FUNDAMENTALS OF THE THEORY OF PLANNING THE SEARCH FOR SPACE OBJECTS

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ABSTRACT

The basic tenets of a new theory of planning the search for space objects, using imprecise a-priori information on their orbit parameters, are developed. This is a second edition, corrected, remade, and enlarged. The main notion and the stem of the theory is the principle of equivalence of the search plan elements for different times. The basis of the theory includes the set-theoretical presentation of the space object current position uncertainty domain and its dynamics, the formulation in these terms of the equivalence principle of the search plan elements for different times. All the main search situations taking place in the space surveillance practice were considered and investigated on the base of this theory. The most search efficiency can be achieved in terms of this theory, first of all, for narrow-angle and narrow-beam sensors and for a weak signal from the sought-for space object. The phenomenon of the search plan degradation in the process of its realization is revealed and mathematically rigorously substantiated and described. This gave the theory a certain finality and completeness. This phenomenon entails the appearance of errors of the 1st and 2nd kind (the formation of gaps between the elements of the plan which implies the probability of the loss of the sought-for space object and the excessive reviewing of the already viewed areas of the space object current position uncertainty domain). A number of constructive ways to mitigate and compensate for the negative consequences of this phenomenon in the construction of search plans are suggested. As a result, the developed theory made it possible to construct mathematically well-founded search plans for space objects (including optimal ones) practically for any character of errors of the initial data on the state vector of the sought-for space object. Some qualitative and quantitative estimates of the proposed search methodology efficiency (obtained both theoretically and by practical realization) are given.

Reviewer's Foreword

The topic of the monograph by the well-known in Russia and abroad expert in this sphere – the search for an SO by rough a priori data on its orbit parameters – is very urgent and timely both at present and in prospect of space research. Namely the imperfectness of usually applied search methods in optical range largely accounts for the limited content of the high orbit SO dynamic catalogs. The latter are referred to as concerning the main products of Space Surveillance Systems (SSS) functioning in the USA, Russia and, in prospect, of the European SSS which is under way. The most important aspect of the problem, in our opinion, is detecting and cataloging SOs of small sizes (microsatellites (cubesats) and elements of space debris) and those having faint brightness.

The problem of searching for SOs with faint brightness is versatile. In this monograph, one of the key moments of the problem is considered – the foundations of the theory of planning the search for an SO with the help of narrow-angle sensors, using imprecise a priori information, which helps not only just reduce the search region but also optimize the process of sounding and sensing this region.

The appearance of this book is very timely, more so because the results presented in it are a pioneering contribution to the field of space research concerned. The significance of them was highly appreciated by the American Association for the Advancement of Science with the international award for 2005 (for the first time for Russians).

Hitherto, such a theory with so high degree of mathematical substantiation did not exist.

A decisive author's choice in his approach to construction of the theory was his suggestion of the set-theoretical representation of the SO current position uncertainty domain and temporal structural transformation of the latter. Namely thanks to such an approach, the success was achieved in suppression of the 1st and 2nd kinds of the search planning errors.

For constructing the theory and practical methods, the principle of equivalence of the search plan elements for different times was introduced in the monograph which has a fundamental importance for solving the problem at hand.

As a very important practical application of the theory, in the monograph there was developed a particular (though having important and highly effective applied significance) conception of planning the search for an SO for the case of primary growth of positional errors down track.

Of special importance in the author's investigations are his discovery and strict mathematical formulation of the phenomenon of the search plan degradation throughout its realization. It is essential that this phenomenon is not only theoretically investigated in detail but also there were worded and formalized some mitigating and compensatory devices and constructive recommendations convenient for use by astronomers-observers in their practice.

From the outset of observations of SOs, a problem of detecting small-sized SOs by imprecise ephemerides arose. At present, this problem became yet more important on account of the necessity of monitoring the near-Earth space with respect to elements of space debris and maneuvering SOs for which propagation of their expected positions is connected with substantial uncertainty. The theory and related practical methods of planning the search for SOs suggested in the monograph raise hope that the progress will be gained in this field because they show efficiency in detection of faint signals.

The idea of accumulating the intelligence signal energy in one point of the sensor's receiver is well known and has been used for a long time in the practice of astronomic observations. However, application of this device was successful only when the state vector was provided with a very high accuracy or through the laborious sorting out of lots and lots of possible state vectors. In this monograph, as an important author's achievement, there developed possible ways for providing the appropriate compensation of the relative motion of the intelligence signal and the receiver for the case of imprecise data on the state vector as well.

So, the work considered in this reference is not only just a monograph, including a compilation of well-known approaches and methods. It contains a lot of new scientific results oriented to enhancing the efficiency of space surveillance and clearing the way for new capabilities in the field of observational astronomy (first of all, we mean the phenomenon of degradation of the search plan throughout its realization).

The author's long-term experience in space surveillance and his participation in the creation of the Russian Space Surveillance System stipulated the wide and profound scope of the problem investigated, proximity of the results to serving the needs of concrete and, chiefly, urgent tasks of space surveillance practice. In fact, all the important search situations present in the space surveillance practice are considered.

Many scientific results obtained in the monograph (in particular, the search methods in an algorithmic form) were implemented in real acting systems as regular dedicated programs. The positive experiences of their operation were repeatedly presented and discussed at several international workshops and conferences (the US/Russian space surveillance workshop, the European Conference on Space Debris and others) and were published in their Proceedings. The principal theses and theoretical propositions of the monograph have been discussed and approved at two scientific seminars in the Institute of Astronomy of Russian Academy of Sciences.

Compared with the author's earlier publication on this topic, the modern edition is considerably expanded, the theory looks more complete, the presentation of the material has become more perfect, didactic, and available for mastering. A number of inaccuracies and misprints have been corrected, and some suggestions of readers have been taken into account. In fact, this is a new independent work.

One could mention some demerits of the monograph, but they do not touch the scientific content and scientific substantiation of the theories presented in it.

We believe advisable to publish the monograph under consideration as a scientific edition in the series of mathematics, astronomy, space surveillance, observations of artificial Earth satellites.

Dr. Sci. in math. and phys. Lydia V. Rykhlova
Dr. Sci. in math. and phys. Alexander V. Bagrov

AUTHOR'S PREFACE

In recent years, in connection with the worldwide trend towards miniaturization of spacecraft, the continuing contamination of near-Earth space (NES), and the growth of the small-sized fraction of space debris, the problem of search for space objects (especially small and weakly-contrasting) has become much more acute and even more urgent.

At the forefront of space researchers experiencing the pressure of this problem are astronomers-observers. At the same time, all specialists involved in space activities, developers, operators, and owners of space assets are interested in its effective solution.

This theory and the monograph describing it were conceived as a result of the observers' numerous unsuccessful attempts to find (using traditional methods) a number of lost objects, characterized by rough available data about their orbits.

This monograph appeared to be a finale of the author's researches in the field of space surveillance and inventory of space objects performed during about the last 40 years. The main results were obtained in the process of the author's participation in creation and perfection of the Russian space surveillance system. Some of them were developed in the frame of the US/Russian Space Surveillance Workshops which exist since 1994. And these workshops as well as participation in the sessions of Inter-Agency Space Debris Coordination Committee (IADC) gave the author an impetus for intensive work on this topic.

The main stimulus for emergence of this monograph was absence of a whole common mathematical theory for planning the search for space objects, using only incomplete and imprecise a-priori information on their state vectors. Such a theory could essentially enhance the efficiency of search and detection of space objects which must make themselves felt in the practice of space surveillance both in Russia and other space-faring nations. For a long time, numerous results in the field of space surveillance, namely, in solving the search problems produced the eclectic sum total of search methods – often empirical ones and not connected theoretically with each other.

During development of such a theory, a certain success was attained in constructing a common methodology of planning the search for a space object by rough a-priori orbital information. This result was obtained owing to use in the planning process of the set-theoretical approach to presentation, analysis, and due regard of the temporal structure transformation of the sought-for space object current position uncertainty domain. At the same time, no less important and fruitful was the formulation of the principle of equivalence of search plan elements for different times, then its generalization, development of the equivalence curves apparatus, and its introduction into the practice of planning the search.

All this together with the developed theory allow constructing search plans (including optimum) on the strict mathematical basis practically for any character of the errors distribution in a-priori data on the sought-for space object’s orbit parameters. The more so, in this monograph, a more general theory was developed that allows planning the search for an abstract object (not only the space one) moving with a given law, some simple restrictions and assumptions being kept. At the same time, permitting some natural and practically justified assumptions on the character of the errors distribution (for example, taking into account prevailing growth of errors only along the track and neglecting the others) leads to essential simplification of the planning procedure and, as a result, to simple distinct visual constructive layouts for getting optimum and suboptimum search plans. This makes it very convenient for astronomers-observers to organize the planning process for the search for space objects. Obvious refinement and natural character of these schemes witness the adequacy and correctness of the developed theory. Practical search methods constructed on the base of this theory were implemented at the real operating electro-optical sensors. They have been successfully running for several years and exposed their high performance. The results of their testing were presented at the international conferences and workshops [1, 2].

Practical background for the theory and methods and the examples of the search plans construction and their application were considered in this monograph predominantly in terms of optical and electro-optical sensors. Despite this, there are no limitations for applying the theory and methods to radars, lasers, and other sensors.

A distinctive feature of the suggested methodology of planning the search for a space object, using imprecise a priori information, is that its optimization scheme appears reiterative, multiform, and rather versatile. These properties allow obtaining optimum solutions not only by different criteria, but also in different forms and in several stages. For example, after having constructed the optimum search plan at a given optimization level and in a certain formulation of the problem or on a given conditions, one can proceed with the optimization process in an alternative form, on alternative conditions, or at the next optimization level, that is, by principle "*optimum optimorum*". And all this is on the set-theoretical base within the frame of the united mathematical apparatus of the equivalence curves.

In addition to the fact that the theory and methods proposed here make it possible to significantly improve the efficiency of the search for space objects in terms of saving the search resource of sensors (both the sensors itself and the employment of the operating personnel), detection of small and weakly-contrasting space objects becomes available. So, the search algorithms turn out to be less critical to the relative velocity of the sought-for objects.

The author expresses his gratitude to Dr. Sci. Prof. O. Yu. Aksenov, Dr. Sci. R. R. Nazirov, Dr. Sci. L. V. Rykhlova, Dr. Sci. A. V. Bagrov, the author's colleagues and other specialists who took part in the discussion of the material of the monograph, made a number of useful comments and suggestions which were gratefully taken into account by the author in finalizing the text of the monograph.

ABBREVIATIONS

ACR – additional cutting rule
BB – “branch-and-bound” (approach, scheme, method (of))
CCD – charge coupled device
CPUD – current position uncertainty domain
DOP – discrete optimization problem
DSO – deep-space object
EC – equivalence curve
ECA – equivalence curves apparatus
ECA – effective checked area
ECS – effective checked surface
ECT – equivalence curves tool
ERS – effective reflecting surface
ESA – European Space Agency
FoV – field of view
GE – generalized ephemeris
GEO – geostationary orbit
GEOSO – geostationary SO (an SO in a GEO)
HEO – highly eccentric orbit
HEOSO – an SO in a HEO
HO – high orbit
HOSO – a high orbital SO, a deep space object
HOSC – a high orbital constellation of SOs
IADC – Inter-Agency Space Debris Coordination Committee
ISS – International Space Station
LEO – low Earth orbit
LEOSO – a low orbital SO
LoT – loss of track
MEO – medium Earth orbit
MEOSO – a medium orbital SO
NASA – National Aeronautics and Space Administration
NES – near Earth space
OD – orbital debris (*US*) = SD (*Eur.*)
OSC – operating spacecraft
OSP – optimum search plan, optimum search planning
OSR – optimum search region
PE – principle of equivalence

PP – picture plane
Q.E.D. – quod erat demonstrandum (*lat.*)
RCS – radar cross section
RF – radio frequency
r.m.s. – root-mean-square (error, criterion)
RSSS – Russian SSS
SC – constellation of SOs, space system
SD – space debris
SMA – short measuring arc
SMAF – small measuring arc factor
SO – space object
SOCPUD – space object current position uncertainty domain
SP – search plan, search planning
SS – space surveillance
SSh – spaceship, spacecraft
SSN – space surveillance network
SSS – space surveillance system
ST – search theory
TOPS – theory of optimum planning the search
TST of SOCPUD – temporal structural transformation of SOCPUD
TV – television

Chapter One

Introduction

Although the volume of space surrounding the Earth has never been empty and permeated with natural objects, in the 60's of the 20th century with the launch of the first Earth satellite, near-Earth space (NES) began to be filled with space objects (SO) of artificial origin.

Domestication of space not only brings progress to humanity. It also has great negative consequences littering NES with space debris (SD) which not only poses a threat to operating spacecraft (OSC), but also violates the ecological balance around the Earth that was set over millions (and possibly billions) of years [3, 66].

The technogenic contamination of NES which has piggy-backed at the good intentions of man is dangerously progressing. And we do not have effective means not only to stop this dramatic process, but even to significantly slow it down. And this is in spite of the fact that a million army of scientists came out to fight against it and the states allocate huge sums for this.

So that, humanity has a new important care (trouble) and even the duty - to keep the process of technogenic "settlement" of NES under close attention and control, that is, to continuously conduct tireless and careful space monitoring [3]. Actually, in technical terms, this means the need to have constantly updated kinetic parameters (positional coordinates and corresponding velocity components) for as many SOs as possible.

Without this, it is impossible to maintain stable communication with existing active satellites and spaceships, to perform operational control from the Earth in real time, to monitor the position and movement of space debris in NES in order to prevent the collision of its elements with operating spacecraft, and predict the possible reentry and fall of large debris to Earth, not to mention a full-fledged comprehensive scientific study of the process of technogenic pollution of NES.

So, to ensure space activities, including its security, close monitoring of the near-Earth environment is essential for which costly professional (dedicated) means of space surveillance are being created.

Two space surveillance systems (SSS) have been built and are in operation for many years – in Russia (RSSS) and in the US (US SSN). The creation of a European SSS under the auspices of the European Space Agency (ESA) is under way [4]. A significant contribution to the implementation of the outer space monitoring function is made by independent (not dedicated) observation facilities (radar, optical, optoelectronic, passive RF) – assets of various countries. The main product of this monitoring is the dynamic catalog of space objects which is maintained and updated due to measurements obtained from the means of observation. The most complete and accurate SOs catalogs containing all principal characteristics of relatively large SOs that are updated in real time are those of space surveillance systems.

A lot of measurements from all the censors serve as a source not only for updating and refining the kinetic parameters of SOs that were previously cataloged, but also for detecting new SOs. The latter function (especially for high-orbit SOs) should be served by special search modes and programs of the surveillance censors (in contrast to the routine modes and programs for observation of specific SOs with the help of precise target designations).

As for the general history of the appearance of the search problem for space objects, it originated mainly with the appearance of high-orbit SOs (HOSOs). To detect low-orbit SOs (LEOSOs) it was irrelevant, because the existing radar network continuously monitors the vast low area of outer space. And this radar network is constantly growing all over the world and their capacity and capabilities are increasing.

The large number of LEOSOs passes through the operation ranges of the radars due to the significant number of the latter, their high productivity, and the short period of an LEOSO circulation. This circumstance easily solves the problem of obtaining a sufficiently dense flow of measurements for all LEOSOs in order to realize the principle of their passive detection (without search).

In accordance with these conditions, the orbit parameters of new SOs are obtained as "production wastes" of the refined parameters of cataloged SOs. And so, special search modes of the dedicated sensors are not required in this case (except the case of very small and faintly reflecting SOs).

However, it should be borne in mind that what has been said applies only to relatively large objects that are available for detection by the network of specialized facilities with a wide operation range. At the same time, in NES, there is a huge population of small and weakly-contrasting SOs (including LEOSOs) inaccessible for detection by radars operating in normal, routine modes. And this population of small-sized objects is constantly growing [3, 65, 66]. In view of the enormous velocities of the motion of LEOSOs (by the way, the relative velocity of head-on collisions can be twice that of each SO), a big danger is the collision not only with large SOs controlled by the radar networks but with small ones as well [3, 63, 64, 67, 71].

Hence, monitoring of the fine fraction of SD is also urgently needed. The existing network of radars cannot provide this working in normal, routine modes. A more or less satisfactory solution to this last problem requires narrow-beam radars (not to mention the use of newest optical telescopes capable of detecting LEOSOs) and special search modes of their operation. In this monograph such search modes are being developed.

In addition to this, it should be borne in mind that the operational slant range of radar is rather limited. Therefore, the situation with the solution of the HOSO detection problem is quite different from that of LEOSO detection. And this is another side of our common large problem. The practical impossibility of creating a sufficiently powerful solid and continuous in time electromagnetic field in deep space and overlapping all possible HOSOs' trajectories (that is, impossibility to provide a complete radar coverage of deep space) leaves the only admissible way for dealing with the HOSO detection problem – namely, the principle of active search for them with the help of optical, electro-optical sensors, and specialized narrow-beam radars.

However, it is not easy to realize this. An effect of some specific high orbits' factors complicates the managing of HOSOs' surveillance. It leads to the appearance of search situations not only at the stage of initial detection of new SOs, but also at the next stages of their motion. Then it stipulates some difficulties in obtaining measurements, and their deficiencies cause excessive energy-consuming and labor-consuming character of deep space surveillance and its high instability.

We list here the main factors:

all operating spacecraft are forced to maneuver both to maintain the correspondence of their orbits to specified flight programs and in order to avoid dangerous collisions with other objects;

technological orbital corrections for geostationary, geosynchronous, and some other HO satellites occur regularly and with a relatively high frequency;

too long periods of circulation of HOSOs cause the factor of a short measuring arc which is expressed in the really unattainable necessity of using very long (in time and space) measuring intervals for attaining acceptable accuracy of the initial orbit determination and renovation of its parameters;

it is extremely important for optical sensors that there are satisfactory meteorological and astroclimatic conditions, and the phase of illumination of the object being observed;

there are a substantial remoteness and a great volume of the space region of high orbits (hundreds and thousands of times larger than those of low orbits);

the level of the intelligence signal is usually very low which may become still fainter because of unfavorable observation conditions (due to the influence of the weather, the phase of illumination, remoteness, and so on);

the output capacity of present sensors is rather limited.

Hence, it is clear that the methodology for detecting and sustaining mass traffic control of HOSOs must be significantly different from that of LEOSOs. Until recently, it was not systematically developed. The search methods used were quite an eclectic collection. The universal mathematical apparatus and the theory of optimum planning the search for HOSOs which should be the basis of this methodology were also absent.

The beginning of the space age which put forward the process of filling the near space with the rapidly growing SO population and, as an inevitable consequence of it, the need for careful surveillance and control of this population posed a whole series of serious theoretical problems. Some of them were quickly and successfully resolved. Others were hard pressed to get their solution. At the same time, in the process of further development and domestication of space, the essences of many problems changed and were refined (sometimes radically). Periodically, there arose also new problems. While one of the basic problems of monitoring SOs' motion – creation of the mathematical model of motion (the propagator) taking into account all substantial perturbations and the determination of their orbits using a set of measurements – has been quite satisfactorily solved in works [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 56, 60], the problem of optimum planning the search for SOs with due regard to imprecise a priori information on their orbits has remained unsolved hitherto [16].

Despite the enormous covering and operational power of the SSSs and the presence of many other facilities for monitoring the SOs in such a rapid space exploration and the catastrophic progress of contamination of

NES given the performance and operational capacity of existing surveillance censors being used and prospective ones, using an out-of-date ideology of searching for and discovering new and lost SOs, will not last long enough. So, the most urgent problem of modernizing this ideology is on the agenda.

All the methods of search for SOs and search situations arising in the space surveillance routine and in astronomer-observers' practice can be conditionally broken up into two large categories:

1) the survey in consecutive order in a given search region (that is, scanning the part of sky allotted to the sensor) and detecting "whatever we come across";
2) the search for a particular SO, using some a priori orbital information (as a rule significantly imprecise), with the help of special individual search programs developed especially for that a priori information.

These categories cover almost all search situations.

In the first case, the sensor carries out scanning a large celestial sphere region which is fixed and motionless in every search cycle. It uses the constant strategy chosen beforehand and detects every SO in the sensor's field of view (FoV) the signal of which has the energy enough for its acquisition by the sensor's receiver (which is determined by the technical performance of the latter). The methods of regularly sequentially scanning the sky region and setting optical and radar fences developed and published in works [17, 18, 19, 20] appeared rather limited for space surveillance practice.

The methods of the first category are useful at the initial stage of cataloging SOs when a priori information on most of SOs' orbits is practically absent and then in a mode of periodical surveys. The application of these methods is limited by

their high energy intensity,
low economy,
long duration and connectedness of the whole scanning cycle and hence low operating rate,
inevitability of obtaining and identification of many unnecessary measurements,
low effectiveness of acquisition of faint intelligence signals,
just unfitness for some important search situations,

significant inefficiency in case of using narrow-angle and narrow-beam facilities.

These methods cannot provide high enough operating rate of detecting a particular SO, high accuracy of initial orbit determination when acquiring the first signal in the end of a cycle of survey, detection of SOs with small effective reflecting surfaces or their radar cross sections (RCS), in cases when the SO is in a poor phase of illumination, and others. They lay high claim to the output capacity of the sensors and altogether leave untouched significant reserves of enhancing efficiency of exploitation of sensors and therefore cannot be considered universal and, moreover, promising.

In contrast to the first case, in the second one, the sensor is checking (by a special search plan) a strictly limited and continuously drifting space domain – the sought-for SO current position uncertainty domain (CPUD) [25, 61, 62]. The size, position, aspect angle, and motion parameters of the SOCPUD in space are determined by given a priori information on the SO orbit and the quality of the latter.

What has been said and a number of additional considerations entail that the necessity of development of optimum search methods on the basis of effective use of imprecise a priori orbital information is stipulated by

the claims of sufficient completeness, high efficiency, and stability to monitoring SOs' motion;
toughening of requirements to operating rate and reliability of detecting new, lost, and maneuvered SOs;
economy reasons and demands for more effective use of sensors;
need of planning provision of observation for deep space objects with the help of narrow-angle and narrow-beam sensors as well as targeting the special lighting-up (backside illumination) facilities;
difficulties of the search, detection, and observation problems when dealing with SOs having faint brightness and rapidly moving across the sensor's FoV;
presence of the problem of guaranteed control of very important SOs' motion during their launch, after the maneuver, during worsening of the observation conditions, and in emergency.

The development of a methodology for optimum and suboptimum search planning and its theoretical validation becomes especially urgent when solving the problem of searching for and detecting small and weakly-contrasting SOs and using narrow-angle optical and narrow-beam radar sensors for these purposes.

The concept of “imprecise orbital information” used above is rather conditional. The same information can be treated as accurate enough for one sensor and imprecise for another. And one need to decide in each case. It depends particularly on the size of FoV or width of the beam. For definiteness, let’s settle to call some a priori orbital information on the sought-for SO precise if FoV of the sensor used for the search completely covers the SOCPUD and imprecise in the opposite case. (The term “completely” here should not be comprehended absolutely. And about this, please, see later.) It is clear that if we have the precise information on the SO’s orbit, its search degenerates into its particular case – observation of the SO by the accurate targeting data. Namely in this sense, one may not make difference of principle between these two modes (see it later in detail).

This monograph is devoted to the investigation of the second category of search methods for SOs – the search, using rough a priori information about their orbits – and the development of the corresponding theory foundations for search planning. It is namely this category that is most relevant and interesting from the mathematical point of view as well.

CHAPTER TWO

THE ESSENCE OF THE PROBLEM

2.1. On available a-priori orbital information on the sought-for space objects

In case of total absence of a priori information about the orbit of the sought-for SO, the observer will prefer the first category of search situations and he has nothing to do but to turn to the survey methods. The theory developed here presupposes the indispensable presence of some, albeit very crude, initial metric information about the sought-for SO. This information which will later be used to construct the search plans may have different forms and origins, for example:

information on the time and place of launch;
designed or expected nominal values of the SO orbit parameters;
statistic data on SOs' orbits of different classes;
estimates of the orbit parameters by imprecise measurements, by few ones, or by measurements at the short measuring arc;
available overaged information on the orbit;
information on the orbit adjustment or on the SO's maneuver;
information on the orbital structure of the SOs' constellation (SC).

The orbit parameters are getting overaged in time due to the evolution of the state vector errors. And if by some reason the influx of measurements for a given SO has been stopped, then its orbital information (both metric and non-metric) gradually loses the accuracy in due course. This is one of the most natural ways of obtaining imprecise orbital information on an SO.

Further, let us take notice that the SO catalog maintained in the SSS serves as another natural source of imprecise orbital information on SOs even if the orbital parameters considered as very accurate from many points of view appears imprecise for narrow-angle and narrow-beam sensors or for carrying out some special operations with the proper SOs, perhaps, using some special auxiliary instruments.

In absence of publications of the orbit parameters values, if there is a need of detection of a HOSO for instance in highly eccentric orbit (HEO), it would be advisable to use the following a priori information.

1. The position of the orbit plane (longitude of ascending node Ω) when the launch site is known can be calculated, using the time and date of launch and regarding the optimum energetic strategy of putting the SO into its orbit.
2. In terms of stableness of the HEO apogee position, its inclination i should be as close as possible to its critical value [13].
3. Based on the structure of the SC, the features of the functional purpose, and the flight program of its spacecraft, the inclinations of their initial orbits have their own specifics for different SCs (see Fig. 2-1-1).
4. In terms of providing the best conditions of radio visibility and the survey of the North hemisphere, the value of argument of perigee ω should be comprised within the interval 270 - 290° [21, 22].
5. Practically all the tasks performed by functional HEOSOs demand periodical repetition of the SO trace on the Earth surface which can be achieved only with multiplicity of the SO nodal period T_Ω to the sidereal day (24 hours) with no regard for the orbit parameters evolution ($T_{sid}/T_\Omega = 1, 2, \ldots$). For reasons of providing more size of the optical and radio visibility zone, long enough duration of covering the particular territory, and better conditions of radio communication, the preferable orbits are those with nodal period $T_\Omega = 11^h 57^m 45^s$ which provides the repetition of the same Earth trace.
6. To ensure that the perigee of the orbit with such a period of circulation is outside the dense layers of the atmosphere, the eccentricity should not exceed the value

$$e_{cr} = 1 - \frac{R_E + h_a}{a} \approx 0.74$$

where e_{cr} is the critical value of eccentricity,
R_E – the Earth radius,
h_a – the height of atmosphere (relative).

For the sake of enlarging the operational part of orbit and reducing energy expenditure for putting the SO into the basic circular orbit, the eccentricity should be as great as possible. That is why for the majority of HEOs with a nearly 12-hour nodal period the designed eccentricity is within the interval 0.7 … 0.74.

7. The value of argument of latitude can also be approximately estimated by a priori information on the program of placing the HEOSO into the operational orbit. It is possible to approximately determine the time t_a of an SO coming to the apogee proceeding from the typical scheme of putting into orbit (with minimum time of phasing the SO in the transfer orbit), the time and site of launch being known:

$$t_a = t_{st} + \Delta t_{act} + \Delta t_{in} + \Delta t_{trans} + T_{in}/2$$

where t_{st} – time of start,
Δt_{act} – duration of active motion (≈ 8 …10 min),
Δt_{in} – duration of motion in the initial (basic) circular orbit,
Δt_{trans} – duration of motion in the transfer orbit,
T_{in} – the initial period of the SO revolution.

Then, based on the available experience, we can suppose that the sum of three items $\Delta t_{act} + \Delta t_{in} + \Delta t_{trans}$ approximately equals 27 min (for example, for HEOSOs started from the Vandenberg base, that is SOs of series SDS, program 711, and so on).

One should mean also that during the launch of HEOSOs, the boosters usually move in the same HEOs. When an SO is put into the geostationary orbit (GEO), its booster remains in the transfer HEO having its inclination equal to the latitude of the launch site and the argument of perigee equal to 0° or 180°. Sometimes, the booster is to be put into GEO. A priori information on the time and site of launch for search of an SO in a circular 12-hour orbit is, as a rule, more definite than that for a HEOSO.

Taking into account the peculiarities of evolution of the state vector errors, detection of HEOSOs and 12-hour circular SOs comes to the search by argument of latitude u. The strict statement of this problem and optimum methods for its solution are given in Chapter 4. If there are appreciable uncertainties in other orbit elements, the problem also comes to the search by argument of latitude u in several passes (section 4.6) or to the search task in a more common statement (Chapter 5).

The search problem with a priori information of a different kind, for example, the out of date state vector, also comes to the search by argument of latitude. Such a search situation very often arises in the space surveillance practice when for some reason the needed measurement information is absent for a long time. In such a case, the SO position error grows most rapidly along the track (that is, in argument of latitude) [11]. One can see this from Table 2-1-1 in which the state vector error evolution

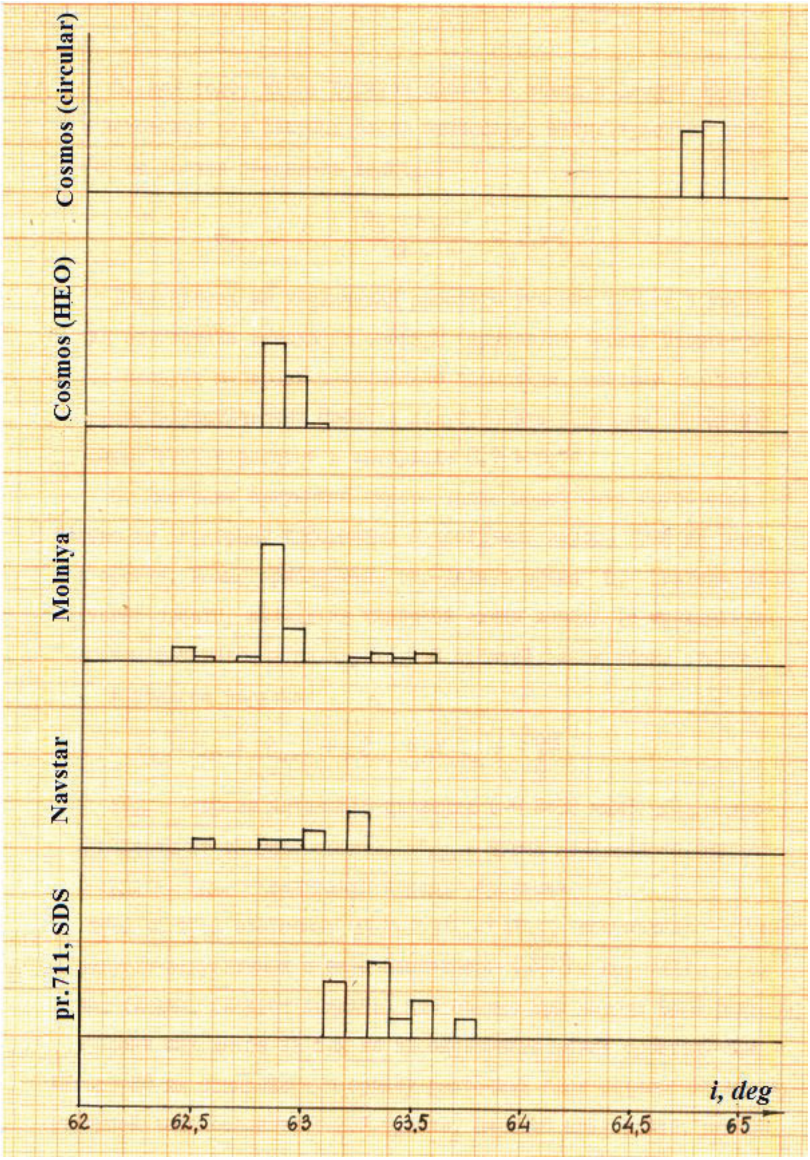

Fig. 2-1-1. Histograms of the initial orbit inclination distributions for some HOSCs

is presented in the radius-vector (r), along the track (l), and in the binormal (b) when propagating the motion of a geosynchronous satellite with a given initial state vector determined by 4 highly precise 3-dimensional measurements (a slant range and two angles) at the measuring arc 360°. The appearance of a problem of search by argument of latitude u, the orbit plane and its position in space being given, in this case is evident.

There is one more source of imprecise a priori information for addressing the search. Typical for space surveillance practice is scheduling the observations of a HOSO by the imprecise ephemeris calculated from the initial state vector determined by measurements obtained on a small measuring arc. Such a situation often appears, for example, because of dependence of successful operation of optical systems upon the time of day and the atmosphere transparency, in case of short-term stay of the SO in the radar zone (for instance, at the edge of the zone), on account of dependence of the passive RF sensor efficiency upon the duration of the irradiating spacecraft operation, and so on. Here the condition for application of the search methods by argument of latitude is still more favorable. This fact is corroborated by the data of Table 2-1-2 where the geosynchronous SO position propagation error evolution is given, the initial state vector being determined by 4 two-dimensional measurements with the root-mean-square (r.m.s.) error 2″ located at the measuring arc 45°.

Table 2-1-1

State vector error evolution in radius-vector (r), along the track (l), and in binormal (b) when propagating the motion of a geosynchronous SO with a given precise state vector (the measuring arc is 360°)

Prediction interval, days		0	1	2	3	10	20	30	60	180
r.m.s. error	in r, km	0.123	0.124	0.124	0.125	0.129	0.134	0.138	0.153	0.207
	in l, km	0.5	0.6	1.0	1.45	4.8	9.7	14.6	29.1	87.3
	in b, km	1.43	1.43	1.43	1.43	1.44	1.46	1.47	1.51	1.69
	in $\dot{r}$, cm/s	3.44								
	in $\dot{l}$, cm/s	0.9								
	in $\dot{b}$, cm/s	10								
	in T_{Ω}, c	0.16								
estimate bias	$M\Delta r$, km	0.004	0.004	0.004	0.005	0.005	0.005	0.005	0.005	0.006
	$M\Delta l$, km	0.017	0.09	0.17	0.25	0.77	1.53	2.29	4.5	13.5
	$M\Delta b$, km	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12
	$M\Delta T_{\Omega}$, s	0.025								

Table 2.1.2
Geosynchronous SO position propagation error evolution (in r, l, b) given an imprecise initial state vector (the corresponding measuring arc is 45°)

Prediction interval, days		0	1	2	3	5	10
r.m.s. error	in r, km	3.67	3.64	3.36	4.36	6.3	9.7
	in l, km	0.08	155	310	465	755	1552
	in b, km	0.38	0.38	0.38	0.38	0.38	0.39
	in $\dot{r}$, cm/s	6.8					
	in $\dot{l}$, cm/s	29.5					
	in $\dot{b}$, cm/s	2.9					

Very often, the search tasks in a similar statement can arise under different circumstances, for example, when from the supposed structure of the SC there is known the plane and the orbit configuration of the sought-for SO included into the SC, but its precise position in orbit is unknown. Or else, the SC orbital structure and the exact position of at least one of its satellites are known. In this case, say, for the NAVSTAR system one can calculate the approximate orbit parameters for all the remaining satellites [22].

Let us consider now some more typical search situations. One of the basic peculiarities of the whole HOSO population dynamics (unlike the LEO population dynamics) is the orbit keeping actions which are performed periodically for every acting satellite. A HOSO is also apt to a sudden maneuver. In section 4.8, one can see that the problem of detection of a HOSO after a maneuver or an orbital correction can be conveniently formulated in terms of the apparatus of the equivalence curves (see sections 3.2 and 4.3). As well as it can be reduced to the search by argument of latitude using some correction operations.

Then, the method of search for an SO by argument of latitude suggested in section 4.2 can be used for performing the task of detecting a rapidly moving SO having faint brightness or/and being in a wrong phase of illumination. As the experience of electro-optical sensors operation shows, the weakening of the intelligence signal registered from such an SO can reach several magnitudes due to relative motion of the signal and the receiver (and, as a consequence, the blurring of its image).

And now we consider a very important situation. The main problem of detecting small and weakly-contrasting SOs is connected with providing favorable conditions for concentration and accumulation of the faint intelligence signal energy in one point of the receiver sufficient for its registration. The compensation of the SO velocity in its relative motion across the receiver (see sections 4.4 and 4.12) is able to significantly enhance the operation potentialities of radar and optical sensors in terms of their sensitivity.

In view of possible high angular velocities of relative motion of the sensors (their axes) and the sought-for SOs, it is appropriate to introduce the search planning optimum algorithms into the sensors' software. Namely owing to a complicated character of the sensor and target relative motion, one should expect to gain the high efficiency of application of these algorithms, their peculiarity being a special form of function $\varphi[\bar{u}(t)]$ (see section 4.3). The latter, by the way, does not touch the received here basic principles of search planning and concerns only the amount of recalculation of the search plan elements into the on-board sensor's targeting data which offers no significant difficulties.

Another very urgent task of search for a HEOSO after its initial detection (more often, by radar) in the vicinity of its orbit perigee can be also formulated as a task of the search by argument of latitude. Such an SO is able to stay in the radar zone for less than a minute (at the edge of zone – only for several seconds).

This fact implies a very low accuracy of the SO's orbit determination. This is especially true of the period of the SO's revolution in orbit. The next passing of the SO through the censor's zone can happen only in about 24 hours or later (that is, in two or more revolutions), the mutual identification of these two consecutive measurements being extremely difficult and sometimes impossible. At the same time, after a HEOSO passes through the radar zone (in the vicinity of perigee), the favorable conditions arise for planning its search in the vicinity of its orbit's apogee at the same revolution with the help of optical sensors using the preceding imprecise state vector obtained by the radar. The preference for such a situation follows from the principle of equivalence (see sections 4.3 and 4.5).

The task of search for a HOSO by argument of latitude arises also within the following strategy of detecting new HOSOs. During the launch of a HEOSO or GEOSO, at first it is put into a low initial (basic) orbit (the so-called phasing orbit) where it is detected by the radar network, and then it is being tracked until it is transferred into the transitory highly eccentric orbit. For a HEOSO, the latter becomes the final one. If the final orbit is assumed to be GEO, then the transition impetus for putting the SO into the transitory orbit is made at one of the nodes of the phasing orbit, the latter becoming a perigee of the transitory orbit.

If the final orbit is assumed to be HEO, then its perigee usually coincides with the south apsidal point of the phasing orbit where the correction impetus is made. To approximately calculate the time of its correction (and to consequently give the concrete expression to it) one should collate the epoch of the last measurement with the time of registration of its last observation by radars. Further, the task can be reduced to the search by argument of latitude in the transitory HEO and then perhaps in GEO. In case of uncertainty due to the limited number of possible options, one can create and then realize the optimum search plan (SP) for every option.

In the last case, since every HOSO during the launch passes through the stage of a basic initial LEO, the described above possible strategy of detecting new HOSOs looks comprehensive. And it is very efficient, both economically (one should not resort to the regular scanning mode, and the search is accomplished in a very limited region of space) and operatively (the detection time would be minimum, the SO being tracked at every stage of its transfer into the final orbit).

Problems of a special kind are those of planning the search for a HOSO or a group of HOSOs with the help of a group of sensors. So, different search situations arise both at the stage of initial detection of HOSOs and during the process of posterior tracking them. The negative statement of the detection problem also makes sense: to guarantee (economically and/or in shortest time) the fact of the absence of the sought-for SO in a given orbit (when monitoring the maneuver, the fragmentation fact, and so on).

2.2. Basic definitions

Since in this sphere of science the basic concepts and definitions in different publications often diverge and even sometimes contradict each other, they will be specified here for elimination of ambiguity in the terminology used in this monograph and for a more accurate understanding of the theory presented in it.

1. In many publications on this subject, there is no unanimity on the definition of a HOSO. Depending on the purpose of consideration and the context, different senses can be put into the notion of a HO. In researches of the atmospheric drag influence on the SO motion, a circular orbit with a height of 500 - 700 km and more can be treated as a HO. In researches of influence of the Moon and the Sun on the SO motion, HOs are those where the mathematical description of the SO motion cannot neglect this influence. These are orbits with heights of more than 10,000 – 20,000 km. Many American experts call the orbits with heights of 150 – 1,500 km (sometimes, up to 2,000 km) as LEOs, those with heights of 1,500 – 35,800 km are considered as medium Earth orbits (MEOs), and those higher than 35,800 km - as HOs.
2. In this monograph, as HOSOs are called the satellites having the nodal period $T_\Omega \geq 3$h where the stable tracking with the help of the ground-based radar network cannot be provided, and there is a need to attract sensors of an alternative kind. Namely in this sense, the HOs acquire the complete complex of specific features that forces one to consider the monitoring of the related SOs as an independent

actual problem. In terms of this definition, a HEOSO with its perigee in the south hemisphere should be considered as a HOSO.
3. The motion of an SO is understood here as the motion of its center of mass (no attitude motion around its center of mass is concerned).
4. The term “acquisition” means detection of a signal from an object.
5. The term “detection” is used here in two different senses. Detection in a wide sense is a positive result of search - that is, ascertainment of the fact of presence of the sought-for SO in the sensor’s FoV.
6. Detection in a narrow sense of the word is introduction of a non-cataloged SO (after its detection in a wide sense and perhaps in some period of tracking and collection of measurements) into the SO catalog with accuracy enough for its stable tracking (or with the accuracy given by the consumer of metric information on it). One should not confuse the so defined notion of detection with the information theory notion of detection (the technological one) which means the process of revealing the intelligence signal against the noisy background, being fully aware of the fact that the sought-for SO is in the sensor’s FoV.
7. The term “search” means here the process of inspecting a limited domain of space with the aim of detecting (in a narrow or wide sense) a certain SO or obtaining the information on its absence there.
8. The monitoring of an SO motion means here a set of measures that allows the consumer of information at any time of his choice to get the values of the monitored SOs’ orbital parameters with the required (by the consumer) accuracy. That practically means maintaining the SO catalog with the required (by the consumer) accuracy.

2.3. Statement of the problem

In the process of searching for a space object (especially by very rough a priori information), using the traditional search methods, there occur errors of the first kind (the formation of "slots" (“chinks”) between already controlled areas of space) and of the second kind (an unjustified repeated survey of some already controlled areas).

As a result of errors of the first kind, it is possible not to find the sought-for SO (really presenting in a given space domain) if it "falls through" into the "slot". Errors of the second kind result in an irrational consumption of the search resource of the observation facility (including power consumption, amortization of equipment, and employment of maintenance personnel).

The matter of fact is that the structures of the 6-dimensional sought-for SO current position uncertainty domain (CPUD), mathematically representing all the initial positional information about it, and its 3- and 2-dimensional projections (namely, the latter are constructively subjected to observation by radar and optical sensors) during the implementation of the search plan undergo a complex nonlinear transformation that leads to a geometric distortion of the already controlled areas and the washing away (blurring) of their boundaries.

This monograph examines the "anatomy" of the occurrence of these errors and suggests approaches to prevent their occurrence or to compensate them fully or partially.

As is received in this monograph, a solution of the problem would be the creation of a methodology allowing in each search situation to choose the sequence of conditional ephemerides optimum, by some efficiency criterion, under the restrictions imposed by astro-weather-ballistic conditions and the technical potentials inherent in the sensors. This sequence of conditional ephemerides will guarantee providing detection of the sought-for SO. One can add to this some operative requirements.

Why the sequence? And why conditional? Since the question is the search under significant uncertainty of a priori information, it is clear that only one ephemeris would be not enough for detection of the SO. And since there are several and may be many ephemerides in the sequence, then they are conditional, perhaps except for one and only thing which allows real detecting the sought-for SO.

In this monograph, such a methodology will be constructed.

The general canonic theoretical approach to the search for a moving target (for example [23]) is notable for superfluous generality and abstraction, neglects some important peculiarities of our problem, operates with redundant dimension of the search space, does not suggest the constructive mechanisms of transfer from one dimension to another, and, generally, does not possess the necessary constructiveness for creation of effective practical methods for planning the search [2, 61].

The contemporary solution of the contemporary search problem should involve the effective use of any available a priori information on the sought-for SO’s orbit, the due regard of dynamics of the SO current position uncertainty domain (SOCPUD) and its structure, and simultaneously taking into account the concrete capabilities and peculiarities of the sensors used which can be very diverse and significantly affect the optimum choice of the search plan design.

If any a priori information on an SO motion is available, it is equal in strength to giving in a 6-dimensional phase space X_6 the CPUD of the SO, i.e., a domain $D_6(t_0)$ of possible values of the SO's motion parameters vector $R_6(t_0)$ (the state vector) at the reference time t_0:

$$R_6(t_0) = \| x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3 \|^T \in D_6(t_0) \subset X_6,$$

and the related probability distribution density function $f_{t_0}(R_6)$ defined on the domain $D_6(t_0)$. The last one practically always can be given as a limited domain by bounding it on a sufficient confidence (credibility) level.

The celestial mechanics laws F define on the domain $D_6(t_0)$ a homeomorphic mapping F_6 transferring each point of D_6 ($R_6(t_0) \in D_6(t_0)$) at time t_0 to another point of the phase space X_6 at another time t_1 (Fig. 2-3-1):

$$R_6(t_1) = F_6[t_0, R_6(t_0), t_1].$$

In other words, domain $D_6(t_0)$ is one-to-one and "to-and-fro" continuously transformed by F_6 into domain $D_6(t_1)$:

$$D_6(t_1) = \bigcup_{R_6 \in D_6(t_0)} F_6[t_0, R_6(t_0), t_1] = \mathrm{F}_6\,[t_0, D_6(t_0), t_1],$$

the function $f_t(R_6)$ being appropriately transformed.

This is the set theory approach to setting the problem on the basis of the search plans equivalence principle [36, 25]. It is worded as follows drawing upon the notations above.

Checking the point $R_6(t_0)$ at time t_0 is equivalent to checking the point $R_6(t_1)$ at time t_1 in the sense that it is not necessary to perform both actions of checking: it is sufficient to check only one of the two equivalent points in the appropriate time. (Let us remember that this is true only in 6-dimensional phase space.)

Similarly, checking the domain $d_6(t_0) \subseteq D_6(t_0)$ at time t_0 is equivalent to checking the domain

$$d_6(t_1) = \bigcup_{R_6 \in d_6(t_0)} F[t_0, R_6(t_0), t_1]$$

at time t_1.

On this basis and in the sense given above, it is admissible not to distinguish between the search plans “with the accuracy up to equivalence of their points”.

By definition, a *search plan* (SP) M_6 is referred to as any set of pairs

$$M_6 = \{(R_6, t)\},\ R_6 \in X_6$$

selected by some arbitrary principle or criterion, each pair meaning the potential check of the point R_6 at time t.

SP M_6 is defined as *non-redundant* if among all the points of this set there are no equivalent ones. SP M_6 is defined as *complete* if the realization of all its pairs by the sensor (or the group of sensors) provides a complete “coverage” of the uncertainty domain D_6 in its dynamics considering the equivalence principle. And so, SP M_6 guarantees the acquisition of the intelligence signal from the sought-for SO and, as a consequence, the detection of the latter.

The equivalence principle and the properties of SP will be considered completely and comprehensively in sections 3.2 and 4.3.

Hitherto, we have dealt with points from 6-dimensional phase space $R_6 \in X_6$, that is, with vectors of orbital (kinetic) state of an SO. However, a real tracking facility is capable to observe not the 6-dimensional domain $d_6(t)$ in the phase space but its real k-dimensional projection (k = 2 for an optical sensor and k = 3 for a radar). With this, the projection set theory equations look as follows:

$$D_k(t_1) = P\, D_6(t_1) =$$

$$= P \bigcup_{R_6 \in D_6(t_0)} F[t_0, R_6(t_0), t_1] = \bigcup_{R_6 \in D_6(t_0)} PF[t_0, R_6(t_0), t_1] =$$

$$= \bigcup_{R_6 \in D_6(t_0)} F_k[t_0, PR_6(t_0), t_1] = \bigcup_{R_k \in D_k(t_0)} F_k[t_0, R_k(t_0), t_1]\ .$$

Here $D_k(t_0) \equiv PD_6(t_0)$, and $P \equiv P_{6\to k}$ is a projection operator from 6-dimensional space to k-dimensional space: $PR_6(t)=R_k(t)$, $k = 2, 3$.

While F_6 is a mapping from 6-dimensional space to 6-dimensional space, F_k is a mapping from k-dimensional space to k-dimensional space. So, F_k can be conditionally (or symbolically) treated as a formal "projection" of F_6 into k-dimensional space (which yields F_k).

The validity of the equations above is proved by the set theory conception of point-by-point projection of any set. But now let us note (and ***it is a very significant remark***) that the main property of the initial mapping F_6 (a homeomorphism) has not been saved (conserved) after its projection. So, F_k is not a homeomorphism any longer because

$$F_k[t_0, R_k(t_0), t_1] = \bigcup_{R_6 : PR_6 = R_k\,(t_0)} PF[t_0, R_6(t_0), t_1] \quad =$$

$$= \quad D_k^*(t_1)^{1)}. \qquad (2.3.1)$$

That means that a point $R_k(t_0) \in D_k(t_0)$ is transferred in time (by mapping F_k) not into a point (like that was in 6-dimensional space) but into a domain $D_k^*(t_1)$, and, at the same time,

$$R_k(t_0) \neq R_k^*(t_0) \not\Rightarrow F_k[t_0, R_k(t_0), t_1] \cap F_k[t_0, R_k^*(t_0), t_1] = \varnothing. \qquad (2.3.2)$$

This relationship means that the F_k-images of different points timed to t_0 may intersect at another moment $t_1 > t_0$ which entails that the mapping F_k is not one-valued in both directions (as that was true for F_6), not to mention the loss of its continuity (Fig. 2-3-1). The symbol $\not\Rightarrow$ means that the left inequality in relationship (2.3.2) does not imply the right equality.

Accordingly, the equivalence principle should be converted for the projected k-dimensional spaces [25]. In terms of the transformed equivalence principle, we deal not with equivalence of the state vector projections for two different times (which is valid only for vectors in 6-dimensional space) but we do with pseudo-equivalence, that is, majorization of checking the k-dimensional point $R_k(t_0)$ at time t_0 by checking the domain $D_k^*(t_1)$ at time t_1. (For more details see section 5.4)

[1]) the starlet is used to distinguish the F_k-image of the point $R_k(t_0)$ from the full uncertainty domain $D_k(t_1) = F_k[t_0, D_k(t_0), t_1]$.

Such a significant topological difference between mappings F_6 and F_k, k < 6, (still more sophisticated by time-dependence of the projection plane

(or projection space for $k = 3$) current position) makes very difficult and complicated the transition from the equivalence of search plan M_6 in 6-dimensional phase space to the pseudo-equivalence of search plan M_k in the real 2- or 3-dimensional search space. This circumstance mostly complicates a constructive projecting, treatment, and construing of the equivalence principle given above. But we will finally overcome this difficulty (see Chapter 5).

In real observation practice, the technical capabilities of a narrow-angle or narrow-beam facility allow it to check every time the compact k-dimensional domain $d_k^{\circ}(t)$ which is practically invariant (as to its area (volume) and form) in time and space.

Nevertheless, during the transit from t_0 to t_1, the pseudo-equivalent (majorizing) domain $d_k^0(t_1) = F_k[t_0, d_k^0(t_0), t_1]$ may be essentially deformed (Figs. 2-3-1 and 2-3-2) due to the known error transformation laws [11, 16, 25]. Besides, according to (2.3.2), some points R^*_k out of domain $d_k^0(t_0)$ ($R^*_k \notin d_k^0(t_0)$) may transit (so to speak, "crawl in") into its F_k-image $d_k(t_1)$:

$$F_k[t_0, R^*_k, t_1] \in d_k(t_1) \text{ (see Fig. 2-3-2).}$$

The formal mathematical cause of these difficulties, theoretically, is the loss of the homeomorphic properties of F_k owing to the in-time transformation of the SOCPUD when projecting it from the 6-dimensional phase space into the real search space (2- or 3-dimensional). The practical consequence of this is the forced admission of a certain redundancy unavoidable even in the optimum search plans. This forced redundancy is to be accurately calculated and accounted for (see also Chapter 5 for details).

And now we consider the difficulties of another sort. For gaining a satisfactory effect when detecting and characterizing a faint intelligence signal from a moving SO as well as solving some problems of targeting a narrow beam to a distant moving object, it is necessary to keep precise direction to the SO during a certain time interval. That is, the accurate enough compensation of its motion relative to the sensor's receiver is needed. But it turns out problematic when only the very rough orbital information on the sought-for SO is available. Hence, the limitation on application of the known scanning or "fence" methods for its detection is evident.

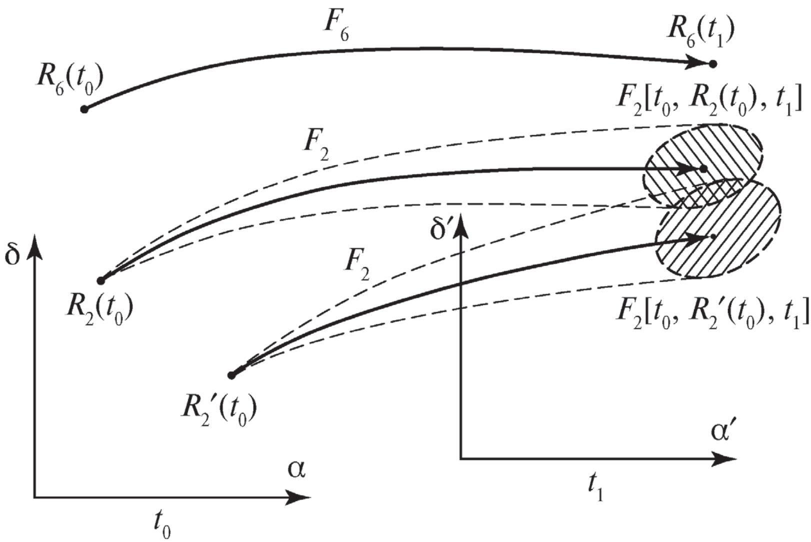

Fig. 2-3-1. F_6- and F_2-transformations of a point

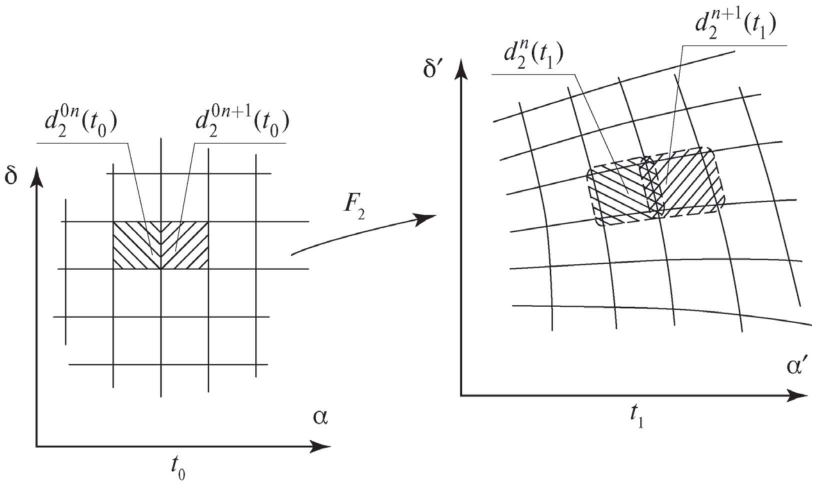

Fig. 2-3-2. Deformation of equivalent (pseudo-equivalent) domains

Strictly speaking, checking of a point $R_k(t_0) \in D_k(t_0)$ is not equivalent (in terms of the equivalence principle) to checking of the domain $D_k^*(t_1) = F_k[t_0, R_k(t_0), t_1]$ in which the point is transferred. Although, due to (2.3.1), check-up of $D_k^*(t_1)$ at time t_1 substitutes (majorizes) for check of the point $R_k(t_0)$ at time t_0 (the reverse is not true in view of (2.3.2)).

In common, the problem consists in choosing the sequence of times

$$T = (t_1, t_2, \dots t_n, \dots t_{\bar{n}}),$$

the corresponding sequences of conditional ephemerides

$$d = (d_k^{01}(t_1), d_k^{02}(t_2), \dots d_k^{0n}(t_n), \dots d_k^{0\bar{n}}(t_{\bar{n}})),$$

and conditional velocities vectors

$$v = (v_k^{01}(t_1), v_k^{02}(t_2), \dots v_k^{0n}(t_n), \dots v_k^{0\bar{n}}(t_{\bar{n}}))$$

for which the value of the adopted criterion is optimum:

$$\Phi(T, d) = extr \qquad (2.3.3)$$

under the restrictions imposed by the technical capabilities of the tracking facilities used (angular size of the field of view or the directional diagram (pattern), maximum speed of retargeting the sensor, etc.), and also by the condition of completeness of the search plan (2.3.6):

$$\psi_m(d_k^{0n}(t_n)) \leq \psi_m^{0n},\ n = 1, 2, \dots \bar{n},\ m = 1, 2, \dots \bar{m}, \qquad (2.3.4)$$

$$t_n - t_{n-1} \leq \Delta t_n,\ \ n = 1, 2, \dots \bar{n}, \qquad (2.3.5)$$

$$\forall R_k(t_0) \in D_k(t_0)\ \exists d_k^{0n(R_k)}(t_n): F_k[t_0, R_k(t_0), t_n] \in d_k^{0n(R_k)}(t_n). \qquad (2.3.6)$$

In these expressions, Φ is an adopted criterion of the search plan quality, (2.3.4) and (2.3.5) are the restrictions imposed by technical characteristics of the sensor, and (2.3.6) is a necessary guarantee of detecting the SO.

A conditional ephemeris $d_k^{0n}(t_n)$ can be presented by right ascension α, declination δ, and time t (for optical sites) or by azimuth A, elevation angle β, slant range D, and time t (for radars), and so on.

The creation of a methodology for concrete statement and solution of the task (2.3.3) - (2.3.6) in different search situations needs preliminary development of some theoretical foundations and a handy mathematical apparatus for equivalent (pseudo-equivalent) transformations, evaluation, and comparative analysis of search plans.

The purpose of development of such a search planning theory is as follows:

- unlike the problem statement in terms of sequentially scanning a large region of space, to word the problem of active search for a specific SO, a priori (rough) information on its orbit being used effectively;
- to create a scientific basis for providing the best conditions for the intelligence signal registration in the receiver and, thereby, to enhance the sensor's operational characteristics;
- to determine the optimum conditions of search for a given SO by important criteria and to provide constructive means of realization of these conditions in the form of optimum search plan (OSP);
- to determine the limits for search planning capabilities taking into account the search region properties, the sensor technical characteristics, and a class of the sought-for SO;
- to avoid the appearance of the first and second kind errors in the search plans implementation;
- to create a single (unified) common theoretical platform for efficiently solving search problems in various search situations arising in space surveillance practice and in many research and routine tasks of astronomers-observers.

2.4. The historic points of the search theory formation

First of all, we will sort out the term and concept of *a search theory*. In the largest sense, the notion of a search theory (ST) is a loose and indefinite concept, its content being very heterogeneous depending on its specific use and the context. Very many problems having nothing in common among them either in a physical sense or in the solution methods can be stated as search problems.

This is due to the large extensibility of concepts *search* and *detection* when they are used for a variety of reasons by very different people. Everything that in numerous publications relates to search theory, strictly speaking, does not represent a unified theory. At best, it is a set of different, by their nature, eclectic theories and methods. And what is more, it seems doubtful whether one can construct a comprehensive all-embracing search theory notable for unity of the axiomatics and a sole method of investigation.

There is no sense to embed all the variety of heterogeneous (physically and mathematically) problems into the Procrustean bed of a single formal scheme. This opinion is supported by the known experts in the field of search tasks, R. Alswede and I. Wegener. Namely for these reasons, they named their book [27] not “The Search Theory” but “Tasks of Search”.

Any theories appear successful if they are developed under a reasonable restriction of the scope of tasks concerned. So, let’s confine ourselves to reviewing publications bearing relation, even if remote, to our problem.

One of the first fundamental works in this oldest branch of operations research belongs to Bernard Koopman and his colleagues from the operations research group on the fight against submarines (Navy, USA) during the Second World War. By irony of fate, in the consequent investigations of search problems the strategies concerned were more often connected with the tasks of search for ships and submarines [28, 29, 17, 27].

B. Koopman and his disciples have developed the search theory as a theory of optimum distribution of search efforts [29, 30, 31, 32, 33, 34].

Most completely, the total of these achievements for 20 years is presented in the monograph by Stone [28]. In Koopman theory, the Bayesian approach was adopted due to which a priori probabilities distribution $p(x)$ of the sought-for SO’s residence at a given region’s point $x \in \Omega$ should be known as well as the total amount of search efforts Φ and the function $q(\varphi)$ connecting the conditional probability of detecting the target deliberately being in the point x with the amount of search efforts $\varphi(x)$ applied here.

The task is to determine the function $\varphi(x)$ which gives the maximum to the total probability of detection of the SO

$$P(\varphi) = \int_{\Omega} p(x)\, q\big(\varphi(x)\big) dx,$$

the sum amount of search efforts to be afforded

$$\Phi = \int_{\Omega} \varphi(x) dx$$

being given under the natural assumption

$$\varphi(x) \geq 0.$$

The paramount significance was paid to search for a motionless target. If a motion sometimes was tolerated, that was the simplest one (linear, as a rule) [28, 29, 30]. This greatly limits the application of the theory in the case of searching for a moving object.

The very hard and obligatory requirement of a priori knowledge of the functions $p(x)$ and $q(\varphi)$ (and in some settings of the problem some other functions) significantly restricts the application of Koopman theory. This shortcoming, in fact, has been acknowledged by the classics of the theory. For example, L. Stone remarks that there are no general means of obtaining any useful information about the function $p(x)$. So, it is usually determined subjectively [28], that is, without appropriate mathematical justification.

Extreme limitations, if not to say scantiness, of the class of problems that can be solved with the help of Koopman theory and the very limited possibility of its generalization step-by-step brought down its originally wide popularity.

However, this fact was understood and realized not immediately. For instance, another eminent researcher J. Danskin tried to apply Koopman theory to solve such a "koopman" problem (on the face of it) of optimum distribution of photographic reconnaissance efforts for the search of rocket sites at a given territory, but he failed. After unsuccessful attempts, he came to the conclusion that Koopman search theory does not work for such problems. As a result, he was forced to develop a new theory for his aim – the theory of reconnaissance [35].

Further development of search methods occurred in the following directions:

- paying regard to the cost of search effort depending on the point of its application [31 and others];
- search for a moving object [16, 17, 28, 36, 37, 38, 44, 57, 61];
- search for a set of moving objects [17, 36, 40, 39];
- Markovian motion of a target [28, 38, 23];
- using a priori and a posteriori information on the sought-for object for enhancing the effectiveness of search [18, 24, 39, 36, 42, 1, 2];
- optimization of data processing during the search and detection [41, 37].

In some works, research is reckoned in terms of the theory of statistic decisions [41, 37, 27].

After development of CCD-matrices, infrared, electro-optical, radio-electronic, TV, and laser techniques, the number of works dedicated to the target selection, detection of intelligence signals (first of all in a technological sense), and enhancing of the anti-interference protection in the search systems has been significantly increased [19, 20, 37, 41].

Going in step with the times, many researches were devoted to investigation and synthesis of the procedure of inspecting the image in a FoV of the sensor and technical realization of such a procedure using modern technologies. Significant attention was paid in the literature to investigation of regularities of search in case of doubtful detection of the object [37, 17, 41, 18], the detection sensors' sensitivity and its relation with stochastic properties of the intelligence signal and the interference [17, 37], optimization of methods for detecting targets making their appearance at different points of a given region by a casual (stochastic) law [28, 24, 38], determination of directions and probability characteristics of the target appearance on the border of the sensor range [17], and so on.

The problem of planning the search for cyclic objects and for those in the real environment, using incomplete or imprecise a priori information on them, was considered only in the last decades [36, 44, 39, 40, 18, 42, 1, 2, 24, 16].

The main reason and impetus for the development of an original theory of planning the search for an SO using imprecise a priori information on its orbit parameters is the fact that the known theories and methods in terms of which it is, in essence, possible to word different tasks of search do not take into account many important peculiarities of our problem – first of all, the cyclic character of motion of a target (often essentially non-linear even in small time intervals), possibility of effectively using a priori information on the sought-for target's motion parameters, the special technical features of the sensor used, etc.

A very significant gap in the methods used to search for SOs was the lack of analysis of the search plan degradation phenomenon during the process of its implementation. Accordingly, there were no methods to compensate for the negative impact of this phenomenon.

A comprehensive analysis of the problem has brought to the conclusion of the necessity of plunging into the more profound theoretical level for significant improvement of effectiveness and reliability of search. This implies, first of all, the necessity of adequately taking into account the temporal structural dynamics of the SOCPUD during the process of realizing the search plan.

CHAPTER THREE

CYCLING OBJECTS AND THE SEARCH FOR THEM

The first and defining feature which essentially distinguishes our problem from numerous alternative search problems (situations) is the motion of the sought-for objects and the cyclicity of this motion. Therefore, it was natural to first develop an auxiliary "pre-theory" of searching for cyclically moving objects which was done in [36]. Cyclic motion of an object along a closed trajectory, not burdened with any particulars, will be used as a transparent abstract model for more evident preliminary identification of the basic features and possibilities of setting and solving search problems put forward by practice. Using this model and making a start from the most general treatment of the search problem, we shall gradually narrow it approaching our concrete one.

One of the most important features of an SO motion is its cyclic recurrence within its orbit.

Historical scientific experience shows that the more specific the peculiarities of the problem concerned that are taken into account when seeking the approach to its solution, the more practically effective the latter turns out.

On the other hand, there is a danger to be “over-diligent”: you can overdo it, because taking into account an excessive number of specific features when creating a solution method greatly narrows its scope. In other words, one should keep the right balance between these two extremes.

A theoretical base of the solution should be of a common enough character for solving not only the problem concerned but also relative and isomorphic ones. At the same time, it should let one easily create constructive and convenient methods for concrete search tasks.

In this chapter, the axiomatics and theoretical basis are developed for a general case of planning the search for a cyclically moving object. This material is notable for a sufficient common character and independence and can serve as a theoretical platform for synthesis and analysis of optimum plans of search for any physical objects the motion laws of which meet the axioms adopted here.

Besides, the mathematical ideology developed here represents the general theoretical basis and apparatus quite suitable as a preliminary platform for constructing, *mutatis mutandis,* a new theory of planning the search for an SO using imprecise a priori information on its orbit parameters. This material includes also voluminous proofs of the necessary theorems. In case if they were placed just into sections devoted to immediately planning the search of an SO, that would violate the visual character and applied harmony of the exposition and narration.

In addition, the introduction of such detailed mathematical evidence is important for more in-depth study and assimilation of a new approach to solving our problem on a more rigorous level.

3.1. Initial concepts and notation

Let's consider a situation in which a sought-for object is cycling along a closed trajectory on which a system of reference is introduced. That means that the law of its motion $s(t)$ is, generally speaking, a non-single-valued (non-one-valued) periodical function modulo S and with a period T:

$$[s(t)]_{\mathrm{mod}S} = [s(t + nT)]_{\mathrm{mod}S} \quad \forall t, \forall s \qquad (3.1.1)$$

where t is time,

s – the object's positional coordinate on its trajectory in the adopted system of reference,

S – the total change of the object's coordinate in one cycle, and

n – is any positive integer [36].

Let us assume now that the function $s(t)$ is continuous, differentiable (more exactly, can be brought to such a form in view of its original right and reverse non-single-valuedness and periodic character), and monotonously increasing (within one cycle). It is known a priori that the object at time t_0 is within the interval $[s_b, s_f]$ which will be called a basic one. The motion law $s(t)$ is given in an explicit form, by an algorithm, or by some differential equation (or a system of equations). And the initial data are known – the value of s at time t_0: $s_0 = s(t_0)$ (more exactly – its mathematical expectation).

Then let us have an instrument or a set of instruments with the help of which one can, by some plan, check the points of the trajectory with the aim of detection of the object, for instance, by moving the telescope's sight axis along the object's track, of course, the necessary restrictions (conditioned by the physical resource for realizing such a check, say, in terms of speed of the instrument retargeting, the size of the trajectory segment checked at once, and so on) being kept.

Theoretically, for detecting the object it is enough, for example, to continuously check the point s_f during the time interval $\tau(s_b, s_f)$ starting since time t_0 where $\tau(s_b, s_f)$ is the duration of the object motion from point s_b to point s_f within a cycle (the search plan M_{s_f}).

At the same time, there is another theoretical way to detect the object. It is enough, for example, to check at once all the points of the arc $[s_b, s_f]$ (the search plan M_{t_0}). But such attractive (because of their simplicity) solutions, besides their non-optimality by many natural and important criteria, can contradict the problem restrictions.

For a stricter statement of the problem and in the interest of presentation convenience and analysis of the plans for checking the trajectory points, let us introduce some useful notions and definitions.

3.2. The equivalence curves

Now we introduce a very important tool for analyzing and synthesizing search plans. Let point $<t, s>$ in the *ts* plane designate the possibility (or an act) of checking the point of the object's trajectory having coordinate s at time t. The check of coordinate s_1 at time t_1 is equivalent to the check of any other coordinate s_2 at time

$$t_2 = t_1 + \tau(s_1, s_2) \pm nT$$

where, the same as above, $\tau(s_1, s_2)$ is a duration of the object motion from point s_1 to point s_2.

The term *equivalence* of points $<t_1, s_1>$ and $<t_2, s_2>$ is used here in the next sense: the object is observed (not observed) in point s_1 at time t_1 in that and only that case if it is observed (not observed) in point s_2 at time t_2, and vice versa.

By definition, points $<t_1, s_1>$ and $<t_2, s_2>$ in the *ts* plane are referred to as equivalent ($<t_1, s_1> \sim <t_2, s_2>$) if

$$[t_2 - t_1]_{\bmod T} = \tau(s_1, s_2) \text{ or } [t_1 - t_2]_{\bmod T} = \tau(s_2, s_1).$$

By definition, any set $M = \{<t, s>\}$ in the *ts* plane can be called a search plan (SP). The trivial SPs, M_{s_f} and M_{t_0}, considered earlier now can be represented as

$$M_{s_f} = \{<t, s_f>: t \in [t_0\ ,\ t_0 + \tau(s_b, s_f)],\ s_f = const\}$$

and

$$M_{t_0} = \{<t_0, s>: t_0 = const,\ s \in [s_b, s_f]\},$$

respectively.

By definition, plans M_1 and M_2 are equivalent ($M_1 \sim M_2$) if

$$\forall < t_1, s_1> \in M_1\ \exists <t_2, s_2> \in M_2 : <t_1, s_1> \ \sim\ <t_2, s_2>$$

and

$$\forall <t'_2, s'_2> \in M_2\ \exists < t'_1, s'_1> \in M_1 : <t'_2, s'_2> \sim < t'_1, s'_1> .$$

That means that, if both plans coincide with an accuracy up to the equivalence of their points, they are equivalent. It is evident that $M_{s_f} \sim M_{t_0}$.

By definition, any plan M is *complete* if for detection of the object it is sufficient to implement only its elements (and no others). Otherwise, it is *incomplete.* Any plan M is *non-redundant* if it does not contain spare, superfluous elements, that is, if elimination of any point $<t, s>$ out of M makes it incomplete. Plans M_{s_f} and M_{t_0}, despite their triviality and non-optimality, nevertheless, possess these important properties – they are complete and non-redundant (although not optimum, perhaps very far from this). It is obvious also that any plan M being in one-to-one correspondence (by the defined above equivalence) with plans M_{s_f} and M_{t_0} ($M \dot{\sim} M_{s_f} \dot{\sim} M_{t_0}$) is complete and non-redundant as well. Further, we will be interested namely in such kinds of SPs.

Of course, the optimum plan should be complete and non-redundant. However, this is necessary but not enough.

By definition, a geometric locus of points in the *ts* plane equivalent to some arbitrary fixed point $<t_0, s_0>$ (a generating point) is called the *equivalence curve* (EC) of that point and designated as $\tilde{s}_{t_0,s_0}(t)$. Correctness of this definition is a consequence of the fact that $\tilde{s}_{t_0,s_0}(t)$ is a motion law of the sought-for object upon the condition that at time t_0 it was at point s_0 (the initial condition).

For a specific fixed time t_0 and for all points of the search interval [s_b, s_f], their ECs form a parametric family $\sum_{t_0}^{s_b,s_f}$ with the parameter $s_0 \in [s_b, s_f]$:

$$\sum_{t_0}^{s_b,s_f} = \{\tilde{s}\ (t)\}_{t_0}^{s_b,s_f}$$

having the following fundamental properties.

1) For any fixed s' and s'', the value of $\tau(s', s'')$ does not depend on the initial condition.
2) Any EC $\tilde{s}(t) \in \sum_{t_0}^{s_b,s_f}$ is monotonically increasing (or can be brought to such a form if it stretches for more than a cycle).
3) Provided $s' \neq s''$, ECs $\tilde{s}_{t_0,s'}(t)$ and $\tilde{s}_{t_0,s''}(t)$ never intersect each other.
4) For any fixed t_0, s', and s'' ($s' < s''$), the quantity

$$\Delta t_{s',s''} = \tilde{s}^{-1}_{t_0 s'}(s) - \tilde{s}^{-1}_{t_0 s''}(s)$$

(where symbol $^{-1}$ denotes the inversion operator of a function) does not depend on s: $\Delta t_{s',s''} = \text{const}\ \forall s$.
5) For any fixed $s*$, the value of the derivative

$$\frac{d\tilde{s}}{dt}\Big|_{t:\tilde{s}\ (t)=s*}$$

is constant for all $\tilde{s}\ (t) \in \sum_{t_0}^{s_b,s_f}$ (does not depend on the specific function selected $\tilde{s}(t)$).

The last two properties are very important practically as one can see later.

Every one of these five properties has a distinct and crisp physical sense.

The first one follows from (2.1.1) and the invariability of the motion law. The second property follows from the monotonicity of function $s(t)$.

The third one means that, if two objects are moving along the same trajectory, the driven one never is going to overtake the leading one which follows from the given motion character.

The fourth property follows from the first one. By that,

$$\Delta t_{s',s''} = \tau(s', s'').$$

On account of this property, any EC of a given family can be considered as a productive (generating) one, so far as any other EC can be obtained by moving this one along the t axis up to absorbing the corresponding productive point (initial condition). The existence of the inverse function for $\tilde{s}_{t_0,s}(t)$ follows from its monotonicity.

The fifth property is a consequence of the independence of the quantity

$$d\tau = \tau(s*, s*+ds)$$

on the initial condition, both $s*$ and ds being fixed.

It is comprehensible that SP M_{t_0} is represented in the ts plane by a segment of a straight line with its ends' coordinates $<t_0, s_b>$ and $<t_0, s_f>$. And SP M_{s_f} is represented by a segment with its ends' coordinates $<t_0, s_f>$ and $<t_0 + \tau(s_b, s_f), s_f>$, respectively (Fig. 3-2-1).

Any arc $\bar{s}(t)$ of a continuous monotonous curve (linear or nonlinear) in the ts plane connecting any point of EC $\tilde{s}_b \equiv \tilde{s}_{t_0,s_b}(t)$ with any point of EC $\tilde{s}_f \equiv \tilde{s}_{t_0,s_f}(t)$ represents some SP equivalent to M_{s_f} and M_{t_0}. This SP $\bar{s}(t)$ is complete and non-redundant because $\bar{s}(t) \dot{\sim} M_{s_f} \dot{\sim} M_{t_0}$. This follows from property 3 of the EC family which allows to easily establish one-to-one correspondence (with the help of ECs) between these sets.

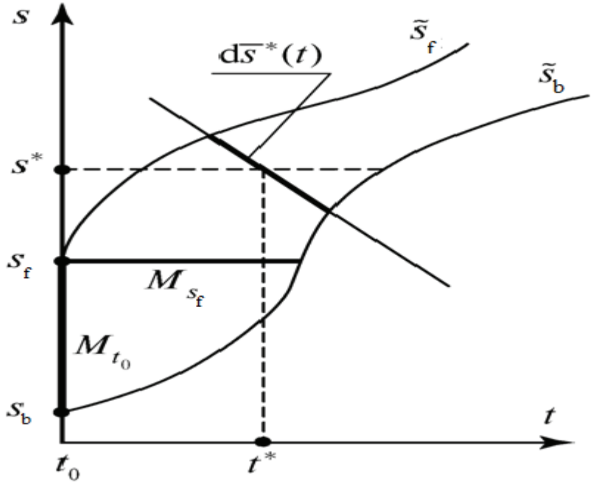

Fig. 3-2-1. Representation of search plans in the *ts* plane

Generally speaking, on the basis of this property and regarding the sense of EC, the more general assertion is true: any set M of points in the *ts* plane which intersects all the EC contained between $\tilde{s}_b$ and $\tilde{s}_f$ represents a complete (perhaps redundant) SP. For its non-redundancy, it is necessary and sufficient that M intersect every EC (between $\tilde{s}_b$ and $\tilde{s}_f$) only at one point. And inversely, any complete and non-redundant SP can be represented in the *ts* plane by a set of points between ECs $\tilde{s}_b$ and $\tilde{s}_f$ (intersecting every EC between $\tilde{s}_b$ and $\tilde{s}_f$ just at one and only one point). So, an important *Theorem* is worded. And, according to it, it is of no sense to construct SPs out of this region of the *ts* plane (a "corridor" between $\tilde{s}_b$ and $\tilde{s}_f$).

These latter properties are important, first of all, from the theoretical point of view. In practice, such generality is superfluous since, as a rule, physically realizable (feasible) SPs used to be continuous and piecewise continuous. Later on, the consideration will be confined with namely such SPs, they being complete and non-redundant. If one does need the inverse, the latter should be specially stipulated.

3.3. The case of search for an object by one censor within a cycle

As it will be posed in this section, it is in terms of the equivalence curves tool that the necessary conditions for the optimality of search plans can be formulated being natural and convenient and, at the same time, mathematically strict. As was shown above, the equivalence principle with its apparatus of curves helps to understand in which region of the *ts* plane and what kind of SP (complete and non-redundant) is reasonable to construct. Any departure from these conditions inevitably involves an increase of the risk of not performing the task or an uneconomical way of action. From the other hand, keeping these conditions reserves the possibility of further increase of search efficiency.

At first, we consider the case when at each moment of time the sensor can check only one point of the trajectory and the law of checking (that is, SP) is given in time as a continuous monotonous differentiable function $\bar{s}(t)$. The practice usually puts the restrictions upon this function, for example, like this:

$$\left|\bar{s}'_t\right| \le c_0,\ c_0 > 0 \tag{3.3.1}$$

(sometimes $\left|\varphi'_t[\bar{s}(t)]\right| \le c_0$) which means limiting the speed of the controlled point movement along the sought-for object trajectory (in terms of real sensors – limiting the re-aiming speed of the sensor).

By definition, SP M is optimum (OSP) if the projection of the curve segment $\bar{s}(t)$ confined between $\tilde{s}_b$ and $\tilde{s}_f$ (in particular, the straight-line segment) upon the t axis, $\bar{s}(t)$ representing M in the *ts* plane, has a minimum length. That means that the search is performed for a guaranteed minimum time.

In view of the apparent uneconomical nature of moving the check point of the trajectory along the direction of the object motion and, at the same time, if there is a possibility of moving it in the opposite direction, it is reasonable to seek the OSP among monotonously decreasing functions. With due regard of this note, the condition (3.3.1) will take the more particular form:

$$-c_0 \le \bar{s}'_t \le 0. \tag{3.3.1′}$$

And now we are ready to formulate and prove some lemmas and a theorem.

Lemma 1. Any solution $\bar{s}(t)$ meeting the restriction (3.3.1′) lies in the *ts* plane under the straight line having the inclination $-c_0$ and coming through the point <$t_{\bar{b}}$,$s_{\bar{b}}$> (which is the meeting of $\bar{s}(t)$ with EC $\tilde{s}_{\mathrm{b}}$) if it does not coincide with its segment.

The proof is obvious. The equation of the straight line above (in the ***Lemma 1*** formulation) is as follows:

$$s^L(t) = s_{\bar{b}} - c_0(t - t_{\bar{b}}) \qquad (3.3.2)$$

and its segment $\bar{s}^L(t)$ confined between ECs $\tilde{s}_b$ and $\tilde{s}_f$ is a complete and non-redundant SP.

Lemma 2. Any solution $\bar{s}(t)$ meeting the restriction (3.3.1′) is not better (in the sense of the adopted earlier optimality criterion) than the linear search plan $\bar{s}^L(t)$ of the form (3.3.2).

Proof. By Lemma 1 and in view of monotonicity of $\tilde{s}_f$ (the EC's property 2 in section 3.2), point <$t_{\bar{f}}$, $s_{\bar{f}}$> of the meeting of $\bar{s}(t)$ with EC $\tilde{s}_f$ is tied with point <$t^L_{\bar{f}}$,$s^L_{\bar{f}}$ > of the meeting of $\bar{s}^L(t)$ with EC $\tilde{s}_f$ by the next relationships:

$$s_{\bar{f}} \leq s^L_{\bar{f}},\ t_{\bar{f}} \leq t^L_{\bar{f}}. \qquad (3.3.3)$$

The projection of SP $\bar{s}^L(t)$ upon the *t* axis has the length $t_{\bar{b}}$ - $t^L_{\bar{f}}$, and the length of the projection of $\bar{s}(t)$ is $t_{\bar{b}}$ - $t_{\bar{f}}$ which in view of (3.3.3) cannot be less than the former.

Consequence of Lemma 2. For obtaining the OSP, it is enough to consider only linear SPs having the inclination $-c_0$.

Later on, the consideration temporarily will be limited by the elementary search interval [s_b, $s_b + ds$]. Let, thus, $\tilde{s}_b$ be equal to $\tilde{s}_{t_0,s_b}(t)$ and $\tilde{s}_f = \tilde{s}_{t_0,s_b+\mathrm{ds}}(t)$ (to be sure, $s_f = s_b + ds$). For the elementary search interval, the next Lemma will be proved.

Lemma 3. Among all linear SPs meeting the restriction (3.3.1′), the best one comes through the point <t_{opt} , s_m> where s_m is a point in the trajectory at which the derivative $\tilde{s}'(t)$ (as well as $s'(t)$) is minimum and

$$t_{opt} = \frac{\tilde{s}_b^{-1}[s_m]+\tilde{s}_f^{-1}[s_m]}{2}.$$

Proof. Let us consider the quantity $\tilde{s}_t{}'[s^*] \equiv \frac{d\tilde{s}}{dt}\Big|_{t:\tilde{s}(t)=s*}$ where $\tilde{s}(t)$ is an arbitrary EC from the family $\sum_{t_0}^{s_b,s_b+ds}$ and $s* \in [0, S]$. On account of property 5 of the ECs family, the value of this quantity does not depend on any concrete $\tilde{s}(t)$ and is determined only by the value of $s*$. And now let us tie with the quantity $s*$ the linear SP $d\bar{s}*(t)$ (Fig. 3-2-1) having the inclination $-c_0$ and coming through the point $<t^*,s^*>$ where

$$t^* = \frac{\tilde{s}_b^{-1}[s*]+\tilde{s}_f^{-1}[s*]}{2}. \qquad (3.3.4)$$

Thus, $d\bar{s}*(t)$ is a segment of a straight line

$$s*(t) = s* - c_0\,(t - t*)$$

confined between ECs $\tilde{s}_b$ and $\tilde{s}_f$. To the elementary interval $[s_b, s_b + ds]$, there corresponds the elementary increment dt_{bf} of the t argument. Here, dt_{bf} is (with accuracy up to the sign) a distance along the t axis between ECs $\tilde{s}_b$ and $\tilde{s}_f$ which, by the property 4 of the ECs family, is constant for all s:

$$dt_{bf} = \tilde{s}_b^{-1}[s] - \tilde{s}_f^{-1}[s] = \tilde{s}_b^{-1}[s*] - \tilde{s}_f^{-1}[s*]\,.$$

Having replaced in a small vicinity of points $t = \tilde{s}_{\text{к}}^{-1}[s*]$ and $t = \tilde{s}_{\text{н}}^{-1}[s*]$ (in which the derivatives of both ECs are equal to the same value $\tilde{s}_t'[s*]$) the arcs of curves $\tilde{s}_b$ and $\tilde{s}_f$ by segments of the tangential lines, one obtains the system

$$dt_{bf} = 2\,(x + y),$$

$$\tilde{s}_t{}'[s^*]\cdot y = c_0 x$$

where x and y are auxiliary variables the sense of which is clear from Fig. 3-3-1. Hence, the length of the elementary SP's projection into the t axis is equal to

$$d\Pi \equiv \Pi[\,d\bar{s}*(t)] = 2x = \frac{dt_{bf}}{1+\frac{c_0}{\tilde{s}_t{}'[s*]}}\,.$$

As the numerator of the right part does not depend on the value of $s*$, then the minimum of $d\Pi$ coincides with the minimum of the derivative $\tilde{s}_t{}'[s*]$. Thus, $s_{opt} = s_m$. By substitution of $s* = s_m$ in (3.3.4), we obtain $t^*=$

$= t_{opt}$. So, Lemma 3 has been proved. And by this, simultaneously, for the elementary search interval, the next very important Theorem has been proved.

Theorem. The optimum SP $\bar{s}_{opt}(t)$ meeting the restriction (3.3.1) is the segment $\bar{s}^{L}_{opt}(t)$ of the straight line $s = s_m - c_0\,(t - t_{opt})$ confined between ECs $\tilde{s}_b$ and $\tilde{s}_f$ where s_m is the point of the trajectory at which the derivative $\frac{ds(t)}{dt}$ is minimum and

$$t_{opt} = \frac{\tilde{s}_b^{-1}[s_m]+\tilde{s}_f^{-1}[s_m]}{2}.$$

On the basis of the proved theorem for the general motion law $s(t)$, one can deduce the next practical recommendation for constructing continuous and discrete SPs. ***The SP or its discrete elements are reasonable to be placed in the vicinity of local minimums of the derivative $\tilde{s}_t'$.***

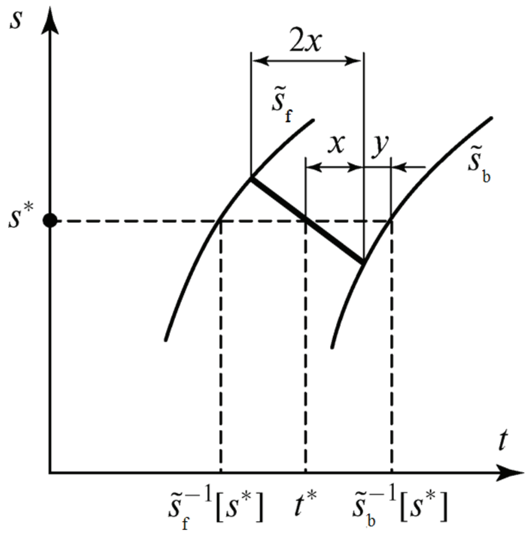

Fig. 3-3-1. Representation of auxiliary parameters x and y

3.4. On an important specific form of the motion law

In the most general case (for an arbitrary form of function $s(t)$), this theorem can be reasonably used for small search intervals (within which variations of the derivative $\frac{ds(t)}{dt}$ are rather limited). Besides that, this theorem is absolutely true for an arbitrary search interval if function $s(t)$ is characterized by the next property:

$$s'_t\,[s_m - \Delta s] = s'_t\,[s_m + \Delta s], \qquad (3.4.1)$$

that is, a velocity of the object's motion along the trajectory is distributed symmetrically around the point s_m.

Let us prove this for the case that the object's velocity monotonously increases when the object moves off the point s_m towards the point with the maximum velocity, and vice versa. Such a situation is notable for orbits of satellites of celestial bodies and generally for passive motion of one material body in the gravitational field of another.

For an arbitrary search interval, the optimum index of SP can be expressed like

$$\Pi[\bar{s}^L(t)] = \int_{s_f^L}^{s_b^L} d\Pi = \int_{s**+\Delta s_2}^{s**-\Delta s_1} \frac{1}{1+\frac{c_0}{\tilde{s}'_t\,[s^*]}} dt* = I(s**)$$

where $<t^*, s^*>$ is a point of reference of the elementary interval, $<t^{**}, s^{**}>$ is a point of reference of the sum interval, and

$$t^{**} = \frac{\tilde{s}_b^{-1}[s**]+\tilde{s}_f^{-1}[s**]}{2},$$

Δs_1 and Δs_2 depending on $s**$.

Let us transfer under the integral sign completely to the variable s^*. As points $<t^*, s^*>$ and $<t^{**}, s^{**}>$ lie in the straight line

$$s^*(t^*) = s^{**} - c_0(t^* - t^{**}),$$

then

$$dt^* = -\frac{1}{c_0} ds^*$$

and

$$I(s^{**}) = -\frac{1}{c_0}\int_{s**+\Delta s_2}^{s**-\Delta s_1} \frac{1}{1+\frac{c_0}{\tilde{s}'_t\,[s^*]}}\, ds^* = \frac{1}{c_0}\int_{s**-\Delta s_1}^{s**+\Delta s_2} \frac{1}{1+\frac{c_0}{\tilde{s}'_t\,[s^*]}}\, ds^*. \qquad (3.4.2)$$

Let us compare the values of the integral (3.4.2) for $s^{**} = s_m$ and $s^{**} \neq s_m$. On account of symmetricity of variation of the derivative $\tilde{s}_t{}'[s^*]$ due to deflection of s^* from s_m in both directions (Fig. 3-4-1), the integration interval for $s^{**} = s_m$ is symmetric as well, and, with due regard to the monotonic increase of the derivative as one is moving away from point s_m (within a cycle), declination of s^{**}, say, to the left from s_m, implies the faster decrease of the left integration limit than the right one. The function under the integral (as well as the derivative $\tilde{s}_t{}'[s^*]$) has the only minimum at point s_m (within the interval $[s_m – S/2,\ s_m + S/2]$). As argument s^* moves away from point s_m, both functions monotonically increase and are symmetric with respect to the vertical $s^* = s_m$ (Fig. 3-4-1).

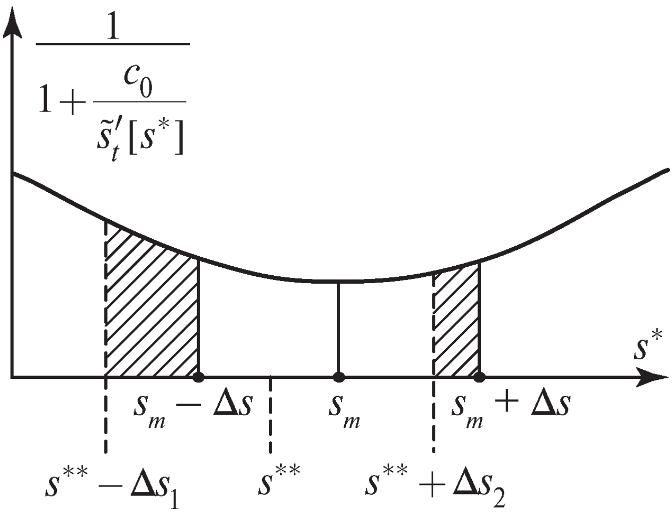

Fig. 3-4-1. The form of the function under the integral (3.4.2)

Let, for determinacy sake, the point s^{**} be shifted to the left from s_m. Then, on account of the above properties of the derivative $\tilde{s}_t{}'[s^*]$ and the function under the integral, the following inequalities take place:

$$(s_m - \Delta s) – (s^{**} - \Delta s_1) > (s_m + \Delta s) – (s^{**} + \Delta s_2),$$

$$P[(s^{**} - \Delta s_1);\ (s_m - \Delta s)] > P[(s^{**} + \Delta s_2);\ (s_m + \Delta s)]$$

where $P[\ldots]$ is the area under the curve over the corresponding interval (in the square brackets) – see Fig. 3-4-1. Thus, $I(s^{**}) > I(s_m)$. So, the OSP is linear and symmetric with respect to the point $<t_{opt}, s_m>$.

So, the Theorem has been proved also for the more general case. While performing the proof, no restrictions were imposed upon the process of checking the trajectory, except the inequalities (3.3.1).

However, generally speaking, in reality a check is not possible at any time. Besides, for real sensors and real time periods, at each time moment only a limited arc of the trajectory is accessible for a check. The first condition is represented in the *ts* plane by vertical strips *Q* over the time intervals within which a check of the trajectory is possible. The second one can be represented by more arbitrary (on their configuration) domains *R* (see Fig. 3-4-2).

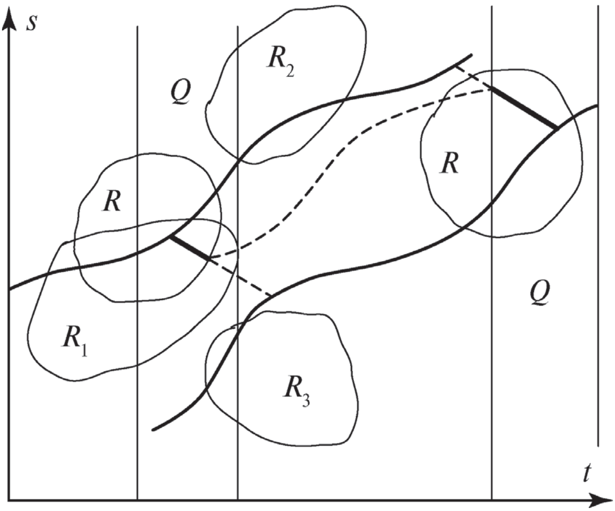

Fig. 3-4-2. Representation, in the plane *ts*, of the possibility conditions for search

The meet Ω of strips Q and domains R ($\Omega = Q \cap R$) represents favorable conditions for checking the trajectory points in the sense that only the part of SP that comes into domain Ω can be productively realized.

If in a given cycle only some part of SP was realized, then in the next one (constructed for the next object revolution or the at next favorable conditions for observations), the mapping of this part to the corresponding new SP should be excluded (see Fig. 3-4-2).

One more practical device can be recommended: sometimes it may be reasonable to embed SP into domain Ω at the cost of some deviation from the optimum SP, and thus to solve the problem during the closest cycle (or favorable conditions for observations), using a suboptimum search plan.

So far, an SP $\bar{s}(t)$ was assumed to be a single-valued and continuous function. However, instruments exist (practically all real facilities) that check at once some arc on the trajectory of size Δs at the discrete moments of time over the time step Δt. In the ts plane, the SP for such an instrument represents a "ladder" connecting ECs $\tilde{s}_b$ and $\tilde{s}_f$ with a constant width and a constant height of its steps Δs and Δt accordingly. The transfer between its steps is carried out along ECs (see Fig. 3-4-3).

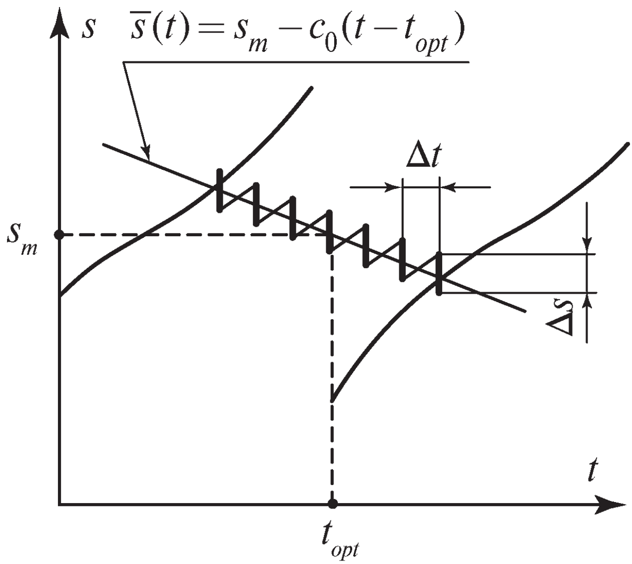

Fig. 3-4-3. Piecewise continuous search plan of the "ladder" type

The OSP is constructed beginning with the middle step having its center at point $<t_{opt}, s_m>$. And then the OSP is to be continued in both directions with the aim of reaching (meeting) the ECs $\tilde{s}_b$ and $\tilde{s}_f$. The inclination of the ladder's directrix (which is accounted for by the limitation on the sensor's re-aiming speed) is determined by the value of parameter c_0 (see the restriction (3.3.1) or (3.3.1′)).

Such a transfer to the piecewise continuous form of function $\bar{s}(t)$ allows further improvement of the SP in the sense that the sum length of its components' projections to the t axis decreases.

Let $\bar{s}_1$ and $\bar{s}_2$ be equal (by their lengths) to sub-segments of the continuous OSP into which it is decomposed by the vertical straight-line $s(t) = s_m$ ($\forall$t). After application of the Theorem of optimality to the segments $\bar{s}_1$ and $\bar{s}_2$ independently, we will improve each of them and obtain the better plans $\bar{s}_{1opt}$ and $\bar{s}_{2opt}$, respectively (see Fig. 3-4-4).

One can continue this process of improvement (in the above economic sense) of the composite SP by means of its further fragmentation (decomposition). By this means, in this process, the controlled region is being narrowed. So that, at some stage, the necessity of re-targeting the search instrument falls away. And in case of a photographic instrument, one can achieve by this a certain economy of photographic materials. For other types of instruments, economy of their search resource is achieved.

At this stage, we came to a search strategy that will be called later "the search on the border (frontier)". Further (see sections 4.4, 4.8 and Figs. 4-4-1, 4-8-2, 4-8-5), some applied advantages (merits) of this strategy (that can be suitable when performing some search tasks with the help of specific observation instruments) will be discussed.

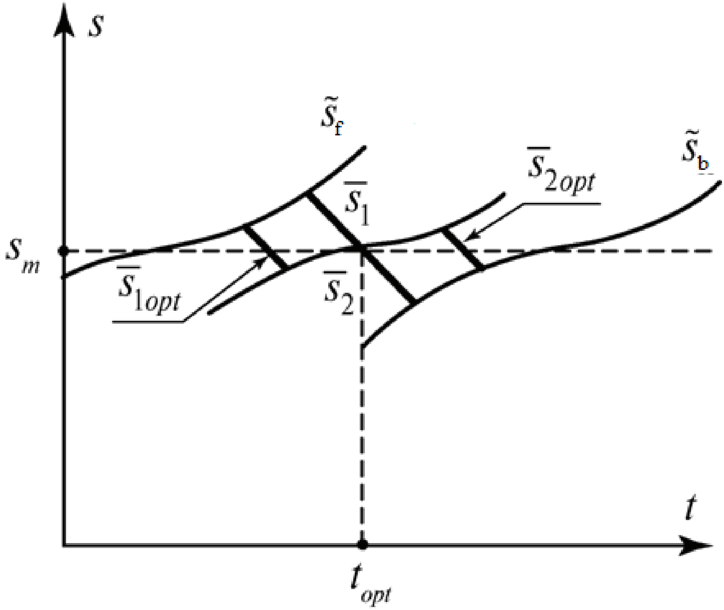

Fig. 3-4-4. The way of further improvement of the OSP by means of its fragmentation

Now, some words on the optimum criteria kinds should be said.

In ideal conditions, that is, if there is a possibility to stop the search process just at once after hitting the target into the FoV of the sensor, the length of the whole SP's projection to the *t* axis is not a satisfactory index of its quality, because practically in almost every case it is enough to realize only a part of the SP for the detection of the sought-for object and to stop after that. Although, given the capabilities of modern sensors and their computing systems, these "ideal" conditions become quite real.

A more adequate criterion for this case would be the mathematical expectation of the search duration. It will be considered in detail in section 3.7. It is easy to see that SPs minimizing this criterion (of no dependence on the object's position distribution density over the search interval) are located between the two straight line segments – OSP in the former sense (segment [*a, b*] in Fig. 3-4-5) and the parallel one between ECs $\tilde{s}_b$ and $\tilde{s}_f$ coming through the point of meeting of $\tilde{s}_f$ with the horizontal line $s(t) = s_m$ (the segment [*d, e*] in Fig. 3-4-5).

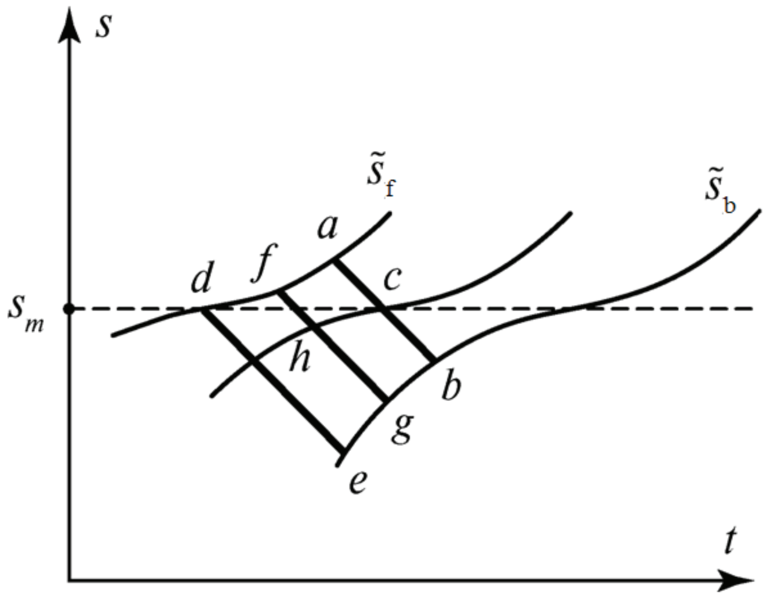

Fig. 3-4-5. Dependence of the OSP position in the plane *ts* on the choice of the optimum criterion

Optimum SPs for symmetric (with respect to the middle of the search interval) distributions of the initial object's position are close to the straight-line segment [*f, g*] equidistant from SPs [*a, b*] and [*d, e*] (see Fig. 3-4-5).

As for the former criterion, then, in the ideal conditions above, it serves as a guarantee index, reckoning the most unfavorable case. At the same time, it is the ideal criterion when we are forced to realize the SP completely (as, for example, in case of using photographic instruments, as well as in case where it is impossible to eliminate the chance of detecting some strange, outside objects along with the sought-for one or when we are not sure that we have found the object we needed).

Anyway, the question of choosing a criterion is always very important. And in terms of the equivalence curves tool, the researcher has the opportunity to make this choice professionally.

3.5. Optimum planning of search for an object with the help of a censor group

Often, there is a task of searching for one cycling object when it is possible to use several search instruments for this. Let one have $\bar{J}$ search instruments with the help of which one should detect the object, $\bar{I}$ cycles of the object motion being allotted for this task. To begin with, for each j-th instrument in every i-th cycle, we construct the OSP or some sub-optimum one taking account of favorable conditions for checking the object's trajectory. Then we project each SP along ECs to the basic (initial) search interval $[s_b, s_f]$ given at time t_0 or to some more conveniently placed interval equivalent to the basic one. Thus, the obtained EC-projections, by definition, are called the capability intervals for checking the trajectory. They will be encoded (designated) with the help of a pair of indices (j/i) according to the proper instrument and cycle, respectively, to be used (see Fig. 3-5-1).

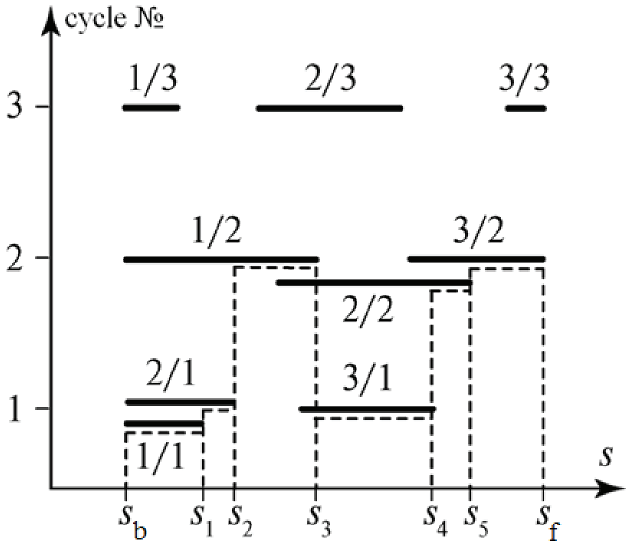

Fig. 3-5-1. The capability intervals as the initial data for planning the search

By that, the problem of the search for an object with the help of $\bar{J}$ instruments during $\bar{I}$ object motion cycles can be reduced to the problem of decomposition of the search interval to sub-intervals, each of them being embedded into some capability interval [39].

Let us introduce some useful notions and notations:

j – a number of the instrument used, $j = 1, 2, \ldots, \bar{j}$;
i – an ordinal number of the object motion cycle, $i = 1, 2, \ldots, \bar{\imath}$;
δ_j – the cost of hiring the j-th instrument;
μ_i – the penalty for work in the i-th cycle;
ω_i – the bonus for work in the i-th cycle;
γ_{ji} – the cost of work of the j-th instrument in the i-th cycle;
$z = ||z_{ij}||$ - a binary matrix in which $z_{ij} = 1$ if and only if the j-th instrument works in the i-th cycle and $z_{ij} = 0$ in the opposite case;
z_j – the j-th component of a column-vector $\bigvee_i z^{(i)}$ that equals a logical sum of all columns of z;
z^i – the i-th component of a row-vector $\bigvee_j z_{(j)}$ that equals a logical sum of all rows of z.

Depending on the concrete conditions of the problem, preferable as a criterion there can be the quickest detection of the object ("at any price"), its detection by the most economic means, or both the former and the latter in different proportions. According to this, all criteria of optimally solving the problem can be divided into three categories.

As a formal criterion in case of preference for quickest detection of the object (with the discreteness being equal to a cycle), the next function can be used:

$$\Phi = \max_i \mu_i z^i$$

or

$$\Phi = \sum_{i=1}^{\bar{i}} \mu_i z^i, \ \mu_1 < \mu_2 < \ldots < \mu_{\bar{i}}, \tag{3.5.1}$$

or, if the object position distribution density $f(s)$ is available over the trajectory at time t_0, then

$$\Phi = \sum_m \int_{I_m} f(s)\omega_{i_m} ds, \ \omega_1 > \omega_2 > \ldots > \omega_{\bar{i}} \tag{3.5.2}$$

where I_m is an element of decomposition of the search interval and i_m is the number of the cycle in which this interval is controlled.

In case of preference for the most economic detection of the object, as an optimum criterion can be used the function

$$\Phi = \sum_{j} \delta_j z_j + \sum_{j,i} \gamma_{ji} z_{ji} \tag{3.5.3}$$

which should be turned into a minimum, and in the compound (hybrid) case – the function

$$\Phi = \sum_{j} \delta_j z_j + \max_{i} \mu_i z^i + \sum_{j,i} \gamma_{ji} z_{ji}, \tag{3.5.4}$$

or

$$\Phi = \sum_{j} \delta_j z_j + \sum_{i+1}^{\bar{i}} \mu_i z^i + \sum_{j,i} \gamma_{ji} z_{ji}. \tag{3.5.5}$$

In both criteria (3.5.4) and (3.5.5), the first term represents the cost of hiring all the instruments used for solving the problem, the second addend is a penalty for the duration of the search, and the third addend is the cost of the current operation of the instruments according to the search plan.

The function (3.5.2) can be rewritten in the form of

$$\Phi = \sum_{m} \omega_{i_m} \int_{I_m} f(s) ds = \sum_{m} \omega_{i_m} p_m \tag{3.5.6}$$

where p_m is the probability of finding the sought-for object at the interval I_m. This function is the mathematical expectation of the bonus that should be maximized. So, the sense of criterion (3.5.6) is completely determined by the ideology of granting bonuses.

For complete strictness of this statement, it is necessary that the probability of the object being out of the search interval be equal to zero, or in (3.5.6), the integral over the search interval complement with the weight $\omega_0 = 0$ should be conditionally included as an addend.

Turning the function (3.5.6) into the maximum is of no difficulty. For this, all capabilities of controlling the trajectory in the first cycle (only except duplicating) should be used. Then, at the complement of the checked part of the search interval, all capabilities of the second cycle should be realized, and so on.

The problem of minimizing functions (3.5.4) and (3.5.5) is not this simple. We will solve it under the next assumption: the end (right one) of the element of decomposition coincides with the end of the capability interval (according to the SP) which contains the former. In case of criteria (3.5.4) and (3.5.5), this assumption is not so restrictive and at the same time it gives additional useful simplification: only the ends of capability intervals can serve as the points of decomposition of the search interval.

One can see that this problem is isomorphic to the classic discrete optimization problem (DOP) of decomposition of a given segment $I = [a, b]$ to sub-segments I_n, each of them being embedded into some interval λ_q from a given interval system $\Lambda = \{\lambda_q\}$ covering the segment I.

Constructively, this problem can be conveniently solved with the help of the "branch-and-bound" (BB) approach [45, 46, 47, 48, 49, 55]. Namely in such methods based upon this approach it is easy to take into account the requirements of belonging of each sub-segment I_n to some interval $\lambda_q \in \Lambda$, whereas in the frame of other methods the formalization of such requirement meets obstacles. Besides, if the structure of the decision tree is chosen so as at the n-th tier of ramification the n-th decomposition element I_n is fixed, then for acceleration of the calculation process one can efficiently use the additional cutting rule (ACR) for cutting perspectiveless branches of the decision tree [48, 49, 55].

So, everything is prepared for the transition to constructing the decision tree which we are now realizing as follows. The first tier of the tree contains vertices representing the capability intervals covering the point s_b. The latter intervals as well as the corresponding vertices are designated by two indices (j/i). A vertex of the first tier designated (j_{k_1}/i_{k_1}) represents a set of solutions having the interval (j_{k_1}/i_{k_1}) as the first (left) element of decomposition. The second tier of the tree contains vertices $(j_{k_1k_2}/i_{k_1k_2})$ ramifying from each vertex (j_{k_1}/i_{k_1}) of the first tier and representing the capability intervals covering the end (right) of the interval (j_{k_1}/i_{k_1}), the second element of decomposition being the interval $(j_{k_1k_2}/i_{k_1k_2})\setminus(j_{k_1}/i_{k_1})$.

A vertex of the n-th tier $(j_{k_1k_2,\dots k_n}/i_{k_1k_2,\dots k_n})$ represents a set of solutions having the fixed composition of elements of the search interval decomposition from the left end (beginning) to the right end of the interval $(j_{k_1k_2,\dots k_n}/i_{k_1k_2,\dots k_n})$, the elements being determined by the vertices' codes lying on the branch of the tree leading from the root of the tree (that represents all possible solutions) to a given vertex of the *n*-th tier (see Fig. 3-5-2):

$(j_{k_1}/\, i_{k_1})$; $(j_{k_1 k_2}/i_{k_1 k_2})\backslash(j_{k_1}/\, i_{k_1})'$;… $(j_{k_1 k_2 \ldots k_n}/\, i_{k_1 k_2 \ldots k_n})\backslash\,(j_{k_1 k_2 \ldots k_{n-1}}/\, i_{k_1 k_2 \ldots k_{n-1}})'$.

Here, the stroke means enlarging the subtracted interval to the left up to superposition of its left end with the point s_b.

The estimate from below ("not less than" = here "not better than") $\widehat{\Phi}$ of the function Φ for the set of solutions represented by some vertex of the tree $(j_{k_1 k_2,\ldots k_n}/i_{k_1 k_2,\ldots k_n})$ is calculated as follows:

$$\widehat{\Phi}\,[\eta(s^0)] = \widehat{\Phi}_1 + \widehat{\Phi}_2;$$

where $\widehat{\Phi}_1$ is an estimate of the part of Φ at the definite part of solution;

$\widehat{\Phi}_2$ is an estimate of the part of Φ at the indefinite part of solution;

$\eta(s^0)$ is a set of already assigned elements of decomposition the last (right) of which terminates in point s^0 (the end of interval $(j_{k_1 k_2,\ldots k_n}/ i_{k_1 k_2,\ldots k_n})$).

Now let us show a way (one of many possible) how to calculate the estimates $\widehat{\Phi}_1$ and $\widehat{\Phi}_2$.

$$\hat{\Phi}_1 = \sum_{j \in J^a} \delta_j z_j + \sum_{i \in I^a} \mu_i z^i + \sum_{\substack{j \in J^a \\ i \in I^a}} \gamma_{ji} z_{ji}; \qquad (3.5.7)$$

where J^a – the set of already assigned instruments;

I^a – the set of already planned cycles.

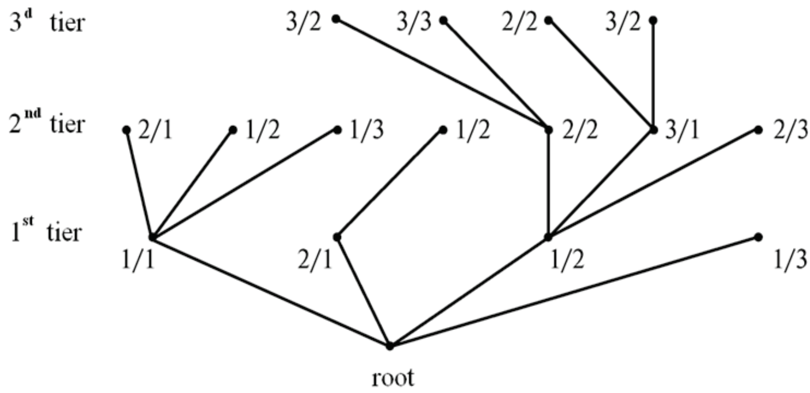

Fig. 3-5-2. The decision tree

$$\hat{\Phi}_2 = \sum_{\nu=1}^{\overline{\nu}^{\delta}} \tilde{\delta}^{\nu} + \sum_{\nu=1}^{\overline{\nu}^{\mu}} \tilde{\mu}^{\nu} + \sum_{\nu=1}^{\overline{\nu}} \gamma^{\nu}; \qquad (3.5.8)$$

$\bar{\nu}$ being an estimate from below of the minimum number of not yet assigned elements of decomposition of the remaining part of the search interval (as, for example, the amount of the longest capability intervals covering the remaining part by one layer);

$$\overline{\nu}^{\delta} = \min \overline{\chi} : \sum_{\chi=1}^{\overline{\chi}} k[j_{\chi}] \geq \overline{\nu};$$

$k\,[j_1]$, $k\,[j_2]$,… - a diminishing sequence of quantities $k\,[j]$;

$k\,[j]$ is a multiplicity of the repetitions of index j of the capability intervals terminating to the right of point s^0;

$$\overline{\nu}^{\mu} = \min \overline{\psi} : \sum_{\psi=1}^{\overline{\psi}} k[j_{\psi}] \geq \overline{\nu};$$

$k\,[i_1]$, $k\,[i_2]$,… - a diminishing sequence of quantities $k\,[i]$;

$k[i]$ is a multiplicity of the repetitions of index i of the capability intervals terminating to the right of point s^0;

$\{\gamma^{\nu}\}$ – a sequence of quantities γ_{ji} arranged in order by their growth having indices *i, j* of capability intervals terminating to the right of point s^0;

$\{\tilde{\delta}^{\nu}\}$ – a growing sequence of quantities $\tilde{\delta}_j$ having indices *j* of capability intervals terminating to the right of point s^0;

$\{\tilde{\mu}^{\nu}\}$ – a growing sequence of quantities $\tilde{\mu}_i$ having indices *i* of capability intervals terminating to the right of point s^0;

$$\tilde{\delta}_j = \begin{cases} 0 \text{ if } j \in J^a, \\ \delta_j \text{ in the opposite case,} \end{cases} \qquad \tilde{\mu}_i = \begin{cases} 0 \text{ if } i \in I^a, \\ \mu_i \text{ in the opposite case.} \end{cases}$$

For application of the cutting rule by the estimate $\widehat{\Phi}$ (using its calculated value), it is necessary to have some kind of a concrete solution (named "record"). To obtain it, we consider an approximate procedure of solving the problem. Let us arrange the capability intervals by layers (see Fig. 3-5-1).

In case the penalty for work in the *i*-th cycle sharply increases with growth of index *i* and the cost of hiring the instruments is relatively small, then the intervals connected with the first cycle are placed in the bottom layer. In the second (next higher) layer, the intervals connected with the second cycle are placed and so on. The simplest approximate solution procedure consists of the following. The content of decomposition is built up from left to right, flattening against the *s* axis as far as possible (that is, on finishing the next interval one transits to the lowest possible one). This solution is remembered and used as a "record" (a dedicated term in the BB approach) for cutting the perspectiveless branches of the decision tree.

One can easily see that this procedure can be improved with the help of additional verification and a stage of correction after the regular transition. The verification consists of revealing the fact (or answering the question): does the next capability interval (into which the transition was made) cover any already assigned elements of the already performed decomposition? If yes, then the correction consists of assimilating such elements by the current capability interval.

The process of getting the precise solution (and the BB approach gives namely the precise one) consists of developing (ramification of) the branches of the tree everywhere where it is possible and cutting all perspectiveless branches with the help of the just formulated rule. At the same time, along with the latter, for this structure of the tree, with the purpose of further cutting down the sorting out of possible solutions, the additional cutting rule [48] can be used. This perfection of the BB approach consists of the following: if two vertices have the same code, then further branching of the vertex with the greater (worse) value of the estimate $\widehat{\Phi}_1$ is reasonable to be stopped.

Indeed, let us suppose for the first vertex $\widehat{\Phi}_1 = \widehat{\Phi}_1^1$ and for the second one $\widehat{\Phi}_1 = \widehat{\Phi}_1^2, \widehat{\Phi}_1^1$ being less than $\widehat{\Phi}_1^2$. In view of the independence of prolongation of the decomposition process on the already constructed part of the whole solution at this tier of the tree, each solution of the first set (represented by the first vertex) corresponds to some solution from the second set (represented by the second vertex) having the same continuation and vice versa.

So, for the first of the mutually corresponding solutions

$$\Phi = \widehat{\Phi}_1^1 + \Phi_2$$

and for the second

$$\Phi = \widehat{\Phi}_1^2 + \Phi_2.$$

Hence, it is clear that the first solution is no worse than the second one. Consequently, the set of solutions represented by the second vertex can be painlessly removed from further consideration.

The process of seeking the precise solution will be finished when no perspective vertices remain (to say nothing of perspectiveless ones).

The method expounded here allows easily taking into account already checked parts of the basic search interval $[s_b, s_f]$.

If it is clear that the sought-for object is absent from the sub-interval $[s^*, s^{**}] \subset [s_b, s_f]$, then the problem ought to be solved for the initial search interval $[s_b, s^*]$ and then, having removed the used capability intervals, one should solve the problem for the initial interval $[s^{**}, s_f]$.

In the next section, a more complicated problem will be solved albeit using the methods already used to solve the previous problem.

3.6. Optimum planning of search for a group of objects with the help of a censor group

In section 3.4, a way for further improvement of the continuous OSP by transfer to a piecewise continuous form of function $\bar{s}(t)$ was shown. There is an opportunity to continue this process of breaking up the SP for each k-th object until the parameter Δs becomes less than Δs_0 where Δs_0 is the size of the FoV of the searching instrument and Δs is the height of the vertical elementary SP equivalent to the optimum elementary SP.

As a result, for each object (within one cycle) a set of time moments $\{t_i\}$ of timing the elementary plans will be obtained, realization of their total sum providing the acquisition of all sought-for objects for a minimum total sum time of exposition (see Fig. 3-6-1).

The set $\{t_i\}$ has the following property:

$$\sum_{\forall i:\mathrm{k}_i=k^*} k_i = k^* m_{k^*}$$

where k_i is an index of the object to which the moment t_i corresponds,

m_k is the number of elements in the piecewise continuous OSP for the k-th object,

and k^* is an index of any concrete fixed object from the set tasked for search.

Each time moment t_i should be embedded into the interval of size Δt which is equal to the duration of a single exposition of the corresponding instrument (see Fig. 3-6-2). The set of the latter intervals (those for realization of the piecewise continuous linear OSPs) together with the data on feasibility of the realization of the intervals with the help of different instruments – all this presents the initial information for compilation of the observation program. The aim is to distribute all the intervals among the instruments capable for their realization, in some optimum way.

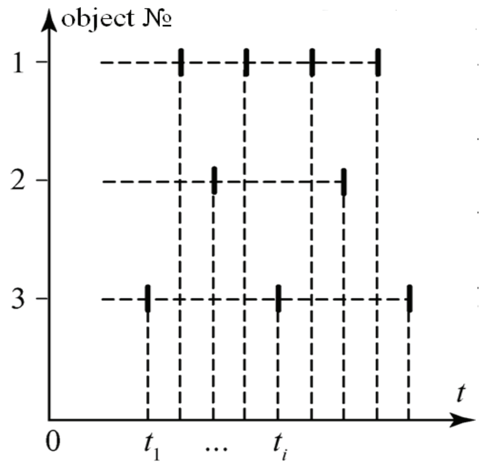

Fig. 3-6-1. Timing of the elementary SPs from the OSP

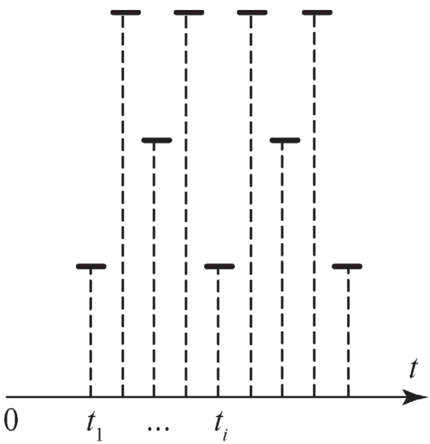

Fig. 3-6-2. Initial data for compilation of the observation program

As an optimality criterion for this task, let us accept a total cost of enlisting the co-operation of instruments:

$$\Phi = \sum_{j \in J} w_j x_j. \qquad (3.6.1)$$

Here, J is a set of indices of instruments,

w_j is the cost of application (operation, hiring, and so on) of the j-th instrument (if necessary, w_j may be worked out in detail);

$$x_j = \begin{cases} 1, \text{ if the j-th instrument is enlisted,} \\ 0 \text{ in the opposite case.} \end{cases}$$

The solution is a matrix $Y = \| y_{ij} \|$,

$$y_{ij} = \begin{cases} 1, \text{ if the } j\text{-th instrument realizes the } i\text{-th interval,} \\ 0 \text{ in the opposite case.} \end{cases}$$

Here, $x_j = x_j(Y)$ is the j-th element of a row-vector $\vee_i Y$ which is a logical sum of all rows of the matrix Y.

The function Φ should be minimized under the conditions:

1) $z_{ij} = 1 \ \forall i,j: y_{ij} = 1$

where

$$z_{ij} = \begin{cases} 1, \text{if } j\text{-th instrument is capable to control } i\text{-th interval,} \\ 0 \text{ in the opposite case;} \end{cases}$$

2) a time distance between adjacent intervals controlled by the j-th instrument should exceed a given quantity Δt^{j}_{adm};

3) all (or selected) intervals should be necessarily controlled as a result.

Without the latter condition the solution would be trivial: $Y = 0$.

The stated task can be brought to the standard problem of servicing $\bar{k}$ subscribers by $\bar{r}$ systems in the next formalization:

$$\sum_{j \in J} w_j x_j (Y) \Rightarrow \min, \qquad (3.6.2)$$

$$\sum\nolimits_{\forall j, \forall i: k_i = k^*} z_{ij} y_{ij} = m_{k^*}, k^* \in K \subseteq \{1, 2, \ldots \bar{k}\}, \qquad (3.6.3)$$

$$|t_i y_{ij} - t_l y_{lj}| \geq y_{ij} y_{lj} \Delta t^{j}_{adm}, \ i \neq l, \ i,l = 1, 2, \ldots \bar{\imath}, \ \forall j. \qquad (3.6.4)$$

If the criterion (3.6.2) affords not a single-valued solution, it is advisable to take an additional optimization step, for example:

$$\sum\nolimits_{i,j} y_{ij} \Rightarrow \max_{Y}. \qquad (3.6.5)$$

If desired, one can use as a figure of merit instead of (3.6.2) the mathematical expectation of the weighed number of acquired objects and introduce additional restrictions from below to the probability of favorable conditions of searching each object.

The accepted matrix-like structure of solution Y prompts a natural and convenient form of the decision tree which is constructed in the next way. Without loss of common character, we assume that the intervals are numbered in chronological order of their centers. The vertices of the n-th tier of the decision tree represent the sets of solutions in which the instruments controlling the first *n* intervals are fixed.

Then, an estimate from below (the lower score) of the function for vertices of the tree is constructed in the following form:

$$\widehat{\Phi} = \widehat{\Phi}_1 + \widehat{\Phi}_2 + \ldots + \widehat{\Phi}_m + \ldots + \widehat{\Phi}_{\overline{m}}.$$

Let the first term be

$$\widehat{\Phi}_1 = \sum_{j \in J_0} w_j$$

where J_0 is the set of already used instruments for realization of the fixed part of SP.

And now the next step. At this stage we introduce some auxiliary notations:

K^1 – the set of not yet fixed intervals (of their numbers), each of them being able to be controlled by only one instrument;

$J(K^1)$ – the set of instruments capable of checking the set of intervals K^1;

$J^{(1)} = J(K^1) \setminus J_0$ (a short notation);

K^2 – the set of numbers of not yet fixed intervals, each of them being able to be controlled by strictly two instruments;

$J(K^2)$ – the set of instruments capable of checking the set of intervals K^2;

$J^{(2)} = J(K^2) \setminus (J(K^1) \cup J_0)$ (a short notation).

By mathematical induction, let us thus define the set $J^{(m)}$ for arbitrary $m \in \{2, 3, \ldots \bar{J}\}$. Then

$$\widehat{\Phi}_2 = \sum_{j \in J^{(1)}} w_j, \quad \widehat{\Phi}_{m+1} =$$

$$= \begin{cases} \min\limits_{j \in J^{(m)}} w_j \text{ if } J^{(m)} \neq \varnothing, \\ 0, \text{otherwise.} \end{cases}$$

Considering this formula, one can arrive at the next practically important conclusion. It can be easily seen that the improvement of the estimate $\widehat{\Phi}$ accuracy can be achieved at the cost of an increase of laboriousness of calculations. So, it is reasonable to stop on some $\overline{m}$ (1 <

$\bar{m} < \bar{j}$) which optimum value is determined, for example, experimentally using some collection of the control versions and considering the times (durations) of their program runs.

And here also as before, along with the traditional cutting rules of the BB approach, the ACR [48] can be applied in the next treating: at the n-th tier of the tree among the vertices with the same composition of the already fixed instruments $\bar{\bar{\bar{G}}}$-comparable (see [48, 49]) are those for which

$$t_n y_{nj} - t_l y_{lj} \geq t_n y_{nj} - t_i y_{ij},\ i < n,\ l < n,\ \forall j$$

or for which the restrictions (3.6.4) in the vicinity of the time moment t_n are not violated. Among all $\bar{\bar{\bar{G}}}$-comparable vertices, only that one is subject to further ramification which has the minimum value of $\widehat{\Phi}_1$ (an estimate from below ("not better than") of Φ at the definite part of solution).

The considered statement of task is natural in absence of strict requirements to drive the search and for high enough probabilities of the SP elements feasibility. Some alternative criteria (other than (3.6.1)) and further modification of the statement of this task are discussed in section 3.8.

3.7. Optimum planning of search by the criterion of detection time

We will gradually expand towards more specialization the set of criteria used. And now we consider the problem of constructing discrete SPs minimizing the mathematical expectation of the object detection time. This criterion (based on the requirement of acquiring the object in the shortest time sparing no economic expense) usually comes into conflict with the earlier considered criteria of a predominantly economic character. For instance, one has to wait for the advent of optimum conditions in terms of criterion (3.4.2) sometimes as long as about the object revolution period T, the average waiting time being as much as $T/2$. If T is great, then the above conflict looks evident and sometimes inadmissible, for instance, when the observer (or the task customer) has no time for waiting handy conditions for the search.

So, the space surveillance reality allows many cases when any SP optimum in the sense of the new criterion, as a rule, is not optimum in the sense of the former criteria. Nevertheless, the equivalence curves apparatus is fit for solving problems of optimum planning the search for SOs practically by any criterion. The more so, it is very convenient for dealing with tasks of such a kind and they are usually mathematically more interesting.

In the former statement of the search problem, the law of the object position distribution was of no importance. It was sufficient to have the initial search interval $[s_b, s_f]$. At the same time, the cyclic character of the object motion was used essentially.

In the current statement of the problem, at the time t_0 along with the basic search interval $[s_b, s_f]$, the object position probability distribution density $f_0(s)$ over the interval $[s_b, s_f]$ for the time t_0 are known (and, of course, the law of the sought-for object motion $s(t)$). The natural purpose is to acquire the object as soon as possible (or in minimum observation steps).

Let t_1 ($t_1 > t_0$) be the earliest time moment when the search can start. That is, since t_1 one can observe the trajectory points, $t_2, t_3, \ldots$ being time moments following t_1 with the time step Δt (embracing the exposition time, re-targeting time, and so on). By definition, $\mu_n = \mu_n(t_n)$ is an element of SP tied to the time moment t_n and which represents a vertical segment of a straight line having the height Δs, the derived element $\mu_n(t_0)$ being its EC-projection to the moment t_0.

Let the quality of SP

$$M = \bigcup_n \mu_n$$

be defined by the criterion

$$\Phi(M) = (t_1 - t_0) + \sum_{n=1}^{\bar{n}} (n-1)\Delta t \int_{\mu_n(t_0)} f_0(s)ds. \qquad (3.7.1)$$

It is the mathematical expectation of the object detection time under the condition of the start of search at time t_1 (more exactly, not earlier than t_1 – see later Fig. 3-8-2).

This figure of merit $\Phi(M)$ should be turned into a minimum.

As soon as in (3.7.1) only the right addend depends on SP, then it will be further considered as the function to be minimized.

The restrictions in the problem statement are presented implicitly:

1) the regions of integration $\mu_n(t_0)$ are obtained by EC-projecting the SP elements μ_n to the time t_0, elements μ_n being strictly tied to the moments t_n , $n = 1, 2, \ldots \bar{n}$, respectively;

2) the SP should intersect every EC from the family

$$\sum\nolimits_{t_0}^{s_b, s_f}.$$

If one assumes the more flexible strategy due to which the variable time step Δt is used (for example, depending on the angle of re-targeting the instrument during the transition from SP element μ_{n-1} to element μ_n), then the function will take the form

$$\Phi(M) = \sum_{n=1}^{\bar{n}} \left(\sum_{i=0}^{n-1} \Delta t_i\right) \int_{\mu_n(t_0)} f_0(s)\mathrm{ds} \qquad (3.7.2)$$

under the condition that $\Delta t_0 = t_1 - t_0$.

The decision tree (if the BB approach is used for solving the task [48]) can have at least the next two natural structures [44].

Structure 1. At the *n*-th tier of the decision tree, elements μ_n of SPs are being fixed considering already fixed parts of SPs at the previous tiers of the tree.

Structure 2. The synthesis of SP is carried out in consecutive order by layers of ECs beginning from the upper, lower, or some middle one (the former two starts provide the sensor with minimum constant re-aiming angle throughout the full search cycle). At the *n*-th tier of the tree, the element μ_{m_n} of SP is fixed which controls the *n*-th layer of ECs at time t_{m_n}.

Each of the above structures has its own merits and demerits. Realization of Structure 1 does not require to concretize in advance (a priori) the number $\bar{n}$ of elements in the SP whereas for Structure 2 the beforehand availability of $\bar{m}$ is necessary. At the same time, the current fixed (already synthesized) part of SP in Structure 2 is represented in the *ts* plane by a compact and simply connected domain which is very comfortable for the following operations. The construction of the next in order SP element μ_{m_n} is carried out from the only possible EC of support (the reference EC) – the lower (higher) boundary of the already constructed part of SP (in terms of EC-layers) if the start was performed from the highest (lowest) EC ($\tilde{s}_f$ or $\tilde{s}_b$ respectively). Or else the construction of μ_{m_n} is carried out from two possible ECs of support – the higher or lower boundary of the already assigned part of SP if the start was performed from some middle layer (see Fig. 3-7-1).

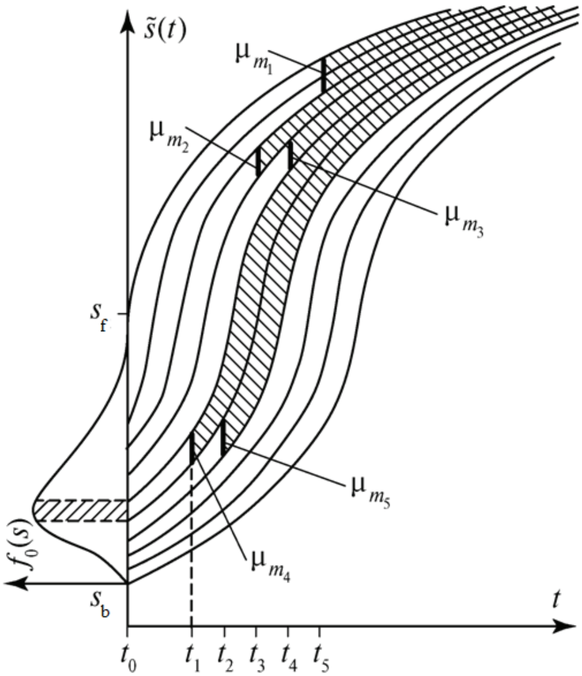

Fig. 3-7-1. Constructing SP in Structure 2

Due to this property, Structure 2 is protected from the appearance of uncomfortable, inconvenient, and irrational systems of ECs having a form of "narrow straits" which usually appear when dealing with Structure 1. The latter can admit a great number of ECs of support in the intermediate partial SP (see Fig. 3-7-2). It is very uncomfortable in the practical construction of SPs. And, of course, it makes trouble for the observer realizing the SP.

The shortcoming of Structure 1 can be overcome if one refuses from the exact solution and favors the approximate. The typical method of synthesis of an approximate (suboptimum) solution will be given below in this chapter.

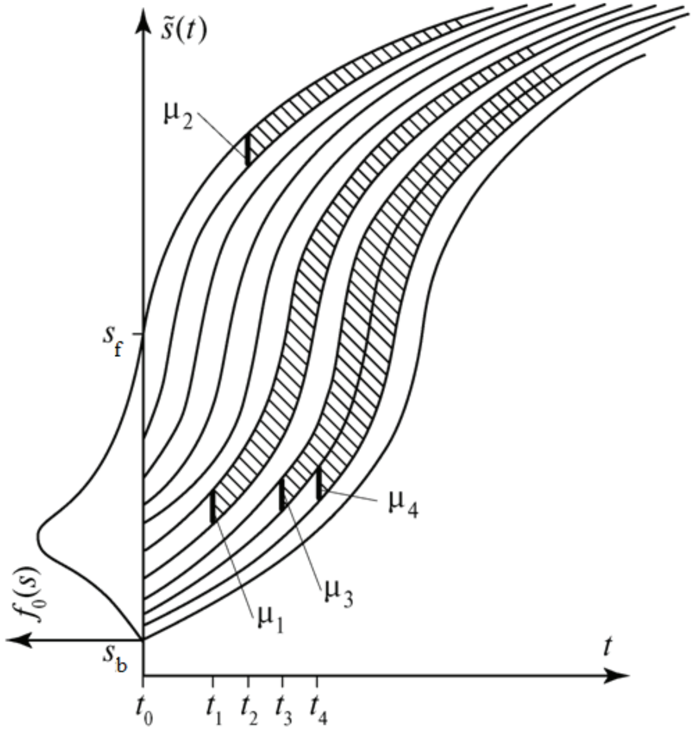

Fig. 3-7-2. Constructing SP in Structure 1

The following should be borne in mind when constructing the SP. The shortcoming of Structure 2, only on the face of it, seems dramatic. The matter of fact is that practically in every situation it is possible to indicate the upper bound for the maximum number of SP elements and assign this value to $\overline{m}$. The SP of high quality is expected to automatically have the real number of elements usually much less than the assigned value of $\overline{m}$ at the expense of the fact that the development of the corresponding branches will be stopped at the earlier tiers than $\overline{m}$-th. The branches having very high indices m_n of elements μ_{m_n} included into the SP at the earlier tiers will be beforehand cut away in the course of a correctly organized calculation process.

Structure 2 as well as Structure 1 assume the variability of the time step Δt_n, although the values of Δt_n for all n should be available in advance. But if the size of step Δt_n can be determined not in advance but only during transition from μ_{n-1} to μ_n, then for solving the problem, Structure 2 does not fit, whereas Structure 1 permits the very comfortable recurrence

$$\Delta t_n = \Delta t_n(\mu_{n-1}).$$

So, here, an intermediate conclusion may be drawn. Structure 2 of the decision tree allows obtaining both an approximate solution and the precise optimum one. However, Structure 1 gives practically only an approximate solution (the versions of approximate procedures suggested below give hope for solutions of high quality). Besides, Structure 1 easily realizes a variable time step of the sequence of SP elements.

A precise method for solving the problem in Structure 2

It is worth recalling that the first tier of the decision tree contains $\overline{m}$ vertices. The first vertex represents the set of solutions in which the first (for definiteness, from above) layer of ECs is controlled at the time moment t_1. The m-th vertex of the first tier represents the set of solutions in which the first layer of the ECs is controlled at time t_m. This scheme is clearly illustrated in Fig. 3-7-1. Vertices of the second tier represent the sets of solutions in which the second layer of the ECs is controlled at times $t_1, t_2, \dots t_{\overline{m}}$, except the moment at which the first layer was controlled in the corresponding vertex of the first tier.

Fixation of an SP element controlling the first layer of the ECs (a vertex of the first tier) sets the EC of support $\tilde{s}_{1\,\mathrm{sup}}(t)$ for constructing the next SP element. Fixation of a SP element controlling the second layer (a vertex of the second tier) determines the EC of support $\tilde{s}_{2\,\mathrm{sup}}(t)$ for constructing the third SP element, and so on. In the n-th tier of the decision tree, the EC of support (the reference EC) is $\tilde{s}_{n\,\mathrm{sup}}(t)$ (for each vertex its own).

After every n-th stage of synthesis of SP, the region for further planning the search (in the ts plane) gets narrow up to the subfamily of ECs

$$\sum\nolimits_{t_0}^{s_b} \tilde{s}_{n\,\mathrm{sup}}(t_0) \quad \subset \sum\nolimits_{t_0}^{s_b} s_f \qquad \text{(see section 3.2)}$$

if the layers are counted from above (which, generally speaking, is not necessary).

The estimate $\widehat{\Phi}$ of functional Φ in the vertex of the n-th tier is calculated as follows:

$$\widehat{\Phi}_1 = \Phi(M_n) = \left(\bigcup_{j=1}^{n} \mu_{m_j}\right) =$$
$$= \sum_{j=1}^{n} (m_j - 1)\Delta t \int_{\mu_{m_j(t_0)}} f_0(s)ds. \qquad (3.7.3)$$

The part of sum (3.7.1) remaining after (3.7.3) which now should be estimated by the addend $\widehat{\Phi}_2$ looks like follows:

$$\widehat{\Phi}_2 = \Delta t((m_{\mathrm{n+1}} - 1)p_{m_{n+1}} + (m_{\mathrm{n+2}} - 1)p_{m_{n+1}} + \ldots + (m_{\bar{n}} - 1)p_{m_{\bar{n}}}) = \Delta t A^T P \qquad (3.7.4)$$

where A is an $(\bar{n} - n)$-vector with components $a_j = (m_{\mathrm{n+j}} - 1)$ and P is an $(\bar{n} - n)$-vector with components $p_{m_{n+j}} \geq 0$.

Without loss of a common character, we suppose that the components a_j of vector A (and as a consequence in (3.7.4)) are arranged in order of growth. The lower estimate $\widehat{\Phi}_2$ for the addend Φ_2 can be obtained, for example, by substitution of some known fictitious vector $\hat{P} \leq P$ (the inequality is meant for every pair of corresponding components) instead of the unknown vector P. Such a vector $\hat{P} = (\hat{p}_1, \ldots \hat{p}_j, \ldots \hat{p}_{\bar{n}-m})^{\mathrm{T}}$ can be constructed as follows supposing $\bar{n} = \bar{m}$.

For all the remaining free (not yet assigned) $\bar{m} - n$ time moments t_{m_j} being chosen from the set

$$\bigcup_{\mathrm{m}=1}^{\bar{m}} t_m \setminus \bigcup_{l=1}^{n} t_{l=1},$$

let us construct (provisionally) fictitious SP elements pseudo-controlling the next in turn $(n + 1)$-th layer of ECs and project them along ECs to the moment t_0, the fictitious element having the maximum length of its projection being denoted as μ^f_{n+1}. Provisionally, let us include the element μ^f_{n+1} in the SP and perform the same operation for the same $\overline{m} - n$ (*sic!*) time moments t_{m_j} in the ($n+2$)-th layer of ECs. Thus, it gives the fake SP element μ^f_{n+2} with maximum length of its projection adjacent to μ^f_{n+1}, and so on. Similarly calculating the elements μ^f_{n+j}, $j = 3, \ldots \bar{j}$, $\bar{j}$: $\mu^f_{n+\bar{j}} \cap \tilde{s}_b \neq \emptyset$, and then for them

$$q_j = \int_{\mu^f_{n+j}(t_0)} f_0(s)ds\,, \qquad (3.7.5)$$

and arranging these quantities (q_j) in order of decrease, we construct as a result the vector $(\hat{p}_1, \ldots \hat{p}_j, \ldots \hat{p}_{\bar{j}})^T$. If $\bar{j} = \overline{m} - n$, the sought-for vector P has been found. If $\bar{j} < \overline{m} - n$, then after adding $\overline{m} - n - \bar{j}$ zeros as the last components to the vector, we obtain what we sought.

Remark 1. The quantity $\bar{j}$ cannot be more than $\overline{m} - n$. Otherwise, $\overline{m}$ was chosen wrong.

Remark 2. From the procedure of constructing the vector $\widehat{P}$, it follows that the unknown component (3.7.4) of the criterion Φ majors the estimate $\widehat{\Phi}_2 = \Delta t A^T \widehat{P}$, Q.E.D.

The structure of function (3.7.1) and the restrictions of the problem permit application of ACR [48] for acceleration of obtaining the solution.

Indeed, if some two vertices of the n-th tier meet the conditions

$$\bigcup_{j=1}^{n} m_j^1 \subseteq \bigcup_{j=1}^{n} m_j^2\,,\ \tilde{s}^1_{n\,sup}(t) \leq \tilde{s}^2_{n\,sup}(t) \text{ и } \widehat{\Phi}^1_1 \leq \widehat{\Phi}^2_1, \qquad (3.7.6)$$

then the vertices are $\bar{\bar{G}}$-comparable (see section 3.6 and [48, 49]) and the second one is eliminated from further consideration.

The theorem below prompts a way of constructing an approximate solution. Its validity will be shown for the case when one can neglect the dependence of the integral in (3.7.1) on the reference time t_n of the SP element μ_n. Taking account of the latter conditions which is relevant, for instance, for $t_{\overline{m}} - t_1 \ll T$ or for very small variation of the derivative $s'_t(t)$ in the region of planning, the optimality condition of the criterion (3.7.1) can be written as follows:

$$\Phi_{min} = \min_P \sum_n (n-1)\,\Delta t\, p_{\nu_n} = \Delta t \min_P \sum_n (n-1)\, p_{\nu_n} \qquad (3.7.7)$$

where $P = (p_{\nu_1}, \dots p_{\nu_{\bar{n}}})$ is a permutation of a given set of $\bar{n}$ numbers $\{p_n\}$,

$$p_n = \int\limits_{\mu_n(t_0)} f_0(s)ds.$$

Theorem. In the permutation P_{min}, turning the value of the function Φ into a minimum, numbers p_{ν_n} should monotonically decrease with the growth of n.

Proof. Suppose the opposite, i. e., there exist $p_{\nu_{n^*}}$ and $p_{\nu_{n^*+1}}$ in P_{min} such that $p_{\nu_{n^*}} < p_{\nu_{n^*+1}}$. Let $p_{\nu_{n^*+1}} = p_{\nu_{n^*}} + \varepsilon$, $\varepsilon > 0$. In such a case,

$$(n^* - 1)\, p_{\nu_{n^*}} + n^* p_{\nu_{n^*+1}} = n^* p_{\nu_{n^*}} - p_{\nu_{n^*}} + n^* p_{\nu_{n^*}} + n^*\varepsilon = (2\, n^* - 1)\, p_{\nu_{n^*}} + n^*\varepsilon$$

whereas

$$(n^* - 1)\, p_{\nu_{n^*+1}} + n^* p_{\nu_{n^*}} = n^* p_{\nu_{n^*}} - p_{\nu_{n^*}} + (n^* - 1)\varepsilon + n^* p_{\nu_{n^*}} =$$

$$= (2\, n^* - 1)\, p_{\nu_{n^*}} + (n^* - 1)\varepsilon.$$

Then, after switching the positions of $p_{\nu_{n^*}}$ and $p_{\nu_{n^*+1}}$ in P_{min}, one can obtain the new permutation P' for which the sum in (3.7.7) is less by ε than for P_{min}. The latter contradicts the definition of P_{min} and consequently disavows the supposition. The Theorem has been proved.

An approximate method of solution in Structure 1

Let us expound an approximate method for the case of a unimodal function $f_0(s)$. For multimodal distribution functions the principle of synthesis of SP remains the same, but sub-families of ECs being controlled by intermediate partial SPs can turn into n-tuply connected domains. By the way, the simple-connectedness of the ECs family equivalent to the resulting SP can be usually achieved at the price of deterioration of its quality (which is true for planning in Structure 1).

Let us denote the EC coming through the point $<t_0, s_0>$ corresponding to the maximum of $f_0(s)$ as $\tilde{s}_0(t)$ and call it the central EC. In the capacity of the first SP element $\mu_1(t_1)$ we choose the vertical segment of length Δs with its center at the point $<t_1, \tilde{s}_0(t_1)>$. The element $\mu_1 = \mu_1(t_1)$ controls the subfamily of ECs

$$\sum\nolimits_{t_2}^{s_b^1, s_f^1}$$

where

$$s_b^1 = \tilde{s}_0(t_1) - \frac{\Delta s}{2}, \quad s_f^1 = \tilde{s}_0(t_1) + \frac{\Delta s}{2}.$$

The boundary ECs $\tilde{s}_b^1$ and $\tilde{s}_f^1$ are those of support (the reference ECs) for constructing the next SP elements. The element $\mu_2(t_2)$ (a segment of length Δs) begins from one of them in the direction where the integral

$$\int_{\mu_2(t_0)} f_0(s)ds$$

has the greater value. The two constructed elements μ_1 and μ_2 control the sub-family of ECs

$$\sum\nolimits_{t_2}^{s_b^2, s_f^2}$$

just what we sought.

If μ_2 was placed upward in the ts plane (if it rests upon the upper boundary EC), then s_b^2 is the coordinate (ordinate) s of the lower end of the element μ_1 (i.e., s_b^1) projected along the EC to the time t_2; and s_f^2 is the ordinate s of the upper end of the element μ_2. If μ_2 was placed downward from the lower boundary EC, then – vice versa (see Fig. 3-7-3).

The next SP element $\mu_3(t_3)$ rests upon one of the (boundary) ECs of support $\tilde{s}_b^2$ and $\tilde{s}_f^2$. So that the integral

$$\int_{\mu_3(t_0)} f_0(s)ds$$

is of the greatest value. And so on.

So, the current partial SP

$$\bigcup_{m=1}^{n} \mu_m$$

controls the simple connected subfamily

$$\sum\nolimits_{t_n}^{s_b^n, s_f^n}$$

of ECs bounded by the most distant one of another ECs intersecting the partial SP - $\tilde{s}_b^n(t)$ and $\tilde{s}_f^n(t)$, the latter being ECs of support for the next SP elements. The synthesis of SP will finish as soon as the next condition is realized:

$$\sum\nolimits_{t_0}^{s_b, s_\mathrm{f}} \subseteq \sum\nolimits_{t_0}^{s_b^n, s_f^n}, \qquad (3.7.8)$$

i.e., as soon as the initial (global) boundary ECs $\tilde{s}_\mathrm{b}(t)$ and $\tilde{s}_f(t)$ embed into the family

$$\sum\nolimits_{t_n}^{s_b^n, s_f^n} \quad (\text{or} \sum\nolimits_{t_0}^{s_b^n, s_f^n} \text{ - which is the same).}$$

The sequence of synthesis is illustrated by Fig. 3-7-3.

The expounded above procedure for obtaining an approximate SP can be used both independently and for getting the initial or intermediate record (as an instrument for cutting off the perspectiveless branches of the decision tree) in the precise method of solution.

Generalized approximate method (in Structure 1)

In the expounded above approximate method, the synthesis of SP started from the central EC. In the important specific case when the range (span) of variations of the derivative s_t' within the search interval is significant and if the object position probability density distribution function $f_0(s)$ is of some specific form, an SP closest to the OSP can be found among those concentrated around some EC distant from the central one.

Thanks to the fast action of an approximate method, one can get several approximate solutions M_i syntheses of which begin from different initial ECs (for example, over the local maximums of the distribution function $f_0(s)$) and finally choose among them the best solution in terms of the criterion $\Phi(M_i)$.

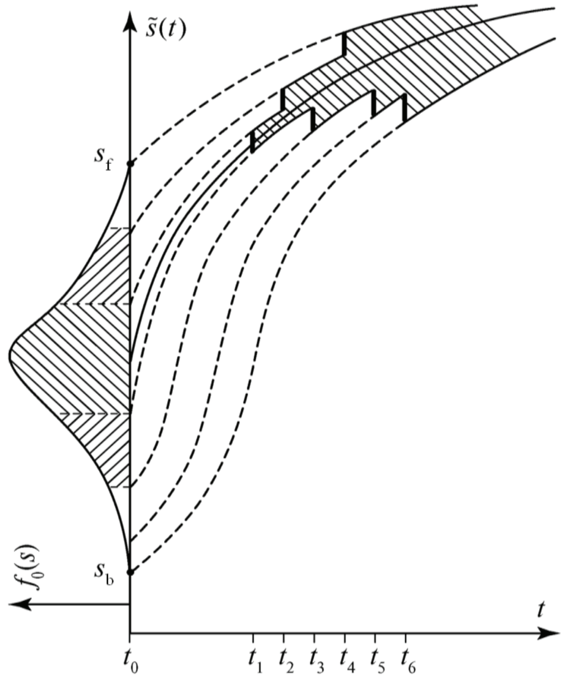

Fig. 3-7-3. Synthesis of a sub-optimum search plan in Structure 1

3.8. The case of unknown probability distribution of the object's position at the search interval

The methods for optimum planning the search suggested in section 3.7 are based on the use of information on the probability distribution of the object's position at the search interval $[s_b, s_f]$. In practice, there are situations when the probability distribution function $f_0(s)$ is often unknown or unauthentic. But these facts should not be dramatized within our theory. In similar cases, the problem of constructing the OSP does not degenerates but requires alternative methods of its solution and/or modification of its criteria, the EC apparatus remaining an appropriate instrument for getting a constructive solution.

A natural criterion in such cases is the number of SP elements providing the guaranteed acquisition of the sought-for object.

The general idea of an approach to the solution of the problem is illustrated by Fig. 3-8-1.

Let's consider an illustrative example. Given the time moments of checking the trajectory t_1 and t_2, the SP $M = \{\mu_1, \mu_2\}$ guarantees acquisition of the object by the time t_2 (see Fig. 3-8-1a). The SP presented in Fig. 3-8-1b provides the equivalent check of only some smaller part of the search interval. This peculiarity leads to the growth of the detection time at least by the step size of the time discreteness Δt. So, the second SP is evidently worse than the first one ***for any probability distribution*** $f(s)$. The latter is one more merit of this approach.

This example shows how essentially the quality of the SP depends on the choice of the ECs layers to be checked at given fixed times.

The rule for forming the search plan. Now, one can word the rule for constructing SP. The element μ_m, $m = 1, 2, \ldots \overline{m}$, is placed in the region of values of coordinate s:

$$s \in \sum\nolimits_{t_0}^{s_b, s_f} \setminus \Sigma\left(\bigcup_{\nu=1}^{m-1} \mu_\nu\right)$$

with the least values of the ECs derivatives $\tilde{s}'_t(t_m)$ at the point t_m.
Here,

$\Sigma(\bigcup \mu_\nu)$ is a subfamily of ECs corresponding to all points of the union of sets $\bigcup \mu_\nu$.

It can be shown that after removing some "natural" restrictions in case of a particular form of the motion law (see section 3.4), this simple rule leads to the optimum SP (with the least number of elements) even if this, at first glance, contradicts the common sense.

One such restriction is the beginning of measurements ***at a given time*** $\boldsymbol{t_1}$ (the "natural" principle: as soon as possible). Fig. 3-8-2 illustrates a "paradoxical" example of SP when removing this restriction (more exactly, turning it into the alternative more general and in our case more literate one: the measurements should begin ***not earlier than*** $\boldsymbol{t_1}$) permits obtaining the SP with less detection time (this is optimum in our case). In this example, for a given discreteness of the measurements time step Δt, the start of the search at a later time moment t'_1 ($t_1 < t'_1$) renders a possibility to decrease the number of SP elements (that is, to enhance the SP quality). And what is more, there are further prospects. The best SP can be found by using the same rule in the iterative scheme by the parameter t'_1.

The methods of optimum planning the search for an object by the detection time criterion suggested in sections 3.7 and 3.8 are applicable not only in case of the cyclic motion of the sought-for object. It is sufficient to know the motion law, the initial position of the search interval $[s_b, s_f]$, the object position probability distribution density $f_0(s)$, and the possible time moment of the start of searching. To be sure, the five main properties of ECs (see section 3.2) should not be violated at the search interval (mandatory requirements).

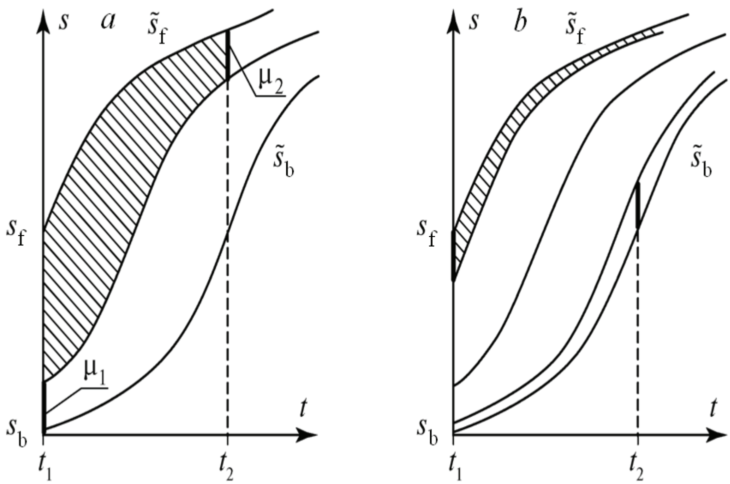

Fig. 3-8-1. An example of different qualities of search plans having two different elements equally timed

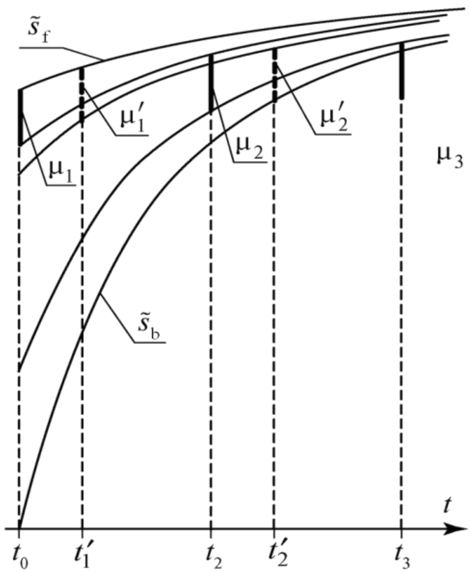

Fig. 3-8-2. A "paradoxical" example of SP

3.9. The influence of an error in the object motion period upon planning its search

In this section, we consider in detail the problem of optimum planning the search in the presence of an error in a given period of the object's motion $T_0 = T + \Delta T$ (T_0 is a given value of the nodal period, T is an unknown real one, and ΔT is an error). By the influence of this error, the true EC of the point $< t_0, s_0 >$ will be distorted (lengthened out or shrunk depending on the error sign) along the t axis by a quantity $\frac{\Delta T}{T}\ \psi(t - t_0)$ where ψ is a monotonically increasing function of time having the following properties:

$$\psi(0) = 0,\ \ \psi(t - t_0 + nT) = \psi(t - t_0) + nT \qquad (3.9.1)$$

where n is a positive integer.

It results from (3.9.1) that $\psi(nT) = nT$. But for a symmetric distribution of the velocity $s'(t)$ with respect to the point s_m ($s_m = s_{min}$) in which the velocity is minimum (a particular form of the motion law - see section 3.4), this equality turns to

$$\psi(n\frac{T}{2}) = n\frac{T}{2}\,.$$

That means that $\psi(t\text{-}t_0)$ coincides with t - t_0 in the nodes s_{min} and

$$s_{max} = s_{min} \pm \frac{S}{2}$$

if, for the sake of determinacy, we assume that t_0 corresponds to s_{min} or s_{max}.

In the investigation of the consequences of the influence of an error in the motion period, let us limit ourselves to the case of pseudo-synphase cyclic motion, i.e., when for two motion models (one with the period $T_1 = T$ and the other with the period $T_2 = T + \Delta T$) the next condition takes place:

$$1) \qquad S_1 = S_2 = S,$$

$$2) \qquad s_1(t_0) = s_2(t_0), \tag{3.9.2}$$

$$3) \qquad \frac{t_1 - t_0}{T_1} = \frac{t_2 - t_0}{T_2} \Rightarrow s_1(t_1) = s_2(t_2).$$

The conditions (3.9.2) mean that with both objects beginning the motion from the same initial phase in equal portions of the periods they will come to the equal phases (not only in the nodes s_{min} and s_{max} but in any point of the trajectory).

If the error ΔT is small as compared with the value of T, then, on account of a little possibility of "deformation" of the model $s(t)$, the restrains (3.9.2) are not so burdensome and, as it can be shown, quite relevant in applications considered below (Chapter 4).

It follows from (3.9.2) that for the same phase it is valid that

$$\frac{t_1 - t_0}{T} = \frac{t_2 - t_0}{T + \Delta T} \text{ and } t_2 - t_1 = \frac{\Delta T}{T}(t_1 - t_0).$$

That means that due to the presence of an error ΔT in T, the EC of the point $<t_0, s>$ will stretch or shrink along the t axis by the value $\frac{\Delta T}{T}(t - t_0)$ to the right or to the left, depending on the sign of ΔT (Fig. 3-9-1).

Consequently, the error of representation of the object′s position in the *ts* plane in different time moments, with the help of the corresponding EC, owing to the error ΔT in the motion period equals

$$\delta s = s_t{}'(t)\,\frac{\Delta T}{T}(t - t_0). \tag{3.9.3}$$

For constructively calculating the extrema of this function, let us turn its derivative with respect to t to zero:

$$\frac{\partial\,\delta s}{\partial t} = \frac{\Delta T}{T}\,(s_t{}''(t)\,(t - t_0) + s_t{}'(t)) = 0.$$

So that, generally speaking, for the determination of regions with the most and the least influence of the error ΔT on the error δs, the roots of equation

$$s_t{}''(t)\cdot(t - t_0) + s_t{}'(t) = 0 \tag{3.9.4}$$

should be found.

For choice, the equation (3.9.4) can be solved analytically or numerically only in case if the concrete expression for $s(t)$ is available. On this account, let us elucidate the general qualitative picture of the phenomenon using, for obviousness, Fig. 3-9-1.

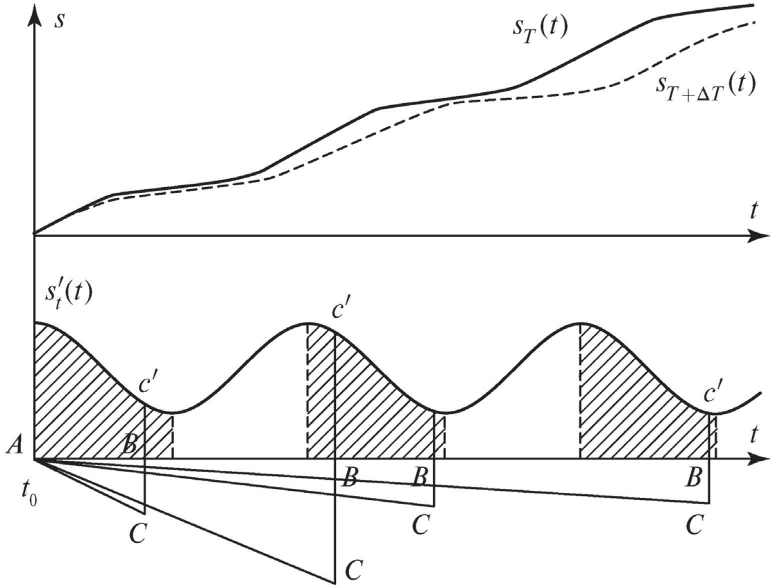

Fig. 3-9-1. Clarification of the calculation of the least ΔT influence on δs

It follows from Eq. (3.9.3) that, with monotonical growth of the derivative $s_t'(t)$ as well as when the derivative $s_t'(t)$ is constant, the error δs monotonically increases. It follows from (3.9.4) that the extrema of the error δs can appear only in the regions of the t axis where $s_t''(t) \leq 0$ (s_t' is always nonnegative on account of monotonicity of function $s(t)$). In Fig. 3-9-1, these regions are hatched. When there are only small variations of the derivative s_t' with respect to its average value, the extrema of δs can appear only in distant (from t_0) cycles. On the contrary, if great variations of s_t' are available, then, within each cycle, the explicit regions of great and small errors δs will appear.

It makes sense to compare formula (3.9.4) and Fig. 3-9-1 and discuss them together. Eq. (3.9.4) in terms of Fig. 3-9-1 can be construed as follows. The value of the second addend $s_t'(t)$ (which is always nonnegative) of the left part of Eq. (3.9.4) should be compensated by the value of the cathetus BC of the triangle ABC in which |AB| = $(t - t_0)$ and the hypotenuse AC is parallel to the tangent to the curve $s_t'(t)$ at point t.

On account of the periodicity of functions $s_t'(t)$ and $s_t''(t)$ and the linear growth of the factor $(t - t_0)$ by s_t'', in the posterior extrema for compensation of the quantities s_t' (limited by the maximum value to the same meaning for all cycles) one ought to choose smaller and smaller values of s_t''.

It is evident that the extrema points of function $\delta s(t)$ remaining in the region of negative values of $s_t''(t)$ with growth of t asymptotically converge to the roots of the equation $\frac{d^2s}{dt^2} = 0$, i.e., to the extrema points of function $s'_t(t)$. The deeper $s(t)$ is “modulated” (i.e., the more non-uniform distribution of the object motion velocity within the cycle takes place) the faster the convergence is going on.

The quantity $max\delta s(t)$ converges to the left edge of the shaded region whereas the quantity $min\delta s(t)$ does to the right edge of that.

Asymptotically, the maximum value of δs is equal to

$$max\delta s(t) = \frac{\Delta T}{T}(t_{max} - t_0)(\max_t s_t'(t,T)),$$

The minimum value of δs is equal to

$$min\delta s(t) = \frac{\Delta T}{T}(t_{opt} - t_0)\, s_t'\left(t_{opt},T\right) =$$

$$= 0{,}5\frac{\Delta T}{T}\; s_t'\left(\frac{\tilde{s}_b^{-1}[s_m]+\tilde{s}_f^{-1}[s_m]}{2}\right)\left(\tilde{s}_b^{-1}[s_m] + \tilde{s}_f^{-1}[s_m] - 2t_0\right). \quad (3.9.5)$$

With that, the following important theorem holds.

Theorem. The point $< t_{opt}, s_m >$ has two remarkable properties: in its vicinity, the search plan not only does reach its optimum, but the effect of an error in the object motion period when planning the search manifests itself and becomes apparent in a minimum way.

As to practical application of the Theorem, it can be added the next. When planning the search for a cyclic object, the influence of an error in the determination of the period becomes apparent in the form of a distortion of ECs comprising the family

$$\sum\nolimits_{t_0}^{s_b,s_f}$$

and this distortion can be compensated at least in two ways:

a) by increasing the exposition time of the sensor when realizing each SP element,

b) by decreasing the calculated size of the trajectory arc Δs_c to be checked as compared with the real one Δs (by providing $\Delta s_c < \Delta s$).

At last, the influence of that error can be minimized if the SP is constructed in the vicinity of point $< t_{opt}, s_m >$ (see the Theorem above).

In the first case (a), the exposition time is to be increased in both sides by the value $\frac{\Delta T}{T}(t - t_0)$.

In the second case (b), if Δs is small, the calculated size of the arc is determined from the next equation:

$$\Delta s \approx 2s_t'(t,T)\frac{\Delta T}{T}(t - t_0) + \Delta s_c\,. \qquad (3.9.6)$$

In case when the size of the arc to be checked by one exposition is significant and such is the amplitude of variations of the derivative s_t' within the element μ_n, the correction should be made proceeding from the next calculation. The search plan element μ_n should be enlarged in the direction of its lower end $< t_n, s_b^{\mu_n} >$ by the value

$$\Delta s_b = \frac{\Delta T}{T}(t_n - t_0)\left(\frac{\mathrm{d}\tilde{s}}{\mathrm{d}t}\,\Big|_{t:\tilde{s}(t)=s_b^{\mu_n}}\right) \qquad (3.9.7)$$

and in the direction of its upper end $< t_n, s_f^{\mu_n} >$ by the value

$$\Delta s_f = \frac{\Delta T}{T}(t_n - t_0)\left(\frac{\mathrm{d}\tilde{s}}{\mathrm{d}t}\,\Big|_{t:\tilde{s}(t)=s_f^{\mu_n}}\right). \qquad (3.9.8)$$

As a result,

$$\Delta s_c = \Delta s_b + \Delta s_f.$$

Such a discrimination of the ends of μ_n is not needed for a uniform character of motion. And then the quantity Δs_c is determined as

$$\Delta s_c = \Delta s - 2S\frac{\Delta T}{(T - \Delta T)T}(t - t_0). \qquad (3.9.9)$$

In all considered cases, ΔT should be taken as the maximum error (for instance, 3 values of the r.m.s. error) of determination of T at the initial time moment t_0. That guarantees the sufficient compensation for the error in question.

In the vicinity of point $< t_{opt}, s_m >$, the application of the second way (b) for compensation of the distortions of the ECs family is more available on account of their least display near the point. If there is no possibility to shift the region of planning to the vicinity of point $< t_{opt}, s_m >$, then in the region of high values of the derivative $s_t^{\prime}$, the first means (a) appears preferable.

And now, the next principal findings can be made from the above in this chapter.

- The suggested theory of search for a cycling object can serve as a basis for development of a convenient constructive methodology of optimum planning of the search for an object, using imprecise a priori information on the law of its motion and initial data. Due to generality of the approach adopted, a great variety of search situations and different types of sensors are admissible.
- In some cases, the construction of optimum search plans is brought to the solution of discrete optimization problems admitting application of ACR [48, 49].
- An optimum (in the sense of economic criteria) search region (OSR) is a vicinity of point $< t_{opt}, s_m >$ having the least value of the derivative $\frac{ds}{dt}$. In this region, the search plan not only reaches its optimum, but the influence of an error in the determination of the motion period upon planning the search becomes apparent in a minimum way.

CHAPTER FOUR

DETECTION OF HIGH ORBITAL SPACE OBJECTS

As was already said in the Introduction and became clearer from the previous chapters, the search problem is mostly urgent for high orbital, small, and weakly-contrasting (having a faint intelligence signal) SOs. That is why this chapter is oriented first of all to such a class of SOs. Nevertheless, if the necessity arises, the methods suggested here for planning the search are quite applicable in case of detection of SOs from alternative classes *mutatis mutandis*.

In this chapter, a set of tasks for monitoring the high orbital region of space related to the appearance of search situations is determined and described.

Drawing upon the analysis of peculiarities of the problem of detecting high orbital space objects (HOSOs) and, as developed in Chapter 2, the principles and methodology of planning the search for cyclic objects as well as the hybrid methods of discrete mathematical programming [48, 49], the scientific basis and methodology of planning the search for HOSOs using imprecise orbital information (including the search for HOSOs after their maneuvers and orbit corrections) are developed here.

The possible ways of enhancing the efficiency of application of the sensors and improving their operational characteristics for performing the function of detecting HOSOs on the basis of the developed approach are considered as well.

4.1. Specifics of the problem

The more specific features inherent in the problem being solved are considered when constructing the methods for solving it, the more effective the methods (for precise solution of this problem). The peculiarities of the problem of detecting HOSOs are determined by those of their orbits, specifics of the related tasks, and capabilities and conditions of the sensors' operation available for observing SOs which are members of this class.

In the Introduction, these peculiarities were partially discussed and the necessity of addressing the principle of an active search was substantiated unlike the principle of passive acquisition of SOs (by simple survey of some space region). The former (active) principle can be realized as methods of two kinds:

1) a regular survey (scanning) of the HO space region (including setting the radar and optical «fences»);
2) the targeted search for specific SOs essentially and effectively using some available incomplete or imprecise a priori orbital information on them.

The principle difference between these approaches consists in that the first methods are oriented to «indifferent» (not connected with concrete SOs) survey of a given orbital region. In contrast to them, methods of the second kind envisage the optimum implementation of the search resource disposed of on the basis of the most efficiently using the available incomplete or imprecise information on the orbital motion of each sought-for SO.

The methods of the first kind having the important merit such as breadth and completeness of surveying the space region, at the same time, are notable for the next demerits:

1) they are not economical;
2) they have low operative efficiency;
3) they do not provide the necessary technological conditions for acquisition of a faint intelligence signal;
4) they are characteristic of a significant connectedness of a whole cycle of scanning, i.e., require the continuity of its realization (and, as a consequence, the continuity of favorable conditions of observation during a rather long period which is sometimes impossible) which deteriorates the capabilities of practical realization of the main advantage of the survey methods;
5) they envisage inevitability of obtaining and identification of a great variety of unnecessary measurements, and these redundant operations deteriorate operative characteristics and reliability of acquisition and detection and complicate all the process of the sought-for SOs detection;
6) the region of regular scanning usually is located near the equatorial plane where the velocity of motion of non-GEO satellites is usually rather high which lowers the accuracy of angular measurements

and deteriorates the conditions of concentration and accumulation of a weak intelligence signal energy at the sensitive element of the receiver.

The optimum planning of search using a priori orbital information on the sought-for SO is tasked to remove these demerits. Even in case of application of regular scanning methods (the survey) at the first stage of detection, further, because of the action of the small measuring arc factor (SMAF), meteorological factors, maneuvers of SOs, and so on, the necessity arises of addressing the optimum search planning.

As a result of application of OSPs, one achieves not only a significant economy of electricity and other kinds of energy, operation resource of the sensors used, and employment of the personnel. Along with this,

the detection process accelerates,
the tracking process failure probability (including that after sudden SOs' maneuvers and orbit corrections) diminishes,
the conditions for identification of measurements are improved,
the sensitivity of all kinds of sensors to acquisition of faint intelligence signals in the search mode is improved.

In other words, along with enhancing the efficiency of exploiting the sensors, the basic performance characteristics of space surveillance are improved. And this is still before the special organization of the SSS's facilities work is proposed which will further improve the performance (see section 5.9).

At the first stage of HOSOs detection, the SMAF, by objective causes, often stipulates insufficient accuracy of initial orbit determination which leads to low accuracy of the acquisition data calculation (the target pointing data) for the next measurement session having the purpose of refining the values of orbit parameters to make them having the sufficient accuracy for putting the detected SO into the main catalog of SOs, for example, or upon request.

Usage of the OSP algorithms and the imprecise initial orbit parameters as a priori data permits to effectively solve this problem.

There is another search situation. After performing unpredictable maneuvers or orbit corrections (station keeping), the problem of search and detection of the SO arises once again. This phenomenon is typical of the class of HOSOs (first of all GEOSOs). And here, the proper application of OSP gives an essential total effect.

And there is one more search situation. Meteorological factors, radar and optical interferences, and other factors can partially or fully violate the SPs constructed beforehand. In such cases, the best decision is optimum re-planning of the search taking into account the already fulfilled part of the former plan.

Detection of an SO in GEO (if its brightness is enough for acquisition) is relieved by some circumstances: a narrow latitudinal belt of such SOs' staying sites, a very limited velocity of their motion with respect to a ground-based observer, and some others. Regular global surveys of the near-equatorial belt by wide-angle optical sensors to an essential degree fulfills this task (by available statistics – for 70 – 80% of GEO satellites). However, this applies only to large and/or bright objects.

At the same time, the maneuvers and regular orbit corrections (station keeping) as well as often explosions and alternative fragmentations involve addressing the OSP methods. Usage of the latter becomes inevitable in case of faint brightness of the sought-for SOs and appreciable deflection of their orbits from strictly geosynchronous one.

The search and detection of non-GEOSOs and, in particular, HEOSOs involve the most substantial difficulties stipulated, for instance, by variable velocity of such SOs, frequent corrections of their orbits, and much more uncertainty of a priori information on their orbits. Effectively solving this problem is possible only on the basis of the OSP methods using a priori orbital information.

Especially sharp is a question of detecting HOSOs over GEO. Acquisition and subsequent monitoring of such super high SOs is practically possible with the help of narrow-angle (up to some portions of one angular minute) optical sensors and narrow-beam radars, using special search programs.

However, after they have been detected, the SMAF action and limitation of the measurement duration period by night time and owing to meteorological factors often stipulate a low accuracy of the initial orbit determination, insufficient for calculation of ephemerides for successfully carrying out the next observation session.

Let us consider an example. If there is a task of detecting an SO in a circular orbit having the height of approximately 140 000 km (its nodal period T_Ω being about 6 days) by 4 two-dimensional angular measurements (consisting of right ascension α and declination δ) with the accuracy $\sigma_{\alpha,\delta}$ (the r.m.s. error) = 2″ obtained uniformly during the 6-hour night (the measuring arc about 15°), then the state vector will be obtained as a result having r.m.s. errors of its parameters given in Table 4-1-1 where

σ_r is the r.m.s. error along the radius-vector of orbit,
σ_l is the r.m.s. error along the tangent to the orbit,
σ_b is the r.m.s. error by the binormal,
σ_T is the r.m.s. error of the motion period T_Ω.

In the same table, the r.m.s. errors of the ephemerides calculated by this state vector (α - right ascension, δ - declination, ρ - slant range) and propagated for the interval Δt_{pr} are given. For reliable detection of an SO, the angular size of the sensor's FoV should exceed 6 r.m.s. errors of the ephemerides (other things being favorable). Then, as can be seen from Table 4-1-1, already in several hours after the detection of an SO and determination of its state vector one will be forced to apply search methods because of the rapid evolution of errors in the detected SO's state vector.

There is one more principle difference between the approaches to rationally organizing the process of detecting LEO and HOSOs. The process of detecting an LEOSO is usually implemented in such a way that the initial orbit is determined by a set of measurements obtained with the help of one or more radars in the mode of independent tracks. Such a principle for LEO is justified by three circumstances:

1) an LEOSO intersects the zone of a radar during a comparatively short time interval;
2) the energy expenses for obtaining measurements of an LEOSO are much less than those of a HOSO;
3) there is no deficit of measurements of LEOSOs (unlike that of HOSOs).

Conditions of detection and observation of HOSOs with the help of radars (narrow-beam and using special frequency bands) are notable for the next circumstances:

1) a HOSO can stay in a radar zone for a comparatively long time, the measuring arc of its orbit often being rather small because of great linear size of the orbit;
2) it is possible to realize an economical mode of expenditure of energy for detecting and then tracking the HOSO by a radar on

account of a specific distribution of power required for supporting the process in time.

Table 4-1-1

Accuracy of calculation of predicted (on the prediction interval Δt_{pr}) parameters α, δ, ρ obtained by 4 measurements distributed on the 6-hour arc ($\sigma_{\alpha,\delta} = 2''$) of angular position of a HOSO having the nodal period T_Ω = 6 days

Accuracy of orbit determination		Δt_{pr}, hour	0	2	4	6	8	12	16	22
σ_r, km	906	σ_α, degree	2″	16″	30″	53″	1.6′	10′	24′	16′
σ_l, km	0.7	σ_δ, degree	2″	6.6″	31″	1.3′	2.6′	5.2′	8.7′	30′
σ_b, km	20.8	σ_ρ, km	857	914	987	1076	1181	1427	1703	2167
$\sigma_{\dot{r}}$, m/s	7.2									
$\sigma_{\dot{l}}$, m/s	8.2									
$\sigma_{\dot{b}}$, m/s	0.12									
σ_{T_Ω}, s	6800									

Fig. 4-1-1 illustrates and qualitatively clarifies the latter remark. It is seen that the greatest expenditure of energy in the detection process falls on the initial stage of the whole process of detection and auto-tracking. Then, thanks to sufficient accumulation of metric information on the detected SO and (what is essentially important) gradually increasing the measuring arc, the accuracy of determination of its trajectory progressively improves, and, consequently, the frequency of emitting the energy pulses batches in the direction of the SO in the course of tracking can be progressively lowered. At last, along with this, the radar beam can be gradually narrowed (if the radar's construction can afford this).

As a result, in the mode of a single track, the gradual "accumulation of accuracy" of the orbit determination will be achieved for a HOSO with a diminution of energy expenditure.

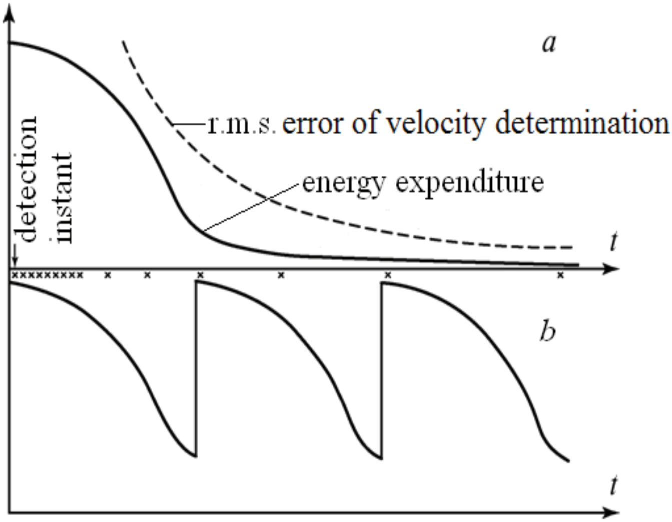

Fig. 4-1-1. Energy expenditure distribution in the modes of a single track (*a*) and multiple independent tracks (*b*) (qualitative comparison)

In the mode of multiple independent tracks, each measurement will be obtained at the greatest energy supply corresponding to the initial stage of the detection and the auto-tracking process (see Fig. 4-1-1, *b*). This fact has to be taken into account when choosing the strategy of detecting HOSOs and the radars usage tactics.

Besides, when planning the tracking of the target in the modes of detection and/or tracking, due regard should be paid to such an effect of SMAF as the inter-wind (inter-revolution) non-uniformity of the SO position errors distribution (Fig. 4-1-2).

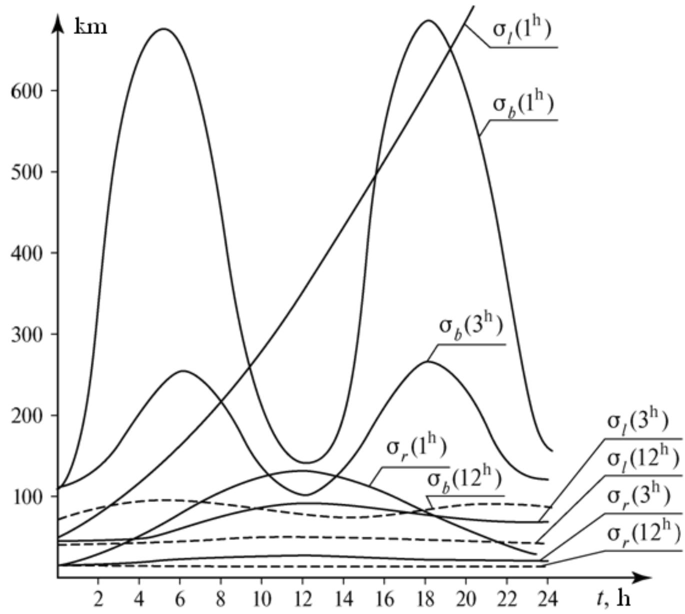

Fig. 4-1-2. Distribution of the accuracy of the GEOSO position parameters propagation within the first revolution for 3 values of the measuring interval

4.2. The search by argument of latitude

The search of a HOSO by argument of latitude is the most typical and actual particular case of search for an SO among the numerous search situations arising in the space surveillance practice and just in the day-to-day astronomer-observer's practice (due to the known property of the state vector errors evolution causing their growth predominantly along the orbit).

Later in this chapter, the treatment of the basic results of Chapter 3 in terms of the problem of search for HOSOs will be given. And then, the relevant methods of optimum search planning for different search situations arising in the space surveillance practice in the high orbit space region will be developed. The search methods by the argument of latitude lie as the basis of the majority of search procedures suggested here (although not all), the probability distribution law of the SO position in orbit being known or not.

4.3. Treatment of the basic concepts of the equivalence curves principle and its practical use

At first, we assume that the sought-for SO orbit is given by six Keplerian elements in the form of the state vector $R_0(t_0)$ and the corresponding covariance matrix $K_0(t_0)$ of errors at the initial time moment t_0 – for certainty and convenience of further consideration – at the apsidal point (although there is no extreme necessity of this assumption). Only errors along the track (in the argument of latitude u) are supposed to be essential (see above). Drawing upon the angular size of the real optical sensor's FoV, this supposition is quite substantiated for the time being. The data of Tables 2.1.1 and 2.1.2 agree with this. Nevertheless, the more general situations admitting alternative errors distribution among the orbit elements will be considered and investigated later (see Chapter 5).

The basic (initial) search interval $[u_b, u_f]$ on account of practical symmetry of the probability distribution of errors in u with respect to the apsidal point can be naturally given as

$$[u_b, u_f] = [u_0(t_0) - k\sigma_u(t_0); u_0(t_0) + k\sigma_u(t_0)] \tag{4.3.1}$$

where $\sigma_u(t_0)$ is the r.m.s. error of the value of argument of latitude u calculated at time t_0. The factor k is determined by the error probability distribution law in u and the accepted confidence probability with which the sought-for SO is supposed to be located within the orbit arc $[u_b, u_f]$ at time t_0.

The ECs apparatus developed in Chapter 3 and all results obtained there are comprehensively transferred to the case of an SO motion with the help of substitution:

$$s \equiv u,\ S = 2\pi,\ T = T_\Omega \text{ (nodal period).}$$

As soon as construing the theory in Chapter 3 in terms of motion and search for an SO is evident, it will be made only for basic notions, concepts, and findings.

The set-theoretic definition of a search plan: an arbitrary set of points $M = \{<t, u>\}$ in the tu plane is treated as a search plan (SP). Further, only continuous, piecewise continuous, and stepwise ("ladder"-like) SPs will be considered as physically realizable (feasible).

Let us denote the EC of point $<t_0, u_0>$ as $\tilde{u}_{t_0,u_0}(t)$. It can be treated as the latitude argument variation law (in time) of the sought-for SO state vector, the SO motion along a given orbit and at a given time t_0 intersecting the latitude u_0. Let us designate as

$$\sum_{t_0}^{u_b,u_f} = \{\tilde{u}(t)\}_{t_0}^{u_b,u_f}$$

the ECs family $\tilde{u}_{t_0,u_0}(t)$ by the parameter $u_0 \in [\ u_b,\ u_f]$ (t_0 being fixed) which meets the properties 1 through 5 from section 3.2 because the motion of an SO is a particular case of the cyclic motion investigated in Chapter 3 (see also [36, 39]).

As it is clear from section 3.2, there is a sense to construct only complete and non-redundant SPs (besides this, continuous, piecewise continuous, and stepwise SPs) in the region of the *tu* plane bounded by ECs $\tilde{u}_\mathrm{b} \equiv \tilde{u}_{t_0,u_b}(t)$ and $\tilde{u}_\mathrm{f} \equiv \tilde{u}_{t_0,u_f}(t)$, the SPs meeting each EC from the family $\sum_{t_0}^{u_b,u_f}$ just in one and only one point. Realization of such a SP guarantees detection of the SO with a probability the same as the confidence probability of its stay within the search interval $[u_b\ ,\ u_f]$ at time t_0 (the other conditions being favorable). And this probability is the maximum one available under the given above task conditions. A special auxiliary case when at each time moment only one point of the orbit can be checked by the sensor and the law of checking the points of the trajectory is represented by a single-valued continuous differentiable function $\bar{u}(t)$ (or it can be reduced to such a function) is first of all of theoretical and methodical interest here. Besides, an almost ideal situation close to the above one arises, for example, when the search is performed with the help of a very narrow-angle sensor with continuously re-targeting the sensor FoV (or its beam). At the same time, by equivalent substitution with conserving optimum properties, it is possible to construe a continuous one-valued SP $\bar{u}(t)$ in terms of a stepwise non-one-valued (in both directions) function (relationship) which is typical for the search of SOs with the help of comparatively wide-angle optical and electro-optical sensors having a discrete (in time) survey of space domains or radars with discrete re-pointing of the sounding beam.

As soon as the speed of the sight axis motion of real sensors is limited from above by some quantity c_0, some restriction should be put upon the function $\bar{u}(t)$:

$$\left|\frac{\mathrm{d}}{\mathrm{d}t}\,\bar{u}(t)\right| \le c_0,\ c_0 > 0, \tag{4.3.2}$$

or

$$\left|\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi[\bar{u}(t)]\right| \leq c_0,\ c_0 > 0 \tag{4.3.3}$$

where the function φ coordinates the actual displacement of the sight axis gliding along the orbit with the corresponding change in the latitude argument u.

This complication is due to the mismatch of the sensor's standing point with the center of the Earth, and there would be no necessity of such an agreement if the sensor were fantastically placed in the center of the Earth. For SOs moving in a distance of several Earth radii from the observer (such SOs are the main population of the considered class of SOs – HOSOs), the function φ is practically close to the identity function ($\varphi[\bar{u}(t)] \approx \bar{u}(t)$).

So, for analysis of ideal continuous SPs the condition (4.3.2) will do. The specification of this condition (i.e., accepting the restriction (4.3.3)) would give more efficiency to the OSP because a non-optimum arc of the orbit is placed nearer to the Earth, and there the restriction upon $\bar{u}(t)$ should be harder than for the more distant optimum arc. So that, the optimum conditions obtained in section 3.3 for search within a cycle taking account of the restriction (4.3.2) instead of (4.3.3) are absolutely guaranteed.

At last, after transfer from idealized continuous SPs to real stepwise and essentially discrete plans, the OSP methods are developed in such a way that they allow to strictly take into account the discrepancy between the angular size of the observable orbit arc $\Delta\gamma = |\varphi(u_2) - \varphi(u_1)|$ and the related increment of argument of latitude $\Delta u = |u_2 - u_1|$. That can be attained by means of the introduction of an appropriate compensation into the computed value of the sensor's field of view.

As it was established in section 3.3, a SP $\bar{u}(t)$ will be called optimum if its projection to the t axis is minimum (here and further – the criterion 1). In agreement with the basic Theorem (sections 3.3 and 3.4), the optimum SP meeting the restriction (4.3.2) is the straight-line segment $\bar{u}^{L}_{opt}(t)$ bounded by the ECs $\tilde{u}_b$ and $\tilde{u}_f$:

$$u^{L}_{opt}(t) = u_a - c_0(t - t_{opt}) \tag{4.3.4}$$

where $$t_{opt} = \frac{\tilde{u}_b^{-1}[u_a] + \tilde{u}_f^{-1}[u_a]}{2}$$

and u_a is the apogee argument of latitude.

It follows from the foregoing that the optimum region for the search is the vicinity of the SO orbit apogee. When constructing and realizing the

SP in the vicinity of apogee, one can achieve the minimum time of exploiting the facilities with no dependence on the availability or absence of the sought-for SO position probability distribution density function *f(u)* over the search interval [u_b, u_f]. For construction of the OSP, it is sufficient to have specified the position of the search interval on the orbit at a given time.

For photographic facilities, there is no use from having known the function *f(u)* as detection of the SO is possible only after the realization of the complete SP (after having developed the exposed photo materials). The same circumstance is valid for any types of sensors in case when one cannot neglect the possibility of the appearance of strange, outside, not the sought-for SOs in the sensor's FoV. If there is a possibility of finishing the search just after the acquisition of the sought-for SO in the FoV and its identification, any available information on the function *f(u)* can serve as a reserve for further perfection of the SP in terms of the reduction of the search time. The relative method will be expounded in section 4.7.

4.4. Transition to discrete search plans

So far, we dealt with an ideal sensor that controls at each time moment only one point of the SO's orbit. However, the real sensor can check at once not only an orbit point but a certain arc of it having size Δu. The acts of checking can follow one after another with the time discreteness $\Delta t \geq \Delta t_{\text{aff}}$ where Δt_{aff} is the minimum affordable value of Δt. The quantity Δt_{aff} includes the sensor re-pointing time, the time for accumulation of the intelligence signal energy at the sensitive element of the receiver, and perhaps some other technological delays. Such a mode is typical of the majority of optical, optical-electronical, radar, and laser facilities for searching and observing HOSOs. And such a mode requires a discrete SP.

In the *tu* plane the act of check is represented by an SP element μ_I in the form of a vertical straight-line segment of size Δu with its center in point $< t_i, u_i >$. A discrete SP is represented by a set of SP elements μ_I following each other with the step $\Delta t_i \geq \Delta t_{\text{aff}}$ along the t axis.

The algorithm for constructing the simplest discrete search plan equivalent to the continuous OSP (in the sense of criterion 1) is as follows. Such an SP is constructed in the form of a "ladder" beginning with the middle step μ_{mid} the center of which is superposed with point $< t_{opt}, u_a >$. The ladder is augmented in both directions up to meeting the ECs $\tilde{u}_b$ and $\tilde{u}_f$, respectively, with the time step Δt_i. Transitions between steps are performed along ECs: the beginning (bottom) of the element μ_I is connected with the end (top) of the element μ_{i+1} along the corresponding EC (coming through both points).

In case if the osculating Keplerian orbit elements are used, it is enough to introduce some easily calculated corrections into the rest 5 components of the Keplerian state vector when performing a transition from one EC to another along the t axis. But if the average Keplerian orbit elements are used, no calculation and introduction of corrections are needed.

And now some elucidation on the search observation acts discreteness ought to be given. The time step Δt between two adjacent SP elements μ_i and μ_{i+1} (in the most organic cases in space surveillance practice $\Delta t_i = \Delta t = const.$) contains as addends the time of exposition, the time of re-pointing of the sensor's sight axis, and, perhaps, some time expenditures for some auxiliary and program (technological) operations. The exposition time includes first of all the time of accumulation of the intelligence signal energy at the same point of the sensitive element of the receiver (for acquiring faint signals).

The size Δu_i of an SP element μ_i can be determined, for example, from the equation

$$\sqrt[2]{[\alpha\left(t_i, u_i + \frac{\Delta u_i}{2}\right) - \alpha\left(t_i, u_i - \frac{\Delta u_i}{2}\right)]^2 + [\delta\left(t_i, u_i + \frac{\Delta u_i}{2}\right) - \delta\left(t_i, u_i - \frac{\Delta u_i}{2}\right)]^2} = \Delta\gamma. \quad (4.4.1)$$

And for a distance between the observer and the checked point of the trajectory about several radii of the Earth one is allowed to put $\Delta u_i = \Delta u \approx \Delta\gamma$. Here, $\Delta\gamma$ is an angular size of the sensor's FoV (or beam), α and δ are the equatorial topocentric coordinates of the beginning and the end of the orbit arc embedded into the sensor's FoV (or beam), the sight axis of the sensor being pointed to u_i at time t_i.

Sometimes, a special search mode is very convenient and even the most effective for a number of criteria (see below) - the so-called "search on the border (frontier) u^*" mode. It is planned on the basis of the procedure for decomposition of the continuous OSP described in section 3.4 and the next transit to the piecewise SP with the purpose of further improvement of its quality in the sense of the criterion 1 (see Fig. 3-4-4). Then continuous elements of the final plan are transformed by the equivalence principle to the vertical segments like μ_i (see the same Fig. 3-4-4).

Practically, an SP of the type “search on the border (frontier) u^*” is conveniently constructed, say, beginning from the last element $\mu_{\bar{\imath}}$ the center of which should coincide with the latitude u^* whereas its beginning should rest upon the EC $\tilde{u}_b$, namely at the point timed by

$$t = \tilde{u}^{-1}_{t_0,u_b}[u^* - \frac{\Delta u_i}{2}].$$

Then the centers of all the rest of the SP elements μ_i are placed at the latitude u^* and the beginning of the element $\mu_{\bar{\imath}-n}$ coincides with the point of intersection of the EC having the end of the element $\mu_{\bar{\imath}-(n-1)}$ as its generating point with the latitude

$$u = u^* - \frac{\Delta u_{\bar{\imath}-n}}{2}.$$

In cases when $\Delta u_i = \Delta u = const.$ (see above), it is enough to construct the single SP element μ_i and to find the point of meeting of its end

$$< \tilde{u}^{-1}_{t_0,u_b}[u^* - \frac{\Delta u}{2}], u^* >$$

with the latitude $u = u^* - \frac{\Delta u}{2}$ having been determined by the step Δt forming a sequence of the SP elements. Now the simplicity of constructing the rest of the SP elements μ_i is obvious (see Fig. 4-4-1).

It is evident that the approach of point u^* to point u_a entails enhancing the efficiency of SPs.

The merits of the “search on the border” strategy are:

- simplicity of the SP construction,
- small variations of the quantity Δu,
- the constancy of the targeting point,
- the greater efficiency of the related SP in comparison with the continuous OSP and the “ladderwise” SP (in the sense of criterion 1) for $u^* = u_a$.

Note also the disadvantage of this strategy: achieving the optimum result (OSP) in it (in the sense accepted above) can lead to too long-time intervals between successive (contiguous) elements of the search plan. This raises the additional problem of rational use of the sensor resource within these intervals. True, one can mitigate the situation somewhat. If the SP envisages several tracks and, at the same time, its elements are located rather sparse (Δt technologically came out great), then the interruptions between them can be usefully utilized for the realization of elements of alternative SOs' SPs having the same or close "border u^*".

The SP constructed in such a way of any of the described (above and below) kinds is not yet a final product of planning the search. After obtaining the latter plan, one should proceed, in the simplest case, with calculation of the conditional ephemerides (the sensor aiming data), using the centers $<t_i , u_i>$ of the SP elements μ_i as the initial data for addressing then the SO motion model.

For this, one uses the conditional orbit state vectors obtained as a result of propagation of the initial (a priori) state vector $R_0(t_0)$ to times t_i, $i = 1, 2, \ldots$, and substitution of argument of latitude values u_i (the necessary linear corrections into the other 5 orbital elements being introduced) instead of the value $u_{R_0}(t_0) = u(R_0(t_0))$.

For the purpose of providing the most favorable conditions for concentration and accumulation of the faint intelligence signal energy in one point of the sensitive element of the receiver, the search facility should realize at the i-th stage of the SP implementation not only one fixed SP element μ_i at time moment t_i but a set of special auxiliary elements during the exposition time interval

$$(t_i^-, t_i^+) \equiv (t_i - \frac{\Delta t_{exp}}{2} ; t_i + \frac{\Delta t_{exp}}{2})$$

having the duration Δt_{exp} (see Fig. 4-4-2).

Timed to the beginning of exposition t_i^-, the conditional ephemeris (α, δ) is calculated drawing upon the center $<t_i^-, u_i^->$ of element μ_i^- which is the EC-projection of the SP element μ_i to time t_i^-. Starting from this moment, the sensor's receiver should be given the certain motion (with the speed determined by the conditional rate of latitude variation u' along the central EC) which will lead the sensor's sight axis in the time Δt_{exp} to the position corresponding to the element μ_i^+, the latter being the EC-projection of the SP element μ_i to time t_i^+ (the right end of the exposition).

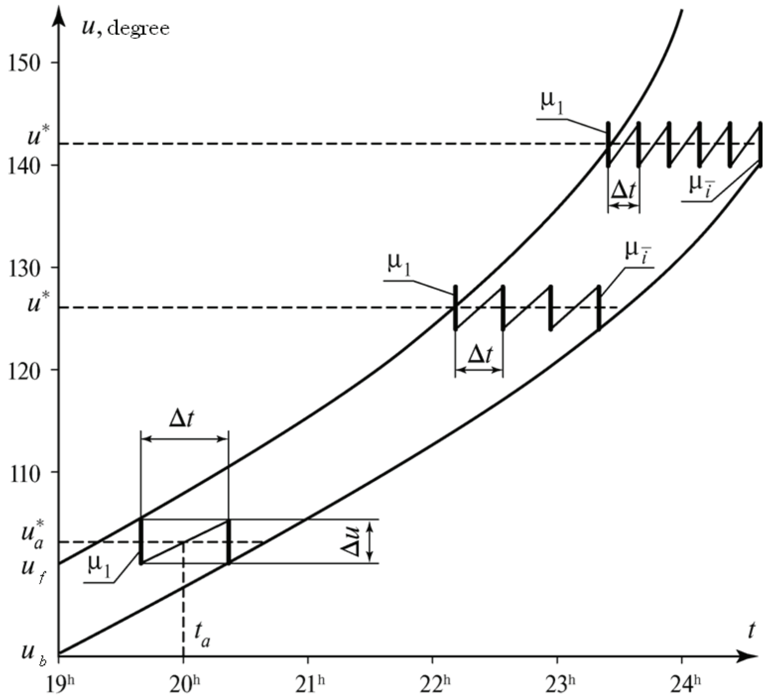

Fig. 4-4-1. Planning the search in the "on the border" mode for a HOSO of the class SDS or "Molniya"

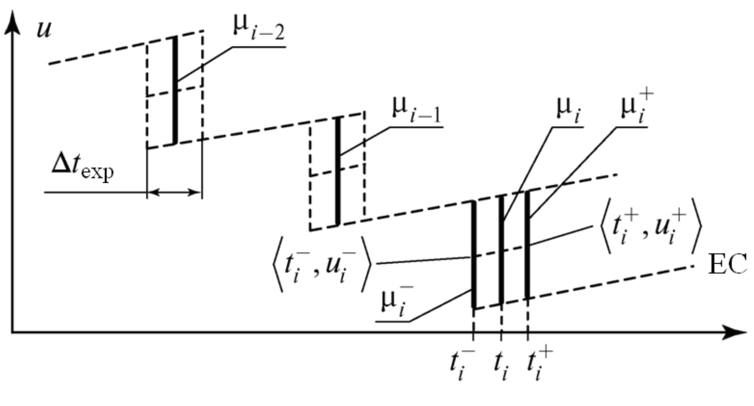

Fig. 4-4-2. Search plan with compensation of the SO motion

For comparatively small values of Δt_{exp}, it is sufficient to calculate the conditional coordinates α and δ referred to the center of element μ_i^- and the rates of their variations $\dot{\alpha}$ and $\dot{\delta}$ in accordance with which the linearized motion of the receiver is given.

So, the SP element μ_i constructed earlier is, sort of, "smeared" (washed away, blurred) along the EC coming through its center. Then the sensor realizes these "smears" ("blurs") by gliding with its sight axis along the conditional orbit (or by the equivalent motion of its receiver).

The technique proposed here can be applied not only for searching for an SO but also for observing an SO with faint brightness and/or moving quickly in the sensor's FoV even if rather exact ephemerides are available.

4.5. Presentation of real observation conditions in the *tu* plane

In the previous sections, the optimum conditions of search for an SO were determined without account of the real restriction upon the observation process, except that of (4.3.2). However, for each site its own astronomic and astroclimatic conditions of visibility exist. Besides, at every time moment only a limited arc of the SO's orbit depending on time is available for the observer. The local twilight and night, for example, are represented in the *tu* plane by vertical stripes Q (Fig. 3-4-2). The conditions of angular visibility of the orbit arc from a given site are represented by more arbitrary and complicated (by their configurations) regions R (see the same Fig. 3-4-2).

If for the time period of planning the search a forecast of favorable and unfavorable meteorological conditions of observations at the certain sky regions is known (and such a forecast is quite reliable if planning the search is accomplished immediately before the observations or in their course), then the favorable conditions of observations can be depicted in the *tu* plane in the form of regions R_1, R_2, and so on (Fig. 3-4-2). Then only the part of the SP should be realized which belongs to the region $P = Q \cap R \cap R_1 \cap R_2 \ldots$ that, additionally, should be corrected with due regard to the SO's phase of illumination.

If the constructed SP at a given revolution of the SO is not completely embedded into the "favorable" region P, the rest (not realized) part of the SP can be transferred by the equivalence to the next revolutions (Fig. 3-4-2). To avoid the latter operation, one can try to forcibly embed the SP into the closest region P at the cost of some deviation from the optimum SP.

4.6. Assumption of presence of errors in all orbit parameters

From the previous research and experience, we know that in the practice of space surveillance the problem of the search for a HOSO often can be reduced to that of the search by argument of latitude *u* at one track. However, in case when the errors in the available (a priori) orbital information are stipulated not only by strictly ballistic causes and/or the SO state vector error evolution, the sought-for SO current position uncertainty domain (CPUD) can have a significant size and a complicated structure. That is caused by the presence of non-neglectable errors not only along the track (that is, in *u*) but in other orbit elements as well.

If such a complicated distribution of the SO state vector errors takes place, the search domain can be split into some orbit "layers" taking into account the size of the sensor's FoV. And then it is possible to perform the search in several tracks (in accordance with the number of "layers"). The simplest way of decomposition of the search domain to layers is with the help of tables like Table 4-6-1. The latter contains the maximum values of errors in ephemerides ($\Delta\alpha$ and $\Delta\delta$) stipulated by different errors in the 5 Keplerian orbit elements:

a – semi-major axis,
e – eccentricity,
i – inclination,
Ω – right ascension of the ascending node,
ω – argument of the perifocal point
for typical orbits of HOSOs.

In such cases when constructing the SP, one ought to admit the intentional diminution of the FoV sizes in order to compensate for the declination of the elements' values owing to those errors at the cost of the unaccounted part of FoV.

Of course, this approach cannot be considered elegant. And it is given here only didactically as one of the possible ways before a special theory is developed that covers all cases of the occurrence and distribution of the state vector errors. An appropriate universal (and more elegant) approach will be presented in Chapter 5.

Table 4-6-1

Maximum errors of the ephemerides calculation for a quasi-GEOSO (a_0 = 42106 km, e_0 = 0.001, i_0 = 0.002°, ω_0 = 0°, Ω_0 = 270°) caused by the errors in orbit elements (here $\Delta \equiv \sqrt{\Delta\alpha^2 + \Delta\delta^2}$)

Δ*a*, km	**Δ*T***	Δ_{max}	**Δ*e***	Δ_{max}	**Δ*i***	Δ_{max}	**Δω**	Δ_{max}	**ΔΩ**	Δ_{max}	$\Delta\delta_{max}$
0.65	2 s	0.5″	$0.0^5 33$	0.1″	4″	4.5″	40′	0″.37	2′	2′15″	1,5″
1.3	4 s	1″	$0.0^4 1$	0.3″	20″	22″.5	1°	0″.55	10′	11′17″	6,5″
2	6 s	1.47″	$0.0^3 1$	3″	40″	44″.7	2°	1′11″	20′	22′30″	13″
3.3	10 s	2.5″	$0.0^3 2$	6″	1′	1′07″	4°	2′23″	40′	44′50″	26″
11	337 s	8.43″	$0.0^3 5$	15″	2′	2′14″	6°	3′35″	1°	1°03′	40″
39	120 s	30″	0.001	30″	10′	11′10″	10°	6′			
110	337 s	1′25″	0.002	1′	30′	33′30″	20°	12′			
220	675 s	2′50″	0.0033	1′39″							
66	34 min	8′30″	0.0075	3′22″							
3300	2.8 h	42′	0.01	4′34″							

Maximum errors of calculation of ephemerides of a HEOSO ($a_0 = 26554$ km; $e_0 = 0.734$; $i_0 = 62^\circ.9$; $\omega_0 = 318^\circ.2$; $\Omega_0 = 102^\circ$) *in its apogee* (*here* $\Delta \equiv \sqrt{\Delta\alpha^2 + \Delta\delta^2}$)

Δ*a*, km	**Δ*T***	Δ_{max}	**Δ*e***	Δ_{max}	**Δ*i***	Δ_{max}	**Δω**	Δ_{max}	**ΔΩ**	Δ_{max}	$\Delta\delta_{max}$
0.411	1 s	0,5″	$0.0^3 33$	6″	4″	4,3″	30″	0.001″	20″	22″	1″
0.822	2 s	1″	0.001	18″.3	10″	11″	1.5′	0.01″	1′	1′06″	3″
1.23	3 s	1.47″	0.002	36.5″″	30″	32.5″	10′	0.35″	5′	5′30″	15″
1.64	4 s	2″	0.004	1′13″	1′	1′05″	30′	3.3″	10′	11′	30″
2.1	5 s	2.45″	0.006	1′40″	2′	2′09″	1°	13.3″	20′	22′	1′
4.1	10 s	5″	0.01	3 ′	10′	11′	1,5°	30″	1°	1°06′	3′
14	34 s	16.7″	0.02	6′	20′	21′.5	2°	53″			
70	170 s	1′23″	0.04	12′	1°	1°05′	3°	2′			
140	341 s	2′47″	0.1	30′			5°	5′32″			
210	8.5 min	4′					10°	22′10″			

Maximum errors of the ephemerides calculation for the same HEOSO outside the apogee vicinity (here $\Delta \equiv \sqrt{\Delta\alpha^2 + \Delta\delta^2}$*)*

	Δ*a*, km	**Δ*T*, c**	Δ_{max}	**Δ*e***	Δ_{max}	**Δω**	Δ_{max}
u_a– 45°	0.411	1	1.15″	$0.0^3 33$	42″	30″	12″
				0.001	2′05″	1,5′	35″
				0.01	22′	10′	4′
u_a– 90°	0.411	1	2.3″	$0.0^3 33$	2′37″	30″	16″
	0.82	2	4.5″	0.001	8′	1.5′	47″
				0.01	1°22′	10′	5.2′

The more laborious although, at the same time, the more flexible way of solving the latter problem is adapting the scheme of the search domain decomposition to layers to concrete conditions of search.

Instead of addressing Table 4-6-1, basing upon the covariance matrix of errors or maximum errors in different orbit elements, the corresponding values of errors in right ascension α and declination δ are calculated taking account of a given class of orbits, a chosen search domain, a given sensor's site coordinates (if it was assigned beforehand), and the sensor's operational characteristics (such as the FoV size, the speed of the sensor re-pointing, and so on).

From the analysis of the influence of errors in the orbit elements a, e, ω upon the accuracy of the calculation of the SO's ephemerides, one more important property of the optimum search region follows: the errors in elements a, e, ω affect the least when planning the search for a HOSO is performed in the vicinity of the orbit apogee. The further the search area in orbit from its apogee, the greater the impact of errors in these elements on the search result.

This is confirmed by at least the following example. From Table 4.6.1 one can see that for a HEOSO like SDS or "Molniya" the error $\Delta\omega = 30''$ in the vicinity of apogee results in errors in α and δ of not more than 0,001″. For deflection from apogee by 45° it leads to the errors about 12″, i.e., 10 000 times more. The errors in the ascending node longitude Ω for a HOSO are practically completely transformed into $\Delta\alpha$, i.e., $\Delta\Omega \approx \Delta\alpha$ (for quasi-GEOSO – into Δu). In the last case, the error in Ω is easily regarded by expansion of the search interval $[u_b, u_f]$ by the value of $\Delta\Omega$ in both directions.

In the common case ($i \neq 0$), an SP is constructed as shown in Fig. 4-6-1. That is, each SP element μ_n is multiplied by the EC-projection to a group $\{\mu_{n,k}\}$, $k = -\bar{k}, \ldots 0, \ldots \bar{k}$. The integer $\bar{k}$ is chosen from the equation

$$(2\bar{k} + 1)\Delta\gamma = \Delta\Omega\,.$$

By the centers $< t_{n,k},\ u_{n,k} >$ of SP elements $\mu_{n,k}$, the basic ephemerides of support (the reference ephemerides $(\alpha_{n,k},\ \delta_{n,k})$), in which the right ascension is then corrected by means of addition of the quantity $k \cdot \Delta\gamma$, are calculated using an ordinary technique (see, for example, section 4.4). The corrected ephemeris of support $((\alpha_{n,k} + k \cdot \Delta\gamma),\ \delta_{n,k})$ is then realized in the form of a "smear" ("blur") (see section 4.4 and Fig. 4-6-1 here).

Let us consider the possibilities of enhancing the economy of a complex (in several tracks) SP when significant errors are available in all orbit elements. For this, we introduce some necessary notations and concepts.

By definition, $M_m \equiv \{\mu_n{}^m\}$ is an arbitrary feasible SP for the m-th track, $m = 0, 1, 2, \ldots \overline{m}$; $n = 1, 2, \ldots$ $\overline{n}_m$; $\overline{n}_m \equiv \overline{n}(m)$.

The set

$$\bigcup_{k=0}^{\bar{k}} \widetilde{M}_k \equiv \bigcup_{k=0}^{\bar{k}} \{\widetilde{\mu}_n^k\}$$

is a complex SP (including $\bar{k}$ tracks) constructed drawing upon independent decompositions of the intervals of the orbit elements possible values (as described above).

By definition, A_t is the plane of equatorial topocentric coordinates (α, δ) corresponding to time t.

By definition, the set $a^M(\mu(t),\ t_0)$ is a set of equatorial topocentric coordinates (α, δ) controlled (checked) by the $EC^{(M)}$-projection of the SP element $\mu \in M$ at time t_0.

By definition, a point $< t, u >_{\widetilde{\mu}}$ of some element $\widetilde{\mu}$ of SP $\widetilde{M}_k$ is co-checked by the element $\mu_n{}^m$ of SP M_m (which is designated as $< t,\ u >_{\widetilde{\mu}} \nearrow \mu_n{}^m$) if

$a^{\widetilde{M}_k}(< t, u >_{\widetilde{\mu}}^{\widetilde{M}_k},\ t_n{}^m) \subseteq a^{M_m}(\mu_n{}^m,\ t_n{}^m)$ where $< t, u >_{\widetilde{\mu}}^{\widetilde{M}_k}$ is the $EC^{\widetilde{M}_k}$-projection of point $< t, u >_{\widetilde{\mu}}$ to time $t_n{}^m$.

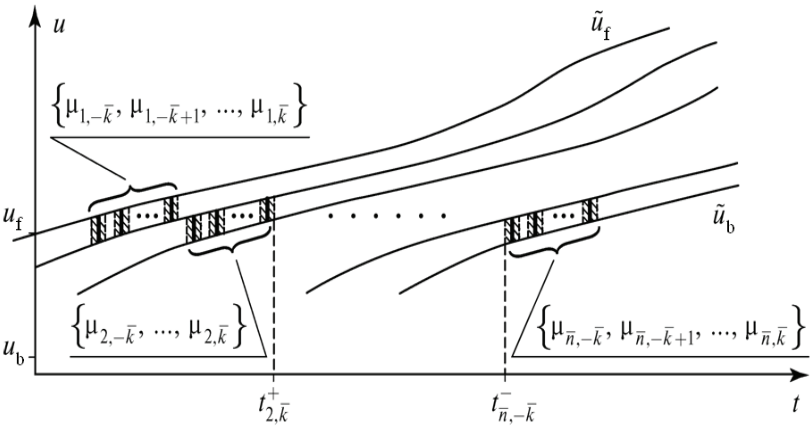

Fig. 4-6-1. A diagram for constructing a complex SP compensating the SO motion and the error in ascending node Ω

By definition, $\widetilde{\mu} \nearrow M$, if $\forall < t, u >_{\widetilde{\mu}} \in \widetilde{\mu}\ \exists\ \mu' \in M$: $< t, u >_{\widetilde{\mu}} \nearrow \mu'$.
By definition, $\widetilde{M} \nearrow M$, if $\widetilde{\mu} \in \widetilde{M} \Rightarrow \widetilde{\mu} \nearrow M$.

By definition,

$$\widetilde{M} \nearrow \bigcup_{m=0}^{\bar{m}} M_m \text{, if } \forall < t, u >_{\widetilde{\mu}} \in \widetilde{\mu} \in \widetilde{M}\ \exists\ M_m : < t, u >_{\widetilde{\mu}} \nearrow M_m$$

and also

$$\bigcup_{k=0}^{\bar{k}} \widetilde{M}_k \nearrow \bigcup_{m=0}^{\bar{m}} M_m \text{, if } \forall k\ \ \widetilde{M}_k \nearrow \bigcup_{m=0}^{\bar{m}} M_m.$$

Now we produce a statement of the basic optimization problem:

$$\sum_{\mathrm{m}=0}^{\bar{m}} |M_m| \Rightarrow \min \tag{4.6.1}$$

under the condition

$$\bigcup_{k=0}^{\bar{k}} \widetilde{M}_k \nearrow \bigcup_{m=0}^{\bar{m}} M_m\ . \tag{4.6.2}$$

Here, $|M_m|$ is the number of SP elements in M_m .

The principle of constructing suboptimum SPs

For obtaining a suboptimum solution of the problem (4.6.1 – 4.6.2), at first, the basic SP $\widetilde{M}_0$ (an SP of support) is constructed using the initial state vector R_0. Each subsequent SP of the k-th track $\widetilde{M}_k$ is corrected in the following way. The element $\widetilde{\mu}_n^k$ is included into the corrected SP

$$\widetilde{M}_k^* = \{ {}^*\widetilde{\mu}_s^k \} \subseteq \widetilde{M}_k$$

if and only if the next condition is violated:

$$\widetilde{\mu}_n^k \nearrow \widetilde{M}_0 \cup \left(\bigcup_{\nu=1}^{k-1} \widetilde{M}_k^* \right) \cup \left(\bigcup_{s=1}^{n-1} \widetilde{\mu}_s^k \right) \equiv M_{k-1,n-1}^*. \qquad (4.6.3)$$

The resulting solution is the union of SPs:

$$\widetilde{M}_0 \cup \left(\bigcup_{k=0}^{\bar{k}} \widetilde{M}_k^* \right).$$

One can offer the more efficient technique that differs from the above in the application of the next "active" operation of checking the condition (4.6.3). In case of its violation, $\widetilde{\mu}_n^k$ is corrected in such a way that its $a^{\widetilde{M}_k}$-image (i.e., $a^{\widetilde{M}_k}(\widetilde{\mu}_n^k(t), t_p{}^\nu)$) as much as possible is shifted (taking account of the whole FoV size) into the region

$$A_{t_p^\nu} \setminus a^{\widetilde{M}_\nu^*}({}^*\widetilde{\mu}_p^\nu(t_p{}^\nu), t_p{}^\nu)$$

on the condition

$$a^{\widetilde{M}_k}(\widetilde{\mu}_n^k(t), t_p{}^\nu) \setminus a^{\widetilde{M}_\nu^*}({}^*\widetilde{\mu}_p^\nu(t_p{}^\nu), t_p{}^\nu) \subseteq a^{\widetilde{M}_k}({}^*\widetilde{\mu}_p^\nu(t), t_p{}^\nu) \qquad (4.6.4)$$

where ${}^*\widetilde{\mu}_p^\nu(t_p{}^\nu)$ is an element of SP $\widetilde{M}_\nu^*$ by which the part $\widetilde{\mu}_p^k$ is co-checked up.

There is a possibility of enhancing the efficiency of such a correction by propagating the condition (4.6.4) to the rest elements of $M_{k-1,n-1}^*$ in an evident way.

The bulkiness of all notations, auxiliary concepts, and terminology applied here should not be referred to as a demerit since such a symbolism accounts for adequacy, crispness, and transparency of deciphering and conceiving of the relationships obtained. However, appreciating the clearness of the sense of the suggested operations one should gain their constructivism. Besides, when planning the search for a HOSO having faint brightness with the help of these techniques along with checking the condition (4.6.3), it is necessary to check also the similar condition with respect to velocity which can, of course, greatly complicate the whole procedure.

However, you should not be upset. In the present monograph, much more advanced methods of solving this and more complex problems are developed. More radically, harmoniously, and in an orderly manner, the search problem (when significant errors are available in all orbit elements) can be solved from the general and united theoretical position, with the help of the general methodology of planning the search for an SO using imprecise a priori information with no restrictions upon the distribution of the initial state vector errors in its components [16, 25, 42, 2, 52, 53, 54] which is given below (see Chapter 5).

The negative results of observations represent the information no less useful than a priori information of any other kind. Albeit it ought to be dealt with correctly and carefully. When planning the search for an SO, it is advisable to consider the negative outcome of observations performed earlier. This can help narrowing the region of subsequent search and enhancing effectiveness of the SP. With this, a posteriori function of the sought-for SO position probability distribution density $f(u)$ may appear piecewise and the basic search interval – n-tuply connected.

In section 3.4, a means was described for considering the information obtained in the preceding sessions of observations when transferring to the next one (see Fig. 3-4-2).

The procedure of planning the search for an SO using rough orbital information described here can be considered as a common technique of planning the search for an SO after a "maneuver" by any one of the 6 orbital elements. Using additional information on the maneuver results in a more economical technique (see sections 4.8 and 4.10).

4.7. The search by the criterion of detection time

Now it's time to move on in the formulation of the search problem from the economic criterion to the operational one. Unlike the economic criterion 1 (see sections 4.3 and 4.4), there exist indices affecting operational properties of SPs, for example, a mathematical expectation of the SO detection time (or duration), the time instant of the possible beginning of the search t_0 being given (here and further – criterion 2).

In some cases, criteria 1 and 2 contradict each other (for example, if the moment t_0 is rather distant from the time when the SO comes through the apogee) which can be accounted for by the different goals of the concrete search problems.

From a given covariance matrix of errors $K_0(t_0)$, one can reproduce the function $f_0(u)$ of the sought-for SO position probability distribution density over the search interval at time t_0. For a given function $f_0(u)$, the problem of OSP construction by the criterion 2 is isomorphic to the problem expounded in detail in section 3.7 where the method of getting the precise solution with the help of the "branch-and-bound" scheme with the application of ACR and some sub-optimum methods are given. That is why there is no sense to repeat them here. It is sufficient to substitute variable *u* instead of variable *s* in section 3.7.

As for detecting an SO in the shortest time, the error distribution law $f_0(u)$ being unknown, one can address section 3.8 *mutatis mutandis* with the similar comments.

In Fig. 4-7-1, a sequence of synthesis of the OSP for a HEOSO like SDS or "Molniya is shown using an optical sensor having an FoV with the size 3° and the discreteness of expositions 15 minutes, the basic search interval having the size 12° in the vicinity of apogee.

And now a very important (both theoretically and practically) remark should be made. The suggested method for constructing OSP by the criterion 2 has an important practical property: implementation of the OSP calculated using too high a value of the variable σ_{0u} (that can happen on purpose, erroneously, or owing to a great uncertainty of the initial data) for the normal law of error distribution in argument of latitude (at the epoch of coming through apogee) guarantees the minimum detection time (as if it were real and much less).

And it looks like a new important theorem. In other words, *an arbitrary increase of the value of the down track error (as compared with the real one) does not violate the optimum property of the SP*. The more so, this theorem has sufficient generality and is valid not only for continuous and piecewise continuous OSPs but, in some important cases, also for those like a "ladder" and some others. *It's universal for application*.

It is hard to overestimate the practical importance of this theorem, which can be called as **Theorem of conservation of optimality**. It allows, with the help of OSP, to neutralize or, at any rate, to mitigate the negative influence of random errors and deformations of a priori information on the sought-for SO and to mitigate the consequences of its unreliability, i.e., it "renders a service" in the hardest and deadlock situations (for example, in cases when we do not know sufficiently accurate characteristics of the sought-for object position errors).

We do not hesitate to recall that a very important result has just been formulated.

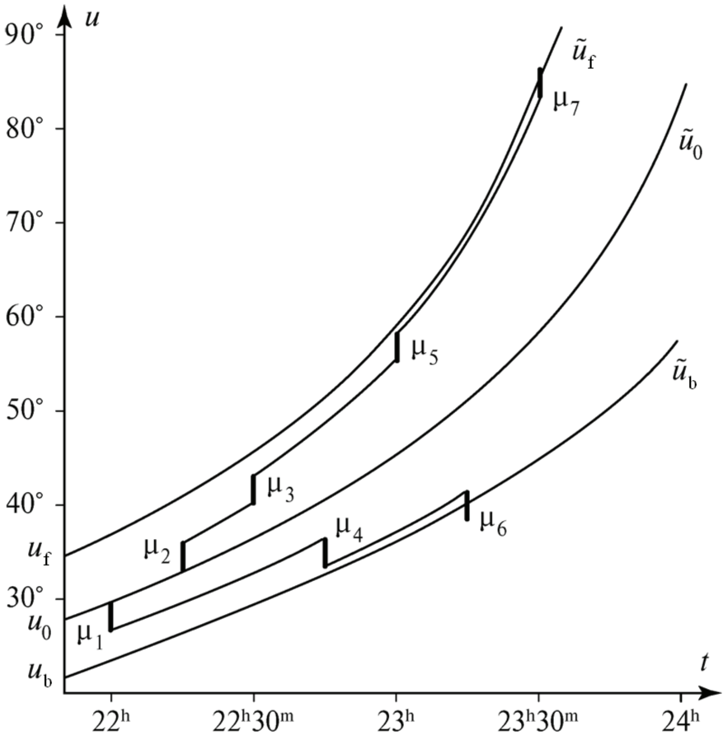

Fig. 4-7-1. OSP for a HEOSO like SDS or "Molniya" by the criterion 2

4.8. The search for an object (a spacecraft) after its maneuver

As the practice confirms, in great majority of cases, a maneuver of the spacecraft motion is performed in the orbit plane by means of a change of the revolution period T_{Ω}. In the absence of information on the value of this change ΔT, one can suggest or compute the possible maximum change ΔT_{max} drawing upon the nominal value of the geographic latitude of the ascending node λ_{nom}, its current value, and the width of range of the admissible values of λ (when the station keeping operation is made). And, for example, for some spacecraft it is possible to use the average statistical value of ΔT if such corrections statistics are available (see also section 4.10).

In case of a maneuver for transferring a GEOSO to an alternative stationary point (this is usually provided operationally in accordance with the spacecraft mission), the quantity ΔT (or ΔT_{max}) can be determined drawing upon the nominal or maximum speed of the SO's affordable drift in latitude and, at any rate, the possible maximum value of the orbit keeping pulse. Theorem of conservation of optimality (see section 4.7) allows us to overstate the value of ΔT_{max} without dangerous consequences.

And now we suppose that the maneuver by the value of period T_Ω leaves all the rest of the Keplerian orbital elements unchanged. For such a situation, the assumptions (3.9.2) and the results of section 3.9 are valid where it is shown that when the deflection of the determined value of period T from the real one is available, the involved deformations of the ECs family in which the SP is being constructed show up in the least way in the vicinity of points with minimum values of the derivative $\tilde{u}_t'$, that is, within the apogee vicinity.

Consequently, the optimum conditions for search for a HOSO after a maneuver or orbit correction take place in a region immediately preceding the orbit's apogee, the optimum of this region on the subsequent revolutions converging to the point with the minimum value of derivative u_t'.

Let the vector of orbit parameters $R_0(t_0)$ at time t_0 be known almost exactly. The HOSO motion is represented in the tu plane by the EC $\tilde{u}_0(t_0, T_0)$ where T_0 is the SO motion period just before the maneuver. Now we consider T as a parameter of the ECs family $\tilde{u}_{t_0,u_0}(t,T)$ and denote the lower boundary of the maneuver beginning time as t_0^{min} and suppose it being equal to the time instant when the SO concerned was observed for the last time in its former (not yet corrected) orbit (if there is no alternative or additional information). In section 4.10, a means is shown how this estimate can be specified.

Now, let us consider the problem for different kinds of available information on the maneuver.

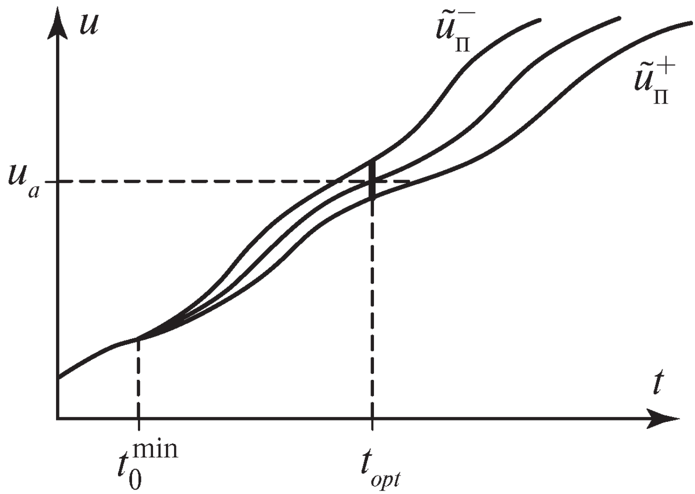

Fig. 4-8-1. Construction of an SP on a pencil of ECs

1. The first kind of such data is when there is minimum information on the maneuver, i.e., the maximum absolute value of the period changes $|\Delta T|_{\max}$ is given. The SP is constructed on the basis of a bunch of ECs coming from the point $<t_0^{min}, \tilde{u}_0(t_0^{min}, T_0)>$ (see Fig. 4-8-1):

$$\sum_{t_0^{min}}^{T_0 \pm |\Delta T|_{\max}} = \{\tilde{u}_{t_0^{min}, \tilde{u}_0(t_0^{min}, T_0),}(t, T)\}_{T \in [T_0 - |\Delta T|\max;\, T_0 + |\Delta T|\max}. \quad (4.8.1)$$

The optimum region of control (according to section 3.9) has an asymptotic center at the point with the minimum value of derivative $\frac{\partial}{\partial t}\tilde{u}_0(t, T_0)$, i.e., at point $<t_{opt}, u_a>$. For this point, the size of the orbit arc to be checked is equal to

$$\Delta u^{\Pi}{}_{opt} = \tilde{u}_{\Pi}^{-}(t_{opt}) - \tilde{u}_{\Pi}^{+}(t_{opt}) = 2\tilde{u}_{0t}^{'}(t_{opt}, T_0)\frac{|\Delta T|_{\max}}{T_0}(t_{opt} - t_0^{min}) \qquad (4.8.2)$$

where

$$\tilde{u}_{\Pi}^{-} \equiv \tilde{u}_{t_0^{min},\ \tilde{u}_0(t_0^{min}, T_0),}(t,\ T_0 - |\Delta T|_{\max}),\ \tilde{u}_{\Pi}^{+} \equiv \tilde{u}_{t_0^{min}, \tilde{u}_0\left(t_0^{min},\ T_0\right),}(t, T_0 + |\Delta T|_{\max}).$$

If the magnitude Δu^{Π}_{opt} exceeds the angular size of the sensor's FoV, one can realize the mode of search "on the border $u* = u_a$" with subsequent fragmentation (decomposition) of the initial SP μ_0 (Fig. 4-8-2).

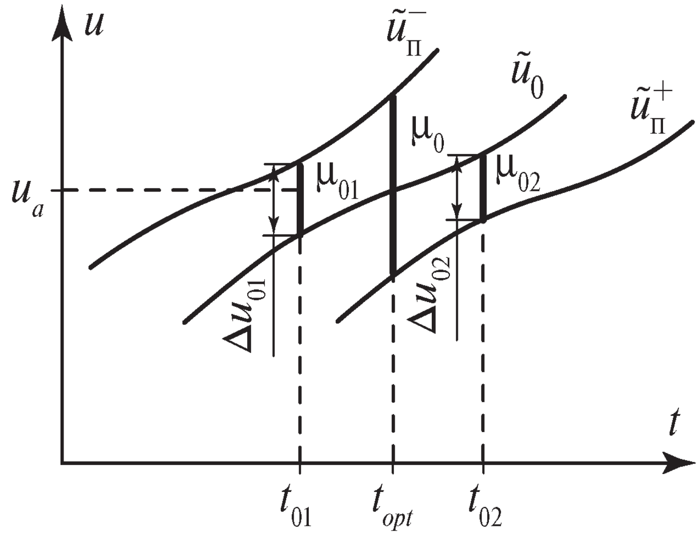

Fig. 4-8-2. The search in the "on the border" mode

At the first stage of such a decomposition, we obtain the SP elements μ_{01} and μ_{02} with the coordinates of their centers equal to

$$u_{01} = u_{02} = u_a\,,\ t_{01} = t_{opt} - 0{,}5\frac{|\Delta T|_{\max}}{T_0}(t_{opt} - t_0^{min}),$$
$$t_{02} = t_{opt} + 0{,}5\frac{|\Delta T|_{\max}}{T_0}(t_{opt} - t_0^{min}),$$

respectively, and the sizes of their SP elements will be

$$\Delta u_{01} = \Delta u_{02} = \tilde{u}^{-}_{\Pi}(t_{01}) - \tilde{u}_0(t_{01}, T_0) < 0{,}5\ \Delta u^{\Pi}_{opt}. \qquad (4.8.3)$$

If the values of Δu_{01} and Δu_{02} are still great, the fragmentation of the SP may be continued. It follows from the inequality in (4.8.3) that the size of a simultaneously checked up orbit arc in the process of fragmentation of the SP decreases faster than the function $2^{-n}\Delta u^{\Pi}_{opt}$ where n is an ordinal number of fragmentation stage. This function can be considered as an upper estimate ("no worse than") of the growth in the original ("optimum") SP discretization process economy depending on the number of its fragmentation stages (the fewer elements, the more economical process). Here, the economy is treated in the sense of the total (summary) orbit arc to be checked.

2. The maximum and minimum possible values of the orbital period change $\Delta T_{\max}$ and $\Delta T_{\min}$ are known and they are of the same sign – for certainty, negative. Of course, $|\Delta T_{\max}| > |\Delta T_{\min}|$). This case is quite typical for GEOSO orbital corrections.

Let t_0^{min} and t_0^{max} be the earliest and latest, respectively, possible time instants of application of the maneuver impulse. In this case, the SP is constructed on a pseudo-bunch of ECs (4.8.4) (see Fig. 4-8-3):

$$\sum_{t_0^{min},\, t_0^{max}}^{T_0+\Delta T_{\min},\, T_0+\Delta T_{\max}} = \{\ \tilde{u}_{t^0,\,\tilde{u}_0(t^0,\,T_0)}\,(t, T)\ \}, \qquad (4.8.4)$$

where

$t^0 \in [t_0^{min}, t_0^{max}]$, $T \in [T_{\min}, T_{\max}]$,

$T_{min} = T_0 + \Delta T_{\max}$, $T_{max} = T_0 + \Delta T_{\min}$.

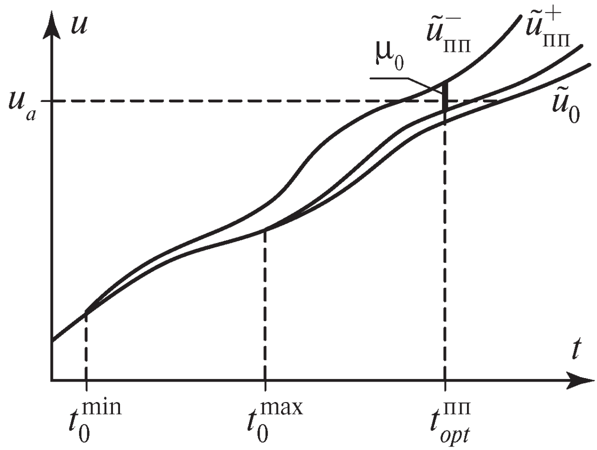

Fig. 4-8-3. Construction of an SP on a pseudo-bunch of ECs

By definition,

$$\tilde{u}^{-}_{\Pi\Pi} = \tilde{u}_{t_0^{min},\tilde{u}_0(t_0^{min},T_0)}\,(t, T_{\min}),$$

$$\tilde{u}^{+}_{\Pi\Pi} = \tilde{u}_{t_0^{max},\tilde{u}_0(t_0^{max},T_0)}\,(t, T_{\max}).$$

Optimum regions of check have their centers at the points

$$<t^{\Pi\Pi}_{opt}(n), u_a + 2\pi n> = \frac{(\tilde{u}^{-}_{\Pi\Pi})^{-1}[u_a+2\pi n] + (\tilde{u}^{+}_{\Pi\Pi})^{-1}[u_a+2\pi n]}{2}, \quad (4.8.5)$$

$$n = 1, 2, \ldots\ldots$$

Further, the addends in the square brackets of the numerator of the right part of the equality (4.8.5) will be omitted, the variable u_a being conceived in a generalized sense.

The size of the initial SP element μ_0 is calculated as

$$\Delta u^{\Pi\Pi}_{opt} = \tilde{u}^{-}_{\Pi\Pi}(t^{\Pi\Pi}_{opt}) - \tilde{u}^{+}_{\Pi\Pi}(t^{\Pi\Pi}_{opt}) =$$

$$\frac{1}{T_0}[|\Delta T_{\max}|\,(t^{\Pi\Pi}_{opt} - t_0^{min})\,\tilde{u}'_{t_0^{min},\tilde{u}_0\left(t_0^{min}\right)}(t^{\Pi\Pi}_{opt}, T_0 + \Delta T_{\max}) -$$

$$-|\Delta T_{\min}|(t^{\Pi\Pi}_{opt} - t_0^{max})\,\tilde{u}'_{t_0^{max},\tilde{u}_0\left(t_0^{max}\right)}(t^{\Pi\Pi}_{opt}, T_0 + \Delta T_{\max})]. \quad (4.8.6)$$

If the $\Delta u^{\Pi\Pi}_{opt}$ magnitude exceeds the angular size of the sensor's FoV, then, the same as in the preceding case, the mode of search "on the border $u* = u_a$" is implemented with the subsequent decomposition of the initial SP. Although here, the realization of such a procedure is somewhat more complicated (owing to the non-triviality of the pseudo-bunch (4.8.4) structure (see also Figs. 4-8-3 and 4-8-4 for explanation)).

Strictly speaking, dividing the pseudo-bunch into two layers (to say more exactly, pseudo-layers) should be accomplished by its EC $\tilde{u}_{t^{\Pi\Pi}_{opt},u_a}(t, T^*)$ with the possible minimum magnitude of T^* of the period contained between $T_0 + \Delta T_{\max}$ and $T_0 + \Delta T_{\min}$, the time instant of period change from T_0 to T^* being located between t_0^{min} and t_0^{max}. However, if it involves some difficulties, practically, one may use the pseudo-bunch's EC more conveniently for constructive calculations.

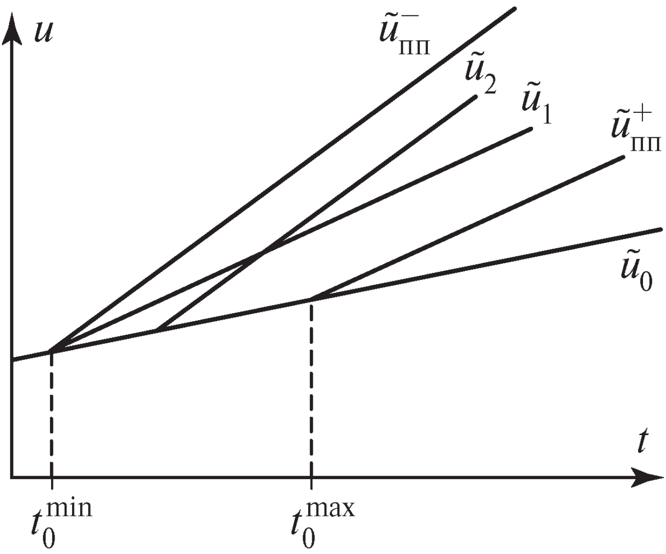

Fig. 4-8-4. Structure of the pseudo-bunch representing a maneuver of an SO in a circular orbit

For such a case, the structure of a pseudo-bunch may be somewhat elucidated by Fig. 4-8-4 which depicts the simplest pseudo-bunch describing a maneuver of the SO in a circular orbit. As one can see, here, all ECs are straight lines. The ECs $\tilde{u}_1$ and $\tilde{u}_2$ belong to the pseudo-bunch although they meet each other inside it (in particular, it is by this fact that the difference between a bunch and a pseudo-bunch is accounted for).

A pseudo-bunch may be decomposed into a set of bunches: structurally, the former represents the union of bunches of ECs having the boundary ECs parallel to $\tilde{u}^{-}_{\Pi\Pi}$ and $\tilde{u}^{+}_{\Pi\Pi}$ coming from point $<t^*,u^*>$, the latter running the segment of the EC $\tilde{u}_0$ beginning from t_0^{min} through t_0^{max}. For the eccentric orbits, the structure of the maneuver bunch is more complicated, of course. However, this fact does not interfere with the application of the OSP method described above thanks to its high generality. For the case of $\Delta T_{min} > 0$, $\Delta T_{\max} > 0$, the corresponding formulae are produced similarly.

3. Now we consider the specific case when the limiting changes of the motion period due to the maneuver ΔT^- and ΔT^+ are known and they are of different signs: $\Delta T^- \leq 0$, $\Delta T^+ \geq 0$. This case is simpler than the previous one because the SP is constructed on the ECs bunch coming from point $< t_0^{min}, \tilde{u}_0(t_0^{min}, T_0) >$. The distinction of this bunch from that of (4.8.1) consists of the absence of its symmetry with respect to the central EC $\tilde{u}_0$. In this connection, the relative formulae will change:

$$\tilde{u}_\Pi^- \equiv \tilde{u}_{t_0^{min},\ \tilde{u}_0\left(t_0^{min}, T_0\right)}\left(t, T_0 + \Delta T^-\right),$$

$$\tilde{u}_\Pi^+ \equiv \tilde{u}_{t_0^{min},\ \tilde{u}_0\left(t_0^{min},\ T_0\right)}\left(t,\ T_0 + \Delta T^+\right),$$

$$\sum\nolimits_{t_0^{min}}^{T_0+\Delta T^-,\ T_0+\Delta T^+} =$$

$$= \{\tilde{u}_{t_0^{min}, \tilde{u}_0(t_0^{min},\ T_0)}\left(t, T\right)\}_{T \in [T_0+\Delta T^-;\, T_0+\Delta T^+]}. \qquad (4.8.7)$$

The optimum region for check-up has its center in point

$$< t^{\Pi}{}_{opt}, u_a >,\ t^{\Pi}{}_{opt} = \frac{(\tilde{u}_\Pi^-)^{-1}[u_a] + (\tilde{u}_\Pi^+)^{-1}[u_a]}{2}.$$

The size of the initial SP element μ_0 is equal to

$$\Delta u_{opt}^{\Pi\Pi} = \frac{T_{avr} - T_{min}}{T_{avr}}(t^{\Pi}{}_{opt} - t_0^{min})\, \tilde{u}'_{0t}(t^{\Pi}{}_{opt},\ T_0 - |\Delta T^-|) +$$

$$+ \frac{T_{max} - T_{avr}}{T_{avr}}\,(t^{\Pi}{}_{opt} - t_0^{min})\, \tilde{u}'_{0t}(t^{\Pi}{}_{opt},\ T_0 + \Delta T^+) =$$

$$= \frac{\Delta T^+ + |\Delta T^-|}{2(T_0 + \frac{\Delta T^- + \Delta T^+}{2})}(t^{\Pi}{}_{opt} - t_0^{min})[\,\tilde{u}'_{0t}(t^{\Pi}{}_{opt},\ T_0 -$$

$$- |\Delta T^-|)] + \tilde{u}'_{0t}(t^{\Pi}{}_{opt},\ T_0 + \Delta T^+), \qquad (4.8.8)$$

$$T_{avr} = 0{,}5\ (\Delta T_{\max} + \Delta T_{\min}).$$

4. Now we consider the case of a circular orbit. We note first of all that for circular orbits there do not exist local optimum regions for constructing SPs which is evident. Nevertheless, the ECs apparatus allows to easily conceive the situation and to construct the complete non-redundant and mostly economical search plan namely for this situation. For circular orbits, only one of the useful properties of the OSP degenerates which is associated with degeneration of the concepts of apogee and perigee for such orbits.

For all considered types of information on a maneuver, the related formulae will take the following form:

$$\Delta u^{\Pi}(t) = 4\pi \frac{|\Delta T|_{\max}}{{T_0}^2}(t - t_0^{min}), \qquad (4.8.2')$$

$$\Delta u^{\Pi\Pi}(t) = \frac{2\pi}{T_0}\left[\frac{|\Delta T|_{\max}}{T_0+\Delta T_{\max}}(t - t_0^{min}) - \frac{|\Delta T|_{\min}}{T_0+\Delta T_{\min}}(t - t_0^{max})\right], \qquad (4.8.6')$$

$$\Delta u^{\Pi}(t) = \frac{\pi}{T_0+\frac{\Delta T^- + \Delta T^+}{2}}(\Delta T^+ + |\Delta T^-|)(t - t_0^{min})\left(\frac{1}{T_0+\Delta T^-} + \frac{1}{T_0+\Delta T^+}\right) =$$

$$= \frac{2\pi(\Delta T^+ + |\Delta T^-|)}{(T_0+\Delta T^-)(T_0+\Delta T^+)}(t - t_0^{min}). \qquad (4.8.8')$$

Fig. 4-8-5 visually illustrates the process of the SP decomposition in the "on the border u^*" mode on a pseudo-bunch for a circular orbit.

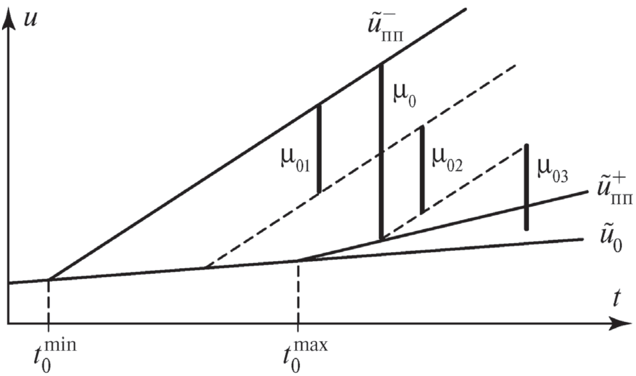

Fig. 4-8-5. Fragmentation of the SP "on the border" on the bunch for a circular orbit

4.9. Accounting for the error in orbital period

Fortunately, for the relevant calculations, it was very convenient that the problem of taking account of the error ΔT in the SO motion period (when planning its search) is formally, constructively, and in its sense close to the problem of the search for an SO after its maneuver. We will make use of this convenience.

The influence of ΔT on deformation of each EC beginning from time instant t_0 is equivalent to the influence of the period correction by the same magnitude at the same time. The main difference is that in the latter problem the uncertainty of the SO's position before a maneuver is neglected whereas in the former problem it is required to take account of the possible deformation of the SP constructed, namely in order to remove this uncertainty.

The appropriate compensation of the consequences of an error in the period is fulfilled in accordance with recommendations of section 3.9 either by increase of the exposition time or by decrease of the calculated size of the SP element μ – for eccentric orbits by formulae (3.9.4) – (3.9.8) and for circular orbits by formula (3.9.9).

In this case, the latter takes a form

$$\Delta u_c = \Delta u - 4\pi\frac{\Delta T}{(T-\Delta T)T}(t - t_0) \tag{4.9.1}$$

where

t_0 is the reference moment of timing the search interval

and t is the moment of timing the SP element μ to be corrected.

4.10. The search for a GEO satellite after its orbit correction

Not deviating from the general methodology of optimum planning the search for an SO after a maneuver expounded in section 4.8, sometimes it is convenient to exploit a special procedure of planning the search for a GEOSO after its orbit correction considering specifics of the latter.

To exactly predict the time of performing the stationkeeping correction is usually impossible owing to a priori uncertainty of the calculation of the stationkeeping impulse and some other causes. But we should never ignore statistics. By statistical data, the error of beforehand predicting the time of the next correction may be as large as some dozen days. However, one ought not to neglect the collected statistical information on the time instants $t^i_{c.o.}$ of already performed orbital corrections for the spacecraft under consideration or for other spacecraft of its family.

The distribution histograms for time intervals $\Delta t^i_{c.o.} = t^i_{c.o.} - t^{i-1}_{c.o.}$ between adjacent stationkeeping corrections for different GEO and HEOSOs are shown in Fig. 4-10-1.

If the distribution of values of interval $\Delta t^i_{c.o.}$ is available, one can calculate the time interval I_{corr} at which the expected orbit correction will happen with a given confidence probability. Proceeding from the requirements to accuracy of keeping the track of the SO and the laws of error propagation on the condition of possible orbit correction one can produce some observation plan for the SO within the interval I_{corr} in order to prevent any failing of its tracking.

The principle of constructing the observation plan is elucidated by Fig. 4-10-2 in which

$$I_{corr} \equiv (a,b),$$

t_0 is a reference epoch of timing the last known state vector of the SO orbit,

$t_1, t_2, \ldots t_i, \ldots$ are time moments of the planned control observations,

$\lambda_{t_0}(t)$ is the mathematical expectation of evolution of the sub-satellite point′s geographical longitude λ (instant ground-trace),

$\{\lambda^{pr}_{t_0}(t)\}$ is a set of trajectories bounded by $\lambda_{t_0}(t) \pm 3\sigma_{pr}$,

σ_{pr} is the r.m.s. error of predicting the magnitude λ for the SO motion not perturbed by the orbit correction,

$\lambda^{c}_{t_i}(t)$ is the expected variation of λ after a hypothetical orbit correction at time t_i,

σ_c is the r.m.s. error of predicting the magnitude $\lambda^{c}_{t_i}(t)$,

$\sum_i^{max}$ is the maximum error of determination of the sought-for satellite angular position owing to an unexpected (non-predicted) orbit correction within the time interval (t_{i-1}, t_i).

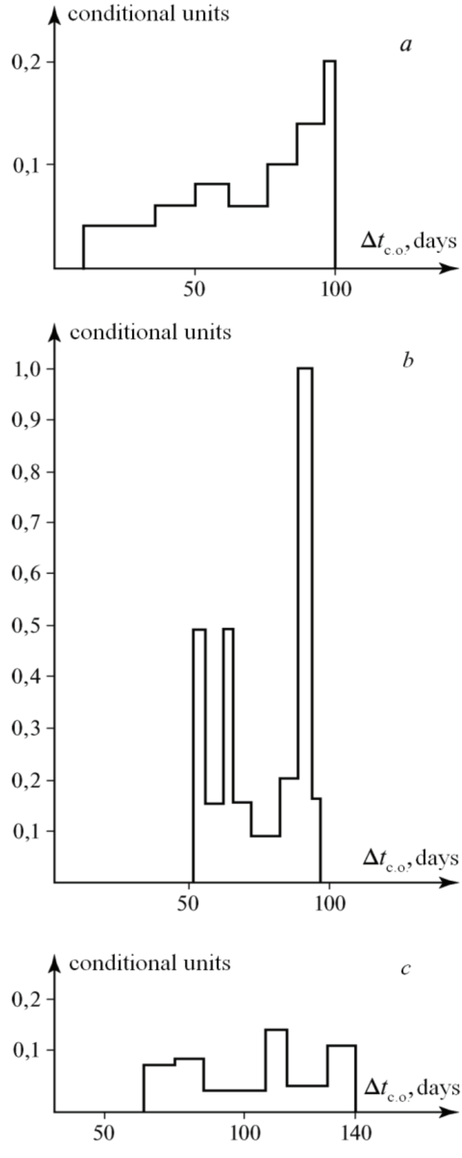

Fig. 4-10-1. Distribution of time intervals $\Delta t^{i}_{c.o.}$ between successive stationkeeping corrections for three real HOSOs

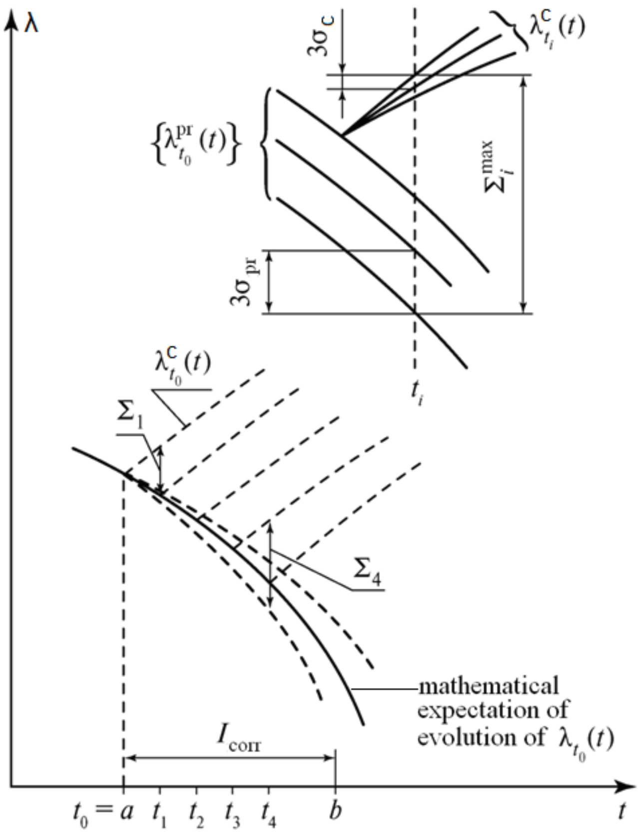

Fig. 4-10-2. The principle of constructing the observation plan for the SO within the correction interval I_{corr}

Given the boundaries of affordable values of the variable λ (they could be determined statistically or from the analysis of the SO′s task (purpose)), for each time moment t_i, one can produce a pseudo-bunch of possible trajectories of the sub-satellite point $\{\lambda_{t_i}(t)\}$ coming from the point $(t_i, \{\lambda^{pr}_{t_0}(t_i)\})$. The time moments are chosen in such a way that the total point-ahead error Σ_i^{max} do not exceed the admissible value.

Provided at some stage of implementing the control observation plan the fact of orbit correction is revealed (for example, the fact of the absence of the sought-for SO in the sensor's FoV is fixed), the search for the SO is performed according to the technique given below.

Then, let us consider the possible variation of parameter λ within the time interval (t_i, t_{i+1}) between two adjacent control observations at which the orbit correction is detected (see Fig. 4-10-3).

On account of the fact that the correction of the period (as well as of the eccentricity) by its sense and with the purpose of the energy economy should be performed in perigee or apogee of the SO orbit, it is easy to calculate the possible time moments of performing corrections $\tau_1, \tau_2, \ldots \tau_j, \ldots \tau_{\bar{J}}$ within the interval $(t_i,\ t_{i+1})$. The exact correction point should be chosen with due regard for the form and mutual configurations of the corrected and nominal orbits [15].

If there is a priori information for the determination of the function $\lambda_{\tau_j}^{k}(t)$ after performing the correction at time τ_j (for example, the upper or the lower boundary of the range of affordable values for variable λ), then for detecting the SO it is sufficient after the control observation with a negative result at time t_{i+1} to make not more than $\bar{J}$ search observations with the time step δt, the latter being determined by technical capabilities of the sensors (Fig. 4-10-3). Via these actions, the SO will be detected during the time that does not exceed the magnitude of $\Delta T_{det}^{max} = \bar{J}\delta t$.

If sufficient a priori information for the determination of function $\lambda_{\tau_j}^{k}(t)$ is absent, its estimate from above (the upper estimate) $\hat{\lambda}_{\tau_1}^{k}(t)$ for the time τ_1 is calculated, and the "pessimistic" plan (unlike the "optimistic" SP depicted in Fig. 4-10-3) is constructed as is shown in Fig. 4-10-4.

The "pessimistic" SP also guarantees detection of the SO where an orbit correction has been performed out of the regular points τ_j. The maximum detection time by this SP is equal to

$$\Delta T_{det}^{max} = (E[\frac{\Delta\lambda - \frac{\delta\lambda}{2}}{\delta\lambda}] + 1)$$

where

$\delta\lambda$ is an angular size of the GEO arc checked by the sensor's FoV,

$E[x]$ is an integer part of x,

and $\Delta\lambda = \Sigma_i^{max}$ (see also Fig. 4-10-2).

Of course, there exists the average compromise SP if one has the possibility to construct both upper and lower estimates of function $\lambda_{\tau_j}(t)$, $j = 1, 2, \ldots \bar{J}$.

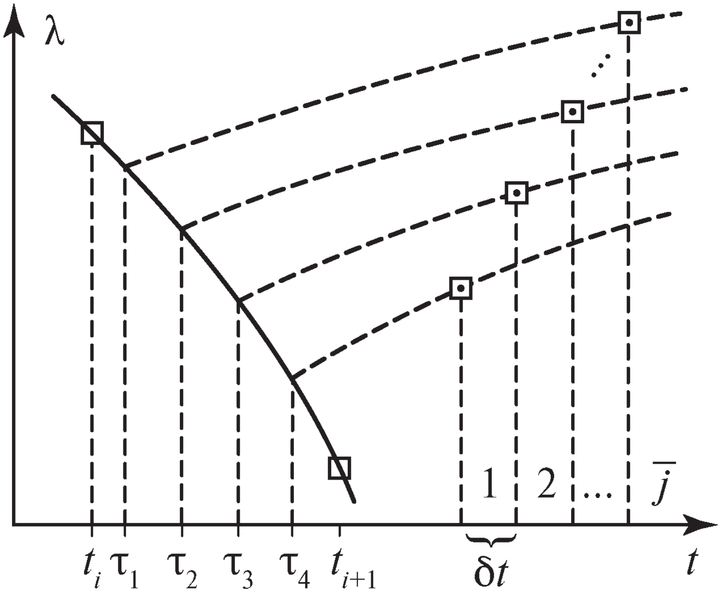

Fig. 4-10-3. The "optimistic" search plan

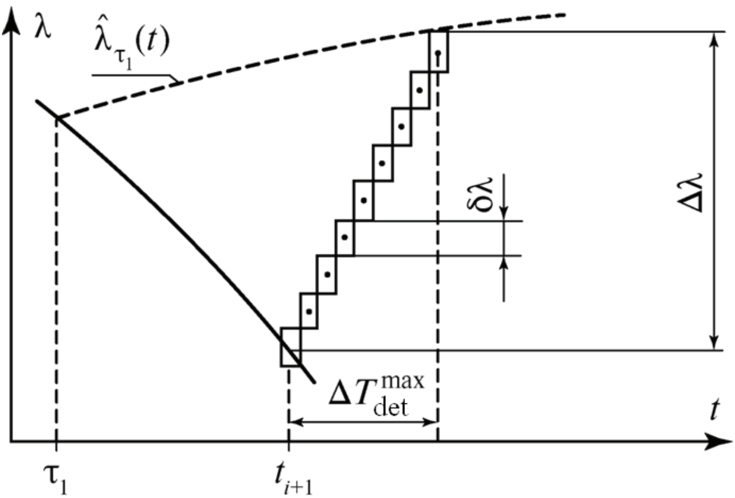

Fig. 4-10-4. The "pessimistic" search plan

It follows from the suggested scheme that for each correction there is a necessity of carrying out a whole series of observations. Nevertheless, all this is much more economical than the application of common non-dedicated search procedures.

Example. Proceeding from the above, for monitoring a geosynchronous SO (the type *b* in Fig. 4-10-1) being kept in a 2° range of longitude with the rate of one observation in 3 days, application of the "optimistic" SP requires on the average 8 control observations during 45 days and 3 search observations after establishing the fact of a maneuver. The "pessimistic" plan would require the same number of control observations and not more than 6…10 search observations when using the sensor having FoV with the size of $6' \times 6'$.

4.11. The search for several space objects with the help of a group of sensors

The situation when a group of sensors are available for conducting observations allows distribution of search efforts among different sensors which is an acknowledged expedient approach in many respects, especially for optical sensors [50].

For the sake of determinacy, optical and optical-electronical sensors will be meant by the sensors used here although it is not a principle. If one admits using alternative kinds of sensors as well, then in this description one should numerate with index *i* not only nights but the revolutions of the SO motion as well.

So, for the search for an SO, let's enlist $\bar{J}$ optical sites during $\bar{\imath}$ nights. The sensors are characterized by their geographic positions, the cost of exploitation (its own for each sensor), the size of FoV, and so on. The problem will be solved in two stages. At the first stage, for each *j*-th site and *i*-th night, in accordance with sections 4.2, 3.3, and 3.4, the optimum or sub-optimum SP is constructed taking into account the physical conditions of the observations. And then, each SP is EC-projected to the basic search interval $[u_b, u_f]$. As a result, the intervals of search capabilities (by definition) will be obtained which, the same as in section 3.5, will be encoded with the help of a pair of indices (j/i) corresponding to the site and night of observations. And these are the initial data for the second stage of solving the problem.

Thus, the problem of search for an SO for $\bar{\imath}$ nights with the help of $\bar{J}$ sensors will be reduced to a problem of the second stage – a problem of decomposition of the basic search interval to sub-intervals, each of them being embedded into some interval of search capabilities [39, 61].

To begin with, let us specify the necessary concepts and notations:

j – a conditional number of the observation site (and/or the sensor if there are several ones at the same site), $j = 1, 2, \ldots \bar{J}$,

i – a conditional number of the night being used for observations, $i = 1, 2, \ldots \bar{\imath}$,

δ_j – the price (cost) of enlisting the j-th sensor or the probability of the clear night sky at the j-th site during the search period taken with an inverse sign (minus),

μ_i – the penalty (fine) for work in the i-th night,

ω_i – the bonus for work in the i-th night (for example, $\omega_i = -\mu_i$),

γ_{ji} – the cost of current operation of the j-th sensor in the i-th night.

The rest of the notations remain the same as in section 3.5 where solutions of the problem are given by several criteria (economic, operative, and compound (hybrid)). These criteria in dependence on the search situation and the task set goal can vary and be attached with different content nuances.

For example, if δ_j is meant as the negative probability of the clear night sky at the j-th site, criteria 3.5.3 – 3.5.6 acquire a probabilistic sense transmitting the property of the reliability of implementing the SP.

If in the criterion 3.5.6 we substitute $-\delta_{j_m}$ (the probability of clear night sky at the j-th site in which the sensor controls the sub-interval I_m of the basic search interval) for ω_{i_m}, then the criterion 3.5.6 will take a sense of the mathematical expectation of the probability with which the sensor detects the SO using a given SP and so on. The solution of the problem will not be this simple. But as soon as it is isomorphic to the former problem, it should be solved by the modified method, using the "branch-and-bound" approach [48, 49] according to the scheme given in section 3.5 for the criteria of the hybrid type. In addition, the calculation procedure for approximately evaluating the figure of merit $\widehat{\Phi}$ will be easier.

The solution of the optimum planning the search problem by criteria 3.5.4 and 3.5.5 is expounded in detail in section 3.5. And so, there is no sense to repeat it here. The assumption accepted in the mathematical statement of the problem appears natural in the case considered here and is justified in particular by the next circumstance. If the j-th site (and its sensor) is enlisted for observations on the i-th night, it is reasonable to exploit all its search resources concerning the sought-for SO during all the night (that may be superposed with some alternative works).

For better clarifying the optimization procedure of the second stage, we consider an example [39]. The initial data are as follows:

$$\bar{j} = 3,\ \bar{\imath} = 3,\ \delta_1 = 10,\ \delta_2 = 15,\ \delta_3 = 20,\ \mu_1 = 10,\ \mu_2 = 50,\ \mu_3 = 100$$

(that is, priority is given to the operative properties of the SP – the detection time), and $\gamma_{ji} = 5$ for all j and i. The intervals of search capabilities are arranged in layers as shown in Fig. 4-11-1.

In accordance with the recommendations given in section 3.5, we construct the approximate solution which is a decomposition of the basic search interval $[u_b, u_f]$ determined by the ends of its elements-subintervals:

(1/1), (1/2), (2/2), (2/1), (3/2), (3/1).

The corresponding embeddings of the decomposition elements into the capability intervals are depicted in Fig. 4-11-1 by bold lines. Then we should fix the obtained solution as a “record” (a special notion – see section 3.5 and [46, 47]) and calculate its value:

$$\Phi_{rec} = (10 + 15 + 20) + (10 + 50) + 6\times5 = 135.$$

Let us recall that when you calculate each new "record" it is necessary to compare its value with the old (fixed) one. If this value is better than the previous one, the "record" is updated and fixed (and only in this case).

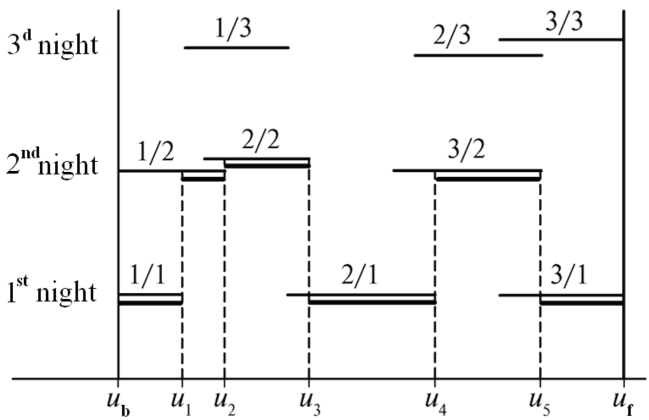

Fig. 4-11-1. The search capability intervals and the search plan

Fig. 4-11-2 depicts the process of development of the decision tree and cutting its non-prospective branches by the application of both the basic cutting rule [46, 47] and the auxiliary additional cutting rule [48, 49]. The use of an additional cutting rule is strongly recommended to improve the efficiency of solving discrete large-dimensional problems.

The super efficiency of ACR in our case is obvious: out of 5 cuttings of branches, 3 were made with the help of ACR and, by the way, 2 of these three were performed at the earliest possible tier of the tree – at the second one (the earlier, the more efficient).

Although our specific problem chosen here as a demonstrative example is not of great dimensionality, the effectiveness of the application of ASC to help her solution looks impressive. This example visibly shows the superiority of ACR over the basic cutting rule when solving the problems of such a class. In large-dimensional problems, the effectiveness of the use of ACR will manifest itself more significantly.

The optimum solution looks as follows:

$$(1/2), (2/2), (2/1), (3/2), (3/1). \tag{4.11.1}$$

The magnitude of the functional is calculated as

$$\Phi_{onm} = (10 + 15 + 20) + (10 + 50) + 5\times5 = 130.$$

The solution (3.7.1) can be deciphered as follows:
interval $[u_b, u_2]$ is controlled by sensor 1 on the second night,
interval $[u_2, u_3]$ is controlled by sensor 2 on the second night,
interval $[u_3, u_4]$ is controlled by sensor 2 on the first night,
interval $[u_4, u_5]$ is controlled by sensor 3 on the second night,
interval $[u_5, u_f]$ is controlled by sensor 3 on the first night.

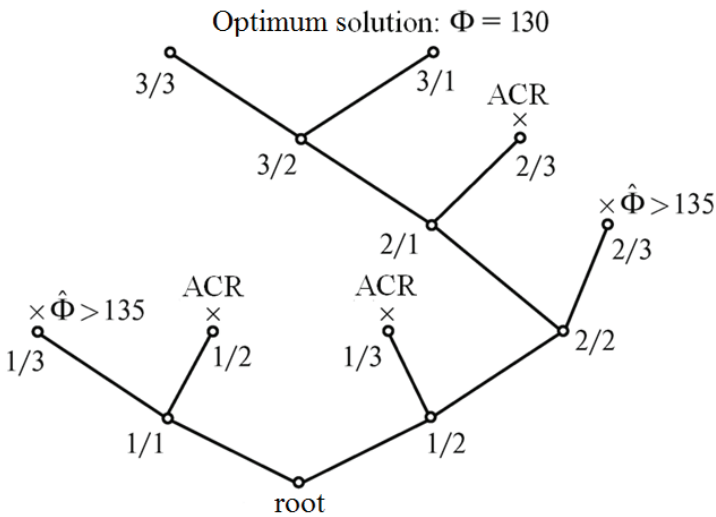

Fig. 4-11-2. The decision tree

And now it is necessary to say a few words about the search for a group of SOs with the help of a group of sensors. The real example of such a problem (by the way, very actual) is a problem of detection of a space system (constellation) (SC). Some recommendations on the first preparatory stage of its solution are given in section 2.1.

The statement of the problem and the solution method suggested in section 3.6 can be considered as a second stage of its whole solution. Here, the same as δ_j in the preceding statement, the magnitude w_j can represent not only the cost of enlisting the *j*-th sensor for observations but also the probability of unfavorable meteorological conditions of observations at the *j*-th site. Then, the criterion 3.6.1 will characterize the reliability of detecting the SC by a given SP (in inverse proportion) depending on the observation conditions at the sites.

As w_j one can accept some index of the functioning capacity of the *j*-th site (or sensor). In this case, a solution of the problem of detecting a set of SCs (ordered by priority) can be very naturally synthesized as a sequence of solutions of the search problems for an SO from the next in turn SC, the parameters $\{w_j\}$ being corrected in accordance with the preceding SCs as follows. If M_k is the SP for the satellites belonging to the *k*-th SC constructed by the values of parameters

$$w^k = (w_1{}^k,\ w_2{}^k, \ldots w_j{}^k, \ldots w_J^k),$$

then the next relation holds $w^{k+1}=f(w^k, M_k)$.

When dealing with a great number of SOs and sensors, and the significant uncertainties in the parameters of their orbits, the dimension of the problem may appear very great. For example, when searching for 30 SOs, the number of variables in the related discrete optimization problem usually exceeds 100. Such a case requires addressing the hybrid “branch-and-bound” scheme using ACR [48]. If, say, every SO is observable on an average by 5 sensors and every SP contains on an average 3 elements, the number of variables reaches 450. In this case, one simply cannot practically solve the problem without the use of additional cutting rule.

4.12. On the efficiency of planning procedure

Let us start with the general qualitative overview of sequentially reducing the region of the sought-for SO possible position which can be obtained using the suggested methodology of the optimum search planning. In this process, one can mark out several stages that (depending on the specific problem) can be realized both independently and in a complex with the others.

1. For each sought-for SO in the *tu*-plane, the region Λ bounded by ECs $\tilde{u}_b$ and $\tilde{u}_f$ is marked out in which and only in which planning the search is justified.
2. Over this region, a special structure is introduced: Λ is represented either as a family of ECs

$$\sum\nolimits_{t_0}^{u_b, u_f}$$

or as a bunch of ECs

$$\sum\nolimits_{t_0^{min}}^{T_0+\Delta T^-,\, T_0+\Delta T^+},$$

or as a pseudo-bunch of ECs

$$\sum\nolimits_{t_0^{min},\, t_0^{max}}^{T_0+\Delta T_{\min},\ T_0+\Delta T_{\max}}.$$

The peculiarity of this structure consists in the following. In order to result in a guaranteed (with the confidence probability associated with the basic search interval) detection of the sought-for SO, it is necessary and sufficient to check each EC $\tilde{u}(t)$ from the family (bunch or pseudo-bunch) in one and only one its arbitrary point.

3. Such a preliminary structuring of Λ clears the way for further optimization of the choice of the specific points for check by different criteria with due regard for the problem's specific character and the sensor's operation capabilities.

4. Further, if the problem at hand envisages detecting an SO, an SC, a group of SOs, or a group of SCs with the help of a group of sensors, then, at first, for every SO and every sensor, the OSP (or a sub-optimum SP) is constructed taking account of the observation conditions. Then, the appropriate optimization problem is solved for planning the search for an SO, a group of SOs, or an SC with the help of a group of sensors. Perhaps, it will be a sequence of search problems for SOs or SCs put in order by priority.

Now, let us address some qualitative and quantitative estimates of the efficiency of the OSP methods suggested in this chapter.

Depending on the specific values of the SO orbit parameters, their given accuracy, the technical and performance characteristics of the sensors used, and so forth, the real gain (in terms of the accepted quality criterion) delivered by the optimum search conditions (of constructing an SP in a special region in orbit) is expected to be different.

For example, for circular orbits, on account of the constancy of the SO velocity down track, the optimum conditions for search for one SO with the help of one sensor within a cycle degenerate. Although the other advantages of the OSP remain. The greater the magnitude of the eccentricity, the higher the efficiency of the optimum solution (by criterion 1), the efficiency being treated here as the degree of reduction of the checked orbit arc and the duration of the search in the vicinity of optimum point.

For HEOSOs from the class of "Molniya" or SDS ($e \approx 0{,}7 \ldots 0{,}73$), the search interval in the vicinity of perigee of size, say, 98° corresponds (by the equivalence principle) to the search interval having the size of 3° in the vicinity of apogee. Taking into account the difference of heights for SOs from this class, the difference of visible sizes of the corresponding search domains can increase still more: 15...17 times as much. In particular, that means that for detecting an SO by photographic or electronic inspection (with the help of TV or CCD technique) of the trajectory in the vicinity of apogee it is enough to make only one exposure with the FoV having the size of 3° whereas in an alternative part of the orbit that may require 10 and many more exposures.

As to the efficiency of OSP in the "on the border" mode, the following data can be brought which are very indicative of the proposed methods effectiveness.

When searching a HEOSO like "Molniya", the deflection of the sight axis of the sensor having a 2° FoV from the optimum direction through an angle, say, 26° augments the laboriousness of the search approximately by 2 times. At the same time, the deflection through an angle 40° brings down the search efficiency by 3 times as much and the 70° deflection − 10 times as much. Taking into account the decrease of the calculated size of the FoV, the real comparison will be still more contrasting.

The SP displacement to a non-optimum region is undesirable not only on account of economic inexpedience, but also regarding the fact that diminution of Δt and Δu involves the danger of the impossibility of physical realization of the SP (one may have no time for its implementation). Of course, this defect can be mitigated to some extent due to the high performance and sensitivity of the sensor (if its performance and sensitivity are really high). But by this, we irrationally use the sensor resource, not to mention the fact that the culture of solving the problem suffers.

When searching a HEOSO like "Molniya" or SDS by the argument of latitude u (the size of a basic search interval at perigee being equal to approximately 170°) with the help of a sensor having the highest affordable speed of its sight axis motion about 100″/s, the transition from a non-optimum SP of the kind M_{u_f} (see M_{s_f} in section 3.2) to the OSP (by criterion 1) of the kind (4.3.4) (both SPs being located in the optimum region of the *tu* plane) reduces the detection time by 12.5 times. Out of the apogee region the gain is less.

At the same time, it is necessary to take account of the weakening the intelligence signal due to the relative (contrary) motion of the sensor's sight axis and the SO. If such a weakening is undesirable, it is possible to pass to a discrete SP and create the favorable conditions for the concentration and accumulation of the intelligence signal energy within each SP element (see Fig. 4-4-2).

The efficiency of the search for a HEOSO in the optimum area in presence of substantial errors in all orbit elements is estimated by a degree of reduction of their influence to the accuracy of the SP elements calculation when transferring from non-optimum area to the optimum one. Table 4.6.1 shows the value this index can reach, for example, for T – by 5 times, for e – 27, for ω – 1600 (*sic!*). It is tantamount to enhancing the efficiency of the SP for sensors with a very small FoV - 216 000 times as much. Along with this, there is something that was not yet taken into account, for instance, enhancing the efficiency due to the increase of the calculated size of an SP element Δu. So, the resulting enhancing of the efficiency can sometimes exceed the magnitude of 1000 000 (in the accepted here calculation).

It is of interest that diminution of the sensor's FoV implies progressive increase of the efficiency of the OSP. This property and a high level of efficiency of the OSP give grounds for positively solving the question of the feasibility of detecting SOs with the help of narrow-angle and narrow-beam space surveillance facilities (in many search situations where they arise).

Example. At a given orbit (with the accuracy $\sigma_T = 2s$, $\sigma_e = 0{,}001$, $\sigma_I = 2''$, $\sigma_\Omega = 2''$, $\sigma_\omega = 20'$, $\sigma_u = 2'$), for a practically guaranteed detection of a HEOSO like "Molniya" or SDS in a non-optimum range (~ u_a - 90°) with the help of a sensor having an FoV of size 36″×36″, the SP should contain ~ 140 000 000 (*sic!*) elements while the OSP (in the vicinity of apogee) requires only 16 (*sic!*) elements, i.e., the latter is quite feasible that cannot be said about the former.

This effect is achieved in only one of the possible aspects and only one form of optimization which is admitted by the present methodology.

One of the distinctive features of the suggested methodology of planning the search for an SO using imprecise a priori information is that its optimization scheme appears multiple, reiterative, multiform, multiprogram, and versatile which allows obtaining optimum solutions not only by different criteria but also in different aspects, forms, and in several stages (with no mutual interference). After having constructed the OSP at one optimization level and in one statement of the problem or upon given conditions, one can proceed with the optimization process in an alternative form and/or upon alternative conditions, i.e., by the "*optimum optimorum*" principle. And all this is on the set-theoretical basis within the frame of the united mathematical apparatus of the ECs.

The practical efficiency of the OSP is determined not only by the economy of energy consumption, reduction of expenditures (employment) of the sensors' and the staff's resources (in this case, the efficiency being estimated by the above introduced time indices (criteria) of the SP realization). Besides, the OSP enhances the capabilities of sensors in alternative concerns as well. In particular, it enlarges the class of detectable SOs (in terms of slant range, brightness, motion velocity, and so forth). It is achieved at the expense of the following properties of the suggested methodology.

If a priori information on the sought-for SO orbit is substantially imprecise, then some special difficulties are connected with creating the favorable conditions for the concentration and accumulation of the intelligence signal energy (which is necessary for detection of fast moving SOs and/or having faint brightness).

The quality of such conditions is determined by the accuracy of the calculation of the SO velocity vector projection to the plane normal with respect to the sensor's sight axis. That is, it depends on the possible variation of the derivative $\frac{du}{dt}$ within each SP element μ_i.

If the sensor's FoV is small, the element μ_i controls a narrow layer of ECs which is notable for the limited possible velocity variations and, as a consequence, provides the high quality of conditions for the concentration and accumulation of the signal energy in one point of the receiver.

The OSP reduces the uncertainty of a priori information on the SO state vector. This reduction is proportional to the number of SP elements, more exactly, inversely proportional to the calculated value Δu of the size of the SP element projected to the *u* axis**.** In the same proportion, it enhances the quality of conditions for concentration and accumulation of the signal energy from the SO in the sensitive element of the receiver.

So, even given the very imprecise a priori information on the SO orbit parameters, in the frame of the suggested methodology of planning the search, we are able to synthesize favorable conditions for the registration of a faint intelligence signal by providing such conditions for each SP element independently.

In case of a large angular size of the counted FoV (perhaps, a portion of the sensor's FoV, if any), the calculated compensation for the unknown motion of the sought-for SO appears to be substantially averaged. So that, for the search of SOs both having faint brightness (or being in a bad phase of illumination) and/or rapidly moving in the picture plane, it is advisable to enlist narrow-angle or narrow-beam sensors or to diminish the exploited part of a wide FoV (*on purpose*). And since the number of SP elements and the possible variations of velocities are minimum in the optimum search range, the most economical synthesis of the optimum conditions for registration of the intelligence signal is implemented by this approach.

For the purpose of practical and visual evaluation of optimization technology, it is of interest to compare the efficiency of the OSP methods and the widespread method of spiral search. The latter is in principle isotropic, that is, the search efforts are distributed uniformly in all directions with no regard for the temporal structural deformation of the

SOCPUD (Fig. 4-12-1). At the same time, in the majority of search situations, it (CPUD) is badly strengthened along the track and sometimes undergoes substantial structural distortion the degree of which is proportional to time.

Taking into account this information (which is accomplished automatically in the OSP) permits, in case of search for a HEOSO, in 24 hours after a 4-fold measurement of its angular position within the 3-hour measurement interval (after recalculation into the Cartesian variables, σ_r = 22 km, σ_l = 140 km, σ_b = 1,2 km) to construct the OSP, at least, 6 to 7 times as economically (7 OSP elements against 47 ones in the spiral method), the size of FoV being $10'\times10'$.

Besides, the spiral search

- is notable for much more integral of the angular motion of the sensor's sight axis (re-targeting),
- does not guarantee detection of the sought-for SO for great magnitudes of σ_u and may appear not feasible,
- becomes still less economical, owing to the necessity of taking account of the reduction of the calculated FoV's size in case of an increase of the time (arc) search interval.

We should add to this (which is very important) that the spiral search does not provide (and does not even set such a goal) the favorable conditions (to say nothing of the optimum ones) for the concentration and accumulation of the faint intelligence signal energy. Besides, the spiral is unrolling with the average speed of the motion of the center of the SOCPUD with no regard of its deformation (that is, with no regard of the SP degradation process – see sections 5.5 and 5.6).

Now, we present some information on the practical evaluation of the here proposed approach to the detection and observation of small and weakly-contrasting SOs, its testing, introduction into the practice of real acting systems, and evaluation of its efficiency. The methods of search by argument of latitude for HEO and GEO at first were implemented in the form of algorithms and experimental programs [1, 2].

After successful testing them at the optical-electronical facilities of the Academy of Sciences of the USSR in the 1980s (Optical station "Sayany" near Irkutsk), the appropriate regular programs were introduced at the other facilities as the staff programs in 2003 - 2004 [1, 2, 42, 52].

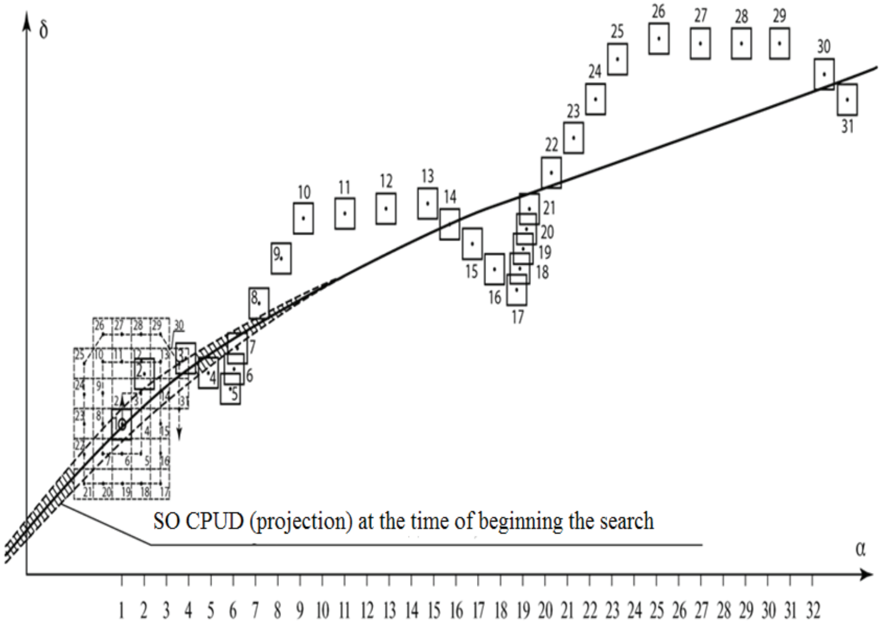

Fig. 4-12-1. Evolvement of the spiral search plan taking account of the sought-for SO motion

As the test results indicated, the new methods appeared at an average 7 – 10 times as operative as the traditional ones (consecutive scanning and the spiral method). And that concerned only the SOs detected with the help of all methods used. Besides (which is more important), the new methods allowed detecting some weakly-contrasting satellites missed by the traditional surveillance practice. Several functional SOs were detected in GEO and HEO ("Molnia"-like) having a poor illumination phase on which no measurements have been collected for a long time, either by their operators or by the SSS. There were cases when an SO became available in 10 – 15 minutes after having been lost for 6 – 9 months. They could not have been detected with the help of the classic methods despite many efforts [24, 52].

The account of the practical results on the new programs operation was presented at the Fourth European Conference on Space Debris [2] and at the Sixth US/Russian Space Surveillance Workshop [1]. Described below is one of the tests after which two of the search methods based on the theory suggested here were accepted as the regular organic programs.

Two proposed methods (both being described in sections 3.7 and 4.3, method #1 being illustrated by Fig. 3-4-3 and method #2 – by Fig. 3-7-3) and the traditional one were tested for determination of their detection reliability and operativeness (drive efficiency). The last property was estimated by the time input Δt_s since the beginning of search t_1 through the time of real acquisition of the sought-for SO signal t_{acq}:

$$\Delta t_s = t_{acq} - t_1.$$

The reliability of acquisition was estimated as the ratio of the number of successfully detected SOs to the whole number of SOs taken for the test on the condition that the sensor technical characteristics and the astro-ballistic conditions provide the sensor with the opportunity of detecting the SO and these conditions are equal for all the methods involved.

For the test, 22 deep space objects in highly elliptical and 12-hour circular orbits were chosen. 10 of them were used for comparative evaluation of the two proposed methods on one hand and the traditional one on the other. 12 other SOs were used for a comparison of the proposed methods between themselves. In the first case, the field of view was 30 angular minutes and in the second one – 10′.

The search was conducted first with the help of the traditional method of sequentially scanning the static search area (in the sensor picture plane) and then with the help of the new methods. All the results of the test are placed in Tables 4.12.1 and 4.12.2. The first one illustrates the comparison of all 3 methods, the second one contains the results of comparatively testing both new methods.

Table 4.12.1

Results of testing the traditional and proposed search methods

International number	Orbit type	Search method	Dimension of Search area (deg.)	Epoch (d.m.y.)	t_s (h:m:s)	t_{acq} (h:m:s)	Δt_s (h:m:s)
01053002	CO	1	2×0.5	13:02:04	20:35:40	20:36:17	0:00:37
		2	2×0.5	13:02:04	20:40:54	20:41:10	0:00:16
		3	2×2	13:02:04	20:43:42	20:44:11	0:00:29
01050001	HEO	1	2×0.5	12:02:04	00:33:11	00:33:57	0:00:46
		2	2×0.5	12:02:04	01:33:00	01:33:16	0:00:16
		3	2×2	12:02:04	01:35:27	01:37:15	0:01:48
01050001	HEO	1	2×0.5	14:02:04	01:42:02	01:42:20	0:00:18
		2	2×0.5	14:02:04	02:21:14	02:21:33	0:00:19
		3	2×2	14:02:04	02:23:40	02:24:52	0:01:12
94051001	HEO	1	2×0.5	13:02:04	19:53:49	19:54:31	0:00:42
		2	2×0.5	13:02:04	19:59:52	20:00:14	0:00:22
		3	2×2	13:02:04	20:02:12	-	-
01004001	CO	1	3.6×0.5	13:02:04	20:08:13	20:09:18	0:01:05
		2	3.6×0.5	13:02:04	20:12:32	20:12:51	0:00:19
		3	3×3	13:02:04	20:17:11	-	-
02017001	HEO	1	2×0.5	13:02:04	20:47:07	20:47:36	0:00:29
		2	2×0.5	13:02:04	21:05:18	21:05:36	0:00:18
		3	2×2	13:02:04	21:09:08	-	-
74026001	HEO	1	2×0.5	13:02:04	21: 14:31	21:15:11	0:00:40
		2	2×0.5	13:02:04	21:19:14	21:19:30	0:00:16
		3	2×2	13:02:04	21:21:32	21:22:02	0:00:30
98054001	HEO	1	2×0.5	14:02:04	01:17:21	01:18:06	0:00:45
		2	2×0.5	14:02:04	01:22:41	01:22:57	0:00:16
		3	2×2	14:02:04	01:39:30	-	-
97015004	HEO	1	2×0.5	14:02:04	01:27:45	01:28:27	0:00:42
		2	2×0.5	14:02:04	01:34:04	01:34:22	0:00:18
		3	2×2	14:02:04	01:36:49	01:37:20	0:00:31
86065001	HEO	1	2.2×0.5	11:02:04	02:02:40	02:03:24	0:00:44
		2	2.2×0.5	12:02:04	01:59:06	01:59:21	0:00:15
		3	2×2	12:02:04	02:05:17	02:07:59	0:02:42

Here:

1 – the search method starting from the edge of search interval,
2 – the search method starting from the center of search interval,
3 – the traditional search method of sequentially scanning the search area,
CO – a 12-hour circular orbit,
HEO – a highly elliptical orbit

Note, please: The shadowed lines in Table 4.12.1 mean that the traditional method could not detect the corresponding SOs.

This test showed the essential superiority of both new methods against the traditional one. The last one failed to detect SOs ## 94051001, 02017001, 98054001, and 01004001. Meanwhile, both new methods detected all space objects involved in the test. This fact can be accounted for by the fact that the traditional method does not consider the proper complex real dynamics of the SO's current position uncertainty domain during the search process.

In fact, it was stated that the new methods decreased the detection process duration at average by 73% (that is, by 3.8 times) and increased the acquisition reliability by 40%.

When the new methods were compared with each other, the second method (starting from the center of search interval) appeared faster by 47.2%. The detection reliability appeared to be the same for both methods, that is, 100%.

Regular application of both methods for several years confirmed their very high efficiency [2].

Table 4.12.2.
Results of comparatively testing the two proposed search methods

International number	Orbit type	Search method	Dimension of search area (deg.)	Epoch (d.m.y.)	t_s (h:m:s)	t_{acq} (h:m:s)	Δt_s (h:m:s)
01050001	HEO	1	1.6×0.5	15.02.04	19:39:32	19:40:38	0:01:06
		2	1.6×0.5	15.02.04	19:44:51	19:45:33	0:00:42
82015001	HEO	1	1.8×0.5	15.02.04	19:50:14	19:51:31	0:01:17
		2	1.8×0.5	15.02.04	20:03:59	20:04:40	0:00:41
80063004	HEO	1	1.2×0.5	15.02.04	19:56:01	19:57:00	0:00:59
		2	1.2×0.5	15.02.04	19:59:36	19:59:52	0:00:16
98054001	HEO	1	2.6×0.5	15.02.04	20:06:36	20:07:58	0:01:22
		2	2.6×0.5	15.02.04	20:12:13	20:12:29	0:00:16
94051001	HEO	1	2.2×0.5	15.02.04	20:18:23	20:19:39	0:00:16
		2	2.2×0.5	15.02.04	20:24:50	20:25:05	0:00:15
01004001	CO	1	2.6×0.5	15.02.04	20:27:33	20:29:00	0:01:27
		2	2.6×0.5	15.02.04	20:33:09	20:33:40	0:00:31
02017001	HEO	1	2.6×0.5	15.02.04	20:30:07	20:31:38	0:01:31
		2	2.6×0.5	15.02.04	20:33:09	20:33:40	0:00:31
74026001	HEO	1	3×0.5	15.02.04	20:53:01	20:54:49	0:01:48
		2	3×0.5	15.02.04	20:56:07	20:57:00	0:00:53
85088001	HEO	1	3.8×0.5	15.02.04	21:06:52	21:08:56	0:02:04
		2	3.8×0.5	15.02.04	21:10:39	21:11:30	0:00:51
86065001	HEO	1	1.8×0.5	15.02.04	21:32:09	21:33:22	0:01:13

		2	1.8×0.5	15.02.04	21:37:25	21:38:54	0:01:29
97015004	HEO	1	2.8×0.5	15.02.04	21:21:23	21:23:07	0:01:44
		2	2.8×0.5	15.02.04	21:26:39	21:27:41	0:00:42
7010001	HEO	1	1.3×0.5	15.02.04	21:56:04	21:57:04	0:01:00
		2	1.3×0.5	15.02.04	22:00:37	22:01:23	0:00:46

CHAPTER FIVE

THE MOST GENERAL STATEMENT AND SOLUTION OF THE SEARCH PROBLEM

As a rule, in all the previous chapters, the search problems in different search situations were solved mainly on the assumption of the availability of a priori information on the orbit parameters of the sought-for SO having a substantial error only down track while all the rest of the errors (in alternative directions) were being neglected (except for some special cases).

Within such a rather strict assumption, a relatively simple, obvious, refined, elegant, and, at the same time, efficient methodology of optimum planning the search for an SO was developed, its principal operations being reduced to convenient and clear constructions in the *tu* plane.

However, as one could see above, there exist some important search situations in the astronomers' and space surveillance practice in which a priori information on the SO orbit is characterized by the presence of substantial errors not only in argument of latitude, but also in all the rest of the orbit elements. And in such cases, adequately taking account of the SOCPUD structure dynamics is much more problematic.

For constructing SPs in similar situations, so far, some rather artificial devices were suggested (see, for example, section 4.6). Those are rather distant from the optimum methods, because of the absence of a united, strict theoretical approach with due regard to the temporal structural deformation of the SOCPUD, in the course of synthesis of an SP for any character of the errors distribution in a priori data on the orbit. The methods of section 4.6 are mainly of only theoretical and didactic interest. These methods, of course, give a solution to the problem (at best suboptimal), but all the methods in themselves look artificial and cumbersome, and they clearly lack the mathematical grace.

Figuratively speaking, in such a serious and actual business like the search for SOs for the benefit of the global monitoring of the near-Earth space, all the search situations arising there have a right to reckon upon the maintenance by methods developed with the help of the profoundly mathematically based search theory, effectively using imprecise a priori information. The foundation of this theory in essentially more general form than above will be suggested in this chapter. To represent the basic concepts used in constructing the theory and methods a set-theoretic approach will be adopted.

5.1. The space object current position uncertainty domain and its temporal transformation

So, after gaining considerable experience in researching our problem, the comprehensive retrospective analysis of all that was done hitherto in this sphere brings the thought that the radical matter of constructing such a theory is a strict mathematical "dissection of the anatomy" of the SOCPUD temporal structural transformation in 6-dimensional phase space with its subsequent projection into the "working spaces" (2- and 3-dimensional) or of other dimensions if required. The more so, this "dissection" should be made by visual demonstration for simplification of assimilating the strict and formal theory, convenience of using it for development of constructive search methods, and mastering its operational procedures by practical observers. In other words, without compromising the rigor of the theory, we will move towards a smooth transition from the theory to practice where it is necessary and possible.

Next, we will propose a method for exposing the "anatomy" of such a complicated process of the 6-dimensional SOCPUD structural transformation in the course of the SO motion in 3-dimensional space and relative to the observer. This method allows constructively considering the real relative (mutual) structural transformation of the already observed CPUD points and of those still not observed for correctly "docking" the search plan elements (all having common borders) – either without any loss or with minimum losses for the observer and the customer.

The complexity of the problem is due to the fact that the implementation of the SP in time superimposes upon the process of the SO motion and deformation of its CPUD in the same space, both the 6-dimensional phase space and the real 3- and 2-dimensional spaces, the latter two ones being the "working" spaces for radar and optical sensors, respectively. Moreover, in the calculation process of planning, one has to transit from one space to another and the images of the points and domains concerned in the course of these transfers should be retraced adequately with due regard for their structural deformation and transformation in time.

In time, both the already checked (by the sensor) points of the SOCPUD and still not checked ones undergo complex mutual dynamics. They get mixed up in the vicinity of the boundaries of the checked domain in the form of the sensor's FoV (to be more specific, in the form of the FoV's image (projection) in the SOCPUD). This is expressed in its geometrical distortion and wash-out (blurring, smearing about, erosion) of the image's boundaries. It is this phenomenon that should necessarily not only be retraced, but also strictly taken into account in the course of planning the search for an SO (as well as during the realization of the search plan).

We add to this that the described process is projected in the course of observation into the picture plane (PP) of the observer which further complicates the situation.

Neglecting this phenomenon (to be stricter – these phenomena) leads to a rise of the search planning errors of the 1st and 2nd kinds, respectively:

- appearance of unconformity or "chinks" between the SP elements (being deformed and transformed in time) where the observer can lose or miss the sought-for SO (1st kind),
- redundant overlapping between the adjacent SP elements which implies non-economical expenditure of the sensor's search resource (2nd kind).

Both are harmful to the solution of the problem. And one of the goals of the new search theory is to prevent these errors from appearing or to significantly suppress their negative impact on the search result.

5.2. Basic notions, concepts, definitions, and notations

Let at some initial time t_1 some imprecise a priori information on the sought-for SO's orbit be available which is tantamount to giving in the 6-dimentional phase space X_6 a domain $D_6(t_1)$ of possible values of the sought-for SO motion parameters vector (the state vector), for example, the mathematical expectation of the latter $R_6(t_1)$:

$$R_6(t_1) = \|X_1,\ Y_1,\ Z_1, Vx_1, Vy_1, Vz_1\|^{\mathrm{T}} \in D_6(t_1) \subset X_6$$

and the corresponding covariance matrix of its errors:

$$K_R(t_1)=\begin{Vmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{XVx} & \sigma_{XVy} & \sigma_{XVz} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} & \sigma_{YVx} & \sigma_{YVy} & \sigma_{YVz} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 & \sigma_{ZVx} & \sigma_{ZVy} & \sigma_{ZVz} \\ \sigma_{XVx} & \sigma_{YVx} & \sigma_{ZVx} & \sigma_{Vx}^2 & \sigma_{VxVy} & \sigma_{VxVz} \\ \sigma_{XVy} & \sigma_{YVy} & \sigma_{ZVy} & \sigma_{VxVy} & \sigma_{Vy}^2 & \sigma_{VyVz} \\ \sigma_{XVz} & \sigma_{YVz} & \sigma_{ZVz} & \sigma_{VxVz} & \sigma_{VyVz} & \sigma_{Vz}^2 \end{Vmatrix}.$$

That also gives the related probability distribution density function $f_{t1}(R_6)$ defined over the domain $D_6(t_1)$.

The availability of such a priori information gives at time t_1 (under some rather tolerable restrictions) the SOCPUD – that is, a set of practically all possible values of the state vector and a function of the state vector probability distribution density over the SOCPUD.

For determinacy sake, let the SOCPUD (further we denote it as $D_6 = D_6(t)$) be a 6-dimensional ellipsoid, each of its semi-axes being equal to a finite number times the related state vector component root mean square error (for example, 3 r.m.s. errors).

In our further reasoning and considerations, we will keep to the same set-theoretical approach, terminology, and mathematical apparatus as in Chapter 3.

In this light, one should get accustomed to almost visually conceive the SOCPUD not only as a whole ellipsoid, but simultaneously also as a set of possible state vector values – by every point (by every vector) both being fixed to a specific time and in its dynamics as well.

We used to predict the expectation of the state vector and the covariance error matrix and usually to deal predominantly only with them. But those are the principal characteristics of the SOCPUD as a whole.

Yet, for adequately solving the problem at hand, such a treatment is insufficient. We should be able to operate with the CPUD literally "by points" – namely, all the time meaning and bearing in mind its every point both in statics and dynamics. And this is the main justification and substantiation of addressing the set-theoretically construing the basic concepts and notions of the problem in our investigation. That's the principle conception which, as we shall see later, turns out to be fruitful.

Theoretically, if by every 6-dimensional vector $R(t) \in D_6(t)$ we calculated the corresponding ephemeris for a sensor, the latter would check whether the sought-for SO was present there or not. Ultimately, with a guarantee (with reliability up to the accepted assumption on the confidence probability of presence of the SO within its CPUD), we would find the SO.

The task of constructing the optimum search plan equivalent to such a theoretical (unreal) survey of the whole 6-dimensional SOCPUD consists in fixing how to do this really and constructively, the search plan being complete and non-redundant (every vector $R(t) \in D_6(t)$ being controlled and none of them twice – see section 2.3).

In other words, the OSP must provide the guaranteed and the most economical detection of the sought-for SO. This is an optimization problem.

To be more specific, this is only the first optimization level. If there are alternatives (but they are available almost always), and/or if it is possible to introduce some additional features of the problem, the higher level of optimization is possible and so forth.

This is an incipient ancillary idealistic formulation and interpretation of the problem. And yet, it is correct and mathematically rigorous.

As a matter of fact, the observers really deal with not a 6-dimensional, but with a 3-dimensional uncertainty ellipsoid and even with its 2-dimensional projection into the observer's picture plane (for example, for optical sensors). By the way, this is the main cause of practical difficulties that concern planning the search for an SO which are the subject of present research.

Let us define the picture plane (PP) as a plane passing through the center of mass of the SO normal to the observer's sight axis.

And now we consider the projection of the 6-dimensional SOCPUD (a 6-dimensional ellipsoid) into PPt, that is, by definition, into the PP which position corresponds to time t. It is an ellipse. Then, let us introduce the Cartesian reference system in PP with the origin in the SO center of mass (coinciding with the center of the uncertainty ellipsoid). The axis α is directed along the observer's axis of sight from him. The axis γ lies in PP and goes up. The axis β is directed rightward along the horizon (Fig. 5-2-1).

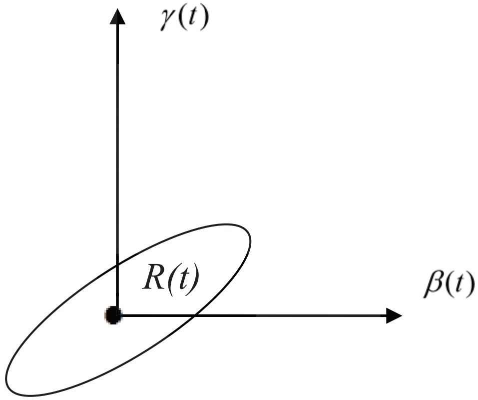

Fig. 5-2-1. Projection of the uncertainty ellipsoid (the SOCPUD) into PP and the coordinate system

For guaranteed detection of an SO, it is necessary to check all the points of its 6-dimensional CPUD. For fulfilling this task, an adequate SP should be compiled. It consists of a sequence of conditional ephemerides $\{\mu_i\}$ covering (after their realization considering the size of the sensor's FoV) the total SOCPUD in its dynamics.

Each SP element μ_i (a conditional ephemeris) is attached to the specific time moment t_i. The magnitude of time interval $\Delta t_i = t_i - t_{i-1}$ between the adjacent SP elements is calculated as a sum of the exposure time (the time for enough accumulation of the intelligence signal energy at the receiver and inner electronic scanning of the sensor's FoV), the time of re-pointing the sensor in accordance with the next ephemeris, and so on.

The task of planning the search for an SO with the help of a given sensor formally consists in choosing a set of conditional ephemerides $\{\mu_i\}$ providing control of all points of the SOCPUD and meeting the given requirements and restrictions (coming from the capabilities and technical characteristics of the sensor and the common conditions of the task). Besides, the set $\{\mu_i\}$ should provide the optimum for the received SP quality criterion $\Phi(\{\mu_i\})$ (see section 2.3).

The ephemeris μ_i for optical sensors represents a set of the time instant t_i and the appropriate angular coordinates (for example, right ascension α_i

and declination δ_i) for setting the sensor's sight axis (for a radar, it may include an azimuth A_i, an elevation ε_i, and a slant range ρ_i). The ephemeris μ_i together with the sensor technical characteristics define the 3-dimensional space domain $d_k^{0i}(t_i)$ being controlled by this sensor at time t_i. Here, k = 2 for optical and electro-optical sensors and then $d_2^{0i}(t_i)$ lies in PP. (For radar and laser sensors, k would equal 3).

Let $D_k(t_i)$ denote the k-dimensional projection in PP of the SOCPUD $D_6(t_i)$, the latter being a region of possible values of the SO motion parameters vector at time t_i which is determined by the mathematical expectation of the SO state vector and the covariance matrix of the initial data errors.

So, by definition, an SP represents a set of conditional ephemerides

$$\{\mu_i\} = \{\mu_i(t_i)\}, i = 1, 2, \ldots \bar{\iota}$$

intended for the realization by the sensor in a consecutive order at the time moments t_i, $i = 1, 2, \ldots \bar{\iota}$.

During the realization of every SP element μ_I, the sensor (both optical and radar) checks some compact 3-dimensional domain $s_i = s(\mu_i(t_i)) \in D_3(t_i)$.

It is understandable that the set s_i and its projection into PP $s_i^{PP} = d_2^{0i}(t_i)$ (the latter equality is by definition) become transformed in time following the dynamics of the SOCPUD and PP itself. It happens due to the change of foreshortening of the set s_i (and the CPUD as a whole) with respect to the observer, evolution of the state vector error distribution, and the change of the PP position in space.

The projection of s_i into PPt_i was denoted earlier as s_i^{PP}. And now, the projection of its transformation with the transfer to the time moment t_j into PPt_j will be denoted as $s_{ij}^{PP} \equiv s_i^{PPt_j}$ (naturally, $i<j$).

It is obvious that the element of SP related to time t_i is an ephemeris μ_i together with the checked domain s_i^{PP}.

The size, form, and position of the set s_i are defined by the observable space region for a given sensor and the ephemeris μ_i. For determinacy sake, let it be a part of a right rectangular pyramid with its peak at the sensor site, its base being located in PPt_i as a square having the angular size $\Delta\psi$. To be more specific, the space region being controlled in one act of observation is a space body which is an intersection (meet) of the pyramid above and the SO position uncertainty ellipsoid. Although, the sensor at time t_i observes its (the controlled body's) projection into PPt_i, that is, the square s_i^{PP}.

Everyone agrees that when planning the search for an SO using imprecise a priori information, a contradictory situation arises. On the one hand, for soonest and economical detection of the SO the current SP element should cover only the unchecked (by the previous elements) points of the SOCPUD. On the other hand, for the guaranteed detection of the SO, all the points of its CPUD should be checked. At the same time, in the course of the SOCPUD motion and due to the evolution of the error distribution, the transformation and deformation of the SOCPUD's internal structure occur and its foreshortening is changing with respect to the observer. All these facts, as one can see below, entail a complicated relative motion and mutual mixing of the checked and unchecked points. Of special interest for the problem at hand is the behavior of the latter in the vicinity of the boundaries of the realized SP elements (continuing to be structurally transformed).

The latter phenomenon does not practically allow avoiding the repeated observation of some checked points of the SOCPUD. Consequently, one can set only a problem (at best, an optimization one) of minimizing these repetitions.

But one should mathematically strictly set and solve this problem making allowance for the above temporal structure transformation of the SOCPUD. Otherwise, it will result in the emergence of errors of the 1st and 2nd kinds (see section 5.1). The former means appearance of displacement between the SP elements and their distortion with time (so, the completeness of SP is violated). Errors of the 2nd kind imply the redundant overlapping of the adjacent SP elements which leads to uneconomical expenditure of the sensor's search resources.

If at time t_1 in PPt_1 the observer checked the point $A^{PPt_1}(t_1) \in D_2^{PPt_1}(t_1) \subset$ PPt_1 and, as a result, has not detected the sought-for SO, then, in order to avoid the repeated check of this point's image at the time t_2 which is unnecessary in conformity with the equivalence principle [25], one should know how its image looks in the picture plane PPt_2 after the transfer to time t_2. That is, the point $A^{PPt_1}(t_1)$ should be propagated by the equivalence to time t_2 and its image (which we will denote $A^{PPt_1}(t_2)$) should be depicted in the picture plane PPt_2.

We used to easily propagate a 6-dimensional state vector of an SO orbit R_6 in the phase space X_6 from any time t_1 to any time t_2 using the celestial mechanics laws F_6:

$$R_6(t_2)= F_6[t_1, R_6(t_1), t_2].$$

But in this case, we should propagate a 2-dimensional point $A^{PPt_1}(t_1)$ from the picture plane PPt_1 associated with time t_1 to another picture plane PPt_2 associated with another time t_2. That means that from the point $A^{PPt_1}(t_1)$ (for short, $A(t_1)$), at first, its 3-dimensional pro-image $R_3^A\ (t_1)$ should be reproduced, and then so should be the 6-dimensional pro-image (that is, the state vector $R_6^A\ (t_1)$). Further, the latter should be propagated to time t_2:

$$R_6^A\ (t_2) = F_6[t_1,\ R_6^A\ (t_1),\ t_2].$$

At last, it should be projected into PPt_2.

However, to do this practically is not so simple as will be shown later (see also [42]), although possible, as also will be shown later.

5.3. Temporal transformation of a point in the picture plane

Let, the same as before, the mathematical expectation of the sought-for SO orbit state vector $R_6(t_1)$ and its covariance matrix of errors $K_R(t_1)$ be given at time t_1.

So, following [42], let us pick up an arbitrary point $A(t_1)$ within the projection of SOCPUD in PPt_1 as if being sighted by the observer (Fig. 5-3-1):

$$A(t_1) \equiv A^{PPt_1}(t_1) \equiv (\beta(t_1),\ \gamma(t_1)).$$

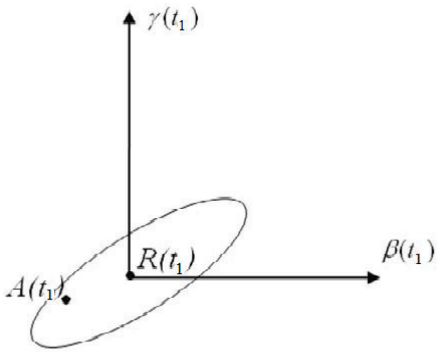

Fig. 5-3-1. Choice of an arbitrary point A in the projection of the SOCPUD in the picture plane

In the 3-dimensional space *XYZ* (the inertial geocentric reference frame), this point corresponds to the straight-line segment given by its ends:

$$[A_b(t_1) \equiv (X_b(t_1), Y_b(t_1), Z_b(t_1)); A_f(t_1) \equiv (X_f(t_1), Y_f(t_1), Z_f(t_1))].$$

This segment is the intersection (meet) of the straight line coming through the point $A(t_1)$ normal to PPt_1 and the uncertainty ellipsoid. In other words, all the points of this segment (which belongs to the SOCPUD) are projected along the sight axis to the point $A(t_1)$.

The beginning and the end of this line segment $A_b(t_1)$ and $A_f(t_1)$ have the phase coordinates, respectively,

$$R_{A_b}(t_1) = \begin{pmatrix} X_{A_b}(t_1) \\ Y_{A_b}(t_1) \\ Z_{A_b}(t_1) \\ \left\{\dot{X}_{A_b}(t_1)\right\} \\ \left\{\dot{Y}_{A_b}(t_1)\right\} \\ \left\{\dot{Z}_{A_b}(t_1)\right\} \end{pmatrix} \text{ and } R_{A_f}(t_1) = \begin{pmatrix} X_{A_f}(t_1) \\ Y_{A_f}(t_1) \\ Z_{A_f}(t_1) \\ \left\{\dot{X}_{A_f}(t_1)\right\} \\ \left\{\dot{Y}_{A_f}(t_1)\right\} \\ \left\{\dot{Z}_{A_f}(t_1)\right\} \end{pmatrix}, \quad (5.3.1)$$

the first three coordinates (positional) of these vectors having definite numerical values as the solution of the joint system of equations: one for the line $[A_b(t_1); A_f(t_1)]$ and the other for the uncertainty ellipsoid surface – in the inertial geocentric coordinates. So, $A_b(t_1)$ is the closest point to the observer of the SOCPUD being projected along the sight axis to the point $A(t_1)$. And $A_f(t_1)$ is the farthest one. $R_{Ab}(t_1)$ and $R_{Af}(t_1)$ are the phase coordinates of those points, respectively.

The other 3 coordinates (velocities – in braces) are indefinite. More exactly, they represent some sets of possible values of these velocities. ***And that is the principal point of the matter! Which is to be reckoned with essentially further.***

The information on the limits of their indefiniteness (with the confidence probability accepted) is contained in the covariance matrix of errors $K_R(t_1)$ and should be extracted from the matrix. But it is not evident how to do this. The more so, namely for the picked point.

Here, the means are proposed for doing this. Let us form two 3-dimensional sub-vectors

$$R_3^{A_b}(t_1) = ||X_{A_b}(t_1), Y_{A_b}(t_1), Z_{A_b}(t_1)||^{T}$$

and

$$R_3^{A_f}(t_1) = ||X_{A_f}(t_1), Y_{A_f}(t_1), Z_{A_f}(t_1)||^{T}$$

from the 6-dimensional phase vectors $R_{A_b}(t_1)$ and $R_{A_f}(t_1)$ (5.3.1) – just "cutting them half-and-half". And let us name and consider them as pseudo measurements of the SO possible positions with practically zero-valued covariance sub-matrices of errors (3-dimensional ones). These sub-vectors are, so to speak, "deprojections", "reprojections", or "retro-projections" of the point $A(t_1)$ from PPt_1 back to the 3-dimensional space.

Such a move is justified by the fact that the primarily selected point $A(t_1)$ in PPt_1 represents a possible (within the SOCPUD projection) position of the sought-for SO in PPt_1 at time t_1.

As the next step, we will solve in a consecutive order the program of orbit refinement for the initial (6-dimensional) orbit state vector $R_6(t_1)$ and its covariance matrix of errors $K_R(t_1)$, using first the 3-dimesional pseudo measurement $R_3^{A_b}(t_1)$ and then $R_3^{A_f}(t_1)$ as additional information on the points. As a result, two refined (pseudo refined) vectors of phase coordinates corresponding to points $A_b(t_1)$ and $A_f(t_1)$, respectively, will be obtained:

$$R^r_{A_b}(t_1) = \begin{pmatrix} X_{A_b}(t_1) \\ Y_{A_b}(t_1) \\ Z_{A_b}(t_1) \\ \left\{\dot{X}^r_{A_b}(t_1)\right\} \\ \left\{\dot{Y}^r_{A_b}(t_1)\right\} \\ \left\{\dot{Z}^r_{A_b}(t_1)\right\} \end{pmatrix} \quad \text{and} \quad R^r_{A_f}(t_1) = \begin{pmatrix} X_{A_f}(t_1) \\ Y_{A_f}(t_1) \\ Z_{A_f}(t_1) \\ \left\{\dot{X}^r_{A_f}(t_1)\right\} \\ \left\{\dot{Y}^r_{A_f}(t_1)\right\} \\ \left\{\dot{Z}^r_{A_f}(t_1)\right\} \end{pmatrix}$$

and their covariance matrices of errors $K^r_{A_b}(t_1)$ and $K^r_{A_f}(t_1)$. The components of the latter matrices are much smaller than those of the initial matrix $K_R(t_1)$ because of the practically absolute accuracy of the pseudo measurements $R_3^{A_b}(t_1)$ and $R_3^{A_f}(t_1)$.

Then, with the help of the model of the SO motion, we will propagate vectors $R^r_{A_b}(t_1)$ and $R^r_{A_f}(t_1)$ at time t_2 and obtain, respectively, new vectors

$$R^{ref}_{A_b}(t_2) = \begin{pmatrix} X_{A_b}(t_2) \\ Y_{A_b}(t_2) \\ Z_{A_b}(t_2) \\ \left\{\dot{X}^{ref}_{A_b}(t_2)\right\} \\ \left\{\dot{Y}^{ref}_{A_b}(t_2)\right\} \\ \left\{\dot{Z}^{ref}_{A_b}(t_2)\right\} \end{pmatrix} \text{ and } \quad R^{ref}_{A_f}(t_2) = \begin{pmatrix} X_{A_f}(t_2) \\ Y_{A_f}(t_2) \\ Z_{A_f}(t_2) \\ \left\{\dot{X}^{ref}_{A_f}(t_2)\right\} \\ \left\{\dot{Y}^{ref}_{A_f}(t_2)\right\} \\ \left\{\dot{Z}^{ref}_{A_f}(t_2)\right\} \end{pmatrix}$$

and the covariance matrices of their errors $K^r_{A_b}(t_2)$ and $K^r_{A_f}(t_2)$.

After projecting these vectors to PPt_2 equipped with the reference system $\beta(t_2),\gamma(t_2)$ (at time t_2 it will be another PP), we will obtain two points $A_b(t_2)$ and $A_f(t_2)$, respectively, that is, the prediction and projection mapping of points $A_b(t_1)$ and $A_f(t_1)$, respectively, to time t_2. To be more specific, the obtained points are only the centers of the corresponding domains (projections of the small uncertainty ellipsoids) in which the points $A_b(t_1)$ and $A_f(t_1)$ are transferred in the course of the period $[t_1, t_2]$.

The question is: what is the whole image of the initially selected point $A(t_1)$ after the transfer to time t_2? This situation is much harder because hitherto we constructed only the ends of the uncertainty segment $[A_b(t_1); A_f(t_1)]$. Speaking strictly, theoretically, the same as we have handled with the ends of the uncertainty segment, we should handle with all its points. So, theoretically, in this way, we could obtain the whole image of propagating and projecting point $A(t_1)$ to PPt_2. But this theoretical way is non-constructive (because of the infinite number of the segment points).

To get away from the endless procedure, practically, albeit approximately, we can represent the segment $[A_b(t_1); A_f(t_1)]$ by several end and intermediate points (that is, to discretize the segment) and apply all the above procedure to each of them. Eventually, we will approximately reconstruct the whole image of the initial point $A(t_1)$ in discrete form (see Fig. 5-3-2). At last, we can envelope the entire domain.

In Fig. 5-3-3, the transformation of an arbitrary point $A(t_1) \in D_2(t_1) \subset$ PPt_1 with time change from t_1 to t_2 is shown (after the appropriate envelopment of the discrete image of the point). So, the mathematical description of the "anatomy" of this transformation is given there. The latter includes the following stages:

- recreation of a 3-dimensional pro-image (inverse image) of point $A(t_1)$: $P_{3\to2}^{-1}[A(t_1)] = d_3^A(t_1)$ (the uncertainty segment);
- recreation of a 6-dimensional pro-image (pro-pro-image) of point $A(t_1)$: $P_{6\to3}^{-1}[d_3^A(t_1)] = d_6^A(t_1)$;
- refining the latter pro-image $d_6^A(t_1)$ using the pseudo measurements: $d_6^{Aref}(t_1) = R_6(t_1) \oplus \{R_3^A(t_1)\}$;
- propagation of a so refined pro-image to time t_2: $d_6^{Aref}(t_2) = F_6[t_1, d_6^{Aref}(t_1), t_2]$;
- projecting the result $d_6^{Aref}(t_2)$ into PPt_2: $P_{6\to2}[d_6^{Aref}(t_2)] = F_2[t_1, A(t_1), t_2]$.

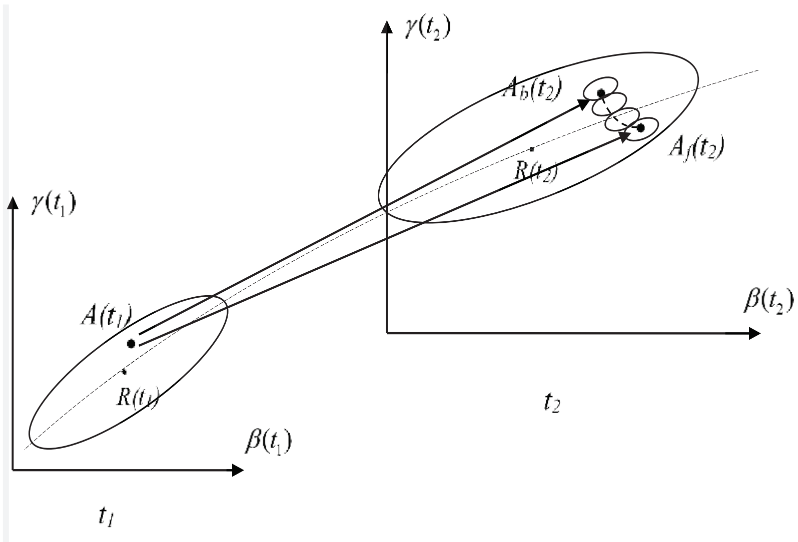

Fig. 5-3-2. Temporal transformation of the initially selected point $A(t_1)$ in PP (its F_2-mapping from t_1 to t_2 in a discrete form)

Here P denotes the projection operator: $P_{m \to n}$ $(m > n)$ means projection from an m-dimensional space to an n-dimensional space and $P^{-1}_{m \to n}$ $(m > n)$ means retro-projection (reproduction) from an n-dimensional space to an m-dimensional space.

The principal finding in the foregoing investigation in this section is that we have managed to clarify that the image of point $A(t_1)$ when transferring from time t_1 to t_2 is not a point, but some set (domain) $F_2[t_1, A(t_1), t_2]$ in picture plane PPt_2. And it was shown there how to depict it in PPt_2. The matter of fact is that to find the image of $A(t_1)$ in PPt_2 just using the SO motion model is not this simple owing to the next. The selected point $A(t_1)$ in PPt_1 is a result of the projection of a rather complicated set of 6-dimensional state vectors from the SOCPUD. And, to say strictly, one should propagate all these state vectors to time t_2 restoring in this process several transitions from a space of one dimension to a space of another dimension in order to find the whole adequate image of point $A(t_1)$ in the picture plane PPt_2.

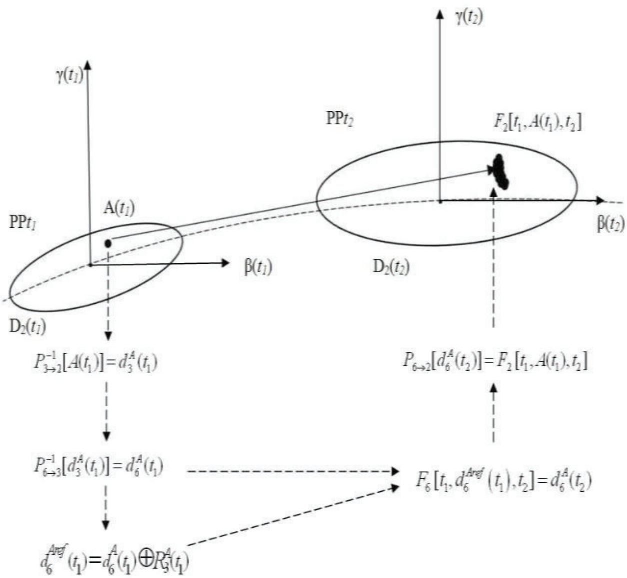

Fig. 5-3-3. Transformation of a point $A(t_1) \in D_2(t_1) \subset$ PPt_1 when transferring to PPt_2 after the proper envelopment of its discrete F_2-image and considering the mathematical "anatomy" of this transformation.

So, we have come to a new important concept in planning the search for an SO by imprecise a priori orbital information – a phenomenon of degradation of a checked point in the picture plane with time change. This phenomenon underlies the more important and fundamental concept having an immediate relation to the problem at hand – a phenomenon of degradation of the search plan of an SO in the course of its realization.

This phenomenon will be discussed in detail in section 5.5.

Although the SOCPUD temporal structure transformation mechanism was so far revealed only for a point, for more complicated geometric figures it can be done by their points discretely.

Now, let us investigate the transition process with the set s_i, its projection $s_i^{\mathrm{PP}t}$ to the picture plane PPt, and the whole SOCPUD, the SO state vector errors being of the most general character.

5.4. Generalization of the equivalence principle of the search plan elements for different times

The main notion and the stem of the theory of optimum planning the search for a space object by its incomplete a priori orbital information is the principle of equivalence of the search plan elements for different times. This theory was developed (up to the real search methods and working algorithms and programs) some years ago for the important case of having respect to the state vector error only along the track [36]. In the latter assumption, the principle is strictly defined in [36, 24] and here, more concisely, in section 3.2.

For this case, practical application of the equivalence principle appeared to be very convenient which makes the search planning procedure practically simple enough. But except this case, there exist many search situations where one cannot neglect the state vector errors in different directions. For such situations, ***the equivalence principle as the main tool of optimum planning the search for an SO should be generalized to the majorizing equivalence principle.***

The generalized equivalence principle can be used for practically constructing optimum plans of search for SOs by rough a priori orbital information in the most common case of the state vector errors.

The same as above (in section 5.2), let the rough (incomplete) a priori orbital information on every SO is available if in the 6-dimentional phase space X_6 there is given a domain $D_6(t_0)$ of possible values of the sought-for SO motion parameters vector (the state vector) on the time moment t_0, i. e., the initial state vector (its mathematical expectation)

$$R_6(t_0) = \left\| x, y, z, v_x, v_y, v_z \right\|^{\mathrm{T}} \in D_6(t_0) \subset X_6$$

and the related covariance matrix

$$K^O(t_0) = \begin{Vmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{XVx} & \sigma_{XVy} & \sigma_{XVz} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} & \sigma_{YVx} & \sigma_{YVy} & \sigma_{YVz} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 & \sigma_{ZVx} & \sigma_{ZVy} & \sigma_{ZVz} \\ \sigma_{XVx} & \sigma_{YVx} & \sigma_{ZVx} & \sigma_{Vx}^2 & \sigma_{VxVy} & \sigma_{VxVz} \\ \sigma_{XVy} & \sigma_{YVy} & \sigma_{ZVy} & \sigma_{VxVy} & \sigma_{Vy}^2 & \sigma_{VyVz} \\ \sigma_{XVz} & \sigma_{YVz} & \sigma_{ZVz} & \sigma_{VxVz} & \sigma_{VyVz} & \sigma_{Vz}^2 \end{Vmatrix}.$$

This is tantamount to giving the related probability distribution density function f_{t0} (R_6) defined upon the domain $D_6(t_0)$.

The celestial mechanics laws define on the domain $D_6(t_0)$ given at a specific time t_0 a homeomorphic mapping F which transfers each point $R_6(t_0)$ of D_6 at time t_0 to another point $R_6(t_1)$ of X_6 at another time t_1. The property of homeomorphism of F (a mapping between two domains – here $D_6(t_0)$ and $D_6(t_1)$) means that the domain $D_6(t_0)$ is one-to-one and to-and-fro continuously transferred by F into the domain $D_6(t_1)$:

$$R_6(t_1) = F[t_0, R_6(t_0), t_1], \quad D_6(t_1) = F[t_0, D_6(t_0), t_1].$$

The equivalence principle primary formulation:

Checking the point $R_6(t_0)$ of $D_6(t_0)$ at time t_0 (to learn if the sought-for SO is present or absent at the point) is equivalent to checking the point $R_6(t_1)=F[t_0, R_6(t_0), t_1]$ of $D_6(t_1)= F[t_0, D_6(t_0), t_1]$ at time t_1 in the sense that it is not necessary to accomplish both acts of checking – it is sufficient to check only one of the two equivalent points.

Similarly, checking the subdomain $d_6(t_0)=F[t_0, d_6(t_0), t_1] \subset D_6(t_0)=F[t_0, D_6(t_0), t_1]$ at time t_0 is equivalent to checking the subdomain $d_6(t_1) \subset D_6(t_1)$ at time t_1 in the same sense.

The search plan M_6 is a set of pairs:

$$M_6 = \{(R_6, t)\}, R_6 \in D_6 \text{ (considering the equivalence principle).}$$

Each pair means checking point R_6 at time t.

The optimum condition for the search plan:

The search plan M_6 is referred to as optimum if it is complete and non-redundant. M_6 is complete if realization of all its pairs $\{(R_6, t)\}$ guarantees a coverage of the sought-for SO in the phase space, that is, $M_6 = \{(R_6, t)\}$ $\forall R_6 \in D_6$ (considering the equivalence principle). M_6 is non-redundant if among all its pairs $\{(R_6, t)\}$ there are no equivalent ones.

For a 6-dimentional uncertainty domain $D_6(t)$ where every its point is the full state vector $R_6(t)$, the F's property of homeomorphism provides the transfer of any point $R_6(t_1)$ to some (exactly one!) point $R_6(t_2)$ and vice versa. This fact greatly simplifies the analysis of temporal structural transformation of the SO position uncertainty domain and application of the equivalence principle for planning the search (if it were in 6-dimentional space!).

But a real sensor sounds not the 6-dimentional phase space but its 2-dimentional (for an optical sensor) or 3-dimentional (for a radar sensor) projection. Simultaneously, the mapping F, so to speak, "is projected" into 2-dimentional (F_2) or 3-dimentional space (F_3), respectively. The projection equations are given in [24, 36]. According to it, the F_k-image (k =2 or 3) of any point $R_k(t)$ is already not a point but a set of points (a domain). So, ***the mapping F_k is not a homeomorphism**,* that is, not one-valued round trip, not to speak about direct and reverse continuity. That means that the real optimum planning process of the search becomes very complicated.

As shown in [24, 36, 44, 1, 2], this process can be essentially simplified (just F_k can be returned to a homeomorphism) for the case of assumption of predominant error propagation only along the track. Under this assumption, one can reduce the process of constructing optimum search plans to simple operations in such a space as the *tu* plane where t is time and u is argument of latitude. And so, application of the equivalence principle becomes constructive and very simple [44, 1, 2].

This case (assuming a predominant error growth along the trajectory) is very important and actual for situations of search for an SO in highly eccentric orbits with the help of narrow-beam radar sensors and narrow-angle optical and electro-optical sensors.

But at the same time, there exist many search situations where one cannot neglect the state vector errors in different directions. For these situations, the equivalence principle as the main tool of optimum planning the search in the preceding case should be generalized to the majorizing equivalence principle.

The main difference of the considered here and below search situations from the simple case mentioned above (the one of predominant state vector error propagation only along the track) consists in the necessity of having respect to a very complicated influence of the different directions errors to the character of structure dynamics of the sought-for SO current position uncertainty domain [42].

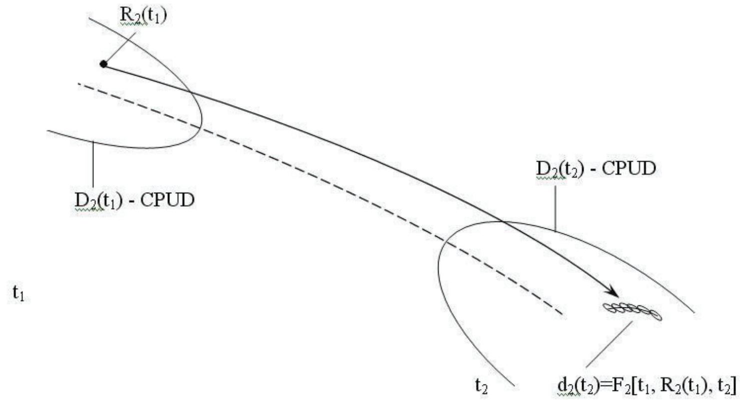

Fig. 5-4-1. Temporal transformation of a point from $D_2(t_1)$

For certainty, let us confine ourselves further by 2-dimentional searching space $D_2(t)$ in the picture plane (PP) which is natural for optical sensors. Unlike the previous case, now any point $R_2(t_1) \in D_2(t_1)$ is transferred by mapping F_2 rather into some domain $d_2(t_2) \subset D_2(t_2)$ than a point $R_2(t_2)$ (see section 5.3 and Fig. 5-4-1).

Then ***the generalized equivalence principle runs as follows:***

Checking the point $R_2(t_1) \in D_2(t_1)$ at time t_1 is equivalent to checking the domain $d_2(t_2) = F\ [t_1, R_6(t_1), t_2] \subset D_2(t_2)$ at time t_2, but not *vice versa* because some other points from $D_2(t_1)$ were also transferred (by F) into $d_2(t_2)$. There was shown in [42] how to calculate the domain $d_2(t_2)$.

This generalized principle together with the related calculating algorithm helps constructing (by points) F_2-image $d_2(t_2)$ of an optical sensor field of view $d_2(t_1)$ in PP (Fig. 5-4-2). For this purpose, a special mathematical model was developed and realized. One can see that the boundary of $d_2(t_2)$ became notably blurred (eroded) due to the above effect of structural transformation of domain $D_2(t_1)$.

In terms of the generalized equivalence principle, the initial field of view $d_2(t_1)$ (fixed at time t_1) is equivalent to, or to be precise, majorizes the internal “volume” of $d_2(t_2)$ – excluding its boundary. Denote this subdomain of subdomain of $d_2(t_2)$ as $d_2^-(t_2)$. As follows from the updated equivalence principle, if you checked $d_2(t_1)$ at time t_1, there is no need to check $d_2^-(t_2)$ at time t_2.

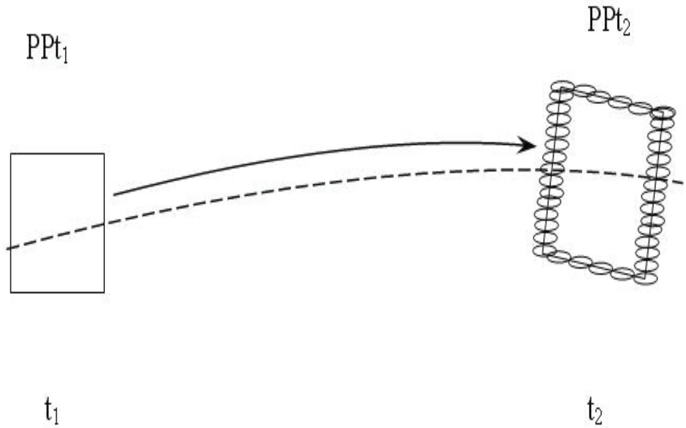

Fig. 5-4-2. Temporal transformation of a field of view

5.5. The search plan degradation during its realization

As for a point $A(t_1) \in D_2(t_1)$ in a picture plane, substantiation of the term “degradation” is not so obvious and demonstrative. This phenomenon can rightfully be called degradation with respect to elements of the search plan (which are domains rather than points). However, this term has a certain emotional shade. So, perhaps more relevant here (with respect to a point) would be using a rather neutral term like “transformation”. But the above described model for the process of transformation of a point picked up from the projection of the SOCPUD to PP is wanted for revealing, reproduction, and investigation of the degradation phenomenon as applied to the whole search plan (which is very important practically). As it will be shown later, planning the search for an SO using imprecise a priori information without due regard to this phenomenon cannot be performed and perceived mathematically and physically correctly (including in terms of counteraction to the negative influence of errors of the 1st and 2nd kinds).

Since the distance from the observer to the SO changes in time, the size of the pyramid's base (the sensor's zone of action), i.e., of the controlled domain s_i^{PPt} of PPt at time t cannot be constant (only the angular size of FoV remains constant).

But, at first, for simplification of understanding and getting the hang of the main process, let us assume that within the SP time span (at time moments $t_1, t_2, \ldots t_i, \ldots t_{\bar{I}}$) one can neglect moving away and drawing near of the PP and as a consequence – neglect the change of the checked domain's size. That means that during realization of the SP elements the latter remain as squares with conservation of the same size of their sides. Generally speaking, such an assumption is somewhat tolerable and approved for small time spans. Further, if a need arises when proceeding to the interactive synthesis of SPs at long time spans, one will have to abandon this assumption which (withdrawing from) implies no difficulties.

In Fig. 5-5-1, the position of the sensor's FoV in the observer's PP bound up with the first SP element (at time t_1) is shown. Let us introduce a notion of the effective checked area (ECA)

$$S_i^+ \equiv S_i^+ (t_i)$$

of the i-th SP element's realization $s_i^{PP_{t_i}}$. By definition, it is an area of the guaranteed controlled domain of the SOCPUD projection (in the i-th act of search, i.e., by the SP element μ_i) – more exactly, a part of the domain where one can confidently assert that there is or there is not the sought-for SO there (with the probability almost 1 under the assumption above).

As Figure 5.5.1 visually depicts, at time t_1 the value of ECA S_1^+ (i.e., $S_1^+(t_1)$) is equal to the area of the square $s_1^{PP_{t_1}}$ (commensurable with FoV). In the notations of Fig. 5-5-1: $s_1^{PP_{t_1}} \equiv s_1^{P(t_1)}$, $s_1^{PP_{t_2}} \equiv s_1^{P(t_2)} \equiv s_{1,2}^{PP}$.

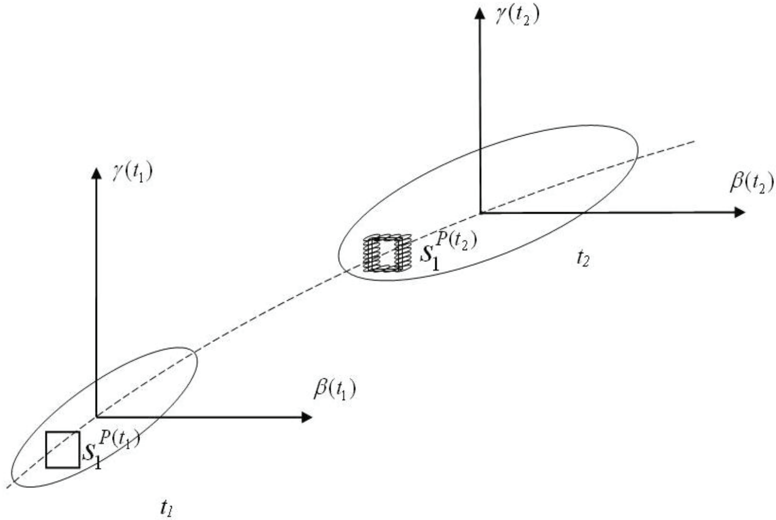

Fig. 5-5-1. Temporal transformation of the first SP element $S_1^{PPt_1}$

At time t_2 (the beginning of realization of the second SP element) due to the above transformation and deformation processes in the already checked domain $s_1 = s(\mu_1(t_1))$, the latter will be mapped into PPt_2 as a rather complicated figure $S_{1,2}^{PP}$ having the blurred (eroded) boundaries (see Fig. 5-5-1).

Due to only the change of the SOCPUD foreshortening with respect to the observer during the transfer from t_1 to t_2 and, as a consequence, the change of the sighting angle through the value $\Delta\varphi$ (even with no regard for the error evolution), the boundary smear (washing away) will be measured as

$$l_{\mu_1} sin\Delta\varphi$$

where l_{μ_1} is the length of a straight line segment which is a part of the μ_1-related sight line limited by the surface of the SO position uncertainty ellipsoid.

However, in view of the above assumption, the mathematical expectation of the resulting figure $s_{1,2}^{PP}$ will remain a square of the same size (although in reality this figure will also be twisted and change its size).

For calculating the transformation of ECA S_1^+ at time t_2 (i.e., $S_1^+(t_2)$), one needs to inscribe into this figure a rectangle*) tangent with its sides to the smeared boundaries of the figure $s_{1,2}^{PP}$, the area of the latter rectangle being equal to

$$S_1^+(t_2) = a(s_{1,2}^{PP}) \cdot b(s_{1,2}^{PP})$$

where $a(s_{1,2}^{PP})$ and $b(s_{1,2}^{PP})$ are the lengths of the horizontal and vertical rectangle sides, respectively.

The same way, one should treat with $s_{1,2}^{PP}$ being propagated to times $t_3, t_4, \dots t_i, \dots t_{\bar{\imath}}$ – the later t_i the larger blurring (erosion) of the boundaries.

So, the very important finding from the above analysis is that the magnitude S_i^+ depends on time (that is, it is variable):

$$S_i^+ = S_i^+(t).$$

The more so, it is monotonously decreasing, this decrease, as the first approximation, being proportional to the change of the sighting angle $\Delta\varphi$. ***And this is an explicit symptom of the degradation phenomenon*** (even within the framework of the accepted simplifying assumption). But really, the decrease of this index is still more complex and progressive if one takes account of the blurring of all 4 boundaries of the square s_i^{PP} and the non-linear deformation of the SOCPUD structure (all the more if the accepted simplifying assumption is abandoned).

This is fundamentally a new postulate as compared to axiomatics of the traditional approaches to planning the search for a SO where the magnitude S_i^+ was supposed (by tacit consent) to be a constant.

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*) here it is a rectangle (not a square) because of different blurring intensity for different boundaries (oriented in different directions)

So, the fundamental (for this investigation) function $S_i^+(t)$ for $t > t_i$ was introduced here which characterizes at the current time t the variable ECA of guaranteed checked part of the SOCPUD since the instant of realization of the i-th SP element μ_i.

Now, let us introduce a notion of the efficiency factor (coefficient) $K_i^{eff}(t)$ for the i-th SP element μ_i at time t. For this, let us circumscribe a rectangle around the figure $s_i^{\mathrm{PP}t}(t)$ and determine its area $S_i^0(t)$.

By definition,

$$K_i^{eff}(t) = \frac{S_i^+(t)}{S_i^0(t)} .$$

And the degradation factor $K_i^{deg}(t)$ of the i-th SP element μ_i at time t is defined as

$$K_i^{deg}(t) = \frac{1}{K_i^{eff}(t)} .$$

A very important problem is to investigate the behavior of the two factors in time. If it occurs that $K_i^{eff}(t) \longrightarrow 0$ when time grows and this process is very fast within the search interval, that means degeneration of the SP (at any rate, of its i-th element). ***So, we have a mathematical tool for calculus of the SP degradation.***

Let us see what it means for the process of planning the search and how it can be correctly taken into account for preventing the degradation process, to say more exactly, for counteraction to the latter or to its negative influence on the SP's quality.

5.6. Available ways for mitigation of the search plan degradation phenomenon negative consequences

Usually and naturally, planning the search for an SO starts with the choice of parameters of the first SP element μ_1 – the time of beginning of its realization t_1 and its location for this time within the SOCPUD projection to $\mathrm{PP}t_1$. The concrete recommendations for this will be given later.

The next steps are the choice of the second and all the following SP elements $\{\mu_i\}$, $i = 2, 3, \ldots \overline{i}$, in increasing succession. (However, in practice there are cases for which it is not so.) On account of blurring the already checked SP element's boundaries, for guarantee of elimination of the 1st kind errors, it is needed that in the process of docking the i-th (already checked) and j-th (still to be checked) elements ($j > i$) the current j-th (square-like) element should touch the blurred boundary of the i-th (rectangle-like) element from the inside of the latter.

In terms of the degradation phenomenon, when planning the search, one ought to dip the current (not yet "virtually realized") SP element with at least one of its sides into the blurred sides of the preceding (already synthesized – "virtually realized") elements. By this way, the already occurred degradation is partially compensated (at the cost of some decrease in the effectiveness of using the sensor's search resource). This is almost the only optimistic factor which could and should be used when planning the search for an SO side by side with any possible ways of acceleration of the search process. Both means to a degree neutralize the negative consequences of the degradation.

On account of different rates of the SO position errors' growth in different directions, the rates of blurring of the SP element different boundaries differ from one another. That is why it is essential to dock the side of the current SP element to the mostly blurred boundary of one of the preceding elements (already checked). This compensatory device is to be regarded when calculating the common (united) degradation coefficient $K_{\Sigma}^{\deg}$ for the SP at time t.

Let us suppose that by this time moment t, several SP elements have been already synthesized (see Fig. 5-6-1).

The reduced summary ECA of the SP by time t is estimated by the following expression

$$S_{\Sigma}^{+}(t) = \sum_{i} S_i^{+}(t) + S_{\Sigma}^{-}(t) = \sum_{i} (S_i^{+}(t_i) - \Delta S_i^{+}(t)) + S_{\Sigma}^{-}(t)$$

where $\Delta S_i^{+}(t)$ is a correction taking into account the blurring of the i-th element's boundaries and the immersion of the current i-th element into the blurred boundary of the adjacent preceding element. In fact, it is an area of the washed away boundaries of the i-th element by time t. The variable $S_{\Sigma}^{-}(t)$ means the total area of all the adjacent washed away boundaries of all already synthesized SP elements by time t (i.e., dipped into the already constructed part of the SOCPUD coverage).

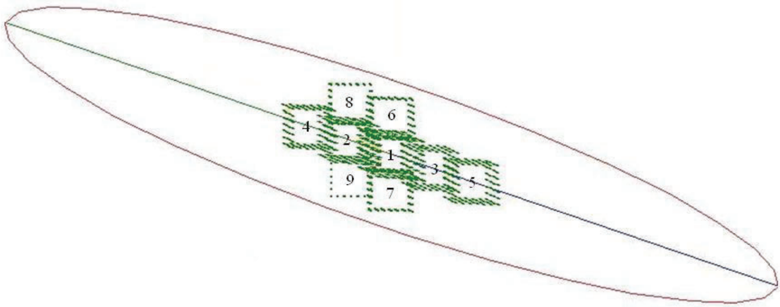

Fig. 5-6-1. Interactive suboptimum accommodation of the SP elements (for the 9 first elements)

From this sum for sure, all the external blurred boundaries should be excluded. Of course, $t_i < t\ \forall i$.

It is the term $S_\Sigma^-(t)$ that determines the degree of compensation of the degradation phenomenon in the planning method used.

By definition, the basic (reference) reduced area of the SP by the moment t (perhaps a part of the SP if $t < t_{\bar{\iota}}$) is the sum of areas of all SP elements having been synthesized (or realized) by time t:

$$S_\Sigma^0(t) = n_t a^2$$

where n_t is the number of synthesized elements by the moment t, and a is the length of the square FoV side in PPt. This index characterizes the whole observation resource spent by time t.

By definition, the total SP efficiency factor by time t equals

$$K_\Sigma^{eff}(t) = \frac{S_\Sigma^+(t)}{S_\Sigma^0(t)}$$

and the total SP degradation factor is

$$K_\Sigma^{deg}(t) = \frac{1}{K_\Sigma^{eff}(t)} .$$

5.7. Planning the search for a space object with partial compensation of the search plan degradation negative consequences

At this stage of learning the SP degradation phenomenon, for mitigating it, we are already able to recommend a rational constructive procedure of synthesizing the plan of search for an SO with due regard for this phenomenon and at least partial compensation of its consequences.

As it was said above, one of the tools for neutralizing the degradation phenomenon can be any means of accelerating the search process because the rate of the boundary blurring is proportional to time.

The updating of the sensor construction (as an alternative mitigation tool) is out of our consideration now. So, only the principles of forming the SP are available. For example, one can choose parameters of the sequence of SP elements so that the rates of the SO detection probability growth should be the greatest. This is an optimization problem.

Consequently, for obtaining the suboptimum solution, the following way is possible. The first element (μ_1, s_1^{PP}) of SP {(μ_i , s_i^{PP})} is placed to the center of the SOCPUD where the SO detection probability density is maximum (for example, for the normal error distribution law of the initial data on the SO motion parameters). Later, all the subsequent SP elements are accommodated at the sites of the SOCPUD projection with the greatest detection probability density within this element. And, if some alternatives are available, with the greatest coverage of the blurred boundaries' area of the preceding (already accommodated) SP elements.

Although, to say strictly, the greatest resultant blur of the boundaries does not witness without fail the greatest rate (speed) of blurring them out. The substantially blurred boundaries may belong to the SP element s_{ij}^{PP} for which the difference $t_j - t_i$ is great. This fact should also be taken into account when accommodating the next SP element (see section 5.5).

The suggested consecutive procedure of synthesizing the SP is illustrated by Fig. 5-6-1. The depicted checked (by the time moment t_j) part of the SOCPUD projection can be expressed as

$$\bigcup_{i=1}^{j} s_{ij}^{PP}$$

where j is the index of the current (being now accommodated) SP element and $\{i\}$, i = 1, 2, …j, are indices of all already accommodated SP elements.

An important note. The lasting degradation process inside the already covered part of the SOCPUD has no negative consequences any longer for the quality of the search plan and the results of its realization. For the latter, only the outer limits of the already constructed part of the SP should be properly respected in the ongoing and being dynamically corrected planning process.

In this case, it is natural to choose as an SP quality criterion the mathematical expectation of the detection time since the beginning of the SP realization:

$$M[T_{\det}] = \Delta t \sum_{i=1}^{\bar{i}} iP_i = \Delta t \sum_{i=1}^{\bar{i}} i \int_{s_i^{PP}} f(s)ds$$

where P_i is the SO detection probability gained by realization of the i-th SP element, $f(s)$ is the probability distribution density function of the event when the sought-for SO is in the point s, s being a point of the figure s_i^{PP} (a square), and Δt is the discrete time step between the consecutive observation time moments.

That was an approximate formula because it neglects the overlapping of the blurred SP element's boundary by the next adjacent element. As a matter of fact, the real detection time is a little more than that given by this formula. The more exact formula looks like this:

$$M[T_{\det}] = \Delta t \sum_{i=1}^{\bar{i}} i[\int_{s_i^{PP}} f(s)ds - \sum_{j<i} \int_{s_i^{PP} \bigcap s_{ji}^{PP}} f(s)ds].$$

But it is harder to calculate it.

At this stage of investigation, some definite findings should be made. The degradation process of the already realized SP elements throughout the performance of the SP is inevitable. But it can be mitigated and to a degree neutralized which can be achieved by rational organization of monitoring the SOCPUD structure transformation and taking it into account when constructing search plans. The possibility and effectiveness of such a compensation depends on the rate of covering the CPUD by the sensor FoVs, the initial data error distribution, and the specific means of planning the search used.

5.8. Generalization of the ephemeris notion for acquisition of a weak intelligence signal

This is really a time of change in astrometry. And these changes touch not only the reference frames and the accuracy levels in the star astrometry. The satellite astrometry is now developing very swiftly. First enhancement of the computer capabilities and electronics on the whole gives room for jump-like increase of productivity and output of the observation facilities. But thereby, the observation technology itself should be perfected as well as that of search for SOs and cataloging them. Some important complicated astrometric procedures can be automated now which was impossible hitherto because of insufficient rapidity and operational capabilities of the former computers [69].

Traditionally, setting the task of keeping track of an SO supposes to maintain the accuracy of its orbital parameter estimates at relatively high level, the "loss of track" (LoT) being received to be taken as "a little or big drama".

The LoT of an LEOSO is much less dramatic than that of a deep-space object (DSO) because the former shortly after having been lost can be easily and even automatically re-detected by the wide network of radars (say, by the US SSN or the RSSS). For a DSO that is usually ruled out.

To solve this problem, there are two distinct directions one could go:

1. To raise the accuracy of keeping track of DSOs to too high a level (as high as possible) to gain a warranty against the LoTs, so to say, "regardless of expenses".
2. To invent something which can remove the "dramatic halo" from the LoT and make it quite tolerable, however paradoxical it may seem.

It is clear that the first way is very laborious, expensive, and may require a significant expansion of the park of surveillance equipment (including the sensors) and intensification of their operation. And this is only to achieve the high metric accuracy of maintaining the SO catalog.

Apparently, the latter is more attractive way as it assumes more flexible and convenient definition of the DSO tracking stability as the system's capability to quickly restore the needed accuracy of the DSO motion parameters every time the first-hand need arises (and not to do this before it). That would alleviate the operation mode of facilities and the personnel in critical circumstances (say, the loss of many DSOs, the loss of a very important DSO, getting several sensors out of order, and so on). This path requires a new, creative approach. How to do this?

In essence, "loss of an SO" (in other words - LoT) means a noticeable (in old habitual conditions called "unacceptable") reduction in the accuracy of an SO motion control to a level where a single target designation (the pointing data) does not allow detecting this SO without searching. LoT is dangerous only due to low search capabilities of facilities used. The bottleneck in traditional setting the task of keeping track of DSO is low effectiveness of search methods used (primitive scanning-like procedures, survey, and so on, which are laborious and cannot provide the acquisition of a faint signal).

To radically increase the catalog maintenance efficiency, one of the ways to achieve this supposes that the accuracy of keeping track of DSO (within the SO catalog) can be decreased with no “fatal” consequences. Then LoTs could be easily resolved and it would be enough to maintain high accuracy only in “emergency” or by wish. In other words, we must make the LoT harmless by using more advanced methods of searching and controlling the motion of SO having rough metric information about its motion.

The second way to alleviate the function of keeping track of SOs (see above in this section) can be realized on the bases of OSPs described here. Using OSPs due to their economy makes some generalization of the conventional notion of an ephemeris expedient. Let us consider this in detail.

Commonly, in absence of data on the accuracy of the angles such as right ascension α and declination δ at time *t,* the appropriate ephemeris renders little (if not no) information about its reliability. Providing its accuracy, one can establish whether the size of a given telescope’s FoV is enough to cover the wanted SO according to this ephemeris. Awareness of the first derivatives’ accuracy of α and δ ($\dot{\alpha}$ and $\dot{\delta}$) allows determining, for example, whether the receiver’s sensitivity (of photo materials, photocathode, TV-tube, CCD-matrix, and so on) is enough to register the signal from DSO having given brightness due to compensation for its motion with the rates of α and δ change. The accuracy of an ephemeris and the corresponding first derivatives $\dot{\alpha}$ and $\dot{\delta}$ may be insufficient for performing successful observations of the wanted DSO (especially when using a sensor with a small field of view and/or if the intelligence signal is weak).

In such a case, the simple decision would be plain enumeration (run over) of all possible (for given uncertainty) conditional ephemerides and rates. And one and only one combination should be correct. But it would be rather practically impossible to realize plain enumeration because of the immense amount of such conditional ephemerides and rates combinations.

At the same time, OSP, carefully and mathematically strictly taking into account a priori information on the DSO motion, the laws of motion and topological deformation of its CPUD, and the technical capabilities of the sensor used cuts off all the infeasible combinations and saves the most economical (minimal) sequence of conditional ephemerides which are necessary and sufficient for detecting the wanted SO according to one and only one conditional ephemeris (and the corresponding rates vector) from the OSP.

Besides, one more disadvantage of the traditional setting of task could be mitigated (when using OSP and taking into account topological deformation of the SOCPUD). The fixed high accuracy of maintaining the DSO catalog was intended for providing the capability of observing any DSO at any time by any sensor (when the DSO is in its zone). But the variety of FoV sizes and directional diagram widths is rather large (from several degrees to a portion of an angular minute) which makes this common accuracy unreasonable. For wide-angle and wide-beam sensors it could be too strict. And for narrow-angle and narrow-beam facilities it could be too low. In the new setting of tasks (in the frame of this section's suggestion), for each sensor, its own OSP utmost economical namely for this sensor is calculated.

So, the new strategy of maintaining the DSO catalog is more flexible in this sense.

At last, in traditional setting the tasks, the process of detecting the new or maneuvered DSOs by a priori information significantly differed from the process of observation by precise ephemerides. They were two essentially different tasks. However, if the OSP elements are treated as generalized ephemerides (GE), the more generalized setting of the tasks of planning and conducting observations and the more generalized strategy of maintaining the DSO catalog are possible.

Instead of two alternative tasks – searching and observation by a precise ephemeris – there will be the joint task – that of planning and implementation of GEs.

In this way, a GE is an OSP [16, 70], a usual (not generalized) ephemeris being just a specific case of a GE when the accuracy of data on orbital parameters of a given SO are exact enough for just observing it with no search.

One can distinguish between the two steps of this generalization:

1. Addition of conditional rates to positional coordinates, that is, transition from just pointing the sensor to the design (calculated) “sweeps” of the site axis [72].
2. Transition to OSP, that is, from one “sweep” to the optimized set of “sweeps”.

Please, note that the generalized statement of the problem of maintaining the DSO catalog

1) simplifies formalization of the problem itself (brings it to the standard program of planning and realization of GEs);
2) enriches the space of options: allows more subtly and by simpler means taking into account and realization of capabilities and peculiarities of the sensors used.

Thus, a strategy of comprehensive use of the facilities at a higher optimization level is developed.

Such a generalization of an ephemeris implies the change of its use. Instead of realization of an ephemeris for aiming the sensor to the point of celestial sphere with coordinates α and δ at time t and waiting for the sought-for SO, now this passive policy will be exchanged for several active movements (“sweeps”) according to the elements of OSP-GE. The current capabilities of electromechanical telescope control and electronic radar-beam control allow easy automation of all the process of the GE setting.

The traditional strategy of maintenance of the DSO catalog consists of conducting observations of every cataloged DSO as soon as the accuracy of its orbit parameters (provided by the propagation programs) becomes less than a given (tolerable) level.

The new strategy consists of conducting the observations as soon as DSO’s parameters errors and the corresponding OSP become too large, are going to be so, or there is some other special urgent reason for observing this object.

It is obvious that the last mode is more economical and more flexible for the facilities operation and the SO catalog maintenance.

By the way, one can see that one more generalization can be made in the space surveillance practice and methodology in the frame of the new search theory. Namely, a notion of the *generalized accuracy* of the DSO catalog maintenance can be introduced which can be measured by a sum or weighed (on some principle) number of elements of OSP representing the GE throughout all the catalog.

As the experience of optical and optical-electronical facilities operation shows, the weakening of intelligence signal due to the disregarded relative motion of the SO image and the sensor's receiver can sometimes reach several magnitudes. As well, it is similar with narrow-beam radars (when accumulation of the return pulse energy is needed). The main problem of detecting DSOs in such situations is connected with providing the favorable conditions for concentrating and accumulating the weak intelligence signal energy at the same receiver spot. The proper compensation of the relative motion should significantly increase the sensitivity of the sensors used.

It is easy to do this when exact metric orbital information on the wanted DSO is available (position and velocity coordinates). In the absence of the latter, we are facing the very hard problem. In such a case, GEs treated as optimum search plans [16, 72] are dedicated to efficiently help solve this problem.

The situation is much more acute when placing a search sensor onboard a spacecraft. Due to possible significant relative angular rates of a target and a space-based sensor's receiver intended for detecting small space objects (including SD), it is very pertinent to introduce the suggested optimum search methods into the on-board software. Namely due to the very sophisticated character of the relative motion of the sought-for SO and the on-board sensor, the GE (OSP-GE) is expected to be very efficient.

In conclusion, it is fair to say that both full absence of any information on the sought-for SO orbital motion (on its state vector) and very rough a priori data make no or little use of GE.

5.9. Challenging aspects of the developed theory impact on the modernization of space surveillance facilities and reorganization of their management

At the present time, one can observe an evident discrepancy between the current Space Surveillance Systems (SSSs) scientific-technical base (which is somewhat overaged) and a high and constantly growing level of complexity of space surveillance tasks put by reality nowadays. The last events in space confirm this.

In February 2009, for the first time in space activity history, two large satellites ("Cosmos 2251" and "Iridium 33") collided in orbit. The event took place at an altitude of about 800 km over Siberia.

The smallest cataloged space objects have their size 5 (very few) – 10 cm while really dangerous for active spacecraft are space debris of size 1 cm and even much less.

In 1996, the French satellite “Cerise” dramatically collided with an Arian’s fragment and was destroyed.

Space debris made a cavern about 4 mm in the porthole of the orbital station “Salute 7”. At the frontal window of one of “Challengers” a dent of 2.5 cm diameter and a half cm deep was found. After 4 years orbiting of the US “Solar Max”, at its heat-protective cover, some two thousand holes and dents were found. This list could be continued for a very long time (see [3]).

The danger of small OD (for space activities) is often undervalued. A striking example of a great danger of collision with a very small OD is the Russian metrological 17 cm sized satellite “Blitz” collided with a microparticle having a mass of ~ 0.035 g, a size of ~ 3 mm, and the relative velocity of collision about 12.3 km/s. As a result, the satellite was destroyed and two fragments were cataloged by both SSSs (Russian and American) and have been tracked. By the way, the density of small OD fluxes at the height of the accident is 4-5 orders of magnitude larger than that of the cataloged SOs [66, 68]. And this is a very eloquent fact.

Accumulation of small SO (including OD) progressively increases and both SSSs don’t cope with the function of informational provision for space activities safety in real time.

There are two evident directions of perfecting the SSS technology:

1) a radical renewal of the space surveillance facilities (sensors, computing complexes, communication lines, and so on),
2) the development of fundamentally new approaches to planning and control operations of the facilities and to space surveillance in general.

The first direction in Russia, USA, Europe, Japan, and China is being developed very actively, but an obvious limit is already visible (economic and technological). The second direction unfortunately lags behind despite the fact that it is very promising.

A possible way to overcome this blind alley consists in revising some approaches to organizing the operation of space surveillance facilities using new principles. As the base for this, one can use the new theoretical approach to optimum planning the search for SOs taking into account temporal structural transformation of the SOCPUD. The application of this theory is most effective for narrow-angle and narrow-beam facilities in the sense that it provides the most economy of their search resource, the detection of the sought-for SOs (including even smaller than those currently available for detection) being guaranteed.

At present, the metric information on SOs is being supplied by radar and optical sensors affiliated with the SSSs. This information is enough for representation of general state of the main space objects population (as far as large SOs are concerned). Special cases (for example, close approaches of SOs especially with small other SOs) need much more precise metric information and that about smaller SOs or fainter signals. For this task, very sensitive predominantly narrow-angle and narrow-beam sensors together with the new or upgraded methods of their operation are needed.

The space surveillance sensors, if being used and controlled traditionally, can metrically maintain a very limited population of SOs. At the same time (which further complicates the situation), there are evident tendency of growth of the number of small SOs that need precise metric information. So, a substantial perfection of the ideology for operating the sensors is necessary.

When planning the search for SOs with the help of narrow-angle and narrow-beam sensors (in case when all the SOCPUD cannot be covered with one field of view of the sensor or/and the intelligence signal is very faint), there occurs a delusion. For constructing the search plan, one usually does not take into account the real temporal structure transformation and deformation of the SOCPUD during its motion. The consequence of such a carelessness is rise of errors of the 1st and 2nd kind - the appearance of "chinks" between the SP elements where one can lose the sought-for SO (1st kind) and redundant overlapping between the adjacent SP elements implying non-economical expenditure of the sensor search resource (2nd kind). This is especially detrimental with a large number of elements of the SP and its long implementation.

For elimination of these errors, a special theory of optimum planning the search (TOPS) was developed in previous chapters. In the light of TOPS (if it would be used at all stages of detection and tracking of SOs), ***the observation of an SO by the assignment (using the one-valued ephemeris) could be considered as a particular (degenerated) case of the search situation and could be struck off from the regular modes of the facility operation as further unnecessary.*** After all, the search plan can consist of one element. Although at present, this is one of the main operation modes. So, such an approach to the monitoring of all space objects motion could become more universal.

After thinking over this idea, one can see that ***namely the search task rather than observation by assignment is the most natural sensor operation mode.*** You just need to get used to this thought.

Firstly, the accuracy of a priori orbital information (including metric information in the SO catalog) is continuously changing in time (getting better – jumpwise (hopping) and worse – monotonously). And one cannot know beforehand that it will be enough or not enough when observation is needed.

Secondly, the same a priori orbital information for different sensors may lead both to observation of the SO by assignment (using precise targeting data) and to the search task – depending on mutual relation of the sizes of FoV and the SOCPUD.

The methods of optimum planning the search allow the most economical disposal of the sensor observation resource and, at the same time, ensure the terms of the guarantee for the SO detection (even if the intelligence signal power is insufficient to acquire it with the traditional approach).

Thereby, the principal characteristics of the SSSs would be enhanced (sensitivity, output capacity, detection reliability, the number of cataloged SOs, and so on) owing to elimination of the errors of the 1st and 2nd kind and possibility of "innocuous" ("wasteless", "cost-free") expansion of the SOCPUD (see the Theorem of conservation of optimality in section 4.7). The SPs constructed in accordance with the TOPS have the property of the "innocuous" expansion of the sought-for SOCPUD in the sense that artificial expansion of CPUD (artificial deterioration of a priori state vector errors) does not involve any increase of the search resource waste because the SO will be detected in due course in accordance with the real size of its CPUD, although unknown in advance. In other words, overstating the unbiased error in giving positional information about the sought-for object is entirely tolerable and does not lead to excessive consumption of the search resource. Moving to optimum search planning eliminates the checkup of an unnecessarily expanded area of the SOCPUD. This is a very useful feature of the proposed approach to search planning.

While one deals with a very faint intelligence signal (a small size of an SO, a poorly reflecting surface material, a wrong illumination phase, an unhappy attitude or foreshortening, and so on) which does not allow detecting the SO using the traditional control of the sensors operation, the TOPS provides the observer with ample opportunity for raising the focal power of the sensor by transfer to the generalized ephemerides (see [70] and section 5.8 here).

For enhancing the focal power by this approach, one needs the compensation of the signal motion on the receiver by the appropriate (conformable) motion of the sensor sight axis. But with the traditional approach it is effective only if the precise data on all metric orbital parameters are available. The peculiarity of our search task is that there is even a very imprecise information on the sought-for SO orbital parameters, including its velocity components. So, the traditional approach to the problem does not give its solution.

At the same time, the TOPS allows beneficially quantizing the whole great orbital data uncertainty into much smaller uncertainties in accordance with a number of the SP elements and their location in the search space. As a consequence, one can achieve lessening the limiting observable size (or brightness) of SO, that is, to enlarge the class of detectable and trackable objects.

With the aim of providing the more favorable conditions for concentrating and accumulating the faint intelligence signal energy at one point of the sensitive element of the receiver, the sensor should realize a set of additional SP elements μ_{ij} during the exposition time period

$$(t_i^-, t_i^+) \equiv (t_i - \frac{\Delta t_{\exp}}{2}; t_i + \frac{\Delta t_{\exp}}{2})$$

rather than only one fixed SP element μ_i of date t_i. By the way, this does not increase the whole-time interval of the SP realization.

For the beginning of exposition t_i^-, the conditional ephemeris (α_i^-, δ_i^-) is calculated by the center $< t_i^-, u_i^- >$ of the element μ_i^- which is a projection of element μ_i along the equivalence curve (see [70] and section 4.3 here) at time t_i^-. Just since this moment, the receiver should be given a compensating motion conformable with the conditional rate of change of argument of latitude which will bring it in time Δt_{exp} to the position corresponding to element μ_i^+. The latter is a projection of element μ_i along the equivalence curve at time t_i^+ (the end of exposition). All this process is demonstratively illustrated by Fig. 5-9-1.

For comparatively small values of Δt_{exp}, it is enough to calculate the conditional coordinates of the SO (α_i^-, δ_i^-) referred to the center of element μ_i^- and the rates of their change (α'^-_i, δ'^-_i) which determine the compensation motion of the sensor sight axis. For relatively great Δt_{exp}, the values of α' and δ' are to be corrected several times during the exposition.

So, the preliminarily constructed SP element μ_i as if is being spread along the equivalence curve coming through its center, and the sensor realizes this “smear” (“stroke”).

This technique can be applied both for the search and observing the small or weakly-contrasting SOs by precise appointments.

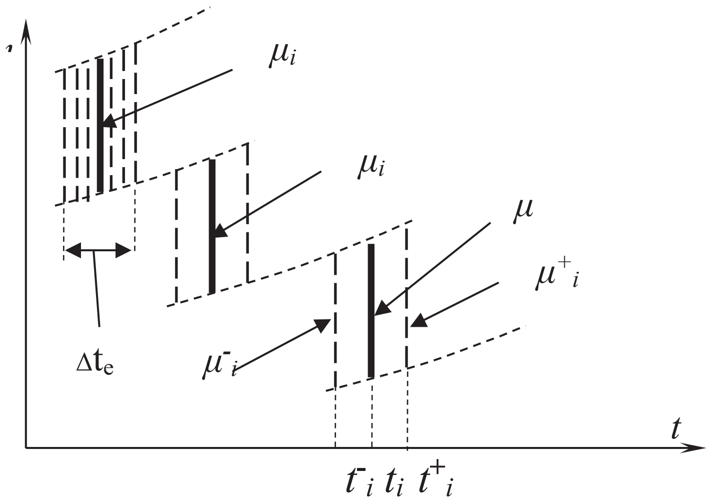

Fig. 5-9-1. A fragment of the search plan in terms of the generalized ephemerides

As an illustration, in Fig. 5-9-1, a fragment of the "generalized" SP is shown. One can see the way how the usual SP is transformed to a set of the generalized ephemerides. One can also notice that initially a simple ephemeris is then being blurred along the equivalence curve into the generalized ephemeris which contains both angular coordinates (α, δ) and their rates (α', δ'). Naturally, both the coordinates and rates here are conditional. That means that each set covers only a part of the whole CPUD (a quantum of all the uncertainty), its particular uncertainty being much less than that of the initial data (***both in coordinates and in velocities***).

In especially complicated cases (very imprecise a priori data), on the base of TOPS, one can substantiate and naturally realize the continuation of this approach when handing over the search task from one sensor to another but with the reduced state vector errors in each transition.

CHAPTER SIX

CONCLUSION

It is the time for making some general findings and conclusions.

Now, when the search theory for SOs is stated, one can discuss its strengths and deal with some emerging doubts and limitations.

In the monograph, a lot of various search situations related to the discovery of space objects that are encountered in the practice of an observer-astronomer and in space surveillance service are considered and studied.

However, special preference is given to the case of searching for an object by rough a priori information on its orbit with the help of a narrow-angle or narrow-beam sensor. Namely, these conditions make the problem highly topical and, at the same time, non-trivial and most interesting mathematically. This subject has been studied most thoroughly in view of its particular relevance and the productivity of methods developed here for this case. The most effective application of the proposed theory will be achieved in case of precisely pointing a laser beam on an SO on which only imprecise metric information is known.

At this point of investigation some definite findings can be made.

The degradation process of already realized SP elements throughout the performance of the SP is inevitable. But it can be mitigated and neutralized (if not completely, then to a degree). It can be achieved by rational organization of monitoring the SOCPUD structure transformation. The possibility and effectiveness of such compensation depend on the rate of sounding the CPUD by the sensor, the initial data error distribution, and the means of planning the search used.

One of the main goals of constructing the new methodology of planning the search for a space object using imprecise a priori information was such as that its optimization scheme would appear reiterative, multi-level (if needed), and rather versatile. These properties allow obtaining optimum solutions not only by different criteria, but also in different forms and in several stages. After having constructed the optimum search plan at one optimization level and in one statement of the problem or on a given conditions, one can proceed with the optimization process in an alternative form and on alternative conditions. And all this is on the set-theoretical base within the frame of the united mathematical apparatus of the equivalence curves and its generalization.

In this case, optimum and suboptimum plans are built dynamically taking into account the structural temporal transformation of the already constructed plan elements and with the appropriate correction of the current element (and all subsequent ones) which is intended to compensate for the ongoing degradation of the whole plan.

The modern level of development of computer technology and programming makes it possible to fully automate the entire process of planning the search for SOs using the methods proposed here and make it more user-friendly.

The real efficaciousness of the optimum search planning is determined not only by economy of the energy consumption and reduction of employment of the sensors' and staff's resources. Besides, the optimum search planning enhances the capabilities of sensors in alternative concerns as well. In particular, it enlarges the class of detectable space objects (in terms of range, size, brightness, motion velocity, and so on). And the latter is not less important than the former.

Moreover, a number of formulated and proven theorems not only help to better understand the essence of the proposed theory and methods, but also provide a lot of convenience in choosing a strategy for constructing search plans and directly in their construction. In particular, the optimality conservation theorem (see section 4.7) makes the search planning process stable against overstating the errors in giving the state vector of the sought-for object. In practice, this means that if you are not sure about the value of the given positional errors, you can safely overstate them. The main thing is not to let them be less than the real ones. And this excess of the true error of the state vector will not lead to an overrun of the search resource.

This is very important in the daily work of space surveillance services. In addition, this capability is easily automated with the help of modern computer technology.

The suggested fundamentals of planning the search for SOs allow from the common point of view systematically embracing all the considered (and some not considered as well) search situations arising in the space surveillance practice and constructing the efficient system of methods for the searching for SOs based on a unified mathematical platform. At the same time, taking into account the specific features of each task increases the effectiveness of the problem solution.

The presentation of the foundations of the search theory is everywhere accompanied by a number of the specific real examples and detailed calculations to demonstrate the practical possibilities of the theory and its methods. This greatly contributes to its assimilation.

It is worth to once again point out the universality of the equivalence principle. The material in sections 3.3 through 3.9 shows that the equivalence principle and its EC apparatus are highly constructive and adaptable for finding optimum and suboptimum solutions in the most diverse and complex search situations.

The key moments of the proposed theory are the set-theoretic interpretation (representation) of the sought-for SO CPUD and the equivalence principle of the elements of the search plan for different instants of time. Moreover, not only the equivalence principle, but also the search theory itself based on it with the set-theoretic treating of the sought-for SO current position uncertainty domain is universal. It allowed generalizing both the principle of equivalence and the concepts of ephemeris and the accuracy of the SO catalog maintenance as well as making more versatile and flexible the maintenance itself and the management of the space surveillance facilities (see sections 5.4, 5.8, 5.9).

So, the new theory of optimum planning the search for SOs affords to enhance the stability of space surveillance under the condition of progressive growth in the number of SOs and their miniaturization. And it is owing to such its merits as elimination of the errors of 1st and 2nd kinds in the SP construction, the possibility of unpunished expansion of the sought-for SO's CPUD, and the possibility of transfer to the generalized ephemerides we have the possibillity to quantize the initial CPUD and increase the sensitivity of the entire system. These merits allow to struggle with the orbital data errors and, as a consequence, mitigate the stiff restrictions on periodicity of observations and release the observation resource. At last, there is no need to consider the loss of an SO as a dramatic event and it can be easily afforded.

We have already said that now there are only two SSSs. The reorganization and modernization of the existing system, especially as unique as the SSS, is a very expensive matter. In the alternative case, when developing and creating a new system, to base its concept on a new, modern methodology for space surveillance it would be more painless. This should be borne in mind by some states (or groups of states) planning to establish their own SSSs in spite of financial and technological difficulties.

In conclusion, it is important to note that the theoretical concept presented here is of great generality. In fact, in this monograph, the foundations of a broader theory of search are developed – the search not only for SOs, but also for cyclically moving objects along a closed trajectory. And the main problem for which the monograph was conceived turned out to be a special case of it. But the development of such a more general theory became an important intermediate (auxiliary) step for constructing the search theory for SOs. This made it possible to identify the main consistent patterns in the formulation and solution of our problem in the most transparent, clear, and comprehensive way. And it made the presentation of the search theory for SOs (from general to particular) more didactic, logical, and understandable (this is what the author has been doing for the last 2 years). At the same time, it brought more mathematical harmony, strength, and persuasiveness to its structure and foundation.

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