Avicenna's Theory of Science

Logic, Metaphysics, Epistemology

Riccardo Strobino

UNIVERSITY OF CALIFORNIA PRESS

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"O degli altri poeti onore e lume . . ."

—Dante, *Inferno*

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Avicenna's Theory of Science

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Riccardo Strobino

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To Tony Street

الفاضل من المتأخرين والمحصل من المنطقيين

μέγα βιβλίον μέγα κακόν

—Callimachus, fr. 465 Pfeiffer

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Medford, 12 January 2021

NOTE ON CITATION, TRANSLITERATION, AND TRANSLATION

Avicenna's works are cited by title, book, chapter, page, and line numbers from the Cairo edition of the *Kitāb aš-Šifāʾ* (*Book of the Cure*), Forget's edition of *al-Išārāt wa-t-tanbīhāt* (*Pointers and Reminders*) (supplemented by Dunyā's edition for Ṭūsī's commentary) and Dānešpažūh's edition of the *Kitāb an-Naǧāt* (*Book of Salvation*).[1] References to the logic of the *Naǧāt* are based on the sequence of chapters printed in Dānešpažūh's edition and correspond to the chapter numbers of Ahmed's English translation. References to the *Išārāt*, unless otherwise noted,

1. Avicenna, *al-Išārāt wa-t-tanbīhāt*, ed. J. Forget (Leiden: Brill, 1892) (hereafter cited as *Išārāt*); Avicenna, *Manṭiq al-Mašriqiyyīn*, ed. M. al-Ḫāṭīb and ʿA. al-Qatlā (Cairo: al-Maktaba al-Salafiyya, 1328/1910) (hereafter cited as *Mašriqiyyūn*); Avicenna, *aš-Šifāʾ, al-Manṭiq, al-Madḫal*, ed. Ǧ. Š. Qanawatī, M. al-Ḫuḍayrī, and A. F. al-Ahwānī (Cairo: al-Maṭbaʿa al-amīriyya, 1952) (hereafter cited as *Madḫal*); Avicenna, *aš-Šifāʾ, ar-Riyāḍiyyāt, Ǧawāmiʿ ʿilm al-mūsīqā*, ed. Z. Yūsuf (Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb, 1956) (hereafter cited as *Mūsīqā*); Avicenna, *aš-Šifāʾ*, *al-Manṭiq*, *al-Burhān*, ed. A. ʿAfīfī (Cairo: al-Maṭbaʿa al-amīriyya, 1956) (hereafter cited as *Burhān*); Avicenna, *aš-Šifāʾ, al-Ilāhiyyāt* (1), ed. Ǧ. Š. Qanawatī and S. Zāyid (Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya, 1960); *aš-Šifāʾ, al-Ilāhiyyāt* (2), ed. M. Y. Mūsā, S. Dunyā, and S. Zāyid (Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya, 1960) (hereafter cited as *Ilāhiyyāt*); Avicenna, *Treatise on Logic: Part One of Danesh-name Alai*; *A Concise Philosophical Encyclopaedia and Autobiography*, ed. and trans. F. Zabeeh (The Hague: Nijhoff, 1971); Avicenna, *aš-Šifāʾ, ar-Riyāḍiyyāt, al-Ḥisāb*, ed. ʿA. Luṭfī (Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb, 1975) (hereafter cited as *Ḥisāb*); Avicenna, *aš-Šifāʾ, ar-Riyāḍiyyāt, Uṣūl al-Handasa*, ed. ʿA. Ṣabra, ʿA. Luṭfī (Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb, 1976) (hereafter cited as *Handasa*); Avicenna, *al-Išārāt wa-t-tanbīhāt*, ed. S. Dunyā (Cairo: Dār al-maʿārif bi-Miṣr, 1970), published with Naṣīr ad-Dīn aṭ-Ṭūsī's commentary at the bottom of the page (note that Dunyā's edition plagiarizes from the Iranian printing of Ṭūsī's commentary [1957–1958]); and Avicenna, *an-Naǧāt min al-ġarq fī baḥr aḍ-ḍalālāt*, ed. M. Dānešpažūh (Tehran: Dāneshgāh Tehran, 1985) (hereafter cited as *Naǧāt*).

are always to the logic. Hence, instead of writing I, 1, 6 for part I, path 1, chapter 6, I write I, 6 (path one, chapter six). Alfarabi's *Kitāb al-Burhān* is always quoted without abbreviation to avoid confusion with Avicenna's *Burhān*.

In all cases, line numbers are counted starting from the first printed line of text on any given page (excluding book and chapter titles), regardless of what numbers appear in the margins.

I use traditional Latin abbreviations for the titles of Plato's and Aristotle's works and the corresponding page and line numbers from the editions of Stephanus and Bekker. Book numbers of Aristotle's works are indicated by a Greek capital letter (for example, *An. Post.* A7, *Top.* E4, *Met.* Δ30). The Greek commentators Alexander of Aphrodisias, Themistius, Philoponus, and pseudo-Philponus are cited according to their standard editions from the *Commentaria in Aristotelem Graeca*.

I have adopted the transliteration guidelines established in 1935 by the German Oriental Society (Deutsche Morgenländische Gesellschaft) with the following exception: *-aw* and *-ay* for the diphthongs instead of *-au* and *-ai*.

In translated texts, I have used the following signs:

(. . .) to indicate a parenthetical remark by the author of the passage

[. . .] to indicate my own explanatory additions or comments to the English translation, including numbers or letters

Introduction

Most of those who pretend to be philosophers learn logic but do not use it, resorting ultimately to natural inclination, galloping about it like one who lets the reins fall slack and does not pull on the bit.

—AVICENNA, *ILĀHIYYĀT* I, 8

Avicenna (ca. 970–1037) is the most influential philosopher in the Arabic-Islamic tradition. His thought is profoundly shaped by the ideal of science and scientific knowledge, and a full appreciation of this fact presupposes an adequate understanding of his views concerning the nature of scientific knowledge and the requirements that a discipline must meet in order to qualify as a science.

At the heart of Avicenna's essentialist epistemology are two central problems. The first is the identification of (i) the conditions under which an assertion may be characterized as certain and therefore be taken to express a scientific truth. The second is the identification of (ii) the conditions under which a complex term may be characterized as an adequate conceptual representation of the essence of an object and therefore be taken to express a real definition. These two sets of conditions determine in turn the nature of the principles assumed in each science for the derivation of its own theorems, the way in which boundaries between sciences are drawn, and more generally, the hierarchical order and arrangement of scientific knowledge in its various domains. At the same time, Avicenna is concerned with central aspects of the logic of scientific reasoning, including the logical form of scientific statements, the structure of demonstrative proofs and the types of admissible argument forms, the conditions for reasoning from an impossibility, the relation between demonstration and definition, and the order of inquiry.

Avicenna's conception of science and scientific knowledge, which lies at the intersection of logic, metaphysics, and epistemology, has never been the object of a systematic study, despite its centrality for his philosophical and scientific work. This is a lamentable gap not just in Avicenna scholarship but also for the history of philosophy as a whole. The primary purpose of this book is to fill this gap by offering the first comprehensive account of Avicenna's theory of science, focusing in particular on his interpretation of the model introduced by Aristotle in the *Posterior Analytics*. The most elaborate version of this interpretation is developed by Avicenna in his *Kitāb al-Burhān* (*Book of Demonstration*), though related texts make important contributions too.

The main thesis of this book may be reduced to two interconnected contentions. The first contention is that Avicenna understands and develops his theory of science as a genuine—if undeniably ambitious—theoretical framework for actual scientific reasoning and for regimenting every suitable domain of scientific inquiry into a properly structured Aristotelian science. This is especially true if we think of an Aristotelian science in its final and canonical form, which ideally reflects the complete set of relations holding between the subject of a science and its attributes. It is possible, and perhaps even plausible, to think that Aristotle himself saw his own theory of science as aiming to accomplish an analogous task. But the *Posterior Analytics* (in fact the two *Analytics* as a whole) can hardly be taken to present a logic of scientific reasoning that is both adequate and ready to use, primarily because of the limited expressive power of the underlying logical system, which focuses on categorical propositions and syllogistic deductions. Moreover, even if we set aside the obscurities and interpretive problems raised by individual passages of the *Posterior Analytics*—a rather difficult work not only for neophytes but also for specialists—many aspects of Aristotle's model are frequently discussed in it only in an embryonic form (a notable case in point is his theory of scientific attributes, as we shall see in chapter 7). Avicenna's *Burhān* can therefore be seen, first and foremost, as an attempt to bring some central aspects of Aristotle's theory of science from potency to act.

The second contention is that, in pursuit of this general strategy, Avicenna develops an extensive battery of conceptual tools and introduces a series of novel ideas in the context of Aristotelian epistemology. While both kinds of innovations are often locally motivated by specific technical needs, they almost invariably presuppose a theoretical framework of much broader significance, which is worthy of being investigated in its own right for its philosophical content.

To put the necessity of this study in perspective, an additional consideration is in order. Avicenna's commitment to the view that philosophical method as well as individual scientific disciplines should somehow conform to the standards set out by Aristotle in the *Posterior Analytics* is a well-established tenet in modern scholarship.[1] And Avicenna's attempt to cast his own metaphysics, of all disciplines, in a mold inspired by the principles of Aristotelian epistemology has been illustrated successfully in recent years (whether the attempt itself is successful is a

1. Gutas (1988), Hasnawi (2012).

separate—and largely irrelevant—issue, at least for our present purposes).[2] This leaves little room for doubt about Avicenna's commitment to the idea that the path charted by the *Posterior Analytics* must always be taken seriously, in metaphysics as well as in every other science (at least relative to the degree of exactness allowed by its subject). But the question of how Avicenna's commitment shapes and affects his reception of Aristotle's theory of science, prompting a sustained effort to turn that theory into a genuine logic of scientific reasoning, has been all but neglected. In other words, the question of the applicability of an Aristotelian scientific method to individual sciences has typically been approached from within the sciences themselves, that is to say, by asking whether (or to what extent) the structure and conceptual vocabulary of a given science conforms to certain logical and epistemological standards. What this book aims to show, by contrast, is that, and how, this ideal of applicability influences Avicenna's theory of science itself, driving an elaborate effort of recalibration of the Aristotelian logic of scientific reasoning.

THE ELEMENTS OF AVICENNA'S THEORY OF SCIENCE

The Arabic term *ʿilm*, just as its Greek and Latin counterparts *epistēmē* and *scientia*, means both (i) scientific knowledge and (ii) science.[3]

Scientific knowledge, according to Avicenna, may be either conceptual or propositional. The first kind is encapsulated by the notion of conception (*taṣawwur*) and is typically associated with definition (*ḥadd*) or description (*rasm*); the second kind is encapsulated by the notion of assertion (*taṣdīq*) and is typically associated with propositions (*qaḍāyā*), especially the premises and conclusions of deductions (*qiyāsāt*).

Scientific knowledge by assertion is connected to the idea of certainty (*yaqīn*), which is the distinctive mark of demonstration (*burhān*). Certainty is in turn an epistemic state characteristically defined in terms of modally firm beliefs (*iʿtiqād*) involving various kinds of necessity (*ḍarūra*). Certain assertions are either self-warranting and self-evident (*bayyin bi-nafsihī*), as in the case of immediate principles (*mabādiʾ*), or else inferentially and causally justified, as in the case of questions (*masāʾil*), that is to say, the theorems of a science.

Scientific knowledge by conception, by contrast, is connected to the idea of an adequate conceptual representation of the essence of an object. An adequate conceptual representation is expressed by a peculiar kind of differentiating or explanatory expression (*qawl mufaṣṣil*), that is to say, by a real definition that captures the complete ordered sequence of essential attributes of the object.

2. Wisnovsky (2003), Bertolacci (2006). Avicenna's putative failure to meet (his own) demonstrative standards, especially in metaphysics, is one of Ġazālī's central contentions in the *Tahāfut al-falāsifa*.

3. On the Arabic root *ʿ-l-m*, see in particular Rosenthal (2007, pp. 6–12 and 196–201).

Modality and explanation are therefore critical aspects of Avicenna's conception of scientific knowledge and play an essential role in his characterization of its nature, both with regard to assertion and demonstration and with regard to conception and definition.

A science is identified by three elements: its subject (*mawḍūʿ*), principles (*mabādiʾ*), and questions (*masāʾil*). Every science is a structured domain of interconnected and (ideally) certain truths, expressed by scientific assertions that may have various logical forms (categorical and hypothetical, conditional or disjunctive). Some scientific assertions are immediate (the principles of a science), while others are non-immediate (the questions of a science). Non-immediate assertions are *proved* either by categorical or by hypothetical deductions.

Scientific questions reflect a variety of types and stages of scientific inquiry (*maṭālib*) concerning, in particular, whether (*hal*) something exists or has a certain attribute, why (*limā*) it exists or has a certain attribute, and what (*mā*) subjects (*mawḍūʿāt*) and attributes (*ʿawāriḍ* or *aʿrāḍ*) are. Scientific explanations account for the relation between causes (*ʿilal*, *asbāb*) and effects (*maʿlūlāt*) and ultimately rest on the essences (*ḥaqāʾiq* or *ḏawāt*), natures (*ṭabīʿāt*), or quiddities (*māhiyyāt*) of the subjects and attributes of a science, which are captured by definitions.

AVICENNA'S *KITĀB AL-BURHĀN*

While the present study is not a commentary, Avicenna's *Burhān* lies at its heart. For a better understanding of the organization of the former, it is therefore worth taking a preliminary look at the structure of the latter. The *Burhān* is the fifth section (*fann*) of the logic (*manṭiq*) of the *Kitāb aš-Šifāʾ* (*Book of the Cure*), that is to say, of Avicenna's most comprehensive philosophical and scientific summa. The text is roughly three times the length of the *Posterior Analytics* and is divided into four treatises. Methodologically, the *Burhān* is not a running commentary but it still follows quite closely the order and arrangement of its source, in conformity with the style of commentary *per modum expositionis* typical of the *Šifāʾ*.[4] If we take the *Posterior Analytics* as its baseline, Avicenna's *Burhān* looks like a sequence of oscillatory movements that deviate more or less significantly from that baseline depending on the difficulty of a locus and the problem under discussion. The amplitude of such oscillations may vary quite dramatically. At the opposite ends of the spectrum are two complementary tendencies. In some cases, Avicenna relies on Aristotle's text merely as the starting point of an independent discussion, frequently introducing new conceptual vocabulary or subtle distinctions. In other cases, he works much more closely on the text and presents an interpretation of problems that directly emerge from it. In between lies a vast array of attitudes

4. Gutas (1988, p. 107); cf. Bertolacci (2006, pp. 607–612, appendix E).

and approaches varying from chapter to chapter, which may result in abbreviations, omissions, reformulations, transpositions, or rearrangements of the material. Sometimes multiple chapters in Aristotle are condensed into a single chapter in Avicenna, while at other times a single chapter in Aristotle is split into multiple chapters in Avicenna. For a detailed synopsis of the content of individual treatises and chapters, the reader is encouraged to consult appendix C ("A Map of *Kitāb al-Burhān*") at the end of the book.

The first of the aforementioned tendencies is most evidently exemplified by the first treatise and by part of the second treatise. The first treatise, which consists of twelve chapters, looks like an extensive gloss on *Posterior Analytics* (*An. Post.*) A1 (on the idea of preexistent knowledge) and A2 (on the definition of scientific and demonstrative knowledge, the conditions on scientific premises, and the taxonomy of scientific principles). The treatise opens with two preliminary chapters (*Burhān* I, 1 and I, 2) on the goal, benefit, and rank of the book, in the tradition of Alexandrian *kephalaia*, introducing the vocabulary of conception and assertion, the notions of definition and deduction, and the idea that the theory of scientific demonstration and definition represents the culmination of logic. In the first treatise, Avicenna appears to exploit particularly significant junctures, problems, and examples in *An. Post.* A1 and A2 almost as pretexts to introduce the full range of theoretically relevant themes that will be later explored, in the subsequent treatises of the book, in closer connection with Aristotle's text. To give a brief illustration of Avicenna's methodology, here are a few examples, which are discussed more extensively in the following chapters. The opening line of the *Posterior Analytics* ("all teaching and all learning involving reason come from preexistent knowledge"; *An. Post.* A1, 71a1–2) becomes a natural starting point, in *Burhān* I, 3, for an elaborate exegetical effort to recast the language of preexistent knowledge in terms of the vocabulary of conception and assertion. In a similar vein, in *Burhān* I, 6, Avicenna turns the problem of the possibility of inquiry, raised by Aristotle's reference to Meno's paradox in *An. Post.* A1, into a discussion of his own foundationalist framework. The classification of the types and order of scientific inquiry, which in the *Posterior Analytics* only appears at the beginning of book B, is presented by Avicenna already in the first treatise (*Burhān* I, 5) as one of the central themes of the work, before being discussed again, in *Burhān* IV, 1, in closer connection with *An. Post.* B1 and B2. Finally, Avicenna examines in detail two critical properties of scientific principles (explanatoriness and priority) in *Burhān* I, 7–10 and I, 11, respectively, before concluding the analysis of the first treatise, in *Burhān* I, 12, with a detailed survey of the types of principles listed by Aristotle in *An. Post.* A2 (axioms, definitions, postulates, hypotheses). *Burhān* I, 7–10 is an especially significant cluster of chapters that collectively represent a fascinating microcosm of insights into Avicenna's understanding of scientific reasoning. The discussion ranges from the definition of demonstration ("a deduction consisting of premises that are certain") and its classification into types associated with different kinds of explanation, to the distinction between causal and noncausal certainty and the analysis of induction and experience. In this case, too, the early and more systematic treatment of an especially significant theme, namely the distinction between demonstration of the fact and demonstration of the reason why in *Burhān* I, 7, foreshadows the more textually oriented presentation of a subsequent chapter (*Burhān* III, 3) in tandem with *An. Post.* A13.

The other notable example of the first tendency is a cluster of four chapters in the second treatise (*Burhān* II, 6–9) dealing with two key issues that are loosely inspired by *An. Post.* A7. The first issue concerns the distinctive elements that characterize the internal structure of a science (*Burhān* II, 6). The second issue concerns the resulting constraints on the interrelations among different sciences, which determine a comprehensive, hierarchical picture of the organization of scientific knowledge as a whole. In this picture, an especially prominent role is assigned to metaphysics, which is supposed to provide the ultimate justification of the non-evident principles of the other sciences (*Burhān* II, 7). In the same context, Avicenna also offers an elaborate account of the conditions under which kind crossing and the transfer of a demonstration from one science to another are possible (*Burhān* II, 8–9). Without being any less rich in innovations, philosophical sophistication, and digressions, the remaining parts of the *Burhān*, namely the first part of the second treatise, the third treatise, and the fourth treatise, proceed almost always in lockstep with the argument and text of the *Posterior Analytics*: in particular, the first part of the second treatise corresponds to *An. Post.* A3–11, the third treatise to *An. Post.* A12–34, and the fourth treatise to *An. Post.* B.

STRUCTURE OF THE BOOK

This is not an easy book to read. Its theme is difficult and its structure complex. Methodologically, it engages simultaneously in philosophical analysis and textual interpretation. Discussions are often technical and presuppose knowledge of Aristotle's *Posterior Analytics*, combined at times with an arguably unhealthy interest in its finer points. Little to nothing has been written on the subject before, especially in comparison to the flourishing industry of modern scholarship on the *Posterior Analytics*. Furthermore, many of the problems and questions addressed in this study have an ambivalent nature. While most are ultimately rooted, one way or another, in Aristotle's text (though more or less recognizably so, depending on the occasion), in Avicenna they take on, more often than not, a fully independent life. This requires a constant balancing act between two impulses, namely the temptation to engage with Avicenna's interpretation of Aristotle's text in the form of a supercommentary on the *Posterior Analytics* and the urge to extract the main philosophical points from Avicenna's analysis in order to identify his contributions as an original theorist.

I have strived to overcome these obstacles in two ways. First, since Avicenna's *Burhān* has never been translated into a Western language (with the exception of a chapter in the twelfth century in Latin, and other excerpts in modern studies), my analysis is frequently accompanied by translations, as I hope to enable a reader with no Arabic to anchor—and verify—my interpretive claims against the evidence on which they are based. Second, I have tried as much as possible to give a clear structure and division to the argument, especially when it does not follow the order and arrangement of Avicenna's text.

The book is divided into five parts, each of which corresponds to a thematic cluster. Four appendices complement the main text: (i) a summary of the conditions of certainty (appendix A), (ii) a brief excursus on the logic of scientific reasoning (appendix B), (iii) a detailed synopsis of the contents of the *Burhān* (appendix C), and (iv) an English-Arabic glossary of technical terms (appendix D). The structure of the book primarily reflects the need to document in detail the original contributions of Avicenna's theory of science, the extent and nature of his innovations, and their broader philosophical significance. It therefore focuses selectively on the parts of the *Burhān* (or related texts) in which those contributions most evidently come to the surface. The order and arrangement of topics depends on their relevance for an adequate reconstruction of Avicenna's theory of science. This is inevitably the product of frequent interpretive choices, rather than being a reflection of the order in which they appear in Avicenna's text.

Part I ("Scientific Knowledge and Scientific Inquiry") identifies, in three chapters, the building blocks of Avicenna's theory of science (conception and assertion), a set of basic types of scientific and non-scientific assertions, and the taxonomy and order of scientific inquiry. The content of part I selectively tracks some of the main themes of *Burhān* I.

Part II ("The Organization of Scientific Knowledge") focuses, in two chapters, on the innovative framework developed by Avicenna in *Burhān* II, 6–7 to account for the way in which scientific knowledge coalesces into different individual domains of interconnected truths (the internal structure of a science, with its principles, subject, and questions) and for the way in which those domains are mutually related (the division and hierarchy of the sciences).

The clusters of themes explored in parts III, IV, and V concern the two key requirements of scientific knowledge (necessity and explanatoriness) and the type of scientific principle that paradigmatically encapsulates them both, that is to say, definition.

Part III ("Modality"), in its three chapters, resumes the thread of the discussion from *Burhān* II, 2–5 as well as drawing on relevant material from two other areas of Avicenna's logic: the theory of the predicables introduced in his *Madḫal* and a fragment of the formal logic developed in his *Qiyās*. The general goal is to illustrate in detail the modal component in the definition of scientific knowledge by looking at how it is deployed in the context of scientific reasoning. The main themes here are the notion of necessity associated with demonstration, the theory of scientific attributes, and various aspects of Avicenna's logic of essence, including the distinction between different kinds of inseparability and his account of reductio proofs in the sciences.

Part IV ("Causality and Explanation") explores the other component associated with the definition of scientific knowledge. Its three chapters, which are textually based on *Burhān* I, 7–10, II, 9, III, 3–5, and IV, 5, 8–9, deal in turn with the distinction between causal demonstration (*burhān limā*) and noncausal demonstration (*burhān anna*), subordination and explanation across different sciences, and Avicenna's interpretation of the four causes, including his understanding of the manners in which the latter are absorbed into the logical structure of demonstrations and definitions.

Part V ("Definition") is devoted to the focal principle toward which both the modal and the explanatory dimensions of scientific knowledge ideally converge. The last two chapters of the book, broadly based on *Burhān* I and especially *Burhān* IV, illustrate in detail the internal structure of definitions, their components, their types and functions, and various heuristic methods for their discovery.

Since this is already a ponderous book, in an attempt to minimize redundancies I have decided to keep the introduction short. The reader should be able to form a sufficiently comprehensive view of its contents and main narrative by reading in sequence the short introductions to the five parts, followed by the conclusion. My hope is that, after taking the shorter route, starting all over again from the beginning and reading the book in its entirety will not appear to be just a futile, Sisyphean task.

PART I

Scientific Knowledge and Scientific Inquiry

Avicenna's theory of science is concerned with two kinds of scientific knowledge, conceptions and assertions, and with a number of fundamental questions that articulate the basic types and the order of scientific inquiry. Each stage in the process of inquiry corresponds to a particular kind of question. What is the meaning of a term? What is the essence of an object? Does a certain subject exist? Does a subject have a certain attribute? Why does a subject have a certain attribute? Answers to such questions come systematically in the form of distinctive types of conceptions or assertions.

Ideas and problems from Aristotle's theory of science are translated by Avicenna into the language of conception and assertion in a dynamic process of transformation of the doctrine of the *Posterior Analytics*. If all teaching and all learning involving reason presuppose some form of preexistent knowledge, conception and assertion are the characteristic elements into which such knowledge may be analyzed. The scientific knowledge of a conclusion acquired by demonstration is equivalent to its justified assertion. And the assertion of a conclusion requires the prior conception and assertion of its premises as well as the prior conception of the conclusion itself.[1] Classical problems of Aristotelian epistemology are also investigated by Avicenna through the lens of conception and assertion. Objections

1. The notion of conception does not apply exclusively to terms but also to relations or connections between terms, including, for example, the predicative nexus between a subject and a predicate, that is to say, the *thought* that something has a certain property. In this regard, it functions in a way that is strikingly similar to the Fregean notions of sense (*Sinn*) and thought (*Gedanke*): the conception of a proposition is independent from its assertion.

against the possibility of inquiry (Meno's paradox) and against the possibility of scientific knowledge (skeptical arguments pointing to the inevitability of infinite regress or circular reasoning) are formulated—and solved—in a language whose constitutive elements are conceptions and assertions. At the same time, new problems arise as a result of Avicenna's interventions on the Aristotelian framework. How does the distinction between potential and actual knowledge play into his broader set of admissible logical forms? Again, if the process of search for principles must inevitably come to a stop, what do primary and immediate conceptions and assertions look like? And how do we come to know them? Conceptions and assertions are the provenance and destination of scientific reasoning. In a science both the starting points and the things that are sought fall in one category or the other. Complex conceptions are acquired by definition and description, starting from simpler, immediate conceptions that are ultimately acquired by abstraction from the domain of perception, while derivative assertions are acquired by demonstration from various kinds of immediate assertions (chapter 1).

The identification of the epistemic character of the basic kinds of immediate assertion is a central component of Avicenna's theory of science. The distinction between certain and non-certain assertions isolates the domain of scientific discourse from other domains of nonscientific or prescientific discourse. Certainty is the distinctive mark of scientific assertions—immediate and non-immediate alike—and a constitutive element in the definition of demonstration (a deduction consisting of premises that are certain and entailing a conclusion that is also certain). Avicenna's account of certainty is in turn dependent on the notion of belief and requires a combination of truth and necessity. And the certainty of immediate assertions that serve as principles of scientific deductions is associated with different kinds of necessity and with different sources. Such necessity may be epistemic or ontological, and its sources may be either internal, as in the case of primary propositions like the law of the excluded middle, or external, as in the case of evident propositions based on perception or experience. A classification of deductive principles based on their epistemic status and the corresponding division of arguments based on the epistemic status of their premises and conclusions allows Avicenna to cast a wide net over different forms of reasoning encountered in the process of scientific inquiry as well as in the rejection of competing theories. In this connection, the classification of nonscientific statements (assertions that may mistakenly be held to be true—or even necessary—just because they are widely accepted) or pseudo-scientific statements (assertions based on estimations that are false but may nonetheless appear compelling) is an integral part of Avicenna's epistemological project (chapter 2).

Scientific inquiry involves three main groups of questions. The first group of questions is concerned with whether something exists or whether a subject has a certain attribute. Do physical qualities exist? Do circles exist? Are humans capable of laughter? Are triangles such that the sum of their internal angles is equal to two right angles? The first two examples fall in the category of what Avicenna calls simple if-questions (*hal basīṭ*), while the last two examples fall in the category of what he calls compound if-questions (*hal murakkab*). The first group is therefore associated with two basic kinds of assertions: existential and predicative. The second group of questions is concerned with what a term means or what constitutes the essence of an object. What is the meaning of "even times even"? What is the meaning of "void"? What is an eclipse? What is a triangle? The first two examples fall in the category of what-questions relative to the meaning of a name (*mā bi-ḥasab al-ism*), while the last two examples fall in the category of what-questions relative to the essence of an object or event. The second group is therefore associated with two different kinds of conceptions: nominal definitions (or descriptions) and real definitions. The third group of questions is concerned with why something exists or why a certain attribute belongs to a subject. Why do circles exist? Why do the four elements exist? Why are broad-leaved plants deciduous? Why does the moon undergo eclipses? The first two examples fall in the category of why-questions relative to the existence of a subject, while the last two examples fall in the category of why-questions relative to whether a subject has a certain attribute. The third group is associated with the same kinds of assertions as the first group (existential and predicative), which in this case answer a why-question rather than just an if-question. The taxonomy of scientific inquiry and its types of questions (if-questions, what-questions, why-questions) is accompanied by a rigorous account of their relative order, ranging from the simple identification of the meanings of terms in a science to the establishment of the existence of subjects, the investigation of the essences of subjects and attributes, the investigation of the necessary attributes of subjects, and the identification of the causes in virtue of which those necessary attributes belong to their subjects (chapter 3).

1

Conception and Assertion

THE TWO PATHS OF SCIENTIFIC KNOWLEDGE

The distinction between conception (*taṣawwur*) and assertion (*taṣdīq*) is a characteristic feature of Arabic logic. Conception is primarily concerned with the sort of knowledge involved in concept formation and in the analysis of concepts, terms, definitions, and descriptions. Assertion is concerned with the sort of knowledge involved in the ascription of truth to propositions and in the analysis of deduction and demonstration.[1]

Avicenna is neither the first nor the last in this tradition to use conception and assertion as building blocks of logic and, more specifically, as basic elements

1. On the distinction between conception and assertion, see *Madḫal* I, 3, p. 17.7–17; *Ilāhiyyāt*, I, 1, pp. 3.5–4.6; I, 5, pp. 29.1–31.4; I, 8, p. 54.3–5; cf. also *Ilāhiyyāt* IX, 7, 429.4–13. The two notions may be regarded as analogues of apprehension (or representation) and judgment in the language of early modern philosophy. The translation of *taṣdīq* is somewhat controversial. A list of plausible alternatives includes (i) assent (arguably the solution preferred by most modern translators); (ii) judgment, Strobino (2010); (iii) belief, Lameer (2006); (iv) acknowledgment of something as true, Gutas (2012); and (v) assertion, Shehaby (1973), Strobino (2018). Assertion seems preferable for two reasons: first, it adequately conveys the idea of a truth-apt judgment based on a belief that may or may not be true and therefore captures the full range of semantic nuances expressed by the other terms without their limitations; and second, it is a more effective rendering of *taṣdīq* in the plural. The translation of *taṣawwur* is, by contrast, less problematic. Besides conception, alternative renderings are concept, conceptualization, and conceptual representation. Since *taṣawwur* may refer both to the process of concept formation and to its outcome, I distinguish between conceptualization in the first case and conception in the second. On *taṣawwur* and *taṣdīq*, see Wolfson (1943) and Van Ess (1966, pp. 95–113); cf. also Maróth (1990) and Lameer (2006).

of a logic of scientific reasoning. Alfarabi before him employs the two notions extensively in his own account of demonstration and definition and understands the internal organization of "material" logic in Aristotle (the five sections of the Arabic *Organon* coming after the *Prior Analytics*) to depend on an underlying classification of different types of assertions.[2] In Avicenna, however, the use of the distinction becomes pervasive and its significance systematic, so much so that in post-Avicennan logic conception and assertion coalesce into a central theme of discussion and, starting in the thirteenth century, are regularly listed among the candidates for the proper subject matter of logic itself.[3]

The distinction between conception and assertion plays a foundational role in Avicenna's theory of science and marks the boundary between two distinct but intimately connected modes of scientific knowledge (*ʿilm*).[4] At the beginning of the section on demonstration in the *Naǧāt*, he writes:

> Text 1.1: *Naǧāt* I, 102 (i)–(ii), pp. 112.5–113.1 (Ahmed 2011, p. 87, transl. modified; cf. also Gutas 2012, p. 395)
> All scientific knowledge is either [(a)] the conception of some notion or [(b)] assertion. Conception may exist without assertion, for example when one has a conception of the statement that void exists without asserting it, or when one has a conception of the notion of human, in which case (as with any simple [notion]) there is no assertion or denial.
>
> Every assertion and every conception are either [(bb)–(ab)] acquired through investigation or [(ba)–(aa)] exist at the beginning. Assertions are acquired [(bba)] through deduction and [(bbb)] through things we have mentioned that resemble it. Conceptions are acquired [(aba)] through definitions and [(abb)] through other things that we will mention.

In Text 1.1, conceptions and assertions are identified as the two fundamental types of scientific knowledge. Each of them is further divided into two classes: conceptions and assertions that are acquired through investigation (*yuktasabu bi-baḥṯ*), as opposed to conceptions and assertions that exist in some primary way at the beginning of the process of inquiry (*wāqiʿ ibtidāʾan*). Investigation is a technical term in Avicenna's logical vocabulary alluding to the articulation of a discursive line of reasoning (*naẓar* is often used in a similar sense). In this passage, it means

2. The Arabic *Organon* traditionally includes Porphyry's (i) *Isagoge*, followed by Aristotle's (ii) *Categories*, (iii) *On Interpretation*, (iv) *Prior Analytics*, (v) *Posterior Analytics*, (vi) *Topics*, and (vii) *Sophistical Refutations*, with the addition of (viii) *Rhetoric* and (ix) *Poetics*. The genesis and characteristics of an interpretive attempt (sometimes referred to as the "context theory") to account for the relation between (v)–(ix) and their place in Aristotelian logic (starting in late antiquity and culminating in the Arabic tradition) are discussed extensively in Black (1990).

3. El-Rouayheb (2012); cf. Sabra (1980).

4. See *Ilāhiyyāt* III, 8 on knowledge as an accident (a quality, state, or disposition of the soul); cf. *Maqūlāt* V, 3, p. 182.7–9.

TABLE 1 Types of scientific knowledge

(a) Conception		(b) Assertion	
(aa) Primary	(ab) Acquired	(ba) Primary	(bb) Acquired
Immediate	Definition	Immediate	Demonstration

two distinct things. In the case of assertions, acquisition through investigation means acquisition through deduction (direct or indirect) and other argument forms such as induction, example, or enthymeme discussed in the treatment of formal logic. In the case of conceptions, acquisition through investigation means acquisition through definition or description, whose rigorous theoretical treatment is the prerogative of the theory of science (as opposed to dialectic). It is important to note that the distinction between what is acquired through investigation and what is not acquired through investigation in Text 1.1 is not a distinction between acquired and innate (Avicenna vigorously rejects innatism) but rather one between different kinds of objects and modes of acquisition. According to this preliminary characterization, illustrated in table 1, scientific knowledge turns out to be one of four things: (aa) a primary conception, (ab) an acquired conception, (ba) a primary assertion, or (bb) an acquired assertion.[5]

The distinction between acquired and non-acquired conceptions and assertions is critical for the formulation of Avicenna's own version of epistemological foundationalism, namely the doctrine that scientific knowledge ultimately presupposes indemonstrable first principles on which everything else depends. In other words, in order for scientific knowledge to be possible, there must be (i) immediate assertions that are not grounded in other assertions and (ii) immediate conceptions that are not in turn dependent on other conceptions. In the continuation of Text 1.1, Avicenna evokes the fatal threat of an infinite regress to argue that the process of acquisition of conceptions and assertions must come to a stop at immediate items of each kind:

5. Conception and assertion are introduced twice in some of Avicenna's logical works: first at the beginning of logic itself (*Naǧāt* I, 1; *Madḫal* I, 1) and then again as a preamble to the treatment of demonstration and definition (*Naǧāt* I, 102; *Burhān* I, 1). A parallel passage establishes a similar point at the opening of the logic of the *Naǧāt* I, 1 (i), p. 7.1–5 (Ahmed 2011, p. 3, transl. modified): "All knowledge (*maʿrifa*) and scientific knowledge (*ʿilm*) are either [(a)] conception or [(b)] assertion. Conception is knowledge that comes first and is acquired by means of definition and whatever is like it. [. . .] Assertion comes about only by means of deduction and whatever is like it. [. . .] Definition and deduction are two tools by means of which one acquires objects of knowledge (*maʿlūmāt*) that are unknown and then become known by means of rational calculation (*rawiyya*)." Avicenna goes on to note that both in the case of definition and in the case of deduction there may be (i) real, (ii) unreal but useful, and (iii) specious cases. One of the primary tasks of logic is to enable us to distinguish among them.

Text 1.2: *Nağāt* I, 102 (iii), p. 113.2–6 (Ahmed 2011, pp. 87–88, transl. modified; cf. also Gutas 2012, p. 395)
Deductions have parts that are asserted and others that are conceptualized, while definitions have parts that are conceptualized. This does not result in an infinite regress, in such a way that knowledge of these parts becomes available only through the acquisition of other parts whose characteristic is this, namely to go on infinitely. Rather, [the process] comes to a stop at things that are objects of assertion and objects of conception immediately (*bi-lā wāsiṭa*).

The two sources of acquired assertions and conceptions are deductions and definitions. Deductions and definitions may both be analyzed into parts. The parts of a deduction are its premises and conclusion, which may in turn be analyzed into terms. Premises and conclusions are objects of assertion, while terms are objects of conception. The parts of a definition are the terms of which it consists, each of which is an object of conception. Both scientific knowledge that is acquired through deduction and scientific knowledge that is acquired through definition must ultimately rest on a (finite) number of immediate assertions and conceptions. The same is true a fortiori of demonstration, which is a particular type of deduction, involving premises and conclusions that are certain.

RANKS OF CONCEPTION AND ASSERTION

In *Burhān* I, 1, Avicenna develops a more precise taxonomy of conception and assertion and discusses in greater detail their relation to definition and deduction. I look more closely at the texts on which the taxonomy is based in chapter 2 for Avicenna's analysis of assertion and deduction and in chapter 12 for his treatment of conception and definition. For the moment, I just wish to give a preliminary characterization of how conception and assertion concretely serve as the main building blocks of Avicenna's theory of science. The taxonomy of *Burhān* I, 1 deals with the two types of acquired scientific knowledge, which may be obtained either by means of thought (*al-ʿilm al-muktasab bi-l-fikra*) or in a manner that does not involve thought (*al-ḥāṣil bi-ġayr iktisāb fikrī*).[6] Thought (*fikra*) is a technical term that designates a faculty of the soul whose peculiar activity is to aid the intellect (*ʿaql*) in combining or separating concepts in propositional compounds that are amenable to truth and falsehood. Scientific knowledge acquired by means of thought is associated by Avicenna with the domain of assertion, and in this capacity, thought indirectly provides the basis of deductive reasoning. Scientific knowledge that is not acquired through thought, by contrast, is associated with the domain of conception. Both domains are arranged in ranks (*marātib*). In the case of assertion, ranks are determined by an underlying series of beliefs (*iʿtiqād*) of

6. A reference to the notion of deductive acquisition (*iktisāb qiyāsī*) is at *Burhān* III, 4, p. 215.10.

decreasing epistemic strength, ranging from certainty (*yaqīn*) to mere supposition (*ẓann*).[7] In the case of conception, ranks are associated with various aggregates of notions or attributes, which may be essential or accidental and jointly proper to something or common to it and other things. In this framework, assertion stands to deduction as conception stands to definition and description. Consequently, the hierarchical arrangement of the different types of assertions and conceptions induces a parallel hierarchical arrangement of deductive arguments and of definitions and descriptions. In particular, Avicenna identifies three ranks of assertion:

(aa) certain (*yaqīn*);
(ab) resembling certain (*šabīh bi-l-yaqīn*); and
(ac) persuasive, based on supposition or opinion (*iqnāʿī ẓannī*).

Each of them is associated, as we shall see in chapter 2, with various types of belief and characterizes the premises and conclusions of a particular kind of deductive argument, that is to say,

(aa) demonstrative (*burhānī*),
(ab) dialectical (*ǧadalī*) or fallacious (*muġāliṭī*), and
(ac) rhetorical (*ḫiṭābī*).

The ranks of conception, by contrast, are four. They are determined by four basic ways of collecting the attributes of an object. These may involve

(ba) an aggregate of essential notions (*maʿānin ḏātiyya*) proper to the object,
(bb) an aggregate of essential notions common to the object and something else,
(bc) an aggregate of accidental notions (*maʿānin ʿaraḍiyya*) proper to the object, or
(bd) an aggregate of accidental notions common to the object and something else.[8]

7. The Arabic root *y-q-n* (*yaqīn*, *yaqīnī*) forms the basis for the canonical translation of the Greek term *bebaios* and its cognates. Aristotle characterizes the Principle of Noncontradiction as the most certain (or firmest) of all principles at *Metaphysics* Γ3, 1005b8–12. Interestingly, *bebaios* is not attested in the *Posterior Analytics*, but *yaqīn* and related terms occur a few times in Abū Bišr Mattā's Arabic translation. In that context they are used, typically in tandem with *ʿilm*, to render the Greek terms *epistēmonikos*, *epistēmēn echein* (*An. Post.* A2, A9, A14) or even *eidenai* (*An. Post.* A2, B2) in the characterization of demonstrative scientific knowledge or knowledge acquired through inquiry.

8. Avicenna does not consider in *Burhān* I, 1 the case of hybrid accounts involving a combination of both essential and accidental attributes, but it seems clear from occasional examples in the corpus that he treats those as cases of descriptions (even a complete, ordered sequence of essential attributes specified at the last step by a nonessential attribute would not count as a complete definition but rather as a complete description).

Each of these four corresponds, as we shall see in chapter 12, to a distinctive kind of differentiating expression (*qawl mufaṣṣil*). A differentiating expression is a complex term involving different ordered sets of essential or accidental attributes, by means of which something is made known (*taʿrīf*) and distinguished (*tamyīz*) from all other things or from some other things only. The four kinds of differentiating expression are:[9]

(ba) complete definition (*ḥadd tāmm*),
(bb) incomplete definition (*ḥadd nāqiṣ*),
(bc) complete description (*rasm tāmm*), and
(bd) incomplete description (*rasm nāqiṣ*).[10]

Conception and assertion are not only the basic ingredients of Avicenna's definition of scientific knowledge, of his account of demonstration and definition, and of his characterization of certainty and other forms of nonscientific assertions. They are also essential elements in his account of the goal (*ġaraḍ*) and utility (*manfaʿa*) of the theory of science, which represents the culmination of logic. The goal of the logic of scientific reasoning is to investigate (i) the conditions under which deductive arguments bring about conclusions that are certain, consisting of epistemically and modally stable beliefs; and (ii) the conditions under which complex linguistic expressions qualify as real definitions that articulate adequately the conception of essences. This requires, in Avicenna's language, the identification of adequate demonstrative and definitional "matters" (*mawādd*), that is to say, suitable assertions or premises, capable of bringing about certainty through demonstration, and suitable conceptions or terms, capable of bringing about the complete conceptualization of an object of inquiry through its definition.[11] The utility of the theory of science lies in the identification of "methods" or "paths" (*ṭuruq*) for the attainment of "certain assertions" and "real conceptions" and in the specification of the criteria of adequacy of both.[12]

9. Explanatory is used here in a weak, nontechnical sense. In fact the term used by Avicenna more neutrally refers to an "expression that provides a differentiation" (*qawl mufaṣṣil*). "Explanatory expression" is the counterpart of Avicenna's preferred term in *Išārāt* I, 4, namely *qawl šāriḥ.*

10. Avicenna's account of complete definition in *Burhān* IV, in addition to coextensiveness, requires that all essential attributes of the *definiendum* be included in its definition to ensure complete conceptual correspondence. The *definiens* must be "equal in conversion" and "equal in notion" to the *definiendum* (*musāwin fī l-ʿaks* or *fī l-inʿikās* and *musāwin fī l-maʿnā*) (for instance at *Burhān* IV, 6, p. 306.8–9). The condition is discussed in more detail in chapter 12, where I present other objections that Avicenna raises against an ill-conceived approach to definition, most likely associated with the view of certain Baġdādī Aristotelians.

11. *Burhān* I, 1, p. 53.15–18.

12. Avicenna's use of *ḥaqīqī* as a qualification of *taṣawwur* or *ḥadd* should not evoke the idea of propositional truth. Rather, it indicates that a conception or a definition adequately corresponds to

TEACHING AND LEARNING INVOLVING REASON

The first actual application of the distinction between conception and assertion in Avicenna's interpretation of the *Posterior Analytics* is linked with the dramatic incipit of Aristotle's work, namely the contention that "all teaching and all learning involving reason come from preexistent knowledge" (*An. Post.* A1, 71a1–2). As noted in the introduction, the first two chapters of the *Burhān* deal with classical problems of Alexandrian exegetical methodology (the goal and rank of the book), and it is only in *Burhān* I, 3 that Avicenna starts to engage directly with Aristotle's text. The chapter is an elaborate gloss on the notion of preexistent knowledge and deals with various questions raised by the latter: (i) what preexistent knowledge consists of, (ii) how preexistent knowledge is compatible with ignorance of what is sought, and more generally (iii) how what is known and what is not (yet) known are related.

A list of alternative modes of teaching and learning isolates a preliminary set of notions against which the sort of teaching and learning Avicenna is interested in for the purposes of scientific discourse must be contrasted:[13]

Text 1.3: *Burhān* I, 3, p. 57.1–9
[(i)] One kind of teaching and learning involves craftsmanship (*ṣināʿī*), like learning carpentry and the art of dyeing, and it is through assiduous practice of the activities [proper to] those arts that it comes about. [(ii)] Another kind involves dictation (*talqīnī*), like the dictation of a certain poem or of [the sounds and utterances of] a certain language, and it is through the assiduous articulation of those sounds and utterances resulting in a habit that it comes about. [(iii)] Another kind involves discipline (*taʾdībī*), and it is through the instruction [imparted by the teacher] to his

an essence. The language of certain assertion and true conception at the end of *Burhān* I, 1 is strongly reminiscent of a critical passage at *Ilāhiyyāt* IX, 7, p. 429.4–13. I return to this point in the conclusion. The investigation of the conditions of possibility of demonstration and definition, or at least the proof of the fact that demonstration and definition are possible, is explicitly acknowledged by Avicenna to be the task of a different discipline (metaphysics), at *Burhān* III, 6, p. 237.6–13, in the context of the reductio argument associated with the proof that demonstrative chains must be finite according to the general trajectory of *An. Post.* A19–23.

13. The list seems to be inspired by Alfarabi's *Burhān* V, 1, pp. 77.5–83.9, which is in all likelihood the proximate source for Avicenna in *Burhān* I, 3 and I, 6. In that context, Alfarabi discusses the notion of teaching (*taʿlīm*), Meno's paradox (V, 1, p. 79.5–21), the definition of teaching, *taṣdīq*, and *taṣawwur*; introduces a digression on impossible entities (for example, void, infinite, and goat-stag, at V, 1, p. 80.16–20) which may be the inspiration for Avicenna's opening of I, 6 and for his own treatment of Meno's paradox. Alfarabi discusses some of the types of nonintellectual teaching and learning listed in Avicenna's Text 1.3 (for example, *talqīnī* and *taʾdībī*), and unsurprisingly devotes some space to the notion of *ḏihn* (V, 1, pp. 78.4–79.4). Alfarabi's discussion of teaching and learning in V, 1 is part of a more general treatment of the four types of demonstrative forms of discourse (*muḫāṭabāt burhāniyya*): (i) teaching and learning (*taʿlīm* and *taʿallum*); (ii) demonstrative conflict (*ʿinād burhānī*); (iii) discovery (*instinbāṭ*); and (iv) scientific eristic reasoning (*imtiḥān ʿilmī*), that is, the demonstrative fallacies (*muġālaṭa burhāniyya*).

> learner that it comes about. [(iv)] Another kind involves the unquestioning adoption of a tradition (*taqlīdī*), which consists in the fact that someone gets used to believing in a certain view, and it is with respect to the trust placed in the teacher that it comes about for him. [(v)] Another kind involves being reminded (*tanbīhī*), as in the case of one who knows that lodestones attract iron but is neglectful of this fact at the right time and does not understand it, when he perceives a lodestone attracting iron; and so he is puzzled by it. But if one says to him: "This is the lodestone whose condition you are familiar with," then at that moment he is reminded of it and ceases to be puzzled.[14] Or as in the case of one who discusses by means of first principles (*awā'il*) without understanding them because of some imperfection in the expression or in his reason, and so one strives to establish them for him. [(vi)] There are other kinds but none of them involves reason or thought.

The significance of Text 1.3 is both exegetical and systematic. On the one hand, it illustrates Avicenna's flexibility in incorporating elements alien to the Greek commentary tradition of the *Posterior Analytics* and peculiar to the Arabic-Islamic tradition (for instance, the notion of *taqlīd*). But on the other hand, it also introduces the notion of reminding or being reminded (*tanbīh*) as a separate mode of knowledge directly associated with first principles. Interestingly, Avicenna seems to take the notion of *tanbīh* as a type of teaching and learning falling outside the scope of discursive reason (which coincides with the domain of acquired conceptions and assertions). Since elsewhere this mode is frequently associated with induction (*istiqrā'*) and the grasp of first principles, which cannot be acquired through more primitive conceptions or assertions, it is tempting to read this remark as part of Avicenna's general understanding of immediate conceptions and assertions as the kind of preexistent knowledge that all knowledge involving reason (in the technical sense of I, 3) is said to presuppose.[15]

More important for our purposes, however, is Avicenna's use of the philosophical vocabulary of conception and assertion in the analysis of the notion of preexistent knowledge itself. In addressing the question of what it means for teaching and learning to involve reason (*ḏihnī*), Avicenna (i) concretely spells out the Aristotelian notion of preexistent knowledge in terms of conception and assertion and (ii) articulates their mutual relations.[16] He writes:

14. On the relation between the essence of lodestone and its capacity to attract iron, see *Ilāhiyyāt* III, 8, p. 141.8–14.

15. At *Ilāhiyyāt* III, 3, p. 106.6–9, while discussing the relation between multiplicity, unity, quantity, number, equality, part, division, and order, Avicenna notes "that all these [characterizations] are reminders (*tanbīhāt*), like the reminders occurring through examples and synonyms, and that some or all of these notions are conceptualized in themselves. They are signified by these things only in order that one may be reminded of them and in order that they may be distinguished."

16. Another *locus classicus* is *Burhān* IV, 1, where Avicenna addresses in general the distinction between conceptual and propositional knowledge as a proxy of the relation between definition and demonstration (in relation to *An. Post.* B3).

Text 1.4: *Burhān* I, 3, p. 57.9–15
[Teaching and learning] involving reason and thought are the ones that are obtained by means of an audible or intelligible discourse whose characteristic it is [(a)] to bring about a *belief* (*iʿtiqād*) or a view (*raʾy*) that did not exist, or [(b)] to bring about a *conception* that did not exist.[17] Such teaching and learning involving reason obtain sometimes between two men and sometimes between a man and himself in two respects. For example, one is a teacher with respect to having the intuition of the middle term in the deduction and a learner with respect to acquiring the conclusion from the deduction. The teaching and learning are one in essence but two according to the point of view. For one and the same thing, namely being driven to the acquisition of what is unknown by means of what is known (*iktisāb al-maǧhūl bi-l-maʿlūm*), is called learning with regard to the one in whom it comes about and teaching with regard to the one from whom it comes about, that is to say the efficient cause, like moving and being moved.

In Text 1.4, Avicenna translates the notion of preexistent knowledge from the opening line of the *Posterior Analytics* into his own language of conception and belief, where belief is in turn the notion in terms of which assertion is typically characterized. In the same passage, Avicenna also incidentally hints at his own definition of logic, which he understands to be a canonical instrument by means of which reason moves from what is known to what is unknown. Teaching and learning involving reason are therefore an expression of what logic itself encapsulates in a more abstract form: a process of cognitive transfer from what is known to what is unknown.[18]

PREEXISTENT CONCEPTIONS AND ASSERTIONS

Teaching and learning involving reason presuppose some form of preexistent knowledge. But what does preexistent knowledge look like? For Avicenna the

17. An interesting exegetical aspect of *Burhān* I, 3, to which I can only refer in passing, is a lengthy gloss on the meaning of *ḏihnī* (which is Abū Bišr Mattā's Arabic for *dianoētikē*) and *fikrī*. A traditional question is why teaching and learning are qualified as "intellectual" by Aristotle, and with what the qualification should be contrasted. It is possible that Avicenna's interest in this question might have been prompted by an analogous digression (in spite of some differences) in Philoponus on *An. Post.* A1. Another question is whether *ḏihnī* is more or less appropriate than *fikrī* as a translation of *dianoētikē*. Avicenna argues that the former ("involving reason") is more appropriate than the latter ("involving thought") because it is more general and covers a broad range of mental activities presupposed by conception and assertion (including thought in its technical sense). At *Burhān* I, 3, p. 60.8–10, he writes: "If every teaching and learning is about intelligible entities (*li-l-umūr al-ʿaqliyya*), then it is either by way of (i) thought (*fikr*), (ii) intuition (*ḥads*), or (iii) comprehension (*fahm*), and this applies not only to assertion but also to conception. All of them involve reason. Thus, [Aristotle's] statement 'teaching and learning involving reason' is more correct."

18. The definition of logic for Avicenna involves a transfer in the mind from what is present or known to what is not present or unknown. The transfer in question applies to both conceptions and assertions. See for instance *Madḫal* I, 2, p. 15.9–17 (cf. also I, 3, pp. 17.17–18.6) and *Išārāt* I, 1, pp. 2.1–3.4.

notion applies to conceptual and to propositional knowledge alike, and its source may be either internal or external. Preexistent knowledge of what is sought must be distinct from the knowledge acquired through teaching and learning but at the same time relevant to it. It must be, in other words, potential knowledge of what is sought without being actual knowledge of it.

In his analysis of preexistent knowledge in *An. Post.* A1, Aristotle illustrates his general contention with a reference to various types of inferential procedures (deduction, induction, example, enthymeme). These inferential procedures are analyzed by Avicenna in terms of relations among assertions. If an assertion is non-immediate, then it must derive from preexistent knowledge, and such preexistent knowledge may be characterized precisely in Avicenna's conceptual vocabulary. His analysis involves three components: (i) the conception of the conclusion, (ii) the conception of the premises, and (iii) the assertion of the premises. He writes:

> Text 1.5: *Burhān* I, 3, p. 58.1–6
> Prior to assertion is knowledge of three things. The first is [(i)] the conception of what is sought (*maṭlūb*), even if it is not yet asserted.[19] The second is [(ii)] the conception of the statement (*qawl*) that precedes [what is sought] in rank. The third is [(iii)] the assertion of the statement that precedes [what is sought] in rank. From knowledge of these three things there follows [(iv)] the assertion of what is sought. Regardless of whether the preceding statement represents a deduction (*qiyās*), an induction (*istiqrāʾ*), an example (*tamṯīl*), an enthymeme (*ḍamīr*), or something else, one or more premises are necessary in order for an assertion that did not exist to be acquired; and knowledge of them comes about in two ways, first with respect to conception and then with respect to assertion.
>
> Prior to conception is the conception of the parts of a definition or a description, and nothing else.

19. I translate *maṭlūb* as "what is sought" and *maṭlūbāt* as "things that are sought." The notion is characterized by a systematic ambiguity: sometimes the term refers to the conclusion of an argument, that is to say, a proposition, and sometimes to its predicate, that is to say, a term. In yet other cases, it refers to a term outside the context of a proposition, for instance when what is sought is a definition. A question that becomes relevant in later Arabic logic is whether an assertion presupposes not only a conception of subject and predicate but also a conception of the nexus (*nisba*) between subject and predicate. This level of analysis seems implicit in Text 1.5 and there seems to be trace of it elsewhere in the *Burhān*, for example at I, 1, p. 53.4–10. In that context, Avicenna suggests that conception is in a sense a principle of assertion because not everything that is conceptualized is asserted (for instance simple and complex terms, which are not amenable to truth and falsehood), but everything that is asserted is conceptualized. Avicenna's explanation is that declarative statements (*aqwāl ǧāzima*) are not only objects of assertion but presuppose a conception associated with the coming about in the soul of *an intelligible form of the nexus obtaining between the two terms* (subject and predicate), in addition to the conception of the two terms taken in isolation.

Any argument form, whether it be inductive or deductive (demonstrative, dialectical, or rhetorical) that aims to produce an assertion requires three kinds of preexistent knowledge, all of which are relevant to what the argument seeks to establish, that is to say, its conclusion. The first requirement is that the conclusion must be an object of conception. The second requirement is that the premises of the argument must be objects of conception. Avicenna refers to them collectively as "the statement that precedes" what is sought, which must refer both to the individual premises (the first two occurrences of *qawl* in Text 1.5) and to their arrangement into an argument (the third occurrence of *qawl* in Text 1.5). The third requirement is that the premises of the argument must be objects of assertion. These are necessary and sufficient conditions for the assertion of the conclusion.

A non-immediate conception, by contrast, presupposes only the conception of the parts of its definition or description. In the last sentence of Text 1.5, Avicenna states the necessary conditions of conception only with regard to the nonpropositional case, even though it is plausible to assume that the kind of conception discussed in (i) and (ii) presupposes in turn the conception of the terms occurring in the premises. The first case illustrates the kinds of preexistent knowledge presupposed when what is sought is an assertion, which involves propositional knowledge. The second case illustrates the kinds of preexistent knowledge presupposed when what is sought is the conception of a complex term. In the former case, an assertion presupposes both the prior *conception* of the nexuses expressed by premises and conclusion (where the conclusion is the assertion itself) and the prior *assertion* of the nexuses expressed by the premises. In the second case, the conception of a complex term such as a definition or a description only presupposes the prior conception of its parts, that is to say, of certain simpler terms, without any prior assertion (and, obviously, without any prior conception of a predicative nexus, given that in principle no such nexus exists in the case of terms).

POSSIBILITY OF INQUIRY AND POSSIBILITY OF SCIENTIFIC KNOWLEDGE

Two problems arise in connection with the view that all teaching and learning involving reason come from preexistent knowledge. First, in what sense is preexistent knowledge not the same as knowledge of what is sought? How do they differ? For example, how is knowledge of the premises of an argument not identical with knowledge of its conclusion? And, if we can only either fully possess knowledge of what is sought or be altogether ignorant of it, how is inquiry possible in the first place? Second, if all preexistent knowledge presupposed by teaching and learning involving reason can only in turn be acquired through teaching and learning involving reason, then the process seems inevitably bound to result either in an infinite regress or in a vicious circle.

The first problem is a version of Meno's paradox and concerns the possibility of inquiry. The second problem consists of two skeptical objections against the possibility of scientific knowledge. Avicenna addresses the first problem and a variant of the first objection from the second problem (the argument from infinite regress) in *Burhān* I, 6. He then deals with another variant of the second problem (including both the argument from infinite regress and the argument from circularity) in *Burhān* II, 1, which is concerned with the analysis of *An. Post.* A3 and where Avicenna specifically substitutes the notions of teaching and learning involving reason with that of demonstrative scientific knowledge.

Meno's Paradox: Knowledge in Potency and Knowledge in Act

Avicenna's solution to the first problem rests on a distinction between potential and actual knowledge, while his solution to the second problem is an expression of his commitment to a foundationalist epistemology. What is especially interesting for our purposes is that in both cases the two problems are framed and solved using the distinction between conception and assertion.

Avicenna's formulation of Meno's paradox may be reconstructed as follows:[20]

(i) For any x, either one knows x in every respect or one is ignorant of x in every respect;
(ii) If one knows x in every respect, one cannot seek x (because the process would be meaningless);
(iii) If one is ignorant of x in every respect, one cannot seek x (because it will be impossible to recognize x).

From (i)–(iii), Avicenna infers the following general conclusions:

(iv) One cannot seek anything (for x in the example is arbitrary), and therefore
(v) All inquiry is impossible.

Avicenna's solution to the paradox consists in rejecting the first premise (i), namely the contention that what is sought is either known in every respect or unknown in every respect. This amounts, in Avicenna's view, to a false dilemma between unqualified knowledge and unqualified ignorance. And it is precisely at this stage that conception and assertion come into play in the analysis of the

20. Avicenna draws a further distinction between two approaches to Meno's paradox, both of which he rejects: the first is supposedly Socrates's approach in the dialogue, which is said to be question-begging (it is not an explanation that neutralizes the paradox, but simply asserts what Meno denies; it is unclear to me what exactly Avicenna means by this, though undoubtedly the fact that he did not have firsthand knowledge of the text does not help); and the second is Plato's solution based on the theory of recollection, which is said to have been refuted already in the *Qiyās.* The reference at *Burhān* I, 6, p. 75.7 is presumably to *Qiyās* IX, 19, p. 545.9–15 (cf. *An. Pr.* B21). For a general account of Meno's paradox in Avicenna, see Marmura (2009).

paradox. For according to Avicenna, one and the same thing that is being sought (*maṭlūb*) may be at the same time known and unknown, provided that this is in different respects. In particular, Avicenna holds that what is sought is known in two respects and unknown in one respect, that is to say, it is

(a) known in act by means of conception,
(b) known in potency by means of assertion, and
(c) unknown in act specifically (but known in act nonspecifically).

A *maṭlūb* (for example, the conclusion of an argument) is known in act by means of conception insofar as its terms and the nexus between them are objects of conception, which is to say that before asserting an affirmative or negative proposition, it is necessary to have a conception of the terms and of their combination or separation (as illustrated in Text 1.5). The conclusion is also known in potency by means of assertion because the premises from which it follows are themselves objects of assertion.

The rejection of premise (i) amounts to the claim that knowing *x* without qualification and being ignorant of *x* without qualification are not the only ways to know *x* or to be ignorant of it; the disjunction is not exhaustive even though the disjuncts are incompatible (for they are presumably contraries). Avicenna takes a group of notions associated with the second disjunct of (i) to be materially equivalent ("being ignorant of *x* in every respect," "*x* being unknown in every respect," and "*x* not being known in any respect") and replaces them with the (correct) notion of "*x* not being known in every respect." The latter rather than (any of) the former is the contradictory of the first disjunct of (i) and therefore yields a disjunction that is both exhaustive and exclusive. On the new reading, the first premise of the argument says that either something is known in every respect or it is not known in every respect. And this proposition is necessarily true, for it is a perfectly legitimate instance of the law of the excluded middle. But then (iii) turns out to be false. For if one does not know *x* in every respect, it does not follow that one does not know *x* in any respect, and if *x* is known at least in some respect, then it is possible to recognize it. Avicenna's solution to the paradox rests on the identification of a problem of negation, quantification, and scope in its original formulation: what we cannot recognize is what is *not* known in *any* respect rather than what is *not* known in *every* respect.[21]

Knowledge in Potency, Knowledge in Act, and Logical Forms

The distinction between knowledge in potency and knowledge in act solves the apparent dichotomy between absolute knowledge and absolute ignorance that gives

21. I am grateful to one of the anonymous readers for pointing out that Avicenna does not appear to be sensitive to the fact that his own response is somewhat problematic, since the paradox seems to put into question our ability to reach partial knowledge, too.

rise to Meno's paradox. We are not absolutely ignorant of what is sought because we have potential knowledge of it, where for Avicenna potential knowledge may be prior knowledge of conceptions as well as prior knowledge of assertions (and sometimes of both at the same time). But the distinction also applies more generally to a set of basic forms of argument employed in Avicenna's theory of science. In *Burhān* I, 3, Avicenna offers an interesting analysis of the sort of preexistent knowledge involved in certain basic types of argument forms:

Text 1.6: *Burhān* I, 3, p. 60.11–20
The thing whose assertion, when it comes about, is an assertion in potency of something else, is either [(i)] its implicant (*malzūm*), [(ii)] its opposite (*muʿānid*), [(iii)] a universal above it, [(iv)] a particular under it, or [(v)] a particular on a par with it (*maʿahū*).

[(i)] When the implicant is known in act, that knowledge is knowledge in potency of its implicate (*bi-lāzimihī*), and that obtains through repetitive deductions from hypothetical conditional [premises] (*šarṭiyyāt muttaṣila*).[22]

[(ii)] When the opposite is known in act, that knowledge is knowledge in potency of its opposite, either because the latter is removed, when the former is posited, or because the latter is posited, when the former is removed, and that obtains through repetitive deductions from hypothetical disjunctive [premises] (*šarṭiyyāt munfaṣila*).[23]

[(iii)] When the existence of an affirmative or negative judgment about the universal is known in act, that is knowledge in potency of the particular that falls under it by way of deduction.[24]

[(iv)] When the existence of an affirmative or negative judgment about the particular is known [in act], that is supposition in potency about the universal above it, if what is known is a judgment about some particulars, and this obtains through incomplete induction (*istiqrāʾ nāqiṣ*); or [it is] knowledge in potency of the universal above it, if what is known is a judgment that is common to every particular, and this obtains through complete induction (*istiqrāʾ tāmm*).[25]

22. The conditions under which knowledge of the consequent is implicit in the knowledge of the antecedent of a true conditional are discussed, along with some difficulties, in *Qiyās* IX, 1. Avicenna has in mind here an inference such as *modus ponens* (see *Qiyās* VIII, 1, on repetitive deductions with a conditional premise). If "If p, then q" is an implicative conditional that is true in fact (where the consequent necessarily follows from the antecedent just in virtue of their respective meanings), then knowledge in act of p is knowledge in potency of q. I briefly return to Avicenna's account of conditionals in chapter 10.

23. On disjunctive statements expressing conflict, see in particular *Qiyās* V, 2; cf. also *Qiyās* VIII, 2, on repetitive deductions with a disjunctive premise. In this case, the hypothetical statement has the form "Either p or q" (where p and q are mutually exclusive and exhaustive). In this case, knowledge in act of p is knowledge in potency of *not-q*, and knowledge in act of *not-p* is knowledge in potency of q.

24. This case corresponds to universal instantiation and is illustrated by Aristotle's example of the right triangle inscribed in a semicircle in *An. Post.* A1. On this topic, see also *Qiyās* IX, 10.

25. On induction (*istiqrāʾ*), see *Qiyās* IX, 22; cf. also *Qiyās* VI, 6 (on divided deduction) and *Burhān* I, 7 and I, 9. Incomplete induction can at best result in potential supposition (*ẓann*), while complete induction is potential knowledge of a universal statement.

> [(v)] When the existence of a judgment about the particular is known [in act], that is supposition in potency about another particular, namely that it is such-and-such, if the former shares in a notion with the latter, and this obtains through example (*tamṯīl*).[26]

In Text 1.6, Avicenna lists a series of inferences (deductive, inductive, or analogical) that fit into the general scheme previously outlined in Text 1.5. The first case is that of a hypothetical conditional statement of the form "If *p*, then *q*," and the contention is that actual knowledge of the antecedent ("the implicant" *malzūm*) implies potential knowledge of the consequent ("the implicate" *lāzim*), in the same way as actual knowledge of the premises of a syllogistic deduction implies potential knowledge of the conclusion. In the second case, a similar line of reasoning applies to certain kinds of oppositions (for instance, disjunctions of incompatible exhaustive alternatives). In a statement of the form "Either *p* or *q*" where *p* and *q* are incompatible, actual knowledge of *p* implies potential knowledge of *not-q*, and actual knowledge of *not-p* implies potential knowledge of *q*. The third case is the one that comes closest to the logic of the example found in *An. Post.* A1, namely that actual knowledge of a universal statement implies potential knowledge of any of its instances (as in "Every A is B; this is A; therefore this is B," where knowledge of the conclusion is said to fall potentially under the broader epistemic scope of the major premise).[27] The fourth case covers induction, namely the process of partial or full universal generalization that leads from actual knowledge of certain particulars to either a potential supposition concerning the universal under which they fall, if the generalization is based on a survey of some (types of) particulars only, or the potential knowledge of that universal, if the generalization is based on a survey of all (types of) particulars falling under that universal. The fifth case involves example, that is to say, an inference based on analogy, whereby actual knowledge of a particular case implies potential knowledge of another particular case falling under the same universal. The basic case of categorical and hypothetical deductions (in which actual knowledge of the premises is knowledge in potency of the conclusion) is tacitly omitted here in favor of additional logical relations (implication and opposition) and of a more detailed account of argument forms encountered before (induction and example).

Infinite Regress and Circularity

Avicenna's answer to the first objection from the second problem (the argument from infinite regress) is formulated in two variants. According to the first variant,

26. On example (*tamṯīl*), see *Qiyās* IX, 23.

27. For the view that something can be known in a sense and unknown in another sense, see *Qiyās* IX, 19 (cf. *Naǧāt* I, 94). The chapter is the counterpart of *An. Pr.* B21, and the discussion is relevant for Avicenna's understanding of the idea of preexistent knowledge because it explores the notion of knowing something in potency through knowledge of a universal premise.

the assumption that all teaching and learning involving reason come from preexistent knowledge leads to an infinite regress, if teaching and learning involving reason are in turn the only possible forms of knowledge. According to the second variant, the assumption that all scientific knowledge is demonstrative also leads to an infinite regress, if demonstrative knowledge is in turn the only possible form of scientific knowledge. In one case the focus is on teaching and learning involving reason, in the other on the reduction of scientific knowledge to demonstrative knowledge. The second objection from the second problem (the argument from circularity) is that the assumption that all scientific knowledge is demonstrative can only alternatively lead to circular reasoning, if instead of postulating a new demonstration at every step, some things are assumed to be mutually demonstrable.

The first variant is addressed at the end of *Burhān* I, 6 after the discussion of Meno's paradox. Teaching and learning involving reason are possible because the regress always comes to a stop at first principles, that is to say, immediate conceptions and assertions:[28]

> Text 1.7: *Burhān* I, 6, p. 77.1–5
> Having established how teaching and learning involving reason come to be and that this is only in virtue of preexistent knowledge, we must then have first principles of conception and first principles of assertion. [For] if all teaching and learning were in virtue of preexistent knowledge, and all knowledge were in virtue of teaching and learning, this would result in an infinite regress, in which case there would be no teaching and learning. Indeed there is no doubt that we have things that are asserted immediately and things that are conceptualized immediately (*bi-lā wāsiṭa*) and that these are the first principles of assertion and conception.

In Text 1.7, Avicenna briefly responds to the first formulation of a skeptical objection against the possibility of knowledge (the argument from infinite regress). This response will then be developed in greater detail in *Burhān* II, 1, where demonstrative knowledge replaces teaching and learning involving reason. Teaching and learning involving reason are possible because the regress always comes to a stop at some first principles. And these first principles are immediate conceptions and immediate assertions.

The discussion at the end of *Burhān* I, 6 is naturally connected to the foundationalist arguments against infinite regress and circularity in *Burhān* II, 1. If scientific knowledge is only obtained by demonstration, how are the premises of a demonstration known? If knowledge of the premises can only in turn become

28. Immediate may be taken in a relative sense or in an absolute sense. Certain conceptions and assertions, for instance the principles of geometry, are immediate with respect to the discipline of which they are principles but may still be proved elsewhere (most of them in metaphysics, in Avicenna's view). Other conceptions and assertions, by contrast, are immediate without qualification.

available through demonstration, then there can be no end to the process, unless we are prepared to accept the view that some things are demonstrated circularly. Avicenna rejects both options. His reconstruction of Aristotle's arguments against infinite regress and circularity in *An. Post.* A3 establishes that there must be things that are known without demonstration.

The second problem, with its two distinct objections pointing to infinite regress and circularity, rests on two assumptions:

1. The principles of a demonstration are more evident (*awḍaḥ*) and better known (*aʿraf*) than its conclusion.
2. Everything becomes evident through demonstration.

If, in order for a principle to be known, there must be a demonstration of it based on more evident principles, then either (aa) the additional principles are conceded in virtue of a proof or (ab) they are not. In the former case, the process leads to an infinite regress and consequently, nothing can truly be known. In the latter case, that is to say, when the putative additional principles are conceded without proof, the original principle turns out not to be evident and therefore cannot serve as a principle to establish anything else. If, by contrast, demonstrations have first principles and everything is demonstrated, (ac) some things must be demonstrated circularly. Circularity is considered by Avicenna to have been adequately refuted by Aristotle in *An. Post.* A3 by means of three arguments: (i) the first argument involves a distinction between "being prior to" and "being better known than," both of which are transitive and irreflexive relations (if something is demonstrated through itself, however many intermediate steps are involved, this ultimately implies that something is both prior to and better known than itself); (ii) the second argument suggests that circular demonstration is just a case of *petitio principii* (and there is a distinction between proving that something is because of other things and trivially proving that it is because it is); and (iii) the third argument refers to the treatment of circular proof in *An. Pr.* B5–7 in terms of convertible and coextensive notions, an occurrence that at best is said to be rare in demonstration (thus, certainly not all principles of demonstration would be at risk of having to be clarified circularly; many principles do not generally satisfy the conditions under which circular proof applies, according to *An. Pr.* B5–7).[29]

Avicenna offers a general argument to establish a priori that self-evident knowledge is possible. The argument involves a disjunctive premise (1) and a conditional premise (2). The antecedent (2a) of the conditional premise is the second disjunct (1b) of the disjunctive premise. The consequent of the conditional premise is another disjunctive statement, encapsulating two basic forms of knowledge. Using disjunctive syllogism and *modus ponens* and then again disjunctive

29. On Avicenna's treatment of circular proof and *petitio principii*, see *Qiyās* IX, 12 and IX, 16.

syllogism, the proof establishes by elimination that some principles must be self-evident, that is to say, known in themselves (disjunct (2ba)):

1. (1a) Either nothing is known or (1b) something is known.
2. If (2a) something is known, then (2b) it is either (2ba) known in itself or (2bb) known by demonstration.

Avicenna contends that (1a) is self-refuting and therefore evidently false. For if nothing is known, then at least (1a) itself, namely the statement that nothing is known, is somehow known as an object of assertion. But if (1a) is false, then (1b) is true. Assuming the truth of (2) to be self-evident, if (1b) is true, then by *modus ponens* the consequent (2b) is also true. Whatever is known is then either known in itself (*bi-ḏātihā*) or by demonstration (*bi-burhān*). The rejection of (2bb), namely that what is known is known (only) by demonstration, proceeds by reductio. Every (syllogistic) demonstration involves a chain of middle terms of some length (bounded at the two extremes by the major term and by the minor term, respectively). Suppose we could enumerate the middle terms in question. In any ordered sequence of terms, whether *finite* or *infinite*, one term must follow another. Avicenna maintains that if there were infinite terms between the first and the last, two impossibilities would follow. (i) First, there would be as (infinitely) many terms between the first term and the last term as there are between the first term and a given intermediate term in the sequence, and this would amount to a violation of the principle that the whole is greater than the part. (ii) Second, in an infinite series of elements (that can be enumerated), between each element and its successor there is no middle; therefore some premises must be immediate, and these are principles of demonstration, but the assumption was that all scientific knowledge is by demonstration (which is to say by means of a middle term). Consequently, the principles of demonstration just identified cannot be known, and this is absurd. At least some known truths must therefore be immediate.[30]

30. One may be tempted to detect a similarity in the argument of *Burhān* II, 1 and the proof of the finiteness of chains of essential causes at *Ilāhiyyāt* VIII, 1, pp. 327.5–329.7 (on which see also Aristotle, *Metaph.* α2, 994a11–19), where the case of infinite intermediate causes is also dismissed as unproblematic. This similarity, however, is merely superficial, for in that context, the claim is that it is irrelevant whether the middle terms are one or many and, if many, whether finite or infinite, so long as there is an upper and a lower bound. In this case, the intermediate terms or causes can be collectively treated as a single term, which is at the same time caused by the first term and a cause of the last (being, in this respect, essentially other than both). In *Burhān* II, 1, by contrast, the point is that even in an infinite series of consecutive terms, there is no intermediate term between any given term and its successor (or predecessor), and hence those relations are immediate and can serve as a proxy of immediate principles. The argument, however, seems to depend on the assumption that the series consists of discrete terms (in a denumerably dense set of terms, between any two terms in the series, there always are infinitely many other terms).

The conclusion of the proof (2ba), namely that something is known in itself, is obtained again by disjunctive syllogism. Scientific knowledge by demonstration is not the only kind of scientific knowledge, for some things are known scientifically "in themselves immediately" (*bi-ḏātihī bi-lā wasaṭ*). Such immediate principles are what we encounter "at the end of the process of analysis" (*ʿinda n-niḥāya fī t-taḥlīl*). What is known in itself is "the principle at which the premises of demonstrations come to a stop" (*Burhān* II, 1, p. 118.18–19).

FIRST PRINCIPLES OF CONCEPTION AND ASSERTION

What do the first principles of conception and assertion look like? Avicenna clearly identifies the latter with various kinds of immediate propositions.[31] The former, by contrast, are more difficult to pin down. In spite of his unquestionable commitment to the existence of first principles of conception, Avicenna rarely discusses the subject in detail, though evidence suggests that they should involve hierarchies of concepts of decreasing generality.

A classification of immediate (or quasi-immediate) propositions, and in particular of those that are necessarily accepted (from different sources, either internal or external), is discussed in detail in *Burhān* I, 4. Most prominently, as we shall see in chapter 2, it includes primary propositions (*awwaliyyāt*) or certain propositions (*yaqīniyyāt*), such as the law of the excluded middle or the principle that equals subtracted from equals result in equals, as well as immediate essential, definitional connections. But it also includes immediate propositions whose evident character is based on perception or other external sources. In the case of first principles of conception, Avicenna does not give explicit examples in the *Burhān*, but a series of remarks about what it means to be prior (to us and in nature) and better known (to us and in nature) in *Burhān* I, 11 and several comments from the *Ilāhiyyāt* indirectly confirm that he considers them to be concepts of the highest generality, such as "existent" and "thing." The latter are first in the order of explanation and do not presuppose any other, more fundamental concept in terms of which they could be explained. In particular, at *Ilāhiyyāt* I, 5, p. 29.1–3, Avicenna

31. Avicenna calls "immediate assertions" the fundamental types of premises (demonstrative and non-demonstrative) that we also encounter in *Burhān* I, 4. In the logic of the *Naǧāt* they are discussed right after the distinction between conception and assertion. These assertions are based not on other assertions (from which they would be obtained inferentially) but rather on perception, experience, sequential testimony, external authority, or another source from Avicenna's canonical list. On immediate conceptions, Avicenna notes, at *Burhān* II, 10, p. 186.3–4, that principles may be doubted as a result of the fact that "their parts are not evidently conceptualized" (*lam takun bayyinat taṣawwur al-aǧzāʾ*). The problem is also mentioned at *Ilāhiyyāt* I, 5, p. 30.1–2, which confirms that Avicenna is explicitly committed to the idea that infinite regress (and circularity) ought to be avoided not only in the case of assertions but also in the case of conceptions.

contends that the conceptions of "thing" (*šayʾ*), "existent" (*mawǧūd*), and "necessary" (*wāǧib*) are impressed in the soul first and primarily, without depending on something better known. This is perhaps his most explicit illustration of what he means by first principles of conception.[32]

The analogy between first principles of conception and first principles of assertion is not merely a matter of superficial similarity for Avicenna. Just as some propositions are asserted in themselves, being necessarily presupposed by anything that comes after them, so much so that if they are not understood, nothing that depends on them can be understood, in the same way, some notions are conceptualized in themselves and serve as principles of conceptualization for other notions. In both cases the investigation of such principles can at best proceed by way of being reminded (*tanbīh*) of their meaning (even if in the process of being reminded we have to rely on propositions or notions that are less evident in themselves). And in both cases, the argument that proves the necessity of first, immediate principles of either kind is a reductio. Thus, the dilemma (infinite regress or circularity) that would result from denying the existence of first principles applies not only to nonnoetic (propositional) knowledge but also to noetic (conceptual) knowledge.

PRIOR AND BETTER KNOWN

In *Burhān* I, 11, Avicenna offers an interpretation of the Aristotelian distinctions between (i) what is prior to us and what is prior in nature, and between (ii) what is better known to us and better known in nature:[33]

> Text 1.8: *Burhān* I, 11, p. 106.13–15
> Prior to us are the things we hit upon first. Prior in nature are the things that, when removed, what comes after them is [also] removed, but not the other way around. What is better known to us is also what is prior to us. Better known in nature are the things to which nature tends in being (*taqṣidu aṭ-ṭabīʿa qaṣdahā fī l-wuǧūd*).

In a hierarchy of being that has individuals or the most specific species (that is to say, species like human, horse, or isosceles, which cannot be divided further into

32. Avicenna is committed to the view that "existent" and "thing" are conceptualized as distinct notions, even if they are inseparable from each other. This is because "existent" is synonymous with "established" (*muṯbat*) and "realized" (*muḥaṣṣal*), while "thing" is the counterpart of "proper existence" (*wuǧūd ḫāṣṣ*), which is in turn identified with the notion of quiddity; see *Ilāhiyyāt* I, 5, p. 31.2–9. On primary notions in Avicenna, see Marmura (1984), Aertsen (2006), and Bertolacci (2008).

33. At *Ilāhiyyāt* I, 5, p. 30.3–4, Avicenna contends: "Notions that are common to all realities—existent, thing, one, and the like—are the ones most worthy of being conceptualized in themselves." At *Ilāhiyyāt* I, 5, p. 36.4, "necessary" is said to be worthier of being conceptualized first than "possible" or "impossible" (because it signifies certainty of existence, and existence is better known than nonexistence).

other subspecies) at the lowest extreme and the most general notions (existent, thing, necessary) at the highest extreme (presumably followed, one level down, by the ten categories), the latter are metaphysically prior.[34] Priority in nature applies to the most general notions. These are first conceptions in themselves, even though they are not prior to us (at least in the sense of priority associated with what we seem to experience first through perception). Text 1.8 is consistent with another claim Avicenna makes in *Ilāhiyyāt* I, 5, according to which the most general notions are prior in nature. General notions such as thing and existent are taken to be prior in nature to anything that is more specific (and in this sense they are absolutely prior in nature), but the relation of "being prior to something in nature" applies at all levels of the hierarchy of being. Avicenna goes as far as to give an explicit test for the identification of what is prior in nature: x is prior to y in nature if and only if the removal of x implies the removal of y, which is to say that x is prior in nature to y if and only if x is implied by y. What is further from the domain of perception is prior in nature and posterior to us. For example, animal, body, and substance are prior in nature to human; body and substance are prior in nature to animal; and substance is prior in nature to body. This notion of priority, however, is not incompatible with the view that individuals and the most specific species in Avicenna's metaphysics have another kind of priority in being. Thus, for example, there is no animal that is not a human, or a horse, or else that does not fall under one of the other most specific species of animal. It is this complementary aspect of Avicenna's classification that is captured by the notion of being better known in nature.

At the same time, Avicenna also holds that what is better known to us is the same as what is prior to us. And what is prior to us seems to be what is more specific, that is to say, what we encounter in our experience first. This might appear to be inconsistent with another passage in *Burhān* I, 11 where Avicenna seems to suggest the opposite to be the case with regard to what is better known to us. In that context, what is better known to us is identified with what is more general (along the lines of Aristotle's contention in *Physics* A that the universal, at least in a way, is what we know first). This is because we first seem to have confused knowledge of more general concepts before discovering and adding determinations to them. An extreme case in Avicenna's language is represented by the notion of "thing": everything (a human, a horse, a palm tree, and so on) may

34. The same distinction between types of priority is made at *Samāʿ ṭabīʿī* I, 1, p. 12.9–18, with regard to the notions of simple and composite (see *Burhān* I, 11, pp. 108.7–109.3). At *Ǧadal* V, 2, p. 249.1–6, Avicenna briefly mentions the two distinctions again while commenting on *Top.* Z4, 141a26–b2. In that context, priority in nature is associated with "the items that are taken in the definition" of something and which "must be constituents of the quiddity of what is being defined." On the distinction between better known to us and in itself in Aristotle, see Mansion (1979).

be correctly designated as “a thing,” even if we know nothing about its essence. This, however, does not imply that what we first encounter in our experience are general concepts. We still encounter humans, horses, and palm trees first (in fact we experience individuals falling under such most specific species), as opposed to things, animals, or bodies in general.

But then how can the two claims be reconciled? Avicenna is tacitly relying on a distinction between (i) what is prior to us and better known to us with regard to perception and (ii) what is prior to us and what is better known to us with regard to the intellect. On the first reading, better known to us (that is to say, perceptually) and better known in nature coincide and are identified with what is more specific. But the parallelism between better known to us and prior to us is lost when we move to the distinction between prior in nature (the more general) and better known in nature (the more specific). On the second reading, better known to us (that is to say, intellectually) and prior to us again coincide, but this time the correspondence with the order of nature obtains at the level of priority. The more specific is what we encounter first in our perceptual experience of the world, while the more general is what we encounter first in our intellectual conceptualization of it. It is in this second sense that we can plausibly claim that what is better known to us with respect to the intellect is the more general. The reason is that, from the perspective of the intellect, we do not typically have all the determinations required to define a most specific species at the beginning of our inquiry. Now, the intellect, unlike perception, tracks the order of priority in nature rather than the order of what is better known in nature. Hence the apparent tension between the two claims is reconciled: what is better known in nature is what is prior and better known to us with respect to perception, and what is prior in nature is what is prior and better known to us with respect to the intellect.

The relations of priority and posteriority, and of being better known or less known, either to us or in nature, are summarized in table 2.

TABLE 2 Prior and better known

		Prior	Posterior	Better known	Less known
In Nature		First notions Highest genera	Individuals Lowest species	Lowest species	First notions Highest genera
To us	Perception	Individuals Lowest species	First notions Highest genera	Individuals Lowest species	First notions Highest genera
	Intellect	First notions Highest genera	Lowest species	First notions Highest genera	Lowest species

ACQUISITION OF IMMEDIATE CONCEPTIONS AND ASSERTIONS

Having established that first principles of conception and assertion are necessary conditions of scientific knowledge, the following question arises: If they cannot be acquired through demonstration, then how do we get to know them? This is in essence the problem of *An. Post.* B19, recast in Avicenna's theoretical framework in the form of two distinct questions concerning on the one hand conceptions and on the other assertions.[35] I only briefly sketch his general strategy here. If anything can serve as a methodological justification for this approach, it is a series of contentions that Avicenna himself puts forward on various occasions in his logical works. Perhaps the most remarkable is in *Burhān* III, 5, the first half of which corresponds to *An. Post.* A18, on the relation between perception and scientific knowledge and how the former is a necessary condition for the latter:

> Text 1.9: *Burhān* III, 5, p. 222.12–16 (McGinnis and Reisman 2007, p. 154, transl. modified)
> This is not the place to know how this works, what faculty does it, and what faculty helps the one that does it; rather this is the province of psychology. But what we shall say here is that perception conveys to the soul things that are mixed and unintelligible, and the intellect makes them intelligible. Once the intellect singles them out as intelligible, it can then compose them in different ways (*yurakkibahā anḥā' at-tarkīb*): some according to the composition proper to the expression that makes the notion of something understood (*at-tarkīb al-ḫāṣṣ bi-qawl mufahhim li-maʿnā š-šāy'*), like definition and description; others in the composition of a declarative proposition (*at-tarkīb al-ǧāzim*).[36]

A detailed analysis of the process of concept formation falls outside the scope of Avicenna's theory of science. The problem is not so much the technical character of his discussion of concept formation as its belonging to the province of a discipline other than logic, namely philosophical psychology. That being said, Avicenna offers three concise accounts of concept formation in *Burhān* III, 5, III, 8, and IV, 10, in correspondence with chapters where relevant issues are raised in the *Posterior Analytics*, most notably A18, A31, and B19. And since the outcome of this process, namely our access to a repository of concepts, is crucially presupposed by Avicenna's theory of definition (especially for the problem of its acquisition in *Burhān* IV, 6–7), in

35. The literature on Avicenna's psychology, and more specifically on his theory of intellection, is vast; see Gutas (2012) for a summary. For further general orientation see Gutas (2001, pp. 1–38; 2006, pp. 351–372); Hasse (2013, pp. 39–72); Davidson (1992); and, more recently, Ogden (2020).

36. The notion of *qawl mufahhim* in Text 1.9 is equivalent to that of *qawl mufaṣṣil* in *Burhān* I, 1. What Avicenna means, in this passage, by the "composition" that is proper to this sort of expression is a method for the acquisition of definition that I discuss in chapter 13.

this section I sketch his account of concept formation in the *Burhān*, focusing on III, 5 and IV, 10.[37] Both chapters emphasize the abstractive character of the process (*Burhān* IV, 10 in particular retraces the trajectory of B19, explicitly translating Aristotle's conceptual vocabulary into the language of abstraction).

The process of acquisition (*iktisāb*) of first principles concerns both conceptions (*taṣawwurāt*) and assertions (*taṣdīqāt*). The fact that immediate conceptions and assertions are *in a sense* acquired is compatible with their being immediate. In the case of assertions, their immediacy lies in the fact that they are not mediated or proved, even though they may still be acquired in some other, nondeductive way. In the case of conceptions, there is a distinction between immediate notions that are used as building blocks of definitions (simple terms like "animal," "rational," "tree," "triangle") and the acquired complexes of terms that articulate essences. The latter are acquired in a different sense, that is to say, through a mechanical procedure of composition and division, which (just like deduction) requires investigation in the technical sense introduced earlier: a method of inquiry involving discursive reasoning. Thus, first principles are acquired through abstraction, which leads to a grasp of universal notions, which are then combined in propositions. But they are not acquired through investigation in the same sense as the conclusion of a deduction or the outcome of a process of composition or division resulting in a definition. Rather, they are the starting points of both.

The acquisition of conceptions is more fundamental at least in the sense that it is a necessary condition for the process of combination and separation involved in the generation of propositions. Avicenna rejects innatism and explains both types of acquisition through a complex process involving a cognitive progression from the five external senses, the internal senses (common sense, imagery, memory, estimation, and thought, the latter being a faculty distinct from the intellect that combines and separates concepts), to the intellect in its four stages (potential,

37. A traditional controversy concerns the question of whether Avicenna's account of the acquisition of principles is more Aristotelian, in its dependence on abstraction and experience, or more Neoplatonic, in its dependence on emanation and illumination. Is scientific knowledge a bottom-up process ultimately grounded in experience and abstraction or a top-down process ultimately dependent on the activity of the Active Intellect that grants access to concepts (and their relations) to the human intellect through emanation and illumination? Historically, the debate has focused on those sections of Avicenna's corpus that primarily deal with his theory of the soul, and there has been comparatively little work on what he says in his logical works about these themes. The latter, however, are significant because they contain extensive discussions of conception, assertion, and the distinction between immediate and acquired knowledge. In his logic, Avicenna tends to focus almost exclusively on abstraction. In the *Burhān*, for example, the only reference to illumination, along with divine emanation, is at III, 8, p. 250.4 (*išrāq fayḍ ilāhī*) (on *An. Post.* A31), and the only other reference to divine emanation is at IV, 10, p. 331.21 (*fayḍ ilāhī*). Emanation and illumination involving the Active Intellect come into play to account for the problem of actual thought and intellectual memory, on which see Gutas (2012) and, more recently, Ogden (2020).

dispositional, actual, acquired). Avicenna's account of principle acquisition aims to explain both (i) how principles are acquired and (ii) the transition from ignorance to knowledge, which is an instance of a traditional problem associated with Meno's paradox.

External and Internal Senses

First principles cannot be known by demonstration. The intellect grasps them through a different process that presupposes the cooperation of other faculties of the soul, at the lower steps of the cognitive ladder, namely the external and the internal senses. The external senses provide an array of basic sensory inputs that are made by the common sense into the objects of our perception. The common sense organizes (in the case of human and nonhuman animals alike) all the sensory information received from the external world into a unit that constitutes the (sensible) form of an object (*ṣūrat al-maḥsūs*). Sensible forms are stored by the imagery or form-bearing faculty (*ḫayāl* or *muṣawwira*), while more complex connotative meanings grasped by the estimative faculty are stored by memory or retention (*ḏikr* or *ḥifẓ*). At this stage, the contents of the imagery are particular forms mixed with accidental features (such as shape, size, and location), not abstract universal forms. A fifth internal faculty called cogitation or thought (*fikr* or *mufakkira*), distinct from the intellect, is responsible for another critical function, namely the combination or separation of items grasped and stored by the other four internal senses. This may result in the generation of more complex terms, but more importantly, is presupposed by the process that generates the content of propositions (combination and separation of concepts are what affirmative and negative predications ultimately depend upon).

Intellect

The intellect operates on the outcome of the process previously described and, taking the forms stored and combined in a basic way with one another by the internal senses as its inputs, is primarily responsible for (i) a process of abstraction that leads to concept formation and for (ii) the combination (separation) of properly abstracted concepts into affirmative or negative assertions.[38] Abstraction in *Burhān* IV, 10 (compare with *An. Post.* B19) is characterized in the following terms:

> Text 1.10: *Burhān* IV, 10, p. 331.5–10 (Strobino 2015c, p. 38, transl. modified)
> Next, the faculty that acquires the first objects of scientific knowledge (*al-quwwa al-muqtaniya li-l-ʿulūm al-ūlā*) [(a)] looks closely at (*tuṭāliʿu*) these internal images, [(b)] discriminates (*tumayyizu*) what is similar and what is different, [(c)] strips

38. On the process of abstraction, see also *Naǧāt* I, 148 (v).

(*tanziʿu*) every form of what is accidental to it, and [(d)] abstracts (*tuğarridu*) what is essential. As a result, what comes about first in that [faculty] is the [(e)] conceptualization of simples (*taṣawwur al-basāʾiṭ*); then [(f)] those simples are combined with one another and separated from one another, with the assistance of a faculty called thought (*mufakkira*), in such a way that [(ga)] some combinations (*tarkībāt*) emerge clearly (*lāḥa*) for [the intellect] with regard to those notions. These turn out to be such that [the intellect] knows them without learning and immediately [. . .] like the fact that the whole is greater than the part, [(gb)] while for many of them, [the intellect] acquires the judgment about their composition and division from the senses by way of experience.

The account is parallel to the one in *Burhān* III, 5 (compare with *An. Post.* A18), with an emphasis on the relation with perception:

Text 1.11: *Burhān* III, 5, p. 222.4–10 (McGinnis and Reisman 2007, p. 154, transl. modified)
We say that the conception of intelligible notions is acquired by means of perception just in one way: perception takes the forms of its objects and delivers them to the imagery, and then those forms are subject to the activity of our theoretical intellect. [In the imagery] there are many forms taken from actual humans as perceived by the senses, which the intellect finds to be different in virtue of accidents. For example, it finds Zayd having a distinctive color, complexion, shape of limbs, and the like, and it finds ʿAmr distinctively characterized by other such things. Thus, the intellect turns to these accidents and strips them off, as though it were peeling away those accidents and setting them to one side until it arrives at the notion shared [by all individuals perceived by the senses] without difference. Thus, [the intellect] obtains [a notion] and conceptualizes it. From the first moment that the intellect inspects the mix in the imagery [faculty], it finds accidental and essential [attributes], and, of the accidental, those that are necessarily implied and those that are not. It separates out one by one the many notions mixed together in the imagery [faculty] and keeps them for itself.

The process of abstraction leads first to the acquisition of concepts and then to the acquisition of assertions.[39] A distinctive feature of this process is that the intellect is somehow capable of recognizing immediately which attributes are essential

39. When Avicenna says that the most general notions such as "thing" or "existent" are conceptualized *first*, he means that they are first logically, or in the order of explanation. Their conceptions, however, are distinct in the soul. At *Ilāhiyyāt* I, 5, p. 31.3–4, he observes that "existent, established [as real], and realized are all synonyms of one single notion, and there is no doubt whatsoever that their notions are already present in the soul of whoever is reading this book." Thing (*šayʾ*) and thingness (*šayʾiyya*) are associated with essence and quiddity. The process described previously, by contrast, seems to apply *first* to most specific universals such as "human" or "horse" (that is to say, the most specific universals whose individuals we encounter first in the domain of experience). The distinction between prior to us and prior in nature and between better known to us and better known by nature is discussed extensively in *Burhān* I, 11.

and which ones are accidental. This is a rather optimistic epistemological assumption that nonetheless seems to be fully in line with Avicenna's characterization of essential relations as facts that are ultimately self-warranting and self-evident. The "accidental attributes that are necessarily implied," as we shall see in chapters 7 and 8, are inseparable in imagination and include the critical category of per se 2 attributes, which truly represent the staple of scientific demonstration.

The intellect inspects the objects of the internal senses, that is to say, images that are still associated with particular characteristics. It distinguishes forms that are similar to one another and forms that are different from one another; strips each form of what is accidental to it (for instance, nonessential qualitative and quantitative features, location, and the like) and obtains by abstraction what is essential to each. It is at this point in the process that conceptualization and assertion come into play. Through this bottom-up process of abstraction, the intellect reaches a stage where it conceptualizes simple notions (*taṣawwur al-basāʾiṭ*).[40] Universality as a property of essences insofar as they exist in the mind applies, according to Avicenna, to the content of the operation of the intellect only after this last step. Such simple abstracted notions are then combined or separated by the intellect with the aid of thought. The contribution of thought, in the technical sense of ratiocination, may consist as noted earlier either in the formation of complex concepts (terms) or in the combination of concepts with one another (or the separation of concepts from one another) in propositions. This process is presupposed by the composition of affirmative and negative predications that are amenable to truth and falsehood (*ṣidq* and *kiḏb*) and by the assertions by means of which truth and falsehood are ascribed (*taṣdīq* and *takḏīb*). In the case of first principles, an assertion may come about in two ways, once the intellect combines or separates simple notions with the aid of thought. Either the combination (separation) of two concepts is self-evident or it is not:

(i) If it is self-evident, then the corresponding assertion is either an axiom or an immediate essential predication.
(ii) If it is not self-evident, then the epistemic strength of the corresponding assertion is based on experience, broadly construed to include experience of what is external to the mind as well as internal experience.

Propositional principles of type (i) may be either common to all sciences or proper to one or more sciences. Some of them encapsulate essential predications grounded in definitions. The necessity of the corresponding assertions ultimately

40. Simples are concepts such as "human," "horse," "animal," and "body," but presumably also, at a higher level of generality, the categories or trans-categorial concepts such as "thing," "existent," "necessary," and the like. A noteworthy list is presented in chapter 2, Text 2.3, in connection with the systematic errors of estimation identified in *Burhān* I, 4.

depends on the internal structure of concepts and the capacity of the intellect to recognize immediate, self-evident essential relations.[41] According to Avicenna, if certain notions are properly conceptualized, the assertions by means of which they or their constitutive parts are combined (in the case of relations of conceptual inseparability) or separated (in the case of relations of conceptual incompatibility) must follow at once and do not require further justification to be evident. Propositional principles of type (ii) are proper to specific domains of scientific knowledge, and their assertion ultimately depends on experience in the general sense introduced earlier.[42]

In the next chapter I explore Avicenna's classification of different types of assertions.

41. Avicenna usually gives the example of a common axiom in this context, but at *Burhān* I, 6, p. 72.16–17 he writes: "A judgment on a universal is either self-evident as 'Every human is an animal' or 'The whole is greater than the part' or evident through induction or experience, in the case of things that are asserted without the aid of deduction." The first example of a self-evident principle is a definitional predication involving a term ("human") and a constituent or per se 1 attribute ("animal"), which provides incontestable evidence that self-evident principles include not only axioms but also assertions based on definitions. Avicenna is also committed to the view that some non-definitional assertions whose predicate is a per se 2 attribute of the subject are among first principles (see, for example, *Burhān* IV, 3, p. 287.7–9). Self-evident principles of assertion are discussed in chapter 4.

42. Such are the assertions based on perception (*maḥsūsāt*) and experience (*muğarrabāt*) discussed in chapter 2.

2

Scientific and Nonscientific Assertions

ASSERTIONS, BELIEFS, DEDUCTIONS

The relation between assertion, belief, and deduction is a pillar of Avicenna's account of scientific reasoning and also serves the negative purpose of determining the perimeter of application of scientific discourse in contrast with other, nonscientific levels of discourse. In *Burhān* I, 1, Avicenna gives a schematic outline of the relations holding between three fundamental kinds of assertion, qualified in terms of decreasing epistemic strength, the beliefs associated with each, and the corresponding types of deductive arguments:

> Text 2.1: *Burhān* I, 1, pp. 51.1–52.2
> Scientific knowledge acquired through thought and the one that is available without an acquisition that involves thought make two divisions, one of which is [(a)] assertion and the other [(b)] conception. [Scientific knowledge] acquired through thought on the basis of assertion becomes available to us by means of deduction, while the one that is acquired on the basis of conception becomes available to us by means of definition. Assertion is arranged, as it were, in ranks. One kind of assertion is certain (*yaqīn*) and along with it a second belief is held (either in act or in a potency which is close to act) that [what is asserted] cannot be otherwise, when it is not possible for this belief in [what is asserted] to cease. Another kind of assertion [(ab)] resembles the one that is certain (*šabīh bi-l-yaqīn*). This is an assertion relative to which only a single belief is held, and either [(aba)] the second belief mentioned above is not held along with the first (neither in act nor in a potency which is close to act)—in fact, if one were to be reminded of it, the soundness of the first assertion would be voided; or [(abb)] if the second belief is held, [the first belief] can nevertheless cease (unless it is stipulated (*mutaqarrir*) and the belief in

> the possibility of the contradictory [of the first belief] is not held in act along with the first belief). Yet another kind of assertion, apart from those, is [(ac)] persuasive or suppositional (*iqnāʿī ẓannī*). This kind is such that the first belief is held and along with it there is a second belief (either in act or in a potency which is close to act) that the contradictory [of the first belief] is possible. If this second belief is not held, it is because it is not present to the mind, in which case it is really only a supposition. For all these reasons, deductions too are arranged in ranks. Thus, one kind of deduction, namely demonstrative deduction, brings about certainty. Another kind, namely dialectical deduction or the kind of sophistical deduction employed in fallacious reasoning, brings about what resembles certainty. And yet another kind, namely rhetorical deduction, brings about persuasion or predominant supposition. Poetic [deduction] does not bring about assertion but rather imagination (*taḫayyul*) that moves the soul to sadness and joy through the imitation of beautiful and repugnant matters.

The analysis of the kinds of assertions that may occur as premises and conclusions of different types of deduction focuses, in *Burhān* I, 1, on whether belief (i) is part of their characterization, as in the case of demonstrative, dialectical, sophistical, or rhetorical assertions and arguments, or (ii) not, as in the case of poetic discourse that involves imagination rather than assertion. The four types of assertions are arranged in a hierarchy of decreasing epistemic strength, ranging from certainty to mere supposition. The arrangement ultimately depends on whether or not the contradictory of a given assertion is believed to be possible and, if so, on how this possibility is formulated. In the case of assertions of certainty, two conditions are given: the belief that something is the case and the belief that it cannot be otherwise. The next category includes propositions that are asserted without being accompanied by a firm belief that their contradictory could not be the case, to the extent that in fact on closer inspection assent to these propositions can always be withdrawn. The third class is defined in terms of a positive belief that something is the case, accompanied by the explicit belief in the possibility of it not being the case (which can be entertained, as it were, with equal probability). The fourth class is introduced to account for the case of assertions based on false beliefs due to various kinds of systematic error.

CERTAINTY AND OTHER EPISTEMIC STATES

Certainty is a central notion in Avicenna's theory of science, and a word is in order to clarify a difficulty raised by the characterization introduced in Text 2.1. Certainty is the combined product of two distinct beliefs. One is *certain* that p if and only if one correctly (a) believes that p and (b) believes that (ba) it is impossible not to believe that p (or at least that the belief in p cannot cease), or that (bb) it is impossible for p not to be the case. The object of the first belief is a fact, captured for instance by the relation between a subject and an attribute in a categorical proposition such as "Every triangle is a plane figure bounded by three sides" or "Every human is capable of laughter." The second belief introduces a modal characterization of the first belief or, more likely, of its content. I give two alternatives for the second clause because there seems to be an ambiguity in Avicenna's characterization. Is the second belief a belief about the fact that the first belief cannot cease (because the first belief is self-evident or established in a demonstrative way that satisfies a required set of explanatory conditions) or a belief about the fact that what the first belief is about—its content—is necessary? In other words, does the modal claim involved in the second belief apply to the first belief and to the way in which it holds or rather to the content of the first belief and to the way things are? In the first case, the modal clause would seem to imply the weaker claim that the certainty of demonstrative discourse concerns the *epistemic* necessity of certain sets of beliefs, while in the second case, it would seem to imply the stronger claim that demonstrative discourse concerns the *ontological* necessity of a privileged class of facts, that is to say, unchanging, eternal, necessary connections holding between subjects and attributes (or more complex facts, of which the latter are the ultimate components).[1]

Avicenna seems occasionally to oscillate between these two positions, but there are also two further interesting dimensions to the distinction. First, his general view seems to be that in demonstration the two aspects, epistemic and ontological, are usually complementary. This is because, as a matter of fact, most sciences (or at least the most exact among them, including the mathematical sciences and presumably metaphysics) do aim at conclusions that turn out to be both epistemically and ontologically necessary. For instance, if one has a demonstration of the fact that triangles have the sum of the internal angles equal to two right angles, the two conditions (epistemic and ontological) are both satisfied: one necessarily believes that the fact holds *and* the fact itself is necessary (for all truths of geometry are necessary). Second, another difficulty with these two extreme alternatives—that certainty should involve either purely epistemic necessity or purely ontological necessity—is that the former requirement seems too weak and the latter too strong. Avicenna is unequivocally committed to the view that there is no space for contingent truths in scientific theories, but at the same time the kind of necessary truths he is willing to admit do not exclusively encompass *eternal* facts, but also facts that hold necessarily but for a limited amount of time, for example only as long as a given object exists or as long as a given characterization is true of it. For instance, material objects whose shape is triangular (bronze or wooden triangles)

1. The ontological view reflects the way in which most of Aristotle's own claims about necessity in the *Posterior Analytics* have traditionally been read, perhaps with the single exception of *An. Post.* A30, which is concerned with scientific claims that hold for the most part.

necessarily have all the properties of triangles as long as they are shaped as triangles, but they may lose some or all of those properties if their shape changes. I return to this problem in chapter 7, but it is important to keep in mind that Avicenna's fine-grained account of necessity is a critical element in this picture, and this may be a helpful consideration in an attempt to bridge the apparent gap between epistemic and ontological necessity.[2]

The relation between certainty, belief, and necessity is analyzed again in *Burhān* III, 9, where Avicenna deals with the distinction between scientific knowledge and opinion discussed in *An. Post.* A33.[3] In this context, Avicenna defines scientific knowledge based on assertion (*al-ʿilm at-taṣdīqī*) and specifies again the modal criterion required for the certainty of scientific assertions. The latter involves "believing that something is such-and-such and believing that it is not possible for it not to be such-and-such, by a belief that results from the impossibility of it ceasing" (*wa-l-yaqīn minhu huwa an yuʿtaqada fī š-šayʾ annahū kaḏā wa-yuʿtaqada annahū lā yumkinu allā yakūna kaḏā iʿtiqādan wuqūʿuhū min ḥayṯu lā yumkinu zawāluhū*) (*Burhān* III, 9, p. 256.5–6). In this passage, the modality associated with the second belief seems to apply unambiguously to the content of the first belief (indicating that whatever the first belief is about cannot be otherwise), but there is still a question concerning the justification of the second belief: Is it supported by the unshakable character of the first belief, or rather by the factual necessity of the content of the first belief? What does "it" refer to in the clause "by a belief that results from the impossibility of *it* ceasing" (*zawāluhū*)? Avicenna sheds some light on the issue by drawing a further distinction. If a belief

2. A further relevant distinction is the one between perpetual certainty and certainty at a time. According to Avicenna, if a simple belief that something is the case is not accompanied by an additional belief encapsulating the required modal condition, then an assertion is not perpetually certain (*yaqīn dāʾim*) but only certain at a given time (*yaqīn waqtan mā*). There seems to be a significant parallel with the terminology in Alfarabi, who distinguishes between necessary certainty (*yaqīn ḍarūrī*) and non-necessary certainty (*yaqīn ġayr ḍarūrī*) in his *Burhān* I, 3 (*On certainty and its divisions*) in analogous temporal terms; cf. also Alfarabi's *Burhān* I, 2 (*On complete assertion*) for the equivalence between complete assertion (*taṣdīq tāmm*) and the notion of certainty (*yaqīn*). The qualification of certainty in temporal terms is not infrequent in Avicenna; see for instance *Naǧāt* I, 134, p. 143.9–10 and *Burhān* I, 8, p. 88.4 on perpetual universal certainty (*yaqīn kullī dāʾim*); other variants are complete perpetual certainty (*yaqīn tāmm dāʾim*, again at *Burhān* I, 8, p. 90.18) and true (or real) certainty (*yaqīn ḥaqīqī*, at *Burhān* I, 8, p. 88.18). Certainty is also used to qualify demonstration (*burhān yaqīnī*) at *Burhān* I, 8, p. 86.19. The qualification may seem redundant given that demonstration is defined as a deduction composed of premises that are certain and accordingly discussed at *Burhān* I, 7, pp. 78.12–79.4. But it can perhaps be charitably understood in light of the fact that even demonstration admits of several ranks, as we shall see in chapter 9. On certainty in Avicenna's epistemology, see Black (2013).

3. The Arabic translation of *An. Post.* A33 is interesting, among other things, because it attests to the use of two key technical terms: *iʿtiqād* for the Greek *hupolēpsis* and *taṣawwur* for the Greek *noēma*.

is self-evident, then it is not possible for it to cease, while if it is not self-evident, then either the belief is accompanied by a sound proof, in which case it is correct to say that it is impossible for it to cease, or it is not accompanied by a sound proof, in which case it is possible for it to cease (for it is neither self-evident nor proved). This seems to suggest that in the analysis of certainty, the second belief (that things cannot be otherwise than they are believed to be) is a result of the unshakable character of the first belief, either because that first belief is self-evident or because it has been demonstrated.

The intermediate degree, quasi-certainty, involves only a single explicit belief about the fact or relation expressed in the proposition, without an additional (explicit) belief involving modality. But at the same time Avicenna contends that the first belief is incompatible with the simultaneous belief in the possibility of its denial (in other words, quasi-certainty about something is incompatible with concurrent doubt).[4] Thus, one is *quasi-certain* of *p* if and only if one (i) believes that *p* and perhaps only implicitly entertains the belief that *p* is necessary, provided that the second implicit belief can be suspended in special circumstances, for example in a thought experiment.

The lowest degree, which corresponds to the level of mere persuasion and supposition (*iqnāʿī ẓannī*), involves a belief about the fact or relation expressed in the proposition but is compatible with a simultaneous belief in the possibility of its denial. Thus, one is merely *persuaded* of *p* if and only if one (i) believes that *p* and (ii) at the same time believes that *not-p* is possible. The resulting taxonomy is summarized in table 3 (where B stands for belief, N for necessity, and M for possibility).

Text 2.1 contains all the main ingredients for the more detailed discussion we encounter in *Burhān* I, 4: a distinction between five types of "attitudes" toward a proposition, four of which involve an assertion, which is then analyzed in terms of belief, and one that does not involve an assertion but rather appeals to imagination.[5]

4. Avicenna's wording seems to suggest that in this case perhaps one may hold a belief and suspend it to raise a doubt, whereas this appears not to be an option in the case of certainty, given that the content-related belief is always accompanied by a belief that it cannot be otherwise and is therefore unshakable. Examples discussed in *Burhān* I, 4 to illustrate and distinguish the case of primary propositions from that of absolute endoxic propositions seem to indicate that while it is *never* possible to raise a legitimate doubt about the former (for Avicenna it is simply inconceivable to doubt that the whole is greater than the part, if the meanings of whole and part are properly understood), it is sometimes possible, under suitable assumptions, to raise a legitimate doubt about the latter (for example, the assertion that justice is beautiful).

5. There is a parallel in Alfarabi, *Burhān* I, 2 (*On the division of complete assertion*); on the *yaqīniyyāt* and the *muqaddamāt al-wāǧib qubūluhā* (followed by a brief account of induction and definition), see Alfarabi, *Burhān* V, 3.

TABLE 3 Assertion, belief, and deduction

Ranks of assertion	Beliefs		Ranks of deduction
	Content	Modality	
Certain	B*p*	B(N*p*) B(N(B*p*))	Demonstrative
Resembling certain	B*p*	None (or defeasible)	Dialectical or sophistical
Persuasive, suppositional	B*p*	B(M(*not-p*))	Rhetorical

TYPES OF ASSERTIONS

In all of his main logical works, Avicenna lists various types of immediate (or quasi-immediate) assertions that can serve as deductive premises. These types are systematically associated with a canonical taxonomy of the different branches of material logic broadly construed (demonstration, dialectic, sophistic, rhetoric, poetics) and of the kinds of argument characteristic of each.[6] The basic divisions are roughly the same, even though terminology, criteria of division, and examples may vary more or less significantly from work to work and from period to period (for instance, the classification developed in the *Nağāt* is somewhat simpler than that of the *Išārāt*). An excellent source of information is the extensive discussion of *Burhān* I, 4. A synopsis of its criteria and internal division is presented in table 4 at the end of the chapter.

At the root of the division is a distinction between deductive principles that involve an assertion (and are therefore amenable to truth and falsehood) and principles that do not involve an assertion. The latter are associated with the eliciting of images not for the purpose of establishing truths (whether in the context of scientific discourse, dialectic, or rhetoric) but rather, one could say, to trigger emotions and feelings and potentially lead to actions motivated by emotions and feelings that prompt imitation.[7] Assertions are classified according to a number of different criteria. In *Burhān* I, 4, Avicenna identifies three main classes: (aa) propositions that have or claim to have some kind of necessity or compelling epistemic force, (ab) propositions that are conceded (where concession means granting a proposition to oneself or to someone else based on a source of conviction that does not have any kind of internal or external necessity or compelling epistemic

6. See Gutas (2012), Hasnawi (2013), and Black (1990). Another brief mention of the five types of deductions is at *Burhān* III, 8, p. 248.11–18, in the context of an argument concerning the question of whether there can only be demonstration of necessary facts or also of for-the-most part facts and, more generally, of anything that involves a modality that may appear to be weaker than necessity.

7. Black (1990).

force), and (ac) propositions that are merely persuasive or object of supposition (the Arabic term *ẓann*, which I translate here as "supposition," is the standard translation of the Greek *doxa*).

Assertions by Way of Necessity

The class of greatest interest from the standpoint of Avicenna's theory of science is the one that includes assertions that have a necessary character. Propositions that are asserted by way of necessity (*ḍarūra*) derive their character from a source that may be either (aaa) external or (aab) internal.[8]

ASSERTIONS BY WAY OF EXTERNAL NECESSITY

Assertions the source of whose necessity is external are based (aaaa) on perception (*ḥiss*); (aaab) on a more elaborate structure involving both empirical observation and deduction, which Avicenna calls experience (*taǧriba*); or (aaac) on sequential testimony (*tawātur*). Examples of assertions whose necessity is based on perception (*maḥsūsāt*) are "The sun is luminous," "Snow is white," as well as more general observations that are qualified as self-evident, such as the existence of sensible qualities.[9] Examples of propositions whose necessity is based on experience (*muǧarrabāt*) are "Scammony purges bile," "Being struck by wood is painful," "Lodestones attract iron," and "Emeralds blind vipers."[10] Examples of propositions whose necessity is based on sequential testimony (*mutawātirāt*) are claims

8. At *Burhān* IV, 10, p. 332.1–4, when an immediate principle is not (internally) self-evident, its source is said to be perception or experience. At *Samāʿ ṭabīʿī* III, 4, p. 188.6, in the context of Avicenna's refutation of atomism, we find a rare example of a claim that is qualified as evidently *false* in itself (*bayyin al-buṭlān fī nafsihī*), namely affirming the existence of the leap (*iṯbāt aṭ-ṭafra*). We will see in chapter 4 that Avicenna typically characterizes the most fundamental scientific principles as self-evident (*bayyina bi-anfusihā*). Another passage, at *Samāʿ ṭabīʿī* III, 2, p. 179.16, qualifies the premises of an argument as "self-evident," adding that "whatever is alleged to refute them refutes premises more general than they are."

9. Avicenna is not at all concerned with (or perhaps is unimpressed by) skeptical objections on the fallibility of perceptual experience. In some cases, the attribute of a physical entity *cannot* be established through observation, for example that bodies are, by nature, divisible (*Ilāhiyyāt* II, 2, p. 65.4–7). In other cases, by contrast, the fact that an attribute belongs to a physical entity is perceptible (*maḥsūs*), for example, the existence of sensible qualities or the fact that corporeal matter can decrease or increase in size due to condensation and rarefaction (*Ilāhiyyāt* II, 3, p. 77.14–15). At *Ilāhiyyāt* III, 4, p. 116.1–3, Avicenna states that the existence of corporeal extended magnitudes (solids) is manifest (*ẓāhir*), that the existence of surfaces and lines depends on the fact that solids are necessarily finite, and that it is possible to cut surfaces in sections and suppose them to have limits. Similarly, at *Ilāhiyyāt* III, 5, p. 119.7–8, Avicenna contends that the existence of number in concrete things is indubitable (*amr lā šakka fīhī*), as a result of the fact that there are multiple things and hence multiple units (that is to say, things that are, individually, one).

10. At *Naǧāt* I, 104, p. 114.1, Avicenna adds claims about the observed movements of the heavens to the class of propositions based on experience.

about the "existence of cities and countries (even though we have not observed them directly)."[11]

ASSERTIONS BY WAY OF INTERNAL NECESSITY

Assertions the source of whose necessity is internal are based (aaba) on the intellect (*ʿaql*) or (aabb) on the faculty of estimation (*wahm*). In the former case, the necessity of an assertion may be based either (aabaa) on the intellect alone or (aabab) on the intellect with the assistance of some additional factor. The additional factor may be either an explicit deduction or an implicit deduction supplied at once by the natural operation of the intellect (*fiṭra*). What is characteristic of this class of assertions is that their necessity is not only (or at least not predominantly) based on an external input but essentially involves the intervention of internal cognitive faculties. Depending on which faculties are involved, and to what extent they are involved, again a number of alternative cases are possible.[12]

Primary and Necessarily Accepted Assertions. Assertions falling in the first category, namely those whose necessity proceeds from the intellect alone, are characterized as primary and necessarily accepted (*al-awwalī al-wāǧib qubūluhū*).[13]

11. Examples are given at *Išārāt* VI, p. 57.16–17 (the existence of Mecca, Galen, and Euclid) and *Naǧāt* I, 105, p. 115.3–4 (the existence of cities and countries). The idea seems to be that it would be impossible for everyone just to agree on the truth of such propositions, if there were no objective basis for them. Avicenna does not seem prepared to entertain the idea that unanimous agreement about factual matters such as the existence of certain particular things may be compatible with falsehood. Rather, he seems to think that propositions based on sequential testimony *cannot* be false because this would require some sort of unlikely secret agreement to lie and perpetuate the lie on the part of all those who are or have previously been involved in the transmission of the relevant information.

12. In *Burhān* III, 5, Avicenna examines four kinds of procedures that have some relevance to the process of acquisition of immediate propositions from sense perception. In the first of them (*Burhān* III, 5, p. 223.8–10), which corresponds to the case of primary premises (*awwaliyyāt*), the role of sense perception is merely accidental. Even if there is an empirical basis to our process of concept formation (we must after all acquire notions such as "whole" and "part" before we can judge that the whole is always greater than the part), the truth, necessity, and self-evident character of primary premises rest on their conceptual content. Knowledge of principles of this sort may be *prompted* by perceptual experience (insofar as the latter is a necessary step for the acquisition of concepts) but is *not grounded* in it.

13. The locution "necessarily accepted" (*al-wāǧib qubūluhā* with a determinate plural noun) is used seven times in Avicenna's *Burhān* to qualify premises (*muqaddamāt*) (once in *Burhān* I, 4, twice in *Burhān* I, 12, once in *Burhān* IV, 1), principles (*mabādiʾ*) (twice in *Burhān* III, 1), objects of knowledge (*ʿulūm*) (once in *Burhān* III, 3). It is also used in the same sense in Alfarabi, *Burhān* V, 3. On Avicenna's classification of principles and the conceptual vocabulary of primary assertions, including the relation to the *yaqīniyyāt* and to what is characterized as *bayyin bi-nafsihī*, see chapter 4.

An account of the *awwaliyyāt* is at *Naǧāt* I, 111, pp. 121.10–122.2 (Ahmed 2011, p. 95 transl. modified): "Primary [assertions] are propositions or premises that come about in someone in virtue of his

Examples of primary assertions (*awwaliyyāt*) are "The whole is greater than the part" and "Magnitudes that are equal to a single magnitude are equal." Avicenna explicitly calls some primary assertions "certain" (*yaqīniyyāt*), most notably his formulation of the law of the excluded middle and its particular applications "Either affirmation or negation truly holds of everything," "Everything is either equal or unequal to another," and "Every magnitude is either rational or irrational" (*Naǧāt* I, 129).[14] But in fact all primary assertions are certain, even when the qualification is omitted.

Quasi-Deductive and Deductive Assertions. Assertions falling in the second category, namely those whose necessity is based on the intellect with the aid of something else, are of two kinds, depending on whether what aids (*al-muʿīn*) the intellect is (aababa) intrinsic to the intellect (*ġarīzī*) or (aababb) not intrinsic to the intellect (*ġayr ġarīzī fī l-ʿaql*). In the former case, the resulting necessity is still regarded by Avicenna as the necessity of a *principle*. In the latter case, by contrast, the resulting necessity simply becomes the necessity of an inferred *conclusion*. This is because, when an assertion is established by means of a proof, the intellect does not merely rely on the internal content of that assertion or on its implicitly deductive character, but explicitly needs to identify further assertions that serve as its premises. And something that is necessarily asserted in this way is an *acquired* conclusion, not a principle.[15]

The intermediate case between purely immediate assertions ("The whole is greater than the part") and purely acquired assertions ("Every triangle has the sum of its internal angles equal to two right angles") is one in which the intellect relies on something intrinsic to it, namely its natural operation (*fiṭra*), to deliver a snap verdict on something without articulating an explicit deduction (or even having to seek and explicitly entertain one or more premises). Quasi-deductive assertions of this kind are called by Avicenna in two ways: "based on a deduction of the natural operation of the intellect" (*fiṭriyyat al-qiyās*) or, alternatively, assertions that are "simultaneous with their own deductions" (*qiyāsātuhā maʿahā*), that is to say, assertions that are the result of an implicit deduction:

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intellectual faculty, with no causes that necessitate their assertion other than themselves and what makes them propositions, namely the faculty of thought, which joins simples by way of affirmation or negation."

14. Avicenna takes these assertions to be instances of the law of the excluded middle, in the same way that the principle that equals subtracted from equals yield equals holds for numbers as well as for extended magnitudes, for instance at *Burhān* II, 6, p. 155.10–14. In addition to axioms, predicative assertions that express essential relations (involving at least certain immediate per se 1 and per se 2 attributes, as explained in chapter 7) also belong to the category of propositions that are certain.

15. The language of acquisition returns in the same sense in the classification of *Išārāt* VI.

Text 2.2: *Burhān* I, 4, p. 64.6–12
What assists the intellect may be part of the natural disposition of the intellect (*ġarīzī fī l-ʿaql*), being present (*ḥāḍir*) in it.[16] This kind of principle is known through a deduction whose middle term is found by the natural operation of the intellect (*mawǧūd bi-l-fiṭra*) and is present to the mind. Thus, whenever what is sought is present as something composed by two terms—a major term and a minor term—such a middle term between them presents itself (*tamaṯṯala*) to the intellect with no need of being acquired, as when we say "Every four is even." For [the notion] that four is even presents itself to anyone who understands "four" and "even," because [the notion] that [four] is divisible into two equals immediately presents itself. Likewise, whenever four presents itself to the mind, and two presents itself to the mind, [the notion] that the former is the double of the latter immediately presents itself [too], because the middle term has presented itself. If one replaces that [example] with thirty-six or another number, the mind needs to seek the middle. Thus, it is best to call this division of deductive premise "based on the natural operation of the intellect."[17]

A variant of the example discussed in Text 2.2 is the assertion "Two is half four." The implicit deductive structure that makes this assertion necessary may be reconstructed, in line with Ṭūsī's analysis of the corresponding passage in *Išārāt* VI, in the following terms:[18]

16. See also Alfarabi, *Burhān* II, 1, p. 23.6 for the use of *ġarīzī* in a similar context.

17. Avicenna develops this division in *Išārāt* VI. What is acquired is acquired either through effort or without effort. What is acquired through effort is not a principle but a theorem (the "effort" presumably being a deductive chain that leads to it as a conclusion). What is acquired without effort are intuited propositions (*ḥadsiyyāt*), a class with no counterpart in *Burhān* I, 4. What is *not* acquired, by contrast, are implicitly syllogistic propositions, which correspond to the quasi-deductive propositions of Text 2.2. Thus, a comparison of the two accounts reveals that (i) in *Išārāt* VI Avicenna uses three categories as opposed to the two he adopts in *Burhān* I, 4; (ii) the category of what is acquired in the *Išārāt* is divided into two sub-cases, one of which accommodates theorems and the other a new type of assertion, not discussed in the *Burhān*, namely intuited propositions; and (iii) intuited propositions are epistemically halfway between theorems and implicitly syllogistic propositions. None of the three types is an absolutely immediate proposition (only primary propositions are). But (standard) theorems are non-immediate propositions that require effort and investigation in the process of discovery of the required middle term(s); intuited propositions are (nonstandard) theorems that can be regarded as principles because their middle term is quickly accessible to the intellect (presumably, at least, of the most excellent people); and implicitly deductive propositions are (nonstandard) theorems that can be regarded as principles because their middle term cannot escape the mind (and it is not acquired because it comes at once when the terms are given).

18. Ṭūsī, *Ḥall muškilāt*, VI, p. 399.2–3: "For two is a number such that four is divided into it and what is equal to it; and everything such that a number is divided into it and what is equal to it is half that number" (in other words, for all x, y, if $y = x + x$, then $x = y/2$). I wish to thank one of the anonymous readers of the manuscript for pointing out that my tentative reconstruction, which supplies a middle term not explicitly given by Avicenna, seems to be at odds with his usual rejection of oblique inferences. Ṭūsī's passage seems to offer some corroborating evidence for the reading proposed here.

1. Every number that a second number divides into along with what is equal to the first number is half the second number (that is, every number *x* such that a second number *y* divides into *x* and *z,* where $z = x$, is half *y*).
2. Two is a number that four divides into along with what is equal to two.
3. Therefore, two is half four.

When the necessity proceeds from the intellect with the aid of its natural disposition, the proposition has a quasi-immediate predisposition to be asserted, which rests on the fact that we grasp at once a term that connects subject and predicate (these are strictly speaking non-immediate assertions, but we never have to *seek* a middle term, for it is evident at once as soon as the terms of the conclusion are understood).

Estimative Assertions. Propositions that are asserted *by way of necessity* include, alongside genuine necessities, also propositions that have a compelling character but which, depending on the domain of application, may turn out to be true or false (and even seriously misleading). Such are the propositions that are based (aabb) on estimation (*wahmiyyāt*). The case of assertions based on estimation occupies a larger portion of Avicenna's discussion, relative to the rest. Such assertions have a limited scope of positive application in scientific reasoning. But their discussion is critical because Avicenna sees in them a source of systematic error when assertions that legitimately apply to the domain of perception are generalized and extended to what can only be grasped by the intellect.

The source of such propositions is one of the most sophisticated internal senses, which may produce correct as well as incorrect assertions but with a common characteristic, namely a compelling epistemic force, which makes them look as necessary as the judgments of the intellect. Estimation, however, is susceptible to certain systematic errors when it reaches beyond its natural scope of application, which is the domain of sense perception. The resulting propositions, for instance, make unwarranted generalizations or dichotomous claims, extrapolating on what empirical observation would seem to suggest in the case of sensible entities. Avicenna writes:

Text 2.3: *Burhān* I, 4, pp. 64.13–65.11
The one that is external to the intellect [consists of] the judgments of the estimative faculty, which judges by means of them decisively and by way of estimative necessity when those judgments are about things on which the intellect does not pass a primary judgment.[19] Besides, those things are outside the realm of perception, and so

19. An example is at *Samāʿ ṭabīʿī* III, 7, p. 211.8–10 (McGinnis 2009, p. 324, transl. modified): "One of the ways in which the estimative faculty has led people to affirm that something is infinite is imagining that everything finite necessarily terminates at something that is rather like an observable thing.

the estimation forces the soul to pass a false judgment of necessity about them, since it takes them [to fall] within [the purview of] judgments concerned with what is perceived. [This is] like the soul judging, when it first becomes capable of discernment (*fī awwali mā yūǧadu mumayyiza*) and before being instructed (*taṯaqqafa*) through views and investigation, that every existent is in a place (*fī makān*) or in an ostensible space (*ḥayyiz mušār ilayhi*); and that something that is neither inside the universe (*fī dāḫil al-ʿālam*) nor outside it does not exist.[20] For the soul passes a judgment of necessity on this, without the intellect being the necessitating [factor] of this (rather, [the intellect] passes over it in silence). Then, when the intellect investigates [the matter] in the way that is proper to it, and composes deductions from premises—shared and accepted by the intellect and other faculties capable of a judgment involving acceptance and concession—it produces [the conclusion] that things in the realm of perception have principles that are different from the things in the realm of perception.[21] Thus, when the investigation comes finally to a conclusion, the faculty that passes the aforementioned judgment rules it out, and so it is known that the conclusion must be false, and that the natural operation and the necessity [of estimation] are other than the necessity of the intellect, even if at the outset it [appeared] to be a strong necessity (*ḍarūratan qawiyyatan*).[22] The first thing that makes one deem [the conclusion] false is that [the conclusion] itself does not fall within [the domain of] estimation. Nevertheless, it is sometimes difficult for us to discern between the two necessities, unless we look at the subject of the conclusion (*maṭlūb*) and its predicate, so that if it is something more general than what is in the realm of perception or falls beyond it,

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From that it necessarily follows that every body terminates at a body and that the accumulation and piling up of bodies goes on infinitely."

20. Reading *mumayyiza* for *mumayyizuhū* (as suggested by ʿAfīfī in the Cairo edition) in the expression "*fī awwali mā yūǧadu mumayyiza.*" The putative nonexistence of what is neither inside nor outside the universe is one of Avicenna's favorite examples of the potentially misleading character of estimative judgments. At *Samāʿ ṭabīʿī* II, 9, p. 144.13–15 (McGinnis 2009, p. 213, transl. modified), Avicenna writes: "Both of these beliefs [that whatever exists is in a place and can be pointed to] are alike in that the common man would give them up once [he sets] aside instinct and the imaginations of the estimative faculty, and consideration and thought prevail upon him. We have already explained the states of these premises *in our discussion on logic* and made clear that they are products of the estimative faculty that fall short of those produced by the intellect, and it is not necessary to consider them." McGinnis (2009, p. 213n26) correctly notes that the reference to the logic is *Burhān* I, 4.

21. The other faculties in question are estimation (*wahm*), imagination (*taḫayyul*), and thought (*fikr*), on which see Black (1993, 2014). Regarding the contention that the principles of things in the realm of perception are not themselves things in the realm of perception and that estimation goes wrong about them, Avicenna seems to be making a similar point at *Naǧāt* I, 107 (iv), p. 117.9–11 (Ahmed 2011, p. 91, transl. modified): "The falsity occurs only with reference to those things that are not objects of perception in themselves but either are the *principles* of the objects of perception, such as prime matter and form, intellect, the creator, or are more general than the objects of perception, such as oneness, multiplicity, finitude, cause, effect, and so forth."

22. This is in spite of the fact that at an early stage this necessity could have looked like a necessity deriving from the intellect.

and the necessity [in question] calls for taking it as a sensible form, then we do not attend to it but rather devote ourselves to the proof [as it should be done].

Existent, thing, cause, principle, universal, particular, end, and so forth, all transcend the realm of perception, and in fact, so too even the essences of what pertains to the realm of species, like the essence of human. For they are the sort of thing that can never become object of imagination or representation in our estimative faculty; rather it is our intellect that attains them. The same applies to every universal essence among the specific essences of sensible things, not to speak of intelligible things, as we shall explain in its proper place.[23] Thus, the principles of demonstrations that belong to the domain of what is apprehended (*ǧins al-mudrakāt*) must be among those [propositions] that are apprehended and asserted by real necessity, and not among these estimative [propositions].

The significance of Text 2.3 (where estimative propositions are discussed after all other types of necessary assertions) lies in the fact that, in this digression, Avicenna distinguishes between assertions concerning real necessity from assertions that have a prima facie compelling character and hence may appear necessary without really being so. The distinction between these two types of *necessities* (only one of which is real, while the other is not) becomes commonplace in Avicenna's logical works.

Another extensive discussion is found at *Naǧāt* I, 107, where estimation is characterized as "a faculty that imagines things only in a sensible fashion." The chapter emphasizes the role of the primary intelligence associated with estimation and its strength or compelling character. Two examples of false judgments based on estimation are "The universe either ends in a void or is an infinite plenum" (*Naǧāt* I, 107, ii), and "Every existent must be localized in some place" (*Naǧāt* I, 107, ii). Counterparts in *Burhān* I, 4 are "Every existent is in a place or in an ostensible space" and "Something that is neither inside the universe nor outside it does not exist." By contrast, an example of a *true* assertion based on estimation is "It is not possible for two bodies to be in a single location or a body in two locations at the same time." The epistemic strength of these judgments is such that they can only be falsified by the intellect, even though they have a natural tendency to stick ("they do not cease to exist in the estimation") and are "not distinguished from the primary [objects] of intelligence (*fiṭra*)." When does the primary intelligence of the faculty of estimation make mistakes? Avicenna gives a rather precise criterion: it goes wrong when it passes judgments on things that are "not sensible themselves but rather principles of the sensibles—such as prime matter and form, intellect and creator, or are more general than the sensibles, such as oneness, multiplicity, finitude, cause, effect, and so forth."

23. At *Ilāhiyyāt* III, 9, p. 148.5–6, estimation is characterized as the standard (*qānūn*) for whatever falls in the domain of perception.

The reason Avicenna devotes so much space to estimative assertions is that they are epistemically dangerous, and it is important for the theorist as well as the practitioner of science to recognize in which category the relevant principle of a discipline should fall.

Scientific Assertions

In summary, Avicenna identifies five genuine types of immediate necessary assertions that can be used in the sciences. Two of them are intellectual—(1) primary assertions and (2) implicitly deductive assertions—and seem to capture analytic and quasi-analytic truths that the intellect grasps on its own or with the assistance of another internal faculty of the rational soul. Even if Avicenna does not use the labels *awwaliyyāt* or *yaqīniyyāt* to qualify immediate essential predications, all definitional truths naturally fall in this category.[24] The other three have their source in the external world, being based directly on (3) perception, (4) experience, or (5) testimony.[25] Such assertion-types are presumably also certain in the sense of *Burhān* I, 1, that is to say, the assertion of their content involves a belief about something and a belief that it cannot be otherwise.

Assertions by Way of Concession

Propositions that involve concession (*taslīm*) are an important class for accommodating a broad range of nonscientific assertions, ranging from dialectical to sophistical. They are not directly relevant for demonstration but play an important role in scientific discourse when competing views are rejected.

CORRECT AND INCORRECT CONCESSION

Their primary division, between (aba) correct and (abb) incorrect concession, is instrumental in carving out a space for commonly held or endoxic (*mašhūrāt*)

24. There may be a simple pragmatic reason why Avicenna does not *explicitly* include such predications in the class of primary propositions, namely that he sees definitions first and foremost as complex terms and not as propositions (and hence trivially not as something that it is within the purview of a theory of assertion to accommodate). Thus, even if definitions or the parts of definitions can occur in propositions that express essential predications, there is still an important distinction for Avicenna between the conception of X and the assertion that expresses the nexus between X and its conception.

25. The role of assertions based on sequential testimony in the sciences is admittedly rather obscure, and Avicenna never comes close to clarifying it. This category might seem to have, on the face of it, a purely taxonomical role. It is not entirely implausible, however, to imagine that, for instance, the collection of facts and observations in a discipline like astronomy could perhaps depend on certain presuppositions about the locations at which the observations were made and other factual assumptions that fall outside the domain of observation. In addition to that, it also seems natural to connect the notion of sequential testimony in the Islamic tradition to the notion of transmission of information through reliable chains of witnesses over time, which is characteristic of the *ḥadīṯ* tradition.

propositions, as opposed to specious propositions (*mušabbihāt*) that are erroneously conceded in the context of a sophistical argument.[26]

The distinction between correct and incorrect concession is only indirectly (and partially) connected to truth. The point is rather to distinguish between the case in which a proposition (whether it be true or not) is legitimately granted because it has no obvious internal inconsistency and at least some degree of plausibility, and the case in which a proposition is granted without reason and possibly as a result of an error. Propositions that are conceded "correctly" in this sense need not be true, even when they are granted by a great number of people (or even by all).

Conceded and Endoxic Assertions. The first category of assertions by way of concession is the broadest and encompasses a number of subdivisions depending on the circumstances under which an assertion is conceded correctly. In a first, narrower sense, an assertion may be conceded to (abaa) an individual for the sake of debate; these propositions receive the technical label of "conceded" (*musallamāt*).[27]

Another type of concession refers not to a single individual but is (abab) common to several. Depending on whether the assertion is conceded (ababa) by all people or (ababb) by a group, and in the latter case, on whether the group is (ababba) a community or (ababbb) a restricted number of trusted authorities, the assertion will be characterized as absolute endoxic (*mašhūra muṭlaqa*), limited endoxic (*mašhūra maḥdūda*), or adopted (*maqbūla*), respectively. Examples of the (ababa) absolute endoxic are the claims "Justice is beautiful" (*Burhān* I, 4, *Naǧāt* I, 108) and "Lying is bad" (*Naǧāt* I, 108).[28]

The (ababba) limited endoxic assertions are not illustrated by an example, but it is plausible to assume that they might encapsulate views that are commonly held within a specific domain of application or inquiry. Thus, at least some of

26. The translation of *mašhūr* as "endoxic" is less than fully satisfactory. The term, however, has such a technical connotation that alternatives like "reputable," "plausible," or "probable" might potentially introduce misleading semantic qualifications. I owe the translation of *mušabbihāt* as "specious" to Tony Street.

27. The label may be confusing because Avicenna refers to the whole class under discussion with a similar term (*taslīm*), while the term specifically used for "conceded" propositions (*musallamāt*) only seems to apply to the case where the concession is made to an individual in a dialectical context.

28. Examples of propositions that reflect a commonly held view (which Avicenna typically wants to contrast with the true view, that is to say, his own) are frequently qualified as endoxic. See, for example, *Samāʿ ṭabīʿī* III, 4, p. 193.1 (McGinnis 2009, p. 292, transl. modified), for a claim that is merely *mašhūr* and not demonstrated: "We ourselves shall discuss this soon, and in fact one should know that, while that there are six directions is a widespread and commonly acknowledged view (*amr mašhūr mutaʿārif*), it is neither true nor demonstrated (*wa-laysa bi-ḥaqq wa-lā ʿalayhi burhān*)." *Samāʿ ṭabīʿī* III, 13, p. 247.12 attests to the use of a *maṣdar* in the VIII form (*ištihār*), with the same meaning, for the characterization of a premise as endoxic.

the occurrences of the term *mašhūr* in the sciences may be understood with this qualification, for they may express things about which ordinary people have no views but which are commonly held by some experts. One may go as far as to say that, in the context of scientific theories, one of the tasks of verification (*taḥqīq*) is precisely to question such deeply rooted convictions and replace them with the truth.[29]

The (ababbb) adopted assertions are characterized, at *Naǧāt* I, 106, p. 115.5–8, as claims whose source is leaders who are experts in religious laws (or perhaps even the prophet, as the reference to the "celestial matter specific to someone" seems to suggest).[30]

In the context of the discussion of absolute endoxic propositions, which are (at least in principle) conceded by everyone and not associated with a community, a group of experts, or an authority, Avicenna presents a short thought experiment that is methodologically reminiscent of the procedure adopted, mutatis mutandis, in his celebrated flying man argument.[31] The aim is to establish that under a specific set of circumstances a given endoxic proposition might *not* be conceded, which ipso facto severs the link with primary propositions. Avicenna considers a situation in which someone who has had no contact with the world, no education or second-order input, considers the proposition "Justice is beautiful." He writes:

> Text 2.4: *Burhān* I, 4, pp. 65.16–66.2
> Or it is a view that is not supported by a group, but rather the acceptance of which is common knowledge among all people (*mutaʿārif fī n-nās*) who, having been drilled in it, do not make room for doubt. If there is something such that, when the discerner considers it, [(1)] he takes himself as if he had come to the world all at once endowed with discernment, [(2)] without the inculcation of any custom, [(3)] without education, [(4)] without having turned to a judge other than the intellect, [(5)] without being subject to shyness or shame (so that his judgment is dependent on a natural disposition (*ḫilqī*) and not intellectual), [(6)] without having looked at what brings about benefit (*mūǧib maṣlaḥatin*) (so that it is through a middle, not by [an immediate] necessity), [(7)] turning away from induction too, which would also be through a middle, [(8)] without having taken into account whether something contradicts his position; then, when he does all this and wishes to raise a doubt concerning [a proposition], doubt is possible for him, as in "Justice is beautiful," "Injustice is repugnant," and "Gratitude to the one who bestows favors is a duty."

29. An interesting note on the dependence of certain endoxic propositions on geography, time, and language is in Alfarabi, *Burhān* V, 2, p. 86.1–13: things that are commonly held at a certain time or place may no longer be commonly held at a different time or place and therefore seem to have an intrinsically relative, context-dependent character.

30. On the *maqbūlāt*, see also *Burhān* II, 5, p. 151.1–2.

31. A similar thought experiment is given at *Naǧāt* I, 108 to illustrate the case of widespread propositions (*ḏāʾiʿāt*) that overlap with the *mašhūrāt* of *Burhān* I, 4.

Unlike primary propositions, which even in the context of the thought experiment envisioned in Text 2.4 would necessarily have to be accepted in virtue of the meaning of the terms involved, endoxic propositions of the sort exemplified here can indeed be subject to doubt. The fact that we do not ordinarily question propositions of this kind is merely a consequence of habit, but there is nothing intrinsic to them that makes them compelling, let alone necessary, in the way that is characteristic of primary propositions.[32]

Specious Assertions. The second category of conceded assertions accommodates those that are (abb) conceded incorrectly. The class includes all assertions that are typically involved in sophistical argument. Specious assertions, quite literally, resemble (*mušabbiha*) other assertions and turn out to be conceded incorrectly in virtue of this misleading resemblance. Avicenna distinguishes between two main types of specious propositions: (abba) those that are incorrectly conceded because of an error involving linguistic expression and (abbb) those that are incorrectly conceded because of an error involving meaning. It is easy to see an immediate connection between this distinction and Avicenna's account of the traditional Aristotelian division between fallacies dependent on language (*de dictione*) and fallacies independent of language (*extra dictionem*).[33]

In *Burhān* I, 4 Avicenna does not discuss them in detail, and the class is altogether absent from the *Naǧāt*:

> Text 2.5: *Burhān* I, 4, p. 66.10–15
> Those that are asserted by way of an erroneous concession [are such that] the one who concedes them concedes something instead of something else because the former resembles the latter and shares with the latter [(abba)] the utterance or [(abbb)] the meaning, as we shall explain in its proper place.[34] These are the specious premises (*al-muqaddamāt al-mušabbiha*), as the one who says "Every ʿ*ayn* sees," which he

32. Avicenna adds that all primary propositions are endoxic, while the converse is not true: "Know that all primary [propositions] are also endoxic while the converse does not hold, just as all [propositions] that are asserted are the object of compositive imagination (*mutaḫayyala*) and move the imagination [that is, the form-bearing faculty] (*muḥarrik li-l-ḫayāl*) while the converse does not hold" (*Burhān* I, 4, p. 66.9–10). At *Ilāhiyyāt* I, 1, p. 8.12–13, Avicenna points out that knowledge of the absolute cause depends, among other things, on the proof of the existence of other causes, which is "*not evident as a primary* [truth] but rather commonly held" (reading *mašhūr* for *mašhūd* with Bertolacci 2006, p. 114). Similarly, "it is not the case that, if [the claim] that things which begin to exist have a principle is close to being self-evident to the intellect (*qarīban ʿinda l-ʿaql*), then it is self-evident, like many geometrical facts proven in the book of Euclid" (*Ilāhiyyāt* I, 1, p. 8.13–15). To bridge the gap between *al-mašhūr* and *al-ḥaqq* is the task of the philosopher.

33. On Avicenna's account of the fallacies, see Strobino (2018) and Ahwani's introduction to Avicenna's *Safsaṭa*.

34. For a general characterization of the specious propositions involved in sophistical argument, see in particular *Safsaṭa* I, 2–4. The identification of this type of proposition is instrumental for the

> concedes because he understands it [according to] one of the meanings of the homonymous name [ʿ*ayn*] (*aḥad maʿānī l-ismi l-muštarak*), and takes another [meaning] instead of the former, thinking that this [other meaning] is the one which is being conceded or intending to deceive by means of it, so much so that he himself or someone else eventually thinks that the *dīnār* sees.[35] Likewise, the one who concedes that every intoxicant is wine and takes instead of it every possible intoxicant. The specious premises are these.

It is worth noting that according to the classification of *Burhān* I, 4, two of the five basic types of propositions associated with the standard five "deductive arts" (demonstration, dialectic, rhetoric, sophistic, and poetic) are in fact subdivisions internal to the category of conceded assertions. The first category (assertion by way of necessity) corresponds to demonstration, the second category (concession, either correct or incorrect) covers dialectic and sophistic, the third corresponds to rhetoric (predominant supposition), and the last falls outside the domain of assertions and corresponds to the domain of poetic discourse.[36] Each of them has its own peculiar epistemic status and type of belief: (i) certainty for demonstration, (ii) a state that only resembles certainty for dialectic and sophistic (in the first case because it is ultimately defeasible, in the second case because it is simply false), and (iii) persuasion for rhetoric.

Assertions by Way of Predominant Supposition

The third main class of assertions includes propositions that are based on predominant supposition.[37] It includes three sub-cases, depending on whether an assertion (aca) appears to have some degree of similarity with an endoxic proposition, except that on closer examination it turns out not to be equally plausible or widely held, (acb) is accepted merely out of trust, or (acc) is accepted in other ways that do not involve an endoxic dimension. Examples of assertions based on predominant supposition are the so-called (aca) prima facie endoxic but unexamined (*mašhūrāt fī bādiʾ ar-raʾy al-ġayr al-mutaʿaqqab*), such as "Help your

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identification of the fallacies. In the practice of scientific reasoning, arguments are sometimes explicitly characterized as sophistical by Avicenna, for example at *Samāʿ ṭabīʿī* IV, 8, p. 296.4.

35. The Arabic term ʿ*ayn* means, among other things, eye, spring of water, and coin (hence the ambiguity with *dīnār*, which also means coin); see also *Naǧāt* I, 150 (vii), pp. 176.14–177.4. In the Arabic tradition this becomes a standard illustration for equivocal terms and typically replaces Aristotle's famous example of a man and the picture of a man set forth at *Cat.* 1, 1a2 to explain homonymy (Wolfson 1938, pp. 168–169).

36. The division is summarized at *Ḫiṭāba* I, 1, pp. 1.1–2.6.

37. According to the classification of *Burhān* I, 1, the class of *maẓnūnāt* is associated with the domain of rhetoric. On rhetorical premises in Avicenna, see Aouad (1999).

brother whether he be oppressor or oppressed."[38] The second case (acb) is not illustrated by any example. The third case (acc) is exemplified by the statement "Whoever sees someone approaching with an ominous look supposes that he is an assailant."

Avicenna clarifies that in the first case the resemblance with endoxic assertions is due to the fact that "this is supposed just as [something] bruited about and one is somehow inclined towards it." As a matter of fact, however, upon examination, one recognizes that what is endoxic is an underlying contention, namely "It is not permissible (*lā yaǧūzu*) to aid the oppressor, even if he is brother or son," which is in fact incompatible with the original statement. The analysis of propositions associated with predominant supposition is consistent, once again, with the general criteria laid out in *Burhān* I, 1. The counterpart of this class of propositions is the kind of assertion that aims at persuasion. This in turn is understood by Avicenna in terms of a belief that something is the case accompanied by a second belief that its contradictory is possible.

> Text 2.6 *Burhān* I, 4, p. 67.2–3
> These assertions based on supposition are beneficial in reasoning (*maqāyīs*) only insofar as a belief [is expressed] by means of them, not because their opposite roils the heart.

The brief comment in Text 2.6 suggests that the point of including this type of propositions in the list is to make room for rhetorical discourse aimed at persuasion, whose goal is only to put forward a belief without necessarily having to exclude the opposite. The extent to which this may have a remote relevance for scientific discourse is unclear, but at the same time it is true that Avicenna himself contends, at the opening of his *Ḫiṭāba*, that the register of persuasion seems to have a role at least in the dialectical discussion of principles. And so, if there is a rhetorical dimension to the teaching of science, there seems to be reason to include a classification of assertions that are typically associated with rhetorical argument in an exhaustive taxonomy of scientific and nonscientific assertions.[39]

38. For this translation, see Gutas (2012). At *Naǧāt* I, 108 (iv), p. 120.3–7, this case is classified as one of the divisions of the widespread (*ḏāʾiʿāt*), namely the prima facie praiseworthy unexamined (*maḥmūda fī badīʾ ar-raʾy al-ġayr al-mutaʿaqqab*). For the example, see *Saḥīḥ al-Buḫārī*, vol. 3, Book 43, Ḥadīṯ 624.

39. As noted in the memoirs of a disciple from Rayy (Gutas 1988, p. 70): "Avicenna has a superbly detailed knowledge, unparalleled in others, of what does not constitute terms of premises in demonstrative, dialectical, and sophistical deductions." The ability to recognize the terms in an argument for what they are is essential for the discrimination of proper scientific reasoning from other forms of reasoning.

CONCLUSION

Avicenna offers two summaries at the end of *Burhān* I, 4. The first is a systematic division, based on epistemic criteria, of the types of deductive principles discussed thus far. This division is prior in the sense that it is relevant to the practitioner of a science ("The one who carries out deductions"). He writes:

> Text 2.7: *Burhān* I, 4, p. 67.13–16
> Hence the principles of deductions are [(1)] image-eliciting (*muḫayyilāt*), [(2)] based on sense perception (*maḥsūsāt*), [(3)] based on experience (*muğarrabāt*), [(4)] based on sequential testimony (*mutawātirāt*), [(5)] primary (*awwaliyyāt*), [(6)] deduced by the natural operation [of the intellect] (*fiṭriyyat al-qiyāsāt*), [(7)] estimative (*wahmiyyāt*), [(8)] absolute endoxic (*mašhūrāt muṭlaqa*), [(9)] limited endoxic (*mašhūrāt maḥdūda*), [(10)] conceded (*musallamāt*), [(11)] adopted (*maqbūlāt*), [(12)] specious (*mušabbihāt*), [(13)] prima facie endoxic but unexamined (*mašhūrāt fī bādiʾ ar-raʾy al-ġayr al-mutaʿaqqab*), [(14)] supposed (*maẓnūnāt*). These are fourteen types.

The second division, by contrast, is based on a contextual criterion in virtue of which something functions as a principle for the learner:

> Text 2.8: *Burhān* I, 4, p. 67.17–20
> There is another division of the principles of deductions (*maqāyīs*), namely the ones that are not principles with respect to the one who carries out the deduction (for the divisions with respect to the one who carries out the deduction are those mentioned above) but rather principles with respect to the teacher. These are such that the teacher forces the learner to concede and posit something in order to base on it the explanation of some other thing (so that [the learner] concedes it and posits it). These things are called hypotheses (*uṣūl mawḍūʿa*) and postulates (*muṣādarāt*).

The second classification is useful for a better understanding of the terminology of *An. Post.* A2 and A10, discussed in chapter 4, and pertains to a different level of analysis of the principles of a science, one that is directly concerned with a terminological problem in the *Posterior Analytics* and its commentary tradition. The classification of the different types of scientific and nonscientific assertions, by contrast, offers, in tandem with the general discussion of conception and assertion presented in chapter 1, a first illustration of Avicenna's original and innovative approach to that tradition and of his concerted effort to expand its philosophical vocabulary.

TABLE 4 Principles of scientific and nonscientific deductions in *Burhān* I, 4

Type and source							Context	Rank
Imitation						1. Image-eliciting	Poetics	(No assertion)
Assertion	By way of necessity	External				2. Based on perception	Demonstration	Certainty
						3. Based on experience		
						4. Based on sequential testimony		
		Internal	Intellect	Alone		5. Primary		
				With aid	Intrinsic (natural operation)	6. Implicitly deductive		
					Non-intrinsic	(Acquired assertion, not a principle)		
			Estimation			7. Estimative	(qualified)	(qualified)
	By way of concession	Correct	Shared	All		8. Absolute endoxic	Dialectic	Resembling certainty
				Group	Community (*umma*) or experts	9. Limited endoxic		
					Restricted number of authorities	10. Adopted		
			Individual			11. (Not useful in the sciences)		
		Incorrect	Expression			12. Specious	Sophistic	
			Meaning					
	By way of supposition	(Widespread, praiseworthy)				13. Prima facie endoxic but unexamined	Rhetoric	Persuasion
		Acceptance out of trust						
						14. Non-endoxic, supposed		

3

The Types and Order of Scientific Inquiry

SCIENTIFIC INQUIRY IN CONTEXT

The analysis of the different types and order of scientific inquiry is a central chapter of Avicenna's theory of science.

The subject is treated at length mainly in two places: in *Burhān* I, 5, where Avicenna presents a first, introductory list of inquiries and their relations, and then in *Burhān* IV, 1, where he develops a second, more elaborate list.[1] The account of I, 5 is terse and focused, in line with the introductory and systematic character of the first book of the *Burhān*. The account of IV, 1 is more directly dependent on the text of *An. Post.* B1–2, but original in its fine-grained analysis of the stages of inquiry and detailed description of their order, captured in a discrete sequence of over a dozen steps. The model laid out in these two chapters describes a comprehensive process involving both noetic (conceptual and definitional) and non-noetic (propositional and demonstrative) aspects of scientific discourse, in close

1. The stages of scientific inquiry are also discussed at *Nağāt* I, 115–18 and *Išārāt* IX, 6. For the brief treatment at *Daneshname*, I, 32, see Achena and Massé (1955, pp. 84–85). Avicenna's account of the order of inquiry in the *Burhān* is in many respects similar to the model recently developed for Aristotle by Bronstein (2016), despite certain structural differences. According to Bronstein, knowledge of the essential attributes of a subject is a necessary condition to discover, by elimination, what its necessary nonessential and demonstrable attributes are (that is to say, its implicates, in Avicenna's vocabulary). In addition to that, according to Bronstein, knowledge of the essences of demonstrable attributes in Aristotle is only attainable by demonstration. Neither of these conditions is required in Avicenna's theory of science. On Aristotle's analysis of questions, see also Gómez-Lobo (1980), Bayer (1997a), and Upton (1991). On Avicenna's logic of questions, see Rescher (1967).

connection with the two modes of knowledge introduced in *Burhān* I, 1, namely conception and assertion.

The theme occupies a central position in the economy of Avicenna's theory of demonstration and definition, as it bears on (i) the distinction between factual and causal demonstration (if- and why-questions concerning demonstrable connections between subjects and attributes, that is to say, the theorems of a science), on (ii) the theory of definition (what-questions concerning the essence of subjects and attributes, as well as the role of nominal definition), on (iii) the structure of scientific theories (if-questions concerning the existence of subjects and what-questions concerning their essence), on (iv) the account of preexistent knowledge and the solution to Meno's paradox, and on (v) Avicenna's foundationalism (especially with regard to the impossibility of an infinite regress in the chain of why-questions).

IF, WHY, AND WHAT

Avicenna's account of scientific inquiry in *Burhān* I, 5 reveals some of the critical problems just introduced. The chapter begins with a brief presentation of the fundamental types of inquiry; it then discusses the relation between nominal definition and real definition and the role of definition in the mathematical sciences; it draws a distinction between simple principles (the attributes of the subject of a discipline) and compound principles (axioms and hypotheses); it establishes the priority of what-inquiries over why-inquiries with respect to both inferential justification and factual explanation; and finally, it discusses the distinction between two roles a middle term can play in a deduction, either as a cause of an assertion or as a cause of the corresponding fact, which is the basis for a further canonical distinction between that-demonstration and why-demonstration. Concerning the fundamental types of inquiry, Avicenna writes:

> Text 3.1: *Burhān* I, 5, pp. 68.1–69.1
> Inquiries are, with respect to what is needed here, three by the first division and six by the second division. The first division consists of the inquiry into the what, the inquiry into the if, and the inquiry into the why. The inquiry into the what has two divisions. The first is the one by means of which one seeks the meaning of the name, as when we say, "What is void?" and "What is the griffin?" The second is the one by means of which one seeks the true reality (*ḥaqīqa*) of the essence, as when we say, "What is motion?" or "What is place?" The inquiry into the if has two divisions. The first is simple, that is to say, the inquiry into whether something exists without qualification (*mawǧūd ʿalā l-iṭlāq*). The other is compound, that is to say, the inquiry into whether something is such-and-such or is not such-and-such, in which case "is" (*mawǧūd*) is a copula, not a predicate, as when you say "Is man an animal?" or "Is man not an animal?" The inquiry into the why has two divisions. For it is either

> concerned with the statement, and in this case it seeks the middle term, namely the cause of the belief in the statement and of its assertion in a deduction that produces what is sought as a conclusion, or it is concerned with the thing itself (*bi-ḥasab al-amr fī nafsihī*), in which case it seeks the cause of the existence of the thing in itself in the way it is (whether it be existence without qualification or being in a certain state).

According to the classification of *Burhān* I, 5, scientific inquiries (*maṭālib*) are divided into three main categories, each of which contains two elements. The first category includes (a) what-questions, either (aa) with respect to the meaning of a name or (ab) with respect to the essence of something.[2] The second category includes (b) if-questions, either (ba) with respect to existence without qualification (where existence is understood as a one-place relation, that is to say, as the equivalent of a monadic predicate) or (bb) with respect to being in a certain state (where existence is understood as a two-place relation and serves as a copula), which corresponds to the case of predication.[3] The third category includes (c) why-questions, either (ca) with respect to assertion alone or (cb) with respect to both assertion and existence (where existence may be understood in either of the two senses associated with if-questions). Certain what-questions and if-questions—namely types (aa) and (ba)—may legitimately apply to existents (for instance motion or place) and non-existents (the void or the griffin) alike.[4]

Avicenna calls the two types of if-question "simple if-question" (*maṭlab hal basīṭ*) and "compound if-question" (*maṭlab hal murakkab*). The distinction and

2. This type is later also referred to as "the what-inquiry with respect to the verification of the thing in itself" (*maṭlab mā llaḏī bi-ḥasab taḥqīq al-amr fī nafsihī*), at *Burhān* I, 5, p. 69.1–2. In this chapter, I use the English terms "inquiry" and "question" interchangeably to refer to the Arabic *maṭlab* (which is in turn the usual translation of the Greek *zētēsis* in Aristotle). "Question" in this book also has another important technical application as a translation of the Arabic *masʾala* (which is in turn the usual translation of the Greek *problēma* in Aristotle). Scientific questions in the latter sense are discussed extensively in chapter 4.

3. Avicenna formulates the distinction, in this context, in terms of two different ways to take the term existent (*mawǧūd*), namely as a predicate (*maḥmūl*) in the existential sense or as a copula (*rābiṭa*) in the predicative sense.

4. This type crucially accommodates the case of non-existent subjects in reductio proofs. At the beginning of *Burhān* I, 6, Avicenna maintains that all scientific inquiries involve existents. What then about impossible things (*al-maʿdūm aḏ-ḏāt al-muḥāl al-wuǧūd*)? How can they be conceptualized? Can what-questions and if-questions legitimately be asked about them? Impossible notions play an important role in the sciences, for we must be able somehow to conceptualize them, at least in order to reject them (as in the case of void, of which we must have a conception, if we are to produce a refutation of its existence; see *Qiyās* V, 5). I return to the issue in chapter 8. The relevant distinction is between two kinds of impossible notions (*muḥāl*): (i) things that have no composition and differentiation (*tarkīb*) and (*tafṣīl*) (for example, the absolutely non-existent); and (ii) things that have *tarkīb* and *tafṣīl* (for example, the griffin, the void, or the atom).

its peculiar vocabulary seem to emerge for the first time in Themistius's paraphrase of the *Posterior Analytics* and are assimilated at an early stage in the Arabic tradition.[5]

The domains of application of why-questions differ depending on whether what is sought is merely an inferential justification of a statement or the cause of the thing itself. In the former case, the question is tantamount to asking whether there is a deductive argument (or a middle term) that entails the conclusion (along with the belief it encapsulates). In the latter case, what is sought is a factual explanation, and the object of inquiry may be either the factual explanation of why something exists without qualification or the factual explanation of why it exists in a certain state, that is to say, the explanation of why a certain attribute holds of a certain subject.[6] If-questions and why-questions correspond to the domain of assertion, while what-questions correspond to the domain of conception.

NOMINAL AND REAL DEFINITION

In addition to the identification of the fundamental types of scientific inquiry, in *Burhān* I, 5 Avicenna also sketches a preliminary picture of the order of inquiry, which is then developed further in *Burhān* IV, 1. Scientific inquiry ideally begins with an account of the meaning of the terms in a given domain of investigation. This has priority over any other inquiry and every subsequent stage presupposes a preliminary answer to a question of type (aa) ("One who seeks whether motion, time, void, or God exist must understand first what these names signify"; *Burhān* I, 5, p. 69.3–4).[7] The next natural step is to seek whether that whose meaning is grasped by such a preliminary account actually exists or not. If the answer to a question of type (ba) is affirmatively established, the essence of the thing in question may then be properly sought, through a question of type (ab), for inquiry concerned with the essence (*ḏāt*), nature (*ṭabīʿa*), or quiddity (*māhiyya*) of something presupposes an answer to the question of whether or not the thing exists without qualification.

Avicenna characterizes this process as a transition from knowledge of a nominal account to knowledge of the real definition and makes it contingent upon establishing the existence (*iṯbāt*) of the notion under investigation. This classification

5. Strobino (2012).

6. A recurrent issue, both in *Burhān* I, 5 and in IV, 1, is the reduction of other putative inquiries to the domain of compound if-questions. Such inquiries include the "which," "how," "how much," "where," or "when," and reflect Avicenna's commitment to the idea that no matter what category a given attribute falls under, the logical form of different types of predicative assertions is the same.

7. This is because we can have knowledge of the meaning of certain terms without having knowledge of whether something actually corresponds to them in reality.

applies to both subjects and attributes, but Avicenna considers the case of attributes to be somehow paradigmatic. The definitions of things whose existence is demonstrated within a science are typically the definitions of attributes, while the definition of the subject(s) of a science is the definition of something whose existence is presupposed by a science, not demonstrated in it (though it may be demonstrated by another science). Thus, the definitions of attributes in a science, strictly speaking, remain purely nominal until such attributes are proved to exist, whereupon nominal definitions become real definitions. And the proof of existence of an attribute, for Avicenna, is a demonstration that the attribute belongs to its subject.[8]

Avicenna seems to imply that knowledge of nominal definitions is characteristic of any competent speaker of a language (*ʿālim bi-l-luġa*) as opposed to someone who has proper technical training in logic (*al-murtāḍ bi-ṣināʿat al-manṭiq*).[9] But only the practitioner of logic (and, derivatively, the scientist) can truly discover real definitions. This is because, first, knowledge of logic, and especially of the logic of scientific discourse, is a necessary condition to recognize a real definition as such, in virtue of its distinctive requirements, namely the essentiality of its constitutive elements, the order and arrangement of such elements in a sequence that reflects the genuine ontological structure of the *definiendum*, and their completeness (Avicenna returns to the theme in *Burhān* IV, 6–7, which I discuss in chapter 13). Second, it is only through the engagement with a given domain of scientific inquiry (after its subject has been proved to exist) that certain notions can be identified as *real* essences and that the corresponding real definitions can be sought. At this more advanced stage, the inquirer is, so to speak, several steps ahead of mere linguistic competence.[10]

8. Avicenna illustrates the point with a geometrical example: the definition of triangle is nominal until a proof of its existence is given. At *Burhān* I, 5, p. 69.8–10, he explicitly refers to Euclid's *Elements* (*Kitāb istiqsāṭ al-handasa*), suggesting that this is what typically happens in geometry: definitions of triangle, square, and other figures are first introduced, and the existence of the relevant objects is then proved at a later stage in the science. At *Burhān* IV, 1, p. 262.5–15, it becomes clear that a proof of existence is, in a case like this, a proof that something belongs, as an attribute or predicate, to a certain subject (in *Burhān* IV, 1 the attribute is equilateral triangle and the subject is the geometrical construction described by the first proposition of *Elements* I). A suggestion along similar lines (illustrated by a simpler, pre-Euclidean example) is already at *An. Post.* B7, 92b15–17, where, in the context of a discussion of the relation between existence and definition, Aristotle maintains that the geometer assumes the definition of triangle and proves that it exists.

9. *Burhān* I, 5, p. 69.11–13.

10. The significance of the distinction between knowing broadly what a term means and knowing in detail its metaphysical structure through appropriate differentiation (*bi-tafṣīl*) is fully acknowledged by Avicenna, who also draws a distinction, in this connection, between a more basic form of knowledge or cognition (*maʿrifa*) and scientific knowledge proper (*ʿilm*), contending that the former stands to sense perception as the latter stands to the intellect.

TRIPARTITE DIVISION OF PRINCIPLES

The preliminary discussion of these basic relations of dependence involving what-questions and if-questions is followed in *Burhān* I, 5 by a digression on the ways in which principles (*mabādiʾ*) in the sciences differ with respect to priority. The theme is related, on the one hand, to the distinction between different types of prior conception and assertion discussed in *Burhān* I, 3 and, on the other, to a canonical tripartite division of preexistent knowledge first introduced by Aristotle at *An. Post.* A1, 71a11–16 (which Avicenna adopts and translates into his own conceptual vocabulary): (i) of certain things one has to know that they are (that is to say, that they are the case or that they exist), (ii) of others one has to know what they are (that is to say, the definition), and (iii) of yet others one has to know both that they are and what they are. The first case is exemplified by (i) the law of the excluded middle and the contention that things that are equal to one and the same thing are equal, both of which are examples of common principles or axioms; the second case by (ii) the notions of triangle and square, both of which are examples of attributes whose definition is assumed but whose existence requires proof; and the third case by (iii) the notions of unit in arithmetic or point in geometry, both of which are examples of subjects whose definition and existence must simultaneously be assumed.

This distinction is in turn associated with another distinction between single (*mufrad*) and compound (*murakkab*) notions and the types of inquiry that distinctively correspond to them. Single notions are terms, which Avicenna explicitly links with the domain of definition (case ii). By contrast, compound notions, in the context of *Burhān* I, 5, correspond to propositions (not to complex terms) and encapsulate either common knowledge or hypotheses (and various forms of existential or predicative claims, however general or specific), all of which are associated with the domain of assertion (case i).[11]

This tripartite division of principles or presuppositions necessary for scientific discourse is then mapped onto the types of inquiry identified at the beginning of *Burhān* I, 5. Avicenna formulates the distinction in terms of two abstract notions corresponding, on the one hand, to if-questions (*haliyya*) and, on the other, to what-questions (*māhiyya*). In the case of compound notions, that is to say, common axioms and hypotheses, the focus is not on definition and quiddity but rather on the proof of something distinct from them, which results from the assumption that they are the case (*bi-l-haliyya*). The class of single notions, by contrast, may be divided into the following subtypes:

11. In this context, assertion is directly linked by Avicenna with the domain of informative composition (*tarkīb ḫabarī*), that is to say, predicating something of something else in a statement-making proposition. On the two different senses of composition (*tarkīb*), see Text 1.9. I discuss the sense of composition that is relevant to definition in chapter 13.

(i) attributes of the subject of the discipline (*āṯār*) and
(ii) what falls in the domain of the subject of the discipline.

Each has its own characteristic requirements. In the first case, an attribute is sought in a scientific discipline in order to determine its existence. For Avicenna, to determine the existence of an attribute is to determine whether or not it belongs to a subject. For an attribute P to exist is for P to belong to S (where S is the subject of the discipline or one of its internal divisions). Avicenna's general contention is that demonstrable attributes cannot be evidently existent (*bayyinat al-wuǧūd*) without being known to belong to the subject of the discipline, for the subject of the discipline is (somehow) assumed in the definition of those attributes and hence presupposed by their existence.[12] In the second case, single notions that fall in the domain of the subject must be both (i) understood and (ii) known to exist. The argument is that without understanding their quiddity, it would be impossible to know anything about their existence, and without positing their existence, it would be impossible to seek whether an attribute belongs to them. Thus, there seems to be an asymmetry between the case of attributes and the case of items that fall in the domain of the subject of a science. In the former case, what is known is their quiddity (*māhiyya*), and what is sought is their existence (*haliyya*) (which is in turn tantamount to seeking whether they belong to the subject of the discipline).[13] In the latter case, by contrast, both the quiddity and the existence must be known in advance, because subjects are fundamental and essentially prior in the constitution of a science and in the order of inquiry.

The upshot of this discussion is that since in demonstrative sciences only simple entities fall in the domain of the discipline, and what falls in the domain of the discipline is either (i) the subject or one of its internal divisions, or (ii) the attributes that are pronounced to belong to the subject, then the principles of a science will include single notions of which one must assume the definitions but not the existence (attributes), and single notions of which one must assume both the definition and the existence (subjects).

The analysis of the interrelations among different kinds of inquiries also indirectly corroborates another fact, namely Avicenna's understanding of assertion and conception as two complementary but independent domains of knowledge. All considerations involving the noetic grasp of a concept and its internal articulation pertain to the domain of conception, that is to say, the domain of definition (or description). Non-noetic, propositional knowledge that is amenable to truth

12. The problem is discussed extensively in *Burhān* II, 2–3. The ways in which the subject of the discipline or one of its constituents may be assumed in the definition of a demonstrable attribute are explored in chapters 4 and 7.

13. *Burhān* IV, 1, p. 262.6–9.

pertains, by contrast, to the domain of assertion (even if the nexus between terms in a proposition is also an object of conception). Conception and assertion are connected, in the taxonomy of scientific inquiry, with the corresponding domains of *māhiyya* and *haliyya*, and the tripartite division of principles is determined by the kind of answers one must have at the initial stage of scientific inquiry. These, as we have seen, involve certain what-questions concerning the demonstrable attributes of a science, if-questions concerning truths expressed by common axioms and hypotheses, or a combination of both what-questions and if-questions concerning the essence and existence of primary subjects and their internal divisions.

WHY-QUESTIONS (I)

The last section of *Burhān* I, 5 is concerned with why-questions and their relative place in the order of inquiry. Why-questions are taken by Avicenna to be posterior both to if-questions and to what-questions.[14] The argument is that if something is not conceptualized first (that is to say, if we cannot answer suitable what-questions about it), it is impossible to ask meaningful why-questions about it (regardless of whether what we seek to establish is why it exists without qualification or why it has a certain attribute). It is also impossible to ask meaningful why-questions about something conceptualized but of which the existence has not been established, either without qualification (we cannot meaningfully ask why S exists if we do not know first that S exists) or in the predicative sense (we cannot meaningfully ask why S is P if we do not know that S is P).

The distinction between two sorts of why-questions introduced at the beginning of *Burhān* I, 5, namely why-questions concerned with the inferential justification of statements and why-questions concerned with the cause of the thing itself, is invoked to suggest that the former may occasionally be prior to the latter.[15] This contention is not illustrated by an example, but it seems clear that it is relevant for the distinction between that-demonstration (the inferential justification of a conclusion) and why-demonstration (the causal explanation of a conclusion, which accounts for the underlying fact). It seems also directly to bear on some of the problems of priority that Avicenna discusses in *Burhān* I, 7 and III, 3. This approach shows once again how a technical distinction in Aristotle may be

14. The fact that Avicenna resumes (and concludes) the classification of the types of inquiry at the end of *Burhān* I, 5 suggests that the lengthy digression on the problem of the three types of presuppositions (what, if, or both) might be an ingenious way to connect the specific problem of the relation between what-questions and if-questions to the standard threefold classification of principles in Aristotle.

15. At *Burhān* III, 7, p. 240.15, Avicenna refers to this inquiry as an investigation of the why (*baḥṯ ʿan al-limā*) in the context of the fourth argument for the superiority of universal demonstration over particular demonstration.

reformulated in Avicenna (at an early stage of the discussion in the *Burhān*) in the vocabulary of scientific inquiries. In this case, the point is to show that it is possible to have a sound argument for the claim that, say, "C is B without knowing the cause of the fact that C is B" (*ʿilla fī nafs al-wuǧūd li-kawn Ǧ B*). In such cases, we have introspective knowledge of two facts, namely (i) that we may not believe that C is B (the belief is not unshakable because it is not grounded in the cause of the fact), and (ii) that we do not know why C is B in reality (*fī nafs al-amr*) (*Burhān* I, 5, p. 71.5–7).

Another claim Avicenna puts forward in connection with why-questions is that why-questions concerned with the cause of the thing itself do not necessarily presuppose why-questions concerned with the inferential justification of a statement. This happens, for instance, when something is self-evident through perception (*bayyin bi-nafsihī bi-l-ḥiss*) and therefore not deductively obtained by means of a middle term. In such cases, the middle term may only be sought with a view to explaining the fact but is not needed in order to justify our belief in the conclusion. If a fact is perceptually evident—for example that lodestones attract iron—or evident on the basis of experience (in Avicenna's technical sense), the corresponding assertion is not deductively proved. The assertion is, as it were, independently established, and as a result of this, the proper why-question to be raised is one that is directly concerned with the cause of the fact, not with the justification of its assertion.[16]

The most common case, however, involves a deduction and a middle term where the latter coincides with the cause of the thing in itself. In this case, the two why-questions seeking the cause of the assertion and the cause of the fact genuinely involve one and the same explanation (*bayān wāḥid*), which is at the same time an explanation of our belief in the conclusion and of the underlying nexus obtaining in reality between subject and attribute.

THE ORDER OF INQUIRY (I)

In summary, according to the conceptual vocabulary of *Burhān* I, 5, a preliminary picture of the order of inquiry involves the following stages:

1. What-question concerning the meaning of the subject (aa).
2. Simple if-question concerning the existence of the subject without qualification (ba).
3. What-question concerning the essence of the subject (ab).

16. The standard example is the fact that lodestones attract iron, on which see also *Ilāhiyyāt* III, 8, p. 141.8–14.

4. What-question concerning the meaning of the attribute (aa).
5. Compound if-question concerning the existence of the attribute in the subject (= simple if-question concerning the existence of the attribute without qualification) (bb).
6. Why-question concerning the existence of the attribute in the subject (= what-question concerning the essence of the attribute) (ca-cb).[17]

At stage 1, the inquirer seeks the meaning of the subject. At stage 2, the inquirer knows the meaning of the subject and seeks the existence of the subject. At stage 3, the inquirer knows that (simple if) the subject exists and seeks what it is (its essence). At stage 4, the inquirer knows that the subject exists and what it is, and seeks the meaning of the attribute. At stage 5, the inquirer knows the meaning of the attribute and seeks whether the attribute exists without qualification (simple if), which is the same as seeking whether the attribute belongs to the subject (compound if). At stage 6, the inquirer knows that the attribute exists without qualification or, to put it another way, that it belongs to the subject, and seeks why it belongs to the subject. This is the same as seeking the essence of the attribute.

ANALYSIS OF ARISTOTLE'S TAXONOMY

In *Burhān* IV, 1, Avicenna presents another taxonomy of scientific inquiry in close connection with the famous opening of the second book of the *Posterior Analytics* (*An. Post.* B1, 89b23–25). The taxonomy establishes a correspondence between a set of paradigmatic types of inquiry, on the one hand, and various types of scientific knowledge acquired at different stages of the process, on the other (noetic and non-noetic, concerning existence or predication). As in the list developed in *Burhān* I, 5, the analysis is ultimately dependent on Aristotle and the commentary tradition, but in IV, 1 there is a characteristic shift in the terminology and a significantly more fine-grained picture of the order of inquiry and of the mutual relations

17. Something seems to be missing from this picture, namely the why-question concerning the existence of the subject without qualification, which presumably has to occur, if at all, right after the subject is established to exist (this may, for example, involve a proof, in a higher science, of the existence of the subject of a lower science). As noted, this question is elsewhere (*Burhān* IV, 1) even associated with a peculiar logical form (an instance of *modus ponens* in which the consequent of the conditional major premise expresses an assertion of existence and the antecedent gives the cause). By contrast, the why-question concerning the existence of the attribute without qualification coincides (in the sense of being identical) with the why-question concerning the existence of the attribute in the subject (or, in other words, with the question of why the attribute belongs to the subject). This ideal picture of the order of inquiry trivially includes the limit case of facts that are perceptually evident and of which we only need to seek the causal explanation (in other words, cases in which the process of inquiry skips stage 5).

holding among if-questions, what-questions, and why-questions.[18] There is also a transition from the streamlined scheme of I, 5, where the number of fundamental types of inquiry is three (or six, if we take into account their internal subdivisions), to the more complex scheme of IV, 1, where Avicenna follows Aristotle more literally in the enumeration of four basic types, if only to refine them even further into several subtypes. The distinctive factor that seems to permeate the analysis of IV, 1 is the systematic association of the different types of inquiry with the basic elements of two deductive structures, most notably first-figure deductions consisting of categorical statements and *modus ponens*.

In *Burhān* IV, 1, the four fundamental inquiries concern (1) existence without qualification, (2) existence in a state (attribute ascription or predication), (3) cause, and (4) essence (or definition). In Avicenna's conceptual vocabulary, the four Aristotelian types are formulated as follows:

Avicenna		Aristotle	
1. unqualified or simple if	(*hal muṭlaq* or *hal basīṭ*)	if-it-is	(*ei esti*)
2. compound if	(*hal murakkab*)	that-it-is	(*hoti esti*)
3. why	(*limā*)	why-it-is	(*dioti/dia ti esti*)
4. what	(*mā*)	what-it-is	(*ti esti*)

Simple if-questions are typically concerned with the existence of a subject, but as we have seen, in a secondary sense they may be brought to bear on the existence of an attribute or even on the inferential justification of a conclusion. Compound if-questions constitute the paradigmatic case of things that are sought to be established demonstratively and typically take the logical form of a categorical statement. Avicenna's compound if-question corresponds to Aristotle's that-question.[19]

Why-questions encapsulate one of the distinctive definitional features of scientific knowledge, that is to say, its explanatory character. What-questions are closely related to why-questions, because the ultimate explanations of the facts established in the conclusions of demonstrative deductions are various kinds

18. Avicenna refers to (and dismisses) a few additional types of inquiries, such as which-questions, how-questions, or how-much-questions. Which-questions deal with essential determinations (differentiae) and can thus be led back to what-questions: they are characterized as "one of the things that follow (*tawābiʿ*)" a what-question (*Burhān* IV, 1, p. 261.6); the other two are to be understood as variants of compound if-questions. The list of these discarded questions surfaces time and again in the earlier Arabic philosophical tradition and can in all likelihood be traced to the ancient commentators Elias and David, on which see Menn (2008, p. 93). A preliminary discussion of this set of issues, with an attempt to connect the analysis with the theory of the categories, is in Rescher (1967).

19. The development of a new vocabulary is crucial for the reconstruction of the textual sources of Avicenna's *Burhān* (with regard to both the translations and the commentary tradition he relies on); see Strobino (2012, pp. 367–371) and cf. also Eichner (2010).

of essential connections between the terms of their premises and conclusions: knowledge of essence and explanation are inextricably codependent.[20]

WHY-QUESTIONS (II)

In *Burhān* IV, 1, Avicenna explicitly distinguishes four types of why-questions on the basis of two parameters. Why-questions may concern either (a) the existence of the subject without qualification or (b) the existence of the subject in a state, that is to say, a case of attribute ascription. In each of these two cases, the cause or explanation that is being sought may be either (aa)-(ba) simply the cause of an assertion, or (ab)-(bb) the cause of a fact as well as of the assertion of that fact. This yields the fourfold scheme of classification of why-questions illustrated in table 5.[21]

The first two why-questions, (aa) and (ab), presuppose affirmative answers to simple if-questions, whereas the last two, (ba) and (bb), presuppose affirmative answers to compound if-questions, depending on whether what is sought is the cause of existence without qualification or the cause of an attribute belonging to a subject. Furthermore, while (aa) and (ba) are merely concerned with the inferential justification of the conclusion that a subject exists or that it has a certain attribute, (ab) and (bb), in addition to being concerned with the inferential justification of an assertion of existence or of a predication, also seek the real cause or explanation of the fact that a subject exists or that it has a certain attribute.

In Avicenna's taxonomy of scientific inquiry, the distinction between inferential and factual why-questions is the counterpart of the distinction between that-demonstration (*burhān al-anna*) and why-demonstration (*burhān al-limā*). The basic elements of this correspondence are already in Aristotle (with a straightforward parallel between the relevant types of inquiry in *An. Post.* B1–2 and the distinction between knowledge of the *hoti* and knowledge of the *dioti* in *An. Post.* A13), but Avicenna's account is more comprehensive, covering as it does not only the predicative case but also the case of existence without qualification. Moreover, the causal explanation of an assertion of existence is the object of a more sophisticated logical analysis in Avicenna, who thinks it is typically encapsulated in the antecedent of a conditional statement that serves as a major premise of *modus*

20. There is a puzzle about the nature of this codependence that has generated something of a subfield in Aristotle scholarship. The puzzle bears an intriguing resemblance to a metaphysical and epistemological version of the Euthyphro dilemma: Are essential relations essential because they are explanatory? Or are they explanatory because they are essential? For Avicenna, the only plausible response is not even worth stating. There are some fundamental facts about the world that cannot be otherwise because of the nature of the things involved, that is to say, because of their essences. Everything derivative is explained in terms of those essences. The explanatory character of an essential feature may at best be a *sign* of the fact that it is an essential feature, but not the reason why it is an essential feature.

21. The distinction is relevant for the analysis of that- and why-demonstration in chapter 9.

TABLE 5 Why-questions

Assertion		Fact and assertion	
(aa) Existence	(ba) Predication	(ab) Existence	(bb) Predication
An antecedent	A middle term	The causal antecedent	The causal middle term

ponens.[22] In other words, the explanation of assertions of existence involves, ideally, an argument such as "If *p*, then *q*; but *p*, therefore *q*," where the cause is expressed by the condition (or antecedent) *p* and the assertion of existence is expressed by the consequent *q*. The suggestion is relevant for at least two reasons. First, the fact that Avicenna associates the causal justification of assertions of existence with a specific argument form distinct from that of "the rest" (that is to say, distinct from the argument form to be used in the case of predications) shows yet again that he sees the whole edifice of formal logic as an integral part of his logic of scientific discourse and that the theory of science must avail itself of its full expressive power. Second, the use of this distinction raises the further question of how Avicenna understands the relation between assertions of existence and predications. A non-immediate predication that requires an explanation is typically expressed by the conclusion of a categorical deduction (or possibly of a mixed deduction combining categorical and conditional or disjunctive statements of the kind discussed in *Qiyās* VI). In this case the cause is expressed by a middle term. A non-immediate assertion of existence, by contrast, is said to be the consequent of a conditional statement, without any further indications concerning

22. At *Burhān* IV, 1, p. 261.9–10, Avicenna explicitly associates a particular argument form with the explanation of assertions of existence. The argument form in question is what he calls hypothetical repetitive deduction (*qiyās šarṭī istiṯnāʾī*). The use of "hypothetical" as a qualification of (or even just in tandem with) repetitive deductions is somewhat problematic. Hypothetical (*šarṭī*) deductions are usually an internal division of connective (*iqtirānī*) deductions in Avicenna's scheme of classification, alongside categorical (*ḥamlī*) deductions (Strobino 2018). And connective deductions, taken collectively, are in turn one of the two main divisions of deductions in general, alongside repetitive deductions. Thus, "hypothetical" would turn out to be the name of one of the two main types of deduction (the one that loosely speaking covers a variant of propositional logic) *and* also the name of a particular type of deduction (the one that is traditionally called hypothetical syllogism) falling under the other main type. As a matter of fact, however, the qualification of repetitive deductions as hypothetical is not uncommon in Avicenna's logical works. This may depend on an ambiguous or contextual understanding of the term *šarṭī*: (i) in the narrower (and literal) sense of "conditional," it may serve to qualify a repetitive deduction whose major premise is in fact a conditional statement (even though conditional statements are usually called *muttaṣil* by Avicenna, even in that context), while (ii) in the broader sense of "hypothetical," it qualifies all non-categorical connective deductions (which include disjunctive as well as conditional statements). The expression is used in a similar sense in Alfarabi, *Burhān* II, 3, p. 28.6–8 (*yubayyana bi-qiyās šarṭī fa-qaṭ*).

its own logical form or that of the antecedent and their relation. That necessary conditional statements may express causal relations is explicitly acknowledged by Avicenna in *Qiyās* V, but why the explanation of an assertion of existence should require a different logical analysis remains an open question. What is plain is that in each case the causal component, whether it be the condition-antecedent of *modus ponens* or the middle term in a categorical deduction, may either merely provide the inferential justification of a conclusion (existential or predicative) or adequately express its causal explanation too.

WHAT-QUESTIONS

In the case of what-questions, Avicenna first identifies the priority of both types of if-questions over what-questions concerning the essence. An inquiry concerning the essence of something presupposes that the object has been established to exist (*An. Post.* B1, 89b 32–34). But as noted in connection with *Burhān* I, 5, according to Avicenna there are two types of what-questions, one aiming at the essence and the other aiming at the meaning of a term.[23] The order of inquiry that emerged in the previous section is confirmed by IV, 1: what-questions concerning the meaning of a term may be (and usually are) prior to if-questions concerning existence without qualification, because a preliminary grasp of the object is necessary if we want to investigate whether it exists or not. And these are in turn prior to the second type of what-questions, which investigate the essence of the object, once the object is known to exist.[24] Thus, the relative order of inquiry, in the case of what-questions and simple-if questions, presupposes a what-question concerning the meaning of the subject, a simple if-question concerning the existence of the subject, and a what-question concerning the essence of the subject.

What about the relation between what-questions and compound if-questions? Is there an ideal order in this case? Avicenna's view is that compound if-questions, too, are somehow prior to what-questions concerning the essence. A demonstrable predicative assertion established by a first-figure categorical deduction involves three terms: a subject-minor term, a middle term, and a predicate-major term. Simple if-questions address the problem of (i) whether the subject exists without qualification; (ii) whether a middle term exists without qualification, that is to say, whether the conclusion is an *explanandum*; and (iii) whether the

23. By "meaning" here one should broadly understand a referent (real or hypothetical) that corresponds to the name and to which all the subsequent questions may be anchored.

24. As a result, all what-questions concerning non-existents (such as the void) are purely nominal, even though they still play a critical role in the context of reductio proofs. The problem is discussed in chapter 8.

predicate exists without qualification (as noted, for Avicenna this is equivalent to the fact that the predicate belongs to the subject).

On this view, one cannot ask what the middle term or the predicate is without prior knowledge of the essence of the subject *and* of the fact that the predicate belongs to the subject (which in turn presupposes knowledge of the existence of a middle term connecting subject and predicate). Thus, what-questions, with regard to middle and major terms, presuppose affirmative answers to compound if-questions. As far as attributes are concerned, Avicenna's identification of simple-if questions (whether an attribute exists without qualification) with the equivalent compound if-questions (whether the attribute belongs to a subject), which was first put forward in *Burhān* I, 5, is reiterated in IV, 1 perhaps in even clearer terms. To ask whether a predicate belongs to a subject (when the predicate expresses a per se attribute of the subject, as is always the case in the context of scientific demonstration) and to ask whether the predicate exists without qualification are one and the same thing (or at least they mutually entail each other).

Per se attributes that belong to the entities falling in the domain of the subject of a science can only exist, according to Avicenna, insofar as they belong to those entities (or to entities that are ontologically more general). In other words, no per se attribute of a subject can at the same time exist without qualification *and* fail to belong that subject (or at least to something that is entailed by that subject, such as its genus, its differentia, the genus of its genus, and so on). If a per se attribute exists, then it exists only qua attribute of that of which it is a per se attribute (or of something more general). Consequently, for Avicenna, to answer the question whether P belongs to S *is* to answer the question whether P exists without qualification, especially if P is a per se attribute of S. The case is illustrated in *Burhān* IV, 1 with an example involving a geometrical construction. The compound if-question is whether the triangle constructed on a segment whose extremes are the centers of two intersecting circles is an equilateral triangle. This question is equivalent, for Avicenna, to the question of whether an equilateral triangle exists.[25]

25. The issue is to establish that for any simple if-question concerning an attribute there is a compound if-question asking whether the attribute belongs to a subject. The example is based on the construction of equilateral triangles in Euclid's *Elements*, Book I, proposition 1. Avicenna presumably inherits the discussion of this particular problem in the context of an analysis of scientific inquiries from the commentary tradition (see for instance ps.-Philoponus, *In An. Post.* B1, p. 343.10–18), and perhaps ultimately from Proclus, *In primum Euclidis Elementorum librum*, pp. 200–210. The proof of existence of an equilateral triangle based on Euclid's construction in *Elements* I is explicitly resolved into four concatenated syllogistic deductions at *Daneshname*, I, 19; see Achena and Massé (1955, pp. 57–59). The same analysis is adopted almost verbatim in Baḥmanyār, *Taḥṣīl* II, 17, pp. 169.1–170.4. The notion that the existence of triangles (and other figures) implies the existence of circles as the most fundamental type of plane figure is discussed by Avicenna in *Ilāhiyyāt* III. On geometrical constructions as existence proofs, see Zeuthen (1896).

What-questions concerning the essence of the attribute are in turn posterior to simple-if questions concerning its existence without qualification. Since the latter are identical with the corresponding compound if-questions, the taxonomy of what-questions and if-questions is the same for both attributes and subjects. The relative order of inquiry presupposes a what-question concerning the meaning of the predicate, a simple if-question concerning its existence without qualification (which is the same as a compound if-question concerning its belonging to a subject), and a what-question concerning the essence of the predicate. Therefore, the two what-questions concerning the predicate, namely what the predicate means and what it essentially is, stand to the compound if-question of whether the predicate belongs to the subject (the existence of the subject in a state, expressed by the predicate, which is in turn required for the predicate to exist without qualification) in the same way as the two what-questions concerning the subject, namely what the subject means and what it essentially is, stand to the simple if-question of whether the subject exists without qualification.

There is a significant structural similarity in the order of inquiry of subjects and predicates of scientific categorical statements. In both cases, questions of existence presuppose knowledge of the meaning of the terms involved and are in turn presupposed in the process that leads to knowledge of the essences and definitions of the entities corresponding to those terms. Furthermore, the two paths are connected in virtue of the fact that simple if-questions concerning the predicate translate into compound if-questions involving both terms (and presuppose in turn the existence of the subject without qualification, among other things).

What-questions concerning the middle term are also posterior to if-questions, but in this case Avicenna introduces a distinction between (i) potential and (ii) actual knowledge and appeals once more to the distinction between merely providing an inferential justification of the conclusion and providing, in addition to that, its factual explanation too.

To possess actual knowledge of the explanatory middle term that shows why an attribute actually belongs to a subject is the ultimate goal of the process of inquiry. But knowledge of a compound if-question for Avicenna presupposes knowledge of the existence of a middle term. To establish a fact as an *explanandum* is to ask whether there is a middle term that necessitates the conclusion expressing that fact. And this can either mean seeking an inferential justification that makes the nexus between minor term and major term *necessary for the mind* (or, to put it otherwise, something that makes the *assertion* necessary) or seeking a causal explanation that also expresses the real underlying cause of the nexus obtaining in reality between subject and attribute.[26]

26. The distinction is formulated, at *Burhān* IV, 1, p. 263.5–7, in terms of seeking "the cause of the deduction relative to the fact that it is a deduction, that is to say, *any* middle term whatsoever, or the

As a result, in the case of middle terms the taxonomy of inquiry is unsurprisingly connected with some of the questions we have already encountered. In particular, a compound if-question asking whether a predicate belongs to a subject entails a simple-if question asking whether there is a middle term connecting subject and predicate in a first-figure deduction (for the inferential justification of the conclusion), and a why-question asking in virtue of what the predicate belongs to the subject entails a what-question concerning the middle term that explains that connection.[27]

The relative order of (i) the compound if-question, which asks whether a predicate belongs to a subject, and (ii) the why-question, which asks why the predicate belongs to the subject, corresponds to the relative order of (iii) the simple if-question concerning the existence of *a* middle term, which justifies the conclusion *that* the predicate belongs to the subject, and (iv) the what-question relative to *the* middle term, which also provides the explanation of why the predicate belongs to the subject. Thus, compound if-questions and why-questions involving a subject and an attribute stand to one another in the same way as simple if-questions and what-questions concerning the middle term. The latter is in fact not a what-question aiming at the essence of the middle term, but rather a question aiming to identify the middle term that provides the answer to the proper what-question relative to the essence of the predicate, which explains why the predicate belongs to the subject. A simple if-question concerning the middle term is entailed by or potentially contained in the compound if-question asking whether the predicate belongs to the subject, whereas a what-question concerning the middle term is entailed by or potentially contained in the why-question asking why the predicate belongs to the subject.

Demonstrative knowledge of the conclusion in a causal case ultimately requires the middle term to be known in act. When the middle term is known, demonstrative knowledge that S is P and why S is P is attained. Thus, the types of scientific questions concerning the middle term are analogous to those previously described for the subject term and the predicate term of demonstrative deductions, the only difference being that what-questions in the case of the middle do not aim at the definition of the middle but at the term itself.[28]

THE ORDER OF INQUIRY (II)

Avicenna's account of the order of inquiry in *Burhān* IV, 1 involves four main types of questions: simple if-questions, compound if-questions, what-questions,

cause of the deduction relative to the fact that it is a demonstration, that is to say, *the* middle term which is the cause of the thing in itself."

27. The "what" in question seems to express a form of knowledge that identifies something among other things rather than knowledge of essence. In other words, the point is to identify a term M, which is the essence of another term P, not to ask a further question about the essence of M.

28. The ambivalence of the Greek *ti* corresponds to the ambivalence of the Arabic *mā*.

TABLE 6 Types and order of inquiry

Order	Inquiry	What is sought	
(1)	What-question (S)	Meaning of the subject	
(2)	Simple if-question (S)	Existence of the subject without qualification	
(3.1)	Why-question (S)	Inferential justification of the existence of the subject without qualification	Assertion (Antecedent)
(3.2)	Why-question (S)	Cause of the existence of the subject without qualification	Fact and assertion (Causal antecedent)
(4)	What-question (S)	Essence of the subject	
(5)	What-question (P)	Meaning of the predicate	
(6.1)	Compound if-question (S-P)	Existence of the predicate in the subject (belonging)	
(6.2 = 6.1)	Simple if-question (P)	Existence of the predicate without qualification	
(7.1)	Why-question (S-P)	Inferential justification of the existence of the predicate in the subject (belonging)	Assertion (Middle term)
(7.2 = 7.1)	Simple if-question (M)	Existence of a middle term without qualification	
(7.3)	Why-question (S-P)	Cause of the existence of the predicate in the subject (belonging)	Fact and assertion (Causal middle term)
(7.4)	What-question (P)	Essence of the predicate	
(7.5 = 7.4)	What-question (M)	Identification of the causal middle term	

and why-questions, instead of the three presented in I, 5 (if-questions, what-questions, and why-questions). The main difference is that in the first list, if-questions are treated as one category with a twofold internal division, rather than as two separate categories. Apart from the need to follow more closely the text of *An. Post.* B1–2, a reason for the discrepancy may be Avicenna's willingness to discuss the relations of priority and posteriority in greater detail in IV, 1. Be this as it may, what emerges in this chapter is a rather impressive account of the process of inquiry, ranging from the simple first grasp of a term's meaning to non-noetic, demonstrative scientific knowledge of predicative assertions based on noetic scientific knowledge of the essences of the terms involved. The relations holding among the four types of scientific inquiry analyzed in IV, 1 and their internal subdivisions are summarized in table 6.

Avicenna's classification of the types of scientific inquiry in *Burhān* IV, 1 ultimately associates two irreducible cases (if-questions and what-questions) to the two modes of scientific knowledge encountered at the beginning of this study, namely assertion and conception. Non-noetic, demonstrative knowledge of existential and predicative claims corresponds to the domain of if-questions, assertion, and deduction. Noetic knowledge of essences corresponds to the domain of what-questions, conceptualization, and definition. While the two domains represent distinct and fundamentally different—albeit complementary—types of scientific knowledge, with different kinds of "objects"—propositions in one case, concepts in the other—they are importantly related to the extent that an adequate causal explanation of a theorem in a science, that is, an adequate answer to certain if- and why-questions, presupposes knowledge of essences and adequate answers to the relevant what-questions.

PART II

The Organization of Scientific Knowledge

Avicenna's theory of science is concerned with two complementary aspects of the organization of scientific knowledge: the internal structure of a science and the division and hierarchy of the sciences with their network of mutual relations. These two problems inspire some of the most significant innovations in his interpretation and reform of the Aristotelian logic of scientific discourse. This area seems to be of such great importance to Avicenna for three reasons. First, as noted, his project of recalibration of Aristotle's *Posterior Analytics* is primarily motivated by its distinctive problems of applicability, namely the limited expressive power of its underlying logic and the insufficient extent to which certain critical details of the theory are developed in that work. Second, the internal structure of the sciences and, more importantly, their architectonic organization as a complex system based on the relative generality of their subjects, are reflections of the hierarchical structure of the universe, the scientific knowledge of which is, in turn, the ultimate goal of the intellectual life and the realization of human flourishing. Third, Avicenna's account of the internal structure of a science is the blueprint he uses, above all else, for his own reform of Aristotelian metaphysics, while his analysis of the hierarchical division of the sciences is the framework for the clarification of the relation of metaphysics to the other sciences.

The internal organization of a science ideally, if not explicitly, conforms to a regimented structure involving three elements: subject, principles, and questions. Avicenna's discussion of principles presupposes multiple levels of analysis, ranging from a systematic translation of Aristotelian notions into the language of conception and assertion, to the reception of the traditional vocabulary of axioms, definitions, hypotheses, and postulates, to the distinction between common and proper principles. The immediate principles of a science include (i) definitions, understood primarily as conceptions of essences like "Mortal rational animal" or "Trilateral plane figure" and, in a derivative sense, as propositional formulations of the latter (for example, "Every human is a mortal rational animal" or "Every triangle is a trilateral plane figure"); (ii) assertions of existence; and (iii) assertions about immediate implicates. Every science presupposes knowledge of the existence as well as of the essence of its subject. The subject of a science may be characterized by different forms of unity (for example as a simple kind or as a complex kind qualified by certain attributes). Avicenna explores in detail the relation between the subject of a science and the subjects of the scientific propositions pertaining to that science. Far from being just a simple foray into uncharted territory of Aristotelian epistemology, his analysis is the consequence of a deliberate attempt to develop systematically a defective aspect of Aristotle's theory. This is even more evident in the case of Avicenna's discussion of the logical form of scientific questions, that is to say, the scientific propositions that constitute the theorems of a science. First, scientific propositions are not restricted to categorical assertions but include conditional and disjunctive statements too. Second, the analysis of the types of subjects and predicates of categorical propositions offers a glimpse of Avicenna's serious commitment to the idea that an adequate theory of science must be worked out down to its last detail, including a fine-grained classification of different types of demonstrations based on the types of terms occurring in their premises and conclusions (chapter 4).

A science may also stand in a variety of relations to the other sciences. The nature of such relations is determined in each case by whether or not two sciences have subjects, principles, or questions in common. Avicenna engages in an exercise of detailed classification of all the ways in which two sciences may be related, ranging from distinctness to parthood, including different kinds of subordination. As a result, there emerges a complex picture of the general architectonic structure of scientific knowledge as a whole, characterized by vertical, horizontal, and "oblique" relations. The *locus classicus* for this analysis is *Burhān* II, 7, which is the only chapter of the book ever to be translated into Latin (it was famously incorporated by Dominicus Gundissalinus [fl. ca 1150] as an independent section titled *Summa Avicennae de convenientia et differentia scientiarum* in his own *De divisione philosophiae*). If taken in isolation, *Burhān* II, 7 might perhaps give the impression of being no more than a self-contained, stand-alone scholastic exercise of classification, with little to no textual connection to the *Posterior Analytics*. This approach, however, would betray a misconception of its role in the general economy of the *Burhān* and of Avicenna's motives for presenting his own views on the interrelation among the sciences precisely at the point where they are found. The chapter fits perfectly in the fabric of the *Posterior Analytics*, placed as it is (almost as an inevitable *ekphrasis*) right after Aristotle—having just introduced, at *An. Post.* A7, 75a38, his famous ban on kind crossing—stops short of exploring its implications in their full scope. The division and hierarchy of the sciences accommodates all major philosophical and scientific disciplines known to Avicenna and culminates in an explicit characterization of the role of metaphysics as a master science to which all other sciences are essentially subordinated. The division of the sciences is based on a hierarchy that governs the corresponding regions of being, a theme that is at the confluence of various other crucial lines of inquiry in Avicenna's theory of science. Most notably, it presupposes a theory of subjects and attributes, which in turn is essentially dependent on his theory of definition. This is because sciences are distinguished first and foremost in virtue of their subjects, and their mutual relations are characterized typically in terms of the relations holding among their subjects. What identifies the subject of a science for Avicenna is a complex question (or rather, a simple question with a complex answer) and some of the most subtle cases of subordination involve combinations of natural kinds and per se attributes. In summary, all of the basic building blocks of Avicenna's theory of science are visibly at work in a sophisticated account of the architectonic structure of scientific knowledge that lays bare its essentialist foundations (chapter 5).

4

The Internal Structure of a Science

THE ELEMENTS OF A SCIENCE

The internal structure of a science is determined by three elements: (i) its principles (*mabādiʾ*), (ii) its subject (*mawḍūʿ*), and (iii) its questions (*masāʾil*).[1] The main source for Avicenna's account of the elements of a science is *Burhān* II, 6, which develops systematically—and in much greater detail—three germane notions introduced by Aristotle at *An. Post.* A7, 75a39–b2.[2] In *Burhān* II, 6, Avicenna first introduces the three elements in general and then analyzes them in turn. Starting from principles (II, 6, pp. 155.10–157.4), he then briefly classifies the types of subjects of various scientific disciplines (II, 6, p. 157.5–19) before dealing extensively with the topic of scientific questions, that is to say, the propositions demonstrated in a science (II, 6, pp. 157.20–161.9). The discussion of scientific questions focuses in particular on the nature of the terms into which categorical scientific propositions may ultimately be analyzed, namely their subjects and predicates. The key issues addressed in the chapter are the classification of different types of principles, the criteria of unity for the subject of a discipline, the logical structure

1. The three elements are also listed at *Burhān* III, 1, p. 191.16–21; cf. also *Ilāhiyyāt* I, 1, p. 5.1–4 (where Avicenna's explicit reference to the "*Book of Demonstration* from the logic," is in all likelihood to *Burhān*, II, 6), and *Išārāt*, IX, 3, pp. 82.11–83.9. For a synopsis of the places where Aristotle discusses the elements of a science, see Barnes (1993, p. 143); on Aristotle's account of the elements of a science, see Gómez-Lobo (1977, 1981), Hintikka (1972), and Frede (1974). For an analysis of the terminology in Avicenna and Alfarabi, see Eichner (2010).

2. See also *An. Post.* A10, 76b11–22; cf. *An. Post.* A28, 87a38–39.

of scientific propositions, and the different types of subjects and predicates of categorical scientific propositions. At the opening of the chapter, Avicenna writes:

Text 4.1: *Burhān* II, 6, p. 155.1–9

We say that all disciplines, especially the theoretical ones, have [(a)] principles, [(b)] subjects, and [(c)] questions.

[(a)] Principles are the premises (*muqaddamāt*) from which a discipline demonstrates without them being demonstrated in that discipline, either [(aa)] because they are evident or [(ab)] because they are of too high a status (*ǧalāla*) to be demonstrated in [that discipline] (it is rather in a science above it that they are demonstrated) or [(ac)] because they are of too low a status to be demonstrated in that science (rather, [they are demonstrated] in a science below it, though this case is rare).[3]

[(b)] Subjects are the items the states pertaining to which (*al-aḥwāl al-mansūba ilayhā*) and the per se accidents of which (*al-ʿawāriḍ aḏ-ḏātiyya lahā*) are just what a discipline investigates.

[(c)] Questions are the propositions (*qaḍāyā*) whose predicates are per se accidents of this subject, of its species, or of its accidents, that is to say those items about which doubts arise and whose state is consequently clarified in that science.[4]

Thus, principles are the items *from* which (*minhā*), questions are the items *of* which (*lahā*), and subjects are the items *about* which (*ʿalayhā*) demonstration is. It is as if the purpose of "that about which" demonstration is, were the per se accidents, while "that for the sake of which" [demonstration is], were the subject, and "that from which" [demonstration is], were the principles.[5]

In Text 4.1, Avicenna draws a series of important preliminary distinctions, setting the stage for the analysis of the internal structure of a science. First, he lists the three types of elements: principles, subject, and questions. Second, he offers a brief description of their own internal divisions. The principles of a science are identified, in this context, as its unproven assumptions, that is to say, the propositions that serve in a science as premises for the demonstration of other propositions. A characteristic feature, which is common to all principles in a science, is the fact that they are not demonstrated in that science. This may be due to different reasons. A principle may be either an indemonstrable proposition without

3. This case is discussed at *Burhān* II, 9, pp. 179.14–180.6 (see Text 10.5); cf. also *Ilāhiyyāt* I, 3, pp. 19.10–20.18. I return to it in chapter 10.

4. Reading *fa-yastabīnu* with S for *fa-yastabara'a*.

5. In *Burhān* II, 10 (the first part of which corresponds to *An. Post.* A10), Avicenna returns to the distinction of the three elements, with a variation: principles, subjects, and per se predicates or attributes. In the transition from the context of *An. Post.* A7 to that of *An. Post.* A10, the shift from questions (*masā'il*) to per se predicates (*maḥmūlāt*) or attributes (*ʿawāriḍ*) in Avicenna tracks a similar shift from conclusion (*sumperasma*) to per se attributes (*pathē*) in Aristotle. See Bertolacci (2006, p. 134) for an alternative translation of Text 4.1 (the major difference is the notion of "essential accident," which I translate as per se, and a slightly different understanding of the last sentence).

qualification, assumed in virtue of its evident character, or a demonstrable proposition that simply turns out not to be demonstrated in the science in which it serves as a principle, in which case it is demonstrated by another science. The rank of the science in which propositions that are principles in this relative sense are demonstrated may be higher or lower than the rank of the science relative to which those propositions are principles. The meaning of this hierarchical distinction will become clearer in chapter 5. Suffice it to say here that Avicenna's view is that every scientific proposition has a natural place in the hierarchical order and arrangement of the comprehensive system of truths that reflect the structure of reality, and that depending on the nature and status of its terms, it may be appropriate for it to be demonstrated in a science that occupies a higher rank in the hierarchy of the sciences, or rather in one that occupies a lower rank. Certain principles are truly fundamental and genuinely indemonstrable. Other principles are, absolutely speaking, theorems of other sciences but can be assumed without proof elsewhere. The question of which science demonstrates them depends roughly on the generality of their terms.

In every science, both principles and propositions that are derived from principles are ultimately concerned with certain entities and their attributes. The perimeter of a science and what the science investigates are defined by its subject. The subject of a science is, broadly speaking, the set of entities of which that science studies the "states" and "per se attributes." The states and per se attributes of the subject(s) of a science are sought and established by demonstration. The object of scientific demonstration in a science is its questions, that is to say, the *propositions* that the science demonstrates on the basis of its principles. Those propositions, as we shall see shortly, may have different logical forms, but the most fundamental kind of scientific question in Avicenna's theory of science is a categorical proposition that expresses a non-immediate, demonstrable nexus between a subject and one of its per se attributes. Epistemically, the questions of a science—unlike principles without qualification, which are self-evident, and principles in a relative sense, which are assumed in that science and demonstrated elsewhere—cannot be asserted with certainty (and are therefore potentially subject to doubt) unless they are established by demonstration. Thus, questions, in and of themselves, encapsulate the set of non-immediate, non-self-evident facts of a science that admit of (and therefore require) causal explanation. Certainty, in this case, is only attained by demonstration.

PRINCIPLES

Avicenna's analysis of the internal structure of a science begins with an account of various types of principles. Generally speaking, Avicenna's treatment of principles involves four levels of analysis and offers a clear example of the hybrid strategy, combining exegetical and systematic contributions, that is characteristic of his approach to the Aristotelian tradition. The first level involves the translation of the Aristotelian notion of principle into the language of conception and assertion in *Burhān* I, 1, discussed in chapter 1. The second level coincides with the elaborate taxonomy of the types of assertions that may serve as deductive principles in *Burhān* I, 4, discussed in chapter 2. The third level is directly concerned with Aristotle's technical vocabulary of axioms, posits, definitions, postulates, and hypotheses. The fourth level addresses the Aristotelian distinction between common and proper principles. The process of appropriation of Aristotle's terminology takes place most prominently in *Burhān* I, 12 and II, 9–10, namely in the counterparts of *An. Post.* A2 and A9–10.[6] The distinction between (i) common and (ii) proper principles, by contrast, is explicitly addressed in *Burhān* II, 6:[7]

> Text 4.2: *Burhān* II, 6, p. 155.10–14
> We say that principles are in two ways: either [(a)] principles proper to an individual science, as [(aa)] the belief in the existence of motion for natural science and [(ab)] the belief in the infinite divisibility of every extended magnitude for mathematics; or [(b)] principles common [to more than one science].[8] The latter fall into two divisions and may be either [(ba)] common without qualification to every science, like our statement "Of everything either the affirmation or the negation is truthfully asserted" or [(bb)] common to a number of sciences, like our statement

6. On principles in Aristotle, see Gómez-Lobo (1977, 1978, 1981), Gotthelf (1987), Irwin (1988), and Landor (1981).

7. The distinction between common and proper principles is adumbrated at *An. Post.* A10, 76a37–b11. Avicenna's discussion seems significantly indebted to Alfarabi, *Burhān* IV, 1 pp. 60.16–61.20. Alfarabi draws a distinction between principles that are proper to a discipline (for example, "Five is an odd number" in arithmetic, though the question arises of whether this is a genuine example of a principle rather than a theorem) and principles that are shared by several disciplines. In the latter case, a principle may be shared by all disciplines or by some disciplines only. Examples of principles shared by all disciplines include the law of the excluded middle and the contention that, when equals are subtracted from equals, the remainders are equal. These may be taken specifically to apply to the subject of a discipline (for instance to extended magnitude in the case of geometry) or without qualification. In particular, Avicenna seems to regard all exhaustive and exclusive oppositions as specific instances of the law of the excluded middle. Alfarabi considers also the case of a shared predicate: "Things that coincide are equal." Coincidence (*inṭibāq*) applies to extended magnitudes (*maqādīr*) only, while equality applies to numbers and magnitudes (*ʿiẓam*) alike. Further examples include "This is either equal or greater or lesser than that line" and "Alternating proportions are proportional" (a : b = c : d implies a : c = b : d; that is to say, if things that are proportional alternate, they remain proportional), *Elements*, V, Prop. 16; the last statement is part of the theory of proportions elaborated by Eudoxus and reworked by Euclid in *Elements* V.

8. On the principles of natural philosophy, see *Samāʿ ṭabīʿī* I, 2, where they are treated "In the manner of postulates and posits" (*ʿalā sabīl al-muṣādara wa-l-waḍʿ*). On the metaphysical proof of the divisibility of body (which cannot be ascertained by mere observation), see *Ilāhiyyāt* II, 2, pp. 65.4–66.17.

"Things that are equal to one and the same thing are equal" (this principle is shared by geometry, arithmetic, astronomy, music, and so forth).[9]

In Text 4.2, Avicenna distinguishes three kinds of principles: proper to a science, common to several sciences, and common to all sciences. The last two are traditional versions of Aristotelian axioms, while the first encapsulates different kinds of assumptions relative to an individual science. In the characterization of principles that are proper to a science, Avicenna tacitly relies on a critical distinction between existential and predicative assumptions, which is the counterpart of the distinction between simple if-questions and compound if-questions discussed in chapter 3.[10] And the language of belief suggests that the focus continues to be on principles of assertion. In the examples, the belief in the existence of motion is the basis for an assertion of existence, while the belief in the infinite divisibility of extended (continuous) quantities is the basis for a predicative assertion, in which infinite divisibility is the attribute and extended quantity the subject.

What is it that ultimately determines whether or not a principle is proper to an individual science? Avicenna's answer indirectly gives us a glimpse of his sophisticated understanding of the internal taxonomy of the terms of a scientific theory and of their mutual relations:

Text 4.3: *Burhān* II, 6, p. 156.3–6
Proper principles whose subjects are either [(i)] the subject of the discipline or [(ii)] the species, [(iii)] the parts, or [(iv)] the proper accidents of the subject of [the discipline] are principles proper to the discipline, regardless of whether their predicates are proper to the subject or proper not to it but to its genus.[11]

9. At *Ilāhiyyāt* I, 8, p. 48.8–14, Avicenna characterizes the law of the excluded middle in the following terms:

> Statements for which it is most right to be true are those that are always true. The truest of all these [statements] is the one whose truth is primary and does not have a cause. The first of all these true statements—the one at which everything comes to a stop in the analysis, so that it is said, in act or in potency, of anything that is proved or that becomes plain by means of it, as we explained in the *Book of Demonstration*—is that there is no middle (*lā wāsiṭa*) between affirmation and negation. This property falls within [the domain of] the accidents of nothing other than the existent qua existent, because it is common to every existent.

A significant part of the discussion concerning "the cure of the perplexed" (at *Ilāhiyyāt* I, 8, pp. 50.8–53.12) involves a discussion of this principle, which is a development of the material in Aristotle's *Metaphysics* IV. The law of the excluded middle is also characterized, at *Ilāhiyyāt* I, 8, p. 53.16–17, as "the first principle of demonstrations" and its defense is said to be the task of the metaphysician.

10. See also *An. Post.* A10, 76b3–11.

11. The case is illustrated by the examples of magnitude for geometry, equality (in number) for numbers, and contraries in natural philosophy.

In Text 4.3, Avicenna briefly introduces various interdependent themes. First, the subject of a principle, that is to say, the subject of a scientific proposition, may be identified with various kinds of items, namely the subject of the discipline itself, a species of the subject of the discipline, a part of the subject of the discipline, or a proper accident of the subject of the discipline (I return later to this set of possible candidates in the discussion of scientific questions). Second, a principle whose subject falls into one of these categories is proper to its discipline. Third, the predicate of a principle proper to a discipline can, but does not have to, be proper to the subject (of the principle), for it can be proper to the genus of the subject, that is to say, the subject of the discipline (if the subject of the principle is a species of the subject of the discipline and presumably in the other cases too). Equality, for example, is an attribute of principles that are proper to arithmetic and geometry, even though it is a primary per se attribute of quantity, which is the genus of the subject of arithmetic (discrete quantity or number) and of the subject of geometry (continuous quantity or extended magnitude). The notion of per se (especially in the sense of per se 2 assumed in the example) and its internal divisions are discussed in detail in chapter 7. In a nutshell, for something to be a per se 2 attribute of a subject, its essence must somehow be dependent on the subject (or on something that is part of the essence of the latter). Moreover, in the proposition "Every even is divisible into two equals," the attribute "divisible into two equals" is proper not to the subject of the proposition (even) but rather to the genus of its subject (number or quantity), but this does not compromise the status of the proposition as a principle proper to arithmetic.

Common principles, by contrast, are used in two ways in scientific discourse, either in potency or in act. In the first sense, they are implicit, suppressed premises of a demonstration.[12] In the second sense, they are effectively employed as one of the elements of a deductive chain. An example of a principle used only in potency is the tacit assumption of the law of the excluded middle to show, for instance, that if *p* is not true, then *not-p* is true. This is not established by an argument in which the law of the excluded middle is explicitly stated as a premise, but simply assumed as a consequence of it (Avicenna takes the law of the excluded middle to be so widely accepted, due to its self-evident character, as to be generally omissible, except in the rejection of sophistical reasoning or in the context of a refutation). An example of a principle used in act, by contrast, is the contention "Every

12. The related contention that principles are not the same for all deductions is discussed at *Burhān* III, 8, pp. 251.1–255.9 (on *An. Post.* A32). In particular, at *Burhān* III, 8, p. 254.17–18, Avicenna draws a rare (if welcome) distinction between principles in the sense of propositions—principles "from which" (*minhā*) being understood as first premises—and principles in the sense of terms—principles "about which" (*fīhā*) being understood as subjects or attributes. The distinction explicitly attests to the use of the term "principle" in a nonpropositional sense. At *Burhān* III, 1, pp. 190.4–191.15, Avicenna discusses three different applications of common principles in act.

extended magnitude is either commensurable or incommensurable," which is a particular instance of the law of the excluded middle involving an exhaustive and exclusive conflict between incompatible terms of geometry.[13] Finally, proper principles may also be proper to the questions of a science, and this in two ways: either with respect to that science as a whole or with respect to one or more of its questions.[14]

The Vocabulary of Principles

Avicenna's conceptual vocabulary for principles includes a variety of expressions. The most common among them are

(i) propositions of certainty (*yaqīniyyāt*): *Naǧāt* I, 129;[15]
(ii) common principles (*mabādiʾ ʿāmma*): *Burhān* II, 6; III, 9; *Naǧāt* I, 129;
(iii) necessarily accepted propositions (*wāǧib qubūluhā*): *Burhān* I, 12;
(iv) primary propositions (*awwaliyyāt*): *Burhān* II, 3; *Naǧāt* I, 111 and I, 122;
(v) self-evident principles (*mabādiʾ bayyina bi-anfusihā*): *Burhān* II, 10; III, 1; I, 11;[16]

13. See *Naǧāt* I, 126 and I, 129; cf. also Alfarabi, *Burhān* IV, 1, p. 61.7: (a) "The diagonal is either incommensurable or commensurable with the side" and (b) "The diagonal cannot be at the same time incommensurable and commensurable," where the examples are discussed in connection with the law of the excluded middle. At *Burhān* II, 6, pp. 156.17–157.2, Avicenna seems to imply that the specification of a proper principle may occur either with respect to the predicate or with respect to the subject (for example, an application of the familiar equality principle to the case of magnitudes on the side of the subject involves a transition from "Everything equal to such-and-such is such-and-such" to "Every magnitude equal to such-and-such is such-and-such"). Avicenna's distinction between two ways in which common principles may be used, that is to say, either in potency or in act, is illustrated by "Every magnitude is commensurable (*mušārik*) or incommensurable (*mubāyin*)." Here the law of the excluded middle is specified to reflect an opposition between incompatible predicates proper to the subject of geometry.

14. Indirectly, this point is also relevant for the discussion, in *Burhān* II, 9 and III, 3, of the case in which a question of a science may be subordinated to (a principle of) another science. I return to the issue in chapter 10.

15. Propositions of certainty include not only common axioms but a broader class of truths, for example propositions that are primary, definitional predicative assertions, or immediate propositions based on experience or perception.

16. Examples of assertions that are qualified as self-evident are not infrequent in Avicenna's corpus. A remarkable example is at *Samāʿ ṭabīʿī* III, 4, p. 196.8–10 (McGinnis 2009, p. 299, transl. modified): "One of the things necessarily forced on them in a way that is obvious to anyone with an ounce of intelligence, is the well-known fact that, when two things are opposite one another and moving toward each other until they meet, and there is absolutely no external obstacle that prevents their meeting, then the two can simultaneously move until they encounter one another [. . . .] When they encounter one another, they can obstruct one another, but before that, there is no obstacle between them. This is something that is *self-evident*." Along similar lines, see also *Samāʿ ṭabīʿī* III, 4, p. 193.5–6 (McGinnis 2009, p. 299): "One thing that is known with certainty, about which there is no doubt or difference of

(vi) immediate propositions: *Burhān* I, 12; and
(vii) first principle (*mabdaʾ awwal*): *Burhān* III, 1.

In *Burhān* II, 10, Avicenna draws a distinction between principles that are self-evident and principles that require proof (in the context of a brief argument aiming to show that principles of science A cannot be proved in science A, because they can only be either self-evident or proved in another science, and hence depend on further principles other than themselves). Principles that are not self-evident and stand in need of proof are proved by a different science, typically one that is hierarchically superordinate to the science of which they are principles. This confirms the general point introduced in Text 4.1. But in *Burhān* II, 10, Avicenna goes on to identify the other possible, though far less frequent, case:

> Text 4.4: *Burhān* II, 10, p. 184.4–6
> An explanatory proof (*bayānuhā*) of the self-evident [principles] (*al-bayyināt bi-anfusihā*) is impossible either in the science [of which they are principles] or in another science. The explanatory proof of those that are not self-evident, by contrast, is possible only in another science, and especially in a higher science. Most of the principles of the most general science, to which the other sciences are subordinated, are self-evident, while some are assumed from some of the particular sciences that fall under it, as we have said (this case is rare).[17]

The crucial contention advanced in Text 4.4 is directly connected to a classification of the types of explanation across sciences and subordination developed in *Burhān* II, 9 and *Ilāhiyyāt* I, 3, which I return to in chapter 10. The "most general science, to which the other sciences are subordinated" is metaphysics, and most of the principles of metaphysics are unsurprisingly taken by Avicenna to be self-evident truths. But even in metaphysics not all principles are self-evident. And when a principle is not self-evident, it must be proved in another science. Since no science (in virtue of the generality of its subject) can be higher than or even of the same rank as metaphysics, any proposition borrowed by metaphysics from another science must be a theorem demonstrated in a science lower than metaphysics, that is to say, in one of the particular sciences.[18]

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opinion, is that there is a certain projected path between any two things having some placement such that when we make a straight line between them, it either fills that projected path or occurs along it."

17. Text 4.4. seems to make the proof of the principles of a science (in the relative sense) the exclusive prerogative of another science, in line with Text 4.1, thereby excluding the case of postulates, which, in *Burhān* I, 12, are said to be potentially subject to proof in the same science but at a later stage (this peculiar characteristic of postulates is confirmed independently at *Naǧāt* I, 128).

18. This case raises a potential problem of circularity, if metaphysics at the same time proves the principles of a subordinate science and borrows conclusions from that subordinate science to use them as principles of its own demonstrations. In *Ilāhiyyāt* I, 3, Avicenna shows in detail how to circumvent the difficulty. I discuss his solution in chapter 10.

Principles of demonstration are also discussed in *Burhān* I, 12, which is the direct counterpart of *An. Post.* A2. In that context, Avicenna distinguishes two ways to characterize the principles of demonstrative scientific knowledge. The discussion fits (almost) perfectly in the scheme of things described thus far. A principle of demonstration may be such either

(i) without qualification, in which case Avicenna calls it an "absolutely immediate premise" (*muqaddama ġayr ḏāt wasaṭ ʿalā l-iṭlāq*), or
(ii) with respect to a given science, in which case the principle is immediate only in a relative sense.

Principles of the first type are such that there is no further explanation (*bayān*) for the affirmative or negative nexus that governs the relation between subject and predicate; they simply hold as such and do not depend on further middle terms or premises.[19] Their acquisition is purely intellectual, and principles of this type are called "common knowledge" (*al-ʿilm al-mutaʿāraf*) or "necessarily accepted premises" (*al-muqaddama al-wāǧib qubūluhā*).[20] Principles of the second type, by contrast, are in themselves non-immediate and depend on further middle terms or premises, because the nexus that governs the relation between subject and predicate is not primitive and uncaused but rather derivative and caused. Principles of this kind, however, are posited in a science and treated as *immediate in that science*, which means that within the logical structure of that science there is *no* middle term or premise on which they depend for their proof (except in the case of postulates, which may be assumed earlier and proved later in the same science, provided that no circularity is involved in their proof). The middle term may be given either previously in another science that is hierarchically above the science in which the principle is assumed, or in another science that is of the same rank as the first, or in the same science but at a later stage (which is the case of postulates).[21] The vocabulary of principles in *Burhān* I, 12 includes the following four main types:

19. An incidental remark at *Burhān* I, 12, p. 110.3—"whether the relation of the predicate to the subject of [an immediate principle] be affirmative or negative (*kānat īǧāban aw salban*)"—shows that Avicenna is explicitly committed to immediate negative principles, in line with the doctrine of *An. Post.* A15 adopted in *Burhān* III, 4.

20. The terminology is used in Alfarabi, *Burhān* V, 3, p. 87.14–16, which lists four types—(i) *yaqīniyya*, (ii) *ḥudūd*, (iii) *uṣūl mawḍūʿa*, and (iv) *muṣādarāt*—and notes that it is customary for the practitioners of logic (*aṣḥāb al-manṭiq*) to call the non-*yaqīniyya* "posits" (*awḍāʿ*) and the *yaqīniyya* "necessarily accepted premises" (*al-muqaddamāt al-wāǧib qubūluhā*).

21. At *Samāʿ ṭabīʿī* I, 5, pp. 30.17–31.6 (cf. *Phys.* A1, 193a3), Avicenna qualifies Aristotle's criticism of the idea that certain principles may be demonstrated. For example, the existence of nature (in the sense of principle of motion or power) is not self-evident, and its proof is a task "only for the metaphysician, whereas the task of the natural philosopher is the study of its essence." The corresponding proof in

(i) definitions
(ii) premises that are necessarily accepted
  (i) immediately by the intellect
  (ii) on the basis of perception or experience
  (iii) by a deduction that is intuitive for the intellect (*badīhī fī l-ʿaql*)
(iii) hypotheses (not used in all sciences but only in some)
(iv) postulates.[22]

The classification in *Burhān* I, 12 reflects more closely the terminology of *An. Post.* A2 and therefore also includes (unlike *Burhān* II, 6) principles that are primarily nonpropositional, like definitions, as well as the distinction between hypotheses and postulates. Interestingly, Avicenna also associates different disciplines with particular classes of principles, presumably based on their degree of abstractness. For instance, arithmetic is said to employ only definitions and primary propositions, while geometry and natural philosophy use in addition all other kinds of propositions. The classification is also translated into the language of assertion: the assertion of principles is not only prior to (*aqdam*) but also firmer (*ākad*) and better known (*aʿraf*) than the assertion of the conclusion, and the assertion of the falsity of the opposite of a principle is, by the same token, stronger than the assertion of the falsity of the opposite of the conclusion.

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metaphysics is at *Ilāhiyyāt* IX, 5. Even in metaphysics it is possible for something assumed at an early stage only to be properly demonstrated at a later stage. For example, the impossibility of an infinite regress in the chain of causes is mentioned at *Ilāhiyyāt* I, 6, p. 39.13–14 as one of the reasons possible existents only exist when they are necessitated by their cause and in relation to their cause. But that impossibility, at such an early stage of metaphysical inquiry, is "still doubtful" (*hāḏā fī hāḏā al-mawḍiʿ baʿdu maškūk fī iḥālatihī*) and will only be established demonstratively at a later stage (*Ilāhiyyāt* VI, 2, VI, 5, and VIII, 1–3).

22. Avicenna discusses the distinction between hypothesis and postulate from *An. Post.* A10, quoting almost verbatim, at *Burhān* I, 12, p. 113.5–10, a sizable passage from Abū Bišr Mattā's translation (A10, 76b26–31). A summary of the distinction between the two types of principles is given at *Burhān* I, 12, p. 114.7–11, where the relevant example is Euclid's parallel postulate. In the same context, Avicenna criticizes a reading of the distinction between hypothesis and postulate that may be traced to Philoponus, *In An. Post.* A10, 127.33–130.9. According to this view the distinction would turn on whether a little reflection (*ta'ammul*) or thought (*fikr*) is or is not required to assert a principle in one sense or the other. Among other things, Philoponus mentions the possibility of using a compass to establish that a certain figure is a circle (because all of its points are equidistant from a center). Avicenna categorically rejects both contentions: the former as a spurious distinction (*Burhān* I, 12, pp. 113.10–114.6), and the latter because it is blatantly circular (no pun intended), as the use of a compass adds nothing to one's knowledge of what a circle is and to the consequent ability to recognize whether a figure is a circle or not (*Burhān* I, 12, pp. 114.16–115.13). Hypothesis is occasionally used in a broader sense. For example, at *Samāʿ ṭabīʿī* II, 8, p. 126.18–19, Avicenna characterizes a counterfactual conditional (if the void, conceived of as a dimension, existed in potency, then before its existence, something would exist in a certain nature that is receptive to the existence of that dimension) in the following terms: "Let the natural philosophers concede this as a hypothesis (*aṣl mawḍūʿ*)."

SUBJECTS

The discussion of subjects is concerned with three different problems: (i) the structure of a scientific proposition (its logical form), (ii) the structure of a scientific discipline (the identification of the domain of a science), and (iii) the structure of scientific knowledge as a whole (the interrelation among the sciences).[23]

The systematic distinction between the subject of a science and the subject of a scientific proposition is crucial for various reasons. First, it clarifies a key requirement of an Aristotelian theory of science in terms that are far more precise than the mere generic acknowledgment of the fact that every science must have its own subject. Second, it offers a workable basis for the theory of per se predication. When we say that the subject is included in the essence or definition of a per se 2 attribute, what exactly do we mean by "subject"? What exactly is supposed to be included in the definition of the attribute? Is it the subject itself? If so, the set of per se attributes of a subject, if not altogether empty, would appear to be in most cases restricted to an exceedingly narrow range of items. For example, what could be a per se attribute of "product of an even by an odd," which is a perfectly legitimate subject term in an arithmetical proposition, if the whole subject has to be taken in the definition of the attribute? How could, for example, "even" be a per se attribute of that subject? Avicenna's idea is that for an attribute to be per se 2 of a subject, what is taken in its definition must always be a term that falls somewhere within a range delimited, at one end, by the subject of the science and, at the other, by the subject of the relevant scientific proposition in which the attribute occurs. Third, the distinction between the subject of a science and the subject of a scientific proposition helps in the ambiguous and vague transition from the general domain of a science (for instance number or extended magnitude) to specific scientific questions pertaining to that science. The subjects of scientific propositions refer to items in the domain of inquiry generally identified by the subject of the science, but what is the relation between the subject of a science and the subjects of certain specific scientific propositions in that science? Are there canonical types of subjects of scientific propositions? And if so, how are they classified? As we shall see shortly, Avicenna introduces a rather precise set of criteria to specify the ways in which the subjects of scientific propositions are related to or fall in the domain of the subject of a science, and examines the constraints such criteria impose on the set of admissible predicates.

23. On subjects, see Alfarabi, *Burhān* IV, 1, pp. 59.9–60.3, especially for his general account ("Subjects are the things that per se accidents belong to") and the internal division according to which three types of notions are relevant to the subject of a discipline, namely (i) the constituents of the definition of its subject, (ii) the species that fall under that subject, and (iii) the per se accidents of that subject.

The Subject of a Science

In his brief, general account of the nature of the subject of a science, Avicenna distinguishes various types of subjects based on different kinds of internal unity that may characterize the domain of inquiry of a given discipline. He writes:

> Text 4.5: *Burhān* II, 6, p. 157.5–14
> A science may have [(a)] a single subject (*mufrad*) like number for arithmetic; or it may have [(b)] [a subject that] is not single, having in reality several subjects that share something in virtue of which they are unified. This may occur in several ways. For [the subjects] may share [(ba)] a genus that serves as the unifying thing, in the way that line, surface, and solid share a genus in virtue of which they are unified, that is, extended magnitude; [(bb)] a continuous ratio holding among them (*munāsaba muttaṣila baynahā*), as in the case of point, line, surface, and solid; for the relation of the first to the second is like the relation of the second to the third, and that of the third to the fourth; [(bc)] a single end, as in the case of the subjects of medicine—I mean the elements, mixtures and mixes, limbs, faculties, and actions (if these subjects of medicine are taken not to be parts of a single subject)—for they [all] share their relation to health, and the subjects of ethics [for they all share] their relation to habit; or [(bd)] a single principle as in the case of the subjects of theology (*ʿilm al-kalām*); for the latter share their relation to a single principle, either obedience to the law (*ṭāʿat aš-šarīʿa*) or their being divine (*ilāhiyya*).[24]

In Text 4.5, Avicenna distinguishes two main cases. The subject of a science may be either *one* (with multiple items or species under it, some of which may in turn be qualified in various ways, without compromising the intrinsic unity of the domain itself) or *many*.[25] If the subject involves a multiplicity of things, there must be a unifying factor, which may be ontologically stronger or weaker, with four distinct sub-cases.

An additional characterization of the subject of a science, which will prove critical for a better understanding of the interrelations among the sciences discussed in chapter 5, involves a distinction between subjects that are genuine kinds and subjects whose identification depends on taking a kind along with one or more qualifications, that is to say, a kind accompanied by attributes of various sorts. He writes:

24. A closely related classification is at *Nağāt* I, 125, p. 135.2–10. On Avicenna's understanding of *kalām* and its place in the hierarchy of the sciences, see Gutas (2005).

25. On the distinction between general sciences (*ʿulūm ʿāmma*) and particular sciences (*ʿulūm ğuzʾiyya*), see Alfarabi *Burhān* IV, 1, p. 62.3–23; in the same work, on sciences whose subjects are specific kinds of existent (mathematics, natural philosophy, and metaphysics), see IV, 1, pp. 62.24–63.1; and on particular disciplines and sciences whose first subject is one (number for arithmetic) or many (point, line, surface, solid for geometry), see IV, 1, p. 63.13.

Text 4.6: *Burhān* II, 6, p. 157.15–19
Again, the subject of a science may be taken either [(a)] without qualification (*ʿalā l-iṭlāq*), insofar as it is what it is and with respect to its nature (*min ğihat huwiyyatihī wa-ṭabīʿatihī*), withtout adding a notion to [that nature] as a condition, like number for arithmetic, and then one seeks its unqualified per se accidents (*ʿawāriḍuhā aḏ-ḏātiyya al-muṭlaqa*); or [(b)] not without qualification but under the condition that a notion be added to its nature (though this notion is not a differentia that turns it into a species), and then one seeks the accidents per se that attach to it in that respect (*ʿawāriḍuhū ḏātiyya talḥaquhū min tilka l-ğiha*), like the investigation of the accidents of the moving spheres.

Text 4.6 identifies two different types of subjects of a science: subjects that are assumed by a science in an unqualified sense and subjects that are assumed with certain qualifications. The former include kinds in an unadulterated form, notions such as number for arithmetic or solid for stereometry, the latter a combination of a kind and an attribute that qualifies the kind without, however, specifying it further into one of its species (which would yield a subject of the first type, only less general), such as moving sphere for astronomy and many other examples from the particular sciences (body insofar as it is subject to motion and rest, body insofar as it is healthy or sick, and so on). In either case, a science investigates the attributes per se of its subject. But the distinction between the two types of subjects is crucial because the set of per se attributes of a subject is determined by the subject in its entirety, and if the latter is identified by a kind accompanied by one or more attributes, it is this whole that determines what is or is not a per se attribute of that subject.[26] An illustration of the types of subjects and attributes investigated by some of the main sciences in Avicenna is given in table 7.

26. At *Ilāhiyyāt* I, 1, pp. 5.18–6.1, Avicenna contends that the subject of a science is something whose existence is conceded (*musallam*) in that science and of which that science only investigates the states, adding that the point has been established "elsewhere." Once more, the implicit reference is in all likelihood to *Burhān*, II, 6. At *Ilāhiyyāt* I, 2, p. 13.11–12, he emphasizes again "the impossibility of establishing the existence of a subject and verifying its quiddity in the same science of which it is the subject," as both are conceded and presupposed in that science. What a science seeks are the states and attributes that belong to its subject. In the case of metaphysics, at *Ilāhiyyāt* I, 8, p. 54.6–7, we learn that "The things that are subjects in other sciences become accidents in this science, because they are states that inhere in the existent and are divisions of it." Such states or attributes of the existent are treated either as its species (substance, quantity, quality, and their divisions) or as its accidents (one, multiple, potential, actual, universal, particular, possible, necessary, cause). Knowledge of the essence of the subjects of the particular sciences is acquired by means of definition in those sciences but it is metaphysics that provides its ultimate validation (*Ilāhiyyāt* I, 8, p. 54.1) and, more generally, "confirmation of the soundness of the principles of the other sciences" (*Ilāhiyyāt* I, 1, p. 5.7–8). On the relation between metaphysics and the principles of the particular sciences in Aristotle, see Gómez-Lobo (1978).

TABLE 7 Subjects and attributes of the sciences (selection)

Science	Subject	Sample attributes
Metaphysics	Existent	Actuality, Potentiality, Priority, Posteriority, Causality, Universality, Particularity, Perfection, Deficiency, Generation, Eternity
Natural philosophy	Body qualified by motion and rest	Change, Motion, Generation, Corruption, Growth, Wilting
Geometry	Extended magnitude (line, surface, solid)	Area, Figure, Circularity, Triangularity, Sphericity, Perpendicularity, Parallelism, Incommensurability, Commensurability, Equality
Arithmetic	Number (1, 2, 3, . . .)	Evenness, Oddness, Squareness, Equality, Defectiveness, Abundancy
Music	Notes qualified by numerical ratios	Consonance, Dissonance
Astronomy	Moving spheres	Uniform motion, Retrograde motion, Conjunction, Opposition, Parallax, Eclipse
Medicine	Human body qualified by health and sickness	Temperaments, Fever, Pleurisy, Phrenitis, White urine, Irregular pulse

Assumptions on the Subject of a Science

In *Burhān* II, 10, Avicenna indulges in a series of pragmatic considerations on what *actually* needs to be assumed as a principle of a science. Even if ideally the constitutive elements are always the same three in every science (subject, principles, and questions or attributes), a contention that echoes Aristotle's *An. Post.* A10, 76b11–22, only principles concerning things that are genuinely unclear must be assumed explicitly. For example, the existence of corporeal magnitudes and of the sensible qualities of bodies need not be assumed explicitly in natural philosophy, because both are manifest (*ẓāhir*) and not subject to doubt (*lā yaqaʿ šakk fī wuǧūdihā*).[27] What needs to be assumed in both cases is their definition. In other cases, as for example with number, one, unit, and point, both the existence and the definition must be assumed.[28] In this context, existential assumptions are

27. See *Ilāhiyyāt* III, 4, p. 116.1 and III, 7, p. 134.1–2, respectively.

28. The existence of circles and curves, by contrast, requires a metaphysical proof. For three proofs of the existence of the circle, see *Ilāhiyyāt* III, 9, pp. 148.14–151.11; on the existence of curves, *Ilāhiyyāt* III, 9, p. 151.11–13. In *Ilāhiyyāt* III, Avicenna is generally very careful to distinguish the different extents to which the quiddity (definition), existence, and type (accidental or not) of a given notion may be evident. For example, it may be evident that a certain kind of thing exists, but whether it is a substance or an accident may be entirely obscure and require investigation. The refutation of the substantiality of quality is primarily the task of natural philosophy. The refutation of the substantiality of quantity is introduced at *Ilāhiyyāt* III, 1 and then developed in the subsequent chapters, where Avicenna argues

characterized as hypotheses:[29] "The assumption of the existence [of the subject] falls in the class of principles of the discipline that are called *hypotheses* (*uṣūl mawḍūʿa*), for it is a shared premise upon which the discipline is built" (*Burhān* II, 10, p. 184.10–11). A hypothesis so defined is an answer to a simple if-question—or question of existence—of the kind discussed in *Burhān* I, 5 (see chapter 3).[30]

SCIENTIFIC QUESTIONS

Scientific questions are the propositions that a science aims to establish by demonstration. According to Avicenna, scientific questions are of two kinds: (i) simple categorical (*basīṭa ḥamliyya*), that is to say, predicative propositions that are ultimately analyzed into their terms (subject and predicate); or (ii) compound hypothetical (*murakkaba šarṭiyya*), that is to say, propositions obtained by combining other propositions (categorical or hypothetical) in conditional or disjunctive statements.[31] He writes:

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for the accidentality of both numbers (discrete quantities) and extended magnitudes (continuous quantities) in great detail.

29. On hypotheses as existential assumptions in Aristotle and the spurious, merely contextual distinction between hypothesis and postulate in *An. Post.* A10, see McKirahan (1992). Every science presupposes the existence of its subject(s), and assertions of existence are frequently justified by the transfer of a demonstration from a higher science in which those assertions are proved. Regrettably, Avicenna does not explicitly address the question of the logical form of this crucial type of principle. On occasion he seems to treat "existent" (*mawǧūd*) just as any other predicate: on this reading, the assertions "Zayd is a man" (*Zayd insān*) and "Zayd exists" (*Zayd mawǧūd*) would have the same logical form. It is tempting, however, to think that assertions of existence might have a distinctive structure, for example one involving a statement of the form "Some A is B" (where A is "existent" and B one of its qualifications or divisions). A proof of the existence of B would then consist in showing that "existent as such" may be qualified or specified as B, much in the same way in which a predicate is proved to exist when it is proved to belong to its subject. This seems to be the way in which Avicenna understands the most fundamental kinds of assertion of existence at *Ilāhiyyāt* I, 8, p. 54.6–8. The question is relevant because at *Burhān* IV, 1, he explicitly identifies the logical form of arguments that purport to establish an existence claim as something other than the canonical logical form of arguments used in the deduction of categorical propositions. And this seems to be a reason to question the view that proofs of existence aim to establish claims of the form "A is B," where A stands for the object whose existence is in question and B is "existent" (*mawǧūd*), as their logical form would be akin to that of ordinary categorical statements. This remains an open problem that requires further investigation.

30. For an analysis of the "so-called question of existence" (*ei esti*) in Aristotle, see Gómez-Lobo (1980).

31. See Alfarabi, *Burhān* IV, 1, pp. 59–65 on scientific questions and their structure. In particular, (I) the subjects of scientific questions are classified as follows: (i) species of the subject of the discipline, (ii) species of its species, (iii) accidents per se of the subject, (iv) accidents per se of the species of the subject or (v) of the species of its species, (vi) accidents per se of accidents, (vii) species of accidents per se or (viii) species of species, or (ix) subject of the discipline itself. (II) The predicates of scientific questions are divided in the same way. Alfarabi also draws a distinction between first-order conclusions

Text 4.7: *Burhān* II, 6, p. 157.20–21
A question is either simple categorical or compound hypothetical. The compound comes after the simple in our presentation. Thus, we say that every simple question is divided into predicate and subject.

The philosophical significance of the distinction made in Text 4.7 cannot be overestimated. This is because it singlehandedly broadens the set of fundamental expressions that may be used in scientific discourse, addressing a classic problem associated with the tradition of the two *Analytics*, namely the question of their expressive power and applicability to actual scientific theories. As noted, even in the putative form of a complete science in its final stage (and leaving aside the fact that no Aristotelian science actually exhibits that form, at least in his surviving works), it would be hard to see how far one could go, in any science, with the sole aid of categorical propositions and categorical syllogistic. Text 4.7 lays one of the foundations for Avicenna's philosophical project of turning Aristotle's theory of science into a viable model. By expanding the logical vocabulary of the *Posterior Analytics*, Avicenna's list of basic types of scientific propositions overcomes, at a single stroke of the pen, one of the most problematic limitations of Aristotle's theory of science in its original formulation.[32] That being said, the fact that Avicenna's basic logical forms include non-categorical propositions does not affect the preeminent role still played by categorical propositions in his logic of scientific discourse. After all, the fundamental truths that every science aims to establish ultimately concern relations between subjects and attributes in its domain of inquiry. And this, for Avicenna, translates into a further requirement for an adequate theory of science, namely, to provide a fine-grained analysis of the logical structure of categorical questions and, in particular, of their types of subjects and predicates.[33]

Subjects of Categorical Questions

Avicenna's commitment to the idea of developing technical aspects of the theory of science in their finest details is plainly revealed by the elaborate characterization,

(*maṭlūbāt uwal*), which are the first non-immediate propositions demonstrated in a science, and second-order conclusions, which are proved with the help of first-order conclusions assumed as premises for their demonstration.

32. For an introductory characterization of the basic types of propositions in Avicenna, see Strobino (2018).

33. Avicenna's main division of scientific questions (*masāʾil*) into two classes based on their logical form seems to be inspired by Alfarabi's. The distinction between categorical (*ḥamliyya*) and hypothetical (*waḍʿiyya*) premises and conclusions is formulated by Alfarabi, *Burhān* II, 3, p. 27.18 in the context of his analysis of absolute demonstration: "Premises are either categorical (*ḥamliyya*) or hypothetical (*waḍʿiyya*), and so are questions (*masāʾil*)." A strong echo of this point in Avicenna is at *Qiyās* V, 1, p. 231.1–5 (see appendix B).

in *Burhān* II, 6, of various types of subjects of categorical scientific questions, which are illustrated by examples drawn from natural philosophy:

> Text 4.8: *Burhān* II, 6, pp. 157.21–158.8
> Let us consider the case of the subject first. We say that the subject in a question that is proper to a science falls either [(a)] in the domain of the subject [of the science] (*dāḫil fī ǧumlat mawḍūʿihī*) or [(b)] in the domain of the per se accidents of the subject [of the science] (*kāʾin min ǧumlat al-aʿrāḍ aḏ-ḏātiyya lahū*).
>
> What falls in the domain of the subject [of the science] is either [(aa)] the subject itself (whether it be one or many), as in "Is body infinitely divisible?" (one of the questions of natural philosophy); [(ab)] a species [of that subject], as in "Does air imprisoned in water spurt upward by nature or due to a compelling pressure (*li-l-inḍiġāṭ al-qāsir*)?" and "Does anger originate in the brain or in the heart?"
>
> What [falls in the domain] of the per se accidents [of the subject] is either [(ba)] a per se accident of the subject of the science, as in "Is such-and-such a motion contrary to such-and-such a motion?" [(bb)] a per se accident of a species of the subject of the science, as in "Is the luminous light of the sun hot?" [(bc)] a per se accident of a per se accident of [the subject of the science], as in "Does time come into existence after rest?" (for time is an accident of motion which is [in turn] a per se accident of body); or [(bd)] a per se accident of the species of an accident of the subject of the science, as in "Is the slowing down (*ibṭāʾ*) of motion due to intervals of rest?" (Slowing down is an accident of some motions but not of others, for some motions, having uniform speed, do not slow down at all.)[34]

While the subject of natural philosophy, generally speaking, is body insofar as it is subject to motion and rest, that science is concerned with a broad array of entities that fall in its domain. Individual scientific questions of natural philosophy—and the same holds for every other science—will therefore not only be about the subject of the science as such, without further qualifications, but also reflect its internal divisions and complexity as well as the division and complexity of its set

34. The divisibility of body in (aa) is relevant for Avicenna's rejection of atomism, especially at *Samāʿ ṭabīʿī* III, 1–5. The question of how air moves upward in water in (ab) seems closely tied to *Samāʿ ṭabīʿī* IV, 14 and, more loosely, to II, 9; the question of the source of anger (*ġaḍab*) is relevant to the discussion of *Nafs* V, 8, pp. 268.20–269.6. The question of the contrariety of motions in (ba) is discussed at *Samāʿ ṭabīʿī* IV, 6. The question of whether the luminous light of the heavenly bodies (the stars, in Aristotle's example) produces heat is raised at *De Caelo* B7, 289a13–34, which may be the remote context for the example in (bb). The question of the relation in (bc) between time and rest is addressed at *Samāʿ ṭabīʿī* II, 4 and II, 10. Avicenna's discussion of time in natural philosophy is primarily at *Samāʿ ṭabīʿī* II, 10–13 (for his account of the essence of time and the proof of its existence, see in particular *Samāʿ ṭabīʿī* II, 11; the accidental character of time and its dependence on motion are concisely summarized at *Ilāhiyyāt* III, 4, p. 117.7–10; cf. also *Maqūlāt* VI, 5, pp. 231.4–232.4). Finally, the question of what causes some motions to be slower in (bd) is a standard problem in Avicenna's discussion of *kalām* arguments, in particular an-Naẓẓām's theory of the "repulsive leap" (*šanāʿat aṭ-ṭafra*) refuted at *Samāʿ ṭabīʿī* III, 4, pp. 194.13–195.7.

of attributes. As a result, the subject of individual questions, while broadly defined by the subject of the science, will logically be identified in each case with a specific type of term. Text 4.8 regiments the range of admissible terms.

The subject of a scientific question may be the subject of the science itself or one of its species. In the case of natural philosophy, body itself is the subject of "Every body is infinitely divisible," but in the case of propositions like "Air imprisoned in water spurts upward by nature" or "Air imprisoned in water spurts upward due to a compelling pressure" (whichever of these is the correct scientific answer to the question in Text 4.8), the subject is air, which is a species of body (just like the other three elements).

The subject of a scientific proposition, however, does not have to coincide with the subject of the science or with one of its species. It may also be an accident of the subject of the science, and even more complex combinations are possible, involving for instance the accident of an accident. Avicenna identifies here four sub-cases: the subject of a scientific proposition may be (ba) an accident per se of the subject of a science (motion, in the example, is an accident of body, which is in turn the subject of natural philosophy), (bb) an accident of a species of the subject of the science (the luminous light of the sun is presumably related with the element of fire which is in turn a species of the subject of natural philosophy, and being hot is a per se accident of fire), (bc) a per se accident of a per se accident (time is an accident of motion and motion is in turn an accident of body), or (bd) a per se accident of a particular kind (a species) of per se accident of the subject of a science (since some motions are uniform, an attribute like slowing down will then have to be a per se accident of some kinds of motions only, that is to say, the nonuniform ones; the latter are characterized here as a species of an accident of the subject of natural philosophy, under the standard assumption that motion is an accident of body).

This is not the only place in the corpus where Avicenna illustrates his own classification of the subjects of scientific questions with examples drawn from the sciences. In another list, at *Naǧāt* I, 126, similar types are elucidated with the help of geometrical terms.[35] Thus, for example in "Every extended magnitude is either commensurable or incommensurable," the subject of the proposition is the same as the subject of the science (extended magnitude).[36] In "Every line is divisible into two halves," the subject is a species of the subject of geometry (as noted, the three species of extended magnitude for Avicenna are line, surface, and solid). These two cases closely correspond to (aa) and (ab) in Text 4.8. The list at *Naǧāt* I, 126

35. See *Naǧāt* I, 126, pp. 135.11–136.3. Yet another list, consisting again of arithmetical and geometrical terms, is at *Daneshname* I, 28 (Achena and Massé 1955, pp. 77–78).

36. As previously noted, according to Avicenna, the statement in question is a specific instance of the law of the excluded middle.

also accounts for the case in which the subject of a scientific question involves a per se accident. For instance, in "Every extended magnitude that is a mean proportional is a side enclosed by two extremes," the subject of the proposition is "extended magnitude that is a mean proportional," that is to say, the subject of geometry qualified by a per se accident (for being a mean proportional is a relevant attribute of certain geometrical entities).[37] Another possibility is for the subject of a proposition to be a species of the subject of the science qualified by an accident, as in "Every line set upon another line determines such-and-such angles," where the subject of the proposition, namely "line set upon a line," is a species (line) of the subject of geometry qualified by an accident (being set upon a line). These last two cases do not seem to have a direct counterpart in Text 4.8, as they both articulate further alternatives in which the subject of certain scientific propositions is the subject of the discipline or one of its species qualified by a per se attribute of the subject of the discipline.[38] The last example in the *Naǧāt* list, by contrast, corresponds to case (bd) in Text 4.8 and is illustrated by the proposition "The angles of every triangle are such-and-such," where the subject "angles" is a per se accident of the subject of geometry. This intriguing characterization of the nature of angles is confirmed independently by extensive discussions of this and other geometrical entities in Avicenna's metaphysics.[39]

37. If $a : b = b : c$, then b is a mean proportional between a and c, also called the geometric mean of a and c. Because $b^2 = ac$ and $b = \sqrt{ac}$, the notion of a mean proportional was traditionally used to express squares and square roots; see Avicenna, *Handasa* VI, 9, p. 188.1–5 (cf. Aristotle, *De Anima* B2, 413a11–20). Geometrically, a simple instance is the perpendicular drawn in a right triangle from the right angle to the hypotenuse, which is the mean proportional between the two segments into which it divides the hypotenuse. Moreover, each short side of the triangle is the mean proportional between the hypotenuse and the adjacent segment of the hypotenuse, which is the projection of the side onto the hypotenuse. This seems to be the context of Avicenna's scientific question, where the subject is "extended magnitude that is a mean proportional" and the predicate is "side enclosed by two extremes." The two extremes are a (the hypotenuse) and c (the projection of the side), and the mean proportional is b (the side).

38. This ultimately depends on how strictly we take the qualification of air as "imprisoned in water" in Text 4.8, case (ab). Since Avicenna does not say anything about the qualification in that passage (and explicitly classifies the example under the rubric "species of the subject of the science"), I am inclined to think that (ab) should fall under the same category as "Every line is divisible into two halves" in *Naǧāt* I, 126 (on this reading, the subject of the proposition would be just a species of the subject of the science in both cases). If, however, "imprisoned in water" is implicitly meant to qualify air in a more robust sense, then the example would arguably be closer to "Every extended magnitude that is a mean proportional is a side enclosed by two extremes" in *Naǧāt* I, 126 (on this alternative reading, the subject of the proposition would be a species of the subject of the science *qualified* by a per se attribute).

39. Commensurable and incommensurable are defined at *Handasa*, X, p. 299.1–2 (*Elements* X, Def. 1). For a definition of mean proportional see *Handasa* VI, p. 179.3–4 (*Elements* VI, Def. 3); for an application, see *Handasa* VI, 9, p. 188.1–5 (the latter is directly concerned with *Elements* VI, Prop. 13,

In summary, the two overlapping classifications of the types of subjects of scientific questions include the following terms:

Burhān II, 6: (aa) the subject of a science, (ab) a species of the subject of a science; (ba) a per se accident of the subject of a science; (bb) a per se accident of a per se accident of the subject of a science; (bc) a per se accident of a species of the subject of a science; (bd) a per se accident of a species of a per se accident of the subject of a science; and

Naǧāt I, 126: (aa) the subject of a science, (ab) a species of the subject of a science; (ac) the subject of a science qualified by a per se accident of the subject of that science; (ad) a species of the subject of a science qualified by a per se accident of the subject of that science; and (ba) a per se accident of the subject of a science.

Three elements out of five in the second list correspond to three out of six in the first, namely (aa), (ab), and (ba). The difference between the remaining cases is that the first list gives three types of per se accidents that may serve as subjects of scientific questions, namely (bb), (bc), and (bd), while the second list gives the subject of the science or one of its species, both qualified by a per se accident of the subject of the science, namely (ac) and (ad), as further options.

The purpose of this detailed account of the two taxonomies is to show Avicenna's unwavering commitment to the applicability of the concepts and distinctions of the theory of science to actual scientific discourse. But a detailed characterization of the nature and types of subjects of scientific questions is also critical for the definition of the set of per se attributes, that is to say, the predicates of categorical questions.

Predicates of Categorical Questions

The predicates of categorical questions are in a sense the most characteristic ingredient of demonstration. Every science ultimately aims to establish demonstratively that certain attributes belong to their subjects. The predicates of scientific questions encapsulate such attributes and therefore come in a variety of types. In *Burhān* II, 6, Avicenna classifies (i) the types of predicates of scientific

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which in turn depends on the corollary to *Elements* VI, Prop. 8). A construction involving a line set upon another line is at *Handasa* I, 17, pp. 36.6–37.2, where it is the subject of a geometrical question (*Elements* I, Prop. 13). The three types of rectilinear angles (right, acute, and obtuse) are also defined by means of it at *Handasa* I, p. 17.3–8 (*Elements* I, Defs. 10–12). Presumably the question whose subject term is "The angles of every triangle" is the conclusion of the famous theorem showing that the sum of the internal angles of a triangle is equal to two right angles at *Handasa* I, 39, p. 52.5–10 (*Elements* I, Prop. 32).The notion of angle is discussed in general terms at *Ilāhiyyāt* III, 4, pp. 116.3–117.6. The priority of right angle over acute and obtuse angle and their definitions are analyzed in greater detail at *Ilāhiyyāt* V 9, pp. 250.6–252.14.

questions and identifies (ii) a series of constraints typically imposed on their combinations in the context of demonstrative arguments. He writes:

> Text 4.9: *Burhān* II, 6, pp. 159.19–160.1
> If the subjects of the items that are sought (*al-maṭlūbāt*) and [the subjects] of the questions (*al-masāʾil*) fall under [(1)] the subject of the discipline, then their predicates fall under [(1.1)] their per se accidents, [(1.2)] the genera of their accidents, [(1.3)] the differentiae of their accidents, and [(1.4)] the accidents of their accidents.
>
> If the subjects [of the items that are sought and the subjects of the questions] fall under [(2)] the per se accidents [of the subject of the discipline], their predicates may fall under [(2.1)] the genus of the subject, [(2.2)] its species, [(2.3)] its differentiae, [(2.4)] its accidents, [(2.5)] the accidents of its accidents, [(2.6)] the genera of other accidents, and [(2.7)] their differentiae, and what plays their role.
>
> Sometimes the predicates of the two types of subjects may be [(3.1)] per se accidents of the genus [of the subject of the discipline] [. . .] or [(3.2)] of what resembles a genus (*šabīh ǧins*).

If the taxonomy of the types of subjects of scientific propositions presented in Text 4.8 seemed elaborate, the taxonomy of the types of predicates given in Text 4.9 is even more complex, including roughly a dozen cases (and, depending on how generously we interpret the last two clauses 3.1 and 3.2, possibly up to twice as many).

Avicenna classifies the predicates of scientific propositions based on their subjects and identifies two main classes with various internal divisions. The subject of a scientific question, if we take only the two main types of entities that may be expressed by it, coincides either with (i) the subject of the discipline or with (ii) a per se attribute of the subject of the discipline. In each of these two cases, the predicate may be a number of different things. In particular, Avicenna contemplates the following possibilities:[40]

1. If the subject of the question coincides with the subject of the discipline, then the predicate may be:
   - 1.1 a per se accident of the subject of the discipline (b)
   - 1.2 a genus of a per se accident of the subject of the discipline (c)
   - 1.3 a differentia of a per se accident of the subject of the discipline (c)
   - 1.4 a per se accident of a per se accident of the subject of the discipline (d)

40. These combinations are instances of the general classification at *Burhān* II, 8, pp. 169.13–170.6. For example, in case (1.3), the predicate (major term) is a differentia of a per se accident (middle term) of the subject of the question (minor term), which is in turn identical with the subject of the discipline. Consequently, a demonstration of this sort has a per se 1 major premise (the major term is a constituent of the middle) and a per se 2 minor premise. This premise pair entails a per se 2 conclusion in which a differentia of a per se accident is proved to belong to its subject. The spirit of Avicenna's analysis seems to be close to the systematic account developed by Alfarabi in his *Burhān* II, 5, on which see Strobino (2019).

2. If the subject of the question coincides with a per se accident of the subject of the discipline, then the predicate may be:
   2.1 a genus of a per se accident of the subject of the discipline (c)
   2.2 a species of a per se accident of the subject of the discipline (c)
   2.3 a differentia of a per se accident of the subject of the discipline (c)
   2.4 a per se accident of a per se accident of the subject of the discipline (d)
   2.5 a per se accident of a per se accident of a per se accident of the subject of the discipline (d)
   2.6 a genus of a per se accident of a per se accident of the subject of the discipline (c)
   2.7 a differentia of a per se accident of a per se accident of the subject of the discipline (c)

Several considerations are in order. First, this account represents a systematic attempt to regiment the theory of scientific attributes, its internal division, and mutual relations with no obvious precedent in the Aristotelian tradition (perhaps with the exception of Alfarabi).[41] Second, it indirectly offers useful insights to test how strictly Avicenna aims to adhere to his own criteria for the classification of per se attributes (so that the requirement of a definitional link between the subjects and attributes of scientific propositions can consistently be met). Third, the account is relevant for a broader taxonomical effort motivated, in *Burhān* II, 8, by the need to identify certain distinctive patterns of demonstration. In that context, Avicenna analyzes various kinds of first-figure syllogistic demonstrations and classifies them on the basis of the relations holding between their terms, calling the resulting configurations "manners of assuming demonstrative matters (*maʾāḫiḏ al-burhāniyyāt*)," that is to say, terms that satisfy the requirements of demonstration. The outcome of that classification is a fourfold division. The middle term, which is the predicate of the minor premise, may be either a per se 1 attribute (PS1) or a per se 2 attribute (PS2) of the minor term, that is to say, of the subject of the scientific question that the demonstration aims to prove. The major term, that is

41. The types of scientific predicates in the two lists may be exemplified by the following terms:

Predicate of scientific proposition	Characterization
1.1 moving	(per se accident of body)
1.2 changing	(genus of motion, which is a per se accident of body)
1.3 divisible into two equals	(differentia of even, which is a per se accident of number)
1.4 contrary, circular	(per se accident of motion)
2.1 quantity	(genus of even)
2.2 even-times-even	(species of even)
2.3 divisible into two equals	(differentia of even)
2.4 slow, fast	(per se accident of motion)
2.5 equal, existing after rest	(per se accident of time, which is a per se accident of motion)

to say, the predicate of the major premise and of the scientific question itself, may be in turn either a per se 1 attribute (PS1) or a per se 2 attribute (PS2) of the middle term. This gives the following four types of demonstrative premise pairs or manners of assuming demonstrative matters (where the major premise is listed first and the minor premise second) and conclusions, illustrated by triplets of terms drawn from Avicenna's theory of per se predication:

(a) PS1-PS1 ⊢ PS1 (human, animal, body)
(b) PS1-PS2 ⊢ PS2 (eight, even-times-even, even)
(c) PS2-PS1 ⊢ PS2 (isosceles, triangle, having the sum of the internal angles equal to two right angles)
(d) PS2-PS2 ⊢ PS2 (body, nonuniform motion, slowing down and speeding up)

For reasons discussed later, (a) is not a standard kind of demonstration, and in most cases this premise pair is considered inadequate by Avicenna. But (b)–(d) all seem to fit squarely within the framework articulated in the preceding outline (the letters next to each item refer to one of these four configurations).[42]

In *Burhān* II, 10, Avicenna briefly discusses the per se predicates of scientific propositions in a rather different context, focusing on what must be assumed about them in a science. Of the predicates of scientific propositions, we usually only assume the definition, if it is not manifest.[43] Their existence, by contrast, cannot be assumed until it is demonstratively established (and hence cannot be assumed *tout court* in a science in which the predicate appears in a scientific question). If the existence of a predicate is assumed in a science, then it is because it has been proved by another science. As noted in chapter 3, such proofs of existence, in the case of predicates, are proofs that the predicate belongs to a subject. Furthermore, in addressing the Aristotelian distinction between knowing, of certain things, that they are (existence) and knowing what they are (essence), Avicenna rephrases it in terms of the distinction between assertion and conception: certain principles must be known with respect to their being the case (*haliyya*), which falls in the domain of assertion. The assertion of principles entails the assertion of questions

42. There seems to be no principled reason for not having a third order (or yet higher order) of accidents. Although Avicenna usually stops at accidents of accidents in his "official" lists, he is committed to more complex hierarchies. For example, equality or divisibility may serve as per se attributes of time in a demonstration. For the view that an accident can be the accident of an accident (even if the real subject of both is ultimately a substance), see *Ilāhiyyāt* II, 1, p. 58.5–9, where velocity is said to be an accident of motion, straightness of line, and being a plane figure of surface. It is interesting to note that the view is characterized by Avicenna as "irrefutable" (and hence, presumably, as one that must be evidently true).

43. Examples of notions whose quiddity is manifest (*ẓāhirat al-māhiyya*) include equality, abundance, and defectiveness in geometry.

(as conclusions of demonstrative arguments) either (i) without qualification (I take it that Avicenna means by *taṣdīqan ḥaqīqiyyan* the assertion of categorical propositions, that is to say, the first-order assertions of a science) or (ii) hypothetically, that is to say, in the form of a conditional or disjunctive proposition (*taṣdīqan waḍʿiyyan*). Both principles and questions, as noted in chapter 1, must be objects of conception before they are asserted. In particular, their subject and predicate must be conceptualized, as a necessary condition for their assertion.[44] While it is a demonstration that proves whether (*haliyya*) the predicate of a scientific question belongs to its subject, its essence or quiddity (*māhiyya*), captured by a definition, must be known in advance. Per se accidents are only assumed with respect to their definition and not with respect to existence, as their existence is proved in the science under which they fall.

Types of Demonstrations and Types of Predicates of Scientific Questions

Two recurrent themes in Avicenna's discussion of the predicates of categorical scientific questions are (i) the identification of certain constraints on the admissible combinations of predicates in a pair of demonstrative premises and (ii) the resulting classification of various types of demonstrations. The reason Avicenna is interested in these two issues is that the nature and relation of the terms in the conclusion of a demonstration unsurprisingly depend on the nature and relation of the terms of its premises. Assuming that the kinds of essential relation (per se 1 or per se 2) holding between the minor term and the middle term and between the middle term and the major term in a first-figure syllogistic demonstration with universal affirmative premises are known, it is in principle possible to determine the kind of essential relation holding between the minor term and the major term, that is to say, between the subject and the predicate of the scientific question (the same will hold, mutatis mutandis, for the other figures and moods).

Avicenna refuses to ascribe the status of a genuine demonstration to a (first-figure) deduction in which both categorical premises are per se 1 predications.[45] The key idea is that per se 1 attributes or constituents naturally fall within the purview of definition rather than demonstration because all these attributes are

44. As noted in chapter 1 (see Text 1.5), Avicenna states in *Burhān* I, 3 that preexistent knowledge involves both assertion and conception, and analyzes it into these elements (conception of premises and conclusion, assertion of premises, conception of the parts of definitions and descriptions).

45. A detailed discussion of the issues raised by the PS1-PS1 premise pair is at *Burhān* II, 6, pp. 158.9–161.9. A similar criticism is formulated at *Naǧāt* I, 127 (iv), p. 137.4–11, where Avicenna contends that "in demonstrative questions [what is] per se in the second sense is what is sought," and at *Išārāt* I, 12, pp. 8.20–9.17, where Avicenna introduces the maxim "the constituent of the constituent is a constituent" in the context of an argument showing that the chain of implicates is finite and that there must be immediate implicates. I return to Avicenna's account of implicates in chapter 8. The predicates of scientific questions and the admissible combinations of premises are also discussed, along the same lines, in *Daneshname* I, 30, see Achena and Massé (1955, pp. 81–82).

self-explanatory and (ought to be) self-evident. It is only in exceptional cases that they may not be recognized as such. Avicenna generally does not give a full argument in support of this view. But he explicitly states its critical assumption, namely the transitivity of the relation of "being a constituent of" or "being a per se 1 attribute of." A reconstruction of the argument runs as follows (where "being a constituent of" is synonymous with "being a per se 1 attribute of"):

(i) All constituents are self-evident.
(ii) If something is self-evident, there is no demonstration of it.
(iii) Therefore, there is no demonstration of constituents.
(iv) If A is a constituent of B, and B is a constituent of C, then A is a constituent of C.
(v) If the major term is a constituent of the middle term, and the middle term is a constituent of the minor term, then the major term is a constituent of the minor term; that is to say,
(vi) if both predicates in the premises of a first-figure demonstration are constituents of their subjects, then the predicate of the conclusion is a constituent of its subject too.
  a. There is a demonstration the predicates of whose premises are constituents of their subjects (assumption for reductio).
  b. If there is a demonstration the predicates of whose premises are constituents of their subjects, then there is a demonstration the predicate of whose conclusion is a constituent of its subject.
  c. Therefore, there is a demonstration the predicate of whose conclusion is a constituent of its subject,
  d. but there is no demonstration of constituents (by iii).
(vii) Therefore, there is no demonstration the predicates of whose premises are constituents of their subjects.

The target of this rejection is what Avicenna considers an atypical and pointless case of pseudo-demonstrative argument, namely one in which an attribute that ought to be in principle self-evident is established by demonstration. Unless what is sought is merely a causal explanation (*limmiyya*) and not a factual assertion (*anniyya*), that is to say, unless what the demonstration seeks to establish is *only why* an attribute belongs to a subject and *not* also *that* it belongs to a subject, the predicate of a simple (categorical) question cannot be a constituent or per se 1 attribute of the subject ("the nature of a genus or differentia or something composed of them"). The reason is that these predicates under normal circumstances evidently belong (*bayyin al-wuǧūd*) to the things of whose definitions they are part (*Burhān* II, 6, p. 158.9–12).[46] But if what is sought is just a causal explanation

46. On the notion of *anniyya*, see Mayer (2009). The restriction applies to a (categorical) question (*mas'ala*) of which one does not know whether it is true, that is to say, whether the predicate belongs

(*limmiyya*) and not also the factual assertion (*anniyya*), then this combination is legitimate, in which case the middle term is a cause of the fact that the major belongs to the minor, as for example in a demonstration of the reason why every human perceives, where the middle term is sensitive (or rational) (*Burhān* II, 6, p. 160.7–9).[47]

There are typically two anomalous cases in which a constituent may fail to be recognized to belong in a self-evident manner to its subject and require a standard demonstration of the fact that as well as of the reason why it belongs to its subject. The two exceptions involve either (i) someone who is not mentally sound and must be reminded (*tanbīh*) of a fact that should otherwise be self-evident or (ii) a case in which the essence or nature of something is known through its accidents, which requires a demonstrative proof for one or more of its constituents (the standard example offered by Avicenna as illustration of this special case is the demonstration that the constituents of the soul belong to its essence).[48]

If, by contrast, the fact that a per se accident belongs to the genus or to the differentia of something is more evident than its belonging to the thing itself, then it is possible to demonstrate that the attribute belongs to the subject using the genus or the differentia as a middle term. Similarly, if the species of an accident belongs to a subject more evidently than the accident itself, then the species of the accident may be used as a middle term to show that the accident belongs to the subject.[49]

In general, if either of the two premises expresses a per se 2 predication, no restriction is imposed on the other premise. To clarify the sense in which genera and differentiae may be used as predicates, Avicenna writes:

> Text 4.10: *Burhān* II, 6, p. 160.10–16
> I say that everything that is not fit to be a predicate in demonstrative questions is not at all fit to be predicate in demonstrative premises, whether they be proper principles or common principles, except for genera and differentiae and what resembles these, for they may be predicated of their species in the premises. For it is possible [(1)] for the major term to be a genus or a differentia of the middle, and for the middle to

to the subject (*mağhūlat al-anniyya*) and of which the *anniyya* is precisely what is sought, but it does not apply to questions of which one merely fails to know *why* they are true (*mağhūlat al-limmiyya*).

47. For this example, see demonstrative combination 4.2 in Alfarabi, *Burhān* II, 5, discussed in Strobino (2019).

48. Interestingly, this case is relevant for the distinction between a real demonstrative deduction and a deduction in which a (complete) definition is used as middle term. Avicenna considers the latter a trivial form of demonstration (*Burhān* IV, 4).

49. At *Burhān* II, 6, p. 158.18–23, Avicenna also draws a distinction between the question of whether A belongs to B (where A is a constituent of B) and the question of whether A is a constituent of B (a definition, a genus, or a differentia). The former falls under the problematic case, but the latter does not. For example, it is legitimate to ask whether (and seek a demonstration of the fact that) sensitive is a genus or a differentia of human, but the fact that every human is sensitive is known by definition, not by demonstration.

be a per se accident of the minor. And just as one may begin with the accident and seek [what follows], in the same way one may begin with its genus or its differentia and seek [what follows]. It is also possible [(2)] for the middle term to be a genus or a differentia of the minor, and for the major to be a per se accident of the middle. Thus, in this respect genera and differentiae fall in the domain of the predicates [of scientific questions].[50]

To exemplify the points addressed in Text 4.10, let us take A, B, and C as major term, middle term, and minor term, respectively. Avicenna considers in this passage the following two combinations of premise pairs in the first figure:

Case (1): the major premise is per se 1 (the predicate-major term is a per se 1 accident of its subject-middle term); the minor premise is per se 2 (the predicate-middle term is a per se 2 accident of its subject-minor term):
- (1.1) A is a genus of B; B is a per se accident of C $\vdash$ A is a genus of a per se accident of C (a more general, non-primary per se accident of C)
- (1.2) A is a differentia of B; B is a per se accident of C $\vdash$ A is a differentia of a per se accident of C (a more general, non-primary per se accident of C)

Case (2): the major premise is per se 2; the minor premise is per se 1:
- (2.1) A is a per se accident of B; B is a genus of C $\vdash$ A is a per se accident of a genus of C (a more general, non-primary per se accident of C)
- (2.2) A is a per se accident of B; B is a differentia of C $\vdash$ A is a per se accident of a differentia of C (a more general, non-primary per se accident of C)[51]

In Text 4.11, Avicenna seems to be suggesting that the criterion that one should adopt in choosing the right middle term is primarily epistemic:

Text 4.11: *Burhān* II, 6, pp. 160.17–161.1
If the fact that the per se accident belongs to the differentia of the thing or to its genus is possibly more evident than its belonging to the thing [itself], then the differentia

50. In the two cases described in Text 4.10, this is either because they are genera or differentiae of a per se accident of the minor (and not of the minor itself), or because, while being genera or differentiae of the minor, the major, which is the predicate in the question-conclusion, is an accident per se of them (and hence of the minor). Thus, in neither case are genera or differentiae of the minor term predicates in the question-conclusion.

51. A straightforward correspondence may be established between the demonstrative moods (*ḍurūb*) in Alfarabi's *Burhān* II, 5, Avicenna's distinctions in Text 4.10, and the types of demonstrations listed in *Burhān* II, 8. Case (1)-type (b) in Avicenna covers Alfarabi's 3.3, 3.4, 3.5, 3.6, 4.3, 4.4, 4.5, 5.2, 5.3, and 5.4, while case (2)-type (c) in Avicenna covers Alfarabi's 6.1, 6.2, 7.1, 7.2, 8.1, 8.2, and 8.3. Leaving aside PS1-PS1 combinations, the other relevant class (PS2-PS2) may be associated with Avicenna's type (d), which covers Alfarabi's 1.4, 1.5, 1.6, 1.7, 2.3, 2.4, 6.3, 6.4, 7.3, 7.4, and 8.4. The full list of the demonstrative moods in Alfarabi is discussed in detail in Strobino (2019).

> or the genus may be used as a middle term. Similarly, since the species of the accident may be better known [to belong] to the thing or what is differentiated (*mafṣūl*) by the accident may be better known [to belong] to the thing, what is better known may be used as a middle term. As for the case in which the major term is a constituent of the minor, it does not occur except in the way defined [previously] (*ʿalā al-wağh al-maḥdūd*).

As noted previously, in the context of an argument concerning the notion of transfer of demonstration in *Burhān* II, 8, Avicenna identifies two paradigmatic schemata of demonstration or manners of assuming demonstrative terms (*maʾāḫiḏ al-burhāniyyāt*).[52] Case (1), from the taxonomy of demonstrative predicates in *Burhān* II, 6, corresponds to type-(b) demonstrations in *Burhān* II, 8 (where the major is a per se 1 attribute of the middle, and the middle a per se 2 attribute of the minor). Case (2) in II, 6, by contrast, corresponds to type-(c) demonstrations in II, 8 (where the major is a per se 2 attribute of the middle, and the middle a per se 1 attribute of the minor). In both contexts, Avicenna rules out PS1-PS1 combinations (*modulo* the two aforementioned exceptions) as a result of his general commitment to the view that the essential constituents of a notion are not generally established to belong to it demonstratively but grasped as those self-evident attributes that are parts of its definition.

The complex apparatus of subjects, principles, and questions finds an immediate and direct application in Avicenna's classification of the various ways in which different sciences may be related to one another, which brings us to the theme of the next chapter.

52. At *Ḥall muškilāt* I, 12, p. 210.13–15, while commenting on Avicenna's treatment of non-constitutive implicates, Ṭūsī explicitly states that these two approaches or manners of assuming the terms (*maʾḫaḏān*) comprise the types of demonstrations (*aṣnāf al-barāhīn*). The discussion is nestled within the broader context of the distinction between immediate and non-immediate implicates and between evident and non-evident implicates, which runs until the end of the commentary on I, 12 (*Ḥall muškilāt* I, 12, pp. 210.1–212.15).

5

Division and Hierarchy of the Sciences

CRITERIA OF CLASSIFICATION

Avicenna's theory of science includes a detailed account of (i) the criteria according to which scientific knowledge is organized in its different domains and (ii) the interrelations among such domains.[1] In this chapter, I analyze two variants of Avicenna's account of the division and hierarchy of the sciences in *Burhān* II, 7.[2] The first classification is concerned with (1) criteria of differentiation (*iḫtilāf*) focusing on the subject and develops a complex taxonomy of relations, while the second classification deals with (2) criteria of commonality (*ištirāk*) focusing on the conditions under which different sciences may partly coincide or overlap with respect to principles, subjects, or questions. The two approaches are complementary, and

1. The classification of the sciences is a classic theme in the Arabic-Islamic intellectual tradition. For a general introduction, see Rosenthal (1975) and Endress (2006a). On Avicenna in particular, see Maróth (1980) and Hugonnard-Roche (1984).

2. For an analysis of the text of *Burhān* II, 7 in relation to its twelfth-century Latin translation, see Strobino (2017). In that chapter, sciences (*ʿulūm*) are typically characterized as general (*ʿāmm*) and specific (*ḫāṣṣ*), or by the comparative terms more general (*aʿamm*) and more specific (*aḫaṣṣ*), or higher (*aʿlā*) and lower (*asfal*). Another standard distinction is the one between universal sciences (*ʿulūm kulliyya*) and particular sciences (*ʿulūm ǧuzʾiyya*) (at the beginning of the treatment of natural philosophy, Avicenna refers to it as a particular science with respect to metaphysics; see *Samāʿ ṭabīʿī* I, 1, p. 7.2–3). A different but equally canonical classification of the sciences in Avicenna, on which see Anawati (1977), is the traditional division into theoretical sciences (*ʿulūm naẓariyya*), whose main branches are natural philosophy, mathematics, and "divine science" or metaphysics, and practical sciences (*ʿulūm ʿamaliyya*), at *Madḫal* I, 2, pp. 12.1–14.18 (*tanbīh ʿalā l-ʿulūm*) and *Ilāhiyyāt* I, 1, pp. 3.5–4.6 (where the discussion is linked with assertion and conception).

the results presented in the first classification are confirmed and validated by the more concise account offered in the second.

In the first part of *Burhān* II, 7, Avicenna addresses the question of what makes two sciences different and systematically identifies a number of possible relations that may characterize an arbitrary pair of sciences. The way in which one science stands to another science is determined in turn by the way in which their underlying subjects are related. Therefore, the task of providing an exhaustive division of the sciences crucially depends on the ready availability of precise criteria for the identification of such subjects. The most general division is between (a) sciences that have two different subjects and (b) sciences that treat one and the same subject in different ways. Subordination nearly always involves, in one form or another, sciences that have different subjects and hence fall into the first category, which accommodates the most common types of interrelations among domains of scientific inquiry.[3]

In the second part of *Burhān* II, 7, Avicenna seems concerned with the same problem but addresses it from a different angle. The focus is, in this case, on the criteria of identity (rather than of distinctness) for an arbitrary pair of sciences and Avicenna's analysis proceeds from the standpoint of their three canonical constitutive elements: subject, principles, and questions (in the first part, only the subject really plays a role in the characterization of the way in which any two sciences may differ). The second classification is a complementary digest of the results obtained in the first, more elaborate classification, and summarizes the basic types of relations and their most notable examples, validating (practically with no exception) the internal structure of the first taxonomy.[4]

THE FIRST CLASSIFICATION

In Avicenna's first classification, the first distinction is between (a) sciences that have different subjects and (b) sciences that have one and the same subject but investigate it in different respects.

3. On subordination in Aristotle, see Gómez-Lobo (1978) and McKirahan (1978, 1992).

4. *Burhān* II, 6–10 are characterized by a strong thematic unity and constitute a cluster of chapters devoted to the organization of scientific knowledge in individual domains and across different domains. A similar thematic unity emerges clearly from a corresponding sequence of chapters in the *Naǧāt*. Avicenna first deals with the internal structure of a science (*Naǧāt* I, 125: subjects of the sciences; *Naǧāt* I, 126: subjects of scientific questions; *Naǧāt* I, 127: predicates of scientific questions, including types, restrictions, and exceptions; *Naǧāt* I, 128: types of principles; *Naǧāt* I, 129: propositions of certitude), then addresses the interrelations among different sciences (*Naǧāt* I, 130: division of the sciences; *Naǧāt* I, 131: explanation across sciences; *Naǧāt* I, 132: transfer of demonstration; *Naǧāt* I, 133: sharing of questions; *Naǧāt* I, 135: summary of the elements of demonstration and transition to explanation).

Sciences with Different Subjects

Sciences that have different subjects may either (ab) overlap or (aa) not.[5] If two sciences overlap, they may overlap (aba) fully or (abb) only in part. If the subjects of two sciences (aa) do not overlap, then the two sciences are genuinely distinct. This is, for instance, the case of arithmetic and geometry, the subjects of which are different kinds of quantity, that is to say, discrete quantity (number) and continuous quantity (extended magnitude).[6]

If, by contrast, the subjects of two sciences (ab) overlap, several cases are possible. These are determined by the way in which the subjects of the two sciences overlap and by the nature of their relation. This class accommodates most of the admissible relations in Avicenna's taxonomy. When two sciences overlap, they may overlap either (aba) fully, if one of the two is entirely subordinated to the other (this is, as we shall see, the paradigmatic type of subordination), or (abb) only in part. If two subjects overlap fully, then one is *more general* than the other (in one of two ways). If they overlap only in part, then the two subjects have something (but not everything) in common. The latter case is illustrated by the relation between medicine and ethics. While medicine is concerned with the investigation of the human body and its parts (and is in turn subordinated to natural philosophy), ethics investigates the rational soul and its practical faculties.

Two fundamentally different kinds of generality may be involved in the case of full overlap, when the subject of one science is taken by Avicenna to be more general than the subject of the other. This is because the more general subject may stand to the less general subject either (abaa) in a genus-like relation or (abab) in an implicate-implicant relation of the kind holding between the notions of one and existent and every other notion or thing. The latter case is introduced to carve

5. Avicenna uses the term *mudāḫala* to indicate that the intersection of two subjects is non-empty. This may be either (i) because one science is part of or subordinated to the other or (ii) because the two sciences simply have something in common.

6. The case of arithmetic and geometry is interesting because it is repeatedly advocated as illustration of different types of relations: in this case, with respect to the subject, as a canonical example of distinctness (the point is indirectly confirmed by what Avicenna says later about principles), and then with respect to the sharing of questions. At *Naǧāt* I, 130 (ii), Avicenna characterizes two sciences as distinct (*mutabāyina*) when they do not share the subject itself or the genus of the subject; the example given there, which is perhaps more straightforward, is the relation between arithmetic and natural philosophy (whose subjects are neither the same nor of the same kind or co-generic). The example of *Burhān* II, 7 is a case of distinctness where the subject itself is not the same, but the genus (quantity) is indeed the same; in the vocabulary of the *Naǧāt*, arithmetic and geometry do not count as distinct sciences but as sciences that are equal in rank because their subjects fall under the same kind. An echo of this language perhaps survives in the second classification of *Burhān* II, 7, where Avicenna describes again arithmetic and geometry in such terms, that is to say, as being equal in rank with respect to their principles (but then uses them again to illustrate a problematic case of apparent kind crossing).

out a space for metaphysics and account for its relation to the other sciences in a way that is consistent with Avicenna's basic ontological tenets: the subject of metaphysics obviously cannot be a genus of the subject of any other science, however qualified (for existent is not a genus of any of the categories or their internal subdivisions). Avicenna first discusses in detail the various sub-cases falling in the category of the genus-like relation between two subjects, deferring the treatment of the other kind of generality to a later stage of the analysis. The peculiar status of metaphysics is discussed right before (b) the case of sciences that differ because their subject, while being one and the same, is investigated in different ways.

FIRST TYPE OF GENERALITY: PARTICULAR SCIENCES

The Genus-Species and the Genus-Accident-of-a-Species Relations: The first alternative (abaa) involves a genus-like relation and is further differentiated into various sub-cases depending on whether the relation between the more general subject and the less general subject is (abaaa) a *real* genus-species relation or involves (abaab) a range of possibilities in which the more general subject stands to the more specific subject in the way a genus stands to an accident of a species.[7]

Parthood and Subordination. Associated with this division is a distinction between two relations of subordination: being part and being under. Avicenna distinguishes between two main types of subordination depending on whether a science is part (*ǧuzʾ*) of another science or merely falls under (*taḥta*) another science. The relation of being part is stronger than the relation of falling under, and the latter includes a variety of cases. The terminology is found in Aristotle in connection with different forms of dependence (*An. Post.* A7), but in Avicenna it is frequently used to provide a more exact categorization of the various pairs of sciences considered in the chapter.[8] The two types of dependence reflect different ontological relations between the underlying subjects. The distinction is also articulated *in nuce* in Alfarabi and, in general, various examples used in *Burhān* II, 7 to illustrate different types of subordination (for example, geometry and the study of cones; geometry and astronomy as the study of moving spheres), as well as the discussion of metaphysics and its relation to dialectic and sophistic, all seem to be themes of direct Farabian derivation.[9]

7. Case (abaaa) is illustrated by two examples, namely the relation between the study of specific kinds of solids, in the example cones (*maḫrūṭāt*), as *part* of the study of solids (*muǧassamāt*) in general, and that of solids as part of (a general theory of extended) magnitudes (*maqādīr*), which presumably coincides with geometry (*handasa*) itself. In other words, the science of cones stands to stereometry in the same way that stereometry stands to geometry.

8. For a detailed list of the numerous Aristotelian expressions used to characterize subordination, see Barnes (1993, pp. 158–159) on *An. Post.* A13.

9. On the distinction between being part (*ǧuzʾ*) and being under (*taḥta*), see Alfarabi, *Burhān* IV, 1, pp. 64.8–65.10.

Returning to the internal division of the first sense of generality, the first type of relation (abaaa) is genuine "parthood:" when the subject of science A is more general than the subject of science B in such a way that the first is a real genus of the second, then science B is *part* of science A. This is because science B is just an internal division or branch of science A, in the way the study of solids (stereometry) is part of the study of extended magnitudes without qualification (geometry).

The other type of relation (abaab) accounts for yet another level of differentiation. Avicenna introduces it to characterize four different types of subordination that do not involve parthood but are, at the same time, distinct from the implicate-implicant subordination previously identified. The second type of generality (abab) discussed later is another type of subordination exemplifying the relation of being under. It is not included by Avicenna in the list of four ways in which a subject is said to fall *under* another with respect to the first sense of generality because the ontological relation it captures is different (it is not a genus-like relation but an implicate-implicant relation).

The division of case (abaa) into a number of sub-cases is motivated by the need to account for two different ways in which a more general subject can be specified to determine the subject of the subordinate science. The question of how a more general subject is specified with the addition of various sorts of qualifications is central for Avicenna, who seems to be interested in providing a systematic justification for the relation between certain salient pairs of scientific disciplines. If the type of subordination is a function of the relation between the subjects of the sciences, then it is essential for the taxonomist to be able to specify exactly what constitutes the subject of each science (which is, by the way, a requirement for the existence of the science in the first place) and, more importantly, to specify the nature of the relation between the two subjects.

The straightforward case of parthood obtains when the more general subject is a genus and the more specific subject is a species of that genus identified by a differentia. In this case only is the lower science said to be *part* of the higher science.

But if parthood is useful to characterize the internal division of a science in its subfields, it fails to capture the complexity of a great many other representative cases of subordination, such as the relation between metaphysics and the other sciences on a general level and, more specifically, the relation between other pairs of sciences, especially in certain "oblique" cases in which the subordination cuts across predicamental lines.

Four Kinds of Subordination. Case (abaab) includes a range of ways in which the more general subject is specified not by means of a dividing differentia but rather through the addition of various kinds of accidental attributes. The more general subject may be specified or become more specific (*muḫaṣṣaṣ*, *aḫaṣṣ*) in association with the following properties: (i) a per se accident, (ii) a foreign non-per se accident that is a disposition in the essence of the subject and not a mere relation, (iii) a foreign non-per se accident that is not a disposition in the essence of the subject but rather a mere relation, or (iv) a foreign non-per se accident of a species of a different subject. The four paradigmatic types of relations, where the ontological connection of the more specific subject with the more general subject becomes gradually weaker as we move from (i) to (iv), are associated by Avicenna with peculiar cases of scientific subordination.

The first case (i) corresponds to the relation between natural philosophy and medicine: medicine as a science is subordinated to natural philosophy because it investigates the subject of a part of natural philosophy (the one concerned with the human body, which is a specific kind of animate body and, more generally, of body insofar as it is subject to motion and rest). The subject of medicine is body insofar as it is qualified by *health* and *sickness*, which are two per se accidents of animate body.[10]

The second case (ii) corresponds to the relation between geometry and astronomy: the subject of geometry is continuous quantity or extended magnitude (*miqdār*)—lines, surfaces, and solids—and part of geometry is concerned with spheres (a species of solid). The subject of astronomy, by contrast, is the moving spheres, that is to say, spheres qualified by a certain attribute. The attribute in question is a foreign non-per se accident of the subject (where "foreign non-per se" must be understood in the technical sense developed in *Burhān* II, 2–3: sphere or continuous quantity is not assumed in the definition of moving). Moreover, the attribute actually inheres in the subject, and even though its ontological connection with it is weaker than in case (i) (where the attribute is per se), it is not merely relational as in case (iii). The subject of astronomy (moving spheres) is less general than the subject of geometry, not simply because it is narrower than extended magnitude as such, but also because it is further qualified by the accident of motion (it is not spheres without qualification that astronomy investigates, but only moving spheres). For this reason, astronomy is *subordinated* to geometry rather than being part of it. Last, astronomy is concerned with moving spheres as geometrical objects and with their geometrical relations, that is to say, with moving spheres insofar as they are *spheres*, rather than insofar as they are characterized by motion. For this reason, astronomy is subordinated to *geometry* and not to natural philosophy.

The third case (iii) corresponds to the relation between geometry and optics. This kind of subordination obtains when the foreign non-per se accident that qualifies the subject of a science has a purely relational connection with it. The subject of optics is visual lines, which are not a genuine *kind* of lines but rather a class (*ṣinf*) of lines characterized by a certain relation to vision (for example, optics

10. On what natural philosophy has in common with other sciences, see in general *Samāʿ ṭabīʿī* I, 8.

studies the properties of the visual cone whose vertex lies in the eye and whose base coincides with the ideal extremes of the observed object).[11] Optics cannot be a part of geometry, because it is concerned with lines in a narrower sense. What is more, its subject is determined by an attribute that is entirely foreign to the nature of lines as such and qualifies them in a purely relational way.

The fourth case (iv) corresponds to the relation between arithmetic and music and represents the most awkward type of subordination. Here the attribute that qualifies the subject of the less general science in a relevant way is a foreign non-per se accident of the species of the subject of a different science. Avicenna cleverly uses this "oblique" relation to account for the ambivalent status of music. The subject of music (notes) is, in and of itself, only a special kind of physical entity (sounds), that is to say, a species of the subject of natural philosophy. But music investigates sounds insofar as they are characterized by numerical properties and relations (focusing especially on numerical ratios). The latter are foreign non-per se attributes with respect to sounds but count as relevant per se attributes of the species of a different subject, namely number (the species of numbers are individual numbers according to Avicenna). Since number is the subject of arithmetic, it is to this science that music is obliquely subordinated. In presenting case (iv), Avicenna explicitly argues that if we were to investigate the subject of music in itself rather than insofar as it is qualified by numerical properties, then music would not be subordinated to arithmetic but rather be a genuine *part* of natural philosophy (and not even merely subordinated to it). The reason is that the subject of music, once stripped of its numerical attachments, *just* is a species of the subject of natural philosophy (a kind of physical entity).[12]

The accidents singled out in (i)–(iv) for the identification of the subject of the subordinate science (whether they be per se or foreign) are distinct from the per se accidents that each science—the superordinate and the subordinate alike—then investigates and seeks to prove to hold of their subjects. Thus, the subject of medicine is the human body insofar as it is healthy or sick, and medicine as a science seeks to establish the per se accidents of the human body under this qualification (and this set will include a broad variety of properties); the subject of astronomy is the moving spheres qua spheres, and astronomy as a science investigates the per se accidents of moving spheres qua spheres; the subject of optics (visual lines) is qualified by an extrinsic property of lines, namely their being related to vision, but optics then investigates the per se accidents of visual lines qua lines. In a similar way, music investigates the per se accidents of notes, namely consonance (*ittifāq*)

11. Examples from *Burhān* II, 9 and III, 3 are discussed in chapter 10.

12. Avicenna's general characterization of music as a discipline, of its subject (sounds and notes), and of its attributes is at *Mūsīqā* I, 1, pp. 9.6–13.17, I, 2, pp. 14.1–17.6, I, 3, pp. 18.12–26.18, and I, 4, pp. 27.1–30.11.

and dissonance (*iḫtilāf*). Thus, while the qualifying accidents together with the subject obviously contribute to determining the set of per se accidents investigated by a science, the two domains are in general not identical.

That Avicenna might have introduced these four cases as mere ad hoc devices to provide a conceptual justification for certain specific claims of subordination is not an outlandish idea, but there seem to be noteworthy systematic aspects to the taxonomy.

The difference between (i) on the one hand and (ii)–(iv) on the other is straightforward: the first case is the only one in which the qualifying accident is a per se accident of the subject of the relevant science; in the three other cases, the accident is foreign non-per se. In case (iv) the foreign accident does not attach directly to the subject, while in cases (ii)–(iii) it does. Thus, (iv) is an oblique form of subordination, while (i)–(iii) (as well as the parthood relation) are vertical, in the sense that it is the science whose subject becomes the subject of the subordinate science through the addition of certain qualifications (healthy or sick *bodies*, moving *spheres*, visual *lines*) that the subordinate science is subordinate to and not *another* science whose subject is different.

Cases (ii) and (iii) differ because the connection between the accident and the subject is stronger in the former and weaker in the latter: in (ii) the accident belongs to the subject as a "real" property or disposition inhering in it, while in (iii) the connection is merely relational.

This is because the moving spheres are objects that are moving (motion is an actually inherent attribute of the spheres), while in the case of visual lines, the link to vision is merely a relation that does not correspond to a property inhering in the subject (lines); hence lines are investigated under a respect that is somehow external.

Last, Avicenna is presumably committed to two further theses that are not stated explicitly but that nevertheless seem to be implied by two comparative observations that he makes. First, (i)–(iii) differ from (iv) because in the former cases, but not in the latter, the subject of the more general science is *predicated* of the subject of the less general science (for the subject of the less general science *is* the subject of the more general science accompanied by a certain qualification). Thus, trivially, human bodies insofar as they are sick or healthy are bodies, moving spheres are spheres, and visual lines are lines. In (iv), by contrast, the subject of the superordinate science is not predicated of the subject of the subordinate science, because the former is not just more general but categorically distinct (notes are not numbers or numerical ratios).[13]

Second, there is also a distinction between the respects in which the relation of subordination obtains. For example, (ii) and (iv) are genuinely distinct cases

13. See *Naǧāt* I, 130 (vii).

because in the former the relation of subordination is determined as a function of the subject (astronomy investigates moving *spheres* insofar as they are spheres and hence is subordinated to geometry), while in the latter it is determined as a function of the qualifying accident (music investigates notes insofar as they are characterized by *numerical ratios*).[14]

SECOND TYPE OF GENERALITY: THE CASE OF METAPHYSICS

We must now resume the discussion of a higher node in the division. Case (aba), in which the subject of one science is more general than the subject of the other science, had two main divisions depending on the kind of generality (*ʿumūm*) involved: a genus-like relation or an implicate-implicant relation. The second type of generality characterizes the unique relation holding between the most general notions (existent, one) and everything that falls under them. A comment Avicenna makes in a different context (*Burhān* II, 3) clarifies the key assumption the division of *Burhān* II, 7 is based on. He writes:[15]

> Text 5.1: *Burhān* II, 3, pp. 139.19–140.4
> Some subjects of per se accidents are real species, intermediate genera, or higher genera, like human for its per se accidents, and likewise animal, body, and quantity. For every one of them has per se accidents, as we have said.
>
> Some [other] subjects resemble genera and species (*yušbihu*) without being [genera and species]: these are the notions that are said of many (though not by being the same) and are implicates (*lawāzim*) that are not included in the quiddity of the things that fall within the categories, like existence and oneness, two [notions] that resemble, in a way, the highest genera (*šābīhāni min ǧiha li-l-aǧnās al-ʿāliya*).

In Text 5.1, Avicenna draws a distinction between the case in which the subjects of per se accidents are genuine species and genera, that is to say, entities in the categories (for example, human, animal, body, or quantity), and the case in which the subjects merely resemble genera (for example, existent or one). Subjects of the former kind are the subjects of the particular sciences, while the second kind of subject is the subject metaphysics (its per se accidents are, for example, potentiality and actuality, cause and effect, necessary and possible, priority and posteriority, universality and particularity, and so on).

14. On the distinction between case (ii) and case (iv), see Strobino (2017, pp. 111–113).

15. At *Naǧāt* I, 130 (vi), the relation between metaphysics and the particular sciences is characterized as one between sciences that are not equal in rank but involve a kind of subordination. Of the two cases of subordination (without parthood) identified as the main divisions of this class, the case of metaphysics is the first, which obtains when "the subject of the higher science is not in reality a genus of the subject of the lower" and the subject of metaphysics is characterized as the existent without qualification (*al-mawǧūd al-muṭlaq*).

The second type of generality (ʿ*umūm*) corresponds to the relation between an implicate (*lāzim*) and that of which it is an implicate, and more specifically to the relation holding between the notions of "one" and "existent" and everything else.[16] According to Avicenna, there is a sense in which "existent" is more general than everything else, and hence the science that investigates it, namely metaphysics, is more general than or superordinate to the sciences that investigate specific domains of the existent.[17] Avicenna avoids any reference to the notion of genus because he is unequivocally committed to the view that the existent is not a genus of anything and that the correct relation is captured by the notion of necessary implication (*luzūm*). This category serves the sole critical purpose of accommodating the case of metaphysics and its relation to all other sciences.

The discussion of the status of metaphysics in *Burhān* II, 7 concisely develops three points. First, Avicenna briefly summarizes various features that distinguish metaphysics from the other sciences and contends that the other sciences are all subordinated to metaphysics without being part of it. Second, he briefly argues that a science more general (in the peculiar sense identified here) than all other sciences is necessary, and that the principles of the particular sciences (at least those that are not independently self-evident) are validated by this more general science and therefore have a conditional character.[18] Third, given the generality of

16. The technical term one would expect is *malzūm*, but Avicenna does not use it in this context. On the fact that the notion of *mawǧūd* is inseparable (*lā yufāriqu luzūm maʿnā l-mawǧūd*) from every *thing* (whether it be a concrete entity or an object of estimation or intellection), see *Ilāhiyyāt* I, 5, p. 32.3–5. Moreover, at *Ilāhiyyāt* V, 6, p. 234.12–18, Avicenna contends that the categories are the most general predicates only in the sense of being genera and constituents of the quiddities of the things of which they are predicated. This implies that they are *not* the most general predicates without qualification. The reason is that there are predicates that are not constituents of the quiddities of the things of which they are predicated, but rather implicates of them. And these predicates are even more general, in a different sense of generality, than the categories themselves. It is clear that Avicenna is referring implicitly to notions such as existent and one.

17. The relation of metaphysics to the other sciences and Avicenna's understanding of it as "the founding discipline" in the system of scientific knowledge are discussed in detail by Bertolacci (2006, especially pt. II, ch. 7). At *Naǧāt* I, 130 (vi), the relation between metaphysics and the particular sciences is characterized as one between sciences that are not equal in rank but involve a kind of subordination. Of the two cases of subordination (without parthood) identified as the main divisions of this class, the case of metaphysics is the first, which obtains when "the subject of the higher science is not in reality a genus of the subject of the lower" and the subject of metaphysics is characterized as the existent without qualification (*al-mawǧūd al-muṭlaq*). On the utility of metaphysics for the particular sciences, see *Ilāhiyyāt* I, 3, p. 18.12–17.

18. On the question of whether all or only some of the principles are proved in metaphysics, Avicenna seems to have a rather cut-and-dried view: some principles (even in natural philosophy) are often described as being self-evident. Moreover, the account of types of assertions in *Burhān* I, 4 seems to show that Avicenna is committed to the view that at least for *some* principles the source of necessity is external, whether based directly on perception or experience. Last, a relevant case is discussed in

this discipline, he addresses the problem of how it is to be distinguished from two other (logical) disciplines—dialectic and sophistic—which seem to have a similarly wide scope of application, and shows that metaphysics differs from dialectic and sophistic with respect to its subject, principles, and goals.[19]

At *Ilāhiyyāt* I, 2, pp. 14.18–15.7, Avicenna explicitly lists among the goals of metaphysics the investigation of the principles of the particular sciences. The principles of more specific sciences are investigated by more general sciences (as in the case of medicine and natural philosophy, geodesy, and geometry), but particular sciences investigate particular kinds of existent, hence their principles are explained by the discipline concerned primarily with the existent as such. Metaphysics investigates the states (attributes) of the existent, its divisions, and its species, identifying and establishing, when necessary, the existence of the subject of natural and mathematical sciences. Once the subject of a science is determined, it then becomes the province of the relevant particular science. In general, metaphysical questions concern (i) the causes of the existent insofar as it is caused, (ii) the accidents of the existent, and (iii) the principles of the particular sciences.[20]

Burhān II, 9, where Avicenna contends that some principles may be borrowed from a lower science to prove theorems of a higher science and claims that those principles may in turn either be self-evident or else proved by (other) claims in the higher science. I return to this problem in chapter 10.

The conditional character associated by Avicenna with the principles of the particular sciences concerns the question of the true logical form of principles in sciences other than metaphysics. Avicenna's contention is that the latter have the form of a conditional statement (a "hypothetical conditional" in his technical terminology) whose antecedent is proved in metaphysics. Such principles will take the form of statements in which the antecedent purports to establish either (i) the existence of a subject (for example, "if there are such-and-such entities, then *p*," where the consequent *p* is a theorem in which a per se accident of the subject is proved to belong to the subject itself) or (ii) a predicative assertion on which the consequent in turn depends. On the role of metaphysics as a discipline that confirms the soundness (*yufīdu taṣḥīḥ*) of the principles of the other sciences, see *Ilāhiyyāt* I, 1, p. 5.7–8.

19. On the aspects that metaphysics has in common with dialectic and sophistic, see also *Ilāhiyyāt* I, 2, p. 16.13–20.

20. At *Burhān* III, 1, p. 194.11–14, Avicenna maintains that the principles of the subordinate sciences are (ultimately) proved by metaphysics. And at *Samāʿ ṭabīʿī* I, 2, p. 14.10, the view could not be expressed in clearer terms: "Let it be posited (*fa-l-yūḍaʿ*) for the natural philosopher that body as such has a principle that is matter and a principle that is form," and again, at I, 2, p. 14.12, "Let it also be posited (*wa-l-yūḍaʿ*) that what is material is never separated from form." These two principles are predicative assertions about physical bodies and about what is material that are assumed by natural philosophy without proof and demonstrated by metaphysics (see *Ilāhiyyāt* II, 2 and 3, respectively). Along the same lines, at *Samāʿ ṭabīʿī* I, 5, p. 30.6–7 (McGinnis 2009, p. 39; emphasis added), Avicenna writes: "We *set it down* as a *posit*, which the *natural philosopher accepts* and the *metaphysician demonstrates*, that the bodies undergoing these motions are moved only as a result of powers in them that are principles of their motions and actions" (the point in question is established at *Ilāhiyyāt* IX, 2). Another example involves one of Avicenna's arguments for the rejection of the existence of void at *Samāʿ ṭabīʿī* II, 8, p. 124.15–16 (McGinnis 2009, p. 179; emphasis added): "Now one of the things would be in its nature an accident, while accidentally being a substance, and so the substantiality would be

Sciences with the Same Subject Investigated in Different Respects

In the first part of *Burhān* II, 7, after a detailed analysis of case (a), namely the various ways in which two sciences may differ when their subjects are different, Avicenna addresses the other primary division of this taxonomical tree, namely the case (b) in which sciences may differ even when their subject is one and the same (*iḫtilāf al-ʿulūm al-muttafiqa fī l-mawḍūʿ*).

According to Avicenna, when the subject of two sciences is one and the same, there may be two reasons to consider them different. Either (ba) one science investigates the subject without qualification and the other investigates it in a specific respect, or (bb) both sciences investigate their shared subject in specific (and different) respects. The purpose of the distinction is, in all likelihood, to account for two significant types of subordination, namely the relation between medicine and natural philosophy, which we have already encountered, and that between astronomy and natural philosophy.

The subject of medicine may at first appear to coincide with the subject of a particular branch of natural philosophy dealing with the human body. According to Avicenna, however, medicine treats the human body *insofar as* it is qualified by the per se accidents of health and sickness, which is different from investigating the properties of a certain kind of body qua body (the latter is the prerogative of natural philosophy).[21]

The other case gives Avicenna the opportunity to elaborate on the subordination relation of astronomy to geometry. Why is astronomy subordinated to geometry and not to natural philosophy? Or, more specifically, why is astronomy neither part of nor subordinated to the branch of natural philosophy corresponding to the *De Caelo*? The reason is that in spite of sharing the same subject, namely the spherical body of the universe, the two sciences investigate this subject from different perspectives. The subject of astronomy is the spherical body of the universe insofar as it is characterized by quantity, and hence as a geometrical object (even with its complex internal divisions, all of which are in one way or another connected with spherical geometry, and in particular with spherical trigonometry, as developed in Ptolemy's *Almagest*, which is the source text for the discipline). Astronomy as a science investigates the per se attributes of the spherical body of

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something accidental to one of the natures—which is impossible, *as will become clear, particularly in first philosophy*." The reference is presumably to *Ilāhiyyāt* II, 1, p. 58.14–16, where entertaining the view that something could be both an accident and a substance is characterized as a serious mistake, already refuted in logic (*Maqūlāt* I, 6).

21. Medicine and natural philosophy were used to illustrate the first type of subordination (i) discussed earlier under case (abaab). An extensive discussion of the structure of medicine as a science occupies the first book of Avicenna's *Qānūn*, where it becomes clear that he understands it as a genuine domain of application of the meta-theoretical guidelines developed in the *Burhān*.

TABLE 8 First classification of the sciences (by subject)

<table>
<tr><th colspan="7">Division of the sciences (I)</th></tr>
<tr><td rowspan="8">(a) Different subjects</td><td colspan="5">(aa) No overlap</td><td>Arithmetic-Geometry</td></tr>
<tr><td rowspan="7">(ab) Overlap</td><td rowspan="6">(aba) Subordination (one subject more general than the other)</td><td rowspan="5">(abaa) First kind of generality (genus-like)</td><td colspan="2">(abaaa) Genus-species (parthood)</td><td>Geometry-Study of cones</td></tr>
<tr><td rowspan="4">(abaab) Genus-accident (subordination)</td><td>i</td><td>Natural philosophy-Medicine</td></tr>
<tr><td>ii</td><td>Geometry-Astronomy</td></tr>
<tr><td>iii</td><td>Geometry-Optics</td></tr>
<tr><td>iv</td><td>Arithmetic-Music</td></tr>
<tr><td colspan="3">(abaa) Second kind of generality (implicate-implicant) (subordination)</td><td>Metaphysics-Particular sciences</td></tr>
<tr><td colspan="4">(abb) Partial overlap</td><td>Medicine-Ethics</td></tr>
<tr><td rowspan="2">(b) Same subject, different respects</td><td colspan="5">(ba) Unqualified-qualified</td><td>Natural philosophy-Medicine</td></tr>
<tr><td colspan="5">(bb) Qualified-qualified</td><td>Astronomy-Natural philosophy</td></tr>
</table>

the universe *in this respect*. The subject of (the relevant part of) natural philosophy, by contrast, is the spherical body of the universe insofar as it is characterized by a peculiar kind of motion (circular motion). Part of natural philosophy as a science investigates the per se attributes of the spherical body of the universe *in this other respect*. In other words, the two sciences deal with the sphericity of the universe from distinct angles: one in terms of its purely geometrical properties, and the other in terms of its physical properties (focusing in particular on the relation between spherical shape and circular motion).[22]

Table 8 illustrates synoptically Avicenna's first classification.

22. At *Naǧāt* I, 130 (iv), Avicenna contends that natural philosophy focuses on the *quality* of motion (or rest) as the salient attribute of the body of the universe, while geometry focuses on its quantity and the attributes that belong to it insofar as it has quantity, for example its sphericity.

THE SECOND CLASSIFICATION

The second part of *Burhān* II, 7 is much shorter, only about a tenth of the whole chapter, but it is nonetheless significant for its complementary character and because it offers an interesting way to confirm and validate the results presented in the first part.

The question that governs the second classification is what sciences may have in common. This requires us to look at the three canonical items that define the perimeter of an Aristotelian science (with a slight variation due to one of the peculiarities of Avicenna's conceptual vocabulary, namely the replacement of the notion of per se attribute with that of question): (1) principles, (2) subjects, and (3) scientific questions (the theorems of a science in which salient properties, or per se accidents of the subject, are demonstratively proved to belong to the subject).[23] He writes:

> Text 5.2: *Burhān* II, 7, pp. 167.11–168.18
> Resuming from the beginning, we say that sciences that share [some of their elements] share either [(1)] principles, [(2)] subjects, or [(3)] questions.
>
> By [sciences] that share principles we do not mean [sciences] that share principles that are common to every science but [sciences] that share principles that are common to some sciences, as the mathematical sciences share the [principles] that things that are equal to one and the same thing are equal. That [kind of] sharing is either [(1.1)] at one and the same rank, like geometry and [the science] of number with regard to the aforementioned principle; or [(1.2)] [such] that the principles belong to one of them first and to the second after [the first], in the way geometry and the science of optics, and indeed arithmetic and the science of music, share this [very same] principle. But geometry is more general than the science of optics as far as the subject is concerned. Therefore, this principle [pertains] to [geometry] first and to optics after [geometry]. Such is also the state of arithmetic with respect to music. Or [(1.3)] what is a principle in a science is a question in another science. This may be in two ways. Either [(1.3.1)] the two sciences have two different subjects in terms of generality and specificity. In such a case, [either] [(1.3.1.1)] a thing is proved in a higher science and assumed as a principle in a lower science (this is a real principle), or [(1.3.1.2)] a thing is proved in a lower science and assumed as a principle in a higher science (this is a principle with respect to us). Or [(1.3.2)] the two sciences are not different in terms of generality and specificity but are like arithmetic and geometry, in which case the questions of one of them are taken as principles for the questions of the other. For many of the principles in the tenth book of the *Elements*

23. The general term for sharing principles, subjects, or questions in this section is *šarika*. Alfarabi, at *Burhān* IV, 2, pp. 65.13–72.11 (*On what sciences share with one another*), distinguishes three kinds of sharing depending on whether what is shared is (i) a premise (principle), (ii) a subject, or (iii) something proved (a question or an attribute). Avicenna's analysis in the second part of *Burhān* II, 7 adopts the same threefold distinction, though instead of focusing on per se attributes, he refers only to the propositions by means of which those attributes are proved to belong to their subjects.

concern numbers and are demonstrated earlier in the books on numbers. This is not possible if the two sciences do not share a subject or the genus of a subject.

The sharing of [(2)] questions is [such] that what is sought in both of them is something predicated of one and the same subject. Otherwise, there is no sharing [of questions] at all. Furthermore, this is not possible unless the two sciences also share the subject.

Therefore, the primary and fundamental [kind of] sharing that sciences may have is the one that necessitates the third division, namely the sharing of [(3)] subject in one of the ways mentioned above, which are three. Either [(3.1)] one of the two subjects is more general and the other more specific, as in natural philosophy and medicine, geometry and the study of cones, and other similar cases.[24] Or [(3.2)] each of the two subjects of the two sciences has something proper [to it] and something shared with the other [science], like medicine and ethics. Or [(3.3)] the subject itself is one and the same for both, but it is considered in two different ways [by the two sciences], in such a way that it becomes a subject for this [science] when considered in a certain way, and [another] subject for that science, when considered in a certain [other] way, just as the body of the heaven and the universe is a subject for astronomy and for natural science. Having discussed the ways in which sciences share subjects, principles, and questions, we must now discuss the transfer of demonstration.

The second classification, which is illustrated in Table 9 at the end of this chapter, offers a concise summary as well as a complementary analysis of the same relations discussed more extensively in the first part of *Burhān* II, 7. Moreover, it sets the stage for the investigation of another critical theme, namely the question of explanation across sciences and the different ways in which claims established in one science may be used as principles for another science (I discuss this in detail in chapter 10). Let us briefly look at the cases identified in this passage and confirm their almost perfect alignment with the elements of the first taxonomy.

Shared Principles

The first criterion is the sharing of principles. Excluding the trivial case of common axioms such as the law of the excluded middle, which are by definition shared by all sciences, sciences may share principles in one of three ways.[25] Either (1.1) the shared principles are of the same rank, or (1.2) in one science the principles are prior and in the other posterior, or (1.3) something is a principle in one science and a scientific

24. On the relation between the subjects of medicine and natural philosophy, see also *Burhān* II, 2, p. 133.5–19.

25. Another relevant source is *Burhān* III, 8. The first part of the chapter (III, 8, pp. 247.1–248.4) corresponds to on *An. Post.* A28 and is concerned with shared subjects, questions, and principles. The last part of the chapter (III, 8, pp. 251.1–255.9) corresponds on *An. Post.* A32 and addresses specifically the sharing of principles to reject the notion that the principles may be the same for all sciences.

question (or theorem) in the other science.[26] Case (1.3) includes in turn various sub-cases. When something is a principle in one science and a question in another science, the two sciences may be such that either (1.3.1) the subject of one is more general than the subject of the other or (1.3.2) their subjects do not differ in terms of their relative generality.

In the first case, namely when one of the two subjects is more general than the other, a proposition may be either (1.3.1.1) a scientific question proved by the more general, superordinate science and a principle assumed by the less general, subordinate science, or (1.3.1.2) a scientific question proved by the latter and assumed by the former. In case (1.3.1.1), presumably the method of transfer should be understood as follows (with a generic example involving syllogistic deductions in the first figure):

Higher science:	AaB; BaC ⊢ AaC
Lower science:	AaC; CaD ⊢ AaD

where the two arguments have two different conclusions, and the conclusion of the syllogism in the higher science becomes a premise of the syllogism in the lower science. Avicenna characterizes this case, in *Burhān* II, 8, as a less interesting type of transfer, presumably because the conclusions are different, and this turns out to be in fact simply a concatenation of two syllogisms.

In case (1.3.1.2), by contrast, the relation is as follows, under the condition that AaB must not be used to prove AaC or CaD (and again with a generic example involving syllogisms in the first figure):

Higher science:	AaD; DaB ⊢ AaB
Lower science:	AaC; CaD ⊢ AaD

According to Avicenna, these two sub-cases account for the distinction between what is a real principle (*mabdaʾ ḥaqīqī*) and what is only a principle according to us (*mabdaʾ bi-l-qiyās ilaynā*).

Case (1.1), in which shared principles are of the same rank, and case (1.3.2), in which subjects do not differ in terms of their relative generality, are interestingly illustrated by the same example, namely by the relation between arithmetic and geometry. As in case (aa) from the first classification, where there was no overlap in subject, in (1.1) too there is a similar distinction concerning principles, which are in a sense of the same rank because they deal with different species of quantities.[27]

26. The Arabic expression *ʿalā martaba wāḥida* captures the idea that the principles of two sciences are of the same rank, as opposed to being related as prior and posterior. At *Naǧāt* I, 130 (iii)–(iv), p. 139.9–11, an analogous expression (*mutasāwiya fī l-martaba*) is used for the characterization of subjects.

27. Avicenna gives as an example for this group the principle that equals being subtracted from equals result in equals, which is no more a principle of geometry than a principle of arithmetic (or

Case (1.3.2) seems more problematic as it involves propositions that are principles in one science and questions in another, in the absence of subordination. This is compatible with the characteristic feature of (1.1) (the principles being one and the same in rank) but raises the following question: How can something be genuinely proved in one science and assumed in another science, if there is no subordination? The example offered by Avicenna as an illustration may be the very reason he feels pressured to introduce this (otherwise suspiciously spurious) case in the first place, namely in order to accommodate in his taxonomy the use, in the tenth book of Euclid's *Elements*, of theorems of arithmetic that are proved in some of the earlier books (presumably *Elements* VII–IX) as principles for certain theorems of geometry.[28]

Case (1.2) encapsulates the relations between geometry and optics and between arithmetic and music. The use of these two examples is unsurprisingly compatible with the first classification, given that they are both instances of subordination not involving parthood. They correspond to (abaab, iii) and (abaab, iv) in the first list, even though Avicenna treats the two pairs separately in that context to reflect a more fine-grained distinction between the types of subjects involved. In this case, the method of transfer should be understood as follows:

Higher science:	AaB; BaC ⊢ AaC
Lower science:	AaD; DaC ⊢ AaC

where the conclusion of the two proofs is the same, but it is established by means of two distinct middle terms of different explanatory power (the middle term B in the higher science is the proximate cause with respect to the predicate of the conclusion, while the middle term D in the lower science is the remote cause with respect to the predicate of the conclusion).[29]

conversely). In this respect the two sciences are the same in rank. Perhaps the criterion can be generalized to every pair of sciences whose subjects are coordinate species under the same kind.

28. Avicenna collectively refers to the relevant books of the *Elements* as the treatises on number (*al-maqālāt al-ʿadadiyya*). The problem is that in case (aa) from the first classification Avicenna seems to suggest that there is no overlap whatsoever between the subjects of arithmetic and geometry, while case (1.3.2) from the second classification ends with the claim that the use of arithmetical theorems as geometrical principles "is not possible if there is no sharing of (i) a subject or (ii) the genus of a subject." Option (i) would be in tension with the general characterization of (aa) and Avicenna's commitment to the specific view that the subject of arithmetic and geometry are distinct. Option (ii), on the other hand, would imply that Avicenna understands the relation in (1.3.2) in rather different terms, that is to say, with respect to a *third* more general science dealing with quantity as such.

29. Presumably the middle term from the higher science must be universally predicated of the middle term of the lower science. In *Burhān* III, 8, Avicenna shows an interest also in the case where one middle term is not predicated of the other, with regard to the question of the criteria of identity and distinctness of two sciences.

Shared Questions

The second criterion is the sharing of questions. It is only briefly mentioned in order to specify a necessary condition, namely that the predicate that is demonstratively proved to belong to the subject of the sciences under consideration be shared. This condition, however, entails an even more fundamental connection between the subjects because two sciences could not be such that one and the same attribute is predicated of their subjects, unless those subjects are identical or overlap in one of the ways described in the first classification and summarized again later in the chapter. If the two subjects were altogether distinct, there could be no sharing of predicates of scientific questions.[30]

Shared Subjects

The third criterion is the sharing of subjects. At *Burhān* II, 7, p. 168.10–11, it is characterized as follows: "the primary and most fundamental [type of] sharing [...] is the sharing of the subject in one of the three aforementioned ways" (*aš-šarika al-awwaliyya al-aṣliyya* [...] *huwa š-šarika fī l-l-mawḍūʿ ʿalā wağh min al-wuğūh al-maḏkūra wa-hiya ṯalāṯa*).

The various cases presented in the first classification are grouped by Avicenna in the second one under three basic headings. Two sciences may have a subject in common in the following ways: either (3.1) the subject of one science is more general than the subject of the other, or (3.2) the two subjects partly overlap, or (3.3) one and the same subject is investigated by one science in one respect and by the other science in another respect. Each of these three relations implies or is identical with one of the relations presented in the first classification, as is confirmed by the various pairs of sciences used by Avicenna to illustrate the second division.

Case (3.1) corresponds to the most conspicuous group in the first list, namely (aba), which includes two main classes determined by the two ways in which sciences are related when the subject of one is more general than the subject of the other (the genus-like relation and the implicate-implicant relation, respectively).

30. At *Nağāt* I, 133, p. 143.3–8, Avicenna distinguishes two ways of sharing a question: in one case, a question is a theorem of a science and a principle of another science; in the other case, a question is a theorem of both sciences, but one science is higher and the other science is lower. In the latter case, both sciences may give a why-demonstration, each in terms of the (partial set of) causes with which it is concerned. For example, the separate causes of physical bodies pertain to a higher science (metaphysics), while the causes that are connected to physical bodies (prime matter and form) pertain to a lower science (natural philosophy). Explanation across sciences and the problem of how many questions are sought on a given occasion are discussed in *Burhān* II, 9 and III, 3, as we shall see in chapter 10.

In the second classification, Avicenna interestingly introduces two complementary pairs to illustrate the two basic cases of hierarchical dependence discussed in the first part of the chapter, namely parthood and subordination (without parthood). The choice of two different pairs—geometry and the study of cones on the one hand, natural philosophy and medicine on the other (see (abaaa) and (abaab, i) in the previous discussion)—as representative examples of the corresponding relations of "being part" and "being under" can hardly be coincidental. In fact, both examples fall within the internal subdivision of the first type of ʿ*umūm* (the genus-like relation) but, as previously noted, the case of subordination without parthood also applies to the second kind of ʿ*umūm*, that is to say, the relation of the particular sciences to metaphysics: the latter is therefore also captured by (3.1).

Case (3.2) coincides with (abb), namely with the case of partial overlap between the subjects of two sciences. It corresponds to the relation between medicine and ethics, as the subject of each has something in common with the subject of the other but also extends further.

Thus, (3.1) and (3.2) taken together coincide with the totality of (ab), namely the collection of cases that involve in various ways overlapping subjects. Case (aa), where there is no overlap and which is exemplified in the first classification by the relation of arithmetic and geometry, is not discussed in the second classification (in connection with the subject) because what is at stake here are cases in which something *is* shared, and trivially there can be no sharing of a subject when the subjects are distinct. But (aa)–(ab) jointly account for all possible ways in which sciences may (a) differ because they have different subjects. Case (3.3) corresponds to (b) in the first classification, namely the case of sciences that have the same subject but investigate it in different ways, such as astronomy and natural philosophy. The example corresponds in fact to (bb), one of the two subcases discussed by Avicenna (the other case, in which one science deals with the subject without qualification and the other with the subject under a particular respect, is not mentioned in the second classification, but presumably falls under (3.3) as well).

The question of the criteria of identity and distinctness for sciences is unsurprisingly rooted in the *Posterior Analytics* not just in virtue of its general conceptual framework, according to which an ontological division of the underlying subjects demarcates the epistemological space of inquiry into distinct domains; it is in fact also addressed explicitly by Aristotle in *An. Post.* A28. Avicenna's treatment of this subject, however, brings it to an entirely different level. While being ultimately inspired by similar principles, it provides an original and much more systematic analysis, illustrated exhaustively, case by case, by a variety of examples that supposedly cover all the basic relations between the most fundamental branches of scientific knowledge.

TABLE 9 Second classification of the sciences (by principles, questions, and subjects)

<table>
<tr><th colspan="5">Division of the sciences (II)</th></tr>
<tr><td rowspan="5">(1) Principles</td><td colspan="3">(1.1) Equal in rank</td><td>Geometry-Arithmetic</td></tr>
<tr><td colspan="3">(1.2) Prior in the more general science, posterior in the less general science</td><td>Geometry-Optics; Arithmetic-Music</td></tr>
<tr><td rowspan="3">(1.3) Principle in one science; question in the other</td><td rowspan="2">(1.3.1) Subjects are different, one being more general than the other</td><td>(1.3.1.1) Question proved in the higher science; principle assumed in the lower science</td><td>No example; Standard case of subordination (first type of transfer)</td></tr>
<tr><td>(1.3.1.2) Question proved in the lower science; principle assumed in the higher science</td><td>Natural philosophy-Metaphysics; Mathematics-Metaphysics</td></tr>
<tr><td colspan="2">(1.3.2) Subjects are different, neither being more general than the other: questions proved in one science are assumed as principles in the other</td><td>Arithmetic-Geometry</td></tr>
<tr><td>(2) Questions</td><td colspan="3">Questions: sharing the predicate term (a shared subject is a necessary condition)</td><td>No example (cf. Burhān II, 9 and III, 3)</td></tr>
<tr><td rowspan="3">(3) Subjects</td><td colspan="3">(3.1) One subject is more general than the other</td><td>(i) Natural philosophy-Medicine (subordination); (ii) Geometry–Study of cones (parthood)</td></tr>
<tr><td colspan="3">(3.2) Subjects have something in common and something distinct</td><td>Medicine-Ethics (partial overlap)</td></tr>
<tr><td colspan="3">(3.3) Subject is one but investigated in two different respects</td><td>Astronomy-Natural philosophy</td></tr>
</table>

PART III

Modality

Avicenna's account of the nature of scientific knowledge involves an extensive use and sophisticated understanding of modality. Modal concepts, especially necessity, play a central role in the definition of scientific knowledge and in the metaphysical characterization of the structure of reality—that reality whose individual regions are locally mirrored by the particular sciences and whose comprehensive order and arrangement are reflected in the hierarchical division and ultimate subordination of the particular sciences to metaphysics. Non-demonstrative scientific knowledge of the principles of a science is necessary and explanatory of the demonstrative scientific knowledge of its questions, which is necessary but subject to explanation. Necessity and explanatoriness are in turn two complementary aspects of essence. Part III addresses the key modal aspects, broadly construed, of Avicenna's theory of science, including his reworking of the notion of necessity; his theory of per se predication, which accounts for the different kinds of essential relations captured by a science; and the elements of his underlying logic of essence. The latter involves distinctions between various types of inseparability; the notion of constituent and implicate; the idea of entailment by correspondence, containment, and implication; and a peculiar understanding of conditional propositions and *reductio ad impossibile* proofs, which critically relies on modal concepts.

The analysis of necessity constitutes an area of great innovation and originality in Avicenna's theory of science. First, the characterization of certainty discussed in chapter 2 seems to combine different kinds of necessity, that is to say, (i) the epistemic necessity of the beliefs expressed by scientific assertions and (ii) the ontological necessity of the underlying connections between subjects and attributes with which scientific assertions are primarily concerned. Second, Avicenna introduces various critical distinctions that expand and develop the modal conceptual vocabulary of Aristotle's *Posterior Analytics*. Two examples are the distinction between the necessity of following from the essence and nature of something and the necessity of being implied by something without being part of its essence, and the distinction between the referential reading and the descriptional reading of necessity propositions. The former is an important factor in Avicenna's classification of scientific attributes and contributes to a better understanding of the Aristotelian notion of per se predication. The latter is a crucial innovation (with independent applications in formal logic) that accounts for a variety of cases of attribute ascription and broadens the legitimate scope of application of scientific reasoning to kinds of necessities that would otherwise be difficult, if not impossible, to accommodate in the logic of the *Posterior Analytics*. Third, Avicenna contends that of whatever has a cause (and is therefore an *explanandum*) there can be no scientific knowledge without knowledge of its cause and offers a terse account of the relation between certainty, necessity, and causal explanation (chapter 6).

The adamantine link between modality and explanation, which is the characteristic trait of the scientific knowledge of principles and questions of a science, rests on the underlying essential nexuses between subjects and attributes expressed by scientific statements. The necessary and the explanatory dimensions plainly emerge as two sides of the same coin in Avicenna's account of per se predication, which regiments the two basic types of admissible attributes in a science. For the paradigmatic kind of scientific question established by demonstration concerns the relation between a subject and its per se attributes. Avicenna's general account of per se 1 and per se 2 is based on the Aristotelian idea of a term being taken in the definition of another term (or something being part of the essence of something else). What determines the type of per se relation between two terms (per se 1 or per se 2) for Aristotle is whether the predicate is included in the definition of the subject or, conversely, the subject is included in the definition of the predicate. Avicenna's definitions of per se 1 and per se 2, especially the latter, constitute a significant development of this part of the *Posterior Analytics*. Avicenna's analysis results in a revision of what he regards as an inadequate definition of per se 2 and in a complex taxonomy of cases (primary and non-primary, proper and non-proper per se) showing how the theory of per se predication finds concrete application in the sciences (chapter 7).

Aspects of the logic of essence that are not directly discussed by Avicenna in the *Posterior Analytics* complex but are nonetheless critically presupposed by it, as its philosophical foundations, include distinctions between three kinds of inseparability that may characterize the relation between a subject and any one of its necessary attributes. The first kind of inseparability is inseparability in conception, which is the distinctive mark of essential, per se 1, necessary and constitutive attributes of something: if a notion is inseparable in conception from another notion, the latter cannot be defined without the former. For example, being a plane figure is inseparable in conception from triangle. The second kind of inseparability is inseparability in imagination (but not in conception), which characterizes a broad array of attributes that belong necessarily to their subjects without being part of their essence. This class includes, but is not limited to, all per se 2 attributes. For example, having the sum of the internal angles equal to two right angles is inseparable in imagination (but not in conception) from triangle and is also a per se 2 attribute of triangle (because something in the essence of triangle is inseparable in conception from that attribute), while (the negative property of) not being a celestial body is inseparable in imagination (but again not in conception) from human, without being a per se 2 attribute of human. The third kind of inseparability is the inseparability in existence (but not in conception or imagination) of attributes that are, so to speak, only contingently necessary for their subjects, such as whiteness for swan or blackness for raven. Avicenna's logic of essence is also connected to his theory of signification and its three fundamental relations—correspondence (*muṭābaqa*), containment (*taḍammun*), and necessary implication (*luzūm* or *iltizām*)—which turn out to be helpful for elucidating the relation between a term and its definition, its proper constituents, and its necessary but non-constitutive attributes. Finally, the notions of necessary implication and conceptual containment are central to Avicenna's understanding of certain kinds of conditional propositions, which are in turn at the heart of his account of *reductio ad impossibile* proofs. The analysis of reductio and of the kind of modal reasoning involved in conditionals with impossible antecedents and consequents reveals, once again, the sophisticated interplay between formal and material logic in Avicenna's theory of science and casts new light on his understanding of the relation between logic and metaphysics (chapter 8).

6

Necessity and Scientific Reasoning

EPISTEMIC NECESSITY AND ONTOLOGICAL NECESSITY

Avicenna's account of certainty, as noted in chapter 1, raises the question of whether, in the realm of scientific knowledge, what cannot be otherwise is primarily (i) a peculiar set of unshakable beliefs, whether self-evident in themselves or obtained by demonstration from other self-evident beliefs, or rather (ii) a specific class of underlying "facts" (that is to say, necessary truths of logic, natural philosophy, mathematics, or metaphysics). By "fact" here I do not mean just states of affairs that obtain in extramental reality, but also relations between essences or quiddities in themselves according to Avicenna's canonical account of the *triplex status naturae*, that is to say, the threefold distinction of essence, nature, or quiddity—in itself, in the mind, and in extramental reality.[1]

Does the mark of necessity that characterizes scientific knowledge concern primarily things themselves or our way of knowing them? In the former case, scientific knowledge would be about things that hold necessarily, and nothing else. In the latter case, even if scientific knowledge turned out to be, in the vast majority of cases, still about things that hold necessarily (for example, the truths of geometry), the

1. This broad notion of fact is captured in Arabic logic by the idea of things being "as they are in themselves" (*fī nafs al-amr*). The expression is common in post-Avicennan logic, but Avicenna himself occasionally employs it too, for instance at *Burhān* I, 1, p. 53.10. On Avicenna's doctrine of the three ways of being, see *Madḫal* I, 2, p. 15.1–5 and *Ilāhiyyāt* I, 5, pp. 31.10–32.2; V, 1, pp. 196.10–197.5; V,1, pp. 200.13–202.2; and VI, 5, p. 292.1–5.

view would be compatible with its being, on occasion, about modally weaker states of affairs, whether actual or merely possible. For example, I could gain demonstrative knowledge (and hence be certain) of the fact that the essence of blackness is in itself compatible with the essence of surface, and that therefore every surface is possibly black. Would demonstrative knowledge of this universal affirmative *possibility* proposition count as scientific knowledge? One might say, perhaps, that this is an assertion of necessity in disguise (if only characterized by a necessity of a higher order), holding that every surface is possibly black not just as a matter of fact, but necessarily so, given that the possibility in question is grounded, after all, in the *essences* of blackness and surface. Yet both the modality and the matter of the original assertion (that is to say, the nature of the underlying relation between its terms, which is mere compatibility rather than inseparability) is possibility rather than necessity. This in turn prompts us to ask whether the notion of necessity in Avicenna's definition of scientific knowledge and in his account of certainty is epistemic, ontological, or a combination of the two.[2]

If an epistemic notion of necessity is at play in Avicenna, it seems to be involved, at least, in his characterization of certainty and of the unshakable character of our beliefs in evident scientific principles and demonstrative conclusions. While it is reasonable to ask (i) whether such an epistemic notion of necessity is a feature of Avicenna's theory, this question is distinct from—though perhaps not entirely independent of—the question of (ii) what types of "real" necessary nexuses scientific assertions may otherwise be about. Some of the language in Avicenna may occasionally seem to suggest that he understands the necessity of scientific knowledge in the epistemic sense (as the necessity of certain sets of beliefs rather than the necessity of certain ontological connections). Ṭūsī seems to take Avicenna along these lines in his commentary on the relevant passages in the *Išārāt*.[3] Further-

2. On the question of ontological and epistemic necessity in Avicenna and, more generally, on the relation between necessity and essence, see Benevich (2018).

3. Avicenna's remarks and Ṭūsī's comments are worth quoting in full. At *Išārāt* IX, 2, pp. 81.10–82.4, Avicenna seems to entertain the idea that scientific conclusions may not be limited to necessity propositions: "Just as what is sought in the sciences may concern the necessity of a judgment (*ḍarūrat al-ḥukm*), the possibility of a judgment (*imkān al-ḥukm*), and actual existence other than absolute necessity (*wuǧūd ġayr ḍarurī muṭlaq*)—in the way one may get to know about the states of planetary conjunctions and disjunctions—every kind [of deduction] having premises and conclusions that are proper to it, in the same way the demonstrator concludes the necessary from the necessary and the non-necessary from the non-necessary, whether mixed or pure." Shortly thereafter he adds that, in the context of demonstration, by "necessary" Aristotle may have "meant that the truth of the premises of a demonstration is, in its necessity, possibility, or absoluteness, a necessary truth." These comments seem to indicate that Avicenna takes seriously the idea that the necessity of demonstrative scientific knowledge may be primarily epistemic (or at least something that is not solely defined by its underlying ontological modality). At *Ḥall muškilāt* IX, 2, pp. 516.5–517.1, Ṭūsī comments: "Just as what is sought in the sciences may be necessary (like the state of the angles of triangle or the infinite

more, as we shall see, the distinction between the referential and the descriptional reading of necessity propositions may be relevant to this problem, especially for question (ii). Avicenna's fine-grained account of temporal and alethic modalities, as well as some of his explicit statements on the distinction between "always" and "for the most part," seem to indicate that scientific statements can (and indeed should) ultimately be understood to express various kinds of ontological necessity, even if such a necessity is subject to various temporal qualifications. Thus, it would be incorrect to infer from the fact that certain attributes necessarily hold of their subjects under a description that the necessity of the corresponding assertions is epistemic rather than ontological. The necessity identified by the alethic component of referential, descriptional, or even just temporal necessity propositions is the same ontological necessity in all cases. What varies in each case is just the relevant time frame at which such ontological necessity applies. Whether an attribute holds of a subject (i) at all times of the latter's continued existence; (ii) at all times at which it is characterized by a certain description; or (iii) at some time only, without further specification, that attribute, *at those times*, holds *necessarily*. And this modal characterization is not just a characterization of our beliefs, but also—and primarily—of a nexus between things or essences.[4]

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divisibility of body), it may also be non-necessary, whether purely possible (like the recovery of someone affected by pulmonary disease) or actually existent (like the eclipse of the moon). Know that if the conclusion is itself the possibility of the judgment, that possibility [conclusion] may also be necessary. In that case, the possibility will be a predicate and not a mode." Ṭūsī seems to think that Avicenna is committed to iterated modalities (and that these are of a different kind: one epistemic, the "predicate," and the other ontological, the "mode"). Again, at *Ḥall muškilāt* IX, 2, p. 520.3–6, Ṭūsī adds: "Avicenna mentioned that [. . .] the demonstrator seeks what is certain in every judgment, whether necessary or not. He seeks to produce every judgment from what is fitting and appropriate to it; except that he only asserts whatever he asserts, whether premise or conclusion, by a necessity that does not cease. The latter is *another necessity* connected to the *certain proposition* (*bi-l-qaḍiyya l-yaqīniyya*), *other than* that which is a *mode* for *some* [propositions]." Thus, *one kind* of necessity would concern a subset only (real necessities) of all things about which there may be demonstration, while the *other kind* of necessity would characterize every demonstration without qualification, irrespective of the facts it is concerned with, whether they be necessary, possible, or actual. The former is ontological, the latter epistemic. Finally, at *Ḥall muškilāt* IX, 2, p. 520.8–13, Ṭūsī concludes: "[Avicenna] said that this could have two meanings. The first is that 'necessary' be construed as that which is a mode of some demonstrative premises and their conclusions. [. . .] The second is that 'necessary' be construed as that which is connected with the truth of all certain premises and conclusions; and it is the second necessity which attaches to the judgment."

4. Concerning the distinction between "always" and "for the most part" in natural science, Avicenna seems to imply, at *Samāʿ ṭabīʿī* I, 13, p. 62.13–16, that the necessity involved in both cases is of the same kind, the only difference being the absence or presence of an obstacle or impediment. At *Burhān* III, 8, pp. 248.5–249.10, he discusses the modality of demonstrative premises (including for the most part, for the least part, necessity, and possibility) with regard to *An. Post.* A30. In his determination (*taḥṣīl*) of the question, Avicenna draws a distinction between two ways of looking at possibilities (*al-umūr al-mumkina*) in demonstration. The first is the perspective of existence, the second the

The general principle that governs necessity in scientific reasoning is that the latter should be concerned with nexuses between subjects and attributes such that the attribute *necessarily* belongs to the subject, either at all times at which the subject exists (referential) or at all times at which the subject is qualified in a certain way (descriptional), whether the subject itself exists or is qualified in the relevant way:

1. Always (*dā'iman*)
2. For the most part (*al-akṯarī*)
3. With equal frequency (*al-musāwī* or *al-mutakāfī'*)
4. For the least part (*al-aqallī*)

Avicenna seems to be distinguishing the following cases:

1. The attribute always belongs to the subject, and when it does (that is to say, always), it belongs necessarily.
2. The attribute does not always belong to the subject, but when it does (that is to say, for the most part), it belongs necessarily.
3. The attribute does not always belong to the subject, but when it does (that is to say, with equal frequency), it belongs necessarily.
4. The attribute does not always belong to the subject, but when it does (that is to say, for the least part), it belongs necessarily.

The question of whether scientific knowledge and demonstration are only about necessary nexuses is therefore an ambiguous one, and some of the conceptual machinery deployed by Avicenna in this area could perhaps be understood as part of his attempt to resolve this ambiguity by identifying different ways or senses in which an attribute may be truthfully asserted to belong to a subject by necessity.[5] For if we were to take the requirement of necessity strictly in the sense of (1),

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perspective of modality. If the focus is on existence (*ḥāl wuǧūdihā*), then the only possibilities with which demonstration may be concerned are things that occur for the most part (not things that happen with equal frequency or for the least part). If, by contrast, the focus is on whether or not something is possible (*ḥāl imkānihā*), then there may be demonstration of any kind of possibilities, because in this case the object of demonstration is their modal status. This indicates, once more, that Avicenna may be committed to the possibility of iterating modalities of different kinds. Finally, at *Burhān* III, 9, p. 257.10–11, he draws another distinction between perpetual necessity and conditional necessity: "The subject of scientific knowledge is the necessary, either in the sense of perpetuity, in which case scientific knowledge is in the sense of perpetuity, or the conditionally necessary, in which case scientific knowledge is also conditional."

5. Three further distinctions make it difficult to determine accurately the modal status ascribed by Avicenna to scientific statements: (i) the distinction between the *matter (mādda*) of a proposition, that is to say, a silent, underlying modality determined merely by the nature of its subject and predicate, and its (explicit) *modality* (or *mode*) (*ǧiha*) (for example, "Every human is possibly an animal" is a

it would appear to be too strong. In other words, if "necessary" only meant perpetually necessary and nothing else, then scientific knowledge and demonstration, at least for Avicenna, would *not* be concerned just with what is necessary. This perhaps explains his rare but unequivocal contentions that scientific knowledge and demonstration are not exclusively about what is necessary but also about what is possible or actual. At the same time, however, Avicenna is certainly not committed to the view that science deals with contingent truths. When an attribute holds actually or possibly, it still holds in virtue of certain underlying essential facts about the nature of its subject and the way its subject is identified (referentially or descriptionally) and about the nature of the attribute itself in relation to the subject or its description. The criteria to consider in the analysis are (i) *when* an attribute holds of a subject (sometimes, always, most of the times, and so on), (ii) *how* it holds when it holds (necessarily, perpetually, actually, possibly, and so on), and (iii) *why* it holds when it holds.[6] Scientific knowledge and demonstration are indeed only concerned with the domain of necessity insofar as that domain is understood in the sense of condition (ii) and, indirectly, of condition (iii) (for the way in which an attribute belongs to its subject depends on the cause in virtue of which it belongs). But according to Avicenna there is no principled reason to restrict the scope of application of scientific reasoning, on the basis of a strict interpretation of condition (i), to the domain of perpetual necessity.[7]

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necessity proposition with respect to its matter, but a possibility proposition with respect to its mode); (ii) the distinction between modality as a *mode* (*ǧiha*) and modality as a *predicate* (*maḥmūl*) (which is, for instance, advocated by Ṭūsī to explain how there may be necessary assertions about possible facts); and (iii) the distinction between the mode of the *quantifier* (*ǧihat as-sūr*) and the mode of the *copula* (*ǧihat ar-rabṭ*) (which roughly amounts to a distinction between *de dicto* and *de re* modality). On (i)–(iii), see in particular *'Ibāra* I, 1, and I, 4.

6. In Avicenna, unless noted otherwise, conditional necessity must always be taken in the sense of *necessitas consequentis* (if *p*, then *necessarily p*), and not just as a form of *necessitas consequentiae* (necessarily, if *p*, then *p*). The latter is acknowledged as a trivial case and deemed altogether irrelevant for the purposes of scientific reasoning.

7. At *Burhān* II, 1 and II, 8, Avicenna understands phenomena such as the eclipse, which occur only rarely, to be nonetheless characterized, when they occur, by the same necessity of things that occur for the most part or at all times without qualification. For there is a distinction between particular eclipses and the eclipse as a permanent, universal nature. The case of the eclipse in fact exemplifies the status of all terms in a science. A term is treated "without qualification as a specific nature which is in itself predicated of particulars" and as "a species which is an intelligible universal nature." This is the object of demonstration and definition (*Burhān* II, 8, p. 172.14–16). A similar point is made at *Burhān* II, 4, p. 145.15–19, where Avicenna clarifies the sense in which "universal" applies to the terms of demonstration. Avicenna's example of the eclipse as a universal nature survives at least until the seventeenth century, as an echo of it is attested in Tehranī, *Naqd al-uṣūl* III, 3, p. 144.4–9.

NECESSITY, ESSENCE, AND EXPLANATION

In *Burhān* II, 5, Avicenna emphasizes the critical role of modal concepts in the domain of demonstration. In the context of his discussion of the conditions under which the necessity of a conclusion follows from the necessity of the premises in a demonstrative argument, he draws an important distinction between two kinds of necessity, which serves as the basis for his account of scientific attributes and the notion of per se predication.[8] The first is a weaker notion of necessity called "necessity in implication." The second is a stronger notion of necessity called "necessity in essence and nature." As will become clear over the course of the next three chapters, necessity in essence and nature is the counterpart of containment (or correspondence in the limit case of definition), of the notion of constituent and per se 1 attribute, and of inseparability in conception. Necessity in implication, by contrast, is the counterpart of the notion of implicate and per se 2 attribute, and of inseparability in imagination. He writes:

> Text 6.1: *Burhān* II, 5, p. 150.4–11
> Things are necessary in two ways: [(a)] necessary in implication (*fī l-luzūm*), without being necessary to one another in essence and nature (*fī l-ǧawhar wa-ṭ-ṭabīʿa*): these are external implicates (*lawāzim ḫāriǧa*) and we have explained earlier that they are not useful for obtaining certain knowledge; and [(b)] necessary in essence and nature, that is to say, the things that belong per se (*al-umūr al-mawǧūda bi-ḏātihā*).[9]
>
> [(ba)] Those that are included in the definition of the subject are necessary to the subject in its essence (*fī ǧawharihī*).
>
> [(bb)] Those in whose definition the subject is included [are such that] the subject is necessary to them in essence, and they in turn are necessary to the subject in implication, either absolutely or by opposition.

8. At *Burhān* II, 5, p. 150.2–3, Avicenna notes that what is "acquired through a middle that can change, is not something stable (*ṯābit*) that does not change; rather, the conclusion necessarily follows (*talzamu*) from necessary premises in which there is no possibility of change." At the end of this chapter I return to a similar argument in *Burhān* I, 8. At *Burhān* II, 5, p. 150.13–16, Avicenna applies a similar temporal analysis to the notions of implicant and implicate: if the implicant (*malzūm*) is not perpetually existent (*dāʾim al-wuǧūd*), what necessarily follows from it—the implicate (*lāzim*)—is not perpetually existent.

9. The use of *ǧawhar* and *ṭabʿ* (pl. *ṭibāʿ*) in connection with the definition of per se 1 and per se 2 is attested in Alfarabi, *Burhān* II, 4, p. 28.13–18. The locution is also close to the characterization of the second sense of *ḏātī*, at *Išārāt* I, 15, p. 10.4–5, as "the predicate that attaches to the subject due to its essence and quiddity (*min ǧawharihī wa-māhiyyatihī*)," which is then illustrated by the standard examples of proportionality and equality for magnitude and quantity, oddness and evenness for number, health and sickness for animals. At *Burhān* II, 5, p. 150.12, Avicenna notes: "We will say later how we rank (*nurattibu*) these [per se predicates], in order for certain, scientific knowledge to come about from them." The reference is, in all likelihood, either to the classification of *Burhān* II, 6 or to that of *Burhān* II, 8, discussed in chapter 4 and again in chapter 10.

In Text 6.1, Avicenna identifies two concepts of necessity that are central for his logic of essence, as we shall see in chapter 8. For the moment, what matters is the distinction between the two levels, one of which does not presuppose any sort of connection between the essences of the terms involved (mere necessity in implication), while the other does. What is characteristic of attributes that belong per se to their subjects is precisely the presence of a connection between their essences.

According to Avicenna, propositions are demonstratively necessary only if their predicates are per se 1 or per se 2, in a sense that is specified in chapter 7, because only essential connections are explanatory in the way required by the definition of scientific knowledge. He writes:

> Text 6.2: *Burhān* II, 5, p. 154.1–5
> [Propositions] are not necessary in the sense presupposed by demonstration unless their predicates are per se, in one of the two senses of per se, in addition to being necessary (*maʿa ḍarūratihā*). For the necessary items that are proper to every genus are either its genera and differentiae or its per se accidents. Anything else is either a necessary [but] foreign [attribute] or a non-necessary [attribute] but rather an accident without qualification. And from this you cannot get to know the causal explanation of anything at all.

Essentiality, causality, and necessity are complementary dimensions in Avicenna's theory of science: if premises are not per se, they cannot be explanatory, but premises must be explanatory, therefore they must be per se; if premises are per se, then they are necessary, because per se predications express essential predications and hence—whether directly or indirectly—necessary predications too.[10]

MODAL ANALYSIS OF PROPOSITIONS

In order to appreciate the significance of Avicenna's understanding of necessity in the *Posterior Analytics* complex, we need to take a step back and look briefly at his general analysis of propositions. Avicenna introduces two radical innovations in the Aristotelian framework. First, in his view, every categorical proposition is modalized, either implicitly or explicitly. The modality may be either temporal (as with sometime or always), alethic (as with possibly or necessarily), or a combination of both. This has interesting repercussions for the traditional square of

10. This is trivial in the case of per se 1 attributes (since they belong to the definition of the subject, they are ipso facto essential attributes and hence necessary attributes of the subject). But Text 6.1 shows that Avicenna is committed to the view that per se 2 attributes are also necessary in virtue of an underlying essential nexus (something in the essence of the subject is involved in the essence of the attribute), and it is this nexus that grounds the *other* type of necessity (in implication) by which they belong to their subjects. On necessity and explanation through the essence in Aristotle, see Angioni (2014). On necessity in general, see Ferejohn (1981), Hagdopoulos (1975b), Lloyd (1981), and Mulhern (1958).

opposition, which Avicenna rejects, and for his account of conversion. Second, and more importantly for our present purposes, every categorical proposition—including explicit necessity propositions—may be interpreted in two different ways, depending on whether we look at the relation between the predicate and the referent identified by the subject (i) as long as such a referent exists or (ii) as long as it is characterized or described by the subject. An example of a necessity proposition true on the first reading is "Everything that walks is necessarily an animal, as long as it exists," which remains true even when the subject of "walks" is not actually walking. An example of a necessity proposition true on the second reading is "Everything that walks necessarily moves while walking," which may or may not remain true when the subject of "walks" is not actually walking (depending on whether the subject moves in some other way).[11]

While the first criterion (temporal or alethic modality) determines the time at which or the way in which the attribute belongs to the referent identified by the subject (sometime, always, possibly, necessarily, and so on), the second criterion (the reading of the subject term) identifies two further distinct time frames with respect to which the attribute belongs to the referent identified by the subject in the manner specified by the first criterion. These two time frames are (i) the continued existence of the referent identified by the subject term (where the latter serves the only purpose of fixing that referent) and (ii) the time at which the referent is actually qualified by the subject term. The two readings are usually called "referential" or "substantial" (*ḏātī*) and "descriptional" (*waṣfī*) in the post-Avicennan tradition. The relevance of all this for Avicenna's theory of science is that he has some interesting things to say about the default reading of necessity in the context of scientific reasoning and about the justification for its adoption.

Referential and Descriptional Necessity in the Sciences

At least three sets of sources unequivocally indicate that Avicenna understands these innovations to be relevant not just in the domain of formal logic but also in the context of scientific reasoning. The first and second illustrations come straight from his own treatment of the *Posterior Analytics*; the third is indirect evidence from the *Prior Analytics*.[12]

The first interesting thing to note is that Avicenna frequently claims, throughout his career and in a variety of contexts, that descriptional necessity is the default

11. An application of the referential reading in metaphysics is at *Ilāhiyyāt* IV, 1, p. 165.10–14, where Avicenna discusses the relation between essential priority, causality, and existence.

12. For a detailed analysis of the senses of necessity in *Burhān* II, 1, see Strobino (2015a). For an overview of the distinction between the two readings in post-Avicennan logic, see Street (2005, 2015a); cf. also Strobino and Thom (2016).

sense of necessity assumed in the theory of science and employed in scientific discourse. For example, he writes:

Text 6.3: *Burhān* II, 1, p. 122.9–14
When we say in the *Book of Deduction* [= *Prior Analytics*] "Every C is necessarily B," we mean that everything that is described as C—however it may be described as C (perpetually, necessarily, at some time, or non-necessarily)—is described at all times or perpetually as B (even when it is not described as C).

In the *Book of Demonstration* [= *Posterior Analytics*] when we say "Every C is necessarily B," we mean that everything that is described as C is necessarily described as B. Nay, the meaning is even more general than this, that is to say, everything that is described as C, *as long as it is described as* C, is [necessarily] described as B, even if it is not as long as it exists.

In formal logic, referential necessity is taken by Avicenna to be the default reading required for the validity of most modal inferences, but in the context of scientific discourse, the descriptional reading seems more natural. What is the reason for this contention? The critical factor is the relative extension of the time frames associated with the two readings. In general, in order for a necessity statement to be true on the referential reading, the predicate must belong to the thing(s) picked out by the subject throughout their continued existence, while in the case of descriptional necessity the temporal requirement is weaker (because the predicate belongs to the underlying item(s) only when the subject-term actually belongs to them too). Thus, Avicenna identifies in the notion of descriptional necessity a broader concept (in the sense of being logically more general than referential necessity) by means of which it is possible to account for an entire category of necessity predications that turn out to be true on the descriptional reading but would otherwise fail to be true on the referential reading. To justify this claim of preeminence for descriptional necessity in scientific discourse, Avicenna appeals to the types of necessary predicates that are standard elements of scientific demonstrations:

Text 6.4: *Burhān* II, 1, p. 122.14–17
This is because the necessary predicates here [in the sciences] are genera, differentiae, and implicate per se accidentals, and these necessarily follow in this sense. For, when something is described by a given species, it need not be described by the genus, the differentia, the definition, or by an implicate of that species perpetually, but rather just as long as it is described by that species.

In Text 6.4, Avicenna justifies the contention that the descriptional reading of necessity is the default sense in the theory of science by appealing to the nature of the terms that typically appear in scientific demonstrations.[13] Suppose we are

13. Compelling evidence that the descriptional reading is central for Avicenna's theory of science comes indirectly from its adoption, as the default sense of necessity, by a member of his immediate

aiming to demonstrate that C, the major term, belongs to A, the minor term ("something" in Text 6.4), by means of a middle term B ("a given species" in Text 6.4). Avicenna's claim is that whatever necessarily belongs to B necessarily belongs to A as long as B belongs to A, or in other words, as long as A is described as B. That is to say, C necessarily belongs to A as long as A is B. Now C may be an essential property of B (a genus or a differentia) or a nonessential necessary property of B (an implicate per se accidental). But what matters in Avicenna's characterization is that the relation between A and B is not specified, and this is what gives the descriptional reading its flexibility. As long as the relevant modal facts are captured by the relation between B and C (the description and what follows from it), then the relation between the description and the underlying referent A (represented in this inferential structure by the minor term) may be as weak as possibility or as strong as perpetual necessity. The only thing that counts is that anything of which B is true, as long as B is true of it, is necessarily C. By the same token, when B ceases to belong to A, there is no guarantee that what follows from B (a genus, a differentia, the definition, or a nonessential necessary attribute) will continue to be true of A.

This critical distinction allows Avicenna to introduce an extremely flexible tool in the logic of scientific discourse. The descriptional reading becomes a vehicle for a broad range of necessity predications that are true only under the condition that certain descriptions apply to their objects. And this is simply impossible with the referential reading alone, because the latter involves a much stronger requirement, namely that the predicate belong to the items picked out by the subject throughout their continued existence, and not (only) as long as they are described by the subject.[14] Avicenna introduces the following terms and predications to illus-

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circle; see Bahmanyār, *Taḥṣīl* III, 5, p. 205.5–9. Bahmanyār's understanding of descriptional necessity seems also to foreshadow the notion of essentialist (*ḥaqīqī*) reading of propositions, which becomes a standard tool of post-Avicennan logic after Rāzī. On the distinction between the essentialist and the externalist reading, see Street (2004, 2005, and 2015a). Propositions that are true on the essentialist reading are true regardless of whether their terms are instantiated or not (at least in extra-mental reality) because their truth depends on the relation between the underlying conceptions or essences. This is roughly the same reason adduced by Bahmanyār for the superiority of the descriptional reading: many necessary truths of geometry, which are perfectly legitimate objects of demonstration, concern figures that may not exist. This in turn implies that the condition of referential necessity would be too strong for propositions of this kind. The descriptional reading, by contrast, is intended to capture precisely the sort of relation between two conceptions that those propositions express.

14. At *Nağāt* I, 122, pp. 132.6–133.4, in the context of his discussion of the meaning of "primary," Avicenna gives an example that illustrates well the main point of Text 6.4. What necessarily follows from triangle, for instance having the sum of the internal angles equal to two right angles, is true of a subject only as long as triangle is true of that subject. For a detailed discussion of Avicenna's examples in *Burhān* II, 1 (and Ṭūsī's analysis of the corresponding passages in the *Išārāt*, which is heavily indebted to *Burhān* II, 1), see Strobino (2015a). At *Samāʿ ṭabīʿī* III, 2, p. 183.16–17, Avicenna considers attributes

trate the fact that the truth and falsehood of a proposition depend on the reading of its terms:

(a) Definition
"Every white thing necessarily has a color that dilates sight."
Descriptional reading ("as long as it is white"): true
Referential reading ("as long as it exists"): false

(b) Differentia (last)
"Every white thing necessarily dilates sight."
Descriptional reading ("as long as it is white"): true
Referential reading ("as long as it exists"): false

(c) Genus
"Every white thing necessarily has a color."
Descriptional reading ("as long as it is white"): true
Referential reading ("as long as it exists"): false, if at least one white thing can become transparent, that is to say, colorless

(d) Genus
"Every white thing necessarily has a color."
Descriptional reading ("as long as it is white"): true
Referential reading ("as long as it exists"): true, if no white thing can become transparent.

Descriptional Necessity and Universal Scientific Predication

The second piece of evidence in support of the view that, according to Avicenna, descriptional necessity is the default notion in the sciences, is a lengthy gloss on *An. Post.* A4, 73a29–30 concerning the meaning of "to be said of every" (*kata pantos*). In that place, Aristotle notoriously addresses the requirements of universal predication in the context of scientific discourse. The distinctive claim is that when a categorical proposition is scientifically known, not only must the predicate belong to everything that falls under the subject, but it must also belong to the subject *at all times*. This gives Avicenna motive and opportunity to put to use once again his own conceptual apparatus. For how are we to interpret the additional temporal

(such as continuity or being together in a place) that "necessarily belong to natural things insofar as the latter have quantity." At *Samāʿ ṭabīʿī* III, 11, pp. 235.13–236.3, he discusses two kinds of necessity, conditional and unconditional. Conditional necessity is the kind of necessity by which the attribute "having the sum of the internal angles equal to two right angles" belongs necessarily to a figure as long as the figure is a triangle. This case seems to encapsulate the notion of descriptional necessity.

condition “at all times” stipulated by Aristotle as a requisite for the truth of universal predications that are object of *epistēmē*? In the last section of *Burhān* II, 1, Avicenna clarifies the meaning associated with the notion of “to be said of every” (*al-maqūl ʿalā l-kull*) in different contexts (formal logic and demonstration), in light of his own distinction between referential and descriptional reading:

> Text 6.5: *Burhān* II, 1, p. 123.3–9
> In the *Book of Deduction* “to be said of every” (*al-maqūl ʿalā l-kull*) just means that the predicate, say B, is true of every one of the things described by the subject, say C, if the universal sentence is affirmative, and that it is denied of it, if the universal sentence is negative. In that context, there is no second condition, namely that the belonging [to the subject] or being denied [of the subject] be at all times. Indeed, in the case of *absolute* propositions, the predicate may belong at some time and not belong at some [other] time to every one of the things that are described by the subject. Here [that is to say in demonstration] “to be said of every” means that everything that is described by the subject and *at all times at which it is described by the subject*—not at all times without qualification—is described by the predicate or the predicate is denied of it.[15]

In Text 6.5, the descriptional reading is once again identified as the default interpretation of certain statements, namely universal affirmative and negative propositions used in scientific reasoning and demonstration. The subsequent discussion involves a rather impressive deployment of some of Avicenna’s most sophisticated logical distinctions for the reading of propositions. Despite its technicality, it is worth looking at it in detail. To gain a better understanding of Avicenna’s arguments, the following abbreviations for various relevant types of modalized propositions will prove useful:[16]

Ne:	No A is possibly B	Na:	Every A is necessarily B
Ae:	No A is ever B	Aa:	Every A is always B
Ao:	Some A is never B	Ai:	Some A is always B
X_1e:	No A is always B	X_1a:	Every A is sometimes B
X_1o:	Some A is not always B	X_1i:	Some A is sometimes B
M_1o:	Some A is not necessarily B	M_1i:	Some A is possibly B
N_{dl}e:	No A is possibly B while A	N_{dl}a:	Every A is necessarily B while A
X_{dl}o:	Some A is not always B while A	X_{dl}i:	Some A is sometimes B while A
M_{dl}o:	Some A is not necessarily B while A	M_{dl}i:	Some A is possibly B while A

15. Avicenna has a distinction between descriptional perpetuity and descriptional necessity, even though in this passage it seems lost.

16. The notation is a variant of Strobino and Thom (2016). N, A, X_1, and M qualify necessity, perpetuity, absolute, and one-sided possibility propositions in the referential reading. N_{dl}, X_{dl}, and M_{dl} are their descriptional variants. The quality and quantity of a proposition is indicated by a, e, i, and o (universal affirmative, universal negative, particular affirmative, and particular negative).

Why does Avicenna understand the additional condition of omnitemporality for universal scientific predications in descriptional terms? His answer involves an elliptical series of objections and replies turning on (i) the meaning of "at all times" on the referential and descriptional readings, and on (ii) what contradicts what.

First, on the referential reading a universal proposition need not be true at all times. A sufficient condition for its truth is that it be true at some time. This condition is met, according to Avicenna, by various types of propositions, in particular by the N (referential necessity), N_{dl} (descriptional necessity), and the X_1 (absolute one-sided) proposition. All these can be called (and indeed are called in *Qiyās* I, 4) "absolutes" because the predicate belongs, at least at some time, to the referent identified by the subject during its continued existence. This is trivially the case with referential necessity propositions, because if the predicate belongs to the referent at all times at which the referent exists, then the predicate also belongs to the referent at some time (regardless of its relation to the subject term). It is also somehow trivially the case with descriptional necessity propositions, because the predicate belongs to the referent at all times at which the referent is described by the subject term, and since those times constitute a subset of the total times at which the referent exists, then the predicate belongs to the referent at some time during its continued existence. In Text 6.5, the sentence "Indeed, in the case of *absolute* propositions, the predicate may belong at some time and not belong at some [other] time to every one of the things that are described by the subject" means that the truth of an absolute proposition *does not require* the predicate to belong at all times. And since "Every A is B" in formal logic is read as an absolute proposition, namely in the sense of X_1a "Every A is sometimes B," its truth is compatible with the truth of X_1o "Some A is not always B." This is the point established by Avicenna in the preamble.

What then of the argument in support of the descriptional reading? The requirement that the predicate belong to the subject "at all times" is justified by looking at the way in which universal propositions are typically denied in scientific discourse. (Avicenna is loosely inspired by Aristotle's line of reasoning on this point.) He writes:

> Text 6.6: *Burhān* II, 1, p. 123.9–13
> This is because these premises are universal necessity propositions and the universality of what is necessary is falsified by two things. Either one says that [(i)] the predicate is judged not to be true of a thing falling under the subject [. . .]; or one says that [(ii)] something that is described by the subject is not, at a certain time, described by the predicate.

For the predicate to belong to everything that falls under the subject is not a sufficient condition for the truth of a universal scientific predication. The predicate must also belong to everything that falls under the subject *at all times*. But what is the time frame with respect to which such propositions must be evaluated: (i) the time of the continued existence of the underlying subject (referential reading) or (ii) the time at which the subject is characterized by the relevant term (descriptional reading)? It is clear from the preamble that Avicenna's answer is (ii). It is *not at all times* at which the underlying things *exist* that the predicate is required to belong to them, but rather *at all times at which* they are *described* by the subject.

What is Avicenna's reason for such a restriction? In a nutshell, as we shall see, his view is that (i) would be too strong a requirement and that in fact the weaker, descriptional interpretation captures more accurately the intended requirement of omnitemporality. Avicenna makes two arguments to justify his position. The first involves a close analysis of contradiction. The second clarifies the different emphasis put on the subject term in the context of formal logic as opposed to demonstration and identifies different temporal frames of reference.

RESOLVING AN AMBIGUITY OF "NECESSARY" IN THE DESCRIPTIONAL SENSE

Avicenna first considers an objection that turns on a problem of terminology. What if someone objected that N_{dl}a-propositions are characterized as *absolute* propositions in formal logic (that is to say, in the *Qiyās*) and that, in that context, no peculiar temporal condition has to be met by their negation? He writes:[17]

> Text 6.7: *Burhān* II, 1, p. 123.14–17
> If one says "You took the necessary that is in the sense of 'as long as the subject is described' to belong to the class of absolutes in the *Book of Deduction*, so that they were universal absolutes there, and their universality was not falsified by a gap occurring with respect to time," the reply is that we took them as absolutes just because the mode of necessity was removed from them. But here we have established the mode of necessity for them in the predicate.

In the context of formal logic, an N_{dl}a-proposition may be treated as an absolute X_la-proposition, if the focus is on the relation between the predicate and the underlying referent, and the time frame is the continued existence of the latter. Such a proposition, however, is contradicted by an Ao-proposition, that is to say, by the assertion that the predicate holds at *no* time, and not just by "a gap with respect to time," for the X_la-proposition is compatible with the X_lo-proposition. And the N_{dl}a-proposition is incompatible with the X_{dl}o-proposition "Some A is

17. At *Burhān* I, 7, p. 78.6–8, there is further evidence for taking "the absolute" in a broad sense "common to [(i)] the necessary as long as the referent exists [the referential sense], [(ii)] the necessary as long as the subject exists in the way in which it is posited [the descriptional sense], and [(iii)] non-necessary existence."

not always B while A" (its contradictory would in fact be an M_{dl}o-proposition). Avicenna writes:

Text 6.8: *Burhān* II, 1, p. 123.17–19
Where we regarded them as absolutes, we did not say that the necessary—as long as the subject is described by what describes it—is absolute with respect to making this necessity a condition in act, but rather absolute with respect to the possibility of making this necessity a condition for it.

The N_{dl}a-proposition is only treated as an X_l a-proposition if we implicitly omit the mode of necessity. This is, as it were, equivalent to a change of target: instead of looking at the relation of inseparability of the predicate from the subject term, we are now looking at the relation between the predicate and the underlying referent. If we specify the condition of necessity, we have a statement, which is contradicted by *one* kind of statement; if we do not specify the condition of necessity, we have another statement, which is contradicted by *another* kind of statement. Different logical properties are at stake depending on the relation on which we are focusing. If the statement is taken strictly without the condition of necessity, that is to say, if we evaluate it with respect to the continued existence of the underlying referent and not with respect to the time at which it is described by the subject, then the statement is an absolute proposition, which is contradicted by a perpetuity proposition negating the attribute at all times (a much stronger condition). If, by contrast, we evaluate the statement with respect to the time at which the referent is described by the subject *and* explicitly add the (true) condition that the attribute belongs to it necessarily at that time, then it becomes a descriptional necessity proposition, which is contradicted by a descriptional possibility proposition.[18] Such a distinction between two complementary perspectives is reinforced in the second argument. Avicenna is keenly aware of the potentially ambivalent status of a proposition, whose evaluation depends on the exact specification of the terms and time frames that are being investigated (with the additional condition, the proposition becomes a descriptional necessity, but without the additional condition, it is just an absolute). He writes:

Text 6.9: *Burhān* II, 1, pp. 123.21–124.2
There is a sharp distinction between the possibility of making something a condition and making it a condition in act.

[(i)] Here [in demonstration], when necessity is made a condition, it is contradicted by negating the judgment at a time (no matter what [that time] is).

[(ii)] There [in formal logic], when necessity is not made a condition, but rather the proposition is absolute with no actual condition, it is not contradicted by negating

18. Avicenna seems oddly to require a stronger contradictory, namely a descriptional absolute proposition.

> the judgment at a time (because it belongs at a time and that the predicate perpetually belong to the subject is not made a condition).[19]

The general point advanced in Text 6.9 may be illustrated by an example that we will encounter again. The proposition "Everything white has a color that dilates sight" may be interpreted in two ways, depending on the relation on which we are focusing and on the corresponding time frames.[20] Both interpretations are equally legitimate, but they also differ significantly. The time frame may be either (i) the continued existence of the referent of "white" (that is to say, the *things* of which "white" is sometime, always, possibly, or necessarily true; the modality is irrelevant, as the only thing that matters is to fix the referent) or (ii) the time at which such a referent is characterized as white. In the first case, the predicate "having a color that dilates sight" belongs to the (unqualified) subject at least at some time of its continued existence (when it is white, for "dilating sight" is the differentia of white). In the second case, the predicate belongs to the referent qualified by the subject at all times at which the subject belongs to it (for those are the times at which it is white, and everything white necessarily has a color that dilates sight at all times at which it is white). In the first sense, the proposition is taken as an absolute, in the second sense as a descriptional necessity proposition. The propositions so interpreted have different contradictories. If the original proposition is taken with the modal qualification and the restriction to the time at which the subject is described as being white, then it becomes an N_{dl}a-proposition:

> "Everything white (necessarily) has a color that dilates sight (as long as it is white)."

The latter is an instance of "Every A is (necessarily) B (as long as it is A)." On the descriptional reading, which is the sense presupposed in the context of demonstration, this modalized proposition is "negated by a gap occurring with respect to time." This means that its contradictory must hold at least at some time at which

19. "Everything white has a color that dilates sight," on the referential reading, is not contradicted by "negating the judgment at a time," because it is an absolute proposition whose meaning is that everything white *sometimes* has a color that dilates sight. The contradictory of such a universal affirmative absolute proposition is a particular negative perpetuity proposition, which negates the judgment *at all times*. But the attribute here "belongs at a time" to the underlying referent (when the referent is described as white) even if it fails to belong at all times of its continued existence. Thus, either the negation at a time correctly contradicts a perpetuity proposition, which our absolute proposition is not, or our proposition is qualified in such a way that the attribute is denied *at some time at which* the description is true of the underlying referent, which requires the proposition to be taken in the referential, not in the descriptional reading.

20. See Philoponus, *In An. Post.* A22, p. 250.25 for the use of *chrōma diakritikon opseōs* (Ar. *lawn mufarriq li-l-baṣar*).

the subject is described as being white. But the N_{dl}a-proposition is incompatible with the X_{dl}o-proposition (or its weaker, possibility counterpart):[21]

> "Every A is necessarily B as long as it is A" is incompatible with "Some A is sometimes not B as long as it is A."

If, by contrast, the proposition is taken in the referential reading, which is the sense presupposed in formal logic, there will be neither a temporal restriction imposed by a descriptional condition nor any resulting necessity (as the necessity was tied inextricably to the interval associated with the description). In this case, our original proposition becomes an X_la-proposition:

> "Everything white sometimes has a color that dilates sight as long as it exists,"

which is an instance of "Every A is sometimes B as long as it exists." But the X_la-proposition is not contradicted by the X_{dl}o-proposition and this is, in all likelihood, what Avicenna means when he says, in Text 6.7, that it is "not falsified by a gap occurring with respect to time." For "Some A is sometimes not B as long as it is A" does not contradict "Every A is sometimes B as long as it exists." In fact it is not even incompatible with it.

In order to contradict an X_la-proposition, a much stronger proposition is required, namely the Ao-proposition "Some A is never B." And at the same time, the fact that the predicate may sometimes fail to belong to the subject does not constitute a counterexample (not even X_lo contradicts X_la). Thus, while Ao, the contradictory of a genuine X_la-proposition, fails to express adequately the intended meaning of negation in the case of universal scientific propositions (for we do not require counterexamples to be true at all times), the contradictory of an N_{dl}a-proposition seems, by contrast, to serve that purpose well. And the fact that the N_{dl}a-proposition may be characterized as an absolute (when the mode of necessity is removed) does not pose a problem either. This is because when an N_{dl}a-proposition is treated as an X_la-proposition, its contradictory is no longer an X_lo proposition (which is compatible with X_la) but a stronger Ao-proposition.

From this complex argument, Avicenna infers that the default sense of universal necessity for scientific propositions is adequately captured by the descriptional reading. This conclusion is drawn (i) by looking at different contextual interpretations of a proposition, each with a different time frame; (ii) by examining the various resulting relations of contradiction; and (iii) by choosing the weaker condition (or alternatively the one for which the failure to belong at a time is relevant). The

21. To be exact, N_{dl}a is contradicted by M_{dl}o, and A_{dl}a by X_{dl}o, but Avicenna here is talking about "denying," and N_{dl}a is indeed incompatible with X_{dl}o (the latter implies the contradictory of N_{dl}a, namely a one-sided descriptional possibility proposition).

adequate proxy of Aristotle's temporal requirement in *An. Post.* A4 must accurately capture the idea of a gap with respect to time, which is to say *a* time at which the predicate does not belong to the subject. And this must be evaluated with respect to the time at which the subject is described in the relevant way, not with respect to the continued existence of the underlying referent.

DESCRIPTIONAL NECESSITY AND RELATIONS BETWEEN CONCEPTIONS

After focusing on the logical relations between different types of modalized propositions and identifying the default temporal characterization of universal scientific predications, Avicenna addresses the problem from a complementary perspective.

The qualification of the subject (the description) serves in the context of demonstration as a criterion for the identification of a certain temporal interval. And it is then with respect to that circumscribed temporal interval that one asks the question, "Does the predicate *always* belong the subject?" that is to say, at all times at which the subject is described in a certain way. If the qualification is removed, the question, "Does the predicate *always* belong to the subject?" implicitly extends further, namely to the time of the continued existence of the subject. Thus, "always" refers in the two contexts to two different time frames. Avicenna writes:

> Text 6.10: *Burhān* II, 1, p. 124.4–12
>
> Let us express this in a different way: that with respect to which the negation at a time and the perpetuity at a time are considered here [in demonstration] is other than that with respect to which the two things are considered there [in formal logic].[22]
>
> There [negation and perpetuity] were considered just with regard to [the relation] holding without qualification between the two terms of the conclusion, that is to say [(i)] "white thing" itself (*ḏāt aš-šayʾ al-abyaḍ*) and [(ii)] "color that dilates sight" itself (*ḏāt al-lawn al-mufarriq li-l-baṣar*). Thus, the state of the predicate was considered next to the subject itself as such (*ʿinda ḏāt al-mawḍūʿ min ḥayṯu ḏātihī*).[23]
>
> Here [negation and perpetuity] are considered with respect to a condition on the subject, that is to say as long as the subject itself is described by the description (*bi-ṣifa*) "white."
>
> There [in formal logic], it was not with this condition but rather without qualification just because [the predicate] was not an accident of the subject itself (*ḏāt al-mawḍūʿ*) perpetually but only at the time of its being so described. Everything described to be white has a color that dilates sight not as long as it exists but rather as long as it is described to be white. Hence, "having a color that dilates sight" is

22. The expression has a coincidental but sinister Aristotelian touch to it, as in "Let us say again what we have just said but not said clearly;" *An. Post.* B19, 100a15.

23. That is to say, the subject insofar as it is what it is, without the qualification "white."

not predicated at all times of what is described to be white itself (*ḏāt al-mawṣūf bi-annahū abyaḍ*), but rather [only] at a certain time.[24]

In Text 6.10, Avicenna shifts the focus on the relata. On the referential reading, the relata are the predicate (*maḥmūl*) and the referent of the subject (*ḏāt al-mawḍūʿ min ḥayṯu ḏātihī*), which is the thing itself of which the subject is true (regardless of whether the subject is true of it at some time, always, necessarily, and so on). On the descriptional reading the relata are the predicate and the subject, or rather their conceptions. The relation between the predicate and the referent of the subject is relevant only indirectly, because the referent is considered here only insofar as it is qualified by the subject. The difference in focus is reflected by a difference in the time frames with respect to which the proposition (in fact two distinct propositions, once the readings are made explicit) must be evaluated. Does everything white always (or necessarily) have a color that dilates sight? The answer depends on the relata. On the descriptional reading, everything white necessarily has a color that dilates sight, because the relevant time is the time at which the subject is white. On the referential reading, by contrast, it is not the case that everything white always (let alone necessarily) has a color that dilates sight, because the relevant time is the time at which the underlying referent of the subject exists, irrespective of the time at which it is white. As soon as the focus shifts from one reading to the other, the proposition must be evaluated differently, for the term "always" picks out different temporal intervals.

Where does that leave us with respect to our original question? What is the role of descriptional necessity in Avicenna's account of the meaning of universal predication in the theory of science? The first argument clarifies that and why the descriptional sense is the default reading. It also solves a terminological ambiguity in the transition between formal logic and the logic of demonstration. The second argument elaborates on the difference between the two contexts at a more general level.

CONVERSION OF UNIVERSAL NEGATIVE PROPOSITIONS

Further corroborating evidence from Avicenna's formal logic supports the view that he is genuinely committed to the centrality of the descriptional reading of necessity propositions for scientific reasoning. Avicenna refers to N_{d1}-propositions (along with N-propositions) on several occasions as "the propositions used in the sciences."[25] The issue is raised primarily in connection with the problem of conversion for absolute e-propositions.

24. I take the Arabic *ḏāt* to be used by Avicenna in a deflationary sense throughout Text 6.10, that is to say, simply to indicate the thing or things (without qualification) designated by the various terms with which *ḏāt* is in *status constructus*.

25. At *Qiyās* VI, 1, p. 300.12, Avicenna also indirectly refers to them as the two negative necessity propositions that convert (*aḍ-ḍarūriyyatayni al-munʿakisatayni*).

Avicenna holds that absolute e-propositions generally do not convert as such ("No A is B" typically fails to convert as "No B is A," which would otherwise be the case if these were Aristotelian assertoric e-propositions). But if we take them in a narrower sense, they do convert. In this case, however, they convert not qua absolutes but qua descriptional or referential necessity e-propositions. (We have seen in the previous sections that the shift is legitimate, as long as we are clear about the relation between the implicit modality and the temporal context of evaluation.)

Avicenna rejects the standard Aristotelian view that assertoric e-propositions convert, based on his own temporal interpretation. There are, however, some exceptions. He writes:

> Text 6.11: *Naǧāt* I, 55 (ii)–(iii), p. 45.3–10 (Ahmed 2011, pp. 37–38, transl. modified)
> It is commonly held (*al-mašhūr*) that the universal negative absolute converts like itself [as a universal negative]. [. . .] The truth (*al-ḥaqq*) is that this conversion is correct not for everything that counts as an absolute, but only for an absolute such that the condition under which necessity correctly attaches to it is not a time that differs for the individuals, but something other than time.

The rather obscure clause in the last sentence of Text 6.11 is immediately illustrated by an example that leaves no room for doubt about the fact that the descriptional reading satisfies the required condition:

> Text 6.12: *Naǧāt* I, 55 (iii), pp. 45.10–46.2 (Ahmed 2011, p. 38, transl. modified)
> An example of a condition under which the mode of necessity correctly attaches to it is "as long as the subject is described by that which is posited with it [as a subject]," as our statement "Everything that moves locally changes." For if you attach the mode of necessity to it, you must say either with your tongue or in your soul "as long as it is described as moving locally."

The reason why, in this case, e-conversion is valid is that the required relations of contradiction are restored, unlike in the case of *genuine* absolute propositions that are not also necessity propositions, for the former are contradicted, as noted, not by absolute propositions of the same kind but by perpetuity propositions of opposite quality and quantity:

> Text 6.13: *Naǧāt* I, 55 (iii), p. 46.2–5 (Ahmed 2011, p. 38, transl. modified)
> This conversion necessarily follows for absolutes of this kind. In their case, when "No [A is B]" is true, "Some [A is B]" is false, and when "Some [A is B]" is true, "No [A is B]" is false, not with the stipulation of a specific time as a condition but rather without qualification. [Propositions] of this kind are *the ones used in the sciences* (*al-mustaʿmilāt fī l-ʿulūm*).

Having identified negative descriptional necessities through the lens of e-conversion as one of the kinds of propositions for which e-conversion is valid, in Text 6.13 Avicenna goes on to draw another explicit connection to the domain of scientific reasoning. Similarly, at *Qiyās* II, 1, in the context of the same discussion of e-conversion, we find another explicit acknowledgment that if "No C is B" is understood to express either a referential necessity or a descriptional necessity, then it converts to "No B is C" and that these propositions are ordinarily used in scientific reasoning:

Text 6.14: *Qiyās* II, 1, pp. 75.4–76.4
If the universal negative, among the absolute, is taken according to the *customary way* of understanding the statement of someone who says "No C is B," which is *the one used in the sciences* (*wa-hiya l-mustaʿmila fī l-ʿulūm*), it converts. If it is taken according to the way it must be in itself, it does not convert. This conversion can be like the original [proposition]. For just as nothing white is black, that is to say as long as it is white, in the same way nothing black is white as long as it is black. And just as no stone is an animal, that is to say as long as it exists, in the same way no animal is a stone as long as it exists. The judgment of the original [proposition] is like the judgment of the converse.

In summary, evidence from at least three contexts—the general characterization of scientific predicates, the interpretation of universal scientific predication, and indirect evidence from outside the theory of science—unequivocally shows that Avicenna imports, in a consistent and conscious manner, the distinction between referential and descriptional necessity developed in the context of formal logic into his own theory of science and sees it as an integral part of the theory and practice of scientific discourse.

NECESSITY, CAUSALITY, AND CERTAINTY

Avicenna explicitly deals with the problem of the relation between necessity, causality, and certainty in two places, namely *Burhān* II, 5 and I, 8. I have addressed elsewhere the discussion of II, 5.[26] An analysis of the temporal and modal conditions of the premises of Barbara NXN in the context of Avicenna's theory of demonstration indicates that the validity—indeed even the perfection—of this mood is not a sufficient condition for it being a proper demonstration. In commenting on *An. Post.* A6, Avicenna considers a case in which the middle term of a first-figure deduction with universal affirmative premises must be interpreted in such a way as to neutralize the effects of what I have called the "intermittence problem," namely the epistemic instability resulting from a middle term that is potentially not always true of the minor. A consequence of the new reading of the premises is that the original Barbara NXN argument turns into a material instance of Barbara

26. Strobino (2015a).

NNN, which Avicenna seems to regard as the paradigmatic case of demonstrative argument with the appropriate epistemic force.

In this section, I briefly analyze the argument developed in the first part of *Burhān* I, 8, where Avicenna establishes a strong link between necessity and certainty whenever there is a cause for the nexus between subject and predicate.[27] In other words, if an attribute belongs or fails to belong to a subject—whether always or at a time—in virtue of a cause, then the nexus (or lack thereof) can only be asserted with absolute necessity in virtue of that cause. The nexus between subject and attribute, in such causal contexts, is not intrinsically necessary and self-explanatory, but only possible. It becomes necessary only when the cause is specified and known, whether it be explicitly stated or just entertained in the mind. Thus, an assertion of necessity relative to a nexus that is dependent on a cause requires knowledge of the cause, without exceptions. In the absence of it, knowledge can at best be of the fact that the nexus is possible, not that it is necessary.[28] Any other type of knowledge is epistemically unstable according to Avicenna, for whenever a causal nexus between a subject and an attribute is believed to obtain in virtue of something other than its cause, there can be no certainty about it.

In that case, one knows only *that* something is the case, not that it cannot be otherwise, because one does not know the cause in virtue of which it cannot be otherwise. In particular, Avicenna rejects the view that using an effect instead of the cause to prove a causal nexus between a subject and an attribute may lead to unconditional certainty about the conclusion.[29] This is because unconditional certainty would require a belief that the contradictory of the conclusion is impossible, and a belief of this kind can only result from an inference that involves the cause of the nexus between subject and attribute.

Certainty requires knowledge of the fact that (i) the cause belongs necessarily to the subject and of the fact that (ii) the effect necessarily follows from that cause (*wuǧūb itbāʿ*). In the absence of such knowledge, the possibility that the nexus between subject and attribute may fail to obtain cannot be ruled out.

27. At *Burhān* I, 8, p. 87.17, by contrast, Avicenna contends that in noncausal contexts "that-demonstration may occasionally give perpetual certainty (*yaqīn dā'im*)." The significance of this claim is discussed in chapter 9.

28. Avicenna's full statement of the view is at *Burhān* I, 8, p. 85.1–9. This epistemological claim is consistent with (and perhaps dependent on) Avicenna's metaphysical characterization of all caused existents (that is to say, everything other than the Necessary Existent) as things that are possible in themselves and necessary through another, on which see in particular *Ilāhiyyāt* I, 6, pp. 37.2–39.16. Just as things that are merely possible in themselves become necessary only in virtue of their cause(s), so knowledge of anything that has a cause is necessary only if the cause is known too.

29. Examples from *Burhān* I, 7, where the middle term is an effect of the major instead of its cause, involve a certain kind of white coagulated urine as a sign or effect of phrenitis (*sarsām*) and the capacity of laughter as a sign or effect of human rationality. I discuss the case in chapter 9.

The central assumption in Avicenna's argument is that when an effect is used as a middle term to prove that the cause belongs to a subject, the intellect can temporarily abandon habit and imagine the nexus between subject and predicate not to obtain always and in all cases. Since it is the effect that in reality belongs to the subject in virtue of the cause and not the other way around, if the effect is used as the middle term to prove that the cause belongs to the subject, the intellect may separate the predicate-cause from the subject at least in the imagination (for it can imagine the predicate not to belong to the subject, at least hypothetically, even if the predicate is inseparable from the subject in existence). And this compromises the necessity of the corresponding assertion, for inseparability is a requirement of necessity.[30]

If, by contrast, the middle term is a cause, there is no room for doubt, as the effect, which is the predicate-major term, must belong to the subject-minor term *in virtue of* the cause-middle term, and no additional condition is required. The demonstration reflects the order and arrangement of cause and effect in reality.

Avicenna's argument may be reconstructed as follows. Let A, B, and C be convertible terms, where A is the minor term and C is naturally prior to B in the order of explanation.[31] Next, suppose one is in doubt about whether "Every A is C" (AaC) necessarily holds. Avicenna's contention is that since B is naturally posterior to C, then AaC cannot be established beyond any reasonable doubt by means of B in the following deduction:

(1) AaB, BaC $\vdash$ AaC.

This is because one of the following must be the case: either AaC was somehow already known or it was not:

(i) If AaC was already known, then seeking it is pointless (for AaC is not truly being sought, as it is already known, and hence it is not really proved by means of B).
(ii) If AaC was not already known, then it is genuinely sought by means of deduction (1).

30. This contention raises a problem because the attributes Avicenna is discussing are all implicates (to which I return in chapter 8), and implicates are inseparable in imagination. Avicenna would therefore be denying here what he usually holds in many other places, namely that the intellect *cannot* separate in imagination an implicate from that of which it is an implicate. I see two interpretive options. Either Avicenna is using the notion of inseparability in a loose sense here, without being seriously committed to the view that the intellect can separate an implicate from its implicant, or he means that an implicate is generally separable in imagination until it is *proved* to belong to that of which it is an implicate, whereupon it becomes inseparable in imagination, because it is associated with its cause (and the corresponding middle terms in the relevant proof). As shown in chapter 8, there are good reasons to believe that the second option is preferable.

31. In the example of *Burhān* I, 8, the terms are human (A), capable of laughter (B), and rational (C).

In the latter case, what is the status of the minor premise AaB, which purportedly contributes to our new knowledge of the conclusion AaC?

(i) Either AaB is in turn sought, and hence *not certain*, in which case one may at least hypothetically imagine its contradictory AoB to be true, whereupon any knowledge obtained by means of AaB, including AaC itself, as the conclusion of (1) will be susceptible to doubt, because it is only obtained through this middle;

(ii) or AaB is known through the cause that necessitates it, in which case it must be known through AaC, and hence AaC itself must have been known all along, and seeking to establish AaC through AaB becomes redundant, for causal knowledge of AaB through a deduction involving C as a middle term makes deduction (1) epistemically irrelevant.

Thus, Avicenna's conclusion is that using an effect as a middle term will ultimately be either epistemically inadequate for the acquisition of certainty (because it cannot serve as a basis for ruling out the possibility of doubt), or, if it is epistemically adequate, this is only because it has previously been established in virtue of its cause, in which case it will be redundant (because knowledge of the cause is one of its requisites). In the second part of *Burhān* I, 8, Avicenna reiterates that in order to attain complete perpetual certainty, demonstrations that proceed from the effect to the cause are insufficient. In other words, one has to provide the genuine cause of the existence of the major term, not just a cause of its belonging to the minor term. With regard to the problem of certainty and time, Avicenna interestingly notes that many of the examples found in the *Posterior Analytics* must be read charitably—or in Avicenna's own terms, with indulgence (*ʿalā s-sabīl al-musāmaḥa*).[32] The examples in question include some notorious cases such as the deciduousness of plants and the attributes of being broad-leaved, having coagulated moisture, or leaf-shedding, and lunar eclipses and the screening of light by the earth.[33] In these cases, Avicenna is concerned that the middle term, and hence derivatively the major term too, may fail to belong at some time to the minor, as all these examples are about phenomena that do not last perpetually, but only at certain times (the general principle that governs the temporal relation between cause and effect is that "if the middle term does not belong perpetually to the minor term, what is necessitated by the middle term need not belong perpetually to the minor term"; *Burhān* I, 8, p. 91.1–2). Strictly speaking, a middle term of this

32. Such occasional remarks are characteristic of Avicenna's style of commentary; cf. also *Burhān* III, 7, p. 246.12–14 and IV, 6, p. 307.13–14.

33. For the botanical vocabulary, see *Nabāt* I, 5, pp. 22.13–25.5.

kind can only provide certainty for as long it belongs to the minor, that is to say, certainty at a time.[34]

CONCLUSION

It is clear from the analysis of necessity in this chapter that Avicenna is committed to the view that modality, explanation and causality, certainty, and time are all parts of one and the same complex picture, and that concepts drawn from logic and metaphysics (so much for the ban on kind crossing) contribute to the fabric of his epistemology in essential and original ways. In particular, Avicenna's most significant contribution to the understanding of necessity and of its role in his recalibration of Aristotle's theory of science is a conscious adoption of the distinction between the referential reading and the descriptional reading, which is in turn the basis for the identification of descriptional necessity as a paradigm in the analysis of relations between essences or conceptions, and for its application to the sciences.

34. At *Burhān* I, 8, pp. 90.18–91.3, Avicenna notes that it is not a sufficient condition for perpetual, certain demonstration (*burhān yaqīn dā'im*) that the middle be only a cause of the major's belonging to the minor. The reason is that "if the middle does not perpetually belong to the minor, what it necessitates and that of which it is a cause need not belong perpetually [to the minor]. As a result, if it is a cause, it only procures certainty at a given time."

7

Scientific Attributes

ESSENCE, NECESSITY, AND SCIENTIFIC PREDICATION

Scientific knowledge is concerned, in every domain of inquiry, with the essences of its subjects and attributes and with their relations. Every science is characterized by its own peculiar set of attributes, and the primary task of a science is to establish by demonstration that those attributes belong to their subjects. These essential relations are ultimately expressed by scientific categorical questions, that is to say, the demonstrable propositions of a science. In chapter 4, I have discussed Avicenna's classification of the types of predicates in the context of his analysis of the logical form of scientific questions. In chapter 6, I have introduced his distinction between two kinds of necessary predicates in scientific demonstrations: (i) predicates that are part of the essence of the subject and (ii) predicates that are necessarily implied by the subject but are not part of its essence (in this case, it is rather the essence of the predicate that is somehow dependent on that of the subject). The relation between these two kinds of necessity is at the heart of Avicenna's account of scientific predication and of his classification of the fundamental types of terms employed in demonstration and definition.

Scientific attributes or predicates—I use the two terms interchangeably—fall under the general heading of per se (*ḏātī*), a notion that serves for Avicenna as the counterpart of Aristotle's concept of *kath' hauto* introduced in *An. Post.* A4.[1] "Per se" has become an entrenched term of art in modern Aristotle scholarship, even

1. On Aristotle's notion of per se, see in particular Wedin (1973), Graham (1975), and Granger (1981).

though it is simply the calque of a common medieval Latin translation. The expression, in more or less ordinary English, means "in itself." In an admittedly much less ordinary sense, it indicates in the present context (and in the Aristotelian tradition at large) all those attributes that somehow belong to something in virtue of it being the kind of thing it is. In particular, two of the four senses identified by Aristotle in *An. Post.* A4 are relevant for the discussion of this chapter: per se 1 and per se 2. In a nutshell, per se 1, in the *Posterior Analytics*, is just any essential predicate belonging to the definition of something (figure and trilateral for triangle, rational for human, body for animal, and so on), while per se 2 is any attribute of a subject such that the subject itself is assumed in the definition of the attribute (even for number, straight for line, lunar eclipse for the moon, and so on).

Even by the standards of an avid Aristotelian readership, the theory of per se predication is a highly specialized and somewhat arcane subject. This is certainly due to the unforgiving, technical nature of the concept, which it would be difficult to situate outside the perimeter of Aristotelian epistemology and metaphysics. But there is a subtler reason for its apparent intractability, namely the fact that Aristotle's treatment of it suffers from a sort of unintended paradox. The notion of per se is undeniably a cornerstone of his theory of science, for all scientific predicates must be of this kind (the case is repeatedly made at *An. Post.* A4, A6, and A22). In particular, the attributes that a science seeks to establish by demonstration to belong to its subject must fall in the category of what is commonly called per se 2. Yet, notwithstanding its centrality, Aristotle's characterization of this critical notion seems to oscillate between incompleteness and inadequacy, for most of the terms that are usually put forward in his works—including the *Posterior Analytics*—as predicates of scientific propositions do not seem to meet the strict definitional requirement set down in *An. Post.* A4 (the examples are legion).

In this chapter, rather than offering a detailed reconstruction of Avicenna's account of per se and its relation to Greek and Arabic sources—for which I refer the reader to Strobino (2016a)—I focus on the novelty and significance of his approach. Avicenna's account of scientific attributes is one of the most interesting areas in his theory of science, in spite (or perhaps because) of its highly technical character. Avicenna's main contributions may be grouped under four headings: (i) a clear, regimented account of per se 1; (ii) a substantive revision of the definition of per se 2; (iii) an account of the internal divisions of both types (primary, non-primary, proper, non-proper); and (iv) a broad array of detailed examples showing how the revised definition of per se 2 is meant to be genuinely applicable to the attributes of all scientific disciplines.

Since in this case Avicenna's original contributions are inseparable from the most exquisitely technical aspects of his analysis, the sense of mystery and historical distance prompted by Aristotle's notion of per se, far from being dispelled, will possibly grow even stronger. But this level of sophistication and conceptual elaboration is just what Avicenna needs to turn Aristotle's account of per se into a real theory of scientific predication (or rather, a theory of scientific predication that deals with a broad range of real scientific predicates and not just with a few simplified examples). This is because an adequate theory of science must provide an elucidation of the nature of the terms and of the types of predicative assertions by which the sciences are populated.

PER SE 1

Avicenna's characterization of per se 1 presented in *Burhān* II, 2 (the counterpart of *An. Post.* A4) identifies and regiments, generally speaking, the same class of essential predicates identified by Aristotle:

> Text 7.1: *Burhān* II, 2, p. 125.7–9
> Per se (*ḏātī*) is said, in one sense, of anything that is predicated of something essentially (*al-maqūl ʿalā š-šayʾ min ṭarīq mā huwa*) and included in its definition (*dāḫil fī ḥaddihī*). Consequently, it is the same to say "per se" or "predicated essentially." This is [(i)] the genus of something, [(ii)] the genus of its genus, [(iii)] its differentia, [(iv)] the differentia of its genus, [(v)] its definition, and [(vi)] every constituent of the essence of the thing (*muqawwim li-ḏāt aš-šayʾ*).

In Text 7.1, Avicenna identifies explicitly five essential relations that illustrate the notion of per se 1 predication. For any subject S and predicate P, P is a per se 1 predicate of S if and only if one of the following is the case: (i) P is the genus of S (as animal for human), (ii) P is the genus of the genus of S (as living being for human), (iii) P is a differentia of S (as rational for human), (iv) P is a differentia of the genus of S (as sensitive for human), (v) P is the definition of S (as rational mortal animal for human), or (vi) P is any constituent of the essence of S. The sixth characterization includes the previous five and extends the notion of per se 1 to any component of the essence of the subject, however general it is (we shall see that Avicenna introduces a restriction in the case of per se 2).

Essences are fully expressed by complete definitions. The latter are complete and ordered sequences of essential terms. Avicenna understands the structure of definitions to conform, ideally, to the model of a Porphyrean tree and always to proceed by genus and differentia. As a result, an essence may be represented as a combination of a proximate genus and a last constitutive differentia:[2]

2. For the sake of simplicity, this reconstruction presupposes a definitional tree involving only a single pair of dichotomous differentiae at each node. But reality is often more complex, and Avicenna is keenly aware of it. His sensitivity to the problem emerges from the complex analysis of the notion of differentia developed in *Madḫal* I, 13, on which see Di Vincenzo (2015), and at *Ilāhiyyāt* V, 4 and V, 6, with some unusually measured statements on our ability to identify differentiae in an accurate and exhaustive manner. Even in his account of the method of division at *Burhān* IV, 7, to which I return

$$E = G_0 D_0$$

If the proximate genus of E is, in and of itself, a complex essence that may in turn be defined in terms of its own proximate genus and constitutive differentia (which is generally true, except in case G_0 is one of the categories), then it may be expressed as follows:

$$G_0 = G_1 D_1$$

The same essence E may then be alternatively captured by a second, equivalent definition that combines this higher genus and the two constitutive differentiae:

$$E = (G_1 D_1) D_0$$

This line of reasoning may be generalized to make explicit any intermediate genus in the definition of E. This is because if, for every G_n such that $n \geq 1$, there is a G_{n+1} and a D_{n+1} such that

$$G_n = G_{n+1} D_{n+1}$$

then it is possible to express the unique definition of E in as many logically equivalent ways as there are orders of nested genera in its definitional tree:

$$\begin{gathered} \mathrm{Def}_1\text{: } E = G_0 D_0 \\ \mathrm{Def}_2\text{: } E = (G_1 D_1) D_0 \\ \mathrm{Def}_3\text{: } E = ((G_2 D_2) D_1) D_0 \\ \cdots \\ \mathrm{Def}_n\text{: } E = (((G_{n-1} D_{n-1}) \ldots D_2) D_1) D_0 \end{gathered}$$

Avicenna's account of per se 1 presupposes an understanding of essences and definitions along these lines, according to which P is a per se 1 attribute of a subject S if and only if P is either (i) a G_i, or (ii) a D_i, or (iii) a Def_i relative to the essence of S.[3]

in chapter 13, Avicenna entertains more complex combinations involving overlapping differentiae. But there is strong textual evidence (for example at *Burhān* IV, 6, p. 306.7–10 and pp. 309.21–310.18, and IV, 7, pp. 312.10–313.22, to mention only a few passages) in support of his commitment to the universal reducibility of definition to the described model. Furthermore, at *Ilāhiyyāt* V, 7, pp. 237.15–238.3 (Marmura 2005, pp. 181–182, transl. modified), Avicenna elegantly summarizes his attitude toward the potential conflict between theory and practice:

> The absence of names and our limited grasp of the differentiae (either one thing or the other) force us away from the true nature of the differentia toward the implicate. As a result, we sometimes derive the name of [the differentia] from its implicate and we mean by "sensitive" what possesses the principle from which sensation and the other things proceed, while some other times the differentia itself is unknown to us, and we only grasp its implicate. *Our discussion*, however, is not concerned with what we ourselves understand, do or manage with regard to these things, but with the manner of their existence *in themselves*.

3. On the terminology of per se 1 or constituent, see Strobino (2016a, pp. 189–190).

Per se 1 attributes are the sole ingredients of definitions and therefore represent the only type of scientific term involved in the process of composition and division by means of which definitions are acquired, as shown in chapter 13. And while demonstrative deductions do not typically aim to establish that a per se 1 attribute belongs to its subject, for the reason examined in chapter 4, this kind of attribute nonetheless plays a critical role in the context of demonstration. This is because there are two kinds of demonstration that make use of per se 1 attributes. The first is (i) a demonstration establishing that a per se 2 attribute of a subject belongs to the subject because the attribute belongs first to a per se 1 attribute of that subject. The second is (ii) a demonstration establishing that a per se 1 attribute of a per se 2 attribute of a subject belongs to that subject because it belongs first to the per se 2 attribute. Both are perfectly legitimate kinds of demonstrations, each proving a different type of per se 2 attribute to belong to its subject (the implicate of a constituent in the first case and the constituent of an implicate in the second case, to put it in the language of chapter 8). Avicenna's clear regimentation of Aristotle's account of per se 1 and the adoption of a strictly hierarchical Porphyrean structure in its analysis are instrumental for a precise clarification of their role in demonstration and definition.

PER SE 2

Avicenna's analysis of the predicates of scientific questions and of the manners of assuming the terms of demonstrative premise pairs established, as noted in chapter 4, that the goal of an affirmative scientific demonstration is to prove that a per se 2 attribute belongs to its subject (with only two clearly identifiable exceptions).

Aristotle's account of per se 2 requires the subject of an attribute to be taken in the definition of the attribute and to be part of its essence. Something belongs per se to a subject in this sense "if what it belongs to itself inheres in the account which shows what it is" (*An. Post.* A4, 73a37) and per se 2 are "the items for which the things to which they belong inhere in what they are" (*An. Post.* A22, 84a13).

This characterization is the source of a traditional difficulty that has vexed interpreters from late antiquity to the present.[4] The reason is that Aristotle's definition of per se 2 appears to be either inadequate or incomplete. Its potential inadequacy

4. For a general analysis of the problem, see Barnes (1993, pp. 112–113, 120–122) and McKirahan (1992, pp. 80–93). Avicenna's strategies for the solution of the difficulties raised by the notion of per se 2 are often similar to (and, if anything, worked out in greater detail than) those advanced by modern interpreters. Per se 2 and per se accidents are taken to be the same class of attributes by Ross (1949, pp. 577, 580), Graham (1975), McKirahan (1992, pp. 98–100), and Bronstein (2016, pp. 43–47), whereas Mignucci (2007, p. 175), Barnes (1993, pp. 112–113), Tiles (1983, pp. 1–16), and Granger (1981, pp. 118–129) argue in favor of keeping the two notions distinct. There is no question that in Avicenna's view the two categories coincide.

lies in the fact that, if Aristotle is to be taken literally, then the definition seems too narrow, as almost none of the scientific predicates encountered in the sciences—or even in the *Posterior Analytics* (including, for instance, the ubiquitous case of the sum of the internal angles of a triangle being equal to two right angles)—could reasonably be thought to meet its central requirement. For, as a matter of fact, in most of Aristotle's own putative examples of per se 2, it is just plain that the subject itself is not what is taken in the definition of the attribute. Its incompleteness, on the other hand, would be a consequence of the fact that the only other plausible alternative is to maintain that Aristotle, by per se 2, implicitly means the broader class of per se accidents (*kath' hauta sumbebēkota*), a notion he frequently invokes in his logical and nonlogical works alike. But if this is what Aristotle has in mind in the *Posterior Analytics*, then the definitions of per se 2 in A4 and A22 are clearly incomplete, because they fail to mention any other item, apart from the subject itself, that could possibly occur in the definition of the attribute.

Avicenna's answer entails, both in theory and in practice, that there is no distinction between per se 2 and per se accidents. All per se 2 attributes are per se accidents, and all per se accidents are per se 2 attributes. The account of per se 2 is revised accordingly to accommodate not only the predicates whose definition includes the subject itself, but also—and more broadly—all predicates whose definition includes something that falls within the essence of the subject.

Subject, however, is said in many ways in Avicenna, as shown in chapter 4. The conceptual vocabulary for the analysis of subject ranges from the subject of a science to the subject of a scientific proposition. The subject of a science may be a simple kind, or rather a kind qualified in various ways by an attribute. These qualifications are part of a whole (comprising the subject *and* its qualifications), and it is this whole that must actually be taken into account in order to determine whether something is or is not a per se 2 attribute of it. Similarly, the subject of a scientific question may be the subject of the discipline (with or without qualification), one of its species, a per se 2 attribute of the subject of the discipline, the species of a per se 2 attribute, or the per se 2 attribute of a species of the subject of the discipline. Avicenna uses a variety of terms to refer to these cases (the substrate, the genus of the substrate, the subject of the substrate, the substrate of the genus, and so on). The important philosophical contribution of these distinctions is that, no matter what we make of the finer points of Avicenna's detailed taxonomy, its overarching goal is to develop a mature theory of per se, one that is capable, at least in principle, of accounting for as many variations of predicates as are found in the sciences. Avicenna's account of per se 2 intentionally reflects this underlying complexity. For the sake of simplicity, however, in the rest of this chapter I use the expression "essential factor" to refer in general to the item which, in each individual case, is understood to be taken in the definition of the per se 2 attribute and to be part of its essence (the expression is neither in Aristotle nor in Avicenna, but it is clear that the latter is tacitly working with a general notion of this kind). The essential factor is the term that ties a per se 2 attribute to its subject and explains why the latter is a per se 2 attribute relative to that subject.

The central question of Avicenna's theory of per se 2 can therefore be formulated as the problem of finding, for any given scientific proposition, the essential factor in virtue of which the predicate is a per se 2 attribute of the subject. Generally speaking, the essential factor is either the subject itself or something in the essence of the subject, that is to say, one of its constituents.

In *Burhān* II, 2, Avicenna presents two lists of terms and examples that supposedly satisfy this criterion. The two lists are not entirely unproblematic (for various systematic and textual reasons). The terminology is at times confusing, and most likely the text of the second list is somewhat corrupt.[5] But the gist of Avicenna's effort is clear. The first list includes three types of terms:[6]

> Text 7.2: *Burhān* II, 2, p. 126.4–8
> What is per se is said in another sense, for if a thing is an accident of another thing (*šayʾ ʿāriḍ li-šayʾ*)—and in the definition of the accident one takes either [(a)] the substrate (*al-maʿrūḍ lahū*), as nose in the definition of snubness, number in the definition of even, and line in the definition of being straight or being curved, [(b)] the subject of the substrate (*mawḍūʿ al-maʿrūḍ lahū*) as the [line] drawn from the two parallels (*al-ḫāriğ min al-mutawāziyayni*) [in the definition] of the equality of its angles on one side to two right angles, or [(c)] the genus of the subject that is the substrate (*ğins al-mawḍūʿ al-maʿrūḍ lahū*) with the aforementioned condition—then each of the above is said to be a per se accident and to be an accident of something by way of it being what it is (*min ṭarīq mā huwa huwa*).

In Text 7.2, the essential factor is designated in three different ways, presumably based on its type. In case (a) the essential factor is identical with the substrate to which the attribute belongs (literally, "that to which the attribute belongs"). In particular, in two of Avicenna's examples the essential factor is also identical with the subject of a science (number) or with the species of the subject of a science (line). In case (b), by contrast, the essential factor is identified as the subject of the substrate (and not as the substrate itself), and the subject of the proposition "Every line falling onto two parallel lines has its angles on one side equal to two right angles" is the species of the subject of a science with an additional qualification (not just line, but line falling onto two parallel lines). While the reason for Avicenna's specific choice of terms is not entirely perspicuous, it is at least possible to see a systematic difference between the two cases. The third case is not illustrated by an example. Presumably Avicenna is referring to higher-order terms

5. For a detailed analysis of these texts, see Strobino (2016a, pp. 197–200).
6. On the terminology of per se 2, per se accident, and implicate, see Strobino (2016a, pp. 191–192).

(where the essential factor is not the substrate or something that plays its role, but a more general term) of the kind discussed in the second list.

The second list includes three further terms. These are said to instantiate cases in which the essential factor is not the subject itself (a first-order term) but one of its constituents (a higher-order term):

> Text 7.3: *Burhān* II, 2, p. 127.5–10
> The manner of taking what constitutes the subject in the definition of the accident consists in [(b)] taking the subject of its substrate (*mawḍūʿ al-maʿrūḍ lahū*), [(d)] the genus of its substrate (*ǧins al-maʿrūḍ lahū*), or [(e)] the subject of its genus (*mawḍūʿ ǧinsihī*). The first is like taking number in the definition of product of an even number by an odd number and triangle in the definition of the equality of the square of its side to the squares of the other two [sides], for the subject of this accident is right triangle, but [it is] triangle [that] is taken in its definition. The second is like taking plane figure in the definition of right triangle, for [the former] is the subject of the genus of the latter. The third is like taking number in the definition of even-times-even. All these are said to be per se accidents.[7]

7. See *Handasa* IX, 28–29, pp. 290.3–291.2 (*Elements* IX, Prop. 28). In Text 7.3, "product of an even by an odd" is a per se 2 predicate (in a scientific proposition such as "Six is the product of an even by an odd"). The essential factor, in this case, is number, which is characterized as the genus of the substrate of "product of an even by an odd" (where the substrate is presumably any individual number that satisfies that description). Alfarabi, *Burhān* II, 4, p. 30.10–12, by contrast, puts the same complex term "product of an even by an odd" in the subject position, and its own per se 2 attribute is "even" (the essential factor is still number, which is the genus of all numbers, including those that happen to be products of an even by an odd).

For the definition of even-times-even (*zawǧ al-zawǧ*), see *Handasa* VII, 6, p. 211.8 (cf. also *Handasa* IX, 32–34, pp. 292.4–293.9, where even-times-even occurs as a predicate). The corresponding account in Euclid is at *Elements* VII, Def. 8. The properties of even-times-even are discussed by Avicenna at *Ḥisāb* I, pp. 27.9–28.20 and p. 30.16–20, the source of which is Nicomachus of Gerasa, *Introduction* I, 8. In Nicomachus, "even" is identified as a genus whose species are said to be even-times-even and even-times-odd, an important point that Avicenna adopts in his classificatory scheme of per se 2. The critical assumption presupposed by these definitions is that the individual integers (2, 3, 4, and so on) are the species of number, while even and odd (and their own species, namely even-times-even, even-times-odd, odd-times-odd, the square of an even, and so on) are all per se 2 attributes of number. Evidence for this view is, for example, at *Ilāhiyyāt* III, 5, p. 119.11–16 (Marmura 2005, p. 91, transl. modified), where Avicenna contends that "each single number is by itself a species; it is one in itself insofar as it is that species; and has, insofar as it is that species, [certain] properties. For something that has no essence (*lā ḥaqīqata lahū*) it would be absurd to have the property of being prime, composite, perfect, abundant, defective, square, cubic, irrational, and the other forms that [numbers] have."

The geometrical examples are concerned with the relation expressed by the Pythagorean theorem: $a^2 = b^2 + c^2$, where a is the hypotenuse and b and c the other two sides. The equality of the square of one side to sum of the squares of the other two sides is a per se 2 attribute of the right triangle, but the essential factor is the genus of right triangle, that is to say, triangle itself.

Text 7.3 is especially problematic for the terminology employed in the characterization of the essential factor, and its examples are not consistently aligned with that terminology. But they are nonetheless extremely interesting and may cast some light on Avicenna's understanding of the array of relations that satisfy the requirements of per se 2 predication. To understand Avicenna's terminology in Texts 7.2 and 7.3, we must bear in mind that more than just two terms are involved in the characterization of per se 2. In fact, at least three terms are required: (i) the per se 2 attribute itself, (ii) the subject of the scientific proposition in which the attribute occurs (attributes do not float in mid air, for they are always tied to the subject of a proposition), and (iii) the essential factor. The reason the terminology is often complicated is the interplay between (ii) and (iii). In particular, the problem is that frequently the subject of the proposition and the essential factor are not the same.[8] As a result, descriptions of the essential factor vary depending on the nature of the subject of the proposition (for example, does the latter coincide with the subject of a science or with an accident of the subject of a science?) and its relation to the attribute, according to the classification discussed in chapter 4. Furthermore, to determine whether a predicate is per se 2 relative to its subject in a scientific question, one must also have a definition of the predicate itself (for example, even is number that is divisible into two equals, and the notion of having the sum of the two angles equal on one side involves a geometrical construction where lines are related in certain peculiar ways). These definitions are almost never explicitly given, let alone discussed, in Avicenna's theory of science, and we must either extract them from his scientific works, or more often than not just reconstruct them on the basis of what he tells us about the essential factor in each particular case. The final step is to determine the nature of the relation between the essential factor and the term that serves as a logical subject of the proposition.

A clear attempt to regiment Avicenna's usage is made by Ṭūsī, who gives an arithmetical example in which one and the same term ("defective") is a per se 2 attribute of four different subjects in virtue of one and the same essential factor ("number"). The terms are illustrated in table 10, where the characterization of the essential factor, in the right-hand column, is relative to the subject of the proposition.

Consider the following propositions: (i) "Some numbers are defective" (or "Every number is defective, equal, or abundant), (ii) "Three is defective," (iii) "The unit is defective," and (iv) "Every even-times-even is defective." The attribute "defective" in arithmetic is true of any integer the sum of whose proper divisors is less than the integer itself. Defective is a per se 2 predicate of several subjects

8. The putative identity of subject and essential factor is the crux of Aristotle's definition of per se 2. As noted previously, if Aristotle is committed to it, then his definition is inadequate; if he is not, then his definition is incomplete.

TABLE 10 Analysis of subject and attribute of a scientific proposition in Ṭūsī's commentary on *Išārāt* I, 15: Identification and characterization of the essential factor

Subject of proposition	Per se 2 attribute	Essential factor	Characterization
Number	Defective	Number	Subject
3			Genus of subject
Unit			Substrate
Even-times-even			Substrate of genus

in arithmetic. For example, along with equal and abundant, it is a predicate that holds disjunctively of number (which is synonymous with positive integer here). Defective is also a per se 2 attribute of number 3 (the only proper divisor of 3 is the unit; in general all prime numbers are defective). Finally, defective also belongs to every number that is even-times-even (the product of an even number by an even number).

In these cases, the essential factor is always number, but the relation between the subject of the proposition and the essential factor is different each time. For example, in the first case the essential factor *is* the subject of the proposition (which is also the subject of arithmetic). In the second case, the essential factor is the genus of the subject of the proposition (which is a species of the subject of arithmetic). In the fourth case, the essential factor is the substrate of the genus of the subject of the proposition. This is because in the proposition "Every even-times-even is defective" (another necessary truth of arithmetic), the subject even-times-even is the species of a per se accident (even) of the subject of the discipline (number). The essential factor in this case is still number, but its relation to the subject of the proposition is such that the former is the substrate of the genus, for the substrate of "even-times-even" can be any number that is the product of an even by an even, that is to say 4, 8, 16, 32, and so on (for all the powers of 2). Thus, the essential factor connecting the per se 2 attribute defective and the subject even-times-even is number, and its characterization tells us how it is related to the essence of the subject of the proposition.

Constraints on the Essential Factor

Avicenna contemplates various restrictions to the level of generality of the essential factor. A per se 2 predicate may be more general than or coextensive with the subject of the proposition in which it occurs, but it can never be more general than the subject of the science. The idea is that whatever falls outside the subject of a science cannot, in principle, play a substantive explanatory role for the truths of that science. He writes:

Text 7.4: *Burhān* II, 2, p. 126.9–22
[With regard to] those [predicates] in whose definition one takes the genus of the subject of the question (*mas'ala*), if that genus is more general than the subject of the discipline, it is not used in the discipline according to the general sense, but rather it is made specific to the subject of the discipline. What falls outside (*ḫāriǧ*) the subject of the discipline is not taken into account, attended to, or used insofar as it falls outside. To be sure, if it falls outside the subject of the question without falling outside the subject of the discipline, [it is not] the subject of the question that is included in the definition [of the predicate], but rather the genus [of the subject], the subject of it[s genus], or something more general. But it is inevitable for the subject of the discipline eventually to be taken in the definition [of the predicate] so that the latter is [counted] among the things that fall within [the domain of] demonstration. For the predicate in "This line is equal to this [other] line" and the predicate in "This [number] multiplied by itself is even" are more general than the subject, and so how could the subject be taken in their definition?

It is not the case that, for every predicate in the premises of demonstration, either the subject itself (*nafs al-mawḍūʿ*) is taken in its definition, or what is taken in the definition of the subject (*wa-immā mā huwa ma'ḫūḏ fī ḥadd al-mawḍūʿ*) [is taken in the definition of the predicate]. At best one should say either that [(a)] the predicates of those premises are either [(aa)] taken in the definitions of their subjects or [(ab)] the subject of the discipline is taken in their definitions, or that [(b)] the predicates of the premises are either [(ba)] taken in the definitions of their subjects, or [(bb)] the subjects or what constitutes [the subjects], in the domain of that discipline, are taken in the definitions of the predicates. The First Teacher goes in this direction, even though he does not state it clearly.

Text 7.4 establishes several important points. The passage reveals what Avicenna thinks is wrong with Aristotle's account of per se 2: in many cases the subject of a scientific proposition cannot be the essential factor because it is too specific. At the same time, however, it is not the case that any constituent of the subject of a scientific proposition can serve as an essential factor, because terms in a science must be modally and explanatorily relevant to the subject of the discipline and cannot fall outside its domain.

In the first section, Avicenna identifies an explicit constraint on the admissible generality of the essential factor. The latter is never allowed, under any circumstances, to extend further than the subject of the discipline, which serves as a sort of natural limit and possibly also as a focal notion in a science, in the sense that every essential factor is ultimately going to be somehow related to it.[9] This is because a term that is too general inevitably fails to be appropriate to the subject

9. At *Ilāhiyyāt* VI, 5, p. 293.2–3, Avicenna contends for example that natural bodies—the species of the subject of natural philosophy—are the cause of the essence (literally the "thingness" or *šay'iyya*) of several forms and accidents and that the latter are only defined by means of them.

of the discipline and hence to have the required explanatory relevance, whereas a term that falls outside the subject of the discipline in an even broader sense is ruled out a fortiori (for the same reason). It is possible, however, for the essential factor to fall somewhere in between the subject of the scientific question and the subject of the discipline.

The second section is a summary of Avicenna's own account of per se 1 and per se 2. It presents two (presumably equivalent) variants. In particular, Avicenna takes issue with Aristotle's characterization of per se 2. The contention that the essential factor must be identical with the subject is rejected because, as a matter of fact, it is not universally true. Avicenna's two proposed definitions revise this criterion to accommodate a broader variety of cases: in one case by generally reducing all essential factors to the subject of the science, in the other by explicitly allowing them to be identified not only with the subject of a proposition but also with one of its constituents (where, again, the most general of such constituents can at best be as general as the subject of the science). Interestingly, Avicenna seems to take this to have been Aristotle's intention, which suggests in turn that rather than considering the Aristotelian definition essentially inadequate for its deliberately narrow scope, he is more inclined to consider it accidentally incomplete, due to its lack of clarity.[10]

Essentiality and Per Se 2

The kind of necessity that characterizes per se 2 attributes is quite specific in nature. As we have seen, per se accidents depend upon the essence of their subjects (however broadly construed and whatever the essential factor is). A further characterization of the notion of per se 2 in *Burhān* II, 2 reveals other critical aspects of Avicenna's understanding of the relation between the essence of the subject and the necessity of its per se 2 attributes:

> Text 7.5: *Burhān* II, 2, pp. 131.11–132.2
> It is because they are proper (*ḫāṣṣa*) to the essence (*ḏāt*) of the thing or to the genus of the essence of the thing that these are called per se accidents (*aʿrāḍ ḏātiyya*). Thus, the essence of the thing or the genus of its essence is never without them (*lā yaḫlū ʿanhā*), either [(i)] without qualification, as the three angles being equal to two right [angles] for triangle, or [(ii)] according to opposition (*bi-ḥasab al-muqābala*), when the subject is never without [the predicate] or its opposite according to contrariety

10. A lengthy digression of two and a half pages in the Cairo edition, at *Burhān* II, 2, pp. 128.15–131.10 (on which I cannot dwell here for reasons of space), concerns a narrow understanding of *ḏātī* that Avicenna vehemently rejects. The opinion is ascribed, in all likelihood, to an anonymous commentator of the *Isagoge*, whose egregious mistake is to take *ḏātī* only in the sense of constituent (*muqawwim*), that is to say, of per se 1. Digressions of such length, devoted to the rejection of competing opinions, are quite rare in the *Burhān*. Di Vincenzo (2018) argues that the views criticized by Avicenna in the gloss may in fact be traced to two Baġdādī Aristotelians whom he frequently criticizes in his logical works.

> or according to the privation which is opposed to it in a proper sense (*al-ʿadam allaḏī yuqābiluhū ḫuṣūṣan*), as line, for it is never without straightness or curvedness, number [for it is never] without evenness or oddness, and thing [for it is never] without being affirmed or being denied.[11] Thus, when it is jointly the case for these accidents that [(i)] the subject is never without them in one of the aforementioned ways, and [(ii)] they do not belong to something other than the subject or its genus, then they are appropriate to its essence (*munāsiba li-ḏātihī*).
>
> If (*law*) the subject were never without them and they were to belong to something other than [the subject] among the things that are foreign (*ġarība*) to its essence or its genus—like blackness to raven—they would not be per se to it in any way at all, since they would not be dependent (*tanʿaliqu*) [(i)] on its essence, [(ii)] on the essence of what constitutes [the subject], or [(iii)] on the essence of something that is constituted through them.[12]
>
> If the subject were without [them] not because the opposite holds of it, but rather because they are merely denied [of the subject], the essence of the subject would not require [them] in the sense of being connected (*fī l-muqārana*) [to them] nor in the sense of being constituted through them (*fī t-taqawwum bihā*).
>
> Thus, when they are among those things [(i)] attaching (*lāḥiqa*) to the subject, [(ii)] that are required by the essence [of the subject], [(iii)] that are proper to its genus, and [(iv)] that are implied by it without qualification or by opposition (*lazimathu muṭlaqan aw bi-ḥasab al-muqābala*), they rightfully come to be called per se accidents.

Text 7.5 identifies the essence of the subject or its genus as that from which a per se 2 attribute is inseparable. As we shall see in chapter 8, Avicenna has different ways to understand inseparability, but something that is clear in this passage is that it is not mere inseparability in existence that serves as the decisive criterion. In Text 7.5, the vocabulary of per se 2 (*ʿarad ḏātī*) is explicitly tied to the notion of essence (*ḏāt*), in the sense that these attributes follow from the essence of something (in virtue of an essential factor) without being part of it. While in the case of per se 1, the necessity of an attribute trivially follows from its being essential to the subject, in the case of per se 2 (where the relation is reversed), the necessity of an attribute follows from a subtler form of connection, but all per se 2 attributes are such that their subjects "can never be without them," which is equivalent for Avicenna to the contention that per se 2 attributes are inseparable from their subjects. Examples involve isolated terms or specific kinds of opposition, where either a term or its opposite necessarily belongs to a subject. If an attribute meets the inseparability condition and falls within the domain of a science, then it is said to be appropriate to it.

11. If A is never without (*lā yaḫlū ʿan*) B, then B is inseparable from A. Other standard expressions for inseparability in Avicenna are discussed in chapter 8.

12. On this example and its dependence on Philoponus and Themistius, see Strobino (2016a, p. 204n43).

Whatever fails to meet these criteria is an extrinsic or foreign accident, which cannot serve as a predicate in scientific questions. The notion of an extrinsic or foreign accident is the negative correlate of appropriate per se accidents. Foreign accidents are not sought in a science partly because they are not sufficiently specific for the subject of that science.[13] But another critical reason is that, if an attribute is foreign to a subject (and hence not per se, as Avicenna shows at *Burhān* II, 9, p. 177.3–4), it cannot encapsulate essential and explanatory relations that are required by the conditions of scientific knowledge. A predicate of this kind simply fails to be a genuine object of scientific knowledge, relative to the domain of a given science.

VARIETIES OF PER SE

A distinctive part of Avicenna's analysis of per se 1 and per se 2 is his painstakingly detailed classification of the internal divisions of both types. In particular, two sets of criteria turn out to be useful for our understanding of this effort of regimentation: (i) the distinction between primary (*awwalī*) and non-primary, and (ii) the distinction between proper (*ḫāṣṣ*) and non-proper per se attributes. The main source for the discussion is *Burhān* II, 3, where Avicenna offers a much more elaborate classification of per se predicates than the one found in *An. Post.* A4. The starting point of *Burhān* II, 3 is the analysis of "primary" (*An. Post.* A4, 73b32–74a4) and its relation to the meaning of "universal" and "as such" in scientific predication.[14]

The resulting permutations are closely related to the complex terminological classification illustrated earlier. The following four passages offer a short illustration of the extent to which Avicenna seems intentionally engaged in a project of classification and regimentation of all sorts of predicates that may be used in scientific discourse. In the first of our four illustrative passages, he writes:

> Text 7.6: *Burhān* II, 3, p. 135.5–7
> Each of the two species of per se (*min nawʿay aḏ-ḏātī*) may be said primarily or not. Thus, when something is predicated of the totality (*kulliyya*) of the subject, as the genus, the species, and the implicate accidental, it is primary to [the subject] only if it is not predicated of something more general [than the subject] first (in which case it would be predicated of [the subject] by putting that thing as a middle).

13. On Avicenna's use of foreign (*ġarīb*) in this context, see Strobino (2016a, p. 207n49).

14. The analysis of primary in *Burhān* II, 3 is introduced by a summary of the notion of universal in demonstration (namely the notion of "to be said of every" encountered in chapter 6). On the mutual relations between universal, as such, and primary in Aristotle, see McKirahan (1992, pp. 95–98). Avicenna's understanding of universal scientific predication is discussed in detail in chapter 6. In this area, Avicenna is profoundly indebted to a remarkable set of distinctions and examples found in Alfarabi, *Burhān* II, 4, pp. 28.13–32.7.

According to Text 7.6, the distinction between primary and non-primary predicates (whether per se 1 or per se 2) depends on whether or not a predicate belongs to its subject first. For example, body is a non-primary per se 1 predicate of human because it belongs to animal first and then to human by virtue of its belonging to animal (*Burhān* II, 3, p. 136.1–2). A non-primary per se 1 predicate of a subject is included in the quiddity (*dāḫil fī l-māhiyya*) of a notion more general than the subject. The case of per se 2 predicates is exemplified by the relation between having the sum of the internal angles equal to two right angles and isosceles, for the predicate belongs first to triangle, which is the genus of isosceles (*Burhān* II, 3, p. 136.2–3). The two examples cover both kinds of per se, but they are especially interesting because they also illustrate another distinction. In the former case, the non-primary (per se 1) predicate of the subject is more general even than that of which it is said primarily (not only of the subject), while in the latter case the non-primary (per se 2) predicate of the subject is coextensive with that of which it is said primarily (in both cases, by definition, the non-primary attribute is more general than the subject itself). An attribute that belongs non-primarily to a subject, in other words, may or may not be primary to its genus.[15]

The oblique relations considered in chapter 4 come into play in Avicenna's characterization of per se too. Having the internal angles equal to two right angles is a primary per se 2 attribute of triangle, which is in turn a constituent of another subject, namely isosceles (*Burhān* II, 3, p. 136.4–6), whereas time, motion, and body are related in such a way that the first is a per se accident of the second, which is in turn a per se accident of the third (*Burhān* II, 3, p. 136.7–8). Finally, it is crucial to recognize that the distinctive condition of a primary attribute is not that it cannot belong to anything other than its subject, but rather that it cannot belong to anything more general than its subject. The latter condition is weaker than the former, and in particular it is compatible with the possibility that a primary attribute may be coextensive with its subject (*Burhān* II, 3, p. 136.8–13).[16]

In Avicenna's view, primary is also distinct from immediate. In particular, being immediate is not a necessary condition for being primary. This is because there are attributes that belong primarily to their subjects without being immediate. For example, having the sum of the internal angles equal to two right angles belongs primarily but not immediately to triangle. Indeed, the assertion that every triangle has the sum of its internal angles equal to two right angles is perhaps the

15. As we shall see in chapter 13, the characterization of primary given in *Burhān* II, 3 is crucial for our understanding of Avicenna's account of the method of composition and division in the process that leads to the discovery of definitions, as discussed in *Burhān* IV, 6 and 7. The regress of primary, primary of primary, primary of primary of primary, and so on comes to a stop at the relevant category or highest genus (in the example of human, animal, and body, the process ends with substance).

16. These characterizations are occasionally employed in the sciences. For example, at *Samāʿ ṭabīʿī* IV, 3, pp. 270.10–271.3, rectilinear and circular are taken as primary accidents of line.

most famous example of a *demonstrable* truth in the *Posterior Analytics*. This makes the ascription of the attribute to its subject, ipso facto, a non-immediate assertion.[17] The reason explicitly adduced by Avicenna is that "between this accident of triangle and triangle there are middle terms and shared definitions, all of which are more proximate accidents than this one" (*Burhān* II, 3, p. 136.14–16). In other words, even though having the sum of the internal angles equal to two right angles is a primary attribute of triangle, because it does not belong to anything more general, the term is not immediate or proximate to triangle, because there are several intervening, coextensive middle terms between the two.[18]

In the second of our four illustrative passages, Avicenna writes:

> Text 7.7: *Burhān* II, 3, p. 138.6–9
> Of the primary predicates that are constituents of the quiddity of something, some are proper like definitions and some differentiae (for example, sensitive for animal); and some are not proper, even if they are primary, like the genus and some differentiae (for example, divisible into two equals for even and rational for human, according to someone who regards rational to be shared by human and angel).

Text 7.7 introduces the distinction between proper and non-proper per se attributes, focusing on per se 1, though the distinction again applies to both kinds of per se. The extension of the terms is involved, albeit in a different sense, in the characterization of this distinction too. A predicate is proper to its subject if there is nothing else (not just if there is nothing more general) to which it belongs. Various combinations are possible: the definition and last differentia of a subject are both primary *and* proper per se 1 predicates of that subject. But in other cases, a notion may be a primary per se 1 predicate of a subject without being proper to it. In such situations, while the predicate is not predicated first of something more general than its subject, it is nonetheless predicated of something other than its subject.

The case of per se accidents is especially interesting. In the third of our four illustrative passages, Avicenna writes:

> Text 7.8: *Burhān* II, 3, p. 139.7–12
> Per se accidents may be [(a)] proper to the subject, as the three angles being equal to two right [angles], for it belongs per se to triangle and is equal (*musāwin*) to it; or they may be [(b)] non-proper and per se, as even, for it is a per se accident of product of odd and even, but it is not proper. That it is not proper is manifest. That it is per se is due to the fact that number, which is the genus of its subject, is taken in its

17. The proof in Avicenna is at *Handasa* I, 39, p. 52.5–10; cf. Euclid, *Elements* I, Prop. 32.

18. If a per se 2 attribute belongs to two coordinate species S1 and S2, then it extends further than S1 and S2 taken severally. In that case, it may or may not belong first to a higher genus. Two instances of the latter case (divisible into two equals and rational) are mentioned in Text. 7.7 under the rubric of "primary non-proper" per se 1 predicates. An analogous case for per se 2 predicates is discussed in Text 7.9, though in that context the sense in which the attribute is non-proper is different.

definition. The proper per se accident may be [(aa)] equal, as the three angles being equal to two right [angles] or [(ab)] narrower (*anqaṣ*), as even for number.

In Text 7.8, Avicenna distinguishes between per se 2 attributes that are proper to their subject and per se 2 attributes that are not. Proper is not taken in the strict sense of coextensive, but rather in the sense of not being predicable of something else. The standard type of proper per se 2 attribute is the one that is coextensive with its subject. By contrast, an attribute is said to be narrower than its subject when it holds disjunctively of the subject as a whole (along with its opposite, as even and odd for number) and properly only of one or more of its species (as even for even number). All these considerations concerning the relative extension of subject and predicate are relevant for the identification and characterization of the essential factor. In Text 7.8, for instance, even is also characterized as a per se accident of the product of odd and even without being proper to it. In this case, the essential factor is number. The subject(s) of product of even and odd are 6, 10, 12, and so on, and their genus is number. The attribute is not proper to the subject because even also belongs to 2 (which is not the product of an even and an odd, as the unit is not a species of number according to Avicenna), as well as to all numbers that are the product of an even by an even.

One last interesting example, which brings us back to the case of parallel lines encountered in Text 7.2, illustrates the notion of primary non-proper per se accident. In the fourth of our illustrative passages, Avicenna writes:

Text 7.9: *Burhān* II, 3, p. 138.11–16
Some [per se accidents] are primary non-proper, in the way that "having the angles on one side equal to two right angles" is primary to [both] "line set upon two lines generating two equal alternate angles" and "line set upon two lines resulting in the external angle being the same as the corresponding internal angle," while not being proper to either. Even if this line is one by essence, it is two in notion and by consideration. Thus, if you have trouble conceptualizing this duality, take instead "line set upon two lines which makes the angles on one and the same side equal to two right angles" and [in] the other [case] "line that makes the two angles [on one and the same side] different but such that the alternate angles are equal."

In Text 7.9, Avicenna considers a more complex situation in which one and the same attribute belongs primarily to two conceptually distinct but coextensive subjects without being proper to either.[19] The example involves a particular kind of line intersecting two parallels, which may be identified in two different ways by means of distinct but coextensional descriptions (depending on the properties of different sets of angles it generates with the parallels). Having the angles on one side equal to two right angles does not belong to anything more general than

19. For a detailed discussion of the parallels example and the relation between Avicenna and Alfarabi on this point, see Strobino (2016a, p. 225n8).

TABLE 11 Per se 2 attributes of numbers and extended magnitudes in *Ilāhiyyāt* III

	Quantity	Quality of Quantity
Discrete	Number (Species: 1, 2, 3, . . .)	Oddness, evenness, perfection, abundance, defectiveness, equality, inequality, greater and less, rationality, irrationality, being a square, being a cube
Continuous	Extended magnitude (Species: line, surface, solid)	Circularity, sphericity, being a cylinder, being a cone, triangularity, rectangularity, squareness, curvedness, straightness

either subject taken in isolation and is therefore a primary per se 2 attribute of both. But it is also not a proper attribute of either because that would require it to be true of one of them only.[20]

In summary, a predicate is non-proper either if it is more general or if it belongs to a coordinate species (or in any case to another term), whereas it is proper if it is either equal or narrower. Every non-primary predicate belongs to something more general than the subject (and is therefore also not proper to it).[21]

Last, it is worth noting that the same attributes may play different roles and be employed by Avicenna for the illustration of different relations. For example, having the internal angles equal to two right angles is a primary attribute of triangle, a non-primary attribute of isosceles, a coextensive primary attribute for triangle (unlike body and animal, where the former is primary but not coextensive with the latter), a per se 2 (as opposed to constituent), and the accident of a constituent (triangle) in contrast to what is an accident of an accident (time and motion). Equal and unequal, by contrast, exemplify the case of per se accidents belonging by opposition (to quantity, number, and magnitude) and the notion of an exhaustive division (just as even and odd for number).

The metaphysical basis for most of the contentions advanced by Avicenna in the texts examined in this chapter with regard to arithmetic and geometry is discussed at *Ilāhiyyāt* III, 4–5 and III, 9. In those chapters, Avicenna argues that the per se accidents of both numbers and extended magnitudes are, ontologically, qualities of quantities, as illustrated in table 11.[22]

20. For the conditional assertion "If a straight line falling onto two straight lines makes the angles on the same side such and such (where 'such and such' may be replaced by various relevant properties), then the two straight lines are parallel" (and for the relation between this conditional and the corresponding categorical assertion), see *Qiyās* V, 3, p. 256.11–15.

21. A set of four per se 2 attributes that belong disjunctively to animal, being primary to each species but not proper to it, includes walking, swimming, flying, and crawling. These are also Avicenna's standard terms for the illustration of a non-dichotomous division.

22. On the notions of equality and inequality, see in general *Ilāhiyyāt* III, 5 and *Maqūlāt* IV, 3–5; on the definition of quantity (*kammiyya*), see *Ilāhiyyāt* III, 4, p. 118.14–15; on the nature of angles, see *Ilāhiyyāt* III, 4, pp. 116.3–117.6. The analysis of *Ilāhiyyāt* VII, 2–3, by contrast, focuses on the discussion

CONCLUSION

Avicenna's account of per se offers a glimpse of his engagement with certain unsolved technical problems in the logic of the *Posterior Analytics*. His main interventions are motivated by two needs, one theoretical, the other practical. First, the systematic account of per se attributes must be developed in greater detail. In Aristotle, it is unclear how exactly the notion of per se 2 (the sort of attribute that, in a science, is characteristically demonstrated to belong to its subject) should be understood, or whether he even has a consistent account of it. Avicenna explicitly addresses this problem by putting forward a revised definition of per se 2. What might seemingly appear to be a minor intervention has in fact far-reaching consequences, for according to the revised definition, the class of per se 2 attributes in Avicenna can include a much broader array of terms than seems to be the case in Aristotle. What is more, a clear systematic justification for this inclusion is offered by way of explicitly linking the definition of the attribute with the subject of the science or with the subject of a scientific proposition, as shown in chapter 4. Second, the classification of the basic types of scientific predicates must have traction on the sciences. It is therefore part of the task of the theory of science to elucidate (and practically show with examples) what terms may serve as per se attributes in as many different domains of investigation as possible. Avicenna's detailed, and admittedly somewhat scholastic, classification and illustration of the subtypes of per se 1 and per se 2 presented in the second part of this chapter serves precisely this purpose. It is a sign of his commitment to the view that the predicates of scientific questions must always be dependent on essential factors ultimately connected to the subject of the discipline. His meticulous effort to work out, case by case, the exact nature of the relation between the essential factor and the subject of scientific propositions is an indication of the extent to which Avicenna takes seriously the language of Aristotle's theory of science, down to its finest distinctions. For Avicenna, a fully developed theory of per se predication is an indispensable part of an adequate theory of science, and for us it is a unique window on its inner workings.

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and criticism of ancient doctrines on mathematical entities. On Avicenna's philosophy of mathematics, see Zarepour (2016).

8

The Logic of Essence

The modal distinctions at the heart of Avicenna's theory of science presuppose an underlying logic of essence. The latter involves a number of ideas developed outside the theory of science itself but silently adopted in it. A first level of analysis concerns the conditions under which an attribute is inseparable (*ġayr mufāriq*) from its subject, whether in conception (*taṣawwur*), imagination (*tawahhum*), or existence (*wuǧūd*), and the distinction between constituent (*muqawwim*) and implicate (*lāzim*). Another level of analysis concerns different kinds of semantic entailment, namely correspondence (*muṭābaqa*), containment (*taḍammun*), and necessary implication (*iltizām* or *luzūm*). Avicenna's insights on inseparability and entailment are in turn crucial for his understanding of the logic of conditional propositions (*šarṭiyya muttaṣila*) and for his account of *reductio ad impossibile* proofs (*qiyās al-ḫalf*). The latter are an essential tool in the sciences, when direct proofs may be in principle unavailable. These different levels of analysis are mutually translatable and constitute integral parts of one and the same philosophical project, each being a manifestation of Avicenna's peculiar version of essentialism.

INSEPARABILITY IN CONCEPTION, IMAGINATION, AND EXISTENCE

In chapters 6 and 7, I have examined different ways in which Avicenna explores the modal component in the analysis of scientific knowledge and identifies the fundamental kinds of essential necessity that are central for his theory of science. In this chapter, I show how a series of notions developed elsewhere, namely in the broader context of what might be called Avicenna's logic of essence, provide the ultimate theoretical underpinning for his account of the role of modality in scientific reasoning. The first family of concepts includes a set of relations that govern the possible nexuses between a subject and a predicate, namely separability, inseparability, compatibility, and incompatibility, and their internal divisions (assuming one of them as primitive, the others may be defined in terms of it).[1] These concepts are central for our understanding of per se predication and the classification of the terms of a science.

Avicenna holds that an attribute B is inseparable from a subject A if and only if B cannot be separated, denied, or removed from A without denying or removing A as well. In other words, B is inseparable from A if and only if A entails B.[2]

The relations of separability or inseparability holding between two terms A and B may be qualified in three different ways depending on whether B is separable or inseparable from A in (i) conception (*taṣawwur*), (ii) imagination (*tawahhum*), or (iii) existence (*wuǧūd*).[3] In particular, in the case of inseparability, B may be inseparable from A in three, progressively stronger senses. The weakest form of inseparability is inseparability in existence. B is inseparable in existence from A if and only if there is no actual instance of A that is not also an actual instance of B (that is to say, if it is never the case that something is A but not B). A standard example in Avicenna is the relation between raven and blackness. This form of modality is the weakest because it is falsified by a simple counterfactual hypothesis: even if all ravens ever found in actual existence are black, it is not impossible to imagine ravens that are not black. An intermediate but much stronger form of inseparability upgrades the modality to the level of imagination: B is inseparable from A in imagination if and only if it is impossible to imagine A without B, that is to say, if not only all actual instances of A are instances of B, but also all possible instances of A are instances of B (at least in some sense of possibility). A great many necessary attributes are accounted for by this relation. In particular, this class accommodates all per se 2 attributes. But these are far from being the only

1. For example, assuming separability to be the primitive relation, we may define the other three as follows: B is inseparable from A if and only if B is not separable from A; B is compatible with A if and only if B is separable from not-A; B is incompatible with A if and only if B is inseparable from not-A. Four Arabic roots are typically associated with the idea of separability in Avicenna: (i) *f-r-q*, to separate; (ii) *s-l-b*, to deny; (iii) *r-f-ʿ*, to remove; and (iv) *n-f-k*, again, to separate. A straightforward example of how the criterion of inseparability is understood by Avicenna is at *Ilāhiyyāt* V, 5, pp. 228.9–229.3, in the context of his discussion of the notion of species.

2. It is worth noting that *r-f-ʿ* is the same root used by Avicenna in the characterization of priority in nature examined in chapter 1, which turns out to be the logical priority of what is implied over what implies it (the former being a necessary condition of the latter). The source of this criterion is *Burhān* I, 11, p. 106.13–14. At *Ilāhiyyāt* IV, 1, p. 169.3–8, Avicenna identifies the removal of the cause as the reason of the removal of the effect, and the removal of the effect as a *sign* of the removal of the cause.

3. On the language of *wahm* and *tawahhum* for inseparability in imagination, see Strobino (2016a, p. 238n100).

attributes that fall under it. For instance, all privative or negative necessity predications in which an attribute is correctly denied of a subject with the essence of which it is incompatible can be construed as cases of inseparability in imagination (for example, nonanimal is inseparable in imagination from stone just as not being odd is inseparable in imagination from even, for no stone is possibly an animal and no even is possibly odd).

Finally, B is inseparable in conception from A if and only if B is part of the essence of A and A cannot be defined (let alone be imagined or exist) without B. In this case, A is quite literally inconceivable without B.

The distinction between inseparability in conception and inseparability in imagination marks the boundary between necessary constitutive attributes (*muqawwimāt*) and necessary non-constitutive attributes (*lawāzim*) (or in more familiar terms, between genuinely essential attributes and necessary nonessential attributes).

The three types of inseparability are characterized by the following logical relations:

(a) If B is conceptually inseparable from A, then B is also inseparable in imagination and in existence from A.
(b) If B is inseparable in imagination from A, then B is also inseparable in existence from A.

In other words, conceptual inseparability entails inseparability both in imagination and in existence but is not entailed by either of them. Inseparability in imagination entails inseparability in existence but is not entailed by it. An attribute B is separable from A without qualification if and only if it is not inseparable in existence (and hence not inseparable in imagination and conception). On the other hand, B may be separable in conception while being inseparable in imagination and existence, or separable in conception and imagination while being inseparable in existence from A.

INSEPARABILITY IN CONCEPTION AND CONTRADICTION

The distinction between inseparability in conception and inseparability in imagination is interestingly related to the idea of contradiction. I return at the end of this chapter to a broader application of these concepts in the context of Avicenna's account of reductio proofs, but it is useful to give an early illustration of the distinction with an example taken from his metaphysics.[4] Being finite is an inseparable

4. Another critical application of the notion of inseparability in conception is the distinction of scope between the assertion that X, insofar as it is X, is Y and the assertion that X is Y, insofar as it is X,

attribute of body without being inseparable in conception from body, that is to say, without being part of the essence or definition of body, as the conception of body does not essentially require the conception of being finite (*Ilāhiyyāt* II, 2, p. 62.3–8). In general, if an essential attribute is hypothetically denied of the subject from which it is conceptually inseparable, a formal contradiction will follow. For Avicenna, this sort of contradiction irrevocably undermines the subject itself. But this is not the case with inseparability in existence, for there is no contradiction in imagining ravens that are not black. Nor is it the case with inseparability in imagination. This is because there is no contradiction in imagining a triangle that does not have the sum of its internal angles equal to two right angles. A formal contradiction arises only in connection with the relevant proof, in which case the denial of the attribute would not just be psychologically impossible, but would eventually result in the denial of one of the principles of geometry or of some previously established assertion. But imagining a triangle that does not have the sum of its internal angles equal to two right angles is not the same as imagining a triangle that is not a triangle, for separating that attribute does not, in principle, turn the concept of triangle right away into a self-contradicting term. In the case of conceptual inseparability, by contrast, one cannot even seriously entertain the notion of a non-trilateral triangle or of a nonanimal human (according to the accepted definition of both). As a result, in the *Ilāhiyyāt* example, imagining an infinite body, which is the same as denying finiteness of body in the imagination, does not result in a formal contradiction. The assertion "Some body is infinite" is undoubtedly false, but not in virtue of an erroneous conceptualization of the terms "infinite" and "body." By contrast, denying "dimension," "length," "height," or "depth" of body would require the conception of a body that is not a body.[5]

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which enables Avicenna to distinguish between attributes that are included in the essence or quiddity of something and attributes that are not, even if they are inseparable. The distinction is crucial for Avicenna's formulation of his account of the *triplex status naturae*: for any property Y that is *not* included in the quiddity of X, it generally holds that X *insofar as it is* X is neither Y nor not-Y (even if one or the other is inseparable from X in imagination or existence). On this problem, see in particular *Ilāhiyyāt* V, 1. See also *Qiyās* II, 3, pp. 100.13–104.4 on the logical behavior of the reduplicative phrases *min ḥayṯu* ("qua," "as such") and *min ǧiha* ("insofar as," "with respect to").

5. The notions of separability and inseparability in conception and imagination (and their counterparts of compatibility and incompatibility) have interesting applications in other areas of metaphysics, especially with regard to higher-order concepts. For example, Avicenna's distinction between universals and particulars, at *Ilāhiyyāt* V, 1, pp. 195.8–196.5, is based on the following idea. A universal (at least in the weakest of the three senses discussed in that chapter) is that whose conception is compatible with it being said of many, whereas a particular is something whose conception is incompatible with it being said of many and which in turn can only be imagined to belong to one thing. At *Burhān* II, 4, p. 145.1–6, Avicenna contends along similar lines that in one sense "universal may be said of something that has neither [the] actual [attribute of] being common (*ʿumūm*) [to many] in existence nor the possibility of being common [to many] in existence, but whose mere conceptualization by the

CONSTITUENTS AND IMPLICATES

At first sight, the distinction between inseparability in conception and inseparability in imagination may seem to have a merely psychological basis and hence to be logically spurious. Nothing could be further from the truth for Avicenna. These notions are in fact closely related to the distinction between necessary essential attributes and necessary nonessential attributes.[6] Two terms of art in Avicenna's logic and metaphysics are constituent (*muqawwim*) and implicate (*lāzim*). A constituent (or constitutive attribute) is nothing other than a genuine essential attribute or a per se 1 attribute in the sense defined in chapter 7, that is to say a genus, a differentia, or—in a derivative sense—one of the logically equivalent definitions of a term. All constituents are inseparable in conception from that of which they are constituents.[7]

An implicate, by contrast, is any necessary attribute of something. While this characterization entails that every constituent is also an implicate, as Avicenna explicitly acknowledges (for every essential attribute is also a necessary attribute), the term "implicate" typically designates non-constitutive necessary attributes.[8] As we have seen in the previous section, non-constitutive necessary attributes may be of two kinds: those that are inseparable in imagination and those that are inseparable in existence. There are accordingly two types of implicates.

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intellect does not prevent it [in principle] from being shared [by many] (*šarika*), even if something prevents it from being shared [by many] in existence and another notion is joined to it and signifies that it always exists only as a single thing." This is the sense of universal that applies to notions like sun or earth (which according to the principles of Avicenna's cosmology are physically bound to be singleton species).

6. A common mistake, according to Avicenna, is to rely solely on the criterion of separability in imagination to determine whether an attribute is essential or accidental (*Madḫal* I, 6, p. 34.1).

7. The relevant textual evidence for the distinction between inseparability in conception and in imagination, and the corresponding notions of constituent and implicate comes primarily from Avicenna's theory of the predicables. For a detailed analysis of it, see Strobino (2016a, pp. 246–255).

8. A characterization of implicate in its proper sense is at *Ǧadal* III, 3, p. 184.10–11 (in the context of Avicenna's discussion of *Top.* Δ5, 125b28–126a2, which is in turn concerned with the Aristotelian notion of *parakolouthoun*): "The implicate is inseparable from the thing but extrinsic to its essence (*ḥaqīqa*) and to its quiddity"; cf. also *Mašriqiyyūn* I, 4, p. 14.5–7 for a similar formulation in the context of the theory of the predicables. That all constituents are implicates is explicitly stated, for example, at *Išārāt* I, 12, pp. 8.19–9.1: "The non-constitutive inseparable [attribute], which is designated by the term 'implicate' (*lāzim*), even though the constituent is also an implicate, is that which accompanies the quiddity without being part of it." At *Burhān* II, 2, p. 130.1–2, Avicenna mentions the extreme case of some scientific disciplines in which *only* non-constitutive implicates are sought: "You know that all inquiries (*maṭālib*) in geometry and arithmetic seek things that are non-constitutive implicates (*umūr lāzima ġayr muqawwimatin*), for you do not find in [those sciences] a deduction seeking a generic or specific predicate (*maḥmūl ğinsī aw faṣlī*)."

The class of implicates that are inseparable in imagination includes a variety of attributes. Avicenna does not distinguish them explicitly, but at least five kinds of non-constitutive necessary attributes may be unequivocally identified from his observations and examples: (i) positive non-constitutive necessary attributes that follow from the essence of their subject, (ii) negative non-constitutive necessary attributes, (iii) trivial non-constitutive necessary attributes, (iv) derivative non-constitutive necessary attributes, and (v) genera relative to their subordinate differentiae.

The first subclass coincides with the class of per se 2 attributes defined in chapter 7. Having the sum of the internal angles equal to two right angles for triangle, being even-times-even for all powers of 2, being capable of laughter for human, being deciduous for broad-leaved plants, being (periodically) eclipsed for the moon, burning for fire: these are all positive necessary attributes that are inseparable in imagination from their subjects. Their additional characteristic, that is to say what makes all of them per se 2 attributes of their subjects, is that there is in each case an essential factor that ties them to the essence of their subjects. In other words, all these attributes are implicates of something *and* essentially related to it, in the sense of necessity specified in chapter 6.[9] According to the distinction drawn in that context, a per se 2 attribute is necessary in implication for its subject *and* tied to the subject by an essential factor that is in turn necessary in essence and nature for the per se 2 attribute in question, regardless of whether the essential factor is the subject itself or one of its constituents.

The second subclass accounts for the truth of certain negative necessity predications. Every human is necessarily not inanimate, not a plant, and not a celestial body. Similarly, every triangle necessarily does not have the sum of its internal angles equal to four right angles. These negative properties are inseparable in imagination from human and triangle, respectively. There is no possible instance of human that is at the same time inanimate, a plant, or a celestial body, not merely in actual existence but in imagination too. And there is no possible instance of triangle the sum of whose internal angles is equal to four right angles, not merely in actual existence but in imagination too. While the sort of inseparability expressed by these predications is, presumably, ultimately a consequence of the nature of the objects in question (for it is because of their essence that human and triangle are necessarily incompatible with certain attributes), all these negative necessary attributes—which, according to Avicenna, are potentially infinite—nonetheless fail to be part of the definitions of the things from which they are inseparable.[10]

9. See in particular Text 6.1, case (bb).

10. Critical evidence in support of the view that Avicenna holds negative attributes to be implicates is at *Qiyās* V, 2, p. 248.2–8. In the context of a discussion of exclusive and non-exhaustive disjunctive propositions, Avicenna characterizes not being inanimate as an implicate (*lāzim*) of plant, alongside

The third subclass accounts for somewhat trivial necessities such as the assertion that everything is identical with itself or distinct from what is other than itself. In this case the predicate is necessarily true of anything of which it is predicated, but this inseparability is merely formal and does not depend on the specific nature or essence of a subject.

The fourth subclass includes another kind of trivial, derivative predicate, which may perhaps be reduced to certain fundamental predicates in the second class. In "Two is half four," the implicate "half four" is in principle interchangeable with potentially infinite equivalent terms ("a third of six," "a quarter of eight," "a fifth of ten," and so on). All of these implicates are equally inseparable in imagination from their subject.[11]

The fifth subclass accounts for the relation between certain differentiae and the genera to which they are subordinated. According to Avicenna, every genus is an implicate (and not a constituent) of a differentia that divides that genus into its subordinate species, as in the case of animal and rational or figure and trilateral. In other words, a genus is inseparable in imagination and existence from its subordinate differentia, even though the genus is not part of the nature of the differentia or even a per se 2 attribute of the differentia.[12]

The other main division of implicates, with regard to inseparability, includes, as noted earlier, attributes that are merely inseparable in existence. These attributes play no role in scientific reasoning.[13]

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other negative implicates (for example, not being a celestial body and not being an angel) as well as affirmative ones (for example, breathing and having roots). The passage also shows Avicenna's commitment to the view that implicates are potentially infinite (a point that should be kept distinct from his commitment to the impossibility of infinite *chains* of implicates). Along the same lines is *Madḫal* I, 13, p. 79.2–5, where the general case of negative implicates is exemplified by nonrational, which is not a differentia but rather an implicate of every species of animal other than human.

11. Ṭūsī's idea, at *Ḥall muškilāt* I, 12, p. 207.9–10, is that implicates are infinite because a potentially endless list of equivalent expressions may replace "two right angles" (for example, "half of four right angles," and so on) in the attribute "having the sum of the internal angles equal to two right angles," that is to say, the standard example of an implicate of triangle. In other words, from a single implicate, a potentially infinite series of logically equivalent implicates could be generated without effort.

12. If genera were per se 2 attributes of their subordinate differentiae, then those differentiae would have to be part of the definition of their genera. But this is absurd because they would then have to be constitutive differentiae of those genera, which by hypothesis they are not. By contrast, it is for a much subtler reason that genera cannot be per se 1 attributes of their subordinate differentiae (for example, animal for rational or mortal). This is a consequence of an independent metaphysical assumption to which Avicenna is committed, namely that differentiae are not, properly speaking, essences or quiddities, or at least not in the way in which genera or species are. Differentiae may be construed as having a *quasi*-essential structure, but in this case their genera are not in the same predicamental line as the genus that they themselves divide. On this point, see in particular *Ilāhiyyāt* V, 6, pp. 232.7–233.19.

13. At *Burhān* II, 2, p. 127.11–13, Avicenna contrasts implicate in the sense of what is inseparable in existence to the two senses of per se that are relevant in the sciences: "Those predicates that are

TABLE 12 Inseparability, constituents, and implicates

		Types of Inseparability					
		Conception		Imagination		Existence	
Constituent	Per se 1	✓	→	✓	→	✓	Triangle—figure Human—rational
Implicate	Per se 2	✗		✓	→	✓	Triangle—2R Human—capable of laughter
	Non per se	✗		✓	→	✓	Plant—non-inanimate Horse—nonrational
		✗		✓	→	✓	Rational-animal
		✗		✗		✓	Raven—black
Common accident		✗		✗		✗	Human—sitting

If an attribute is neither a constituent nor an implicate of a subject, then it is separable from that subject. An attribute of this kind is called a "common accident" or an "accident without qualification" in Avicenna's vocabulary.[14]

The main relations between types of inseparability and types of attributes are illustrated in table 12.

Immediate and Non-immediate Implicates

Avicenna is committed to another crucial distinction, which turns out to be especially relevant for our understanding of his classification of the fundamental types of principles in a science. Non-constitutive implicates (including the subset of non-constitutive implicates that coincide with per se 2 attributes) may be of two kinds: (i) immediate and (ii) non-immediate. An example of a non-immediate implicate, which we have encountered as a per se 2 attribute in chapter 7, is having the sum of the internal angles equal to two right angles for triangle. This attribute belongs primarily to triangle (for it does not belong to anything more general than triangle) but not immediately, because it is a demonstrable attribute connected to its subject by means of several middle terms (roughly the terms involved in the

not taken in the definition of the subject or in whose definition the subject or its constituents are not taken are not per se. Rather, they are accidents without qualification that do not fall in the discipline of demonstration, as whiteness for swan, even if it is an implicate (*lāzim*)."

14. The distinction is expressed in lapidary terms at *Išārāt* I, 13, p. 9.18–19: "All predicates that are neither constituents nor implicates may be separated from the subject."

proof given by Euclid at *Elements* I, Prop. 32). The existence of immediate implicates is a logical consequence of the existence of non-immediate implicates and other assumptions (including the general thesis that infinite chains of constituents and infinite chains of implicates are impossible). It is worth reconstructing Avicenna's argument in detail, as it usually comes in a rather condensed form.[15]

Immediate implicates are by definition attributes that follow necessarily *and* immediately from the essence of their subjects. They must also be, according to Avicenna, self-evident and inseparable in imagination. How can such implicates be proved to exist in principle? Starting from the assumption that something has at least a non-constitutive implicate, the argument runs as follows:

1. B is a non-constitutive implicate of A.
2. B is either an immediate and evident implicate of A or a non-immediate and non-evident implicate of A.
    - 2.1. If B is an immediate and evident implicate of A, then the existence of one such predicate has been established.
    - 2.2. If B is a non-immediate and non-evident implicate of A, then there must be another term between A and B by means of which B will necessarily be proved to belong to A (and therefore be known evidently to belong to A as its implicate). Let C be this additional term, that is to say, the factor that explains why A necessarily implies B (an explanation required by the fact that B is a non-evident implicate of A). C may be either a constituent or an implicate of A.
        - 2.2.1 If C is a constituent of A, then what is the relation between C and B? Suppose that B may be in turn either a constituent or an implicate of C.
            - 2.2.1.1 B cannot be a constituent of C, if C is a constituent of A,

15. Avicenna presents a short version of the argument at *Išārāt* I, 12, p. 9.3–17. A more detailed treatment of the issue is developed in *Burhān* III, 6 in correspondence with *An. Post.* A19–23. Another compressed variant is at *Madḫal* I, 6, p. 36.4–8 in the context of a classification of different types of implicates. Immediately before presenting the argument, at *Madḫal* I, 6, p. 36.3–4, Avicenna points out that both types of implicates are indispensable elements in the vocabulary of scientific reasoning: immediate implicates must be postulated in order to avoid an otherwise inevitable infinite regress; and non-immediate implicates are, among other things, a feature of our epistemic makeup, for as a matter of fact there are things we do not know, and since immediate implicates are in principle self-evident, if all implicates were immediate then all implicates would be self-evident, and it would be impossible to account for our ignorance of certain facts. At *Burhān* IV, 3, p. 287.7–9, Avicenna contends that certain attributes belong to their subject in themselves without a cause (other than their being what they are and their subject being what it is). These are either (i) non-accidental attributes or (ii) accidental attributes that belong to their subjects first and not in virtue of a cause. Scientific principles involve attributes of both kinds. Their distinctive feature is not to be established deductively because they are evident, even though the definitions of these attributes may still have to be acquired (in ways that will be clarified in chapter 13).

because B would then also be a constituent of A, in virtue of the transitivity of the relation "being a constituent of" encountered in chapter 4. But B was assumed to be a non-constitutive implicate of A; therefore it cannot be a constituent of C.

2.2.1.2 B must, therefore, be an implicate of C. And if B is an implicate of C, then B will be in turn either an immediate and evident implicate of C or a non-immediate and non-evident implicate of C.

2.2.1.2.1 If B is an immediate and evident implicate of C, again, the existence of one such predicate has been established.

2.2.1.2.2 If B is a non-immediate and non-evident implicate of C, then there must be another term between B and C. Applying the same line of reasoning to the new term, the process will either come to a stop, sooner or later, at an immediate and evident implicate, or never cease to require additional terms, which will result in an infinite regress.

2.2.2 If C is an implicate of A, then C is either an immediate and evident implicate of A or a non-immediate and non-evident implicate of A.

2.2.2.1 If C is an immediate and evident implicate of A, then the existence of one such predicate has been established.

2.2.2.2 If C is a non-immediate and non-evident implicate of A, then there must be another term between C and A. Applying the same line of reasoning to the new term, the process will either come to a stop, sooner or later, at an immediate and evident implicate, or never cease to require additional terms, which will result in an infinite regress.

Since there cannot be an infinite regress in the chain of implicates, the conclusion of the argument is that there must consequently be at least a pair of terms such that one is an immediate and evident implicate of the other and inseparable in imagination from it.[16]

16. At *Madḫal* I, 6, pp. 35.18–36.3, Avicenna gives an example of one such implicate:

Some accidents are necessarily implied by the quiddity in a primary and evident way without the mediation of another accident, in which case it is impossible to deny them of the quiddity while seeking to affirm the quiddity and entertaining them together in the mind, if [the quiddity]

The pairwise combinations of the two basic types of attribute considered in this argument (constituent-constituent, implicate-constituent, constituent-implicate, implicate-implicate) correspond exactly to the "manners of assuming demonstrative" terms in the premises of a deduction discussed in chapter 4, namely (a) PS1-PS1, (b) PS1-PS2, (c) PS2-PS1, and (d) PS2-PS2.[17]

Inseparability in Imagination, Immediate and Non-immediate Implicates

Two questions arise concerning the relation between inseparability in imagination and the two types of implicates just discussed. Are both immediate and non-immediate implicates inseparable in imagination? If so, are they inseparable in the same way or in different ways? Avicenna's answer is that, properly speaking, only immediate implicates are inseparable in imagination without qualification. Non-immediate implicates, by contrast, are inseparable in imagination only if they are supported by a demonstration. In other words, while an immediate implicate cannot possibly be imagined not to belong to its subject, a non-immediate implicate taken in isolation, that is to say, demonstratively untethered from its subject, can be removed in imagination from the latter. But this is true, as it were, only as long as the question of whether or not the non-immediate implicate belongs to its subject is still open. For once it is demonstratively established that the implicate does belong to its subject, then its real nature as a necessary attribute is revealed, and the possibility of entertaining the thought that the implicate might not belong to its subject is extinguished. This is not to say, again, that the distinction between the two kinds of implicate is primarily psychological. Rather, the point is that in the order of explanation, which for Avicenna must track the order of reality, those non-immediate implicates

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> has [the accident] not in virtue of a middle between [the quiddity] and [the accident]. This is like the possibility in imagination to extend one of the sides of a triangle in a straight line, or another notion resembling the latter from the domain of the accidents of [triangle]. Sometimes the existence of the accident is mediated, and if that middle is not present to the mind, it is possible to deny it, as [the sum of] any two angles of a triangle being less than two right angles.

On the geometrical example, see *Handasa* I, 23, p. 40.6–10 (cf. *Elements* I, Prop. 17).

17. At *Ilāhiyyāt* V, 4, pp. 225.11–226.2, Avicenna identifies different ways in which implicates may be related to a genus G : (i) as implicates of the genera of G (if there are any), (ii) as implicates of the constitutive differentiae of the genera of G, (iii) as implicates of G itself due to its own (constitutive) differentia, (iv) as implicates of some differentiae falling under G, or (v) as implicates of the matter (or of the accidents of the matter) of the genera of G. Cases (i)–(iii) and (v) involve implicates of notions that are more general than G. Consequently, these implicates are also implicates of G (and of whatever falls under G). By contrast, in case (iv), the implicate is an implicate of something less general than G. It is therefore not an implicate of G itself. In stating case (v), Avicenna also contends explicitly, in line with the doctrine of *Burhān* II, 6 discussed in chapter 4, that it is perfectly legitimate for accidents to have their own implicates.

are genuinely inseparable, though not without qualification but only in virtue of the full chain of terms that connect them to their subjects. For example, an essence E may have an immediate implicate I_1, I_1 may have its own immediate implicate I_2, I_2 may in turn have its own immediate implicate I_3, and so on. Every implicate I_i (with $i > 1$) in the chain belongs to E non-immediately in virtue of its predecessors. Implicates may also be related in such a way that I_1 is an immediate implicate of E and then both I_2 and I_3 are coordinate implicates of I_1 (several other combinations are possible, including terms that are mutually implicative).[18] Immediate implicates that are held to belong to their subjects in a self-evident manner are, for example, being equal to a certain number for a given number and the possibility for any of its sides to be extended in a straight line for triangle.[19]

CORRESPONDENCE, CONTAINMENT, AND IMPLICATION

A second family of critical concepts for Avicenna's logic of essence emerges from his analysis of signification (*dalāla*). The relation between a linguistic expression and the meanings or concepts it necessarily signifies as a result of its imposition,

18. At *Taʿlīqāt* p. 180.14–26, Avicenna first explains what it is to be an implicate and then gives a brief but clear account of the sort of relations he envisions:

> The meaning of implicate is that something is necessarily implied by something without the mediation of something or that something necessarily implies something without the mediation of something, or that something necessarily implies something immediately. The first implicates of their principle [(i)] are implicates of it, [(ii)] have their source [in it], [(iii)] are not implicates of it in virtue of something else, [and] belong to it [. . .]. Those implicates and those attributes (*ṣifāt*) are necessarily implied by [the principle] itself because of what it is, that is to say nothing else is their cause. Implicates that are necessarily implied by something else are necessarily implied not because [their principle] is what it is; rather they may be mediated by something else and by another implicate (whereas if it is not by mediation of something [else], then it is an implicate of [its principle] because of what [the latter] is). The true nature of all implicates is to be necessarily implied by something because of what it is. The implicate of the first can only be one single simple [notion], for only one single [notion] necessarily follows from the one. Next, the other implicate is the implicate of its implicate. And similarly, the third implicate is the implicate of its implicate, and the process continues in this way. Multiple implicates of the first are like this. This is just as when we say "The existent is something, [that something] in turn necessarily implies that it is a point, point in turn necessarily implies that it is another thing.

19. At *Qiyās* IX, 1, pp. 416.12–421.10, Avicenna draws a further distinction between (i) something that is necessarily implied in an evident manner and (ii) something that is necessarily implied by something evident. In that context, Avicenna is concerned with the identification of the constraints under which the consequent of a conditional may be said to follow evidently from its antecedent (though the distinction is applied to other kinds of deduction too). It is not inconceivable that Avicenna's discussion of the distinction in *Qiyās* IX, 1 might be the initial source of an intense (but then largely self-sustaining) debate in post-Avicennan logic concerning the notion of an evident implicate.

which in turn reflect the ontological structure of reality, can naturally be mapped onto the sets of notions and distinctions encountered thus far in our account of Avicenna's essentialist framework. In particular, a close connection may be established between Avicenna's account of signification and the two types of necessity of scientific predication examined in chapter 6, the notions of per se 1 and per se 2 discussed in chapter 7, and the different types of inseparability identified in his chapter. Avicenna distinguishes three ways in which a term may signify a meaning or concept: (i) by correspondence (*muṭābaqa*), (ii) by containment (*taḍammun*), and (iii) by implication (*luzūm* or *iltizām*).[20]

A concise formulation of the distinction is offered in the *Išārāt*:

> Text 8.1: *Išārāt* I, 6, pp. 4.19–5.7
>
> An expression signifies a notion either [(i)] by correspondence (*muṭābaqa*), in that the expression is imposed for that notion as its [exact] counterpart (*bi-izāʾihī*), as "triangle" signifies figure bounded by three sides; or [(ii)] by containment (*taḍammun*), in that the notion [signified] is part of the notion corresponding to the expression, as "triangle" signifies figure not by being a name for figure but by being a name for a notion of which figure is a part; or [(iii)] by way of following (*istitbāʿ*) and implication (*iltizām*), in that the expression signifies by correspondence a notion and that notion necessarily implies a notion other than itself as something extrinsic that comes along with it, not as a part but rather as something that accompanies it and is necessarily implied by it (*muṣāḥib mulāzim*), as the expression "roof" signifies wall, and "human" signifies receptive of the art of writing.[21]

20. For a detailed analysis of the three types of signification, see Kalbarczyk (2018) and Street (2020).

21. In addition to Text 8.1, standard loci for Avicenna's distinction between the three types of signification are *Naǧāt* I, 12, p. 15.1–2; *Madḫal* I, 8, pp. 43.12–44.2; and *Mašriqiyyūn* I, 5–6, pp. 14.15–16.5. In general, Avicenna's reliance on the distinction between correspondence, containment, and implication is ubiquitously attested in the preliminaries to his discussion of the predicables. Further evidence, in particular for the distinction between containment and implication is at *Naǧāt* I, 9, p. 13.1–2, where Avicenna considers "a simple expression [. . .] that contains (*yataḍammanu*)—I do not say, 'necessarily implies' (*lastu aqūlu yaltazimu*)—all the essential notions constitutive of a thing."

For a more detailed discussion of the relevant textual sources, see Strobino (2016a). The language of containment is frequently attested in Avicenna's *Burhān* too, where it is consistently used to characterize the notion of per se 1. The most striking example is a lengthy discussion at *Burhān* IV, 6, pp. 309.21–310.18, where Avicenna lists various constituents of the definition of animal (as part of the process that leads to the definitions of human and horse), calling them the "items that are contained" (*muḍammanāt*) in that notion. The term *lāzim*, which falls in the same semantic area as *iltizām* and *luzūm*, is (almost) synonymous with per se 2 in Avicenna's theory of science, as we have seen in chapter 7. Even in his treatment of the *Topics*, Avicenna appeals to the notion of containment. At *Ǧadal* V, 1, p. 248.5–9, the genus is explicitly said to be "contained in the nature of the species" (*muḍamman fī ṭabīʿat an-nawʿ*) (for example, animal is contained, in this intensional sense, by human); cf. also *Ilāhiyyāt* V, 6, p. 232.16 for the view that the genus is "predicated of the species insofar as it is *part* of the quiddity [of the species]." Finally, at *Ilāhiyyāt* V, 7, p. 237.11–12, Avicenna qualifies the way in which sense perception may be taken as a differentia of animal in the context of an argument that turns

Once an expression is imposed to signify a complete concept, as in the case of "triangle" and trilateral plane figure or "human" and mortal rational animal, various semantic relations are determined as a result. In Text 8.1, Avicenna identifies the most fundamental of such relations with correspondence, which captures the connection between a term and its definition, that is to say, the full expression of the underlying essence. An intermediate relation is containment, which characterizes a term and any part of its definition or essence. Finally, a weaker relation is the one between a term and what is necessarily implied by its definition or essence, without being part of it.

Correspondence, containment, and necessary implication encapsulate different kinds of necessity. All of these relations express necessary nexuses holding between essences, their parts, and their implicates. What matters from the standpoint of Avicenna's theory of science is that this tripartition is (almost) perfectly aligned with the distinction between per se 1 and per se 2, and with the distinction between inseparability in conception and inseparability in imagination. In particular, the relations in the first class, from the following list, are mutually equivalent (assuming the notion of constituent to cover definitions as well as proper constituents, that is to say, genera and differentiae):

1.1 B is a per se 1 attribute of A.
1.2 B is a constituent of A.
1.3 B is inseparable in conception from A.
1.4 B is contained by or corresponds to A.

The second class of relations is slightly different, as its elements are not all mutually equivalent:

2.1 B is a per se 2 attribute of A.
2.2 B is a non-constitutive implicate of A.
2.3 B is inseparable in imagination from A.
2.4 B is necessarily implied by A.

If (2.1) B is a per se 2 attribute of A, then B is (2.2) a non-constitutive implicate of A, (2.3) inseparable in imagination from A, and (2.4) necessarily implied by A. But the converse claims do not generally hold. In particular, as we have seen, Avicenna is committed to the existence of non-constitutive implicates that are not per se 2. The distinctive feature of a per se 2 attribute is the existence of an essential factor, connected with the subject, that is conceptually inseparable from the attribute (and

on the distinction between containment and implication: "Or else one may confine oneself to sense perception, where it would signify all [the internal and external senses]—not by containment but by implication." This passage is only one of many in Avicenna's corpus where the distinction appears to be used as an actual tool for scientific reasoning.

hence necessary to it in essence and nature and part of its definition). The other three relations—(2.2), (2.3), and (2.4)—by contrast, are mutually equivalent, if "inseparable in imagination" and "necessarily implied" are taken in a strict sense, that is to say, excluding inseparability in conception, containment, and correspondence.

The notions of correspondence, containment, and necessary implication may be defined with the aid of a primitive notion of inseparability. B is necessarily implied by A if and only if B is inseparable from A, whereas containment and correspondence are defined in terms of inseparability and an additional condition. In particular, B is contained by A if and only if B is inseparable from A and B is a proper part of A's essence, that is to say, either a genus or a differentia of A; and B corresponds to A if and only if B is inseparable from A and B is a definition of A:

(1) Correspondence (*muṭābaqa*): B is inseparable from A and B is Def_i ($n \geq i \geq 1$) of A,
(2) Containment (*taḍammun*): B is inseparable from A and B is G_i or D_i ($n\text{-}1 \geq i \geq 0$), and
(3) Necessary implication (*iltizām*, *luzūm*): B is inseparable from A,

where A's essence is expressed by one of its logically equivalent definitions (bearing in mind that these logically equivalent definitions are simply different expressions of one and the same unique real definition of the unique essence of A):

$$\mathrm{Def}_1\text{: } E = G_0D_0$$
$$\mathrm{Def}_2\text{: } E = (G_1D_1)D_0$$
$$\mathrm{Def}_3\text{: } E = ((G_2D_2)D_1)D_0$$
$$\ldots$$
$$\mathrm{Def}_n\text{: } E = (((G_{n\text{-}1}D_{n\text{-}1}) \ldots D_2)D_1)D_0$$

This analysis can also accommodate the important distinction between immediate and non-immediate implicates, if we assume the primitive notion of inseparability to express immediate implication. Non-immediate implicates can then be defined in terms of arbitrarily long finite concatenations of immediate implicates or constituents. Such chains will ultimately reduce to finite subsequences of immediate nexuses. In general, the simplest kind of non-immediate implicate will be either (i) the immediate implicate of an immediate implicate, (ii) the immediate constituent of an immediate implicate, or (iii) the immediate implicate of an immediate constituent. For example, a non-immediate implicate of A may be such because it is an immediate implicate of B, which is in turn an immediate implicate of A, or because it is an immediate implicate of B, which is in turn an immediate constituent of A, and so on.[22]

22. Examples of immediate constituents are (i) the proximate genus of something (as animal for human, triangle for isosceles, or even for even-times-even) and (ii) its constitutive differentia (as rational for human, having two equal sides for isosceles, or multiplied by an even for even-times-even).

INSEPARABILITY AND PER SE

The proposed understanding of inseparability as a relation in terms of which correspondence, containment, and implication may be defined seems to find indirect confirmation in a passage at *Burhān* III, 5. Avicenna writes:

> Text 8.2: *Burhān* III, 5, p. 227.4–8
> [(i)] Predicate per se is said of that whose characteristic it is not to be separable from the thing and, at the same time, to be a constituent of its quiddity, not having come to it as a foreign [attribute]. Its counterpart is what is generally understood as the accidental predicate. Thus, being white is predicated by accident of the surface.
>
> [(ii)] Predicate per se is said of everything whose characteristic it is for it to be taken in the definition of something or for the thing to be taken in its definition. In general, [predicate per se is said in this sense of] what is appropriate (*munāsib*) to that thing in virtue of the definition of one of the two.[23] What falls outside the scope of these two is predicated by accident.

Text 8.2 reproduces the final part of an extensive discussion of the distinction between per se predication and accidental predication, which Avicenna introduces right before addressing the proof of the finiteness of demonstrative chains (in correspondence with *An. Post.* A19–23).[24] In that context, he examines eight different characterizations of the distinction between per se predication and accidental predication.[25] The last two are associated with per se 1 and per se 2 and generally characterized in terms of inseparability. In particular, case (i) is a narrow interpretation of per se, which only captures the notion of per se 1 and is formulated in terms of inseparability and the additional condition that the attribute be a constituent of the essence of the subject. Case (ii), by contrast, is a broader sense of per se that covers both per se 1 and per se 2. Here the notion of inseparability introduced in (i) is implicitly presupposed (again, without internal differentiation between inseparability in conception and inseparability in imagination) and accompanied by the characteristic conditions examined in chapter 7 (even though the second condition only mentions the subject). Correspondence and containment may be expressed accordingly by means of an unqualified notion of inseparability accompanied by the assumption that a term be a definition, a genus, or a differentia of its subject.

23. *Ilāhiyyāt* V, 6 is a critical source for the metaphysical distinction between constituent and implicate. In particular, at *Ilāhiyyāt* V, 6, p. 231.14–16, Avicenna distinguishes (i) the case of essential predication, in which the predicate is a constituent of the quiddity of the subject, from (ii) the case of nonessential predication, in which the predicate is an implicate of the subject without being a constituent of its quiddity.

24. The conclusion established in *Burhān* III, 6, namely that infinite chains of constituents and infinite chains of implicates are impossible, is philosophically, if not textually, relevant for the proof of the existence of immediate implicates discussed earlier.

25. Strobino (2016a, p. 236n98).

INSEPARABILITY, NECESSARY IMPLICATION, AND *REDUCTIO AD IMPOSSIBILE*

Another glimpse of Avicenna's logic of essence may be obtained from his account of (i) hypothetical conditional propositions (*qaḍāyā šarṭiyya muttaṣila*) and (ii) the structure of *reductio ad impossibile* proofs (*qiyās al-ḫalf*). In the present context, I can only deal with the problem in outline, adopting, so to speak, Avicenna's own method of pointers. But to proceed without at least an overview of his analysis of reductio and its philosophical significance would mean to ignore an important dimension of Avicenna's theory of science. I say dimension, rather than part, because Avicenna's explicit treatment of reductio in the *Posterior Analytics* complex is limited to isolated observations and to a brief exegetical response to *An. Post.* A26 (the chapter in which Aristotle discusses the superiority of direct demonstration over indirect demonstration). Avicenna's corresponding treatment of the topic at *Burhān* III, 7 is interesting but not particularly innovative. In fact, it is aligned with the Greek commentary tradition: reductio (or indirect demonstration) is inferior to direct demonstration (*burhān mustaqīm*) because (i) it presupposes an additional (hypothetical) deduction, (ii) it does not proceed according to the natural hierarchical order governing premises and conclusion (including certain part-whole, inferior-superior relations), (iii) its premises are not better known than the conclusion, and (iv) it fails to provide a causal explanation of the conclusion because it proceeds from something extrinsic to it (namely the falsehood of its contradictory, which necessitates something impossible). Consequently, according to Avicenna, indirect demonstration is merely a that-demonstration and not a genuine why-demonstration (the distinction between these two kinds of demonstration is discussed in detail in chapter 9).[26] But *outside* the *Posterior Analytics* complex, Avicenna's account of reductio is extraordinarily interesting and, among other things, depends crucially on a peculiar understanding of conditionals and of the logic that governs propositions and inferences involving essences, constituents, and implicates. Furthermore, it is legitimate to import what we learn from the treatment of reductio in formal logic back into the theory of science because Avicenna himself repeatedly disseminates comments and remarks in his works implying that this is precisely what we should be doing.[27]

26. Avicenna's most detailed discussion of *reductio ad impossibile* (*qiyās al-ḫalf*) in the *Posterior Analytics* complex is at *Burhān* III, 7, pp. 244.14–245.17 (on *An. Post.* A26), where he analyzes the first three reasons, namely (i)–(iii); reason (iv) is mentioned at *Burhān* I, 8, p. 90.15–17. Avicenna's analysis of reductio in the *Prior Analytics* complex is at *Qiyās* VIII, 3, pp. 408.1–411.5 (on *An. Pr.* A23, 41a22–b5) and IX, 14, pp. 518.1–523.11 (on *An. Pr.* B14). For an illuminating interpretation of Aristotle's account of indirect demonstration in the context of the *Posterior Analytics*, see Malink (2020).

27. For a formal analysis of reductio in Avicenna, see Hodges (2017).

In general, Avicenna's reliance on reductio proof as a method of argument in his philosophical and scientific works is unsurprisingly extensive. Just to mention a few examples—outside the obvious domains of arithmetic and geometry—reductio proofs in metaphysics range from the first sketch of the proof of the necessary existent (*Ilāhiyyāt* I, 6) to the characterization of the structure of the universe and the motions of the heavenly bodies (*Ilāhiyyāt* IX), including the rejection of atomism (*Ilāhiyyāt* III) or the more elaborate arguments concerning the finiteness of all chains of essential causes (*Ilāhiyyāt* VIII). In natural philosophy, prominent cases are the reductio arguments against the existence of void (*Samāʿ ṭabīʿī* II, 8), atoms (*Samāʿ ṭabīʿī* III, 3–5), and an actual infinite (*Samāʿ ṭabīʿī* III, 7–9). Finally, reductio is also a critical tool for establishing the validity of different types of inferences in logic. In other words, reasoning from an impossibility is an integral part of Avicenna's philosophical method, and it is an indispensable instrument of investigation in every science.

Conditionals

For Avicenna, schematically, a proposition *p* can be established by reductio in three steps: (i) the assertion that a false (or impossible) proposition *q* follows from the assumption for reductio, namely from the contradictory of *p*; (ii) the assertion of *not-q*, which is true (or necessary) because of *q*'s falsehood (or impossibility); and (iii) the assertion, by contraposition, of the original proposition *p* (assuming implicitly that double negation is equivalent to affirmation). The conditional relation between *not-p* and *q*, that is to say, between the assumption for reductio and the false or impossible proposition that follows from it (possibly along with other assumptions), is at the heart of Avicenna's understanding of reductio. But what does "following from" mean in this context? The originality of Avicenna's account of reductio lies both (i) in the fact that he seems to be among the first logicians to have raised this question and (ii) in the details of his answer.

Conditional propositions are, in Avicenna's taxonomy, one kind of hypothetical proposition (the other kind being disjunctives). A distinctive feature of his analysis of hypotheticals is that these propositions, just like categoricals, may have quality and quantity. In the case of conditionals, the relation between antecedent and consequent is embedded within the scope of a quantifier that may express a universal affirmative (a-C), a universal negative (e-C), a particular affirmative (i-C), or a particular negative (o-C) conditional proposition.[28] General forms of quantified conditionals (with quantified antecedents and consequents) are the following:

28. For a short introduction to Avicenna's account of conditionals in the context of his logic, see Strobino (2018).

(a-C)aa	"Always, if every A is B, then every C is D"
(e-C)ia	"Never, if some A is B, then every C is D"
(i-C)oe	"Sometimes, if not every A is B, then no C is D"
(o-C)ee	"Not always, if no A is B, then no C is D"

A universal affirmative conditional statement, (a-C), asserts that the consequent of the conditional is either true along with or inseparable from the antecedent at all times and under all circumstances. A particular affirmative conditional, (i-C), asserts that the consequent is either true along with or inseparable from the antecedent at some time and under some circumstances. A universal (particular) negative conditional statement denies that the consequent of the conditional is true along with or inseparable from the antecedent at all (some) times and under all (some) circumstances.

Avicenna's account of conditionals draws a further distinction between two types of following, or rather between a relation that may be legitimately characterized as genuine following, in one case, and a relation of (mere) coincidence or agreement in truth, in the other case. The former is typically associated with a distinctive syntactic marker, as in "Always, if *p*, then it necessarily follows that *q*." The two relations express the idea of following (*ittibāʿ*) or of being-the-case-together (*maʿiyya*) of an antecedent and a consequent, respectively. The first type of conditional is called an implicative (*luzūmī*) conditional, while the second type is called a coincidental (*ittifāqī*) conditional. With implicative conditionals there must always be an underlying semantic or metaphysical connection of some kind between antecedent and consequent. Coincidental conditionals, by contrast, express the coincidence of the actual truth of the consequent with the actual or supposed truth of the antecedent. Examples of true coincidental conditionals are

(1) "Always, if every human is rational, then every donkey brays," and
(2) "Always, if every human is rational, then void does not exist,"

whose respective antecedents and consequents are true propositions that are entirely irrelevant to each other. The following conditional is, by contrast, a true coincidental whose antecedent is (necessarily) false and whose consequent is (necessarily) true:

(3) "Always, if every donkey is rational, then every human is rational."

Examples of implicative conditionals are

(4) "Always, if every human is an animal, then every human is sensitive,"

where both antecedent and consequent are (necessarily) true and the consequent is genuinely implied by the antecedent (because the predicate of the consequent is contained in the predicate of the antecedent, and both are predicated of the same subject); and

(5) "Always, if 4 is not divided into two equals, then 4 is odd,"

where both antecedent and consequent are (necessarily) false, but the consequent nonetheless necessarily follows from the antecedent in virtue of its conceptual content (the predicate of the antecedent is the definition of the predicate of the consequent, and both are predicated of the same subject), which makes the conditional true in the sense of implication (*luzūm*) but false as a coincidental.[29]

Implicative Conditionals and Inseparability

Avicenna explicitly characterizes the notion of following involved in implicative conditionals in terms of inseparability. The concept is broad enough to cover a variety of relations that may hold between an antecedent and a consequent when the former necessarily implies the latter. These relations include (a) the dependence (*ʿalāqa*) of a categorical proposition on another categorical proposition; (b) an explicit relation involving a real connection (*iḍāfa ḥaqīqiyya*) between the terms; or (c) a relation of implication involving a connection such that the antecedent stands to the consequent (ca) as a cause stands to its effect (where cause is taken in the sense of a sufficient rather than a necessary condition), (cb) as a whole stands to its parts, (cc) as a universal stands to it particulars, or (cd) similar relations involving a necessary connection (*iḍāfa lāzima lahū*) between two terms neither of which is a constituent of the other. Discussing in detail what Avicenna means by each of these alternatives lies beyond the scope of this chapter. What is immediately clear is that some of them, for example (cb) and (cc), express relations of containment or correspondence involving constituents, while others, for example (cd), express relations of necessary implication that do not involve constituents.[30]

Truth Conditions

Both coincidental and implicative conditionals are false if the antecedent is true and the consequent is false.

Implicative conditionals are true if and only if there exists a necessary connection between antecedent and consequent such that the consequent is inseparable from the antecedent, regardless of whether (i) the antecedent and the

29. In *Qiyās* V and VI, Avicenna discusses countless examples for each type, displaying more complex combinations involving negation and modality.

30. See, in particular, *Qiyās* V, 1, p. 237.3–9; cf. also *Qiyās* V, 1, pp. 233.17–234.9.

consequent are both true, (ii) the antecedent is false and the consequent is true, or (iii) the antecedent and consequent are both false.

Coincidental conditionals, by contrast, are true if and only if the consequent is true, regardless of whether both antecedent and consequent are true or the antecedent is false and the consequent is true. A critical point is that coincidental conditionals, according to Avicenna, are not true when both the antecedent and the consequent are false (which, among other things, immediately severs the link with the notion of material implication). Moreover, if the antecedent and consequent of a true implicative conditional are both true, then the implicative conditional is also (trivially) true as a coincidental conditional (because its consequent is true).

Avicenna draws a further distinction between implicative conditionals whose antecedents and consequents are both true, and implicative conditionals whose antecedents and consequents are both false (including the specific case in which they are both impossible). The former are called "true in fact" (*fī nafs al-amr*), while the latter are called "true by implication only" (*bi-l-ilzām*). This distinction is central for Avicenna's understanding of reductio.

Impossible Antecedents and Impossible Consequents

The general distinction between (i) implicative conditionals and coincidental conditionals, and the specific distinction between (ii) implicative conditionals that are true in fact and implicative conditionals that are true by implication only are essentially presupposed by Avicenna's account of reductio proofs. The structure of a reductio proof, as noted earlier, involves for Avicenna an explicit conditional step. And since the conditional step, in every reductio proof, is encapsulated by a conditional proposition with a false (or impossible) antecedent—the assumption for reductio—and a false (or impossible) consequent—what the assumption for reductio necessarily implies—the only kind of conditional that can adequately capture such a relation of following is, by definition, an implicative conditional that is true by implication only. This is because no coincidental conditional is true if its consequent is false, and no implicative conditional that is true in fact can have a false antecedent and a false consequent.

The truth of an implicative conditional holding by implication only is dependent on an underlying connection between the terms of its antecedent and consequent. Falsehoods or impossibilities that entail other falsehoods or impossibilities can only be accommodated by this kind of conditional. Since a reductio proof is in turn an argument that establishes the (necessary) truth of p, based on the conditional relation between two impossibilities, namely the impossible antecedent $\neg p$ (the assumption for reductio) and an impossible consequent q that necessarily follows from $\neg p$, a reductio proof can only be based on an implicative conditional holding by implication only, namely "if ¬*p*, then *q*." The contrapositive conditional, namely "if ¬*q*, then ¬¬*p*" (from which we ultimately derive *p* itself), is also an implicative conditional, but in this case, its antecedent ¬*q* and its consequent ¬¬*p* (or, equivalently, *p*) are both necessary. This implicative conditional is therefore true in fact, not by implication only.

In the context of a scientific theory, it is essential to be able to draw genuine (impossible) conclusions from impossible antecedents, as this may sometimes be the most efficient, if not the only, way to prove a theorem. Avicenna seems to have identified a relevant characteristic of the logical form of reductio arguments that enable us to move inferentially from an impossibility to another impossibility, in a nontrivial sense, and to have consciously associated the kind of conditional proposition involved in reductio proofs with a peculiar relation of necessary implication.

But there is an even deeper connection between Avicenna's account of reductio and his logic of essence, which emerges in his analysis of quantified conditionals. Before we get to it, however, we must look at the logical structure of reductio itself to appreciate the sophistication of Avicenna's analysis.

The Structure of Reductio

A reductio proof is a compound deduction (*qiyās murakkab*)—that is to say, a concatenation of (two) deductions. Every reductio involves a connective (hypothetical) deduction and a repetitive deduction (an instance of *modus tollens*). Both categorical and hypothetical propositions may be proved by reductio. An example of a reductio proof of the proposition "Every A is B" (AaB), inferred directly by the categorical deduction (*) CaB; AaC ⊢ AaB, is as follows:

1. CaB; Always, if ¬AaB, then AoB ⊢ Always, if ¬ AaB, then AoC
2. Always, if ¬AaB, then AoC; AaC ⊢ AaB

The first deduction is what Avicenna calls a "connective" (*iqtirānī*) deduction, with a conditional minor premise and a categorical major premise:

i. CaB	(major premise of the initial deduction (*) = major premise of 1)
ii. Always, if ¬ AaB, then AoB	(minor premise of 1, square of opposition)
iii. Always, if ¬AaB, then AoC	(from i and ii, by Hyp. Baroco Cat-C-C, *Qiyās* VI, 4, p. 328.5–9)

The second deduction is what Avicenna calls a "repetitive" (*istiṯnāʾī*) deduction. The latter takes the conditional conclusion of the first deduction as its major premise and the minor premise of the direct deduction (*), whose conclusion we are aiming indirectly to prove, as its minor premise:

iv. Always, if ¬AaB, then AoC	(iii)
v. AaC	(minor premise of the initial deduction (*) = minor premise of 2)
vi. AaB	(from iv and v, by *modus tollens*) QED

The relevant relation between falsehoods (impossibilities) expressed by an implicative conditional that holds by implication only, that is to say, the key conditional step in the proof, is encapsulated by the conditional proposition (iii) = (iv). In this conditional, the false (impossible) consequent AoC necessarily follows from the false (impossible) antecedent ¬AaB by way of implication only (their falsehood or impossibility is determined in conjunction with AaC and CaB).

To illustrate the role of an implicative conditional holding by implication only within the context of a reductio proof, let us consider an example involving the terms "plant," "stone," and "living being." The following direct deduction in the third figure is an instance of the categorical mood Felapton:

(Major premise)	No plant is a stone.
(Minor premise)	Every plant is a living being.
(Conclusion)	Not every living being is a stone.

A standard indirect proof would assume the two premises of the direct proof and then rely on a subsidiary proof based on the assumption for reductio (the contradictory of the conclusion of the direct proof) to derive an impossible conclusion:

1. No plant is a stone.	(Major premise)
2. Every plant is a living being.	(Minor premise)
3. Every living being is a stone.	(Assumption for reductio)
4. Every plant is a living being.	(Iterated from 2)
5. Every plant is a stone.	(3, 4, Barbara)
6. Not every living being is a stone.	(reductio: 1, 3–5)

In Avicenna's formulation, this argument would be analyzed in terms of two deductions. The first deduction is a connective deduction with a conditional minor premise expressing the assumption for reductio (as its consequent) and one of the premises of the direct deduction as its major premise. The conclusion of this connective deduction is the implicative conditional holding by implication only, which involves an impossible antecedent and an impossible consequent. The antecedent of this inferred conditional is the assumption for reductio; its consequent is the impossibility that follows necessarily from the assumption for reductio. The second deduction is the instance of *modus tollens*, which takes the conditional conclusion of the first deduction as its major premise and the other premise of the direct deduction as its minor premise. The conclusion of this second deduction is the conclusion of the direct deduction. The same example can now be expressed in the following (Avicennan) terms:

(a) Every plant is a living being; Always, if not (not every living being is a stone), then every living being is a stone ⊢ Always, if not (not every living being is a stone), then every plant is a stone.
(b) Always, if not (not every living being is a stone), then every plant is a stone; no plant is a stone ⊢ Not every living being is a stone.

The conditional statement that serves in this example as the conclusion of (a) and the major premise of (b) tells us something about the relation between an impossible antecedent, namely that every living being is a stone (via double negation), and an impossible consequent that necessarily follows from it, all other things being equal, namely that every plant is a stone. The connection between these two propositions exemplifies one of the relations mentioned earlier among various candidates that satisfy the requirements of implicative conditionals. This is because the subjects of antecedent and consequent stand in a whole-part relation: extensionally, plant falls under living being, and conceptually, living being is inseparable from plant. As a result, everything that is universally true of living being—including stone, by virtue of the assumption for reductio—would be true of plant too.

Direct and Indirect Proof, Monotonicity, and Reasoning from an Impossibility

Avicenna's understanding of the relevance of his account of conditionals and of reductio proof for his theory of science emerges from certain details of his discussion of the meaning of quantification. What does it mean to say that a true universal affirmative conditional must be true at all times and under all circumstances (or in all states, as Avicenna frequently puts it)? The temporal condition is self-explanatory, but what about the quantification over circumstances or states? Avicenna's answer raises a series of questions that from a modern perspective could fall under the broad heading of the distinction between monotonic and non-monotonic reasoning.[31] In particular, a true universal affirmative conditional must remain true no matter what additional conditions may be assumed along with its antecedent, that is to say, no matter how its antecedent is strengthened. He writes:

31. Monotonicity is a property of a consequence relation in virtue of which if *p* follows or is derivable from a set of assumptions Γ, then *p* still follows or is derivable from Γ in conjunction with one or more (arbitrary) additional assumptions. In other words, if *p* follows from Γ, then *p* still follows from Γ no matter how Γ is strengthened.

Text 8.3: *Qiyās* V, 4, pp. 262.11–263.2 (Shehaby 1973, p. 61, transl. modified)
If the connection (*ittiṣāl*) is asserted under any condition and circumstance (*ʿalā kull ištirāṭ wa-waḍʿ*) [at which the antecedent] is posited, then the hypothetical conditional proposition is universal.

The otherwise potentially cryptic meaning of Text 8.3 is spelled out a little more clearly in Text 8.4:

Text 8.4: *Qiyās* V, 4, p. 265.1–5 (Shehaby 1973, p. 63, transl. modified)
In the statement "Always, if C is B, then H is Z" the expression "always, if" is not just meant to generalize what is intended [by the statement] (*taʿmīm al-murād fa-qaṭ*) in such a way that it is as if one said "Every time at which C is B, H is Z." Rather, [the expression "always, if"] is meant to include in general every state (*ḥāl*) which may be added to the statement "Every C is B," in such a way that there is no state (*ḥāl*) or condition (*šarṭ*) whatsoever that makes C to be B without making H to be Z, when connected with [the antecedent].

We gather even more explicit information from yet another text that goes along the same lines and offers the most elaborate account in Avicenna's *Qiyās* of what it means for a conditional to be universal:

Text 8.5: *Qiyās* V, 4, pp. 272.13–273.5 (Shehaby 1973, pp. 69–70, transl. modified)
We say that the hypothetical [conditional] universal proposition is universal only if the consequent follows in every [case] the antecedent is posited (*kull waḍʿ al-muqaddam*) not just with respect to what is intended [by it] (*fī l-murād fa-qaṭ*), but with respect to the states [that may be added to it]. And which states are these? They are the states that are necessarily implied by the assumption of the antecedent or [the states] that may be assumed [as antecedents] for it, [as well as those that may] follow from it or be [true] together with it. [This is] either [(i)] in virtue of [terms that are] predicated of the subject of the antecedent, if [the latter] is categorical, or [(ii)] in virtue of other premises connected with [the antecedent], if [the latter] is not categorical—I mean the premises that may be true when [the antecedent] is true, and *which do not give rise to an impossibility together with* [the antecedent], even if [the latter] is impossible in itself—or [(iii)] in virtue of the concession of something that makes [the antecedent] necessary or possible, even if it is in itself impossible. In all these cases, the antecedent may not only be in itself true (*ḥaqq*): it may in fact also be false (*bāṭil*), being assumed [merely] hypothetically (*ʿalā sabīl al-waḍʿ*). For even the latter will have implicates (*lawāzim*) and accidents (*ʿawāriḍ*); or one may assume that, if (*law*) it were to exist, [something] would belong to it or necessarily follow [from it].

Concerning universal affirmative conditionals, one may raise the question of whether it is legitimate to add arbitrary assumptions to the antecedent in such a way that the consequent would cease to follow. In Texts 8.4 and 8.5, Avicenna suggests that one of the functions of universal quantification (in the case of affirmative conditional statements) is precisely to rule out this possibility. This is true, in Avicenna's view, regardless of whether the antecedent of a universally quantified affirmative conditional statement is categorical or not. The main difference is that in the former (and more fundamental) case, the "states" over which the quantifier ranges are attributes (that is to say, things that may be predicated of the subject of the categorical antecedent), while in the latter case they are propositions (that is to say, things that may be held to be true together with the propositional antecedent). The tacit assumption seems to be that the "states" must be the of the same kind as the items into which the antecedent of the conditional can first be analyzed, that is to say, terms if the antecedent is categorical, and propositions if the antecedent is hypothetical (because the elements into which a hypothetical proposition can first be analyzed are, by definition, propositions and not terms).

Essence and Focal Meaning

The case of universal affirmative conditionals is analyzed in detail at *Qiyās* V, 4.[32] Avicenna is concerned with two problems. The first is to determine whether or not there can be, in principle, any true universal affirmative implicative conditional. The second is to determine whether or not there can be, in principle, any true universal affirmative implicative conditional that holds by implication only. The two problems arise because one might argue that under certain circumstances, that is to say, if suitable assumptions are added to the antecedent of a conditional, the consequent may seem no longer to follow from it. For example, the truth of the universal conditional

(1) Always, if this is a human, then this is an animal

would be contradicted by the truth of the following conditional:

(2) If this is a human *and* has no sensation or movement, then this is not an animal.

Avicenna thinks that if the antecedent of (1), namely "This is a human," is strengthened by adding an inconsistent assumption to it, as in (2), then its consequent no longer follows always and under all circumstances. This is because, under the assumption that something is human but has no sensation or movement, the contradictory of the original consequent, namely that this is not an animal, would

32. In the case of universal negative implicative conditionals (whose intended meaning is to deny that a consequent follows from its antecedent), the question is whether it is legitimate to add arbitrary assumptions to the antecedent in such a way as to make the consequent follow. Avicenna's treatment of the negative case, at *Qiyās* V, 5, pp. 279.1–283.9, is parallel (in terms of argument, objections, replies, and examples) to his analysis of the affirmative case in *Qiyās* V, 4.

follow from the strengthened antecedent. A similar example is the universal affirmative conditional

(3) Always, if this is a pair, then this is not odd,

which Avicenna takes to be falsified by the following counterexample, in which an inconsistent assumption is added to the antecedent:

(4) If this is a pair *and* is not divided into two equals, then this is odd.

A putative strategy of response might consist in the rejection of both counterexamples, based on the idea that they involve impossible antecedents. But, according to Avicenna, there would be something intrinsically wrong in rejecting conditionals with impossible antecedents (and consequents). First, the truth of an implicative conditional does not depend on the truth of its parts but on the relation between its parts. Second, most reductio proofs are essentially dependent on the truth of a universal conditional with an impossible antecedent and an impossible consequent. He writes:

> Text 8.6: *Qiyās* V, 4, p. 273.12–17 (Shehaby 1973, pp. 70–71, transl. modified)
> We cannot say [of an implicative conditional proposition] that it is false because the antecedent is impossible. For the truth of a hypothetical [implicative conditional proposition] is not the truth of its antecedent or [the truth of] its consequent but rather [depends on] the state of the implication (*luzūm*). Most hypothetical [conditional propositions] used in the sciences, where reductio proof is employed, have this characteristic, for their antecedents are impossible. [But] they are not said to be false due to the impossibility of their antecedents and consequents. Similarly, if one were to say "If (*law*) this were a pair and were not divisible into two equals, then it would be odd" this would be true even if the antecedent is impossible.

Text 8.6 clarifies that the truth or falsehood of an implicative conditional is determined solely by the *relation* between its two parts ("the state of the implication"), not by the truth or falsehood of the parts themselves. More importantly for our purposes, Avicenna explicitly recognizes reductio proofs as part of the repertoire of deductive tools used in scientific discourse and notes in turn that these inextricably depend on certain implicative conditionals, that is to say, the only kind of conditionals that can accommodate impossible antecedents and consequents.

As previously noted, a distinctive feature of Avicenna's account of hypotheticals is quantification, where the quantifiers range over times and circumstances or states. In the present context, Avicenna gives us what is probably the clearest illustration of what he means by those abstract requirements:

> Text 8.7: *Qiyās* V, 4, pp. 273.17–274.6 (Shehaby 1973, p. 71, transl. modified)
> Therefore, there are states (*aḥwāl*) that are not impossible in [the realm of] assumption (*fī l-farḍ*), even though they are impossible in [the realm of] existence (*fī*

l-wuǧūd), relative to which, if the antecedent is assumed, the consequent does not follow from it.

For example, it is not the case that always, if this is assumed to be a pair, then it necessarily follows that it is even. Rather, [this is the case only] if nothing that contradicts [the antecedent] is assumed with it. If something that contradicts [the antecedent] is assumed with it, then that [consequent] will be contradicted (even if [the antecedent] is impossible in [the realm of] existence, as its being impossible in [the realm of] existence does not prevent it from being something that can be assumed). Thus, it is not the case that whenever something is assumed to be a pair, then it follows from this that it is even. Rather, there are impossible assumptions (*furūḍ muḥāla*) that prevent it.

In Text 8.7, Avicenna indicates explicitly that the states or conditions that seem to pose an inherent threat to the truth of any universal affirmative conditional must be identified with inconsistent conditions that could putatively be added to the antecedent. In other words, if all additional conditions or assumptions incompatible with the antecedent could be excluded, this would seem enough to preserve the possibility of true universal affirmative conditionals. And this is where the distinction between implicative conditionals that are true in fact and implicative conditionals that are true by implication only comes into play. For Avicenna notes that the safeguard against the putative counterexamples discussed earlier is already built into implicative conditionals that are true in fact. This is because if one were to add an inconsistent assumption to the antecedent of an implicative conditional that is true in fact, the new antecedent would ipso facto be false, and the new conditional would no longer count as a conditional that is true in fact. Conditionals that are true in fact are simply immune to counterexamples involving impossible antecedents. But this is not so in the case of implicative conditionals that are true by implication only:

Text 8.8: *Qiyās* V, 4, p. 274.9–11 (Shehaby 1973, p. 71, transl. modified)
This [objection] is true if [the implicative conditional is true] by implication only (*bi-ḥasab al-ilzām*), but it is not true [if the implicative conditional is true] in fact (*fī nafs al-amr*) and the universal [conditionals that are true] in fact are not demolished by this [objection]. This only demolishes the universal [conditional] in the sense of [what is true by] implication only.

Thus, it seems that the distinction between implicative conditionals that are true in fact (those whose antecedents and consequents, in addition to being governed by a relation of necessary implication, are also true) and implicative conditionals that are true by implication only (those whose antecedents and consequents are only governed by a relation of necessary implication, even though they are both false) provides a satisfactory solution to the first problem. For the objection that no implicative conditional could be universally true, if inconsistent assumptions could be freely added to its antecedent, simply does not apply to the first category.

What remains to be discussed is the second problem, namely whether or not there can be, in principle, any true universal affirmative conditionals that hold *by implication only*. The same line of argument and battery of counterexamples involving impossible antecedents may now be directed against this particular type of conditional. And in this case the definitional safeguard that insulated implicative conditionals that are true in fact from the objection is no longer available, because the mark of implicative conditionals that are true by implication only is to have false or impossible antecedents and consequents. The addition of an inconsistent assumption to the antecedent is not ruled out in principle by the nature of the conditional itself and by its truth conditions. But then it is legitimate to ask whether there can really be any true universal conditionals of this kind at all. Avicenna writes:

> Text 8.9: *Qiyās* V, 4, pp. 274.11–275.3 (Shehaby 1973, pp. 71–72, transl. modified)
> Next, one may say that we will therefore never find a [true] universal affirmative [conditional] proposition that holds by implication only.
>
> We say that we do. That is when we add to the antecedent (in its meaning) the requirement that we ought to discard any condition that would force [the antecedent] necessarily to imply a consequent that is not already necessarily implied [by that antecedent] in itself, as when you say "Always, if this pair, in the sense in which [a pair] can be, then it is even" or "Always, if this is a void—in the sense in which the void is assumed to exist by the existence according to which it is assumed (or to be an inevitable consequence of the existence according to which it is assumed, or to be something whose assumption is inevitable, if it is possible), with no condition that contradicts the concept of void—then it is a dimension."

In Text 8.9, Avicenna rejects the claim that there cannot be true universal affirmative implicative conditionals that hold by implication only and provides a criterion to make up for their apparent vulnerability. In this case, the safeguard that is not built into their definition must be introduced by stipulation. Avicenna's strategy of response imposes a restriction on the domain of quantification. In particular, the quantifier "every" (as in "every state or condition") of a *true* universal affirmative implicative conditional that holds by implication only cannot range over any state or condition without qualification, but only over those states and conditions that are *compatible* with the antecedent. Avicenna's point is that whenever we evaluate conditionals with impossible antecedents, for example one that turns on a metaphysically empty term like "void" (no pun intended) as in Text 8.9, we must keep within the conceptual perimeter of the notions involved and use their definitions as a criterion to rule out incompatible assumptions. The qualifications (i) "in the sense in which *x* can be," (ii) "in the sense in which *x* is assumed to exist" (or similar clauses), and above all (iii) "with no condition that contradicts the concept of *x*," when attached to an antecedent that contains *x*, play exactly this role and crystallize the focal meaning of the relevant term. Once that independent variable is fixed, then it is *with respect to it* (and to nothing other than it) that the compatibility or incompatibility of additional assumptions must be assessed. The domain of quantification will be defined accordingly, and so will be the totality of circumstances or states relative to which the conditional is true.

Direct and Indirect Proof

Avicenna's argument in support of the view that universal affirmative implicative conditionals that hold by implication only can be true involves the requirement that any such conditional be (implicitly or explicitly) accompanied by a clause that rules out by stipulation any additional assumption incompatible with its antecedent. In other words, conditionals of this kind are universally true only with an additional stipulation (*ziyāda*) setting precise semantic boundaries relative to the antecedent, that is to say, a clause of the form "if *x* is P in the sense in which something genuinely is P" or "if *x* is P *qua* P" (and not, for instance P and R, where R is incompatible with P). By filtering potential inconsistencies generated by incompatible conditions added to the antecedent, this criterion enables Avicenna to isolate genuine relations of necessary implication determined exclusively by the meanings and essences of the terms involved. The standard on the basis of which all inconsistent assumptions are to be assessed and excluded is the essence of something. It is the set of constituents of a concept that defines what is compatible or incompatible with it. And it is especially in the case of reductio proofs involving non-existents such as atom or void that we can only reason conditionally from an impossibility if we have an independent criterion to establish what is compatible with certain assumptions (for in this case it cannot be their actual truth). Hence, definition (even in the sense of nominal definition, which is the only kind of definition to which pseudo-entities like atom and void can aspire) and essence are again central concepts that play a critical role in Avicenna's account of what it means to reason from an impossibility.

The role of reductio proofs is emphasized specifically with regard to scientific reasoning, when Avicenna tells us that implicative conditionals that are true by implication only, that is to say, the *only* kind of conditional that enables reductio to function in the first place, are used in the sciences exclusively in the context of indirect proofs, and that the default kind of conditional used in direct proofs is the implicative conditional that is true in fact. This is not surprising, because all premises and conclusions of direct proofs in the sciences must be true (among other things), and the only falsehoods or impossibilities that have any citizenship in the sciences are the antecedents and consequents of true implicative conditionals holding by implication only, namely the engine of indirect proofs.[33] Avicenna writes:

> Text 8.10: *Qiyās* V, 4, p. 275.5–14 (Shehaby 1973, p. 72, transl. modified)
> This sort of confusion only arises insofar as the universal [implicative conditional] is taken in the sense of [an implicative conditional holding by] implication only (*bi-ḥasab al-ilzām*), not if it is taken in the sense of [an implicative conditional that is true in] fact (*bi-ḥasab al-amr*). Such universal [implicative conditionals] taken in the sense of [implicative conditionals holding by] implication only are [found] just in deductions where the argument leads to the impossible. Direct deductions do not need them.
>
> Thus, if you use these propositions when the matter is unclear to you, make it a condition for yourself that you will drop the contradicting conditions [that might be added to the antecedent], leaving it the way it ought to be. For if you use "Always, if this is a human, then this is an animal," you [should only] be aware of *it*, without assuming any impossible condition that contradicts the judgment of the antecedent and prevents it from being true in itself, whereupon you may concede [that it is] universal.
>
> [(i)] If the antecedent is true in existence (*ṣaḥīḥ al-wuğūd*), the [additional] considerations (*iʿtibārāt*) will be things and propositions that are true (*ṣaḥīḥa*) [with it].
>
> [(ii)] If [the antecedent] is impossible, the [additional] considerations will be things that [would] be true together with that impossibility and follow from it, not things that are incompatible with it—namely, with that impossibility—and indeed contradict it and remove it, whether they be true or false.[34]

The confusion mentioned in Text 8.10 concerns the distinction between the two types of implicative conditionals and the relative strength of the objections raised against them. If we keep them distinct, then the objections are easily solved: in one case the counterexamples simply do not apply, and in the other case the solution is to add a stipulation that restricts quantification to whatever is compatible with the essences of the terms involved in the antecedent.

Interestingly, Avicenna offers advice on what to do with conditionals about whose status we may be unsure, especially with regard to the question of whether

33. Such impossibilities could never be asserted in a science. What may be asserted in a science is something about the *relation* between such impossibilities, in the form of conditional statements that hold by implication only, in which those impossibilities are embedded as antecedents and consequents. Conditionals of this kind are perfectly legitimate truths of a science.

34. At *Qiyās* V, 5, p. 283.1–7, Avicenna characterizes conditionals with impossible antecedents as "the hypothetical [conditional] propositions used in reductio proofs," reiterating the point made at the opening of Text 8.1. In that context, Avicenna notes again that the impossibility of the antecedents of such conditionals (which are true by implication only) is by no means a reason to reject them as false. Examples are "If void existed, it would be a dimension" and "If two were not divisible into two equals, it would be odd," both of which are "true propositions even though their antecedents are impossible" (*wa-takūnu qaḍiyyatāni ṣādiqatayni wa-in kāna muqaddamuhumā muḥālan*).

they hold universally or not. In particular, (i) if the antecedent of the conditional is true in fact, then a safe rule is at best potentially to add only further truths to it ("things and propositions that are true along with it"). As long as no falsehoods are assumed together with the antecedent, the truth of a universal affirmative conditional cannot be compromised. By contrast, (ii) if the antecedent is impossible, then the indication is that nothing incompatible should be assumed together with it (regardless of whether the additional assumption is itself true or false). Truth is in and of itself a sufficient condition for compatibility in the first case, but not in the second. With impossible antecedents, there is no rejecting an additional assumption simply because it would make the antecedent false, for the antecedent was false all along. In such cases, one must look instead at the relation between any putative additional assumption and the antecedent of the conditional under consideration and determine whether any inconsistency would arise. That relation is, once again, intrinsically dependent on the essences of the terms involved.

In conclusion, even some admittedly technical aspects of Avicenna's account of conditionals (their quantification, taxonomy, and role in the economy of reductio proofs) seem to be inspired by, or at least to be relevant for, the question of the applicability of formal logic to scientific reasoning. And if the question of non-monotonicity raised previously is, as I suspect, also inseparable from Avicenna's account of the sort of implicative conditionals required by reductio proofs, then a notable property of Avicenna's formal logic would be determined at least partly (but no less significantly) by one of its intended uses, that is to say, as a logic of scientific discourse built on essentialist foundations.

PART IV

Causality and Explanation

Avicenna's account of the nature of scientific knowledge, just as we have seen in detail with regard to its modal components, involves an equally extensive use and sophisticated understanding of the notions of causality and explanation. In this area, however, what is striking about his approach are not so much the individual innovations as the systematic character of the analysis and the comprehensive attempt to import the Aristotelian theory of the four causes into the theory of demonstration and definition. In Avicenna's necessitarian metaphysics, every existent is either necessary in itself—as the Necessary Existent—or possible in itself and necessary through another, that is to say, through a cause—as in the case of every other being, whose essence and existence are contingent upon formal, material, final, or efficient causes. As a result, the causal dimension is ubiquitous in Avicenna's thought, and his theory of science is profoundly influenced by this theoretical framework. Moreover, while modality and explanation might appear, at first sight, to play a perfectly symmetrical and complementary role in Avicenna's account of scientific knowledge, the former seems in fact to be dependent on the latter. This is because everything that has a cause is necessitated only by its cause, and scientific knowledge of everything that has a cause (and is therefore not an immediate principle either without qualification or with respect to a given science) can be known with certainty only by means of its cause. Knowledge of the cause(s) in virtue of which certain attributes belong to their subjects is a necessary condition for the epistemic stability of our beliefs relative to those nexuses.

From abstract causal relations in metaphysics to mathematical explanations, from the causal analysis of the physiological structure of animals and plants to the account of emotions, from the study of the motion of the heavens to sublunar physics, from the causes of winds, clouds, thunders, and rain, to those of earthquakes, eclipses, eruptions, and rainbows, nothing escapes the yoke of causal necessity in Avicenna's chain of being. Consequently, an adequate theory of science must be applicable, at least in principle, to every segment of reality and capable of articulating its internal ontological structure with its modal and causal qualifications. For Avicenna, this requires a refinement of various conceptual tools developed by Aristotle in the *Posterior Analytics* to ensure that they may concretely gain traction on the sciences.

Avicenna adopts the technical Aristotelian distinction between demonstration of the fact (that-demonstration) and demonstration of the reason why (why-demonstration) and deploys it in various sciences in dialogue with the commentary tradition on *Posterior Analytics* A13. Two corresponding kinds of explanation are defined with regard to the internal explanatory structure of a single science and with regard to the possibility of explanation across multiple sciences, with various subtypes of why-demonstration and that-demonstration being identified on the basis of the terms involved and their causal relations. The classification yields a ranking of different argument forms of decreasing strength, among which the most powerful kind of demonstration is the one that explains that and why an attribute belongs to a subject in virtue of the relation between the essence of the attribute and the essence of the subject (chapter 9).

The application of the distinction between why-demonstration and that-demonstration to the problem of explanation across multiple sciences is closely connected with Avicenna's account of subordination and the hierarchical division of the sciences. What is, for instance, the difference between the physical proof and the mathematical proof of the sphericity of the earth? In what sense are some explanations more fundamental than others? Why is it that most of the principles of the particular sciences have a conditional status? What is the logical form of proofs that encapsulate different kinds of explanations? Can there be multiple, epistemically equivalent demonstrations of one and the same conclusion, or must there always be a privileged causal demonstration to which all other demonstrations are subordinated? Is the transfer of demonstration from one science to another possible at all? And, if so, under what conditions? Avicenna's methodical approach to these questions and the level of detail that characterizes his answers to them shed light on the extent to which he is committed to the idea that an Aristotelian logic of scientific reasoning must be one that actually works in the practice of scientific discourse (chapter 10).

An adequate Aristotelian theory of science must also be able to account for the ways in which each of the four Aristotelian causes—formal, material, final, and efficient—may be absorbed in the inferential structure of a demonstration and in the formal structure of a definition. Avicenna focuses in particular on three tasks.

The first is the identification of various lists of causes or, rather, of the same list of four kinds of causes, illustrated by different examples and characterized by different levels of detail, including the distinction between essential and accidental, universal and particular, common and proper, actual and potential, and proximate and remote causes for each class. The second is an analysis of the relation between causes, demonstration, and definition. The third is a discussion of the relative extension of causes and effects and of the way in which it affects the level of generality of a demonstration. What kinds of causes are characteristic of the most explanatory type of demonstration? What constraints does the essence of an *explanandum* place on the sort of terms that are admissible in its demonstration? Are all four kinds of causes involved in the process of definition or only some of them? Avicenna's discussion of these themes is ultimately instrumental for a precise characterization of the notions of complete demonstration and of complete definition: a complete demonstration is one that proceeds from essential, universal, proper, actual, and proximate causes, while a complete definition is one that fully reflects the internal structure of an essence and assumes the relevant kinds of causes depending on the realm of being—metaphysical, mathematical, or physical—to which that essence pertains (chapter 11).

9

Causal and Noncausal Demonstration

THAT-DEMONSTRATION AND WHY-DEMONSTRATION

Cause (*ʿilla*, *sabab*) and explanation (*bayān*) are two central notions in Avicenna's theory of science.[1] The first context in which the causal dimension of scientific knowledge is explored in detail is the distinction between that-demonstration (*burhān al-anna*, *burhān anna*; less frequently, *burhān annī*, *burhān ʿalā anna*) and why-demonstration (*burhān al-limā*, *burhān limā*; less frequently, *burhān limmī*). This distinction is central for Avicenna's philosophical vocabulary and serves as an indispensable tool for the assessment of the epistemic character of different kinds of arguments used in the sciences.[2] Even though the two families of concepts may ultimately be traced back to the Aristotelian notions of knowledge

1. On causality and explanation in Aristotle, especially with regard to the *Posterior Analytics*, see Robin (1909–10), Evans (1958–59), Wilkins (1970), Brody (1972), Jope (1972), Kosman (1973), Hagdopoulos (1975b), McKirahan (1978), Sorabji (1980), and Patzig (1981); on the notion of explanation in ancient Greek thought more generally, see Hankinson (1998).

2. At *Burhān* II, 2, p. 134.3–13, Avicenna explicitly contends that it would be too restrictive to assume that the theory of demonstration should only be about why-demonstration (*limā*) to the exclusion of that-demonstration (*anna*). In that context, there is a brief but interesting digression on knowledge of the existence of the creator (*al-bāriʾ*). This is a paradigmatic case in which noncausal knowledge is the optimal state, since the existence of the creator has no cause (other than itself), and there can be no why-demonstration of it. The opposite view, that "there is only one species of demonstration," namely why-demonstration, is wrongly attributed to Avicenna by Galileo in his questions on the *Posterior Analytics*, on the basis of misleading observations made by Averroes. See Wallace (1992, D3.1, p. 173 and p. 210n6); cf. Averroes, *Quaesita* VI, f. 114vb; *Expositio magna* f. 129v and f. 209r (*Šarḥ al-Burhān* p. 275.4–8 and p. 348.18–20, on *An. Post.* A6 and A13, respectively).

of the fact (*to hoti*) and knowledge of the reason why (*to dioti*, *dia ti*), in Avicenna the distinction is more fine grained and comes with a number of internal subdivisions that may at best be partly—and, if so, only implicitly—found in Aristotle.

Avicenna provides a first taxonomy of causal and noncausal demonstrations in *Burhān* I, 7, where he develops his own systematic treatment of the topic. He then resumes the thread of the discussion in *Burhān* III, 3, while commenting on *An. Post.* A13, and offers a more textually oriented analysis (as noted previously, brief considerations are also in *An. Post.* A9, but I return in chapter 10 to its considerably more extensive counterpart, namely *Burhān* II, 9). In *Burhān* I, 7, Avicenna writes:

> Text 9.1: *Burhān* I, 7, p. 79.13–16
> If a deduction gives the assertion that such-and-such is such-and-such, without giving the cause [of the fact that] such-and-such is such-and-such in [the realm of] existence in the same way in which it gives the cause of the assertion, then it is a that-demonstration. If it gives at the same time the cause of the two things—in such a way that, just as the middle term in [the deduction] is a cause of the assertion that the major belongs to the minor or is denied of it *in the proof*, so it is a cause of the fact that the major belongs to the minor or is denied of it *in existence itself*—then this demonstration is called a why-demonstration.[3]

In Text 9.1, Avicenna characterizes a that-demonstration as an argument that merely proves the predicative nexus between the two terms of its conclusion. A that-demonstration does not explain the underlying nexus between subject and attribute but just provides an inferential justification for an assertion, that is to say, "the cause of the coming together of the two extremes of the conclusion in the mind" (*Nağāt* I, 113, p. 126.6).[4]

A why-demonstration, by contrast, proves the predicative nexus between the two terms of its conclusion *and* explains the underlying nexus between subject and attribute. In this respect, in addition to providing an inferential justification for an assertion, a why-demonstration supplies a (or *the*) cause in virtue of which that nexus obtains in reality: it does not give merely the cause of the coming together of the extremes of the conclusion in the mind but also "the cause of the

3. At *Burhān* I, 5, p. 71.1–5, Avicenna frames the distinction in terms of (i) the cause of something in itself (*ʿillat al-amr fī nafsihī*) and (ii) the cause of the assertion of something (*ʿillat at-taṣdīq bihī*). Alternative formulations of the problem include the notion of an "investigation of the why" (*baḥṯ al-limā*) and, in connection with the classification of scientific inquiries, the distinction between knowing the *haliyya* of something (knowing that it is the case) and knowing its *limmiyya* (knowing why it is the case). For the distinction between *anniyya* and *limmiyya*, see also *Burhān* II, 6, p. 158.9–10. On the distinction between why-demonstration and that-demonstration, see also *Išārāt* IX, 5, pp. 84.8–85.9.

4. In this case, the benefit consists in knowing that an assertion necessarily follows from certain premises, regardless of any further modal characterization of the underlying nexus.

coming together of the two extremes of the conclusion in existence" (*Nağāt* I, 113, p. 126.8).[5] In Text 9.1, Avicenna explicitly holds the distinction to apply to both affirmative and negative scientific propositions.

A why-demonstration, in most cases, is also trivially a that-demonstration. This is because by providing a factual justification of the conclusion, it also provides an inferential justification of it. Avicenna, however, contemplates exceptional cases in which one may be seeking merely to establish the cause in virtue of which a certain nexus obtains, whereas the fact that it obtains is already independently granted.[6]

Types of That-Demonstration and Why-Demonstration

Both (a) that-demonstrations and (b) why-demonstrations have their own internal division. That-demonstrations are of two types. The first type is a demonstration such that (aa) the middle term is neither a cause nor an effect of the major term and is called by Avicenna "absolute that-demonstration" (*burhān al-anna al-muṭlaq*). In this case, either (aaa) the major term and the middle term are *both* effects (or consequences) of an unstated cause or (aab) their nexus with the minor term is immediate and does not involve a cause.[7] The second type, by contrast, is a demonstration such that (ab) the middle term is not a cause of the major term but rather its effect. This demonstration is called "sign" (*dalīl*, which translates the Greek *sēmeion*).[8]

5. In the same context (*Nağāt* I, 113, p. 126.9–10), the distinction is also cast in terms of whether the middle term is (i) a cause of the assertion of the conclusion (*ʿilla li-taṣdīqika bi-n-natīğa*) or (ii) a cause of the existence of the conclusion (*ʿilla li-wuğūd an-natīğa*).

6. At *Burhān* I, 5, p. 71.7–13, Avicenna gives one of his favorite examples of something that may require a reminder (*tanbīh*), namely the fact that lodestones attract iron. This phenomenon is self-evident through perception (*bayyin bi-nafsihī bi-l-ḥiss*), and what is sought is its explanation only. Cf. also *Samāʿ ṭabīʿī* I, 5, p. 30.1–6 and *Ilāhiyyāt* III, 8, p. 141.8–14.

7. If the middle term and the major term are both immediate effects of the minor, Avicenna does not consider the nexus to be causal. This is because the two terms belong in themselves to the minor, as he shows at *Burhān* I, 8, p. 87.2–16, on which see Strobino (2016b). The case in which the middle and the major are both effects of an unstated cause is, by contrast, a genuine causal case (see, for example, *Burhān* I, 8, pp. 89.11–90.7, where Avicenna discusses the case of correlative terms). This kind of argument, however, is not a *dalīl*, as the cause is not part of the proof (a *dalīl* is a that-demonstration in which the cause is part of the proof, but it appears in the wrong explanatory order, for example as major term rather than middle term in a first-figure deduction). The notion of convertible and mutually inseparable effects of one and the same cause is a frequent theme in Avicenna, from metaphysics (*Ilāhiyyāt* II, 4, pp. 81.3–83.3) to logic (*Qiyās* V, 1, p. 234.2–4), where it is listed as an instance of the relation (*ʿalāqa*) of following expressed by necessary (implicative) conditionals. On the possibility of two attributes being immediate implicates of the same thing (and non-immediate implicates of another thing of which the first is in turn an immediate implicate), see also *Taʿlīqāt* p. 180.22–26.

8. At *Ilāhiyyāt* I, 1, p. 6.11–12, the proof of God's existence is characterized as a *dalīl*, that is to say, as a kind of that-demonstration. The reason is that there is no cause for God's existence; see Bertolacci (2006, p. 144n23). In addition to the fact that God's existence does not have a cause, according to

Why-demonstrations are also of two types. The first type is a demonstration such that (ba) the middle term is (i) a cause of the existence of the major term without qualification and (ii) a cause of the fact that the major term belongs to the minor term.[9] The second type is a demonstration such that (bb) the middle term is a cause of the fact that the major term belongs to the minor term but not of the existence of the major term without qualification. Avicenna uses existence (*wuǧūd*) both as a one-place and as a two-place attribute, that is to say, in the sense of existence without qualification and in the sense of attribute ascription (*wuǧūd li* or *wuǧūd fī*). In summary, Avicenna contemplates the following types of demonstrations in *Burhān* I, 7:

- (a) That-demonstration (*burhān al-anna*)
  - (aa) Middle term neither cause nor effect of the major term: absolute that-demonstration (*burhān al-anna al-muṭlaq* or ʿ*alā l-iṭlāq*)
    - (aaa) Middle term and major term effects of an unstated cause that connects them with the minor term (non-immediate, causal nexus)
    - (aab) Middle term and major term immediate effects of the minor term (immediate, noncausal nexus)
  - (ab) Middle term effect of the major term: sign (*dalīl*)
- (b) Why demonstration (*burhān al-limā*)
  - (ba) Middle term cause of the existence of the major term and of its belonging to the minor term (*burhān al-muṭlaq*)
  - (bb) Middle term not a cause of the existence of the major term but only of its belonging to the minor term.[10]

The distinction between the two types of that-demonstration (in fact three, if we consider the internal subdivision of the first type) cannot be traced to Aristotle, nor does it appear in the extant Greek commentary tradition on the *Posterior Analytics*.

Avicenna, this proof of existence is a *dalīl*, in the terminology of the *Burhān*, because it is bound to proceed from effect to cause, which is the distinctive mark of this particular type of that-demonstration.

9. A useful illustration is at *Burhān* IV, 4, p. 288.12, where, in the context of a discussion of definition, the three sides of a triangle are said to be the causes of triangle. Thus, having three sides is at the same time a cause of the fact that triangle is what it is and of its belonging to anything that has that attribute.

10. Avicenna often illustrates the case of a middle term that is the cause of the fact that the major belongs to the minor but not of the existence of the major without qualification through the species-genus relation. In this case, either an intermediate genus is used as a middle term to prove why a higher genus belongs to a species (for example, human, animal, body) or an infima species is used as a middle term to prove why a genus belongs to an individual (for example, Zayd, human, animal) (*Burhān* I, 7, p. 81.2–5). In the former case, the intermediate genus is the reason why the species falls under the higher genus; in the latter case, the species is the reason why the individual falls under the genus.

TABLE 13 Types of demonstration in *Burhān* I, 7

That-demonstration			Why-demonstration	
Absolute that-demonstration		Sign	Absolute demonstration Existence and belonging	Belonging
Middle term and major term (i) both effects or implicates of an unstated cause	Middle term and major term (ii) both effects of the minor term	Middle term effect of the major term	Middle term cause (i) of the existence of the major term and (ii) of its belonging to the minor term	Middle term cause (i) of the major term's belonging to the minor term

And it does not seem to be part of Alfarabi's interpretive machinery either. Avicenna may indeed have been the first to introduce it. By contrast, the genesis of the distinction between the two types of why-demonstration is more uncertain. It does not appear before Avicenna in these terms, but its basic conceptual ingredients might be implicitly present in Aristotle.[11] Avicenna, however, seems to be the first interpreter (at least in the Arabic-Islamic tradition, and possibly in the premodern commentary tradition of the *Posterior Analytics* as a whole) to have identified explicitly different types of causal demonstrations based on their distinct explanatory roles. Moreover, Avicenna's approach is not confined to a passive reception of the distinction implicit in Aristotle, but translates into a frequent, and consistent, association of the two types of why-demonstration with their respective contexts of application (especially in connection with *An. Post.* A8 and B16–17). Last, Avicenna develops the required conceptual vocabulary not merely to acknowledge the existence of different types of why-demonstrations but also to provide a clear philosophical account of their differences.[12] The taxonomy of that-demonstration and why-demonstration in *Burhān* I, 7 is illustrated in table 13.

11. The two types are identified by modern interpreters by means of different labels: type A and type B demonstration in Lennox (1987); subject-attribute demonstration and application arguments in McKirahan (1992); and, with some qualifications, Model 1 and Model 2 demonstrations in Bronstein (2016).

12. For Avicenna, two argument forms may qualify at best as that-demonstrations, in virtue of their own internal structure, which prevents them from being able to deliver causal explanations. The first is *reductio ad impossibile* (*qiyās al-ḫalf*), discussed in chapter 8. The second is circular proof (*bayān ad-dawr*). Avicenna remains unequivocally committed to the rejection of circular demonstration (as shown in chapter 1), but circular proof may have, under certain conditions, a heuristic value in scientific discourse (on the relation between circular proof and scientific reasoning, see *Qiyās* IX, 12, p. 507.2–5). In a nutshell, reductio is explanatorily inadequate because the falsehood of the contradictory of an assertion is never, in and of itself, prior to or better known than the truth of the assertion itself (it is not because *not-p* is false that *p* is true), while circular proof is explanatorily inadequate because priority and being better known, in particular, are nonreflexive and antisymmetric relations.

ABSOLUTE THAT-DEMONSTRATION

Examples of absolute that-demonstration are rare, but it is clear that the notion constitutes a genuine division for Avicenna and that it is not just introduced in passing in *Burhān* I, 7. This is confirmed by the fact that it is mentioned again in other contexts, for example in *Burhān* I, 8 and *Burhān* III, 3.[13] In *Burhān* I, 8, Avicenna makes a significant theoretical point about circumstances falling under the heading of absolute that-demonstration (of type (aab), to be precise). For in such cases, assertions of certainty may be attained without exhibiting a cause (trivially, because there is no cause to begin with).[14] In *Burhān* III, 3, the two types of that-demonstration introduced in I, 7 are reconsidered in light of the criteria presented by Aristotle in *An. Post.* A13. In that context, absolute that-demonstration is *not* discussed under its name but seems nonetheless at play (in its (aaa) variant) as one of the two main divisions of that-demonstration in which the middle and major terms are convertible, the other division being the so-called sign (*dalīl*).

KINDS OF EXPLANATION

The two types of why-demonstration both provide at the same time (i) the inferential justification of an assertion and (ii) the real causal justification for the underlying fact expressed by that assertion (where it is worth bearing in mind that fact is understood in the broad sense elucidated at the opening of chapter 6). In other words, the premises and conclusions of why-demonstrations encapsulate real explanatory nexuses, and their terms are arranged accordingly.[15] The two types of why-demonstration, however, express *different kinds* of explanation. Avicenna writes:

> Text 9.2: *Burhān* I, 7, p. 80.19–20
> Everything that is a cause of the existence (*wuǧūd*) of what is sought (*al-maṭlūb*) may be either [(a)] a cause of [(aa)] the major term itself while being at the same

13. An example is the case of phrenitis at *Burhān* I, 7, pp. 79.20–80.4 (cf. also *Burhān* I, 8, p. 85.9–11). This seems to coincide with the case illustrated in general terms (without example) at *Burhān* III, 3, p. 204.3–8, where Avicenna mentions a type of that-demonstration in which middle and major may both be effects of a cause not stated in the deduction.

14. On the notion of noncausal certainty in Avicenna, see Strobino (2016b).

15. Further support for the distinction between that-demonstration and why-demonstration of the second type comes from *Burhān* IV, 9, p. 329.9–13. To clarify his interpretation of *An. Post.* B18, Avicenna specifies that by "cause of the conclusion" he means not the cause of the assertion (of the conclusion), which is a distinctive element of a that-demonstration, but rather the cause of existence in itself, which is by contrast a distinctive element of a why-demonstration. In Avicenna, the distinction between cause of the conclusion and cause of the fact is, in all likelihood, an echo of Themistius, *In An. Post.* A13, p. 27.15–17. The latter contends that any middle term is an inferential cause of the conclusion (*aition tēs sunagōgēs tou sumperasmatos*), in contrast to a middle term that is also the cause of the thing being proved (*dia tēs tou pragmatos aitias tou deiknoumenou*). The distinction seems to have been drawn first, if only *in nuce*, by Alexander of Aphrodisias, *In An. Pr.* A1, p. 21.10–24. Cf. also Philoponus, *In An. Post.* A13, pp. 166.15–169.27.

time a cause of [(ab)] the fact that the major term belongs to the minor (*sabab li-nafs al-ḥadd al-akbar maʿa kawnihī sababan li-wuǧūdihī li-l-aṣġar*), or [(b)] [a cause] only of the fact that the major term belongs to the minor, without being a cause of the major term itself (*aw lā yakūnu sababan li-wuǧūd al-ḥadd al-akbar fī nafsihī wa-lākinna li-wuǧūdihī li-l-aṣġar faqaṭ*).

In Text 9.2, Avicenna distinguishes two types of why-demonstration. The first type, (a), is generally called by Avicenna absolute demonstration (*burhān muṭlaq*) and shows that the major term belongs to the minor term by means of a middle term that is the cause not only of the predicative nexus but also of (the existence of) the major term.[16] The second type, (b), also expresses a causal nexus, except that in this case it only shows why the major term belongs to the minor term, without giving at the same time the cause of (the existence of) the major. This is typically because this type of why-demonstration involves a major term that genuinely belongs to something other than the minor term first and then to the minor term in virtue of the former (regardless of whether the middle term is something more general than the minor term and under which the minor term falls or something coextensive with the minor term).

The distinctive feature of the second type of why-demonstration is that it does not show why the attribute-major term belongs primarily to that to which it is shown to belong. Rather, this demonstration simply applies a more general truth to a specific (or derivative) case (for this reason, borrowing a term used by R. McKirahan for Aristotle, I call this type of why-demonstration an "application argument").[17] For example, having the sum of the internal angles equal to two right angles is proved to belong to isosceles because it belongs to triangle first, and triangle is the genus of isosceles.[18]

In absolute why-demonstrations, by contrast, the middle term accounts for (i) the existence of the major term (by expressing its nature either fully or in part) and (ii) the fact that the major term belongs to the minor. This is because in such demonstrations (iii) the middle term ideally expresses the definition, or part of the definition, of the major term and, at the same time, (iv) the minor term falls under the middle term. Conditions (i) and (iii) are proper to absolute why-demonstrations only.[19]

16. Absolute demonstration (*burhān muṭlaq*) is not to be confused with absolute that-demonstration (*burhān al-anna al-muṭlaq*). The former is the paradigmatic type of why-demonstration and in fact of demonstration tout court (it corresponds to the notion of *demonstratio potissima* in the medieval Latin tradition), while the latter is simply a type of that-demonstration.

17. On application arguments, see McKirahan (1992, ch. XIV).

18. In the exposition, unless otherwise noted, I always refer for the sake of simplicity to first-figure deductions with a universal affirmative conclusion. Avicenna, however, shows in numerous places that he takes similar considerations to apply, mutatis mutandis, also to negative predications, as well as to the case of the other two figures.

19. Application arguments are relevant in the context of the analysis of primary attributes in *Burhān* II, 3, namely with regard to the idea that certain attributes belong first to a genus and then, in

The perspicuity of the Arabic terminology is partially lost in English, especially with regard to cause (*sabab* or *ʿilla*) and effect (*maʿlūl* or *musabbab*). Assuming that A, B, and C stand for major, middle, and minor term, respectively, in an absolute demonstration, A is an effect of (*maʿlūl li*) B absolutely (*ʿalā l-iṭlāq*) and an effect of B in its belonging to C (*maʿlūl li-B fī wuǧūdihī li-Ǧ*), while in an application argument A is predicated of (*maḥmūl ʿalā*) C through the mediation of its being predicated of (*bi-tawassuṭ ḥamlihī ʿalā*) B. In the latter case, B is a cause of C's being A (*ʿilla li-wuǧūd Ǧ A*), because A is predicated first of (*maḥmūl awwalan ʿalā*) B, and B is predicated of C, and hence A is predicated of C.[20]

The two types of why-demonstration are illustrated by useful examples. For instance, in the case of absolute demonstration, fever on alternate days (major term) is both (1.1.) an effect of the putrefaction of the bile (middle term) without qualification and (1.2) an effect of the fact that fever belongs to an individual (which is to say that the middle term "putrefaction of the bile" is a cause of the major term belonging to the minor). In the case of application arguments, animal (major term) belongs to Zayd, because it belongs to human, and human in turn belongs to Zayd. Similarly, in the case of general terms, body (major term) belongs to human (minor term) because it belongs to animal (middle term) first, and then because animal belongs in turn to human. In neither case, however, is the middle term a cause of the major term without qualification.

Avicenna is keenly aware of the fundamental difference between these two kinds of explanation and devotes an entire chapter of his work, namely *Burhān* I, 10, to the clarification of what it means for a less general term to be the cause of the fact that a more general term belongs to what falls under the less general term.[21] The leading principle is that "the genus is the cause of the fact that the [constitutive] differentia of the genus is predicated of the species (*ʿilla li-n-nawʿ fī ḥamli faṣli l-ǧinsi ʿalayhi*), just as [the genus] is the cause of the fact that the genus of the genus is predicated of the species" (*kamā huwa ʿilla lahū fī ḥamli ǧins al-ǧins ʿalayhi*).[22]

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a derivative sense, to the species falling under that genus. At *Burhān* II, 3 pp. 135.14–136.6, the relation is illustrated by the standard example of isosceles, triangle, and having the sum of the internal angles equal to two right angles. A demonstration of the fact that every isosceles has the sum of the internal angles equal to two right angles involves a straightforward PS2–PS1 premise pair ("Every triangle has the sum of the internal angles equal to two right angles" and "Every isosceles is a triangle"), which is explicitly recognized as such by Avicenna.

20. *Burhān* I, 7, pp. 80.19–81.5.

21. In *Burhān* I, 10, Avicenna quite literally settles a debt contracted with his readers at *Burhān* I, 7, p. 81.8–9: "Let the answer to this [question] be a duty (*farḍ*) and a debt (*dayn*) for us to pay [later]." The question in I, 7 is what Avicenna describes as "the doubt concerning whether the differentia of the genus belongs first to the species or to the genus." In *Burhān* I, 10 Avicenna analyzes the sense in which animal is a *cause* of the fact that body and sensitive belong to human (as remote genus and differentia, respectively).

22. See *Burhān* I, 7, p. 81.10, immediately after which, Avicenna reiterates that the claim will be confirmed later in solving the aforementioned doubt (*nubayyinu taḥqīqa ḏālika min ḥall aš-šakk*

Examples of That-Demonstration and Why-Demonstration

The two main types of demonstration discussed in *Burhān* I, 7 and their divisions are illustrated by a series of examples.

(aa) Absolute that-demonstration is exemplified by a set of terms taken from the domain of medicine. Avicenna considers a middle term and a major term that are both effects of the same cause: having coagulated white urine at the peak of the fever (middle term-effect 1) and having phrenitis (major term-effect 2). Their cause, which is unstated in the demonstration, is identified with the violent movement and eruption of the humors in the direction of the head.[23] In this case, neither term is a cause or an effect of the other. A demonstration that uses them (interchangeably as middle or major) and ascribes them to a subject who has fever (*hāḏā l-maḥmūm*) will merely produce a true assertion but not an explanation of the nexus between the terms of the conclusion. This is because an effect of an unstated cause is used to show that another effect of the same cause belongs to a subject.[24]

(ab) Signs, namely the type of that-demonstrations in which the middle term is an effect of the major term, are illustrated by four different examples. The first example is again taken from medicine and looks like a variation on the one just given, except that in this case the middle term and major term are related in the way an effect is related to its cause: having fever in alternate days (middle term), having fever due to the putrefaction of the bile (major term).[25]

The second and third examples come from the domain of astronomy. The second, in particular, corresponds to one of the famous examples in *An. Post.* A13. The three terms are moon (minor), receiving light in a certain way, that is to say, waxing and waning (middle-effect), and being spherical (major-cause). A that-demonstration of the fact that the moon is spherical will use the attribute waxing as a middle term to show that its cause (namely the fact that the moon is spherical) is an attribute of the moon. The third example is the common case of the lunar eclipse frequently discussed in the *Posterior Analytics*, involving the terms moon (minor), eclipse (middle-effect), and interposition of the earth between the moon and the sun (major-cause). The fourth example is from natural philosophy and

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al-maḏkūr baʿdu), which is again a reference to the discussion of *Burhān* I, 10, where he addresses the question in full. It is also interesting to note that *Burhān* I, 10, pp. 99.13–102.8 coincides verbatim with the text of *Ilāhiyyāt* V, 3, pp. 214.2–217.7.

23. On urine, its signs, and its symptoms, see *Qānūn* III, 1, 3.

24. On *sarsām* or *sirsām* (*Qānūn* III, I, 3), see Jacquart (1992) and Carpentieri (2017). The term translates the Greek *phrenitis* (an inflammation of the meninges or a form of encephalitis). The general characterization of this type of demonstration, without example, seems to correspond to the case discussed at *Burhān* III, 3, p. 204.3–8.

25. At *Burhān* IV, 5, p. 296.2–6, the proximate efficient cause of fever is identified with the putrefaction of the bile, while the remote efficient cause is said to be the obstruction of the pores. On obstruction as a cause of illness in general, see *Qānūn* I, II, 10.

deals in particular with a phenomenon of alteration, involving the terms wood (minor), burning (middle-effect), and being in contact with fire (major-cause).[26]

In all these cases the cause is proved starting from the effect. What is established is undoubtedly a true conclusion, but one that is not explained through its proper cause. Demonstrations of this kind merely provide an inferential justification of the conclusion and of the coming together of the two extremes in the mind. The reason is that the cause is proved to belong to the subject, inverting the natural order of explanation. According to Avicenna, this kind of demonstration fails to provide certainty, because whenever there is a cause for the nexus between a subject and an attribute, certainty can only be attained by taking the cause as middle term. This point is central to the argument of *Burhān* I, 8, analyzed in chapter 6, by which Avicenna establishes an unbreakable link between certainty, necessity, and causality in order to address the intermittence problem. These notions are parts of a unified conceptual framework attempting to establish the conditions for assertions with the highest epistemic strength, namely complete perpetual certainty (*al-yaqīn at-tāmm ad-dā'im*).

Absolute demonstration (why-demonstration) is illustrated by the same four sets of terms employed in case (ab), but here the appropriate order of explanation is reflected by the use of causes as middle terms and effects as major terms. In absolute demonstration, the middle term is not only the cause of the fact that the major term belongs to the minor term, but also of the nature and existence of the major term itself. The distinctive characteristic of these examples is that they all "give the assertion of what is sought (*taṣdīq bi-l-maṭlūb*) while giving at the same time the cause of the existence of what is sought in itself (*ʿillat wuǧūd al-maṭlūb fī nafsihī maʿan*)," where the latter should be understood here not generically as the fact expressed by the conclusion but, more specifically, as the major term that is proved to belong to the minor term by means of a middle term that is at the same time the genuine cause of the major term.[27]

26. The example is not in Aristotle but frequently occurs in Avicenna (on putrefaction and combustion, see *Afʿāl wa-infiʿālāt* I, 6, pp. 223.1–225.8). At *Naǧāt* I, 113, pp. 126.10–127.1 (cf. also *Burhān* IV, 1, p. 264.17–18), combustion is the main example illustrating the notion of an absolute (why-) demonstration. In that chapter, Avicenna contends that the middle term is a cause of the major term either without qualification or (only) relative to its belonging to the minor. Examples involve (i) a cause and an effect in the correct explanatory order (for example, something having intense heat transformed this piece of wood; everything that is transformed by something having intense heat burns; therefore, this piece of wood burns); and (ii) a species as the middle term and its genus, differentia, or inseparable accident as the major term (in the latter case, the major is first predicated of the middle and then, as a result of this, it is also predicated of the minor). The second case is illustrated by the terms isosceles, triangle, and having the sum of the internal angles equal to two right angles.

27. At *Burhān* I, 7, p. 80.17–18, Avicenna notes that "a detailed and exhaustive discussion (*tafṣīl*) of the types of causes (*aṣnāf al-asbāb*) and of how they may be taken as middle terms (*kayfa yumkinu an*

Why-Demonstration and Complete Perpetual Certainty

In the second part of *Burhān* I, 7, Avicenna seems concerned with two problems, namely (i) to clarify the distinction between being a cause of something and being a cause just of its belonging to something else and (ii) to determine under what conditions certainty may be attained, that is to say, what terms are adequate middle terms. In this context, he considers various objections and replies.

In particular, at *Burhān* 1.7, pp. 81.11–82.17, Avicenna discusses the status of causes and explanations in connection with the notion of why-demonstration and complete perpetual certainty. He lists the ideal conditions under which the latter may be attained, even though in the actual practice of demonstration those conditions are often not met.[28] The epistemic state produced by a term that is only partially explanatory is not sufficiently strong to qualify as complete perpetual certainty. Paradigmatic examples involve either instances of the intermittence problem, in which an effect is used to prove its cause, or noncausal cases, in which a differentia is used as a middle term for a genus that is the implicate of that differentia, as in the case of sensitive and animal. The details cannot be analyzed here, but it is at least worth noting that Avicenna uses two of his standard argumentative techniques to show what kinds of causes must ultimately be assumed in order for complete perpetual certainty concerning a given predicative nexus to come about.

Proximate and Remote Causes in Why-Demonstration

In *Burhān* I, 8, Avicenna notes that giving the proximate cause in act is not a necessary condition of why-demonstration in general and that an argument may count as a why-demonstration as long as a real explanatory nexus is expressed in it (even when that nexus is not, as it were, explanatory in the most fundamental way) and as long as the argument relies on premises that are certain. The chapter ends with a reference forward (*sa-naqūlu*) to the discussion of the

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tuʾḫaḏa ḥudūdan wusṭā) will be provided at a later stage." The reference is most likely to *Burhān* IV, 4 and, especially, to IV, 5, which jointly correspond to *An. Post.* B11. I return to this theme in chapter 11.

28. At *Burhān* I, 7, p. 81.12–14, Avicenna writes: "Everything that is a cause of the major term can be a middle term for it, even if it is not evident that it is a cause of it. But the resulting deduction is not a why-demonstration yet. Thus, until that is proven [too], complete certainty is not acquired by means of it (*fa-lā yuktasabu bihī al-yaqīn at-tāmm*)." He then goes on to note that, if something else is required to prove that the middle term is a cause, complete certainty can only be obtained by taking into account the additional middle term(s): "If it becomes clear through an argument [. . .] it is not through that middle term alone that certainty becomes complete, but through *the other middle term* [too], namely the one that proves that the cause is a *cause in act*." The crucial point is that proper why-demonstration and complete perpetual certainty are only attained when all terms that are required for a fully adequate explanation are supplied.

claim that a demonstration supplying the remote cause of the major term is not a why-demonstration (in a strict sense). The issue is central to the identification, in *Burhān* III, 3, of one of the two ways in which why-demonstration and that-demonstration differ within one and the same science, as discussed in the next section. He writes:[29]

> Text 9.3: *Burhān* I, 8, pp. 91.17–92.5
> Why-demonstration is not only the one that gives the proximate cause in act. Indeed, [something may be] a why-demonstration even if it does not do that, provided that it proves what it proves by means of the cause and of what is certain, and [that] the proof in it is resolved into the causes.
>
> What we will say about this is that when a demonstration gives the remote cause of the major term, it is not a why-demonstration. Let for example the minor term be C, the middle term be B, and the major term be A, where B, however, is not the proximate cause of the fact that C is A.
>
> [B] is only a cause of the fact [that C is A] because [B] is D. If we give the fact that B is A, one of the following must be the case: either the fact that B is A is certain for us and accepted by us or it is not. If it is not accepted, this deduction will not be a demonstration, let alone a why-demonstration.[30] If it is accepted not with respect to D, we will not be completely certain that every B is A, and our conclusion that every C is A because it is B will not be certain by complete perpetual certainty.
>
> If the knowledge of the fact that every B is A because D is A is previously available to us, or comes after, that will be known, for in that case, the demonstration will not be merely a that-demonstration (*burhān anna muğarradan*).

Consider the sequence of terms A, B, C, D, where A is the minor term, B a remote cause (*ʿilla baʿīda*) (of D, for A), C a proximate cause (*ʿilla qarība*) (of D, for A), and D the major term. The demonstration described in Text 9.3 is, according to one of the criteria of *Burhān* III, 3, a that-demonstration. In this demonstration, the term closer to the major is a cause of the major, while the term closer to the minor is a cause of the conclusion (not in the sense that it is merely a cause of the assertion of the conclusion, but a cause of the fact that the major belongs to the minor). Avicenna consistently understands by proximate cause the term closest to the major (the ultimate cause, with respect to the minor). This characterization fits well in the taxonomy of the cases of subordination discussed in chapter 10, where the greatest explanatory power is associated with a middle term coming from a higher science (and hence with the ultimate cause).

29. Compare with *Burhān* IV, 9, p. 329.9–13, on *An. Post.* B18.

30. Reading *limā* for *anna*, with Ms. Qum, Sayyid Muḥammad Ḥusayn Ġiyaṯ ʿAlawī, private collection (no number), f. 142r, accessed 30 June 2020, www.avicennaproject.eu.

DIFFERENCE BETWEEN THAT-DEMONSTRATION AND WHY-DEMONSTRATION IN ONE SCIENCE

In *Burhān* III, 3, Avicenna deals with *An. Post.* A13, which contains the most elaborate Aristotelian analysis of the difference between knowledge of the fact and knowledge of the reason why in the *Posterior Analytics*. The distinction is understood to apply to (i) a single science and (ii) multiple sciences. Avicenna engages with Aristotle's text more closely in this context, but it soon becomes clear that his approach is characterized by several elements of originality and that he is indebted in various ways to the Greek commentary tradition. *Burhān* III, 3 is, among other things, interestingly concerned with several subtle problems of exegesis, classification, and application.

The first question (i) concerns the ways in which one may demonstrate or fail to demonstrate causally within one and the same science and aims to identify the kinds of arguments that meet these conditions. The second question (ii) concerns explanation across different sciences. Avicenna's answer to the first question includes two alternatives that seem to correspond, at least in outline, to the cases examined by Aristotle in *An. Post.* A13. But his account of the relevant distinctions is more detailed and linked to the systematic treatment of *Burhān* I, 7. The answer to the second question identifies several types of relations and is directly connected, on the one hand, to Avicenna's original analysis of the hierarchy and division of the sciences, in *Burhān* II, 7, and on the other to an elaborate interpretation of some remarks from *An. Post.* A9, in *Burhān* II, 9, which is discussed in chapter 10.

At *Burhān* III, 3, p. 202.1–2, Avicenna raises two questions: What is the difference (*farq*) between that-demonstration and why-demonstration in one and the same science? How can there be that-demonstration and why-demonstration of one and the same thing (*kayfa yakūnu ʿalā šayʾ wāḥid burhān anna wa-burhān limā*)?[31]

The distinction between that-demonstration and why-demonstration within a science depends on the way in which the terms employed in a demonstration are related and on whether and how their arrangement in a proof reflects or fails to reflect the correct explanatory order. In particular, there are two possible ways in which that-demonstration and why-demonstration may differ within a science,

31. The question of whether or not that-demonstration and why-demonstration are about one and the same conclusion is explicitly addressed by Avicenna at the end of this discussion. His considered view is that both that-demonstration and why-demonstration aim at the same conclusion in different forms and in a different order but without circularity. Furthermore, the question arises with respect to a single science as well as with respect to multiple sciences. A summary of the different options is offered at the end of chapter 10.

depending on whether the terms involved are (i) non-immediate or (ii) immediate (in the latter case, the terms may in turn be either convertible or nonconvertible).[32]

Non-Immediate Terms

The first case is introduced at *Burhān* III, 3, p. 202.3–9, though its treatment is immediately deferred to a later stage in the chapter. A detailed account (*tafṣīl*) of it is at *Burhān* III, 3, pp. 204.15–205.18. The distinction between that-demonstration and why-demonstration applies, in this sense, (1) when a conclusion is proved by means of two deductions, (1.1) the first of which uses a remote cause (*ʿilla baʿīda*) of the conclusion and (1.2) the second of which uses a proximate or primitive cause of the conclusion (*ʿilla qarība* or *ʿilla ūlā*).[33] Both deductions are explanatory of the conclusion, but the first is characterized as a that-demonstration, while only the second deserves the title of why-demonstration. The two deductions differ not only because they express distinct types of causes, but also because the deduction using the remote cause does not in fact consist of immediate premises. A schematic illustration may be given in terms of a chain of four terms A—D—B—C, involving three immediate nexuses (A—D, D—B, and B—C) and three non-immediate nexuses (A—B, D—C, and A—C). The conclusion AaC ("Every A is C") is proved by means of two distinct middle terms, B and D, where B is the proximate cause of C for A and D is the remote cause of C for A.[34] Thus, from left to right, A is the minor term and C is the major term in both deductions, while B (the proximate cause) is the middle term in one deduction and D (the remote cause) is the middle term in the other:[35]

32. One of the interpretive problems raised by *An. Post.* A13 is the question of how many cases Aristotle contemplates for the distinction between knowledge of the fact and knowledge of the reason why in one science. Avicenna's first dichotomous division corresponds to two general cases, which, according to most interpreters, are also the ones identified by Aristotle (in *Burhān* III, 3, Avicenna deliberately follows the argument of the *Posterior Analytics* more closely). When it comes to their internal divisions, however, the details of their accounts begin to diverge. Avicenna's contribution, besides its systematic significance, may in fact also be regarded as an effort to clarify some of the problems raised by the first part of *An. Post.* A13.

33. The latter is characterized more precisely as the cause that is "proximate to the thing and necessitates it in itself" (*al-qarība li-l-amr al-mūǧiba lahū li-ḏātihī*) at *Burhān* III, 3, p. 202.4–5.

34. On the question of whether proximity and remoteness should be understood to be relative to the minor term or the major term, Avicenna is clearly committed to the idea that proximate means closest to the major term, that is to say, to the demonstrable attribute. The question naturally arises for Aristotle, too, who seems to oscillate between the two alternatives. A locus classicus for the competing view, namely the identification of proximate with what is closest to the minor term ("the one which is primitive in the direction of the particular" or "nearest to what it is explanatory for") is *An. Post.* B18, 99b10–12. On this issue, see Barnes (1993, pp. 155–156 and 257).

35. On the kind of demonstration that involves a remote cause, Avicenna notes that "the [non-immediate] premise does not give the real why-it-is (*al-limā al-muḥaqqaq*)" (*Burhān* III, 3, p. 202.8). Corroborating evidence is at *Naǧāt* I, 136 (i), p. 145.1–2, where Avicenna contends that when the remote

1.1 AaD; DaC ⊢ AaC
1.2 AaB; BaC ⊢ AaC

According to Avicenna's characterization, (1.1) is a that-demonstration of AaC, while (1.2) is a why-demonstration of the same conclusion, where (1.2) expresses the proximate cause of the conclusion and (1.1) its remote cause. Furthermore, the major premise of (1.1), namely DaC, is not an immediate premise. For in order to be established, DaC requires B and a supplementary deduction with the following structure:

1.3 BaC, DaB ⊢ DaC.[36]

Avicenna returns to this case right before moving on to the discussion of the distinction between that-demonstration and why-demonstration in two sciences. In this, he follows the order and arrangement of *An. Post.* A13, where the distinction is famously illustrated by means of a negative conclusion ("No wall breathes") and a deduction in the second figure. The schematic illustration just provided is in the first figure only for the sake of simplicity, but the same considerations apply to Avicenna's actual examples later in the chapter.[37] The discussion is connected with a notoriously problematic passage at *An. Post.* A13, 78b13, where Aristotle introduces a variant of the distinction concerned with "the things where the middle term is put outside." The expression has been variously interpreted since antiquity. Avicenna reports two famous interpretations with which he may have been familiar directly from the commentary of Philoponus or, perhaps, indirectly from a gloss to the Arabic translation of the *Posterior Analytics* preserved by Ms. BNF Ar. 2346 (if we assume that the translation and the glosses circulated together).

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cause is given "the investigation of the why remains open, in such a way that the [process] of giving the why has not been completed yet (*fa-yakūnu iʿṭāʾ al-limā lam yastakmil baʿdu*)." At *Burhān* I, 8, p. 91.13–16, the question of certainty when multiple middle terms are involved is linked to the analysis of concatenated deductions:

> One could say that when a judgment is established concerning a minor [term] and the conclusion is correct, [if] we want to make it the major [premise] of another deduction, what will that deduction be like with respect to [its ability] to procure certainty? We say that when the minor [term] becomes the middle [term], with the major term evidently belonging to it by means of a cause, that cause becomes in itself a cause of everything that is described by the minor, hence it becomes a cause of the second minor as well, except for the fact that it is a cause of the second minor in a mediated way, while [being a cause] of the first [minor] in an immediate way.

36. Avicenna does not mention a third deduction, but that he must have one in mind is implied by the contention, at *Burhān* III, 3, p. 202.7–8, that "a premise in one of the two deductions needs an intermediary which is the proximate cause." The proximate cause is B, that is to say, the term closest to the major term C.

37. In addition to the standard negative Aristotelian example (wall, not being an animal, not having lungs, not breathing), at *Naǧāt* I, 136 Avicenna gives an affirmative example where the four terms are feverish individual (minor), having obstructed pores (remote cause), having putrefied mixture (proximate cause), and fever (major-effect).

According to a first more literal (*aẓhar*) interpretation (*ta'wīl*), "outside" (*ḫāriğ*) means that the middle term occupies an external position with respect to the minor term and the major term. In this case, the resulting argument would be a deduction in the second figure (which happens to be the example of that-demonstration given in *An. Post.* A13 to illustrate this division). This reading works well with negative demonstrations, concerning which Avicenna presumably believes that the use of non-proximate causes shows in a more evident manner the reason why a demonstration may fail to encapsulate a proper causal explanation.

According to a second interpretation (*tafsīr*), which Avicenna takes to be "the most correct (*al-awṣab*) even though it is not the most literal (or, alternatively, the most manifest) (*wa-in lam yakun al-aẓhar*)" (*Burhān* III, 3, p. 205.4), the "middle term" is the proximate cause and converts with its effect, and "outside" does not refer to the position of the terms in a syllogistic figure but rather to the fact that such a cause has been left out altogether. On this reading, the argument is a that-demonstration because the attribute of not having lungs (which is the real cause of the fact that walls do not and cannot breathe) does not appear in it, as a remote and more general cause (namely, not being an animal) is used in its place.

Immediate Terms (Convertible and Nonconvertible)

The second case is more complicated.[38] It partly corresponds to the case of immediate terms in *An. Post.* A13, and its most significant sub-case is uncontroversially meant to be the counterpart of the type of that-demonstration called *sign* (which is confirmed by the use of either the same or structurally identical examples). Avicenna, however, introduces a number of further distinctions, most likely absorbing comments from Themistius and Philoponus.[39]

That-demonstration and why-demonstration differ in the same science when (2) one of two deductions fails to express a cause and its middle term is related to the middle term of the other deduction in one of the following ways. Either (2.1) the two middle terms convert, in which case either (2.1.1) the middle term of the first deduction is an effect of the middle term of the second deduction or (2.1.2) it is not (that is to say, neither the middle term nor the major term is a cause

38. The discussion of the second case is at *Burhān* III, 3, pp. 202.9–204.14 and is articulated as follows: general division at pp. 202.9–203.1, illustration of (2.1.1) at pp. 203.3–204.2, illustration of (2.1.2) at p. 204.3–8, and summary of the discussion at p. 204.9–14.

39. These include the case of nonconvertible terms exemplified by smoke and fire (smoke implies fire, but fire does not imply smoke). An example that occurs in both Themistius and Philoponus but not in Avicenna suggests that the discussion might presumably be traced back to the *Prior Analytics* and to another notion of sign (*'alāma*). One example involves the terms woman, lactating (effect-sign), and giving birth (cause); another example (of nonconvertible terms) involves the terms woman (minor), giving birth (effect-sign), and having had intercourse (cause). On the notion of sign in the Greek commentary tradition, see Morrison (1997) and, especially, Bellucci and Marmo (2018).

of the other, as they are both effects of an unstated cause or immediate implicates of the minor term); or (2.2) the two middle terms do not convert, and the effect (presumably the middle term of the first deduction) is either (2.2.1) more general or (2.2.2) more specific than the middle term of the second deduction.[40]

In (2.1.1), the first deduction is (2.1.1.1) a that-demonstration (a sign), and the second deduction is (2.1.1.2) a why-demonstration. In (2.1.2), both deductions are that-demonstrations, but a why-demonstration can come about if one of the terms is suitably replaced by a genuine cause.

(2.1.1) is illustrated by means of two examples from *An. Post.* A13 (after completing the division, Avicenna returns to them to clarify a number of points in greater detail). The first example is what Barnes (1993, p. 156) has called a "planetary syllogism." The three terms are planets, not twinkling, and being near. The attribute of not twinkling for a celestial object is at the same time convertible with and an effect of the attribute of being near. A demonstration proving the latter through the former is a that-demonstration (of the fact that the planets are near, based on an observable effect, namely the fact that they do not twinkle). A demonstration proving the former through the latter is, by contrast, a genuine why-demonstration (of the fact that the planets do not twinkle, based on the real cause, namely the fact that they are near). A similar example involves the moon, the way it receives light from the sun (its phases, or the attributes of waxing and waning), and its spherical shape. The latter is the real cause of the phenomenon of waxing, and this is again an example in which, with suitable restrictions to the relevant domain, the cause is convertible with the effect (*wa-huwa mimmā yanʿakisu ʿalā l-ʿilla*) (*Burhān* III, 3, p. 202.12–13).

A slightly less straightforward example involves the claim that a halo (*hāla*) indicates that rain is pouring from the clouds in which the halo comes about.[41] What matters for our purposes is that Avicenna contends that while the latter might be a convertible effect or sign (*ʿalāma*) of the former, it need not necessarily be. This is because the effect in some cases is more general than its cause (for example, a house being lit by lightning, as lightning could cause something else to be lit) and in some cases more specific (for example, smoke being produced by fire, presumably because smoke could be caused by something else).[42]

40. Convertible and nonconvertible causes and effects are mentioned again at *Burhān* IV, 3, pp. 286.16–287.4.

41. For Aristotle's account of the halo as a sign of rain (and other causes), see *Meteor.* Γ3, 372b15–18 (Barnes 1984, transl. Webster): "Sight is reflected in this way when air and vapor are condensed into a cloud and the condensed matter is uniform and consists of small parts. Hence it is a sign of rain, but if it fades away, of fine weather, if it is broken up, of wind."

42. It is unclear why the case is illustrated first by this example and then discussed again in general terms. The discussion may reflect examples found in the commentators or implicitly foreshadow the problem of the relative extension of cause and effect in *Burhān* IV, 8 and IV, 9.

TABLE 14 Types of demonstration in *Burhān* III, 3

That-demonstration				Why-demonstration		
Non-immediates	Immediates			Non-immediates	Immediates	
	Convertible		Nonconvertible		Convertible	Nonconvertible
	[Absolute that-demonstration]	Sign			[Absolute demonstration]	
Middle term remote cause of the major term	Middle term and major term both effects or implicates of an unstated cause or of the minor term	Middle term effect of the major term	Middle term (more specific) effect of the major term	Middle term proximate cause of the major term	Middle term cause of the existence of the major term and of its belonging to the minor term	Middle term (more specific) cause of the major term

The characteristic feature of (2.1.1) is that whether an argument is a that-demonstration or a why-demonstration depends on what plays the role of middle term and major term in it. For a schematic illustration, let us take three terms, A, B, C, where B and C are convertible and B is the cause of C. Depending on whether B or C is the middle term, we will have the following two deductions:

(i) AaC; CaB ⊢ AaB (that-demonstration)
(ii) AaB; BaC ⊢ AaC (why-demonstration)

According to Avicenna, (i) is a that-demonstration, while (ii) is a why-demonstration. But why should a that-demonstration such as (i) be of any interest at all? Avicenna stresses the fact that the choice of the middle term may depend on the relation that turns out to be better known to us (showing a concern, in this case, not only for the structure of a science in its final stage, but also for what may actually occur at intermediate stages). If the effect is better known to belong to the minor than the cause, then it is legitimate—and indeed, more appropriate (*awlā*)—to start with a demonstration in which the middle term is the effect and the major term is the cause (*Burhān* III, 3, p. 203.4–6). The corresponding why-demonstration is obtained simply by inverting the order of the terms.[43] The taxonomy of that-demonstration and why-demonstration in *Burhān* III, 3 is illustrated in table 14.

One Conclusion or Two Conclusions?

In deductions (i) and (ii), however, the conclusion is different: (i) proves *that* the middle term-cause belongs to the minor term, whereas (ii) proves *why* the major term-effect belongs to the minor term. Thus, while Avicenna's identification of (i) as a that-demonstration and (ii) as a why-demonstration is not in question, the two arguments seem to be demonstrations *of different things*, that is to say, of different conclusions. Is this a general truth about the relation between sign (*dalīl*) and its cognate why-demonstration obtained by switching middle term and major term?

Avicenna seems to believe that cases of this sort may be understood in terms of a concatenation of deductions that involve, at different stages, knowledge of the same thing but in different respects. At the beginning, under the assumption that the effect is better known to us, we may have only *factual knowledge* of AaC,

43. The examples illustrating this case are all from *An. Post.* A13: (i) the fixed stars, twinkling, and being far; (ii) the planets, not twinkling, and being near; and (iii) the moon, receiving light in phases, and being spherical. According to Avicenna, the distinctive feature of (i)–(iii) is that the middle term is better known (Ar. *aʿraf*; Gr. *gnōrimoteron*) to belong to the minor term than the major term is. Otherwise, the real cause could be taken as a middle term instead, and this would result in a why-demonstration in all three cases. Avicenna notes that, in the domain of celestial bodies, twinkling and not twinkling are effects of distance and proximity. If the received text is correct (but note that the term is missing in ms. S, as recorded in the Cairo edition), it bears witness to a rare occurrence, in Avicenna's *Burhān*, of *musabbab* for "effect," alongside the far more frequent *maʿlūl*.

that is to say, of the minor premise of (i). From AaC and CaB, we can prove AaB. But AaB, which is now inferentially justified (on the basis of factual knowledge of AaC, which is one of the assumptions), can in turn be used to establish demonstratively *causal knowledge* of AaC, that is to say, of the conclusion of (ii) (B and C by hypothesis are convertible terms; therefore, if CaB is assumed to be true, BaC can be assumed to be true too).

Avicenna realizes that this line of reasoning looks suspiciously question-begging but rejects a potential objection to its apparent circularity by pointing out that the minor premise AaC of (i) is used exclusively to show *that* AaB is the case, while AaB is in turn used exclusively to show *why* AaC is the case.[44] There is no denying that AaC is assumed in (i) to prove AaB, and that AaB is in turn assumed in (ii) to prove AaC. But according to Avicenna the strong impression of circularity may be dispelled if we appeal to a distinction between two different *kinds* of knowledge. For what is it that is being proved by the two arguments individually? Deduction (i) assumes factual knowledge of AaC to prove AaB, while deduction (ii), which ultimately relies on that factual knowledge of AaC, aims to provide causal knowledge of AaC. The kind of knowledge we have of AaC as an assumption in (i)—to prove something, which is then in turn assumed to prove AaC as the conclusion of (ii)—is not the same kind of knowledge we have of AaC as the conclusion of (ii).

Two Effects

After the discussion of (2.1.1) (sign and the corresponding type of why-demonstration), Avicenna offers another characterization of (2.1.2), namely the case in which the middle term and the major term are both effects or implicates of an unstated cause (or of the minor term).

The general analysis of the second way in which that-demonstration and why-demonstration differ in one and the same science ends with a summary, which seems to provide indirect corroboration for the reconstruction put forward thus far (bearing in mind that the discussion of the first type had been deferred). If we look at the way in which the premises are related, (2.1.1) and (2.1.2) may be characterized as follows:

44. "In the first proof, the why-it-is is not sought at all, while in the second proof the that-it-is is not sought at all" (*Burhān* III, 3, p. 203.17–18). Avicenna draws an interesting distinction between the pure that (*anna ṣirf*) and the pure why (*limā ṣirf*), which implies that the two dimensions can be investigated separately. Note also the reference to the possibility of doubts arising, not about the *anniyya*, but about the *limmiyya* only (*Burhān* III, 3, pp. 203.20–204.2). The language seems relevant to the constraints imposed in *Burhān* II, 6 on the admissible combinations of predicates of scientific questions. At *Samāʿ ṭabīʿī* I, 1, pp. 11.15–12.3, Avicenna refers to the same distinctions introduced in *Burhān* I, 7 and *Burhān* III, 3 (as "what has been clarified in the discipline of demonstration").

(i) A premise is shared by two deductions (based on the convertibility of middle term and major term).
(ii) The premises are different.

The first case corresponds to (2.1.1), the second to (2.1.2). In (2.1.1), a premise may be shared because the terms are convertible. Thus, arguments (i) AaC; CaB ⊢ AaB (that-demonstration) and (ii) AaB; BaC ⊢ AaC (why-demonstration), in a sense, may be regarded as sharing a premise (CaB, BaC). In (2.1.2), by contrast, the premises are different because their middle terms are different. Thus, if B and C are two effects of an unstated cause D, (1) AaB; BaC ⊢ AaC and (2) AaC; CaB ⊢ AaB are both that-demonstrations, while (3) AaD; DaC ⊢ AaC and (4) AaD; DaB ⊢ AaB are both why-demonstrations. In this case, no premise in the first two deductions is a premise in the last two deductions. In summary, Avicenna's contention is that "if one of the causes is made to be the middle term, it is a why-demonstration and a that-demonstration at the same time (*burhān limā wa-anna maʿan*); if one of the implicates or the effects is made to be middle term, it is a that-demonstration only (*burhān anna faqaṭ*)" (*Burhān* III, 3, p. 204.8–9). Avicenna also notes that this reading enables him to account for the fact that an argument is both a that-demonstration and a why-demonstration of one and the same thing, while others are committed to the view that there is a that-demonstration of one thing and a why-demonstration of something distinct (*Burhān* III, 3, p. 204.9–14).

SUMMARY

The two taxonomies of that-demonstration and why-demonstration in *Burhān* I, 7 and *Burhān* III, 3 are related as follows:

Burhān I, 7

- [a] That-demonstration
    - [aa] Absolute that-demonstration
        - [aaa] Middle term and major term both effects of an unstated cause = [2.1.2]
        - [aab] Middle term and major term both immediate effects of the minor term = [2.1.2][45]
    - [ab] Sign = [2.1.1.1]
- [b] Why-demonstration
    - [ba] existence and belonging = [1.2], [2.1.1.2]
    - [bb] belonging only = [1.1]

45. It is unclear whether case [aab] in *Burhān* I, 7 corresponds to one of the divisions of *Burhān* III, 3. By elimination, it would presumably have to be a variant of case [2.1.2].

Burhān III, 3

- [1] Non-immediates (first way in which that-demonstration and why-demonstration differ)
  - [1.1] That (belonging only) (middle term remote cause) = [bb][46]
  - [1.2] Why (existence and belonging) (middle term proximate cause) = [ba]
- [2] Immediates (second way in which that-demonstration and why-demonstration differ)
  - [2.1] convertible terms
    - [2.1.1] middle term cause or effect
      - [2.1.1.1] That (sign) (middle term effect of the major term) = [ab]
      - [2.1.1.2] Why (existence and belonging) (middle term cause of the major term) = [ba]
    - [2.1.2] Absolute that (middle term and major term both effects of an unstated cause or implicates of the minor term) = [aaa], [aab][47]
  - [2.2] nonconvertible terms
    - [2.2.1] middle term more general cause or effect
    - [2.2.2] middle term more specific cause or effect

For Avicenna, this fine-grained classification of arguments allows us to identify different kinds of scientific knowledge (causal and noncausal) or different stages in the process of acquisition of scientific knowledge (distinction between sign and proper why-demonstration from cause to effect) within the perimeter of a single domain of investigation. This, however, is just half of the story. As shown in chapter 7, the hierarchical dimension of the organization of scientific knowledge is a key aspect of Avicenna's theory of science, and the analysis of how explanation works across multiple sciences is, therefore, a central part of it too.

46. Case [1.1] cannot coincide with either of the two cases of that-demonstration from the list of *Burhān* I, 7. It could perhaps be construed as a case of [bb], but this would imply that Avicenna has an equivocal notion of that-demonstration. For it seems that, in this case, there would be a kind of that-demonstration that not only gives an inferential justification of an assertion but also involves a genuine causal explanation, even though the cause involved is a remote cause and not a proximate cause.

47. In Themistius, *In An. Post.* A13, pp. 28.30–29.3, we find an interesting example of convertible terms that are both effects of one and the same cause: fever (cause), a certain unstable state of the veins (an effect of fever), and a change in the internal heat (another effect of fever). As noted earlier in this chapter, Avicenna frequently employs examples from medicine, and fevers are identified either as causes of certain effects or as effects of certain causes. Another interesting example, at *Qiyās* V, 3, p. 255.11–12, involves a true (implicative) conditional whose antecedent encapsulates a conjunction of symptoms, signs, or effects, and the consequent expresses their cause (a particular disease): "If this man has chronic fever, dry cough, shortness of breath, sharp pain, and high pulse rate, then he has pleuritis (*ḏāt al-ğanb*)." On Avicenna's account of different kinds of respiratory disease (*Qānūn* III, 10), see Hashemi and Raza (2009).

10

Explanation across Sciences, Subordination, and the Transfer of Demonstration

The Aristotelian ban on kind crossing (*metabasis eis allo genos*) is compatible with various forms of interdependence. Avicenna frequently discusses this theme under the heading of "aid" or "contribution" (*taʿāwun*) that a science can bring to another. If two sciences share subjects, principles, or questions in the way described in chapter 5, different sorts of complementarity are possible. Avicenna's discussion constitutes a sizable contribution to the subject and is shaped, as in Aristotle, by the idea of looking at the respective contributions of two related sciences to our knowledge of the *that* and of the *why*. Aristotle discusses the relation between that-demonstration and why-demonstration in two sciences at various places in the *Posterior Analytics*, more prominently in A9 and in the second part of A13. In Avicenna, the most elaborate counterparts to these chapters are *Burhān* II, 9 and III, 3, where the distinction between the two kinds of demonstration is closely connected to his account of the hierarchy of the sciences and subordination.

In particular, in *Burhān* III, 3, Avicenna identifies three cases (which seem partly to overlap with Aristotle's classification). The three types are (1) full subordination, (2) partial subordination, and (3) subordination relative to an individual question. In each case, the subordinate science (or the science to which the individual question pertains) provides knowledge of the *that*, while the superordinate science provides knowledge of the *why*. In this context, Avicenna also clarifies the relation between metaphysics and the particular sciences, which is discussed in detail in *Burhān* II, 9 and presupposed by the corresponding discussion in *Ilāhiyyāt* I, 3. Finally, Avicenna's account of explanation across sciences and subordination is connected with the notion of transfer of demonstration (*naql al-burhān*) from one science to another.[1]

THAT-DEMONSTRATION AND WHY-DEMONSTRATION IN TWO SCIENCES

The distinction between that-demonstration and why-demonstration in a single science, as noted in chapter 9, depends in one case on whether a demonstration gives a proximate cause or a remote cause of the nexus between subject and attribute. The same criterion also applies to the distinction between that-demonstration and why-demonstration in different sciences: the science that gives the proximate cause is also the science capable of producing the why-demonstration of a given question, while the science that gives the remote cause can only produce a that-demonstration of it.

The most detailed source for the discussion of subordination across multiple sciences is the second part of *Burhān* III, 3, pp. 205.19–209.17, which corresponds to *An. Post.* A13, 78b34–79a15. In this chapter, subordination is understood primarily in terms of the explanatory power of a science relative to another science, whereas in the classification of the sciences of *Burhān* II, 7 examined in chapter 5, the emphasis was on the kinds of elements (especially the subjects) that two sciences may have in common and on their possible relations. The two dimensions, however, are interdependent, and several pairs of sciences analyzed in *Burhān* II, 9 and III, 3 are the same ones encountered in II, 7.[2]

Subordination of a Science to Another Science

At *Burhān* III, 3, pp. 205.19–207.8, Avicenna discusses the most common case, namely the subordination of one science *as a whole* to another science (which clearly corresponds to the case identified by Aristotle first, at *An. Post.* A13, 78b32–79a10). The characteristics of this relation are discussed in detail in the context of *Burhān* II, 7, where the notion of "being under" (Ar. *taḥta*, Gr. *hupo*) a science is articulated in its specific senses. This first category covers a variety of types of subordination, depending on the underlying relations between the subjects of the two sciences under consideration.[3]

1. For a summary of Aristotle's vocabulary of subordination and the examples of *An. Post.* A13, see Barnes (1993, pp. 158–159). On subordination in Aristotle, see Jope (1972) and McKirahan (1978, 1992).

2. The scheme of classification seems inspired not only by the examples found in *An. Post.* A9 and A13, but presumably also by the general structure of Alfarabi's *Iḥṣāʾ al-ʿulūm*.

3. This is the most common case also in the classification of principles in *Burhān* II, 7, as noted in chapter 5; cf. also Hugonnard-Roche (1984) and Strobino (2017).

The examples are the same as in Aristotle: (a) geometry and optics, (b) solid geometry (*ʿilm al-muǧassamāt*) and mechanics (*ʿilm al-ḥiyal*), (c) arithmetic (*ʿilm al-ʿadad*) and harmonics (*ʿilm taʾlīf al-luḥūn*), and (d) astronomy—introduced as the "judgments concerning the stars" (*aḥkām an-nuǧūm*) and immediately glossed as the "judgments of astronomy" (*aḥkām ʿilm al-hayʾa*)—and the observation of the stars (*ẓāhirāt al-falak*).[4]

Avicenna examines the Aristotelian claim that in each of these pairs, the higher science and the lower science are almost synonymous. They seem to be synonymous because in each respective pair the two sciences are somehow related to the same object. Star-gazing (*ẓāhirāt*) and astronomy (*al-hayʾa*) investigate the properties of celestial bodies and their distances; mathematical astronomy (*an-nuǧūm at-taʿlīmī*) and nautical astronomy (*nuǧūm aṣḥāb al-malāḥa*) investigate the positions of the stars (*mawāḍiʿ an-nuǧūm*) (Avicenna returns to this characterization at the end of the chapter with some important remarks); mathematical harmonics (*taʾlīf al-luḥūn at-taʿlīmī*) and acoustic harmonics (*taʾlīf al-luḥūn as-samāʿī*) investigate the properties of notes, that is to say, of certain kinds of sound; optics and geometry investigate the properties of shapes, lines, and magnitudes; and mechanics and stereometry investigate the properties of three-dimensional magnitudes.[5] The elements of each pair are not properly synonymous for two reasons. First, they do not perfectly share their subjects; for instance, music and optics investigate their subjects—numerical ratios and lines, respectively—in a given respect (in relation to notes and vision), while their superordinate sciences, arithmetic and geometry, investigate those subjects without qualification (*Burhān* III, 3, p. 206.8–12). Second, their subjects stand in a relation of priority and posteriority, to the effect that one science gives the *why*, while the other gives the *that* (*Burhān* III, 3, p. 206.13–18).[6]

4. Stereometry (or solid geometry) is used in *Burhān* II, 7 to illustrate two distinct types of relation. It is subordinated to geometry as a *part* of it, because its subject is a genuine species of the subject of geometry, and astronomy is subordinated to (without being part of) it because the former deals with the moving spheres, that is to say, with a species of solid qualified by the attribute of motion. Avicenna is presumably contrasting mere observations (*ẓāhirāt*) to mathematical judgments (*aḥkām*) involving inferential connections about the position of stars and planets (observational astronomy is usually designated by the Arabic term *rasḍ*). Abū Bišr Mattā translates Aristotle's *astrologikē* just as *ʿilm an-nuǧūm*. At *Madḫal* I, 1, p. 11.8, Avicenna refers to the relation between observational judgments (*al-aḥkām ar-raṣdiyya*) and the canons of natural philosophy (*al-qawānīn aṭ-ṭabīʿiyya*). For the use of *aḥkām an-nuǧūm*, see also Alfarabi's *Burhān* IV, 2, p. 71.21 and IV, 3, p. 75.14.

5. The expression *taʾlīf al-laḥn* designates, in Avicenna's compendium of music (*Ǧawāmiʿ ʿilm al-mūsīqā*), a specific subject (the topic of *Mūsīqā* VI, 1, pp. 139.1–142.9, to be precise), while *ʿilm al-mūsīqā* (or simply *al-mūsīqā*) is a more general expression referring to the science as a whole.

6. The fact that the higher science provides a causal demonstration while the lower science provides a factual demonstration has been made clear already (*ʿalā naḥw mā kunnā naḥnu anfusinā awḍaḥnā fī mawḍiʿihī*). The reference seems to be, in all likelihood, to *Burhān* II, 9.

Avicenna then goes on to comment on a remark made by Aristotle concerning the role of empirical observation, namely that knowledge of the *that* (*ʿilm bi-anna huwa*) pertains to the practitioner of the lower science, and knowledge of the *why* (*ʿilm bi-limā huwa*) pertains to the practitioner of the higher science (*Burhān* III, 3, p. 206.19), as with the nautical astronomer (*al-mallāḥ*) and the astronomer (*al-munağğim*), or the expert of practical music (*al-mutadarrib fī ṣināʿat al-mūsīqā al-ʿamaliyya*) and the practitioner of mathematical harmonics (*ṣāḥib ʿilm at-taʾlīf at-taʿlīmī*).[7] The practitioners of the higher sciences may have knowledge of the cause (*aṣḥāb al-ʿulūm al-ʿāliyya ʿindahum as-sabab*) of a phenomenon without having perceptual awareness of the particulars or without having practical experience. Avicenna's own example is of someone who knows the theory of musical intervals but cannot tell whether a particular interval is consonant with another interval, even if he knows what the cause of consonance is. Something similar happens in connection with other abstract disciplines, such as geometry. There may be a purely theoretical (mathematical) study of magnitudes and bodies, abstracted from matter, even if those only exist in matter and not as separate forms: "the geometer strips them of their matter and investigates them in themselves (*li-ḏātihā*) and not for what belongs to them as an accident in virtue of the fact that they exist in matter, as we have explained earlier" (*Burhān* III, 3, p. 206.7–8).[8]

7. An analogous remark is at *Burhān* I, 8, p. 87.17–21:

> In this respect we say that the mathematician does not possess certainty in many things related to astronomy because he assumes them with respect to what is found by astronomical observation (*bi-r-raṣd*). Such is his way to operate, for instance, when he deduces the apogee (*awğ*) of the sun on the basis of the fact that the motion of the sun is not uniform in all parts of the ecliptic (*falak al-burūğ*) with respect to speed and slowness, so that it is slowest at the apogee and fastest at the perigee (*ḥaḍīḍ*), without giving the cause of anything in this matter ([the cause] is only given by the natural philosopher).

The comment is relevant for the relation between observational astronomy and natural philosophy, and concerns one of the anomalies of the solar theory in the Ptolemaic system (the apparent nonuniform motion of the sun, which slows down at the apogee, during the summer months, and speeds up at the perigee, during the winter months). According to Avicenna, the explanation of this fact is the task of natural philosophy. On this problem, see also *Samāʾ wa-ʿālam* I, 6, pp. 46.16–48.4 and *Hayʾa* III, 3, pp. 162.1–164.7, where the apparent difference in speed of the sun traversing the ecliptic is accounted for, mathematically, by an epicycle and an equant.

8. "Earlier" (*min qabli*) seems to be, once again, a reference to *Burhān* II, 9 (see in particular II, 9, p. 181.8–14). In that context, Avicenna engages in a significant digression on the sciences that depend on abstraction (*ʿulūm intizāʿiyya*), which appears to be rather close to Alfarabi, *Burhān* IV, 2, pp. 68.25–70.2. On Avicenna's account of the nature of mathematical objects and the role of abstraction, see Zarepour (2016).

Subordination of Part of a Science to Another Science

At *Burhān* III, 3, p. 207.9–17, Avicenna discusses what he takes to be a less common case, namely the subordination of *part* of a science to another science. After giving a brief summary of the first case—which he characterizes once again as the most frequent division (*hāḏā l-qism huwa l-akṯar*)—he suggests that there may be a second way (*wa-qad yakūnu ʿalā waǧh ṯānin*) for a science to fall under another. In this case, it is not a science as a whole but only *part of it* that is subordinated to the other science. The examples could hardly be more interesting. Avicenna seems to be introducing a separate class to accommodate relations such as those between optics and the study of the rainbow, which Aristotle, at *An. Post.* A13, 79a10–13, treats merely as an additional example of the first case of subordination examined earlier. For Avicenna, the distinctive feature of this case is that between the respective domains of the two sciences there is only partial overlap (and hence only part of the subordinate science falls under the superordinate science).

The study of the rainbow and of other analogous phenomena, that is to say, of "images resulting from the reflection (*inʿikās*) [for the Greek *anaklasis*], of sight on something shiny or colored that is not smooth (*amlas*) and polished (*ṣaqīl*)," is (i) a part of natural philosophy, but (ii) its subject is ultimately subordinated to geometry via optics. The science as a whole, however, is not subordinated to optics (or to geometry), even though Avicenna does not give a reason for this claim.[9]

Another example is the study of (a) certain visual angles involving the mean position (*wasaṭ*) and the real position (*muqawwam*) of the celestial bodies and of (b) the angles of parallax (*zawāyā l-inḥirāfāt*).[10] This investigation, according to Avicenna, is (i) a part of astronomy—namely, the science developed in the *Almagest* (*ʿilm al-maǧisṭī*)—while being at the same time (ii) subordinated to optics. It is only the study of those particular kinds of angles, however, and not astronomy

9. These phenomena do not involve the reflection of shape and color on a polished surface. Aristotle briefly discusses this kind of reflection at *Meteor.* Γ2, 372a29–b8 in connection with the investigation of halos, rainbows, mock-suns, and rods (on the rainbow as a reflection, see also *Meteor.* Γ4, 373a32–34). Avicenna briefly mentions the rainbow (*qaws*) at *Burhān* IV, 8, p. 319.9–14 (cf. *An. Post.* B15), and the halo at *Burhān* III, 3, pp. 202.14–203.2; for Avicenna's more extensive treatment of both phenomena, see *Maʿādin wa-āṯār ʿulwiyya* II, 3, pp. 47.1–56.2.

10. The problem concerns the way in which angles may be used to determine distance or size. The expression *inḥirāfāt al-manẓar* is uncommon for parallax, which is usually rendered in Arabic by *iḫtilāfāt al-manẓar.* The context, however, leaves no room for doubt (even though the reference to "that with respect to which a celestial body or an epicycle are observed in terms of proximate or remote distance" remains rather obscure). Parallax is discussed by Avicenna in his account of the *Almagest*, at *Hay'a* V, 8 (lunar parallax); V, 9 (distance of the moon); V, 10 (size of the moon); V, 11 (distance of the sun); V, 12 (moon-sun parallax); and V, 13 (equation of parallax). I am grateful to Robert Morrison for his valuable comments on this passage.

as a whole that falls under optics. In similar cases (for example with regard to the claim that the rainbow has such-and-such properties because of such-and-such a cause), the natural philosopher does not possess the most determinate and proximate explanation (*sabab muḥaṣṣal muqarrib*), and it is the task of the practitioner of optics to provide this kind of cause (*Burhān* III, 3, p. 207.16–17). The result is confirmed at the end of the chapter, where Avicenna returns to the question and explains the role of these examples in Aristotle's argument in *An. Post.* A13.[11]

It is worth noting that a distinction between the relative frequency of a first and of a second type of subordination is also drawn in *Burhān* II, 9, but its scope there is rather different. In that context, as we shall see later, Avicenna qualifies the two main cases under consideration as "more frequent" and "less frequent." The less frequent case, however, is not (or at least it is certainly not presented as) a case of partial subordination as in III, 3, for it involves a superordinate science borrowing questions proved by a subordinate science and using them as principles for the demonstration of some of its own questions. This constitutes a critically important case in Avicenna's epistemology, as it encapsulates the occasional relation between metaphysics and the particular sciences (especially various branches of mathematics and natural philosophy). While the less frequent type in the classification of *Burhān* III, 3 involves a specific case of *partial* subordination, the one in *Burhān* II, 9 discussed in detail later in this chapter maintains unaltered the standard relation of genuine subordination (of mathematical and natural sciences to metaphysics) but accounts for the possibility of occasional "loans" from a lower science to a higher science. This is to say that even if science A (metaphysics) can, under appropriate conditions, use as a principle something proved in science B, it does not follow that A is partially subordinated to B.[12]

Subordination of a Question of a Science to Another Science

At *Burhān* III, 3, p. 208.1–10, the third case concerns the subordination of an individual question of a science to another science. This is the last example considered by Aristotle at *An. Post.* A13, 79a13–16, namely a situation in which there is no systematic relation of dependence between two sciences, and the connection is confined to an isolated question (*masʾala*). The relation is further characterized by

11. At *Samāʿ ṭabīʿī* II, 8, p. 125.14–15 (McGinnis 2009, p. 181, transl. modified), in the context of a series of elaborate arguments against the void, Avicenna introduces an assumption that is scientifically proved elsewhere, namely in logic: "Differentiae are those things by which the essence of something is qualified, whether it is assumed to exist in concrete particulars or that is not taken into consideration. The scientific account of this (*hāḏā l-ʿilm*) is fully given in another discipline" (McGinnis 2009, p. 181n6 suggests that the source might be *Madḫal* I, 13, where Avicenna discusses the concept of differentia; other plausible candidates might be *Madḫal* I, 5 and I, 6).

12. The subordination of all particular sciences to metaphysics is a leitmotif of *Burhān* II, 7 and *Ilāhiyyāt* I.

Avicenna as one that results from adding to the subject of a science an accidental attribute that is foreign to it, in the technical sense of chapters 5 and 7. Once the subject is qualified in this manner, it is possible to investigate the per se accidents that belong to it insofar as it is characterized by that attribute. In similar cases, the task of providing a why-demonstration comes within the purview of the science under which the attribute falls.

The case covers Aristotle's famous example of circular wounds, which are said to heal more slowly than any other kinds of wound. Avicenna deploys the same example and explains it in the language of his theory of per se. The scientific question in need of demonstration is the contention that circular wounds heal with greater difficulty. The conception of wound belongs to the subject of medicine, which is body qualified by health and sickness. Circularity, by contrast, is an accidental attribute of wound, one that belongs to another discipline, namely geometry. The attribute of healing with difficulty is a per se accident that necessarily follows from being circular (inasmuch as the latter is taken to be a property of wound).[13] As a result, "the subject (wounds), through the connection with the foreign accident (*bi-qtirān ʿāriḍ ġarīb*), becomes determinate and disposed for a per se accident to follow necessarily (*li-ltizām ʿāriḍ ḏātī*)" (*Burhān* III, 3, p. 208.3–4). The why-demonstration comes from the science associated with the foreign accident, in this case geometry.[14]

Interestingly, Avicenna introduces a further complication, suggesting that a proper account of this phenomenon may in fact involve a compound cause (*sabab murakkab*), which combines a physical explanation with a geometrical explanation. The two components are captured by a bridge principle of the following sort: in the case of wounds, "healing requires a movement [of the growing skin]

13. At *Burhān* I, 11, p. 106.8–12, while commenting on the notion of appropriateness (*munāsaba*), Avicenna notes:

> In dialectical persuasion one often assumes false endoxic [premises] by means of which a true [conclusion] is produced; and in deductions one often assumes true non-appropriate [premises] by means of which true [conclusions] are produced, as when the physician argues that circular wounds heal more slowly because what is circular has a larger area (*akṯar iḥāṭa*). Cases of this kind are signs (*dalāʾil*), not real demonstrations, because [the premises] are not appropriate. For one uses a major premise [from the domain] of geometry intending to prove by means of it a conclusion (*maṭlūb*) [in the domain] of natural science, without making manifest the appropriate cause.

14. See *Burhān* III, 3, p. 208.5, where Avicenna explicitly characterizes it as the demonstration that gives the why (*al-burhān al-muʿṭī li-limā*). Geometry supplies the explanatory premise "because the circle is the widest figure by perimeter" (that is to say, it has the greatest area-to-perimeter ratio). The explanation is not explicitly given by Aristotle, but there is an interesting discussion in Philoponus, *In. An. Post.* A13, pp. 182.10–183.2, to which Avicenna may be indebted (especially for the idea of an angle that makes it easier for the extremities of a wound to meet).

towards the center; thus, if there is a fixed angle for the direction of the movement [of the growing skin], it is easier [for the extremities] to meet; if there is no [such] angle, the movement on the whole perimeter occurs at the same time, the parts mutually struggle against one another, and the healing is slow" (*Burhān* III, 3, p. 208.8–10).

EXEGETICAL PROBLEMS: AVICENNA THE INTERPRETER

The analysis of *Burhān* III, 3 ends with a remarkable digression, divided into two sections (*Burhān* III, 3, pp. 208.11–209.17).[15] In the first section, Avicenna discusses an example from optics and geometry that he takes to be problematic, while in the second section he concludes the discussion of that-demonstration and why-demonstration with further examples from various disciplines and addresses again the distinction between proximate and remote causes.

In the first section, Avicenna laments the fact that "in the commentaries" (*fī š-šurūḥ*) confusion arises concerning the way in which two sciences are supposed to cooperate and criticizes the use of an unfortunate (and perhaps irremediably ill-conceived) example. The contention under consideration is that the practitioner of optics establishes that the visual cone (that is to say, the bundle of visual rays connecting the eye of an observer to the object seen) disappears when the object is far, but it is the geometer that provides a causal justification for this fact. The putative geometrical principle adduced as an explanation involves a straightforward version of the parallel postulate. But according to Avicenna, it is not even clear how the claim that the visual cone disappears with distance should be understood, and hence, the explanatory role of the geometrical principle is at best vague.[16]

15. The last part of the chapter is discussed in Eichner (2010).

16. The geometer knows that if two lines intersect a third line in such a way that the sum of the internal angles on one side is less than two right angles, then the two lines meet (assuming that they are produced on that side). It is not easy to see how exactly this might be a reason for the fact that, when an object is distant, the visual cone disappears. Avicenna seems to impute the error to an incorrect reading of a diagram that inverts the orientation of the visual cone. The correct way of interpreting the visual cone is by placing its vertex in the observer's eye. This, according to Avicenna, would imply that with distance, the cone becomes larger rather than smaller. But the way in which the parallel postulate is put to use in the argument seems to suggest that the cone is wrongly assumed to have its base in the eye and its vertex in the object. In that case, presumably, with the distance increasing indefinitely, the lines would converge less and less toward the object, and in this sense the cone might be said to disappear. Eichner (2010, p. 93) correctly notes that this is not an example from Themistius or Philoponus, even though the problem bears a remote resemblance to the optical principle (which is, as a matter of fact, discussed by Philoponus, *In An. Post.* A13, pp. 178.18–179.12) that things seen at a nearer distance appear larger because they are seen under a larger angle. The principle that things seen under a larger

Avicenna reiterates his dissatisfaction with the commentary tradition on *An. Post.* A13 before putting forward, in the second section, his own solution to the difficulty. The examples found in Aristotle and in the commentators (*fī t-taʿlīm al-awwal wa-fī š-šurūḥ*) present two problems. Aristotle (and the commentators), in particular, fail to deliver on the promise of showing "two deductions, of the that and of the why, in two different sciences" (*qiyāsayni ʿalā anna wa-limā fī ʿilmayni muḫtalifayni*) (*Burhān* III, 3, p. 209.1–2). According to Avicenna, this is due to the fact that "the approaches adopted in the commentaries just show us two things" (*wa-maʾāḫiḏ at-tafāsīr lahā innamā turīnā amrayni*), both of which fail to go to the heart of the problem. The first is (1) a conflation of the distinction between knowledge of the *that* and knowledge of the *why* with the distinction between observational knowledge and inferential knowledge.[17] According to this unsatisfactory view, in one science the *why* is known by means of a deduction, whereas in the other science the *that* is known on the basis of perception. The second is (2) the view that knowledge of the *that* and knowledge of the *why* in the two sciences concern two different questions, rather than one and the same question.

According to Avicenna, however, the interesting case is not one in which the two sciences are either both concerned with the deduction of different conclusions or one science is concerned with deduction and the other with perception, but rather one in which *one and the same question* is *proved* factually (*that*) in one science and causally (*why*) in the other science. In solving the difficulty, Avicenna explains what he believes Aristotle's meaning to be, when the latter maintains that the lower science is concerned with factual knowledge based on perception. This should not be taken to imply that the practitioner of the lower science has perceptual knowledge of the conclusion (namely, that it is the case), but rather that he makes use of premises that are based on perception and that he proves that the conclusion obtains in virtue of such premises: "The practitioners of an applied science (*aṣḥāb al-ʿamal*) possess deductions that involve premises based on experience and testing (*taǧrībiyya wa-imtiḥāniyya*), and there is hesitation amongst them in establishing or rejecting [a conclusion] on that basis" (*Burhān* III, 3, p. 209.7–8).[18]

Avicenna seems tacitly to presuppose that the interesting philosophical problem is to identify the criteria that allow us to distinguish between why-demonstration and that-demonstration in two sciences when (i) *both sciences* are concerned with

angle appear larger is the fourth definition of Euclid's *Optics*. Avicenna's example remains as interesting as its details are obscure.

17. The point is discussed at *Burhān* III, 3, pp. 206.19–207.8.

18. The distinction between question for testing (*masʾala imtiḥāniyya*) and scientific question (*masʾala ʿilmiyya*) is introduced at *Burhān* III, 1, pp. 192.18–193.15.

deductions (against 1) and (ii) those deductions are of *one and the same* question or conclusion, which they prove through different terms (against 2).[19]

To solve the first problem, Avicenna takes the reference to perception to mean that a deduction involves *premises* based on perception. In this case, there may be two deductions of the same conclusion, one of which makes use of a premise based on perception and gives a that-demonstration while the other gives a why-demonstration. To solve the second problem, Avicenna invokes the distinction between proximate and remote cause. In this case, too, there may be two deductions of the same conclusion, one of which relies on a remote cause and gives a that-demonstration, while the other relies on a proximate cause and gives a why-demonstration. The two solutions are illustrated by the following examples:

(1) Arithmetic—mathematical harmonics—acoustic harmonics
Question: "This note (*naġma*) is not consonant (*muwāfiqa*) with this [other] note."
Explanation: "Because a given chord (*al-watar al-fulānī*) is such-and-such, and because a given note is such-and-such (*an-naġma al-fulāniyya*)" (*Burhān* III, 3, p. 209.9–10).
Both premises are perceptual (*ḥissiyya*), and hence the conclusion derived from them is also perceptual, merely establishing the fact that something has or does not have such-and-such a property.

(2) Geometry—mathematical astronomy—nautical astronomy
Question: "This is not the time for such-and-such a star to be in that position."
Explanation: "Because such-and-such [other] star has not yet risen" (*Burhān* III, 3, p. 209.12).

(3) Optics—natural philosophy—theory of the rainbow
Question: "This arc [of a rainbow] is not a semi-circle."
Explanation: "Because the sun is not on the horizon" (*Burhān* III, 3, p. 209.13–14).

In the first two examples, (1) and (2), the premises are based on testing (*imtiḥāniyya*) and experience. The third example (3) turns on the distinction between remote and proximate causes. The remote cause is given by natural philosophy, while the proximate cause is from the domain of optics. How is the latter identified, and why does it fall under optics? Let us designate the terms as follows: the arc being a semicircle (q), the axis (or pole) of the arc falling on the horizon (p), and the sun being on the horizon (r). Avicenna, following Aristotle, maintains that

19. Avicenna makes a similar point concerning the distinction between that-demonstration and why-demonstration in one science, in an attempt to explain how they may be both about one and the same question. As we have seen in chapter 9, however, this requires a peculiar stipulation to avoid circularity.

if there is a rainbow, then *q*, *p*, and *r* are mutually entailing and causally related. In particular, *q* is the effect, while *p* and *r* are the proximate cause and the remote cause, respectively. Thus, the denial of *q* ("This arc is not a semicircle") entails, as an explanation, the denial of *r* ("The sun is not on the horizon"). But this is only a remote cause of that fact. The real explanation, expressed by the proximate cause, is the negation of *p* ("The axis or pole of the arc does not fall on the horizon"). The premise associated with the proximate cause belongs to optics because its predicative nexus concerns the position of a point whose identification depends in turn on the notion of a visual cone and on the position of the observer (the point in question is called the anti-solar point and is located opposite the sun on the line that connects the eye of the observer to the center of the circle of which the visible part of the rainbow is a circular segment).[20]

According to Avicenna's reconstruction, in all three examples, both sciences proceed deductively in demonstrating one and the same thing (that is to say, they both aim at the same conclusion), showing *that* and *why* it is the case, respectively. In the first two cases, the proof involves premises of different kinds (perceptual and non-perceptual). In the third case, the proof involves premises that are both causal but express different degrees of causality, encapsulated by proximate and remote causes. Thus, *Burhān* III, 3 ends in a way analogous to the way it started, that is to say, with a characterization of the distinction between knowledge of the *that* and knowledge of the *why* in two sciences that is aligned with the first characterization of the distinction between knowledge of the *that* and knowledge of the *why* in one science, where the key distinction is the one between proximate cause (why) and remote cause (that).

VARIETIES OF SUBORDINATION AND EXPLANATION ACROSS THE SCIENCES

In *Burhān* II, 9, Avicenna addresses the theme of subordination from a complementary perspective. Aristotle briefly addresses the relation between *that* and *why* and the hierarchical arrangement of two sciences in *An. Post.* A9. The general principle is that the lower science proves *that* something is the case, while the higher science proves *why* it is the case. When Aristotle resumes the thread

20. The shape of the rainbow is semicircular when the sun is low, that is to say, when it is *on* the horizon (whether rising or setting). By contrast, when the sun is higher in the sky and *above* the horizon, the shape of the arc is less than a semicircle, even though the circle (most of which becomes progressively invisible, hidden below the horizon) is larger. The pole of the arc is the anti-solar point on the axis of the visual cone connecting the eye to the locus of points that reflect the parallel rays of the sun at the same angle. See *Meteor.* Γ2, 371b26–372a10 on the shape, size, and colors of the rainbow; Γ5, 375b16–19 for the claim that the (visible part of the) rainbow is never greater than a semicircle; and Γ5, 375b20–29 on its relation to the rising or setting of the sun.

of the discussion in *An. Post.* A13, he does not seem to add much to the analysis of A9 (especially with regard to the first case, which as noted previously is the most representative of the three). In *Burhān* II, 9, by contrast, Avicenna engages in a discussion that is remarkable both for its extent and for its level of detail.[21] The focus, in particular, is on the different ways in which two sciences aid or contribute (*maʿūnat al-ʿilmayni*) to the knowledge of the *why* and the knowledge of the *that*.[22]

The taxonomy is introduced by a series of comments that are worth examining. First, the subordination of optics to geometry and of music to arithmetic depends on the fact that the two pairs of sciences have the relevant subjects in common, a fact that has been previously established elsewhere (*mušāraka fī l-mawḍūʿ bi-waǧhin ʿalā mā qīla min qabli*).[23] Second, subordination presupposes that the lower science provides a that-demonstration (*burhān anna*), while the higher science provides a why-demonstration (*burhān limā*). The premises in the lower science are "taken as conceded" (*maʾḫūḏa musallama*) but are not known with respect to their causes; hence the conclusions derived from those premises are not genuinely certain. This is because the essential causes (*ʿilal wa-asbāb ḏātiyya*) are only given by the higher science. It is therefore only by virtue of a peculiar type of transfer from the higher science to the lower science that certainty can be attained (*Burhān* II, 9, p. 177.8–14).

What is especially interesting about this discussion, however, is Avicenna's rejection of an interpretation (*taʾwīl*) according to which the sense of the claim is that the that-demonstration in the lower science and the why-demonstration in the higher science are of one and the same question (*al-masʾala bi-ʿaynihā*). In principle, this is not an ideal way to proceed but rather a "license required by the hardship and limitedness of man in his disposition for what is needed before and at the right time for his soul" (*Burhān* II, 9, p. 177.15–23). It is only because the human intellect has its limitations that we cannot always explain in a proper manner. For if we could, "it would be better to demonstrate the states of lines assumed in optics and numbers assumed in music not in optics and music but in geometry

21. See especially *Burhān* II, 9, pp. 178.8–183.8, as the first part of the chapter is more closely concerned with the analysis of the example of the method for the squaring of the circle ascribed by Aristotle to Bryson at *An. Post.* A9, 75b40–76a2.

22. See *Burhān* II, 9, p. 178.8; cf. also *Naǧāt* I, 131, where the same theme is addressed under the heading of *taʿāwun*.

23. This is plainly a reference to the discussion of subordination in *Burhān* II, 7. In the same passage, Avicenna notes that in the case exemplified by these two pairs, the middle term comes not from the genus (that is to say, the subject) of the discipline but from a higher genus (*ǧins aʿlā*), and "it is transferred from it to what is below it," namely to the lower science (*Burhān* II, 9, p. 177.7–8). The transfer of demonstration is the theme of the preceding chapter (*Burhān* II, 8), where Avicenna identifies two types of transfer and investigates the second type as the one relevant for the theory of science.

and arithmetic, and adjust their conclusions for the practitioners of optics and music" (*Burhān* II, 9, p. 178.1–3).[24] Avicenna writes:

> Text 10.1: *Burhān* II, 9, p. 178.3–6
> Since human strengths are limited with regard to knowledge of all the premises that happen to be needed, in optics and music—for those are a great many—it is not possible to work them out exhaustively (*lam yumkin iʿdāduhā iʿdādan mustawfan*). Rather, in that domain one can work out what is needed in terms of fundamental principles without theorems (*al-uṣūl dūna l-furūʿ*) or what is needed in terms of fundamental principles that one can be aware of without [other] principles that one [only] becomes aware of at a later stage.[25]

Explanation across multiple sciences is inevitable only because there are intrinsic limitations to the power of the human intellect, which simply cannot entertain all the required nexuses in order to establish scientific facts on the basis of their proper causes.

THE TAXOMONY OF *BURHĀN* II, 9

The cases of subordination identified in *Burhān* II, 9 are again three, but there is a critical difference from the classification of III, 3. The first case involves two sciences that differ with respect to their higher or lower hierarchical status (*fī l-ʿuluww wa-d-dunuww*). In line with the corresponding (first) case of *Burhān* III, 3, this is regarded as the most frequent type (*al-akṯar*). The second case, however, does not correspond to the second case of *Burhān* III, 3. Rather, it introduces a less common kind of relation, which proves to be especially interesting with regard to

24. The need to rely on multiple proofs from different sciences is a consequence of certain limitations of our intellectual powers (*quṣūr*). The reference in *Burhān* II, 9 is not just a throwaway remark. In fact, it is not at all uncommon in Avicenna's works. A similar point is made, for example, at *Ilāhiyyāt* I, 3, p. 21.6–10: "Due to the weakness of our soul we cannot follow this demonstrative method, which consists in proceeding from principles to secondary things and from causes to effects, except in some ranks of the existents and in a non-detailed manner. This science, therefore, [that is to say, metaphysics] ought to be prior to all sciences, even if it is posterior to all of them with regard to us." But the examples are many, ranging from the definition of logic (a canonical instrument for the attainment of knowledge in a measure consistent with the power of the human mind) to some of Avicenna's comments on the power of his own intellect (or lack of power, in the case of some of his sources and interlocutors) disseminated (i) in the autobiography, (ii) in the letter to an anonymous disciple, (iii) in the letter to Kiyā, and indirectly (iv) in the memoirs of a disciple writing from Rayy; on (i)–(iv), see Gutas (1988).

25. Avicenna adds that negligence often generates confusion. This, along with the number of required premises, makes it difficult to identify the relative contribution of different sciences to the proof of one and the same conclusion: "Since the effort in the discovery (*al-imʿān fī l-istinbāṭ*) brings about the need for other premises, one neglects to identify [the relative contribution of] the two sciences and to attach to a science what comes from it" (*Burhān* II, 9, p. 178.7–8).

metaphysics. The third case is only mentioned in passing and corresponds to the third case of *Burhān* III, 3, namely the subordination of an individual question of a science to another science.

First Case: Subordination of a Lower Science to a Higher Science

The most frequent case (*Burhān* II, 9, pp. 178.10–179.14) appears to include two sub-cases: (i) the first involves a question proved by a higher science and assumed as a principle by a lower science, while (ii) the second involves one and the same question proved both by the lower and by the higher science but through different proofs, that is to say, by means of different middle terms. The crucial distinction is that in the first sub-case, which corresponds to the first type of transfer of demonstration developed in *Burhān* II, 8, the use of the conclusion of a demonstration belonging to the higher science as a premise in the lower science ipso facto implies that the two conclusions are distinct: the conclusion of the higher science becomes a premise used by the lower science to prove another conclusion.[26] The second sub-case, by contrast, corresponds to the second type of transfer developed in *Burhān* II, 8 (both types of transfer are discussed later), which paradigmatically involves the use of a proximate cause in the higher science and of a remote cause in the lower science to prove one and the same conclusion. The latter case is illustrated by a series of detailed examples.[27] Avicenna writes:

> Text 10.2: *Burhān* II, 9, p. 178.10–11
> In most cases, the demonstration that gives the why (*al-burhān al-muʿṭī li-limā*) from the higher science to the lower is completed only in virtue of the fact that the higher supplies the lower with premises that are taken as principles of demonstration [by the lower science].[28]

According to Text 10.2, to give the *why* means in most cases simply to provide the lower science with demonstrated truths, which the lower science assumes as premises. The higher science proves a conclusion, which is then assumed, as a principle, by the lower science. The higher science indirectly contributes to the proof of *why* the conclusion of the lower science holds, because it supplies an explanatory premise to the lower science. Avicenna writes:

26. This case corresponds to type (1.3.1.1) from the discussion of *Burhān* II, 7 in chapter 5.

27. Proximate and remote are defined with respect to the attribute or major term. Thus, in a sequence of causally explanatory terms, where the minor term belongs to the lower science, the proximate cause (which is proximate relative to the major term and remote relative to the minor term) belongs to the higher science, whereas the remote cause (which is remote relative to the major term and proximate relative to the minor term) belongs to the lower science.

28. At *Burhān* II, 9, p. 179.14, Avicenna notes: “Most often this is the way in which the higher science assists [the lower science] with regard to the why.”

Text 10.3: *Burhān* II, 9, p. 178.11–13
Besides, in cases of this kind, in one science the middle term of a demonstration is one cause, and in the second science the middle term of another demonstration is another cause prior to the former (*qabla tilka l-ʿilla*), that is to say, a cause of the cause, and so the lower science does not give the cause in a complete way (*bi-t-tamām*).

The higher science gives the proximate cause (the one closest to the attribute and which expresses the cause of the attribute at the highest possible level of generality), while the lower science gives the remote cause (the one furthest from the attribute and closest to the subject).

The examples by means of which Avicenna illustrates this case are of great interest. (i) The first example involves natural philosophy and metaphysics, and concerns the relation between different proofs of the homogeneity (*tašābuh*) and stability (*ṯabāt*) of the first (heavenly) motion (*ḥaraka ūlā*) (*Burhān* II, 9, pp. 178.14–179.3).[29] Avicenna notes that, in cases of this sort, questions are often *repeated*, that is to say, they are proved in different ways by different sciences, which provide different kinds of explanations. The reason adduced for this otherwise seemingly redundant multiplication of proofs—that is to say, proofs of "repeated questions" (*masāʾil muraddada*), in "their repeated occurrence" (*tardīduhā*)—is once again the limited strength of the human intellect and the extent to which it is capable of drawing distinctions (*quṣūr munan an-nās ʿan al-mubālaġa fī t-tamyīz*).[30] Natural philosophy proves that the first motion has the attribute of being homogeneous (*tašābuh*) in virtue of the fact that such a motion presupposes "a nature that has no contrary" (formal cause) and "a simple matter that is undifferentiated, such that it is impossible for it to undergo corruption or change" (material cause). Metaphysics, by contrast, proves that the first motion has the attribute of being homogeneous in virtue of the fact that it tends toward the pure good (an efficient

29. The attribute of being homogenous (or being characterized by mutual resemblance, that is to say, by a certain kind of uniformity) (*tašābuh*) is different from the canonical notion of assimilation (*tašabbuh*) employed in *Ilāhiyyāt* IX, 2–3. A reference to homogeneous motion having no contrary (*mutašābih al-ḥaraka lā muḍādda fīhā*) is at *Burhān* II, 6, p. 160.4–5, on which see also *Samāʿ ṭabīʿī* IV, 3, p. 271.4–5. The concept is used in this sense also in the cosmological section of the *Išārāt*, *Namaṭ* 6, on ends and principles; see Goichon (1951, pp. 406–407) for a translation (*se rassembler*) implying the same meaning. A variant of this example is also discussed in Abū l-Barakāt al-Baġdādī, *Muʿtabar* IV, 2, pp. 211.16–212.2. Avicenna's contention in this section should be understood against the backdrop of *Ilāhiyyāt* VIII, 1, on the first cause; *Ilāhiyyāt* VIII, 3, on the finiteness of chains of final causes; and *Ilāhiyyāt* VIII, 6, on perfect existence. On the proximate cause of the first motion, see also *Ilāhiyyāt* IX, 1, p. 381.10–11.

30. A question is repeated if it is the conclusion of two distinct demonstrations (in more than one science).

cause), the pure intellect (another efficient cause), and pure existence (the final cause).[31] The proofs in the two sciences are different because they involve different middle terms. This is in line with the requirements laid down in *Burhān* III, 3 and serves another important purpose. For it is, among other things, on account of the fact that Aristotle does not provide different demonstrations for different sciences that Avicenna believes the model illustrated in *An. Post.* A13 is inadequate. Natural philosophy fails to provide a fully explanatory demonstration, a why-demonstration without qualification (*burhān limmī muṭlaqan*). The reason is that a physical proof involving the middle terms mentioned earlier shows merely that the first motion has a certain attribute as long as a certain kind of matter and a certain kind of nature exist, but it does not prove without qualification the perpetual existence of either of those necessary conditions. This task is accomplished by metaphysics, which is therefore the science that supplies a fully explanatory demonstration—"the perpetual why-demonstration without qualification" (*al-burhān al-limmī ad-dā'im muṭlaqan*)—of the fact that and the reason why the attribute in question belongs to the first motion.[32]

(ii) The second example also involves natural philosophy and metaphysics. The question at stake is whether the earth does not have a perfectly spherical shape because its surface is characterized by certain irregularities. The physical proof shows that water falls into depths and crevasses, covering the surface of the earth in many regions.[33] Once again, what differentiates the two proofs in the two sciences is the nature of the *explanantia*. The causes alluded to in the physical proof are material and efficient (*Burhān* II, 9, 179.4-11). Avicenna writes:

> Text 10.4: *Burhān* II, 9, p. 179.4–11
> Water flows by nature into cavities, earth is dry and is not shaped per se and preserves accidental shapes; if it suffers generation or destruction, what remains where the corruption took place is not aggregated in a spherical shape and what remains where generation took place is an elevation; the same applies to other causes of the transfer of a part from its place; water and air on the other hand are aggregated according to their own shape, whether one adds or removes from them. This shape is the simple spherical shape, and a simple nature cannot require anything else.[34]

31. See *Ilāhiyyāt* VIII, 6, p. 355.6–9. At *Ilāhiyyāt* I, 3, p. 20.9–11, Avicenna notes in particular that only metaphysics provides explanatory knowledge concerning remote final causes.

32. The same applies to natural philosophy and mathematics with regard to the proofs of the sphericity of the earth, of water, and of the celestial bodies (note that this is a different problem from the one discussed next, which concerns the fact that the body of the earth is not *really*, that is to say not perfectly, spherical).

33. The specific problem of *Burhān* II, 9 seems to be directly related to *Samāʿ ṭabīʿī* IV, 11, where Avicenna discusses dryness and the imperfectly spherical shape of the earth; see Freudenthal (2018).

34. See *Ilāhiyyāt* III, 9, pp. 148.14–151.11, where Avicenna demonstrates the existence of the circle and refers to a physical proof that establishes first the existence of the sphere (for there is a simple

Metaphysics, by contrast, provides an explanation in terms of the final cause, that is to say, the fact that the elements ultimately come to a stop in their natural place.[35] In this case, the middle terms are different in the two demonstrations, while the conclusion is one and the same. The subordinate science provides an incomplete cause, and the ultimate explanation falls within the purview of the superordinate science.

Second Case: A Principle of a Higher Science Is Proved in a Lower Science

The second case (*Burhān* II, 9, pp. 179.14–180.18) represents the least common situation but constitutes nonetheless a critical instance of subordination.[36] It obtains when a higher science assumes principles from a lower science. Avicenna writes:

> Text 10.5: *Burhān* II, 9, pp. 179.14–180.6
> The least frequent case consists in the fact that sometimes the higher science assumes the principles of the *why* from the lower science, provided that those principles [borrowed from the lower science] are not dependent for their soundness on the soundness of principles that are only proved in the higher science or that will be proved by means of principles from the higher science (indeed [what is] proved at a second stage in the higher science by means of [those questions of the lower science assumed as principles in the higher science] are only questions [of the higher science] that are [in turn] not principles for [those same questions of the lower science assumed as principles by the higher science] or for the part to which they belong in this lower science).[37]
>
> Rather, it is as if some questions of one science were principles with regard to some [other] questions of it, in virtue of the mediation of questions from it that are closer to the principles than the former are. Thus, it is not implausible that the questions of a given [lower] science should be [(a)] proved by means of principles from another [higher] science, and that those questions [of the lower science] then become principles [of the higher science] for other questions of that other science without circularity. Thus, this is the state of some questions that are proved in the lower science (by means of principles from the higher science) and by means of which some questions of the higher science are proved.[38]

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body, every simple body has a natural shape, the natural shape of simple bodies is undifferentiated in its parts, and every undifferentiated shape is circular).

35. *Burhān* II, 9, p. 179.12–13.

36. This case corresponds to type (1.3.1.2) from the discussion of *Burhān* II, 7 in chapter 5 (a question proved in the lower science becomes a principle assumed in the higher science); cf. also *Burhān* III, 8, p. 247.3–11 (on *An. Post.* A28) and *Burhān* II, 10, p. 184.6.

37. The first qualification is meant to rule out circularity. The parenthetical claim introduces the legitimate case, which is then explained in the second half of the passage.

38. See *Ilāhiyyāt* I, 3, p. 20.12–17.

> Or [(b)] these principles assumed [by the higher science] from the lower science are in no way proved by means of principles from the higher science. Such is the case when they are proved by means of principles that are evident [(ba)] by themselves, [(bb)] by perception, or [(bc)] by experience.

In Text 10.5, the key problem is how to avoid circularity. If a higher science borrows questions proved by a lower science to use them as principles, and then goes on to prove conclusions that are in turn used as principles by the lower science, how can the process not result in a *petitio principii*? It would seem that conclusions proved in the lower science cannot be safely assumed as premises in the higher science, if they themselves were ultimately to depend on premises of the lower science that are proved in the higher science. To circumvent the problem, Avicenna clarifies that the questions proved in the lower science and assumed as principles by the higher science must be either independently established (that is to say, proved by means of axioms or principles that are self-evident or based on empirical observation), or—if they are dependent on principles of the higher science—they must depend on distinct principles that are not in turn proved by means of them.

Thus, assuming that

1. A is the higher science and B the lower science;
2. PA_i is a principle of A;
3. QA_j is a question of A, proved in A;
4. PB_k is a principle of B; and
5. QB_l is a question of B, proved in B,

Avicenna's contention amounts to the following. If $PA_i = QB_l$, then any PB_k used to prove QB_l must be either (i) independent of A (because it is self-evident or based on empirical observation) or (ii), if it comes from A, then it must be distinct from PA_i and from any QA_j in whose proof PA_i is contained. To put it otherwise, questions proved in the lower science and borrowed by the higher science as principles cannot be in turn dependent for their own proof on the principles of the higher science without qualification. Rather, they must either be independently available or, if they are established by means of principles of the higher science, the latter cannot be among those that are proved, in the higher science, by the principles borrowed from the lower science.

The problem is also discussed at *Ilāhiyyāt* I, 3, pp. 19.10–20.11, where Avicenna confirms that circularity is neutralized in three ways.[39] The questions of the

39. In that context, Avicenna refers to what has already been established, in all likelihood, at *Burhān* II, 9. On the nature of the principles and questions of metaphysics in general, see *Ilāhiyyāt* I, 2, pp. 14.3–15.1.

lower science assumed as principles in the higher science are either (a) principles of different things; (b) self-evident; or (c) such that the lower science gives the *that*, and the higher science gives the *why*.[40] The tripartite classification implies that (a) A-principles are not principles of every B-question but only of some B-questions (otherwise assuming any B-question as an A-principle would result in a *petitio principii*); (b) some B-principles may be immediate (axioms that are either self-evident or immediately grounded in perception or experience); and (c) A provides causal knowledge, while B provides just factual knowledge.

In summary, when a question from a lower science is assumed as a principle by a higher science, that is to say, when a B-question is assumed as an A-principle, one of the following must obtain:

(a) Either the B-question is proved in B from B-principles that are in fact A-questions; but those A-questions are not in turn proved in A from principles that include that B-question
(b) Or the B-question is not proved in B from B-principles that are in fact A-questions; in which case it is proved in B from principles that are either
  (ba) self-evident,
  (bb) based on perception, or
  (bc) based on experience.

Third Case: Subordination of a Question

The last case (*Burhān* II, 9, p. 180.10–14) corresponds to the third case of *Burhān* III, 3, where the distinction between that-demonstration and why-demonstration refers to an individual question. A series of further remarks specify conditions that we will encounter later to strengthen the connection between subordination and subjects. Avicenna reiterates that frequently one science gives a that-demonstration (*muʿṭiyan fī masʾala wāḥida bi-ʿaynihā burhān al-anna*) while another science gives a why-demonstration of the same question (*Burhān* II, 9, p. 180.10–11), but adds that no two particular sciences (*ʿulūm ğuzʾiyya*) can both give distinct why-demonstrations of one and the same question (*Burhān* II, 9, p. 180.17–18).[41]

40. For the claim that natural philosophy and mathematics occasionally supply a that-demonstration without a why-demonstration (especially in the case of remote final causes), see *Ilāhiyyāt* I, 3, p. 20.9–11.

41. Examples include the distinction between that-demonstration obtaining through a sign (*burhān anna bi-d-dalīl*) in mathematics and why-demonstration in natural philosophy concerning the sphericity of water, the fact that the earth is spherical and at the center of the universe (on the latter claim, see also *Burhān* II, 4, p. 144.4) and the sphericity of the celestial bodies (*ağsām samāwiyya*). Another standard example is the relation between arithmetic and geometry in *Elements* X. Cf. also Alfarabi, *Burhān* IV, 2.

THE TRANSFER OF DEMONSTRATION

In *Burhān* II, 8, Avicenna examines in detail the ways in which an explanatory premise or term from one science may be passed on to another science.[42] This process is designated by the technical expression "transfer of demonstration" (*naql al-burhān*).

Avicenna identifies two types of transfer. In the first case, (i) the conclusion of a demonstration in a science becomes the premise of another demonstration in another science (in other words, one science proves a nexus, which becomes part of the proof of a different nexus established in the other science). In the second case, (ii) the conclusion of a demonstration in a science is proved by means of a middle term taken from another science (in other words, there is one nexus, which the two sciences prove separately by means of middle terms carrying a different explanatory power). The notion encapsulates the ways in which explanation across sciences works. He writes:

> Text 10.6: *Burhān* II, 8, p. 169.1–4
> Transfer of demonstration is said in two ways. In the first way, it is said of [(i)] the case in which something is assumed as a premise in a science while its demonstration is in another science (whereupon it is conceded in this science and its demonstration is transferred to that science, which is to say that it is passed on to [that] science by means of it). In another way, [transfer of demonstration] is said of [(ii)] anything assumed in a science on condition of it being sought, then it is demonstrated by means of a demonstration whose middle term is from another science, in which case the parts of the deduction, that is to say the terms, can fall under the two sciences.

Although the two types introduced in Text 10.6 are equally legitimate and, as a matter of fact, are both employed in the practice of scientific reasoning, Avicenna seems to think that the second type is somehow more fundamental, and this is the focal sense in which the notion of transfer should be understood. In particular, he identifies the following requisite:

> Text 10.7: *Burhān* II, 8, p. 169.9–12
> We mean here by transfer of demonstration what is according to the second division. This [means that] it is not possible for one of the two sciences not to fall under the other. In general, they must share the subject in order to share the attributes, either without qualification or in a certain way. This sense [of transfer presupposes] that one [science] falls under the other, whereupon it is possible to transfer the

42. A simplified account of transfer is at *Naǧāt* I, 132: (i) something is a question in science A, and a principle in science B; (ii) something is a question both in science A and in science B, but the middle terms are different, in which case the why-demonstration takes the minor term from the lower science and the explanatory middle term from higher science.

demonstration from the general [science] to the specific. Thus, the general [science] gives the cause to the specific, as we will clarify later.[43]

In Text 10.7, Avicenna establishes that the focal meaning of transfer in the theory of science is the one that presupposes a genuine relation of subordination. It is in the context of this discussion that Avicenna explicitly identifies two fundamental manners of taking the terms of a demonstration, depending on the nature of the per se relations holding between them, as we have seen in chapters 4, 5, and 7 from different angles. He writes:

Text 10.8: *Burhān* II, 8, pp. 169.13–170.6
If [the two sciences] share the subject in one of the other ways, they may coincide in the deduction. For [(i)] if the middle term is a genus of the minor [term] or a constitutive differentia or one of these constituents, and the major [term] is a [per se] accident of that genus or of that constituent—which is the first manner of taking demonstrative [terms] (*wa-huwa al-maʾḫad al-awwal min maʾāḫiḏ al-burhāniyyāt*)—or [(ii)] [if] the middle [term] is a per se accident of the minor [term] and the major [term] is another per se accident or the genus of a [per se] accident or its differentia or a constituent of it—which is the second manner of taking demonstrative [terms] (*wa-huwa al-maʾḫad aṯ-ṯānī min al-burhāniyyāt*)—(and we have explained that there is nothing other than these two), then the mode of investigation in the two sciences is one and the same. Otherwise the deduction is not demonstrative either in [at least] one of the [two sciences] or in both at the same time. Rather, it may be demonstrative in one and non-demonstrative in the other, or non-demonstrative in both, since it is evident that demonstration must be [according to] one of these two manners of taking [the terms], and we have said enough about this.[44]

The condition for the second (and more significant) case of transfer is that one science be *under* the other (the theme is extensively treated in *Burhān* II, 7, where Avicenna gives a full account of the pairs that satisfy this condition). If two sciences share attributes or affections (*aṯār*), then they share the subject with or without qualification, and in this case a transfer of demonstration is possible.

But when a question proved in one science is assumed as a principle of the other science, how should the condition of a shared subject be understood? The scheme Avicenna has in mind is as follows, where S1 and S2 are sciences, A is a shared subject, and AaB, BaC, CaD are immediate predications:

43. The reference is to *Burhān* II, 9 (and indirectly to III, 3).

44. Text 10.8 is relevant to the classification of the terms of scientific propositions and the taxonomy of demonstrations (the combination of premises) of *Burhān* II, 6 discussed in chapters 4 and 7. In fact, the reference ("we have explained that there is nothing other than these two") is, in all likelihood, to the discussion of the constraints that govern the admissible predicates of scientific questions in II, 6 examined earlier.

S1: AaB; BaC ⊢ AaC
S2: AaC; CaD ⊢ AaD

Avicenna's argument is that in such cases a coincidence in deduction is a consequence of the fact that there are only two basic manners of taking the terms in a demonstration, and both of them presuppose a coincidence in subject. This is because according to the first manner (in a first-figure demonstration), the middle term is a per se 1 attribute of the minor term and the major term a per se 2 attribute of the middle term (bearing in mind that the major term cannot, in this case, be a per se 1 attribute of the middle term); according to the second manner, the middle term is a per se 2 attribute of the minor term, while the major term is either a per se 1 or a per se 2 attribute of the middle term. In either case, the essential relations expressed by the premises require a shared subject, with or without qualification, and the mode of investigation (*naḥw an-naẓar*) in the two sciences is said to be one and the same. The proof is by reductio: if this were not the case, the deduction would not be a demonstration in both sciences but rather a demonstration in one science and not in the other, or not a demonstration in either science. In other words, the types of terms in the described schemata of demonstration impose certain constraints based on the idea that two sciences that have genuinely distinct subjects cannot coincide in the structure of a demonstration because the set of admissible attributes indirectly requires coincidence in subject. Thus, Avicenna proves the required identity of the subject based on the logical structure of demonstration and the admissible types of premise pairs, which are in turn determined by the types of predicates involved.

Finally, an interesting point concerns a constraint on the minor premise. If the subject of a scientific question is the same as the subject of the science, a species of the subject of the science, or an accident of the subject of the science, then it follows that a minor premise involving such a subject cannot be shared across different sciences (*Burhān* III, 8, p. 255.7–9). The minor premise is different both when the questions are different, as in the following case:

S1: AaB; BaC ⊢ AaC
S2: AaC; CaD ⊢ AaD

and when the question is one and the same (that is to say, repeated or shared), as in the following case:

S1: AaB; BaD ⊢ AaD
S2: AaC; CaD ⊢ AaD.[45]

45. At *Burhān* III, 8, p. 252.13–15 (a gloss on *An. Post.* A32, 88a30–36), Avicenna notes that two sciences, neither of which is subordinated to the other, cannot share proper principles.

This raises an interesting question that we have encountered already in connection with *Burhān* III, 3 with regard to the distinction between that-demonstration and why-demonstration, both within one and the same science and within different sciences: whether it is one and the same thing that that-demonstration and why-demonstration are about, or rather different things.

ONE QUESTION OR TWO QUESTIONS?

When a science is subordinated to another science, are there identical questions that the two sciences demonstrate in different ways (that is to say, through different middle terms), or are the two sciences always concerned with different questions (one of which is a question in one science and a principle in the other)? This is a characteristic problem of explanatory subordination for Avicenna, which seems to involve not only the practical issue of determining the conditions under which one procedure or the other is being used on a given occasion, but also the more general question of whether both procedures are equally legitimate.

Avicenna discusses the problem in various contexts, specifying a number of constraints. As noted, he explicitly rules out the possibility of multiple genuine why-demonstrations in different particular sciences: "It does not happen in the case of particular sciences that the two sciences simultaneously give the why for one and the same question" (*Burhān* II, 9, p. 180.17–18).[46] But when there is subordination (in one of the senses introduced earlier), it is possible for two sciences to give different kinds of why-demonstrations (one of which, however, Avicenna would characterize as a that-demonstration, trading on an ambiguity that we have identified already in *Burhān* III, 3 when looking at the distinction from the perspective of a single science). Such cases either involve (1) the subordination of a particular science to another particular science, in which case the higher science gives the explanation; (2) the subordination of a particular science to metaphysics, in which case metaphysics gives the ultimate cause, and the particular science gives a proximate cause; or (3) the (partial) subordination of certain questions in metaphysics to particular disciplines, which is an admissible, if rare, circumstance.

There seems to be a parallel with the case of definition, for just as there is only one real definition of an essence, in the same way there may be only one real causal demonstrative chain connecting an attribute to its subject. The frequent case

46. See also *Burhān* III, 8, pp. 247.10–248.4 (on *An. Post.* A29). The question is whether there may be different demonstrations of one and the same thing, and in particular whether there may be different demonstrations whose middle terms are not related in such a way that one is predicated of the other. The triplets of terms discussed in III, 8 are (i) human, animal, nutritive; (ii) human, sleeping, nutritive; (iii) receptive of pleasure, moving, changing; (iv) receptive of pleasure, resting, changing; (v) human, capable of laughter, capable of wonder; and (vi) human, bashful, capable of wonder.

where there are multiple why-demonstrations involving different middle terms that are more or less remote presupposes one form of subordination or another (the two typical cases being subordination of one particular science to another, like optics to geometry, and subordination of a particular science to metaphysics).

Distinct Questions

If the questions are distinct in the two sciences, then the two demonstrations may be represented as follows:

S1: BaC; CaD ⊢ BaD A—B—C—D (why) (S2 is only completed by S1)
S2: AaB; BaD ⊢ AaD A—B—C—D (that) (BaD is demonstrated by S1)

Assume S1 is the higher science, S2 the lower science. The conclusions are different, and the major premise of the demonstration in the lower science is the conclusion of the demonstration in the higher science.[47] When the major premise of the demonstration in the lower science is proved by a higher science, the major term may be the same in the two (different) conclusions. Another problem, discussed in *Burhān* III, 8, concerns the relation between the middle terms, which in this case are sequentially ordered in such a way that one is universally predicated of the other.

Repeated Question

If a question is the same in the two sciences, then the two demonstrations may be represented as follows:

S1: AaC; CaD ⊢ AaD A—B—C—D (why)
S2: AaB; BaD ⊢ AaD A—B—C—D (that)

In this case, the conclusion is the same in both sciences. What differs are the middle terms by means of which that conclusion is demonstrated in the two sciences. This arrangement raises a few questions. What is the relation between the middle terms? Is one predicated of the other or not? And how is this pair of demonstrations related to the least common case discussed in *Burhān* II, 9? In fact, this seems to correspond to the first type of distinction between that-demonstration and why-demonstration discussed at the beginning of *Burhān* III, 3, which involves non-immediate terms. For in such cases, both middle terms are genuine causes, even though one of them is a cause that is not caused, while the other is a cause that is itself caused, at least relative to the context under consideration.

47. The same case can be made concerning the minor premise of the deduction in the lower science.

TABLE 15 Why-demonstration and that-demonstration of one question and of two questions, in one science and in two sciences

		Scientific questions	
		One question	Two questions
Sciences	One science	(i) A—B—C—D (C = proximate cause; B = remote cause; D = effect) 1. AaC; CaD ⊢ AaD (why) 2. AaB; BaD ⊢ AaD (that)	(i) B = cause; C = effect of B 1. AaB; BaC ⊢ AaC (why) 2. AaC; CaB ⊢ AaB (that) (sign) (ii) D = cause; B and C = effects of D 1. AaB; BaC ⊢ AaC (that) 2. AaC; CaB ⊢ AaB (that) 3. AaD; DaC ⊢ AaC (why) 4. AaD; DaB ⊢ AaB (why)
	Two sciences	(i) A—B—C—D (if BaC is true) (ii) A—B—D and A—C—D (if BaC is not true) (C = proximate cause; B = remote cause; D = effect) Transfer (type 2) S1. AaC; CaD ⊢ AaD (why) S2. AaB; BaD ⊢ AaD (that)	(i) A—B—C—D (C = proximate cause; B = remote cause; D = effect) Transfer (type 1) S1. BaC; CaD ⊢ BaD (why) S2. AaB; BaD ⊢ AaD (that)

Avicenna's classification of the distinction between that-demonstration and why-demonstration in one science and in multiple sciences is summarized in table 15.

After discussing in detail the problem of subordination and explanation across sciences, at *Burhān* II, 9, pp. 180.18–183.8, Avicenna concludes his analysis with a first account of the four types of Aristotelian causes (formal, material, efficient, final). The four causes encapsulate the fundamental types of explanation in Avicenna's metaphysics and epistemology and are therefore a critical ingredient of his theory of science, both for demonstration and for definition.

11

The Four Causes in Demonstration and Definition

CLASSIFICATION OF THE FOUR CAUSES

The four Aristotelian causes (formal, material, efficient, and final) are key ingredients of Avicenna's theory of science.[1] The explanatory premises of demonstration encapsulate causal nexuses that may ultimately be traced to the four causes. And the features of the primitive items in a science, which are captured by definitions, explain their derivative features in the same four ways.[2]

A sign of the centrality of the four causes for Avicenna's theory of science, in addition to their being consistently discussed throughout the *Posterior Analytics* complex, is the fact that he offers no fewer than three different classifications of them in the *Burhān*. The three classifications analyze the same four kinds of causes but approach them from different perspectives. The first classification is introduced, at *Burhān* II, 9 pp. 180.18–183.8, as a digression on *An. Post.* A9, after the extensive discussion of subordination and explanation across the sciences examined in

1. The main sources for Avicenna's discussion of the four causes outside the *Posterior Analytics* complex are *Ilāhiyyāt* I, 1 and VI, 1 (on causality in general); V, 3 (on efficient causality); V, 4 (on material and formal causality); and the extensive discussion of VI, 4 (on final causality) and *Samāʿ ṭabīʿī* I, 9–12 (cf. also *Samāʿ ṭabīʿī* I, 15, in particular, on the way in which causes contribute to the investigation of why-questions). For a detailed analysis of the four causes in Avicenna's natural philosophy, see Lammer (2018). On material and formal causality in Avicenna's metaphysics, see Bertolacci (2002).

2. At *Burhān* IV, 4, p. 294.9–10, Avicenna notes that knowledge of the number of causes is required by the fact that "all real demonstrations and some or most definitions are only completed by means of the causes."

chapter 10.[3] The second classification, at *Burhān* IV, 4, pp. 294.9–295.16, marks the transition from *An. Post.* B10 to B11 (that is to say, from the taxonomy of definition to the inclusion of the four causes in the structure of demonstration) and serves as a brief summary of B11, focusing mainly on Aristotle's examples.[4] The third and most detailed classification, at *Burhān* IV, 5, pp. 296.1–297.9, is effectively an independent and more elaborate analysis of B11, characterized by several original distinctions and examples.

In *Burhān* II, 9, the four causes are designated as (i) the principle of motion (efficient cause), (ii) the subject (material cause), (iii) the form (formal cause), and (iv) the end or perfection for the sake of which something is (final cause). The focus of the first discussion is the relation among the four causes and the requirement that the principles of demonstration be appropriate (*munāsiba*) to their conclusions. As we have seen, this has already led to an extensive digression on the distinction between that-demonstrations and why-demonstrations in different sciences. The treatment of the four causes, in *Burhān* II, 9, is another digression of the same sort. Its main goal is to establish, in particular, which causes are relevant for which subjects (metaphysical, natural, mathematical) and whether all four causes or only some of them are employed in the demonstrations and definitions of a given science. The decisive criterion for these two questions is the nature of the subject of the science, which may generally be a composite of matter and form (that is to say, a physical entity), a separate substance, or an abstract object. In some sciences, for example in natural philosophy and its internal divisions, all four causes are required to provide adequate explanations (*Burhān* II, 9, p. 181.15–18).[5] In other sciences, not all causes may be relevant. For example, the study of purely separate intellects only depends on efficient and final causes. And among the sciences whose subjects are abstracted from matter (*ʿulūm intizāʿiyya*), some investigate entities that are separate only in definition, such as arithmetic

3. At *Burhān* II, 9, p. 180.18–19, Avicenna explicitly refers to a later discussion: "For we will clarify later how many the causes are and how they are middle terms and, if they are middle terms, how they are such as to give a complete demonstration (*muʿṭiyat al-burhān at-tāmm*)." This is, in all likelihood, a reference forward to the more systematic treatment of *Burhān* IV, 5.

4. On *An. Post.* B11, see Novak (1978), Sorabji (1980), Friedman (1984), Leunissen (2010a), and Ferejohn (2013).

5. The interplay of the four causes is illustrated by the case of man: (i) the efficient cause is either a man, the sperm, or a potency in the sperm (these may in turn be three things: a species of the subject of the science, a form, or a per se accident belonging to the subject of natural science, that is to say, body insofar as it is qualified by motion and rest); (ii) the material cause is either the elements (*arkān*) or the mixtures (*aḫlāṭ*) (both of which would fall under the subject of natural science too); (iii) the formal cause is the soul; and (iv) the final cause is a certain kind of perfection proper to the soul. Another example is the explanation of the structure of molars, on which see also *Naǧāt* I, 145 (v).

and geometry, and are therefore exclusively concerned with formal causes.[6] These criteria are taken by Avicenna to be principles (*uṣūl*) that regiment the relation between causes, demonstrations, and the subjects of the sciences. For example, if some of the causes of a fact fall outside the domain of the science concerned with that fact (and are part of the domain of another science), then there may be different demonstrations of one and the same conclusion in the two sciences. But if all four causes of a fact are associated with the same science, a why-demonstration may be produced in that science only (*Burhān* II, 9, p. 183.3–5).

In *Burhān* IV, 4, the four causes are designated as (i) the form (*ṣūra*) of something in the true reality of its existence in itself (*fī ḥaqīqat wuğūdihī fī nafsihī*) (formal cause), (ii) the thing or the things that are needed in order for something to be existent and receptive of the form of its being (material cause), (iii) the principle of motion (efficient cause), and (iv) the perfection (*tamām*) for the sake of which the matter and the form of an entity are joined (final cause). In this context, Avicenna's focus shifts to the specific problems and terminology of *An. Post.* B11. The goal, in particular, is to establish how the four causes may be used as middle terms in a demonstration.

Finally, in the classification of *Burhān* IV, 5, Avicenna applies five distinctions to the traditional framework.[7] Each of the four causes may be (i) remote or proximate, (ii) essential or accidental, (iii) in potency or in act, (iv) proper or common, and (v) universal or particular.[8] Only the first two distinctions are exemplified in detail.[9] The last two turn on the relative extension of the terms involved and are addressed in *Burhān* IV, 8–9 (I return to them later).

6. For a similar point, see *Ilāhiyyāt* VI, 5, pp. 298.19–299.5. The notion of a science based on abstraction (*ʿilm intizāʿī*), that is to say, a science whose subject is determined by abstraction from matter, is discussed by Alfarabi at *Burhān* IV, 2, pp. 68.25–69.8. According to Avicenna, sciences that only rely on formal causes as middle terms cannot have why-demonstrations in common, presumably because essential causes in that context are understood in a strict sense as causes of the quiddity, and the quiddity is unique for any given subject or attribute.

7. The analysis of the four causes is followed by a series of remarks and further conditions, possibly in an attempt to systematize the problematic examples of *An. Post.* B11. The problems concern primarily the relation of formal and final cause to the effect and the role of time, which offers a natural transition to the theme of *An. Post.* B12.

8. A remote source for these distinctions is presumably Aristotle's discussion in *Phys.* B3. The proximate source, however, is almost certainly Alfarabi, *Burhān* II, 3, pp. 26.15–27.13, which lists four out of five criteria summarized by Avicenna in *Burhān* IV, 5, namely, (i) proximate or remote, (ii) essential or accidental, (iii) common or proper, and (iv) in potency or in act. The distinction between universal and particular causes is omitted, but Alfarabi discusses it elsewhere in connection with a certain kind of incomplete definition (the conclusion of a demonstration). According to Alfarabi, all deductions making use of causes included in (i)–(iv)—except for the accidental—may be called demonstrations. Just as in Avicenna, the strongest kind of demonstration is based on essential, proximate, proper, actual causes.

9. The criteria discussed at *Nağāt* I, 145 (i) are the same, but the examples are different. For the efficient principle, Avicenna gives the carpenter for the chair and the father for the child; for matter,

Each cause provides the answer to a why-question of a different kind and is explicitly characterized as a principle (*mabdaʾ*). At *Burhān* IV, 5, pp. 296.1–297.9, Avicenna examines an array of possible causal answers to various why-questions. The questions introduced to illustrate the distinction between proximate cause (*ʿilla qarība*) and remote cause (*ʿilla baʿīda*) are as follows:

Question	Kind of cause
1. Why does someone walk?	(Final)
2. Why does someone have a fever?	(Efficient)
3. Why do animals die?	(Material)
4. Why is a certain angle right?	(Formal)

The first two examples are drawn from medicine, broadly construed, which is concerned with the human body insofar as it is qualified by health and sickness. A remote final cause of walking is the fear of bad digestion (*tawaqqī sūʾ al-haḍm*). A proximate final cause of walking is the fear of the congestion (*iḥtiqān*) of the mixture and of a seizure (*istīlāʾ al-bard*). A remote efficient cause of fever is the obstruction of the pores.[10] A proximate efficient cause of fever is the putrefaction (*ʿafūna*) of the bile.[11] The third example is from natural philosophy, and in particular from the branch of natural philosophy that deals with animal physiology. A remote material or elemental (*ʿunṣūrī*) cause of the death of animals is the contrariety of the elements (*taḍādd al-arkān*).[12] A proximate material cause of the

wood and menstrual blood; for the final cause, shelter for the house. Only the examples of proximate and remote efficient causes are the same (putrefaction and obstruction for fever). Avicenna also gives an example of proper and common causes, namely building (proper) and the builder (common) for the house. Further examples of essential and accidental causes are heating (essential) or cooling (accidental) for scammony and water.

10. Reading *sudda* for *šadda* with Ms. Istanbul, Nuruosmaniye Kütüphanesi, 2710, f. 94v. The reading is not only more plausible but also indirectly supported by the authority of the *Qānūn*, where the obstruction or engorgement of pores and vessels is invoked as an explanation for a variety of phenomena, most notably fever.

11. See also *Burhān* IV, 5, p. 300.7–8 and IV, 8, p. 323.1.

12. See *Ilāhiyyāt* VI, 1, p. 257.5, for another characterization of the material cause as "elemental" (*ʿunṣūrī*). At *Burhān* IV, 4, p. 295.14, Avicenna asks why *humans* (rather than animals) die. Neither example is in *An. Post.* B11. The ultimate source is, in all likelihood, Themistius, *In An. Post.* B11, p. 52.9–11, which identifies "being composed of contraries as the matter and middle term explaining why bodies are subject to corruption (*to dē sunkeisthai ex enantiōn hulē te kai mesos horos tou ta sōmata einai phtharta*)." But see also Alfarabi, *Burhān* II, 7, p. 42.11–13, where the death of humans is causally explained in terms of their being composites of contraries (*murakkab min al-aḍḍād*). At *Burhān* IV, 5, p. 296.3, Avicenna raises the more general question of why *animals* die. The remote cause is the contrariety of the elements (*arkān*), whereas the proximate cause is that dryness ultimately becomes predominant over moisture in the body. On dryness and moisture, see also *Afʿāl wa-infiʿālāt* I, 9. Another question, which bears only a remote (and somewhat indirect) but interesting relation to the cause of death of animals, is what makes some species live longer than others. Aristotle famously thinks there is

death of animals is the fact that dryness becomes predominant over moisture in the mixtures of the body. The fourth example is from geometry. A remote formal cause of the fact that a certain angle is right is the fact that a line stands on a line (*qiyām ḫaṭṭ ʿalā ḫaṭṭ*). A proximate formal cause is the fact that a line stands on a line at two equal adjacent angles.

The questions introduced to illustrate the distinction between essential cause (*ʿilla bi-ḏ-ḏāt*) and accidental cause (*ʿilla bi-l-ʿaraḍ*), by contrast, are as follows:

Question	Kind of cause
5. Why does someone walk before eating?	(Final)
6. Why does a wall collapse?	(Efficient)
7. Why is a figure reflected on a surface?	(Material)
8. Why is a certain line perpendicular to another?	(Formal)

The fifth example seems to be a variant of the first and presumably also pertains to medicine. An essential final cause (*tamāmī*) of walking before a meal is health. An accidental final cause of walking is becoming tired (*kalāl*) or discovering a treasure.[13] The sixth example is not clearly reducible to a science (unless we either consider the art of house-building a science or take the example to illustrate something about weights). An essential efficient cause of the collapsing of a wall is weight (*ṯiql li-inhidām al-ḥāʾiṭ*). An accidental efficient cause is the failing of what supports it (*diʿāma*). The seventh example is presumably relevant for optics (or applied catoptrics). An essential material cause of the reflection of a figure on a surface is the fact that it is polished (*ṣiqāla li-ʿaks aš-šabḥ*).[14] An accidental cause is the fact that the surface is made of iron (*ḥadīdiyya*), which is a naturally reflective material. The eighth example is concerned again with a geometrical problem. An essential formal cause of the perpendicularity of a line to another line is the fact

a correlation between life span and the presence or absence of a gall bladder. The example of animals lacking a gall bladder appears also in Avicenna, at *Burhān* IV, 9, p. 328.13–17 (cf. *An. Post.* B17, 99b4–7 and *De Part. An.* Δ2, 677a11–b10). For an account of the function of the gall bladder in animals and the claim that its presence, size, or absence is typically correlated with the length of life associated with a given species, see *Ḥayawānāt* XIV, 1, pp. 325.1–326.15; cf. also *Ḥayawānāt* XIII, 7, pp. 320.1–321.7.

13. In the first question of *Burhān* IV, 5 (but also in *Burhān* IV, 4, as well as in the *Qānūn*), Avicenna adopts the standard Aristotelian example of walking *after* a meal for the sake of being healthy. In the fifth example of IV, 5, however, the question seems to be concerned with the implications of walking *before* a meal. One of the answers seems somewhat plausible: an accidental cause of walking might be the desire of getting tired (perhaps as a form of preprandial exercise). The treasure example is more difficult to make sense of. The source might be Themistius, *In An. Post.* A4, p. 11.14–15, where the discovery of a treasure is used to illustrate the fourth sense of per se in *An. Post.* A4. The latter also concerns the distinction between essential and accidental causes, but in that case there seems to be no relation with walking (as "if one found a treasure while digging the vine: for it is not because of the gold that he was digging, but it just happened"). Cf. also Philoponus, *In An. Post.* A4, p. 64.8–10.

14. On reflection and mirrors, see *Nafs*, III, 5–6.

that the two angles generated on both of its sides are equal.[15] An accidental cause of perpendicularity is the fact that the angle generated by a first line standing on a second line and by the line parallel to the second line on which the first line stands is right.

Various other combinations are possible. For example, essential causes may be remote or proximate; remote causes may be essential or accidental, and so on. Avicenna's analysis seems driven by the need to cast as wide a net as possible for the multiple kinds of explanations that may be encountered in the sciences. And as shown by the second list, even accidental causes may not be extraneous to scientific reasoning. This is because, with the exception of the purely coincidental relation between walking and stumbling upon a treasure, which is presumably mentioned just for illustrative purposes, the other relations may be construed as involving per se accidents. Consequently, they represent a legitimate, if not the ultimate, sort of explanatory nexus that can be investigated in a science. For example, the attribute of a certain geometrical construction identified in answer to the question of why a line is perpendicular to another line, while not being part of the definition of perpendicular, may still be necessarily true of it and explain other necessary nonessential attributes of perpendicular.

Avicenna's identification of multiple levels of causality and explanation in *Burhān* IV, 5, and his analysis of the interplay of the four causes in different domains of inquiry are instrumental to the question of what counts as a complete demonstration and a complete (causal) definition.

CAUSES, DEMONSTRATION, AND DEFINITION

After the elaborate account of causes presented in the first part of *Burhān* IV, 5, Avicenna begins to examine the ways in which those causes may be absorbed into the logical structure of demonstrations and definitions.[16] He writes:

15. Perpendicularity is discussed at *Handasa* I, 17, pp. 36.6–37.2; cf. also Euclid, *Elements*, I, Def. 10: "When a straight line standing on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands." The example is used again at *Burhān* IV, 5, p. 300.6–7. The discussion of the nature of angles plays an important role in Avicenna's metaphysics, both for the characterization of quantities and in the analysis of definition, on which see *Ilāhiyyāt* III, 4, pp. 116.3–117.6, and *Ilāhiyyāt* V, 9, pp. 250.6–252.14, respectively.

16. The discussion of *Burhān* IV, 4–5 continues in *Burhān* IV, 8–9, loosely tracking the order and arrangement of *An. Post.* B11 and B15–18. Similarly, at *Naǧāt* I, 145, Avicenna presents a classification of the four causes (i)–(ii), discusses the relation between causes and demonstration (iii)–(vii), and finally deals with causes and definitions (viii)–(x), whereas at *Naǧāt* I, 147, he returns to the more specific issues raised by B15–18. The structure of Avicenna's discussion in the *Naǧāt* makes it slightly easier to appreciate the thematic unity of these problems.

Text 11.1: *Burhān* IV, 5, p. 299.8–9
It is clear from all this that a demonstration is a complete demonstration (*burhān tāmm*) only when it gives the proximate, proper, essential, and actual cause.

The characterization of complete demonstration in Text 11.1 could not be easier to state (and its conditions harder to meet): a complete demonstration is one that proceeds through a middle term (or a sequence of middle terms) that express essential causes that are proper and proximate as well as being causes in act.

The case of complete definition is more complicated (I return to it in greater detail in chapter 12). Suffice it to say here that Avicenna first states a general criterion of completeness governing the inclusion of causes in definition and then addresses a series of specific questions concerning the definition of various types of entities. He writes:

Text 11.2: *Burhān* IV, 5, p. 299.10–16
The complete definition (*ḥadd tāmm*) is the one that includes causes of this kind, with regard to things that have causes of the quiddity, so that they are completely stated without any one of them missing, if it is essential. For we have already said before that the purpose of defining (*taḥdīd*) is not to discriminate (*tamyīz*) by means of essential attributes that are equal in convertibility to what is being defined, but rather [to discriminate by means of essential attributes that are] equal to it in notion, in such way that none of the essential attributes of what is being defined fails to be contained and included in the definition (*illā wa-qad taḍammanahū l-ḥadd wa-štamala ʿalayhi*). If [a definition] lacks [any] one of them, being limited to discrimination (*tamyīz*), it does not signify the quiddity [of what is being defined]. This is because the quiddity [of what is being defined] is not [just] some of its constituents and essential attributes, but rather the collection of all of its essential notions (*huwa bi-ǧtimāʿ ǧamīʿ maʿānīhī ḏ-ḏātiyya*). Hence, someone who knows some of them without knowing the others does not know the essence of [what is being defined] in a complete manner (*mā ʿarafa ḏātahū bi-t-tamām*). The purpose of defining is the realization in the soul of a form that corresponds (*ṣūra muwāziya*) to the quiddity of the thing in its perfection (*bi-kamālihā*).

Text 11.2 establishes that a complete definition—that is to say, a complete *definiens*—must include all essential causes that are relevant to its *definiendum*. This is because the purpose of the activity of defining is not just the identification of a notion that is coextensive with the *definiendum* (where equal in convertibility means that the *definiens* is true of all and only the things of which the *definiendum* is true), but rather one that conceptually corresponds to the *definiendum* (equal in notion) and is, for the intellect, the counterpart of a complete essence and of its internal structure. And since the essence of an object is not identical to some of its constituents only but to their complete collection, a complete definition must contain them all (and in the correct order, as we shall see in chapter 13).

The notion of an essential cause, however, is potentially ambivalent. This is because essential may be taken, in this context, either in a narrow sense, as equivalent to formal, or in a broader sense, to include all four kinds of causes as long as they are causal factors in an essential rather than in an accidental manner. In this connection, Avicenna considers two potential objections that enable him to clarify important aspects of the relation between causes and definition.

First, (i) if a definition makes the essence of something known, how can causes that seem to be external to the essence be included in it (*Burhān* IV, 5, p. 300.15–16)? Avicenna's answer is based on his canonical distinction between causes of the quiddity (*ʿilal al-māhiyya*) and causes of the existence (*ʿilal al-wuǧūd*) of something: while the causes of the essence as such are distinct from the causes of the existence, and the notions included in the essence are distinct from factors that are external to it, a definition may nonetheless be completed by specifying the causes that are associated with the existence of the thing (*anniyya*) "in such a way that its quiddity may be conceptualized in the way it exists (*ḥattā yutaṣawwara māhiyyatuhū kamā huwa mawǧūd*)" (*Burhān* IV, 5, pp. 300.16–301.12).[17] Thus, for example, while efficient or final causes may not necessarily be part of the essence of an entity in the way in which formal and material causes are, it may still be appropriate to incorporate them in a complete characterization of the entity in question, if the goal is to explain its existence as well as its essence.

Second, (ii) if something is defined not by means of its causes but rather by means of concomitant factors (*lawāḥiq*) that are external to its quiddity, is the resulting account a definition or a description (*Burhān* IV, 5, p. 301.13–14)? Attributes such as the capacity of laughter, of being ashamed, or of crying, for example, are all defined by means of certain extrinsic actions (*afʿāl*). Avicenna's answer, in this case, is that if an account includes extrinsic actions that are implicates of what is being characterized, then it is a description, whereas if those actions are constituents of the essence of the capacity in question, then it is a definition, "for the definition requires making known the essence of something" (*Burhān* IV, 5, pp. 301.15–302.10).[18] The purpose

17. An essence, nature, or quiddity may be investigated in itself, regardless of the conditions that necessarily follow once it is investigated as an existent (whether in the mind as a universal concept or in extramental reality as an individual thing). At *Ilāhiyyāt* V, 8, p. 245.7–13, Avicenna notes that the definition of a composite should not merely include the form, because it is part of the nature of every composite to consist of both matter and form.

18. The use of passions or ways to be acted upon (*infiʿālāt*) instead of differentiae (understood presumably as qualities) is identified as an error at *Top.* Z6, 145a3–12; cf. *Ǧadal*, V, 2, p. 262.5–11. In general, Avicenna is keenly aware of the risk of using implicates instead of constituents in a definition. This is part of a broader discussion of certain systematic errors associated with the discovery of definitions, which includes several issues already discussed in the *Topics* and some additional material. In the *Šifāʾ*, the theme is addressed at various places in *Ǧadal* III, V, and VI (especially with regard to *Top.* Δ2, Δ5–6,

of these observations, in the broader context of Avicenna's discussion of causality, is to establish criteria for the inclusion of the four causes in the conceptual characterization of an entity, depending on the kind of entity it is and on what is being investigated about it, that is to say, only its nature or its existence too.[19]

PROBLEMS OF GENERALITY

In *Burhān* IV, 8 and IV, 9, Avicenna shows a significant interest in a set of problems discussed by Aristotle in *An. Post.* B15–18. His approach to the material suggests that he takes these chapters to be concerned again with a variety of relations between causes and effects and with the ways in which such relations are absorbed into demonstrations and definitions. In particular, B15 raises the question of whether (i) *multiple effects* may have one and the same cause. In a demonstration, this translates into the question of whether *multiple questions* (or problems, in Aristotle's vocabulary) may all be proved through one and the same middle term.[20] The problem of B16, by contrast, is (ii) how to avoid circularity when causes and effects are coextensive, that is to say, mutually convertible. Avicenna circumvents the difficulty by pointing out that causes and effects are always asymmetrical: a demonstration using the cause as a middle term is a why-demonstration, while a

and Z6). The most concise (and perhaps straightforward) account of such systematic errors is at *Naǧāt* I, 149, where Avicenna notes that a common source of error is to take implicates instead of genera or differentiae as if they were genuine constituents of a *definiendum*.

19. Further issues discussed by Avicenna in his excursus on B11 concern (i) the inclusion of the four causes in the definition of artifacts (for example, ring or sword) and in the definition of products of will or desire; (ii) the question of whether positing an effect in act requires positing all of its causes; and (iii) the conditions under which the existence of a plurality of causes is not a sufficient condition for the effect. With regard to (i), in particular, Avicenna's intention seems to be to offer a typological analysis of all the fundamental kinds of notions that may count as *definienda* and the constraints on the inclusion of causes in their definitions. We have already seen some of the criteria for physical objects, separate substances, and the abstract objects of mathematics (which jointly cover the entities investigated in the three main divisions of the theoretical sciences). The addition of artifacts and products of will or desire might be a way for Avicenna to bring his survey to an ideal completion by covering the sort of entities with which practical and productive sciences are primarily concerned. The discussion of artifacts also offers another paradigm of what Avicenna calls unifying definitions (*ḥudūd muttaḥida*), that is to say, definitions that rely on all four causes.

20. In this context, Avicenna refers to various phenomena of attraction involving lodestones, amber, and cupping glasses. The source is probably Themistius, *In An. Post.* B15, p. 60.1–2, on which see Strobino (2012, pp. 375–376). Avicenna considers two cases: (i) multiple questions unified as one question as a result of the fact that there is a genuine middle term for all of them (one by species); and (ii) multiple questions that share a cause without being in reality one question. The latter case requires the relevant middle terms to be ordered hierarchically depending on their proximity or remoteness (in which case the multiple questions do not reduce to one single question but are still explained by a cause, more or less proximately or remotely) (*Burhān* IV, 8, pp. 319.9–320.7).

demonstration using the effect as a middle term is a that-demonstration (or sign). In this case, the two demonstrations typically have different conclusions, but Avicenna is committed to the view that they may genuinely be about one and the same question or conclusion, without circularity, as we have seen in the analysis of *Burhān* III, 3 in chapter 9.[21] The problem of B17 is in a way the mirror image of that of B15: What if (iii) *one effect* (or, again, *one question*) has *multiple causes* (middle terms) that are not coextensive with it? Avicenna discusses it under the heading of how to include in a demonstration causes that are more specific (*aḫaṣṣ*) than their effects.[22] Finally, B18 is concerned with the question of (iv) what it means for a cause to be proximate or remote.

The analysis of these problems, which may once again appear, on a superficial reading, an exceedingly technical aspect of Avicenna's theory of science, is in fact motivated by his conscious attempt to regiment a series of philosophically significant questions concerning the relation between causality, demonstration, and definition, and to amalgamate problems that are apparently only loosely interconnected in the *Posterior Analytics* under one and the same rubric.

Avicenna is committed to the view that the four causes may be used as middle terms in demonstrations in order to prove conclusions whose predicates are per se accidents of their subjects. And just as demonstration may suitably incorporate all four types of causes for the proof of a scientific assertion (stating that and why an attribute-effect belongs to its subject), so definition, too, may include all four types of causes for the characterization of the quiddity of the *definiendum* (either in itself or when it is conceptualized as an existent). But what is the relation between the two? Are all causes that may serve as terms of a demonstration also eligible to be part of a definition? The answer to this question seems to depend for Avicenna on the relative generality of the terms involved and is relevant for his classification of different types of explanation and the corresponding demonstrations.

MULTIPLE CAUSES

The analysis of the problem at *Naǧāt* I, 145 and I, 147 offers some useful insights (bearing in mind that definitions are always understood by Avicenna to consist

21. The argument is that, in order for circularity to arise, a proof must obtain *through one and the same* thing (*Burhān* IV, 8, pp. 320.16–321.5). According to Avicenna, one of the factors that determines sameness (and hence circularity in this context) is *how* a premise is used in an argument; thus, if *p* is used to prove that *q* is the case, and *q* is used to prove why *p* is the case, it does not follow that *p* is used to prove, circularly, why *p* is the case.

22. *An. Post.* B16–17 and B18 are generally concerned with the causal relation of a subject and an attribute when the two are connected by multiple (seemingly equivalent) predicative paths, which involve several middle terms. As noted in chapter 10, this issue arises also in *An. Post.* A29 and is discussed by Avicenna in *Burhān* III, 8.

of genera and differentiae, even in extreme or counterintuitive cases).[23] Avicenna maintains, at *Naǧāt* I, 145 (viii), p. 163.8–11, that essential, constitutive causes must clearly be included in the definition of a thing. But what about causes that may be more specific than the *definiendum*? These could at best be causes that are coextensive with something that falls under the *definiendum*. And in this case they would only be sufficient, but not necessary, conditions for it. For example, fever may be caused by (i) the putrefaction of the bile, (ii) the violent movement of the spirit, or (iii) a flare-up due to a cause other than putrefaction. Similarly, noise may be caused by (iv) the quenching of fire in the clouds (which results in thunder), (v) the breaking of a bottle, or (vi) the knocking of a stick.

Avicenna contends that these specific causes cannot be part of the definition of the attribute in general (fever or noise), even though they may legitimately be used in a demonstration that proves why an attribute belongs to a subject. These specific causes may only play a role in definition if a single unifying causal notion is discovered under which they can all be subsumed. Such a cause (the unifying factor or cause, common to all specific causes) would indeed be part of the definition of the attribute. Unless a unifying factor exists and can be identified, the specific causes of an effect are definitionally associated not with the effect itself but rather with its *species*. Thus, for instance, the quenching of fire is not a cause that may be included in the definition of noise in general, but it can (and in fact must) be included in the definition of a particular kind of noise, namely thunder.[24]

Avicenna returns to the analysis of the status of causes that are more specific than their effects and of the way in which they may be used as middle terms in demonstrations in *Naǧāt* I, 147, where he discusses the problem of *An. Post.* B17, albeit in slightly different terms. The following examples involve causes that are more specific than their effects (or, to put it otherwise, of middle terms that are more specific than the major terms in the corresponding demonstrations):

(a) existence of cloud (aa) due to the condensation of the air because of coldness and (ab) due to the coagulation of vapor;[25]

23. The problem is first raised at *Naǧāt* I, 145 (ix) and then discussed more extensively at *Naǧat* I, 147.

24. To the definition of thunder as noise due to the quenching of fire in the clouds (a canonical example from *An. Post.* B8), Avicenna devotes a significant amount of attention, especially in *Burhān* IV, 4. And he repeats on numerous occasions that the genus in such a definition is noise. This incomplete definition of thunder falls under the kind of definition called by Avicenna (and Aristotle) the "conclusion of a demonstration," as we shall see in chapter 12.

25. On clouds, see *Maʿādin wa-āṯār ʿulwiyya* I, 2, pp. 10.1–12.11, and II, 1, pp. 35.1–36.15.

(b) existence of earthquake (ba) due to the occurrence of winds, (bb) the crushing of the heights down into a deep ditch, or (bc) the bursting forth of a flood under the earth;[26]
(c) existence of thunder (ca) due to wind and (cb) due to the quenching of igneous fumes in the clouds;[27] and
(d) existence of fever (da) due to the putrefaction of the bile and (db) due to the heat of the spirit in the absence of putrefaction.

In these four cases, each attribute (or phenomenon) may be separately explained by more than a single cause. These causes, taken severally, are more specific than their effect.[28] For example, a specific kind of fever may be caused by the putrefaction of the bile, but this is not universally true of all kinds of fever. If there exists a general notion, coextensive with the effect, under which all specific causes can be subsumed, then that notion is the proximate cause of the effect.[29] And it is only when a cause of this kind, which is coextensive with or equal (*musāwin*) to its effect, is used as a middle term that an absolute why-demonstration may be produced, that is to say, one that reveals the cause of the existence of the attribute-major term and of its belonging to the subject-minor term.

If, by contrast, no general notion can be identified, and it is then impossible to find a cause that is extensionally and conceptually equivalent to the major term-effect, then specific causes cannot be used as middle terms in the demonstration of the general attribute without qualification, or at least not to produce the strongest kind of why-demonstration. But they can still be used to produce a weaker kind of why-demonstration that proves the attribute to *belong* to the subject. In other words, a middle term that is more specific than the major term—in the absence of

26. On earthquake, see *Maʿādin wa-āṯār ʿulwiyya* I, 4, pp. 15.1–19.15.

27. On thunder, see *Maʿādin wa-āṯār ʿulwiyya* II, 5, pp. 67.1–72.9 (including a discussion of lightning); on wind, see *Maʿādin wa-āṯār ʿulwiyya* II, 4, pp. 58.1–61.17.

28. Further examples of single natures that may be due to multiple, more specific causes, at *Burhān* IV, 8, pp. 322.17–323.1, are (a) thunder due to (aa) wind in the cloud or (ab) quenching of fire, and (b) cloud due to (ba) the upward movement of steam and (bb) the cooling of air; similarly, (c) heat spreading from the heart (*al-ḥarāra al-muntašira min al-qalb*) in the limbs, which results in fever, may be due to (ca) the flaring up of the spirit (*rūḥ*), (cb) the putrefaction of the mixture, or (cc) the inflammation of a limb.

29. This family of problems is directly relevant for Avicenna's discussion of generality in the account of the scientific notions of "universal" and "primary" in *Burhān* II, 3 and II, 4. In particular, *Burhān* II, 4 examines the three cases set forth by Aristotle in *An. Post.* A5 to explain why we sometimes erroneously think that we are giving a universal demonstration when in fact we are not. This is the case, for instance, when we lack a name for an attribute under which a series of more specific attributes may be subsumed, and which in turn is coextensive with its subject, belonging to it universally in the sense of *An. Post.* A4.

a general notion under which it may be subsumed and which is in turn coextensive with and conceptually equivalent to the major term—can at best be the vehicle of a demonstration showing why the major term (attribute) belongs to the minor term (subject), but not of a fully explanatory demonstration.[30] At *Naǧāt* I, 147 (iv), p. 168.5–6, Avicenna makes it clear (in retrospect) that his primary concern is to distinguish the cases in which the middle term-cause is convertible with the major term-effect from the cases in which it is not. And this is in turn a decisive criterion for establishing whether or not an argument is an absolute why-demonstration.

The convertibility of a cause with its effect, the notion of proximate cause, the identification of the correct level of generality for a middle term relative to its *explanandum*-major term, and the idea of subsuming several specific causes under one notion that corresponds to the effect, are all aspects of the same problem. For Avicenna, the significance of this analysis lies in the fact that it brings to the surface the conditions under which different kinds of explanations are possible, when reasoning about causes and effects.

CAUSE OF THE EFFECT AND CAUSE OF THE CONCLUSION

At *Burhān* IV, 8, pp. 321.9–323.2, Avicenna addresses the problem of B17, that is to say, the case in which multiple demonstrations of one and the same question may come about through different middle terms-causes. The interplay of different causes is illustrated once again by the example of death. Assuming death to be an effect whose explanation (*bayān*) may involve two different causes, namely (i) the heat dissipating the moisture on which life depends (efficient cause) and (ii) the principle that every matter subject to generation is subject to corruption (material cause, or rather, a general principle concerning material entities), what is the relation between demonstrations using one middle term or the other?[31] Avicenna

30. At *Naǧāt* I, 147, Avicenna gives the following example: fever is not an effect of putrefaction without qualification; only the fever of *this* man or a particular kind of fever is (for instance tertian fever). The general underlying principle is that the species is not the cause of the existence of the genus, but rather of the fact that the genus belongs to what falls under the species, by means of the species (the genus may belong to a lower species, if the middle term is an intermediate species, or an individual, if the middle term is an infima species). The problem is analyzed in detail in *Burhān* I, 10.

31. The justification of this principle is that if something has matter, then it necessarily has some arrangement or configuration, and this requires an efficient cause from which that arrangement or configuration necessarily derives. In this case, Avicenna is committed to the view that regardless of whether the material or the efficient cause is explicitly stated, the other cause is implicitly presupposed, "because matter is not actualized except in virtue of the efficient [cause], while the efficient [cause], in material beings, does not act unless [there is] a matter" (*Burhān* IV, 8, p. 321.15–16). As noted previously, another example of material cause is being composed of contraries.

notes that "the cause which necessitates the conclusion [that humans are mortal] is *one* thing, that is to say the *aggregate* (*maǧmūʿ al-ǧumla*) of all relevant causes, not just of one or the other taken separately (*Burhān* IV, 8, p. 322.6). In other words, an adequate explanation of an attribute-effect must include all the relevant explanatory factors that account for its essence and for its existence: "It must be known, concerning the way in which several causes are given as middle terms, that they are potentially one single cause in reality. The process of giving [the cause] is not complete and does not necessitate [the conclusion] as long as it fails to point to the aggregate" (*Burhān* IV, 8, p. 322.8–10).

According to Avicenna, multiple specific causes may be used as middle terms to prove that and why an attribute-effect belongs to a subject. But if a cause-middle term is more specific than the effect-major term, the resulting demonstration is not a why-demonstration of the strongest kind but rather one that (causally) proves the major to belong to the minor. He writes:

> Text 11.3: *Burhān* IV, 8, p. 323.8–13
> Every single intermediate species is a cause of the fact that its genus belongs to the species and to the individuals falling under it. It is not a necessary requirement that the cause in demonstrations be always coextensive [with the effect], in such a way that, if the middle term is more specific than the major term, then this is not a demonstration. Rather, one must know that [(i)] some of the causes are included in the definition (those are undoubtedly coextensive, whether they be material or efficient), while [(ii)] some are more specific than the nature of the thing [that is being defined] and [(iii)] some are more general. The more specific [causes] are not included in the definition because the nature of the thing does not contain them with respect to its essence, in such a way that the existence of that nature depends on the existence of that cause.

Text 11.3 expresses, in general terms, the idea examined earlier through the examples of *Naǧāt* I, 147. Causes that are more specific cannot be part of an account of the nature and existence of an attribute without qualification, and hence cannot be included in its definition. They are sufficient conditions for proving that the attribute belongs to a subject, but not necessary conditions of its nature or of its existence. The strongest kind of demonstration is the one that explains the attribute by revealing its essence and establishing a nexus with the subject. And as noted, a complete demonstration is one that proceeds, among other things, through the proximate cause, which is in turn a cause coextensive with its effect. In order to do so, the strongest kind of demonstration cannot rely on terms that only explain part of the nature of the attribute (or fully explain something that is in fact more specific than the attribute). There is therefore a clear parallel between this sort of explanation and the requirements that causes must meet in order to be included in a real definition. Both must be complete, proximate, and explanatory of the nature of the attribute, not something more specific than the attribute.

An explanation in terms of a cause more specific than its effect is still an explanation but not one that fully reveals the nature of the effect. Avicenna writes:

> Text 11.4: *Burhān* IV, 8, p. 323.15–20
> If there are causes that are more specific [than the attribute-major term] without being causes of the nature of the major term without qualification, they are not included in its definition, notwithstanding the fact that they are causes and give the why of the conclusion. These causes are causes of the conclusion essentially, and of the major term accidentally (if the latter is [understood] without qualification and not in relation to the minor). We have already explained before that among the middle terms that are causes, some are causes of the conclusion only, without being causes of the major term, as heat (*suḫūna*) in the spirit, for it is a cause of the existence of fever in this body, not of the existence of fever without qualification.

Text 11.4 draws a critical distinction, which we have already encountered, between being a cause of the conclusion and being a cause of the major term. This is another way to express the distinction between being a cause of the fact that the major term belongs to the minor term and being in addition also the cause of the nature and existence of the major term itself.[32] Note that here to be cause of the conclusion should not be taken in the sense of being cause of an assertion only (the coming together of the extremes in the mind, which, as we have seen in chapter 9, is the distinctive characteristic of a that-demonstration), but rather in the sense of being cause of the fact expressed by the conclusion, without explaining the nature of the attribute itself. Triangle is the cause of the conclusion that every isosceles has the sum of the internal angles equal to two right angles, but it is not in and of itself the cause of that attribute (the cause in this case will be rather a complex series of terms used in a separate demonstration that explains why every triangle has that attribute).

If there is a general notion common (*amr ʿāmm*) to all specific causes, and such a common notion is a cause that corresponds to the effect (*ʿilla muṭābiqa li-š-šayʾ al-maʿlūl*) and is convertible with it (*munʿakisa ʿalayhi*), then that common notion may be taken in the definition of the attribute-effect. Anything less general, including the specific causes subsumed under the common notion (and which are causes of the fact that the common notion-cause, and hence the attribute-effect, belong to their subjects), will not, by contrast, be included in the definition. For example, in the case illustrated by figure 1, A is the attribute-effect (major term); H is the subject (minor term); C, D and E, F, G are two chains of specific causes (middle terms); and B is the common notion (the most general middle term), which subsumes the more specific causes and is equivalent in extension (and concept) to the attribute-effect, being a cause that corresponds to that effect. The two

32. See *Burhān* IV, 9, p. 329.9–13.

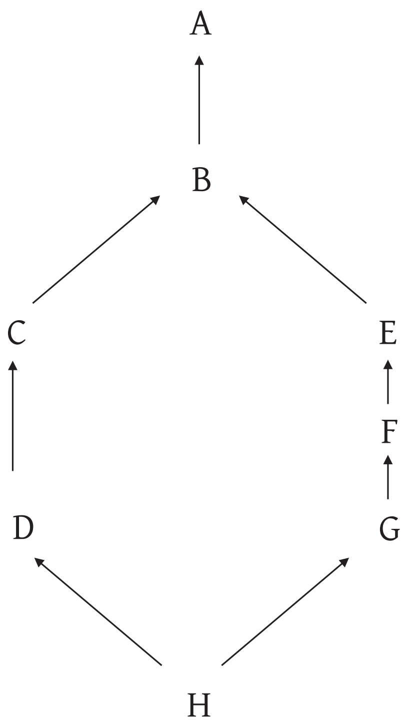

FIGURE 1. Generality and explanation with multiple chains of middle terms

predicative chains are both sufficient to prove that every H is A: (1) HaD, DaC, CaB, BaA; and (2) HaG, GaF, FaE, EaB, BaA. But the real explanatory middle term for HaA is B. When a term of this kind exists, then it can function both as a middle term in the demonstration and as part of the definition of A, while C, D, and E, F, G can at best serve as explanations of the fact that A belongs to H, but not of its quiddity or of its existence without qualification.

If there are multiple causal paths involving immediate predications that connect a subject H to an attribute A (the effect) through different causes, then at some point the chains may converge, in which case the proximate cause of A will be coextensive with A. The common notion B is at the same time an effect of the specific causes and a coextensive, proximate cause for the attribute. An example discussed in *Burhān* IV, 8 involves the following terms:

A = effect: cloud

B = aggregate of all specific causes: strength of condensation of the upper air

C = a specific cause of B: coagulation of vapor

D = another specific cause of B: cold

According to Avicenna, DaB and CaB are immediate predications (no further middle term can be found between D and B or between C and B). D and C are also both causes of B, each of them being more specific than B. Furthermore, BaA is another immediate predication, as nothing else falls by hypothesis between the two terms. And B is a cause that corresponds to A (it is coextensive and accounts for the nature of A). Avicenna's contention is, under these assumptions, that D and C are not included in the definition of A, but B is.

COEXTENSIVENESS, PROXIMITY, AND REMOTENESS

The analysis of causes that are more specific than their effects prompts Avicenna to formulate the principle of correspondence between cause and effect, which requires the subsumption of specific causes under a common notion. But what about the case of coextensive causes? Can the same effect be associated with multiple coextensive causes? And, if so, how? In *Burhān* IV, 9, which is a continuation of the discussion of B17, Avicenna addresses these questions from the standpoint of the distinction between proximate cause and remote cause in the case of coextensive terms, examining the well-known Aristotelian example of leaf-shedding. While the details of the discussion must be taken with a grain of salt, by Avicenna's explicit admission, the goal of his analysis is clear.[33] Consider the following chains of predication:

(a) fig—being broad-leaved—congealing of the moisture—shedding leaves[34]
(b) vine—being broad-leaved—congealing of the moisture—shedding leaves

In (a) and (b) leaf-shedding (*intiṯār*) is an attribute of fig and vine, respectively. The attribute is more general than each of the subjects, but it is coextensive with the middle terms. In both cases, two causes serve as middle terms: being broad-leaved (that is to say, being a particular kind of plant) and the congealing of the moisture, which is assumed to be an attribute of all broad-leaved plants. The example shows that the proximate or remote character of a cause does not necessarily depend on

33. In the context of his discussion of certainty, causality, and necessity, at *Burhān* I, 8, pp. 90.18–91.3, Avicenna maintains that this example should be treated with indulgence. The other case he mentions is the eclipse. The problem is that these are causal phenomena that only occur at a certain time (even though, when they occur, they occur necessarily). Moreover, these examples are only concerned with an explanation of why the major term belongs to the minor, and hence they cannot serve as illustration of the notion of complete perpetual certainty (*yaqīn tāmm dāʾim*). At *Burhān* IV, 9, p. 326.9–12, Avicenna openly says that the essential cause of the shedding of leaves is only the fact that they fall in virtue of their natural weight (*ṯiql ṭabīʿī*); shriveling (*inqišāš*) and the congealing of the moisture are causes of the absence of a connecting cause, and the absence of a connecting cause is the essential cause of the separation of the leaves from their stem.

34. On the nature and function of leaves, see *Nabāt* I, 5, pp. 22.13–25.5.

its relative generality (though it certainly can, as we have seen). The proximate cause of leaf-shedding is the congealing of the moisture at the junction (where the stem attaches to the tree). This cause is coextensive with its effect: everything that sheds its leaves undergoes congealing of the moisture, and everything that undergoes congealing of the moisture sheds its leaves. But it is clear that these two contentions are not explanatorily equivalent; only the second expresses a genuine causal nexus. At *Burhān* IV, 8, p. 320.9–11, being broad-leaved is also said to be a coextensive cause of leaf-shedding "even if it is a remote cause." This is because everything that is broad-leaved undergoes congealing of the moisture and everything that undergoes congealing of the moisture is broad-leaved. There are, therefore, three coextensive terms, one of which is an effect of a proximate cause *and* of a remote cause, where the former is a kind of explanation that captures more accurately the reason the attribute is what it is (why things that shed their leaves shed their leaves). But then what makes a cause proximate or remote cannot be just its relative extension with respect to its effect. For the example shows that there may be multiple causes, all coextensive with their effect, which are, however, explanatory of the effect in different ways. Proximity and remoteness are rather dependent on the immediate or non-immediate character of the relation of a cause to its effect. Broad-leaved plants shed their leaves because the moisture is congealed and not the other way around. Plants that undergo congealing of the moisture may well do so because they are broad-leaved, but that is a different nexus. With respect to the attribute, the genuinely explanatory factor is the cause that is proximate to it, and since attributes are, in general, the things whose explanation is sought in a science, the focal meaning of proximate cause in Avicenna's theory of science is that of a cause closest to and most explanatory of the nature and existence of attributes. Remote causes, by contrast, connect the proximate cause and the attribute-effect to its subject.

Before turning to Avicenna's account of definition, it is worth looking at one last passage from *Burhān* IV, 9, which concludes the discussion of the relation between causes and demonstration with a brief comment on *An. Post.* B18. He writes:

> Text 11.5: *Burhān* IV, 9, p. 329.9–13
> Again, if between two extremes there are convertible middle [terms] some of which are causes of others, the cause of the minor [term] is the one among them which is closest to [the minor] because it is the cause of the fact that the second cause, which is closer to the predicate, belongs to them. The cause of the major [term] is the one closest to the major. You know the difference between the cause of the conclusion and the cause of the major alone. For [reading *fa-inna* with S for *bi-anna*] the first is a cause of the conclusion, hence what is closer to the minor [term] is more fitting as a cause of the conclusion, while the second is the cause of the major [term] alone. I do not mean by "cause of the conclusion" in this place the cause of the assertion but the cause of the existence [of the thing] in itself.

Text 11.5 addresses a question that we have already encountered in chapter 9, in the analysis of *Burhān* I, 8 and III, 3. Avicenna explains that cause of the conclusion in IV, 9 is not to be understood in the sense of a mere inferential justification, but rather as the cause of the fact that the predicate belongs to the subject (as in an application argument) as opposed to the cause of the nature and existence of the attribute itself. The passage offers corroborating evidence in support of the view that Avicenna's theory of science envisions two distinct kinds of explanation, the strongest of which is intimately connected with the essence and definition of the attributes that a science seeks to establish.

Far from being an exercise of scholastic pedantry, Avicenna's attempt to collect and classify a broad array of examples from actual sciences, in order to clarify how the four causes contribute to the structure of demonstration and definition, is another sign of his commitment to the idea that an adequate theory of science must be applicable to concrete scientific theories. This is clear not only from the examples, but also from the implications of the general distinctions in terms of which those examples are categorized. Just as the theory of per se required a painstaking analysis and classification of the kinds of scientific attributes used in scientific reasoning, so the theory of causes requires an equally tenacious effort in identifying what plays the critical role of cause (and in what sense) for a broad array of phenomena and facts investigated by the sciences, from biology to meteorology, and from mathematics to metaphysics.

PART V

Definition

Definitions are a paradigmatic type of scientific principle in Avicenna's theory of science. Their centrality is largely a consequence of the essentialist character of Avicenna's metaphysics. Definitions encapsulate essential facts about the subjects and attributes of any given domain of scientific inquiry—facts about substance and accident, act and potency, motion and time, numbers and horses, triangles and planets, trees and fevers, leaf-shedding and laughter, eclipses and thunders—in sum about every nook and cranny of metaphysical, mathematical, and physical reality where essences are to be found and can adequately be conceptualized as universals. The essences of subjects and attributes are in turn the sources of the modal and explanatory connections captured by the premises and conclusions of scientific demonstrations.

Definitions and demonstrations are the two fundamental paths to scientific knowledge. In particular, definitions are vehicles of scientific knowledge through conception. As a result, they are primarily complex terms that reflect the internal metaphysical structure of essences and articulate them as ordered sequences of attributes according to certain rules. But indirectly, definitions also play a role in the domain of assertion, as their content may be fully or partly absorbed in the structure of propositional scientific principles. For instance, while the definition of triangle is, properly speaking, the complex term "Trilateral plane figure," that is to say, its *definiens*, the universal affirmative assertion "Every triangle is a trilateral plane figure," in which the *definiens* is predicated of the *definiendum*, may also be treated as a definition, in an auxiliary sense.

Avicenna's account of definition focuses on two sets of problems. The first set of problems concerns the general characterization of definition and description, the identification of their constitutive elements, and the classification of their types. Definitions and descriptions make something known (*taʿrīf*) and distinguish it (*tamyīz*) from other things. Both are kinds of linguistic expressions whose aim is to differentiate and specify their objects (*qawl mufaṣṣil*). Depending on whether or not there is an exact correspondence between an expression and its object, and on whether the expression consists of essential or of accidental attributes, four different kinds of complex terms are possible: complete definitions, incomplete definitions, complete descriptions, and incomplete descriptions. The highest kind of scientific principle, the one that truly reflects the essence of something and displays the strongest explanatory character, is complete definition, but in Avicenna's theory of science the other notions play an important role too. Description is the accidental counterpart of definition. In general, any account of something that characterizes it in terms of non-constitutive accidental attributes is a description and, as we have seen in chapter 8, attributes in this class may range from common accidents that are separable from their subjects to various kinds of inseparable accidents. Moreover, if the notion of description generally applies to any account or complex term that is not a definition (in virtue of the fact that it contains accidental attributes), it then also specifically applies to the particular class of inseparable accidental attributes that correspond to per se 2 predicates. And since the ultimate goal of scientific demonstration is to establish that and why per se 2 attributes belong to their subjects, Avicenna is then committed to the view that a particular kind of description, that is to say, a term like "Having the sum of the internal angles equal to two right angles," must be the staple of demonstrative reasoning. A science must therefore presuppose definitional principles as well as immediate descriptional principles and typically aims to demonstrate that non-immediate scientific descriptions are true of their subjects. The distinction between different forms of immediacy (or rather, the genuine immediacy of certain definitions as opposed to others) emerges also in connection with the division of real definitions into causal and noncausal ones. The former are typically definitions of demonstrable attributes that may be cast into demonstrative form, or conversely may be extracted from a demonstration and cast into the form of a definition by genus and differentia. The latter are the most fundamental principles in a science, namely the definitions of indemonstrable essences (chapter 12).

The second set of problems in Avicenna's account of definition concerns the question of how this special kind of principle, without which scientific knowledge would be altogether impossible, can be acquired. The question of how definitions are discovered is central for any foundationalist theory of science that postulates the necessity of first principles. If definitions are not innate (a view the possibility of which Avicenna hardly deems worth entertaining), how do we come to know them? Empirically, by means of abstraction, or by means of some form of intuition? Are they available through divine emanation or illumination? Is the process of discovery of definitions such as “Mortal rational animal” or “Trilateral plane figure” distinct from the process of concept formation by means of which we acquire their internal components, that is to say, simple universal terms like “Mortal,” “Animal,” “Trilateral,” or “Figure”? What are, in short, the characteristics of an adequate epistemology of essence? Avicenna is categorically opposed to the idea that a definition may be established as the conclusion of an argument, whether it be demonstration, induction, or division. He explores and develops in detail a series of difficulties and objections already raised against these methods by Aristotle in the *Posterior Analytics*. What is especially original is Avicenna’s approach to induction, which he vehemently rejects as a method for establishing that a complex term may be the definition of something, on pain of inconclusiveness, circularity, or infinite regress. His positive account of the process of acquisition of definition systematically reshapes the discussion of *Posterior Analytics* B13. The heuristic procedures identified in this context include primarily a series of instructions and rules called by Avicenna the “method of composition” (*ṭarīq at-tarkīb*), which is supplemented by the traditional method of division (*ṭarīq al-qisma*). Definitions are acquired primarily by means of a bottom-up process of composition and abstraction, which is then validated by a complementary top-down process of division to ensure that the complete ordered sequence of essential attributes of a *definiendum* is adequately captured by the relevant linguistic expression (chapter 13).

12

Definition and Description

Structure and Types

DEFINITION IN CONTEXT

In Avicenna's theory of science, definitions are one of the fundamental kinds of principle, along with common axioms and assertions of existence. Definitions are the ultimate expression of the specific content of a science, providing conceptual knowledge of the essences of its primitive subjects and attributes, and the set of basic relations that constitute the foundation for all scientific knowledge subsequently obtained by demonstration. In Avicenna's modal epistemology, as we saw in chapters 6–8, the account of necessity and per se attributes, the notions of constituent and implicate, and the vocabulary of inseparability in conception and inseparability in imagination are all critically dependent on essence and definition. In particular, the set of assumptions of a science must include the definitions of its subject(s) as well as the definitions of (some of) its attributes, especially the immediate implicates of the subject of that science.[1]

Avicenna is committed to a clear division of intellectual labor with regard to definition. From a passage at the beginning of the dialectical treatment of definition, at *Ǧadal* V, 1, p. 241.1–7, we learn that he envisions two distinct approaches to the subject. A first, more specific investigation (*baḥṯ aḫaṣṣ*) is the *scientific* investigation (*baḥṯ ʿilmī*) of definition, while a second, more general investigation

1. The definitions of *both* subjects and per se attributes are identified as a requirement for a science at *Išārāt* IX, 3, pp. 82.18–83.2.

(*baḥṯ aʿamm*) is its *dialectical* investigation (*baḥṯ ǧadalī*).[2] In particular, the task of the theory of science is to establish (i) how definitions are structured (*taʾlīf*), that is to say, their criteria of well-formedness, and (ii) how definitions are acquired (*iktisāb*), that is to say, the procedures by means of which we come to discover them. The task of dialectic, by contrast, is to determine whether or not the conditions identified in response to (i) are met, and the methods identified in response to (ii) are correctly applied.[3]

The priority of a theory of definition developed within the broader context of the theory of science lies in the fact that the latter specifies a set of necessary and sufficient conditions that a complex term must satisfy in order to qualify as a real definition of an essence, and identifies the stages an inquirer must go through in order to acquire it (the methods of composition and division). Dialectic, by contrast, provides a series of cross-checks, tests, and criteria to verify whether or not a proposed account is an adequate definition.[4] The privileged status of the investiga-

2. By "scientific investigation" Avicenna means the investigation of the nature of definition and of the conditions for its acquisition as opposed to its dialectical treatment. This should not be confused with the use of the term "scientific" in the context of the (dialectical) distinction between scientific commonplace rules or loci (*mawāḍiʿ ʿilmiyya*) and nonscientific ones. In dialectic, the term "scientific" is used to identify the privileged *status* of certain commonplace rules (and their legitimate *use* in the sciences) due to their being general logical or metaphysical principles that are necessarily true (for example, relations governing genera and species). Other commonplaces, by contrast, are merely endoxic (*mašhūrāt*), that is to say, principles that are commonly held but may fail to be true. The latter are of no use in the positive construction of a science but may nonetheless play an instrumental role for the purposes of refutation. Thus, when Avicenna calls a commonplace rule scientific, this tells us something about the status of the commonplace rule itself and about its domain of application, even if these designations are part of the vocabulary of dialectic. Another kind of commonplace is the demonstrative commonplace (*mawḍiʿ burhānī*), an important example of which is the locus for the determination of whether something is an accident discussed at *Ǧadal* II, 1. As we shall see in chapter 13, Avicenna refers to it as part of the repertoire of techniques involved in determining (indirectly) whether an attribute is essential to a subject, which is in turn one of the three conditions for the adequacy of a definition.

3. A similar point is reiterated at *Ǧadal* VI, 1, p. 297.3–4, where Avicenna contends that questions concerning the relation between definition and deduction and the way in which definition is hunted are investigated in the section of logic that precedes dialectic, namely demonstration. For Avicenna's dialectical treatment of definition, see in particular *Ǧadal* V and VI (on *Topics* Z and H). On definition in Aristotle, see Ackrill (1981), Gómez-Lobo (1981), Deslauriers (2007), and Charles (2010c). For an introduction to the theme in Avicenna, see Strobino (2010). At *Ilāhiyyāt* I, 8, pp. 53.18–54.2, Avicenna contends that "knowledge of the essence of the subjects (*maʿrifat ǧawhar al-mawḍūʿāt*), which is obtained in the previous [disciplines] by means of definition only, is something the [first] philosopher must determine and validate" in metaphysics, and then goes on to reject the view that the task of the metaphysician would be, in this respect, the same as that of a particular scientist. For a discussion of this objection and of Avicenna's reply, see Bertolacci (2006, pp. 223–225nn337–347).

4. If this is the role of dialectic, there is perhaps a hint of circularity in the assumption, discussed in chapter 13, that dialectic provides criteria to determine whether an attribute is essential (genus or differentia) or not (accident). But the difficulty is only apparent, because for Avicenna the identification

tion of definition undertaken by the theory of science is also indirectly confirmed by another passage, at *Madḫal* I, 9, p. 48.1–3. In that context, Avicenna suggests that the verification (*taḥqīq*) of some general contentions concerning definition and description that are briefly introduced in his discussion of the predicables is deferred to a later stage of inquiry, namely the one concerned with the logic of scientific reasoning.[5]

GENERAL CHARACTERIZATION OF DEFINITION AND DESCRIPTION

Avicenna gives a general characterization of definition and description in *Burhān* I, 1.[6] The two notions are associated with the domain of conception (*taṣawwur*), in contrast with propositions and deductions, which fall in the domain of assertion (*taṣdīq*). He writes:

> Text 12.1: *Burhān* I, 1, p. 52.3–20
> Acquired conception is arranged in ranks. One kind is the conception of something [(aa)] by means of accidental notions whose aggregate is proper solely to [the thing], or [(ab)] in a way that is common to [the thing] and what is other than it. Another kind is the conception of something [(ba)] by means of essential notions in a way that is proper solely to [the thing], or [(bb)] in a way that is common to [the thing] and to what is other than it. The conception consisting of essential notions that is proper solely to the thing either [(baa)] includes the perfection of the reality of the

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of an attribute as essential or accidental does not ultimately *depend* on protocols or rules, which force a conclusion on us (in the way demonstration does). It is rather something that ultimately depends on immediate relations that we either see or fail to see (intellectually). In other words, the identification of an attribute as essential rests on recognizing that it belongs to its subject in a self-evident and self-warranting manner, which cannot, in principle, be established by inference (deductive or inductive). These dialectical methods are ladders that may help us reach the stage at which that intuitive intellectual grasp becomes possible, but they are not genuine grounds for it.

5. Definition is also discussed in detail in *Ilāhiyyāt* V, 8. In particular, definition (in the sense of *definiens*) is characterized at *Ilāhiyyāt* V, 8, p. 247.7 in the following terms: “Every definition is an intellectual conception that is truthfully predicated of what is defined” and again at V, 8, p. 247.11: “Definition belongs with certainty to what is defined in a real sense (*al-maḥdūd bi-l-ḥaqīqa*).” In fact, *Ilāhiyyāt* V may be regarded in its entirety as a treatise concerned with definition and other concepts presupposed by definition (the notion of universal, genus, differentia, species, and so on). One of its most notable aspects is the ubiquitous distinction between constituents (notions that are part of the essence of something) and implicates (notions that are inseparable from or necessarily implied by something without being part of its essence).

6. The Greek antecedent for description is *hupographē* (cf. *diagraphē*). The term is attested already in Aristotle (for example at *De Int.* 13, 22a22), but most prominently occurs in the Greek commentary tradition on the *Categories*, starting with Porphyry, in the characterization of entities that cannot in principle be defined by genus and differentia, such as individuals or the categories themselves (that is to say, the highest genera).

> thing's existence (*kamāl ḥaqīqat wuǧūdihī*) and results in an intelligible form equal to the thing's existent form, if none of its essential notions is left aside; or [(bab)] it encompasses (*yatanāwala*) part of its reality but not its perfection. Similarly, the discrimination (*tamyīz*) [obtaining] by means of a differentiating expression (*qawl mufaṣṣil*) used to discriminate and determine (*taʿrif*) something may be, with respect what is being determined, a discrimination of it from some [things] but not from others—in which case, if it is by means of accidental notions, it is [(ab)] an *incomplete description*, and, if it is by means of essential notions, it is [(bb)] an *incomplete definition*—while sometimes it may be a discrimination of [something] from all [other things], in which case, if it is by means of accidental notions, it is [(aa)] a *complete description*, especially if the genus in it is proximate (*qarīb*); and if it is by means of essential notions, then, for the literalist logicians (*ʿinda ẓ-ẓāhiriyyīn min al-manṭiqiyyīn*), it is [(ba)] a complete definition, while for those who validate and determine (*muḥaṣṣilīn*), if it contains all of the essential notions in such a way that nothing is left aside, then it is [(baa)] a *complete definition*; but if something is left aside, it is not a complete definition.[7]

In Text 12.1, Avicenna establishes that a complex term is either a definition or a description depending on whether the notion it aims to characterize is conceptualized by means of essential or by means of accidental attributes. Moreover, a definition or a description is either complete or incomplete depending on how many attributes are employed in the process, and on how the notion is distinguished from other notions (extensionally and conceptually). In general, a definition is a complex term or expression that (i) signifies the essence (*ḏāt*, *ǧawhar*, *ḥaqīqa*), nature (*ṭabīʿa*), or quiddity (*māhiyya*) of something and (ii) produces in the soul an intelligible form that corresponds to the essence of the thing.[8]

The analysis of definition and description in *Burhān* I, 1 is part of Avicenna's broader characterization of scientific knowledge and of its modes of acquisition. If conception (*taṣawwur*) and assertion (*taṣdīq*) are the two fundamental vehicles of scientific knowledge (*ʿilm*), and assertions are the building blocks of deductive reasoning and demonstration, conceptions are the building blocks of definition and description.[9] In particular, definitions and descriptions are types of expres-

7. The term *taḥṣīl*, in the technical sense of validation or determination, is understood in contrast to an exceedingly narrow (and ultimately superficial) interpretation of Aristotle's text ascribed to the literalists (*aẓ-ẓāhiriyyūn*); see Gutas (1988, p. 25n13 and p. 26n16).

8. At *Ḥudūd* Par. 5, p. 3.6–8, Avicenna characterizes definition in terms that are strongly reminiscent of Aristotle's account at *Top.* A5, 101b39: "A statement signifying the quiddity of the thing, that is to say the perfection of its essential being." A description is, by contrast, a statement composed of a genus and of one or more accidents, which provides nonessential knowledge and discrimination (*tamyīz*). On Avicenna's understanding of the way in which universals exist in the soul, see in particular *Ilāhiyyāt* V, 2.

9. Should definitions and descriptions be understood as propositions or terms? In Avicenna's *Burhān*, definitions and descriptions are primarily complex terms, objects of conceptual representation

sions that provide a differentiation or detailed characterization (*qawl mufaṣṣil* or *mufaṣṣal*) of a given notion in terms of a complete or incomplete list of essential or accidental attributes, typically arranged in a certain order. The essentiality, completeness, and correct order of a given set of attributes are necessary and (jointly) sufficient conditions for that set of attributes to qualify as a real definition.[10]

Depending on the type of attribute (essential or accidental) and on the level of generality of the characterization (proper or common), four types of differentiating expression are possible. A definition characterizes its object (the *definiendum*) in terms of *essential* notions (*maʿānin ḏātiyya*) (per se 1 attributes in the language of chapter 7), while a description characterizes its object in terms of *accidental* notions (*maʿānin ʿaraḍiyya*). Such characterizations may be complete (*tāmm*) or incomplete (*nāqiṣ*). A complete definition distinguishes—an operation usually called discrimination (*tamyīz*)—its object from all other objects, and the conjunction of all the attributes it contains is proper to that object alone; an incomplete definition distinguishes its object from some other objects only, and is common to its object and to something else. The same applies, mutatis mutandis, to description.

Consequently, Avicenna's scheme of classification contains four types of differentiating expression (*qawl mufaṣṣil*): (i) complete definition (*ḥadd tāmm*), (ii) incomplete definition (*ḥadd nāqiṣ*), (iii) complete description (*rasm tāmm*), and (iv) incomplete description (*rasm nāqiṣ*).[11] The taxonomy is summarized in table 16.

(*taṣawwur*), even though they may be employed as subjects or predicates, that is to say, as parts of a proposition. A definition in the propositional sense is a proposition in which the *definiens* is predicated of its *definiendum*.

10. One and the same notion may be identified by multiple descriptions, but there can only be one complete real definition corresponding to its unique essence. Such a definition may be formulated in different ways, depending on how many differentiae are made explicit, but that is a separate issue, and it does not compromise the uniqueness of the definition in question. At *Ilāhiyyāt* III, 5, pp. 121.9–122.10, Avicenna illustrates the distinction with an example involving the definition of number ten. While multiple (in fact potentially infinite) characterizations are equally true of ten—for instance the sum of five and five, six and four, three and seven, and so on—these are all descriptions (*rusūm*) and inseparable attributes (*lawāzim* and *ʿawāriḍ*) of ten. The real definition of a number is the sum of a unit, and another unit, and so on, mentioning all required units (*Ilāhiyyāt* III, 5, p. 121.9–10). It is therefore important not to confuse the unique account that gives the real definition of something with an implicate or a property (ten as the sum of four and six), which would merely be a description of it.

11. Description (*rasm*) is a technical term in Avicenna, especially when it is contrasted with definition (*ḥadd*). The term *ṣifa*, by contrast, is more general and covers both constituents and accidents (separable and inseparable). *Waṣf* is often used in connection with the idea of descriptional necessity (and I usually translate it as description or characterization in the relevant contexts). Descriptions in the sense of *rasm* may be relevant for scientific reasoning in two ways: either (i) because they involve per se attributes (it is hard to imagine that Avicenna would seriously contemplate the possibility that separable accidents, that is to say, merely adventitious properties, would play a role in the sciences)

TABLE 16 Conception, differentiating expression, definition, and description

Ranks of acquired conception		Ranks of differentiating expression	
Notions	Aggregate	Discrimination	Account
(a) Essential	(aa) Proper	From all other notions	Complete definition*
	(ab) Common	From some other notions	Incomplete definition
(b) Accidental	(ba) Proper	From all other notions	Complete description
	(bb) Common	From some other notions	Incomplete description

*An aggregate of essential notions that is proper to the *definiendum* and distinguishes it from all other notions is a complete definition only according to the "literalists." Avicenna requires in addition that no essential attribute of the *definiendum* be omitted from its account.

In the case of definition, Avicenna understands the notion of completeness in terms of both extensional and conceptual equivalence. This is plain from his discussion of two erroneous claims concerning definition, which he consistently rejects in various contexts on the grounds that the primary aim of a definition is to capture the essence of the *definiendum* in a complete way. The first claim is that definitions must be *concise* statements of the essence, which according to Avicenna they need not be; rather, definitions must articulate quiddities in their full scope (*Nağāt* I, 140 iv, vi). The second claim is that definitions aim *only* to *distinguish* things from other things (*tamyīz*), whereas, once again, for Avicenna their epistemological function is to reflect fully the conceptualization of an essence in its complete content and structure (*Nağāt* I, 140 v, vi). In *Burhān* IV, 6, we learn that complete definitions must not only be coextensive with their objects but also conceptually equivalent to them in terms of representational content. A definition is incomplete if it fails to express its essence in a complete way, even when it provides a sufficient basis for distinguishing a kind from all other kinds.[12]

Incompleteness, by contrast, may indicate two different ways in which a definition (or a description) is defective. The account of *Burhān* I, 1 refers primarily to extensional incompleteness: an incomplete definition of a species, for example, is

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or, if they do not involve per se attributes, (ii) because they are part of a preliminary account of a notion until they are replaced later by a set of inseparable accidents (a genuine scientific description) or constituents (a definition). At *Burhān* IV, 1, pp. 264.2–265.10, Avicenna is explicit about the fact that descriptions (for example, accounts that include a reference to efficient and final causes) are legitimate terms in scientific discourse. They are used, as we have seen in chapter 11, when a quiddity is conceptualized as existent and hence requires an indication of the causes of its existence.

12. For the contention that discrimination is not the only end of the process of defining, see also *ʿIbāra* I, 1, p. 11.12–13 (where the reference to what will be taught "in its right place" is possibly a reference to the *Burhān* exposition of definition). At *Ḥall muškilāt* II, 7, p. 252.1–2, Ṭūsī associates the opposite view, namely that the only distinctive function of definition is discrimination (*tamyīz*), with the "literalists," that is to say, a group of logicians in the Aristotelian circle of Baġdād.

a definition that does not identify the species as such but stops at its genus (something common to the species and something else, which distinguishes one kind from some other kinds but not from all other kinds). Another characterization, which is briefly mentioned in Text 12.1 and becomes the main focus of various passages in *Burhān* IV, is based instead on conceptual incompleteness: even a definition that expresses the essence of a kind and is proper to that kind alone may still be incomplete, if it fails to capture the full range of attributes that are part of its essence and to reflect its internal structure (the standard example is the definition of human as mortal rational body rather than mortal rational animal).

THE LOGICAL STRUCTURE OF DEFINITION

The definition of an essence E, according to Avicenna, always has (at least ideally) the structure discussed in chapter 7, namely that of an ordered sequence of essential attributes, captured by a combination of a genus and one or more differentiae. The same essence may be expressed by equivalent definitions depending on what terms are made explicit, which is in turn a consequence of how remote or proximate the chosen genus is and of the number of differentiae that are required to specify it. If G_0D_0 is a definition of E by proximate genus and last differentia, and G_1D_1 is in turn the definition of G_0, then E may be expressed interchangeably by both G_0D_0 and $(G_1D_1)D_0$, each of which is coextensive with and conceptually equivalent to E. At each step, the genus may be resolved into a combination of a higher genus and a differentia, until the relevant category is reached in this process of upward regress. All such definitions are equivalent and encapsulate different ways to express the complete structure of E, which includes all the essential predicates in their unique, correct order. The simplest case is one in which at each step of the division a node is identified by its proximate genus and by one differentia only, but in principle essences may be identified in more complex ways, for example through multiple coextensive differentiae (as in the case of animal, which is defined by specifying its proximate genus, namely "living (or animate) body," by means of the differentiae "sensitive" and "moving voluntarily").[13] An incomplete definition either stops at a higher node in the sequence (even if up to that node it contains all the essential predicates in the correct order) or fails to include at least one of the essential predicates.

This schematic representation may also be used to account for the case of descriptions. Even if descriptions cannot be regimented in the same way as definitions to reflect a unique metaphysical structure (for definitions are subject to the

13. The two differentiae in the example are dividing differentiae of "living body" and constitutive differentiae of "animal." Strictly speaking, however, even these two differentiae are ultimately just signs of more complex differentiae, according to *Ilāhiyyāt* V, 7, pp. 237.5–238.3.

constraint of uniqueness relative to the essence of the *definiendum*, while descriptions can consist of a broad range of accidental attributes), it is still true that (i) in many cases descriptions have a similar syntactic structure (where the role of genera and differentiae is played, at least in part, by inseparable accidental attributes, instead of essential ones) and that (ii) in fact it is even possible for some descriptions to consist of fragments of a real definition combined with one or more accidental differentiae to identify a species.

The hierarchical structure of definitions is manifest in Avicenna's account of the process by means of which they are acquired, first and foremost through composition from below, but also through division from above. A detailed example is given at *Burhān* IV, 6, pp. 309.21–310.18 for the definition of the species human and horse, including the explicit identification of certain rules to avoid redundancy in the concatenation of genera and differentiae.

TYPES OF DEFINITION

Avicenna's theory of science is concerned with different types of entities, most prominently primitive subjects, derivative subjects, and their attributes, as we learn in particular from *Burhān* II, 6. All these items have essences but, since some are more fundamental than others, the corresponding definitions are related in different ways. While the general considerations introduced earlier apply to all types, definitions are further differentiated depending on the sort of entities of which they are definitions. In particular, what significantly varies from case to case is the way in which a definition may be acquired and its relative place in the order of explanation. The main source for Avicenna's classification of the different types of definition is *Burhān* IV, 4 (on *An. Post.* B10), where he first distinguishes nominal definition from real definition, and then discusses various kinds of real definitions. In particular, real definitions may be either causal or noncausal, depending on whether their *definienda* are demonstrable or indemonstrable entities of a science.[14]

Causal and Noncausal Definition

In *Burhān* IV, 3 and IV, 4, Avicenna deals with Aristotle's positive treatment of definition at *An. Post.* B8–10, having addressed the *aporiai* of B3–7 in *Burhān* IV, 1 and IV, 2 (where the focus is on the ways in which definitions cannot be acquired). In particular, in *Burhān* IV, 3, Avicenna offers (i) an interpretation of B8 with a

14. The terms causal and noncausal are not Avicenna's. I use them to distinguish definitions based on the nature of their *definienda*, not to suggest that one type is explanatory while the other is not. In fact, what I call noncausal definitions are in a sense the most explanatory of all in a given science, because they articulate the essences of the most fundamental entities in that science.

systematic account of the paradigmatic case of causal definitions, whereas in *Burhān* IV, 4 he clarifies (ii) the elements of the taxonomy of B10. Let us start from the latter and then examine Avicenna's account of causal definition. Aristotle's analysis in B10 consists of a first list of four types (93b29–94a10) followed by a summary, which apparently seems to refer to three of them only (94a11–14). The first list includes

1. an account of the meaning of a name (traditionally qualified as nominal definition);
2. a (causal) definition that differs from demonstration in virtue of the arrangement (*thesei*) of the terms;[15]
3. a (causal) definition qualified as the "conclusion of a demonstration of the what-it-is"; and
4. a (non-causal) indemonstrable statement of the essence (*thesis tou ti estin anapodeikto*s), that is to say, the definition of an immediate.

Aristotle's summary notoriously appears to mention only 4, 2, and 3 (in this order), which raises questions about the consistency of the two lists and about the role of nominal definition. For Avicenna, an additional problem is that both lists leave out another type of definition mentioned by Aristotle elsewhere, namely the one that is called the "principle of a demonstration" (which is introduced in passing at *An. Post.* A8, 75b31–32 but altogether omitted from the two lists of B10).[16]

Avicenna's positive treatment of definition begins, at *Burhān* IV, 4, p. 288.1–4, with a short preamble, which serves as a basis to connect his own distinction between complete and incomplete definition to Aristotle's account in B10:

15. At *An. Post.* B10, 94a6–7, Aristotle notes "The same account is said in another way" (*ho autos logos allon tropon legetai*) and characterizes it as a "continuous demonstration" (*apodeixis sunechēs*) with regard to the arrangement of the terms (the case is exemplified by the famous definitions of thunder as noise due to the quenching of fire in the clouds and of eclipse as loss of light due to the interposition of the earth between the moon and the sun).

16. The notion of definition as principle of a demonstration is discussed in the Greek commentary tradition and plays an important role in Avicenna's account of causal definition. In particular, it seems to be identified (or at least associated) with the definition of an indemonstrable by Themistius, *In An. Post.* B10, p. 51.19–22: "The fourth is the definition of those things that are first and immediate (*prōtōn kai amesōn*), which [is] neither [the one that] differs from demonstration in virtue of the arrangement [of the terms] [. . .] nor the conclusion of a demonstration, but rather the *principle of a demonstration* (*archē tēs apodeixeōs*) and an indemonstrable statement of the what-it-is (*thesis tou ti estin anapodeiktos*)." Concerning the putative inconsistency of Aristotle's classification, Ross (1949, p. 634) suggests that there are really just three kinds of definition in the first list, because 1 is in fact the same as 3 (the conclusion of a demonstration is simply a nominal account). Themistius and ps.-Philoponus, by contrast, understand the first list in B10 to include four genuine types. Ps.-Philoponus, in particular, explains away the discrepancy between the fourfold classification and the threefold summary by ruling out nominal definition; Avicenna seems to be following this line too.

(i) complete definition (*ḥadd tāmm*),
(ii) incomplete definition (*ḥadd nāqiṣ*) in the sense of principle of a demonstration (*mabdaʾ al-burhān*), and
(iii) incomplete definition (*ḥadd nāqiṣ*) in the sense of conclusion of a demonstration (*natīǧat al-burhān*),

where (i)–(iii) are real definitions that express the essence of the *definiendum* (*ḥadd ḥaqīqī bi-ḥasab aḏ-ḏāt*), either completely or incompletely. An additional type is

(iv) figurative definition concerned with the name (*ḥadd maǧāzī bi-ḥasab al-ism*).

The latter is also characterized as "what explains the name and makes its intended meaning understood in itself" (*yušarriḥu l-ism wa-yufahhimu l-maʿnā llaḏī huwa maqṣūd bi-ḏ-ḏāt*).[17]

A fifth type, which had been introduced in the previous chapter, at *Burhān* IV, 3, p. 287.7–9 (a brief paragraph corresponding to *An. Post.* B9) is

(v) noncausal definition.

At *Burhān* IV, 4, p. 292.1–7, Avicenna addresses the apparent inconsistency between the two lists of B10. His view is that (i) complete definition is somehow included in both lists; (ii) the definition that is called principle of a demonstration is left out of both lists because it is obvious, and a learner would have no trouble understanding it; (iii) if definition in the sense of principle of a demonstration were to be included in the list, then real complete definition would become the fourth type (presumably after nominal, conclusion, and principle); but (iv) since, as a matter of fact, it is not included in the list, Aristotle takes the noncausal definition of immediates as the fourth type instead.[18]

17. The case of nominal definition is discussed at length, with an explicit recognition of the fact that in the nominal case the definition provides no indication of the existence (or of the cause of the existence) of the *definiendum*, contrary to what happens in the case of real causal definitions (since void is impossible, a nominal definition of void cannot in any sense specify genuine conditions of existence). At *Samāʿ ṭabīʿī* III, 7, p. 209.8, Avicenna explicitly refers to the notion of infinite taken "in a figurative sense" (*ʿalā l-maǧāz*) to introduce his discussion of various arguments against the possibility of an actual infinite.

18. The number varies from work to work, depending on how real definitions are counted and on whether nominal definition is included in the final list. For example, a classification that explicitly contains five types is given by Avicenna at *Naǧāt* I, 144, pp. 159.7–160.6, where the types of real definitions are all counted separately (complete, principle, conclusion, immediates) in addition to nominal definition. At *Muḫtaṣar awsaṭ fī l-manṭiq* (*Burhān*) II, 9, pp. 249.1–250.12, Avicenna raises the question of whether the types of definition are really four or five in the broader context of a discussion of the relation between definition and demonstration. In *Burhān* IV, 3, however, Avicenna is directly

TABLE 17 Types of definition

<table>
<tr><td>(a) Name</td><td colspan="5">(b) Essence</td></tr>
<tr><td rowspan="3"></td><td colspan="2">(ba) Noncausal</td><td colspan="3">(bb) Causal</td></tr>
<tr><td rowspan="2">(baa)
Complete</td><td rowspan="2">(bab)
Incomplete</td><td>(bba) Complete</td><td colspan="2">(bbb) Incomplete</td></tr>
<tr><td>Concatenation
of conclusion
and principle</td><td>(bbba)
Principle of a
demonstration
Cause</td><td>(bbbb)
Conclusion of a
demonstration
Effect</td></tr>
<tr><td>(aa) First stage
of inquiry
(ab) Impossible
entities</td><td colspan="2">Subjects,
Immediate implicates</td><td colspan="3">Demonstrable attributes</td></tr>
<tr><td>Nominal</td><td colspan="5">Real</td></tr>
</table>

Avicenna first draws a general distinction between nominal and real definitions, and then classifies real definitions into two categories: causal definitions, which are paradigmatically definitions of demonstrable attributes, and noncausal definitions, that is to say, definitions of entities that do not have a cause other than themselves. Causal definitions are further divided into three types: the complete definition of a notion and two kinds of incomplete definitions (which are proper parts of the complete definition), called the "principle of a demonstration" and the "conclusion of a demonstration," respectively. The characteristic of complete causal definitions is that their parts (principle and conclusion) stand in a relation of cause and effect, and are employed as terms in a sequence of demonstrative deductions that prove the attribute-*definiendum* to belong to its subject.

In the case of complete noncausal definitions, by contrast, no term or component plays the role of cause or effect, and the distinction between principle and conclusion of a demonstration does not apply. The types of definition discussed by Avicenna in *Burhān* IV, 4 are illustrated in table 17.

In summary, Avicenna's classification includes the following kinds of definition:

(a) Name: (aa) preliminary grasp of the meaning of a term corresponding to a real essence (which is discovered at a later stage) or (ab) nominal account of nonexistent, especially impossible, entities (terms such as void, infinite, and atom, which are used in the context of reductio proofs)

(b) Essence:

 (ba) Noncausal

concerned with the difficulty raised by Aristotle's text, which puts some constraints on the systematic character of his own classification.

- (baa) Complete: definition of point or unit
- (bab) Incomplete: partial definition of point or unit that captures a genus only or perhaps a definition that fails to express the essence fully, even if it is extensionally equivalent

(bb) Causal

- (bba) Complete (eclipse as loss of light due to the interposition of the earth; thunder as noise in the cloud due to the quenching of fire)
- (bbb) Incomplete:
  - (bbba) Principle (cause) (interposition of the earth; quenching of fire)
  - (bbbb) Conclusion (effect) (loss of light; noise in the cloud)

CAUSAL DEFINITION

Avicenna usually discusses the structure of causal definitions under the general heading of the mutual participation of definition and demonstration. This is, in fact, the theme of *Burhān* IV, 4, where he gives a systematic account of the examples discussed by Aristotle in *An. Post.* B8.[19] He writes:

> Text 12.2: *Burhān* IV, 4, pp. 289.16–290.5
> Definition is said in another sense of what gives the cause of the existence of the notion of what is being defined (*al-maḥdūd*) and is assumed itself in the demonstration as a middle term, in which case it is the principle of a demonstration. If in assuming this definition one adds to it its perfection (*kamālahū*), which consists in adjoining it to the effect (*iḍāfatuhū*), and what is being defined is set down as a subject (*wuḍiʿa*), then three things are put together in it, namely [(i)] what is being defined, [(ii)] a definition that gives the cause, and [(iii)] its perfection in giving the cause, which consists in mentioning the effect. These three things convert with one another. Otherwise, there will be nothing defined, no definition, and no perfection of a definition, for what is being defined and the definition are mutually equivalent, and the perfection of the definition is the effect of the definition and only exists because of it, belongs to all of what is being defined, and is in turn equivalent to the first two. These three things are posited in order for a demonstration to come about from them concluding, by means of two deductions, that the perfection of the definition [belongs] to a given subject.

In Text 12.2, Avicenna articulates in detail his understanding of the logical structure of causal definition and its relation to demonstration.[20] Let P be a

19. Avicenna's textual analysis of *An. Post.* B8 is at *Burhān* IV, 3, pp. 284.3–287.6; cf. also *Naǧāt* I, 143, pp. 157.3–159.6. On *An. Post.* B8 and its place in Aristotle's theory of definition, see von Kirchmann (1878), Vailati (1903), Bolton (1976, 1987), Granger (1976), Ackrill (1981), Brunschwig (1981), Gómez-Lobo (1981), Guariglia (1982), Landor (1985), DeMoss and Devereux (1988), Bayer (1995, 1997a), Goldin (1996, pp. 101–136), Charles (2000, pp. 23–56 and pp. 198–204), and Deslauriers (2007).

20. At *Burhān* IV, 3, p. 287.5–6, Avicenna interestingly suggests that a causal definition is an object of *tanbīh*: "But let us be concerned here with our goal and say that the First Teacher shows that

demonstrable attribute-*definiendum* (what is being defined, *al-maḥdūd*). An incomplete causal definition of P, which gives (ii) the cause of the existence of the notion of the *definiendum* (*ʿillat wuǧūd maʿnā l-maḥdūd*), is taken as a middle term in a demonstration of the fact that (and of a partial reason why) attribute P belongs to its subject S. If one adds to such an incomplete causal definition of P another supplementary notion, namely (iii) its perfection (or completion, *kamāl*), and connects it to the attribute-effect (*iḍāfat al-maʿlūl*), it is possible to extract the complete definition of the demonstrable attribute P from two concatenated deductions by means of which the attribute is causally proved to belong to its subject S.[21] To illustrate how the procedure works, Avicenna writes:

> Text 12.3: *Burhān* IV, 4, p. 290.6–14
> The criterion that governs the arrangement of the terms of the demonstration [consists in] the inversion of the arrangement of the parts of the definition. An example of this is [as follows]. Let cloud (*ġaym*) be the subject of the three terms, let this definition, which is the cause, be the quenching of fire in the cloud, and let its perfection be the coming about of noise. We shall then say that cloud is moisture in which fire is quenched; every moisture in which fire is quenched [is such that] noise comes about in it; therefore, cloud [is such that] noise comes about in it; but every noise coming about in a cloud is thunder (*raʿd*); therefore cloud [is such that] thunder comes about in it.
>
> Thus, these *three things* have become *parts of two demonstrations*: the minor [term] twice (among the terms of the two [demonstrations]), that is to say cloud, [as] the subject of the three things. The quenching of fire is the first of these three to be mentioned, then the coming about of noise. The coming about of noise is proved in the *conclusion* of the first demonstration (the quenching of fire is not proved [here] but is rather the *principle* of a demonstration, not a conclusion). What is being defined, namely thunder, is the last of the three things to be mentioned in the second demonstration, being mentioned in the second conclusion.

In Text 12.3, Avicenna shows how the complete demonstrative proof of the fact that and of the reason why a demonstrable attribute belongs to its subject structurally involves four terms arranged in two deductions: the subject, the attribute, and two partial and causally related accounts of the attribute. The terms that appear in this demonstration, which involves a concatenation of two deductions, are the same terms that appear in the definition of the attribute. The attribute itself is the *definiendum*; the two partial accounts are parts of its complete definition (one being the incomplete definition known as the principle of a demonstration,

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demonstrations that involve causes give, in a way, a reminder of definitions (*tanbīh ʿalā l-ḥudūd*). This [is the case] with things that are accidents of something and belong to something (*fī šayʾ*) in virtue of the kind of causes assumed in definitions." The same procedure described by Avicenna in *Burhān* IV, 3 to account for the relation between definition and demonstration (principle, conclusion, perfection) is also discussed by Alfarabi, *Kitāb al-Burhān* III, p. 47.6–25.

21. On the notion of perfection in this context, see also *Mubaḥāṯāt* Par. 761, p. 262.6–11.

the other being the incomplete definition known as the conclusion of a demonstration); and the subject is a sort of anchor that determines, as it were, in the background what the attribute is an attribute of, tying it to a specific domain. Thus, if we take four terms S—C—E—P with immediate nexuses SaC, CaE, and EaP (where C and E stand for cause and effect), they may be demonstratively arranged in the following two deductions:

(1) First demonstration
  (1.1) CaE
  (1.2) SaC
  (1.3) SaE
(2) Second demonstration
  (2.1) EaP
  (2.2) SaE
  (2.3) SaP

where the same proposition SaE is the conclusion of (1) and the minor premise of (2). The first demonstration is a why-demonstration in which an incomplete definition of the attribute (the effect or conclusion of a demonstration) is proved to belong to the subject through another incomplete definition of the attribute (the cause of that effect or principle of a demonstration). The second demonstration is a why-demonstration in which the attribute is proved to belong to its subject. Jointly, the two middle terms convey the full causal explanation of the attribute as well as of the fact that the attribute belongs to the subject. Avicenna reinterprets Aristotle's example in *An. Post.* B8 as follows:

S = cloud (the subject of the three terms)
P = thunder (the predicate, *maḥdūd*, *definiendum*)
E = coming about of noise in the cloud (the perfection *kamāl*, effect) (incomplete definition: conclusion)
C = quenching of fire in the cloud (definition *ḥadd*, cause) (incomplete definition: principle)

The subject is that to which the three other terms—the attribute and the two middle terms—ultimately belong. The attribute and the middle terms are convertible and definitionally related. In particular, the two middle terms are incomplete definitions of the attribute. Leaving the subject aside, demonstration and definition differ in this case only in virtue of the relative arrangement of these other three terms. The order of the terms in the demonstration is inverted with respect to the order of the parts of the definition. In the definition, P is the *definiendum*, and the complete definition is the concatenation of the two incomplete definitions E and C (the conclusion of a demonstration and the principle of a demonstration).

Hence the order is P, E, C. In the demonstration, C is the first of the three definitionally interconnected terms to occur in the proof. E comes right after C and is proved by means of C to belong to S (in the conclusion of the first demonstration). C is not proved to belong to S but only used as a principle, hence the name of this type of incomplete definition. P occurs last, as the major term in the conclusion of the second demonstration. Hence the order is C, E, P. What the two deductions show in demonstrative form may be recast in the form of a complete definition in the following way:

Def(P) = (E | because of C)

Thunder (in the cloud) = Noise (in the cloud) | due to the quenching of fire.

A complete definition of a demonstrable attribute is obtained by means of two (incomplete) notions (the incomplete parts of a complete definition). Only the incomplete parts of a complete definition appear in the two deductions, not the complete definition itself. The latter is obtained by concatenating the two parts and merging them into a single expression that corresponds to the essence of the attribute. At the extremes are S and P, the subject and the demonstrable attribute. The middle terms are E and C. E is an account of P that expresses part of the essence of P. C is another part of the essence of P that is causally related to E as a cause is related to its effect. C is the causally relevant part of the essence of P, because the other part, E, is dependent on C as its effect. Thus, C is the principle of the demonstration not only because it appears as a middle term in the premises of the first deduction, but also, and more importantly, because it grounds the nexus between S and P by mediating between S and E (which is in turn connected to P). E is the conclusion of the demonstration because it appears in the conclusion of the first deduction, but more importantly, because the nexus between E and S is causally dependent on C.[22]

The arrangement of the terms in the definition (*definiens*) is called by Avicenna "definitional composition" (*taʾlīf ḥaddī*) (*Burhān* IV, 4, p. 290.15).[23] In the definition, the principle of a demonstration becomes the last term, while the conclusion of a demonstration becomes the first term. If the complete definition is formulated in propositional form, the *definiendum* (the demonstrable attribute) becomes the subject of the complete definition resulting from the combination of the two incomplete definitions (conclusion and principle, in this order).[24]

22. At *Burhān* IV, 3, p. 286.3–15, Avicenna explicitly contends that (i) this procedure is only legitimate when a demonstration involves genuinely causal terms and is not merely a deduction from "accidents and implicates," as in the example of lunar eclipse at *An. Post.* B8, 93a36–b3 (for the latter is just a that-demonstration) and that (ii) even why-demonstrations that simply give the cause of the major term's belonging to the minor term are inadequate for the discovery of definition.

23. On *taʾlīf* in this sense, see also *Ǧadal* V, 1, p. 241.1–7.

24. At *Naǧāt* I, 143, Avicenna contends that a complete definition is not a single term in a demonstration but a complex term consisting of two parts, each of which occurs in a premise, and that

Avicenna gives two further examples to illustrate causal definitions, namely the notions of anger and eclipse. The relevant definitional terms for anger (attribute-*definiendum*) are heart (subject), ebullition of blood (conclusion of demonstration, first term in the complete definition), and desire for revenge (principle of demonstration, second term in the complete definition). The relevant definitional terms for (lunar) eclipse (attribute-*definiendum*) are moon (subject), loss of light (conclusion of demonstration, first term in the complete definition), and interposition of the earth (principle of demonstration, second term in the complete definition).[25]

In the case of noncausal definitions, the distinction between principle and conclusion of a demonstration does not apply. This does not mean that complete noncausal definitions have no parts. The logical structure by genus and differentia is the same in the causal as well as in the noncausal case. The difference is that genera and differentiae in the former case are understood to be related as causes and effect.[26] In other words, according to Avicenna, the distinction between complete and incomplete definition applies to both cases, but in different ways. In the noncausal case, an incomplete definition is simply one that lacks at least one of the essential constituents of the *definiendum*. In the causal case, there is a further distinction between different kinds of incomplete definitions based on their underlying causal relations and the respective role they consequently play in the corresponding demonstrations.[27]

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the order of the terms, in a definition and in the corresponding demonstration, is inverted: what is predicated of the subject first in the demonstration is predicated second of the attribute-*definiendum* in the definition (taking definition here in the propositional sense), while what is predicated second in the demonstration is predicated first in the definition.

25. On Aristotle's definition of anger, see Aubenque (1957) and Harris (1997); cf. also ps.-Philoponus, *In An. Post.* B8, pp. 364.30–365.36.

26. At *Burhān* IV, 4, p. 291.5–10, Avicenna seems to suggest that the role of genus should always be played by the term identified as the conclusion of a demonstration (and presumably, even if the point is not explicitly stated, that the role of differentia should be played by the term identified as the principle of a demonstration). This is reflected in the relative order of appearance of the two terms in a complete definition. In particular, the conclusion of a demonstration (effect) gives a generic account of the attribute-*definiendum*, which is then determined further by the principle of a demonstration (cause). In the same context, Avicenna criticizes the opinion of a group (*qawm*) for its identification of definition in the sense of conclusion of a demonstration with matter and of definition in the sense of principle of a demonstration with form. The view rejected by Avicenna bears a remote resemblance to ps.-Philoponus, *In An. Post.* B10, p. 375.2–8, where two kinds of definition are called *eidikos* (formal) and *hulikos* (material), respectively. The two terms occur several times in ps.-Philoponus on B8–10.

27. At *Ilāhiyyāt* I, 5, p. 31.6–7, Avicenna states the general principle that every (possible) being has an essence (*ḥaqīqa*) in virtue of which it is what it is. Interesting examples of definitions are unsurprisingly scattered throughout the corpus, but a good general starting point is Avicenna's own *Book of Definitions*, which contains a list of definitions of a broad array of entities, ranging from the real definitions of mathematical entities and attributes (for example, circle, triangle, line, angle, solid, number, odd,

NONCAUSAL DEFINITION

Noncausal definition is briefly discussed by Avicenna at *Burhān* IV, 3, p. 287.7–9 based on a distinction between (i) items for which there is a cause other than themselves and (ii) items for which there is no cause other than themselves, introduced by Aristotle at *An. Post.* B9, 93b21–22. The essences of the former are captured by causal definitions, while the essences of the latter are captured by noncausal definitions. In particular, while the essences of items of type (i) may be exhibited by demonstrative deductions in the way just examined, the essences of items of type (ii) are, by contrast, "immediates and principles," which must be "presupposed or made clear in another way," that is to say, in a non-demonstrative manner. Aristotle calls their account, at *An. Post.* B10, 94a8–12, "the definition of immediate items," which is "the indemonstrable posit of the essence" and "the indemonstrable account of the essence."

According to Avicenna, the class of items for which there is no cause other than themselves and whose definitions are consequently noncausal definitions of immediate principles includes two kinds of entities. He writes:

> Text 12.4: *Burhān* IV, 3, p. 287.7–9
> What does not have a cause of its own existence without qualification or of its belonging to something—because [(i)] it is not an accident of something or because [(ii)] it is a first accident [that belongs to something] without a cause (the principles of the sciences are of this kind)—sometimes is asserted with no deduction at all to show that it is (*haliyya*); rather, the fact that it is, is evident. Still, definitions of these [entities] may be acquired.

Text 12.4 identifies such items with the primitive subjects of a science ("what is not an accident of something") and its immediate implicates ("a first accident without a cause"). Nothing that falls in one category or the other may be explained in terms of something more fundamental within a science. In particular, there is no reason why an immediate implicate belongs to that of which it is an immediate implicate other than things being just what they are.

Immediate facts about these entities and their nexuses may be asserted without a deduction, whether it be because the existence of a subject is evident or because

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even, abundant, perfect, deficient, product, and so on) to nominal definitions of non-existents. Several definitions are also given in the *Ilāhiyyāt*: for example, quantity, at *Ilāhiyyāt* III, 4, p. 118.14–15; body, at *Ilāhiyyāt* II, 2, p. 64.6–7 ("true corporeality is a form of continuity in which the three dimensions can be assumed"); and motion, at *Ilāhiyyāt* III, 8, pp. 140.17–141.8. In the case of non-existents, by contrast, Avicenna considers pseudo-essences admissible for the purposes of reductio proofs, as we have seen in chapter 8. At *Burhān* I, 6, p. 72.1–15, he discusses two basic types of non-existent. The first type includes non-existents such as the void or the contrary of God, which do not involve composition and differentiation, and can only be understood with negative reference to an existent. The second type includes non-existents that involve composition and differentiation, and can hence be analyzed into real essences. In the latter case, the putative essence of a non-existent object is just what results from the combination of the real essences of its components into a complex that is not instantiated, either just as a matter of fact or because it is genuinely impossible.

one of its immediate implicates belongs to the subject in an evident manner.[28] Both primitive subjects and immediate implicates, however, must be defined, and their definitions must be assumed as principles in a science. The primitive status of such entities and the immediacy of the nexuses obtaining between them does not absolve us from the task of providing an account of *how* their definitions may be acquired.

28. The notion of *haliyya* may be understood in Text 12.4 either as simple or as compound, as noted in chapter 3.

13

The Epistemology of Essence

DIVISION OF LABOR

Definition is a notion at the intersection of logic, metaphysics, epistemology, and philosophical psychology. In order to understand the nature, scope, and limits of Avicenna's discussion of the problem of the acquisition of definition, it is especially critical not to conflate the following three questions. First, is the sort of knowledge encapsulated by real definitions at all possible? Second, how are definitions acquired? Third, how does the process of concept formation work? For Avicenna, the first is a metaphysical question; it depends on the assumption that the world to which the theory of science applies is a world populated by essences and governed by relations between essences, and that those essences and their relations are accessible to the human intellect. To establish all this is to show that definition and demonstration are in themselves possible, which is something that squarely falls within the purview of metaphysics.[1] The second is a proper question for the theory of science. The answer to this question consists in the identification of the methods discussed in this chapter, namely composition and division, by means of which we discover the complete ordered sequence of essential attributes that constitute those particular kinds of complex terms that we call definitions. These methods presuppose in turn that we have a certain conceptual vocabulary of simple terms and tell us how to put them together (they also indirectly ensure

1. See, for instance, *Burhān* III, 6, p. 237.9–10, where Avicenna notes that the fact that definitions and demonstrations exist is set down as an assumption (*mawḍūʿ*) for the logician (including the branch of logic concerned with the theory of science) but proved elsewhere.

that the resulting complex terms are the sort of expressions that adequately correspond to essences, because they consist of all and only the essential attributes of a *definiendum*, arranged in the correct order). The third question, namely how we acquire the conceptual vocabulary of simple terms, is a question for philosophical psychology, perhaps in tandem with other areas of logic. The distinction between essential and accidental attributes is something that Avicenna expressly describes as part of the process of concept formation, that is to say as something the intellect is aware of when it operates an abstraction, but it is at the same time a distinction with regard to which certain procedures examined in the logical theory of commonplaces—most notably, of genus, differentia, propria, and accidents—may prove helpful.

By his own explicit acknowledgment, Avicenna's theory of science is not required to establish that definitions are possible: epistemology owes its optimism to metaphysics. Nor is it required to establish that, and how, the basic ingredients employed in the process of acquisition of definition become available to us: epistemology owes its alphabet to philosophical psychology. What Avicenna's theory of science must do instead is to provide rules of construction and criteria of adequacy for the complex terms that serve as the ultimate foundation of scientific reasoning.

HOW DEFINITION IS *NOT* ACQUIRED

Three approaches or methods may be taken as prima facie plausible paths for the acquisition of definitions: demonstration, division, and induction.[2] All three are vigorously rejected by Avicenna. In general, the fundamental problem is the idea of establishing a definition to belong to its *definiendum* through some kind of inferential procedure. In particular, the three methods are shown to involve an infinite regress, or to be either too strong (and hence circular) or too weak (and hence inconclusive) to prove conclusions of the form "S is P" or "Every S is P," where P is the *definiens* of S. Some of the puzzles and objections are familiar *aporiai* from *Posterior Analytics* B4–7, while others are introduced independently by Avicenna.

In the case of demonstration, Avicenna's criticism is a development of the argument of *An. Post.* B4: definitions cannot be deductively proved qua definitions on pain of circularity, infinite regress, or a violation of the requirement of uniqueness for real definitions. The indemonstrability of real definitions is established

2. The point is stated clearly at *Naǧāt* I, 139, where Avicenna establishes concisely three main claims, namely that it is not possible to acquire a definition (i) as a conclusion of a demonstration, (ii) as a conclusion of a process of division cast in deductive form (the arguments are the same as in *Burhān* IV, 2 and IV, 3), and (iii) as a conclusion of a process of induction. In the same chapter, Avicenna also rejects the idea of obtaining the definition of an opposite from the definition of its opposite. The arguments are broadly inspired by Aristotle but, especially in the case of induction, Avicenna's approach is highly original.

by reductio. A middle term M, involved in a putative demonstration by means of which a major term P is shown to be the *definiens* of a minor term S, could only be (by elimination) a *definiens* of the minor. But then, either the definition is unique, in which case it is being *circularly* assumed for its own proof, or it is not unique. In the latter case, what about the relation between S and M? If it turns out that M is an additional *definiens* of S, it then stands in need of another proof, involving another middle term. But in order to prove that M is the *definiens* of S, the new middle term will have to be yet another *definiens* of S (otherwise it will not be sufficiently strong to show that M is the *definiens* of S), and this would lead to an infinite regress.[3]

In the case of division, which is discussed in *Burhān* IV, 2 and IV, 3 (on *An. Post.* B5–6), the main difficulty lies in the fact that each step is *assumed* rather than proved (division presupposes knowledge of certain facts), and the assertion of a contradictory in a dichotomous division (for example, the assertion that something is not nonrational to establish that it is rational) cannot be probative because it involves at best something that is just as evident (if not less evident) than what needs to be established, and one of the requirements of a definition is that it should consist of notions that are better known than the *definiendum*.

In the case of induction, Avicenna's arguments deserve a separate excursus, (i) because the notion is potentially ambiguous and (ii) because it is often associated, in Aristotle, with the acquisition of first principles. It is therefore necessary to specify what Avicenna means by induction in the context of his theory of science and to see why it is not helpful in the acquisition of first principles, especially definitions. This will be instrumental for a better understanding of the nature of the procedure by which Avicenna replaces induction, namely the method of composition.[4]

INDUCTION AND INFERENCE

Avicenna discusses induction in two contexts that are relevant for our present purposes. At *Burhān* I, 9, he gives an elaborate argument for the rejection of

3. On Aristotle's argument for the indemonstrability of definition, see Schröder (1984). For a detailed reconstruction of Avicenna's argument in *Burhān* IV, 2, with translations of the relevant texts, see Strobino (2010).

4. For a general characterization of induction (*epagōgē*) in Aristotle, see Ross (1949, pp. 481–485) and Bayer (1997b). On induction as a mode of establishment of a universal proposition, that is to say, as a type of proof or justification through the observation of particulars, along the lines of *An. Pr.* B23, see Hamlyn (1976), Engberg-Pedersen (1979), and McKirahan (1983). On induction as a mode of discovery—or "generative psychological account of how we acquire concepts or universal scientific principles" (McGinnis 2003)—along the lines of *An. Post.* B19 or *Met.* A1, see Upton (1981), Hintikka (1980), and McCaskey (2007). An excellent recent account of induction in the second sense, in the context of the *Posterior Analytics*, is Bronstein (2016, pt. III).

induction as a general inferential procedure for establishing immediate noncausal propositions, that is to say, the first principles of a science (including definitions in their propositional form).[5] At *Burhān* IV, 3, Avicenna gives a shorter argument for the rejection of induction as a method specifically intended for proving or acquiring definitions. These arguments have no parallel in the *Posterior Analytics*, even though they may be traced, directly or indirectly, to a brief remark at *An. Post.* B7, 92a37–b1.

Induction is primarily understood by Avicenna as a kind of inference whose aim is to establish predicative assertions of the form "Every S is P." The argument form that encapsulates inductive reasoning is a hypothetical deduction involving a complex categorical major premise and a categorical minor premise with a disjunctive term—Avicenna frequently refers to the latter as a disjunctive (*munfaṣila*) premise.[6] The process relies on (i) the enumeration of all or some of the particulars falling under the subject term (the minor premise of the induction) and (ii) the ascription of the predicate to each and every one of those particulars (the major premise of the induction). Depending on whether the enumeration covers all or only some of the particulars falling under the subject term, the induction is either complete (*istiqrāʾ tāmm* or *mustawfan*) or incomplete (*istiqrāʾ nāqiṣ*).[7]

5. This may be an echo, however remote, of *An. Pr.* B23, 68b13–14, where Aristotle suggests that there are only two types of persuasion, one that proceeds by demonstration and the other by induction. At the end of the chapter (*An. Pr.* B23, 68b30–32), Aristotle then draws a connection between the two methods and the presence or absence of a (causal) middle term: "This is the sort of deduction that is possible of a primary and immediate premise: for the deduction of those premises of which there is a middle term is by means of the middle term; but the deduction of those of which there is not a middle term is by means of induction." At *An. Post.* A31, 88a5–9, Aristotle again appears to be making a similar point: even though in that passage there is no explicit reference to induction, we encounter again the distinction between the causal case, in which knowledge obtains only through the cause, and the noncausal case of first principles, for which "a different account" is needed. The focus of *An. Post.* A31 is perception, but perception must be accompanied by some form of inductive reasoning, if it is to offer any sort of justification of scientific principles; the progression from perception to induction as a putative path to immediate principles is confirmed in *An. Post.* A18. Last, as we have seen in chapter 12, Aristotle notes at *An. Post.* B9, 93b21–24 that the items for which there is no cause other than themselves must be made clear "in another way," that is to say, by induction rather than demonstration. It is precisely this line of reasoning that Avicenna seems to be attacking in *Burhān* I, 9.

6. Under suitable conditions, induction is equivalent to what Avicenna calls "divided deduction" (*qiyās mufaṣṣal*), that is to say, a mixed hypothetical connective deduction consisting of a premise with a disjunctive predicate and of a categorical premise that is equivalent to a conjunction of as many categorical premises as the particular instances of the subject term that are being enumerated. Avicenna discusses the formal properties of divided deductions at *Qiyās* VI, 6, pp. 349.1–354.12. The notion appears in his other logical works, too, and is explicitly acknowledged to have applications outside the domain of formal logic.

7. Incomplete induction is sometimes characterized as endoxic (*mašhūr*), when it is based on most but not all particulars. Induction is explicitly linked with endoxic premises at *Daneshname*, I, 25; see

Thus, if $s_1, s_2, \ldots, s_n$ are the particulars falling under S (where those particulars may be either less general terms or singular terms), a complete induction may be analyzed as follows:

1. Every S is either s_1 or s_2 or . . . or s_n.
2. Every s_1 or s_2 or . . . or s_n is P (which is equivalent to "Every s_1 is P" and "Every s_2 is P" and, . . ., and "Every s_n is P").
3. Therefore, every S is P.

The schema is illustrated by the following example (where the minor premise is stated first): "Every animal is a body and every plant is a body and every mineral is a body; everything that moves is either an animal or a plant or a mineral; therefore, everything that moves is a body (*Nağāt* I, 82 (xvii), p. 88.5–12). In particular, if the terms are general rather than singular, we have a universal categorical proposition with a disjunctive subject, which is equivalent to a conjunction of universal categoricals with simple subjects. The inference is by cases rather than being through a cause, and its structure is in fact equivalent to a categorical deduction, given that a conjunction of universal propositions sharing the same predicate is equivalent to a categorical having that predicate and a disjunctive term for subject.

The analysis of the logical form of inductive arguments offers useful insights into Avicenna's motives for the rejection of induction in the broader context of his epistemology of essence.

INDUCTION AND IMMEDIATE, NONCAUSAL ASSERTIONS

Scientific knowledge of a causal nexus between a subject and an attribute can only be obtained by demonstration and requires knowledge of the cause of that nexus. But when no such cause exists and the nexus between a subject and an attribute is immediate, what is the epistemic justification of its assertion based upon? Since the justification of immediate, noncausal assertions cannot (in principle) depend on demonstration, those assertions must be either self-evident or, as Aristotle frequently suggests, established by induction. Avicenna's analysis is intended to show that induction can in no way serve as a method for establishing noncausal assertions. The latter can therefore only be self-evident, self-warranting, and self-explanatory. This is true of every kind of immediate principle, but for our present

Achena and Massé (1955, p. 71). In the same work, Avicenna openly reveals his negative attitude toward this method when he contends that induction is "the kind of reasoning to which theologians and dialecticians give credit" *Daneshname* I, 21, see Achena and Massé (1955, p. 61).

purposes, it is especially true of the nexus between a *definiendum* and its *definiens* (or one of its constituents).[8]

The goal of the argument against induction in *Burhān* I, 9 is precisely to show that this method is inadequate for establishing noncausal immediate universal predications, and that the latter must be self-evident. The argument has a complex structure but, in a nutshell, it runs as follows. In the immediate, noncausal assertion "Every S is P," the nexus between S and P is either self-evident or not. If it is, the desired conclusion is established. If it is not, then let us assume that it can only be established inductively. This, as we have seen in the previous section, would presuppose knowledge of the nexus between P and each and every one of the particulars falling under S (for the sake of argument, let us assume the latter to be less general terms rather than singular terms). The question then is whether the nexus between P and the particulars falling under S is in turn either self-evident or not.

Avicenna offers an elaborate proof to show that the nexus between P and the particulars under S cannot be self-evident (the subsidiary proof argues that no matter what type of attribute P is—essential, inseparable accidental, or separable accidental)—various kinds of impossibilities would inevitably follow. But if the nexus between P and the particulars under S is not self-evident, then it must be amenable to explanation. Such an explanation can be either deductive or inductive. If the nexus between P and the particulars falling under S is explained through a deduction, this will involve a middle term, which will then be the real cause of the nexus, and *this* rather than the overall inductive process of which this step is a part would be the ultimate justification of "Every S is P." If, by contrast, the nexus between P and the particulars under S is not explained through a deduction, but rather by means of another induction, this will eventually lead to an infinite regress.

On closer inspection, there are in fact two separate problems with induction, according to Avicenna. The first is the traditional question of the justification of the advance from a set of particulars to a universal. How can we be certain that there is no potential counterexample, or in other words that there is no particular that has not been accounted for, and to which the predicate fails to belong? Have all the particulars been exhaustively enumerated and examined? This turns out to be an objection against incomplete induction. A possible answer is that if the induction is complete, that is to say, if the premises of the above argument, namely

8. McGinnis (2003) draws a useful connection with the distinction between induction as a mode of discovery and induction as a mode of establishment. Avicenna's argument, at *Burhān* I, 9, pp. 93.7–94.21, has two parts. In the first part (I, 9, pp. 93.10–94.11), Avicenna rejects the case in which the nexus between P and the particulars under S is assumed to be self-evident. In the second part (I, 9, p. 94.12–18), he rejects the case in which the nexus is assumed not to be self-evident but subject to explanation. McGinnis (2003) offers a reconstruction of the first part of the argument. An English translation of Avicenna's argument is in McGinnis and Reisman (2007, pp. 147–149).

(1) "Every S is either s_1 or s_2 or . . . or s_n (enumerating all the particulars under S)" and (2) "s_1 is P and s_2 is P and . . . and s_n is P," are known to be true, then counterexamples are ruled out by hypothesis. The second question is subtler. Avicenna argues that even complete induction can at best lead to knowledge of the fact *that* every S is P, but not to the recognition of the self-evident and self-warranting character of the nexus between S and P in its necessity. The latter is something we ultimately either see or fail to see. Incomplete induction is obviously inadequate for the justification of a noncausal universal assertion, because it is by hypothesis open to potential counterexamples (regardless of whether the nexus involves a subject and one of its immediate constituents, its complete definition, or one of its immediate implicates). But incomplete induction is inadequate, too, because even if it is not open to potential counterexamples, it still cannot justify the advance from particulars to a universal, for the reason illustrated in *Burhān* I, 9. Under no circumstances can a procedure based on particulars provide the required epistemic justification for the corresponding universal assertion, because it is always bound to be either too weak, and hence inconclusive, or too strong, and hence question-begging, in assuming what it has to prove.

The argument of *Burhān* I, 9 shows, in general, that immediate, noncausal nexuses cannot be established inductively. Induction, in such cases, can at best serve as a reminder (*tanbīh*). The rejection of induction effectively amounts to another argument for the self-evident and self-warranting character of immediate scientific principles, including definitions (in their propositional form). And this does not simply mean that scientific principles are indemonstrable but also that there is, in principle, *no other method of justification* for them. Their status of immediate, primitive truths can only be recognized. This might still in the end require considerable effort, in the form of empirical observation and abstraction, but Avicenna seems to be committed to the view that even the putative need for a psychological ladder to reach the point where we can recognize immediate principles as such does not affect their entirely self-sufficient metaphysical status, and that certainty is only genuinely attained when our epistemic states are fully aligned with the latter.[9] While this is true of all immediate principles, the paradigmatic case is the relation between a *definiendum* and its *definiens*.

9. At *Burhān* III, 5, pp. 223.11–224.5, Avicenna examines four different procedures (including induction) for the acquisition of certain kinds of immediate propositions from perception, but it is clear that the role of induction, even in this case, is rather limited. As noted, induction may serve at best as a reminder (*tanbīh*) of something that is in principle self-evident. According to Avicenna, one may need to be reminded of something that should otherwise be self-evident when the latter is not recognized as such. But *tanbīh* does *not*, properly speaking, *prove* or *establish* anything, in the sense of providing a justification for it. The object of *tanbīh* here is the belief in a universal (statement). An example taken from the domain of primary propositions (*awwaliyyāt*), at *Burhān* III, 5, p. 223.11–15, is the contention that if two things are both in contact with a third thing, without being in contact with

INDUCTION AND DEFINITION

At *Burhān* IV, 3, pp. 280.19–281.12, Avicenna argues, specifically, that induction cannot prove a conclusion of the form S is P, where P is the *definiens* of S. The argument has a close parallel at *Nağāt* I, 139 (iv), p. 149.2–9. The textual basis for the discussion is *An. Post.* B7, where Aristotle briefly notes that definition cannot be established inductively, but Avicenna develops a more elaborate argument with two objections.

Induction fails as a mode for establishing definitions for the following reasons. First, if it is inadequate to establish the general kind of immediate universal predications discussed in the previous section, then a fortiori it must also be inadequate to establish the specific kind of immediate universal (affirmative) predication by means of which a *definiens* is asserted to belong to its *definiendum*.

Second, even in the specific case of definition, induction is open to the same dichotomous objection raised before against immediate noncausal predications. If the assumptions of the inductive process are sufficiently strong to establish that a complex term is a definition of something, then they turn out to be too strong and ultimately result in circular reasoning. If they are not sufficiently strong, circularity is avoided but the process is inconclusive.

The argument in *Burhān* IV, 3 consists of two objections. The first appears to be merely a dialectical objection (real induction is based on sensible particulars of which there is no definition). The second, by contrast, comes in the form of a more sustained argument. If one tries to establish inductively that P is the definition of S, Avicenna maintains that one of the following must be the case. Either (i) P is taken to be a definition of each and every one of the individuals falling under S, or (ii) P is taken to be a definition of the species of those individuals.

The first alternative (i) must be rejected because two impossibilities would follow from it. The first absurd consequence is that if P were a definition proper to each and every one of the individuals, it could not be shared by any two of them. Therefore, either it would be impossible to ascend to the species and predicate the definition of the species as a whole (which is the purpose of the inductive argument), or what is being transferred from the set of individuals to their species would in fact be a plurality of different definitions rather than a single definition. The second absurd consequence is that if P were a definition proper to each and every one of the individuals (even assuming for the sake of argument that such a definition could exist), it could not consist of essential attributes because, as we

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each other, then the third thing must be divisible. A similar principle is used in an argument against indivisibles at *Samāʿ ṭabīʿī* III, 4, p. 189.16–17, where Avicenna characterizes it as evident in itself (*bayyin fī nafsihī*). For a use of *tanbīh* in the context of a metaphysical discussion of general concepts such as thing, existent, and one, see *Ilāhiyyāt* I, 5, pp. 30.3–31.4; cf. also *Ilāhiyyāt* III, 3, p. 106.6–9.

know from the *Isagoge*, the "definition" of an individual can only be given in terms of accidental attributes whose aggregate is not true of any other individual and which are therefore proper to that individual alone.[10] But accidental attributes are not included in the essence of a thing; hence an inductive argument of this sort would inevitably fail to deliver a real definition.

The second alternative (ii) must be rejected because the inductive argument presupposes what it has to prove, namely that P is *a definition of the species* of the individuals falling under S. But the species of the individuals falling under S is, by hypothesis, S itself, and hence the inductive argument aiming to establish that P is the definition of S relies, circularly, on prior knowledge of the fact that P is the definition of S. The crucial assumption in case (ii) is that P is predicated of every s_i qua definition of their species, that is to say, of S, since "for none of the individuals does the fact that a notion belongs to it show that it is a definition of its species unless its species is known first along with the fact that the definition belongs to it" (*Burhān* IV, 3, p. 281.9–10). In this case, the inductive argument fails, again, due to circularity.[11] The reasons why induction is an inadequate method for establishing definitions are neatly summarized in the following passage from the *Naǧāt*. Avicenna writes:

> Text 13.1: *Naǧāt* I, 139 (iv), p. 149.2–9 (Ahmed 2011, p. 116, transl. modified)
> Induction cannot provide universal knowledge. [But] then how can it provide definitions?
>
> [(i)] For if you were to carry out an induction [assuming] that the definition is a definition of every individual in order to make it a definition of the species, you will have made an error.
>
> [(ii)] If you were to say that the definition is predicated of every individual without adding [anything else], this will not necessitate its being a definition of the species.
>
> [(iii)] If you were to say that the definition is a definition of the species of every one of those individuals, you will have begged the question.

It is not difficult to recognize, in the first and third arguments of Text 13.1, condensed versions of the points raised in *Burhān* IV, 3. The inductive basis of a proof that putatively shows a predicate to be the *definiens* of a subject must consist of a sequence of predications involving particulars that fall under that subject. Each of those predications assumes the predicate to belong to an individual either as its

10. See Porphyry, *Isagoge*, 2, p. 7.21–23. In this context, Avicenna does not question the validity of this principle.

11. At *Burhān* IV, 3, p. 281.10–12, Avicenna notes: "One cannot say 'Since this is the definition of the species of this individual, and the definition of the species of that individual [and so on], then it is the definition of the species of all these individuals' because [the conclusion] is already known (if it is known that it is the definition *of the species* of the first individual)" (reading with S *iḏā ʿurifa annahū ḥadd nawʿ aš-šaḫṣ al-awwal* for *iḏ ʿurifa annahū ḥadd ḥadd li-nawʿ aš-šaḫṣ al-awwal*).

own definition or as the definition of its species. In the former case, the principle that the definition of an individual is proper to it presumably entails that there can be no transfer to the species, leaving aside the question of how an account so obtained could even qualify as a genuine definition rather than being simply a description, given that a collection of attributes proper to an individual can only involve accidental attributes. In the latter case, the argument begs the question because, in order to prove P to be the definition of S by reducing this claim to a series of predications the subjects of which are individuals falling under S, P is already assumed to be the definition of S (for P is assumed to be the *definition of the species* of the individuals falling under S, which is nothing other than S itself). Finally, (ii) suggests that if P is simply true of the individuals under S and no other assumption is made, this will perhaps be enough to prove that every S is P, but not to prove that P is the definition of S.

THE ACQUISITION OF DEFINITION

After showing that definition cannot be established by demonstration, division (*Burhān* IV, 2), or induction (*Burhān* IV, 3), what remains to be seen is how definition may be acquired. This is the task accomplished by *Burhān* IV, 6 and IV, 7. The two chapters, taken together, are an elaborate analysis—both philosophical and textual—of *An. Post.* B13. The latter is a notoriously difficult text, whose internal division has been the object of various interpretations from antiquity to the present, and which contains a genuine interpretive crux in the Aristotelian corpus. In the following, I only address Avicenna's philosophical analysis.[12]

Avicenna discusses two methods for the acquisition of definition (*iktisāb al-ḥadd*): composition (*tarkīb*) and division (*qisma*).[13] Composition proceeds from the bottom up, that is to say, from less general terms, and may yield the definition

12. Due to constraints of space, I cannot address here the problems associated with the internal structure of B13 or situate Avicenna in the historical debate. A detailed analysis of the Greek, Arabic, and Latin reception of *An. Post.* B13 is discussed in Strobino (forthcoming), as part of a volume on essence and definition from Aristotle to Kant edited by D. Bronstein and P. Anstey. Avicenna's interpretation of B13, especially with regard to the method of composition, belongs to a family of interpretations that originates with Themistius and survives at least until the twelfth century in Greek and until the seventeenth century in Latin and Arabic. On *An. Post.* B13 more generally, see Ross (1949, pp. 653–662), Barnes (1993, pp. 240–250), Bolton (1987), Bolton and Pellegrin (1993), Falcon (1997, 2000), Charles (2000, pp. 221–239), and Bronstein (2016, pp. 189–222).

13. On Faḫr ad-Dīn ar-Rāzī's critique of Avicenna's epistemology of essence, see Bilal (2013). The epistemological optimism presupposed by the methods illustrated in the present chapter appears, on occasion, to run counter to some of Avicenna's own measured statements, for example at *Ḥudūd* Par. 4, pp. 2.12–3.5 and Par. 9, pp. 6.12–7.4, concerning the possibility of attaining real definitions (even incomplete ones).

of (i) lowest species and (ii) intermediate genera.[14] For various reasons, Avicenna holds this procedure to be more reliable and regards it as the privileged method for hunting definitions. Division is a complementary method that proceeds from the top down, that is to say, from more general terms, and validates the results obtained by composition. It is useful, in particular, because it provides cross-checks to ensure that nothing is omitted from the set of essential attributes of a *definiendum* and that the order in which those attributes are arranged is correct.

Avicenna presents two separate accounts of the method of composition, the first of which comes in two variants. The first variant of the first account is discussed at *Burhān* IV, 6, pp. 306.4–308.3 (on *An. Post.* B13, 96a24–b14). In that context, the method is primarily intended for the definition of the lowest species, but with a simple adjustment, it can be made to work in the case of intermediate genera too. The second variant of the first account is discussed at *Burhān* IV, 6, pp. 308.4–311.9 (on the obscure passage at *An. Post.* B13, 96b15–25). In this case, the method is exclusively intended for the definition of intermediate genera. The second variant presupposes the first because it identifies the definition of the genus by abstraction from the definitions of its subordinate species, and the process is ultimately bound to come to a stop at the definitions of the lowest species, which must therefore be acquired independently. The second account of the method of composition is presented at *Burhān* IV, 7, pp. 315.18–317.2 (on *An. Post.* B13, 97b7–25).[15]

The method of division is discussed at *Burhān* IV, 7, pp. 312.1–315.17 (on *An. Post.* B13, 96b25–97b6).[16]

First Account of the Method of Composition

The method of composition (*ṭarīq at-tarkīb*) is the first of the two approaches to the acquisition of definition examined by Avicenna.[17] In *Burhān* IV, 6, he gives

14. Highest genera (that is to say, the Aristotelian categories) and individuals are, strictly speaking, undefinable.

15. For the corresponding section in *An. Post.* B13, Barnes (1993, p. 248) suggests that this method may be "alternative or complementary to division" and that it consists in a procedure of abstraction. It is clear that for Avicenna (and several ancient and medieval commentators) this is (i) the same method or a variant of the same method discussed in *Burhān* IV, 6 (corresponding to the first and second sections of B13); that it is (ii) an instance of composition; and that it is (iii) complementary to, though more fundamental than, the method of division. At *Burhān* IV, 7, p. 317.19–20 (see Text 13.11), the process whose starting points are lowest species or individuals, and by means of which the definition of an intermediate genus (in the first case) or of a lowest species (in the second case) is obtained by way of composition, is characterized as better or preferable (*afḍal*) and closer to caution (*aqrab ilā l-iḥtiyāṭ*).

16. The account of division includes Avicenna's replies to the Speusippean objections at *Burhān* IV, 7, pp. 314.1–315.4 (on *An. Post.* B13, 97a6–22).

17. Composition is used in two different senses in Avicenna's logical works. The kind of composition relevant to definition and description is what Avicenna calls "notional" or "conceptual composition" (*tarkīb maʿnawī*) at *Burhān* I, 8, p. 87.15. This is the "composition proper to an expression

two variants: one for the definition of a lowest species or an intermediate genus, and the other for the definition of an intermediate genus only.[18]

In its first variant, composition starts from what is prior to us, that is to say, the essences, natures, or quiddities of the lowest species or the individual essences of particulars falling under them. The *definiendum* may be an intermediate genus or a lowest species itself. The method consists of surveying these items and identifying essential attributes which, taken separately, are more general than the *definiendum*, but whose conjunction is extensionally and conceptually equivalent to it. Avicenna writes:

> Text 13.2: *Burhān* IV, 6, p. 306.4–9
> We attend to the essences and the indivisible items (*aḏ-ḏawāt wa-l-umūr allatī lā tanqasimu*) from the domain of what is being defined (regardless of whether what is being defined is a genus or a species). We then take [(i)] the essential items predicated of [those essences and indivisible items] that are [(ii)] more general than them without falling outside their first genus (that is to say, substance, quantity, quality, and so on) or their proximate genus (that is to say, a genus which is like number for odd). We then take, from all this, what is included in their quiddity and [(iii)] collect it all together until something comes about out of [those predicates] that is both [(1)] equal to what is being defined in terms of convertibility—even if each single [predicate] is wider than [what is being defined] in terms of generality—and [(2)] equal in notion to what is being defined, in such a way that there is no constituent left which is not contained (*muḍammana*) in it.

The procedure described in Text 13.2 consists of starting with indivisible items from the domain of the *definiendum* (whether it be a lowest species or an intermediate genus) and searching for attributes that are

(i) essential,
(ii) more general, but
(iii) not more general than the first genus (category or proximate genus, presumably whichever is closest, where the reference to the category identifies the upper limit),
(iv) and whose collection is equal in extension and concept to the *definiendum*.

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that makes something understood" (*tarkīb ḫāṣṣ bi-qawl mufahhim*), which is contrasted, at *Burhān* III, 5, p. 222.14–16, with another kind of composition, namely the "statement-making composition" (*tarkīb ǧāzim*) of subject and predicate in a proposition. On the distinction between the two senses of composition, see also *Nafs* V, 5, p. 237.12–15. The notion of definitional composition is alternatively characterized, in the case of real kinds, as "natural composition" (*tarkīb ṭabīʿī*) consisting of genus and differentia, at *Mašriqiyyūn* I, 15, p. 36.8.

18. In *Burhān* IV, 6, Avicenna identifies by composition the definition of three (a lowest species), the definition of animal and line (both intermediate genera), and the definition of human and horse (both lowest species, with their complete definitional trees).

If the *definiendum* is a lowest species, the process requires us to take every predicate that is (i) constitutive of the essence, (ii) necessary, (iii) said of everything of which the *definiendum* is said, and (iv) primary.[19] Avicenna writes:

> Text 13.3: *Burhān* IV, 6, p. 306.10–11
> Thus, if we want to define the species without going beyond it to define the genus, we take every predicate which is at the same time [(i)] a constituent of the quiddity of [what is being defined], [(ii)] belongs necessarily to it, [(iii)] is said of every, and [(iv)] is primary.

In Text 13.3, Avicenna only specifies the criteria that the terms of a definition should meet, without giving instructions on how to determine whether a predicate is constitutive of the quiddity, necessary, said of every (in the demonstrative sense elucidated in chapter 6), or primary. In *Burhān* IV, 7, as we will see, Avicenna indicates that certain topical principles may be helpful in the process. But it should be clear at this point that this is not really something that Avicenna sees as a problem, in light of his commitment to the view that the constituents of the quiddity are ultimately self-evident. In this respect, composition presupposes knowledge of the essential attributes of a *definiendum* (as noted at the opening of this chapter, the capacity of the intellect to attain such knowledge and to discriminate between essential and accidental attributes is a separate question, which is addressed elsewhere, not in the theory of science): what composition contributes are specific criteria for the organization of essential attributes into complex essences.

The first variant of the method of composition discussed in *Burhān* IV, 6 may be adjusted to work in the case of intermediate genera too. If the *definiendum* is a genus, then in addition to essential, universal, and primary predicates of the items that we are surveying, we must also include non-primary predicates, that is to say, predicates that are primary of the primary, proceeding to increasing levels of generality.[20] Avicenna writes:

19. On the definition of three as odd prime number (prime in two senses), discussed at the beginning of *An. Post.* B13, Avicenna notes that “the fact that this expression is equal (*musāwāt hāḏā l-qawl*) to three-ness (*ṯalāṯiyya*) is something manifest (*amr ẓāhir*).” The self-evident character of definitional statements is central to Avicenna’s essentialism. Concerning Aristotle’s example, and the fact that it blatantly uses notions that according to Avicenna are not per se 1 but per se 2 (such as odd and prime, in both senses), he notes that it is pointless to take issue with it (or with examples of the same sort). Avicenna discusses this specific case just for the sake of argument and to follow the characterization of the procedure in Aristotle’s own terms. As we have seen, however, Avicenna’s own definition of individual numbers (including three) is altogether different and somewhat closer to the modern idea of generating integers through the notion of a successor.

20. This is because what is primary to a species is its proximate genus, and if we want to define that genus, we must assume predicates that are in turn more general than the genus itself, in the same way that we assume predicates that are more general than the lowest species when defining the latter, according to the first variant of the method of composition.

> Text 13.4: *Burhān* IV, 6, p. 306.11–12
> If we want to go beyond [the species] to define the genus, we do not confine ourselves to the universal predicates that are primary [to the species] but rather take all of them, including [not only] those that are primary to [the species] but also those that are not primary to it. In this case, if we find [something], it is possible to define the genus, for if we drop from the definition of the species its most specific predicates, what remains is the definition of the genus.

In Text 13.4, Avicenna briefly shows how to generalize the first variant of the method of composition to obtain the definition of a genus. The procedure requires all the terms identified for the definition of the lowest species and an additional set of more general terms, namely those that are not primary to the species itself. The definition of the genus is then obtained by omitting from the definition of the species its most specific attributes, namely those that are proper to the species. Hence, the definition of the genus is obtained residually by removing what belongs exclusively to the definition of the species.

In summary, the process of composition that leads to the definition of a lowest species requires the identification of the following types of attribute:

(i) constitutive of the essence,
(ii) necessary,
(iii) said of every, and
(iv) primary to the species (that is to say, its proximate genus and constitutive differentia).

The process of composition that leads to the definition of an intermediate genus requires the identification of the same attributes (i)–(iv) *and* of those that are primary to the primary attributes of the lowest species. For example, living being (or animate) is a primary attribute of animal (as its proximate genus), and animal is in turn a primary attribute of human (again, as its proximate genus). Living being is therefore a primary attribute of a primary attribute of human. Thus, to say that in order to define animal one must seek the attributes that are primary to the primary attributes of the species of animal simply means the following: just as in the definition of human one must take the primary attributes of human (for example, animal), so in the definition of animal one must take the attributes that are primary to animal (for example, living being). But the latter are in turn the same as the attributes that are primary to the primary attributes of the species of animal, including human. Thus, living being may be characterized equivalently as a primary attribute of the genus-*definiendum* and as a primary attribute of a primary attribute of the species falling under that genus.

A test of the fact that the set of attributes is *equal* to the *definiendum* requires the following conditions to be satisfied: the attributes must not be jointly predicated (1) of the genus of the *definiendum* (otherwise the definition would be too general) or (2) of something other than the *definiendum* that falls under the same genus (that is to say, a coordinate species), and (3) their conjunction, proceeding from the top down, must be the last complex term encountered in the predicamental line of the *definiendum* (if we proceed in the opposite direction, namely from the bottom up, the conjunction of these predicates is the first such collection we encounter).[21]

The second variant of the method of composition discussed in *Burhān* IV, 6 gives instructions on how to acquire the definition of an intermediate genus by abstraction from the definition of its lowest subordinate species. It is, in a way, unsurprisingly equivalent to the adjusted version of the first variant that has just been discussed. In this case, the starting point is the complete definitions of the subordinate species of an intermediate genus (for example, the definitions of individual numbers for number; the definitions of different kinds of line, such as straight, circular, crooked, and so on, for line; the definitions of different kinds of animal, such as human, horse, bull, and so on, for animal). These definitions are independently obtained by composition in the way outlined earlier. The definitions of their genera are then simply obtained by abstraction, that is to say, by dropping any essential attribute that is proper to a lowest species and by retaining only those that are common to all.[22]

Second Account of the Method of Composition

In the second part of *Burhān* IV, 7, after addressing the method of division, Avicenna returns to the method of composition and gives a second account of it (in line with the transition between the corresponding sections in *An. Post.* B13).

The procedure identified in *Burhān* IV, 7 involves the following steps, where S is a general term for species:[23]

1. Survey of individuals that are described as S (that is to say, individuals falling under S)
2. Identification of their essential predicates
3. Identification of essential predicates that are common to all individuals (a "first-order" intersection: definition of the lowest species)
4. If what is sought is the definition of a genus, repetition of steps 1–3 for a different set of individuals (falling under a different species of the same genus)

21. By the same token, the second collection (or complex term), proceeding from the bottom up, or the penultimate collection (or complex term), proceeding from the top down, would be the definition of the genus of the *definiendum*.

22. Avicenna's approach to the finer textual details of Aristotle's problematic characterization of this procedure at *An. Post.* B13, 96b15–25 is fascinating. For a detailed discussion of the problem and of Avicenna's dependence on the commentary tradition on this point, see Strobino (2012).

23. Indivisible terms falling under S may be again either individuals or lowest species, which suggests that this variant too is intended for the definition of lowest species as well as intermediate genera.

5. Identification of essential predicates that are common all species (a "second-order" intersection: definition of the genus)

The second account of the method of composition is also a bottom-up procedure, in the sense that it does not proceed from a whole by division but rather from lower-order terms (individuals or lowest species) to attain the definition of higher-order terms. It involves a process of abstraction that (i) sets aside features belonging exclusively to certain individuals (accidental attributes) or to certain species (proper essential attributes) and (ii) identifies the collection of those that are essentially common to all.

As noted in the first section of this chapter, the fact that knowledge of essential attributes is presupposed in this process (or at least the ability to identify them as such) is not a problem for Avicenna, and this is perhaps the reason why he does not even mention the issue in this context. This is partly due to the division of labor mentioned at the beginning of this chapter: the theory of science is neither required to justify the legitimacy of the distinction between essential and accidental attributes nor to show how we may actually distinguish between them. It can take both things for granted and focus on what to do with essential attributes in order to produce definitions. *This* is what the criteria of composition and division are all about, and there is no reason for Avicenna to give another set of criteria here for the identification of essential attributes. But if these considerations were not enough, Avicenna's comments on concept formation in *Burhān* III, 5 and IV, 10 seem to suggest explicitly that essential and accidental attributes are "recognized" as such in the process of concept formation, and suitably identified already at that level. If, at an early stage of the process of concept formation, we have the ability to identify and distinguish essential attributes from accidental ones, that same ability could serve us well at the corresponding early stage of the discovery of definitions, which presupposes that essential attributes have been identified correctly and exhaustively, before we can start combining them according to the rules of composition.[24]

Comparison of the Two Accounts of the Method of Composition

The two procedures described in *Burhān* IV, 6 and IV, 7 (in fact three, if we take the cases of lowest species and intermediate genera in IV, 6 to be genuinely distinct) are

24. Avicenna's reference to the "book of dialectical arguments" (his own *Ǧadal*) in the section on division in *Burhān* IV, 7 (see Text 13.8)—which mirrors an analogous reference to the *Topics* in *An. Post.* B13—seems to suggest that for the identification of essential attributes one can rely, positively, on the relevant commonplaces for genera and differentiae and, negatively, on the commonplaces for common accidents. At *Ǧadal* III, 1, p. 166.1, Avicenna contends that "most commonplaces mentioned under the rubric of the genus are scientific," presumably to emphasize the significance of their application in the scientific theory of definition.

both characterized as instances of the method of composition (*ṭarīq at-tarkīb*) and share various structural features. They are both bottom-up processes that are said to start from indivisible terms, by which in this context Avicenna means either lowest species (the least general of the general terms) or individuals. In both cases, one is supposed to identify attributes that are essential to and more general than what is being defined (in the first case, following Aristotle, Avicenna adds the condition that such attributes should not extend further than the genus, even though his understanding of genus here includes a broader range of options, from the proximate genus of the *definiendum* to the relevant category). In the first account, the next step is the conjunction of all common attributes. In the second account, the language is that of removing proper attributes and retaining common ones, which is for all intents and purposes equivalent to the first. Furthermore, the second account is explicitly characterized as a test for the detection of homonymy (again, in line with Aristotle's example of magnanimity in B13). Surveying subjects and ascribing attributes, proceeding through various levels of generality, from lower-order to higher-order terms, are steps presupposed by both procedures. And so is the requirement of coextensiveness with the *definiendum*. In other words, both may be characterized as residual methods that are somehow ultimately based on abstraction.

The Method of Division

Avicenna discusses the method of division (*ṭarīq al-qisma*) in two contexts within the *Posterior Analytics* complex.[25] The first is concerned with the question of whether a definition can be *proved* by means of division, raised in chapters B5–6 of the *Posterior Analytics*. The second is part of the complex discussion of B13, where Aristotle addresses the question of how definitions are acquired.[26] As we have seen, Avicenna broadly follows the scheme of B13, but interestingly divides the discussion into two separate chapters (*Burhān* IV, 6 and IV, 7), one mainly devoted to composition (the abstractive method that proceeds from the bottom up, starting from individuals or most specific species, and identifying their shared constituents), the other mainly devoted to division. Overall, Avicenna seems to believe that one should adopt a flexible approach, even though the method of composition is generally preferable and perhaps less likely to generate errors.[27]

25. For Avicenna's treatment (and dismissal) of division in *Burhān* IV, 2, see Strobino (2010); cf. also *Qiyās* IX, 5. On Aristotle's account of division and definition, see Lloyd (1961), Balme (1987), Lennox (1987), Bolton and Pellegrin (1993), and Falcon (1997, 2000).

26. Division is a method that tracks the metaphysical structure of reality, and in particular the articulation of being into the categories and their internal subdivisions. Avicenna discusses an interesting application of division in metaphysics, concerning in particular the relation between genera and differentiae, at *Ilāhiyyāt* V, 4, pp. 220.10–222.7.

27. At *Naǧāt* I, 141, Avicenna identifies the same three Aristotelian criteria discussed in this section: order, essentiality, and completeness, where the last condition implies that we should not stop

While division is categorically rejected as a method for proving definitions—because it can at best only assume (*iqtiḍāb*), at each step, the relevant node of a dichotomously branching tree, without showing its necessity (*luzūm*) and without showing that the collection of such nodes belongs to the *definiendum*—it can nonetheless be a useful method in the process of defining (*taḥdīd*). In particular, in the first section of *Burhān* IV, 7, Avicenna identifies three ways in which division may be useful for the discovery of definitions. He writes:

Text 13.5: *Burhān* IV, 7, pp. 312.3–313.7
The first is that division shows what is more general and what is more specific, from which one discovers how the parts of the definition [must] be ordered, taking what is more general first and what is more specific second. Thus, for example, in defining human one says "animal, two-footed, terrestrial," not "two-footed, animal, terrestrial," because there is a difference between those two things. [This is] due to the fact that in the expression "two-footed, animal, terrestrial," when one says "two-footed," one has already said "animal" and so, when one says "animal" after that, it is a repetition and the order is wrong. If one says "animal" first (without saying it after "two-footed," whether in act or in potency in the sense in which items that are contained [in a notion] are said [in potency]), then when one says "two-footed" after "animal," there is no deficiency.

The second is that division shows you that every differentia is connected with a genus above it, making the latter a genus for what is under it. Thus, the order of the differentiae proceeds in accordance with the consequents (*at-tawālī*). This way, the collection of the differentiae will be put together only in accordance with their consequents, so that none of them goes in the middle. Thus, if one wants to compose the definition from species to genera, he will not jump from a species to a remote (*abʿad*) genus, but to the genus that follows it [immediately].

The third is that, if [the division] is complete in the way it should be, then it will include all of the essential differentiae, and none of those that are included in the quiddity of the thing will be left out (unless it is already [implicitly] contained in it). Thus, we give all the differentiae in accordance with their consequents lengthwise as well as giving them in their completeness, even if crosswise. For it is possible to divide the genus by two divisions one of which is not under the other, as "animate body" [may be divided] once into "moving voluntarily" and "non-moving voluntarily," and another time into "sensitive" and "nonsensitive." Thus, one must observe this in dividing crosswise just as one observes it [in dividing] lengthwise in order not to neglect one of the differentiae that divide into overlapping (*mutadāḫila*) or exhaustive (*mutawāfiya*) essential differentiae. The overlapping are like "mortal"

in the middle but only with "essentials that, if divided further, lead to accidentals or individuals." For example, human is a lowest species in the category of substance because any further division (such as writing and illiterate, justly successful and usurper, and so on) involves accidental attributes. There is no further essential attribute by means of which human can be differentiated.

and "non-mortal," "rational" and "nonrational." The exhaustive are like "sensitive" and "nonsensitive," "moving voluntarily" and "non-moving-voluntarily."

Division, in Text 13.5, is said to be useful because

(i) it helps us identify the extension of a certain notion relative to other notions ("it shows what is more general and what is more specific," IV, 7, p. 312.3) and is therefore relevant for the identification of the correct order (*tartīb*) in which attributes appear in the definitional structure of a given concept;

(ii) it connects each differentia with a higher genus and identifies the latter as a genus for what falls under it, and this is in turn important to ensure that there is no gap in the sequence of attributes, and that the concatenation of genera and differentiae contains all necessary links; and

(iii) if the division is complete, then it contains all the essential differentiae, and none of the items that are part of the essence of a notion is omitted from a definition obtained in this manner. In particular, if the division is complete it includes all vertical (*ṭūlan*) and horizontal (*ʿarḍan*) nodes, including differentiae of various types, that is to say, both overlapping (*mutadāẖila*) and exhaustive (*mutawāfiya*) essential differentiae.[28]

Special attention must be paid to the question of order and completeness. The language used by Avicenna in the section devoted to division reflects its complementarity with respect to the method of composition. The essentiality of attributes

28. The first two pairs of essential differentiae (exemplified by rational and nonrational, mortal and non-mortal), which Avicenna calls "overlapping" differentiae, identify three kinds: non-mortal rational (angel), mortal rational (human), mortal nonrational (nonhuman animal or nonhuman living being). The sense in which these two pairs overlap is that their respective positive and negative differentiae are not fully aligned: there are things that are mortal and rational, things that are mortal and nonrational, and things that are non-mortal and rational, so rational covers non-mortal and part of what is mortal, while mortal covers nonrational and part of what is rational. By contrast, the other two pairs of essential differentiae (exemplified by sensitive and nonsensitive, moving voluntarily and non-moving voluntarily), which Avicenna calls "exhaustive" differentiae, identify two kinds only: what is sensitive and moving voluntarily (animal) and what is neither sensitive nor moving voluntarily (plant, if we take the negative differentiae to be a division of living being). The sense in which these two pairs are exhaustive is that their respective positive and negative differentiae are fully aligned: something is sensitive if and only if it moves voluntarily, and something is nonsensitive if and only if it does not move voluntarily. The critical difference is that with overlapping essential differentiae, a single positive differentia from either pair is not sufficient for the identification of a kind (for example, rational will be further divisible into mortal and non-mortal, and mortal will be further divisible into rational and nonrational), whereas with exhaustive essential differentiae, a single positive differentia from either pair is sufficient for the identification of a kind, and the corresponding differentia from the other pair always follows suit.

is, unsurprisingly, a critical feature of division too, but Avicenna insists on using the vocabulary of primary attributes, which is a proxy for the idea of proceeding through consecutive steps of increasing (composition) or decreasing (division) generality. For example, a primary division of a genus is a division such that there is no intermediate species into which the genus is divided, in the same way as, mutatis mutandis, a primary attribute of a species is such that there is no genus of which the attribute is predicated first. This requirement is a measure taken against potential omissions (and also, in a less clear way, against the accidentality of the predications involved). Avicenna writes:

> Text 13.6: *Burhān* IV, 7, p. 313.8–14
> The rule for observing the second and third [conditions of division], so that they may be useful, is that division [should be] by means of the essential constituents of the species, and that division [should be] a primary division of the genus, that is to say [reading *wa-huwa* for *wa-huwa fī* with S] the division that is of the genus as such. For example, it is into flying, swimming, crawling, and walking, that animal must be divided first, and [only] then [must] walking be divided into two-footed and many-footed, and flying into whole-winged and split-winged. Thus, if there is a gap in this [process] and animal is divided first into whole-winged and split-winged, animal is not divided as such but rather insofar as it is flying. Similarly, if animal is divided into many-footed and two-footed, animal is not divided as such, but rather insofar as it is walking.

In Text 13.6, Avicenna emphasizes again that a proper division must involve only essential constituents of the *definiendum*, and that it must proceed step by step from the top down by dividing the genus insofar as it is a genus. The procedure is such that it must cover all possible differentiae that qualify the genus, whether vertically (lengthwise) or horizontally (crosswise). The qualification "insofar as" indicates that the main concern is to make sure that a definition captures real ontological units in their genuine structure. For example walking animal is a genuine division of animal, but two-footed and many-footed, while being in turn genuine divisions of walking animal, are by no means genuine divisions of animal as such. In the process of definition, constant attention must be paid to these otherwise seemingly subtle differences, if we want our definitions to reflect the real structure of the entities we are defining and not to be mere linguistic constructs that extensionally identify the relevant objects while failing to express their essences in a complete and adequate manner.

The complementarity of division and composition emerges clearly in Text 13.7, where Avicenna refers to the process of collecting the attributes identified by division into an ordered sequence using the same term he employs elsewhere for his characterization of composition as a distinct method for the discovery of definition. He writes:

Text 13.7: *Burhān* IV, 7, p. 313.15–22
Thus, one must investigate first whether the genus needs to have a nature added to its [own] generic nature in order to receive this division or rather this division applies to [the genus] first, in which case the division that applies to [the genus] first is prior, and the division that does not [apply to the genus] first is posterior. If [the genus] is divided by a primary division, what is divided (*al-maqsūm*) and the differentiae are collected. Next, [the resulting notion] is divided by another primary division, and so on until one comes to a stop at what is not divisible any further, except numerically. Next, the extremes of the division are assumed as predicates of the species and adjoined to the composition (*li-t-tarkīb*). Thus, if you have divided something once by means of a primary division, you must prolong your effort and investigate whether it has another primary division different from this one. If you find one, you divide again until the division is exhaustively completed lengthwise and crosswise, and so you exhaustively [cover] all of the predicates. The dividing differentiae must be essential. We have already explained how this is accomplished in the first section [of the logic].[29]

In Text 13.7, Avicenna focuses on the notion of a primary division. In order to observe criteria (ii) and (iii) from Text 13.5, the method of division must proceed by means of essential attributes that are constitutive of the species and identify terms that are *primary* divisions of the genus. It is interesting to note that primary can be spelled out in two complementary senses here. A division of a genus is primary if the relevant species into which the genus is divided are the first ones we encounter in our downward journey. But this can be the case only if there is nothing more general than those species to which the genus belongs first. Thus, A and B are two primary divisions of a genus C if and only if C is a primary essential (generic) constituent of A and B, in the sense of not belonging first to something more general than either A or B. In this sense, two-footed and many-footed are not primary divisions of animal—because animal belongs to walking first—and animal in turn is not a primary attribute of two-footed or many-footed—because walking, to which animal belongs first, is more general than two-footed and many-footed.

Condition (iii) examined earlier, in particular, is complementary to the requirement identified in the case of composition for the definition of the genus, namely that not only the predicates that belong primarily to its subordinate species but also those that belong primarily to the genus itself be included in the list of attributes of the *definiendum*.

29. The reference is, presumably, to *Madḫal* I, 13 and II, 1. On the analysis of differentia in Avicenna's theory of the predicables, see Di Vincenzo (2015).

Essentiality, Order, Completeness

Avicenna summarizes the three conditions, already specified by Aristotle in B13, that division must satisfy if it is to be an effective instrument for the acquisition of definitions. These conditions are

1. the essentiality of the attributes (which must be identified independently),
2. the order in the arrangement of the attributes (which is validated by division), and
3. the completeness of the set of attributes (the set is complete when division cannot proceed any further because it has reached the level of individuals).[30]

As far as essentiality is concerned, Avicenna notes that the commonplaces of genus and differentia (for a positive characterization) and accident (for a negative characterization) may be useful for the identification of the correct set of attributes to be used in a definition. He writes:

> Text 13.8: *Burhān* IV, 7, p. 315.5–10
> One must observe three goals in the choice of the division that is useful for defining. The first is to make sure that the division falls within the quiddity, I mean that it is by means of differentiae that are essential to the species. It is possible to use in this area [of inquiry] the commonplaces (*mawāḍiʿ*) mentioned in the book of dialectical arguments (*kitāb al-ḥuğağ al-ğadaliyya*) where we mention the commonplaces [for establishing] whether or not something is a genus or a differentia, and to take from that [book] what is not based on purely endoxic [commonplaces].[31] One may also use the commonplaces showing that something is a non-constitutive accident of the quiddity of something, in order to guard against the possibility that the division might consist of accidental differentiae.

The reference in Text 13.8 is to the discussion of the commonplaces for genus and accident in Avicenna's *Ğadal*, especially books II and III, following a similar suggestion in Aristotle's B13. There is little doubt that, according to Avicenna, this is a legitimate way to acquire knowledge of essential attributes (or rather, knowledge of the fact that certain attributes are essential). By contrast, the broader question of whether and how effectively dialectic can offer adequate criteria (in the form of a set of necessary and sufficient conditions) to determine whether a given attribute P is essential or accidental for a subject S is a separate problem. Trying to offer even a preliminary answer would be in and of itself a daunting task, but more importantly, it is one that certainly lies outside the scope of this study. Suffice it to say, to conclude, that Avicenna seems to believe that many commonplaces in the

30. On omission as a reason for the incorrectness of a definition, see *Ğadal* V, 2, p. 253.13–15 and V, 4, p. 274.9–10.

31. In Text 13.8, Avicenna hints at a distinction, on which he frequently relies in his *Ğadal*, between scientific commonplaces and endoxic commonplaces, as noted in chapter 12.

Ǧadal are genuinely scientific (*ʿilmī*) or demonstrative (*burhānī*), most of them in book II and III. But for better or worse, as has been repeatedly emphasized, Avicenna is independently committed to the view that the intellect has the ability to detect essentiality and accidentality even when it performs abstraction, in the process of concept formation. Thus, whatever we make of the contribution of dialectic, the intuitive grasp of the distinction between essential and accidental attributes that our intellect seems naturally capable of (in Avicenna's view) remains the ultimate source of our knowledge of the basic ingredients of definitions.

Division also plays an important role for the validation of the correct order in which essential predicates must be arranged. Avicenna writes:

> Text 13.9: *Burhān* IV, 7, p. 315.11–14
> The second goal is to derive the order from the division. Thus, what is first in the order of the division is made to be first in the order of the definition in such a way that what is more general [according to the division] is taken first [in the definition], whereas what is more specific is taken second. If two differentiae are equal in generality and specificity, what resembles the matter goes before [in the definition] and what resembles the end goes after [in the definition]. If they do not differ in that [respect], you can put before and after any of the two, in whatever way you prefer.

Order is determined on the basis of the generality and specificity of concepts, and hence division is especially well suited for detecting the only correct arrangement of the predicates in the structure of a definition.

Last, division aims at completeness. Avicenna writes:

> Text 13.10: *Burhān* IV, 7, p. 315.15–17
> The third [goal] is that you continue to divide until you reach the thing which is being defined, if it is an intermediate species (*nawʿ mutawassiṭ*), or stop at the last division by means of essential [attributes]—after which there is only a division by means of accidental [attributes]—if you aim to define the ultimate species (*al-anwāʿ al-aḫīra*).

In Text 13.10, Avicenna emphasizes again that the process of division is complementary to the process of composition and applies to the same types of entities. In particular, division may be used to define lowest species as well as intermediate genera (or species, as they are qualified in this passage). Composition comes to a stop when it reaches the first suitable collection of essential attributes that corresponds to the *definiendum*, both extensionally and conceptually. In the same way, division continues until it reaches (the same collection of attributes that characterize) the *definiendum*. The *last* division, without qualification, is the one that corresponds to a lowest species and captures the full definitional structure of it, starting at the top from the relevant category and proceeding all the way down to the items that can only be divided (numerically) by means of accidental attributes, that is to say, the individuals that fall under that species.

COMPARISON OF THE METHODS OF COMPOSITION AND DIVISION

What are we to say about the relation between composition and division in Avicenna's account of the acquisition of definition? While composition is in a sense the most fundamental method, it must be complemented by the top-down approach encapsulated by division, which in a way provides a validation and a series of cross-checks on the results obtained by composition. He writes:

> Text 13.11: *Burhān* IV, 6, p. 311.6–9
> One need not believe that the First Teacher confines himself, for the acquisition of definition, to the method of assuming from below, collecting (*lāqiṭ*) arbitrary characteristics, as if he only contemplated the method of composition and nothing else. Rather, to that method he adjoins the observance of [the requirements of] the genus, and the observance of the primary predicates and of those that are primary to the primary. Furthermore, this is a [context] in which division is sometimes needed alongside the observance of [the method of] composition.

Text 13.11 confirms that Avicenna sees only two main candidates in the process of acquisition of definition: composition and division. The procedures other than division discussed by Aristotle in *An. Post.* B13 all fall, according to him, under the same heading. Their distinctive feature is to proceed "from below," that is to say, from less general terms (or from individuals) to more general terms. But composition does not operate by collecting random attributes of the *definiendum*. Rather, it observes "the genus," that is to say, it identifies attributes within the logical space determined at one end by the proximate genus of the *definiendum* and at the other by the category under which the *definiendum* falls. On this path, composition proceeds rigorously by discrete increments of generality, first with the identification of attributes that are primary to the species, then with the identification of attributes that are primary to what is primary to the species, and so on. Division, as we have seen, has complementary features, but there is one critical difference. Since division does not proceed from what is prior to us, relying exclusively on it as the sole method for the acquisition of definitions could potentially be risky and certainly less safe than starting with composition. Avicenna explicitly indicates that the bottom-up nature of the process of composition makes it the most prudent option for the discovery of definitions. He writes:

> Text 13.12: *Burhān* IV, 7, p. 317.19–20
> Thus, it is clear that starting [the process of] defining from the species and then composing them with one another in order for the definition of the genus to become manifest is the best [approach] and the one closest to caution.

Avicenna's remark concerning caution (*iḥtiyāṭ*) in Text 13.12 isolates an essential dimension of this process. Our epistemology of essence must adhere to the

TABLE 18 Notions defined in Avicenna's *Kitāb al-Ḥudūd*

1. Definition	19. Star	37. Dimension	55. Soft
2. Description	20. Sun	38. Place	56. Loose
3. Intellect	21. Moon	39. Void	57. Friable
4. Soul	22. Genie	40. Plenum	58. Transparent
5. Form	23. Fire	41. Privation	59. Rarefaction and condensation
6. Prime matter	24. Air	42. Rest	60. Aggregation
7. Subject	25. Water	43. Speed	61. Contact
8. Second matter	26. Earth	44. Slowness	62. Interpenetration
9. Element (*ʿunṣūr*)	27. Universe	45. Thrust and inclination	63. Continuous
10. Element (*usṭuqus*)	28. Motion	46. Lightness	64. Unity
11. Element (*rakm*)	29. Duration (timeless)	47. Weight	65. Successive
12. Individual nature	30. Time	48. Heat	66. Consecutive
13. Nature in general	31. Instant (now)	49. Cold	67. Cause
14. Body	32. Limit	50. Moisture	68. Effect
15. Substance	33. Infinite (limitless)	51. Dryness	69. Creation (*ibdāʾ*)
16. Accident	34. Point	52. Rugged	70. Creation (*ḫalq*)
17. Angel	35. Line	53. Smooth	71. Bringing about (*iḥdāṯ*)
18. Celestial sphere	36. Surface	54. Hard	72. Anteriority (pre-eternity)

strictest safety protocols, because there can be no scientific knowledge or science in the absence of proper definitions. The process of discovery by means of which we find answers to the fundamental what-questions in a given domain of scientific discourse is what everything else in that domain ultimately depends on (in addition to some assumptions of existence). It is therefore prudent, and indeed necessary, at the same time to rely on all adequate methods for the identification of essences and their internal structure *and* to identify which of those methods is less likely to generate errors. Avicenna is undoubtedly convinced, at a general level, that our intellect is naturally capable of accessing the structure of reality and conceptualizing it fully and accurately. But he is also convinced, more specifically, that this is not just the result of an intuitive, immediate grasp by means of which we see at once the structure of reality in its internal articulation (not in the case of ordinary people at least, or of ordinary philosophers, for that matter). The process, which requires work and effort, depends on certain protocols for the organization of our conceptual knowledge of the essences of the subjects and attributes that are investigated in the sciences. Table 18 is a synopsis of seventy-three privileged philosophical terms defined by Avicenna in his *Kitāb al-Ḥudūd*. These may be regarded, in a way, as the symbolic building blocks of Avicenna's thought.

Conclusion

I care not to perform this part of my task methodically; but shall be content to produce the desired impression by separate citations of items [. . .] and from these citations, I take it—the conclusion aimed at will naturally follow of itself.

—HERMAN MELVILLE, *MOBY DICK*, CH. 45

The goal of this book is to document Avicenna's originality, both as a theorist and as an interpreter, in the area of logic that he himself sees as the genuine culmination of the discipline, namely the logic of scientific discourse developed by Aristotle in the *Posterior Analytics*. The most elaborate expression of Avicenna's views on the subject is found in his *Kitāb al-Burhān*, which offers a paradigmatic example of his usual strategy of active appropriation.[1]

GENERAL OUTCOMES

The key contentions of this study can be broadly formulated under four headings: (i) Avicenna's theory of science aims to recalibrate the model of the *Posterior Analytics*, extend the range of its applicability, and endow it with real traction on the sciences; (ii) in order to overcome certain internal limitations of the Aristotelian model, Avicenna introduces a broad array of innovations to improve its theoretical framework; (iii) such innovations, which lie at the intersection of logic, metaphysics, and epistemology often have a much broader philosophical significance; and (iv) this shift of perspective indirectly confirms and, perhaps more importantly, provides a fresh foundation for an established tenet of Avicenna scholarship, namely the centrality of the ideal of demonstration for his

1. The category of active appropriation, as opposed to passive reception, is applied by Wisnovsky (2003) to Avicenna's metaphysics and extensively documented by Bertolacci (2006). The category also applies mutatis mutandis to Avicenna's theory of science. On the use of the concept of active appropriation to characterize the Arabic philosophical and scientific tradition see Sabra (1987).

thought. While such a centrality is widely accepted, the complementary question of whether and how Avicenna's methodological commitment to the ideal of demonstration might have influenced his understanding of the *Posterior Analytics* and translated into action has not been raised before. I have tried in this book to answer the question in detail, showing that Avicenna's theory of science is shaped by its intended use and application. The fact that it is so also casts light on the relation between the theory and practice of scientific reasoning from a new and previously unexplored angle.

If Avicenna's theory of science is fundamentally an effort to reassess the model of the *Posterior Analytics* with a view, at least in principle, to its ideal applicability, what makes this effort necessary in the first place? There are chiefly two general problems, which pull in somewhat different directions, and then a third, more practical problem. The first problem is that some of the Aristotelian requirements for scientific knowledge may reasonably appear to be too strict and ultimately to set an unattainable standard for most scientific theories (for instance, that genuine scientific knowledge should always require knowledge of the cause, or that demonstration in the strongest sense should always proceed from appropriate principles). The second problem is that the expressive power of the logical system presupposed by the *Posterior Analytics* is limited, as Aristotle predominantly—if not solely—relies on categorical propositions and categorical syllogisms. The third problem is that many areas of Aristotle's theory are not fully developed and require further elaboration to work out their details.

Avicenna is not particularly concerned about the first problem. In fact, his interventions on the general requirements of scientific knowledge result, if anything, in the stipulation of even stricter conditions. The difficulty is mitigated by the abundant resources afforded by Avicenna's unwavering metaphysical essentialism and staunch epistemological optimism. In light of these two factors, the prospect that a suitable body of knowledge might satisfy the criteria of scientific knowledge and count as a science is much less unrealistic than may otherwise appear to the mind of a modern reader (or even of any skeptics among Avicenna's immediate readership).

Assuming the ideal applicability of an Aristotelian model of scientific reasoning to be Avicenna's ultimate motive, then the second and third problems that afflict the *Posterior Analytics*—the limited expressive power of its logic and a certain degree of incompleteness of detail—obviously represent his opportunity. Avicenna's interventions to address the issue of expressive power consistently move in the direction of expanding the conceptual vocabulary, methods, and procedures of his main source. And even his way of handling the finer points invariably seems to suggest that he is driven not so much by a tedious exegetical urge, but rather by a genuine theoretical need. In other words, all the evidence in this book supports the view that Avicenna is tacitly engaged in a project of reform, adjustment, and recalibration of the model of the *Posterior Analytics* aimed at improving its general applicability. Demonstration is, for Avicenna, a living discipline.

If we adopt, as a leading interpretive principle, the idea that this is Avicenna's general goal, then the number and types of innovations he introduces can be understood as part of a coherent project, and their philosophically sophisticated character can be appreciated in its full significance, that is to say, in light of the compounding effect collectively produced for the theory rather than merely as a series of disconnected individual episodes.

As noted in the introduction, Avicenna's theory of science presupposes two distinct but complementary kinds of scientific knowledge (*ʿilm*): conception (*taṣawwur*) and assertion (*taṣdīq*). The first is the basic ingredient of definitional, non-demonstrative knowledge of the essences, natures, or quiddities of the subject(s) of a science—for example, numbers, triangles, plants, celestial bodies, humans, minerals, sounds, and so on—and their attributes. The second is the basic ingredient of demonstrative knowledge of the fact that and the reason why certain inseparable non-constitutive attributes (which Avicenna calls implicates)—for example, oddness, deciduousness, perpetual motion, the ability to laugh—belong to their subjects. Every science is identified by its subject, principles, and questions, and is in turn a structured body of beliefs characterized by a peculiar modal stability and by explanatory relations holding between primitive definitional truths and derivative demonstrable truths. Such stable beliefs are obtained by demonstration from first principles or from previously demonstrated conclusions. First principles include assertions of existence, definitions (conceptions of the primitive notions of a science), and immediate assertions ascribing certain implicates to their subjects. If we leave aside the basic formal requirements satisfied by any valid deduction to focus instead only on what turns a valid deduction into a *demonstration*, we can isolate two distinctive groups of properties. Out of several conditions (truth, immediacy, primariness, priority, explanatoriness, being better known, necessity, per se, appropriateness) discussed by Aristotle at different places (*An. Post.* A2, A4, A6, A9), two general headings—modality and causality—are especially useful to classify some of Avicenna's more technical interventions. These two notions are also central for our understanding of the most fundamental type of first principle in Avicenna's theory of science, that is to say, definition. All sciences, theoretical and practical, fit into a hierarchical system characterized by the logic of a complex Porphyrean tree. The division of the sciences reflects the structure of the universe as a whole and in its internal articulation. Since, as we shall see at the end, according to Avicenna the ultimate goal of human life is the attainment of happiness through the acquisition of scientific knowledge of the universe, an adequate theory of science turns out to be a necessary condition for the full realization of the rational nature of human beings.

SPECIFIC OUTCOMES

What are then the individual contributions made by Avicenna's theory of science, and how are they organized? The first cluster of innovations, discussed in part I ("Scientific Knowledge and Scientific Inquiry"), concerns the general characterization of scientific knowledge in its different forms. There are three key interventions in this area: the introduction and extensive use of the vocabulary of conception and assertion (chapter 1), an elaborate classification of different types of assertions and particularly the identification of the ones that are employed in demonstrative reasoning (chapter 2), and a fine-grained account of the fundamental types of scientific inquiry and their order (chapter 3).

Conception and assertion are at the heart of Avicenna's epistemology. Chapter 1 showed how both his idea of preexistent knowledge and his commitment to foundationalism (the rejection of infinite regress and circularity in explanation) are formulated in terms of these two notions: scientific knowledge presupposes primary conceptions and primary assertions at which the search for principles comes to a stop. Moreover, one of the difficulties traditionally associated with *An. Post.* B19 (Is the acquisition of principles primarily concerned with concepts or propositions?) is also explicitly resolved with a distinction between the domains of conception and assertion: the stages of concept formation are prior to the operation of combination and separation into propositional compounds that constitute the objects of assertion, and every science presupposes a blend of both.

The taxonomy of assertions analyzed in chapter 2 rests on the distinction between different types of premises of scientific deductions and identifies their kind of necessity based on the source with which those types of premises are associated, whether it be internal (the intellect for primary propositions) or external (perception, experience, or testimony). It also serves as a useful tool of classification for the identification of the premises of nonscientific deductions (commonly held or endoxic, estimative, and specious propositions) in the refutation of competing arguments and theories.

The distinctive feature of Avicenna's account of scientific inquiries is their rearrangement into three basic types corresponding to if-, what-, and why-questions. Following Themistius, he regards existential and predicative assertions as species of a single kind: simple and compound if-questions. Aside from a gain in simplicity, this move allows Avicenna to make another relevant contribution. Compound if-questions (predicative assertions) are treated as conclusions of categorical syllogisms, and the structure of their proof translates into the need to search for a middle term of a categorical syllogism. By contrast, simple if-questions (existential assertions) are the conclusions of repetitive syllogisms with a conditional major premise (argument forms like *modus ponens*), and their proof requires the identification of the appropriate antecedent as a cause. The distinction illustrates the need to articulate in greater detail the logical form of arguments associated with different types of inquiry and possibly to reflect the conditional nature of the existence of those sciences the existence of whose subjects is not self-evident. For in these cases the existence of the subject must be proved by a higher science (or, ultimately, by metaphysics) and must therefore be expressed in conditional form. Finally, Avicenna's elaborate account of the order of inquiry represents a substantial enrichment of the brief sketch offered by Aristotle in *An. Post.* B1–2.

These moves (i) cast light on some critical but implicit presuppositions in the *Posterior Analytics*, most notably the distinction between noetic and non-noetic knowledge; and (ii) offer a key to understanding the source of the necessity of scientific assertions (whether it be internal, as in the case of primary propositions, or external, based on self-warranting empirical evidence or reliable testimony) and criteria for the identification and rejection of nonscientific assertions, in particular commonly held but not genuinely necessary assumptions. The discussion of scientific inquiries and their order (iii) offers a unifying perspective on the relation between noetic and non-noetic knowledge and on how the stages of inquiry correspond to the logic of demonstration and definition.[2]

The second cluster of innovations in part II ("The Organization of Scientific Knowledge") involves two especially significant aspects of Avicenna's theory of science: (i) a coherent framework showing in greater detail what a given body of interconnected truths must look like, if it is to count as a science (chapter 4); and an ideal blueprint of the whole complex of scientific knowledge mapping various kinds of interrelations among the sciences (chapter 5).

In his analysis of the internal structure of a science, Avicenna makes two key contributions. The first is (i) a revision of the list of the constitutive elements of a science, that is to say, its principles, subject, and questions (Aristotle's per se predicates are subsumed under this heading); the second is (ii) a detailed account of the internal division of each group, an aspect that is altogether absent from Aristotle's discussion. Why does this matter? With regard to subjects, because it is one thing, for instance, to hold that a science is identified by its subject (for example, number), and quite another to have, in addition to this general criterion, a detailed account of the relation between the subject of that science and the subjects of various types of scientific statements that are made in it (for example, statements about three, about squares, or about the product of an even number by an odd number, all of which are particular kinds of number). Avicenna is unsatisfied with Aristotle's general characterization of the domain of a science just as a function of its subject: a viable theory of science must be capable of providing a more

2. The resulting picture is remarkably close to the interpretation of the *Posterior Analytics* recently offered in Bronstein (2016).

fine-grained analysis and of specifying precisely a set of constraints under which the subject of a science can translate into the subject(s) of scientific statements.

A similar strategy informs Avicenna's account of scientific questions. What looks like an imperceptible move—namely the replacement of per se attributes with questions—is in fact a critical transition from terms to propositions, which opens the door to the introduction of different types of propositional forms in the logic of scientific discourse and significantly increases its expressive power. Scientific propositions can now be hypothetical (conditional or disjunctive) as well as categorical. As a result, an Aristotelian science no longer has to consist exclusively of concatenated categorical syllogisms (though these still play a central role). But if conditional and disjunctive statements are admissible types of propositions in the context of scientific reasoning, this makes room for arguments of a more complex nature. The investigation of various classes of inferences in Avicenna's *Qiyās* VI–VIII (a pure system of hypotheticals, a mixed system of hypotheticals and categoricals, and other forms such as *modus ponens*, *modus tollens*, or disjunctive syllogism, among others) illustrates the scale of the project and the significance of allowing more complex logical forms (and derivatively the associated argument forms) to be imported from formal logic into the theory of science. Even the potential attempt to reconstruct (at least some of) Avicenna's demonstrative proofs in his own scientific or philosophical works, all of a sudden, ceases to look like a foolish endeavor. And while a word to the wise is undoubtedly in order (for Avicenna expressly warns his students that he does not always care to cast every argument in deductive form in his works), it is also true that he himself occasionally makes concrete use, in the sciences, of the expanded vocabulary of logical forms investigated in the *Qiyās* and presupposed in the *Burhān*. This suggests that the distinctions and classification of types of propositions are not purely artificial constructs disconnected from scientific practice, but genuine tools that find application in scientific discourse. The identification of a larger set of logical forms for scientific propositions, which crucially allows for the introduction of conditional and disjunctive propositions, is possibly the single most significant step systematically promoted by Avicenna in his theory to overcome the problem of expressive power of the underlying logic of the *Posterior Analytics*.

The classification and division of the sciences discussed in chapter 5 makes an equally important but different sort of contribution. Aside from anything else, this was for centuries the only portion of Avicenna's theory of science to be readily available outside the boundaries of the Arabic-Islamic world. This factor alone cannot count as a mark of significance, as we do not know enough about the circumstances that led to the translation of *Burhān* II, 7 into Latin in the twelfth century (especially whether or not the rest of the work was partly or fully available). But it is quite possible that the remarkable originality of Avicenna's contribution in the area was the reason behind this preferential treatment. In developing a series of indications in Aristotle, mostly from *An. Post.* A9 and A13, Avicenna identifies and develops in detail a broad array of relations that may characterize any given pair of sciences (distinctness, partial and complete overlap, parthood and subordination). The result of his analysis is a synoptic view of the structure of scientific knowledge as a whole, in all of its domains, sub-domains, and articulations. The level of detail of Avicenna's analysis is a tangible sign of his commitment to the view that human reason is capable of grasping the structure of the universe as a whole in all of its internal, hierarchical divisions. And to account for such divisions is squarely within the purview of an adequate theory of science, as individual scientific disciplines are mirrors of different regions of being, and the epistemological hierarchy of science as a whole reflects the order and arrangement of the universe and the mind that produced it.

The next three clusters of innovations concern different manners in which Avicenna's theory of science is shaped by his own flavor of essentialism, focusing in particular on how the latter bears on modality (part III), causality and explanation (part IV), and definition (part V).

The requirement of firm epistemic stability for the body of beliefs that constitute a scientific theory is inextricably related to the idea of necessity. But as we know from other areas of his logic, necessity can mean different things for Avicenna. As discussed in chapter 6, the combined temporal and alethic components in Avicenna's original analysis of modal concepts, which is one of the hallmarks of his formal logic, find a philosophically relevant application in his theory of science. The distinction between referential necessity and descriptional necessity, and the acknowledgment that the latter (as the more general notion) is to be taken as a default sense in scientific discourse, play a critical role in the characterization of different cases of essential predication, depending on whether an attribute belongs to a subject at all times of its continued existence or at all times at which the subject is described in a certain manner. The distinction enables Avicenna to accommodate a broader range of legitimate necessity predications in the context of scientific discourse and to offer an analysis of their differences.

The distinction between necessity in essence or nature and necessity in implication is another crucial revision of the concept of necessity, on the basis of which Avicenna builds his theory of per se. The elaborate account of per se predication discussed in chapter 7 significantly expands the scope of a technical distinction internal to the Aristotelian model and grants it a broader theoretical relevance. Three levels of discourse are linked in an original way here: (i) the theory of per se itself; (ii) a distinction between necessities of different strengths, captured by the notions of containment and necessary implication; and (iii) a distinction between types of inseparability, which grounds the other two. All three levels express in different ways the dichotomy between essential and nonessential necessities. First, the notion of a per se attribute is ultimately linked with the idea that in a scientific theory the admissible terms must be salient properties of the objects under investigation. Avicenna holds that it is only through this regimented conceptual vocabulary that the two defining conditions of scientific knowledge, namely its necessity and its explanatory character, can be met by a theory. In doing so, he provides a systematic account of per se that elaborates on Aristotle's sketchy suggestions in *An. Post.* A4 and A22. The Aristotelian analysis of per se (*kath' hauto*) is based on the relations of being included in or being part of the definition or the essence of something. A term is per se 1 with respect to another if and only if it is part of the definition of the other term, and a term is per se 2 of another if and only if that other term is part of the definition of the first. Avicenna links the definition and classification of per se 1 and per se 2 to the notions of constituent (*muqawwim*) and implicate (*lāzim*), two key ingredients of his conceptual vocabulary associated with different kinds of ontological necessity. These different kinds of necessity are the counterparts of two types of entailment: containment (*taḍammun*) and necessary implication (*luzūm* or *iltizām*). The latter express necessary connections of different strength between terms: containment is the relation holding between something and its intensional parts (the constituents of its essence or the parts of its definition), while necessary implication is a weaker form of necessity holding between two notions one of which is inseparable from the other.

The distinction between per se 1 and per se 2 and that between containment and necessary implication are further associated by Avicenna with another distinction, in this case between two types of inseparability, developed in his commentaries on the *Isagoge*. The first type is inseparability in conception (*taṣawwur*), and its counterparts are the notions of per se 1 and of containment; the second type is inseparability in imagination (*tawahhum*), and its counterparts are the notions of per se 2 and of necessary implication. In Avicenna these three levels of analysis are intimately connected, and his understanding of necessity appears to offer a sophisticated, comprehensive interpretation of *An. Post.* 4 (on per se) and A6 (on the necessity of scientific premises), developing elements that at best remain at an inchoate stage in Aristotle. This is because the *Burhān* displays a comprehensive account of various possible sub-cases of per se 1 and, more importantly, of the problematic notion of per se 2. For instance, it is far from clear what can truly be achieved by any science with the notion of per se 2 presented in *An. Post.* A4, if Aristotle is to be taken literally. In particular, if it is a requirement of per se 2 that the subject itself always be taken in the definition of the predicate, it is not even clear whether and how some of the recurrent, basic examples of the *Posterior Analytics* could work (How does triangle enter in the definition of having the sum of the internal angles equal to two right angles?). Avicenna's identification of a broader set of terms that can satisfy the relation of being part of the definition of a term (not only the subject but also some of its constituents, up to a certain level of generality) solves this problem. What is more, for our purposes, it shows yet again his commitment to the ideal applicability of demonstration, for there can be no Aristotelian science without an adequate theory of per se predication, and an adequate theory of per se predication requires a more powerful account of all possible relations between subjects and per se 2 predicates in scientific statements than is otherwise to be found in Aristotle.

Other original aspects of Avicenna's analysis, such as the aforementioned distinction between containment and implication, only indirectly contribute to the overarching goal of turning Aristotle's theory of science into a model fit for application. Yet they represent philosophically critical developments and serve as independent but nonetheless indispensable foundations for other concepts that are directly relevant for that project. A particularly significant example is the distinction between inseparability in conception and inseparability in imagination, which is the basis for a distinction, relevant in the sciences, between attributes that belong to the definition of a subject (its constituents) and attributes that are necessarily true of a subject without being part of its definition (its implicates).

Part IV illustrates Avicenna's reworking of causality and explanation, that is to say, of the other component associated with the definition of scientific knowledge. In particular, chapter 9 deals with the distinction between why-demonstration (*burhān limā*) and that-demonstration (*burhān anna*) in the context of one and the same science and provides a more fine-grained account of different types of demonstration than is to be found in Aristotle. More importantly, Avicenna's effort of classification and conceptual analysis casts a new light on the somewhat scattered and episodic treatment of the subject in the *Posterior Analytics*. The outcome is a rigorous taxonomy that regiments various types of demonstrations and identifies different levels of explanation, corresponding largely to the different kinds of that- and why-questions in Avicenna's classification of scientific inquiries delineated in chapter 3. But the distinction between explanations of different strength, which in turn express more or less fundamental causal links, is also central from the viewpoint of the structural organization of scientific knowledge in its domains. This is because, when certain constraints are met, facts that pertain to a science may serve as explanations of facts that pertain to another science. Avicenna's extensive analysis of the manners in which explanation works across different sciences, of subordination, and of the notion of transfer of demonstration (*naql al-burhān*), discussed in chapter 10, is indicative of his unwavering commitment to the idea of a hierarchical arrangement of the different bodies of scientific knowledge and to the unity of science (notwithstanding the fact that rigid boundaries separate sciences whose principles, subjects, and questions are not related in one of the ways identified in chapter 5).

The causal component in the definition of scientific knowledge is also behind Avicenna's search for a mechanism to absorb the standard Aristotelian account of the four causes (material, formal, final, and efficient) into his theory of science. In

An. Post. B11, which Ross (1949) describes as "a series of jottings for further consideration," Aristotle begins a process that prompts Avicenna to offer three separate accounts of the four causes in the context of scientific reasoning and to analyze in detail how they can be imported into the logical structure of demonstrations and definitions. Avicenna's original contribution here is twofold. On the one hand, the simple account of the four causes in B11 is replaced by a more robust set of distinctions, assuming that each type of cause may be essential or accidental, universal or particular, common or proper, proximate or remote, actual or potential. This multiplication of cases is a reflection of Avicenna's attempt to regiment the theory of causes in the context of demonstration and definition in a way that is more inclusive and accounts for different levels of discourse and explanation in the sciences. Different kinds of explanation correspond to different kinds of causes that may be used as middle terms in demonstrations, and this set of distinctions enables us to pinpoint more clearly what constitutes complete demonstrations and complete definitions, that is to say, arguments and complex terms that appeal to essential, universal, proper, proximate, and actual causes rather than to their counterparts.

Avicenna's world is a world of essences. His theory of science is one that presupposes a world of essences, and its purpose is to provide tools suitable for the investigation of a world of essences. But essences are captured by definitions. Avicenna's theory of science is therefore as much a theory of demonstration as it is a theory of definition. In this area, too, Avicenna introduces relevant innovations, in the form of various conceptual clarifications, a translation of the notion of definition into the vocabulary of conception, and a sophisticated account of one of the basic Aristotelian methods for the acquisition of definition. The analysis of definition and description in chapter 12 identifies their general characteristics and discusses Avicenna's classification of various kinds of definitions (nominal, noncausal, causal) in connection with *An. Post.* B10 (description is a general term under which different types of complex terms involving accidental attributes can fall, ranging from per se 2 to the merely adventitious). Definitions and descriptions are characterized as ordered sets of essential or accidental attributes, which may be either complete or incomplete. A complete definition must express the full essence of an object in its exact internal articulation, and depending on the nature of the object in question, a definition may be an indemonstrable principle or a complex term that can be cast in the form of a deduction. Aristotle's discussion of this procedure in *An. Post.* B8 is notoriously problematic, and Avicenna devises an ingenious way to account for it by offering complementary procedures to resolve a definition into its terms and rearrange them into a demonstration, and to extract a definition from a demonstration.[3]

3. In general, a causal definition of P that has the form GD (where G stands for a genus that expresses the sort of thing P is and D for a differentia that somehow captures the cause of P) can be

Since definitions cannot in principle be established by demonstration, division, or induction, heuristic criteria and methods for the discovery of real definitions must necessarily be identified to secure the foundations of a science. The theme traditionally falls under the general heading of "acquisition" (*iktisāb*) of definition and is ultimately inspired by *An. Post.* B13, the longest (and one of the most difficult) chapters of Aristotle's work. Chapter 13 discusses in detail two procedures, composition (*tarkīb*) and division (*qisma*), associated with the criteria of essentiality, order, and completeness that a set of attributes must satisfy in order to be a definition. Avicenna's account of composition is especially interesting, as it falls squarely within an uninterrupted exegetical tradition with extraordinary ramifications that originates in all likelihood with Themistius.

SCIENTIFIC KNOWLEDGE, HAPPINESS, AND THE REALIZATION OF HUMAN NATURE

Why, in the end, is a proper logic of scientific reasoning so important in Avicenna's thought? To answer this question, we need to look at the two endpoints of the spectrum of his philosophical and scientific investigation and at their mutual relation: (i) Avicenna's definition of logic and (ii) his account of the ultimate goal of human life. The implications of this relation lie far beyond the scope of this study, and I cannot even begin to explore them in detail here, but to put things in context, it will be useful to look at one last batch of relevant texts.

Logic is an indispensable instrument for the full realization of human rationality (it is arguably also a science for Avicenna, but this is a story for another time). Such a process of realization requires knowledge of the truth, in the theoretical sciences, and knowledge of the good, in the practical sciences. Since knowledge of both is limited, most of what we come to know is the object of a process of acquisition. The instrumental value of logic comes into play at this stage, as logic is the tool that enables the acquisition of what is unknown from what is known by regimenting various processes of transition from the latter to the former. The goal of these processes is to acquire scientific knowledge of what is unknown starting from what is known. This trajectory is neatly summarized at the beginning of the *Šifāʾ*, where Avicenna writes:

> C1: *Madḫal* I, 3, pp. 16.15–17.6
> Since [(1)] the process of seeking *perfection* for humans insofar as they possess an intellect consists, as we shall clarify in its proper place, in knowing the truth for its own sake, and the good for the sake of acting by it and adopting it (*iqtibās*); and

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cast in demonstrative form to show that and why P belongs to a subject S. A first deduction shows that every S is D, and every D is G; therefore every S is G; a second deduction shows that every S is G, and every G is P; therefore every S is P.

> since [(2.1)] the first natural operation and internal disposition of humans by itself is characterized by little knowledge of either, and [(2.2)] most of what is available to them with regard to the truth and the good only becomes available by acquisition (*iktisāb*), and [(2.3)] this acquisition is the acquisition of what is unknown (*iktisāb al-mağhūl*), and [(2.4)] what enables the acquisition of what is unknown is what is known; then humans must start first by learning [(3.1)] *how what is unknown comes to be acquired from what is known*, and [(3.2)] the state of things that are known and their structure in themselves, [(3.3)] in order to procure *scientific knowledge of what is unknown*, that is to say in order that, when the order in the mind is the necessary order, the form of those known things is established firmly in [their mind] according to the necessary order, the mind transfers from them to what is sought and unknown (*maṭlūb mağhūl*), and then knows it.

To acquire what is unknown from what is known is either to derive unknown assertions (conclusions) from known assertions (premises) or to obtain unknown conceptions from known conceptions. Logic studies the structure of both processes. More importantly, logic also investigates the content and conditions of what is known, the starting points or principles of any investigation, by looking at their matter, that is to say, their constitutive terms and their relations. Thus, the central role of an adequate theory of science is built into Avicenna's own definition of logic and characterization of its instrumental value: the discipline is primarily designed to teach us how to attain scientific knowledge, through mastery of both formal and material aspects of reasoning.

If the definition of logic presented at the beginning of the *Šifāʾ* contains an explicit reference to the realization of the rationality of human nature, this notion is more clearly articulated again toward the end of the *Ilāhiyyāt*, as the perfect conclusion of the all-encompassing process of scientific and philosophical investigation embodied by Avicenna's most comprehensive summa.[4]

At the end of his metaphysics (*Ilāhiyyāt* IX, 7), Avicenna discusses the Neoplatonic notion of "return" or "destination" (*maʿād*) to illustrate the condition of the human soul in the afterlife, after it separates from the body. In this context, he distinguishes between two kinds of destination, that is to say, between two kinds of conditions in the afterlife: the happiness and misery described and transmitted by revelation (*manqūl min aš-šarʿ*) and the happiness and misery grasped intellectually by demonstrative argument (*mudrak bi-l-ʿaql wa-l-qiyās al-burhānī*). There is a dimension of the transition from mortal life to the afterlife that is not conveyed

4. Since medicine is one of the sciences for which Avicenna has a systematic place in his own taxonomies, the *Qānūn* should perhaps be included in the list. More importantly, however, Avicenna himself tells us in the letter to Kiyā (Badawī 1947, p. 121.7; Gutas, 1988, p. 63) that his "comprehensive book, *The Cure*, [. . .] contains all the sciences of the ancients."

by religious law but is purely the object of rational investigation and in itself subject to demonstrative deduction. Avicenna writes:

> C2: *Ilāhiyyāt* IX, 7, p. 423.6–9 (Marmura 2005, pp. 347–348, transl. modified)
> Another kind of destination (*maʿād*) is grasped by the intellect and demonstrative deduction (this prophethood has confirmed). It consists of the happiness and misery [established] by deduction and which belong to the souls, even though our imagination falls short of conceiving them now for reasons we shall explain. The desire of divine wise people (*al-ḥukamāʾ al-ilāhiyyūn*) for attaining this happiness is greater than the desire to attain bodily happiness.[5]

This kind of destination as well as the types of happiness and misery associated with it are the object of intellectual knowledge and of demonstrative proofs. They are established deductively and cannot be truly grasped through the image-eliciting discourse of religious revelation. But they also depend on deduction and scientific reasoning in another essential way. This form of intellectual happiness, or lack thereof, is directly associated with demonstrative argument, that is to say, with the most sophisticated form of application of logic to the study of reality. Avicenna is not simply committed to the demonstrability of the immortality of the soul or the demonstrability of certain features of the most dignified kinds of happiness and misery in the afterlife (he certainly is committed to both, but this is beside the point); rather, those kinds of happiness and misery, in and of themselves, directly depend on the amount of scientific knowledge humans accumulate during their mortal lives. And this scientific knowledge can only be acquired through demonstration and definition.

What does this form of happiness, which is the ultimate goal of rational life, look like? It consists in becoming a mirror of the universe, in reflecting on (and ultimately becoming a reflection of) the structure of the universe and of its hierarchical order of causes and effects. This order is the object of scientific knowledge in its various forms and is captured by the content of the individual sciences as well as by the comprehensive hierarchy of the sciences. Avicenna could not be more explicit on this point:

> C3: *Ilāhiyyāt* IX, 7, pp. 425.16–426.4 (Marmura 2005, p. 350, transl. modified)
> The perfection proper to the rational soul consists in *becoming an intelligible world in which there is impressed the form of the universe*; the intelligible order found in

5. The notion of *maʿād* is the counterpart of the Neoplatonic concept of *epistrophē* (return or reversion), which traces the order of being back to its origin. "On Origin and Destination" is famously also the title of one of Avicenna's philosophical compendia, at the end of which he presents the same ideas sketched in this conclusion concerning intellectual knowledge, certainty, and the happiness of the rational soul (*Mabdaʾ wa-maʿād* III, 14, p. 110.13–14, III, 15, p. 114.10–13, III, 15, p. 115.15–18).

> the universe; and the good that emanates on the universe from the principle of the universe, proceeding then to the noble, spiritual, separate substances, then to the spiritual [substances] that in a way depend on bodies, then to bodies that are superior by qualities and faculties, and so on *until* [the soul] *exhaustively receives in itself the structure of existence in its entirety*. It thus *becomes transformed into an intelligible world that is the counterpart of the existing world in its entirety*, experiencing that which is absolute perfection, absolute good, [and] true absolute beauty, becoming united with them, imprinted with their image and structure, part of them, and becoming part of their substance.

It is difficult to resist the temptation of seeing in Avicenna's characterization of the division and hierarchy of the sciences, illustrated in chapter 6, an attempt to flesh out in broad strokes the ideal presented in this passage of the *Ilāhiyyāt*. The panoptic view of *Burhān* II, 7 offers a comprehensive account of the interrelations among the sciences and a way to see how each individual science, in reflecting the internal structure of the domain of being corresponding to it, contributes to our knowledge of the whole.

The centrality of the theory of science for Avicenna is therefore evident from his characterization of the nature of intellectual pleasure in the afterlife. Happiness is only attainable in proportion to the degree of scientific knowledge acquired during an individual's mortal life. That determines a soul's capacity (as a measure of containment) for its continued happiness in the afterlife, which is ultimately a form of contemplation of the structure of the universe. Such a capacity can only grow and be nurtured through the sort of intellectual development associated with scientific inquiry and the acquisition of scientific knowledge.

The role of the logic of scientific discourse developed in the *Burhān*, as a necessary condition for the realization of human rationality and the attainment of the highest form of happiness available to it, is illustrated by a striking parallel between two passages. In *Ilāhiyyāt* IX, 7, Avicenna raises the question of just how much scientific knowledge is required to attain intellectual happiness. His educated guess explicitly reminds us of the essential role played by the two modes of knowledge investigated in this book:

> C4: *Ilāhiyyāt* IX, 7, p. 429.4–13 (Marmura 2005, pp. 353–354, transl. modified)
> But how much conceptualization of the intelligibles (*taṣawwur al-maʿqūlāt*) must come about in the human soul, so that it may go beyond the limit before which this misery lies, and in crossing it and going beyond it, it may hope for this happiness?
>
> This is something I can only determine with approximation. I believe that this requires that the human soul should have [(1)] a *true conception* (*taṣawwur ḥaqīqī*) of the separate principles, [(2)] having *assertions* about them that are *certain* (*taṣdīq yaqīnī*) because they exist for it *through demonstration*; that it should know the final causes of things occurring in universal motions, not the particular ones which are infinite; that there should become established for it the structure of the universe

(*tataqarrara ʿindahā hayʾat al-kull*), the relation of its parts to each other, and the order deriving from the first principle down to the most remote of the existents that fall within its arrangement; that the soul should conceive providence and the manner thereof; that it should verify what proper existence and what proper unity belongs to the essence that precedes the whole, and the manner in which [this essence] knows (whereby neither multiplicity nor change attaches to it in any way) and the manner in which the existents are ordered and related to it.

The pinnacle of Avicenna's Neoplatonic eschatology is shaped by a revised Aristotelian epistemology that has been translated into the vocabulary of Arabic logic. *True conception*s and *certain assertions* relative to the structure of the universe and its internal articulation and its principles are necessary conditions for the attainment of happiness. The same two paths for the ideal attainment of this ambitious goal appear, verbatim, at the beginning of Avicenna's *Burhān* in the characterization of the goal and benefit of the book:[6]

C5: *Burhān* I, 1, p. 53.15–18
Having mentioned the goal of the book, which is to procure the methods that bring about certain assertion (*taṣdīq yaqīnī*) and true conception (*taṣawwur ḥaqīqī*), its benefit is manifest, namely the attainment of certainty in the sciences and of true conceptions. These are useful, or rather necessary, to us if we begin to use this instrument, that is to say logic, and assess by means of its scales (*mīzān*) both the theoretical sciences and the practical sciences.[7]

If logic tout court enables us to discriminate between good and bad reasoning, the logic of scientific reasoning articulated in the *Burhān* upgrades this function to the discrimination between scientific and nonscientific reasoning. The scales of logic, particularly in the form of an adequate theory of science that combines a proper theory of demonstration and a proper theory of definition, are the basis for a critical evaluation of the content of the theoretical sciences as well as of the practical ones. True conceptions and certain assertions are the essential vehicles by means of which we may ideally come to know "the truth for the sake of itself" and "the good for the sake of acting by it," whereupon our nature as human beings is realized in its full perfection, along with the attainment of the highest form of intellectual happiness.

6. At *Burhān* III, 9, p. 260.1–4, Avicenna offers a similar characterization of wisdom (*ḥikma*), while glossing on the term *sophia* at *An. Post.* A34, 89b32: "Wisdom is the process that leads the human soul to the perfection that is possible for it, within the limits of scientific knowledge and action. In the domain of scientific knowledge this [requires the soul] to conceptualize the existents as they are and to assert propositions as they are; in the domain of action, it [requires] the realization in it of the trait of character called justice. Wisdom is sometimes said of the process of seeking the perfection of the rational soul with respect to containing theoretical and practical intelligibles, even if does not result in a trait of character."

7. For the expression "scales" or "balance" of logic (*mīzān*), see also the preface and first section of the logic of the *Daneshname* (Achena and Massé 1955, p. 21 and p. 25).

APPENDIX A

Conditions of Certainty

The certainty of scientific assertions acquired by demonstration depends on the *form* (*ṣūrā*) of the valid deductive arguments by means of which conclusions are necessarily derived from their premises, as well as on their *matter* (*mādda*), which in Avicenna's technical vocabulary stands for the kinds of terms and relations expressed by premises and conclusions.

Demonstrative premises are identified in Avicenna by a characteristic set of features. They must be (i) true, (ii) immediate, (iii) primary, (iv) prior to, (v) explanatory of, and (vi) better known than the conclusion; (vii) appropriate; (viii) necessary; and (ix) per se.[1] These conditions are discussed at different places (partial lists are in *Burhān* I, 11, I, 12, and II, 9).[2] A way to read Avicenna's *Burhān* is as a sustained effort to investigate these conditions in turn.

The standard loci where they are first introduced (or discussed more extensively) are

(i) true, *Burhān* I, 11;
(ii) immediate, *Burhān* I, 12;
(iii) primary, *Burhān* II, 3 (with a distinction between primary premise and premise whose predicate is primary);[3]

1. This set may ultimately be traced to a celebrated combination of conditions identified by Aristotle in *An. Post.* A2 (the first six), A4, A6, and A9 (some of them, for example truth, are somewhat redundant once stronger ones, for example necessity, are postulated). The treatment of such conditions varies from author to author in the commentary tradition, and it occasionally results in special treatises devoted to the subject, such as Alfarabi's *Šarā'iṭ al-yaqīn* (*The Conditions of Certainty*) (in *Kitāb al-Burhān wa-Kitāb šarā'iṭ al-yaqīn*), on which see Black (2006).

2. A partial summary is at *Burhān* II, 9, p. 174.1–4 (premises must be true, primary, immediate, universal in the sense of "said of every," and appropriate).

3. A primary premise is an immediate premise. The assertion of an immediate categorical proposition does not require a middle term between its subject and predicate. A premise whose predicate is

(iv), (vi) prior and better known, *Burhān* I, 11 (with the canonical distinctions between what is prior to us and what is prior in nature, and between what is better known to us and what is better known in nature);
(v) explanatory, *Burhān* I, 11;
(vii) appropriate, *Burhān* II, 9;
(viii) necessary, *Burhān* II, 1 and 5 (also I, 8); and
(ix) per se, *Burhān* II, 2–3.

These conditions are typically inferred on the basis of various arguments inspired by the *Posterior Analytics*. Avicenna's focus is on the epistemic and modal stability ascribed to scientific assertions, whose two critical components are necessity (associated with the non-transitory character of scientific knowledge) and causality (associated with the idea that the required necessity of a scientific assertion, when it is not the necessity of a self-evident principle, must ultimately depend on the knowledge of its causes).[4] From these conditions, Avicenna derives in turn the other epistemic and proof-theoretic conditions (truth, immediacy, priority, appropriateness). In general, the conditions of certainty of demonstrative scientific knowledge are polarized around the two fundamental dimensions of necessity and explanatoriness and constitute, as it were, two complementary families.[5]

Thus, for example, in *Burhān* I, 11, Avicenna addresses conditions (iv), (v), (vi), and contends that if the premises of a demonstrative argument must *explain* the conclusion (not

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primary, by contrast, is a proposition whose predicate does not belong first to something more general than its subject (*Burhān* II, 3 deals with "primary" in the second sense). Avicenna contends that in the second case, unlike the first (by definition), "several middle terms" may be needed (*Burhān* II, 3, p. 137.12–13). The claim is best understood in light of the example he offers only a few lines above at *Burhān* II, 3, p. 136.14–16, namely the relation between having the sum of the internal angles equal to two right angles and triangle. The attribute belongs in a primary way to triangle, because it is not predicated on anything more general than triangle, but it is not immediate because several middle terms are required to prove that it belongs to triangle (in this case all predicates must be coextensive, and the sequence involves a series of horizontal predications).

4. At *Samāʿ ṭabīʿī* IV, 8, p. 294.1–3 (McGinnis 2009, p. 453), Avicenna offers a good illustration of the epistemic state he has in mind: "These and similar things are the basis upon which the two groups argue, but the argumentation of neither one of them is outstanding (even if the second school of thought is true). The fact is that they have entrusted us with no demonstration such that it either completely satisfies us or brings us to a level of understanding that *removes all doubts*."

5. At *Burhān* I, 11, p. 106.1–7, Avicenna summarizes the main conditions as follows: "Since the premises of demonstration are *causes* (*ʿilal*) of the conclusion and the cause is essentially prior, the premises of demonstration are essentially *prior* (*aqdam bi-ḏ-ḏāt*). Similarly, they are prior to the conclusion for us in time (*aqdam min an-natīǧa ʿindanā fī z-zamān*) and prior for us in knowledge (*aqdam ʿindanā fī l-maʿrifa*) with respect to the fact that the conclusion is known only through them, and they must be *true* (*ṣādiqa*) in order to produce truth. If these premises are causes, they must be *appropriate* (*munāsiba*) to the conclusion and fall in the domain of the science in which the conclusion falls or [in the domain] of a science that shares [something] with the former (*yušārikuhū*) as we shall explain later, and the principles of their demonstration (*awāʾil barāhīnihā*) must be *first* (*uwal*), *self-evident* (*bayyina bi-nafsihā*) premises that are *better known* (*aʿraf*) and prior to every premise [that comes] after them. If they do not satisfy these conditions, they are not demonstrative premises."

just provide its inferential justification), and causality and explanation presuppose essential *priority* (*aqdam bi-ḏ-ḏāt*), then the premises of demonstrative arguments must be prior, better known, and more manifest.[6] On the other hand, if the premises of a demonstrative argument are causes of the conclusion, they must be *appropriate* to the conclusion, where the condition of appropriateness means that they must fall in the domain of the science to which the conclusion belongs or in that of a science that has common elements with it (*Burhān* I, 11, p. 106.4–6).[7] The appropriate character of a scientific attribute (or of the premises or principles of which the attribute is part) may be triangulated from different perspectives but ultimately depends on whether the attribute belongs to the same genus as the subject; hence the appropriateness of an attribute may be framed at the same time as a problem of modality (what type of per se attribute it is), as one of causality (how proximate or remote it is in the order of explanation, that is to say, in terms of generality), or as one of interrelation between sciences (how the attribute is related to subjects of different generality).

If necessity is directly associated with the non-transitory character of scientific knowledge, what ensures that scientific premises and conclusions have the required modal strength is in fact the essential character of the relations that are the object of scientific investigation. The latter are encapsulated in Avicenna's theory of science by the notion of per se.[8] In particular, for Avicenna, the per se character of demonstrative premises seems to be a consequence of the fact that premises cannot be foreign to the conclusion, because if they were, they would in turn fail to be explanatory.[9]

The strongest case for the relation between necessity and appropriateness is made at *Burhān* II, 5. When strengthening one of the requirements that must be satisfied for the mere formal validity of inferences involving necessity (for instance Barbara NXN), Avicenna argues that if demonstrable scientific assertions are necessary, then *all* of the premises from which those assertions are derived must be necessary too, again as a result of the non-transitory epistemic status that characterizes them by definition.[10]

6. Principles must be clearer (*awḍaḥ*) and better known (*aʿraf*) (*Burhān* II, 1, p. 117.1–3).

7. A similar point is made at *Burhān* II, 5, p. 154.7–8: an explanation may be transferred (*bayān manqūl*), as opposed to being foreign (*bayān ġarīb*), only when there is some sort of sharing of subject, questions, or principles (as explained in *Burhān* II, 7), in which case premises are appropriate to the conclusion.

8. Thus, for instance, "since the premises of demonstration procure scientific knowledge that is non-transitory (*lā yataġayyaru*) and such that what is known cannot be otherwise, the premises of demonstration cannot be otherwise. This is what 'necessary' means (*ḍarūrī*)" (*Burhān* II, 1, p. 120.13–15).

9. In *Burhān* II, 2, Avicenna seems to argue by reductio that if the terms were not per se, then they would be foreign, and if they were foreign, then they would not be explanatory. But since, by definition, scientific knowledge is explanatory, then by contraposition, the terms must be per se, which implies in turn that they must be definitionally connected and belong to the same kind. This is what it means for a principle to be appropriate. On the relation between appropriate and per se, see also the brief remark at *Burhān* II, 9, p. 177.3–4.

10. For, if x is obtained from a middle that can change, then x is epistemically unstable and transitory; and since scientific conclusions are stable and non-transitory, the premises from which they are obtained cannot be subject to change and must therefore be necessary. Thus, a necessary conclusion necessarily follows from necessary premises that cannot change (*lā yaqiʿu fīhā imkān taġayyur*)

On the other hand, true premises that are non-appropriate are extrinsic and foreign to the conclusion, and the resulting knowledge lacks the explanatory character required for it to count as *certain* scientific knowledge (*Burhān* II, 5, p. 151.5–6), as causes are by definition appropriate in the sense of being co-generic or at least somehow essentially connected to that of which they are causes.[11]

The boundaries between epistemic, ontological, and proof-theoretic conditions are sometimes blurred. Conditions that may prima facie seem to be uncontroversially epistemic (for example, the property of being better known, which plays a significant role in the arguments against circularity mentioned in chapter 1) may acquire an ontological connotation (as in the distinction between better known to us and better known in nature). Other notions, such as immediacy or priority, may be taken to have a proof-theoretic connotation (for example, immediacy plays a role in the argument against infinite regress in the search for premises), even though they also carry a heavy metaphysical baggage. The same holds of necessity, which as we have seen in chapter 6, seems to be at times an epistemic notion that reflects certain requirements about the stability of our beliefs and, on other occasions, an ontological notion that captures the status of certain factual relations between subjects and attributes.[12]

(*Burhān* II, 5, p. 150.1–3). In the same chapter, Avicenna also briefly discusses the relation between necessity and appropriateness (*Burhān* II, 5, p. 154.1–5).

11. Non-appropriate premises may only provide an inferential justification of the truth of the conclusion, not "the necessity of its truth or the causality of its truth" (*Burhān* II, 5, p. 151.7–8). Avicenna argues again for the close connection between modality and explanation in *Burhān* I, 8, where he contends that knowledge of any claim such that the relation between subject and attribute presupposes one or more causes (and hence is non-immediate) can only be certain in virtue of our knowledge of the cause(s).

12. Proofs are deductive chains, whose premises are either (i) primitive truths or (ii) previously established truths (in a logical but not necessarily in a temporal sense). Part of the investigation pertaining to the theory of science concerns proof-theoretic questions such as whether there is a limit to the length of demonstrative chains or whether certain kinds of demonstrations are preferable (for example, universal to particular, affirmative to negative, direct to indirect demonstration). Straightforward proof-theoretic aspects include the examination of certain metalogical properties of demonstration in *Burhān* III, 5 (*An. Post.* A19–22); the discussion of figures and the preliminary discussion of the treatment of ignorance in *Burhān* III, 4 (*An. Post.* A14–15); and the preferability of universal, affirmative, and direct demonstration in *Burhān* III, 6 (*An. Post.* A24–26). For reasons of space, these themes were reluctantly but deliberately omitted from this study.

APPENDIX B

The Logic of Scientific Reasoning

One of the major contributions of Avicenna's theory of science is the explicit acknowledgment (corroborated by compelling evidence of sustained and consistent use) of the idea that logic, and in particular demonstration, is meant to find a direct application in the sciences. While being an instrument of scientific reasoning is not the sole purpose of logic, that seems undoubtedly to represent its ultimate domain of intended application, in the form of a theory of demonstration and definition.[1]

A notorious problem of expressive power is traditionally associated with Aristotle's categorical syllogistic, which both as a matter of fact (that is to say, in Aristotle's own practice) and presumably also in principle seems to be fundamentally inadequate to articulate complex scientific demonstrations, let alone scientific theories in their full deductive structure.[2]

1. On the ambivalent status of logic, especially the theory of deduction or syllogism (*qiyās*), which is at the same time an independent inquiry but is also driven by an ultimate end, namely to serve as an instrument for the acquisition of demonstrative knowledge, see for instance *Qiyās* I, 1, pp. 3.1–4.3: the study of deduction is primarily for the sake of demonstration and secondarily for the sake of other disciplines such as dialectic, rhetoric, or sophistic. At *Ḫiṭāba* I, 1, pp. 1.1–2.6, Avicenna notes that persuasion plays an important role in the discussion of first principles. The point is reiterated at *Ilāhiyyāt* I, 8, where he contends that the most fundamental principles of assertion cannot be defended demonstratively.

2. This is in part due to a fundamental inability to treat relations, a problem that for all intents and purposes is presumably shared by Avicenna's logic, but see El-Rouayheb (2010) on the *fortuna* of the theme in the later Arabic tradition. Perhaps more importantly, however, this is also due to the fact that Aristotle's logic is primarily a logic of terms without a developed propositional component, and with a limited set of inference patterns, being mostly confined to the theory of categorical syllogism. Neither of these is a limitation of Avicenna's theory. The problem is part of a larger question concerning the consistency of Aristotle's logic with his own scientific practice: "a hoary old chestnut indeed" in the words of Lloyd (1990, p. 371). On this issue, see in general Barnes (1969, 1981), Mueller (1974), Kullmann (1981), Bolton (1987), Gotthelf (1987), Lennox (1987), Wians (1989), and Tuominen (2010).

Avicenna's logic, by contrast, while fundamentally Aristotelian in character and language, includes a much broader set of logical forms and inference patterns. In particular, the logical form of scientific statements is not restricted to subject-predicate propositions, and consequently, the theory of inference used in scientific discourse is not limited to the categorical syllogistic.

Avicenna states on various occasions that scientific statements can have different logical forms involving categorical propositions (*ḥamliyyāt*) and hypothetical propositions (*šarṭiyyāt*).[3] The latter can be either conditional (*muttaṣilāt*) or disjunctive (*munfaṣilāt*) propositions. Categoricals have a privileged status, because they are the simple nodes, expressing fundamental predicative relations, in the network of logically interconnected statements that constitute the body of a science.[4] But compound propositions with other logical forms are also useful (and indeed necessary) to connect various categorical statements with one another. Compound propositions may express relations of following (*ittibā*ʿ) (especially inseparability) or conflict (ʿ*inād*) (incompatibility). Further, Avicenna's account of reductio proofs explicitly involves a combination of propositions at least some of which must be non-categorical, and his understanding of the relation between antecedent and consequent in conditional statements, which involves a distinction between coincidental (*ittifāqī*) and implicative conditionals (*luzūmī*), is often explicitly associated with scientific reasoning.[5]

3. I discuss the logical form of scientific statements in chapter 4. Avicenna gives an excellent general characterization at the beginning of the treatment of hypotheticals at *Qiyās* V, 1, p. 231.1–5 (Shehaby 1973, p. 35, transl. modified), in a passage that summarizes in a few lines four key aspects of his understanding of the application of logical forms to scientific reasoning: "Just as (i) some premises are categorical (*ḥamliyya*) and others are hypothetical (*šarṭiyya*), similarly some of the things that are sought are categorical and others hypothetical. And just as (ii) some categorical propositions are asserted without deduction while others need a deduction [to be established], so also with hypotheticals. For (iii) many claims in mathematics, natural philosophy, and metaphysics are conditional or disjunctive hypotheticals. (iv) Categorical propositions may be proved by categorical deductions and by hypothetical deductions, while hypothetical propositions are deduced either from pure or from mixed hypothetical deductions, but not from categoricals." Other loci address specific cases (for instance, *Qiyās* V, 5, p. 286.15–16 on the default type of disjunctive proposition used "in the sciences and where real truth is concerned"). The definition of logic at *Išārāt* I, 1, pp. 2.1–3.4 emphasizes its instrumental and normative character (*āla qānūniyya*), its focus on the correct form (*hay'a*) and order (*tartīb*) of reasoning for the acquisition of knowledge, and its discrimination from incorrect reasoning.

4. Another class of statements includes assertions of existence. Avicenna goes so far as to associate them with a specific kind of argument, namely repetitive deduction. The view is put forward in *Burhān* IV, 1, where Avicenna says that proofs whose conclusion is a simple if-question (an assertion of existence) are best represented by repetitive deductions, where the middle term or cause is the condition. By this, Avicenna means that the antecedent of the conditional "if *p*, then *q*," which is "repeated" or asserted to infer *q*, is the cause of the existential claim expressed by *q*. The view is close to the one expressed by Alfarabi, *Burhān* I, 3, p. 28.6–8: "[Such assertions of existence] are proved by hypothetical deductions only" (*yubayyana bi-qiyās šarṭī fa-qaṭ*).

5. In particular, conditional statements with impossible antecedents and consequents, which are the standard type involved in reductio proofs, can only be a specific type of implicative conditionals (those that are true by implication only and not in fact), as we have seen in chapter 8.

Avicenna's use of conditional and disjunctive propositions in the nonlogical corpus is extensive, and if this may appear to be a trivial observation, he also *explicitly* argues in favor of or against certain views based on considerations having to do with logical form, which shows that he consciously regards this conceptual vocabulary as an integral part of his philosophical method, and that the logic *investigated* in the logical corpus is genuinely meant to be the same logic *used* everywhere else in his nonlogical works.[6]

Avicenna's positive characterization of the logical form of scientific statements and the admissible inference patterns in scientific discourse is confirmed by two different types of application in his works (logical and nonlogical). On the one hand, if we look at his philosophical corpus through this lens, it seems clear that many of Avicenna's arguments are cast precisely in a form that at least ideally meets the standards of his own logic. If it remains true that not all arguments in Avicenna's sciences involve exclusively categorical, disjunctive, and conditional statements, still the sense in which it is often possible accurately to reconstruct an argument, whether in metaphysics or natural philosophy, according to these standards is much more robust than the sense in which one might try to match Aristotle's arguments with his syllogistic.[7]

Commentators had already set the model for this by trying to bring out the logical form of passages in Aristotle, but what is peculiar about Avicenna's approach is the systematic effects this has on his understanding of the logic of scientific reasoning. Moreover, in Aristotle, the attempt to reconstruct arguments in syllogistic form often results in a painful exercise of rhabdomancy, at the end of which the most one can hope for is usually just a few microscopic traces of categorical syllogistic reasoning here and there. In Avicenna, by contrast, the gap between theory and application is significantly narrower. Avicenna's arguments frequently turn out to be precisely the sort of complex structures involving many of the fundamental forms he identifies in his logic (most notably conditional, disjunctive, and categorical statements).[8] Evidence of the fact that Avicenna really means what he says

6. *Ilāhiyyāt* I, 8, pp. 49.8–50.4 contains a brief but interesting digression on the notion of deduction (*qiyās*) in the context of Avicenna's account of the task of the first philosopher in the refutation of a "sophist." Such a putative interlocutor is characterized as a radical skeptic who denies the law of the excluded middle (namely "that there is no middle between affirmation and negation"), which for Avicenna is "the proposition that is most worthy of being true" and "which is always true [. . .] primarily and without a cause," "the one in which the analysis of everything comes to a stop." Avicenna characterizes it also as "a property which is an accident just of existent as such." Deduction is also discussed at *Ilāhiyyāt* VI, 5, pp. 291.8–292.5 in the context of Avicenna's analysis of final causes.

7. At the same time, according to the memoirs from a disciple from Rayy (Gutas 1988, p. 70), Avicenna himself regards the analysis of the terms and their relations rather than the formulation of arguments in a proper deductive form to be the real priority of scientific reasoning: "We constantly used to hear him say: 'In analysis, do not spend too much time taking into account the forms of deductions for that is one of the easy parts and a sound instinct rarely makes a mistake about it; you should rather practice examining in detail the matters [of deductions].'"

8. The extent to which such reconstructions appear compelling may vary, depending on specific arguments and works. In some cases, the structure emerges rather clearly, as for example in the proofs of the existence of the circle at *Ilāhiyyāt* III, 9, pp. 148.14–151.11, or some of the sub-proofs in the refutation of the existence of void and atoms, at *Samāʿ ṭabīʿī* II and III, respectively. All these proofs evidently

concerning, for example, the logical form of propositions and arguments in scientific discourse, may also be gleaned from passages of his works in which he explicitly identifies the logical form of a step in an argument he is intent on rejecting. A non-exhaustive list of examples that illustrate this approach includes various contentions in *Qiyās* V about disjunctive statements or the negation of conditionals; counterexamples in *Qiyās* VI involving terms drawn from metaphysics or natural philosophy which show the non-productivity of certain hypothetical deductions; remarks on the meaning of logical particles such as "since" (*lammā*) in *Qiyās* IX, 1 and their role in the sciences; and a list of argument forms in *Burhān* I, 3 (examined in chapter 1) that encapsulate in different ways the notion of potential knowledge (with a reference to conditional and disjunctive statements).[9] But explicit references of this kind are not found in Avicenna's logical works only. For example, at *Ilāhiyyāt* IV, 2, in rejecting a certain definition of capacity, Avicenna argues that it is inadequate because the characterization contains conditional statements and therefore cannot establish certain attributes to hold (categorically) of the *definiendum*.[10] At *Samāʿ ṭabīʿī,* II, 11, he criticizes an opponent's interpretation of an argument in natural philosophy, which (wrongly) turns on an ascription of circularity in the definition of motion.[11] Another note-

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involve several sequences of nested disjunctive and conditional statements (the possibility of nesting is explicitly entertained by Avicenna in his characterization of both types of statements at *Qiyās* V, 3, pp. 253.1–256.10, where he maintains that the parts of a hypothetical statement, whether conditional or disjunctive, may be either categorical or hypothetical).

9. On scientific disjunctive propositions, see, for instance, *Qiyās* V, 5, p. 289.10–16: "Sometimes every line is either equal to or less than any other line;" "Always either every line is equal to or less than the diameter of the cosmos;" and *Qiyās* V, 5, p. 290.12–13: "Every fire moves upwards or downwards." Concerning the conditional particle "since" (*lammā*), Avicenna notes that it introduces hypothetical premises not used in full deductive form, that is to say, without making the complete inferential structure explicit, as in "since *p*, *q*," which is an abbreviated form for "If *p*, then *q*; and *p*; therefore *q*." The assertion of the antecedent, preceded by *lammā*, suggests that the conditional major premise is implicit (usually because it is taken to be either self-evident or previously established: the context of the discussion in *Qiyās* IX, 1, 419.9–15 is the evident character of the major premise of certain arguments).

10. At *Ilāhiyyāt* IV, 2, p. 173.2–12, (Marmura 2005, pp. 132–133, transl. modified) Avicenna analyzes an argument in terms of conditionals: "This is because [according to this definition of capacity it] would be true for the [agent] to act if he wills and not to act if he does not will. Both these [statements] are conditionals, that is, 'If he wills, he acts,' and, 'If he does not will, he does not act.' The two are included in the definition of capacity only inasmuch as they are conditionals. It is not a condition for the truth of a conditional that there should be in any way a repetition of [antecedent or consequent] or a categorical truth. [. . .] From this it would correctly follow that, if he did not will, he did not act, and, if he did not act, he did not will. But there is nothing in this to imply necessarily that at some time he did not will. This is clear to anyone who knows logic." On the view that the truth of a conditional does not require the truth of its parts, see chapter 8. Another interesting example along the same lines in *Samāʿ ṭabīʿī* III is Avicenna's reply to an argument containing a "professed demonstration of the infinite," where he contends: "The response is to recall what we stipulated at the outset of the account, namely that this depends upon a hypothetical proposition based upon a supposition, not an existence claim."

11. *Samāʿ ṭabīʿī* II, 11, p. 156.6–7 (McGinnis 2009, p. 232): "Time is the number of motion when it is differentiated into earlier and later parts—not by time, but, instead, with respect to distance; otherwise,

worthy example is at *Samāʾ wa-ʿālam* I, 2, where Avicenna attacks a view on the basis of an incorrect use of contraposition with the wrong type of proposition.[12] Finally, at *Samāʿ ṭabīʿī* II, 10, p. 154.7–11, Avicenna rejects an argument that purports to show that time is the celestial sphere because its proponents "reason from two affirmative premises in the second figure," that is to say, by means of an invalid categorical mood.

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the definition would be circular. (This is what one of the logicians believed—namely, that a circle occurred in this explanation—but he believed wrongly, since he did not understand this)."

12. The passage is worth quoting in full; see *Samāʾ wa-ʿālam* I, 2, p. 14.2–17 (emphasis added):

> Furthermore, he produced a *faulty deduction*. For he said that "It is possible for simple bodies the species of whose nature is not a single species to move according to a simple motion whose species is a single species" is *converted by contraposition* (*inʿikās an-naqīḍ*), and so it is possible for things that do not move according to a natural motion that is one by species, to be [simple bodies] of a single natural species. Thus, in his view the contrapositive is a consequent of an antecedent that is its contrapositive. He erred in this conversion precisely because he took this to be a possibility proposition and supposed it as an existence or necessity [proposition], which necessarily converts. *This kind of conversion by contraposition [however] is incorrect for possibility premises, when the possible is taken as a mode and not as part of the predicate.* It is as if one were to say "If it were possible [for] different substances, the nature of whose species is not one and the same nature, to share in one and the same common essence or description, it would be possible for the nature and species of things, which do not share in one and the same common essence or description, to be one and the same." Since this conversion is incorrect, know that what he said is not necessary. If the possible is made part of the predicate, the conversion is correct, but he does not get what he wants. The contrapositive conversion of that premise is "What cannot move according to a single simple motion whose species is one and the same, is not a simple body the species of whose nature is not a single species." This is true. From this, then, one learns that the heavenly nature differs from these natures with regard to the principles of motions.

In this passage, Avicenna discusses the question of whether a statement of the form "S is P" converts to "not-P is not-S" (or, conditionally, "if *p*, then *q*" converts to "if *not-q*, then *not-p*"). This kind of conversion, which Avicenna calls "conversion by contraposition," does not hold in the case of possibility propositions, unless possibility is taken as a predicate rather than a mode. The example shows how a rule of formal logic finds explicit application in the context of a scientific discussion where the wrong contrapositive principle is supposedly used to make a point about the relation between certain kinds of bodies and their characteristic type of motion.

On conversion, the role of subject and predicate, and the relation between necessity and possibility propositions, see also *Qiyās* II, 3, pp. 100.13–105.14.

APPENDIX C

A Map of *Kitāb al-Burhān* (*Book of Demonstration*)

The chapter headings are not intended to be translations of the transmitted titles.

FIRST TREATISE

Chapter I, 1. Goal and benefit of the *Book of Demonstration*

1. Types of assertion and types of deduction (pp. 51.1–52.2)
2. Types of conception and types of definition and description (p. 52.3–20)
3. The goal of the *Book of Demonstration*: identification of the conditions for demonstrative deductions (certainty in assertion) and for real definitions (complete conception) (p. 53.1–3)
4. Relation between conception and assertion (p. 53.4–14)
5. The benefit of the *Book of Demonstration* (p. 53.15–18)

Chapter I, 2. Rank of the *Book of Demonstration*

1. Classification of the sections of logic (p. 54.1–13)
2. Demonstration and dialectic (pp. 54.14–56.12)
3. Demonstration and the other deductive arts: rhetoric, poetics, sophistic (p. 56.13–21)

Chapter I, 3. All teaching and learning involving reason come from preexistent knowledge (compare with *An. Post.* A1, 71a1–2)

1. Classification of the types of teaching and learning (p. 57.1–19)
   - 1.1 Preliminary list (p. 57.1–9)
   - 1.2 Definition of teaching and learning involving reason (p. 57.9–15)
   - 1.3 Teaching and learning involving reason and thought come from preexistent knowledge (p. 57.16–19)

2. Preconditions of assertion and conception (p. 58.1–9)
   2.1 Preconditions of the assertion of a conclusion: conception of the conclusion; conception of the premises; assertion of the premises (p. 58.1–5)
   2.2 Preconditions of the conception of a complex term: conception of the parts of definition and description (p. 58.6–9)
3. Gloss on "involving reason" (pp. 58.10–59.7)
4. Classification and meaning of "involving reason," "involving thought," "intuitive," "involving comprehension" (pp. 59.7–60.10)
5. Assertion in act and assertion in potency with different argument forms (conditional, disjunctive, universal, inductive, and analogical inference) (p. 60.11–20)
6. Temporal priority and essential priority (pp. 61.1–62.5)

Chapter I, 4. Classification of deductive principles (scientific and nonscientific assertions)

1. Division of the principles of deduction: based on assertion and based on imitation; description and dismissal of the second type (principles of poetical deduction) (p. 63.1–13)
2. Division of the principles of deduction based on assertion (pp. 63.13–67.12)
   2.1 Assertion by way of necessity (pp. 63.14–65.10)
   2.2 Assertion by way of concession (pp. 65.10–66.15)
   2.3 Assertion by way of predominant supposition (pp. 66.16–67.12)
3. Summary of fourteen types of assertion; distinction between hypothesis and postulate (p. 67.13–20)

Chapter I, 5. Types and order of scientific inquiry; scientific principles and middle terms (compare with *An. Post.* B1 and A13; cf. also *Burhān* IV, 1)

1. Three inquiries: what, if, why (with two subtypes each) (pp. 68.1–69.13)
2. Classification of principles: (i) existence, (ii) meaning, (iii) existence and meaning (p. 69.14–19)
3. Simple and compound notions (pp. 69.20–71.1)
   3.1 Compound notions: axioms, hypotheses (*haliyya*) (pp. 69.20–70.3)
   3.2 Simple notions: (i) accidents of the subject of the discipline or (ii) notions falling within the domain of the subject of the discipline (*māhiyya*) (pp. 70.4–71.1)
4. The status of why-questions: (i) with respect to the thing itself or (ii) with respect to the statement; the middle term is the cause of a deduction, either (i) of the assertion only or (ii) of the assertion and of the fact (p. 71.1–13)

Chapter I, 6. The acquisition of what is not known starting from what is known (compare with *An. Post.* A1, 71a17–72b8; cf. also *An. Pr.* B21)

1. Conception of non-existents: (i) without composition (*tarkīb*) or (ii) with composition (p. 72.1–15)
2. Knowledge in act and knowledge in potency: how a universal judgment comes about (solution of the problem of *de re* knowledge in *An. Post.* A1, example of pairs) (pp. 72.16–74.12)

3. Meno's paradox and its solution; transition to the discussion of the principles of certain assertion (pp. 74.13–77.6)

Chapter I, 7. Kinds of demonstration and kinds of explanation

1. Definition of certainty (senses of necessity; distinction between absolute, referential necessity, and descriptional necessity) (p. 78.1–11)
2. Definition of demonstration (pp. 78.12–79.4)
3. On (complete) induction and divided deduction (p. 79.5–12)
4. That-demonstration and why demonstration (pp. 79.13–84.10)
   - 4.1 Definitions (p. 79.13–16)
   - 4.2 That-demonstration: absolute that-demonstration and sign (*dalīl*) (pp. 79.17–80.10)
   - 4.3 Why-demonstration (pp. 80.11–84.5)
     - 4.3.1 Cause of (i) the existence of the major and of its belonging to the minor and cause of (ii) the major's belonging to the minor only; on differentiae, genera, causality, conceivability and modality (pp. 80.19–82.17)
     - 4.3.2 Middle term cause of the major's belonging to the minor but effect of the existence of the major (pp. 82.18–83.8)
     - 4.3.3 Summary of why-demonstration; objection and reply (pp. 83.9–84.5)
   - 4.4. Summary of why-demonstration and that-demonstration (p. 84.6–10)

Chapter I, 8. Certainty, necessity, and causality; relations between major, middle, and minor terms

1. Causal and noncausal certainty; that-demonstration and why-demonstration; necessity; cause and essence (*ḏāt*); immediate implicates (pp. 85.1–87.21)
2. Example of composite (*muʾallaf*) and the case of relative terms (pp. 87.22–90.7)
3. Repetitive deduction and *reductio ad impossibile* (cases of that-demonstration) (p. 90.8–17)
4. Complete perpetual certainty (*yaqīn tāmm dāʾim*) (pp. 90.18–92.5)
   - 4.1. Additional relations middle term-minor term in demonstration
     - 4.1.1 Minor term cause of middle term (p. 91.4–9)
     - 4.1.2 Minor term proprium of middle term (p. 91.10–12)
   - 4.2 Why-demonstration and types of causes (in act, proximate, or remote) (pp. 91.17–92.5) (cf. *Burhān* II, 9, IV, 4, and IV, 5)

Chapter I, 9. Noncausal certainty; induction and experience

1. Subject-predicate nexus: self-evident (no cause) or only known through the cause; rejection of induction (pp. 93.1–94.21) (cf. *An. Pr.* B23, *An. Post.* B5)
2. Experience (pp. 95.1–98.6)
   - 2.1 Introduction; first objection and reply (example of scammony) (p. 95.1–15)
   - 2.2 Second objection (the representativeness of a sample and the problem of justification) (pp. 95.16–96.3)
   - 2.3 Reply to the second objection: observation combined with a deduction, universal conditional certainty (p. 96.4–11)

2.4 Note on experience and two further objections (pp. 96.12–97.15)
2.5 Summary of experience and of the type of certainty it conveys (pp. 97.16–98.3)
3. Difference between objects of sensation, induction, and experience (p. 98.4–6)

Chapter I, 10. Generality and explanation (compare with *Burhān* I, 7 and III, 3)

1. Doubt: how can the less general be the cause of the fact that the more general belongs to what falls under the less general? (p. 99.1–8)
2. Reply: distinctions between matter and genus, and between form and differentia (p. 99.9–12)
3. Example: body, animal, human (pp. 99.13–101.4) [beginning of identical passage at *Ilāhiyyāt* V, 3, p. 214.2]
4. General criterion (p. 101.5–14)
5. Priority (body as matter and body as genus) (pp. 101.15–102.8) [end of identical passage at *Ilāhiyyāt* V, 3, p. 217.7]
6. Types of nexus: species—genus, differentia of genus, genus of genus; *bayān burhānī*, *bayān yaqīnī*, *bayān wuǧūdī*; example of sensitive and animal (pp. 102.9–105.10)

Chapter I, 11. Characteristics of demonstrative premises (compare with *An. Post.* A2, 71b16–72a5; cf. also *Burhān* IV, 6–7 on composition)

1. Demonstrative premises: causes of the conclusion; prior to the conclusion essentially; prior temporally and in terms of knowledge; true; appropriate; primary; self-evident; better known (p. 106.1–12)
2. Prior to us and prior in nature; better known to us and better known in nature; universals (generic and specific); intelligibility (pp. 106.13–108.6)
3. Simple and compound notions, causes, method of composition (pp. 108.7–109.3)
4. Objection and reply (p. 109.4–12) (with a reference to the *Physics*)

Chapter I, 12. Classification of principles (compare with *An. Post.* A2, 72a15–24; cf. also *Burhān* II, 6–7, and 10 on the types of principles)

1. Types of principles: immediate without qualification or relative to a science (p. 110.1–10)
2. Premise and definition; terminological issues (posit; hypothesis, postulate, primary premises) (pp. 110.11–112.6)
3. Assertion of the principles (premises) must be prior, firmer, and worthier than assertion of the conclusion (pp. 112.7–113.3)
4. Distinction between hypothesis and postulate (pp. 113.4–114.15) [p. 113.5–10, quote from Abū Bišr Mattā's translation of *An. Post.* A10, 76b26–31]
5. Rejection of a claim (on the use of a pair of compasses to "verify" that a given construction is a circle) and conclusion of the discussion (pp. 114.16–116.3) [Reference to the proof existence of circle at *Ilāhiyyāt* III, 9]
6. Demonstration and dialectic; deductive premises, demonstration and fallacy (p. 116.4–18) (with a note on the matter of the premises: *mawādd wāgiba ḍarūriyya* and *mawādd mumtaniʿa ḍarūriyya*)

SECOND TREATISE

Chapter II, 1. Universality and necessity of the principles of demonstration (compare with *An. Post.* A3 and A4; on circular proof, see also *An. Pr.* B5–7)

1. Against circular proof and infinite regress (pp. 117.1–120.12)
   1.1 First view (circular proof) (p. 117.4–10)
   1.2 Second view (infinite regress) (pp. 117.11–118.2)
   1.3 Preliminary refutation of a premise shared by both views (everything is known by demonstration) (p. 118.3–17)
   1.4 Some things are known immediately and the regress in the search for principles comes to a stop (p. 118.18–21)
   1.5 Three arguments against circular proof (pp. 119.1–120.12)
      1.5.1 First argument: priority of the premises (p. 119.4–13)
      1.5.2 Second argument: triviality (if *p*, then *p*) (p. 119.14–21)
      1.5.3 Third argument: convertibility of the terms (p. 120.1–12)
2. Scientific premises and the six types of necessity: (1) non-predicative necessity; predicative (2.1) absolute necessity; (2.2) referential necessity; (2.3) descriptional necessity; (2.4) trivial conditional necessity; (2.5) temporal necessity (pp. 120.13–122.20)
3. On "to be said of every" (*maqūl ʿalā l-kull*) (pp. 123.1–124.16)

Chapter II, 2. Per se attributes (compare with *An. Post.* A4 and A10)

1. Per se character of demonstrative premises (p. 125.1–6)
2. The four senses of per se (pp. 125.7–128.2)
   2.1 Per se 1 and difference between *al-maqūl fī ǧawāb mā huwa* and *al-maqūl min ṭarīq mā huwa* (pp. 125.7–126.3)
   2.2 Per se 2 (pp. 126.4–127.17)
   2.3 Per se 3 (p. 127.18–20)
   2.4 Per se 4 (pp. 127.21–128.2)
3. Summary and introduction of a difficulty concerning the identification of per se with constituent (p. 128.2–14)
4. Discussion and rejection of the opinion of unidentified commentators who erroneously identify per se predicates with constituents only (pp. 128.15–131.10)
   4.1 First statement (with a reference to a wrong interpretation of the *Isagoge*) (pp. 128.15–129.4)
   4.2 Second statement (with a reference to "those who are associated with knowledge") (p. 129.5–21)
   4.3 Two implausible claims (p. 130.1–16)
   4.4 The meaning of per se (*ḏātī*), necessary (*ḍarūrī*), and universal (*kullī*) is not the same in the *Isagoge*, in the *Prior Analytics*, and in the *Posterior Analytics* (pp. 130.17–131.10)
5. Explanation of the term *ḏātiyya* (relative to the *ḏāt*) as a qualification of per se attributes; appropriate and foreign attributes (pp. 131.11–132.14) (cf. *An. Post.* A4, 73b15–25)

6. Division of the sciences into universal and particular, depending on their subjects (pp. 132.15–134.20) (cf. *Burhān* II, 7, 9 and *Burhān* III, 3)

Chapter II, 3. Universal, primary, and per se attributes

1. Distinction between *maqūl ʿalā l-kull* in formal logic (said of every) and in demonstration (said of every and at all times); distinction between *maqūl ʿalā l-kull* (said of every and at all times) and *kullī* (said of every, at all times, and primary) in demonstration (p. 135.1–6)
2. Per se 1 and per se 2: primary and non-primary, proper and non-proper (pp. 135.6–137.10)
   2.1 Predicated of the whole subject (pp. 135.6–136.16)
   2.2 Not predicated of the whole subject (pp. 136.17–137.10)
3. Distinction between "primary premise" (immediate) and "premise whose predicate is primary" (p. 137.11–14)
4. Taxonomy of per se (pp. 137.14–143.11)

Chapter II, 4. Three errors concerning universal demonstration (compare with *An. Post.* A5)

1. Preamble (p. 144.1–5)
   1.1 First reason, with a digression on the distinction between universal and particular (cf. *Ilāhiyyāt* V, 1 on the definition of universal) (pp. 144.6–146.5)
   1.2 Second reason (p. 146.6–16)
   1.3 Third reason (pp. 146.17–149.9)
      1.3.1 First case (pp. 146.18–148.7)
      1.3.2 Second case (pp. 148.7–149.9)

Chapter II, 5. Necessity and appropriateness (compare with *An. Post.* A6)

1. Necessity of demonstrative premises; two types of necessity—in essence and in implication—and their relation to per se predicates (p. 150.1–12)
2. Truth, necessity, causality, appropriateness, and the intermittence problem (pp. 150.13–152.22)
3. Three analogies (on truth and falsehood, necessity and nonnecessity) (p. 153.1–11)
4. Accidental premises: two uses (pp. 153.12–154.1)
5. Per se, necessary, and appropriate character of demonstrative premises (p. 154.1–8)

Chapter II, 6. Subjects, principles, and questions of a science (compare with *An. Post.* A7 and A10; cf. *Burhān* I, 12, *Burhān* II, 7–8 and 10)

1. Three elements: introductory characterization (p. 155.1–9)
2. Principles: proper and common (pp. 155.10–157.4)
   2.1 Proper to the subject of a discipline (types) (p. 156.3–13)
   2.2 Common (in potency or in act) (pp. 156.14–157.2)
   2.3 Proper to the subject of a question (p. 157.3–4)
3. Subjects (of a science) (p. 157.5–19)
   3.1 Single subject and multiple subjects (unifying factor) (p. 157.5–14)
   3.2 Subject unqualified (p. 157.15–16)
   3.3 Subject qualified (p. 157.16–19)

4. Questions (pp. 157.20-161.9)
    4.1 Simple categorical (subject-predicate) and compound hypothetical (p. 157.20–21)
    4.2 Classification of subjects (of questions) (pp. 157.21–158.8)
    4.3 Classification of predicates (pp. 158.9–161.9)
        4.3.1 When what is sought is the *anniyya* (pp. 158.9–160.6)
            4.3.1.1 General remarks on the types of predicates-attributes (with two exceptions involving per se 1) (pp. 158.9–159.18)
            4.3.1.2 Taxonomy of predicates of *maṭlūbāt* and *masāʾil* depending on their subjects (pp. 159.19–160.6)
        4.3.2 When what is sought is the *limmiyya* (p. 160.7–9)
        4.3.3 Demonstrative premise pairs and their per se predicates; rejection of PS1-PS1 demonstration (and exceptions) (pp. 160.10–161.9)

Chapter II, 7. Division of the sciences

1. First classification: difference with respect to subject(s) (pp. 162.1–167.10)
    1.1 Difference with respect to multiple subjects (pp. 162.3–166.15)
        1.1.1 Distinctness; partial overlap; full overlap; parthood and subordination (pp. 162.3–165.2)
        1.1.2 Case of metaphysics (pp. 165.3–166.15)
    1.2 Difference with respect to one and the same subject (pp. 166.16–167.10)
2. Second classification: shared principles, subjects, or questions (pp. 167.11–168.18)
    2.1 Principles (pp. 167.12–168.7)
    2.2 Questions (p. 168.8–9)
    2.3 Subjects (p. 168.10–16)
3. Conclusion and transition to the transfer of demonstration (*naql al-burhān*) (168.17–18)

Chapter II, 8. Transfer of demonstration and the domain of application of demonstration (eternal as opposed to perishable, individual entities) (compare *Burhān* II, 8, pp. 169.1–170.16 with *An. Post.* A7, and *Burhān* II, 8, pp. 170.17–173.18 with *An. Post.* A8)

1. Two types of transfer of demonstration (p. 169.1–12)
2. Manners of assuming the terms in demonstrative premise pairs (pp. 169.13–170.6) (cf. Alfarabi, *Burhān* II, 5)
3. Remarks on *An. Post.* A7, 75b12–20 (p. 170.7–16)
4. Perpetual universal premises and conclusions; individual insofar as it is characterized by the nature of the species (compare with notion of descriptional necessity) (pp. 170.17–171.23)
5. Three objections and replies (pp. 172.1–173.18)
    5.1 Two senses of "universal" (p. 172.1–11)
    5.2 Demonstration and definition are concerned with species, conceived of as intelligible, universal, essential, perpetual, and certain natures (pp. 172.12–173.7)
    5.3 Relation between the universality of demonstrative premises and the domain of what is perishable (p. 173.8–18)

Chapter II, 9. Appropriateness of demonstrative and dialectical premises to their conclusions; explanation across sciences (compare with *An. Post.* A9, with two significant digressions at *Burhān* II, 9, pp. 178.7–183.8)

1. Premises must not only be true and said of every but also primary, immediate, and appropriate (*munāsiba*) to their conclusions (p. 174.1–4)
2. Bryson's proof of the squaring of the circle and Avicenna's interpretation (pp. 174.5–177.2)
3. Middle terms required for appropriateness: per se predicates and accidents; the lower science gives the that, while the higher science gives the why (pp. 177.3–177.14)
4. Rejection of an opinion (pp. 177.15–178.6)
5. Explanation across sciences: why and that (pp. 178.7–180.17)
   - 5.1 Most common case: examples from natural philosophy and metaphysics (homogeneous character of the first motions; imperfect sphericity of the earth) (pp. 178.10–179.13)
   - 5.2 Least common case; principles and questions (how metaphysics borrows principles from lower sciences without circularity) (pp. 179.14–180.17)
6. The four causes (formal, material, efficient, and final) as middle terms (pp. 180.17–183.8)

Chapter II, 10. Principles of demonstration (compare *Burhān* II, 10, pp. 184.1–187.5 with *An. Post.* A10; and *Burhān* II, 10, pp. 187.6–189.14 with *An. Post.* A11, 77a5–25)

1. Principles that are self-evident and principles that require proof (p. 184.1–6)
2. Subject of a science, assertion and conception (p. 184.7–15)
3. Per se predicates (p. 185.1–6)
4. Principles (*min ṭarīq al-haliyya*, with regard to *taṣdīq*; *mawḍūʿat al-anniyya fī nafsihā*) (p. 185.7–11)
5. Summary: subjects, accidents, and principles (pp. 185.12–186.4)
6. How definition differs from both hypothesis and postulate (pp. 186.5–187.5)
7. Rejection of separate Forms as subjects; the discussion pertains to metaphysics, not to logic (pp. 187.6–188.14)
8. What is required in demonstration is the universal; minor terms as *aʿyān al-mawǧūdāt* (pp. 188.15–189.14)

THIRD TREATISE

Chapter III, 1. Appropriate and non-appropriate principles (*mabādiʾ*) and questions (*masāʾil*) (compare *Burhān* III, 1, pp. 190.1–192.17 with *An. Post.* A11, 77a26–35, and *Burhān* III, 1, pp. 192.18–195.21 with *An. Post.* A12, 77a36–b15. On the three elements of a science, cf. also *Burhān* II, 6)

1. On the law of the excluded middle: three uses (pp. 190.1–191.21)
   - 1.1 First use (pp. 190.4–191.5)
   - 1.2 Second use (p. 191.6–8)
   - 1.3 Third use (p. 191.9–15)

2. Common, necessarily accepted principles are shared by the sciences (p. 191.16–21)
3. Dialectic is not limited in subjects, questions, and principles (p. 192.1–17) (with a reference to *Topics* and *Rhetoric*)
4. Scientific question and testing question (*masʾala ʿilmiyya* and *masʾala imtiḥāniyya*) (pp. 192.18–193.15)
5. Interpretation of a problematic passage (on *An. Post.* A12, 77b4–9) (pp. 193.16–194.10)
6. The principles of all sciences are proved in metaphysics (p. 194.11–14) (cf. *Burhān* II, 7; *Ilāhiyyāt* I, 1–3 and 8)
7. The geometer is concerned with geometrical questions (on *An. Post.* A12, 77b10–15) (pp. 194.15–195.21)

Chapter III, 2. Mathematical sciences and dialectic; error; analysis and synthesis (compare with *An. Post.* A12, 77b16–78a21)

1. Ignorance that is contrary to scientific knowledge (p. 196.1–6)
   - 1.1 First type of error (homonymy, example of circle) (pp. 196.7–198.6)
   - 1.2 Second type of error (syllogistic figures; Caeneus) (p. 198.7–17)
2. Difference between dialectic and mathematics: questions and terms (pp. 198.18–201.12)
   - 2.1 Analysis (*taḥlīl*) (p. 199.10–15)
   - 2.2 Synthesis (*tarkīb*) (pp. 199.15–200.4)
   - 2.3 Increase (*tazayyud*); two interpretations of a difficult passage (pp. 200.5–201.12)

Chapter III, 3. Why-demonstration and that-demonstration (compare with *An. Post.* A13; cf. also *Burhān* I, 7 and 10, *Burhān* II, 9, *Burhān* IV, 5, and 8–9)

1. Distinction between why-demonstration and that-demonstration in one science (pp. 202.1–205.18)
   - 1.1 First case (non-immediate premises; remote and proximate cause) (p. 202.3–9)
   - 1.2 Second case (immediate premises, convertible terms, cause and effect; discussion of *dalīl*; two effects) (pp. 202.9–204.14)
   - 1.3 Two interpretations of a difficult passage ("the case in which the middle is placed outside") based on Philoponus and Alexander of Aphrodisias, possibly mediated through a gloss on Abū Bišr Mattā's translation in BNF Ar. 2346) (pp. 204.15–205.18)
2. Distinction between why-demonstration and that-demonstration in two sciences (pp. 205.19–209.17)
   - 2.1 Most common case: one science falls under the other (full subordination) (pp. 205.19–207.8)
   - 2.2 Part of one science falls under the other (optics and astronomy) (p. 207.9–17)
   - 2.3 A question of one science falls under the other (example of circular wound) (p. 208.1–10)
   - 2.4 An interpretive problem, with references to the commentaries (*šurūḥ*, *tafāsīr*) (pp. 208.11–209.17)

Chapter III, 4. Superiority of the first figure; ignorance (compare with *An. Post.* A14–17)

1. Three arguments for the superiority of the first figure (the three arguments correspond to Aristotle's, but the third one refers in particular to its perfection and self-evident character) (pp. 210.1–212.2) (*An. Post.* A14)
2. A doubt concerning Aristotle's claim that deductions in the second and third figure can be analyzed into immediate premises in the first figure (with Avicenna's comment that his "copy" of the text allegedly replaces predicate with subject: *wa-ammā lafẓ al-kitāb fī nusḫatinā*, p. 213.9) (pp. 212.3–214.3) (*An. Post.* A15)
3. Kinds of ignorance (pp. 214.4–219.16)
   - 3.1 Simple ignorance (p. 214.4–13) (*An. Post.* A16)
   - 3.2 Compound ignorance (pp. 214.14–219.16)
     - 3.2.1 How compound ignorance comes about (pp. 214.14–215.14)
     - 3.2.2 Through syllogistic deduction, with immediate premises (pp. 215.15–217.14)
       - 3.2.2.1 a-proposition, first figure (pp. 215.15–216.10)
       - 3.2.2.2 e-proposition, first figure (pp. 216.11–216.18)
       - 3.2.2.3 e-proposition, second figure (pp. 216.19–217.14)
     - 3.2.3 Through syllogistic deduction, with non-immediate premises (pp. 217.15–219.16) (*An. Post.* A17)
       - 3.2.3.1 First figure (pp. 217.15–218.10)
       - 3.2.3.2 Second figure (pp. 218.11–219.16)

Chapter III, 5. Acquisition of intelligibles; elements of a science; distinction between essential and accidental (compare *Burhān* III, 5, pp. 220.1–224.11 with *An. Post.* A18; and *Burhān* III, 5, pp. 224.12–227.10 with *An. Post.* A19, 81b10–30; cf. also *Burhān* III, 8 and IV, 10 on *An. Post.* A31 and B19, respectively)

1. Acquisition of intelligibles, with a note on the relation between logic and psychology and an excursus on induction and experience (pp. 220.1–224.11) (*An. Post.* A18)
2. Three terms in a syllogistic deduction; chains of predication (pp. 224.12–225.10) (*An. Post.* A19, 88b10–30)
3. Eight ways predicates are said to be per se (*bi-ḏ-ḏāt*) or by accident (*bi-l-ʿaraḍ*) (preliminary to the discussion of the finiteness of demonstrative chains) (pp. 225.11–227.10)

Chapter III, 6. Finiteness of demonstrative chains (compare *Burhān* III, 6, pp. 228.1–229.17 with *An. Post.* A19, 81b30–82a21; and *Burhān* III, 6, pp. 229.18–237.5 with *An. Post.* A20–23)

1. Upward and downward chains (pp. 228.1–229.17) (*An. Post.* A19)
2. Middle terms (pp. 229.18–230.15) (*An. Post.* A20) (*ḥaml ḥaqīqī* and *ḥaml ʿaraḍī*)
3. Negative chains (pp. 230.16–231.7) (*An. Post.* A21)
4. Finiteness of affirmative upward and downward chains (pp. 231.8–235.14) (*An. Post.* A22)
   - 4.1 Distinction between essential predication (*ḥaml bi-l-ḥaqīqa*) and accidental predication (*ḥaml bi-l-ʿaraḍ*) (pp. 231.12–232.19)
   - 4.2 Rejection of Platonic forms (p. 233.1–2) (cf. *Burhān* II, 10)
   - 4.3 Argument for finiteness (pp. 233.3–234.11)

   4.4 Another argument for the finiteness of per se 1 and per se 2 predicates (reductio), objection and reply (pp. 234.12–235.10)
   4.5 Summary of the main results (p. 235.11–14)
5. Predication in virtue of something common (pp. 235.15–237.5) (*An. Post.* A23)
   5.1 An exegetical question: Avicenna agrees with Philoponus on the meaning of *epi dē tou tritou tropou* at *An. Post.* A23, 85a10–11: "third" refers not to a third way to prove an e-conclusion but to a mood of the third figure (pp. 236.15–237.5)
6. Summary with observations on the need to prove in another discipline that demonstration and definition are possible (not in logic but in metaphysics) (p. 237.6–13)

Chapter III, 7 Universal, affirmative, and direct demonstration; exactness (compare with *An. Post.* A24–27)
1. Universal demonstration (pp. 238.1–242.9) (*An. Post.* A24)
   1.1 Two inadequate arguments (pp. 238.4–239.7)
   1.2 Arguments in favor of universal demonstration
      1.2.1 As such (p. 239.8–13)
      1.2.2 Single account (pp. 239.14–240.10)
      1.2.3 That and why (p. 240.11–14)
      1.2.4 Regress of causes stops at what is more universal (pp. 240.15–241.9)
      1.2.5 Particulars are infinite and unlimited (p. 241.10–14)
      1.2.6 Know "this" and something else (p. 241.15–18)
      1.2.7 Nearer to the principles (p. 241.19–21)
   1.3 Summary (p. 242.1–9)
2. Affirmative demonstration (pp. 242.10–244.13) (*An. Post.* A25)
3. Direct demonstration (pp. 244.14–245.17) (*An. Post.* A26)
4. Exactness (pp. 245.18–246.14) (*An. Post.* A27)
   4.1 That and why (p. 245.18–19)
   4.2 Abstraction from matter (p. 246.1–4)
   4.3 Simple subject with addition (p. 246.5–11)
   4.4 Avicenna's approach to Aristotle's text: a methodological remark (p. 246.12–14)

Chapter III, 8. Difference and agreement of the sciences in principles and subjects (compare with *An. Post.* A28–32; cf. *Burhān* II, 6–7 and *Burhān* III, 5)
1. Shared subject, per se accidents, shared principles (of the subject; or its parts; or its species) (p. 247.1–9) (*An. Post.* A28; cf. *Burhān* II, 6–7)
2. Different demonstrations of the same thing (pp. 247.10–248.4) (*An. Post.* A29; cf. *Burhān* I, 7, *Burhān* II, 8–9, and *Burhān* III, 3)
3. The modality of demonstrative premises and conclusions, with a reference to the commentators (*mufassirūna*) (pp. 248.5–249.10) (*An. Post.* A30)
4. Sense perception is not a sufficient condition for demonstration (pp. 249.11–250.20) (*An. Post.* A31; references to *išrāq* and *fayḍ ilāhī*, *tağriba*, and the previous discussion in *Burhān* I, 9 and III, 5)
5. Refutation of the view that the principles of everything are the same (pp. 251.1–255.9) (*An. Post.* A32)

Chapter III, 9. Scientific knowledge (*ʿilm*), opinion (*ẓann*), understanding (*fahm*), intuition (*ḥads*), acumen (*ḏakāʾ*), discipline (*ṣināʿa*), wisdom (*ḥikma*) (compare with *An. Post.* A33–34; cf. also *Burhān* I, 1 on certainty and belief)

1. Scientific knowledge by assertion and opinion (pp. 256.1–259.11) (*An. Post.* A33)
   1.1 Certainty (*yaqīn*), belief (*iʿtiqād*), and opinion (*ẓann*) (pp. 256.1–257.9)
   1.2 Subject of scientific knowledge is what is necessary (*ḍarūrī*): either (i) by perpetuity (*bi-d-dawām*) or (ii) with a condition (*bi-š-šarṭ*) (p. 257.10–14)
   1.3 Three characterizations of opinion (pp. 257.15–258.6)
   1.4 Difference between scientific knowledge and opinion (pp. 258.7–259.11)
2. Definitions of *ḏihn*, *ṣināʿa*, *fahm*, *ḥikma*, *ḏakāʾ*, *ḥads* (pp. 259.12–260.4) (*An. Post.* A34)

FOURTH TREATISE

Chapter IV, 1. Scientific inquiries, middle terms, and preliminary discussion of the relation between definition and demonstration (compare with *An. Post.* B1–3)

1. Types of scientific inquiries (*maṭālib*) (pp. 261.1–262.18) (*An. Post.* B1; cf. *Burhān* I, 5)
2. Role of the middle term (pp. 263.1–267.19) (*An. Post.* B2)
   2.1 Preliminary analysis: (i) if and (ii) what (pp. 263.1–20)
   2.2 Discussion of the requirement that the middle term of a demonstration be (only) a definition of the major term (pp. 264.1–267.3) (possibly, an attack against ps.-Philoponus)
   2.3 Conclusion of the analysis of the relation between (iii) that and (iv) why (pp. 267.3–267.19)
3. Differences between definition and demonstration (pp. 267.20–269.17) (*An. Post.* B3)

Chapter IV, 2. Definition is not acquired through demonstration or division (compare with *An. Post.* B4–5)

1. Definition is not acquired through demonstration (on pain of circularity or infinite regress) (pp. 270.1–274.15) (*An. Post.* B4)
2. Definition is not acquired through division (pp. 274.16–278.17) (*An. Post.* B5 and B6, 92a6–19)
   2.1 Analogy with circular induction (p. 275.2–11)
   2.2 Three problems with division (pp. 275.12–276.14)
   2.3 Two purported deductions of division and their rejection (pp. 276.15–278.17)
3. Conclusion (p. 278.18–19)

Chapter IV, 3. Division, induction; types of definition; what and why (compare with *An. Post.* B6, 92a20–30 and B7–9)

1. Hypothetical deduction (pp. 279.1–280.18) (*An. Post.* B6, 92a20–30)
   1.1 Four objections (with a reference to Avicenna's *Ǧadal*) (pp. 279.6–280.18)
2. Two arguments against induction as a method for establishing a definition (pp. 280.19–281.12) (*An. Post.* B7)
   2.1 First argument (two impossibilities) (p. 281.3–8)
   2.2 Second argument (circularity) (p. 281.8–12)

3. Types of definitions with various remarks (pp. 281.13–284.2) (*An. Post.* B7)
   3.1 Definition and existence (pp. 281.13–282.11)
   3.2 Definition does not prove existence; "existent" (*mawǧūd*) is not a genus or a differentia of any quiddity; existence as an implicate (*wuǧūd maḥmūl lāzim*) (pp. 282.12–283.5)
   3.3 Method of real definition (*ma'ẖaḏ al-ḥadd al-ḥaqīqī*) (pp. 283.6–283.17)
   3.4 Structural similarity of induction and syllogism (induction only establishes *haliyya basīṭa* or *haliyya murakkaba*, but cannot establish definition) (p. 284.1–2)
4. Why and what; complete definition (pp. 284.4–286.9) (*An. Post.* B8)
5. Causal and noncausal definition (pp. 286.10–287.20) (*An. Post.* B9; cf. *Burhān* I, 8–9)

Chapter IV, 4. What definition and demonstration have in common; preliminary discussion of the four causes (compare *Burhān* IV, 4, pp. 288.1–292.8 with *An. Post.* B10; and *Burhān* IV, 4, pp. 294.9–295.16 with *An. Post.* B11; cf. also *An. Post.* A8 and *Burhān* II, 8 on the terminology of principle and conclusion of a demonstration)

1. Real (*ḥaqīqī)* and figurative (or nominal) (*maǧāzī*) definition; causal and noncausal definition; complete and incomplete definition (principle and conclusion of a demonstration) (pp. 288.1–294.8) (*An Post.* B10)
   1.1 Nominal definition (pp. 288.5–289.15)
   1.2 Causal definition; three elements: (i) *definiendum* (*maḥdūd*), (ii) definition *definiens (ḥadd*), (iii) perfection of the definition (*kamāl li-l-ḥadd*); two demonstrations; definition as principle of a demonstration (pp. 289.16–290.16)
   1.3 Rejection of the view that definition in the sense of conclusion of a demonstration always expresses the matter (p. 291.1–10)
   1.4 Definition of what does not have a cause; account of the problematic classification in Aristotle (pp. 291.11–292.8)
   1.5 Epistemic states and syllogistic figures (pp. 292.9–294.8)
2. The four causes: introduction (pp. 294.9–295.16) (*An. Post.* B11)

Chapter IV, 5. The four causes and how they are included in the structure of demonstration and definition (compare with *An. Post.* B11–12; cf. *Burhān* IV, 4)

1. The four types of causes (efficient principle, elemental or material principle, formal principle, perfecting principle), with five criteria (pp. 296.1–303.5) (*An. Post.* B11)
   1.1 Five distinctions (pp. 296.1–297.9)
      1.1.1 Remote and proximate (p. 296.1–8)
      1.1.2 Essential and accidental (pp. 296.9–297.4)
      1.1.3 Potential and actual (p. 297.5–7)
      1.1.4 Proper and common (p. 297.8)
      1.1.5 Particular and universal (p. 297.8–9)
   1.2 Discussion of their properties (pp. 297.10–299.7)
   1.3 Complete demonstration and complete definition (p. 299.8–16)
   1.4 Notes, objections and replies on how the four causes are assumed in definitions or descriptions; distinction between causes of the quiddity and causes of existence;

how many causes may be included in the quiddity, depending on the type of entity (pp. 299.17–303.5)

2. Time, correlatives, cyclical events, and demonstration (pp. 303.6–305.20) (*An. Post.* B12)

Chapter IV, 6. Acquisition of definition through the method of composition (*tarkīb*); first account (compare with *An. Post.* B13, 96a20–b25)

1. First variant: definition of a lowest species, definition of an intermediate genus (pp. 306.1–308.3)
2. Second variant: definition of an intermediate genus (p. 308.4–21)
3. Avicenna's interpretation of the difficulties at *An. Post.* B13, 96b15–25 (reference to the "commentators" and the "translator"); account of containment (*taḍammun*); definitional tree of human and horse; rules to avoid redundancy and repetition (pp. 309.1–311.3)
4. Conclusion (p. 311.4–9)

Chapter IV, 7. How the method of division is useful in defining; second account of the method of composition (compare with *An. Post.* B13, 96b25–97b39)

1. Division (*qisma*) (pp. 312.1–315.17)
   - 1.1 Three ways in which division is useful for defining (*taḥdīd*) (pp. 312.3–313.7)
     - 1.1.1 First way (p. 312.3–9)
     - 1.1.2 Second way (p. 312.10–14)
     - 1.1.3 Third way (p. 313.1–7)
   - 1.2 A rule concerning the use of the primary divisions of a genus; lengthwise and crosswise division; overlapping and exhaustive essential differentiae (with a reference to Avicenna's *Madḫal*) (p. 313.8–22)
   - 1.3 Comments on *An. Post.* B13, 97a7–24 (with two refutations, at p. 314.11–14 and at pp. 314.15–315.4)
   - 1.4 Summary of the three criteria (division and definition) (p. 315.5–19) (*An. Post.* B13, 97a24–b5)
     - 1.4.1 Essentiality (with a reference to the commonplaces of genus, differentia, and accident) (p. 315.6–10)
     - 1.4.2 Order (p. 315.11–14)
     - 1.4.3 Completeness (p. 315.15–17)
2. Method of composition (by abstraction); its relation (and overall superiority) to the method of division (pp. 315.18–317.20)

Chapter IV, 8. Division of a whole into its parts; middle terms as convertible and non-convertible causes (compare with *An. Post.* B14–16)

1. Division and hierarchies of concepts (pp. 318.1–319.8) (*An. Post.* B14)
2. Multiple questions unified by one and the same causal middle term (pp. 319.9–320.7) (*An. Post.* B15)
3. Order of explanation and middle terms; why and that; the four causes as middle terms; application arguments; types of explanation; *ʿilla muṭābiqa* (pp. 320.8–324.7) (*An. Post.* B16)

Chapter IV, 9. Conditions for the use of causes as middle terms (*tawsīṭ al-ʿilal*) (compare with *An. Post.* B17–18)

1. The relative extension of causes and effects; questions involving multiple middle terms and solution of various doubts; causality and the types of demonstration (pp. 325.1–329.6) (*An. Post.* B17)
2. Proximate and remote causes (p. 329.7–13) (*An. Post.* B18; cf. *Burhān* I, 10, II, 9, and III, 3)

Chapter IV, 10. Abstraction, concept formation, and principles (compare with *An. Post.* B19; cf. *An. Post.* A18, A31; *Burhān* III, 5 and 8)

1. Status of scientific principles: a notorious dilemma; rejection of innatism, description of cognitive progression: from sense perception to retention of images (pp. 330.1–331.4)
2. Abstraction and the role of the intellect (p. 331.5–10)
3. Account of our ignorance of principles, experience, divine emanation (pp. 331.11–332.4)
4. Example of rout in a battle (p. 332.5–15)
5. Intellect and the other faculties (pp. 332.16–333.5)
6. Transition to the *Topics*; usefulness of demonstrative commonplaces (*mawāḍiʿ burhāniyya*) (p. 333.6–7)

APPENDIX D

English-Arabic Glossary

absolute	(*muṭlaqa*); said of a proposition
absolutely	(*ʿalā l-iṭlāq*); cf. without qualification
abstracted	(*muǧarrad*)
abstraction	(*taǧrīd*)
absurd	(*muḥāl*)
accident	(*ʿaraḍ*, pl. *aʿrāḍ*); cf. substance
accidental	(*ʿaraḍī*); said of what is not per se 1
accidental attribute	(*ʿāriḍ,* pl. *ʿawāriḍ*); cf. essential
accidentally	(*bi-l-ʿaraḍ*); cf. essentially
acquisition	(*iktisāb*); said of knowledge, both assertion and conception
actually	(*bi-l-fiʿl*)
adopted on authority	(*maqbūlāt*); said of premises
affirmation	(*iṯbāt*); in the sense of establishing the existence of something; cf. disproving
affirmation	(*ṯubūt*); cf. negation
affirmative	(*mūǧiba*); said of a proposition
agent	(*fāʿil*)
aggregate	(*maǧmūʿ*)
aid	(*taʿāwun*); said of the contribution of a science to another science
air	(*hawāʾ*)

analysis	(*taḥlīl*)
angle	(*zāwiya*)
animal	(*ḥayawān*)
answer	(*ǧawāb*)
antecedent	(*matbūʿ*, followed by *tābiʿ*; *muqaddam*, followed by *tālin*; *šarṭ*, followed by *ǧāzim*); cf. consequent
apprehension	(*idrāk*)
appropriate	(*munāsib*); said of a demonstrative term or premise
argument	(*huǧǧa*); cf. proof and explanation
arithmetic	(*ʿilm al-ḥisāb*)
as long as	(*mā dāma*)
ask, one may	(*li-sāʾil an yasʾala*); standard way to introduce a question; cf. say, one may
asserted	(*muṣaddaq bihī*)
assertion	(*taṣdīq*)
assumed	(*muqaddar*); opposed to verified as existent (*muḥaqqaq*)
assumption	(*taqdīr*)
astronomy	(*ʿilm an-nuǧūm*; *ʿilm al-hayʾa*)
belief	(*iʿtiqād*)
belonging to	(*wuǧūd li* or *fī*); said of the relation of an attribute to its subject (attribute ascription)
belonging to	(*mawǧūd li* or *fī*)
better known	(*aʿraf*); to us (*ʿindanā*) and in nature (*ʿinda ṭ-ṭabīʿa*)
blood	(*dam*)
body	(*ǧism*) preferred for sublunar bodies; (*ǧirm*) preferred for celestial bodies
capable of laughter	(*ḍaḥḥāk*)
capable of wonder	(*mutaʿaǧǧib*)
categorical	(*ḥamliyya*); said of a proposition; cf. hypothetical
causality	(*ʿilliyya*)
cause	(*ʿilla, sabab*)
caution	(*iḥtiyāṭ*)
ceasing	(*zawāl*)
certain	(*yaqīn*, *yaqīnī*)
certainty	(*yaqīn*)
certainty, premises of	(*yaqīniyyāt*)

chain	(*silsila*)
characteristic	(*ṣifa*)
characterization	(*waṣf*); cf. description
circle	(*dawr*); said of an argument form
circle	(*dā'ira*); said of a geometrical figure
circular proof	(*bayān ad-dawr*)
circumstances	(*awḍā'*); said of one of the domains of quantification of hypothetical propositions
co-implication	(*talāzum*)
co-implicative	(*mutalāzim*)
coincident	(*muwāfiqa*); said of a proposition; cf. opposing
coincidental	(*ittifāqī*); said of a kind of conditional proposition; cf. implicative
commensurable	(*muštarak*); said of magnitude; cf. incommensurable
commentaries	(*šurūḥ, tafāsīr*)
commentators	(*mufassirūna*)
common	(*'āmm*)
common accident	(*'araḍ 'āmm*)
common people	(*ğumhūr*)
complete	(*tāmm*); said of definition and description; cf. incomplete
completeness	(*tamām*)
composition	(*tarkīb*); said of a method for the acquisition of definition; cf. division
compound	(*murakkab*); said of an expression, a notion, or a deduction; cf. single and simple
conceded	(*musallamāt*); said of premises
conceived	(*muṣawwar*)
concept	(*mafhūm*); cf. understanding
conception	(*taṣawwur*); cf. assertion
conceptualization	(*taṣawwur*)
concession	(*taslīm*)
conclusion	(*natīğa*)
conclusion of a demonstration	(*natīğat al-burhān*); said of one of the types of definition; cf. principle of a demonstration
concomitant	(*lāḥiq*); cf. implicate
condition	(*šarṭ*)
conditional	(*muttaṣila*); said of a hypothetical conditional proposition; cf. disjunctive

connective	(*iqtirānī*); said of a deduction; cf. repetitive
consequent	(*tābiʿ*, following *matbūʿ*; *tālin*, following *muqaddam*; *ğāzim*, following *šarṭ*); cf. antecedent
constituent	(*muqawwim*); cf. per se
constitutive of	(*muqawwim li*)
contact	(*ittiṣāl*)
containment	(*taḍammun*); said of the signification of a notion in relation to another notion contained in its quiddity; cf. correspondence and implication
contradiction	(*tanāquḍ*)
contradictory	(*naqīḍ*)
contraposition	(*ʿaks an-naqīḍ*)
copy	(*nusḫa*)
corporeal	(*ğismī*)
correlation	(*taḍāyuf*)
correspondence	(*muṭābaqa*); said of the signification of a notion in relation to another notion equal to its quiddity; cf. containment and implication
correspondent	(*muṭābiq*); said of a true proposition and of a complete definition
Creator	(*al-bāriʾ*)
deduction	(*qiyās*)
deductive	(*qiyāsī*)
defining	(*taḥdīd*)
definition	(*ḥadd*); cf. description
definition	(*taʿrīf*); general term for real definition and description
demonstration	(*burhān*)
demonstration, absolute	(*burhān muṭlaq*); cf. that-demonstration (*burhān anna*) and why-demonstration (*burhān limā*)
demonstrative	(*burhānī*)
demonstrative commonplace	(*mawḍiʿ burhānī*)
described by	(*mawṣūf bi*); said of a subject (standard expression for the descriptional reading)
description	(*rasm*); cf. definition
destination	(*maʿād*); cf. origin
determination	(*taḥṣīl*)
dialectic	(*ğadal*)
dialectical	(*ğadalī*)

difference	(*farq*)
differentia	(*faṣl*, pl. *fuṣūl*)
differentiating	(*mufaṣṣil*); said of an expression (*qawl*)
difficulty	(*šubha*)
discipline	(*ṣināʿa*); cf. science
discrimination	(*tamyīz*)
disjunctive	(*munfaṣila*); said of a hypothetical disjunctive proposition; cf. conditional
disproving	(*ibṭāl*); cf. affirmation
distinct	(*mubāyin*); said of the subject of a science
division	(*taqsīm*)
division	(*qisma*); said of a method for the acquisition of definition; cf. composition
doubt	(*šakk*)
earth	(*arḍ*)
earthquake	(*zalzala*)
effect	(*maʿlūl*); cf. cause
efficient cause	(*ʿilla fāʿila*)
emanation	(*fayḍ*)
end	(*ġāya*); cf. final cause
endoxic	(*mašhūrāt*); said of premises; limited (*maḥdūda*) and absolute (*muṭlaqa*)
equal	(*musāwin*)
equivalent	(*mutasāwin*)
equivocal	(*muštarak*); said of name
error	(*ġalaṭ*, *ḫaṭāʾ*)
essence	(*ḥaqīqa*, *ḏāt*, *ğawhar*); cf. quiddity
essential	(*ḏātī*); said of per se 1 and of per se 2; cf. per se accidents
essentially	(*bi-ḏ-ḏāt*); cf. acccidentally
essentially	(*fī ṭarīq mā huwa*); said of per se 1 predicates
estimation	(*wahm*)
estimative	(*wahmiyyāt*); said of premises
even	(*zawğ*); cf. odd
even-times-even	(*zawğ az-zawğ*)
evident	(*bayyin*); cf. self-evident
exactness	(*istiqṣāʾ*)

example	(*tamṯīl*)
existence	(*wuğūd*)
existential	(*wuğūdī*)
experience	(*tağriba*)
experience, premises based on	(*al-muğarrabāt)*
explanation	(*bayān*)
explanatory statement	(*qawl šāriḥ*)
expression	(*lafẓ*, *qawl*)
extensionally	(*bi-l-ʿaks*)
external	(*ḫāriğī*)
extrinsic	(*ḫāriğī*)
faculty	(*quwwa*)
fallacy	(*muġālaṭa*)
false	(*bāṭil, kāḏib*)
figurative	(*mağāzī*); cf. real
figure	(*šakl*)
final cause	(*ʿilla tamāmiyya*); cf. end
fire	(*nār*)
first	(*awwal*); said of a principle; said of a syllogistic figure (*šakl awwal*)
First Teacher	(*al-muʿallim al-awwal*), that is to say, Aristotle
First Teaching	(*at-taʿlīm al-awwal*); said of the works of Aristotle, especially *An. Post.* in the *Burhān*, to introduce a literal quotation or the paraphrase of a passage
foreign	(*ġarībī*); cf. per se
form	(*ṣūra*)
formal cause	(*ʿilla ṣūriyya*)
frequent testimony	(*kaṯrat aš-šahādāt*); cf. premises based on sequential testimony
generic	(*ğinsī*); cf. specific
genus	(*ğins*)
geometer	(*muhandis*)
geometry	(*handasa*)
goal	(*ġaraḍ*)
group	(*qawm*); said of the supporter(s) of a view

happiness	(*saʿāda*)
here	(*hā hunā*); frequently used in contrast with "there" (*hunāka*) to distinguish the use of an expression or concept in different contexts
higher	(*aʿlā*); said of a superordinate science; cf. lower
hitting upon	(*iṣāb*)
homonymy	(*ištirāk al-ism*)
human	(*insān*)
hunting out	(*iqtināṣ*); said of the process by means of which definitions are sought
hypothesis	(*aṣl mawḍūʿ*)
hypothetical	(*šarṭiyya, waḍʿiyya*); said of a proposition; cf. categorical
if	(*hal*); said of scientific inquiry; simple if (*hal basīṭ*) for existence without qualification and compound if (*hal murakkab*) for existence in a state (predication)
if-ness	(*haliyya*); said both in the existential and in the predicative sense as an abstract noun to indicate whether (i) something exists, whether (ii) something is something, or whether (iii) something is the case (in the sense of being true); simple if-ness (*haliyya basīṭa*) and compound if-ness (*haliyya murakkaba*)
if-question	(*maṭlab al-hal*)
ignorance	(*ǧahl*); simple (*basīṭ*) and compound (*murakkab*)
illumination	(*išrāq*)
image-eliciting	(*muḫayyilāt*); said of premises
imagery	(*ḫayāl*); said of one of the internal senses
immediate premise	(*muqaddima lā wasaṭa lahā* or *muqaddama ġayr ḏāt wasaṭ*)
immediately	(*bi-lā wāsiṭa*)
implicant	(*malzūm*)
implicate	(*lāzim*, pl. *lawāzim*)
implication	(*luzūm, iltizām*); cf. containment and correspondence
implicative	(*luzūmiyya*); said of a conditional hypothetical proposition in which the relation between the antecedent and the consequent is one of implication rather than mere coincidence; cf. coincidental
implicitly deductive	(*qaḍāyā qiyāsātuhā maʿahā*); said of premises
impossibility	(*imtināʿ*)
impossible	(*muḥāl*, *ḫulf*)
included in	(*dāḫil fī*)

incommensurable	(*mubāyin*); cf. commensurable
individual	(*šaḫṣ*, pl. *ašḫāṣ*)
induction	(*istiqrāʾ*); complete induction (*istiqrāʾ tāmm*), incomplete induction (*istiqrāʾ nāqiṣ*), circular induction (*istiqrāʾ dāʾir*), and fallacious induction (*istiqrāʾ muġāliṭī*)
infinite regress	(*ḏahaba ilā mā lā nihāyata lahū*)
informative	(*ḫabarī*); said of propositional composition
inseparable accident	(*ʿaraḍ ġayr mufāriq*); cf. concomitant and implicate
instrument	(*āla*)
intellect	(*ʿaql*)
intellectual	(*ʿaqlī*)
intelligible form	(*ṣūra maʿqūla*)
interposition	(*tawassuṭ*)
interpretation	(*taʾwīl*)
intuition	(*ḥads*); said of the ability to grasp middle terms
intuitive	(*ḥadsiyyāt*); said of premises
intuitive	(*badīhī*)
investigation	(*baḥṯ*, *naẓar*)
judgment	(*ḥukm*)
justice	(*ʿadl*)
knowledge	(*maʿrifa*)
known	(*maʿlūm*)
language	(*luġa*)
light	(*nūr*)
line	(*ḫaṭṭ*)
literalists	(*ẓāhiriyyūna*); said of a group of logicians; opposed to validating scholars (*muḥaṣṣilūna*) and verifying scholars (*muḥaqqiqūna*)
logic	(*manṭiq*)
logicians	(*manṭiqiyyūna*)
lower	(*asfal*); said of a subordinate science; cf. higher
magnitude, extended	(*miqdār*)
major premise	(*muqaddama kubrā*)
major term	(*ḥadd akbar*, occasionally *ḥadd aʿẓam*)
material cause	(*ʿilla māddiyya, ʿilla unṣūriyya*)

matter	(*mādda*); cf. proposition
matter	(*mādda*); cf. form
matter, prime	(*hayūlā*)
meaning	(*maʿnan*)
medicine	(*ṭibb*)
memory	(*ḏikra*)
middle term	(*ḥadd awsaṭ*)
minor premise	(*muqaddama ṣuġrā*)
minor term	(*ḥadd aṣġar*)
mode	(*ǧiha*)
mood	(*ḍarb*)
more general	(*aʿamm*); said of the extension of a term and of the subject of a science
more specific	(*aḫaṣṣ*); said of the extension of a term and of the subject of a science
motion	(*taḥarruk*)
moving	(*mutaḥarrik*)
moving voluntarily	(*mutaḥarrik bi-l-irāda*)
music	(*mūsīqā*, *taʾlīf al-luḥūn*)
name	(*ism*)
natural	(*ṭabīʿī*)
natural operation of the intellect	(*fiṭrat al-ʿaql*)
naturally	(*bi-ṭ-ṭabʿ*)
nature	(*ṭabīʿa*)
necessarily accepted	(*wāǧib qubūluhā*); said of premises
necessary	(*ḍarūrī*, *wāǧib*)
necessity	(*ḍarūra, wuǧūb*)
negative	(*sāliba*); said of proposition
nexus	(*nisba*); said of the relation or connection between subject and attribute
noise	(*ṣawt*)
notion	(*maʿnan*)
null	(*bāṭil*)
number	(*ʿadad*)

observation, based on	(*mušāhadāt*); said of premises
odd	(*fard*); cf. even
opinion	(*ẓann*); cf. supposition
opposing	(*muqābila*); cf. coincident
order	(*tartīb*)
origin	(*mabdaʾ*); cf. destination
overlap	(*mudāḫala*); said of one of the relations between two sciences
part	(*ǧuzʾ*, pl. *aǧzāʾ*)
particular	(*ǧuzʾī*); cf. universal
per se	(*ḏātī*, *li-ḏātihī*)
perpetuity	(*dawām*)
persuasion	(*iqnāʿ*)
persuasion, based on	(*iqnāʿī*); said of a type of assertion associated with rhetorical premises
phrenitis	(*sarsām* or *sirsām*)
poetical	(*šiʿrī*); said of a type of deduction that does not produce an assertion
poetics	(*šiʿr*)
point	(*nuqṭa*)
posit	(*waḍʿ*, pl. *awḍāʿ*); cf. circumstance
positing	(*waḍʿ*)
possibility	(*imkān*)
posteriority	(*taʾaḫḫur*); cf. priority
predicate	(*al-maḥkūm bihī*); said of that with which judgment is passed
predicate	(*maḥmūl*)
predication	(*ḥaml*)
premise	(*muqaddama*)
primary	(*awwalī*); said of an attribute
primary	(*awwaliyyāt*); said of premises
principle	(*mabdaʾ*, pl. *mabādiʾ*)
principle of a demonstration	(*mabdaʾ al-burhān*); said of one of the types of definition; cf. conclusion of a demonstration
priority	(*taqaddum*); cf. posteriority
product	(*maḍrūb*); said of a quantity multiplied by a quantity
proof	(*bayān*); cf. also argument, demonstration, explanation
proper	(*ḫāṣṣ*); cf. common

proposition	(*qaḍiyya*, pl. *qaḍāyā*)
proprium	(*ḫāṣṣa*)
proximate	(*qarīb*); cf. remote
quality	(*kayfiyya*)
quantity	(*kammiyya*)
question	(*masʾala*, pl. *masāʾil*)
quiddity	(*māhiyya*); cf. essence and reality
rational	(*nāṭiq*)
reality	(*ḥaqīqa*, pl. *ḥaqāʾiq*); cf. essence and quiddity
realization	(*taḥaqquq*)
reductio ad impossibile	(*qiyās al-ḫalf*)
reflection	(*taʾawwul*); cf. thought
reflection	(*inʿikās*); said of optical or acoustic phenomenon
reminder	(*tanbīh*); said of a method used by the intellect to grasp first principles of conception and assertion that should otherwise be self-evident (not a method of proof); cf. induction
reminding	(*tanbīh*)
remote	(*baʿīd*); cf. proximate
removal	(*irtifāʿ*); cf. positing
repetition	(*istiṯnāʾ*)
repetitive	(*istiṯnāʾī*); said of a deduction; cf. connective
resemblance	(*mušābaha*)
return	(*maʿād*); cf. destination
rhetoric	(*ḫiṭāba*)
rhetorical	(*ḫiṭābī*)
rule	(*qānūn*)
said, it is	(*qīla*); often used to introduce a lemma or quotation
said of every, to be	(*maqūl ʿalā l-kull*)
say, one may	(*li-qāʾil an yaqūla*); standard way to introduce an objection; cf. answer (*ǧawāb*); cf. ask, one may
science	(*ʿilm*)
scientific	(*ʿilmī*)
scientific commonplace	(*mawḍiʿ ʿilmī*)
scientific knowledge	(*ʿilm*)

scientific question	(*masʾala ʿilmiyya*)
second	(*ṯānin*); said of a syllogistic figure (*šakl ṯānin*)
self-evident	(*bayyin bi-ḏātihī, bayyin bi-nafsihī*); said of a proposition
sense perception	(*ḥiss*)
sense perception, based on	(*maḥsūsāt*); said of premises
sensitive	(*ḥassās*)
separable accident	(*ʿaraḍ mufāriq*)
sequential testimony	(*tawātur*)
sequential testimony, based on	(*mutawātirāt*); said of premises
sharing	(*šarika*); said of subjects, principles, and questions
signification	(*dalāla*); cf. correspondence, containment, implication
simple	(*basīṭ*)
single	(*mufrad*)
solid	(*muǧassam*)
solution	(*ḥall*); of a doubt (*šakk*) or difficulty (*šubha*)
sought, what is	(*maṭlūb*); said of the predicate of a scientific question or of the question itself
sound	(*ṣawt*)
species	(*nawʿ*)
specific	(*nawʿī*)
specious	(*mušabbiha*); said of premises involved in fallacious reasoning
sphere	(*kura*)
spherical	(*kurī*)
sphericity	(*kuriyya*)
spirit	(*rūḥ*)
star	(*naǧm, kawkab*)
statement	(*qawl*)
statement-making	(*ǧāzim*); said of a proposition
stripping off	(*tanzīʿ*); said of the process of abstraction of accidental attributes
structure	(*taʾlīf*); cf. composition; definitional composition (*taʾlīf ḥaddī*)
subject	(*al-maḥkūm ʿalayhi*); said of that on which judgment is passed
subject	(*mawḍūʿ*)
subjects	(*mawḍūʿāt*); said of the subjects of the sciences

substance	(*ğawhar*); cf. essence and quiddity
supposed	(*maẓnūnāt*); said of premises
supposition	(*ẓann*); cf. opinion
syllogism	(*qiyās*)
synonymous	(*murādif*); said of an expression
systematically ambiguous	(*mušakkak*)
that-demonstration	(*burhān al-anna*)
thing	(*šay'*, *amr*)
thing in itself	(*al-amr fī nafsihī; fī nafs al-amr*)
thinking	(*fikr*)
third	(*ṯāliṯ*); said of a syllogistic figure (*šakl ṯāliṯ*)
thought	(*fikra*)
thunder	(*ra'd*)
time	(*waqt, zamān*)
transfer	(*naql*)
transfer of demonstration	(*naql al-burhān*)
translator	(*mutarğim*)
true	(*ḥaqq*, *ṣādiq*)
under	(*taḥta*); said of a type of subordination
understanding	(*fahm*)
unit	(*waḥda*)
unity	(*ittiḥād*); said of the subject of a science
universal	(*kullī*); cf. particular
universal nature	(*ṭabī'a kulliyya*)
universal premise	(*muqaddama kulliyya*)
univocal	(*mutawāṭi'*)
unknown	(*mağhūl*)
validation	(*taḥṣīl*)
verification	(*taḥqīq*)
verified	(*muḥaqqaq*)
view	(*ray'*)
water	(*mā'*)
what	(*mā*)
what-question	(*maṭlab al-mā*)

whole	(*kull*); cf. part
why	(*limā*)
why-demonstration	(*burhān al-limā*)
why-question	(*maṭlab al-limā*)
wind	(*rīḥ*)
without qualification	(*ʿalā l-iṭlāq*)
wonder	(*taʿağğub*)
world	(*ʿālam*)

REFERENCES

PRIMARY SOURCES

Alexander of Aphrodisias. (1883) *In Aristotelis Analyticorum Priorum librum I commentarium*. Edited by M. Wallies. CAG II, 1. Berlin: Reimer [= *In An. Pr.*].

———. (1891) *In Aristotelis Topicorum libros octo commentaria*. Edited by M. Wallies. CAG II, 2. Berlin: Reimer [= *In Top.*].

Alfarabi. (1949) *Iḥṣāʾ al-ʿulūm*. Edited by ʿU. Amīn. Cairo: Dār al-Fikr al-ʿArabī.

———. (1987) *Kitāb al-Burhān wa-Kitāb šarāʾiṭ al-yaqīn*. Edited by M. Faḫrī. Beirut: Dār al-Mašriq.

Anonymous. (1909) *In Analyticorum Posteriorum alterum commentarium*. Edited by M. Wallies. CAG III, 3. Berlin: Reimer.

Aquinas, Thomas. (1989) *Expositio libri Posteriorum*. Edited by R.-A. Gauthier. *Opera Omnia (editio Leonina)* t. I-2. Roma: Commissio Leonina; Paris: Vrin.

Aristotle. (1948–1952) *Manṭiq Arisṭū*. Edited by ʿA. Badawī. 3 vols. Cairo: Maktabat Dār al-Kutub al-miṣriyya [= *Manṭiq Arisṭū*].

———. (1949) *Aristotle's Prior and Posterior Analytics*. A Revised Text with Introduction and Commentary by Sir D. W. Ross. Oxford: Clarendon Press.

———. (1954) *Analytica Posteriora Gerardo Cremonensi interprete*. Edited by L. Minio-Paluello. Aristoteles Latinus, IV, 3. Bruges: Desclée de Brouwer.

Averroes (Ibn Rušd). (1562a) *Primi Voluminis Pars II Aristotelis Stagiritae Posteriorum Resolutoriorum Libri Duo cum Averrois Cordubensis Magnis Commentariis triplici interpretatione distinctis*. Venetiis: apud Junctas. ff. 1r-668v. Reprint Frankfurt am Main: Minerva, 1962. Vol. I, pt. 2a [= *Expositio Magna*].

———. (1562b) *Quaesita varia in Logica*. In *Aristotelis Opera cum Averrois commentariis*. Venetiis: apud Junctas [= *Quaesita*].

———. (1984) *Šarḥ al-Burhān li-Arisṭū wa-Talḫīṣ al-Burhān*. Edited by ʿA. Badawī. Kuwait: al-Mağlis al-waṭanī li-ṯ-ṯaqāfa wa-l-funūn wa-l-ādāb [= *Šarḥ al-Burhān*].
Avicenna (Ibn Sīnā). (1892) *al-Išārāt wa-t-tanbīhāt*. Edited by J. Forget. Leiden: Brill [= *Išārāt*].
———. (1328/1910) *Manṭiq al-Mašriqiyyīn*. Edited by M. al-Ḫaṭīb and ʿA. al-Qatlā. Cairo: al-Maktaba al-Salafiyya [= *Mašriqiyyūn*].
———. (1947) *Risāla ilā Abī Ğaʿfar Ibn al-Marzabān al-Kiyā*. In Badawī 1947, 119–122.
———. (1951) *Danišnāmah-yi ʿAlāʾī*. Edited by M. Muʿīn. Tehran: Dānišgāh-yi Tihrān [= *Daneshname*].
———. (1952) *aš-Šifāʾ, al-Manṭiq, al-Madḫal*. Edited by Ğ. Š. Qanawatī, M. al-Ḫuḍayrī, A. F. al-Ahwānī, and S. Zāyid. Cairo: al-Maṭbaʿa al-amīriyya [= *Madḫal*].
———. (1954a) *al-Burhān min Kitāb al-Šifāʾ*. Edited by ʿA. Badawī. Cairo: Maktaba al-nahḍa al-miṣriyya.
———. (1954b) *aš-Šifāʾ, al-Manṭiq, al-Ḫiṭāba*. Edited by M. Sālim. Cairo: al-Maṭbaʿa al-amīriyya [= *Ḫiṭāba*].
———. (1955) *Avicenne: Le Livre de Science*. Vol. I, *Logique, Métaphysique*. Traduit par M. Achena et H. Massé. Paris: Les Belles Lettres.
———. (1956a) *aš-Šifāʾ, al-Manṭiq, al-Burhān*. Edited by A. ʿAfīfī. Cairo: al-Maṭbaʿa al-amīriyya [= *Burhān*].
———. (1956b) *aš-Šifāʾ, ar-Riyāḍiyyāt, Ğawāmiʿ ʿilm al-mūsīqā*. Edited by Z. Yusūf. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb [= *Mūsīqā*].
———. (1958) *aš-Šifāʾ, al-Manṭiq, as-Safsaṭa*. Edited by A. F. al-Ahwānī. Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Safsaṭa*].
———. (1959a) *aš-Šifāʾ, al-Manṭiq, al-Maqūlāt*. Edited by Ğ. Š. Qanawatī, M. al-Ḫuḍayrī, A. F. al-Ahwānī, and S. Zāyid. Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Maqūlāt*].
———. (1959b) *Avicenna's De Anima (Arabic Text), Being the Psychological Part of the Kitāb al-Shifāʾ*. Edited by F. Rahman. London: Oxford University Press [= *Nafs*].
———. (1960a) *aš-Šifāʾ, al-Ilāhiyyāt* (1). Edited by Ğ. Š. Qanawatī and S. Zāyid, Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Ilāhiyyāt*].
———. (1960b) *aš-Šifāʾ, al-Ilāhiyyāt* (2). Edited by M. Y. Mūsā, S. Dunyā, and S. Zāyid, Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Ilāhiyyāt*].
———. (1963) *Avicenne: Livre des définitions*. Edité, traduit, et annoté par A.-M. Goichon. Cairo: Publications de l'Institut Français d'archéologie orientale du Caire [= *Kitāb al-Ḥudūd*].
———. (1964) *aš-Šifāʾ, al-Manṭiq, al-Qiyās*. Edited by S. Zāyid. Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Qiyās*].
———. (1965a) *aš-Šifāʾ, al-Manṭiq, al-Ğadal*. Edited by A. F. al-Ahwānī. Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Ğadal*].
———. (1965b) *aš-Šifāʾ, aṭ-Ṭabīʿiyyāt, al-Maʿādin wa-l-āṯār al-ʿulwiyya*. Edited by ʿA. Muntaṣir, S. Zāyid, and ʿA. Ismaʿīl. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-t-taʾlīf wa-n-našr [= *Maʿādin wa-āṯār ʿulwiyya*].
———. (1966) *aš-Šifāʾ, al-Manṭiq, aš-Šiʿr*. Edited by ʿA. Badawī. Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Šiʿr*].
———. (1969) *aš-Šifāʾ, aṭ-Ṭabīʿiyyāt, as-Samāʾ wa-l-ʿālam, al-Kawn wa-l-fasād, al-Afʿāl wa-l-infiʿālāt*. Edited by M. Qāsim, Cairo: Dar al-kitāb al-ʿarabī li-ṭ-ṭibāʿa wa-n-našr [= *Samāʾ wa-ʿālam, Kawn wa-fasād, Afʿāl wa-infiʿālāt*].

———. (1970a) *al-Išārāt wa-t-tanbīhāt*. Edited by S. Dunyā in four volumes, with Ṭūsī's commentary at the bottom of page. Cairo: Dār al-maʿārif bi-Miṣr.

———. (1970b) *aš-Šifāʾ, al-Manṭiq, al-ʿIbāra*. Edited by M. al-Ḫuḍayrī. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-t-taʾlīf wa-n-našr [= *ʿIbāra*].

———. (1970c) *aš-Šifāʾ, aṭ-Ṭabīʿiyyāt, al-Ḥayawānāt*. Edited by ʿA. Muntaṣir, S. Zāyid, and ʿA. Ismaʿīl. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-t-taʾlīf wa-n-našr [= *Ḥayawānāt*].

———. (1971) *Avicenna's Treatise on Logic: Part One of Danesh-name Alai; A Concise Philosophical Encyclopaedia and Autobiography*. Edited and translated from the original Persian by F. Zabeeh. The Hague: Nijhoff.

———. (1973) *Taʿlīqāt*. Edited by ʿA. Badawī. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb [= *Taʿlīqāt*].

———. (1975a) *aš-Šifāʾ, ar-Riyāḍiyyāt, al-Ḥisāb*. Edited by ʿA. Luṭfī. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb [= *Ḥisāb*].

———. (1975b) *aš-Šifāʾ, aṭ-Ṭabīʿiyyāt, an-Nabāt*. Edited by ʿA. Muntaṣir, S. Zāyid, and ʿA. Ismaʿīl. Cairo: al-Hayʾa al-ʿāmma li-šuʾūn al-maṭābiʿ al-amīriyya [= *Nabāt*].

———. (1976) *aš-Šifāʾ, ar-Riyāḍiyyāt, Uṣūl al-Handasa*. Edited by ʿA. Sabra and ʿA. Luṭfī. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb [= *Handasa*].

———. (1980) *aš-Šifāʾ, ar-Riyāḍiyyāt, ʿIlm al-Hayʾa*. Edited by M. Madwār and I. Aḥmad. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb [= *Hayʾa*].

———. (1983) *aš-Šifāʾ, aṭ-Ṭabīʿiyyāt, as-Samāʿ aṭ-ṭabīʿī*. Edited by S. Zāyid. Cairo: al-Hayʾa al-miṣriyya al-ʿāmma li-l-kitāb [= *Samāʿ ṭabīʿī*].

———. (1984a) *al-Mabdaʾ wa-l-maʿād*. Edited by A. Nūrānī. Institute of Islamic Studies, McGill University, in collaboration with Tehran University. Tehran: Dānišgāh Tehran.

———. (1984b) *Remarks and Admonitions Part One: Logic*. Translated from the original Arabic with an introduction and notes by S. C. Inati. Toronto: Pontifical Institute of Mediaeval Studies.

———. (1985) *an-Naǧāt min al-ġarq fī baḥr aḍ-ḍalālāt*. Edited by M. Dānispažūh. Tehran: Dānishgāh Tehran [= *Naǧāt*].

———. (1992) *Mubāḥaṯāt*. Edited by M. Bīdārfar. Qum: Maṭbaʿat-i Amīr [= *Mubāḥaṯāt*].

———. (1413/1993) *al-Qānūn fī ṭ-ṭibb*. Edited by I. al-Qašš and ʿA. Zayʿūr. Beirut: ʿIzzaddīn [= *Qānūn*].

———. (2009) *al-Muḫtaṣar al-awsaṭ fī l-manṭiq*. Edited by H. Takci. Haziran: Sakarya University.

———. (2011) *The Deliverance: Logic*. Translated by A. Ahmed. Karachi: Oxford University Press.

al-Baġdādī, Abū l-Barakāt. (2015) *Kitāb al-Muʿtabar*. Edited by M. ʿUṯmān. Cairo: Maktabat aṯ-Ṯaqāfah ad-Dīniyya [= *Muʿtabar*].

Bahmanyār b. al-Marzubān. (1997) *at-Taḥṣīl*. Edited by M. Muṭahharī. Tihrān: Dānishgāh-i Tihrān [= *Taḥṣīl*].

Euclid. (1908) *The Thirteen Books of the Elements*. Translated with introduction and commentary by Sir T. Heath. Vol. 1. Cambridge: Cambridge University Press.

al-Ġazālī, Abū Ḥāmid. (1997) *The Incoherence of the Philosophers, Tahāfut al-falāsifa: A Parallel English-Arabic Text, Translated, Introduced, and Annotated by M. E. Marmura*. Provo, UT: Brigham Young University Press.

Gundissalinus, D. (1903) *De divisione philosophiae*. Edited by L. Baur. Münster: Aschendorff.
al-Ḫūnağī, Afḍal ad-Dīn. (2010) *Kašf al-asrār ʿan ġawāmiḍ al-afkār*. Edited by Kh. El-Rouayheb. Tehran-Berlin: Iranian Institute of Philosophy & Institute of Islamic Studies–Free University of Berlin.
al-Isfarāyīnī, M. (2004) *Šarḥ kitāb an-Nağāt li-Ibn Sīnā: Qism al-Ilāhiyyāt taʾlīf Faḫr ad-Dīn al-Isfarāyīnī an-Nīsābūrī; taqdīm wa-taḥqīq Ḥāmid Nāğī Iṣfahān*. Tihrān: Anjuman-i Āṯār va Mafākhir-i Farhangī.
al-Kātibī, Nağm ad-Dīn. (1367/1982) *ar-Risāla aš-Šamsiyya (Logic for Šams ad-Dīn)*. In *Taḥrīr al-qawāʿid al-manṭiqiyya fī šarḥ ar-Risāla aš-Šamsiyya*, by Quṭb ad-Dīn ar-Rāzī Taḥtānī. Cairo: Dār Iḥyāʾ al-kutub al-ʿarabiyya, Muṣṭafā al-Bābī al-Ḥalabī.
Philoponus. (1909) *Iohannis Philoponi in Aristotelis Analytica Posteriora commentaria cum anonymo in librum II*. Edited by M. Wallies. CAG, XIII, 3. Berlin: Reimer.
Porphyry. (1887) *Porphyrii Isagoge et in Aristotelis Categorias Commentarium*. Edited by A. Busse. CAG IV, 1. Berlin: Reimer.
ps.-Philoponus. (1909) *Iohannis Philoponi in Aristotelis Analytica Posteriora commentaria cum anonymo in librum II*. Edited by M. Wallies. CAG XIII, 3. Berlin: Reimer.
ar-Rāzī, Faḫr ad-Dīn. (1355/1936) *Lubāb al-Išārāt wa-t-tanbīhāt*. Edited by A. ʿAṭiyya. Cairo: Maktabat al-Khānğī.
———. (2002) *Manṭiq al-Mulaḫḫaṣ*. Edited by A. F. Qaramaleki and A. Asgharinezhad. Tehran.
———. (1384 solar/2005) *Šarḥ al-Išārāt*. Edited by ʿA. R. Najafzādeh. Tihrān: Anjuman-i Āṯār va Mafākhir-i Farhangī.
Tehrānī, Muḥammad Yusuf. (1389/2010) *Naqḍ al-uṣūl wa-talḫīṣ al-fuṣūl*. Edited by A. F. Qaramaleki. Zanjān: Dānešgāh-e Zanjān.
Themistius. (1900) *Themistii Analyticorum Posteriorum paraphrasis*. Edited by M. Wallies. CAG, V. Berlin: Reimer.
———. (1956) *Themistius' Paraphrasis of the Posterior Analytics in Gerard of Cremona's Translation*. Edited by J. R. O'Donnell. Mediaeval Studies, 18: 239–315.
aṭ-Ṭūsī, Naṣīr ad-Dīn. (1970) *Ḥall muškilāt al-Išārāt wa-t-tanbīhāt*. Edited by S. Dunyā in Ibn Sīnā, *al-Išārāt wa-t-tanbīhāt* (at the bottom of the page). 2nd ed. Cairo: Dār al-maʿārif bi-Miṣr [= *Ḥall muškilāt*].
Zabarella, J. (1597) *In duos Aristotelis libros Posteriores Analyticos commentarii*. In J. Zabarella, *Opera logica*. Cologne: Zetzner.

SECONDARY SOURCES

Achena, M., and H. Massé (trans.). (1955) *Avicenne: Le Livre de Science*. Vol. I, *Logique, Métaphysique*. Paris: Les Belles Lettres.
Ackrill, J. L. (1981) "Aristotle's Theory of Definition: Some Questions on *Posterior Analytics* II 8–10." In Berti 1981, 359–384.
Adamson, P. (ed.). (2013) *Interpreting Avicenna: Critical Essays*. New York: Cambridge University Press.
Aertsen, J. (2008) "Avicenna's Doctrine of the Primary Notions and Its Impact on Medieval Philosophy." In Akasoy and Raven 2008, 21–42.
Ahmed, A. Q. (2003) "Avicenna's Reception of Aristotle's Modal Syllogistic." In Reisman 2003, 3–24.

———. (2008) "The *Jiha/Tropos-Mādda/Hulē* Distinction in Arabic Logic and its Significance for Avicenna's Modals." In Rahman, Street, and Tahiri 2008, 229–253.

———. (2011) *Avicenna's Deliverance: Logic*. Translation and notes by Asad Q. Ahmed. Karachi: Oxford University Press.

Akasoy, A., and W. Raven (eds.). (2008) *Islamic Thought in the Middle Ages: Studies in Text, Transmission and Translation, in Honour of Hans Daiber*. Leiden: Brill.

Alonso Alonso, M. (1963) "Accidente, accidental y número según Avicena." *Al-Andalus* 28: 117–154.

Alwishah, A., and J. Hayes (eds.). (2015) *Aristotle and the Arabic Tradition*. Cambridge: Cambridge University Press.

Anagnostopoulos, G. (ed.). (2009) *A Companion to Aristotle*. Oxford: Blackwell.

Anawati, G. C. (1977) "La division des sciences intellectuelles d'Avicenne." *Mélanges de l'Institut dominicain d'études orientales* 13: 323–335.

Angioni, L. (2014) "Aristotle on Necessary Principles and on Explaining X Through the Essence of X." *Studia Philosophica Estonica* 7, no. 2: 88–112.

Anstey, P., and D. Bronstein (eds.). (forthcoming) *Essence and Definition from Aristotle to Kant*. n.p.: n.p.

Aouad, M. (1999) "Les prémisses rhétoriques selon les *Išārāt* d'Avicenne." In Büttgen, Diebler, and Rashed 1999, 281–304.

Asztalos, M., J. Murdoch, and I. Niiniluoto (eds.). (1990) *Knowledge and the Sciences in Medieval Philosophy: Proceedings of the Eighth International Congress of Medieval Philosophy (S.I.E.P.M.)*. Vol I. Helsinki: Yliopistopaino.

Aubenque, P. (1957) "Sur la définition aristotélicienne de la colère." *Revue philosophique de la France et de l'étranger* 147: 300–317.

Aydede, M. (1998) "Aristotle on *Episteme* and *Nous*: The *Posterior Analytics*." *Southern Journal of Philosophy* 36: 15–46.

Bäck, A. (1992) "Avicenna's Conception of the Modalities." *Vivarium* 30, no. 2: 217–255.

Badawī, ʿA. (ed.). (1947) *Arisṭū ʿinda l-ʿArab*. Cairo: Maktaba an-nahḍa al-miṣriyya.

Balme, D. M. (1987) "Aristotle's Use of Division and Differentiae." In Gotthelf and Lennox 1987, 69–89.

Barnes, J. (1969) "Aristotle's Theory of Demonstration." *Phronesis* 14: 123–152.

———. (1975) "Aristotle's Theory of Demonstration." In Barnes, Schofield, and Sorabji 1975, 65–87. Originally published in *Phronesis* 14 (1969): 123–152.

———. (1981) "Proof and the Syllogism." In Berti 1981, 1–59.

———. (1984) *The Complete Works of Aristotle*. 2 vols. Princeton, NJ: Princeton University Press.

———. (1993) *Aristotle: Posterior Analytics*. Translated with a commentary. 2nd ed. Oxford: Clarendon Press.

———. (2014a) *Proof, Knowledge, and Skepticism: Essays in Ancient Philosophy*. Vol. 3. Edited by M. Bonelli. Oxford: Oxford University Press.

———. (2014b) "Aristotle on Knowledge and Proof." In Barnes 2014a, 73–94.

Barnes, J., M. Schofield, and R. Sorabji (eds.). (1975) *Articles on Aristotle Vol. I: Science*. London: Duckworth.

Bayer, G. (1995) "Definition through Demonstration: The Two Types of Syllogisms in *Posterior Analytics* II.8." *Phronesis* 40: 241–264.

———. (1997a) "The What-Is-X? Question in the *Posterior Analytics*." *Ancient Philosophy* 17: 317–334.

———. (1997b) "Coming to Know Principles in *Posterior Analytics* II 19." *Apeiron* 30: 109–142.

Bellucci, F., and C. Marmo. (2018) "Sign and Demonstration in Late-Ancient Commentaries on the *Posterior Analytics*." *Cahiers de l'Institut du Moyen-Âge grec et latin* 87: 1–33.

Benevich, F. (2018) *Essentialität und Notwendigkeit*. Leiden: Brill.

Berti, E. (ed.). (1981) *Aristotle on Science: The Posterior Analytics*. Proceedings of the Eighth Symposium Aristotelicum. Padua: Antenore.

Bertolacci, A. (2002) "The Doctrine of Material and Formal Causality in the *Ilāhiyyāt* of Avicenna's *Kitāb al-Šifā'*." *Quaestio* 2: 125–154.

———. (2006) *The Reception of Aristotle's Metaphysics in Avicenna's "Kitāb al-Šifā'": A Milestone of Western Metaphysical Thought*. Leiden: Brill.

———. (2008) "'Necessary' as Primary Concept in Avicenna's Metaphysics." In Fioravanti and Perfetti 2008, 31–50.

Biard, J. (ed.). (2015) *Raison et démonstration: Les commentaires médiévaux sur les Seconds Analytiques*. Turhnout: Brepols.

Bilal, I. (2013). "Faḫr ad-Dīn ar-Rāzī, Ibn al-Hayṯam and Aristotelian Science: Essentialism versus Phenomenalism in Post-Classical Islamic Thought." *Oriens* 41: 379–431.

Black, D. (1990) *Logic and Aristotle's "Rhetoric" and "Poetics" in Medieval Arabic Philosophy*. Leiden: Brill.

———. (1993) "Estimation (*Wahm*) in Avicenna: The Logical and Psychological Dimensions." *Dialogue* 32: 219–258.

———. (2006) "Knowledge (*'ilm*) and Certitude (*yaqīn*) in Al-Fārābī's Epistemology." *Arabic Sciences and Philosophy* 16: 11–45.

———. (2013) "Certitude, Justification, and the Principles of Knowledge in Avicenna's Epistemology." In Adamson 2013, 120–142.

———. (2014) "How Do We Acquire Concepts? Avicenna on Abstraction and Emanation." In Hause 2014, 126–144.

Bolton, R. (1976) "Essentialism and Semantic Theory in Aristotle: *Posterior Analytics*, II, 7–10." *Philosophical Review* 85: 514–544.

———. (1987) "Definition and Scientific Method in Aristotle's *Posterior Analytics* and *Generation of Animals*." In Gotthelf and Lennox 1987, 120–166.

———. (1997) "Aristotle on Essence and Necessity in Science." *Proceedings of the Boston Area Colloquium in Ancient Philosophy* 13: 113–138.

———. (2014) "Intuition in Aristotle." In Osbeck and Held 2014, 39–54.

Bolton, R., and P. Pellegrin. (1993) "Division, définition et essence dans la science aristotélicienne." *Revue philosophique de la France et de l'étranger* 2: 197–222.

Brentjes, B. (ed.). (1980) *Avicenna/Ibn Sīnā*. Vol. II, *Wissenschaftsgeschichte*. Wittemberg: Martin Luther Universität Halle.

Brody, B. A. (1972) "Towards an Aristotelian Theory of Scientific Explanation." *Philosophy of Science* 39: 20–31.

Bronstein, D. (2010) "Meno's Paradox in *Posterior Analytics* 1.1." *Oxford Studies in Ancient Philosophy* 38: 115–141.

———. (2012) "The Origin and Aim of *Posterior Analytics* II.19." *Phronesis* 57: 29–62.
———. (2015) "Essence, Necessity, and Demonstration in Aristotle." *Philosophy and Phenomenological Research* 90: 724–732.
———. (2016) *Aristotle on Knowledge and Learning*. Oxford: Clarendon Press.
Brunschwig, J. (1981) "L'objet et la structure des *Seconds Analytiques* d'après Aristote." In Berti 1981, 61–69.
Burnett, C. (ed.). (1993) *Glosses and Commentaries on Aristotelian Logical Texts: The Syriac, Arabic and Medieval Latin Traditions*. London: The Warburg Institute.
Burnyeat, M. F. (1981) "Aristotle on Understanding Knowledge." In Berti 1981, 97–139.
———. (2011) "Episteme." In Morison and Ierodiakonou 2011, 3–29.
Butler, T. (2003) "*Empeiria* in Aristotle." *Southern Journal of Philosophy* 41: 330–342.
Büttgen, P., S. Diebler, and M. Rashed (eds.). (1999). *Théories de la phrase et de la proposition de Platon à Averroès*. Paris: Éditions Rue d'Ulm.
Carpentieri, N. (2017) "On the Meaning of *Birsām* and *Sirsām*, a Survey of the Arabic Commentaries on the Hippocratic Aphorisms." *Mélanges de l'Institut dominicain d'études orientales* 32: 81–92.
Carruth, A., S. C. Gibb, and J. Heil (eds.). (2018) *Ontology, Modality, Mind: Themes from the Metaphysics of E. J. Lowe*. Oxford: Oxford University Press.
Charles, D. (2000) *Aristotle on Meaning and Essence*. Oxford: Clarendon Press.
———. (ed.). (2010a) *Definition in Greek Philosophy*. Oxford: Oxford University Press.
———. (2010b) "The Paradox in the *Meno* and Aristotle's Attempts to Resolve it." In Charles 2010a, 115–150.
———. (2010c) "Definition and Explanation in the *Posterior Analytics* and *Metaphysics*." In Charles 2010a, 286–328.
Chiba, K. (2010) "Aristotle on Essence and Defining-Phrase in his Dialectic." In Charles 2010a, 204–251.
———. (2012) "Aristotle on Heuristic Inquiry and Demonstration of *What It Is*." In Shields 2012, 171–201.
Cleary, J. J. (1985) "On the Terminology of 'Abstraction' in Aristotle." *Phronesis* 30: 13–45.
Corcoran, J. (ed.). (1974) *Ancient Logic and Its Modern Interpretations*. Dordrecht: Reidel.
Couloubaritsis, L. (1980) "Y-a-t-il une intuition des principes chez Aristote?" *Revue internationale de Philosophie* 133–134: 440–471.
Correia, F. (2013) "On the Reduction of Necessity to Essence." *Philosophy and Phenomenological Research* 84, no. 3: 639–653.
Cresswell, M. (ed.). (2018) *The Logic of Modalities from Aristotle to Carnap: The Story of Necessity*. Cambridge: Cambridge University Press.
Davidson, H. (1992) *Alfarabi, Avicenna, and Averroes on Intellect: Their Cosmologies, Theories of the Active Intellect, and Theories of Human Intellect*. New York: Oxford University Press.
De Haas, F., M. Leunissen, and M. Martijn (eds.). (2010) *Interpreting Aristotle's Posterior Analytics in Late Antiquity and Beyond*. Leiden: Brill.
Demoss, D., and D. Devereux. (1988) "Essence, Existence, and Nominal Definition in Aristotle's *Posterior Analytics* II 8–10." *Phronesis* 33: 133–154.
Deslauriers, M. (2007) *Aristotle on Definition*. Leiden: Brill.

Detel, W. (1993) *Aristoteles, Analytica Posteriora: Übersetzung und Erläuterung*. 2 vols. Berlin: Akademie Verlag.

Devereux, D., and P. Pellegrin (eds.). (1990) *Biologie, logique et métaphysique chez Aristote*. Paris: Éditions du Centre National de la Recherche Scientifique.

Di Liscia, D. A., E. Kessler, and C. Methuen (eds.). (1997) *Method and Order in Renaissance Philosophy of Nature: The Aristotle Commentary Tradition*. Aldershot: Ashgate.

Di Vincenzo, S. (2015) "Avicenna against Porphyry's definition of Differentia Specifica." *Documenti e studi sulla tradizione filosofica medievale* 26: 129–183.

———. (2018) "Avicenna, *Book of the Healing*, *Isagoge* ('*Madḫal*'). Edition of the Arabic Text, English Translation and Commentary." PhD diss., Scuola Normale Superiore (Filosofia), École Pratique des Hautes Études (Études arabes et civilization du monde musulman).

D'Ooge, M. L. (1926) *Nicomachus of Gerasa: Introduction to Arithmetic*. London: Macmillan.

Dutilh Novaes, C., and S. Read (eds.). (2016) *The Cambridge Companion to Medieval Logic*. Cambridge: Cambridge University Press.

Dutilh Novaes, C., and O. Thomassen Hjortland (eds.). (2012) *Insolubles and Consequences: Essays in Honour of S. Read*. Milton Keynes: College Publications.

Ebbesen, S. (2015) "The *Posterior Analytics* 1100–1400 in East and West." In Biard 2015, 11–30.

Eichner, H. (2010) "Al-Fārābī and Ibn Sīnā on 'Universal Science' and the System of Sciences: Evidence of the Arabic Tradition of the *Posterior Analytics*." *Documenti e studi sulla tradizione filosofica medievale* 21: 71–95.

El-Rouayheb, Kh. (2009) "Impossible Antecedents and Their Consequences: Some Thirteenth-Century Arabic Discussions." *History and Philosophy of Logic* 30, no. 3: 209–225.

———. (2010) Introduction to al-Ḫūnağī 2010, iii–lix.

———. (2012) "Post-Avicennan Logicians on the Subject Matter of Logic: Some Thirteenth- and Fourteenth-Century Discussions." *Arabic Sciences and Philosophy* 22, no. 1: 69–90.

El-Rouayheb, Kh., and S. Schmidtke (eds.). (2015) *The Oxford Handbook of Islamic Philosophy*. Oxford: Oxford University Press.

Endress, G. (2006a) "The Cycle of Knowledge: Intellectual Traditions and Encyclopaedias of the Rational Sciences in Arabic Islamic Hellenism." In Endress 2006b, 103–134.

———. (ed.). (2006b) *Organizing Knowledge: Encyclopaedic Activities in the Pre-Eighteenth Century Islamic World*. Leiden: Brill.

Engberg-Pedersen, T. (1979) "More on Aristotelian *Epagoge*." *Phronesis* 24: 301–319.

Evans, M. G. (1958–1959) "Causality and Explanation in the Logic of Aristotle." *Philosophy and Phenomenological Research* 19: 466–485.

Everson, S. (ed.). (1990) *Epistemology*. Cambridge: Cambridge University Press.

Falcon, A. (1997) "Aristotle's Theory of Division." In Sorabji 1997, 127–146.

———. (2000) "Aristotle, Speusippus, and the Method of Division." *Classical Quarterly* 50: 402–414.

Federici-Vescovini, G., and A. Hasnawi (eds.). (2013) *Circolazione dei saperi nel Mediterraneo: filosofia e scienze (secoli IX-XVII)*. Firenze: Cadmo.

Ferejohn, M. T. (1981) "Aristotle on Necessary Truth and Logical Priority." *American Philosophical Quarterly* 18, no. 4: 285–293.

———. (1982) “Definition and the Two Stages of Aristotelian Demonstration.” *Review of Metaphysics* 36: 375–395.
———. (1988) “Meno’s Paradox and *De Re* Knowledge in Aristotle’s Theory of Demonstration.” *History of Philosophy Quarterly* 5: 99–117.
———. (1991) *The Origins of Aristotelian Science*. New Haven, CT: Yale University Press.
———. (2013) *Formal Causes: Definition, Explanation, and Primacy in Socratic and Aristotelian Thought*. Oxford: Oxford University Press.
Fine, K. (1994) “Essence and Modality.” *Philosophical Perspectives* 8: 1–16.
Fioravanti, G., and S. Perfetti (eds.). (2008) *Conoscenza e contingenza nella tradizione aristotelica medievale*. Pisa: ETS.
Fischer, B., and F. Leon (eds.). (2012) *Modal Epistemology after Rationalism*. Cham, Switzerland: Springer.
Fleet, K., G. Krämer, D. Matringe, J. Nawas, and E. Rowson (eds.). (2009) *Encyclopaedia of Islam 3*. http://dx.doi.org/10.1163/1573-3912_ei3_COM_22817.
Frede, D. (1974) “Comment on Hintikka’s Paper ‘On the Ingredients of an Aristotelian Science.’” *Synthese* 28: 79–89.
Freudenthal, G. (2018) “The Medieval Hebrew Reception of Avicenna’s Account of the Formation and Perseverance of Dry Land: Between Bold Naturalism and Fideist Literalism.” In Hasse and Bertolacci 2018, 269–311.
Friedlein, G. (1873) *Proclus: In primum Euclidis Elementorum librum commentarii*. Leipzig: Teubner.
Friedman, R. G. (1984) “Matter and Necessity in Aristotle’s Logical, Physical, and Biological Works.” PhD diss., University of Virginia.
Fritz, K. (1971a) “Die ΕΠΑΓΩΓΗ bei Aristoteles.” In Fritz 1971b, 623–676. Originally printed in *Sitzungsberichte des Bayerischen Akademie der Wissenschaften, phil.hist. Kl.* 3, München, 1964.
———. (ed.). (1971b) *Grundprobleme der Geschichte der antiken Wissenschaft*. Berlin: De Gruyter.
Gabbay, D. M., and J. Woods (eds.). (2004) *Handbook of the History of Logic: Greek, Indian and Arabic Logic*. Amsterdam: Elsevier.
Germann, N., and M. Najafi. (eds.). (2020) *Philosophy and Language in the Islamic World*. Berlin: De Gruyter.
Gerson, L. P. (1999) *Aristotle: Critical Assessments*. Vol. 2. New York, NY: Routledge.
———. (2009) *Ancient Epistemology*. Cambridge: Cambridge University Press.
Gifford, M. (2000) “Lexical Anomalies in the Introduction to the *Posterior Analytics*, Part 1.” *Oxford Studies in Ancient Philosophy* 19: 163–223.
Goichon, A.-M. (1938) *Lexique de la langue philosophique d’Ibn Sīnā*. Paris: Desclée de Brouwer.
———. (1951) *Ibn Sīnā (Avicenne), Livre des Directives et Remarques (Kitāb al-ʾIšārāt wa l-tanbīhāt)*. Traduction avec introduction et notes par A.-M. Goichon. Beirut-Paris: Commission internationale pour la traduction des chefs d’oeuvre.
Goldin, O. (1996) *Explaining an Eclipse: Aristotle’s Posterior Analytics 2.1–2.10*. Ann Arbor, MI: University of Michigan Press.
———. (2004) “Atoms, Complexes, and Demonstration: Posterior Analytics 96b15–25.” *Studies in History and Philosophy of Science* 35: 707–727.

———. (2009) *Philoponus(?): On Aristotle Posterior Analytics 2*. London: Duckworth.
Gómez-Lobo, A. (1977) "Aristotle's Hypotheses and the Euclidean Postulates." *Review of Metaphysics* 30: 430–439.
———. (1978) "Aristotle's First Philosophy and the Principles of Particular Disciplines." *Zeitschrift für philosophische Forschung* 32: 183–194.
———. (1980) "The So-Called Question of Existence in Aristotle, *An. Post. 2.1–2*." *Review of Metaphysics* 34: 71–91.
———. (1981) "Definitions in Aristotle's *Posterior Analytics*." In O'Meara 1981, 25–46.
Gotthelf, A. (1987) "First Principles in Aristotle's *Parts of Animals*." In Gotthelf and Lennox 1987, 167–198.
Gotthelf, A., and J. Lennox (eds.). (1987) *Philosophical Issues in Aristotle's Biology*. Cambridge: Cambridge University Press.
Gourevitch, D. (ed.). (1992) *Maladie et maladies: Histoire et conceptualisation; Mélange en l'honneur de Mirko Grmek*. Geneva: Droz.
Graham, W. (1975) "Counterpredicability and Per Se Accidents." *Archiv für Geschichte der Philosophie* 57: 182–187.
Granger, G. G. (1976) *La théorie aristotélicienne de la science*. Paris: Aubier.
Granger, H. (1981) "The Differentia and the Per Se Accident in Aristotle." *Archiv für Geschichte der Philosophie* 63: 118–129.
Gregorić, P., and F. Grgić. (2006) "Aristotle's Notion of Experience." *Archiv für Geschichte der Philosophie* 88: 1–30.
Guariglia, O. N. (1982) "La definición y la explicación causal según Aristóteles." *Revista Latinoamericana de Filosofia* 8: 119–146.
Guerriere, D. (1975) "The Aristotelian Concept of Episteme." *Thomist* 39: 341–348.
Gutas, D. (1988) *Avicenna and the Aristotelian Tradition*. Leiden: Brill.
———. (1993) "Aspects of Literary Form and Genre in Arabic Logical Works." In Burnett 1993, 29–76.
———. (2001) "Intuition and Thinking: The Evolving Structure of Avicenna's Epistemology." In Wisnovsky 2001, 1–38.
———. (2003) "Medical Theory and Scientific Method in the Age of Avicenna." In Reisman 2003, 145–162.
———. (2005) "The Logic of Theology (*Kalām*) in Avicenna." In Perler and Rudolph 2005, 59–72.
———. (2006) "Imagination and Transcendental Knowledge in Avicenna." In Montgomery 2006, 337–354.
———. (2012) "The Empiricism of Avicenna." *Oriens* 40, no. 2: 391–436.
Hagdopoulos, D. J. (1975a) "Demonstration and the Second Figure in Aristotle." *New Scholasticism* 49: 62–75.
———. (1975b) "A Note on Aristotle's Notions of Universality and Necessity." *Logique et Analyse* 18: 171–174.
———. (1976) "Reduction to Immediate Premises." *Apeiron* 10: 29–37.
———. (1977) "διὰ μέσων or δι᾿ἀμέσων: *Posterior Analytics* II, viii, 93a36." *Apeiron* 11, no. 1: 32–39.
Hamlyn, D. W. (1976) "Aristotelian *Epagoge*." *Phronesis* 21: 167–184.

Hankinson, R. J. (1998) *Cause and Explanation in Ancient Greek Thought*. Oxford: Clarendon Press.

———. (2011) "Avant *nous* le déluge: Aristotle's Notion of Intellectual Grasp." In Morison and Ierodiakonou 2011, 30–59.

Harari, O. (2004) *Knowledge and Demonstration: Aristotle's Posterior Analytics*. Dordrecht: Kluwer.

Harris, W. V. (1997) "Saving the φαινόμενα: A Note on Aristotle's Definition of Anger." *Classical Quarterly* 47: 452–454.

Hashemi, S. M., and M. Raza. (2009). "The Traditional Diagnosis and Treatment of Respiratory Diseases: A Description from Avicenna's *Canon of Medicine*." *Therapeutic Advances in Respiratory Disease* 3, no. 6: 319–328.

Hasnawi, A. (2013) "L'âge de la démonstration: Logique, science et histoire; Al-Fārābī, Avicenne, Avempace, Averroès." In Federici-Vescovini and Hasnawi 2013, 329–364.

Hasper, P. S., and J. Yurdin. (2014) "Between Perception and Scientific Knowledge: Aristotle's Account of Experience." *Oxford Studies in Ancient Philosophy* 47: 120–150.

Hasse, D. (2001) "Avicenna on Abstraction." In Wisnovsky 2001, 39–72.

———. (2013) "Avicenna's Epistemological Optimism." In Adamson (2013), 109–119.

Hasse, D. N., and A. Bertolacci (eds.). (2018) *The Arabic, Hebrew and Latin Reception of Avicenna's Physics and Cosmology*. Berlin: De Gruyter.

Hause, J. (ed.). (2014) *Debates in Medieval Philosophy*. London: Routledge.

Heath, T. (1921) *A History of Greek Mathematics*. 2 vols. Oxford: Clarendon Press.

———. (1925) *The Thirteen Books of Euclid's "Elements"*. 2nd ed. 3 vols. Cambridge: Cambridge University Press.

———. (1949) *Mathematics in Aristotle*. Oxford: Clarendon Press.

Henry, D. (2011) "Aristotle's Pluralistic Realism." *The Monist* 94: 197–220.

Hesz, W. (1970) "Erfahrung und Intuition bei Aristoteles." *Phronesis* 15: 48–81.

Hintikka, J. (1972) "On the Ingredients of an Aristotelian Science." *Nous* 6: 55–69.

———. (1974) "Reply to Dorothea Frede." *Synthese* 28: 91–96.

———. (1980) "Aristotelian Induction." *Revue Internationale de Philosophie* 34: 422–439.

Hodges, W. (2017) "Ibn Sīnā on *reductio ad absurdum*." *Review of Symbolic Logic* 10, no. 3: 587–601.

Hugonnard-Roche, H. (1984) "La classification des sciences de Gundissalinus et l'influence d'Avicenne." In Jolivet and Rashed 1984, 41–75.

Husserl, E. (1900) *Logische Untersuchungen. Band I, Prolegomena zur reinen Logik*. Halle: Niemeyer.

Inwood, B. (1979) "A Note on Commensurate Universals in the *Posterior Analytics*." *Phronesis* 24: 320–332.

Irwin, T. H. (1988) *Aristotle's First Principles*. Oxford: Clarendon Press.

Jacquart, D. (1992) "Les avatars de la phrénitis chez Avicenne et Rhazès." In Gourevitch 1992, 181–192.

Janssens, J. L. (2004) "Experience (*tağriba*) in Classical Arabic Philosophy (al-Fārābī–Avicenna)." *Quaestio* 4: 45–62.

Jolivet, J. (1996) "Classifications of the Sciences." In Rashed and Morelon 1996, III: 1008–1025 (Avicenna 1017–1022).

Jolivet, J., and R. Rashed (eds.). (1984) *Études sur Avicenne*. Paris: Les Belles Lettres.
Jope, J. (1972) "Subordinate Demonstrative Science in the Sixth Book of Aristotle's *Physics*." *Classical Quarterly* 22: 279–292.
Kalbarczyk, A. (2012) "The *Kitāb al-Maqūlāt* of the *Muḫtaṣar al-awsaṭ fī l-manṭiq*: A Hitherto Unknown Source for Studying Ibn Sīnā's Reception of Aristotle's *Categories*." *Oriens* 40: 305–354.
———. (2018) *Predication and Ontology: Studies and Texts on Avicennian and Post-Avicennian Readings of Aristotle's Categories*. Berlin: De Gruyter.
Kalbarczyk, N. (2018) *Sprachphilosophie in der islamischen Rechtstheorie*. Leiden: Brill.
Kahn, C. (1981) "The Role of *Nous* in the Cognition of First Principles in *Posterior Analytics* II 19." In Berti 1981, 385–414.
Kal, V. (1988) *On Intuition and Discursive Reasoning in Aristotle*. Leiden: Brill.
Kapp, E. (1975) "Syllogistic." In Barnes, Schofield, and Sorabji 1975, 1–35.
Karimullah, K. (2014) "Avicenna (d. 1037), Logical Theory, and the Aristotelian Tradition." PhD diss., McGill University.
Kato, M. (1987) "Aristoteles über den Ursprung wissenschaftlicher Erkenntnis." *Phronesis* 32: 188–205.
Key, A. 2018. *Language between God and the Poets: Maʿnā in the Eleventh Century*. Oakland, CA: University of California Press.
Knuuttila S., R. Työrinoja, and S. Ebbesen (eds.). (1990) *Knowledge and the Sciences in Medieval Philosophy: Proceedings of the Eighth International Congress of Medieval Philosophy (S.I.E.P.M.)*. Vol. II. Helsinki: Yliopistopaino.
Koslicki, K. (2012) "Essence, Necessity, and Explanation." In Tahko 2012, 187–206.
Kosman, L. A. (1973) "Understanding, Explanation, and Insight in Aristotle's *Posterior Analytics*." In Lee, Mourelatos, and Rorty 1973, 374–392.
Kullmann, W. (1975) "Zur wissenschaftlichen Methode des Aristoteles." In Seeck 1975, 301–338.
———. (1981) "Die Funktion der mathematischen Beispiele in Aristoteles' *Analytica Posteriora*." In Berti 1981, 245–270.
Kullmann W., and S. Follinger (eds.). (1997) *Aristotelische Biologie: Intentionen, Methoden, Ergebnisse*. Stuttgart: Franz Steiner Verlag.
Kung, J. (1977) "Aristotle on Essence and Explanation." *Philosophical Studies* 31: 361–383.
LaBarge, S. (2004) "Aristotle on 'Simultaneous Learning' in *Posterior Analytics* 1.1 and *Prior Analytics* 2.21." *Oxford Studies in Ancient Philosophy* 27: 177–215.
———. (2006) "Aristotle on *Empeiria*." *Ancient Philosophy* 26: 23–44.
Lameer, J. (1994) *Alfarabi and Aristotelian Syllogistics*. Leiden: Brill.
———. (2006) *Conception and Belief in Ṣadrā l-Dīn Shirāzī*. Tehran: Iranian Institute of Philosophy.
Lammer, A. (2018) *The Elements of Avicenna's Physics: Greek Sources and Arabic Innovations*. Berlin: De Gruyter.
Landor, B. (1981) "Definitions and Hypotheses in *Posterior Analytics* 72a19–25 and 76b35–77a4." *Phronesis* 26: 308–318.
———. (1985) "Aristotle on Demonstrating Essence." *Apeiron* 19: 116–132.
LeBlond, J. M. (1975) "Aristotle on Definition." In Barnes, Schofield, and Sorabji 1975, 63–79.

Lee, H. D. P. (1935) "Geometrical Method and Aristotle's Account of First Principles." *Classical Quarterly* 29: 113–124.

Lee, E. N., A. P. D. Mourelatos, and R. M. Rorty (eds.). (1973) *Exegesis and Argument*. Assen: Van Gorcum.

Lennox, J. (1987) "Divide and Explain: The *Posterior Analytics* in Practice." In Gotthelf and Lennox 1987, 90–119.

———. (1994) "Aristotelian Problems." *Ancient Philosophy* 14: 53–77.

———. (2001) *Aristotle's Philosophy of Biology*. Cambridge: Cambridge University Press.

———. (2004) "Getting a Science Going: Aristotle on Entry Level Kinds." In Wolters 2004, 87–100.

Lennox, J., and R. Bolton (eds.). (2010) *Being, Nature, and Life in Aristotle: Essays in Honor of Allan Gotthelf*. Cambridge: Cambridge University Press.

Lesher, J. H. (1973) "The Meaning of Nous in the *Posterior Analytics*." *Phronesis* 18: 44–68.

———. (2001) "On Aristotelian *Epistēmē* as 'Understanding.'" *Ancient Philosophy* 21: 45–55.

———. (ed.). (2010a) "From Inquiry to Demonstrative Knowledge: New Essays on Aristotle's *Posterior Analytics*." Special issue, *Apeiron* 43.

———. (2010b) "Just as in Battle: the Simile of the Rout in *Posterior Analytics* II 19." *Ancient Philosophy* 30: 95–105.

———. (2011) "A Note on the Simile of the Rout in the *Posterior Analytics* II 19." *Ancient Philosophy* 31: 1–5.

Leszl, W. (1980) "Unity and Diversity of the Sciences: The Methodology of the Mathematical and of the Physical Sciences and the Role of the Nominal Definition." *Revue Internationale de Philosophie* 34: 384–421.

———. (1981) "Mathematics, Axiomatization and the Hypotheses." In Berti 1981, 271–328.

Leunissen, M. (2007) "The Structure of Teleological Explanation in Aristotle: Theory and Practice." *Oxford Studies in Ancient Philosophy* 33: 145–178.

———. (2010a) *Explanation and Teleology in Aristotle's Science of Nature*. Cambridge: Cambridge University Press.

———. (2010b) "Aristotle's Syllogistic Model of Knowledge and the Biological Sciences: Demonstrating Natural Processes." In Lesher 2010a, 31–60.

———. (2015) "Comments on Malink's *Aristotle's Modal Syllogistic*." *Philosophy and Phenomenological Research* 90: 733–741.

Lloyd, A. C. (1981) "Necessity and Essence in the *Posterior Analytics*." In Berti 1981, 157–171.

Lloyd, G. E. R. (1961) "The Development of Aristotle's Theory of the Classification of Animals." *Phronesis* 6: 59–81.

———. (1990) "Aristotle's Zoology and His Metaphysics: The Status Quaestionis; A Critical Review of some Recent Theories." In Devereux and Pellegrin 1990, 7–35.

Malink, M. (2013) *Aristotle's Modal Syllogistic*. Cambridge, MA: Harvard University Press.

———. (2020) "Aristotelian Demonstration." In Raven 2020, 33–48.

Mansion, S. (1979) "'Plus connu en soi,' 'plus connu pour nous': Une distinction épistémologique importante chez Aristote." *Pensamiento* 35: 161–170.

Marmura, M. (1984) "Avicenna on Primary Concepts in the Metaphysics of his *al-Shifāʾ*." In Savory and Agius 1984, 219–239.

———. (1990) "The *Fortuna* of the *Posterior Analytics* in the Arabic Middle Ages." In Asztalos, Murdoch, and Niiniluoto 1990, 85–103.

———. (2005) *Avicenna: The Metaphysics of the Healing. A Parallel English-Arabic Text Translated, Introduced, and Annotated by M. E. Marmura*. Provo, UT: Brigham Young University Press.

———. (2009) "Avicenna on Meno's Paradox: On Apprehending Unknown Things through Known Things." *Mediaeval Studies* 71: 47–62.

Maróth, M. (1980) "Das System der Wissenschaften bei Ibn Sīnā." In Brentjes 1980, 27–32.

———. (1990) "*Taṣawwur* and *taṣdīq*." In Knuuttila, Työrinoja, and Ebbesen 1990, 263–274.

———. (1994) *Die Araber und die antike Wissenschaftstheorie*. Leiden: Brill.

Mayer, T. (2009) "Anniyya." In Fleet et al. 2009.

McCaskey J. (2007) "Freeing Aristotelian *Epagōgē* from *Prior Analytics* II 23." *Apeiron* 40, no. 4: 345–374.

McGinnis, J. (2003) "Scientific Methodologies in Medieval Islam: Induction and Experimentation in the Philosophy of Ibn Sīnā." *Journal of the History of Philosophy* 41, no. 3: 307–327.

———. (2006) "Making Abstraction Less Abstract: The Logical, Psychological, and Metaphysical Dimensions of Avicenna's Theory of Abstraction." *Proceedings of the American Catholic Philosophical Association* 80: 169–183.

———. (2008) "Avicenna's Naturalized Epistemology and Scientific Method." In Rahman, Street, and Tahiri 2008, 129–152.

———. (2009) *Avicenna: The Physics of the Healing. A Parallel English-Arabic Text Translated, Introduced, and Annotated by Jon McGinnis*. 2 vols. Provo, UT: Brigham Young University Press.

McGinnis, J., and D. Reisman. (2007) *Classical Arabic Philosophy: An Anthology of Sources. Translated with introduction, notes, and glossary by Jon McGinnis and David Reisman*. Indianapolis, IN: Hackett.

McKirahan, R. (1978) "Aristotle's Subordinate Sciences." *British Journal for the History of Science* 11, no. 3: 197–220.

———. (1983) "Aristotelian Epagoge in *Prior Analytics* 2.21 and *Posterior Analytics* 1.1." *Journal of the History of Philosophy* 21: 1–13.

———. (1992) *Principles and Proofs: Aristotle's Theory of Demonstrative Science*. Princeton, NJ: Princeton University Press.

Mendell, H. (1984) "Two Geometrical Examples from Aristotle's *Metaphysics*." *Classical Quarterly* 34, no. 2: 359–372.

———. (1998) "Making Sense of Aristotelian Demonstration." *Oxford Studies in Ancient Philosophy* 16: 161–225.

Menn, S. (2008) "Al-Fārābī's *Kitāb al-Ḥurūf* and his Analysis of the Senses of Being." *Arabic Sciences and Philosophy* 18, no. 1: 59–97.

Michot, Y. (J. R.). (1994) *Avicenne, Kitāb al-mabdaʾ wa-l-maʿād. Livre de la genèse et du retour*. Traduction française intégrale par J. R. Michot, version exploratoire. Bruxelles: n.p.

Mignucci, M. (1965) *La teoria aristotelica della scienza*. Firenze: Sansoni.

———. (1975) *L'argomentazione dimostrativa in Aristotele: Commento agli "Analitici Secondi"*. Padova: Antenore.

———. (1981) "Ως ἐπὶ τὸ πολύ et nécessaire dans la conception aristotélicienne de la science." In Berti 1981, 173–203.

———. (2007) *Aristotele. Analitici secondi*. Traduzione e commento di Mario Mignucci. Roma-Bari: Laterza.

Modrak, D. (2010) "Nominal Definition in Aristotle." In Charles 2010a, 252–285.

Montgomery, J. E. (ed.). (2006) *Arabic Theology, Arabic Philosophy. From the Many to the One: Essays in Celebration of Richard M. Frank*. Leuven: Peeters.

Moraux, P. (1979) *Le Commentaire d'Alexandre d'Aphrodise aux "Seconds Analytiques" d'Aristote*. Berlin: De Gruyter.

Moravcsik, J. (1974) "Aristotle on Adequate Explanations." *Synthese* 28: 3–17.

Morison, B. (2012) "An Aristotelian Distinction between Two Types of Knowledge." *Proceedings of the Boston Area Colloquium in Ancient Philosophy* 37: 29–63.

Morison, B., and K. Ierodiakonou (eds.). (2011) *Episteme, Etc.: Essays in Honour of Jonathan Barnes*. Oxford: Oxford University Press.

Morrison, D. (1997). "Philoponus and Simplicius on Tekmeriodic Proof." In Di Liscia, Kessler, and Methuen 1997, 1–22.

Mulhern, J. J. (1958) "Aristotle on Universality and Necessity." *Logique et Analyse* 12: 288–299.

Mueller, I. (1974) "Greek Mathematics and Greek Logic." In Corcoran 1974, 35–70.

———. (1990) "Aristotle's Doctrine of Abstractionism in the Commentators." In Sorabji 1990, 463–480.

Novak, J. A. (1978) "A Geometrical Syllogism: *Posterior Analytics* II, 11." *Apeiron* 12: 26–33.

Ogden, S. (2020) "Avicenna's Emanated Abstraction." *Philosophers' Imprint* 20, no. 10: 1–26.

O'Meara, D. J. (ed.). (1981) *Studies in Aristotle*. Washington, DC: Catholic University of America Press.

Osbeck, L. M., and B. S. Held (eds.). (2014) *Rational Intuition: Philosophical Roots, Scientific Investigations*. Cambridge: Cambridge University Press.

Patzig, G. (1981) "Erkenntnisgründe, Realgründe und Erklärung (zu *An. Post.* A 13)." In Berti 1981, 141–156.

Pellegrin, P. (1987) "Logical Difference and Biological Difference: The Unity of Aristotle's Thought." In Gotthelf and Lennox 1987, 313–338.

———. (2005) *Aristote: Seconds Analytiques*. Paris: Flammarion.

———. (2010) "Definition in Aristotle's *Posterior Analytics*." In Lennox and Bolton 2010, 122–146.

Perelmuter, Z. (2010) "*Nous* and Two Kinds of *Epistêmê* in Aristotle's *Posterior Analytics*." *Phronesis* 55: 228–254.

Perler, D., and U. Rudolph (eds.). (2005) *Logik und Theologie: Das Organon im Arabischen und im Lateinischen Mittelalter*. Leiden: Brill.

Peters, F. (1968) *Aristoteles Arabus: The Oriental Translations and Commentaries on the Aristotelian Corpus*. Leiden: Brill.

Rahman, F. (1959) *Avicenna's De Anima (Arabic Text), Being the Psychological Part of Kitāb al-Shifā'*. London: Oxford University Press [= *Nafs*].

Rahman, S., T. Street, and H. Tahiri (eds.). (2008) *The Unity of Science in the Arabic Tradition: Science, Logic, Epistemology and their Interactions*. Dordrecht: Springer.

Rashed, R., and R. Morelon (eds.). (1996) *Encyclopedia of the History of Arabic Science*. London: Routledge.

Raven, M. (ed.). (2020) *The Routledge Handbook of Metaphysical Grounding*. New York: Routledge.

Reisman, D. (ed.). (2003) *Before and after Avicenna: Proceedings of the First Conference of the Avicenna Study Group*. Leiden: Brill.

Rescher, N. (1967) "Avicenna's Logic of Questions." *Archiv für Geschichte der Philosophie* 49, no. 1: 1–6.

Robin, L. (1909–1910) "Sur la conception aristotélicienne de la causalité." *Archiv für Geschichte der Philosophie* 23: 1–28 and 184–210.

Rolland-Gosselin, M. S. (1912) "Les méthodes de la définition d'après Aristote." *Revue des Sciences Philosophiques et Théologiques* 6: 236–252 and 661–675.

Rosenthal, F. (1975) *The Classical Heritage in Islam*. London: Routledge and Kegan Paul.

———. (2007) *Knowledge Triumphant: The Concept of Knowledge in Medieval Islam*. Leiden: Brill.

Ross, W. D. (1924) *Aristotle's Metaphysics: A Revised Text with Introduction and Commentary*. 2 vols. Oxford: Clarendon Press.

———. (1949) *Aristotle's Prior and Posterior Analytics: A Revised Text with Introduction and Commentary*. Oxford: Clarendon Press.

Ruben, D.-H. (1990) *Explaining Explanation*. London: Routledge.

Rudolph, U. (ed.). (Forthcoming) *Grundriss der Geschichte der Philosophie: Philosophie in der Islamischen Welt. Band II, 11.–12. Jahrhundert*. Basel: Schwabe.

Sabra, A. I. (1980) "Avicenna on the Subject Matter of Logic." *Journal of Philosophy* 77: 746–764.

———. (1987) "The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement." *History of Science* 25: 223–243.

Salmieri, G. (2010) "*Aisthēsis*, *Empeiria*, and the Advent of Universals in *Posterior Analytics* II 19." In Lesher 2010a, 155–185.

———. (2014) "Aristotelian *Epistēmē* and the Relation between Knowledge and Understanding." *Metascience* 23: 1–9.

Savory, R. M., and D. A. Agius (eds.). (1984) *Logos Islamikos: Studia Islamica in Honorem Georgii Michaelis Wichens*. Toronto: Pontifical Institute of Mediaeval Studies.

Sayili, A. M. (1939) "The Aristotelian Explanation of the Rainbow." *Isis* 30, no. 1: 65–83.

Schiaparelli, A. (2011) "Epistemological Problems in Aristotle's Concept of Definition: *Topics* vi, 4." *Ancient Philosophy* 31: 127–143.

Schröder, J. (1984) "Eine aristotelische Argument zur Unbeweisbarkeit von Definition." *Archiv für Geschichte der Philosophie* 66: 225–242.

Seeck, G. A. (ed.). (1975) *Die Naturphilosophie des Aristoteles*. Darmstadt: Wissenschaftliche Buchgesellschaft.

Shehaby, N. (1973) *The Propositional Logic of Avicenna*. Dordrecht: Reidel.

Shields, C. (ed.). (2012) *The Oxford Handbook of Aristotle*. Oxford: Oxford University Press.

Smith, R. (1982) "The Relationship of Aristotle's Two *Analytics*." *Classical Quarterly* 32: 327–335.

———. (2009) “Aristotle’s Theory of Demonstration.” In Anagnostopoulos 2009, 51–65.

Sorabji, R. (1980) *Necessity, Cause and Blame: Perspectives on Aristotle’s Theory*. Ithaca, NY: Cornell University Press.

———. (1981) “Definitions: Why Necessary and in What Way?” In Berti 1981, 208–244.

———. (ed.). (1990) *Aristotle Transformed: The Ancient Commentators and Their Influence*. London: Bloomsbury.

———. (ed.). (1997) *Aristotle and After*. Bulletin of the Institute of Classical Studies, supplement 68. London: ICS.

Street, T. (2002) “An Outline of Avicenna’s Syllogistic.” *Archiv für Geschichte der Philosophie* 84: 129–160.

———. (2004) “Arabic Logic.” In Gabbay and Woods 2004, 523–596.

———. (2005) “Logic.” In Adamson and Taylor, 247–265.

———. (2015a) “Arabic and Islamic Philosophy of Language and Logic.” In Zalta 2015.

———. (2015b) “Kātibī, Taḥtānī and the Shamsiyya.” In El-Rouayheb and Schmidtke 2015, 348–374.

———. (2020) “The Reception of *Pointers* 1.6 in Thirteenth-Century Logic: On the Expression’s Signification of Meaning.” In Germann and Najafi 2020, 101–128.

———. (Forthcoming) “Avicenna: Logic (Ibn Sīnā: Logik).” In Rudolph forthcoming.

Strobino, R. (2010) “Avicenna on the Indemonstrability of Definition.” *Documenti e studi sulla tradizione filosofica medievale* 21: 113–163.

———. (2012) “Avicenna’s Use of the Arabic Translations of the *Posterior Analytics* and the Ancient Commentary Tradition.” *Oriens* 40, no. 2: 355–389.

———. (2015a) “Time and Necessity in Avicenna’s Theory of Demonstration.” *Oriens* 43: 338–367.

———. (2015b) “What If That (Is) Why? Avicenna’s Taxonomy of Scientific Inquiries.” In Alwishah and Hayes 2015, 50–75.

———. (2015c) “Principles of Scientific Knowledge and the Psychology of (Their) Intellection in Avicenna’s *Kitāb al-Burhān*.” In Biard 2015, 31–45.

———. (2016a) “Per se, Inseparability, Containment, and Implication: Bridging the Gap between Avicenna’s Theory of Demonstration and Logic of the Predicables” *Oriens* 44: 181–266.

———. (2016b) “Avicenna on Knowledge (*ʿilm*), Certainty (*yaqīn*), Cause (*ʿilla/sabab*) and the Relative (*muḍāf*).” *British Journal for the History of Philosophy* 24: 426–446.

———. (2017) “Avicenna’s *Kitāb al-Burhān* II.7 and Its Latin Translation by Gundissalinus: Content and Text.” *Documenti e studi sulla tradizione filosofica medievale* 28: 105–147.

———. (2018) “Ibn Sina’s Logic.” In Zalta 2018.

———. (2019) “Varieties of Demonstration in Alfarabi.” *History and Philosophy of Logic* 40, no. 1: 42–62.

———. (Forthcoming) “Definition by Composition: Epistemology of Essence in the Greek, Latin, and Arabic Aristotelian Tradition.” In Anstey and Bronstein forthcoming.

Strobino, R., and P. Thom. (2016) “The Logic of Modality.” In Dutilh Novaes and Read 2016, 342–369.

Tahko, T. E. (ed.). (2012) *Contemporary Aristotelian Metaphysics*. Cambridge: Cambridge University Press.

———. (2017) "Empirically-Informed Modal Rationalism." In Fischer and Leon 2012, 29–45.
———. (2018) "The Epistemology of Essence." In Carruth, Gibb, and Heil 2018, 93–110.
Taylor, C. C. W. (1990) "Aristotle's Epistemology." In Everson 1990, 116–142.
Thom, P. (2003) *Medieval Modal Systems: Problems and Concepts*. Aldershot: Ashgate.
———. (2008) "Logic and Metaphysics in Avicenna's Modal Syllogistic." In Rahman, Street, and Tahiri 2008, 361–376.
———. (2012) "Necessity- and Possibility-Syllogisms in Avicenna and Ṭusī." In Dutilh Novaes and Thomassen 2012, 239–248.
———. (2018) "Necessity in Avicenna and the Arabic tradition." In Cresswell 2018, 91–112.
Tierney, R. (2001) "Aristotle's Scientific Demonstrations as Expositions of Essence." *Oxford Studies in Ancient Philosophy* 20: 149–170.
Tiles, J. E. (1983) "Why the Triangle Has Two Right Angles *Kath'Hauto*." *Phronesis* 28: 1–16.
Tuominen, M. (2010) "Alexander and Philoponus on *Prior Analytics* I 27–30: Is There a Tension between Aristotle's Scientific Theory and Practice?" In De Haas, Leunissen, and Martijn 2010, 137–156.
———. (2010) "Back to *Posterior Analytics* II 19: Aristotle on the Knowledge of Principles." In Lesher 2010a, 115–144.
Upton, T. (1981) "A Note on Aristotelian Epagoge." *Phronesis* 26: 172–176.
———. (1983) "Aristotle on Hypothesizing the Genus and Scientific Explanation." *Nature and System* 5: 161–168.
———. (1991) "The If-It-Is Question in Aristotle." *Ancient Philosophy* 11: 315–330.
Vailati, G. (1903) "La teoria aristotelica della definizione." *Rivista di filosofia e scienze affini* 5, no. 2: 4–18.
Van Ess, J. (1966) *Die Erkenntnislehre des ʿAḍudaddīn al-Īǧī: Übersetzung und Kommentar des Ersten Buches seiner Mawāqif*. Wiesbaden: Franz Steiner Verlag.
Van Fraassen, B. (1980) "A Re-examination of Aristotle's Philosophy of Science." *Dialogue* 19: 20–45.
Von Kirchmann, J. H. (1878) *Erläuterungen zu den zweiten Analytiken des Aristoteles*. Leipzig: Dürr.
Wallace, W. (1992) *Galileo's Logical Treatises: A Translation, with Notes and Commentary, of His Appropriated Latin Questions on Aristotle's "Posterior Analytics"*. Dordrecht: Springer.
Wedin, V. E. (1973) "A Remark on Per Se Accidents and Properties." *Archiv für Geschichte der Philosophie* 55: 30–35.
Weil, E. (1975) "The Place of Logic in Aristotle's Thought." In Barnes, Schofield, and Sorabji 1975, 88–112.
Wheeler, M. (1999) "Concept Acquisition in *Posterior Analytics* II.19." *Hermathena* 167: 13–34.
Wians, W. (1989) "Aristotle, Demonstration, and Teaching." *Ancient Philosophy* 9: 245–253.
Wilkins, B. (1970) "Aristotle on Scientific Explanation." *Dialogue* 9: 337–355.
Wilson, M. (1997) "Speusippus on Knowledge and Division." In Kullmann and Follinger 1997, 13–25.
Wisnovsky, R. (ed.). (2001) *Aspects of Avicenna*. Princeton, NJ: Markus Wiener.
———. (2003) *Avicenna's Metaphysics in Context*. Ithaca, NY: Cornell University Press.

Wolfson, H. A. (1938) "The Amphibolous Terms in Aristotle, Arabic Philosophy and Maimonides." *Harvard Theological Review* 31, no. 2: 151–173.

———. (1943) "The Terms *taṣawwur* and *taṣdīq* in Arabic Philosophy and their Greek, Latin, and Hebrew Equivalents." *Muslim World* 33, no. 2: 114–128.

Wolters, G. (ed.). (2004) *Homo Sapiens und Homo Faber*. Berlin: De Gruyter.

Zalta, E.N. (ed.). (2015) *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/spr2015/entries/arabic-islamic-language/.

———. (ed.). (2018) *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/fall2018/entries/ibn-sina-logic/.

Zarepour, S. (2016) "Avicenna on the Nature of Mathematical Objects." *Dialogue* 55, no. 3: 511–536.

Zeuthen, H. G. (1896) "Die geometrische Construction als 'Existenzbeweis' in der antiken Geometrie." *Mathematische Annalen* 47: 222–228.

Zimmermann, F. (1981) *Alfarabi's Commentary and Short Treatise on Aristotle's "De Interpretatione"*. Oxford: Oxford University Press.

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