cdot (m'_1, u'_1)) = p_{d_F}(m_1 m'_1, u_1 u'_1)$$

$$= p'_{d_F}(f(m_1 m'_1), f(u_1 u'_1))$$

$$= p'_{d_F}(f(m_1)f(m'_1), f(u_1)f(u'_1))$$

$$= p'_{d_F}((f(m_1), f(u_1)) \cdot (f(m'_1), f(u'_1)))$$

$$\geq \min\{p'_{d_F}(f(m_1), f(u_1)),\ p'_{d_F}(f(m'_1), f(u'_1))\}$$ (As p'_{d_F} is a FSG)

$$= \min\{p_{d_F}(m_1, u_1),\ p_{d_F}(m'_1, u'_1)\}$$

Also,

$$p_{d_F}(m_1, u_1)^{-1} = p_{d_F}(m_1^{-1}, u_1^{-1})$$

$$= p'_{d_F}(f(m_1^{-1}), f(u_1^{-1}))$$

$$= p'_{d_F}(f(m_1)^{-1}, f(u_1)^{-1})$$

$$= p'_{d_F}(f(m_1), f(u_1))^{-1}$$

$$\geq p'_{d_F}(f(m_1), f(u_1))$$

$$= p_{d_F}(m_1, u_1)$$

So, by and p_{d_F} is a FSG. Again, as p'_{c_F} is a FS its preimage under f i.e. p_{c_F} is a FS and hence, P_s is a PFSG of *U*.

Theorem 4.3 Homomorphic image of a PFSG is a PFSG (provided for FDAF and FDCF supremum property holds).

Proof: Let ψ be an attribute and V_ψ be the corresponding range of attribute's values and

$$P_s = (P, \psi, V_\psi,\ p_{d_F}, p_{c_F})$$

be a PFSG of a group *U*. Also, let *f* be a homomorphism defined on $U \cup V_\psi$.

Then the image of P_s is denoted as

$$P'_s = (P', \psi, V'_\psi, p'_{d_F}, p'_{c_F}),$$

where $p'_{d_F} : P' \times V'_\psi \to [0,1]$ is defined as

$$p'_{d_F}(m_2, u_2) = \sup_{\substack{m_1 \in f^{-1}(m_2) \\ u_1 \in f^{-1}(u_2)}} p_{d_F}(m_1, u_1)$$

and $p'_{c_F} : V'_\psi \times V'_\psi \to [0,1]$ is defined as

$$p'_{c_F}(u_3, u_4) = \sup_{\substack{u_1 \in f^{-1}(u_3) \\ u_2 \in f^{-1}(u_4)}} p_{c_F}(u_1, u_2).$$

Then

$$\forall (f(m_1), f(u_1)), (f(m'_1), f(u'_1)) \in P' \times V'_\psi,$$

$\exists\, m_0 \in f^{-1}(f(m_1))$ and $\exists\, u_0 \in f^{-1}(f(u_1))$ such that

$$p_{d_F}(m_0, u_0) = \sup_{\substack{m_t \in f^{-1}(f(m_1)) \\ u_t \in f^{-1}(f(u_1))}} p_{d_F}(m_t, u_t).$$

Also, $\exists\, m'_0 \in f^{-1}(f(m'_1))$ such that

$$p_{d_F}(m'_0, u'_0) = \sup_{\substack{m_t \in f^{-1}(f(m'_1)) \\ u_t \in f^{-1}(f(u'_1))}} p_{d_F}(m_t, u_t).$$

So,

$$p'_{d_F}(f(m_1), f(u_1)) \cdot (f(m'_1), f(u'_1)) = p'_{d_F}(f(m_1) \cdot f(m'_1), f(u_1) \cdot f(u'_1))$$

$$= \sup_{\substack{m_z \in f^{-1}(f(m_1) \cdot f(m_1')) \\ u_z \in f^{-1}(f(u_1) \cdot f(u_1'))}} p_{d_F}(m_z, u_z)$$

$$\geq \min\{p_{d_F}(m_0, u_0), p_{d_F}(m_0', u_0')\}$$

$$= \min\{p'_{d_F}(f(m_1), f(u_1)), p'_{d_F}(f(m_1'), f(u_1'))\}$$

Again,

$$p'_{d_F}(f(m_1), f(u_1))^{-1} = p'_{d_F}(f(m_1)^{-1}, f(u_1)^{-1})$$

$$= \sup_{\substack{m_z \in f^{-1}(f(m_1)^{-1}) \\ u_z \in f^{-1}(f(u_1)^{-1})}} p_{d_F}(m_z, u_z)\ p_{c_F}$$

$$= p_{d_F}(m_0{}^{-1}, u_0{}^{-1})$$

$$= p_{d_F}(m_0, u_0)^{-1}$$

$$\geq p_{d_F}(m_0, u_0)$$

$$= p'_{d_F}(f(m_1), f(u_1))$$

So, by and p'_{d_F} is a FSG. Also, as p_{c_F} is a FS its image under f i.e. p'_{c_F} is a FS and hence, P_s' is a PFSG of $f(U)$.

In Definition 4.1 only FDAF (p_{d_F}) and FDCF (p_{c_F}) has been used. Instead of using a fuzzy environment, intuitionistic fuzzy or neutrosophic environments can also be used. In that case, the degree of appurtenance functions will be IFDAF ($p_{d_{IF}}$) or NDAF (p_{d_N}). Also, different degrees of contradiction functions like IFDCF ($p_{c_{IF}}$) or NDCF (p_{c_N}) can be used. In this chapter, for simplicity, only FDCF (p_{c_F}) has been considered. However, one may always use some complicated degree of contradiction functions, like, $p_{c_{IF}}$ or p_{c_N} to increase the level of accuracy as well as complexity as per their requirements. In the next section, using $p_{d_{IF}}$ and the notion PIFSG has been introduced and some of their homomorphic characteristics have been mentioned.

4.2. Plithogenic Intuitionistic Fuzzy Subgroup

Definition 4.2 Let

$$P_s = (P, \psi, V_\psi, p_{d_{IF}}, p_{c_F})$$

be a PS of a group *U*. Where ψ is an attribute, V_ψ is a range of all attribute's values,

$$p_{d_{IF}} : P \times V_\psi \to [0,1]^2$$

is the corresponding IFDAF and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

is the corresponding FDCF. Then P_s is called a PIFSG of *U* iff

$$p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_\psi\}$$

is an IFSG i.e. in other words iff

$$\forall (m_1,u_1),(m_2,u_2) \in P \times V_\psi$$

the subsequent conditions are fulfilled:

(i) $\alpha(p)_{d_{IF}}(m_1,u_1)\cdot(m_2,u_2) \geq \min\{\alpha(p)_{d_{IF}}(m_1,u_1), \alpha(p)_{d_{IF}}(m_2,u_2)\}$

(ii) $\alpha(p)_{d_{IF}}(m_1,u_1)^{-1} \geq \alpha(p)_{d_{IF}}(m_1,u_1)$

(iii) $\beta(p)_{d_{IF}}(m_1,u_1)\cdot(m_2,u_2) \leq \max\{\beta(p)_{d_{IF}}(m_1,u_1), \beta(p)_{d_{IF}}(m_2,u_2)\}$

Table 7. IFDAF with membership

$\alpha(p)_{d_{IF}}$	*m*	*u*	*mu*	*e*
1	0.4	0.4	0.8	0.2
-1	0.4	0.4	0.5	0.2
i	0	0	0	0
-*i*	0	0	0	0

(iv) $\beta(p)_{d_{IF}}(m_1,u_1)^{-1} \leq \beta(p)_{d_{IF}}(m_1,u_1)$

A set of all PIFSG of a group *U* is denoted as PIFSG(*U*). Note that in Definition 4.2 for simplicity, the attribute value contradiction function p_{c_F} has been chosen. But to generalize Definition 4.2 one may use $p_{c_{IF}}$.

Example 4.3 Let

$$P_s = (P, \psi, V_\psi, p_{d_{IF}}, p_{c_F})$$

bet a PS of a group *U,* where $P = \{1, -1, i, -i\}$ be a cyclic group, ψ be an attribute, $V_\psi = \{m, u, mu, e\}$ be the Klein four-group. Also, let

$$p_{d_{IF}} : P \times V_\psi \to [0,1]^2 \,(\, p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_\psi\}\,)$$

and $p_{c_F} : V_\psi \times V_\psi \to [0,1]$ are respectively the corresponding IFDAF and FDCF, which are given in Table 7, and Table 9.

Table 8. IFDAF with nonmembership

$\beta(p)_{d_{IF}}$	*m*	*u*	*mu*	*e*
1	0.6	0.6	0.2	0.8
-1	0.6	0.6	0.5	0.8
i	1	1	1	1
-*i*	1	1	1	1

Table 9. FDCF

c_{d_F}	*m*	*u*	*mu*	*e*
m	0	0.2	0.9	0.5
u	0.2	0	0.4	0.3
mu	0.9	0.4	0	0.2
e	0.5	0.3	0.2	0

Then $P_s \in$ PIFSG(*U*).

Example 4.4 Let

$$P_s = (P, \psi, V_\psi, p_{d_F}, p_{c_F})$$

bet a PS of a group *U,* where $P = \{m, u, mu, e\}$ be the Klein four-group, ψ be an attribute and

$$V_\psi = \{1, -1, i, -i\}$$

be a cyclic group. Also, let

$$p_{d_{IF}} : P \times V_\psi \to [0,1]^2 \,(\, p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_\psi\}\,)$$

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

are the respective IFDAF and FDCF, which are given in Table 10, Table 11 and Table 12.

4.2.1. Homomorphism on PIFSG

Let ψ be an attribute and V_ψ be the corresponding range of attribute's values and

$$P_s = (P, \psi, V_\psi, p_{d_{IF}}, p_{c_F})$$

Table 10. IFDAF with membership

$\alpha(p)_{d_{IF}}$	**1**	**-1**	*i*	**-***i*
m	0.3	0.4	0.4	0.4
u	0.3	0.4	0.4	0.4
mu	0.3	0.4	0.4	0.4

Table 11. IFDAF with nonmembership

$\beta(p)_{d_{IF}}$	**1**	**-1**	*i*	**-***i*
m	0.7	0.6	0.6	0.6
u	0.7	0.6	0.6	0.6
mu	0.7	0.6	0.6	0.6
e	0.8	0.8	0.8	0.8

$\alpha(p)_{d_{IF}}$	**1**	**-1**	*i*	**-***i*
e	0.2	0.2	0.2	0.2

Table 12. FDCF

c_{d_F}	**1**	**-1**	*i*	**-***i*
1	0	0.5	0.3	0.8
-1	0.5	0	0.7	0.2
i	0.3	0.7	0	1
-*i*	0.8	0.2	1	0

be a PS of a group *U*, where

$$p_{d_{IF}}: P \times V_{\psi} \to [0,1]^2 \,(\, p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_{\psi}\}\,)$$

is the corresponding IFDAF and

$$p_{c_F}: V_{\psi} \times V_{\psi} \to [0,1]$$

is the corresponding FDCF. Also, let *f* be a function defined on $U \cup V_{\psi}$. Then the image of P_s is denoted as

$$P'_s = (P', \psi, V'_{\psi}, p'_{d_{IF}}, p'_{c_F}),$$

where

$$p'_{d_{IF}}: P' \times V'_{\psi} \to [0,1]^2 \,(\, p'_{d_{IF}} = \{((m,u), \alpha(p')_{d_{IF}}, \beta(p')_{d_{IF}}) : (m,u) \in P' \times V'_{\psi}\}\,)$$

is defined as

$$\alpha(p')_{d_{IF}}(m_2, u_2) = \sup_{\substack{m_1 \in f^{-1}(m_2) \\ u_1 \in f^{-1}(u_2)}} \alpha(p)_{d_{IF}}(m_1, u_1),$$

$$\beta(p')_{d_{IF}}(m_2, u_2) = \inf_{\substack{m_1 \in f^{-1}(m_2) \\ u_1 \in f^{-1}(u_2)}} \beta(p)_{d_{IF}}(m_1, u_1).$$

Again, $p'_{c_F}: V'_{\psi} \times V'_{\psi} \to [0,1]$ is defined as

$$p'_{c_F}(u_3, u_4) = \sup_{\substack{u_1 \in f^{-1}(u_3) \\ u_2 \in f^{-1}(u_4)}} p_{c_F}(u_1, u_2).$$

Also, if

$$P'_s = (P', \psi, V'_{\psi}, p'_{d_{IF}}, p'_{c_F})$$

is a PS of $f(U)$ then the preimage of P'_s will be denoted as

$$P_s = (P, \psi, V_\psi, p_{d_{IF}}, p_{c_F}).$$

Here

$$p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_\psi\},$$

where

$$\alpha(p)_{d_{IF}}(m_1, u_1) = \alpha(p')_{d_{IF}}(f(m_1), f(u_1)),$$

$$\beta(p)_{d_{IF}}(m_1, u_1) = \beta(p')_{d_{IF}}(f(m_1), f(u_1)) \forall (m_1, u_1) \in P \times V_\psi$$

and p_{c_F} is defined as

$$\forall (u_1, u_2) \in V_\psi \times V_\psi,\ p_{c_F}(u_1, u_2) = p'_{c_F}(f(u_1), f(u_2)).$$

Theorem 4.4 Homomorphic preimage of a PIFSG is a PIFSG.

Proof: Let ψ be an attribute and V'_ψ be the corresponding range of attribute's values and

$$P'_s = (P', \psi, V'_\psi, p'_{d_{IF}}, p'_{c_F})$$

be a PFSG of a group $f(U)$, where f is a homomorphism on $U \cup V_\psi$. Here,

$$p'_{d_{IF}} : P' \times V'_\psi \to [0,1]^2$$

is defined as

$$p'_{d_{IF}} = \{((m,u), \alpha(p')_{d_{IF}}, \beta(p')_{d_{IF}}) : (m,u) \in P' \times V'_\psi\}.$$

Then preimage of P'_s is denoted as

$P_s = (P, \psi, V_\psi, p_{d_{IF}}, p_{c_F})$.

Here

$p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_\psi\}$,

where

$\alpha(p)_{d_{IF}}(m_1, u_1) = \alpha(p')_{d_{IF}}(f(m_1), f(u_1))$,

$\beta(p)_{d_{IF}}(m_1, u_1) = \beta(p')_{d_{IF}}(f(m_1), f(u_1)) \forall (m_1, u_1) \in P \times V_\psi$

and p_{c_F} is defined as

$\forall (u_1, u_2) \in V_\psi \times V_\psi,\ p_{c_F}(u_1, u_2) = p'_{c_F}(f(u_1), f(u_2))$.

Then

$\forall (m_1, u_1), (m'_1, u'_1) \in P \times V_\psi$,

$\alpha(p)_{d_{IF}}((m_1, u_1) \cdot (m'_1, u'_1)) = \alpha(p)_{d_{IF}}(m_1 m'_1, u_1 u'_1)$

$= \alpha(p')_{d_{IF}}(f(m_1 m'_1), f(u_1 u'_1))$

$= \alpha(p')_{d_{IF}}(f(m_1) f(m'_1), f(u_1) f(u'_1))$

$= \alpha(p')_{d_{IF}}((f(m_1), f(u_1)) \cdot (f(m'_1), f(u'_1)))$

$\geq \min\{\alpha(p')_{d_{IF}}(f(m_1), f(u_1)),\ \alpha(p')_{d_{IF}}(f(m'_1), f(u'_1))\}$

$$= \min\{\alpha(p)_{d_{IF}}(m_1, u_1),\ \alpha(p)_{d_{IF}}(m_1', u_1')\}$$

Also,

$$\alpha(p)_{d_{IF}}(m_1, u_1)^{-1} = \alpha(p)_{d_{IF}}(m_1^{-1}, u_1^{-1})$$

$$= \alpha(p')_{d_{IF}}(f(m_1^{-1}), f(u_1^{-1}))$$

$$= \alpha(p')_{d_{IF}}(f(m_1)^{-1}, f(u_1)^{-1})$$

$$= \alpha(p')_{d_{IF}}(f(m_1), f(u_1))^{-1}$$

$$\geq \alpha(p')_{d_{IF}}(f(m_1), f(u_1))$$

$$= \alpha(p)_{d_{IF}}(m_1, u_1)$$

So, by and $\alpha(p)_{d_F}$ satisfies conditions (i) and (ii) of Definition 4.2.

Again,

$$\forall (m_1, u_1), (m_1', u_1') \in P \times V_\psi,$$

$$\beta(p)_{d_{IF}}((m_1, u_1) \cdot (m_1', u_1')) = \beta(p)_{d_{IF}}(m_1 m_1', u_1 u_1')$$

$$= \beta(p')_{d_{IF}}(f(m_1 m_1'), f(u_1 u_1'))$$

$$= \beta(p')_{d_{IF}}(f(m_1) f(m_1'), f(u_1) f(u_1'))$$

$$= \beta(p')_{d_{IF}}((f(m_1), f(u_1)) \cdot (f(m_1'), f(u_1')))$$

$$\leq \max\{\beta(p')_{d_{IF}}(f(m_1), f(u_1)),\ \beta(p')_{d_{IF}}(f(m_1'), f(u_1'))\}$$

$$= \max\{\beta(p)_{d_{IF}}(m_1, u_1),\ \beta(p)_{d_{IF}}(m_1', u_1')\}$$

Also,

$$\beta(p)_{d_{IF}}(m_1, u_1)^{-1} = \beta(p)_{d_{IF}}(m_1^{-1}, u_1^{-1})$$

$$= \beta(p')_{d_{IF}}(f(m_1^{-1}), f(u_1^{-1}))$$

$$= \beta(p')_{d_{IF}}(f(m_1)^{-1}, f(u_1)^{-1})$$

$$= \beta(p')_{d_{IF}}(f(m_1), f(u_1))^{-1}$$

$$\leq \beta(p')_{d_{IF}}(f(m_1), f(u_1))$$

$$= \beta(p)_{d_{IF}}(m_1, u_1)$$

So, by and $\beta(p)_{d_{IF}}$ satisfies conditions (iii) and (iv) of Definition 4.2.

Hence,

$$p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_\psi\}$$

forms an IFSG. Again, as p'_{c_F} is a FS its preimage under i.e. p_{c_F} is a FS and hence, P_s is a PIFSG of *U*.

Theorem 4.5 Homomorphic image of a PIFSG is a PIFSG (provided for $\alpha(p)_{d_{IF}}$ and p_{c_F} supremum property hold and for $\beta(p)_{d_{IF}}$ infimum property holds).

Proof: Let ψ be an attribute and V_ψ be the corresponding range of attribute's values and

$$P_s = (P, \psi, V_\psi, p_{d_{IF}}, p_{c_F})$$

be a PFSG of a group *U* and *f* be a homomorphism defined on $U \cup V_\psi$.

Here,

$$p_{d_{IF}} : P \times V_{\psi} \rightarrow [0,1]^2$$

is defined as

$$p_{d_{IF}} = \{((m,u), \alpha(p)_{d_{IF}}, \beta(p)_{d_{IF}}) : (m,u) \in P \times V_{\psi}\}.$$

Then the image of P_s is denoted as

$$P'_s = (P', \psi, V'_{\psi}, p'_{d_{IF}}, p'_{c_F}).$$

Here

$$p'_{d_{IF}} = \{((m,u), \alpha(p')_{d_{IF}}, \beta(p')_{d_{IF}}) : (m,u) \in P' \times V'_{\psi}\},$$

where

$$\alpha(p')_{d_{IF}}(m_2, u_2) = \sup_{\substack{m_1 \in f^{-1}(m_2) \\ u_1 \in f^{-1}(u_2)}} \alpha(p)_{d_{IF}}(m_1, u_1)$$

and

$$\beta(p')_{d_{IF}}(m_2, u_2) = \inf_{\substack{m_1 \in f^{-1}(m_2) \\ u_1 \in f^{-1}(u_2)}} \beta(p)_{d_{IF}}(m_1, u_1).$$

Also, p'_{c_F} is defined as

$$p'_{c_F}(u_3, u_4) = \sup_{\substack{u_1 \in f^{-1}(u_3) \\ u_2 \in f^{-1}(u_4)}} p_{c_F}(u_1, u_2).$$

Then

$\forall\, (f(m_1), f(u_1)), (f(m_1'), f(u_1')) \in P' \times V_\psi',$

$\exists\, m_0 \in f^{-1}(f(m_1))$ and $\exists\, u_0 \in f^{-1}(f(u_1))$

such that

$$\alpha(p)_{d_{IF}}(m_0, u_0) = \sup_{\substack{m_t \in f^{-1}(f(m_1)) \\ u_t \in f^{-1}(f(u_1))}} \alpha(p)_{d_{IF}}(m_t, u_t).$$

Also,

$\exists\, m_0' \in f^{-1}(f(m_1'))$ and $\exists\, u_0' \in f^{-1}(f(u_1'))$

such that

$$\alpha(p)_{d_{IF}}(m_0', u_0') = \sup_{\substack{m_t \in f^{-1}(f(m_1')) \\ u_t \in f^{-1}(f(u_1'))}} \alpha(p)_{d_{IF}}(m_t, u_t).$$

So,

$$\alpha(p')_{d_{IF}}(f(m_1), f(u_1)) \cdot (f(m_1'), f(u_1')) = \alpha(p')_{d_{IF}}(f(m_1) \cdot f(m_1'), f(u_1) \cdot f(u_1'))$$

$$= \sup_{\substack{m_z \in f^{-1}(f(m_1) \cdot f(m_1')) \\ u_z \in f^{-1}(f(u_1) \cdot f(u_1'))}} \alpha(p)_{d_{IF}}(m_z, u_z)$$

$$\geq \min\{\alpha(p)_{d_{IF}}(m_0, u_0), \alpha(p)_{d_{IF}}(m_0', u_0')\}$$

$$= \min\{\alpha(p')_{d_{IF}}(f(m_1), f(u_1)), \alpha(p')_{d_{IF}}(f(m_1'), f(u_1'))\}$$

Again,

$$\alpha(p')_{d_{IF}}(f(m_1), f(u_1))^{-1} = \alpha(p')_{d_{IF}}(f(m_1)^{-1}, f(u_1)^{-1})$$

$$= \sup_{\substack{m_z \in f^{-1}(f(m_1)^{-1}) \\ u_z \in f^{-1}(f(u_1)^{-1})}} \alpha(p)_{d_{IF}}(m_z, u_z)$$

$$= \alpha(p)_{d_{IF}}(m_0{}^{-1}, u_0{}^{-1})$$

$$= \alpha(p)_{d_{IF}}(m_0, u_0)^{-1}$$

$$\geq \alpha(p)_{d_{IF}}(m_0, u_0)$$

$$= \alpha(p')_{d_{IF}}(f(m_1), f(u_1))$$

So, by and $\alpha(p')_{d_{IF}}$ satisfies conditions (i) and (ii) of Definition 4.2.

Again, let

$$\forall\, (f(m_1), f(u_1)), (f(m_1'), f(u_1')) \in P' \times V_\psi',$$

$\exists\, m_0 \in f^{-1}(f(m_1))$ and $\exists\, u_0 \in f^{-1}(f(u_1))$

such that

$$\beta(p)_{d_{IF}}(m_0, u_0) = \inf_{\substack{m_t \in f^{-1}(f(m_1)) \\ u_t \in f^{-1}(f(u_1))}} \beta(p)_{d_{IF}}(m_t, u_t).$$

Also,

$\exists\, m_0' \in f^{-1}(f(m_1'))$ and $\exists\, u_0' \in f^{-1}(f(u_1'))$

such that

$$\beta(p)_{d_{IF}}(m_0', u_0') = \inf_{\substack{m_t \in f^{-1}(f(m_1')) \\ u_t \in f^{-1}(f(u_1'))}} \beta(p)_{d_{IF}}(m_t, u_t).$$

$$\beta(p')_{d_{IF}}(f(m_1), f(u_1)) \cdot (f(m_1'), f(u_1')) = \beta(p')_{d_{IF}}(f(m_1) \cdot f(m_1'), f(u_1) \cdot f(u_1'))$$

$$= \inf_{\substack{m_z \in f^{-1}(f(m_1) \cdot f(m_1')) \\ u_z \in f^{-1}(f(u_1) \cdot f(u_1'))}} \beta(p)_{d_{IF}}(m_z, u_z)$$

$$\leq \max\{\beta(p)_{d_{IF}}(m_0, u_0), \beta(p)_{d_{IF}}(m_0', u_0')\}$$

$$= \max\{\beta(p')_{d_{IF}}(f(m_1), f(u_1)), \beta(p')_{d_{IF}}(f(m_1'), f(u_1'))\}$$

Again,

$$\beta(p')_{d_{IF}}(f(m_1), f(u_1))^{-1} = \beta(p')_{d_{IF}}(f(m_1)^{-1}, f(u_1)^{-1})$$

$$= \inf_{\substack{m_z \in f^{-1}(f(m_1)^{-1}) \\ u_z \in f^{-1}(f(u_1)^{-1})}} \alpha(p)_{d_{IF}}(m_z, u_z)$$

$$= \beta(p)_{d_{IF}}(m_0{}^{-1}, u_0{}^{-1})$$

$$= \beta(p)_{d_{IF}}(m_0, u_0)^{-1}$$

$$\leq \beta(p)_{d_{IF}}(m_0, u_0)$$

$$= \beta(p')_{d_{IF}}(f(m_1), f(u_1))$$

So, by and $\beta(p')_{d_F}$ satisfies conditions (iii) and (iv) of Definition 4.2.

Hence,

$$p'_{d_{IF}} = \{((m,u), \alpha(p')_{d_{IF}}, \beta(p')_{d_{IF}}) : (m,u) \in P' \times V'_\psi\}$$

forms an IFSG. Again, as p_{c_F} is a FS its preimage under f i.e. p_{c_F} is a FS and hence, P_s is a PIFSG of U.

Notice that in Definition 4.2 IFDAF ($p_{d_{IF}}$) and FDCF (p_{c_F}) has been used. But, here instead of using an intuitionistic fuzzy setting one may wish to use a neutrosophic environment. In that case, the degree of appurtenance functions will be NDAF (p_{d_N}). Also, one may wish to use other degrees of contradiction functions like IFDCF ($p_{c_{IF}}$) or NDCF (p_{c_N}). In the next section using p_{d_N} and p_{c_F} the notion PNSG has been introduced and some of their homomorphic characteristics have been discussed.

4.3. Plithogenic Neutrosophic Subgroup

Definition 4.3 Let

$$P_s = (P, \psi, V_\psi, p_{d_N}, p_{c_F})$$

be a PS of a group *U*. Where ψ is an attribute, V_ψ is a range of all attribute's values,

$$p_{d_N} : P \times V_\psi \to [0,1]^3$$

is the corresponding NDAF and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

is the corresponding FDCF. Then P_s is called a PNSG of *U* iff

$$p_{d_N} = \{((m,u), t(p)_{d_N}, i(p)_{d_N}, f(p)_{d_N}) : (m,u) \in P \times V_\psi\}$$

is a neutrosophic subgroup i.e. in other words iff

$$\forall (m_1, u_1), (m_2, u_2) \in P \times V_\psi,$$

the subsequent conditions are fulfilled:

(i) $t(p)_{d_{IF}}((m_1,u_1)\cdot(m_2,u_2)^{-1}) \geq \min\{t(p)_{d_{IF}}(m_1,u_1), t(p)_{d_{IF}}(m_2,u_2)\}$

Table 13. NDAF with truth

$t(p)_{d_N}$	m	u	mu	e'
a	0.4	0.4	0.5	0.2

Table 14. NDAF with indeterminacy

$i(p)_{d_N}$	m	u	mu	e'
a	0.3	0.3	0.3	0.2
e	0.4	0.4	0.5	0.2

$t(p)_{d_N}$	m	u	mu	e'
e	0.4	0.4	0.6	0.2

(ii) $i(p)_{d_{IF}}((m_1, u_1)\cdot(m_2, u_2)^{-1}) \geq \min\{i(p)_{d_{IF}}(m_1, u_1), i(p)_{d_{IF}}(m_2, u_2)\}$

(iii) $f(p)_{d_{IF}}((m_1, u_1)\cdot(m_2, u_2)^{-1}) \leq \max\{f(p)_{d_{IF}}(m_1, u_1), f(p)_{d_{IF}}(m_2, u_2)\}$

A set of all PNSG of a group *U* is denoted as PNSG(*U*). Note that in Definition 4.3 for simplicity the attribute value contradiction function p_{c_F} has been chosen. But to generalize Definition 4.3 one may use p_{c_N}.

Example 4.5 Let

Table 15. NDAF with falsity

$f(p)_{d_N}$	m	u	mu	e'
a	0.6	0.6	0.5	0.8
e	0.6	0.6	0.4	0.8

Table 16. FDCF

p_{c_F}	m	u	mu	e'
m	0	0.1	0.9	1
u	0.1	0	0.5	0.7
mu	0.9	0.5	0	0.2
e'	1	0.7	0.2	0

$$P_s = (P, \psi, V_\psi, p_{d_N}, p_{c_F})$$

bet a PS of a group *U,* where $P = \{a, e\}$ ($a^2 = e$) be a group, ψ be an attribute, $V_\psi = \{m, u, mu, e'\}$ be the Klein's four group. Also, let

$$p_{d_N} : P \times V_\psi \to [0,1]^3 \,(\, p_{d_N} = \{((m,u),\, t(p)_{d_N}, i(p)_{d_N}, f(p)_{d_N}) : (m,u) \in P \times V_\psi\}\,)$$

and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

are respectively the corresponding NDAF and FDCF mentioned in Table 13, Table 14, Table 15 and Table 16.

Then $P_s \in$ PNSG(*U*).

4.3.1. Homomorphism on PNSG

Let ψ be an attribute and V_ψ be the corresponding range of attribute's values and

$$P_s = (P, \psi, V_\psi, p_{d_N}, p_{c_F})$$

be a PS of a group *U,* where

$$p_{d_N} : P \times V_\psi \to [0,1]^3 \,(\, p_{d_N} = \{((m,u), t(p)_{d_N}, i(p)_{d_N}, f(p)_{d_N}) : (m,u) \in P \times V_\psi\}\,)$$

is the corresponding NDAF and

$$p_{c_F} : V_\psi \times V_\psi \to [0,1]$$

is the corresponding FDCF. Also, let *g* be a function defined on $U \cup V_\psi$. Then the image of P_s is denoted as

$$P'_s = (P', \psi, V'_\psi, p'_{d_N}, p'_{c_F}),$$

where

$$p'_{d_N} : P' \times V'_\psi$$
$$\to [0,1]^3 \, (p'_{d_N} = \{((m,u), t(p')_{d_N}, i(p')_{d_N}, f(p')_{d_N}) : (m,u) \in P' \times V'_\psi\})$$

is defined as

$$t(p')_{d_N}(m_2, u_2) = \sup_{\substack{m_1 \in g^{-1}(m_2) \\ u_1 \in g^{-1}(u_2)}} t(p)_{d_N}(m_1, u_1),$$

$$i(p')_{d_N}(m_2, u_2) = \sup_{\substack{m_1 \in g^{-1}(m_2) \\ u_1 \in g^{-1}(u_2)}} i(p)_{d_N}(m_1, u_1),$$

$$f(p')_{d_N}(m_2, u_2) = \inf_{\substack{m_1 \in g^{-1}(m_2) \\ u_1 \in g^{-1}(u_2)}} f(p)_{d_N}(m_1, u_1)$$

and $p'_{c_F} : V'_\psi \times V'_\psi \to [0,1]$ is defined as

$$p'_{c_F}(u_3, u_4) = \sup_{\substack{u_1 \in g^{-1}(u_3) \\ u_2 \in g^{-1}(u_4)}} p_{c_F}(u_1, u_2).$$

Also, if

$$P'_s = (P', \psi, V'_\psi, p'_{d_N}, p'_{c_F})$$

is a PS of $\mathrm{g}(U)$ then the preimage of P'_s will be denoted as

$P_s = (P, \psi, V_\psi, p_{d_N}, p_{c_F})$.

Here

$$p_{d_N} = \{((m,u), t(p)_{d_N}, i(p)_{d_N}, f(p)_{d_N}) : (m,u) \in P \times V_\psi\},$$

where

$$\forall (m_1, u_1) \in P \times V_\psi, t(p)_{d_N}(m_1, u_1) = t(p')_{d_N}(\mathrm{g}(m_1), \mathrm{g}(u_1)),$$

$$i(p)_{d_N}(m_1, u_1) = i(p')_{d_N}(\mathrm{g}(m_1), \mathrm{g}(u_1)),$$

$$f(p)_{d_N}(m_1, u_1) = f(p')_{d_N}(g(m_1), g(u_1))$$

and p_{c_F} is defined as

$$\forall (u_1, u_2) \in V_\psi \times V_\psi, p_{c_F}(u_1, u_2) = p'_{c_F}(\mathrm{g}(u_1), \mathrm{g}(u_2)).$$

Theorem 4.6 Homomorphic preimage of a PNSG is a PNSG.

Proof: Using Theorem 4.2 and Theorem 4.4 this can be easily proved.

Theorem 4.7 Homomorphic image of a PNSG is a PNSG (provided for $f(p)_{d_N}$, $i(p)_{d_N}$ and p_{c_F} supremum property hold and for $f(p)_{d_N}$ infimum property holds).

Proof: Using Theorem 4.3 and Theorem 4.5 this can be easily proved.

CONCLUSION

Group theory is a fundamental part of abstract algebra. To study algebraic characteristics of any object we need to understand functions which preserve its algebraic characteristics i.e. we need to study homomorphism. The notion of plithogenic subgroup is nothing but generalization of crisp subgroup, fuzzy subgroup, intuitionistic fuzzy subgroup, and neutrosophic subgroup. Hence, plithogenic subgroups have been introduced and the effects of homomorphism on them have been studied. Most of the definitions mentioned in this chapter can further be generalized by using general T-norm and T-conorm. Again, most of the theorems, propositions mentioned here can also be proved by using those triangular norms. In future, one may extend research by introducing normal forms of different plithogenic subgroups i.e. normal plithogenic fuzzy subgroup, normal plithogenic intuitionistic fuzzy subgroup, and normal plithogenic neutrosophic subgroup. Again, one can study their fundamental properties and homomorphic characteristics. In addition, one may introduce the notion of soft set theory in plithogenic subgroup and further generalize them.

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Chapter 9
Minimal Spanning Tree in Cylindrical Single-Valued Neutrosophic Arena

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ABSTRACT

In this chapter, the concept of cylindrical single-valued neutrosophic number whenever two of the membership functions, which serve a crucial role for uncertainty conventional problem, are dependent to each other is developed. It also introduces a new score and accuracy function for this special cylindrical single valued neutrosophic number, which are useful for crispification. Further, a minimal spanning tree execution technique is proposed when the numbers are in cylindrical single-valued neutrosophic nature. This noble idea will help researchers to solve daily problems in the vagueness arena.

1. INTRODUCTION

The idea of vagueness theory was first invented by (Zadeh, 1965) in his paper. After that, the paper of (Atanassov, 1986) presents the remarkable concept of an intuitionistic fuzzy set in the field of uncertainty theory in which the perception of membership function and non-membership function both are considered. Day by day, as development goes on, researchers invented triangular (Chen S., 1994) and (K.K.Yen, S.Ghoshray, & G.Roig, 1999), trapezoidal (Chen & Chen, 2007) and (S.Abbasbandy & T.Hajjari, 2009), pentagonal (R.Helen & G.Uma, 2015)and (Chakraborty A., et al., 2018) and Later (Maity, Chakraborty, Dey, Mondal, & Alam,

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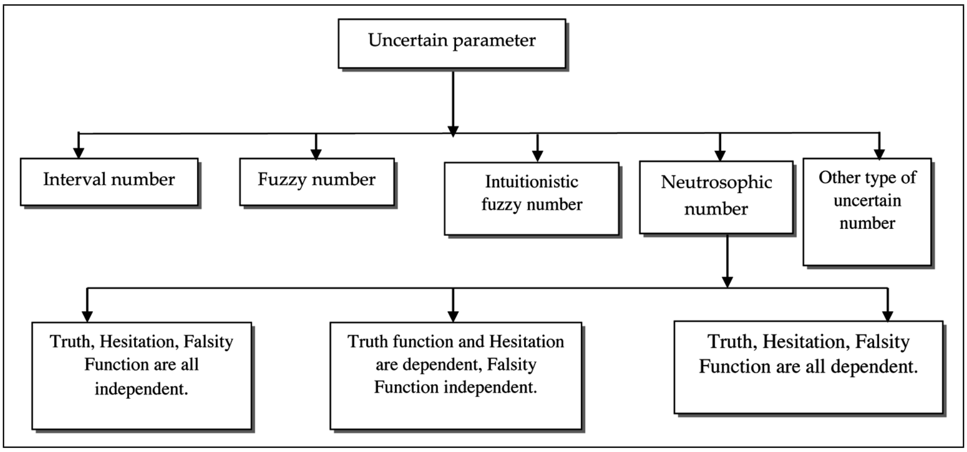
2018) invented heptagonal fuzzy number which are useful and plays a key role in mathematical modeling and statistical computational problem. Further, (F & XH, 2007), developed the concept of triangular intuionistic fuzzy set and (Li & Chen, 2015) introduced the idea of trapezoidal intuinistic fuzzy set. Later, (Smarandache, 1998)) invented the concept of neutrosophic set where there are three disjunctive components are considered namely i) truth, ii) indeterminacy, iii)falsity. Neutrosophic concept is a very effective & an useful idea in real life problems. Later, (Chakraborty A., Mondal, Ahmadian, Senu, Alam, & Salahshour, 2018) developed different form of triangular neutrosophic number and its application. Recently, (Chakraborty A., et al., 2019) introduced the concept of triangular bipolar neutrosophic number and its application in multi criteria decision making problem. It was defined that in case of neutrosophic set, the sum of the truth function, falsity function and hesitation function is less than or equal to 2, whenever any two of them are dependent, while the third one is independentfrom them. Now, the conception of cylindrical neutrosophic single-valued number has been imported which is fur more advanced than accustomed conception of dependence neutrosophic number.

All those neutrosophic numbers whose addition of the squares of two dependent membership functions are less than or equals to 1^2 and independent membership function less or equals to 1 in the 1st quadrant of three dimensional co-ordinate axes are considered as cylindrical single-valued neutrosophic number. Graphically it signifies the $\frac{1}{4}$ th part of a total unit radius cylinder. Thus, the idea of cylindrical single-valued neutrosophic number or cylindrical dependence fuzzy set in neutrosophic domain is being established.

Obviously it will contain all general neutrosophic number which will satisfy cylindrical single-valued neutrosophic number. In this article, classification and operation on cylindrical single-valued neutrosophic fuzzy number and a new important score and accuracy function is being developed which plays useful roles in vagueness theory. Later, it can be applied in various fields of science and engineering problems.

Minimal spanning tree is a very crucial topic in graph theory domain. (Ye, 2014) introduced Single valued neutrosophic minimum spanning tree and its clustering method. (Mandal & Basu, 2016) proposed an approach based on similarity measure for searching the optimum spanning tree problems in a neutrosophic environment considering the inconsistency, incompleteness and indeterminacy of the information. (Mullai, Broumi, & Stephen, 2017) discussed about the minimum spanning tree problem in bipolar neutrosophic environment. Later, (Broumi, Talea, Smarandache, & Bakali, 2016) (Broumi S., Bakali, Talea, Smarandache, & Kishore Kumar, 2017) discussed on Shortest path problem on single valued neutrosophic graphs. Also, (Broumi, Smarandache, Talea, & Bakali, 2016) developed Decision-making method based on the interval valued neutrosophic graph and after that (Kandasamy, 2016)

Box 1.



introduceddouble-valued neutrosophic sets, their minimum spanning trees. Recently, (Broumi S., Bakali, Talea, Smarandache, & Vladareanu, 2016) developed Dijkstra algorithm for solving neutrosophic shortest path problem is being invented.

But if the edges of the graph are in cylindrical single-valued neutrosophic domain then what is the process to solve this kind of problem ? The main purpose of this paper is to find out a minimal spanning tree from the graph where all the edges exhibit cylindrical single valued neutrosophic number. The proposed algorithm will described the usefulness of score and accuracy function value to solve this interesting problem. The classification of uncertainty parameters are shown in Box 1.

2. MATHEMATICAL PRELIMINARIES

2.1 Definition: Neutrosophic Set A set $\widetilde{N_{neu}}$ in the universal discourse *X*, symbolically denoted by *x*, it is called a neutrosophic set if

$$\widetilde{N_{neu}} = \left\{x;\left[\alpha_{\widetilde{N_{neu}}}\left(x\right),\beta_{\widetilde{N_{neu}}}\left(x\right),\gamma_{\widetilde{N_{neu}}}\left(x\right)\right]:x \in X\right\},$$

where

$$\alpha_{\widetilde{N_{neu}}}\left(x\right): X \to \left[0,1\right]$$

is said to be the truth membership function, which represents the degree of belongingness,

$$\beta_{\widetilde{N_{neu}}}\left(x\right): X \to \left[0,1\right]$$

is said to be the indeterminacy membership, which represents the degree of uncertainty, and

$$\gamma_{\widetilde{N_{neu}}}\left(x\right): X \to \left[0,1\right]$$

is said to be the falsity membership, which represents the degree of non-belongingness of the decision maker.

$$\alpha_{\widetilde{N_{neu}}}\left(x\right), \beta_{\widetilde{N_{neu}}}\left(x\right) \,\&\, \gamma_{\widetilde{N_{neu}}}\left(x\right)$$

exhibits the following relation:

$$0 \le \alpha_{\widetilde{N_{neu}}}\left(x\right) + \beta_{\widetilde{N_{neu}}}\left(x\right) + \gamma_{\widetilde{N_{neu}}}\left(x\right) \le 3\,.$$

2.2 Definition: Single-Valued Neutrosophic Set: A Neutrosophic set $\widetilde{N_{neu}}$ in the definition 2.1 is said to be a single-Valued Neutrosophic Set $\left(\widetilde{sigN_{neu}}\right)$ if *x* is a single-valued independent variable.

$$\widetilde{SigN_{neu}} = \left\{x;\left[\alpha_{\widetilde{SigN_{neu}}}\left(x\right), \beta_{\widetilde{SigN_{neu}}}\left(x\right), \gamma_{\widetilde{SigN_{neu}}}\left(x\right)\right]: x \in X\right\},$$

where

$$\alpha_{\widetilde{SigN_{neu}}}\left(x\right), \beta_{\widetilde{SigN_{neu}}}\left(x\right) \,\&\, \gamma_{\widetilde{SigN_{neu}}}\left(x\right)$$

denoted the concept of truth, indeterminacy and falsity memberships function respectively.

If there exist three points $a_0, b_0 \,\&\, c_0$, for which

$$\alpha_{\widetilde{SigN_{neu}}}\left(a_0\right)=1,\beta_{\widetilde{SigN_{neu}}}\left(b_0\right)=1\,\&\,\gamma_{\widetilde{SigN_{neu}}}\left(c_0\right)=1,$$

then the $\widetilde{sigN_{neu}}$ is called neut-normal.

$\widetilde{signeuS}$ is called neut-convex, which implies that $\widetilde{sigN_{neu}}$ is a subset of a real line by satisfying the following conditions:

1. $\alpha_{\widetilde{SigN_{neu}}}\Omega a_1+\left(1-\Omega\right)a_2\geq\min\alpha_{\widetilde{sigN_{neu}}}\left(a_1\right),\alpha_{\widetilde{sigN_{neu}}}\left(a_2\right)$
2. $\beta_{\widetilde{sigN_{neu}}}\Omega a_1+\left(1-\Omega\right)a_2\leq\max\beta_{\widetilde{sigN_{neu}}}\left(a_1\right),\beta_{\widetilde{sigN_{neu}}}\left(a_2\right)$
3. $\gamma_{\widetilde{sigN_{neu}}}\Omega a_1+\left(1-\Omega\right)a_2\leq\max\gamma_{\widetilde{sigN_{neu}}}\left(a_1\right),\gamma_{\widetilde{sigN_{neu}}}\left(a_2\right)$,

where

$$a_1\,\&\,a_2\in\mathbb{R}\;and\;\Omega\in\left[0,1\right]$$

2.3 Definition: Single Valued Neutrosophic Number: Single Valued Neutrosophic Number $\left(\tilde{z}\right)$ is defined as

$$\tilde{z}=\left[\left(e^1,f^1,g^1,h^1\right);\theta\right],\left[\left(e^2,f^2,g^2,h^2\right);\vartheta\right],\left[\left(e^3,f^3,g^3,h^3\right);\varphi\right]$$

where $\theta,\vartheta,\varphi\in\left[0,1\right]$, the truth membership function $\left(\alpha_{\tilde{z}}\right):\mathbb{R}\rightarrow\left[0,\theta\right]$, the indeterminacy membership function $\left(\beta_{\tilde{z}}\right):\mathbb{R}\rightarrow\left[\vartheta,1\right]$ and the falsity membership function $\left(\gamma_{\tilde{z}}\right):\mathbb{R}\rightarrow\left[\varphi,1\right]$ is given as:

$$\alpha_{\tilde{z}}\left(x\right)=\begin{cases}\tau_{\tilde{z}l}\left(x\right) & e^1\leq x\leq f^1\\ \theta & f^1\leq x\leq g^1\\ \tau_{\tilde{z}u}\left(x\right) & g^1\leq x\leq h^1\\ 0 & otherwise\end{cases}$$

$$\beta_{\tilde{z}}(x) = \begin{cases} \iota_{\tilde{z}l}(x) & e^2 \le x \le f^2 \\ \vartheta & f^2 \le x \le g^2 \\ \iota_{\tilde{z}u}(x) & g^2 \le x \le h^2 \\ 1 & otherwise \end{cases}$$

$$\gamma_{\tilde{z}}(x) = \begin{cases} \varepsilon_{\tilde{z}l}(x) & e^3 \le x \le f^3 \\ \varphi & f^3 \le x \le g^3 \\ \varepsilon_{\tilde{z}u}(x) & g^3 \le x \le h^3 \\ 1 & otherwise \end{cases}$$

2.4 Definition: **Neutrosophic Set: When two membership functions are dependent:** A set $\widetilde{N_{neu}}$ in the universal discourse *X*, symbolically denoted by *x*, it is called a neutrosophic set if

$$\widetilde{N_{neu}} = \left\{x; \left[\alpha_{\widetilde{N_{neu}}}(x), \beta_{\widetilde{N_{neu}}}(x), ^3{}_{\widetilde{N_{neu}}}(x)\right] \vdots x \in X\right\},$$

where

$$\alpha_{\widetilde{N_{neu}}}(x) : X \to [0,1]$$

is said to be the truth membership function, which represents the degree of belongingness,

$$\beta_{\widetilde{N_{neu}}}(x) : X \to [0,1]$$

is said to be the indeterminacy membership, which represents the degree of uncertainty, and

$$\gamma_{\widetilde{N_{neu}}}(x) : X \to [0,1]$$

is said to be the falsity membership, which represents the degree of non-belongingness of the decision maker.

$$\alpha_{\widetilde{N_{neu}}}(x), \beta_{\widetilde{N_{neu}}}(x) \,\&\, \gamma_{\widetilde{N_{neu}}}(x)$$

exhibits the followingrelation:

$$0 \le \alpha_{\widetilde{N_{neu}}}(x) + \beta_{\widetilde{N_{neu}}}(x) + \gamma_{\widetilde{N_{neu}}}(x) \le 2\,.$$

2.5 Definition: Cylindrical Neutrosophic single-valued Set: When two membership functions are dependent: A set $\widetilde{CylS}$ in the universal discourse *X*, symbolically denoted by *x*, it is called a cylindrical neutrosophic set if

$$\widetilde{CylS} = \left\{x; \left[\pi_{\widetilde{CylS}}(x), \vartheta_{\widetilde{CylS}}(x), Æ_{\widetilde{CylS}}(x)\right] \vdots\, x \in X\right\},$$

where

$$\pi_{\widetilde{CylS}}(x) : X \to [0,1]$$

is called the truth membership function,

$$\vartheta_{\widetilde{CnS}}(x) : X \to [0,1]$$

is called the hesitation membership and

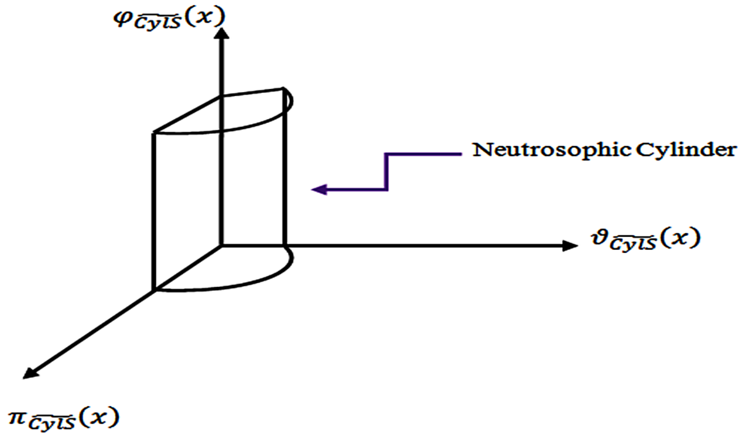
$$\varphi_{\widetilde{CylS}}(x) : X \to [0,1]$$

known as falsity membership function.

$$\pi_{\widetilde{CylS}}(x), \vartheta_{\widetilde{CylS}}(x) \,\&\, \varphi_{\widetilde{CylS}}(x)$$

exhibits the following relation:

Figure 1. Cylindrical single valued neutrosophic fuzzy set



$$\left(\pi_{\widetilde{CylS}}\left(x\right)\right)^2 + \left(\vartheta_{\widetilde{CylS}}\left(x\right)\right)^2 \leq 1^2, \varphi_{\widetilde{CylS}}\left(x\right) \leq 1 .$$

For convenience,

$$Cyl_n = \left[\pi_{\widetilde{CylS}}\left(x\right), \vartheta_{\widetilde{CylS}}\left(x\right), \varphi_{\widetilde{CylS}}\left(x\right)\right]$$

is defined as a Cylindrical neutrosophic single- valued number ($CylNFN$), generally denoted as,

$$Cyl_n = \left(\pi_{Cyl}, \vartheta_{Cyl}, \varphi_{Cyl}\right).$$

3. PROPOSED SCORE AND ACCURACY FUNCTION

In this section, a score function is defined as follows.

For any $CylNFN$,

$$Cyl_{N_{neu}} = \left(\alpha_{CylN_{neu}}, \beta_{CylN_{neu}}, \gamma_{CylN_{neu}}\right)$$

$$S_{Cyl_{N_{neu}}} = \frac{2\left(\alpha_{CylN_{neu}}\right)^2 - \left(\beta_{CylN_{neu}}\right)^2 - \left(\gamma_{CylN_{neu}}\right)^2}{2}$$

Where $S_{Cyl_{N_{neu}}} \in [-1,1]$ and Accuracy function is described as,

$$A_{Cyl_{neu}} = \frac{2\left(\alpha_{CylN_{neu}}\right)^2 + \left(\beta_{CylN_{neu}}\right)^2 + \left(\gamma_{CylN_{neu}}\right)^2}{2},$$

where $A_{Cyl_{N_{neu}}} \in [0,2)$

Here,

If $Cyl_{N_{neu}} = (1,0,0)$ then, $S_{Cyl_{N_{neu}}} = 1$ and $A_{Cyl_{N_{neu}}} = 1$

If $Cyl_{N_{neu}} = (0,1,1)$ then, $S_{Cyl_{N_{neu}}} = -1$ and $A_{Cyl_{N_{neu}}} = 1$

If $Cyl_{N_{neu}} = (0,0,0)$ then, $S_{Cyl_{N_{neu}}} = 0$ and $A_{Cyl_{N_{neu}}} = 0$

For any two Cylindrical neutrosophic fuzzy number

$$Cyl_{neu1} = \left(\alpha_{CylN_{neu}1}, \beta_{CylN_{neu}1}, \gamma_{CylN_{neu}1}\right)$$

and

$$Cyl_{N_{neu}2} = \left(\alpha_{CylN_{neu}2}, \beta_{CylN_{neu}2}, \gamma_{CylN_{neu}2}\right)$$

if,

1. $S_{Cyl_{N_{neu}1}} > S_{Cyl_{N_{neu}2}}$, then $Cyl_{N_{neu}1} > Cyl_{N_{neu}2}$
2. $S_{Cyl_{N_{neu}1}} < S_{Cyl_{N_{neu}2}}$, then $Cyl_{N_{neu}1} < Cyl_{N_{neu}2}$
3. $S_{Cyl_{N_{neu}1}} = S_{Cyl_{N_{neu}2}}$, then
   a. $A_{Cyl_{N_{neu}1}} > A_{Cyl_{N_{neu}2}}$, then $Cyl_{N_{neu}1} > Cyl_{N_{neu}2}$

b. $A_{Cyl_{N_{neu}1}} < A_{Cyl_{N_{neu}2}}$, then $Cyl_{N_{neu}1} < Cyl_{N_{neu}2}$

c. $A_{Cyl_{N_{neu}1}} = A_{Cyl_{N_{neu}2}}$, then $Cyl_{N_{neu}1} \sim Cyl_{N_{neu}2}$

4. MINIMAL SPANNING TREE PROBLEM IN CYLINDRICAL SINGLE- VALUED NEUTROSOPHIC ENVIRONMENT

- **Spanning Tree:** A tree is T said to be a spanning tree of a connected graph G if T is a subgraph of G and T contains all the vertices of G.
- **Minimal Spanning Tree:** A spanning tree having the smallest weight in G is called a minimal spanning tree.

Let, a graph in a cylindrical single-valued neutrosophic environment. Now, proposed algorithm is to find out the minimal spanning tree from a graph, whose weights are cylindrical single valued neutrosophic nature.

Algorithm:

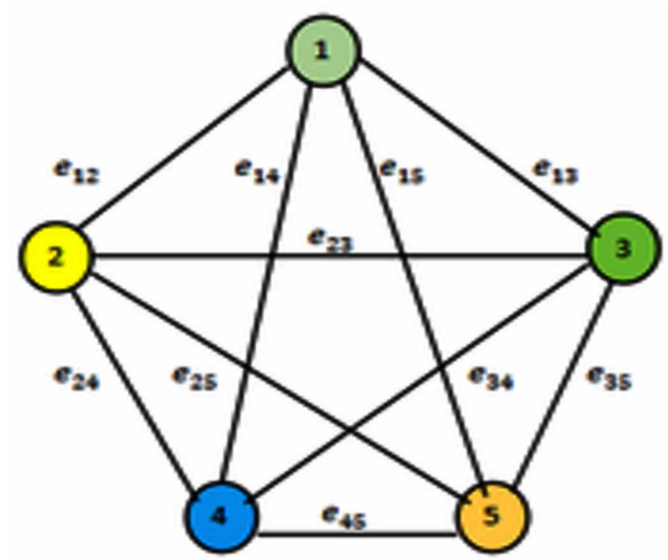
- Consider the adjacency matrix of the graph.
- To find out the minimum spanning tree, we utilize the score function (1) concept and construct our score matrix.
- Select the smallest weight and if more than one edge having smallest weight then uses accuracy function (2) and choose the smallest one.
- Out of all remaining edges choose an edge having the smallest one which doesn't form a circuit with the edges which are already included.
- Continue this process until all the vertices were included.
- Whenever all vertices are included then stop.

4.1 Illustrative Example

Obtain a minimal spanning tree of the graph shown in Figure 2.

Step 1: The associated adjacency matrix of figure 1 is given as follows:

Figure 2. A Graph with Cylindrical Single-Valued Neutrosophic Weight Edges



$$\begin{bmatrix} \langle 0;0;0\rangle & \langle 0.6;0.7;0.4\rangle & \langle 0.7;0.8;0.25\rangle & \langle 0.5;0.3;0.4\rangle & \langle 0.7;0.5;0.2\rangle \\ \langle 0.6;0.7;0.4\rangle & \langle 0;0;0\rangle & \langle 0.65;0.6;0.3\rangle & \langle 0.7;0.7;0.2\rangle & \langle 0.6;0.6;0.4\rangle \\ \langle 0.7;0.8;0.25\rangle & \langle 0.65;0.6;0.3\rangle & \langle 0;0;0\rangle & \langle 0.8;0.3;0.2\rangle & \langle 0.7;0.6;0.3\rangle \\ \langle 0.5;0.3;0.4\rangle & \langle 0.7;0.7;0.2\rangle & \langle 0.8;0.3;0.2\rangle & \langle 0;0;0\rangle & \langle 0.8;0.5;0.15\rangle \\ \langle 0.7;0.5;0.2\rangle & \langle 0.6;0.6;0.4\rangle & \langle 0.7;0.6;0.3\rangle & \langle 0.8;0.5;0.15\rangle & \langle 0;0;0\rangle \end{bmatrix}$$

Step 2: Using the score function, the associated score matrix is

$$\begin{bmatrix} 0 & 0.035 & 0.139 & 0.125 & 0.345 \\ 0.035 & 0 & 0.198 & 0.225 & 0.100 \\ 0.139 & 0.198 & 0 & 0.575 & 0.265 \\ 0.125 & 0.225 & 0.575 & 0 & 0.504 \\ 0.345 & 0.100 & 0.265 & 0.504 & 0 \end{bmatrix}$$

Table 1. The values of weights related with edges

Edges	Cylindrical Single-Valued Neutrosophic Weights
e_{12}	$\langle 0.6; 0.7; 0.4 \rangle$
e_{13}	$\langle 0.7; 0.8; 0.25 \rangle$
e_{14}	$\langle 0.5; 0.3; 0.4 \rangle$
e_{15}	$\langle 0.7; 0.5; 0.2 \rangle$
e_{23}	$\langle 0.65; 0.6; 0.3 \rangle$
e_{24}	$\langle 0.7; 0.7; 0.2 \rangle$
e_{25}	$\langle 0.6; 0.6; 0.4 \rangle$
e_{34}	$\langle 0.8; 0.3; 0.2 \rangle$
e_{35}	$\langle 0.7; 0.6; 0.3 \rangle$
e_{45}	$\langle 0.8; 0.5; 0.15 \rangle$

Figure 3. Graphical Representation of Step 3

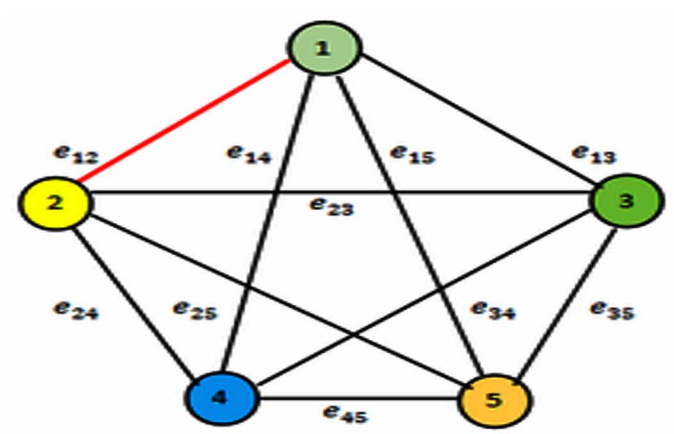


Figure 4. Graphical Representation of Step 4

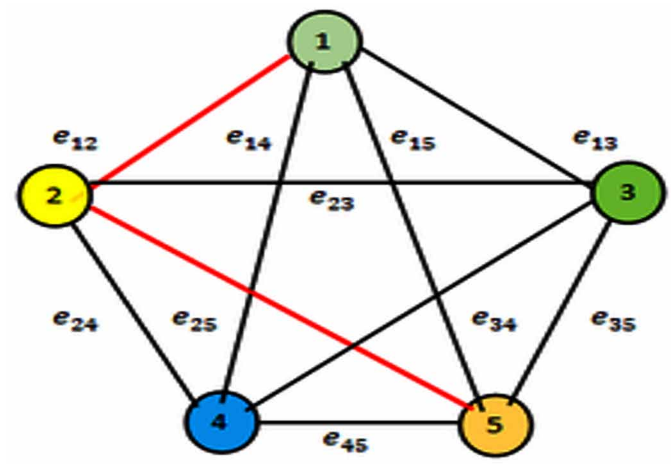


Figure 5. Graphical Representation of Step 5

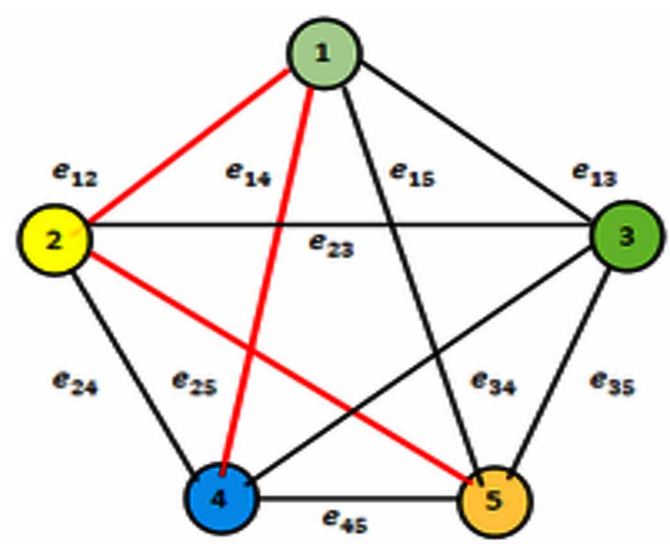


Figure 6. Graphical Representation of Step 6

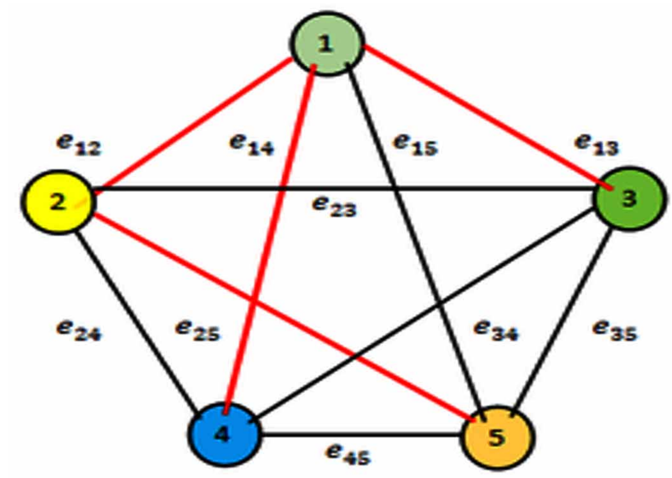


Figure 7. Graphical Representation of final minimal spanning tree

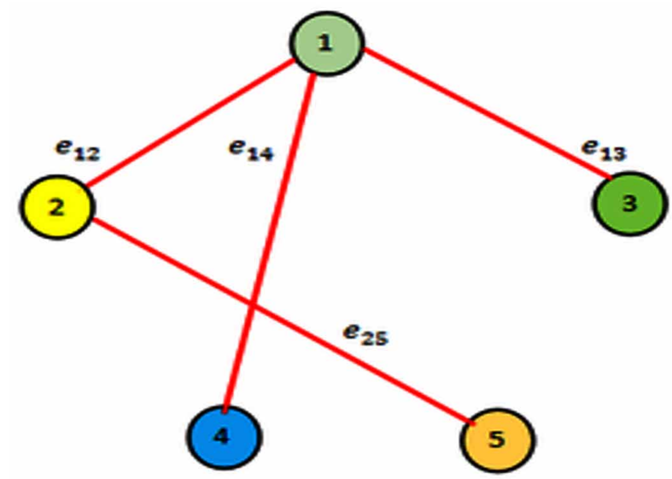
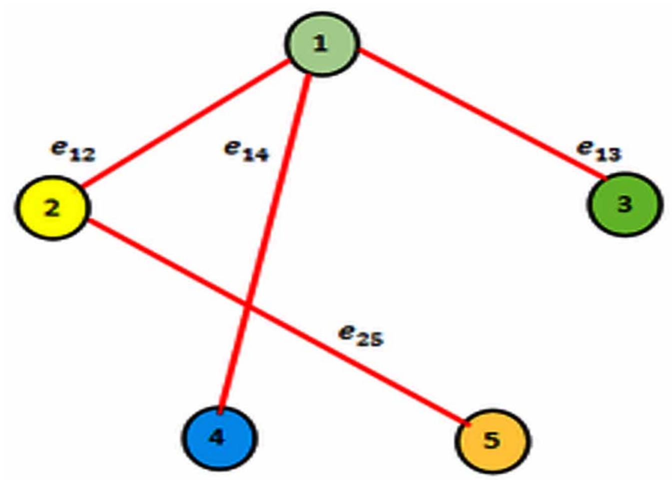


Figure 8. Graphical Representation of minimal spanning tree



Step 3: Examine that the minimum value is 0.035. Thus, select the edge connected with the nodes in between (1,2) and marked it. Repeat the procedure unless and until someone get the final spanning tree.

Step 4: Examine that the minimum value is 0.1 among the rest weighted edges. Thus, select the edge connected with the nodes in between (2,5) and marked it.

Step 5: Observed that the minimum value is 0.125 among the rest weighted edges. Hence, select the edge connected with the nodes in between (1,4) and marked it.

Step 6: Observed that the minimum value is 0.139 among the rest weighted edges. Hence, select the edge connected with the nodes in between (1,3) and marked it.

Step 7: It is examined that all the nodes are connected and further addition of edges will create circuit in the required figure. According to the definition of spanning tree it doesn't contain any circuit but all the nodes must be connected. Thus, the final minimum spanning tree is as follows:

The minimum weight of this required graph is –

$\left(0.035 + 0.125 + 0.139 + 0.1\right) = 0.399$ units.

5. COMPARISON

In case of comparison of our work we take the established work of (Mullai, Broumi, & Stephen, 2017) algorithm

Step 1: Let $S_1 = \{1\}$ then $\overline{S_1} = \{2,3,4,5\}$
Step 2: Let $S_2 = \{1,2\}$ then $\overline{S_2} = \{3,4,5\}$
Step 3: Let $S_3 = \{1,2,5\}$ then $\overline{S_3} = \{3,4\}$
Step 4: Let $S_4 = \{1,2,5,4\}$ then $\overline{S_1} = \{5\}$

The required spanning tree is shown in Figure 8.

- **Discussion:** The difference between our method and Mullai's method is that Mullai's method is related on the comparison of edges in each iteration process of the algorithm which leads to step by step high computation and time taking method whereas proposed method related with Matrix approach can be controlled by applying Matlab software.

CONCLUSION

In this paper, the concept of cylindrical neutrosophic number from different viewpoints and aspects for analyzing its true character has been constructed. Development of the score and accuracy function concept used in converting a neutrosophic number into a crisp number finding its significance for minimal spanning tree problem giving a far ambidextrous result.

The work is done on cylindrical single valued neutrosophic number. But in future researchers can take the generalized or interval valued cylindrical single valued neutrosophic number by different techniques.

Thus, it can be concluded that the approach for taking the cylindrical single valued neutrosophic number is very helpful for the researchers who are involved in dealing the mathematical modeling with impreciseness in various fields of sciences and engineering.

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KEY TERMS AND DEFINITIONS

Membership Function: Let, A be a set, then membership function on A is a mapping from A to real interval [0,1]. The membership function of a fuzzy set $\tilde{A}$ is generally denoted by $\mu_{\tilde{A}}(x)$. For any element x in $\tilde{A}$, the value of $\mu_{\tilde{A}}(x)$ generally called the membership degree of x in $\tilde{A}$.

Minimal Spanning Tree: A spanning tree having the smallest weight in G is called a minimal spanning tree.

Multi-Criteria Decision-Making Problem: Multi criteria decision making problem is concerned with structuring and solving decision and planning problems involving multiple criteria. The purpose is to support decision-makers facing such problems. Typically, there does not exist a unique optimal solution for such problems and it is necessary to use decision-maker's preferences to differentiate between solutions.

Spanning Tree: A tree is T said to be a spanning tree of a connected graph G if T is a subgraph of G and T contains all the vertices of G.

Tree: A connected graph G having no circuit is called a tree.

Uncertainty: The word uncertainty means vagueness. It occurs only when the boundary of a piece of information is not clear-cut. The concept of uncertainty was first introduced by L.A. Zadeh in his research paper in 1965 as an extension of classical notion of set.

Chapter 10
Two Centroid Point for SVTN-Numbers and SVTrN-Numbers:
SVN-MADM Method

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ABSTRACT

In this chapter, some basic definitions and operations on the concepts of fuzzy set, fuzzy number, intuitionistic fuzzy set, single-valued neutrosophic set, single-valued neutrosophic number (SVN-number) are presented. Secondly, two centroid point are called 1. and 2. centroid point for single-valued trapezoidal neutrosophic number (SVTN-number) and single-valued triangular neutrosophic number (SVTrN-number) are presented. Then, some desired properties of 1. and 2. centroid point of SVTN-numbers and SVTrN-numbers studied. Also, based on concept of 1. and 2. centroid point of SVTrN-numbers, a new single-valued neutrosophic multiple-attribute decision-making method is proposed. Moreover, a numerical example is introduced to illustrate the availability and practicability of the proposed method. Finally, since centroid points of normalized SNTN-numbers or SNTrN-numbers are fuzzy values, all definitions and properties of fuzzy graph theory can applied to SNTN-numbers or SNTrN-numbers. For example, definition of fuzzy graph theory based on centroid points of normalized SVTN-numbers and SVTrN-numbers is given.

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1. INTRODUCTION

In practical applications including multi-atttribute decision-making(MADM) problems, many decision makers have different uncertain or ambiguous natural events and their modelling and solution involves the use of mathematical systems. Therefore, a number of theories have been introduced for dealing with such systems in an effective way such as; fuzzy set theory (Zadeh, 1965) intuitionistic fuzzy set theory (Atanassov, 1996), neutrosophic set theory (Smarandache, 1998) and so on. Then, many extended forms of the theories have been studied on fuzzy set theory [(İbrahim, 2004), (Rao & Shankar, 2011), (Wang et al., 2006), (Wang, 2009)], intuitionistic fuzzy set theory [(Dong et al. 2015), (Chan & Kumar, 2007), (Das & Guha, 2013), (Das & Guha, 2016), (Esmailzadeh, 2013), (Gani & Mohamed, 2015), (Gautamet al. 2016), (Hajek & Olej, 2014), (Kumar & Kaur, 2013), (Li, 2010), (Li, 2014), (Li & Yang, 2015), (Liu & Li, 2017), (Nayagam et al. 2016), (Nehi, 2010), (Prakash et al. 2016), (Rezvani, 2013), (Roseline & Amirtharaj, 2015), (Varghese, & Kuriakose, 2012)] and neutrosophic set theory [(Broumi et al., 2014), (Broumi et al., 2014a), (Broumi et al., 2014b), (Deli et al., 2014), (Liu et al.,2014a), (Liu et al.,2014b), (Peng et al., 2014), (Peng et al., 2016), (Wang et al., 2010)].

Since ranking neutrosophic numbers is a difficult task many methods have been proposed for ranking of neutrosophic numbers in [(Basset et al., 2018), (Biswas et al., 2016a), (Biswas et al., 2016b), (Broumi et al., 2016a), (Broumi et al., 2016b), (Deli & Subası, 2016), (Deli & Subası, 2017), (Öztürk, 2018), (Liang et al., 2017), (Liang et al., 2017b), (Subaş, 2015), (Ye, 2015), (Ye, 2017)]. In this study we have ranked single valued neutrosophic numbers (SVN-numbers) using the new centroid centroid point concepts. The remainder of this paper is organized as follows: Section 2 introduces some basic definitions and properties deal with fuzzy set, intuitionistic fuzzy set and neutrosophic set. Section 3 proposes two centroid point are called 1. and 2. centroid point for single valued neutrosophic numbers (SVN-numbers). Also, the section contain some desired properties of 1. and 2. centroid point of SVN-numbers. Section 4 developes a new single valued neutrosophic multiple attribute decision making method based on concept of 1. and 2. centroid point. Moreover, a numerical example is presented to show the feasibility and superiority of the method in the section. Section 5 give an application in fuzzy graph because centroid points of normalized SNTN-numbers or SNTrN-numbers are fuzzy values we can applied all definitions and properties of fuzzy graph theory to SNTN-numbers or SNTrN-numbers. The conclusions and innovations are summarized in Section 6. The present expository paper is a condensation of part of the dissertation (Öztürk, 2018).

2. BACKGROUND

This section reviews some basic concepts related to fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets that are used throughout this paper.

Definition 1. [(Zadeh, 1965)] A fuzzy set A on universe set E is given by

$$A = \{\mu_A(x) / x : x \in E\}$$

where the functions $\mu_A : E \to [0,1]$ define the degree of membership membership $x \in E$.

Definition 2. [(Wang et al., 2006)]A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let $\hat{A}$ be a fuzzy number, whose membership function $\mu_{\hat{A}} : R \to \left[0, w_{\hat{A}}\right]$ can generally be defined as

$$\mu_{\hat{A}}(x) = \begin{cases} f_{\mu l}(x) & a \le x < b \\ w_{\hat{A}} & b \le x < c \\ f_{\mu r}(x) & c \le x < d \\ 0 & otherwise \end{cases}$$

where $w_{\hat{A}} \in \left[0,1\right]$ is a constant,

$$f_{\mu l}(x) : [a,b] \to [0, w_{\hat{A}}]$$

and

$$f_{\mu r}(x) : [c,d] \to [0, w_{\hat{A}}]$$

are two strictly monotonical and continuous mappings from R to the closed interval $[0, w_{\hat{A}}]$. If $w_{\hat{A}} = 1$, then $\hat{A}$ is a normal fuzzy number; otherwise, it is said to be a non-normal fuzzy number(or generalized fuzzy number). If the membership function $\mu_{\hat{A}}(x)$ is piecewise linear, then $\hat{A}$ is referred to as a trapezoidal fuzzy number and is usually denoted by $\hat{A} = \left(a,b,c,d; w_{\hat{A}}\right)$ or $\hat{A} = \left(a,b,c,d\right)$ if $w_{\hat{A}} = 1$.

Also, Wang [(Wang, 2009)] introduce a new definition is called centroid point of the fuzzy number $\hat{A} = \left(a,b,c,d; w_{\hat{A}}\right)$ for X-axis as;

$$C\left(\hat{A}\right)=\frac{\int_a^d x\mu_{\hat{A}}\left(x\right)dx}{\int_a^d \mu_{\hat{A}}\left(x\right)dx}$$

Definition 3.[(Atanassov, 1996)]A intuitionistic fuzzy set K on universe set E is given by

$$K=\{<x,\mu_K(x),v_K(x)>:x\in E\}$$

where

$$\mu_K(x):E\to[0,1]$$

and

$$v_K(x):E\to[0,1]$$

satisfy the condition

$$0\le\mu_K(x)+v_K(x)\le 1\,,$$

for every $x\in E$. The value $\mu_K(x)$ and $v_K(x)$ defines the value of membership and non-membership, respectively.

Definition 4. [(Wang et al., 2010)] A single valued neutrosophic set A on universe set E is given by

$$A=\{<x,T_A(x),I_A(x),F_A(x)>:x\in E\}$$

Where

$T_A(x):E\to[0,1], I_A(x):E\to[0,1]$ and $F_A(x):E\to[0,1]$

satisfy the condition

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$$

for every $x \in E$. The value $T_A(x)$, $I_A(x)$ and $F_A(x)$ defines the value of truth-membership, indeterminacy-membership and falsity-membership, respectively.

Definition 5. [(Subaş, 2015)] Let $a_1 \le b_1 \le c_1 \le d_1$ such that $a_1, b_1, c_1, d_1 \in [0,1]$. A single valued trapezoidal neutrosophic number (SVTN-number)

$$\tilde{A} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$$

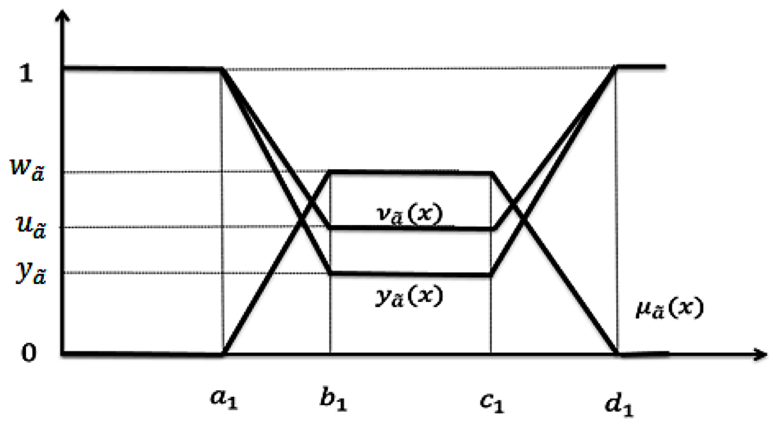
is a special neutrosophic set on the real number set R, whose truth-membership function $\mu_{\tilde{A}} : R \to [0, w_{\tilde{A}}]$, indeterminacy-membership function $\nu_{\tilde{A}} : R \to [u_{\tilde{A}}, 1]$ and falsity-membership function $\lambda_{\tilde{A}} : R \to [y_{\tilde{A}}, 1]$ are given as follows;(An example of SVTN-number is given in Fig. 1)

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - a_1) w_{\tilde{A}} / (b_1 - a_1), & a_1 \le x \le b_1 \\ w_{\tilde{A}}, & b_1 \le x \le c_1 \\ (d_1 - x) w_{\tilde{A}} / (d_1 - c_1), & c_1 \le x \le d_1 \\ 0, & otherwise \end{cases},$$

$$v_{\tilde{A}}(x) = \begin{cases} b_1 - x + u_{\tilde{A}}(x - a_1), & a_1 \le x \le b_1 \\ u_{\tilde{A}}, & b_1 \le x \le c_1 \\ x - c_1 + u_{\tilde{A}}(d_1 - x), & c_1 \le x \le d_1 \\ 1, & otherwise \end{cases}$$

$$\lambda_{\tilde{A}}(x) = \begin{cases} b_1 - x + \lambda_{\tilde{A}}(x - a_1), & a_1 \le x \le b_1 \\ y_{\tilde{A}}, & b_1 \le x \le c_1 \\ x - c_1 + \lambda_{\tilde{A}}(d_1 - x), & c_1 \le x \le d_1 \\ 1, & otherwise \end{cases}$$

Figure 1. Example of a SVTN-number



$$If\ \tilde{A} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$$

and

$$\tilde{B} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{B}}, u_{\tilde{B}}, y_{\tilde{B}} \rangle$$

be two SVTN-numbers and $\tilde{a} \neq 0$, then we have

1. $\tilde{A} + \tilde{B} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle$
2. $\tilde{A}\tilde{B} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle\, d_1 > 0\, d_2 > 0 \\ \langle (a_1d_2, b_1c_2, c_1b_2, d_1a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle\, d_1 < 0\, d_2 > 0 \\ \langle (d_1d_2, c_1c_2, b_1b_2, a_1a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle\, d_1 < 0\, d_2 < 0 \end{cases}$
3. $\gamma\tilde{A} = \begin{cases} \langle (\gamma a_1, \lambda b_1, \lambda c_1, \lambda d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle\, \gamma > 0 \\ \langle (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle\, \gamma < 0 \end{cases}$
4. $\tilde{A}^{\gamma} = \begin{cases} \langle (a_1^{\gamma}, b_1^{\gamma}, c_1^{\gamma}, d_1^{\gamma}); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle\, \gamma > 0 \\ \langle (d_1^{\gamma}, c_1^{\gamma}, b_1^{\gamma}, a_1^{\gamma}); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle\, \gamma < 0 \end{cases}$

Definition 6. [(Subaş, 2015)] Let $a_1 \leq b_1 \leq c_1$ such that $a_1, b_1, c_1 \in [0,1]$. A single valued triangular neutrosophic number (SVTrN-number)

$$\tilde{A} = \langle (a_1, b_1, c_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$$

is a special neutrosophic set on the real number set R, whose truth-membership function $\mu_{\tilde{A}} : R \to \left[0, w_{\tilde{A}}\right]$, indeterminacy-membership function $\nu_{\tilde{A}} : R \to \left[u_{\tilde{A}}, 1\right]$ and falsity-membership function $\lambda_{\tilde{A}} : R \to \left[y_{\tilde{A}}, 1\right]$ are given as follows ;(An example of SVTN-number is given in Fig. 2)

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(x - a_1\right) w_{\tilde{A}} / \left(b_1 - a_1\right) & a_1 \le x < b_1 \\ w_{\tilde{A}} & x = b_1 \\ \left(c_1 - x\right) w_{\tilde{A}} / \left(c_1 - b_1\right) & b_1 < x \le c_1 \\ 0 & otherwise \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \left(b_1 - x + u_{\tilde{A}}\left(x - a_1\right)\right) / \left(b_1 - a_1\right) & a_1 \le x < b_1 \\ u_{\tilde{A}} & x = b_1 \\ \left(x - b_1 + u_{\tilde{A}}\left(c_1 - x\right)\right) / \left(c_1 - b_1\right) & b_1 < x \le c_1 \\ 1 & otherwise \end{cases}$$

and

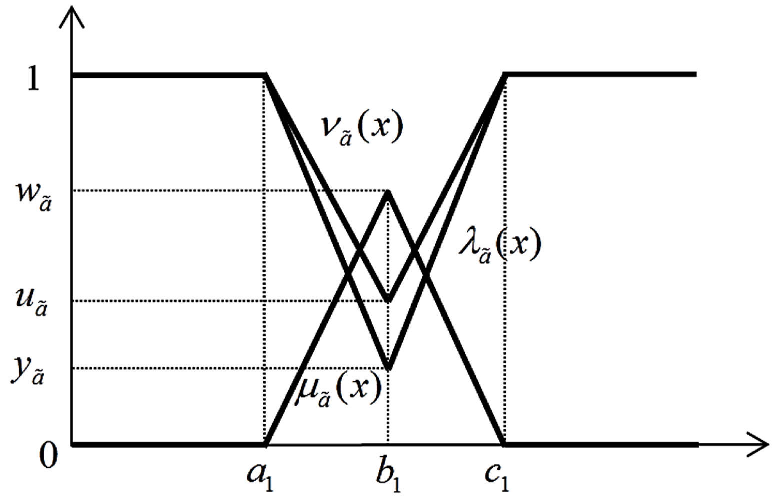
$$\lambda_{\tilde{A}}(x) = \begin{cases} \left(b_1 - x + y_{\tilde{A}}\left(x - a_1\right)\right) / \left(b_1 - a_1\right) & a_1 \le x < b_1 \\ y_{\tilde{A}} & x = b_1 \\ \left(x - b_1 + y_{\tilde{A}}\left(c_1 - x\right)\right) / \left(c_1 - b_1\right) & b_1 < x \le c_1 \\ 1 & otherwise \end{cases}$$

$$If\ \tilde{A} = \langle (a_1, b_1, c_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$$

and

$$\tilde{B} = \langle (a_2, b_2, c_2); w_{\tilde{B}}, u_{\tilde{B}}, y_{\tilde{B}} \rangle$$

Figure 2. Example of a SVTrN-number



be two SVTrN-numbers and $\tilde{a} \neq 0$, then we have

1. $\tilde{A} + \tilde{B} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle$

2. $$\tilde{A}\tilde{B} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle c_1 > 0 \quad c_2 > 0 \\ \langle (a_1 c_2, b_1 b_2, c_1 a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle c_1 < 0 \quad c_2 > 0 \\ \langle (c_1 c_2, b_1 b_2, a_1 a_2); w_{\tilde{A}} \wedge w_{\tilde{B}}, u_{\tilde{A}} \vee u_{\tilde{B}}, y_{\tilde{A}} \vee y_{\tilde{B}} \rangle c_1 < 0 \quad c_2 < 0 \end{cases}$$

3. $$\gamma \tilde{A} = \begin{cases} \langle \left(\gamma a_1, \gamma b_1, \gamma c_1 \right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle \gamma > 0 \\ \langle \left(\gamma c_1, \gamma b_1, \gamma a_1 \right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle \gamma < 0 \end{cases}$$

4. $$\tilde{A}^{\gamma} = \begin{cases} \langle \left(a_1^{\gamma}, b_1^{\gamma}, c_1^{\gamma} \right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle \gamma > 0 \\ \langle \left(c_1^{\gamma}, b_1^{\gamma}, a_1^{\gamma} \right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle \gamma < 0 \end{cases}$$

Definition 7. [(Subaş, 2015)] Let

$$\tilde{A}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{A}_j}, u_{\tilde{A}_j}, y_{\tilde{A}_j} \rangle$$

are collection of some SVTN-numbers

$$(or\ \tilde{A}_j = \langle (a_j, b_j, c_j); w_{\tilde{A}_j}, u_{\tilde{A}_j}, y_{\tilde{A}_j} \rangle$$

are collection of some SVTrN-numbers.) Then,

1. SVTN (or SVTrN) weighted aritmetic operator $N_{ao}(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n)$ is defined as;

$$N_{ao}(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n) = \sum_{j=1}^{n} w_j \tilde{A}_j$$

2. SVTN (or SVTrN) weighted geometric operator $N_{go}(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n)$, is defined as;

$$N_{go}(\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_n) = \prod_{j=1}^{n} \tilde{A}_j^{w_j}$$

where, $w = (w_1, w_2, ..., w_n)^T$ is a weight vector associated with the N_{ao} and N_{go} operator, for every j $(j = 1, 2, ..., n)$ such that $w_j \in [0,1]$ ve $\sum_{j=1}^{n} w_j = 1$.

3. 1. AND 2. CENTROID POINT FOR SVN-NUMBERS

In the subsection we present two centroid point, are called 1. and 2. centroid point, for single valued neutrosophic numbers (SVN-numbers).

Definition 8. Let

$$\tilde{A} = \left[\left(a, b, c, d\right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right]$$

be an SNTN-number, which is shown in Fig. 3, with the functions

$$f_{\tilde{A}}^{L} : \left[a, b\right] \to \left[0, w_{\tilde{A}}\right],$$

$$f_{\tilde{A}}^{R} : \left[c, d\right] \to \left[0, w_{\tilde{A}}\right],$$

$$h_{\tilde{A}}^{L} : \left[a, b\right] \to \left[0, u_{\tilde{A}}\right],$$

$$h_{\tilde{A}}^{R} : \left[c, d\right] \to \left[0, u_{\tilde{A}}\right],$$

$$g_{\tilde{A}}^{L}:\left[a,b\right]\to\left[0,y_{\tilde{A}}\right]$$

and

$$g_{\tilde{A}}^{R}:\left[c,d\right]\to\left[0,y_{\tilde{A}}\right]$$

as;

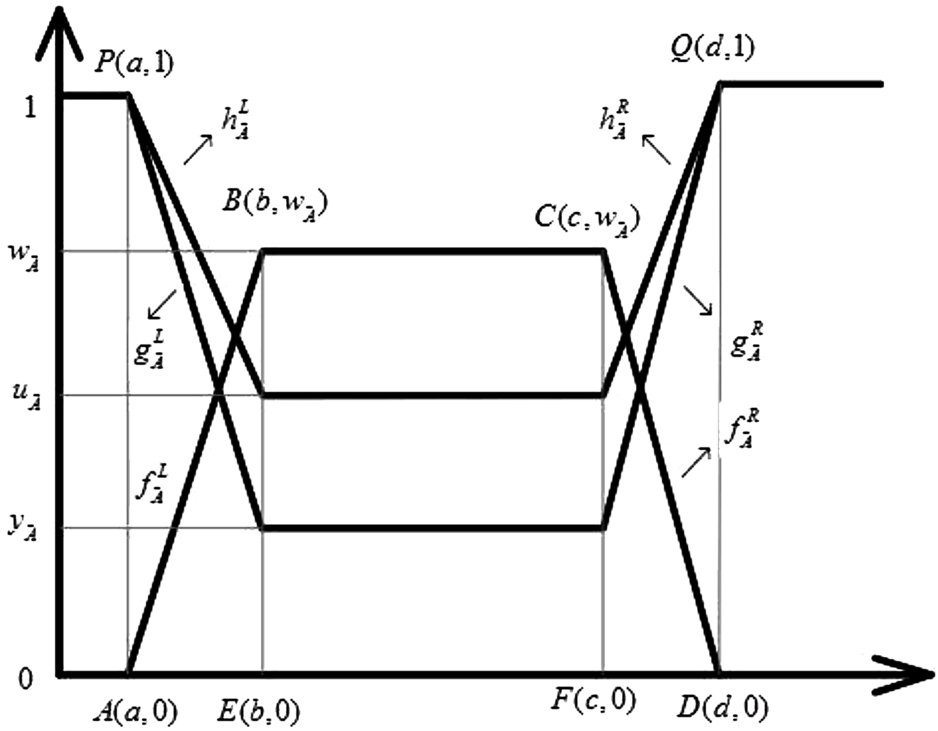
$$\mu_{\tilde{A}}(x)=\begin{cases} f_{\tilde{A}}^{L}(x)=\dfrac{w_{\tilde{A}}(x-a)}{\left(b-a\right)}, & a\le x\le b \\ w_{\tilde{A}}, & b\le x\le c \\ f_{\tilde{A}}^{R}(x)=\dfrac{w_{\tilde{A}}\left(d-x\right)}{\left(d-c\right)}, & c\le x\le d \\ 0, & otherwise \end{cases}$$

$$v_{\tilde{A}}(x)=\begin{cases} h_{\tilde{A}}^{L}(x)=\dfrac{\left(x-b\right)+u_{\tilde{A}}\left(a-x\right)}{\left(a-b\right)}, & a\le x\le b \\ u_{\tilde{A}}, & b\le x\le c \\ h_{\tilde{A}}^{R}(x)=\dfrac{\left(x-c\right)+u_{\tilde{A}}\left(d-x\right)}{\left(d-c\right)}, & c\le x\le d \\ 1, & otherwise \end{cases}$$

$$\lambda_{\tilde{A}}(x)=\begin{cases} g_{\tilde{A}}^{L}(x)=\dfrac{\left(x-b\right)+y_{\tilde{A}}\left(a-x\right)}{\left(a-b\right)}, & a\le x\le b \\ y_{\tilde{A}}, & b\le x\le c \\ g_{\tilde{A}}^{R}(x)=\dfrac{\left(x-c\right)+y_{\tilde{A}}\left(d-x\right)}{\left(d-c\right)}, & c\le x\le d \\ 1, & otherwise \end{cases}$$

In order to find the centroid point of the SVTN-number $\tilde{A}$, the area under the the degree of truth-membership functions, indeterminacy-membership function and falsity-membership functions is consider together. Then,

Figure 3. The SVTN-number



1. centroid point of $\tilde{A}$, is denoted by $X'_{\tilde{A}}$, is defined as;

$$X'_{\tilde{A}} = \frac{\int_a^b x f_{\tilde{A}}^L dx + \int_c^d x f_{\tilde{A}}^R dx + \int_b^c x w_{\tilde{A}} dx - \int_a^b x g_{\tilde{A}}^L dx - \int_c^d x g_{\tilde{A}}^R dx - \int_b^c x y_{\tilde{A}} dx + \int_a^b x h_{\tilde{A}}^L dx + \int_b^c x u_{\tilde{A}} dx + \int_c^d x h_{\tilde{A}}^R dx}{\int_a^b f_{\tilde{A}}^L dx + \int_c^d f_{\tilde{A}}^R dx + \int_b^c w_{\tilde{A}} dx - \int_a^b g_{\tilde{A}}^L dx - \int_c^d g_{\tilde{A}}^R dx - \int_b^c y_{\tilde{A}} dx + \int_a^b h_{\tilde{A}}^L dx + \int_b^c u_{\tilde{A}} dx + \int_c^d h_{\tilde{A}}^R dx}$$

2. centroid point of $\tilde{A}$, is denoted by $X'_{\tilde{A}}$, is defined as;

$$X''_{\tilde{A}} = \frac{\int_a^b x f_{\tilde{A}}^L dx + \int_c^d x f_{\tilde{A}}^R dx + \int_b^c x w_{\tilde{A}} dx - \int_a^b x g_{\tilde{A}}^L dx - \int_c^d x g_{\tilde{A}}^R dx - \int_b^c x y_{\tilde{A}} dx - \int_a^b x h_{\tilde{A}}^L dx - \int_b^c x u_{\tilde{A}} dx - \int_c^d x h_{\tilde{A}}^R dx}{\int_a^b f_{\tilde{A}}^L dx + \int_c^d f_{\tilde{A}}^R dx + \int_b^c w_{\tilde{A}} dx - \int_a^b g_{\tilde{A}}^L dx - \int_c^d g_{\tilde{A}}^R dx - \int_b^c y_{\tilde{A}} dx - \int_a^b h_{\tilde{A}}^L dx - \int_b^c u_{\tilde{A}} dx - \int_c^d h_{\tilde{A}}^R dx}$$

Theorem 9. Let

$$\tilde{A} = \left[\left(a,b,c,d\right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right]$$

be an SNTN-number. Then,

1. centroid point of $\tilde{A}$, is denoted by $X'_{\tilde{A}}$, is computed as;

$$X'_{\tilde{A}} = \frac{d^2 + dc + c^2 - a^2 - ab - b^2}{3\left(d + c - a - b\right)}$$

2. centroid point of $\tilde{A}$, is denoted by $X''_{\tilde{A}}$, is computed as;

$$X''_{\tilde{A}} = \frac{\dfrac{\left(d-c\right).\left[-a^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) - 6a^2b + b^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)\right]}{3\left(d-c\right).\left[-a^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) - 4ab + b^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)\right]} + \left(a-b\right).\left[-c^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right) + d^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) + 6d^2c\right]}{+3\left(a-b\right).\left[-c^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right) + d^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) + 4dc\right]}$$

Proof. Assume that

$$\tilde{A} = \left[\left(a,b,c,d\right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\right]$$

be an SNTN-number.

1. centroid point $X'_{\tilde{A}}$ of $\tilde{A}$ is given by

$$X'_{\tilde{A}} = \frac{\int_a^b x f_{\tilde{A}}^L dx + \int_c^d x f_{\tilde{A}}^R dx + \int_b^c x w_{\tilde{A}} dx - \int_a^b x g_{\tilde{A}}^L dx - \int_c^d x g_{\tilde{A}}^R dx - \int_b^c x y_{\tilde{A}} dx + \int_a^b x h_{\tilde{A}}^L dx + \int_b^c x u_{\tilde{A}} dx + \int_c^d x h_{\tilde{A}}^R dx}{\int_a^b f_{\tilde{A}}^L dx + \int_c^d f_{\tilde{A}}^R dx + \int_b^c w_{\tilde{A}} dx - \int_a^b g_{\tilde{A}}^L dx - \int_c^d g_{\tilde{A}}^R dx - \int_b^c y_{\tilde{A}} dx + \int_a^b h_{\tilde{A}}^L dx + \int_b^c u_{\tilde{A}} dx + \int_c^d h_{\tilde{A}}^R dx}$$

If we write the functions

$f_{\tilde{A}}^{L}, f_{\tilde{A}}^{R}, g_{\tilde{A}}^{L}, g_{\tilde{A}}^{R}, h_{\tilde{A}}^{L} \ ve \ h_{\tilde{A}}^{R}$

in $X'_{\tilde{A}}$, then we have

$$X'_{\tilde{A}} = \frac{\int_a^b \frac{w_{\tilde{A}}(x-a)}{(b-a)} x dx + \int_b^c x w_{\tilde{A}} dx + \int_c^d \frac{w_{\tilde{A}}(d-x)}{(d-c)} x dx - \int_a^b \frac{(x-b)+y_{\tilde{A}}(a-x)}{(a-b)} x dx - \int_b^c x y_{\tilde{A}} dx}{\int_a^b \frac{w_{\tilde{A}}(x-a)}{(b-a)} dx + \int_b^c w_{\tilde{A}} dx + \int_c^d \frac{w_{\tilde{A}}(d-x)}{(d-c)} dx - \int_a^b \frac{(x-b)+y_{\tilde{A}}(a-x)}{(a-b)} dx - \int_b^c y_{\tilde{A}} dx}$$

$$\frac{-\int_c^d \frac{(x-c)+y_{\tilde{A}}(d-x)}{(d-c)} x dx + \int_a^b \frac{(x-b)+u_{\tilde{A}}(a-x)}{(a-b)} x dx + \int_b^c x u_{\tilde{A}} dx + \int_c^d \frac{(x-c)+u_{\tilde{A}}(d-x)}{(d-c)} x dx}{-\int_c^d \frac{(x-c)+y_{\tilde{A}}(d-x)}{(d-c)} dx + \int_a^b \frac{(x-b)+u_{\tilde{A}}(a-x)}{(a-b)} dx + \int_b^c u_{\tilde{A}} dx + \int_c^d \frac{(x-c)+u_{\tilde{A}}(d-x)}{(d-c)} dx}$$

And then,

$$X'_{\tilde{A}} = \frac{\left[\frac{w_{\tilde{A}} x^3}{3(b-a)} - \frac{a w_{\tilde{A}} x^2}{2(b-a)}\right]_a^b + \left(\frac{w_{\tilde{A}} x^2}{2}\right)\Big|_b^c + \left[\frac{w_{\tilde{A}} d x^2}{2(d-c)} - \frac{w_{\tilde{A}} x^3}{3(d-c)}\right]_c^d}{\left[\frac{w_{\tilde{A}} x^2}{2(b-a)} - \frac{a w_{\tilde{A}} x}{(b-a)}\right]_a^b + \left(w_{\tilde{A}} x\right)\Big|_b^c + \left[\frac{w_{\tilde{A}} d x}{(d-c)} - \frac{w_{\tilde{A}} x^2}{2(d-c)}\right]_c^d}$$

$$\frac{-\left[\frac{x^3\left(1-y_{\tilde{A}}\right)}{3(a-b)} + \frac{\left(a y_{\tilde{A}} - b\right) x^2}{2(a-b)}\right]_a^b - \left(\frac{y_{\tilde{A}} x^2}{2}\right)\Big|_b^c - \left[\frac{x^3\left(1-y_{\tilde{A}}\right)}{3(d-c)} + \frac{\left(d y_{\tilde{A}} - c\right) x^2}{2(d-c)}\right]_c^d}{-\left[\frac{x^2\left(1-y_{\tilde{A}}\right)}{2(a-b)} + \frac{\left(a y_{\tilde{A}} - b\right) x}{(a-b)}\right]_a^b - \left(y_{\tilde{A}} x\right)\Big|_b^c - \left[\frac{x^2\left(1-y_{\tilde{A}}\right)}{2(d-c)} + \frac{\left(d y_{\tilde{A}} - c\right) x}{(d-c)}\right]_c^d}$$

$$\frac{+\left[\frac{x^3\left(1-u_{\tilde{A}}\right)}{3(a-b)} + \frac{\left(a u_{\tilde{A}} - b\right) x^2}{2(a-b)}\right]_a^b + \left(\frac{u_{\tilde{A}} x^2}{2}\right)\Big|_b^c + \left[\frac{x^3\left(1-u_{\tilde{A}}\right)}{3(d-c)} + \frac{\left(d u_{\tilde{A}} - c\right) x^2}{2(d-c)}\right]_c^d}{+\left[\frac{x^2\left(1-u_{\tilde{A}}\right)}{2(a-b)} + \frac{\left(a u_{\tilde{A}} - b\right) x}{(a-b)}\right]_a^b + \left(u_{\tilde{A}} x\right)\Big|_b^c + \left[\frac{x^2\left(1-u_{\tilde{A}}\right)}{2(d-c)} + \frac{\left(d u_{\tilde{A}} - c\right) x}{(d-c)}\right]_c^d}$$

Thus,

$$X'_{\tilde{A}} = \frac{\dfrac{-a^3\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{6\left(a-b\right)}+\dfrac{b^3\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{6\left(a-b\right)}-\dfrac{c^3\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{6\left(d-c\right)}+\dfrac{d^3\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{6\left(d-c\right)}}{\dfrac{-a^2\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{2\left(a-b\right)}+\dfrac{b^2\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{2\left(a-b\right)}-\dfrac{c^2\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{2\left(d-c\right)}+\dfrac{d^2\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)}{2\left(d-c\right)}}$$

$$= \frac{\left(d-c\right).\left[-a^3.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)+b^3.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)\right]}{3\left(d-c\right).\left[-a^2.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)+b^2.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)\right]}$$

$$\frac{+\left(a-b\right).\left[-c^3.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)+d^3.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)\right]}{+3\left(a-b\right).\left[-c^2.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)+d^2.\left(w_{\tilde{A}}-y_{\tilde{A}}+u_{\tilde{A}}\right)\right]}$$

$$= \frac{d^2+dc+c^2-a^2-ab-b^2}{3\left(d+c-a-b\right)}$$

2. centroid point $X''_{\tilde{A}}$ of $\tilde{A}$ is given by

$$X''_{\tilde{A}} = \frac{\int_a^b xf^L_{\tilde{A}}dx+\int_c^d xf^R_{\tilde{A}}dx+\int_b^c xw_{\tilde{A}}dx-\int_a^b xg^L_{\tilde{A}}dx-\int_c^d xg^R_{\tilde{A}}dx-\int_b^c xy_{\tilde{A}}dx-\int_a^b xh^L_{\tilde{A}}dx-\int_b^c xu_{\tilde{A}}dx-\int_c^d xh^R_{\tilde{A}}dx}{\int_a^b f^L_{\tilde{A}}dx+\int_c^d f^R_{\tilde{A}}dx+\int_b^c w_{\tilde{A}}dx-\int_a^b g^L_{\tilde{A}}dx-\int_c^d g^R_{\tilde{A}}dx-\int_b^c y_{\tilde{A}}dx-\int_a^b h^L_{\tilde{A}}dx-\int_b^c u_{\tilde{A}}dx-\int_c^d h^R_{\tilde{A}}dx}$$

If we write the functions

$f^L_{\tilde{A}}, f^R_{\tilde{A}}, g^L_{\tilde{A}}, g^R_{\tilde{A}}, h^L_{\tilde{A}}\ and\, h^R_{\tilde{A}}$

then we have

$$X''_{\tilde{A}} = \frac{\int_a^b \frac{w_{\tilde{A}}(x-a)}{(b-a)} x dx + \int_b^c x w_{\tilde{A}} dx + \int_c^d \frac{w_{\tilde{A}}(d-x)}{(d-c)} x dx - \int_a^b \frac{(x-b)+y_{\tilde{A}}(a-x)}{(a-b)} x dx - \int_b^c x y_{\tilde{A}} dx}{\int_a^b \frac{w_{\tilde{A}}(x-a)}{(b-a)} dx + \int_b^c w_{\tilde{A}} dx + \int_c^d \frac{w_{\tilde{A}}(d-x)}{(d-c)} dx - \int_a^b \frac{(x-b)+y_{\tilde{A}}(a-x)}{(a-b)} dx - \int_b^c y_{\tilde{A}} dx}$$

$$\frac{-\int_c^d \frac{(x-c)+y_{\tilde{A}}(d-x)}{(d-c)} x dx - \int_a^b \frac{(x-b)+u_{\tilde{A}}(a-x)}{(a-b)} x dx - \int_b^c x u_{\tilde{A}} dx - \int_c^d \frac{(x-c)+u_{\tilde{A}}(d-x)}{(d-c)} x dx}{-\int_c^d \frac{(x-c)+y_{\tilde{A}}(d-x)}{(d-c)} dx - \int_a^b \frac{(x-b)+u_{\tilde{A}}(a-x)}{(a-b)} dx - \int_b^c u_{\tilde{A}} dx - \int_c^d \frac{(x-c)+u_{\tilde{A}}(d-x)}{(d-c)} dx}$$

And then,

$$= \frac{\left[\frac{w_{\tilde{A}} x^3}{3(b-a)} - \frac{a w_{\tilde{A}} x^2}{2(b-a)}\right]_a^b + \left(\frac{w_{\tilde{A}} x^2}{2}\right)_b^c + \left[\frac{w_{\tilde{A}} d x^2}{2(d-c)} - \frac{w_{\tilde{A}} x^3}{3(d-c)}\right]_c^d}{\left[\frac{w_{\tilde{A}} x^2}{2(b-a)} - \frac{a w_{\tilde{A}} x}{(b-a)}\right]_a^b + \left(w_{\tilde{A}} x\right)_b^c + \left[\frac{w_{\tilde{A}} d x}{(d-c)} - \frac{w_{\tilde{A}} x^2}{2(d-c)}\right]_c^d}$$

$$\frac{-\left[\frac{x^3\left(1-y_{\tilde{A}}\right)}{3(a-b)} + \frac{\left(a y_{\tilde{A}} - b\right) x^2}{2(a-b)}\right]_a^b - \left(\frac{y_{\tilde{A}} x^2}{2}\right)_b^c - \left[\frac{x^3\left(1-y_{\tilde{A}}\right)}{3(d-c)} + \frac{\left(d y_{\tilde{A}} - c\right) x^2}{2(d-c)}\right]_c^d}{-\left[\frac{x^2\left(1-y_{\tilde{A}}\right)}{2(a-b)} + \frac{\left(a y_{\tilde{A}} - b\right) x}{(a-b)}\right]_a^b - \left(y_{\tilde{A}} x\right)_b^c - \left[\frac{x^2\left(1-y_{\tilde{A}}\right)}{2(d-c)} + \frac{\left(d y_{\tilde{A}} - c\right) x}{(d-c)}\right]_c^d}$$

$$\frac{-\left[\frac{x^3\left(1-u_{\tilde{A}}\right)}{3(a-b)} + \frac{\left(a u_{\tilde{A}} - b\right) x^2}{2(a-b)}\right]_a^b - \left(\frac{u_{\tilde{A}} x^2}{2}\right)_b^c - \left[\frac{x^3\left(1-u_{\tilde{A}}\right)}{3(d-c)} + \frac{\left(d u_{\tilde{A}} - c\right) x^2}{2(d-c)}\right]_c^d}{-\left[\frac{x^2\left(1-u_{\tilde{A}}\right)}{2(a-b)} + \frac{\left(a u_{\tilde{A}} - b\right) x}{(a-b)}\right]_a^b - \left(u_{\tilde{A}} x\right)_b^c - \left[\frac{x^2\left(1-u_{\tilde{A}}\right)}{2(d-c)} + \frac{\left(d u_{\tilde{A}} - c\right) x}{(d-c)}\right]_c^d}$$

Thus,

$$X''_{\tilde{A}} = \frac{\dfrac{-a^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) - 6a^2b}{6\left(a-b\right)} + \dfrac{b^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)}{6\left(a-b\right)}}{\dfrac{-a^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) - 4ab}{2\left(a-b\right)} + \dfrac{b^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)}{2\left(a-b\right)}}$$

$$\frac{\dfrac{-c^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)}{6\left(d-c\right)} + \dfrac{d^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) + 6d^2c}{6\left(d-c\right)}}{\dfrac{-c^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)}{2\left(d-c\right)} + \dfrac{d^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) + 4dc}{2\left(d-c\right)}}$$

Finally, we have

$$X''_{\tilde{A}} = \frac{\left(d-c\right).\left[-a^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) - 6a^2b + b^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)\right]}{3\left(d-c\right).\left[-a^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) - 4ab + b^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)\right]}$$

$$\frac{+\left(a-b\right).\left[-c^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right) + d^3\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) + 6d^2c\right]}{+3\left(a-b\right).\left[-c^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right) + d^2\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) + 4dc\right]}$$

Result 10. Let

$$\tilde{A} = \langle\left(a,b,c\right); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}}\rangle$$

be an SNTrN-number. Then

1. Centroid point, is denoted by $X'_{\tilde{A}}$, is defined as;

$$X'_{\tilde{A}} = \frac{c^2 + bc - a^2 - ab}{3\left(c-a\right)}$$

2. Centroid point, is denoted by $X''_{\tilde{A}}$, is defined as;

$$X''_{\tilde{A}} = \frac{\left(c-b\right).\left[-a^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) - 6a^2b + b^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)\right]}{3\left(c-b\right).\left[-a^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) - 4ab + b^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right)\right]}$$
$$\frac{+\left(a-b\right).\left[-b^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right) + c^3.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 4\right) + 6c^2b\right]}{+3\left(a-b\right).\left[-b^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} + 2\right) + c^2.\left(w_{\tilde{A}} - y_{\tilde{A}} - u_{\tilde{A}} - 2\right) + 4bc\right]}$$

Definition 11. Let $\tilde{A}$ and $\tilde{B}$ be any two SNTN-numbers or SNTrN-numbers. Then, the comparison method is given as;

1. If $X'_{\tilde{A}} > X'_{\tilde{B}}$ then $\tilde{A} > \tilde{B}$
2. If $X'_{\tilde{A}} < X'_{\tilde{B}}$ then $\tilde{A} < \tilde{B}$
3. If $X'_{\tilde{A}} = X'_{\tilde{B}}$ then
   a. If $X''_{\tilde{A}} > X''_{\tilde{B}}$ then $\tilde{A} > \tilde{B}$
   b. If $X''_{\tilde{A}} < X''_{\tilde{B}}$ then $\tilde{A} < \tilde{B}$
   c. If $X''_{\tilde{A}} = X''_{\tilde{B}}$ then $\tilde{A} = \tilde{B}$

Example 12. Assume that two SNTN-number be

$$\tilde{A} = \langle\left(1,3,5,7\right);0.3,0.5,0.8\rangle$$

and

$$\tilde{B} = \langle(2,5,6,9);0.8,0.6,0.3\rangle$$

Then, we have

$$X'_{\tilde{A}} = \frac{7^2 + 35 + 5^2 - 1^2 - 3 - 3^2}{3\left(7+5-1-3\right)}$$
$$= 4$$

$$X'_{\tilde{B}} = \frac{9^2 + 54 + 6^2 - 2^2 - 10 - 5^2}{3\left(9+6-2-5\right)}$$
$$= 5,5$$

Since $X'_{\tilde{A}} < X'_{\tilde{B}}$, we have $\tilde{A} < \tilde{B}$.

Theorem 13. Let $\tilde{A}$ and $\tilde{B}$ be any two SNTN-numbers or SNTrN-numbers. Then the equations is not always valied.

1. $X'_A + X'_B = X'_{A+B}$
2. $X''_A + X''_B = X''_{A+B}$
3. $X'_A . X'_B = X'_{A.B}$
4. $X''_A . X''_B = X''_{A.B}$

4. A NEW SVN-MADM METHOD ON 1. AND 2. CENTROID

In this section, 1. and 2. Centroid point of SNTrN-numbers are applied to MADM problems. For a MADM problem with SNTrN-numbers, suppose that the set of alternatives is $A = \{x_1, x_2, ..., x_m\}$ and the set of criteria is $C = \{u_1, u_2, ..., u_n\}$. Based on the alternates($x_i \left(i = 1, 2, ..., m\right)$ with respect to the criteria $u_j \left(j = 1, 2, ..., n\right)$, evaluation information

$$\left(x_{ij}\right)_{mxn} = \left\langle (a_{ij}, b_{ij}, c_{ij}); w_{ij}, u_{ij}, y_{ij} \right\rangle$$

which is described by SNTrN-numbers, is given by a expert based on Table 1. Also, $\omega = (\omega_1, \omega_2, ..., \omega_n)$ be weighting vector of the criteria set $C = \{u_1, u_2, ..., u_n\}$ given by expert based on Table 2.

Now we give the decision-making procedures as follows;

Step 1: Give the evaluation matrix $\left(x_{ij}\right)_{mxn}$;

Step 2: Give the weighting vector $\omega = (\omega_1, \omega_2, ..., \omega_n)$ of the criteria set.

Step 3: Find of the normalized weights of the criteria set as;

$$w_j = \frac{X'(\omega_j)}{\sum_{j=1}^{n} X'(\omega_j)} \left(j = 1, 2, ..., n\right)$$

Table 1. The linguistic values of the SVTrN-number for the evaluation matrix

Linguistic Values	SVTrN-Number Values
Very Poor (VP)	$\langle(0.0, 0.0, 0.1); 0.1, 0.7, 0.8\rangle$
Poor (P)	$\langle(0.1, 0.1, 0.2); 0.1, 0.6, 0.7\rangle$
Medium Pool (MP)	$\langle(0.2, 0.3, 0.4); 0.3, 0.6, 0.7\rangle$
Bad(W)	$\langle(0.3, 0.4, 0.5); 0.4, 0.5, 0.6\rangle$
Medium Good (MG)	$\langle(0.5, 0.6, 0.7); 0.7, 0.5, 0.4\rangle$
Good (G)	$\langle(0.7, 0.8, 0.9); 0.8, 0.4, 0.3\rangle$
Very Good (VG)	$\langle(0.8, 0.9, 1.0); 0.9, 0.4, 0.2\rangle$

Step 4: Compute the weiğhted matrix

$$M_{ij} = w_j \times k_{ij}, \left(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\right);$$

Step 5: Compute the $S_i = \sum_{j=1}^{n} M_{ij}$ (i=1,2,…,m) for $x_i \left(i = 1, 2, \ldots, m\right)$

Step 6: X'_{S_i} (or X''_{S_i}) for the values $S_i \left(i = 1, 2, \ldots, m\right)$;

Step 7: Rank all alternatives $x_i \left(i = 1, 2, \ldots, m\right)$, by using the X'_{S_i} (or X''_{S_i}) and determine the best alternative.

Example 14. The board of an automobile factory plans to select an automobile (or car) model design from the alternatives set for the Middle East countries for the purpose of achieve the highest profit. For this, there are five automobile (or car) model design indicated by $x_i \left(i = 1, 2, \ldots, 5\right)$. For this automobile design the board have a criteria set $C = \{u_1 =$ political developments in the countries where sales will be made $u_2 =$ taxation, $u_3 =$ car's spare parts price, $u_4 =$ suitability of the car to the climate and physical conditions of the countries $\}$. The board use the linguistic

Table 2. The linguistic values of the SVTrN-number for the criteria weights

Linguistic Values	SVTrN-Number Values
Very Low (VL)	$\langle(0.0, 0.0, 0.1); 0.1, 0.0, 0.0\rangle$
Low (L)	$\langle(0.0, 0.2, 0.3); 0.2, 0.1, 0.1\rangle$
Medium Low (ML)	$\langle(0.2, 0.3, 0.4); 0.4, 0.3, 0.2\rangle$
Medium (M)	$\langle(0.4, 0.5, 0.6); 0.5, 0.4, 0.3\rangle$
Medium High (MH)	$\langle(0.5, 0.6, 0.7); 0.6, 0.5, 0.4\rangle$
High (H)	$\langle(0.7, 0.8, 0.9); 0.8, 0.7, 0.6\rangle$
Very High (VH)	$\langle(0.9, 1.0, 1.0); 1.0, 0.9, 0.8\rangle$

terms shown in Table 1 to represent the characteristics of the potential areas with respect to different criteria. Also, same board use the linguistic terms shown in Table 2 to represent criteria weights. Using the factory management data, the algorithm will select the automobile model design for the most profitable investmen.

Step 1: We gave the evaluation matrix $\left(x_{ij}\right)_{5x4}$ for automobile(or car) model design as Table 3,

Step 2: The weighting vector

$$\omega = (\omega_1 = M, \omega_2 = MH, \omega_3 = H, \omega_4 = VH)$$

of the criteria set is given by expert based on Table 2 according to j. attribute $u_j\left(j = 1, 2, \ldots, 4\right)$

Step 3: We found normalized weights of the criteria set as a weighting vector;

$$\omega = (\omega_1 = 0{,}173913043, \omega_2 = 0{,}217391304, \omega_3 = 0{,}260869565, \omega_4 = 0{,}347826087)$$

Table 3. Evaluation matrix for automobile (or car) model design

	u_1	u_2
x_1	$\langle(0.2,0.3,0.4);0.3,0.6,0.7\rangle$	$\langle(0.3,0.4,0.5);0.4,0.5,0.6\rangle$
x_2	$\langle(0.5,0.6,0.7);0.7,0.5,0.4\rangle$	$\langle(0.8,0.9,1.0);0.9,0.4,0.2\rangle$
x_3	$\langle(0.7,0.8,0.9);0.8,0.4,0.3\rangle$	$\langle(0.7,0.8,0.9);0.8,0.4,0.3\rangle$
x_4	$\langle(0.1,0.1,0.2);0.1,0.6,0.7\rangle$	$\langle(0.5,0.6,0.7);0.7,0.5,0.4\rangle$
x_5	$\langle(0.3,0.4,0.5);0.4,0.5,0.6\rangle$	$\langle(0.2,0.3,0.4);0.3,0.6,0.7\rangle$
	u_3	u_4
x_1	$\langle(0.7,0.8,0.9);0.8,0.4,0.3\rangle$	$\langle(0.1,0.1,0.2);0.1,0.6,0.7\rangle$
x_2	$\langle(0.1,0.1,0.2);0.1,0.6,0.7\rangle$	$\langle(0.3,0.4,0.5);0.4,0.5,0.6\rangle$
x_3	$\langle(0.3,0.4,0.5);0.4,0.5,0.6\rangle$	$\langle(0.5,0.6,0.7);0.7,0.5,0.4\rangle$
x_4	$\langle(0.5,0.6,0.7);0.7,0.5,0.4\rangle$	$\langle(0.8,0.9,1.0);0.9,0.4,0.2\rangle$
x_5	$\langle(0.0,0.0,0.1);0.1,0.7,0.8\rangle$	$\langle(0.7,0.8,0.9);0.8,0.4,0.3\rangle$

Step 4: We computed the weiğhted matrix

$$M_{ij} = \omega_j \times k_{ij}, \left(i = 1,2,...,5; j = 1,2,3,4\right)$$

as Table 4,

Step 5: We computed the $S_i = \sum_{j=1}^{4} M_{ij}$ for $x_i\left(i = 1,2,...,5\right)$ as;

$$S_1 = \langle(0.31739, 0.38260, 0.48260);0.1,0.6,0.7\rangle$$
$$S_2 = \langle(0.39130, 0.46521, 0.56521);0.1,0.6,0.7\rangle$$
$$S_3 = \langle(0.52608, 0.62608, 0.72608);0.4,0.5,0.6\rangle$$
$$S_4 = \langle(0.53478, 0.61739, 0.71739);0.1,0.6,0.7\rangle$$
$$S_5 = \langle(0.33913, 0.41304, 0.51304);0.1,0.7,0.8\rangle$$

Step 6: We found X'_{S_i} (or X''_{S_i}) for the values $S_i \left(i = 1, 2, ..., 5\right) as;$;

$$X'_{S_1} = 0.39420 \quad X''_{S_1} = -0,01814177$$
$$X'_{S_2} = 0.47391 \quad X''_{S_2} = -0,016503165$$
$$X'_{S_3} = 0.62608 \quad X''_{S_3} = -0,01374502$$
$$X'_{S_4} = 0.62318 \quad X''_{S_4} = -0,013236678$$
$$X'_{S_5} = 0.42173 \quad X''_{S_5} = -0,020969049$$

Step 7: All alternatives $x_i \left(i = 1, 2, ..., 5\right)$, by using the X'_{S_i} (or X''_{S_i}) for the values $S_i \left(i = 1, 2, ..., 5\right)$ is ranked as;

$$X'_{S_3} > X'_{S_4} > X'_{S_2} > X'_{S_5} > X'_{S_1}$$

Since we have

$$x_3 > x_4 > x_2 > x_5 > x_1$$

the best alternative is x_3.

5. APPLICATION OF 1. AND 2. CENTROID POINT TO FUZZY GRAPHS THEORY

(SUNITHA, 2001) said "A graph is a symmetric binary relation on a nonempty set V. Similarly, a fuzzy graph by introduced (Rosenfeld, 1975) and (Kaufmann, 1977) is a symmetric binary fuzzy relation on a fuzzy subset.

Definition 15. (Harary, 1988) A graph $G = (V, E)$ is a pair of vertex set (V) and an edge set (E) where $E \subseteq V \times V$ V i.e. E is a relation on V.

Definition 16. (Rosenfeld, 1975) A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ with

$\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for $\forall u, v \in V$,

Table 4. $M_{ij} = \omega_j \times k_{ij}, \left(i = 1,2,...,5; j = 1,2,3\right)$ *matrix*

	u_1	u_2
x_1	$\langle\left(0.03478, 0.05217, 0.06956\right); 0.3, 0.6, 0.7\rangle$	$\langle\left(0.06521, 0.08695, 0.10869\right); 0.4, 0.5, 0.6\rangle$
x_2	$\langle\left(0.08695, 0.10434, 0.12173\right); 0.7, 0.5, 0.4\rangle$	$\langle\left(0.17391, 0.19565, 0.21739\right); 0.9, 0.4, 0.2\rangle$
x_3	$\langle\left(0.12173, 0.13913, 0.15652\right); 0.8, 0.4, 0.3\rangle$	$\langle\left(0.15217, 0.17391, 0.19565\right); 0.8, 0.4, 0.3\rangle$
x_4	$\langle\left(0.01739, 0.01739, 0.03478\right); 0.1, 0.6, 0.7\rangle$	$\langle\left(0.10869, 0.13043, 0.15217\right); 0.7, 0.5, 0.4\rangle$
x_5	$\langle\left(0.05217, 0.06956, 0.08695\right); 0.4, 0.5, 0.6\rangle$	$\langle\left(0.04347, 0.06521, 0.08696\right); 0.3, 0.6, 0.7\rangle$
	u_3	u_4
x_1	$\langle\left(0.18261, 0.2087, 0.23478\right); 0.8, 0.4, 0.3\rangle$	$\langle\left(0.03478, 0.03478, 0.06956\right); 0.1, 0.6, 0.7\rangle$
x_2	$\langle\left(0.02609, 0.02609, 0.05217\right); 0.1, 0.6, 0.7\rangle$	$\langle\left(0.10435, 0.13913, 0.17391\right); 0.4, 0.5, 0.6\rangle$
x_3	$\langle\left(0.07826, 0.10435, 0.13043\right); 0.4, 0.5, 0.6\rangle$	$\langle\left(0.17391, 0.20869, 0.24347\right); 0.7, 0.5, 0.4\rangle$
x_4	$\langle\left(0.13043, 0.15652, 0.18261\right); 0.7, 0.5, 0.4\rangle$	$\langle\left(0.27826, 0.31304, 0.34782\right); 0.9, 0.4, 0.2\rangle$
x_5	$\langle\left(0.0, 0.0, 0.02609\right); 0.1, 0.7, 0.8\rangle$	$\langle\left(0.24348, 0.27826, 0.31304\right); 0.8, 0.4, 0.3\rangle$

where V is a finite non empty set and ∧ denote minimum.

Note that centroid points of normalized SNTN-numbers or SNTrN-numbers are fuzzy values. Therefore, all definitions and properties of fuzzy graph theory are valid for centroid points of normalized SNTN-numbers or SNTrN-numbers. For example, we can give a definition of fuzzy graph theory, in (Rosenfeld, 1975) based on Centroid points of normalized SNTN-numbers or SNTrN-numbers.

Definition 17. Let E be a universe set and $\tilde{A}(x), x \in E$ be an normalized SNTN-number or SNTrN-number. Then, an SNTN or SNTrN set NS on universe set E is given by

$$NS = \{\tilde{A}(x) \,/\, x : x \in E\}$$

If $C(x)$ is centroid point of $\tilde{A}(x), x \in E$, then we use centroid points of normalized SNTN-numbers or SNTrN-numbers to reduce the SNTN-numbers or SNTrN-numbers to fuzzy values.

SNTN or SNTrN set NS_c based on centroid point is given by

$$NS_c = \{C(x) \,/\, x : C(x) \in [0,1], C \in \{X', X''\}, C(x) \text{ is centroid point of } \tilde{A}(x), x \in E\}$$

Now, we give some definitions based on Def. 15-16.

Definition 18. SNTN or SNTrN set NS_c based on centroid point graph $G_c = (\sigma, C)$ is a pair of functions $\sigma : E \to [0,1]$ and

$$C : E \times E \to [0,1] C(x,y) \le \sigma(x) \wedge \sigma(y) \text{ for } \forall x, y \in E,$$

where E is a finite non empty set and $\wedge$ denote minimum.

CONCLUSION

In this study, we presented two centroid point, are called 1. and 2. centroid point, for SNTN-numbers or SNTrN-numbers. Then, we studied some desired properties of 1. and 2. centroid point for SNTN-numbers or SNTrN-numbers. Also, we proposed based on concept of 1. and 2. centroid point, a new single valued neutrosophic multiple attribute decision making method. Finally, a numerical example is given and application of 1. and 2. centroid point to fuzzy graphs theory is proposed. This study can be extended by using other type of neutrosophic multi-attribute approaches, including interval valued neutrosophic numbers, such as; TOPSIS, VIKOR, PROMETHEE and so on.

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KEY TERMS AND DEFINITIONS

Centroid Point: It's area under the the degree of truth-membership functions, indeterminacy-membership function and falsity-membership functions which is consider together.

Graph Theory: It is mathematical structures used to model pairwise relations between objects.

Multi-Criteria Decision Making: It is a mathematical method which deal with decisions involving the choice of a best alternative from alternatives set.

SVTN-Number: A single valued trapezoidal neutrosophic number (SVTN-number) $\tilde{A} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ is a special neutrosophic set on the real number set R.

SVTrN-Number: A single valued triangular neutrosophic number (SVTrN-number) $\tilde{A} = \langle (a_1, b_1, c_1); w_{\tilde{A}}, u_{\tilde{A}}, y_{\tilde{A}} \rangle$ is a special neutrosophic set on the real number set R.

Chapter 11
Distinguishable and Inverses of Neutrosophic Finite Automata

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ABSTRACT

This chapter focuses on neutrosophic finite automata with output function. Some new notions on neutrosophic finite automata are established and studied, such as distinguishable, rational states, semi-inverses, and inverses. Interestingly, every state in finite automata is said to be rational when its inputs are ultimately periodic sequence that yields an ultimately periodic sequence of outputs. This concludes that any given state is rational when its corresponding sequence of states is distinguishable. Furthermore, this study is to prove that the semi-inverses of two neutrosophic finite automata are indistinguishable. Finally, the result shows that any neutrosophic finite automata and its inverse are distinguished, and then their reverse relation is also distinguished.

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INTRODUCTION

Automata are abstract machines for solving computational problems (Kafle & Gallagher 2017; Pan et.al. 2015). Fuzzy automata emerge from the inclusion of fuzzy logic into automata theory. Fuzzy finite automata are beneficial to model uncertainties which inherent in many applications (Doostfatemeh & Kremer 2005). Fuzzy finite automata with output offers further inclination in providing output compares to fuzzy finite automata. For each assigning input, the machine will generate output and its value is a function of the current state and the current input. Generally, in most applications and systems there is some kind of final output or decision. In the realm of deterministic and non-deterministic finite automata, there are no explicit outputs beyond the concept of acceptance attributed to each state. Hence, it is significant to assign output values to the states of fuzzy finite automata. In order to enhance the membership value of fuzzy automata into general algebraic strucutres, Li and Pedrycz (2005) study the concept of fuzzy automata based on lattice-ordered monoids. Jin et al. (2013) investigated the algebraic study of fuzzy automata based on po-monoids. Abolpour and Zahedi (2017) utlize the categorical concepts in the study of automata with membership values in different lattice structures.

In 1955, from the seminal paper by Mealy (1955) introduced Mealy automaton is a finite state machine with output, in which the state transitions are uniquely determined by the current state and input state. Further, Moore (1956) introduced a sequential machine based on output states, say, Moore automata. Mordeson and Nair (1996) have been introduced and studied a new case of automata that is so-called fuzzy Mealy machine. Verma and Tiwari (2017) recently introduced and studied the concepts of state distinguishability, input-distinguishability and output completeness of states of a crisp deterministic fuzzy automaton with output function. Huang et al. (2017) established the concepts of weak commutativity of a Mealy-type fuzzy finite state machine and studied its characterizations and properties. In 2018, Mealy-type and Moore-type weighted finite automata with output with respect to various semantics were studied by in (Ignjatovic et al. 2018).

The notions of neutrosophic sets was proposed by Smarandache (1999; 2006), generalizing the existing ordinary fuzzy sets, intuitionistic fuzzy sets and interval-valued fuzzy set in which each element of the universe has the degrees of truth, indeterminacy and falsity and the membership values are lies in $]0^{-},1^{+}[$, the nonstandard unit interval (Rivieccio 2018) it is an extension from standard interval [0,1]. It has been shown that fuzzy sets provide a limited platform for computational complexity but neutrosophic sets are suitable for it. The neutrosophic set is an appropriate mechanism for interpreting real-life philosophical problems but not for scientific problems since it is difficult to consolidate. In neutrosophic sets, the degree of indeterminacy can be defined independently since it is quantified explicitly which led to different from intuitionistic fuzzy sets. As an application neutrosophic sets have been applied in many disciplines such as traffic control management (Nagarajan et al. 2019a; Nagarajan et al. 2019 b), graph theory (Tan et al. 2019; Jan et al. 2019; Broumi 2018) and decision-making (Abdel-Basset et al. 2019; Fahmi et al. 2018).

The concept of single-valued neutrosophic finite state machine and switchboard state machine has been introduced and studied by (Tahir 2018) and the interval neutrosophic finite switchboard state machine in (Tahir & Khan 2016). Recently, Kavikumar et al. (2019b) studied the concepts of neutrosophic general fuzzy automata and neutrosophic general switchboard automata based on the work of Kavikumar et al. (2019a). Some generalized form of the finite state machines can found in the literature, for instance, bipolar finite state machines (Jun & Kavikumar 2011), N-structures of fuzzy finite state machines (Kavikumar et al. 2013) and intuitionistic fuzzy finite machines (Jun 2005). However, the realm of the automata with fuzzy output in the neutrosophic environment has not been studied yet in the literature so far. Hence, it is possible to study the algebraic structure of the neutrosophic finite automata with output function. Inspired by the work of (Ginsburg 1960; Smarandache 1999; Smarandache 2006) the concept of neutrosophic finite automata is introduced in this paper. We study the distinguishability and rational states of neutrosophic finite automata in Section 3; specifically, we introduce the definition of neutrosophic finite automata. Section 4 focused on the reverse action of the machine, which we called as inverses of neutrosophic finite automata.

PRELIMINARIES

Let us recall basic concepts and notation.

Definition 2.1. (Smarandache 1999). Let *X* be a universe of discourse. The neutrosophic set is an object having the form

$$A = \{\prec x, \delta_1(x), \delta_2(x), \delta_3(x) \succ \mid \forall x \in X\}$$

where the functions can be defined by

$$\delta_1, \delta_2, \delta_3 : X \to \left]0,1\right[$$

and δ_1 is the degree of membership or truth, δ_2 is the degree of indeterminacy and δ_3 is the degree of non-membership or false of the element $x \in X$ to the set *A* with the condition

$$0 \leq \delta_1(x) + \delta_2(x) + \delta_3(x) \leq 3.$$

Let *X* be a universe of discourse and λ is a neutrosophic subset of *X*. A map $\lambda : X \to L$, where *L* is a lattice-ordered monoid. The definition of lattice-ordered monoid is as follows:

Definition 2.2. (Guo 2009). An algebra $L = (L, \leq, \wedge, \vee, , 0, 1)$ is called a **lattice-ordered monoid** if

1. $L = (L, \leq, \wedge, \vee, 0, 1)$ is a lattice with the least element 0 and the greatest element 1.
2. (*L*,•, 1) is a monoid with identity $1 \in L$ such that $a, b, c \in L$.
   a. $a0 = 0a = 0$,
   b. $a \leq b \Rightarrow \forall x \in L, ax \leq bx$ and $xa \leq xb$.
   c. $a(b \vee c) = (ab) \vee (bc)$ and $(b \vee c)a = (ba) \vee (ca)$.

Throughout, we work with a lattice-ordered monoid *L* so that the monoid (*L,*•, 1) satisfies the left cancellation law.

A finite *fuzzy automaton with outputs*, or *Mealy-type*, known also as a *fuzzy finite state machine*, is a tuple $M = (Q, \Sigma, Z, \delta, \sigma)$ (Petkovic 2006), where *Q* is a finite set of *states*, Σ is a finite set of *input* symbols, *Z* is a finite set of *output* symbols, δ is a fuzzy subset of $Q \times \Sigma \times Q$ representing *transition mapping*, σ is a fuzzy subset of $Q \times \Sigma \times Z$ representing *output mapping*, and the following condition is satisfied:

$$(q_i \in Q)(\forall x \in \Sigma)\Big(\big((\exists q_j \in Q)\delta(q_i, x, q_j) > 0\big) \Leftrightarrow \big((\exists y \in Z)\sigma(q_i, x, y) > 0\big)\Big).$$

The alphabets Σ and *Z* will be fixed throughout the paper. By Σ* and *Z** free monoids, i.e., the sets of all words with letters from Σ and *Z*, respectively are denoted. The empty word is denoted by Λ. As proved in [8], mappings δ and σ can be extended to the sets $Q \times \Sigma^* \times Q$ and $Q \times \Sigma^* \times Z$, respectively, by

$$\delta\left(q_i, \Lambda, q_j\right) = \begin{cases} 1 & if\, q_i = q_j \\ 0 & if\, q_i \neq q_j \end{cases}$$

$$\delta\left(q_i, x_1 x_2, q_j\right) = \bigvee_{r \in Q} \left\{ \delta\left(q_i, x_1, r\right) \wedge \delta\left(r, x_2, q_j\right) \right\}$$

$$\sigma\left(q_i, x, q_j, y\right) = \begin{cases} 1 & if\, x = y = \Lambda \\ 0 & if\, x = \Lambda, y \neq \Lambda\, or\, x \neq \Lambda, y = \Lambda \end{cases}$$

$$\sigma\left(q_i, x_1 x_2, q_j, y_1 y_2\right) = \bigvee_{r \in Q} \left\{ \sigma\left(q_i, x_1, q_j, y\right) \wedge \delta\left(q_i, x_1, r\right) \wedge \sigma\left(r, x_2, q_j, y_2\right) \right\}$$

for all

$q_i, q_j \in Q, x_1 \in \Sigma, y_1 \in Z, x_2 \in \Sigma^*, y_2 \in Z^*$.

DISTINGUISHABLE AND RATIONAL STATES

Throughout the paper, a neutrosophic finite automaton with outputs (in short ; neutrosophic finite automata) has considered with neutrosophic transition function and neutrosophic output function.

The first part of this section deals with the concept of distinguishable and indistinguishable states which between the neutrosophic finite automata. According to Ginsburg (1960), every state in finite automata is said to be rational when its inputs are ultimately periodic sequence, which yields an ultimately periodic sequence of outputs. Thus, we now consider the properties of the rational states in neutrosophic finite automata.

Definition 3.1. A neutrosophic finite automaton is a five-tuple $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$, where Q is a finite non-empty set of states, Σ is a finite set of input alphabet, Z is a finite set of output alphabet, δ is a neutrosophic subset of $Q \times \Sigma \times Q$ which represents neutrosophic transition function, and σ is a neutrosophic subset of $Q \times \Sigma \times Z$ which represents neutrosophic output function.

Definition 3.2. Let $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$ be a neutrosophic finite automata.

1. $Q = \left\{q_1, q_2, \ldots, q_n\right\}$, is a finite set of states
2. $\Sigma = \left\{x_1, x_2, \ldots, x_n\right\}$, is a finite set the input symbols,
3. $Z = \left\{y_1, y_2 \ldots, y_n\right\}$, is a finite set of output symbols,
4. Let $\delta = \prec \delta_1, \delta_2, \delta_3 \succ$ is a neutrosophic subset of $Q \times \Sigma \times Q$ such that the neutrosophic transition function
5. $\delta : Q \times \Sigma \times Q \to L \times L \times L$
6. is defined as follows: for all $q_i, q_j \in Q$ and $x_1, x_2 \in \Sigma$,

$$\delta_1\left(q_i, \Lambda, q_j\right) = \begin{cases} 1 & if\, q_i = q_j \\ 0 & if\, q_i \neq q_j \end{cases}$$

$$\delta_2\left(q_i, \Lambda, q_j\right) = \begin{cases} 0 & if\, q_i = q_j \\ 1 & if\, q_i \neq q_j \end{cases}$$

$$\delta_3\left(q_i, \Lambda, q_j\right) = \begin{cases} 0 & if\, q_i = q_j \\ 1 & if\, q_i \neq q_j \end{cases}$$

and

$$\delta_1\left(q_i, x_1x_2, q_j\right) = \bigvee_{r \in Q}\left\{\delta_1\left(q_i, x_1, r\right) \wedge \delta_1\left(r, x_2, q_j\right)\right\}$$

$$\delta_2\left(q_i, x_1x_2, q_j\right) = \bigwedge_{r \in Q}\left\{\delta_2\left(q_i, x_1, r\right) \vee \delta_2\left(r, x_2, q_j\right)\right\}$$

$$\delta_3\left(q_i, x_1x_2, q_j\right) = \bigwedge_{r \in Q}\left\{\delta_3\left(q_i, x_1, r\right) \vee \delta_3\left(r, x_2, q_j\right)\right\}$$

7. Let $\sigma = \prec \sigma_1, \sigma_2, \sigma_3 \succ$ is a neutrosophic subset of $Q \times \Sigma \times Z$ such that the neutrosophic output function

$$\sigma : Q \times \Sigma \times Z \to L \times L \times L$$

is defined as follows: for all $q_i, q_j \in Q$, $x_1, x_2 \in \Sigma$ and $y_1, y_2 \in Z$,

$$\sigma_1(q_i, x_1, y_1) = \begin{cases} 1 & if\, x_1 = y_1 = \Lambda \\ 0 & if\, x_1 = \Lambda, y_1 \neq \Lambda\, or\, x_1 \neq \Lambda, y_1 = \Lambda \end{cases}$$

$$\sigma_2(q_i, x_1, y_1) = \begin{cases} 0 & if\, x_1 = y_1 = \Lambda \\ 1 & if\, x_1 = \Lambda, y_1 \neq \Lambda\, or\, x_1 \neq \Lambda, y_1 = \Lambda \end{cases}$$

$$\sigma_3(q_i, x_1, y_1) = \begin{cases} 0 & if\, x_1 = y_1 = \Lambda \\ 1 & if\, x_1 = \Lambda, y_1 \neq \Lambda\, or\, x_1 \neq \Lambda, y_1 = \Lambda \end{cases}$$

and

$$\sigma_1(q_i, x_1x_2, y_1y_2) = \bigvee_{r\in Q} \left\{\sigma_1(q_i, x_1, y_1) \wedge \delta_1(q_i, x_1, r) \wedge \sigma_1(r, x_2, y_2)\right\}$$
$$= \sigma_1(q_i, x_1, y_1) \cdot \bigvee_{r\in Q} \left\{\delta_1(q_i, x_1, r) \wedge \sigma_1(r, x_2, y_2)\right\}$$

$$\sigma_2(q_i, x_1x_2, y_1y_2) = \bigwedge_{r\in Q} \left\{\sigma_2(q_i, x_1, y_1) \vee \delta_2(q_i, x_1, r) \vee \sigma_2(r, x_2, y_2)\right\}$$
$$= \sigma_2(q_i, x_1, y_1) \cdot \bigwedge_{r\in Q} \left\{\delta_2(q_i, x_1, r) \vee \sigma_2(r, x_2, y_2)\right\}$$

$$\sigma_3(q_i, x_1x_2,, y_1y_2) = \bigwedge_{r\in Q} \left\{\sigma_3(q_i, x_1, y_1) \vee \delta_3(q_i, x_1, r) \vee \sigma_3(r, x_2, y_2)\right\}$$
$$= \sigma_3(q_i, x_1, y_1) \cdot \bigwedge_{r\in Q} \left\{\delta_3(q_i, x_1, r) \vee \sigma_3(r, x_2, y_2)\right\}$$

Definition 3.3. Let $M = (Q, \Sigma, Z, \delta, \sigma)$ and

$$M' = (Q', \Sigma, Z, \delta', \sigma')$$

be a neutrosophic finite automata. A pair of states $(q_i, q_i^{'})$ is said to be **indistinguishable**, if

$$\sigma(q_i, x, y) = \sigma'(q_i^{'}, x, y),$$

for every $q_i \in Q, q_i^{'} \in Q'$ and $\forall x \in \Sigma, y \in Z$.

Remark 3.1. If a pair of states $\left(q_i, q_i^{'}\right)$ is not indistinguishable, then it is said to be **distinguishable**. A neutrosophic finite automata *M* is said to be **indistinguished** if each pair of distinct states in *M* are indistinguishable.

Definition 3.4. A neutrosophic finite automata $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$ is called **free** if for each $q_i \in Q$ and for each $x \in \Sigma$ there exists $y \in Z$ such that $\sigma\left(q_i, x, y\right) > 0$.

Proposition 3.1. Let

$$M = \left(Q, \Sigma, Z, \delta, \sigma\right) \text{and } M' = \left(Q', \Sigma, Z, \delta', \sigma'\right)$$

be neutrosophic finite automata. If a pair of states $\left(q_i, q_i^{'}\right)$ is indistinguishable, then $\delta\left(q_i, x, q_j\right)$ is indistinguishable from $\delta'\left(q_i^{'}, x, q_j^{'}\right), \forall x \in \Sigma$.

Proof: Since the Definition 3.3, for all $x_1 \in \Sigma$ and $y_1 \in Z$, a pair of states $\left(q_i, q_j^{'}\right)$ is indistinguishable, we have

$$\sigma\left(q_i, x, q_j\right) = \sigma'\left(q_i^{'}, x, q_j^{'}\right).$$

Now, for $x_2 \in \Sigma$ and $y_2 \in Z$,

$$\{\sigma_1\left(q_i, x_1, y_1\right) \cdot \bigvee_{r \in Q} \left\{\delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_2, y_2\right)\right\}$$

$$= \bigvee_{r \in Q} \{\sigma_1\left(q_i, x_1, y_1\right) \wedge \delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_2, y_2\right)\}$$

$$= \sigma_1\left(q_i, x_1 x_2, y_1 y_2\right) = \sigma_1^{'}\left(q_i^{'}, x_1 x_2, y_1 y_2\right)$$

$$= \bigvee_{r' \in Q'} \left\{\sigma_1^{'}\left(q_i^{'}, x_1, y_1\right) \wedge \delta_1^{'}\left(q_i^{'}, x_1, r'\right) \wedge \sigma_1^{'}\left(r', x_2, y_2\right)\right\}$$

$$= \sigma_1^{'}\left(q_i^{'}, x_1, y_1\right) \cdot \bigvee_{r \in Q} \left\{\delta_1^{'}\left(q_i^{'}, x_1, r'\right) \wedge \sigma_1^{'}\left(r', x_2, y_2\right)\right\},$$

this implies that

$$\delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_2, y_2\right) = \delta_1^{'}\left(q_i^{'}, x_1, r'\right) \wedge \sigma_1^{'}\left(r', x_2, y_2\right),$$

as *L* satisfies the left cancellation law. Thus, $\delta_1\left(q_i, x_1, r\right)$ is indistinguishable from $\delta_1^{'}\left(q_i^{'}, x_1, r'\right)$.

$$\sigma_2\left(q_i, x_1, y_1\right) \cdot \bigwedge_{r\in Q}\left\{\delta_2\left(q_i, x_1, r\right) \vee \sigma_2\left(r, x_2, y_2\right)\right\}$$

$$= \bigwedge_{r\in Q}\left\{\sigma_2\left(q_i, x_1, y_1\right) \vee \delta_2\left(q_i, x_1, r\right) \vee \sigma_2\left(r, x_2, y_2\right)\right\}$$

$$= \sigma_2\left(q_i, x_1x_2, y_1y_2\right) = \sigma_2^{'}\left(q_i^{'}, x_1x_2, y_1y_2\right)$$

$$= \bigwedge_{r\in Q}\left\{\sigma_2^{'}\left(q_i^{'}, x_1, y_1\right) \vee \delta_2^{'}\left(q_i^{'}, x_1, r'\right) \vee \sigma_2^{'}\left(r', x_2, y_2\right)\right\}$$

$$= \sigma_2^{'}\left(q_i^{'}, x_1, y_1\right) \cdot \bigwedge_{r\in Q}\left\{\delta_2^{'}\left(q_i^{'}, x_1, r'\right) \vee \sigma_2^{'}\left(r', x_2, y_2\right)\right\},$$

this implies that

$$\delta_2\left(q_i, x_1, r\right) \vee \sigma_2\left(r, x_2, y_2\right) = \delta_2^{'}\left(q_i^{'}, x_1, r'\right) \vee \sigma_2^{'}\left(r', x_2, y_2\right),$$

as *L* satisfies the left cancellation law. Thus, $\delta_2\left(q_i, x_1, r\right)$ is indistinguishable from $\delta_2^{'}\left(q_i^{'}, x_1, r'\right)$.

$$\sigma_3\left(q_i, x_1, y_1\right) \cdot \bigwedge_{r\in Q}\left\{\delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_2, y_2\right)\right\}$$

$$= \bigwedge_{r\in Q}\left\{\sigma_3\left(q_i, x_1, y_1\right) \vee \delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_2, y_2\right)\right\}$$

$$= \sigma_3\left(q_i, x_1x_2, y_1y_2\right) = \sigma_3^{'}\left(q_i^{'}, x_1x_2, y_1y_2\right)$$

$$= \bigwedge_{r'\in Q'}\left\{\sigma_3^{'}\left(q_i^{'}, x_1, y_1\right) \vee \delta_3^{'}\left(q_i^{'}, x_1, r'\right) \vee \sigma_3^{'}\left(r', x_2, y_2\right)\right\}$$

$$= \sigma_3^{'}\left(q_i^{'}, x_1, y_1\right) \cdot \bigwedge_{r' \in Q'} \left\{\delta_3^{'}\left(q_i^{'}, x_1, r'\right) \vee \sigma_3^{'}\left(r', x_2, y_2\right)\right\},$$

this implies that

$$\delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_2, y_2\right) = \delta_3^{'}\left(q_i^{'}, x_1, r'\right) \vee \sigma_3^{'}\left(r', x_2, y_2\right),$$

as *L* satisfies the left cancellation law. Thus, $\delta_3\left(q_i, x_1, r\right)$ is indistinguishable from $\delta_3^{'}\left(q_i^{'}, x_1, r'\right)$. Thus $\delta\left(q_i, x, q_j\right)$ is indistinguishable from $\delta'\left(q_i^{'}, x, q_j^{'}\right)$ for $q_j \in Q, q_j^{'} \in Q'$ and $\forall x \in \Sigma$. ν

Theorem 3.1. Let $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$ be a neutrosophic finite automaton, for all

$$q \in Q,\ x \in \Sigma, y \in Z,\ \left|x\right| \neq \left|y\right| \Rightarrow \sigma\left(q_i, x, y\right) = 0.$$

Proof: Let

$$q_i \in Q, x \in \Sigma, y \in Z$$

and $\left|x\right| \neq \left|y\right|$. Suppose $\left|x\right| > \left|y\right|$ and let $\left|y\right| = n$. We prove the result by induction on *n*. If $n = 0$, then $y = \Lambda$ and $x \neq \Lambda$. Hence by definition, we have $\sigma\left(q_i, x, y\right) = 0$. Suppose the result is true for all $x \in \Sigma, y \in Z$ such that $\left|u\right| > \left|v\right|$ and $\left|v\right| = n-1$. Suppose $n \geq 1$. Write $x = ua$, $y = vb$, where

$$u \in \Sigma^*, a \in \Sigma, v \in Z^*$$

and $b \in Z$. Then $\left|u\right| > \left|v\right|$ and $\left|v\right| = n-1$. Now by the induction hypothesis, for all $\backslash r \in Q$, $\sigma\left(r, x, y\right) = 0$. Thus

$$\sigma_1\left(q_i, x, y\right) = \sigma_1\left(q_i, ua, vb\right) = \bigvee_{r \in Q}\left\{\sigma_1\left(q_i, u, v\right) \wedge \delta_1\left(q_i, u, r\right) \wedge \sigma_1\left(r, a, b\right)\right\} = 0$$

$$\sigma_2\left(q_i, x, y\right) = \sigma_2\left(q_i, ua, vb\right) = \bigwedge_{r \in Q}\left\{\sigma_2\left(q_i, u, v\right) \vee \delta_2\left(q_i, u, r\right) \vee \sigma_2\left(r, a, b\right)\right\} = 0$$

$$\sigma_3\left(q_i,x,y\right)=\sigma_3\left(q_i,ua,vb\right)=\bigwedge_{r\in Q}\left\{\sigma_3\left(q_i,u,v\right)\vee\delta_3\left(q_i,u,r\right)\vee\sigma_3\left(r,a,b\right)\right\}=0.$$

Hence for all $q_i\in Q, x\in\Sigma^*$ and $y\in Z^*$, if $\left|x\right|>\left|y\right|$, then $\sigma\left(q_i,x,y\right)=0$. Similarly, by induction, we can show that for all $q\in Q, x\in\Sigma^*$ and $y\in Z^*$, if $\left|x\right|<\left|y\right|$, then $\sigma\left(q_i,x,y\right)=0$. ν

Every state in finite automata is said to be rational when its inputs are ultimately periodic sequence which yields an ultimately periodic sequence of outputs. We now consider the properties of the rational states in neutrosophic finite automata.

Definition 3.5. Let $M=\left(Q,\Sigma,Z,\delta,\sigma\right)$ be a neutrosophic finite automata. A state $q\in Q$ is said to be **rational** if for each ultimately periodic sequence of input $\left\{x_n\right\}\in\Sigma$ there exists ultimately periodic sequence of outputs $\left\{y_n\right\}\in Z$ such that

$$\sigma(q,\{x_n\},\left\{y_n\right\})>0\Rightarrow\left\{\sigma\left(q_n,x_n,y_n\right)\right\}>0,$$

where $q_1=q$ and for $n\geq 2$, $\delta\left(q_{n-1},x_{n-1},q_n\right)>0$.

Remark 3.2.

1. It is clear that if *q* is a rational state of a neutrosophic finite automata and *p* is indistinguishable from *q*, then *p* is rational.
2. To check the given $q\in Q$ is rational state it is enough to assume that the sequence $\left\{x_n\right\}\in\Sigma$ is an infinite.

Theorem 3.2. Let $M=\left(Q,\Sigma,\delta,\sigma\right)$ be a neutrosophic finite automata, let $q\in Q$ and *X* be a generating subset of Σ. If $\sigma(q,\{x_n\},\left\{y_n\right\})>0$ is ultimately periodic for every ultimately periodic infinite sequence $\left\{x_n\right\}\in X$, then *q* is a rational state.

Proof: Straightforward.

Theorem 3.3. Let $M=\left(Q,\Sigma,Z,\delta,\sigma\right)$ be a neutrosophic finite automata. For every positive integer *r*, the associated sequence of distinguished states $\left\{q_i\right\}$ contains an element, say p_r, which occurs at least *r* times. Then *q* is rational.

Proof: Given that $q\in Q$ and $\left\{x_n\right\}$ an ultimately periodic infinite sequence of inputs. For some integers *m* and m_0, $x_{n+m}=x_n$ for all $n\geq m_0$. Assume that the sequence of distinguishable states $\left\{q_i\right\}$ contains an element p_r where

$$r = (m_0 + m)(m_0 + m - 1)$$

which occurs *r* times. Moreover, the subscripts *n* of q_n for which $p_r = q_n$ which gives the sequence $a(1), a(2), \cdots, a(r)$. There are at most $m_0 + m - 1$ different x_i in $\{x_i\}$, since $\{x_n\}$ is ultimately periodic and $x_{n+m} = x_n$ for all $n \geq m_0$, which gives a finite sequence of strictly increasing integers $s_1, s_2, \cdots, s_{m_0+m}$. Similarly, we can get $m_0 + m - 1$ different y_i in $\{y_i\}$. Therefore, we have

$$\left(q_{a(s_i)}, x_{a(s_i)}, y_{a(s_i)}\right) = \left(q_{a(s_j)}, x_{a(s_j)}, y_{a(s_j)}\right).$$

Since $a(s_i) \geq m_0$ for $i \geq m_0$, there are at least $m + 1 a(s_i)$. Therefore there exist two integers t_1 and t_2 in $\{s_i\}$, $m_0 \leq a(t_1) < a(t_2)$, so that $a(t_2) - a(t_1)$ is divisible by *m*, i.e., $a(t_2) - a(t_1) = km$. Now we can get,

$$\left(q_{a(t_1)+n}, x_{a(t_1)+n}, y_{a(t_1)+n}\right) = \left(q_{a(t_1)+km+n}, x_{a(t_1)+km+n}, y_{a(t_1)+km+n}\right)$$

for all $n \geq 0$.

Hence

$$\sigma(q_j, x_j, y_j) = \sigma(q_{j+km}, x_{j+km}, y_{j+km})$$

for all $j \leq a(t_1)$. Therefore

$$\sigma(q_j, \{x_j\}, \{y_j\}) \geq 0$$

is an ultimately periodic sequence so that *q* is a rational state.

INVERSES

This section is interested with a neutrosophic finite automaton, which have inverses. Consider *M* and M' be neutrosophic finite automata. If M' is inverse of *M* which reverse the action performed by *M*.

Definition 4.1. Let $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$ be a neutrosophic finite automata. Let

$M' = \left(Q', \Sigma', Z', \delta', \sigma'\right)$

is a **semi-inverse** of *M* is a neutrosophic finite automata such that:

1. $Z = \Sigma'$ and $Z' = \Sigma$;
2. For each state $q \in Q$,

$\exists g\left(q\right) \in Q', \forall x \in \Sigma, x' \in \Sigma'$

such that

$\sigma'(q\left(q0, \sigma\left(q, x, x'\right), x\right) = \sigma\left(q, x, x'\right)$;

3. For each state $q' \in Q'$,

$\exists h\left(q'\right) \in Q, \forall x \in \Sigma, x' \in \Sigma'$

such that

$\sigma\left(h\left(q'\right), \sigma'\left(q', x', x\right), x'\right) = \sigma'\left(q', x', x\right)$.

Remark 4.1. It is clear that *M* is a semi-inverse of M' whenever M' is a semi-inverse of *M*. In other words, *M* and M' are semi-inverse of each other.

Lemma 4.1. Let $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$ and $M' = \left(Q', \Sigma', Z', \delta', \sigma'\right)$ be semi-inverse of each other. Them the following statements are true:

1. $hg(q)$ is indistinguishable from *q* and $gh\left(q'\right)$ is indistinguishable from q'.
2. If M' is distinguished, then $g\left(h\left(q'\right)\right) = q'$, where $g : Q \to Q'$ and $h : Q' \to Q$ are onto and one-to-one function. If *M* is also distinguished, then *g* and *h* are inverse functions.

3. If

$$\sigma\left(q,\sigma'\left(q\left(q\right),x',x\right),x'\right)=\sigma'\left(q',x',x\right)$$

and

$$\sigma'\left(q',\sigma\left(h\left(q'\right),x,x'\right),x\right)=\sigma\left(q,x,x'\right),$$

$\forall x\in\Sigma$, $x'\in\Sigma'$.

4. For fixed *q*, $\sigma:\Sigma\to Z$ is a one-to-one and onto function.
5. If *M* be a neutrosophic finite automata, then M' also be a neutrosophic finite automata.

Proof:

1. 1. It is enough to show that the pair $\left(q,hq\left(q\right)\right)\in Q$ are indistinguishable. Now, for any $x\in\Sigma$,

$$\sigma\left(q,x,x'\right)=\sigma\left(hg\left(q\right),\sigma'\left[g\left(q\right),\sigma\left(q,x,x'\right),x\right],x'\right)=\sigma\left(hg\left(q\right),\sigma\left(q,x,x'\right),x'\right)=\sigma\left(hg\left(q\right),x,x'\right).$$

This implies that, *q* and $hg\left(q\right)$ are indistinguishable. In similar way, by remark 4.1, we can have q' and $gh\left(q'\right)$ are indistinguishable.

2. Since M' is distinguished and $gh\left(q'\right)$ is indistinguishable from q' by (1). Now define the functions $g:M\to M'$ and $h:M'\to M$ such that $g\left(q\right)=q'$ and $h\left(q'\right)=q$. By composite function we can have

$$g\left(h\left(q'\right)\right)=q',\forall h\left(q'\right)\in Q$$

and $q' \in Q'$. From distinguished of M', it follows that *g* is onto. Again, let $h(q'), h(p') \in Q$ then $h(q') = h(p')$ if and only if

$$q' = gh(q') = gh(p') = p', \forall q', p' \in Q'.$$

Thus *h* is one-to-one. In similar case, we will get $gh(q') = q'$ and $hg(q) = q$ when *M* is distinguished.

3. Since q' and $gh(q)$ are indistinguishable and *q* and $hg(q)$ are indistinguishable, we have

$$\sigma\left(q, \sigma'\left(g(q), x', x\right), x'\right) = \sigma\left(hg(q), \sigma'\left(g(q), x', x\right), x'\right) = \sigma\left(hg(q), \sigma'\left(q', x'x\right), x'\right) = \sigma'\left(, x', x\right)$$

and

$$\sigma'\left(q', \sigma\left(h(q'), x, x'\right), x\right) = \sigma'\left(gh(q), \sigma\left(h(q'), x, x'\right), x\right) = \sigma'\left(gh(q), \sigma\left(q, x, x'\right), x\right) = \sigma\left(q, x, x'\right).$$

4. By definition 4.1 and lemma 4.1(3), it is easy to see that σ is onto.
5. 5. Assume that *M* be a neutrosophic finite automata and let $x_1x_2 = x_1x_3$ where $x_1, x_2, x_3 \in £$. Consider $q_i \in Q$ is a fixed state. Then

$$\sigma_1\left(q_i, x_1, y_1\right) \cdot \left\{\bigvee_{r\in Q} \delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_2, y_2\right)\right\}$$

$$\sigma_1\left(q_i, x_1, y_1\right) \cdot \left\{\bigvee_{r\in Q} \delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_2, y_2\right)\right\}$$

$$= \bigvee_{r\in Q}\left\{\sigma_1\left(q_i, x_1, y_1\right) \wedge \delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_2, y_2\right)\right\} = \sigma_1\left(q_i, x_1x_2, y_1y_2\right) = \sigma_1\left(q_i, x_1x_3, y_1y_2\right)$$
$$= \bigvee_{r\in Q}\left\{\sigma_1\left(q_i, x_1, y_1\right) \wedge \delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_3, y_2\right)\right\} = \sigma_1\left(q_i, x_1, y_1\right) \cdot \left\{\bigvee_{r\in Q} \delta_1\left(q_i, x_1, r\right) \wedge \sigma_1\left(r, x_3, y_2\right)\right\}$$
$$\sigma_2\left(q_i, x_1, y_1\right) \cdot \left\{\bigwedge_{r\in Q} \delta_2\left(q_i, x_1, r\right) \vee \sigma_2\left(r, x_2, y_2\right)\right\}$$

$$= \bigwedge_{r\in Q} \left\{\sigma_2\left(q_i, x_1, y_1\right) \vee \delta_2\left(q_i, x_1, r\right) \vee \sigma_1\left(r, x_2, y_2\right)\right\} = \sigma_2\left(q_i, x_1x_2, y_1y_2\right) = \sigma_2\left(q_i, x_1x_3, y_1y_2\right)$$
$$= \bigwedge_{r\in Q} \left\{\sigma_2\left(q_i, x_1, y_1\right) \vee \delta_2\left(q_i, x_1, r\right) \vee \sigma_2\left(r, x_3, y_2\right)\right\} = \sigma_2\left(q_i, x_1, y_1\right) \cdot \left\{\bigwedge_{r\in Q} \delta_2\left(q_i, x_1, r\right) \vee \sigma_2\left(r, x_3, y_2\right)\right\}$$
$$\sigma_3\left(q_i, x_1, y_1\right) \cdot \left\{\bigwedge_{r\in Q} \delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_2, y_2\right)\right\}$$

$$= \bigwedge_{r\in Q} \left\{\sigma_3\left(q_i, x_1, y_1\right) \vee \delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_2, y_2\right)\right\} = \sigma_3\left(q_i, x_1x_2, y_1y_2\right) = \sigma_3\left(q_i, x_1x_3, y_1y_2\right)$$
$$= \bigwedge_{r\in Q} \left\{\sigma_3\left(q_i, x_1, y_1\right) \vee \delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_3, y_2\right)\right\} = \sigma_3\left(q_i, x_1, y_1\right) \cdot \left\{\bigwedge_{r\in Q} \delta_3\left(q_i, x_1, r\right) \vee \sigma_3\left(r, x_3, y_2\right)\right\}$$

Define a relation ~ on Σ by $x_2 \sim x_3$ if and only if

$$\sigma\left(q_i, x_2, y_2\right) = \sigma\left(q_i, x_3, y_2\right)$$

and

$$\sigma\left(q_i, x_1x_2, y_1y_2\right) = \sigma\left(q_i, x_1x_3, y_1y_2\right),\ \forall q_i \in Q, x_1 \in \Sigma$$

and $y_1, y_2 \in Z$. It is clear that ~ is an equivalence relation on Σ. Thus, $x_2 = x_3$. Therefore M' is also a neutrosophic finite automata.

Lemma 4.2. Let

$$M_S = \left(Q_S, \Sigma, Z, \delta_S, \sigma_S\right) \text{and } M_T = \left(Q_T, \Sigma, Z, \delta_T, \sigma_T\right)$$

be neutrosophic finite automata and the state $q_S \in Q_S$ is indistinguishable from $q_T \in Q_T$. If $M_S^{'}$ and $M_T^{'}$ are semi-inverse of M_S and M_T respectively, then $g_{M_S}\left(q_S\right) \in M_S^{'}$ is indistinguishable from $g_{M_T}\left(q_T\right) \in M_T^{'}$.

Proof: Since q_S and q_T are indistinguishable and define the map $\sigma_S : \Sigma \to Z$ is a one-to-one function and onto, there exists $x \in \Sigma, y \in Z$ and so that

$$\sigma_S\left(q_S, x, y\right) = \sigma_T\left(q_T, x, y\right).$$

Thus, we have

$$\sigma_T\left(g_{M_T}\left(q_T\right), \sigma_S\left(q_S, x, y\right), x\right) = \sigma_S\left(q_S, x, y\right) = \sigma_T\left(q_T, x, y\right) = \sigma_S\left(g_{M_S}\left(q_S\right), \sigma_T\left(q_T, x, y\right), x\right).$$

Since $M_S^{'}$ and $M_T^{'}$ are semi-inverse of M_S and M_T, we consider that

$$\sigma_S^{'}\left(g_{M_S}\left(q_S\right), y, x\right) = \sigma_S^{'}\left(g_{M_S}\left(q_S\right), \sigma_T^{'}\left(q_T, y, x\right), y\right)$$
$$= \sigma_T^{'}\left(g_{M_T}\left(q_T\right), \sigma_S^{'}\left(q_S, y, x\right), y\right) = \sigma_T^{'}\left(g_{M_T}\left(q_T\right), y, x\right).$$

Thus, $g_{M_S}\left(q_S\right)$ and $g_{M_T}\left(q_T\right)$ are indistinguishable states. ν

Theorem 4.1. Let $M = \left(Q, \Sigma, Z, \delta, \sigma\right)$ be a neutrosophic finite automata. If M' is a semi-inverse of *M*, then $\bar{M}$ is a semi-inverse of *M* if and only if $\bar{M}$ is equivalent to M'.

Proof: Given that *M* and M' be semi-inverse of each other. Define the maps $g: M \to M'$ and $h: M' \to M$ such that $g\left(q\right) = q'$ and

$$h\left(q'\right) = q, \forall q \in Q, q' \in Q'.$$

It is trivial to show that both maps are well-defined. Assume that $\bar{M}$ is equivalent to M'. Now, define the functions $k: \bar{M} \to M'$ and $l: M' \to \bar{M}$ such that

$$k\left(\bar{q}\right) = \bar{q}, \forall \bar{q} \in \bar{Q} \text{ and } l\left(q'\right) = q', \forall q' \in Q'.$$

Again, let

$$\bar{q}, \bar{p} \in \bar{Q}\left(q', p' \in Q'\right).$$

Then $\bar{q} = \bar{p}\left(q' = p'\right)$ if and only if

$$\bar{\sigma}\left(\bar{q}, x, y\right) = \bar{\sigma}\left(\bar{p}, x, y\right)\left(\sigma'\left(q', x, y\right) = \sigma'\left(p', x, y\right)\right), \forall x \in \Sigma$$

and $y \in Z$. Thus *k* and *l* are well-defined and one-to-one. From distinguishedness of $\bar{M}$ and M', it follows that *k* and *l* are onto. Again define a function $lg: M \to \bar{M}$ such that

$\lg\left(q\right) = \overline{g}\left(q\right), \forall q \in Q$ and $hk : \bar{M} \to M$

such that

$hk\left(\overline{q}\right) = \overline{h}\left(\overline{q}\right), \forall \overline{q} \in \bar{Q}$.

In order to show that *M* and $\bar{M}$ are semi-inverses of each other, it is enough to show that the conditions (b) and (c) of Definition 4.1 hold. Then,

$$\sigma_{\bar{M}}\left(\overline{g}\left(q\right), \sigma_M\left(q, x, \overline{x}\right), x\right) = \sigma_{\bar{M}}\left(\lg\left(q\right), \sigma_M\left(q, x, \overline{x}\right), x\right)$$

Since $g\left(q\right)$ and $\lg\left(q\right)$ are indistinguishable,

$$\sigma_{\bar{M}}\left(\overline{g}\left(q\right), \sigma_M\left(q, x, \overline{x}\right), x\right) = \sigma_{M'}\left(g\left(q\right), \sigma_M\left(q, x, x'\right), x\right) = \sigma_M\left(q, x, \overline{x}\right),$$

and also

$$\sigma_M\left(\overline{h}\left(\overline{q}\right), \sigma_{\bar{M}}\left(\overline{q}, \overline{x}, x\right), \overline{x}\right) = \sigma_M\left(hk\left(\overline{q}\right), \sigma_{\bar{M}}\left(\overline{q}, \overline{x}, x\right), \overline{x}\right)$$

Since $\overline{q}$ and $k\left(\overline{q}\right)$ are indistinguishable.

$$\sigma_M\left(\overline{h}\left(\overline{q}\right), \sigma_{\bar{M}}\left(\overline{q}, \overline{x}, x\right), \overline{x}\right) = \sigma_M\left(hk\left(\overline{q}\right), \sigma_{M'}\left(k\left(\overline{q}\right), \overline{x}, x\right), \overline{x}\right) = \sigma_{\bar{M}}\left(k\left(\overline{q}\right), \overline{x}, x\right) = \sigma_{\bar{M}}\left(\overline{q}, \overline{x}, x\right).$$

Hence, *M* and $\bar{M}$ are semi-inverses of each other.

Conversely, assume that *M* and $\bar{M}$ are semi-inverses of each other. Define the maps $\overline{g} : M \to \bar{M}$ and $\overline{h} : \bar{M} \to M$ such that $\overline{g}\left(q\right) = \overline{q}$ and

$\overline{h}\left(\overline{q}\right) = q, \forall q \in Q, \overline{q} \in \bar{Q}$.

It is trivial to show that both maps are well-defined. It is clear that

$h\left(q'\right) \in M, \forall q' \in M'$.

By Lemma 3.2, $gh(q')$ and $\overline{g}h(q')$ are indistinguishable states. By Lemma 3.1(1), q' is indistinguishable from $gh(q')$. In a similar way, we can see that for every $\overline{q} \in \overline{M}$, there exists a corresponding state $g\overline{h}(\overline{q}) \in M'$ which is indistinguishable from $\overline{q}$, Consequently, *M* and $\overline{M}$ are equivalent. ν

Definition 4.2. A neutrosophic finite automata M' is said to be **inverse** of *M* if

1. M' is a semi-inverse of *M*, and
2. $g\left(\delta_M\left(q,x,p\right)\right) = \delta_{M'}\left(g\left(q\right),\sigma_M\left(q,x,y\right),p\right), \forall q,p \in Q, x \in \Sigma$ and $y \in Z$.

Theorem 4.2. If M' is a distinguished neutrosophic finite automata which is a semi-inverse of *M* then M' is an inverse of *M*.

Proof: Since M' is a semi-inverse of *M*, we define a map $g : M \to M'$ such that $g(q) = q', \forall q \in Q$. To show that M' is an inverse of *M*, it is necessary to show that for all $y \in Z$,

$$g\left(\delta_M\left(q,x,p\right)\right) = \delta_{M'}\left(g\left(q\right),\sigma_M\left(q,x,y\right),p\right),$$

$\forall q \in Q$ and $x \in \Sigma$. Now, for $x, x_1 \in \Sigma$,

$$\sigma_1\left(q, xx_1, x'x_1'\right)$$

$$= \sigma_1'\left(g\left(q\right), \sigma_1\left(q, xx_1, x'x_1'\right), xx_1\right)$$

$$= \sigma_1'\left(g\left(q\right), \bigvee_{r\in Q}\left[\sigma_1\left(q,x,x'\right) \wedge \delta_1\left(q,x,r\right) \wedge \sigma_1\left(r,x_1,x_1'\right)\right], xx_1\right)$$

$$= \sigma_1'\left(g\left(q\right), \sigma_1\left(q,x,x'\right) \cdot \left[\bigvee_{r\in Q}\delta_1\left(q,x,r\right) \wedge \sigma_1\left(r,x_1,x_1'\right)\right], xx_1\right)$$

$$= \bigvee_{s\in Q'}\left\{\begin{array}{l}\sigma_1'\left(g\left(q\right),\sigma_1\left(q,x,x'\right),x\right) \wedge \delta_1'\left(g\left(q\right),\sigma_1\left(q,x,x'\right),s\right) \\ \wedge\sigma_1'\left(s, \bigvee_{r\in Q}\left\{\delta_q\left(q,x,r\right) \wedge \sigma_1\left(r,x_1,x'\right)\right\}, x_1\right)\end{array}\right\}$$

$$= \sigma_1^{'}\left(g\left(q\right), \sigma_1\left(q, x, x'\right), x\right) \cdot \bigvee_{s \in Q'} \begin{Bmatrix} \delta_1^{'}\left(g\left(q\right), \sigma_1\left(q, x, x'\right), s\right) \\ \wedge \sigma_1^{'}\left(s, \bigvee_{r \in Q}\left\{\delta_q\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}, x_1\right) \end{Bmatrix}$$

$$\bigvee_{r \in Q}\left[\sigma_1\left(q, x, x'\right) \wedge \delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x_1^{'}\right)\right]$$
$$= \sigma_1\left(q, x, x'\right) \cdot \bigvee_{r \in Q}\left[\delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x_1^{'}\right)\right]$$

$$= \sigma_1^{'}\left(q, x, x'\right) \cdot \bigvee_{s \in Q'} \begin{Bmatrix} \delta_1^{'}\left(g\left(q\right), \sigma_1\left(q, x, x'\right), s\right) \\ \wedge \sigma_1^{'}\left(s, \bigvee_{r \in Q}\left\{\delta_q\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}, x_1\right) \end{Bmatrix}$$

$$\bigvee_{r \in Q}\left[\delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x_1^{'}\right)\right]$$
$$= \bigvee_{s \in Q'}\left\{\delta_1^{'}\left(g\left(q\right), \sigma_1\left(q, x, x'\right), s\right) \wedge \sigma_1^{'}\left(s, \bigvee_{r \in Q}\left\{\delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}, x_1\right)\right\}$$

$$\bigvee_{r \in Q}\left[\delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x_1^{'}\right)\right]$$
$$= \bigvee_{s \in Q'}\left\{q_1^{'} \wedge \sigma_1^{'}\left(s, \bigvee_{r \in Q}\left\{\delta_q\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}, x_1\right)\right\},$$

where

$$q_1^{'} = \delta_1^{'}\left(g\left(q\right), \sigma_1\left(q, x, x'\right), s\right).$$

Since M' is neutrosophic finite automata, we have

$$\sigma_1^{'}\left(g\left(\delta_1\left(q, x, p\right)\right), \bigvee_{r \in Q}\left\{\delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}, x_1\right)$$
$$= \bigvee_{r \in Q}\left\{\delta_1\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}$$
$$= \bigvee_{s \in Q'}\left\{q_1^{'} \wedge \sigma_1^{'}\left(s, \bigvee_{r \in Q}\left\{\delta_q\left(q, x, r\right) \wedge \sigma_1\left(r, x_1, x'\right)\right\}, x_1\right)\right\}$$

Since M' is distinguished, we have $g\left(\delta_1\left(q, x, p\right)\right) = q_1^{'}$, thus

$$g\left(\delta_1\left(q, x, p\right)\right) = \delta_1^{'}\left(g\left(q\right), \sigma_1\left(q, x, x'\right), s\right).$$

Similarly, we have

$$g\left(\delta_2\left(q,x,p\right)\right)=\delta_2^{'}\left(g\left(q\right),\sigma_2\left(q,x,x'\right),s\right)$$

and

$$g\left(\delta_3\left(q,x,p\right)\right)=\delta_3^{'}\left(g\left(q\right),\sigma_3\left(q,x,x'\right),s\right).$$

Hence M' is an inverse of *M*. ν

Theorem 4.3. If *M* and M' are both distinguished neutrosophic finite automata and if M' is an inverse of *M*, then *M* is an inverse of M'.

Proof: Let *M* and M' be semi-inverses of each other, so that define the maps $g:M\to M'$ and $h:M'\to M$ such that $g\left(q\right)=q'$ and

$$h\left(q'\right)=q,\forall q\in Q,q'\in Q'$$

which gives

$$\sigma_{M'}\left(q',x',x\right)>0 \text{ and } \sigma_M\left(h\left(q'\right),x,x'\right)>0.$$

It is trivial to show that both maps are well-defined. Consider

$$g\left(\delta_M\left(q,x,p\right)\right)=\delta_{M'}\left(g\left(q\right),\sigma_M\left(q,x,y\right),p\right),$$

since M' is an inverse of *M*. Now, define a function $hg:M\to M$ such that $hg\left(q\right)=q$ and *M* is distinguished, we have

$$\begin{aligned}&h(\delta_{M'}\left(g\left(q\right),\sigma_M\left(q,x,x'\right),p\right)=hg\left(\delta_M\left(q,x,p\right)\right)=\delta_M\left(q,x,p\right)\\&=\delta_M\left(h\left(q'\right),\sigma_{M'}\left(q',x',x\right),p\right)=h(\delta_{M'}\left(q',x',p\right)\end{aligned}.$$

Hence *M* is an inverse of M'.

CONCLUSION

This paper introduced and developed a finite automaton with output in the context of neutrosophic sets. The definition of indistinguishable, distinguishable, rational state of inputs and outputs, semi-inverse and inverses of the neutrosophic finite automata are examined and discussed their properties. The future will focus on finite automata with output respond to input strings based on *N*-structure (c.f., Jun et al. 2010 & Kavikumar et al. 2013) fuzzy finite automata.

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KEY TERMS AND DEFINITIONS

Fuzzy Finite Automata: Fuzzy automata are used for handling the system uncertainty problem because classical automata cannot deal with system uncertainty.

Indistinguishable States: It is one of method to reducing the states of a finite automaton is based on finding and combing indistinguishable states.

Inverses: It considers the situation of one machine "undoing" the work of another.

Lattice Ordered Monoid: It is generated by the natural numbers and satisfied the left cancellation law.

Mealy Machine: It is a finite-state machine whose output values are determined both by its current state and the current inputs.

Neutrosophic Output Function: Machine generates an output on every input.

Rational States: It is a state where each ultimately periodic sequence of inputs yields an ultimately periodic sequence of outputs.

Chapter 12
Neutrosophic Soft Digraph

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ABSTRACT

Neutrosophic soft sets are an important tool to deal with the uncertainty-based real and scientific problems. In this chapter, the idea of neutrosophic soft (NS) digraph has been developed. These digraphs are mainly the graphical representation of neutrosophic soft sets. A graphical study of various set theoretic operations such as union, intersection, complement, cross product, etc. are shown here. Also, some properties of NS digraphs along with theoretical concepts are shown here. In the last part of the chapter, a decision-making problem has been solved with the help of NS digraphs. Also, an algorithm is provided to solve the decision-making problems using NS digraph. Finally, a comparative study with proposed future work along this direction has been provided.

1. INTRODUCTION

Prof. (L. A. Zadeh, 1965) introduced the concept of fuzzy sets in 1965. Soft Set theory was introduced by (Molodtsov, 1999) as a parametric tool to deal the uncertain data which is present in many mathematical problems. In his paper, several applications of this theory are shown by him. Also (Maji et al., 2003; 2004) have further studied the theory of soft sets and the concept of fuzzy soft set and intuitionistic fuzzy soft set, vague set (Maji et al., 2001; 2001a; 2002; 2003a; 2004, Gau et al., 1993), a more

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generalized concept was introduced by them. Gradually research in soft set theory (SST) are grown up in many areas like algebra, entropy calculation, applications (see Atanasov, 1986; R. Kumar et al., 2018; 2019a; 2019b; 2019c; P.K. Maji, 2013), for example).

On the other hand based on neutrosophic logic and sets, a new theory has been developed. (Prof F. Smarandache, 2006) invented the theory of Neutrosophic Set (NS) in which every set has truth member ship (T), indeterminacy membership (I) and falsity membership (F).

In neutrosophic sets, all the membership degree lie in] 0, 1[. Unlike in IFS in which uncertainty is a function of the degree of belongingness as well as degree of non-belongingness, here the indeterminacy factor is present in the form of uncertainty. Clearly truth and falsity values is not a function of indeterminacy. Works on soft sets and neutrosophic sets are progressing very rapidly (Ali et al., 2016; 2017; Ansari et al., 2013; Aydodu, 2015; Broumi et al., 2014, 2016a, Smarandache, 2011, 2015c). In 2013, (P.K. Maji, 2013) gave the idea of Neutrosophic Soft Set theory.

In current years, mathematicians use graph theory in large scale to solve many open problems in modern mathematics. As a consequence, research in fuzzy graph theory, fuzzy digraph theory, intuitionistic fuzzy graphs, soft digraphs (Broumi et al., 2019; Gani et al., 2003; 2010; 2012; Karaaslan, 2015a; Majumdar et al., 2008; 2010a; 2013b; Majumdar, 2015b; Rosenfield, 1975; Sinha et al., 2016b; 2018a; 2019d) are being carried out very rapidly. However (Samarandache; 2015c, Broumi et al., 2014, 2016a) introduced the concept of Neutrosophic graphs firstly in their papers.

In this paper a new theory is introduced i.e. Neutrosophic Soft Digraph theory for the first time. The organization of the paper are done in the following manner: In Section 2, some preliminary ideas and results regarding Neutrosophic Soft set are given. Section 3 introduces the notion of Neutrosophic soft digraph and studies some properties of a NS digraph. In Section 4 the application of Neutrosophic soft digraphs are shown by solving a decision making problem.

2. PRELIMINARIES

Neutrosophic sets has several applications in different areas of physical systems, biological systems etc. and even in daily life problems. In (Majumdar, 2015b; Smarandache, 2006; 2011), most of the preliminary ideas can be found. However a discussion containing some definitions and terminologies regarding neutrosophic sets is given here.

Definition 2.1. (Majumdar, 2015b) Let X be a universal set. A neutrosophic set A on X is characterized by a truth membership function tA, an indeterminacy function iA and a falsity function fA, where t_A, i_A, f_A : X → [0, 1], are functions and

$$\forall x \in X, x = x(t_A(x), i_A(x), f_A(x)) \in A$$

is a single valued neutrosophic element of A.

Grammatically the difference between indeterminacy and uncertainty is that indeterminacy is the condition of being indeterminate while uncertainty is basically doubt. They might be slightly different from each other philosophically but practically speaking they are often used synonymously. If something is indeterminate then it is uncertain. One may think that indeterminacy is a cause of which uncertainty is an uncertainties occurs due to several reasons like randomness, haziness, vagueness, imprecision or even indeterminacy. So to us indeterminacy is a cause of uncertainty.

(Molodtsov, 1999) firstly introduced the concept of soft sets. After almost 4 years, (Maji and Roy, 2003a) have defined operations on soft set along with their properties.

Definition 2.2. (Maji et al., 2001) Consider an initial universal set U and a set of parameters E. Let P (U) denote the power set of U and A ⊆ E. A pair (F, A) is called a soft set over U if and only if F is a mapping given by F: A → P (U).

Throughout this paper, we consider U and E as a finite set.

Example 2.3. Suppose a soft set (F, E) narrates the choice of places, which the authors are planning to visit with his family.

U = the under considerable places = $\{x_1, x_2, x_3, x_4, x_5\}$.

E = {Desert, forest, mountain, sea-beach} = $\{e_1, e_2, e_3, e_4\}$.

Let

$F(e_1) = \{x_1, x_2\}$,

$F(e_2) = \{x_1, x_2, x_3\}$,

$F(e_3) = \{x_4\}$,

F (e_4) = { x_2, x_5 }.

So, the soft set (F, E) is a family {F (e_i); i = 1, . . ., 4} of U.

In 2013, (P.K. Maji, 2013) gives the idea of Neutrosophic Soft Set in his paper as mentioned below:

Definition 2.4. Consider an initial universal set U and a set of parameters E. Consider A ⊆ E. Let N(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A → N(U). Throughout this paper, we consider U and E to be a finite set.

Example 2.5. Suppose X and E be the set of buses and condition of buses, i.e. the parameters set. Consider E = {beautiful, eco-friendly, costly, very costly, luxurious, good seating arrangement, well conditioned, worst conditioned, cheap, expensive}. Suppose, there are four buses in the universe X given by, U = { h_i ; i = 1, 2, 3, 4} and the parameters set A = { e_i ; i = 1, 2, 3, 4}, where e_1 denotes the beautiful parameter, e_2 denotes the parameter eco-friendly, e_3 denotes the parameter costly and the parameter e_4 denotes the good seating arrangement. Let

$$F\left(beautiful\right) = \left\{\left(h_{1,} 0.4, 0.7, 0.3\right), \left(h_2, 0.3, 0.6, 0.2\right), \left(h_3, 0.3, 0.6, 0.2\right), \left(h_4, 0.3, 0.6, 0.2\right)\right\}$$

$$F\left(eco\text{-}friendly\right) = \{(h_1, 0.6, 0.7, 0.8), \left(h_2, 0.5, 0.5, 0.1\right), (h_3, 0.2, 0.3, 0.6), (h_4, 0.6, 0.1, 0.8)\}$$

$$F\left(costly\right) = \left\{\left(h_{1,} 0.7, 0.9, 0.1\right), \left(h_2, 0.3, 0.3, 0.4\right), \left(h_3, 0.5, 0.4, 0.8\right), \left(h_4, 0.8, 0.7, 0.8\right)\right\}$$

$$F\left(good\, seating\right) = \{(h_1, 0.4, 0.1, 0.4), \left(h_2, 0.3, 0.7, 0.4\right), (h_3, 0.1, 0.3, 0.2), (h_4, 0.9, 0.6, 0.8)\}$$

Then (F, E) is a neutrosophic soft set (NSS) over X.

The most of the terminologies regarding NSS can be found in (Maji, 2013). Thus it is requested to the readers to follow the article (Maji, 2013) thoroughly for terminologies, operations etc. of NS set.

Graphs and Digraphs are played an important role in the various applications (say shortest path problem, Transportation problem etc.) of modern mathematics. Many real life problems, which contain uncertainty, can be easily solved with the help of graph theory. To see the details terminology regarding graphs and digraphs one can follow any standard book say (Chartrand et al., 2005; Harary,1969)

3.NEUTROSOPHIC SOFT DIGRAPH

In this paper NS digraph will be introduced.

Definition 4.1. Let (F, E) be a NS set over an universe U. We introduce a dummy parameter ea such that F(e_a) = φ where (t_{e_a} , i_{e_a} , f_{e_a}) = (0.5, 0.5, 0.5). Consider D = (V_D, A_D) be any digraph with vertex set V_D and arc set A_D such that

1. $V_D = E \cup \{e_a\}$
2. $A_D = \{(e_i, e_j) : h_j \in F(e_i)$ and j≤ |E|} $\cup\{(e_i, e_a) : h_j \in F(e_i)$ and j> |E|} and the functions

$$t_{A_D} : A_D \rightarrow [0,1],\ i_{A_D} : A_D \rightarrow [0,1],\ f_{A_D} : A_D \rightarrow [0,1]$$

are defined by

$$t_{A_D}(e_i, e_j) = \left[t_{F(e_i)}(h_j)\right],$$

$$i_{A_D}(e_i, e_j) = \left[i_{F(e_i)}(h_j)\right],$$

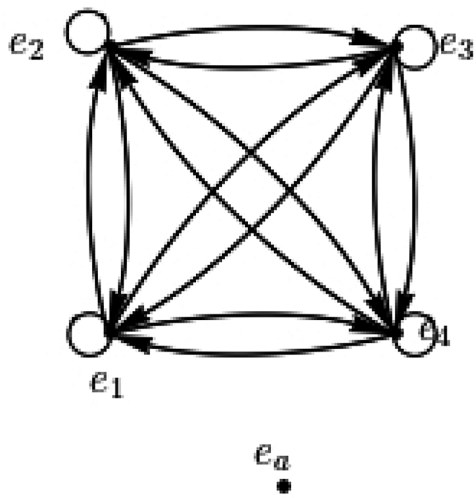
$$f_{A_D}(e_i, e_j) = \left[f_{F(e_i)}(h_j)\right],$$

Where

$$t_{A_D}(e_i, e_j), i_{A_D}(e_i, e_j), f_{A_D}(e_i, e_j)$$

denotes a truth membership function, a inderterminacy function and falsity function of the arc $(e_i, e_j) \in A_D$ respectively and

Figure 1. The NS digraph D_1



$$0 \leq t_{A_D}\left(e_i, e_j\right) + i_{A_D}\left(e_i, e_j\right) + f_{A_D}\left(e_i, e_j\right) \leq 3,$$

for all

$$\left(e_i, e_j\right) \in A_D, i, j \in \left\{1, 2, \ldots, n, a\right\}.$$

Then D is called a NS digraph corresponding to the NS set (F, E). The vertex e_a is called the universal vertex for any soft digraph D.

Example 4.2. Consider the digraph $D_1 = (V_{D_1}, A_{D_1})$ corresponding to the NS set (F, E) in Example 2.5. Here the vertex set $V_{D_1} = \{e_1, e_2, e_3, e_4, e_a\}$ and arc set

$$A_{D_1} = \{(e_1, e_2), (e_2, e_1), (e_1, e_3), (e_3, e_1), (e_2, e_3), (e_3, e_2), (e_3, e_4),$$

$$(e_4, e_3), (e_4, e_1), (e_1, e4), (e_4, e_2), (e_2, e_4)\}$$

with one loop at each vertex. Here each arc of D1 has the following neutrosophic value:

$$\left(e_1, e_1\right) = \left(0.4, 0.7, 0.3\right), \left(e_1, e_2\right) = \left(0.2, 0.3, 0.6\right),$$

$\left(e_1, e_3\right) = \left(0.4, 0.4, 0.2\right), \left(e_1, e_4\right) = \left(0.6, 0.5, 0.4\right),$

$\left(e_2, e_1\right) = \left(0.6, 0.7, 0.8\right), \left(e_2, e_2\right) = \left(0.5, 0.5, 0.1\right),$

$\left(e_2, e_3\right) = \left(0.2, 0.3, 0.6\right), \left(e_2, e_4\right) = \left(0.6, 0.1, 0.8\right),$

$\left(e_3, e_1\right) = \left(0.7, 0.9, 0.1\right), \left(e_3, e_2\right) = \left(0.3, 0.3, 0.4\right),$

$\left(e_3, e_3\right) = \left(0.5, 0.4, 0.8\right), \left(e_3, e_4\right) = \left(0.8, 0.7, 0.8\right),$

$\left(e_4, e_1\right) = \left(0.4, 0.1, 0.4\right), \left(e_4, e_2\right) = \left(0.3, 0.7, 0.4\right),$

$\left(e_4, e_3\right) = \left(0.1, 0.3, 0.2\right), \left(e_4, e_4\right) = \left(0.9, 0.6, 0.8\right)$

It is clear that D_1 is a NS digraph by Definition 4.1.

Remark 4.3: We call V_D as the vertex set of D,A_D as the arc set of NS digraph D. An arc x=

$\left(e_{i,} e_i\right)$ in NS digraph D includes a loop at the vertex e_i. If $x = \left(e_i, e_j\right)$ is an arc in NS digraph

D, we say that x is associated with e_j and e_j, e_i is adjacent to e_j and e_j is adjacent from $e_{i.}$ The out degree od(e_j) (resp. in degree id(e_j)) of a vertex e_j in a NS digraph D is the number of vertices of D adjacent from (resp. to) e_j.

Remark 4.4: The following properties of a neutrosophic soft digraph *D* are quite from the Definition 4.1:

1. The digraph *D* cannot be strongly connected. It is always weakly connected. It is clear that for NS digraph *D*, it always contain a universal vertex e_a such that $F\left(e_a\right) = \varphi$. Hence there is no vertex which is adjacent from e_a. Thus a NS digraph *D* may be weakly connected, but it cannot be strongly connected and hence the following remarks are also valid for the same reason.
2. A NS digraph *D* cannot contain an Eulerian Cycle.
3. A NS digraph *D* cannot contain a cycle and any NS digraph is always acyclic.

Definition 4.5. Suppose that (*G,B*) and (*F,E*) are two NSSs over a common universe U.

Consider

$H=(V_H,A_H)$ & $D=(V_D,A_D)$

be two NS digraphs corresponding to NS set (*G,B*) and (*F,E*) respectively. Therefore,

$V_H = B \cup \{e_a\}$ and $V_D = E \cup \{e_a\}$.

Then *H* is a NS subdigraph of

D if $V_H \subseteq V_D$ and $A_H \subseteq A_D$.

Note that $V_H \subseteq V_D$ implies $B \subseteq E$. Also $A_H \subseteq A_D$ implies

$\forall e \in B, G(e) \subseteq F(e)$,

i.e.

$t_{H(e)}(x) \leq t_{F(e)}(x)$;

$i_{H(e)}(x) \leq i_{F(e)}(x)$;

$f_{F(e)}(x) \leq t_{H(e)}(x)$;

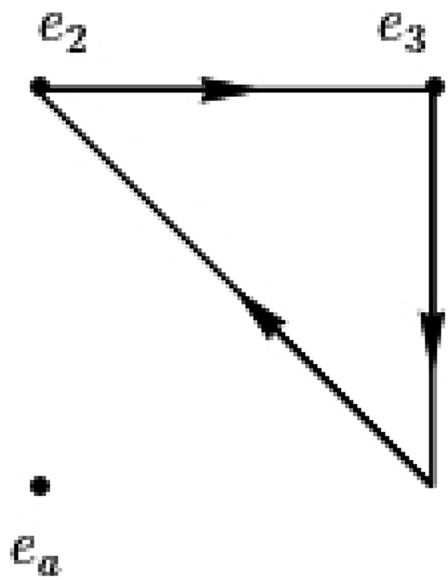
$\forall e \in A; x \in U$.

Thus, if *H* is a soft subdigraph of *D*, then (*G,B*) is a soft subset of (*F,E*).

Example 4.6. Suppose $B = \{e_2, e_3, e_4\} \subseteq E$. Let (*G,B*) be NS set over the same universe U in Example 2.5 as considered that

$G(e_2) = \{(h_3; 0.1, 0.3, 0.7)\}$,

Figure 2. The NS diagraph D_2



$$G\left(e_4\right) = \left\{\left(h_2; 0.3, 0.7, 0.4\right)\right\}$$

$$G\left(e_3\right) = \left\{\left(h_4; 0.6, 0.6, 0.8\right)\right\},.$$

Therefore, $\left(G, B\right) \subseteq \left(F, E\right)$. Now we draw the NS digraph D_2 which is Ns subdigraph of D_1.

Definition 4.7. Suppose

$$H = (V_H, A_H) \text{ and } D = (V_D, A_D)$$

be two NS digraphs corresponding to two equal soft sets (*G,B*) and (*F,E*) respectively. Then *H* and *D* are said to be equal NS digraph if $V_H = V_D$ and $A_H = A_D$ respectively.

In this case also, we see that (G, B) and (F, E) are subsets of each other.

Definition 4.8. Suppose (*F,E*) is a NS set over a universe U and $D = \left(V_D, A_D\right)$ be a NS digraph corresponding to it. Consider a digraph $D^c = (V_{D^c}, A_{D^c})$ as follows:

$$V_D = V_{D^c},$$

$A_{D^c} = \{(e_{i,} e_j) : h_j \in F(e_i)$ and j≤|E|} $\cup\{(e_i, e_a) : h_j \notin F(e_i)$ and j>|E|},

and the functions

$t_{A_D} c : A_D c \rightarrow [0,1]$,

$i_{A_D} c : A_D c \rightarrow [0,1]$,

$f_{A_D} c : A_D c \rightarrow [0,1]$,

are defined by

$tA_D c(e_i, e_j) = f_F c(e_i)(h_j)$

$iA_D c(e_i, e_j) = i_F c(e_i)(h_j)$

$fA_D c(e_i, e_j) = t_F c(e_i)(h_j)$

Then the NS digraph *D* is called a complement of the NS digraph D. It can be easily seen that the NS set corresponding to *D* is the complement NS set (F^c, E) of (*F,E*) where F^c :E$\rightarrow N(U)$ is a mapping given by

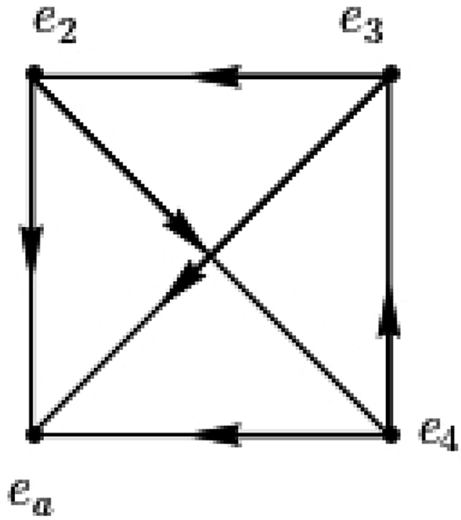
$F^c(\alpha) = U - F(\alpha)$for all $\alpha \in E$.

Example 4.9. Consider the digraph in Figure 3. Here we see that the NS digraph in Figure 3 represents the complement set (G^c, E) of (*G,E*) as follows:

$G^c(e_2) = \{(h_4; 0.4, 0.7, 0.3), (h_5; 1, 0, 0)\}$,

$G^c(e_4) = \{(h_3; 0.8, 0.6, 0.6), (h_5; 1, 0, 0)\}$,

Figure 3. The NS diagraph $\overline{D_2}$



$$G^c\left(e_3\right)=\left\{\left(h_2;0.7,0.3,0.6\right),\left(h_5;1,0,0\right)\right\}$$

and it is quite clear that the digraph in Figure 3 represents the complement digraph of D_2.

Definition 4.10. A NS digraph D is self-complementary if D is isomorphic to D^c.

Definition 4.11. Let (*F,E*) be a null NS set over a common universe *U*. Then, we have $\forall e \in A$,

$F\left(e\right)=\varphi$ i.e.

$$t_F\left(e\right)=0, i_F\left(e\right)=0, f_F\left(e\right)=0.$$

A digraph $D=\left(VD, AD\right)$ is said to be a null NS digraph corresponding to the NS set (*F,E*) if the following holds:

1. $V_D = E \cup \left\{e_a\right\}$
2. $A_D = \{(e_i, e_j) : h_j \in F\left(e_i\right)$ and $j \leq |E|\} \cup \{(e_i, e_a) : h_j \in F\left(e_i\right)$ and $j > |E|\}$ and the functions

$t_{A_D} : A_D \rightarrow [0,1], i_{A_D} : A_D \rightarrow [0,1], f_{A_D} : A_D \rightarrow [0,1]$

are defined by

$t_{A_D}\left(e_i, e_j\right) = 0,$

$i_{A_D}\left(e_i, e_j\right) = 0,$

$f_{A_D}\left(e_i, e_j\right) = 0,$

where $t_{A_D}\left(e_i, e_j\right)$, $i_{A_D}\left(e_i, e_j\right)$, $f_{A_D}\left(e_i, e_j\right)$ are usual meaning.

Example 4.12. Consider a null NS set (*G,E*) over a universe $U = \left\{h_1, h_2, h_3, h_4, h_5\right\}$. Consider $E = \left\{e_1, e_2, e_3\right\}$. Suppose

$G\left(e_1\right) = \left\{\left(h_3, 0, 0, 0\right), \left(h_4, 0, 0, 0\right)\right\}$

$G\left(e_2\right) = \left\{\left(h_1, 0, 0, 0\right), \left(h_4, 0, 0, 0\right)\right\}$

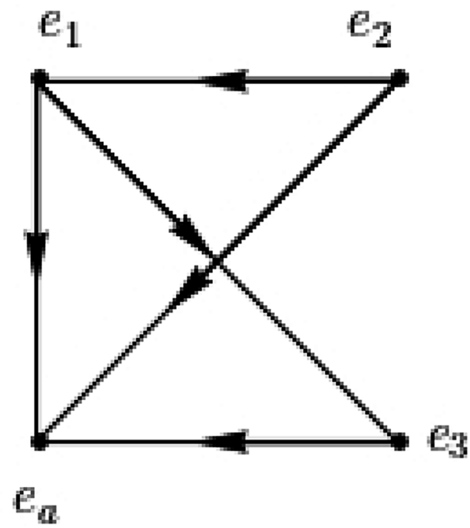
$G\left(e_3\right) = \{\left(h_4, 0, 0, 0\right)\}$

Clearly (*G,E*) is a null NS set. Consider the digraph D_3 in Figure 4. One can easily verify that D_3 is a null NS digraph. Here each arc of D_3 has neutrosophic value (0,0,0).

Definition 4.13. Suppose $H = \left(V_H, A_H\right)$ and $K = \left(V_K, A_K\right)$ are two NS digraphs corresponding two NSSs (*F,A*) and (*G,B*) respectively. Consider a digraph *D* by taking the union of two soft digraph *H* and *K* respectively as follows:

1. $V_D = V_H \cup V_K = A \cup B \cup \left\{e_a\right\}$, where e_a is the universal vertex for a NS digraph
2. $A_D = A_H \cup A_K \cup \left\{S\right\}$, where

Figure 4. The NNS diagraph D_3



$S = \{\{(e_i, e_j)\} \setminus \{(e_i, e_a)\}$

if

$h_j \in F(e_i), e_i \in A \setminus B, e_j \in B \setminus A\}\, or\, S = \{\{(e_i, e_j)\} \setminus \{(e_i, e_a)\}$

if

$h_j \in G(e_i), e_i \in B \setminus A, e_j \in A \setminus B\}\, or\, S = \varnothing\, in\, all\, other\, cases.$

3. $t_{A_D}(e_i, e_j) = [t_{F(e_i)}(h_j)], (e_i, e_j) \in H \setminus K$

$= [t_{G(e_i)}(h_j)], (e_i, e_j) \in K \setminus H$

$= \max[t_{G(e_i)}(h_j), t_{F(e_i)}(h_j)], (e_i, e_j) \in H \cap K$

$$i_{A_D}\left(e_i,e_j\right)=[i_{F\left(e_i\right)}(h_j)],\left(e_i,e_j\right)\in H\setminus K$$

$$=[i_{G\left(e_i\right)}(h_j)],\left(e_i,e_j\right)\in K\setminus H$$

$$=[(i_{G\left(e_i\right)}\left(h_j\right)+i_{F\left(e_i\right)}\ f(h_j))\,/\,2],\left(e_i,e_j\right)\in H\cap K$$

$$f_{A_D}\left(e_i,e_j\right)=[f_{F\left(e_i\right)}(h_j)],\left(e_i,e_j\right)\in H\setminus K$$

$$=[f_{G\left(e_i\right)}(h_j)],\left(e_i,e_j\right)\in K\setminus H$$

$$=\min[f_{G\left(e_i\right)}\left(h_j\right),f_{F\left(e_i\right)}(h_j)],\left(e_i,e_j\right)\in H\cap K$$

Example 4.14. Consider (*F,A*) be a NS set over the universal set $U=\left\{x_1,x_2,x_3,x_4,x_5\right\}$ as follows:

$$A=\left\{e_1,e_2\right\},F\left(e_1\right)=\left\{\left(x_1,0.7,0.1,0.3\right),\left(x_2,0.4,0.6,0.2\right)\right\},F\left(e_2\right)$$

= { (x_1, 0.4, 0.4, 0.6), (x_2, 0.6, 0.3, 0.9), (x_5, 0.5, 0.1, 0.8)}

Again we take the NS set (*G,B*) over the same universal set *U* as follows:

$$B=\left\{e_3\right\},G\left(e_3\right)=\ \left\{\left(x_5,0.3,0.4,0.2\right)\right\}.$$

Now as per Definition 4.1 we draw the NS digraph D_4 and D_5 specifying the NSSs (*F,A*) and (*G,B*) respectively.

It is quite clear that the NS digraph D_6 in Figure 6 is the union of two NS digraphs D_4 and D_5 respectively. In that case the neutrosophic value of each arc of D_6 remains unchanged and it is same as the corresponding arcs of D_4 and D_5 respectively.

Corollary 4.15. If D_1, D_2 are two NS digraph, then $D_1\cup D_2$ is also a NS digraph. The proof of the Corollary 4.15 is quite clearly follows from the definition of union of two NS digraph.

Figure 5. The NS diagraphs D_4 and D_5

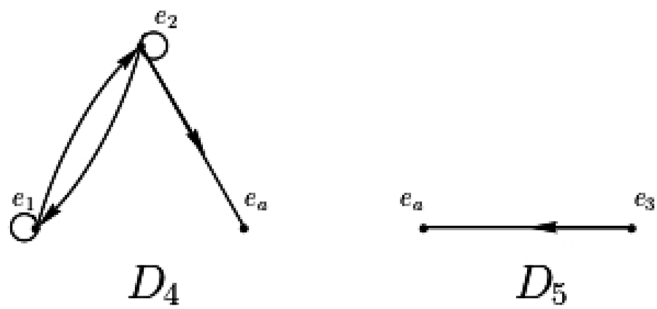
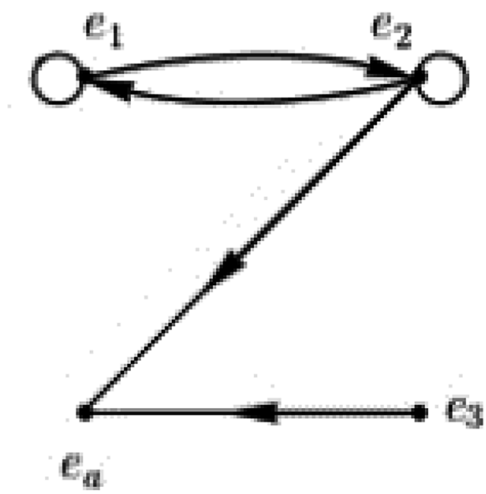


Figure 6. The NS diagraph D_6



Definition 4.16. Suppose (F,A) and (G,B) are two NSSs over a common universe U. Let H and K are two NS digraphs corresponding to the soft set (F,A) and (G,B) respectively. Then the NS digraph D is called the intersection of two soft digraphs H and K if the following holds:

1. $V_D = V_H \cap V_K = A \cap B \cap \{e_a\}$, where e_a is the universal vertex for a NS digraph
2. $A_D = \{(e_i, e_j) : (e_i, e_j) \in A_H \cap A_K\} \cup \{(e_i, e_a) : h_j \in F(e_i) \cap G(e_i), e_i \in A \cap B$ and $\notin$ A ∩ B}
3. $t_{A_D}(e_i, e_j) = [t_{F(e_i)}(h_j)],\ (e_i, e_j) \in H \setminus K$

$$= [t_{G(e_i)}(h_j)], (e_i, e_j) \in K \setminus H$$

$$= \min[t_{G(e_i)}(h_j), t_{F(e_i)}(h_j)], (e_i, e_j) \in H \cap K$$

$$i_{A_D}(e_i, e_j) = [i_{F(e_i)}(h_j)], (e_i, e_j) \in H \setminus K$$

$$= [i_{G(e_i)}(h_j)], (e_i, e_j) \in K \setminus H$$

$$= [(i_{G(e_i)}(h_j) + i_{F(e_i)}(h_j)) / 2], (e_i, e_j) \in H \cap K$$

$$f_{A_D}(e_i, e_j) = [f_{F(e_i)}(h_j)], (e_i, e_j) \in H \setminus K$$

$$= [f_{G(e_i)}(h_j)], (e_i, e_j) \in K \setminus H$$

$$= \max[f_{G(e_i)}(h_j), f_{F(e_i)}(h_j)], (e_i, e_j) \in H \cap K$$

Example 4.17. Consider the NS digraph D_4 in Figure 5. Now we take the NS set (*H,C*) over the same universal set *U* as follows:

$$C = \{e_1, e_2\}, H(e_2) = \{(x_1 1, 0.5, 0.4, 0.7)\}.$$

Figure 7. The NS diagraph D_7

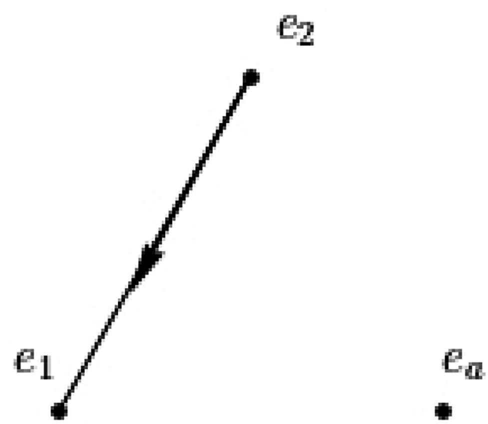
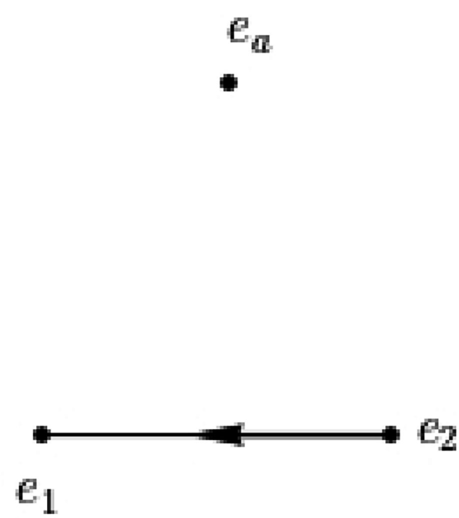


Figure 8. The NS diagraph D_8



Now as per Definition 4.1 we draw the NS digraph D_7 specifying the $NSS(H,C)$ respectively. It is quite clear that the NS digraph D_8 in Figure 8 is the intersection of two NS digraphs D_4 and D_7 respectively.

The neutrosophic value of the arc (e_2, e_1) of D_8 is $\left(0.4, 0.4, 0.7\right)$.

Corollary 4.18. If D_1, D_2 are two NS digraph, then $D_1 \cap D_2$ is also a NS digraph. The proof of the Corollary 4.18 is quite clearly follows from the definition of intersection of two NS digraph.

Definition 4.19. Suppose

$H = \left(V_H, A_H\right)$ and $K = \left(V_K, A_K\right)$

be two NS digraphs corresponding two NSSs (*F,A*) and (*G,B*) over a universal set $U = \{h_1, h_w, ...h_r\}$. Let $A = \{e_i; i \in I\}$ and $B = \{f_i; i \in I\}$ be two parameter sets of order m and n respectively. Consider a NS digraph $D = \left(V_D, A_D\right)$ of $H \wedge K$ as follows:

1. $V_D = V_H \times V_K = \{\left(A \cup \left\{e_a\right\}\right) \times (B \cup \{f_a)\}$
2.

$$A_D = \{\left(\left(e_i, f_k\right), \left(e_j, f_l\right)\right) \mid f_k = f_l \, and \, h_j \in F\left(e_i\right), j < m) \, or \, (e_i = e_j \, and \, h_l \in G\left(f_k\right), l < n)\} \, or$$

$$\cup\{\left(\left(e_i, f_k\right), \left(e_a, f_l\right)\right) \mid f_k = f_l \, and \, h_j \in F\left(e_i\right), j > m)\}$$

$$\cup\{\left(\left(e_i, f_k\right), \left(e_j, f_a\right)\right) \mid e_j = e_j \, and \, h_l \in G\left(f_k\right), l > n)\}$$

3.

$$t_{D\left(e_i, f_k\right), \left(e_j, f_l\right)}\left(m\right) = \min[t_{F\left(e_i\right)}\left(h_j\right), t_{G\left(f_k\right)}(h_l)], i_{D\left(e_i, f_k\right), \left(e_j, f_l\right)}\left(m\right) = [(i_{G\left(f_k\right)}\left(h_l\right) + i_{F\left(e_i\right)}(h_j)) / 2]$$

$$f_{D\left(e_i, f_k\right), \left(e_j, f_l\right)}\left(m\right) = \max[f_{F\left(e_i\right)}\left(h_j\right), t_{G\left(f_k\right)}(h_l)],$$

for all $e_i \in A, f_k \in B.$

Clearly *D* corresponds the digraph $\left(F, A\right) \wedge \left(G, B\right)$.

Corollary 4.20. If D_1,D_2 are two NS digraph, then $D_1 \wedge D_2$ is also a NS digraph.

Definition 4.21. Suppose H=(V_H, A_H) and $K = \left(V_K, A_K\right)$ be two NS digraphs corresponding two NSSs (*F,A*) and (*G,B*) over a universal set $U = \{h_1, h_w, ...h_r\}$. Let $A = \{e_i; i \in I\}$ and $B = \{f_i; i \in I\}$ be two parameter sets of order m and n respectively. Consider a NS digraph $D = \left(V_D, A_D\right)$ of $H \vee K$ as follows:

1. $V_D = V_H \times V_K = \{\left(A \cup \left\{e_a\right\}\right) \times (B \cup \{f_a\})\}$

2.

$$A_D = \{\left(\left(e_i, f_k\right), \left(e_j, f_l\right)\right) \mid f_k = f_l \, and \, h_j \in F\left(e_i\right), j < m) \, or \, (e_i = e_j \, and \, h_l \in G\left(f_k\right), l < n)\} \, or$$

$$\cup\{\left(\left(e_i, f_k\right), \left(e_a, f_l\right)\right) \mid f_k = f_l \, and \, h_j \in F\left(e_i\right), j > m)\}$$

$$\cup\{\left(\left(e_i, f_k\right), \left(e_j, f_a\right)\right) \mid e_j = e_j \, and \, h_l \in G\left(f_k\right), l > n)\}$$

3.

$$t_{D\left(e_i, f_k\right),\left(e_j, f_l\right)}\left(m\right) = \max[t_{F\left(e_i\right)}\left(h_j\right), t_{G\left(f_k\right)}(h_l)], i_{D\left(e_i, f_k\right),\left(e_j, f_l\right)}\left(m\right) = [(i_{G\left(f_k\right)}\left(h_l\right) + i_{F\left(e_i\right)}(h_j)) / 2]$$

$$f_{D\left(e_i, f_k\right),\left(e_j, f_l\right)}\left(m\right) = \min[f_{F\left(e_i\right)}\left(h_j\right), t_{G\left(f_k\right)}(h_l)] \text{ for all } e_i \in A, f_k \in B.$$

early, the soft digraph D corresponds the soft set $\left(F, A\right) V \left(G, B\right)$.

Corollary 4.22. If D_1, D_2 are two NS digraph, then $D_1 \vee D_2$ is also a NS digraph.

3. SOME TERMINOLOGY ON NS DIGRAPH

Definition 5.1. The degree and the total degree of a vertex e_i of a NS digraph $D = (V_D, A_D)$ are denoted by

$$d_D\left(e_j\right) = \left(t_V\left(e_i\right), i_V\left(e_i\right), f_V\left(e_i\right)\right) = \left(\sum_{j,i\neq j} t_A\left(e_i, e_j\right), \sum_{j,i\neq j} i_A\left(e_i, e_j\right), \sum_{j,i\neq j} f_A\left(e_i, e_j\right)\right)$$

$$Td_D\left(e_j\right) = \left(\sum_{j,i\neq j} t_A\left(e_i, e_j\right) + t_V\left(e_i\right), \sum_{j,i\neq j} i_A\left(e_i, e_j\right) + i_V\left(e_i\right), \sum_{j,i\neq j} f_A\left(e_i, e_j\right) + f_V\left(e_i\right)\right)$$

Example 5.2. The degree and total degree of the vertex e_2 of the digraph D_1 in Example 2.5 are $d_D\left(e_2\right) = \left(0.8, 1, 1.4\right)$ and $Td_D\left(e_2\right)$ = (1.3, 1.5, 1.5).

Definition 5.3. A NS digraph $D=(V_D, A_D)$ is called a *k*-regular NS digraph if $d_D\left(e_i\right) = \left(k, k, k\right)$ $\forall e_i \in V_D$.

Definition 5.4. A NS digraph D = (V_D , A_D) is called a totally regular NS digraph of degree (k_1, k_2, k_3) if $Td_D\left(e_2\right)$= (k$_1$, k$_2$, k$_3$) $\forall e_i \in V_D$.

In the next definition, we will define the concept of degree and total degree of an arc of a NS digraph.

Definition 5.5. The degree and the total degree of an arc $\left(e_i, e_j\right)$ of a NS digraph are denoted by

$$d_D\left(e_i, e_j\right) = \left(d_t\left(e_i, e_j\right), d_i\left(e_i, e_j\right), d_f\left(e_i, e_j\right)\right)$$

and

$$Td_D\left(e_i, e_j\right) = \left(Td_t\left(e_i, e_j\right), Td_i\left(e_i, e_j\right), Td_f\left(e_i, e_j\right)\right)$$

respectively and are defined as follows:

$$d_D\left(e_i, e_j\right) = d_D\left(e_i\right) + d_D\left(e_j\right) - \frac{1}{2}\left(t_A\left(e_i, e_j\right), i_A\left(e_i, e_j\right), f_A\left(e_i, e_j\right)\right),$$

$$Td_D\left(e_i, e_j\right) = d_D\left(e_i, e_j\right) + \left(t_A\left(e_i, e_j\right), i_A\left(e_i, e_j\right), f_A\left(e_i, e_j\right)\right).$$

One can easily verify that the above three concepts are completely different to each other.

Example 5.6. Consider the NS digraph D_1 in Figure 1. Here the degree and total degree of the vertices { e_1, e_2, e_3, e_4 } of D_1 as following:

$d_D(e_1)$ = (1.2, 1.2, 1.2), T $d_D\left(e_1\right)$ = (1.6, 1.9, 1.5),

$$d_D\left(e_2\right) = \left(0.8, 1, 1.4\right), Td_D\left(e_2\right) = \left(1.3, 1.5, 1.5\right),$$

$$d_D\left(e_3\right) = \left(1.1, 1, 1.2\right), Td_D\left(e_3\right) = \left(1.6, 1.4, 2\right),$$

$$d_D\left(e_4\right) = \left(0.7, 0.8, 0.8\right), Td_D\left(e_4\right) = \left(1.6, 1.4, 1.6\right),$$

$d_D\left(e_a\right) = \left(0,0,0\right), Td_D\left(e_a\right) = \left(0.5,0.5,0.5\right).$

Now we calculate the degree and total degree of an arc say (e_1, e_2) of A_{D_1} of D_1 as follows:

$d_D\left(e_1,e_2\right) = \left(1.9,2.05,2.3\right), Td_D\left(e_1,e_2\right) = \left(2.1,2.35,2.9\right).$

Definition 5.7. The maximum degree of a NS digraph $D = \left(V_D, A_D\right)$ is defined as

$$\Delta(\mathbf{D}) = \left(\Delta_t\left(D\right), \Delta_i\left(D\right), \Delta_f\left(D\right)\right)$$

where

$$\Delta t(D) = \max\left\{d_t(e_i) : e_i \in V_D\right\},$$

$$\Delta_i(D) = \max\left\{d_t(e_i) : e_i \in V_D\right\},$$

$$\Delta_f(D) = \max\left\{d_t(e_i) : e_i \in V_D\right\}.$$

Definition 5.8. The minimum degree of a NS digraph $D = \left(V_D, A_D\right)$ is defined as

$$\delta\left(D\right) = \left(\delta_t\left(D\right), \delta_i\left(D\right), \delta_f, \left(D\right)\right)$$

where

$$\delta_t\left(D\right) = \min\{d_t\left(e_i\right) : e_i \in V_D\},$$

$$\delta_i\left(D\right) = \min\{d_i\left(e_i\right) : e_i \in V_D\},$$

$$\delta_f\left(D\right) = \min\{d_f\left(e_i\right) : e_i \in V_D\},$$

Figure 9.

$$a_{ij} = \begin{cases} 1, & \text{if } (i,j) \in A_D \\ 0, & \text{if } (i,j) \notin A_D. \end{cases}$$

Definition 5.9. Suppose $D = (V_D, A_D)$ be a NS digraph corresponding to a NS set V_D. Then *D* is said to be arc regular NS digraph if every arc in D has the same degree (k_1, k_2, k_3). equally arc regular NS digraph if k_1=k_2=k_3. totally arc regular NS digraph if every arc in D has the same total degree (k_1, k_2, k_3).

4. ADJACENCY MATRIX OF A NEUTROSOPHIC SOFT DIGRAPH

An $n \times n$ matrix $A = [a_{ij}]$ is said to be an adjacency matrix of a NS digraph *D* of order *n* if $D = (V_D, A_D)$ with $|V_D| = n$, and for $1 \leq i, j \leq n, (i, j) \in A_D$ if and only if the entry a_{ij} of *A* is specified. See Figure 9.

Example 6.1. Suppose a matrix $A = [a_{ij}]$ specifying the neutrosophic soft digraph D_1 of Figure 1 as shown in Figure 10.

Here, we take the first rows as { e_1, e_2, e_3, e_4, ea} and columns as { e_1, e_2, e_3, e_4, e a}. It can be easily seen that B is the matrix representation of the NS digraph D_1 of Figure 1.

Figure 10.

$$A = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.1 Weight Matrix of a NS Digraph

An $n \times n$ matrix $B = [b_{ij}]$ is said to be an adjacency neutrosophic matrix of a NS digraph D of order n if it is an neutrosophic matrix such that

$$b_{ij} = < t\left(e_i, e_j\right), i\left(e_i, e_j\right), f\left(e_i, e_j\right) > \text{if } \left(e_i, e_j\right) \in A_D$$

$$= \left\langle 0, 0, 0 \right\rangle > \text{if } \left(e_i, e_j\right) \notin A_D .$$

We say a weight w_{ij} of an arc $\left(e_i, e_j\right)$ is defined as

$$t\left(e_i, e_j\right) + i\left(e_i, e_j\right) + f\left(e_i, e_j\right)$$

for all $\left(e_i, e_j\right) \in A_D$. A weight matrix C = [c_{ij}] of adjacency neutrosophic matrix are defined as follows:

$$c_{ij} = b_{ij} \times w_{ij} \text{ if } \left(e_i, e_j\right) \in A_D ,$$

= 0 otherwise.

We will use the concept of above matrices for solving a decision making problem in the next section.

5. A DECISION MAKING PROBLEM

Several authors (Karaaslan, 2015a; R. Kumar et al., 2018, 2019a, 2019b, 2019c) have used the neutrosophic digraph theory for solving decision making problem, shortest path problems, transportation etc. under neutrosophic environment. In this section, a decision making problem will be settled by using neutrosophic soft digraph theory.

Example 7.1. Consider the following problem: The author wish to build a house. He has parameters set i.e. E = {beautiful, lake-view, garden view, good communication, good construction, cheap, expensive}. Suppose, the author has four choice of houses U = {h_i; i = 1, …., 4} among a set of houses X and the set of parameters A = {e_i; i = 1,…, 4}, where e_1, e_2, e_3, e_4 denotes good communication, good construction, expensive and garden-view parameters respectively. Suppose a set decision makers

Figure 11.

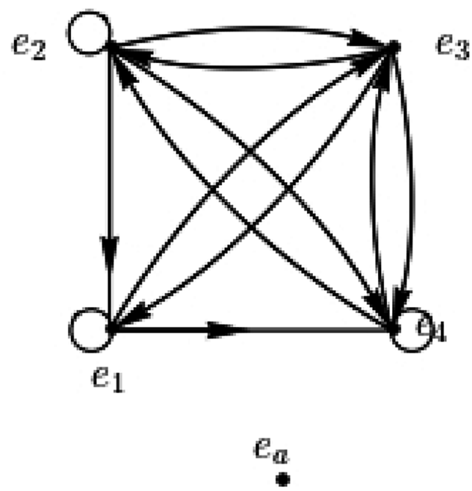


Figure 12.

$$A_{D_9} = \begin{bmatrix} 2 & 0 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Figure 13.

$$\begin{bmatrix} (0.5,0.6,0.3) & (0,0,0) & (0.6,0.1,0.4) & (0.2,0.7,0.1) & (0,0,0) \\ (0.5,0.2,0.7) & (0.4,0.4,0.2) & (0.5,0.6,0.4) & (0.5,0.9,0.2) & (0,0,0) \\ (0.3,0.3,0.2) & (0.6,0.2,0.8) & (0,0,0) & (0.4,0.4,0.3) & (0,0,0) \\ (0,0,0) & (0.3,0.3,0.5) & (0.6,0.4,0.2) & (0.4,0.5,0.5) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \end{bmatrix}$$

Now we compute the weight matrix $C = C_{D_9}$ *as following:*

$$\begin{bmatrix} 1.6 & 0 & 0.3 & 0.8 & 0 \\ 0 & 1.2 & 0.7 & 1.2 & 0 \\ 0.4 & 0 & 0 & 0.5 & 0 \\ 0 & 0.1 & 0.8 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

have evaluated those houses w.r.t the given choice of parameters and the whole situation is expressed as a NS set as follows:

$$F\left(good\, communication\right) = \left\{\left(h_1, 0.5, 0.6, 0.3\right), \left(h_3, 0.6, 0.1, 0.4\right), \left(h_4, 0.2, 0.7, 0.1\right)\right\}$$

$$F\left(good\, construction\right) = \left\{(h_1, 0.5, 0.2, 0.7), \left(h_2, 0.4, 0.4, 0.2\right), (h_3, 0.5, 0.6, 0.4), (h_4, 0.5, 0.9, 0.2)\right\}$$

$$F\left(expensive\right) = \left\{\left(h_1, 0.3, 0.3, 0.2\right), \left(h_2, 0.6, 0.2, 0.8\right), \left(h_4, 0.4, 0.4, 0.3\right)\right\}$$

$$F\left(garden\, view\right) = \left(h_2, 0.3, 0.3, 0.5\right), (h_3, 0.6, 0.4, 0.2), (h_4, 0.4, 0.5, 0.5)\}$$

Now (*F,E*) is a NS set over *X*. Now in Figure 9 we draw the NS digraph D_9 specifying the NS set (*F,E*) in Example 7.1. Now we find out the adjacency matrix A_{D_9} of the NS digraph D_9 as shown in Figures 11 and 12.

After that we will find out the adjacency neutrosophic matrix B_{D_9} of the NS digraph D_9 as shown in Figure 13.

Finally in matrix C, the row sum of each row is as follows; $C_1 = 2$, $C_2 = 1.3$, $C_3 = 1.8$, $C_4 = 3.3$. Therefore max $C_i = C_2 = 3.3$, ∀i. Here the maxi- mum of column sums of the matrix C is considered as a optimal solution. It is quite clear that the weight of the column C_4 is maximum which indicates that every choice of parameter e_i, i = 1, 2, 3, 4 meets the house h_4 with respective weights w_i, i = 1, 2, 3, 4 respectively with a higher value than others.

***Decision**: The author can buy the house h_4.*

Based on the above example, in any decision making problem, we have to convert the problem into a NS set. After that we draw the NS digraph which corresponds our NS set. Then we compute the adjacency matrix, adjacency neutrosophic matrix and weight matrix respectively. After computing this matrices, we consider maximum of the column sum (C_i) of weight matrix. The algorithm is as following:

1. Input the NS set *(G, F)*.
2. Input *P,* as choice parameters' set.
3. Draw the NS digraph *D* corresponding to the soft set *(G, P)*.
4. Find out the adjacency matrix *A*, adjacency neutrosophic matrix *B* and weight matrix *C* according to the NS digraph *D*.
5. Find out each column sum C_i, $\forall\ i$ of the matrix *C*.
6. Choose k, for which $x_k = maxC_i$, *if* $i \neq a$ (according to the problem).

Then is the desired solution. If more than one desired solution exists, then one can take any solution.

CONCLUSION

To cope up with real and practical life situations contains uncertainty, (Molodtstov, 1999) invented the soft set. After that Prof P.K. Maji and several authors introduced NSS theory and have shown the properties and application of NSS (Maji, 2013). Current chapter contains the notion of NS digraphs as well as some of its important operations, terminologies and finally with the help of NS digraphs a decision making problem is solved. For instance, firstly a digraph corresponding to each NS set can be easily obtained. Secondly, for pictorial representation of each NS set, the calculation is much easier than previous all techniques. In future, the properties of NS digraph in details may be studied and may be applied it to some real life practical problems.

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