Handbook of Research on

Advances and Applications of Fuzzy Sets and Logic

Said Broumi

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Handbook of Research on Advances and Applications of Fuzzy Sets and Logic

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Crisp Set vs. Fuzzy Set Qualitative Comparative Analysis, Technical Differences, and Much More.... 1
Jean Michel Rocchi, Sciences Po Aix, France

The idea of a mathematical formalization of logic that dates back to antiquity was only formalized in the 19th century by Charles Boole who proposed a binary system or clear sets. Although the fuzzy set theory is sometimes considered as a simple extension of Boolean algebra, the practical consequences for social science research are fundamental. The QCA has progressively seen the emergence of five increasingly sophisticated variants, the two main ones being the csQCA and the fsQCA. Is what opposes the crisp set qualitative comparative analysis (csQCA) to the fuzzy set qualitative comparative analysis (fsQCA) simply a technical improvement? The answer will be no; beyond different technical tools, there is a paradigm shift affecting in depth the research including epistemological choices, either explicit or implicit. In addition, part of these differences, which at first glance seem essentially methodological, are in fact partly due to a socio-cultural environment and religious values linked to the representation of the world, which is only rarely conscious.

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Sustainable development is among the emerging priorities in the European Union (EU) in the last decade. Waste management plays an important role in ensuring sustainable development. In this study, waste management performances of EU countries were evaluated using fuzzy data envelopment analysis (DEA) method. The data set for the period 2014-2018 was collected from Eurostat, and five input and five output variables were determined for the analyses. Then, the waste management performances of 28 EU countries were analyzed using the Kao-Liu model. Efficiency scores were calculated for 0, 0.5, and 1 α-cut levels, while triangular fuzzy numbers were used in the model. In the analysis, the output-oriented DEA model was preferred, and overall, technical and scale efficiency values were determined. According to the results of the analysis, only Luxembourg and Cyprus are efficient at the lower and upper bounds for 0, 0.5, and 1 α-cut levels in terms of overall efficiency. When the value given for the α-cut is increased, the number of efficient countries also increases.

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Alicia Feldman, Queensland University of Technology, Australia
Deanna Grant-Smith, Queensland University of Technology, Australia

Qualitative comparative analysis allows for the breadth of analysis associated with quantitative data and the depth of case knowledge provided by qualitative analysis. In particular, fuzzy-set qualitative analysis (fsQCA) provides a nuanced analysis of data and offers actionable insights. Despite this, fsQCA has been underutilized in the social sciences. This chapter explores the application of fsQCA in the social sciences. The authors advance an argument for its wider adoption due to fsQCA's ability to disentangle the causal complexity involved in person- and policy-based contexts by applying a set-theoretic understanding of causation. The chapter provides an introduction to fsQCA for readers unfamiliar with the approach and advocates for its suitability within a broad range of social science studies.

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Mehmet Ali Taş, Department of Industrial Engineering,Turkish-German University, Turkey
Esra Çakir, Department of Industrial Engineering, Galatasaray University, Turkey

Health tourism is the set of activities that patients perform to be treated and to feel good. Each health activity should have its own conditions, and gathering these activities under one building requires expert views. Therefore, choosing the site for health tourism in a sustainable environment is a complex problem. It is also becoming more difficult in terms of developing sustainable health tourism as it involves environmental conditions. This study proposes a strategy that helps to appraise sustainable health tourism areas by combining fuzzy SWARA and fuzzy MARCOS methods. This approach weighs specified criteria via fuzzy SWARA according to the expert evaluations and ranks the alternatives via fuzzy MARCOS. This study aims to select sustainable health tourism site according to the criteria by decision makers. Sensitivity analysis is performed by creating different scenarios, and the results are discussed. According to the results of the proposed approach, it is thought that it can be used in the solution of the problem of sustainable health tourism site selection.

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Florentin Smarandache, University of New Mexico, Gallup, USA
Sultan Saleh Alodhaibi, Qassim University, Saudi Arabia
Hamiden Abd El-Wahed Khalifa, Qassim University, Saudi Arabia

Scheduling problem is a problem concerned with searching for an optimal or near optimal schedule subject to a number of constraints. Neuromas approaches have been investigated to solve the problem of scheduling with an aim to determine the sequence of processing jobs on a given set of machines. Its importance and relevance to industry has been promoted. Scheduling requires a variety of activities to accomplish a particular purpose within time and budget. Flow shop is the most classical study manufacturing environment in scheduling literature. This chapter deals with a special structured n-job of three machine flow shop scheduling with piecewise quadratic fuzzy processing times. One of the best

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approximate intervals, namely close interval approximation of PQFNs, is introduced. A fuzzy approach with the help of a heuristic algorithm is developed for minimizing the rental cost of machines under the specified rental cost. A numerical example is given to demonstrate the computational efficiency of the proposed algorithm.

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Mostafijur Rahaman, Indian Institute of Engineering Science and Technology, Shibpur, India
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Banashree Chatterjee, Dr. B. C. Roy Engineering College, India
Shariful Alam, Indian Institute of Engineering Science and Technology, Shibpur, India

This chapter explores the possible application of fractional calculus in the field of operation research, more specifically inventory control problem. The sense of memory can be implemented in a dynamical system with the mathematical manipulation through fractional calculus. In this chapter, some recently published papers on generalized lot-sizing models described by fractional differential equation in crisp as well as uncertain environments are reviewed. The intuitional applicability, obstacles, and challenges for studying the inventory management problems under fractional differential equation (in Riemann-Liouville and Caputo approaches) are discussed in this chapter.

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Sumera Naz, Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan
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Aatifa Saeed, School of Mathematics, Minhaj University, Lahore, Pakistan

The concept of complex q-rung orthopair fuzzy sets (Cq-ROFSs) serves as an extension of the current complex orthopair fuzzy sets, including complex intuitionistic fuzzy sets and complex Pythagorean fuzzy sets. Hamy mean (HM) aggregation operator is a useful tool to model the interrelationship between attributes. In this contribution, the traditional HM is generalized to aggregate complex q-rung orthopair fuzzy information, and a multi-attribute decision-making (MADM) approach is proposed to solve decision-making problems with complex q-rung orthopair fuzzy information. Firstly, the complex q-rung orthopair fuzzy Hamy mean (Cq-ROFHM) operator and the complex q-rung orthopair fuzzy dual Hamy mean (Cq-ROFDHM) operator along with its weighted forms are proposed. Moreover, to deal with MADM problems, a novel approach is developed based on the proposed HM operators under complex q-rung orthopair fuzzy framework. Lastly, a numerical model is described for the selection of enterprise resource planning (ERP) systems to demonstrate the use of the generated technique and exhibit its adequacy.

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Paul Augustine Ejegwa, University of Agriculture, Makurdi, Nigeria
Idoko Charles Onyeke, University of Agriculture, Makurdi, Nigeria

This chapter introduces some new distance and similarity measures for PFSs which resolve the inaccuracies observed in the existing measures. The objectives of this work are to introduce a new distance and similarity measures for PFSs, encapsulate the measures in an algorithm, and explore their applicability in some decision-making problems. Numerical experiments are supplied to ascertain the superiority of the new distance and similarity measures over the existing measures. Some decision-making problems such as students' career placement and pattern recognition represented in Pythagorean fuzzy pairs are addressed using the new distance and similarity measures coded with JAVA programming language.

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Said Broumi, Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco & Regional Center for the Professions of Education and Training (CRMEF), Casablanca-Settat, Morocco

A green environment is one of the sustainability goals, and all nations are striving hard to make the environment free from pollution. The overwhelming corporate sectors should devise strategies towards promoting and preserving the environment from getting degraded by manufacturing activities. To control the environmental degradation, the industries should devise strategies to attain the goal of the green globe through the sub-goals such as livelihood sustainability, community progression, energy conservation, consumer compatibility, and resources revamp. This chapter explores the possible ways of handling the environmental mishaps, and it throws light on the possible contributions of corporate environmental responsibility and citizen science that involves the participatory approaches of the citizens as a whole. To make a decision on 16 proposed strategies, the hybrid decision-making method on multi goals cum strategies is proposed, and the integrated approach of FUCOM (full consistency method fuzzy additive ration assessment) and fuzzy ARAS (fuzzy additive ratio assessment) is applied.

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Characterization of Pythagorean Q-Fuzzy Ideal of Near-Ring 229
Amal Kumar Adak, Ganesh Dutt College, Lalit Narayan Mithila University, Begusarai, India

Pythagorean fuzzy sets are an extension of the intuitionistic fuzzy sets, and they also overcome the limitations of the intuitionistic fuzzy sets. The theory of Pythagorean fuzzy sets possesses significant advantages in handling vagueness and complex uncertainty. Additionally, Pythagorean fuzzy information is used to simulate the ambiguous nature of subjective judgements and measure the fuzziness and impression more flexibly. This chapter presents some useful concepts of Pythagorean fuzzy sub-ring, upper, and lower cut seta. Also, the authors derive some useful properties of Pythagorean Q-fuzzy ideals of near-rings.

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Sunny Joseph Kalayathankal, Jyothi Engineering College, India
Joseph Varghese Kureethara, Christ University, India

One of the most ubiquitous constructions in functional analysis is the spectral measure. The aim of this chapter is to develop the possibility to use the fuzzy spectral measure to define spectral representation theorems. Firstly, the authors introduce the fuzzy spectral measure and examine some of its properties. Furthermore, they elucidate fuzzy Hilbert space, the fuzzy normal, bounded, and adjoint operator on it. Finally, they establish the main result of this study which states that for a given fuzzy integral representation of C(X), there is a unique fuzzy spectral measure E on the Borel subsets of X.

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Hüseyin Kamacı, Yozgat Bozok University, Turkey
Subramanian Petchimuthu, University College of Engineering, Nagercoil, India

In this chapter, the concepts of bipolar q-rung picture fuzzy set and bipolar q-rung picture fuzzy number are introduced. Moreover, the fundamentals of bipolar q-rung picture fuzzy sets and bipolar q-rung picture fuzzy numbers are studied. Relatedly, some basic operations, aggregation operators, and relations on the bipolar q-rung picture fuzzy sets/numbers are derived. The intuitive definition of Tanimoto's similarity coefficient is proposed to measure the similarity between two bipolar q-rung picture fuzzy sets. Finally, it is shown that the proposed Tanimoto similarity measures can be used to deal with the problems of medical diagnosis and pattern recognition under the bipolar q-rung picture fuzzy environment.

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Didem Guleryuz, Bayburt University, Turkey

This chapter aims to propose prediction models to estimate Turkey's manufacturing sector's capacity utilization rate between 2008-2019 monthly basis using the adaptive neuro-fuzzy inference system (ANFIS) with genetic algorithm (GA) and particle swarm optimization (PSO) via determined indicators. The model's accuracy will be tested using some of the performance evaluation criteria, namely mean absolute error (MAE), root mean square error (RMSE), and the coefficient of determination (R2) values were used to compare the prediction ability. The coefficient of determination for GA-ANFIS, PSO-ANFIS, and ANFIS models are 0.9787, 0.9786, and 0.9679 in the training phase and 0.9591, 0.7677, and 0.7264 in the testing phase, respectively. The study results showed that the GA-ANFIS model showed better predictive ability with the least prediction error among other models. As a result, ANFIS, whose parameters are adjusted with GA, can predict the Turkish capacity utilization rate in the manufacturing industry with high accuracy.

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K. Bhargavi, Siddaganga Institute of Technology, India

Uncertainty is the difference between actual information and obtained information which can exists in two different forms: one is randomness, and another is impreciseness. The fuzzy soft sets are widely used in a variety of applications in the fields of medical science, automation industry, engineering, economics, share market, social science, game theory, operational research, and so on. Chemical engineering, electrical engineering, mechanical engineering, computer science engineering, and telecommunication engineering are prone to a variety of uncertainty in terms of system uncertainty and parameter uncertainty. Hence, with the application of fuzzy soft set, the uncertainty in the mentioned engineering types can be handled very well. In this chapter, fuzzy soft set is applied for handling the uncertainty in the mentioned engineering domains. The performance achieved by the fuzzy soft set-enabled computer science and engineering and biotechnology is found to be good towards the performance metrics like response time, throughput, resource utilization rate, and error rate.

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Reeta Bhardwaj, Department of Mathematics, Amity University Haryana, Gurugram, India
Amit Sharma, Department of Mathematics, Amity University Haryana, Gurugram, India
Naveen Mani, Chandigarh University, Chandigarh, India
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Fuzzy sets, as well as their extension intuitionistic fuzzy sets (IFS), are more effective and appealing tools for expressing quantitative complexity throughout the decision-making process, and they have gotten greater attention from researchers in recent years for new study directions. Keeping the advantages of IFS, this chapter proposes a novel information measure to measure the fuzziness of IFS known as entropy measure (EM). The various notable features of the proposed EM are also presented. EM is a very useful tool to determine the attribute's weight during the multi-attribute decision-making (MADM) process. Therefore, in this chapter, the authors propose a new MADM approach by using the proposed EM where the attribute's weights are absolutely unknown. Finally, the proposed MADM methodology is applied to solve the real-life MADM examples. Comparative studies are also given to show the advantages of the proposed MADM methodology. The proposed MADM methodology can overcome the weaknesses of the existing MADM approaches.

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Nimet Yapıcı Pehlivan, Selçuk University, Turkey
Yasemin Günter, Selçuk University, Turkey

Fuzzy set theory proposed by Zadeh is introduced to cope with the imprecision and uncertainty. Intuitionistic fuzzy sets (IFS) introduced by Atanassov are a generalization of fuzzy sets characterized by a membership function and a non-membership function. Various multi-criteria decision-making (MCDM) methods based on IFSs have been proposed. Global competitiveness index (GCI) provides information about the competitiveness of the countries based on data collected by the World Economic Forum in a

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report. Competitiveness of the country is the economic strength necessary for an economy to upgrade its economic prosperity and standard of living. In the report, three main criteria—factor, efficiency, and innovation—and related 12 sub-criteria are covered for the evaluation of the countries. In this chapter, MCDM methods of TOPSIS, WASPAS, and COPRAS based on IFSs are applied to data obtained from the GCI report, and ranking of the countries is obtained to investigate relationship.

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Pairote Yiarayong, Pibulsongkram Rajabhat University, Thailand

Theory of hesitant fuzzy sets is an important tool in algebraic systems, fuzzy sets theory, partially ordered sets, data mining, etc. The aim of this chapter is to apply prime hesitant fuzzy ideal for dealing with several kinds of theories in semigroups. The concepts of prime hesitant fuzzy, weakly prime hesitant fuzzy, completely prime hesitant fuzzy, and weakly completely prime hesitant fuzzy ideals on semigroups have been introduced. The authors deduce some fascinating properties and characterizations for these hesitant fuzzy ideals. Finally, they obtain sufficient conditions of a prime hesitant fuzzy (weakly prime hesitant fuzzy) in order to rephrase a completely prime hesitant fuzzy (weakly completely prime hesitant fuzzy) ideal in a semigroup.

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S. Amanathulla, Raghunathpur College, India
Madhumangal Pal, Vidyasagar University, India

A picture fuzzy set is an extension of intuitionistic fuzzy set. In this set, a new parameter called neutral value of an object is incorporated. Based on this set, the picture fuzzy graph is defined and investigated by several scientists. Picture fuzzy graph is a new type of fuzzy graph, and it is used for solving many real life problems. In this chapter, the concept of picture fuzzy set is presented. It also introduced the picture fuzzy graph and investigated some useful properties of picture fuzzy graph. Some fundamental terms like degree of a vertex, order and size of picture fuzzy graph, strong picture fuzzy graph, complete picture fuzzy graph, independent strong and independent weak picture fuzzy edge, bipartite picture fuzzy graph, complement of a picture fuzzy graph, path, strength of a path, connectedness, homomorphism, isomorphism, automorphism, Cartesian product, composition, etc. are defined and presented some properties. An application is provided to select best airline route among multiple paths.

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Adem Yolcu, Kafkas University, Turkey
Taha Yasin Öztürk, Kafkas University, Turkey

The fuzzy parameterized fuzzy soft (FPFS) set structure was defined in 2010 as a different combination of fuzzy sets and soft sets. In 2016, the topological structure on this structure was defined. By using this set structure and topological concepts, different studies have been made and continue to be done on many decision-making problems and daily life problems. In this chapter, the authors define and investigate some properties of fuzzy parameterized fuzzy soft (FPFS) Ti (i=0,1,2,3,4) spaces using fuzzy parameterized fuzzy soft points. They also discuss some hereditary properties of these spaces. Further, they introduce fuzzy parameterized fuzzy soft connected spaces. Moreover, they explore some basic properties of these spaces.

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Mamoni Dhar, Science College, Kokrajhar, India

From the beginning of the theory of fuzzy sets, it has been a matter of concern for many researchers in the field to find an appropriate connection between probability and possibility. One such effort can be found in the randomness-fuzziness consistency principle where a fuzzy number of the type [α,β,γ] is expressed as a combination of two probability laws. The main intention of this work is to show that a fuzzy number of the type [α,β,β] or [β,β,γ] can be expressed by one probability law with the help of superimposition of uniformly fuzzy intervals.

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Vassilis C. Gerogiannis, University of Thessaly, Greece
Dimitrios Tzimos, Hellenic Open University, Greece
Le Hoang Son, Vietnam National University, Vietnam

Requirements prioritization (RP) is a crucial process which aims to evaluate candidate software requirements to be implemented in the next release of a software system. RP is often applied iteratively, according to various criteria, by multiple stakeholders, who may have different roles and needs. The problems encountered during the RP of large sets of requirements can be addressed by following a systematic large-scale decision-making (LSDM) process. The aim of this chapter is to propose an LSDM RP process which applies multi-criteria evaluation based on intuitionistic fuzzy sets (IFSs). IFSs are suitable to consider stakeholders hesitation for criteria importance and requirements ratings. The proposed process is supported by a recommender system which provides suggestions to stakeholders and also guide them to collaboratively reach consensus for the final RP. The proposed process is tested using an illustrative example. The results are promising since they demonstrate that the suggested process can effectively support multiple stakeholders to prioritize large requirements sets.

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Aynur Yonar, Selçuk University, Turkey
Nimet Yapıcı Pehlivan, Selçuk University, Turkey

Multi-criteria decision making (MCDM) is a process of evaluating and ranking the various alternatives with respect to the criteria using subjective judgments of decision maker(s). Since the classical MCDM methods cannot deal with imprecise and uncertain information in the decision-making process, MCDM methods based on Type-1 fuzzy sets are suggested. Afterwards, Type-2 fuzzy set characterized by a fuzzy membership function was introduced as an extension to Type-1 fuzzy sets. Interval Type-2 fuzzy sets (IT2FSs) are the most widely used Type-2 fuzzy sets. Various MCDM methods based on IT2FSs have been proposed and applied on different fields. In this chapter, two integrated methods are introduced by using AHP, COPRAS, and VIKOR methods based on IT2FSs. The proposed integrated methods are carried out to evaluate life quality of 28 European Union countries and another six countries, Iceland, Macedonia, Montenegro, Serbia, Turkey, and Kosovo with respect to 30 subjective criteria.

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Ijeoma Abigail Onyeozili, University of Abuja, Nigeria
Ahmed Yusuf Omeiza, Federal University, Dutsinma, Nigeria

A weighted intuitionistic fuzzy soft set (WIFSS) is an important generalization of intuitionistic fuzzy soft set (IFSS). In this research work, the authors define the basic binary set operations, such as union, restricted union, restricted intersection, extended intersection, restricted difference, sum, AND-product, OR-product, and some concepts on weighted intuitionistic fuzzy soft set. They also present some properties of the operations and established some fundamental results including various De Morgan's type of results in weighted intuitionistic fuzzy soft set context.

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Juanjuan Ding, Shanxi University, China
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Deyu Li, Shanxi University, China
Said Broumi, Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco

Neutrosophic theories and methods are suitable for handling a variety of uncertain information, especially in complex decision-making situations. Nowadays, how to study multi-attribute group decision-making (MAGDM) problems in neutrosophic environments is a vital task for scholars and practitioners. Granular computing is a popular mathematical framework for uncertain problems, and it has been proved as a useful tool for solving MAGDM problems. In this chapter, a new neutrosophic multigranulation model for MAGDM is proposed, and some theoretical models and decision-making algorithms are presented. Finally, some case studies and comparative analysis are performed to show the merit of the presented models and algorithms. In general, this chapter is conducive to neutrosophic decision-making communities and intelligent decision-making methods.

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Suriana Alias, Universiti Teknologi MARA, Malaysia
Daud Mohamad, Universiti Teknologi MARA, Malaysia
Adibah Shuib, Universiti Teknologi MARA, Malaysia

The effect of uncertainty in information raises human logical thinking due to many variabilities from expert decision making. The generalization of rough neutrosophic multisets theory tries to overcome this problem. The rough neutrosophic multisets allow the multiplicity of information represented by truth membership sequences, indeterminacy membership sequences, and falsity membership sequences. The lower and upper approximations belong to this triple membership sequence, making the information result more valuable since the relationship among this membership sequence makes sense. This chapter introduces the generalization of rough neutrosophic multisets with some operators and an illustrative example. The rough neutrosophic multisets basic operators, algebraic operators, and "AND" and "OR" operators are defined with some property propositions.

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Saliha Karadayi-Usta, Fenerbahce University, Turkey

Interpretive structural modeling is of paramount importance in barrier/risk/challenge analysis as a hierarchical visual map by emphasizing the root cause of the problems. It asks expert opinions to evaluate the causal relationships of identified variables. However, in many cases experts cannot determine a relationship or can be doubtful about stating an idea. In the meantime, the neutrosophic cognitive mapping provides a step-by-step guidance in order to deal with the indeterminate relationships. Therefore, this study aims to propose a neutrosophic ISM approach and to implement it for the medical tourism services' barriers during the COVID-19 pandemic as an illustrative example. In order to do that, text mining was conducted to the medical tourism-related tweets written in English from January to December 2020 via RapidMiner software, and the barriers in medical tourism were identified. Next, the relationships between these barriers were examined via expert evaluations, and the proposed neutrosophic ISM was applied to construct a structural model.

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Saliha Karadayi-Usta, Fenerbahce University, Turkey

The circular economy movement has paved the way for the sustainable supply chains with its nature-friendly structure via recycling and repeated use of products. Hence, this study aims to determine the factors affecting the paper packaging industry in the circular economy in order to provide a conceptual model of the cause-and-effect relationships through neutrosophic cognitive mapping. Paper packaging machinery manufacturer Windmöller & Hölscher's representatives have been in-depth interviewed to gather their opinions. Since there are some relations that cannot be determined (i.e., existence of indeterminacy), neutrosophy became a necessity in this chapter. Delphi technique is implemented to obtain a single neutrosophic adjacency matrix. As a result, the sustainable supply chain structure based on environmental awareness and recycling has been modeled visually, and the causal relationships were determined.

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Nouran M. Radwan, Sadat Academy for Management Sciences, Egypt
Wael K. Hanna, Sadat Academy for Management Sciences, Egypt

In recent years, e-learning has become a revolutionary competitive method. Adapting the content according to learner knowledge is a current challenge in e-learning systems. Currently, most of the e-learning systems evaluate the learner's knowledge level according to crisp responses that are taken during the learning process. Therefore, one of the most significant challenges in e-learning is how to improve the course adaptation in order to achieve high-quality interaction for all learners. Adaptation is an efficient way to help learners to learn their learning activities in easy and a suitable ways. However, there are many factors that lead to uncertainty about the learner evaluation process. This chapter presents a novel approach to handle imprecision, vagueness, ambiguity, and inconsistency in the learner evaluation process to recommend the suitable learning material according to the learner's knowledge level.

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Florentin Smarandache, University of New Mexico, USA

The advance payment of acquiring cost is mostly encouraged by both supplier and retailer in current situations. This chapter discusses the economic order quantity model with multiple payments of acquiring cost in advance at equal intervals. The current research examines two-warehouse economic order quantity model with imperfect items under neutrosophic environment by considering the proportion of defective product as triangular fuzzy neutrosophic number. An inventory model is framed, and the proposed model is formulated under triangular neutrosophic environment. The expected total profit and the optimal order quantity have been computed under both the crisp and the neutrosophic sense. De-neutrosophication of the proposed model has been done using expected value of triangular fuzzy neutrosophic number. The numerical illustration is analyzed in both crisp and neutrosophic situation along with the comparative studies. The sensitivity analysis is performed to show the effect of neutrosophication on defective rate along with the changes in parameters.

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Irvanizam Irvanizam, Department of Informatics, Universitas Syiah Kuala, Indonesia
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Many prestigious researchers have been exploring the issues of imprecision, inconsistency, and uncertain information in decision making, which are still challenges in developing a decision support system that is more feasible and efficient. This chapter proposes a new multicriteria group decision-making (MCGDM) strategy to overcome those issues. This strategy integrates the original MABAC method with trapezoidal fuzzy neutrosophic numbers (TrFNNs). First, the proposed method converts independent judgments from experts in the form of linguistic variables into TrFNNs and aggregates them using some aggregation operators. Next, it utilizes the functions of score and accuracy to rank the evaluated alternatives. By using the distance measure between the alternatives and the border approximation area, the proposed MABAC selects the best solution. Finally, the chapter illustrates an example of COVID-19 vaccine selection and a comparative analysis to show that the proposed MABAC has benefits to support indeterminate information and is more reasonable and practicable in handling MCGDM problems.

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Neutrosophic TOPSIS Method for Sustainable Supplier Selection in a Discount Market Chain..... 692
Nimet Yapıcı Pehlivan, Selçuk University, Turkey
Neşe Yalçın, Adana Alparslan Türkeş Science and Technology University, Turkey

Sustainable supplier selection is one of the most important decisions for companies in sustainable supply chain management. Therefore, sustainable supplier performance evaluation and selection are of great importance in terms of economic, environmental, and social aspects of sustainable development. Sustainable supplier selection problem can be considered as a multiple-criteria decision-making (MCDM) problem. In the MCDM problems, decision-makers evaluate conflicting criteria and alternatives according to their own preferences/judgments. One of the most well-known MCDM methods is the technique for order preference by similarity to an ideal solution (TOPSIS). Neutrosophic set (NS) is an extension of fuzzy set where each element of the universe has the degrees of truth, indeterminacy, and falsity. A single-valued neutrosophic set (SVNS) is a special case of the neutrosophic set. This chapter aims to evaluate sustainable supplier selection in a Turkish discount market chain using the single-valued neutrosophic TOPSIS method based on normalized Euclidean and Hamming distances.

Chapter 32
Inverse Single-Valued Neutrosophic Soft Set and Its Application..... 716
Somen Debnath, Umakanta Academy, India

Single-valued neutrosophic set (SVNS) can be viewed as an instance of the neutrosophic set (NS) proposed by Smarandache in 2005 to solve real decision-making problems that contain imperfect knowledge of the domain. Embedding the idea of SVNS with the soft set (SS), a new mathematical structure known as a single-valued neutrosophic soft set (SVNSS) is introduced to model the vague concept parametrically. In this chapter, the authors introduce a new soft model, namely inverse single-valued neutrosophic soft set (ISVNSS), that can be perceived as another mathematical model which is used to solve the purposes that are just opposite in the context of SVNSS. Also, they define different properties and operators associated with ISVNSSs. Finally, by using the ISVNSSs, an algorithm has been introduced that will serve the purpose of decision-makers to solve various real decision-making problems, and it will enable to unveil another type of uncertainty domain hidden in nature. This new approach surely provides more information to the decision-makers before they make any decisions on various attributes.

Chapter 33
Neutrosophic Fuzzy Soft Matrix Theory and Its Application in Group Decision Making 741
Somen Debnath, Umakanta Academy, India

In 2020, Sujit et al. introduced the notion of the neutrosophic fuzzy set (NFS) to deal with uncertainty under a fuzzy environment where the fuzzy membership grade of every element of the field of the domain is associated with three independent neutrosophic components, namely truth, indeterminacy, and falsity membership grade. The main focus of the chapter is to introduce a neutrosophic fuzzy soft set (NFSS) to deal with uncertainty parametrically, and it gives the approximate solution to the problem. NFSS is a mixture of a neutrosophic set, fuzzy sets, and soft sets. So, it can be treated as a hybrid structure that

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gives more flexibility to solve multi-criteria decision-making problems under a fuzzy environment. NFSS evolved as an extension of fuzzy set, soft set, neutrosophic set, etc. Then the authors propose an algorithmic approach for group decision-making (GDM) problems using a neutrosophic fuzzy soft matrix (NFSM) and its related properties. Finally, an illustrative example shows the applicability of the proposed approach.

Chapter 34

Quadripartitioned Neutrosophic Soft Graphs 771
Satham Hussain S., Jamal Mohamed College, India
Jahir Hussain R., Jamal Mohamed College, India
Isnaini Rosyida, Universitas Negeri Semarang, Indonesia
Said Broumi, Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco & Regional Center for the Professions of Education and Training (CRMEF), Casablanca-Settat, Morocco

The neutrosophic quadripartitioned soft model is a hybrid model by combining neutrosophic soft sets with quadripartitioned sets. This work concerns with the quadripartitioned neutrosophic soft graphs for treating neutrosophic soft information by employing the theory of quadripartitioned neutrosophic soft sets with graphs. Operations like Cartesian product, cross product, lexicographic product, and strong product of quadripartitioned neutrosophic soft graphs are established. The proposed concepts are explained with examples.

Chapter 35

Priority Weighted Neutrosophic Refined Soft Set: A Multi-Criteria Decision-Making Approach 796
C. Antony Crispin Sweety, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, India
Sumathi I. R., Department of Mathematics, Amrita School of Engineering, Coimbatore, India
Aishwaryapriyadharshini G., Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, India

This chapter devises a new concept of priority weighted neutrosophic refined soft set (PWNRSS) by combining neutrosophic refined sets and soft sets implemented with prioritized universal elements and weightage-imposed parameters. The concepts of PWNRS subset, PWNRS null set, and PWNRS universal set are defined. Based on the definitions of n-norm and n-co-norm, the theoretical operations of PWNRS sets such as union, intersection, and complement are defined. AND-product and OR-product between two priority-weighted neutrosophic refined soft sets are introduced. Furthermore, priority weights neutrosophic refined soft set is expanded to MCDM technique to handle decision-making issues. TOPSIS has been examined more thoroughly for the PWNRSS decision-making issue. This proposed method might be extremely valuable in large-scale decision-making situations. Numerical examples are also provided to demonstrate the methodologies' dependability and applicability.

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Chapter 36
Neutrosophic Contra RW-Continuous Functions in Neutrosophic Topological Spaces..... 833
D. Savithiri, Sree Narayana Guru College, Coimbatore, India
Saeid Jafari, Mathematical and Physical Science Foundation, Denmark
C. Janaki, L. R. G. Government Arts College, Tirupur, India

The aim of this chapter is to introduce a new class of contra continuous functions, namely neutrosophic contra RW-continuous functions, in neutrosophic topological spaces. Further, the authors introduce neutrosophic almost RW-continuous function, neutrosophic almost contra RW-continuous function, and some of its properties have been discussed. Relationships with some other existing neutrosophic contra continuous functions have been analyzed. Furthermore, NRW-T0, NRW-T1, NRW-T2, and NRW-normal spaces have been introduced.

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Foreword

It was a pleasure to be invited to write the foreword for *Handbook of Research on Advances and Applications of Fuzzy Sets and Logic*. Learning about fuzzy sets opened my mind to a new range of possibilities in analysis. It changed the way I view the world.

The concepts of fuzzy logic and fuzzy sets were introduced through mathematician Lotfi Zadeh's fuzzy set theory in 1965. Unlike classical logic, which allows for only true or false conclusions, fuzzy logic deals with vagueness and imprecision. Similarly, unlike Boolean sets, which allow for only figures of 0 or 1, fuzzy sets embrace a range of measures falling between 0 and 1. Rather than forcing binary membership in sets, fuzzy sets allow for membership anywhere on a continuum from 0 to 1. Harnessing the power of fuzzy sets and fuzzy logic allows researchers to explore a range of scenarios where information is incomplete or imprecise.

Due to increasing attention in recent years working with fuzzy logic and sets is a dynamic and exciting field as the theory underpinning these ideas continues to develop and grow. Notable developments include the introduction of intuitionistic fuzzy sets, Pythagorean fuzzy sets and neutrosophic sets, which have allowed for neutrality, a broader range of uncertainty, and indeterminate membership in sets.

Fuzzy logic has moved far beyond the realm of mathematics and continues to grow traction in many fields. In fields where probability theory has been the dominant paradigm for decades, fuzzy set theory offers a new complementary approach. As a result, fuzzy sets and logic are being applied in increasingly diverse applications and contexts to great effect.

This book provides a unique offering by bringing together the work of scholars of numerous disciplines from across the globe. Topics range from discussions of the paradigmatic and epistemological underpinnings of fuzzy set theory, to theoretical and methodological presentations of fuzzy set adaptations and methods. Other contributors have presented empirical applications harnessing fuzzy sets, applying fuzzy sets and logic to a range of contexts including waste management practices, sustainable health tourism, capacity utilisation rates in the manufacturing industry, engineering applications, economic competitiveness of countries, group decision making, the paper packaging industry, eLearning, and supply chain management. The methods applied in these chapters are also diverse, demonstrating the flexibility and adaptability of fuzzy sets and logic.

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Researching in this space, particularly in applying fuzzy set Qualitative Comparative Analysis, I have found an incredibly supportive group of researchers, with a genuine sense of community, despite being from diverse backgrounds and levels of expertise. This book provides additional insights for those already familiar with fuzzy sets and logic. I am confident that this collection of edited chapters will also encourage and support others to delve into theories and methods that are new to them.

Across the globe there is a tight-knit community of people who are passionate about fuzzy sets and logic. We encourage you to join us.

Alicia Feldman
Queensland University of Technology, Australia

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Preface

The notion of fuzzy sets has been widely utilized to depict various imprecise and vague information existing in the real world and their realistic applications are appearing in all walks of life nowadays. Originated from the fuzzy set theory, fuzzy logic serves as a dedicated logic of imprecision and approximate reasoning, and it may be viewed as an attempt at formalization/mechanization of these two remarkable human capabilities. During the past decades, fuzzy sets and logic have become an important research branch in fuzzy mathematics.

Handbook of Research on Advances and Applications of Fuzzy Sets and Logic studies recent theoretical advances of fuzzy sets and numbers, fuzzy systems, fuzzy logic and their generalizations, extensions, and much more.

Neutrosophy, as a new branch of philosophy and a generalization of dialectics, is based on the dynamics of opposites and the neutralities between them, and it has been extended to Refined Neutrosophy, and consequently the Neutrosophication was extended to Refined Neutrosophication. Whence, Regret Theory, Grey System Theory, and Three-Ways Decision are particular cases of Neutrosophication and of Neutrosophic Probability. We have further extended the Three-Ways Decision to n-Ways Decision, the last one is a particular case of Refined Neutrosophy.

Neutrosophy is also an extension of the international movement called Paradoxism (based only on contradictions in science and literature) .

Neutrosophic Set, defined on three components {membership (T), indeterminacy (I), and nonmembership (F)} is a generalization of Crisp Set, Fuzzy Set, Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set, q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set. Neutrosophic Set has been further extended to Refined Neutrosophic Set. Further on, as extension and alternative there was defined the Plithogenic Set based on MultiVariate Analysis.

This book also explores the applications of fuzzy sets and logic applied to science, technology, and everyday life to further provide research on the subject. This book is ideal for mathematicians, physicists, computer specialists, engineers, practitioners, researchers, academicians, and students who are looking to learn more about fuzzy sets, fuzzy logic, and their applications.

We believe that this book provides useful guidance for learning the current progress on advances and applications of fuzzy sets and logic. In total, 36 chapters have been presented to reflect the call of the thematic vision. The contents of these studies are briefly summarized as follows.

In the first chapter entitled “Crisp Set vs. Fuzzy Set Qualitative Comparative Analysis, Technical Differences, and Much More,” Rocchi discussed qualitative comparative analysis, technical differences and many other relations between crisp sets and fuzzy sets.

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In Chapter 2 entitled "A Fuzzy Data Envelopment Analysis-Based Performance Assessment of European Union Countries' Waste Management Practices," Altin explored waste management performances of European Union countries via using the fuzzy data envelopment analysis method.

In Chapter 3, Feldman and Grant-Smith present "Fuzzy Set Qualitative Comparative Analysis: Application of Fuzzy Sets in the Social Sciences," which provides an introduction to fuzzy set qualitative analysis for readers who are unfamiliar with the technique/ approach, and demonstrates its suitability within a broad range of social science studies.

In Chapter 4, entitled "A Hybrid Fuzzy MCDM Approach for Sustainable Health Tourism Sites Evaluation," Taş and Çakir put forward an innovative strategy that helps to appraise sustainable health tourism areas by combining fuzzy SWARA with fuzzy MARCOS methods, which can be used in the solution of the sustainable health tourism site selection problems.

In Chapter 5, entitled "A Fuzzy Approach for Minimizing Machines Rental Cost for a Specially Structured Three-Stages Flow-Shop Scheduling Problem in a Fuzzy Environment," Khalifa et al. explored a special structured n-job of three machines flow shop scheduling with piecewise quadratic fuzzy processing times.

In Chapter 6, Rahaman et al. presented "Application of Fractional Calculus on the Crisp and Uncertain Inventory Control Problem," which explores the possible application of fractional calculus in the field of operation research, especially the inventory control problem.

In Chapter 7, "A Hybrid Multiple Attribute Decision-Making Model Under Complex Q-Rung Orthopair Fuzzy Hamy Mean Aggregation Operators," Naz et al. generalized the traditional Hamy mean aggregate operators to the context of complex q-rung orthopair fuzzy sets, and a corresponding multiple attribute decision-making approach is proposed based on the proposed aggregation operators.

In Chapter 8 entitled "Some New Distance and Similarity Algorithms for Pythagorean Fuzzy Sets With Application in Decision-Making Problems: Some New Distance and Similarity Algorithms for PFSs," Ejegwa and Onyeke introduced some new distance and similarity measures for PFSs which resolve the inaccuracies observed in the existing measures, and they further explored their applicability in some decision-making problems.

In Chapter 9, "Fuzzy Decision Making on the Impacts of Corporate Environmental Responsibility and Citizen Science Approaches Towards Green Globe," Martin et al. explore the possible ways of handling the environmental mishaps and it throws light on the possible contributions of corporate environmental responsibility and citizen science that involves the participatory approaches of the citizens as a whole.

In Chapter 10, "Characterization of Pythagorean Q-Fuzzy Ideal of Near-Ring," Adak presented some useful concepts of pythagorean fuzzy sub-ring, upper and lower cut set, some useful properties of pythagorean q-fuzzy ideals of near-rings are derived as well.

In Chapter 11, "Fuzzy Integral Representation Associated With Spectral Measure," George et al. developed the possibility to use the fuzzy spectral measure to define spectral representation theorems.

In Chapter 12, Kamacı and Petchimuthu presented "Tanimoto Similarity Coefficients Measuring Bipolar Q-Rung Picture Fuzzy Information and Their Applications," which investigates the concepts of bipolar q-rung picture fuzzy set and bipolar q-rung picture fuzzy number from various aspects, and the presented theoretical models are further used for solving the problems of medical diagnosis and pattern recognition under the bipolar q-rung picture fuzzy environment.

In Chapter 13, "Prediction of Capacity Utilization Rate for Turkey Using Adaptive Neuro-Fuzzy Inference System With Particle Swarm Optimization and Genetic Algorithm," Guleryuz proposed prediction models to estimate Turkey's manufacturing sector's capacity utilization rate between 2008-2019

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monthly basis using the adaptive neuro-fuzzy inference system with genetic algorithm and particle swarm optimization via determined indicators.

In Chapter 14, Bhargavi presented "Fuzzy Soft Set and Its Engineering Applications." In specific, the fuzzy soft set is applied for handling the uncertainty in engineering domains. The performance achieved by the fuzzy soft set enabled computer science and engineering and biotechnology is found to be good towards the performance metrics like response time, throughput, resource utilization rate and error rate.

In Chapter 15, "An intuitionistic Fuzzy Entropy Measure and Its Application in Multi-Attribute Decision Making With Incomplete Weights Information," Bhardwaj et al. mainly proposed a new multiple attribute decision-making approach by using the proposed EM where the attribute's weights are absolutely unknown.

In Chapter 16, "Evaluation of the Global Competitiveness Index (GCI) by Multi-Criteria Decision-Making Methods Based on Intuitionistic Fuzzy Sets: Comparative Analysis," Pehlivan and Günter provided MCDM methods of TOPSIS, WASPAS and COPRAS based on IFSs which are applied to data obtained from the GCI report and ranking of the countries is obtained to investigate relationship between.

In Chapter 17, Yiarayong presented "On Prime Hesitant Fuzzy Ideals in Semigroups," which mainly applies prime hesitant fuzzy ideal for dealing with several kinds of theories in semigroups.

In Chapter 18, Amanathulla and Pal have defined some fundamental terms like, degree of a vertex, order and size of picture fuzzy graph, strong picture fuzzy graph, complete picture fuzzy graph, independent strong and independent weak picture fuzzy edge, bipartite picture fuzzy graph, complement of a picture fuzzy graph, path, strength of a path, connectedness, homomorphism, isomorphism, automorphism, Cartesian product, composition, etc. Also, an application is provided to select best airline route among multiple paths.

In Chapter 19, Yolcu and Öztürk presented "Separation Axioms and Connectedness on Fuzzy Parameterized Fuzzy Soft Topological Spaces." The authors investigated some properties of fuzzy parameterized fuzzy soft spaces via using fuzzy parameterized fuzzy soft points and also discussed some hereditary properties of these spaces. Moreover, they introduced fuzzy parameterized fuzzy soft connected spaces with some basic properties of these spaces.

In Chapter 20 entitled "Probability and Fuzziness: A Case Study," Dhar showed that a fuzzy number of the combination of two probability laws can be expressed by one probability law with the help of superimposition of uniformly fuzzy intervals.

In Chapter 21 entitled "Intuitionistic Fuzzy Sets in Large-Scale Software Requirements Prioritization," Gerogiannis et al. explored a large-scale decision-making requirements prioritization process which applies multi-criteria evaluation based on intuitionistic fuzzy sets.

In Chapter 22, "Comparison of Integrated Multi-Criteria Decision-Making Methods Based on Interval Type-2 Fuzzy Sets: An Application to Life Quality Evaluation," Yonar and Pehlivan put forward two integrated methods by using AHP, COPRAS, and VIKOR methods based on IT2FSs. The proposed integrated methods are carried out to evaluate life quality of 28 European Union countries and other six countries with respect to 30 subjective criteria.

In Chapter 23, Msirali et al presented "Some Results on Weighted Intuitionistic Fuzzy Soft Set," which mainly defines the basic binary set operations, such as union, restricted union, restricted intersection, extended intersection, restricted difference, sum, AND-product, OR-product and some concepts on weighted intuitionistic fuzzy soft sets.

In Chapter 24, "A New Neutrosophic Multigranulation Model for Multi-Attribute Group Decision Making," Ding et al. proposed a new neutrosophic multigranulation model for MAGDM. This chapter

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describe both some theoretical models and decision making algorithms. Finally, some case studies and comparative analysis are discussed to show the merit of the proposed models and algorithms. In general, this chapter is conducive to neutrosophic decision making communities and intelligent decision making methods.

In Chapter 25, "Rough Neutrosophic Multisets: Operations Over Rough Neutrosophic Multisets," Alias et al. introduced the generalization of rough neutrosophic multisets with some operators and an illustrative example. The rough neutrosophic multisets basic operators, algebraic operators, and a "AND" and "OR" operators are defined with some property propositions.

In Chapter 26, "A Novel Neutrosophic Interpretive Structural Modeling Approach: Hierarchical Visual Graphs of Indeterminate Causalities," Karadayi-Usta proposed a neutrosophic ISM approach and implemented it for the medical tourism services' barriers during the COVID-19 pandemic as an illustrative example.

In Chapter 27, Karadayi-Usta presented "The Role of the Paper Packaging Industry in the Economy: The Causal Relationship Analysis via Neutrosophic Cognitive Maps," which determines the factors affecting the paper packaging industry in the circular economy to provide a conceptual model of the cause-and-effect relationships through neutrosophic cognitive mapping.

In Chapter 28, Radwan and Hanna presented "An Adaptive eLearning Sequence Based on Neutrosophic Logic," which puts forward a novel approach to handle imprecision, vagueness, ambiguity, and inconsistency in learner's evaluation process as to recommend the suitable learning material according to learner's knowledge level.

In Chapter 29, "Application of Triangular Fuzzy Neutrosophic Number to an Effective Economic Order Quantity Model," Rajeswari et al. examined two-warehouse economic order quantity model with imperfect items under neutrosophic environment by considering the proportion of defective product as triangular fuzzy neutrosophic number.

In Chapter 30, "Improved MABAC Method for Multicriteria Group Decision-Making With Trapezoidal Fuzzy Neutrosophic Numbers," Irvanizam and Zi proposed a new multicriteria group decision-making strategy to overcome those issues, which integrates the original MABAC method with trapezoidal fuzzy neutrosophic numbers.

In Chapter 31, Pehlivan and Yalçın presented "Neutrosophic TOPSIS Method for Sustainable Supplier Selection in a Discount Market Chain," which aims to evaluate sustainable supplier selection in a Turkish discount market chain using the single-valued neutrosophic TOPSIS method based on normalized Euclidean and Hamming distances.

In Chapter 32, "Inverse Single-Valued Neutrosophic Soft Set and Its Application," Debnath introduced a new soft model namely inverse single-valued neutrosophic soft sets that can be perceived as another mathematical model which is used to solve the purposes that are just opposite in the context of single-valued neutrosophic soft sets.

In Chapter 33, Debnath presented "Neutrosophic Fuzzy Soft Matrix Theory and Its Application in Group Decision Making," which introduces a neutrosophic fuzzy soft set to deal with uncertainty parametrically and it gives the approximate solution to the problem.

In Chapter 34, Hussain presented "Quadripartitioned Neutrosophic Soft Graphs," which concerns with the quadripartitioned neutrosophic soft graphs for treating neutrosophic soft information by employing the theory of quadripartitioned neutrosophic soft sets with graphs. Operations like Cartesian product, cross product, lexicographic product and strong product of quadripartitioned neutrosophic soft graphs are established. The proposed concepts are explained on examples.

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In Chapter 35, "Priority Weighted Neutrosophic Refined Soft Set: A Multi-Criteria Decision-Making Approach," Sweety et al. devised a new concept of priority weighted neutrosophic refined soft set by combining neutrosophic refined sets and soft sets implemented with prioritized universal elements and weightage imposed parameters.

In Chapter 36, entitled "Neutrosophic Contra RW-Continuous Functions in Neutrosophic Topological Spaces," Savithiri et al. introduced a new class of contra continuous functions namely neutrosophic contra RW-Continuous functions in neutrosophic topological spaces. Further the authors introduced neutrosophic almost RW-continuous function, neutrosophic almost contra RW-continuous function and some of its properties.

This book would not have been possible without the skills and efforts of many people. First, the advisory board who guided the editors through the editorial process. Second, the contributors who have provided perspectives of their valuable works on advances and applications of fuzzy sets and logic. Third, the reviewers for their kind services to critically review the book chapters.

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Chapter 1
Crisp Set vs. Fuzzy Set Qualitative Comparative Analysis, Technical Differences, and Much More

Jean Michel Rocchi
Sciences Po Aix, France

ABSTRACT

The idea of a mathematical formalization of logic that dates back to antiquity was only formalized in the 19th century by Charles Boole who proposed a binary system or clear sets. Although the fuzzy set theory is sometimes considered as a simple extension of Boolean algebra, the practical consequences for social science research are fundamental. The QCA has progressively seen the emergence of five increasingly sophisticated variants, the two main ones being the csQCA and the fsQCA. Is what opposes the crisp set qualitative comparative analysis (csQCA) to the fuzzy set qualitative comparative analysis (fsQCA) simply a technical improvement? The answer will be no; beyond different technical tools, there is a paradigm shift affecting in depth the research including epistemological choices, either explicit or implicit. In addition, part of these differences, which at first glance seem essentially methodological, are in fact partly due to a socio-cultural environment and religious values linked to the representation of the world, which is only rarely conscious.

INTRODUCTION

The differences between Boolean mathematics and fuzzy set logic may seem at first glance to be essentially technical, yet researchers, particularly in the social sciences, must be fully aware of all the consequences, including epistemological ones, of their choices.

Aristotle's great ambition of a mathematical formalization of logic will take more than two thousand years to be realized, it will then take a little more than a hundred years to go from the "clear sets" (Boole, 1854) to the "fuzzy sets" (Zadeh 1965).

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Section 1 will remind the long journey of the mathematical formalization of logic. Section 2 explains that the dream comes true with both Boolean algebra and the introduction of crisp set qualitative comparative analysis in social sciences. Section 3 deals with the emergence of fuzzy set qualitative comparative analysis and the appearance of a simple technical improvement. Section 4 will argue that fuzzy set qualitative comparative analysis is a constituent of a new paradigm shift. Section 5 will cope with the introduction of a second level of complexity for a better understanding of researchers choices: Small Ns Vs Large Ns qualitative comparative analysis. We will propose a matrix which summarizes the explicit and implicit choices of the researchers.

FORMALIZING LOGIC MATHEMATICALLY HAS LONG BEEN A DREAM OF BOTH PHILOSOPHERS AND MATHEMATICIANS

According to Bocheński (1961) the history of formal logic begins with Aristotle (384-322 BC). Logic is a mathematical discipline stemming from Aristotelian philosophy; this was self-evidence since, according to the Greeks, philosophy was the science that encompassed all the others.

Nevertheless, an intrinsic difficulty in Greek thought in matters of logic, be it Aristotle or other contemporary thinkers or mathematicians, lies in the fact that they did not know the theory of probability (Bernstein, 1996), The probabilities will only be discovered 2,000 years later, notably by two mathematicians the French Pierre-Simon de Laplace (1749-1827) and the Swiss Daniel Bernouilli (1700-1782). Indeed, Sambursky (1956) in the last sentence of his seminal paper express clearly the limits of the ancient Greeks: "*Neither the technique of experimentation nor the theory of probability, two cornerstones of modern science, could develop in ancient Greece*" (p.48)

Conversely, Aristotle as well as other Greek philosophers were able to develop a rich logical corpus. For McKeon (1941/2001) Aristotle works, notably in "Organon" (the collection of its logical treatises), consist in the development of an inference schemes system (syllogisms) for deduction and widely for reasoning. Organon is composed of 7 logical treaties: *Categoriae* (Categories), *De Interpretatione* (On Interpretation), *Analytica Priora* (Prior Analytics), *Analytica Posteriora* (Posterior Analytics), *Topica* (Topics), *De Sophisticis Elenchis* (On Sophistical Refutations). For Belohlavek et al. (2017): " I*n developing his syllogistic, Aristotle fully subscribed to the principle of bivalence*". (p.6) The syllogism is a deductive reasoning based on three propositions which link the premises to a conclusion: for instance, Socrates is a man, men are mortal, therefore Socrates is mortal. The conclusion is therefore always binary because it is or it is not. As stated McKeon (1941/2001), this issue is discussed in the treaty *De Interpretatione* (On Interpretation): "*Everything must either be or not be*".

Nevertheless, in one another treatise *Categoriae* (Categories) Aristotle recognizes for instance for human classification that sharp boundaries are not always relevant: " *Qualities admit of variation of degree. Whiteness is predicated of one thing in a greater or less degree than of another …*" These limitations in Aristotle *stricto sensu* binary reasoning are known as possible contingencies. In conclusion, it clearly appears that the syllogisms described in the treatise *On Interpretation* are explicitly linked to the logic of future Boolean mathematics, whereas in the treatise *Categories* the "variation of degree" is implicitly linked to the future logic of fuzzy sets. According to Perlovski (2007): "*Fuzzy logic invented by Zadeh is usually counterposed to the usual logic invented by Aristotle. This paper argues that Aristotle did not consider logic as a fundamental mechanism of the mind, rather he considered the working of the mind*

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to be more similar to fuzzy logic" (p.30). Hence, the author considers that Aristotle, according to the considered treatise of logic, can be associated to both ways of thinking.

One of the rare attempts to go beyond Aristotle's thinking by trying to integrate possible contingencies into binary reasoning lies in the analysis proposed by the English philosopher and Franciscan William of Ockham (1287-1347) in his *Treatise on Predestination, God's Foreknowledge, and Future Contingents* (~ 1324 AD) which is proposing a three-value logic instead of the Aristotle's two-value logic. Hence, to the possible results true or false, Ockham adds a third one: "undeterminated". Ockham's work is known through another Franciscan Boehner (1945) which presented its analyses. Moreover, Boehner (1945) coined the term "Neuter" as representing the case of neither true nor false and developed a truth function of three-value implication which implied a table with nine possible results. Belohlavek et al. (2017) emphasizes that even if Ockham did take seriously its own idea: "*As a consequence, his idea of three-valued logic had virtually no influence on the development of logic*". (p.8)

In two thousand years since Aristotle, logic hardly evolved before German mathematician Leibnitz discussed, in a manuscript of 1679, the possibility to build "*a mechanical binary calculator which would use moving balls to represent binary digits*.", but such machine was never built. Then, a French Jesuit named Joachim Bouvet living in China wrote to Leibniz about similarities of his ideas on binary algebra with Fuxi ancient Chinese hexagrams. Perkins (2004) explains " *Bouvet was convinced that the hexagrams represented a binary arithmetic. And after he announced this to Leibniz in 1701, Leibniz was also convinced*" (p.117). Then, Leibniz (1705), in a manuscript presented in 1703 to the Royal Academy of Science of France explains his binary algebra, published two years later. Leibniz sees an analogy between his system of binary arithmetic and the creation of the world by God, which had only two stages: nothingness and the created world. Hence, Liebniz will speak several times of a "*marvelous order* " ("*ordre merveilleux"*) in his works initially published in French to qualify its binary arithmetic.

Both Aristotle and Leibniz thought that logic could be formalized mathematically, but they reach an impasse, such as other mathematicians did. Indeed, as explained by Hawking (2007): *"All of the previous attempts to render logic in algebraic form had tried to force it into the algebra of real numbers. Boole realized that another algebra was needed*" (p.953).

THE DREAM COME TRUE: FROM BOOLEAN ALGEBRA TO CRISP SET QUALITATIVE COMPARATIVE ANALYSIS

From a conceptual point of view, the deep roots of the qualitative comparative analysis (QCA) can be found in the seminal book of the British economist and philosopher J.S. Mill entitled *A System of Logic: Raciocinative and Inductive* (1843). According to Scarre (1987) his seminal book embodied "*the greatest revolution in logical studies since Aristotle*". Mill (1843) proposes the "*canons of induction*" useful for social science. Ducheyne (2008) is proposing a synthetic presentation of its work through a kind of decision tree that he calls an "*algorithm*" (p.374).

But the real breakthrough came essentially from the founding work of Charles Boole who formalized logic mathematically for the first time by creating his own new algebra. According to Boole (1854): "*For greater distinctness of conception let x represent the class men, and let us express, according to the last Proposition, the Universe by 1; now if from the conception of the Universe, as consisting of "men" and "not-men," we exclude the conception of "men," the resulting conception is that of the contrary class, "not-men." Hence the class "not-men" will be represented by 1 - x. And, in general, whatever class of*

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objects is represented by the symbol x, the contrary class will be expressed by 1 – x." (p.32) In other words, the class x of "*men*" according to him represents the "*Universe*" express by the number 1 and conversely, the class "*non-men*" is expressing "*Nothing*" represented by 0 (the solution to the formula 1 – x). It is therefore perfectly obvious that Boole, clearly a successor of Leibniz, is only interested in the extreme values 0 ("*Nothing*") and 1 ("*Universe*").

Social science researchers have long been marked by the often rude opposition between proponents of qualitative and quantitative research. The sub-title of Charles Ragin's first seminal book on QCA (Ragin 1987), namely, "*Moving Beyond Qualitative and Quantitative Strategies*" suggests that by design QCA can be perceived as an attempt to occupy a new and intermediary position while avoiding a neverending debate between both other research methods. Verweij (2015) in table 1 below presents the differences among the three alternative methods.

Table 1. QCA vs. qualitative & quantitative research

	Requirements			
Approach	In-depth vs. generalization	Case-based vs. comparative	Attention to context	Attention to time
Variable-oriented	Generic patterns	Comparative, not case-based	Limited	Limited
Case-oriented	In-depth, focus on idiosyncrasies	Not comparative, case-based	Yes	Yes
QCA	Iterations between generic patterns and idiosyncrasies	Case-based comparison as main feature	Yes	Yes, but weak

Source: Verweij (2015), p.62.

In addition, Verweij (2015) underlines that variable-oriented (quantitative) research has a linear causality and focus on large N and its aim is to objectify; whereas QCA is dedicated to medium N and has a complex causality while the objective is both systematic and transparent comparison; lastly, case-based (qualitative) method has a holistic (only explicable by the whole) causality, is dedicated to small N and is trying to be interpretative.

Today, there are a handful of QCA variants that will be presented in Box 1 below. In the other developments of our chapter we will focus only on the comparison between the crisp set and the fuzzy set QCA. It is important to indicate that more recently Rihoux & Ragin (2007) have coined a new term "*Configurational Comparative Methods*" as an alternative name for QCA.

The author will focus on csQCA and fsQCA for two main reasons: they are very appropriate (while opposed) cases for a comparison, and the dominating ones in peer review journals as demonstrated in figure 1 below.

According to Rihoux (2006) the three variants of QCA should not be chosen randomly or according to the taste of the researcher, but because of their intrinsic characteristics that make them appropriate to be used: " *In conclusion, one could say that dichotomous QCA is more specifically designed to address small-N situations, say: less than 30–40 cases, with a key emphasis laid on case-based knowledge.*

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Conversely, fuzzy sets are more targeted at larger-N situations, as a challenge to mainstream statistical data treatment. Finally, MVQCA lies in some sort of middle ground between QCA and fuzzy sets – it is more powerful in medium-N situations" (p.686). Herrmann & Cronqvist (2005; 2009) are also proposing a kind of best practices which are more precise when they emphasize two dimensions: the first is the number of cases (and consequently the implied size of the data set); the second relies in the imperious necessity to preserve the maximum of the richness of the data information in the used raw data set. The figure 2. below (Rihoux, 2006) clearly shows the universe of appropriate choices depending on these two dimensions.

Box 1. The five main variants of QCA: csQCA, mvQCA, fsQCA tQCA & tsQCA

When Ragin (1987) introduced the csQCA it was at that time a synonym of QCA as it was the first and only existing method, it will be presented in depth hereafter in this chapter.

With the Multi-Value qualitative comparative analysis (or mvQCA) developed by Cronqvist (2004, 2005, 2006, 2007a & b), see also Cronqvist & Berg-Schlosser (2006, 2009), we face an extension of csQCA. The difference on both methodologies relies on the fact that if csQCA allows only binary (or dichotomous) conditions, conversely mvQCA can perform multi-value variables. But anyway, the outcome must remain dichotomous in mvQCA, or in other words there is no multi-value outcome, for the time being. In mvQCA it is possible to code non-binary "real" conditions: for instance trafic lights having three-value conditions (red, yellow and green), four value conditions of marital status (single, married, divorced, widowed) or some important religions (Christian, Muslim, Jewish, Buddhist), the five-value of the colors of the Olympic flag (blue, black, red, yellow, green) or the five Oceans, etc … As mvQCA is a generalization of csQCA, the Boolean operators are identical in both methods, as well in the minimization rule and the use of logical remainders (that will be explained hereafter when presenting csQCA. Cronquist & Berg-Schlosser (2009) who specifies the "best practices" specific to mvQCA emphasizes that "*Multi-value can be used to create a more genuine representation of multi-categorical nominal data, ordinal data and interval data* " (p.84). But they added a very important advice to the researchers: "*Include multi-value "à la carte", when needed. If at all possible, keep the number of values low (e.g., if you have the choice between three or four categories, use three); more than five categories must be avoided" (p.84). They also recommend in a mvQCA: "if justifiable from an empirical and / or theoretical perspective, use a preponderance of dichotomous conditions*"(p.84). Hence, even if the mvQCA could be considered as an extension of csQCA, it remains a rather emergent technique based on the criteria of published articles in academic reviews.

FsQCA introduced by Ragin (2000) will be presented in depth hereafter in this chapter.

The temporal QCA (tQCA) methodology was proposed by Caren & Panovsky (2005) in the following terms: " *a modification that is capable of capturing the temporal nature of causal interactions. This modification involves a hybrid of Boolean algebra and sequence analysis to create a parsimonious set of solutions. This technique is referred to as temporal qualitative comparative analysis, or TQCA*" (p.147). As with the appearance of each new version of the QCA method, it is a question of pushing back the limits of the method; in this case the fact that QCA was "atemporal"and that "*QCA ignores the order of events by treating combinations of attributes as though they occur simultaneously rather than as unfolding over time*" (p.147). The tQCA was built to solve these limits which are obvious in some specific contexts, especially in historical social science research as mentioned by the authors. It is established that tQCA is a valuable extension of QCA but it is manageable only if there is a limited number of sequences. Ragin & Strand (2008) in their comment on the method are recommending rather than doing the logical minimization by hand, to use the software fsQCA 2.5, and advise that they consider it as an extension to fsQCA. According to Schneider & Wageman (2012): "*So far, tQCA is the most formalized strategy to include time*"(p.273). The main weakness of the tQCA remains its still confidential nature and the very low number of published articles and moreover the scarce availability of case studies. The tsQCA is the last version of QCA. The two-step QCA was introduced by Schneider & Wagemann (2006): "In sum, we argue that the distinction between remote and proximate factors reflects the (implicit) structure of most social scientific theories and opens up the possibility for a two-step fs/QCA approach. The first step examines the contextual conditional combinations, under which a given outcome is more likely to occur than in other contexts. The second step leads to the precise formulation of causal paths which have provoked the outcome" (p.761). According to Schneider & Wagemann (2013): "remote factors are relatively stable over time. […] As a consequence, remote factors cannot usually be altered by actors and are treated as exogenously given". (p.253) Proximate factors have opposite characteristics. On step one, the truth table is construct in using only "remote factors", but in step two "proximate factors" are reintroduced. A practical result of splitting the conditions in two groups lies in the substantial decline of logical remainders. (Schneider & Wageman, 2006). Schneider (2019) revisited the tsQCA and proposes a new protocol: " I therefore argue that the first of the two steps in the protocol should be redefined as an analysis of necessity and only step 2 understood as an analysis of sufficiency" (p.1109). Having in mind that in the initial protocol (Schneider & Wagemann 2006) step 1 was an analysis of inconsistent sufficiency. Schneider (2019) emphasizes that in fact, it is not a slight amendment to the methodology previously proposed but a full aggiornamento; only the two-step principle is unchanged as shows the "comparison of updated and old two-step protocol" provided on table 1 on page 1119.

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Figure 1. Number of articles using the different QCA methodologies
Source: Roig-Tierno et al. 2017, p.20.

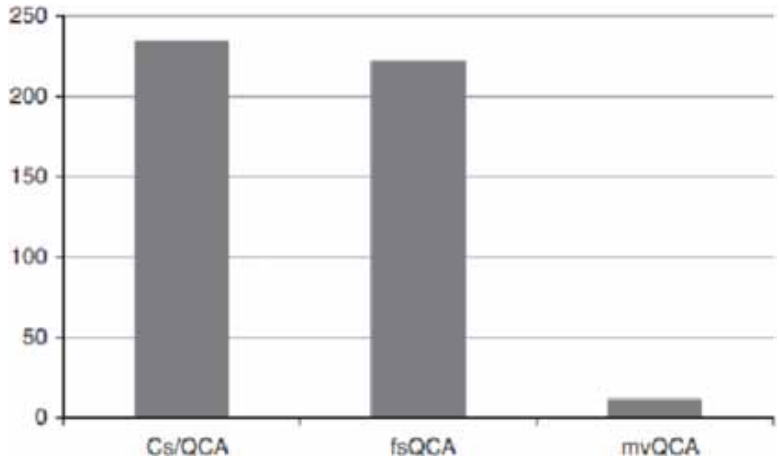
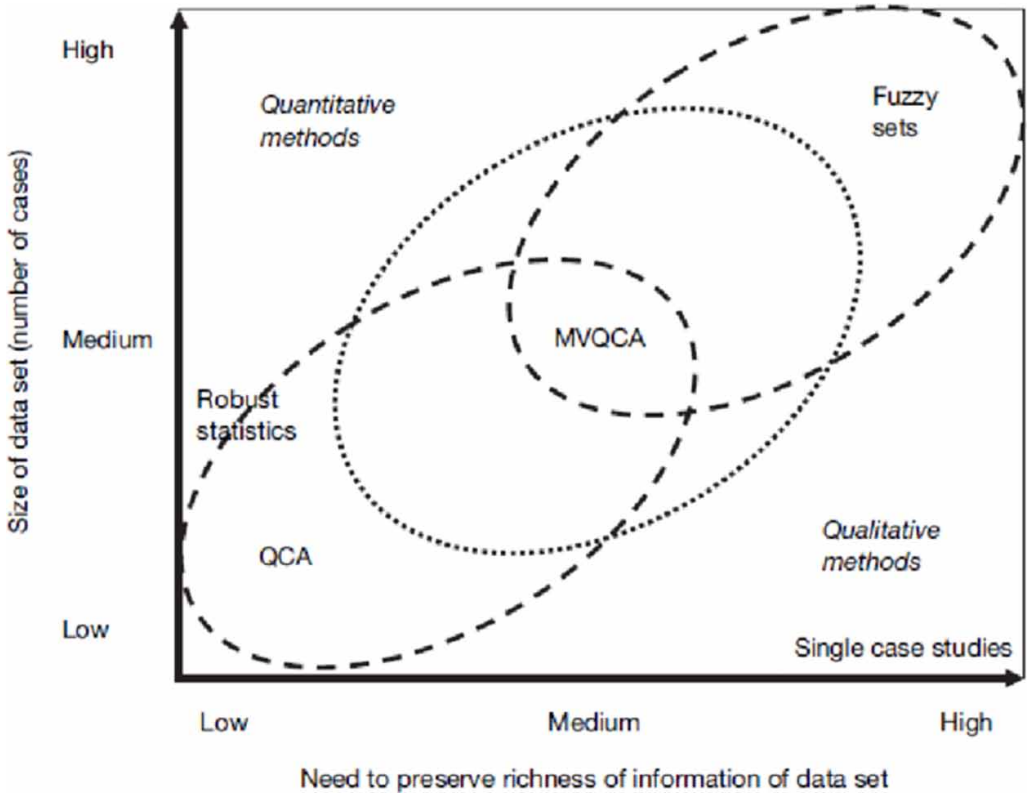


Figure 2. Best Use of QCA, MVQCA and Fuzzy Sets
Source: Rihoux (2006), p.686.



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The csQCA In Practice

The basis of the QCA is to test, whether or not, conditions allowed to obtain an outcome. In regards of the csQCa, the cases retained in the sample, or the census (because we are in the small N universe) will have their data binarized into 1 or 0 depending on whether the condition tested is met or not. QCA is considered by scholars as relevant to study populations in the range of 5 to 50 (less often 100), that is to say a size deemed insufficient to conduct quantitative research.

Step 1: Elaborating a Dichotomous Data Table

To establish a data table, it is obvious that preliminary work will be necessary, starting by elaborating its research design and in particular by selecting with rigor and relevance its cases and conditions. As this method is aimed at small Ns, the use of a census rather than a sample will often be a wise choice. The number of columns in the table will consist of the case name plus the number of conditions and one column for the outcome. Thus, a csQCA with 4 conditions will have 6 columns. With the exception of the column containing the case names, the number of columns will be 1 (present) or 0 (not present). In the table each case will be a row and represents a configuration of conditions themselves rated between 0 and 1 as well as the outcome of the considered case.

Step 2: Building a "Truth Table"

The "Truth Table" (or table of "configurations") is composed of combinations of conditions plus the outcome. There are five types of possible outcome:

- Configuration with a "1" in observed cases (on the field of research).
- Configurations with a "0" in observed cases.
- Configurations with a " – " ("don't care")
- Configurations with a " C" (contradictory configuration)
- Configurations with a " R" (logical remainders) not observed on the field during the previous empirical study of existing cases.

Step 3: Eliminating "Contradictory Cases" (If Any)

The challenge consisting of contradictory cases must be solved. The non-acceptance of the phenomenon of contradiction was already ruled out by Boole (1854) and was reaffirmed by Ragin (1987). It consists of the situation where two different cases having the same configuration of conditions leads at the same time to the observation and non-observation of the studied phenomenon. This contradiction tends to invalidate any attempt at demonstration. The simplest and most used way to remove a contradictory case is to add a new condition, nevertheless Rihoux & De Meur (2009) propose nine different methods to do it. Sometimes it is not possible to delete them all (Ragin et al., 1996).

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Step 4: Boolean Minimization

The result of an QCA study will be given by a truth table that will provide a solution in the form of a formula, which will then become a parsimonious formula through Boolean minimization in using Quine (1952, 1955) and (or) Mc Cluskey (1956) algorithms initially invented for simplifying electrical circuits. These algorithms are also known as a method of prime implicants or tabulation method as they are a two-step method for minimization of Boolean functions: firstly, identification of prime implicants, secondly establishing a prime implicants chart. In this step, the minimization of the configuration is made without logical remainders, usually the formula provided is longer (more complex) than the next formula described hereafter.

Step 5: Including "Logical Remainders" Cases

Due to the very limited diversity of organizations in social science, there is often a "reservoir" (De Meur & Rihoux, 2002) of non-observed cases or logical remainders that could be included in the analysis. Doing so is a way to achieve a better parsimony. In csQCA this new formula obtained is called "*parsimonious formula*". Beyond this Boolean algebra formula, a software like TOSMANA is providing a Venn diagram which is representing all configurations: observed cases + logical remainders.

Due to the "*asymmetry*" of causal relations in social science (Lieberson, 1985), firstly a complete analysis of [1] configurations had to be made and secondly a complete analysis of [0] configurations, the result of them is called "full report" in TOSMANA. Consequently, there will be four complete minimization procedures to be run by the software:

- [1] configurations without logical remainders
- [1] configurations including logical remainders
- [0] configurations without logical remainders
- [0] configurations including logical remainders

At this stage the coverage of formulas is available.

Step 6: Interpretation of the Outcome

This step is to be emphasized in several respects, it is not the least important and likewise it is not the easiest to drive rigorously. QCA is a "*case-oriented technique"* (Ragin, 1987) interpretating the formulas provided by the software requires very often a "return to the cases". The principle of "*equifinality*" is often encountered in QCA's outcome, in other words, several different configurations can give the same result (*"All roads lead to Rome*"). QCA is also a back and forth between "*maximum complexity* " and "*maximum parsimony*" as well as cases comparisons and case by case analysis (C-B-C) as are showing Rihoux & Lobe (2009) in figure 3.

Profile of the Formula: INUS vs. SUIN

In QCA analysis most the time the outcome, in parsimonious form after minimization, obtained through a set of conditions reached (or not), will take an INUS form and more rarely a SUIN form.

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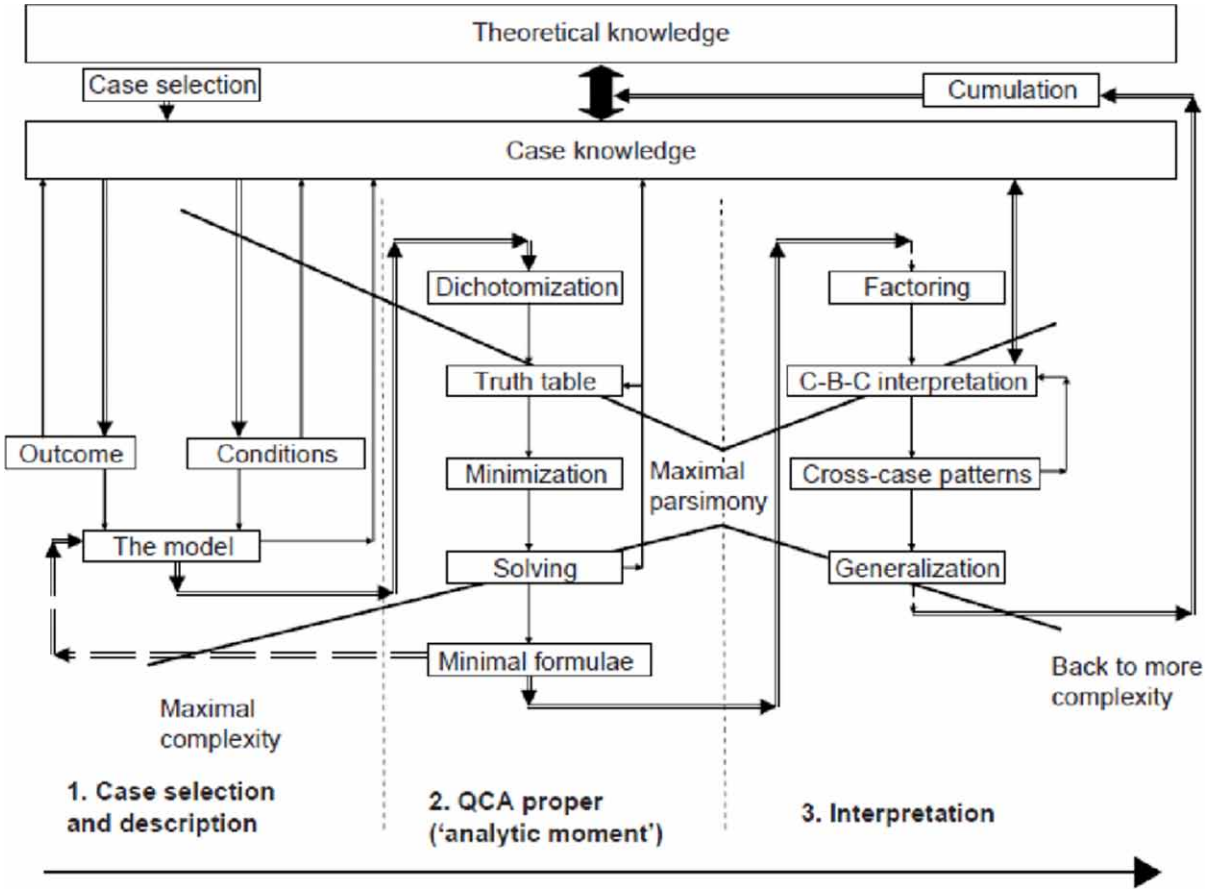
Figure 3. QCA and the funnel of complexity with concrete step and feedback loops
Source: Rihoux & Lobe (2009), p.238.



John Mackie (1965) explained:

In this case, then, the so-called cause, is and is known to be, an **insufficient** *but* **necessary** *part of a condition which is itself* **unecessary** *but* **sufficient** *for the result. […] In view of the importance of conditions of this sort in our knowledge of and talk about causation, it will be convenient to have a short name for them: let us call such a condition (from the initial letters of the words italicized above), an INUS condition (Words in italics put in bold by the author, p.245).*

Schneider & Wagemann (2012: 79) are providing the following example:

INUS Boolean formula: A*B + ~ B*C + D*~ F + → Y, (A, B, ~B, C, D and ~F are INUS conditions).
Where, ~ express a negative condition (the condition is not observed).

Schneider & Wagemann (2012:79) concluded: "*In other words, single conditions are often not sufficient on their own, but need to be combined with others*". The equifinality principle is observed there are three alternatives (**A*B, ~ B*C, and D*~ F)** leading to the same result (or outcome).

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Mahoney et al. (2009) have identified a new configuration of conditions described by the acronym SUIN: " *Sufficient but unnecessary part of a factor which is insufficient but necessary for the result". (p.126).* Schneider & Wagemann (2012: 80) give the following example of a SUIN configuration:

SUIN Boolean formula: (A+B) * (C+~D) ← Y, (A, B, C, ~D, are SUIN conditions).

Schneider & Wagemann (2012:80) concluded " *There are two necessary conditions: the unions A+B and C+-D. Each union taken alone is insufficient for producing Y.*"

The csQCA answers a question such as "how" (and not "why" as explains Curchod,2003). For example, let's suppose the question *how* a room is illuminated? The answer lies in a set of 4 necessary and not sufficient conditions: being connected to an electrical network, having a working electrical installation, having a working light bulb and finally pressing the switch. The room can only be turned off or on, the result can only be binary, we are in the presence of a pure dichotomy. The success of the csQCA was growing, but it could only process binary data, which was seen as a significant work limitation for some social researchers. For them the appearance of fuzzy set qualitative comparative analysis (fsQCA) was seen as the solution to their problem. If we take up the metaphor of electricity again, an on or off position corresponds if we refer to the nature of the extreme cases of full day and full night, but there are in fact a multitude of variations in brightness. A halogen lamp describes the new possibilities inherent in csQCA, the owner of the lamp can indeed choose his level of brightness, just as a researcher can calibrate his conditions with regard to the observed level of grade of membership (different level of brightness of darkness related to the time of the day or night).

The Conditions of Validity of a csQCA Analysis

- The absence of contradictory case is the first issue;
- Compliance with Marx criterion is the second.

From the random generation of 1,500 truth tables, Marx (2010) recommends a minimum number of cases according to the number of conditions to avoid spurious correlations, we present its matrix in table 2 below.

Table 2. Minimum number of cases according to the number of conditions

Minimum number of cases required	**≥ 7**	**≥ 10**	**≥ 13**	**≥ 18**	**≥ 25**	**≥ 34**	**≥ 49**
Number of conditions	2	3	4	5	6	7	8

Source: adapted from Marx (2010), Marx & Dusa (2011).

According to Marx (2010), with a low ratio, for instance in a study including seven cases testing two conditions (2/7), the probability of not encountering a contradictory set is low, which should be interpreted as the probable absence of spurious correlations. In a spurious correlation scenario, the absence of contradictory cases may indeed be due to chance. Many researchers consider that the universe of small N is a number less than or equal to 50 (and more rarely to 100), on the basis of the smallest threshold

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it can be seen that in practice it is not possible to rigorously test more than 8 conditions. Marx & Dusa (2011) explain: "*Under certain circumstances csQCA does not generate contradictions for a model even on random data. Whether this problem arises or not is a function of the number of conditions and cases which are included in the analysis. Given any particular number of cases, there is a ceiling to the number of conditions which can be included safely in an analysis*" (p.121). The Ocham's razor rule dear to economists also applies to fsQCA, keep it simple by not testing too many conditions.

- Coverage ratios: they will be described hereafter when we will present the conditions of validity of fsQCA.

THE EMERGENCE OF FUZZY SET QUALITATIVE COMPARATIVE ANALYSIS AND THE APPEARANCE OF A SIMPLE TECHNICAL IMPROVEMENT

Where Did fsQCA Come From? Understanding Basic Principle

Zaddeh (1965) proposed the following definition: "*A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one*" (p.338). In other words, it deals with the space voluntarily ignored by the Boolean algebra. Dubois and Prade (2015) are adding that fuzzy set is: "*fuzzy sets just introduce grades to soften boundaries of an uncertainty set*" (p. 26).

Hence, for Zadeh (1965), a fuzzy set is a solution to "*ambiguity*" and he explains it's usefulness in crystal clear words: "*More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. […] Clearly, the "class of all real numbers which are much greater than 1," or "the class of beautiful women," or "the class of tall men," do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction*" (p.338).

Dubois and Prade (2015) explain: "*In a nutshell, the main idea behind fuzzy sets is to make membership to sets gradual rather than abrupt. For instance, in the case of totally ordered universes, changing sharp membership thresholds into soft ones*" (p.23). Presented that way fuzziness is related neither to uncertainty nor vagueness. Nevertheless, fuzziness doesn't mean vagueness as are explaining Dubois and Prade (2015): "*Vagueness refers to uncertainty of meaning (the membership function is ill-known), which is distinct from gradualness (membership is a matter of degree)*" (P.23).

Fuzzy-set analysis is according to Dernoncourt (2013), an extension of the Boolean crisp set as "*the classical set theory is a subset of the theory of fuzzy sets*" (p.5).

As underlines Dernoncourt (2013) the expression "fuzzy sets" dominant in the literature is in fact *"an abuse of language*" because it would be much more appropriate to use the terminology of "*fuzzy subsets"* (p.5). Schneider and Wagemann (2012) complementarily explain: "*A crisp set is nothing else than a very special case of a fuzzy set, one that only allows for full membership and full non-membership"* (p.14).

Fuzzy-set analysis was introduced in social sciences by Smithson (1987), who points out that the emergence of fuzzy set theory at the beginning was relatively slow, because its appearance subject to debates "*over whether fuzziness is merely probability in disguise*" (p.9). Notwithstanding, the influence on social sciences was initially also slight (Smithson, 1988). Zadeh (2000) responded to these criticisms

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about a new name for old knowledge by demonstrating that the fuzzy set was a new concept that was really distinct from the logic of probabilities. However, on closer inspection, this criticism was strange because from the original works (Zadeh 1965) there was no ambiguity in the writings between fuzzy sets and probabilities since: "*In fact, the notion of a fuzzy set is completely non statistical in nature*" (p.340). The outcome of this bad trial made to the fuzzy sets was thus acquired, nevertheless one can be astonished by the time it took to recognize the true progress that they constituted.

It should be emphasized that it is very unfortunate that the precursory works of Kaplan & Scott (1951) devoted to the "*empirical classes*" tend to be forgotten and remain unjustly unrecognized, since they unquestionably herald the future development of "fuzzy sets", even if it is not a matter of shared authorship of the concept. In their paper they describe "indicators" which are a function to [0,1]. To a certain extinct, the primary idea can go back 1920 with the Lukasiewicz 's presentation to the Polish Philosophical Society. As emphasized by Belohlavek et al. (2017): " *In this paper, Lukasiewicz allows each proposition to have three values. Two of them are the classical values 1 and 0, which represent truth and falsity, respectively. The third value represents neither truth nor falsity and is interpreted as "possibility." This value is denoted in this early paper by 2, but later on, the convenient symbol ½ was adopted*" (p.11). An ancient root could also be find in the French Physicist and Philosopher Duhem (1906/1954) who opposed in his work "*approximately*" and "*exactness*".

Finally, when Ragin (2000) introduced fsQCA he opened new frontiers to researchers: "*Fuzzy sets offers researchers an interpretative algebra, a language that is half-verbal-conceptual and half-mathematical-analytical. Thus, the greatest value of fuzzy sets for social scientists is their potential for enlivening, intensifying, and extending "the dialogue" between ideas and evidence in social research*" (p.4).

The Specifics of fsQCA in Practice

According to Ragin (2009), for its part, the fsQCA, unlike the csQCA, is part of a continuum between "*three-value fuzzy set*" (that he also name "*rudimentary fuzzy set*") to the other end the "*continuous fuzzy set"*, and in between the two are interspersed the "*four- value*" and the "*six-value*" fuzzy set, all versions are presented in table 3 hereafter.

As explained by Ragin (2008) there are two methods to calibrate in fsQCA: "*The first, the direct method, is based on researcher-specified benchmarks for full membership, full nonmembership, and the crossover point. The second, labeled the indirect method, is based on the researcher's, sorting of cases into six categories and the use of a regression estimation procedure to translate raw scores into fuzzy membership scores*" (p.84). The crossover point which is 0.5 is expressing the "*maximum ambiguity*".

Another specific of fsQCA is explained in the following words by Ragin (2009): "*A multidimensional vector space constructed from fuzzy sets as 2^k corners, just as a crisp truth table has 2^k rows (where k is the number of conditions)*" (p.104). Hence, there is a correspondence between vector spaces corners and truth table raws (Ragin, 2000).

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Table 3. Crisp set QCA versus fuzzy set QCA

Crisp set	*Three-value fuzzy set*	*Four-value fuzzy set*	*Six-value fuzzy set*	*"Continuous" fuzzy set*
1 = fully in 0 = fully out	1 = fully in 0.5 = neither fully in nor fully out 0 = fully out	1 = fully in 0.67 = more in than out 0.33 = more out than in 0 = fully out	1 = fully in 0.9 = mostly but not fully in 0.6 = more or less in 0.4 = more or less out 0.1 = mostly but not fully out 0 = fully out	1 = fully in Degree of membership is more "in" than "out": $0.5 < X_i < 1$ 0.5 = cross-over: neither in nor out Degree of membership is more "out" than "in": $0 < X_i < 0.5$ 0 = fully out

Source: Ragin (2009), p.91, in Rihoux et Ragin (Eds).

The fsQCA In Practice

Step 1: The Definition and the Labelling of the Fuzzy Sets

For each condition, a fuzzy set has to be defined as well as the degree of membership (which range between 0 and 1) of each case has to be chosen. Due to the asymmetry in set-theoretic analysis, two fuzzy sets will be defined for conducting two different analysis (like in csQCA).

Step 2: Calibration

In order, to transform an interval-scale measure into fuzzy sets, a direct (or an indirect) method of calibration will be used. For instance, in a direct calibration, specifying the three qualitative anchors that structure a fuzzy set will be an important operation. These anchors will include the threshold for full membership (for instance a fuzzy score = 0.95), and the threshold for full non-membership (for example fuzzy score = 0.05), and lastly the cross-over point or maximum ambiguity point (classically fuzzy score = 0.5). These three thresholds will be defined based on both the theoretical and empirical in-depth knowledge of the cases.

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Step 3: Test of Necessity

Based on defined set-theoretic relationships of the analysis, in the case where the fuzzy-set scores of any condition or any configuration are consistently equal to or higher than the fuzzy-set scores for the outcome, a relation of necessity is therefore indicated (Schneider & Wagemann, 2012). Therefore, this means that if a causal condition is necessary for an outcome then instances of the outcome constitute a subset of instances of the causal condition. According to Ragin (2000), the test of necessity must be performed before performing the sufficiency test.

Step 4: Test of Sufficiency

In the case where the fuzzy-set scores of a condition or configuration are consistently equal to or smaller than their fuzzy-set membership scores on the outcome, it means that the condition or configuration is a fuzzy subset of the outcome (Schneider & Wagemann, 2012). Such a set-relationship indicates that in this case the condition is sufficient for the outcome. The truth table is a tool that identifies all configurations sufficient for the outcome.

Step 5: Boolean Minimization

Boolean minimization will produce three possible solutions: a complex solution based on observed cases, a parsimonious solution obtained by integrating local remainders and lastly, an intermediate solution which permits the incorporation of only remainders that are supposed to influence the outcome, on the basis, of a prerequisite empirical in-depth research. Hence, the intermediate solution is case knowledge centric one.

Step 6: Interpretation of the Outcome

Like in csQCA it is a very important part of the csQCA study.

The Conditions of Validity of a fsQCA: Consistency and Coverage

The method was provided notably by Ragin (2006) which describes notably calculation formulas and best practices and Fiss (2011) who describes how to use it in a study, and is followed in significant studies as a standard (Vergne & Depeyre, 2016).

“Consistency”:

Empirically, the sets of necessary and sufficient conditions are imperfect, hence there is a need of an indicator (consistency - between 0 and 1 the closest to 1 the better) to assess in which measures the conditions are related to the result, the “fit of the empirical evidence” (Mello, 2017). For instance, in an ideal world reaching 1 would be perfect, a threshold of 0.9 would be excellent and of 0.75 is considered usually as the minimum acceptable threshold.

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"Coverage":

According to Mello (2017) The measure of coverage is used to determine "*the relevancy of a condition in empirical terms*" (p.128). More, precisely as explain Mello (2019): "*For sufficient conditions, coverage indicates "how much" of the empirical evidence is explained by a given condition. […] For necessary conditions, coverage helps to distinguish relevant from trivial necessary conditions*" (p.7). Here again, the closer to 1 will be the ratios, the better because a high coverage value indicates a "*close fit between the condition and the outcome"*. Classically there are three levels of coverage ratios: solution coverage, raw coverage, and unique coverage. As indicated by Mello (2019): "*While solution coverage indicates how comprehensively the observed phenomenon is accounted for with a given solution, raw coverage designates how much any single path accounts for the outcome. Moreover, unique coverage measures how much is only covered by a single path and not by any overlapping paths*" (p.8).

Researcher perception as a basis for choosing the most suitable QCA variant: a practical example

Rocchi (2019; 2020) used the Crisp set QCA to study the survival conditions of digital banks in France over the period 1994-2019, the choice of this method seems a priori obvious: the firms in the sample survive (coded 1) or not (coded 0), the result is a priori obviously binary. Nevertheless, some companies have survived after being bought out, and the theory explains (Sutton, 1987) that a company that has been bought out does not constitute an "*organizational death*" because it has not lost its entire identity. Hence, in this study living firms include, indistinctly, firms that have been created and bought out.

Conversely, a second researcher could have considered this to be a case of a "*three-value fuzzy set*" QCA by coding the case of acquired firms at 0.5. However, one could object to this choice that it does not cover all the reality of the cases as sometimes only some of the assets of the firm are acquired. In the latter case, a third researcher could have chosen to use a "*four-value fuzzy set*" QCA and the coding will now be 0.67 for a company buyout and 0.33 for the acquisition of certain assets. We can see that the two possible fsQCA studies are different from the first one, it is no longer really a question of studying survival but rather the preservation of its identity by the company, i.e. the measurement of a "*grade of membership*". In other words, the research question is not the same in the different studies.

FUZZY SET AS A NEW PARADIGM SHIFT

This innovation, sometimes perceived as a simple technical improvement to increase the field of use of QCA, is in fact a fundamental change of perspective from Boolean algebra to fuzzy logic. Thinking of fsQCA as an improvement of the older, more crude csQCA is a relatively simplistic. It seems to the author that csQCA and fsQCA belong to different paradigms that seem to us relatively incompatible. Morgan (1979): "*The term "paradigm" is therefore used here in its metatheoretical or philosophical sense to denote an implicit or explicit view of reality*" (p.607).

Zadeh (2006) who is also a proponent of the generalized theory of uncertainty (GTU) explains that they are fundamentally two ways to measuring in research, firstly, "*Measurement-based*" or "*numerica*l" the classical way, secondly, "*perception-based"* or "l*inguistic*" (the new way based on GTU). For Zadeh (20O6): "*Equating information to a generalized constraint is the fundamental thesis of GTU*" (p.15). In the figure hereafter 2 we can observe the three possible logical systems.

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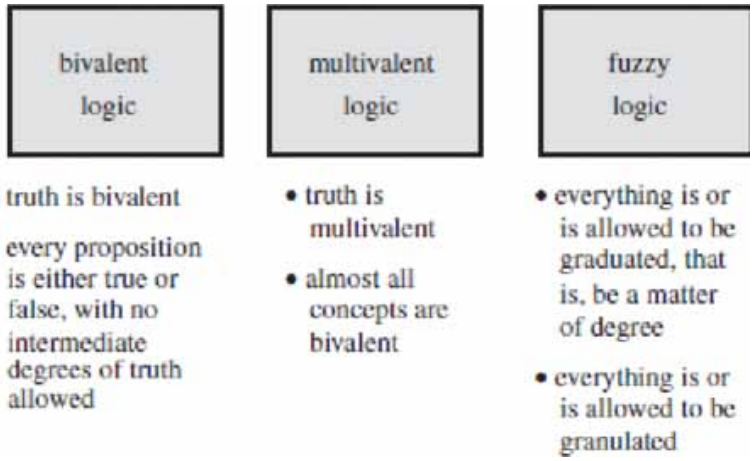
Figure 4. Logical systems
Source: Zadeh (2006), p.17.



The csQCA (bivalent logic) and the fsQCA (fuzzy logic) do not relate to the same logic of thought, nor of perception of the world by the researcher. Unlike csQCA, fsQCA, by asking the researcher to propose a calibration of the conditions tested, confers a greater degree of subjectivism. Nevertheless, as Curchod (2003) points out, one should not be a "*naive positivist*", since all forms of QCA involve a back-and-forth movement between theory and field cases. Moreover, as Morgan and Smircich (1980) have shown, epistemology can be thought of as a *continuum*, which is why we use the prefix "*rather*" below in the table 2 proposed as a synthesis to separate csQCA from fsQCA.

We observed that from a conceptual point of view the fuzzy set and the crisp set diverged strongly, nevertheless to have a clear vision of all the possibilities offered to the researcher, it is now also necessary to consider the impact of the sample (or census) size on the practice of QCA and the interpretation that it should be given.

Table 4. csQCA vs. fsQCA

	CsQCA	**fsQCA**
Date of QCA	Ragin (1987)	Ragin (2000)
Inspiration	Boole (1854)	Zadeh (1965,1968)
Method	Inductive: Mill (1843)	Inductive (or Deductive)
Nature	"*clear sets*"	"*fuzzy sets*" (or "*vague sets*")
Logic	"*bivalent*" or "*binary*"	"*fuzzy*"
Type of membership	"*Full*" membership only	"*Partial*" membership is possible
QCA software	No calibration (Boolean)	"*Calibration*" (between 1 and 0)
Measurement sense	Rather "measurement-based"	Rather "perception-based"
Compatibility with GTU	No	Yes
Epistemology	Rather objectivist	Rather subjectivist

Source: The author.

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INTRODUCING A SECOND LEVEL OF COMPLEXITY FOR A BETTER UNDERSTANDING OF RESEARCHERS CHOICES: SMALL N QCA Vs. LARGE N QCA

Small N QCA, indicatively 12 to 49 cases, allow testing fewer conditions than large N QCA, which is a consequence of the Marx criterion presented above. According to Greckhammer et al (2013) Small N QCA are inductive while Large N QCA could be either inductive or deductive. As explained by Van de Ven (2013) for empiricists like Bacon, Locke and Hume:

"*Induction starts with the enumeration of past and present empirical observations to draw inferences about the physical world. For example, if all the swans we observe are white, then we conclude that all swans are white*" (p.41). Conversely, he added that Descartes's method changed: "*He relied on logical deductive rules, beginning with a set of axioms or premises considered to be true to infer valid conclusion about the world. For example, if all swans are white, and the particular bird we observe is a swan, then we deduce that the bird is white* (p.41).

Table 5. Small-N QCA vs. large-N QCA

	Small-N QCA	Large-N QCA
Objectives		
Reasoning	Mostly inductive	Inductive or deductive
Primary Goal	Theory building	Theory building and testing
Sample and causal model		
Number of cases	12–50	50+
Relationship to cases	Relatively close, based on knowledge of each case	Relatively distant, based on knowledge of conceptual relationships
Sample/case selection	Theoretical sampling based on theoretical relevance or significance of the case	Theoretical or random sampling; random sample may be inappropriate for large-N research primarily interested in diversity
Number of causal conditions	Typically 4–8	Typically 6–12
Analyses processes		
Consistency	Consistency = 1 (i.e., "Always sufficient") is plausible (though minimum threshold consistency of .80 can be used).	Consistency ≥ .80 (i.e., "Almost always sufficient") is convention.
Resolving contradicting observations	Various strategies; intimacy with cases may benefit some while small-N may limit others	Various strategies; large-N may benefit some while distance from cases may limit others
Coverage	Typically high – all cases accounted for after iterations of building the model based on in-depth case knowledge	Relatively lower – large coverage desirable but not necessary
Frequency threshold	Minimum typically one or two cases	Minimum typically higher (3+); tradeoff between potential for deductive analysis and inclusion of rare configurations
Interpretation of findings	Results of necessity and sufficiency are interpreted by returning back to cases; case knowledge is used to build theory	Results of sufficiency and necessity are interpreted primarily as patterns across many cases without returning back to cases; statistical inferences are possible

Source: Greckhamer et al. (2013), p.54.

In Small N QCA or comprehensive QCA, the researcher will want to achieve a detailed, not to say intimate knowledge of each of the cases in which he aims to become a specialist: the QCA is secondary to the cases, "QCA is at the service of the case". As a result, the qualitative outweighs the quantita-

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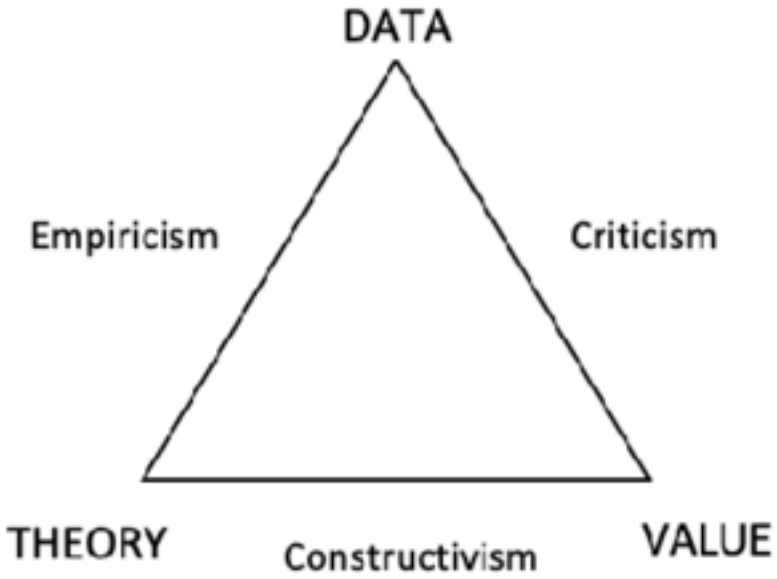
tive. Conversely, in the Large N CAQs, what will be privileged are the results of inter-case studies and therefore these will naturally be reinforced by a large sample or even a census. The knowledge of these cases becomes elementary because it is sufficient to be able to code the boxes of the truth table, "the cases are now at the service of the QCA". Hence, this time the quantitative outweighs the qualitative. The reciprocal importance of both operations has been inverted.

Clarifying the Debate in the Light of Galtung's Triangle

Galtung (1968) has proposed an integrated conception of social science which is represented on figure 3 below. We will discuss the following question: How does the QCA relate to this analytical framework?

Figure 5. Galtung's Integrated Conception of Social Science
Source: Alker (1998), p.797.



First of all, it should be considered that in principle the csQCA is more empiricist than the fsQCA which confers more power to the researcher through the calibration phase, and therefore conversely fsQCA is more impregnated with constructivism and criticism. Moreover, with regard to the dimension of the number of cases, in a situation where there is a high number of cases the data dimension will be more preeminent due to a quantitative point of view, but the intrinsic knowledge of the cases will be summary; on the contrary, with a smaller number of cases one will be entitled to aim at a fine, intimate knowledge of each one of them and therefore the dimension of criticism and constructivism will be more determining. Nevertheless, it should be kept in mind that the QCA is by nature difficult to strictly confine in a category because it is carried out by multiple back and forth between the cases in the field (data dimension) and the theory to be built or validated.

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Connecting the Dots: A Proposed Synthesis in the Form of a Matrix

Our approach consists in constructing a matrix based on two distinctions: on the one hand, between csQCA and fsQCA; on the other hand, between a QCA of small Ns and large Ns, in order to be able to show that, in our opinion, choices that may seem purely technical have important methodological and even epistemological consequences.

Table 6. QCA's Matrix: matching the type of QCA and the size of the sample

Algebra / Number of cases	*Comprehensive QCA (12 <50)*	*Configurational QCA* (≥ **50**)
Fuzzy set (Zadeh 1965.) Fuzzy set theory.	Rather Qualitative, Inductive & Part of constructivism. Criticism.	Impure Quantitative Deductive (or Inductive) Impure Constructivist. Criticism.
Crisp-Set (Boole 1854.) Boolean algebra.	Impure Qualitative, Inductive Empiricist. Inductive. Naive Positivist.	Rather Quantitative & Inductive (or Deductive). Naive Positivist. Empiricist.

Mahoney & Goertz (2006) note that "*Comparisons of the quantitative and qualitative research traditions sometimes call to mind religious metaphors*" (p.227). Moreover Brady (2006) statement is even more explicit: "*while this debate is not in any sense about religion, its dynamics are best understood as though it were about religion. We have always known that, it just needed to be said*" (p.53). Conversely, Mahoney & Goertz (2006) stated: "*We prefer to think of the two traditions as alternative cultures. Each has its own values, beliefs, and norms. Each is sometimes privately suspicious or skeptical of the other though usually more publicly polite. Communication across traditions tends to be difficult and marked by misunderstanding*" (p.227).

If for Brady (2006) the argumentation between qualitativists and quantitativists can resemble a religious anathema, he rules out any religious dimension. Conversely, the author believes that with regard to the distinction between the crisp set and the fuzzy QCA, cultural considerations inspired by transcendental visions and therefore having a religious component can at least unconsciously influence the researcher. Moreover, the author considers that if one is interested in intra-QCA differences and debate, these two visions are not exclusive of each other but, on the contrary, complementary. Cultural differences are explicit (due to different tools and practices) as for example in Greckhamer et al. (2013) and therefore perfectly conscious, on the contrary other implications of a superior nature whatever the qualifier chosen of quasi-religious, metaphysical, ontological or epistemological appear more implicit and rarely fully conscious.

The idea of the formalization of logic finds its sources in Aristotelian philosophy. Binary mathematics from Leibniz to Boole had a profound impact on the world, the most obvious of which was computer language. The crisp set QCA is related to this current of thought.

The fuzzy set QCA created by Zadeh (1965, 1968), on the other hand, has detached itself from crisp set analysis, which is sometimes considered to be too crude because it does not offer enough flexibility to the researcher to account for the complexity of the situations to be analyzed. Proponents of fsQCA are tempted to place the debate under a technical angle, since fsQCA is more sophisticated and would

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surpass csQCA, which would have become outdated. The author thinks that this vision is simplistic, the question is not technical but epistemological, philosophical and even at the extreme of the conscious or unconscious aspects of cosmology.

The csQCA confers a minor role to the researcher who, in the course of his analysis, observes the existence of dichotomies that pre-exist his research, the relationship to knowledge is rather objectivist. For Leibniz the "*marvelous order*" provided by binary algebra is only the rediscovery of a divine order that surpasses men and preexists them because it is a gift of God. It is also partly a continuation of Galileo Galilei (1564-1642) explaining in *Il Saggiatore* (The Essayist): *"... philosophy is written in this great book that is continually open before our eyes (I say the universe), but it cannot be understood if one does not first learn to understand the language, and know the characters in which it is written. It is written in mathematical language …* " (Author translation, p.264) Yet, God having created the universe, mathematics are implicitly the language of God.

Conversely, the fsQCA confers a more pre-eminent position to the researcher who will carry out a more or less fine calibration between bounds 0 and 1 and participates more actively in the construction of the outcome, the inspiration of this method is rather constructivist. The fuzzy vision set in Zadeh (1965) is of more Promethean inspiration: the researcher must be able to apprehend all the human complexity by its work. His ability to calibrate the conditions will make the quality of the researcher, like a great chef who will cook with passion the ingredients so that they restore the greatest flavor. Zadeh (2006) through the generalized theory of uncertainty (GTU) reaffirms and reinforces his "perception-based" or "linguistic" vision. But the sophistication of the research opens the Pandora's box and the famous warning of Wittgenstein (1921/2017) should be taken into account: "*language disguise the thought"*. Two years later, another great contribution was made by Russell (1923) with his famous article "*vagueness*": " *I propose to prove that all language is vague, and that therefore my language is vague*" (p.84). Conversely, it clarifies that treating language in using csQCA doesn't make sense from a philosophical point of view. Brion and Cholet (2015) have studied the organizational tensions in dedicated innovation projects. Firstly, they coded the verbatim of actors in using Nvivo and secondly, they proceed to a fsQCA. Such a research design is not only technically interesting but seems from a philosophical point of view the only acceptable one.

For the author to a certain extinct cosmology matters even unconsciously. Hence, our way of thinking is neither mechanical nor disembodied, but takes place in a given socio-cultural context where references to the past are often implicit but very real.

Old religions like Zoroastrianism, The Lord Ahura Mazda has two sons Ormazd (the good) and Ahriman (the evil) permanently in conflict until the end of times; and Manicheism founded on the struggle between the "light" and the "darkness" are binary cosmologies. For instance, both religions also oppose material and non-material, called *gētīg* and *mēnōg* in Zoroastrianism. As explain König (2020): "*Historiography of Iranian religion has always emphasized that Zoroastrianism and Manichaeism represent two variants of a dualistic worldview.* " (p.20). As mentioned at the beginning of the chapter, the vision of the ancient Greeks was also fundamentally binary. With on one side the gods installed in Olympus who decide the destiny of men who only undergo it on earth (i.e. Ulysses in *the Odyssey*, or the Greeks and the Trojans in *the Iliad*). The researcher can also evoke the binary opposition between the Greeks (the civilized world) and the barbarians (those who express themselves only by the borborygmas "bar" and "bar").

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Christianity, if it is also based on the opposition between "good" and "evil", or the duality of Adam and Eve in the book of *Genesis*, is nevertheless not a pure binary cosmology since ternary sets ("trinity") play a very important role: the father, the son and the holy spirit; hell, paradise and purgatory; or the three wise men.

Manicheism has not entirely disappeared, it still exists in Iran nowadays, and also it has left in everyday language the word Manichean, which is tainted in a rather pejorative way since it is according to the context the expression of a thought, crude, simplistic, dogmatic, way of thinking. Behind the existence (or not) of a binary order would thus hide a global conception of the world ("*Weltanschauung*" in German philosophy). In philosophical terms the crisp set and the fuzzy set analysis have a different "essence" (founding principle). Hence, the debate between the crisp set and the fuzzy set qualitative comparative analysis is also partly constituted by a metaphysical debate.

DISCUSSION AND FUTURE RESEARCH DIRECTIONS

Our comparative study between csQCA and fsQCA is inherently simplifying because it does not consider all the variants of QCA. Nevertheless, it seems to retain a rather good degree of relevance and validity if we consider that mvQCA is an extension of the Boolean approach (csQCA) while tQCA and tsQCA are related to fuzzy sets (fsQCA).

The lack of knowledge of the five QCA variants is all too often a source of methodological errors, based on the erroneous assumption that the choice of part of the analysis is limited to choosing between the csQCA or the fsQCA, whereas other variants would appear to be more appropriate depending on the nature of the research being conducted. As Rihoux (2006) reminds us, the initial choice of the variant used should be based more on the intrinsic characteristics of the study to be carried out than on the researcher's tastes and presuppositions. Therefore, no matter how good the research design, a poor initial choice will result in an original methodological weakness that will persist. Moreover, these variants did not arise by chance but were attempts to respond to the inadequacies of the two dominant variants. The mvQCA tried to remedy the pure binary analysis inherent in the csQCA. The tQCA was intended to improve on the csQCA by allowing the temporal dimension to be considered. Finally, the tsQCA claimed to correct a methodological inaccuracy of the csQCA by distinguishing between "remote" and "proximate" factors, which in practice resulted in excessive complexity that ultimately dissuaded many researchers from using it. Too much complexity has killed the method and Schneider (2019) has proposed a new tsQCA. The author nevertheless places some hope in the new version of tsQCA (Schneider, 2019), but it is much too recent to make a first assessment. The author also believes that the mvQCA should continue to develop, while conversely the tQCA continues to stagnate.

With the notable exception of Curchod (2003), the academic literature has shown little interest in discussing the epistemology inherent to the QCA, so much remains to be done. This seems to be essentially the consequence of the fact that Ragin (1987) voluntarily placed himself in the field of technique and methodology, choice confirmed when he extended it to fuzzy sets (Ragin, 2000). Rihoux & Lobe (2009) are describing in the same spirit: "*QCA as an approach"* (p.223) and "*QCA as a set of techniques*" (p.224).

This lack of interest by researchers in the study of the ontology and epistemology of QCA may appear in many ways very surprising. No one disputes that qualitative analysis, whether it is based on a case or cases, has an epistemology. In the same way, quantitative analysis, which is based on statistics, is part of epistemological choices. Why should QCA based on mathematics (clear sets or fuzzy sets) not

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have an ontological or epistemological dimension? We can only encourage researchers to investigate in this direction.

Belohlavek et al (2017) have made great progress in the study of the genesis of fuzzy sets, it certainly remains to pursue this historical path, which can greatly help to highlight and clarify the ontological and epistemological dimensions of the various forms of QCA. Finally, it seems to us that considering the socio-cultural environment, both secular and sacred, is essential to grasp the foundations of Boolean mathematics and the fuzzy set theories that support the forms of QCA. Very little has been done in this direction, although the author considers the prospects very promising.

Table 7. Perception of the world and main steps leading from csQCA to fsQCA and their latest developments in social science

Date	Name	Vision of the world or logical-math breakthrough
2019	Schneider	tsQCA revisited (or tsQCA V2).
2006	Zadeh	Generalized Theory of Uncertainty (GTU).
2006	Schneider & Wagemann	Two-step qualitative comparative analysis (tsQCA).
2005	Caren & Panovsky	Temporal qualitative comparative analysis (tQCA).
2004	Cronquist	Multi-value qualitative comparative analysis (mvQCA)
2000	Ragin	Fuzzy set qualitative comparative analysis (fsQCA).
1987	Ragin	Crisp set qualitative comparative analysis (csQCA).
1987	Smithson	Introduction of fuzzy sets in social science.
1965	Zadeh	Fuzzy sets.
1956	McCluskey	Boolean minimization algorithm.
1952, 1955	Quine	Boolean minimization algorithm.
1951	Kaplan & Scott	Concepts of “empirical classes” and “indicators”.
1945	Boehner	Three-value: 1, 0 and “Neuter” (Nine cases truth table).
1923	Russell	Article “vagueness” (about language).
1921	Wittgenstein	Tractarus logico-philosophicus: no “perfect” language.
1920	Luckasiewicz	Three-value: 1, ½, 0
1906	Duhem	“approximately” versus “exactness”.
1854	Boole	Invention of the full Boolean algebra.
1843	Mill	A System of Logic: Canons of induction in social science.
1679, 1705	Leibniz	Basic binary mathematics.
Est. 1324	Okham	Three-valued logic (or 3VL): True, False, neither true nor false (or undetermined).
Mani (216-274 AD)	Manichaeism (Prophet Mani)	Pure binary vision. Some limited differences with Zoroastrianism.
AD (or 0)	Christianism	Impure binary vision (God & Evil, Adam & Eve). Some trinities: Father, Son, Holy Spirit, Wise Men. (etc…)
384-322 BC	Aristotle	The “Organon” (7 treatise on logical reasoning).
	Ancient Greece	Pure binary vision. Olympus Vs Earth, Gods Vs Men, Greeks Vs Barbarians.
600 BC – 650 AD	Zoroastrianism (Prophet Zarathustra)	Pure binary vision. In decline since 650 AD but still existing in Iran.

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CONCLUSION

"We don't think much about ontology and epistemology so that we can get of with the craft of doing research instead of talking about it. But underlying any form of research is a philosophy of science that inform us about the nature of the phenomenon examined (ontology) and methods for understanding it (epistemology). Whether explicit or implicit, we rely on a philosophy of science to interpret the meanings, logical relations, and consequences of our observational and theoretical statements. Many of us inherit the philosophy of science that underlies the research practices of our teachers or mentors."

"It is better to choose a philosophy of science than to inherit one by default."

Andrew Van de Ven (2013: 37)

The author pointed out that the methodology and rigor applied to the QCA has continued to progress over time, both for the csQCA (and its extension the mvQCA) and the fsQCA (and its extensions the tQCA and the tsQCA). Nevertheless, proposing a new version gets on with the craft of QCA to try to resolve the limitations of previous ones but is far from being a guarantee of success. Hence, emerging as a new mainstream methodology is a long and rough journey.

In return, there is a real danger, not always perceived, that the sophistication of the tools used by the method may obscure its deep purpose, its "essence" as philosophers would say. A purely technical view of the QCA may also lead, where appropriate, to perceive the distinction between csQCA and fsQCA only from the angle of different tools when it comes to fundamental choices on the part of researchers, particularly epistemological choices. Similarly, beyond the QCA methodologies, the size of the sample will have a very strong influence on the analysis conducted, it is important to be fully aware of this. Therefore, the researcher has proposed a matrix to explain the 4 main possible environments during a QCA analysis.

In a nutshell, the researcher who conducts a csQCA has a relatively limited role assigned to him, he seeks to establish a solution from data that are preexistent and exploited from Boolean mathematics. The academic who chooses on the contrary the fsQCA arrogates to himself a more Promethean function: he will have to build a solution by processing data by means of the theory of fuzzy sets. We assume that the first approach is rather positivist, while the second is more constructivist, but obviously the debate is open.

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Chapter 2
A Fuzzy Data Envelopment Analysis-Based Performance Assessment of European Union Countries' Waste Management Practices

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ABSTRACT

Sustainable development is among the emerging priorities in the European Union (EU) in the last decade. Waste management plays an important role in ensuring sustainable development. In this study, waste management performances of EU countries were evaluated using fuzzy data envelopment analysis (DEA) method. The data set for the period 2014-2018 was collected from Eurostat, and five input and five output variables were determined for the analyses. Then, the waste management performances of 28 EU countries were analyzed using the Kao-Liu model. Efficiency scores were calculated for 0, 0.5, and 1 α-cut levels, while triangular fuzzy numbers were used in the model. In the analysis, the output-oriented DEA model was preferred, and overall, technical and scale efficiency values were determined. According to the results of the analysis, only Luxembourg and Cyprus are efficient at the lower and upper bounds for 0, 0.5, and 1 α-cut levels in terms of overall efficiency. When the value given for the α-cut is increased, the number of efficient countries also increases.

1. INTRODUCTION

The concept of sustainable development has evolved over 50 years and has become one of the most important issues of our age. United Nations (UN) Conference on the Human Environment held in Stockholm in 1972 emphasized the importance of environmental protection in terms of sustainable development and

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contributed to the development of this issue. Later, the United Nations Environment Program Secretariat was established to encourage international environmental cooperation (Rogers et al., 2008:9).

In 1987, the World Commission on Environment and Development published a report entitled "Our Common Future", also known as the Brundtland Report. The report was not a prediction of increasing environmental problems, poverty, and challenges in a more polluted world with increasingly scarce resources. Instead, it emphasized that environmental problems are an integral part of sustainable development policies and that environmental problems should be integrated as part of economic considerations, energy decisions, social issues and development efforts (WCED, 1987).

Sustainable development consists of activities, aimed at protecting and preserving the natural environment of the world and ensuring international social and economic equality (Blewitt, 2015:16). Therefore, sustainable development approach basically tries to understand the relationships of three complex systems; the global economy, society and the physical environment. On the other hand, in order to make the world a more livable place, sustainable development offers societies a holistic framework by setting economic, social and environmental goals (Sachs, 2015:4).

Smart, sustainable and inclusive growth constitutes a vision of Europe's social market economy for the 21st century. Sustainable growth has also been one of the three main priorities of Europe 2020 (European Commission, 2010):

- Intelligent growth: building an economy based on knowledge and innovation.
- Sustainable growth: Creating a greener and more competitive economy where resources are used more efficiently.
- Inclusive growth: Creating an economy with high employment rate for economic, social and regional cohesion.

At the United Nations General Assembly in September 2015, countries from all over the world agreed on the 2030 Agenda for Sustainable Development and its 17 Sustainable Development Goals. The EU was one of the most important supporters of the United Nations 2030 Agenda (European Commission, 2019).

In the report, published by the European Environment Agency in 2013, the main objectives and targets of the EU environmental policy for the 2010-2050 period were explained in detail. The report focused specifically on selected environmental and resource policies. One of these was the issue of waste management (European Environment Agency, 2013). The aim of the EU waste policy is to contribute to a sustainable economy by recycling as much as possible from waste. While only 38% of waste is recycled in the EU, five tons of waste is generated each year by an average European. In some EU countries, more than 60% of household waste still goes to landfill. For this reason, the EU aims to protect the environment and human health by waste management and help the EU transition to a circular economy (European Commission, 2021). The EU waste management is based on the five-stage "waste hierarchy" specified in the Waste Framework Directive. The waste hierarchy was created to show a preference for waste management and disposal. According to Figure 1, preventing waste is the most preferred alternative. On the other hand, the disposal of waste should be seen as a last resort.

The aim of this study is to evaluate the waste management performances of European Union countries (EU-28) using Data Envelopment Analysis (DEA) method. The rest of this study is organized as follows: Literature review, Research methodology, Analysis of the data and Conclusion.

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Figure 1. Waste hierarchy
Source: *European Commission, 2021.*

2. LITERATURE REVIEW

When the literature is searched, it is seen that there are many studies examining the waste practices of EU countries. The most recent studies in which the waste management activities of the EU countries are evaluated using the DEA method are summarized below.

Rios and Picazo-Tadeo (2021) assessed the environmental performances of EU-28 countries for the treatment of municipal waste using DEA. The data (recycling, recovery, landfill and incineration) regarding the desired and undesirable treatment of municipal solid waste were determined to calculate the performance scores. In the study, the data of the countries in 2017 were used. According to the results of the analysis, most of the countries with the best performance were Nordic and Central European countries, while the performances of Eastern European and Southern European member countries were lower. In addition, it has been determined that there is a positive relationship between the environmental performance of the countries in municipal waste treatment and their economic development levels.

Giannakitsidou et al. (2020) evaluated the performances of 26 EU countries in the management of Municipal Solid Waste (MSW) by two models: environmental performance and circular economy performance. In the models, input-output variables for the years between 2014-2017 were used and environmental and circular economy performances were measured using DEA. According to the analysis results, it is seen that there are significant differences between the performances of European countries. Older EU members such as Spain or France perform worse than newer members such as Slovenia or Poland, in terms of both environmental efficiency and circular economy efficiency. On the other hand, Belgium has been determined to be the best performing country according to both models.

Pais-Magalhaes et al. (2020) aimed to examine the waste management efficiencies of 15 EU countries using DEA. The analyzes were conducted for five-year time intervals (2001-2005, 2006-2010 and 2011-2015) and three input and two output variables were used in the study. In the study, different efficiency levels of environmental targets realized within the scope of Kyoto commitments were evaluated. According to the analysis results, Luxembourg was the most efficient country in most of the years between 2001 and 2015. On the other hand, countries such as France, Portugal and Austria showed the worst performance in terms of eco-efficiency scores for all periods analyzed.

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Halkos and Petrou (2019) assessed the environmental efficiencies of EU-28 countries in national waste generation using DEA method. The data for the years between 2008 and 2014 were used in the study. In addition, three different perspectives and eight variables were determined. The environmental efficiencies of the countries have been measured using three different models and different input-output combinations. Based on the analysis results, the most effective countries were shown as Germany, Ireland and England according to the three models. Then, the findings were reviewed according to the recycling rate of each country for the relevant periods. It has been noticed that countries with higher efficiency scores reach higher recycling rates.

In addition, the issue of waste management performance evaluation using DEA method is an area that has been examined by many researchers for a long time and still has not become outdated. Yeh et al. (2020) proposed a two-stage analysis of the relative efficiencies of the reverse logistics channels of twenty-three municipalities using DEA method in a fuzzy environment. Romano and Molinos-Senante (2020) assessed the eco-efficiencies of municipal waste services of municipalities in the Tuscan region by different management models. Sarra et al. (2019) evaluated both environmental and economic performances of municipal waste management systems in the Abruzzo region in Italy using DEA method, taking the effects of spatial proximity into account. Perez-Lopez et al. (2018) aimed to examine the solid waste disposal service in Spain using temporal scale efficiency in DEA panel data estimates. Albores et al. (2016) designed a DEA model consisting of four inputs and one output to measure the efficiencies of 20 incineration plants in China. De Jaeger and Rogge (2014) proposed to evaluate the costs of 35 municipal waste joint ventures in Belgium for 2010 using the BoD model-based DEA method. Chang et al. (2013) evaluated the relative performances of MSW recycling systems of 23 local governments in Taiwan for 2009 using the DEA Slacks-Based Measure model. Simoes et al. (2010) evaluated the performances of 29 solid waste facilities in Portugal by comparing DEA and DEA bootstrap scores. Zhang et al. (2008) developed DEA-based eco-efficiency models for regional industrial systems of 30 cities in China. Moore et al. (2003) used DEA to measure the 6-year (1993-1998) performances of 11 municipal services in the largest 46 cities in the United States.

3. RESEARCH METHODOLOGY

In the globalizing world, the rapid population growth by the effect of industrialization and urbanization causes unconscious consumption of resources and the formation of waste. The importance of waste management is increasing for EU (28) countries for a livable world and sustainable development. The aim of this study is to evaluate the periodic waste management of the EU countries using a fuzzy DEA method.

In this section, information was given about the Fuzzy Logic, Data Envelopment Analysis and Kao-Liu Model, used in the study. The methodological framework of the study is shown in Figure 2.

3.1. Fuzzy Logic

The concept of fuzzy logic was first used by Zadeh (1965) and brought to the literature. According to fuzzy logic, factors and criteria can be classified without strict limitations. For this reason, the concept of fuzzy is used to define ambiguous and uncertain situations (Klir and Yuan, 1996:5). Then, Mamdani and Assilian (1975) provided the control of a steam engine using a fuzzy logic system. This developed fuzzy logic control system has attracted great attention from both academic and industrial circles (Feng,

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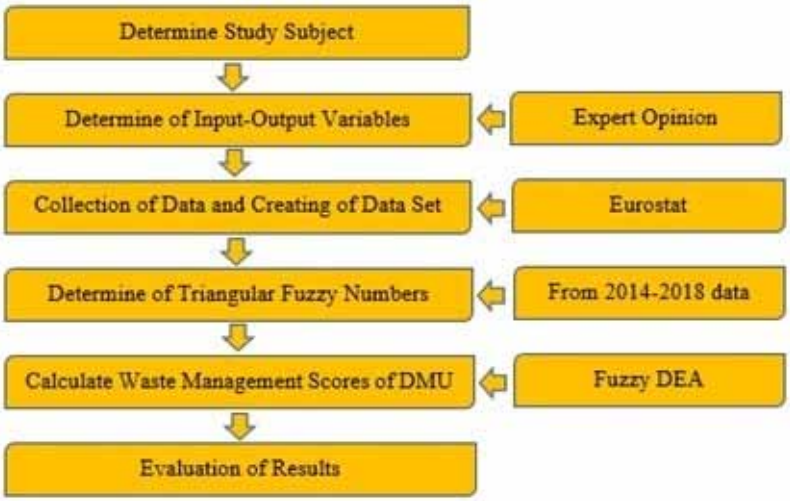
2010:1). For this reason, fuzzy logic algorithms have gained popularity to solve many real engineering application problems, especially nonlinear systems that involve many parameters and many of these parameters changing in time (Li and Gupta, 1995: XIII).

Figure 2. Methodological framework



Fuzzy logic is based on a fuzzy set logic. Fuzzy set refers to the cluster whose degree of belonging to the cluster is defined by the value of membership (Chen and Pham, 2001:1). A fuzzy set is a class of objects that can be expressed in an infinite number of membership functions. In such a class, each object is expressed as a continuous membership function that can take values ranging from zero to one (Zadeh, 1965:338). Where, the value 0 indicates that the object is not a member of the set, and the value 1 indicates that the object is a full member of the set. For the fuzzy set, values between 0-1 are expressed by the concept of partial membership (Zimmermann, 1987:11).

Fuzzy expressions such as long, hot and cold can be expressed in the form of fuzzy sets. A fuzzy set of $\tilde{A}$ can be represented as (Ibrahim, 2003:31):

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in E\} \quad (1)$$

where, an x element belonging to the set $\tilde{A}$ takes a value between 0-1 by the degree of membership $\mu_{\tilde{A}}(x)$.

When the universe X is continuous and infinite, this mapping is shown as follow (Bai et al., 2006:23)

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x)}{x} \quad (2)$$

When the universe X is discrete and finite, the fuzzy A set can be represented as (Ross et al., 2002:32):

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$$\tilde{A} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \ldots + = \sum_{i=1}^{n} \frac{\mu_{\tilde{A}}(x_i)}{x_i} \tag{3}$$

The symbols $\int, /, \sum$ and $+$ in the above expressions are not algebraic integrals, divisions and arithmetic summation symbols. Integral and summation symbols indicate the community of the cluster members, the division symbol indicates the degree of membership, and the summation symbol indicates the unification of the cluster members.

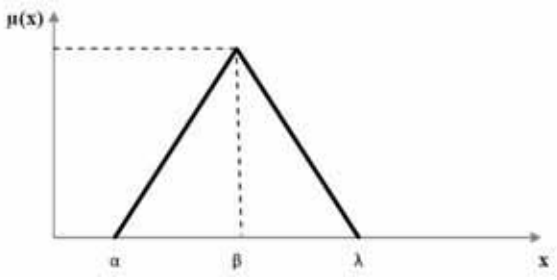
The membership functions of fuzzy sets can be expressed as fuzzy numbers or fuzzy intervals. In order for the fuzzy set $\tilde{A}$ defined in the set of real numbers (R) to be a fuzzy number, it must have at least three properties below (Klir and Yuan, 1995:97):

- $\tilde{A}$ must be a normal and convex fuzzy set.
- $\tilde{A}$ must a closed interval for every $\alpha \in (0\text{-}1]$.
- Support set of $\tilde{A}$ must be a bounded set.

The membership function is a curve that defines how an element maps to a membership value between 0 and 1. In a fuzzy set, the fuzziness is determined by the membership function and is usually represented by μA. The μA (x) value is called the membership degree of x in the fuzzy set and takes values between 0 and 1 (Adil et al., 2015:76-77). The type of membership function should first be determined based on the character of the restrictions (Kim and Whang, 1998:618).

A membership function can be represented in different shapes. Commonly used membership functions can be classified as triangular, trapezoidal, Gaussian, bell, sigmoidal and polynomial (Zhao and Bose, 2002:229). The triangular membership function is frequently used in studies due to its simple form and ease of use. On the other hand, triangular fuzzy numbers are a special case of trapezoidal fuzzy numbers (Wang et al., 2006:920). The triangular membership function is shown in Figure 3 (Monicka et al., 2011:9).

Figure 3. Triangular membership function



In Figure 2, the elements of the triangular membership function are defined as $\tilde{A} = (\alpha,\beta,\gamma)$. Where, β is the most probable value, α is the smallest value or lower bound, and γ is the biggest value or upper bound. The triangle membership function is shown in equation (4).

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$$\mu_{\tilde{A}}(x;\alpha,\beta,\gamma)=\begin{cases}0, & x\langle\alpha\ \ veya\ x\rangle\gamma\\ \dfrac{x-\alpha}{\beta-\alpha}, & \alpha\times\beta\\ \dfrac{\gamma-x}{\gamma-\beta}, & \beta\times\gamma\end{cases} \tag{4}$$

In a fuzzy $\tilde{A}$ set whose elements are defined as $\mu_{\tilde{A}}(x)>0$, the crisp set consists of elements whose membership degree is equal to or greater than α. The α-cut set of $\tilde{A}$ is denoted as $\tilde{A}_\alpha$ (Zimmermann, 2010:319).

$$\tilde{A}_\alpha=\{xE|_{\tilde{A}}(x)\geq\alpha \tag{5}$$

The strong α-cut set when the membership degree consists of elements greater than α (6) and the weak α-cut set occurs when the membership degree consists of elements greater and equal to the α value (7) (Ibrahim, 2004:35).

$$\tilde{A}_\alpha=\{xE|_{\tilde{A}}(x)>\alpha \tag{6}$$

$$\tilde{A}_\alpha=\{xE|_{\tilde{A}}(x)\geq\alpha \tag{7}$$

3.2. Data Envelopment Analysis

Over the past 40 years, Data Envelopment Analysis (DEA) has become a powerful quantitative and analytical tool, used to measure and evaluate performance. DEA has been successfully implemented for organizations operating in a wide variety of activities around the world (Cooper et al., 2011:1). DEA is a non-parametric method used to evaluate the comparative efficiency of homogeneous decision-making units (dmu) such as schools, hospitals, utility companies or points of sale (Thanassoulis, 2001:1). DEA is a linear programming-based method used to measure the relative performance of units in organizations where multiple input and output variables make it difficult to compare (Boussofiane et al., 1991:1).

The first efficiency measurement studies using multiple inputs and single output were carried out by Farrell (1957). In the following years, the studies of Boles (1966), Aigner and Chu (1968), Shephard (1970) and Afriat (1972) provided the necessary foundations for the development of DEA. Then, Charnes et al. (1978) measured the efficiency of an educational program using more than one input-output.

The basis of DEA is based on Farrell's production frontier approach and has been developed to measure the efficiency of public organizations (Kneip et al., 1998:783). There are two DEA models commonly used in the literature. The first is the CCR model based on the assumption of constant return to scale developed by Charnes, Cooper, and Rhodes (1978), the other is the BCC model based on the assumption of variable returns to scale developed by Banker, Charnes and Cooper (1984) (Charnes et

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al., 1994:23). DEA models are linear programming-based models that aim to maximize or minimize the objective function under certain constraints (Angiz et al., 2010:5153).

It is possible to develop models that provide input-oriented or output-oriented projections for both CCR and BCC models. In the output-oriented model, it is aimed to maximize the proportional increase in output variables while the amount of input variables is constant. (Dinc and Haynes, 1999:475). Due to the use of output-oriented models in this study, information about the output oriented CCR and BCC models is given below.

The CCR Model

The production possibility set (P_C) of the CCR model is expressed as follows (Tone, 2001:32):

$$P_C = \{(x, y) \mid x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}, \tag{8}$$

Where λ is a semipositive vector in R^n.

The overall efficiencies of the DMUs are calculated using the CCR model. In order for DMU to be efficient according to the CCR model, it must have both technical and scale efficiencies (Charnes et al., 1994:23). The output-oriented dual (envelopment) CCR model under a constant-returns to scale assumption is shown below (Luptacik, 2010:150):

$$\max h_0 = \phi + \varepsilon \left(\sum_{i=1}^{m} s_i^- + \varepsilon \sum_{r=1}^{s} s_r^+ \right) \tag{9}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \beta_j - x_{i0} + s_i^- = 0$$

$$-\sum_{j=1}^{n} y_{rj} \beta_j + \phi y_{r0} + s_i^+ = 0$$

$$\beta_j, s_i^-, s_r^+ \geq 0 \text{ , j=1,2,…,n , r=1,2,…,s, i=1,2,…,m}$$

In the dual model, DMU is efficiency when ϕ=1 and $s_i^-, s_r^+ = 0$. The DMU, which is efficiency according to the input-oriented CCR model, is also efficiency according to the output oriented CCR model. There is a relationship ϕ=1/θ between the two models. Therefore, while it is always θ≤1 in the input oriented CCR model, it is always $\phi \geq 1$ in the output oriented CCR model.

The BCC Model

By the BBC model, both technical and scale efficiencies of the DMU are calculated. The production possibility set (P_B) of the BCC model is expressed as follows (Tone, 2001:34):

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$$P_B = \{(x, y) \mid x \geq X\lambda,\ y \leq Y\lambda,\ e\lambda = 1,\ \lambda \geq 0\}.$$

The output-oriented dual (envelopment) BCC model under a variable-returns to scale assumption is shown below (Luptacik, 2010:163):

$$\max h_0 = \varphi + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \tag{10}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij}\beta_j - x_{i0} + s_i^- = 0 \text{ i=1,2,…,m}$$

$$\sum_{j=1}^{n} y_{rj}\beta_j - \varphi y_{r0} + s_i^+ = 0 \text{ r=1,2,…,s}$$

$$\sum_{j=1}^{n} \beta_j = 1$$

$$\beta_j, s_i^-, s_r^+ \geq 0 \text{ , j=1,2,…,n, r=1,2,…,s, i=1,2,…,m}$$

In the model DMU_0 is efficiency for φ=1 and $s_i^-, s_r^+ = 0$, it is not efficiency for other cases and is φ>1. The difference between CCR and BCC models is that the convexity constraint $\left(\sum_{j=1}^{n} \beta_j = 1\right)$ is added to the BCC model (Thanassoulis, 2001:129-130).

3.3. Kao-Liu Model

In case some variables are fuzzy numbers, a method has been developed by Kao and Liu to find fuzzy membership functions. In the model, the fuzzy DEA model is transformed into a convectional DEA model using the α-cut method and Zadeh's expansion principle. Then, the convectional DEA model is solved by the linear programming model (Leon et al., 2003:408).

In the model: The set DMUs is represented by $\tilde{X}_{ij}$ inputs and $\tilde{Y}_{ik}$ outputs and fuzzy sets with membership functions $\tilde{X}_{ij}$ and $\tilde{Y}_{ik}$. Hence, the fuzzy DEA model (11) can be shown as (Kao-Liu, 2000:431):

$$\tilde{E}_r = \max \sum_{k=1}^{t} u_k \tilde{Y}_{rk} \Big/ \left(v_0 + \sum_{i=1}^{s} v_j \tilde{X}_{ij}\right) \tag{11}$$

Subject to:

$$\sum_{k=1}^{t} u_k \tilde{Y}_{rk} \Big/ \left(v_0 + \sum_{i=1}^{s} v_i \tilde{X}_{ij}\right) \leq 1$$

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$u_k, v_j \geq \varepsilon > 0,\ j=1,\ldots,s,\ k=1,\ldots,t,\ i=1,\ldots,n$

The α-cuts of $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$ can be represented as (Kao-Liu, 2000:431):

$$(\tilde{X}_{ij})_\alpha = \{x_{ij} \in S(\tilde{X}_{ij}) |_{\tilde{X}_{ij}} \left(x_{ij}\right) \geq \alpha\}, \qquad \forall_i, j \tag{12}$$

$$\left(\tilde{Y}_{ik}\right)_\alpha = \{y_{ik} = S\left(\tilde{Y}_{ik}\right) |_{\tilde{Y}_{ik}} \left(y_{ik}\right) \geq \alpha\}, \qquad \forall_i, k \tag{13}$$

According to Zadeh's expansion principle, the membership function of the efficiency value of r DMU can be shown as (Kao-Liu, 2000:431):

$$\mu_{\tilde{E}_r}\left(z\right) = \sup_{x,y} \min\left\{\mu_{\tilde{X}_{ij}}\left(\tilde{X}_{ij}\right), \mu_{\tilde{Y}_{rk}}\left(\tilde{Y}_{rk}\right),\ \forall i, j, k \mid z = E_r\left(x, y\right)\right\} \tag{14}$$

The purpose of creating the $\mu_{\tilde{E}_r}$ membership function in this model is to determine the α-cuts of $\mu_{\tilde{E}_r}$. The lower and upper bounds of the α-cut of $\tilde{E}_r$ to find the membership function $\mu_{\tilde{E}_r}$ can be expressed as (Liu, 2014:637):

$$(E_r)_\alpha^U = \max E_r\left(x, y\right) \tag{15}$$

Subject to:

$$(X_{ij})_\alpha^L \leq x_{ij} \leq (X_{ij})_\alpha^U, \qquad \forall i, j$$

$$(Y_{ik})_\alpha^L \leq y_{ik} \leq (Y_{ik})_\alpha^U, \qquad \forall i, k$$

$$(E_r)_\alpha^L = \min E_r\left(x, y\right) \tag{16}$$

Subject to:

$$(X_{ij})_\alpha^L \leq x_{ij} \leq (X_{ij})_\alpha^U, \qquad \forall i, j$$

$$(Y_{ik})_\alpha^L \leq y_{ik} \leq (Y_{ik})_\alpha^U, \qquad \forall i, k$$

The lower bound of the efficiency value in any α-cut of a DMU; It is obtained by taking the output level at the lowest value, the input level at the highest value, and the output levels of the other DMUs at the highest value, and the input levels at the lowest value in the same α-cut of that DMU. In this case, the lower bound of the efficiency value of a DMU is as (17):

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$$(E_r)_\alpha^L = \max \sum_{k=1}^{t} u_k (Y_{rk})_\alpha^L / (v_o + \sum_{i=1}^{m} v_j \left(X_{rj}\right)_\alpha^U \right) \quad (17)$$

Subject to:

$$\sum_{k=1}^{t} u_k (Y_{rk})_\alpha^L / \left(v_o + \sum_{j=1}^{s} v_j (X_{io})_\alpha^U\right) \leq 1$$

$$\sum_{k=1}^{t} u_k (Y_{k})_\alpha^U / (v_o + \sum_{J=1}^{s} v_j (X_{ij})_\alpha^L \leq 1 \quad i = 1,2,\ldots,n \quad i \neq$$

The upper bound of the efficiency value of a DMU is as (18):

$$(E_r)_\alpha^U = \max \sum_{k=1}^{t} u_k (Y_{rk})_\alpha^U / (v_o + \sum_{i=1}^{m} v_j \left(X_{rj}\right)_\alpha^L \right) \quad (18)$$

Subject to:

$$\sum_{k=1}^{t} u_k (Y_{rk})_\alpha^U / \left(v_o + \sum_{j=1}^{s} v_j (X_{io})_\alpha^L\right) \leq 1$$

$$\sum_{k=1}^{t} u_k (Y_{k})_\alpha^L / (v_o + \sum_{j=1}^{s} v_j (X_{ij})_\alpha^U \leq 1 \quad i = 1,2,\ldots,n \quad i \neq r$$

In the model, as the determined α value increases, the algebraic difference between the upper and lower bounds of the efficiency value will decrease.

4. ANALYSIS OF THE DATA

In this section, information was given about the input-output variables and the fuzzy DEA scores.

4.1. The Data

A data set for the years 2014-2018, was obtained from the Eurostat database. Expert opinions were taken for the waste management performance activities of the countries and ten variables (five inputs and five outputs) were determined. Descriptions of the input-output variables, used in the study are given in Table 1 (European Commission, 2021).

4.2. Calculation of the Efficiency Scores of EU Countries using the Fuzzy DEA

In the study, triangular fuzzy numbers were obtained using the five-year data set. Fuzzy DEA model, developed by Kao and Liu (2000) was used to calculate efficiency scores. Max of the values in the five-year period for the upper value of the fuzzy numbers, the average of the values in the five-year period

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for the middle value, and min of the values in the five-year period for the lower value were used and triangular fuzzy numbers were obtained.

Table 1. Definitions of the input and output variables

Code	*Variable Name*	*Unit*	*Definition*
I1	Packaging Waste	Tonne	The packaging is all products made of all kinds of materials used for the containment, protection, handling, delivery and presentation of goods. Packaging input consists of paper and cardboard packaging, plastic packaging, wooden packaging, metallic packaging, glass packaging, and other packaging subcategories.
I2	Waste Electrical and Electronic Equipment	Tonne	The amount of waste electrical and electronic equipment (e-waste) generated every year in the EU is one of the fastest growing waste streams. Electrical and Electronic Equipment (EEE) refers to items that are designed for use not exceeding 1000 volts with alternating current and 1500 volts with direct current, and that depend on electric current or electromagnetic field for proper operation and items that are used for the production, transfer and measurement of these currents and fields.
I3	End-of-Life Vehicles	Tonne	The End of Life Vehicles Directive regarding the end of life of automotive products has been prepared by the European Union. This Directive sets out measures aimed primarily at the prevention of waste from vehicles, in addition to the reuse, recycling and other forms of recovery of end-of-life vehicles and their components, thereby reducing disposal.
I4	Collection of Portable Batteries and Accumulators	Tonne	The purpose of batteries and accumulators is to ensure the proper operation of many products and devices that people use on a daily basis. For this reason, they constitute an indispensable source of energy in our lives. Every year, approximately 800.000 tons of automotive batteries, 190.000 tons of industrial batteries, and 160.000 tons of consumer batteries enter the European Union.
I5	Municipal Waste	Tonne	Municipal waste is expressed as waste collected and processed by municipalities. Municipal waste includes household wastes and wastes whose nature and composition are similar to household wastes.
O1	Recycling Rates of Packaging	Rate (%)	Recycling rate is defined as the total amount of recycled packaging waste divided by the total amount of collected packaging waste. According to the Packaging Waste Directive, between 55% as a minimum and 80% as a maximum by weight of packaging waste should be recycled. The distribution of this recycling rate should be as follows; minimum 60% for glass, 60% for paper and cardboard, 50% for metal, 22.5% for plastic and 15% for wood.
O2	Recycling Waste Electrical and Electronic Equipment	Tonne	Recycling of Waste from electrical and electronic equipment (WEEE) is a specific part of the waste and recycling industry. WEEE includes a wide variety of devices such as computers, refrigerators and cell phones at the end of their useful life. Waste from modern electronic equipment contains rare and expensive resources that can be recycled and reused if managed effectively.
O3	Recycling Rates of Batteries and Accumulators	Rate (%)	Batteries have special recycling processes that differ from each other due to the various component metals from which they are made. The Batteries Directive focuses on recycling processes to ensure recycling efficiency. The efficiency of the recycling process is calculated by dividing the mass of output fractions by the mass of the waste batteries and accumulators input fraction expressed as a percentage.
O4	Recycling Rates of End-of-Life Vehicles	Rate (%)	The recycling of End-of-Life Vehicles varies depending on the material used in vehicle manufacturing and the assembly process of the parts. For end-of-life vehicles, recycling means energy recovery as well as any reprocessing of waste materials in a production process. It is estimated that Europe can reach a recycling rate of 89% in 2030 for end-of-life vehicles.
O5	Recycling Rates of Municipal Waste	Rate (%)	The recycling rate of municipal waste gives information about the efficiency of the municipal waste management of the countries. The recycling rate is calculated by dividing the amount recycled from municipal waste by the total municipal waste collected. The municipal solid waste recycling rate increased from 6% in 1960 to about 10% in 1980, to about 29% in 2000 and to 32.1% in 2018.

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Then, using the obtained fuzzy numbers, the efficiency values for 0, 0.5 and 1 α-cuts were calculated. Output oriented model was preferred in analysis and the overall efficiency, the technical efficiency and the scale efficiency scores were calculated. EMS 1.3 package program was used to calculate efficiency scores. When $\alpha = 0$, the efficiency scores of the countries are shown in Table 2.

Table 2. Efficiency Scores when α= 0

Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)	Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)
Belgium	Lower bound	154.60	100.00	154.60	**Lithuania**	Lower bound	245.20	105.9	231.54
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Bulgaria	Lower bound	213.00	105.60	201.70	**Luxembourg**	Lower bound	100.00	100.00	100.00
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Czechia	Lower bound	278.50	108.40	256.90	**Hungary**	Lower bound	113.70	100.00	113.70
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Denmark	Lower bound	184.00	100.00	184.00	**Malta**	Lower bound	131.30	100.00	131.30
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Germany	Lower bound	209.60	100.00	209.60	**Netherlands**	Lower bound	201.50	108.60	185.54
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Estonia	Lower bound	166.30	110.90	150.00	**Austria**	Lower bound	189.90	102.90	184.55
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Ireland	Lower bound	214.50	108.70	197.30	**Poland**	Lower bound	282.60	107.70	262.40
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Greece	Lower bound	344.40	135.20	254.70	**Portugal**	Lower bound	279.20	106.20	262.90
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Spain	Lower bound	413.90	113.10	366.00	**Romania**	Lower bound	358.10	118.30	302.70
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00

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Table 2. Continued

Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)	Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)
France	Lower bound	252.90	108.80	232.40	**Slovenia**	Lower bound	185.20	106.60	173.73
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Croatia	Lower bound	167.40	107.90	155.10	**Slovakia**	Lower bound	256.40	109.10	235.01
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Italy	Lower bound	309.40	114.30	270.70	**Finland**	Lower bound	159.30	108.70	146.55
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Cyprus	Lower bound	100.00	100.00	100.00	**Sweden**	Lower bound	111.60	100.00	111.60
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Latvia	Lower bound	270.20	112.50	240.20	**UK**	Lower bound	232.50	109.70	211.94
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00

According to Table 2, all 28 EU countries are efficient at the upper bound in terms of overall, technical and scale efficiency. On the other hand, it is seen that only Luxembourg and Cyprus are efficient at both the upper and lower bounds. When $\alpha = 0.5$, the efficiency scores of the countries are shown in Table 3.

Table 3. Efficiency scores when α=0.5

Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)	Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)
Belgium	Lower bound	128.30	100.00	128.30	**Lithuania**	Lower bound	193.7	101.3	191.21
	Upper bound	100.00	100.00	100.00		Upper bound	100	100	100.00
Bulgaria	Lower bound	155.00	100.00	155.00	**Luxembourg**	Lower bound	100	100	100.00
	Upper bound	100.00	100.00	100.00		Upper bound	100	100	100.00
Czechia	Lower bound	175.50	104.70	167.62	**Hungary**	Lower bound	100	100	100.00
	Upper bound	100.00	100.00	100.00		Upper bound	100	100	100.00

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Table 3. Continued

Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)	Countries		Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)
Denmark	Lower bound	166.00	105.40	157.50	**Malta**	Lower bound	104.2	100	104.20
	Upper bound	100.00	100.00	100.00		Upper bound	100	100	100.00
Germany	Lower bound	158.70	100.00	158.70	**Netherlands**	Lower bound	164.5	106.2	154.90
	Upper bound	100.00	100.00	100.00		Upper bound	100	100	100.00
Estonia	Lower bound	127.70	107.50	118.79	**Austria**	Lower bound	140.1	100	140.10
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Ireland	Lower bound	158.80	104.10	152.55	**Poland**	Lower bound	180.80	102.40	176.56
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Greece	Lower bound	194.10	116.00	167.33	**Portugal**	Lower bound	197.40	100.00	197.40
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Spain	Lower bound	234.90	109.60	214.32	**Romania**	Lower bound	218.50	109.70	199.18
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
France	Lower bound	174.90	105.10	166.41	**Slovenia**	Lower bound	143.60	100.00	143.60
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Croatia	Lower bound	100.00	100.00	100.00	**Slovakia**	Lower bound	181.10	103.40	175.15
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Italy	Lower bound	197.90	111.30.	177.81	**Finland**	Lower bound	127.10	102.30	124.24
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Cyprus	Lower bound	100.00	100.00	100.00	**Sweden**	Lower bound	100.00	100.00	100.00
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00
Latvia	Lower bound	191.50	105.40	181.69	**UK**	Lower bound	160.70	100.40	160.06
	Upper bound	100.00	100.00	100.00		Upper bound	100.00	100.00	100.00

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As in Table 2, according to Table 3, all countries are efficient at the upper limit in terms of overall, technical and scale efficiency. However, when $\alpha = 0.5$, the number of efficient countries both at the lower and upper bound increased to five (Luxembourg, Cyprus, Croatia, Hungary and Sweden). When $\alpha = 1$, the efficiency scores of the countries are shown in Table 4.

Table 4. Efficiency scores when α=1

Countries	Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)	Countries	Overall Efficiency Scores (CCR)	Technical Efficiency Scores (BCC)	Scale Efficiency (CCR / BCC)
Belgium	107.50	100.00	107.50	Lithuania	115.00	100.00	115.00
Bulgaria	100.00	100.00	100.00	Luxembourg	100.00	100.00	100.00
Czechia	105.60	100.00	105.60	Hungary	100.00	100.00	100.00
Denmark	111.90	100.00	111.90	Malta	100.00	100.00	100.00
Germany	109.30	100.00	109.30	Netherlands	117.20	100.80	116.27
Estonia	100.00	100.00	100.00	Austria	103.70	100.00	103.70
Ireland	111.20	100.00	111.20	Poland	116.00	100.00	116.00
Greece	112.70	100.00	112.70	Portugal	116.40	100.00	116.40
Spain	113.90	100.00	113.90	Romania	126.20	103.30	122.17
France	116.30	100.00	116.30	Slovenia	108.30	100.00	108.30
Croatia	100.00	100.00	100.00	Slovakia	105.30	100.00	105.30
Italy	109.60	100.00	109.60	Finland	100.00	100.00	100.00
Cyprus	100.00	100.00	100.00	Sweden	100.00	100.00	100.00
Latvia	105.90	100.00	105.90	UK	107.20	100.00	107.20

From Table 4, it has been determined that nine (Bulgaria, Estonia, Croatia, Cyprus, Luxembourg, Hungary, Malta, Finland and Sweden) countries are efficient in terms of overall, technical and scale efficiency. In terms of overall efficiency, the countries by the worst efficiency values ranked as Romania (126.20%), Netherlands (117.20%), Portugal (116.40%), France (116.30%), Poland (116.20%) and Lithuania (115.00%). The α variable takes values between 0-1 and as the value given to the α variable increases, it is seen that the efficiency scores improve. Only Luxembourg and Cyprus are efficient for all three values (α=0, α=0.5 and α=1) of the α variable. On the other hand, when α=1, it is concluded that the efficiency scores calculated using fuzzy DEA and the efficiency scores calculated using classical DEA were equal to each other.

When α=1, the scores of Finland and Sweden from the Northern European countries are efficient in terms of overall efficiency, while Denmark (111.90%) is in the second rank. When the Southern European countries are examined, Malta and Cyprus' scores are efficient, while Italy (109.60%) is in the second rank. On the other hand, the efficiency rates of old EU members, Greece (112.70%), Spain (113.90%) and Portugal (116.40%) are significantly lower.

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Among the Western European countries, only Luxembourg's score is efficient in terms of overall efficiency. The efficiency rates of the Netherlands (117.20%) and France (116.30%) were significantly worse, compared to the efficiency rates of Austria (103.70%), Belgium (107.50%) and Germany (109.30%). Among the Eastern European countries, Bulgaria, Estonia, Hungary and Croatia are the countries that are efficient in terms of overall efficiency. Romania has the worst efficiency score among both the Eastern European and the EU countries. In addition, the efficiency scores of Poland (116.00%) and Lithuania (115.00%) are significantly worse compared to the efficient countries.

Technical efficiency shows the success of a country in producing maximum output by a certain amount of input. According to Table 4, it is seen that all EU countries are efficient in terms of technical efficiency. Scale efficiency means that a country operates at the most efficient scale size. Scale efficiency is calculated by dividing the total efficiency score by the technical efficiency score. In addition, a country that is efficient in terms of total efficiency is also efficient in terms of technical and scale efficiency.

CONCLUSION

Due to both economic and environmental factors, the importance of waste management is increasing day by day all over the world. The aim of this study is to evaluate the efficiencies of waste management activities of the EU countries. For this purpose, a five-year data set covering the period of 2014-2018 and five input and five output variables were determined. While the input and output variables were expressed as triangular fuzzy numbers, analyzes were made using the fuzzy DEA model developed by Kao-Liu. Waste management efficiencies of 28 EU countries were measured for 0, 0.5 and 1 α-cut levels.

Based on the results of the analysis, it is seen that there are significant differences between EU countries in terms of efficiency scores. For example, unexpectedly, the efficiency values of Eastern European countries are better than the efficiencies values of Western European countries. In addition, the performances of Northern European countries are better than Southern European countries.

On the other hand, the waste management performances of the new EU members, such as Croatia and Bulgaria, are much better than the EU founding members, such as France and Italy. Only Luxembourg and Cyprus are efficient for all α-cut levels used in the study, while Romania's performance is the worst among EU countries.

In the study, waste management activities of the EU countries were evaluated in a general framework on the basis of determined input-output variables. However, more detailed studies can be done for the input-output variables. The results obtained in the analyzes are valid within the scope of the 2014-2018 period and the used input-output variables. For this reason, analysis results can be examined using recent data and different input-output combinations. Especially during the Covid-19 pandemic period, it can be researched whether there is a change in the waste management performances of the EU countries. In addition, different fuzzy DEA models can be tried and the findings can be developed by different approaches.

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Chapter 3
Fuzzy Set Qualitative Comparative Analysis:
Application of Fuzzy Sets in the Social Sciences

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ABSTRACT

Qualitative comparative analysis allows for the breadth of analysis associated with quantitative data and the depth of case knowledge provided by qualitative analysis. In particular, fuzzy-set qualitative analysis (fsQCA) provides a nuanced analysis of data and offers actionable insights. Despite this, fsQCA has been underutilized in the social sciences. This chapter explores the application of fsQCA in the social sciences. The authors advance an argument for its wider adoption due to fsQCA's ability to disentangle the causal complexity involved in person- and policy-based contexts by applying a set-theoretic understanding of causation. The chapter provides an introduction to fsQCA for readers unfamiliar with the approach and advocates for its suitability within a broad range of social science studies.

INTRODUCTION

Qualitative Comparative Analysis (QCA) was developed to bridge case based and traditional quantitative approaches to research, and harness the best of both approaches (Ragin, 2014). QCA is commonly used to guide public policy and has been applied to multiple policy contexts. Examples include intersectional (Ragin & Fiss, 2017) and multidimensional (Neff, 2013) understandings of poverty, organizational structures (Florea, et al. 2019), outcomes of social organizations (Cress & Snow, 2000), pro-environmental

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behaviors (Schmitt, Grawe & Woodside, 2017), and entrepreneurial decision making (Pappas & Brown, 2020). It has been used to assess policy performance in relation to issues such as policy accountability (Pérez-Durán, 2013), funding (Vis, 2011), service delivery (Gasparro & Walters, 2017) public opposition to policy proposals (Kirchherr, Charles & Walton, 2016), and the development of new policy approaches (Feldman, 2021). It can also be used to conduct cross-jurisdictional comparisons which can generate policy learnings and contribute to effective policy transfer (Ingrams, 2018).

Despite its continued uptake in public policy application, QCA has largely been underutilized in the social sciences. Probability-based null hypothesis significance testing remains the dominant approach (Woodside, 2017). This chapter explores the application of fuzzy-set Qualitative Comparative Analysis (fsQCA) to the social sciences and advances an argument for its wider adoption. This chapter provides an overview of fsQCA and its application as both a methodology and analysis technique. The authors argue that fsQCA should be more widely adopted within the social sciences due to i) the prevalence of set-theoretic rather than correlational relations within social phenomena (Ragin, 2014), and ii) the ability of fsQCA to unravel the causal complexity often involved in person- and policy-based contexts (Rihoux, Rezöhazy & Bol, 2011). In advancing this argument the chapter explores the key underpinning principles of fsQCA, namely; conjunctural causation, equifinality, asymmetry and multifinality (Gerrits & Verweij, 2013). Using illustrative examples, the utility of fsQCA is demonstrated. The chapter includes a discussion of the key characteristics and benefits of fsQCA analysis with comparisons made to linear/correlational techniques and the dominance of symmetric theory construction and null hypothesis statistical testing in the social sciences. To assist those intending to use fsQCA, the chapter also describes the analytical steps required to perform fsQCA analysis, including calibration of set memberships, truth table construction, and analyses of necessity and sufficiency. Finally, the chapter considers the strengths and limitations of fsQCA. The aim is to provide a middle ground between journal articles that explore a single aspect of QCA, and books that provide highly detailed discussions.

BACKGROUND

Qualitative Comparative Analysis (QCA) in its original form is referred to as crisp-set Qualitative Comparative Analysis (csQCA) because membership in condition sets was limited to binary measures of 1 "full membership" or 0 "full non-membership" and relied on Boolean logic. Ragin later developed fuzzy-set Qualitative Comparative Analysis (fsQCA) which allows membership on a continuous scale between 0 and 1 (Ragin, 2000, 2008; Zadeh, 1965). This development substantially progressed QCA as a research approach and an analysis technique, making it applicable to a broad range of applications (Rihoux et al., 2013). As a research approach, QCA is able to uncover the five possible types of causes common to "historical explanations" of causality (Mahoney, Kimball, & Koivu, 2009). These are: causes which are necessary but not sufficient for the outcome; causes which are sufficient but not necessary for the outcome; SUIN and INUS causes (discussed further in this chapter); and causes which are both necessary and sufficient for the outcome (note that while theoretically and logically possible, causes that are both necessary and sufficient are rarely, if ever, uncovered in social sciences).

The research approach and theorizing aligned with QCA are configurational, rather than correlational. This allows for the incorporation of heterogeneity and diversity in data. Unlike techniques such as regression, QCA does not identify which constructs have the most predictive power. Rather, QCA attempts to identify conditions- or more commonly combinations of conditions- deemed necessary or sufficient

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for changes in the outcome condition. This approach is valuable as it allows researchers to examine the predictive validity of a range of competing antecedent conditions in a sophisticated yet parsimonious, asymmetric and non-linear fashion. QCA uses sophisticated techniques to examine numerical data but does so based on case analysis and case based modelling (instead of variable analysis). Rutten (2020, p.10) notes that "QCA's complex causality approach and its explicitly distinguishing between empirically observed cross-case regularities (the truth-table analysis findings) and causal mechanisms (the substantive interpretation of these findings) connect extremely well to critical realism". Further discussion of these links is available in Rutten (2020), Byrne (2009), and Gerrits and Verweij (2013). As aptly noted by Schneider & Wagemann (2010, p. 2) "QCA is not just another (computer-based) data-analysis technique. In order to do justice to its underlying epistemology, it needs to also be understood – and applied – as a research approach in a broad sense".

A Happy Medium: QCA as a Research Approach

The choice of any research method is driven by ontological and epistemological assumptions. For example, epistemologically positivistic research typically assumes that underlying relationships in the phenomena studied are symmetric and creates models that fit this assumption. Gigerenzer (2004) notes that null hypothesis statistical testing has been institutionalized in psychological and behavior research since the 1950s, and is often presented as the only quantitative approach available to researchers. The research questions and/or designs employed in social sciences do not typically reflect the case based nature of the phenomena under examination (Fiss, 2007). Despite theorizing in social sciences typically being case-based (Fiss, 2007) the quantitative study designs employed in many fields do not typically reflect this and are often variable-centric. Continued use of symmetric variable-centric analysis techniques has contributed to confusion and contradictory findings in the literature of many disciplines (Woodside, 2017). Ragin (2008) has noted that these 'conventional' data analysis techniques (such as regression) are adopted due to their perceived rigor, however this rigor is limited to the analysis of 'net-effects' of 'independent' variables. Most social sciences studies have however applied linear-based models to populations of interest and aimed to uncover variables key to outcome prediction based on their net-effects. There is growing support for the notion that application of linear models to such complex phenomena are often ineffective due to underlying complex configurational, asymmetric and equifinal relationships, hence persistent attempts to generate predictive models based on linear relationships contributes to confusion and inconsistencies (Feldman, 2021). Symmetric testing is limited to a focus on the relative "importance" of variables when holding all else constant. In reality, symmetrical relationships are rare in real life contexts and this "net-effects" approach has many limitations in social science applications (Ragin, 2008). Relationships between constructs in many social sciences contexts are "frequently better understood in terms of set-theoretic relations rather than correlations" (Fiss, 2011, p. 395).

QCA presents a novel theoretical perspective and methodological approach which incorporates "the best features of the case-oriented approach with the best features of the variable-oriented approach" (Ragin, 2014, p. 84). QCA assesses both breadth and depth, uncovering which conditions are related to a particular outcome and the variation in an outcome (Rihoux & Ragin, 2009; Rihoux & Marx, 2013; Ragin, 2008). In his key work *The Comparative Method* published in 1987 and re-released in 2014, Ragin suggests QCA does not diminish either the complexity of causation or the variation in social phenomena. In this way, QCA is a mid-point between complexity and generality (Ragin, 2014). Adopting a case based rather than variable based approach enhances the understanding of the context

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in which individual conditions are important. In doing this, QCA actually welcomes the complexities that are often ignored in studies utilizing traditional quantitative modelling techniques. Ragin (2008, p. 177) states: "It is not that conventional analytic techniques are flawed—in fact, they are powerful and rigorous. Rather, the argument is that they are not well suited for analyzing causal complexity. Indeed, the assessment of net-effects requires that the researcher assume that causation is uncomplicated." QCA on the other hand assumes underlying complexity.

Complex Causality

QCA has the capacity to analyze complex causation (Ragin, 2008) and the complexity of reality described by critical realism and complexity theory (Gerrits & Verweij, 2013). QCA has been acknowledged as a better choice for analysis when the links are complex and as highly suitable for answering research questions regarding social phenomena due to the complex, configurational links between data that are expected, based on the complexity and intertwined nature of conditions (Schneider & Eggert, 2014). Indeed, Ragin (2014, p. 105) suggests "one of the primary goals of the qualitative comparative approach is to allow maximum causal complexity—to avoid making simplifying assumptions about causes at the outset, as is done in most conventional statistical analyses". Because the focus of QCA is to highlight the interplay of conditions, rather than isolating single variables as commonly seen in variable-based statistical analyses, it is not only suited to analyzing complexity but is also complexity-informed itself (Gerrits & Verweij, 2013). QCA identifies only those conditions that are necessary and/or sufficient to predict an outcome, rather than incorporating all conditions measured (Fiss, 2011; Gabriel et al., 2018). Thus, despite embracing complexity, QCA offers parsimony.

The key principles of QCA include conjunctural causation, equifinality, asymmetry and multifinality (Gerrits & Verweij, 2013). Unlike more commonly recognised methods of quantitative analysis, QCA does not average differences between cases in a sample. Instead it uses the heterogeneity in data to identify distinctly different configurations based on how conditions operate conjointly, known as conjunctural causation (Ragin, 2014; Gabriel et al., 2018, p. 878). Additionally, this method does not search for a single combination of conditions to predict an outcome. Instead QCA allows for multiple different configurations to predict the same outcome, known as equifinality (Fiss, 2011; Kane, et al., 2014). QCA also allows for asymmetry in data and for asymmetric solutions. The antecedent configurations sufficient for the presence or high scores of the outcome are not necessarily the opposite of the configurations sufficient for the absence or low scores of the outcome. Similarly, a single condition may be present in one solution recipe, negated in another and absent from a third recipe solution, all leading to the same outcome condition. This principle of multifinality means that conditions may have a different effect in different contexts or when assessed in combination with different conditions. This contrasts with traditional linear analysis which is unifinal (Gerrits & Verweij, 2013, p. 175).

Illustrative Example: Understanding Intersectional Inequality Through fsQCA

Researchers may get different results, depending on the quantitative research approach selected. Consider Ragin and Fiss's (2017) studies of poverty. Poverty is a complex and wicked social policy issue (Spicker, 2016). Inequalities experienced by individuals that have been linked to poverty include family background, education, test scores and more. Typical modelling has been used to estimate the independent net effects of these factors, seeking to identify the 'best' predictor of poverty later in life. As a result, Hernstein &

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Murray (1994) suggested that intelligence (as measured by standardized testing) is more important than other proposed predictors of poverty.

Ragin and Fiss observed that due to the nature of social structures many factors often coincide. They noted that the Matthew effect of accumulated advantage suggests that it is not realistic to hold all else constant while measuring an "independent" variable in this context. Ragin and Fiss instead examined set-theoretic relations using the same data analyzed by others contributing to the research into poverty. Ragin and Fiss successfully demonstrated that social advantages overlap, as do social disadvantages. In other words, the set of individuals who are highly educated tend to have high levels of set coincidence with the set of individuals who have high test scores, the set of individuals with high parental income and the set of individuals whose parents are highly educated, and therefore the set of individuals who avoid poverty. This is known as multiple set coincidence. Because set theory is asymmetric and configurational this analysis differed substantially from correlational approaches, uncovering previously unseen relationships of consistently sufficient pathways to avoid poverty for different groups. For example, Ragin and Fiss found that the 'recipe solutions', or sets of conditions, for avoiding poverty for Black females were a subset of the recipes for white males, indicating that the conditions for avoiding poverty for Black females are tougher.

SET THEORY

QCA adopts a set theoretic approach and sets are key to the methodology. Duşa (2019) describes sets as a category / collection of objects that share a common property. Based on existing theory, membership in sets is conceptualized and then applied to the data through a process known as calibration (Gabriel, et al., 2018; Ragin & Pennings, 2005). While crisp sets are simply calibrated based on full membership (1) or full non membership (0), fuzzy sets allow for degrees of membership within a set. Set membership definition involves the use of adjectives, while variables are often labelled using verbs. Investigating set relationships instead of linear relationships is based on the idea that social science relationships are often based on explicit connections that are set-theoretic relations rather than correlations (Ragin, 2014). This demonstrates the link to complexity as set-relations are asymmetric and "complexity focuses on intricate causal patterns that progress non-linearly" (Gerrits & Verweij, 2013, p. 168).

QCA investigates relationships of necessity and sufficiency. These relationships are represented by set relations in the form of supersets and subsets. QCA asks "what conditions—alone or in combination with other conditions—are necessary or sufficient to produce an outcome" (Kane et al., 2014, p. 202). A relationship of sufficiency is represented where Y is a superset of X, while a necessity relationship involves X being a superset of Y (Fiss, 2007; Kane et al., 2014; Ragin, 2013). Accordingly, a sufficiency relation indicates that X is a subset of Y and a necessity relation indicates that Y is a subset of X (Ragin, 2008). In contrast, correlation techniques are symmetrical- searching for linear relationships of causal changes leading to an outcome. Demonstrating explicit connections of necessity and sufficiency is only possible through set-theoretic relations and cannot be assessed with correlational methods.

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Illustrative Hypothetical Example: Understanding Environmental Stewardship Action Through fsQCA

The mathematical underpinning of QCA is Boolean and fuzzy algebra. While not all Boolean principles/ principles of formal logic are adopted in fsQCA, many basic operations developed by Boole (1854) are applicable. The following sections outline key operations of fsQCA. Each concept is introduced, followed by a hypothetical, simplified example examining an outcome of environmental stewardship action, with a large-n individual-level design.

Environmental stewardship actions are "intentional and conscious civic behaviors that are focused on systemic causes of environmental problems and the promotion of environmental sustainability" (Alisat & Riemer, 2015, p. 14). These behaviors are conscious, intentional and focused on creating positive impacts; and often involve specific competence (Alisat & Riemer, 2015; Bramston, Pretty, & Zammit, 2010; Ridings, 2017). In an attempt to ameliorate significant damage to natural environments, policy makers are interested in encouraging environmental stewardship behaviors, and identifying individuals who perform or are likely to perform them (Gregory, 2018).

Three common predictors of environmental stewardship behaviors are i) holding liberal / progressive political views, ii) belonging to an environmental organization, and iii) indicating intent to perform the behavior. Ortega-Eaga (2014) claimed political views are one of the most consistent correlational measures of positive environmental behaviors. Individuals holding more progressive / liberal views are typically more likely to perform these behaviors (Dunlap & Van Liere, 1978; Sánchez et al., 2016; Taniguchi & Marshall, 2018). Membership in an environmental organization has also been linked to higher levels of positive environmental behavior such as environmental stewardship, as this is often a key activity undertaken by these groups (Cho & Kang, 2016; McDougle et al., 2011). Intention is widely accepted as a key indicator of any behavior performance (Morren & Grinstein, 2016) including pro-environmental behavior (de Leeuw et al., 2015). Behavioral intention is the most proximal and strongest factor influencing behavior in Ajzen's model of planned behavior (Ajzen, 1985; Ajzen, 1991, 2012; Howard, 2019).

The three antecedent conditions of environmental stewardship action are utilized in the illustrative example below. As per QCA convention, sets are labelled with short, capitalized descriptors to indicate presence / membership (sets in lowercase or with a preceding tilde i.e., "~" indicate negated sets). In this example, the set of individuals belonging to an environmental organization are labelled with ENVORG, the set of individuals holding liberal / progressive political views are labelled as LIBERAL, the set of individuals with intent to perform the behavior are labelled as INTENT, and finally the set of individuals who perform environmental stewardship behaviors are labelled ENVSTEW.

Set Intersection/Logical AND

Set intersection represents the conjunction of two or more sets, which can be operationalised in multiple ways. Typically, set intersection is represented with multiplication symbols or no symbol: either A*B, AB, or A•B. This is understood and verbalised as "set A and set B". As seen in Figure 1, the intersection of multiple sets reduces the area covered. To calculate the membership score of set intersection, the minimum, or the lesser of the two values is taken. This is operationalised as $A \cdot B = min(A_i, B_i)$.

Set intersection is not limited to only two sets and can incorporate many sets. Each additional set will reduce the membership score to the minimum of all sets which are intersected e.g., $A \cdot B \cdot C \cdot D = min(A_i, B_i, C_i, D_i)$. Applying the environmental stewardship example, Figure 2 shows a hypothetical intersec-

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tion of the three antecedent condition sets. The intersection of these three sets is understood as the set of individuals who are a member of an environmental organisation AND possess liberal political views AND report intention to perform stewardship behaviours.

Figure 1. Set intersection

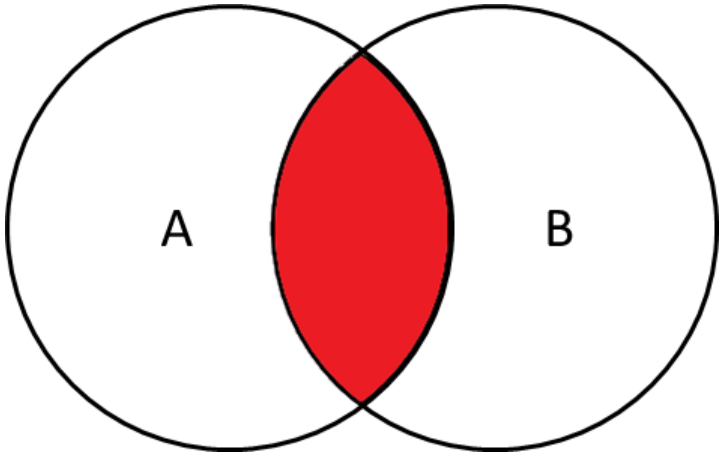
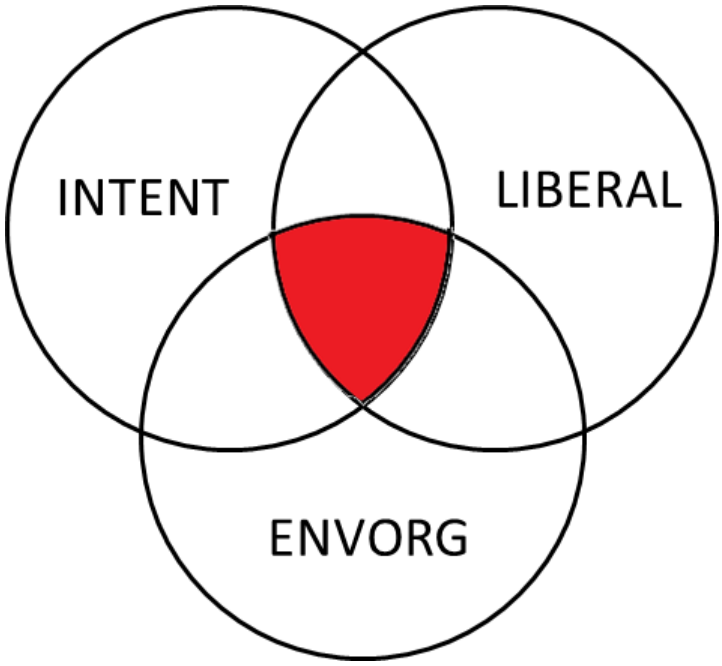


Figure 2. Multiset intersection: Case example



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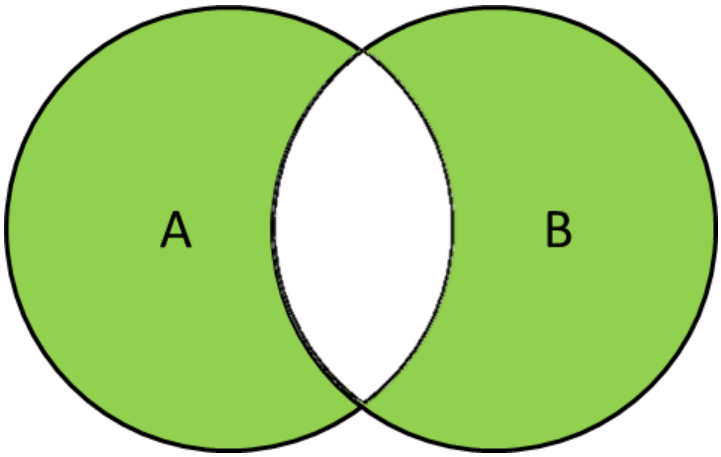
Set Union/Logical OR

The union of two sets (or disjunction of sets) is operationalised as A + B. In verbal form this is understood as "either set A or set B". As sets are combined in relationships of union, the area covered increases, as shown in Figure 3. To calculate the membership score of a set union, the maximum, or the greater of the two values of the sets is taken. Hence, A+B= max(A_i,B_i).

Figure 3. Set union



As in set intersection, set union is not limited to only two sets. Multiple set union takes the maximum of all sets included e.g., $A+B+C+D= max(A_i, B_i, C_i, D_i)$. Applying the environmental stewardship example, Figure 4 shows a hypothetical union of the three antecedent condition sets. The union of these three sets would be understood as the set of individuals who are a member of an environmental organization OR the set of individuals who possess liberal political views OR the set of individuals who report intention to perform stewardship behaviors.

Set Coincidence

Set coincidence is the number of cases found in the intersection of two sets, relative to the number of cases in their union i.e., set coincidence is low when the set intersection is a small fraction of the set union. High set coincidence of three sets is depicted in Figure 5. Set coincidence is calculated using $\sum(A_i,B_i,C_i)/\sum max(A_i,B_i,C_i)$. Recall that in the illustrative example of *Intersectional Inequality* earlier in this chapter Ragin and Fiss (2017) uncovered high levels of set coincidence.

Figure 6 uses the stewardship behavior example to depict a scenario where the set intersection varies for the three antecedent sets. As shown, there is high set coincidence between the set of individuals who are a member of an environmental organization and the set of individuals who indicate intent to perform environmental stewardship actions, but lower set coincidence between the set of individuals who possess liberal political views and those who are a member of an environmental organization.

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Figure 4. Multiset union: Case example

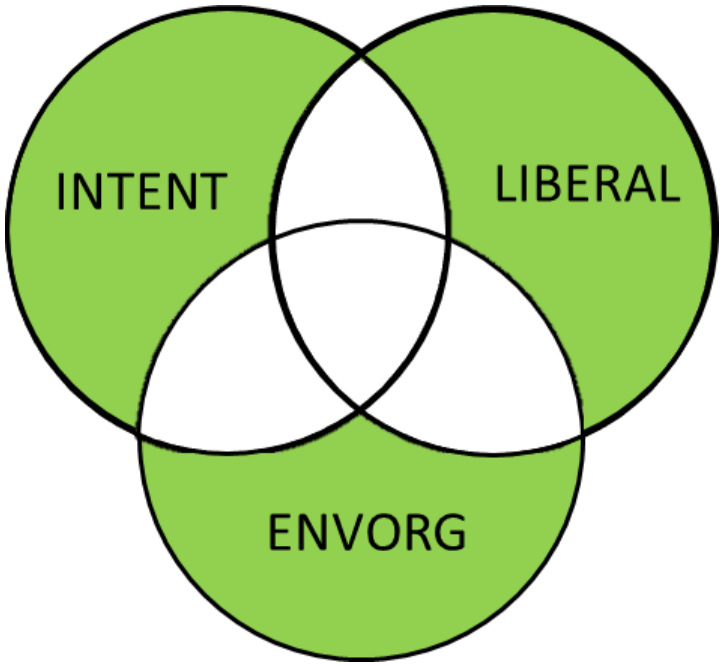

Figure 5. High set coincidence

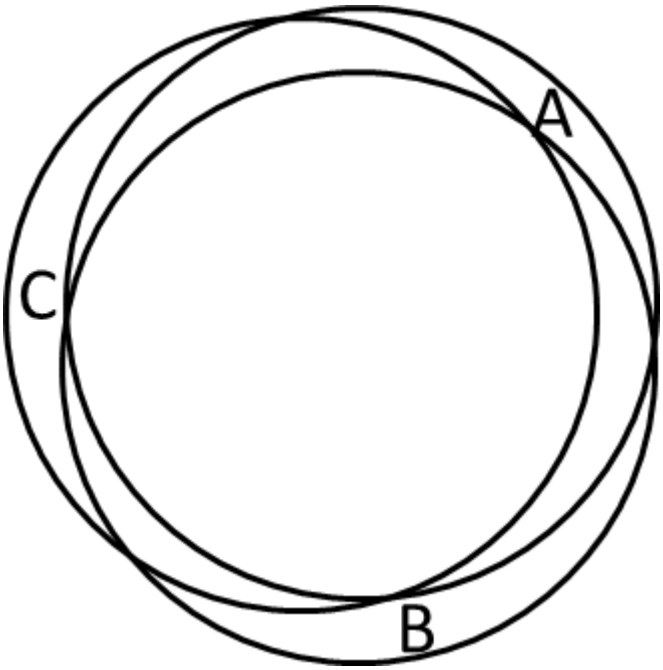

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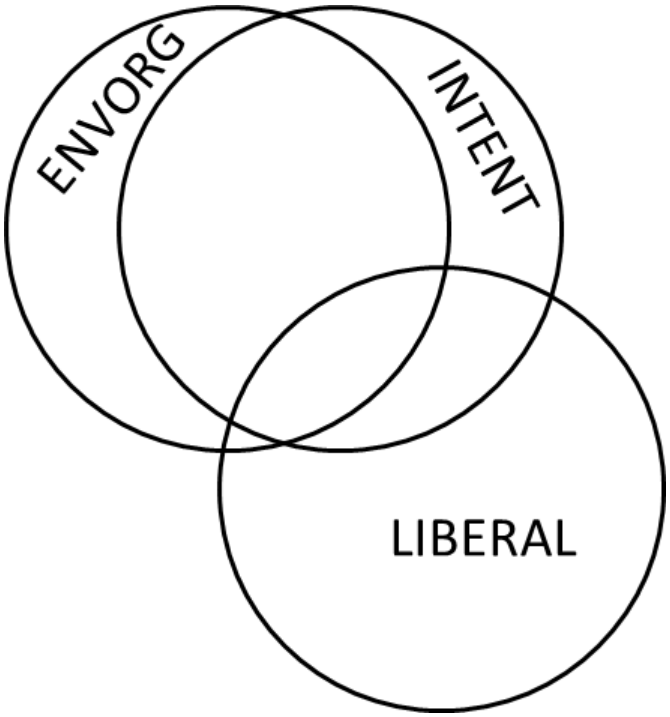
Figure 6. Multiset coincidence: Case example



Set Negation

Set negation is all instances of the absence of the set, or 1-"set". Set negation is represented with lower case letters or with a tilde i.e. "a" or "~A". In a verbal sense this is simply "not A", as represented by the shaded area in Figure 7. In Figure 8, this would be understood as the set of individuals who are not a member of an environmental organization. Negated and non-negated sets can both be part of a single recipe solution, e.g., ~A•B or a•B.

Subset Relations

A subset relationship is seen when a causal or antecedent condition is a subset of the outcome condition. This is represented in Figure 9 and is expressed as $X_i \leq Y_i$ or X ⇒ Y, where X is a subset of Y.

Figure 10 depicts a subset relationship, where the antecedent conditions of behavioral intent, environmental organization membership and possessing liberal political views are all subsets of the group of individuals who perform environmental stewardship behaviors e.g. the set of individuals who are members of an environmental organization are a subset of the set of individuals who perform environmental stewardship actions.

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Figure 7. Set negation

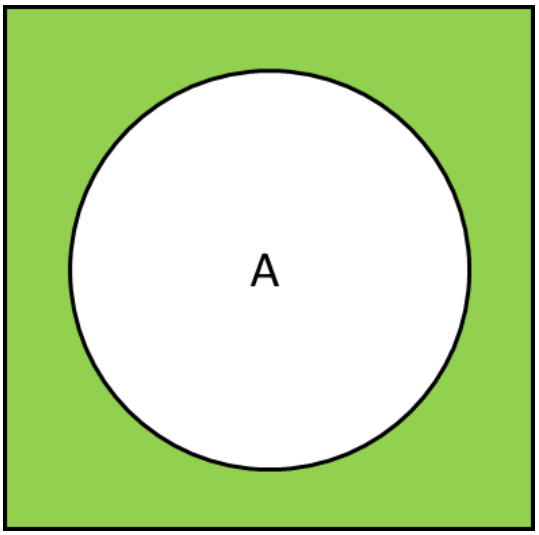


Figure 8. Set negation: Case example

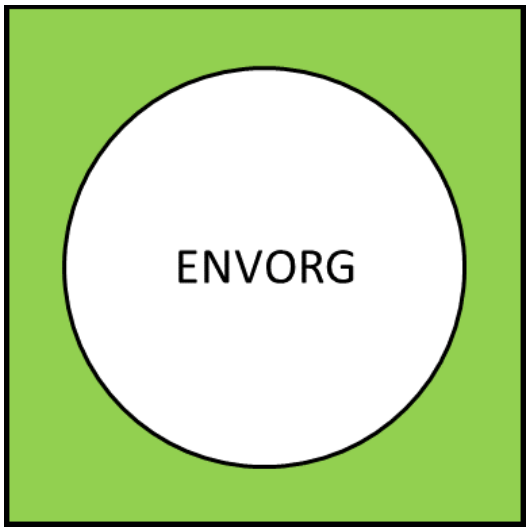
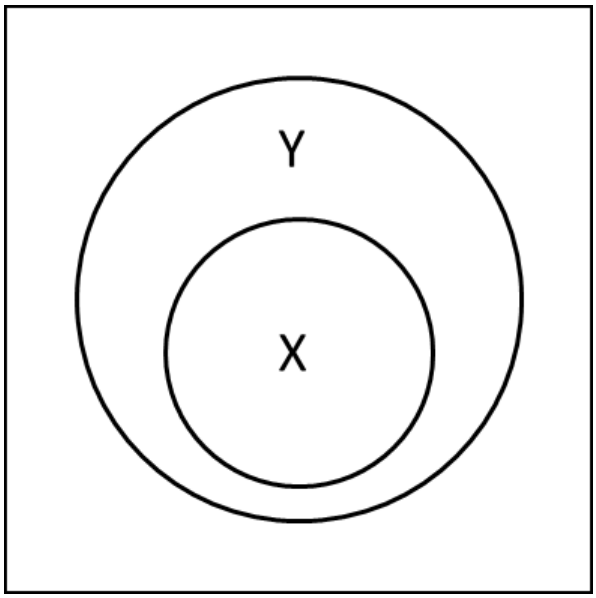


Figure 9. Subset relationship



Superset Relations

On the other hand, a superset is seen when Y is a subset of X, i.e. the outcome condition is a subset of the causal condition, or the causal condition is a superset of the outcome condition. This is shown in Figure 11 and represented as $X_i \geq Y_i$ or X ⇐ Y.

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Figure 10. Multisubset relationship: Case example

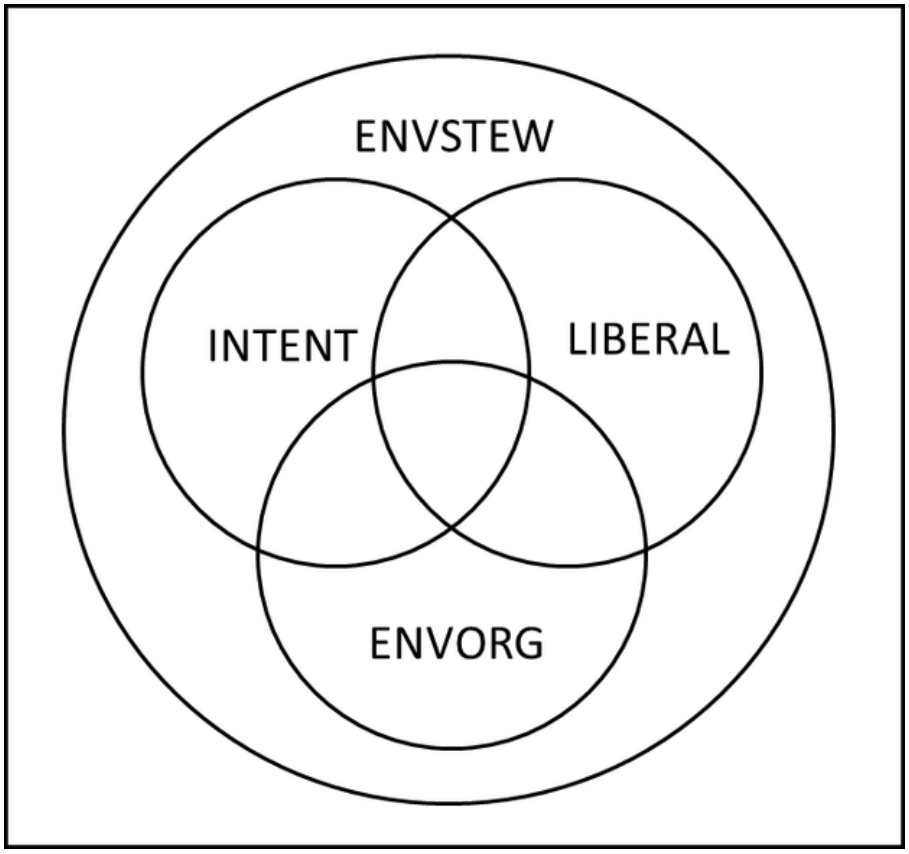
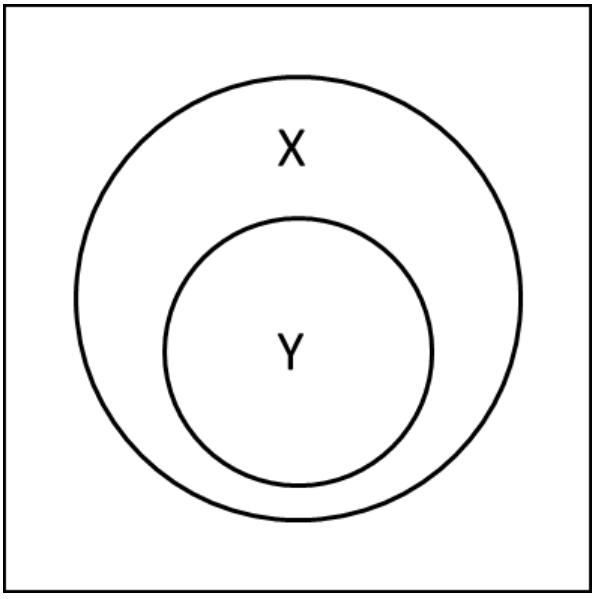


Figure 11. Superset relationship



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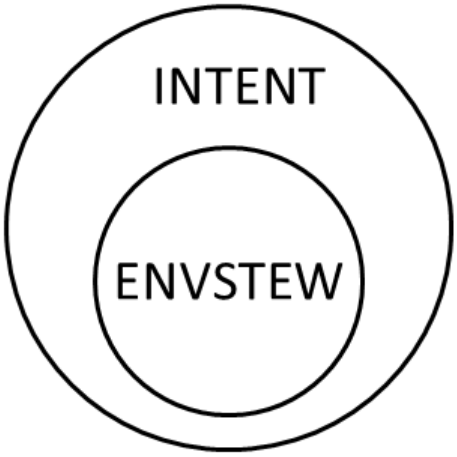
With environmental stewardship, the researcher should be open to the idea that not everyone who expresses intent will perform the behavior. Figure 12 shows a relationship where intent is a superset of environmental stewardship behaviors i.e. those performing such behaviors are only a subset of the individuals who indicate intent to perform them.

Figure 12. Superset relationship case example



TECHNICAL STEPS: QCA AS A TECHNIQUE

Given the authors promote the increased adoption of QCA in social science, what follows is a discussion of the analysis procedure. The following sections summarize the technical steps, with reference to more detailed guidelines where appropriate. Many software packages are available for fsQCA analysis, all of which differ in capability and commands. The fs/QCA software produced by Ragin & Davey (2009) remains the most popular. It is an excellent tool for beginners wishing to familiarize themselves with the technical application of QCA. Additional capability is available in other programs such as the QCA package (Dusa & Thiem, 2012) in R (R Development Core Team), and TOSMANA (Cronqvist, 2011). Thiem & Dusa (2013) review these three commonly used software packages, detailing the technical differences of each. The explanation of technical steps outlined in this chapter are applicable regardless of the software utilized. Guides for conducting each step in software packages are available for fs/QCA (Ragin, 2018), R (Dusa, 2019) and TOSMANA (Cronqvist, 2016). The present authors offer general, introductory coverage.

Case Selection and Research Design

After a research question is defined (and fsQCA is identified as an appropriate research method due to the configurational nature of the underlying mechanisms) it is important to select cases that are suitable for analysis. The most common approach is to select an outcome of interest and then identify both cases that exemplify this outcome, as well as cases which should theoretically display this outcome but do not.

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Following this, relevant causal conditions are identified based on theoretical and substantive knowledge. Prior literature may indicate important causal conditions but it is also important to use case knowledge to confirm all important causal conditions are included in the analysis. Despite the ability of QCA to simplify complexity and identify only those conditions necessary or sufficient for an outcome, selection of potential causal conditions should be limited. Rather than a broad inclusion of many conditions, a tightly specified set of causal conditions is always best. Data sets are constructed typically, with a separate row for each case and columns measuring condition scores. As set membership scores may only fall between 1 and 0, data requires calibration. This will ensure that sets are accurately representing their given label.

Calibration of Conditions

Social science measures are often "uncalibrated" measures of distributions of cases relative to each other. Calibration allows for precise measures of set membership based on external criteria from existing knowledge or knowledge uncovered from in depth examination of cases. Like case selection, calibration is a crucial step of fsQCA which impacts the entire analysis. The process of calibration assigns this level of membership to data points within sets. Most calibration is completed using computer software but the user carefully specifies the point of full membership, full non membership and the cross over point (or point of maximum ambiguity) in a set. Note that leaders in the field warn against the calibration of sets using the mean or median. Following suggestions by Ragin (2008) and Ragin and Pennings (2005) selection of the qualitative anchors for calibration (full membership, full non-membership, and the cross-over point or point of maximum ambiguity) must instead be selected based on theoretical reasoning rather than properties of the data. The choice of full membership and full non membership determine which variation in the data is relevant and which is irrelevant (Ragin & Fiss, 2017). This differs from conventional quantitative techniques which treat all equal unit increases in a measure as the same.

"Verbal" labels used for calibrated sets should adequately and accurately reflect the data and cut off values (Fiss, 2007). Different labels may be used to calibrate the same data in different ways, however sets with all scores above or below 0.5 should be reconsidered and relabeled. Calibrated sets do not need to be symmetrical and ideally range from close to 0 or 0, to close to 1 or 1. Correct calibration of conditions can be confirmed by plotting the raw variable on the X axis and the calibrated variable on the Y axis. This should display an S curve around the point of full membership and full non membership, with point of maximum ambiguity at 0.5. Irrelevant variation is displayed as truncated data. This is seen in Figure 13, where continuous data measuring age of respondents has been calibrated to a measure of the set of old individuals. As the set of interest is those of a relatively high age, data below ~50-years-old is irrelevant variation.

Analysis of Necessity

Necessity relationships are uncovered with the presence of superset relationships. A condition (or disjunction of conditions) is deemed necessary when the presence of the outcome condition is only attained with the presence of this condition (Fiss, 2007, 2011; Schneider & Eggert, 2014). In other words, the outcome is a subset of the antecedent condition, or values of the outcome are less than values of the cause (Dusa, 2019). Uncovering necessity relations indicates that the outcome does not occur without the presence of this necessary condition (or set of conditions). Alternatively, this means that the necessary condition (or set of conditions) is always present or almost always present when the outcome occurs (Braumoeller

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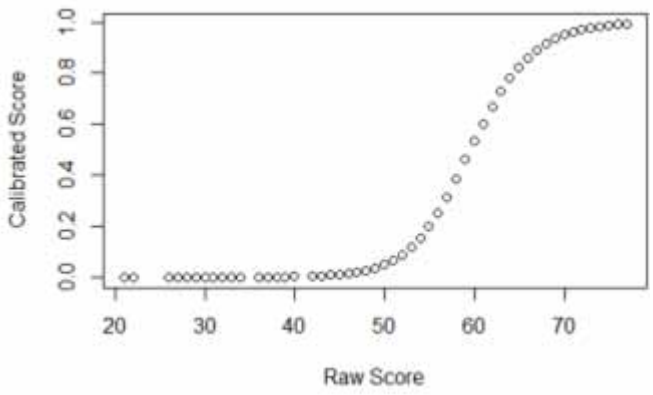
& Goertz, 2000). Analysis of necessity should always be conducted prior to construction of truth tables and sufficiency analysis, as recommended by Rihoux and Ragin (2009), and Schneider and Wagemann (2010). There may be inductively expected necessary conditions but a deductive confirmation of which conditions are necessary should be performed (Dusa, 2019).

Figure 13. Calibrated set membership for old individuals



Rather than a correlational-based relationship that implies a higher level of a condition leads to a higher level of an outcome, a necessity relationship indicates the minimum level of a condition (or a set of conditions) that is necessary for the outcome to occur. Analysis of necessity is computed with appropriate software packages using simple commands. This analysis may uncover no necessary conditions, a single necessary condition, or SUIN conditions. SUIN conditions are a Sufficient but Unnecessary part of a condition set that is itself Insufficient but Necessary for the outcome (Mackie, 1965). A SUIN condition is not necessary by itself but in configuration with other conditions forms part of a necessity recipe solution. Solution output will also indicate measures of solution consistency, coverage and relevance of necessity, which are discussed further in the following sections. As well as examining output figures for consistency, coverage and relevance of necessity, researchers should consider empirical and theoretical relevance and conceptual meaningfulness of results (Schneider, 2018; Dusa, 2019). Solutions should also be plotted graphically to check for deviant cases. Interested readers are directed to Schneider and Wagemann (2012) for further discussion and examples.

Consistency

Consistency of necessity signifies how well or how perfectly a relationship indicates necessity, and ranges from 0 to 1. Consistency of set X as a superset of set Y is calculated using $\sum min(X_i, Y_i) \sum (Y_i)$. Unlike the universally accepted p-values used in significance testing ($p < 0.05$ and $p < 0.01$), Ragin (2014) suggests avoiding a clear-cut threshold for consistency measures. The context of each study may lead to differing appropriate thresholds. Indeed, an overreliance on universal thresholds for p-values in null hypothesis statistical testing has contributed to significant concerns surrounding the credibility of many published claims (Goodman, 2016). Figure 14 shows a fairly consistent but not perfect necessity

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relationship. This is also seen in Figure 15, where intention is a mostly consistent necessary condition for environmental stewardship behaviors.

Figure 14. Necessity consistency

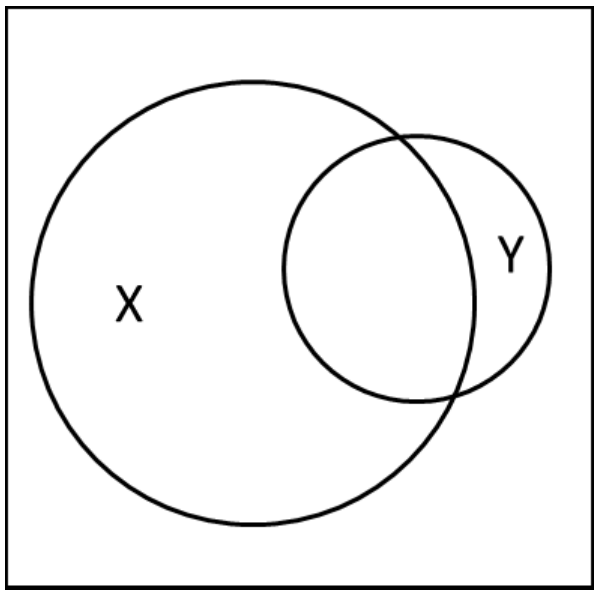
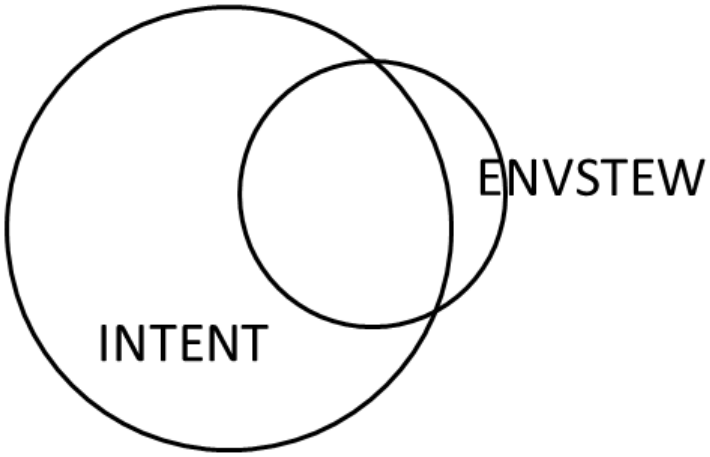


Figure 15. Intention as a highly consistent necessary condition for environmental stewardship behaviour: Case example



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Coverage and Relevance of Necessity

Trivial necessary conditions do not vary across the outcome; they are universally (or almost universally) present or absent (Goertz, 2006). Trivial necessary conditions occur either i) when the condition(s) set is far greater in size than the outcome set, or ii) when there is little to no evidence of the negated condition set to the point that the condition set is constant or almost constant. Necessity coverage scores indicate the proportion of cases that fit the necessity relationship, while also highlighting the first form of trivialness if present (Ragin, 2006). Coverage of a necessary condition is calculated using $\Sigma min(X_i, Y_i)/\Sigma(X_i)$. Relevance of necessity (RoN) is a measure to ensure the second form of trivialness is not present. A low RoN score indicates this form of trivialness and invalidates claims of necessity (Schneider & Wagemann, 2012). Relevance of necessity is calculated using $\Sigma(1-x_i)/\sum(1-min(X_i = Y_i)$.

Analysis of Sufficiency

Sufficiency is established when it can be shown that instances of the causal recipe constitute a reasonably consistent subset of instances of the outcome. A condition (or condition set) is deemed sufficient when the presence of the causal condition always, or almost always, results in the presence of the outcome condition (Fiss, 2011; Schneider, & Eggert, 2014). In other words the causal condition is a subset of the outcome condition, or values of the cause are equal to or less than values of the outcome (Dusa, 2019). Analysis of sufficiency involves multiple steps, beginning with the construction of a truth table.

Truth Table Construction

Truth tables originate from formal logic and represent the "2k logically possible combinations, where k is the number of attributes under consideration" (Fiss, 2007, p. 402) summarizing the "property space" of these attributes from the data (Fiss, 2011). A row is constructed for every theoretically possible combination of conditions (truth tables are constructed automatically by various software packages) and cases are grouped based on the causal conditions exhibited. In fsQCA the rows with empirical instances of membership above 0.5 are included. Each case will be represented with a membership above 0.5 in only one row of a truth table. It should be noted that if a case has membership in a configuration of exactly 0.5 this will split its membership across more than one row and will not be included in the list of cases in the truth table (which is why selection of the cross over point / point of maximum ambiguity during calibration procedures should avoid exact alignment with points in the data). Rows without cases are evidence of limited diversity (limited diversity exists in almost all measures of naturally occurring phenomena) and these rows are labelled "remainders" or "logical remainders".

The researcher is able to apply a frequency threshold and incorporate counterfactual cases into the analysis. A frequency threshold indicates the number of cases required to be empirically present in a row displaying the outcome for this recipe to be considered sufficient. This threshold is chosen based on observations and to ensure at least 80% of all cases are retained as recommended by Ragin (2008). This is usually set at one or two cases for small n studies however there may be reasons to increase this threshold for larger n studies. "Easy" counterfactual cases are incorporated into the analysis by the researcher by incorporating information that is known about the expected influence of a causal condition on the outcome. This allows the (intermediate) solution to be constructed based on plausible solutions, with counterintuitive aspects removed. A subset of "difficult" counterfactuals (untenable assumptions)

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can also be incorporated in the analysis. Excluding logical remainder rows that would lead to untenable assumptions prior to logical minimization ensures no solution terms are based on untenable assumptions (Schneider & Wagemann, 2012). Untenable assumptions are a subset of difficult counterfactuals including incoherent counterfactuals (counterfactuals contradicting necessity statements and/or contradictory simplifying assumptions) and implausible counterfactuals (logical impossibilities). See Schneider & Wagemann (2013; 2012, Ch. 8) for a discussion and worked examples regarding untenable assumptions.

Truth Table Minimization

Logical minimization involves a bottom-up comparison and incremental elimination to specify combinations linked to the outcome. This is conducted automatically by algorithms built into the various software packages available. Logical minimization results in three solutions of sufficiency for the outcome condition: conservative, parsimonious and intermediate. Not all three solution formulas are used extensively for substantive interpretation (Schneider & Wagemann, 2010). The conservative solution (sometimes also referred to as the complex solution) is based on no use of remainders to reduce conditions; only empirical instances are used. The parsimonious solution allows any remainders that result in fewer, simpler recipes (i.e., includes both difficult and easy counterfactual cases, but may then include logically meaningless solutions as it does not rely on theory) but may exclude important conditions. Meanwhile the intermediate solution relies on counterfactual analysis using theoretical and substantive knowledge that is input into the software. This only utilizes remainders aligned with the counterfactual analysis (i.e., only remainders with clear directionality are included). The parsimonious and conservative solution are steps in the process to calculate the intermediate solution. However, all three should be examined and compared. While the parsimonious and conservative solutions are still considered in interpreting the intermediate solution, often only the intermediate solution is presented (Ragin, 2008, p. 175).

Consistency

Set-theoretic consistency is an indication of the proportion of cases for which an outcome is present among a set of cases that share a combination of conditions. This is calculated using $(X_i \leq Y_i)=\sum(min(X_i,Y_i))/\sum(X_i)$. A highly consistent sufficiency relationship is depicted in Figure 16. This is also shown in Figure 17 where environmental organization membership is a highly consistent sufficient condition for environmental stewardship behavior.

As with consistency of necessity measures, thresholds for consistency of sufficiency are dependent upon setting. However, researchers are warned that a consistency score below 0.75 is often inconsistent. A generally accepted threshold is 0.80 (Misangyi et al., 2016; Ragin, 2008; Misangyi & Acharya, 2014; Ragin, 2008; Ragin, 2014; Dusa, 2019). As with the analysis of necessity relations, multiple consistency thresholds should be applied and the solutions compared. This should be undertaken along with examination of the truth table for "natural breaks" in consistency scores. This examination should also be paired with consideration of the measures of proportional reduction in inconsistency (PRI), to ensure claims of sufficiency relations are robust (Schneider & Wagemann, 2010, 2012).

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Figure 16. Consistency of sufficiency

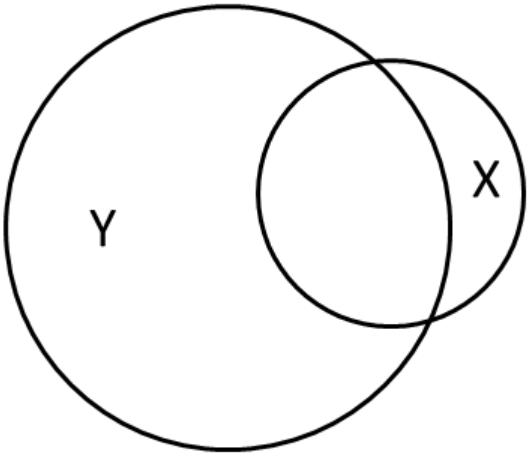
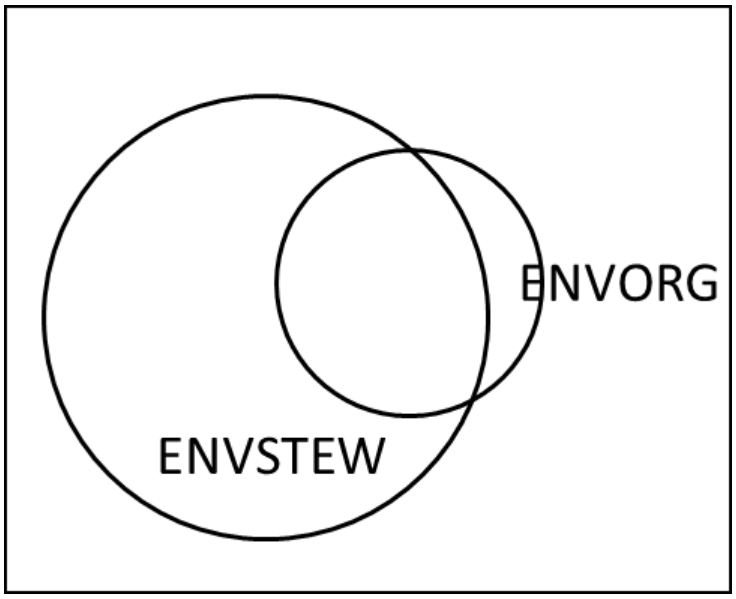


Figure 17. Environmental organization membership as a highly consistent sufficient condition for environmental stewardship behavior: Case example



There are measures of both raw consistency and Proportional Reduction in Inconsistency (PRI). Usually these are very similar scores however PRI is a more rigorous measure. While raw scores are more descriptive of the data, PRI is relevant for fuzzy sets as there is a possibility for a condition to be a subset of both the outcome and the negation of the outcome (for condition scores lower than 0.5). Highly consistent scores can occur despite not truly reflecting a sufficiency relationship (as X is below 0.5).

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Coverage

Sufficiency recipe coverage is an indication of empirical relevance or importance. It does not imply better performance in terms of achieving the outcome but is an indication of being more prevalent (Ragin, 2008). Coverage should only be calculated and inspected after confirming that a set is consistently sufficient for the outcome. Raw coverage is calculated by taking the number of cases with the causal condition and the outcome, divided by the number of cases with the outcome, expressed as $(X_i \leq Y_i) = \sum(min(X_i, Y_i))/\sum(Y_i)$. This is an indication of the proportion of cases with the outcome that are included in that recipe. Coverage does not have a cut off and is used as a relative measure. It is important to examine both consistency and coverage together. Often the highest consistency scores can be so narrowly defined that they do not apply to many cases and therefore have low coverage. While a very consistent recipe is a good indicator of the causal conditions leading to the outcome, its practical use / interpretation is limited if it applies in very few instances.

A Note on INUS Conditions

While the goal of QCA is to uncover sufficient conditions which lead to a causal outcome, results rarely indicate a single condition is sufficient for an outcome. Application of the procedure often uncovers INUS solutions. INUS conditions are those which are an "Insufficient but Necessary part of a condition [set] which is itself Unnecessary but Sufficient" for the result (Mackie, 1965, p. 247). Almost all conventional statistical procedures are based on the analysis of bivariate correlations. If INUS causes are involved, these analyses are not helpful in uncovering them. Inconsistent findings in many areas of social science research may in fact be explained by constructs which are often claimed to be predictors of outcomes, but are in fact INUS conditions and only sufficient in conjunction with the presence and/or absence of multiple other INUS conditions. As the QCA methodology embraces complexity and configurations, when examining and interpreting technical QCA results, the recipe solution should be considered as a whole. As with SUIN conditions in necessity solutions, the presence or absence of a single condition in a solution recipe, should not be over-interpreted as it is the interplay with other conditions in the recipe, that together explain the outcome (Ragin, 2008).

Repetition of Steps for Negated Outcome

Following the technical steps outlined above, researchers should also conduct a full test of necessity and sufficiency for the negation of the selected outcome. Because set relations are asymmetric, the configurations which lead to the absence of an outcome are not always the inverse of those which lead to its presence.

Presentation of Results

QCA results are presented in various ways, depending on context and suitability. However, the most typical approach is to present a summary of necessity and intermediate sufficiency solutions in a single table. This table is known as a concov (consistency-coverage) table. Investigators typically use Venn diagrams, similar to those presented earlier in this chapter, to visualize the superset and subset relationships. See Rubinson (2019) for a summary of the various ways to present QCA results.

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CRITIQUES AND LIMITATIONS OF FSQCA

First and foremost, the authors acknowledge that fsQCA is not a universal method. While this chapter presents arguments for the use of QCA in social science, fsQCA may not always be the most appropriate method or technique. All research should be conducted holistically, with the methodological and analytical approach chosen to most suitably answer the research question(s) at hand. QCA is best applied as an interpretive case-based method. It should be selected on those grounds and not as a wholesale alternative to regression. The research question and approach should be configurational and congruent.

Despite developments and increased uptake, QCA is still a niche method due to its age and the long-established dominance of other approaches. It should be noted, however, that many of the critiques of fsQCA are derived from incorrect or inappropriate application of the technique by individual researchers, rather than flaws with the technique itself (Schneider, 2018). Other, more valid critiques of fsQCA have been addressed in recent times. For example, measures of proportional reduction in inconsistency (PRI) and the enhanced standard analysis (ESA) procedure have resolved many issues surrounding difficult counterfactuals (Schneider & Wagemann, 2013). Perceived limitations surrounding temporality and use outside of cross-sectional data analysis have been addressed by Ragin & Strand (2008). While QCA was originally designed for medium-n (10-50) case analysis, the QCA research community continues to focus on ensuring the robustness of QCA when applied to large-n studies (Rubinson, Gerrits, Rutten & Greckhamer, 2019).

Despite QCA's ability to reduce complexity, results uncovered by QCA are often complex. Meaningful interpretation can be challenging. Correlational, linear techniques often violate statistical principals when too many variables are included in a model. While fsQCA does not suffer from this limitation, Schneider and Wagemann (2010) suggest restricting the number of conditions to a moderate level. As a result, researchers employing QCA need to find an appropriate balance between the number of causal conditions and the number of cases. For every causal condition added, the truth table is increased substantially. Too many causal conditions and too few cases, results in many truth table rows with no empirical cases present. This can severely limit the ability to adequately conduct analysis. As Vaisey (2014) highlights, the ability of QCA to simplify complexity relies on counterfactual assumptions, which requires coverage of many truth table rows. Patterns of missing rows can easily lead to misinterpretation, especially by the novice QCA user. Vaisey (2014, p.110) warns "Naïve use of QCA can lead to spurious results".

Along with the additional reading suggestions appended, a resource of great value to researchers interested in QCA is COMPASSS (COMParative methods for Systematic cross-caSe analySis, see compasss.org) which includes details of events, training opportunities, resources and more.

Fiss, Marx & Rihoux (2014) acknowledge that all data analysis techniques are a craft and "QCA is no different: to do it well requires not mechanical application of formal rules but both good practice and the "wisdom" that comes with continued use." (p.98)

CONCLUSION

This chapter has detailed the key principles and concepts of an underutilized but sophisticated analytical technique; fuzzy-set Qualitative Comparative Analysis. The authors hope we have encouraged the wider adoption of fsQCA within social science disciplines, given its ability to disentangle the causal complexity involved in person- and policy-based contexts. The authors encourage fsQCA analysis in areas where

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the extant literature is based on traditional quantitative techniques, contains contradictory findings, and is likely to include configurational causal mechanisms. This is exemplified in Ragin and Fiss's (2017) study of *Intersectional Inequality* as discussed throughout. In this way fsQCA has the potential to provide further insights in areas of social science dealing with complex and wicked policy problems.

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KEY TERMS AND DEFINITIONS

Causal Asymmetry: The observation that antecedent configurations sufficient for the presence or high scores of an outcome are not necessarily the opposite of the configurations sufficient for the absence or low scores of an outcome.

Equifinality: The observation that a given end state or goal can be reached by many potential means or ways, thus a diversity of pathways may lead to the same outcome.

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Fuzzy Set: A class of objects with a continuum of grades of membership ranging between 0 and 1. Differ from Boolean or crisp sets which are limited to only values of 1 or 0.

INUS Condition: An **I**nsufficient but **N**ecessary part of a condition set which is itself **U**nnecessary but **S**ufficient

Multifinality: The observation that one component may function differently depending on the organization of the system in which it operates thus leading to different outcomes. Challenges the concept of causality in relationships.

Necessity: A situation in which the presence of an outcome condition is only attained with the presence of a condition. Seen when the outcome is a subset of the antecedent condition, or values of the outcome are less than values of the cause.

Qualitative Comparative Analysis (QCA): An analytical technique developed by Charles Ragin that is a means to analyse causal contribution of conditions to an outcome using a configurational approach.

Sufficiency: A situation in which the presence of a causal condition always, or almost always, results in the presence of the outcome condition. Seen when the causal condition is a subset of the outcome condition, or values of the cause are equal to or less than values of the outcome.

SUIN Condition: A **S**ufficient but **U**nnecessary part of a condition set that is itself **I**nsufficient but **N**ecessary.

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Chapter 4
A Hybrid Fuzzy MCDM Approach for Sustainable Health Tourism Sites Evaluation

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ABSTRACT

Health tourism is the set of activities that patients perform to be treated and to feel good. Each health activity should have its own conditions, and gathering these activities under one building requires expert views. Therefore, choosing the site for health tourism in a sustainable environment is a complex problem. It is also becoming more difficult in terms of developing sustainable health tourism as it involves environmental conditions. This study proposes a strategy that helps to appraise sustainable health tourism areas by combining fuzzy SWARA and fuzzy MARCOS methods. This approach weighs specified criteria via fuzzy SWARA according to the expert evaluations and ranks the alternatives via fuzzy MARCOS. This study aims to select sustainable health tourism site according to the criteria by decision makers. Sensitivity analysis is performed by creating different scenarios, and the results are discussed. According to the results of the proposed approach, it is thought that it can be used in the solution of the problem of sustainable health tourism site selection.

INTRODUCTION

Since the first definition of the phenomenon of tourism was revealed at the beginning of the 20th century, various definitions have continued to be given by many institutions and organizations over the years (Netto, 2009). It is possible to explain tourism as traveling for a certain period of time (Franklin, 2003). Tourism has become to express a broader meaning than these definitions due to its strong economic and social impacts (Song et al., 2012). Today, tourism is one of the largest industries in the world, with

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numerous employment opportunities and the huge economy it creates directly or indirectly. Factors such as the changing socio-economic structure of the world population, increasing urbanization and the formation of tourism culture have contributed to the development of tourism (Beeton, 2006). Moreover, the cheaper, easier, and more convenient travel opportunities form one of the driving forces in the growth of the tourism economy (Medlik, 2012). Tourism and travel provided an economy of $ 9170 billion and jobs for 334 million people worldwide in 2019 (WTTC, 2021).

In order to categorize the concept of tourism, many different parameters can be considered. For instance, it can be classified according to destination, number of people and season (Tureac & Turtureanu, 2010). Tourism can also be divided into many types according to the purpose of the participants such as sea tourism, congress tourism, yacht tourism, golf tourism, cultural tourism, etc. (Becken & Simmons, 2002). One of them is health tourism, which is one of today's tourism trends.

Health (or healthcare) tourism is a rapidly growing field all over the world. It is defined by World Tourism Organization (UNWTO, 2018) as "…tourism which have as a primary motivation, the contribution to physical, mental and/or spiritual health through medical and wellness-based activities which increase the capacity of individuals to satisfy their own needs and function better as individuals in their environment and society." There is another definition (Hudson, 2012) as "...traveling overnight away from the normal home environment for the express benefit of maintaining or improving health, and the supply and promotion of facilities and destinations which seek to provide such benefits." Although wellbeing and medical tourism are often used instead of health tourism, it has a content that encompasses both of these (Carrera & Bridges, 2006). It is also an inclusive concept that includes medical, preventive and truth tourism (Izadi et al., 2012).

In 2019, it is estimated that the economy created by health tourism activities was $ 65-87.5 billion (Salehi-Esfahani et al., 2021). Many countries are working on strategies that prioritize health tourism opportunities in their country promotions and increase their share in the health tourism market by serving a large number of people (Khan et al., 2020). In order to achieve this trend, countries emphasize and advertise their natural resources (spa, thermal water, etc.) and climate advantages (Mrčela et al., 2015). Since traveling for diagnosis, treatment and surgery is also included in health tourism, health investments and workforces such as doctors are also highlighted (Connell, 2006).

The expectation of high economic returns provided by mass tourism activities caused the issue of sustainability to remain in the background. It is known that natural resources are negatively affected in destinations such as coastlines, islands and riversides where tourism activities are generally very intense (Weaver et al., 1999). Furthermore, these activities cause widely economic, social, and cultural damages. Sustainable tourism practices make tourism continue by conducting protective planning and activities in the destination and the region where it is located (Swarbrooke, 1999). The concept of sustainable health tourism has emerged to maintain activities without damaging resources that provide health tourism (Liu, 2003).

As mentioned above, it is necessary to take into account many dimensions, especially economic, environmental, social, and cultural sides for sustainable health tourism management (Abed et al., 2011). Various criteria should be implemented to determine and compare the performances of destinations (sites) in this regard. However, the sites cannot be directly compared according to the criteria under different dimensions. This makes the selection of sustainable health tourism site a complex problem. In this view, the multi-criteria decision making methods (MCDM) are appropriate to handle this difficulty on selection problems.

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Multi-criteria decision making methods are employed to make evaluations that contain conflicting criteria (Opricovic & Tzeng, 2004). Numerous well-known MCDM methods have been used in the literature such as Analytical Hierarchy Process (AHP) (Saaty, 1980), TOPSIS (Hwang & Yoon, 1981), PROMETHEE (Brans et al., 1986), Analytical Network Process (ANP) (Saaty, 1996), VIKOR (Opricovic, 1998), MOORA (Brauers & Zavadskas, 2006) and others (Stojčić et al., 2019). In real-life applications, the methods performed individually or in combination with another one. These methods can also be combined with fuzzy logic to reflect human behavior. Thus, it is possible to use the uncertainty of subjective evaluations in solving the problem (Kahraman et al., 2004).

In the 1960s, uncertainty and probability theory were reexamined and criticized. Therefore, studies have been intensified to develop methods that can be used instead of probability theory. The definition of fuzzy sets was made by Lotfi A. Zadeh, who is the theoretician of this subject, in his original article "Fuzzy Sets" published in 1965 (Zadeh, 1965). A set is defined as fuzzy if there is no sharp boundary between elements (or objects) belonging to and not belonging to the related class. A crisp set allows only full or no membership, while fuzzy sets allow partial membership. A fuzzy set can be characterized by a membership function that assigns membership values to each of the elements from 0 to 1. Members not included in the cluster are assigned a membership value of 0, and those included in the cluster are assigned membership values. of 1. Uncertain elements are assigned values between 0 and 1 based on the uncertainty state (Kahraman et al., 2004). In the literature so far, fuzzy sets have been included in many decision-making problems to express uncertainty such as fuzzy AHP (Lyu et al., 2020), fuzzy TOPSIS (Rashidi & Cullinane, 2019), and fuzzy SWARA (Taş et al., 2021) and so on. The methods are also performed on real-life applications that handle vagueness and uncertainty in different fields (Çağman & Deli, 2012; Ashraf et al., 2019; Mittal et al., 2020; Deli & Karaaslan, 2021; Deli, 2021).

This study contributes the literature by combining two multi-criteria decision making methods, fuzzy SWARA (Step-Wise Weight Assessment Ratio Analysis) and fuzzy MARCOS (Measurement of alternatives and ranking according to compromise solution). This integrated method is utilized to select the best site for sustainable health tourism in Turkey. A literature review is made to determine the criteria to be considered. A group of decision makers is consulted to appraise the ten criteria determined. Linguistic evaluations are transformed into triangular fuzzy numbers. With the fuzzy SWARA-MARCOS method, six alternative sites are evaluated, ranked and the best site is selected. Finally, a sensitivity analysis is carried out to check the results.

Background

In the literature, site selection problems are frequently examined by MCDM methods. Önüt et al. (2010) employed the fuzzy AHP and fuzzy TOPSIS methods in choosing a place for a shopping mall in Turkey. Ashrafzadeh et al. (2012) searched the best location for warehouse using fuzzy TOPSIS. Aydin et al. (2013) selected the optimal sites for solar and wind energy using ordered weighted averaging and Geographic Information System (GIS). Sennaroglu and Celebi (2018) utilized many MCDM methods in determining the best location for a military airport (AHP, PROMETHEE and VIKOR) and comparing the results. In addition to these, Many types of site selection problems were solved with various MCDM methods such as selecting of garage site selection (Baušys & Juodagalvienė, 2017), solid waste disposal site selection (Kahraman et al., 2017), petrol station site selection (Ayyildiz & Gumus, 2020), and electric vehicle fast-charging station (Guler & Yomralioglu, 2020) site selection.

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MCDM studies are also related with the content of sustainable health tourism.. Levary (2011) evaluated the medical tourism locations using the AHP method. Ranking of five countries was made with four criteria reflecting cost, foreign language ability, distance and political situation. As a result of the method, the most important criterion was "Ability to speak the patient's language" by far, and the closest criterion to this was "country's political stability". In the country ranking made, India was the most suitable country for medical tourism. Hung et al. (2014) proposed a methodology that includes DANP (DEMATEL with ANP) and VIKOR methods to appraise its services in medical tourism. Karami et al. (2017) used AHP to score the criteria for the development of medical tourism in Iran. Because of the method, the leading criterion for domestic tourists was obtained as "advanced medical facilities and equipment". Nilashi et al. (2019) conducted a study of the determinants for the medical tourism adoption of hotels in Malaysia with DEMATEL and fuzzy TOPSIS. The DEMATEL method was selected to examine the relationship between dimensions and criteria, while fuzzy TOPSIS was used to determine the order of priority. "Medical technology" in the technological dimension, "investment" in the environmental dimension, "specialists" in the human dimension, and "infrastructure" in the organizational dimension were found to be the most important factors. Ghasemi et al. (2021) established a hybrid method that combines fuzzy SWARA and PROMETHEE to evaluate countries for sustainable medical tourism. It was concluded that the most important of the sustainable medical tourism criteria is "Abilities of skilled staff" and the best alternative country is India.

SWARA method was introduced into the literature by Keršulienė et al. (2010). In the study, the application steps of the SWARA method were shown and the dispute resolution methods in Lithuania were evaluated by the method. Zolfani and Saparauskas (2013) utilized the SWARA method to weigh the factors for energy systems in terms of sustainability. Stanujkic et al. (2015) used the SWARA method to determine the most suitable options for packaging. As a result of the comparison of the results of the application with the AHP method, they stated that SWARA can be preferred because it contains fewer pairwise comparisons. In the literature, there are studies in which the method is implemented with other MCDM methods in recent years, instead of using it as the only MCDM method. Karabašević et al. (2016) made a study on choosing the best candidate sale manager among four alternatives for a company. The SWARA method was employed to calculate the weight of seven criteria, and the best alternative candidate was selected by the WASPAS method. Shukla et al. (2016) applied SWARA for the calculation of criterion weights in the selection of enterprise resource planning (ERP) systems, and PROMETHEE for the evaluation of alternative systems. Valipour et al. (2017), on the other hand, determined the criteria weights with SWARA and the ranking of alternatives with COPRAS. Stanujkic et al. (2017) also sought a solution to the personnel selection problem with an approach that includes SWARA and ARAS methods. In addition to these approaches, there are studies in which fuzzy logic and SWARA are integrated. Mavi et al. (2017) conducted a case study examining alternatives for reverse logistics providers using fuzzy SWARA and fuzzy MOORA. The weights of the 23 criteria in the study were calculated using the fuzzy SWARA method. Ghorabaee et al. (2018) used the fuzzy SWARA method in integration with the fuzzy CRITIC and fuzzy EDAS method. Petrović et al. (2019) compared the alternative assessment results achieved with fuzzy ARAS, fuzzy TOPSIS and fuzzy WASPAS by calculating the criteria for supplier selection with fuzzy SWARA. Ren et al. (2019) studied the charging station locations for electric vehicles using hesitant fuzzy linguistic term with SWARA (HFL-SWARA) and WASPAS (HFL-WASPAS). Agarwal et al. (2020) ranked the suggested strategies to cope with the barriers in the humanitarian supply chains using fuzzy SWARA and fuzzy WASPAS. Ghasemi et al.

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(2021) proposed a methodology that combines fuzzy SWARA and PROMETHEE methods to evaluate countries for sustainable medical tourism.

Although the MARCOS method is relatively recent, many studies can be found in the literature. Stević et al. (2020) introduced MARCOS method to the literature for the first time. Eight alternatives are included in the study, in which the selection of sustainable suppliers for medical supplies in a polyclinic in Bosnia and Herzegovina is evaluated according to 21 sub-criteria that are below economic, social, and environmental criteria. The authors concluded that the results calculated using MARCOS method are compatible with other MCDM methods and can be selected for decision-making problems. Ilieva et al. (2020) applied fuzzy MARCOS to select for ordering cloud storage service. In the study using symmetric triangular fuzzy numbers, ten alternative cloud storage services were appraised using five criteria. Alternatives were divided into three groups according to the results of the fuzzy MARCOS method. Chattopadhyay et al. (2020) conducted a supplier selection study for the iron and steel industry in India. D-MARCOS (D numbers with MARCOS) method was employed to select the best supplier. In the study, five suppliers were examined against seven criteria, and alternatives were ranked. Stević and Brković (2020) integrated FUCOM and MARCOS methods to evaluate the human resources of the transportation company in Bosnia and Herzegovina and to select the employee of the month. The weights of the five criteria determined for the evaluation of 23 drivers of the transport company were computed using FUCOM. MARCOS was implemented to evaluate the alternatives, and the results obtained were compared with other MCDM methods such as SAW, EDAS, ARAS in the sensitive analysis study. Stanković et al. (2020) conducted a study of the risks of the main road in Bosnia and Herzegovina. Fuzzy PIPRECIA (PIvot Pairwise RElative Criteria Importance Assessment) method with triangular fuzzy numbers was used to determine the weights of six criteria. 38 sections of the main road were ranked in terms of risk using the fuzzy MARCOS method. Badi and Pamucar (2020) used MARCOS method with gray numbers (Gray-MARCOS) in the supplier selection study of LISCO (Libyan Iron and Steel Company) in Libya. The weights of the four criteria were calculated with gray theory, and the ranking of six alternative suppliers was created using MARCOS method. Vesković et al. (2020) assessed possible solutions to problems in railway transportation in the Republic of Srpska using Fuzzy PIPRECIA and Fuzzy EDAS hybrid. F-MARCOS was performed as one of the MCDM methods used to compare the results. Mijajlović et al. (2020) used FUCOM and fuzzy MARCOS in their study to examine the competition of spa centers in Bosnia and Herzegovina. FUCOM was used for criteria weights, and fuzzy MARCOS with triangular fuzzy numbers were used to rank alternative spas. Ulutaş et al. (2020) executed manual stacker selection for small warehouses, using CCSD (Correlation Coefficient and the Standard Deviation), ITARA (Indifference Threshold-based Attribute Ratio Analysis Method), and MARCOS. MARCOS method was employed for alternatives evaluation in this article. Puška et al. (2020) also used MARCOS method to choose between project management software in the United Arab Emirates. Bakır and Atalık (2021) conducted a study to rank airline companies according to the quality of online service. Fuzzy MARCOS method was used in ranking the alternatives according to the criteria weighted with fuzzy AHP in this study.

MAIN FOCUS OF THE CHAPTER

A methodology is proposed to find a solution for location selection practices of sustainable health tourism. This study contributes to the literature by establishing a hybrid SWARA-MARCOS method under

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uncertainty. To express the vague information, triangular fuzzy numbers are considered. In addition to the existing literature of the methods, the combined version of fuzzy SWARA and fuzzy MARCOS methods are applied on the sustainable health tourism site selection problem. It is aimed to guide future studies on sustainable health tourism site selection.

Issues, Controversies, Problems

Health tourism has become a rising trend all over the world. Marketing strategies can be strengthened by highlighting destinations with high performance according to sustainability criteria. The results obtained with multi-criteria decision-making methods are suitable for being used in persuading managers. The value of the results increases when the assessments are based on the opinions of industry experts. By using fuzzy numbers, the evaluations are able to reflect more accurately than the classical 1-9 scale evaluation system. The contribution can be made to the literature of fuzzy logic with real-life applications in different sectors.

METHODOLOGY

Triangular Fuzzy Numbers

Fuzzy sets was introduced by Zadeh (1965). With the fuzzy set theory, the ability to represent vague expressions and data is obtained (Kahraman et al., 2004). $\tilde{A}$ is a fuzzy set of X universal set and it can be defined as ($x \in X$ and $\mu_{\tilde{A}}(X) \to [0,1]$):

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \middle| x \in X \right\} \tag{1}$$

A fuzzy number is a subset of the fuzzy set with some properties. Linguistic expressions are often used to evaluate in real life. Fuzzy numbers make it possible to express these linguistic evaluations numerically. While triangular and trapezoidal fuzzy numbers are the most used types of fuzzy numbers, triangular fuzzy numbers offer more convenience in calculations (Grzegorzewski & Mrówka, 2005; Moniri et al., 2020). A triangular fuzzy number is constructed by three notations and can be expressed as $\tilde{a} = (l, m, u)$ where $u \geq m \geq 1$ (Figure 1).

The membership function ($\mu_{\tilde{a}}$) of $\tilde{a}$ is shown in Eq. (2):

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & x < l, \\ \dfrac{x-l}{m-l}, & l \leq x \leq m, \\ \dfrac{u-x}{u-m}, & m \leq x \leq u, \\ 0, & x > u. \end{cases} \tag{2}$$

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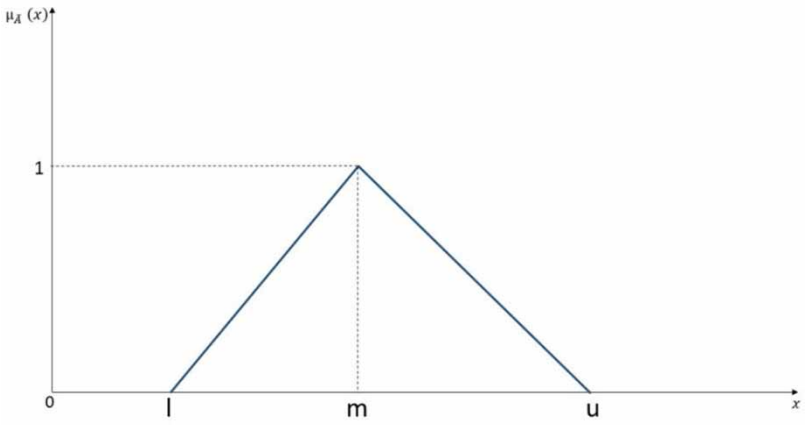
Figure 1. A triangular fuzzy number



The basic algebraic operations of two triangular fuzzy numbers, which are $\tilde{a}_1 = (l_1, m_1, u_1)$ and $\tilde{a}_2 = (l_2, m_2, u_2)$, are shown in Eq. (3)-(7) (Liu et al., 2013).

Fuzzy addition:

$$\tilde{a}_1 \oplus \tilde{a}_2 = (l_1 + l_2,\ m_1 + m_2,\ u_1 + u_2) \tag{3}$$

Fuzzy substraction:

$$\tilde{a}_1 \ominus \tilde{a}_2 = (l_1 - u_2,\ m_1 - m_2,\ u_1 - l_2) \tag{4}$$

Fuzzy multiplication:

$$\tilde{a}_1 \otimes \tilde{a}_2 = (l_1 * l_2,\ m_1 * m_2,\ u_1 * u_2) \tag{5}$$

Fuzzy division:

$$\tilde{a}_1 \oslash \tilde{a}_2 = \left(\frac{l_1}{u_2},\ \frac{m_1}{m_2},\ \frac{u_1}{l_2}\right) \tag{6}$$

Multiplying by a coefficient:

$$k * \tilde{a}_1 = (k * l_1, k * m_1, k * u_1) \tag{7}$$

SWARA Method

SWARA method is an easy-to-apply method that includes few comparisons and allows experts' evaluations to be included in the calculations with less effort (Stanujkic et al., 2015). Fuzzy SWARA follows

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the steps of the SWARA method, which consists of five steps. The steps are given as follows (Zarbakhshnia et al., 2018):

Step 1: Ranking the alternatives (criteria). The most important alternative takes the first place in the ranking, while the least important one takes the last place.

Step 2: Obtaining the "comparative importance of average value". This step includes the comparison of the alternative *(j-1)* with alternative *j*. The process that starts with the comparison of the second alternative with the first and continues until the end of the list created in the first step. The relative importance values in comparisons is named by (Keršuliene et al., 2010) as "comparative importance of average value" and denoted by "$\tilde{s}_j$". Linguistic expressions and their fuzzy number equivalents used in comparison are in Table 1.

Table 1. Scale of linguistic responses for rating criteria (Zarbakhshnia et al., 2018)

Linguistic Responses on Importance	Abbreviation	Response Scale
Equally	E.	(1, 1, 1)
Moderately less	Mod. L.	$\left(\frac{2}{3},1,\frac{3}{2}\right)$
Less	L. I.	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$
Very less	V. L.	$\left(\frac{2}{7},\frac{1}{3},\frac{2}{5}\right)$
Much less	M. L.	$\left(\frac{2}{9},\frac{1}{4},\frac{2}{7}\right)$

Step 3: Computing the $\tilde{k}_j$ values. This fuzzy coefficient is calculated as follows (Eq. (8)):

$$\begin{cases} (1,1,1) & j=1 \\ \tilde{s}_j+(1,1,1) & j>1 \end{cases} \tag{8}$$

Step 4: Computing the $\tilde{q}_j$ values. This fuzzy importance vector value is calculated using Eq. (9):

$$\begin{cases} (1,1,1) & j=1 \\ \dfrac{\tilde{x}_{j-1}}{\tilde{k}_j} & j>1 \end{cases} \tag{9}$$

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Step 5: Determining the $\tilde{w}_j$ values. The fuzzy relative weights of the alternatives ($\tilde{w}_j$) are calculated with the following Eq. (10):

$$\tilde{w}_j = \frac{\tilde{q}_j}{\sum_{k=1}^{n} \tilde{q}_k} \tag{10}$$

Eq. (11) is used to aggregate the calculated criteria weights for each decision-maker (Pak et al., 2015; Pamucar et al., 2018):

$$w_j^l = \min_i \left\{ w_{ij}^l \right\} \quad w_j^m = \frac{1}{n} * \sum_{1}^{n} w_{ij}^m \quad w_j^u = \max_i \left\{ w_{ij}^u \right\} \tag{11}$$

MARCOS Method

The MARCOS method can be used in many problem types due to its flexible structure (Stević et al., 2020). This method produces solutions according to the best and worst alternatives. The steps of MARCOS method (Stanković et al., 2020):

Step 1: Establishing an initial decision matrix. A decision matrix is created by *m* alternatives and *n* criteria. The fuzzy values of the elements in this matrix $\left(\tilde{x}_{ij}\right)$ are determined by the decision makers who evaluate alternatives against the criteria using linguistic responses in Table 2 (i=1,2,…,m and j=1,2,…,n).

Table 2. Scale of linguistic responses for evaluating alternatives (Stević et al., 2020)

Linguistic Evaluation	Abbreviation	Response Scale
Extremely good	E.G.	(7,9,9)
Very good	V.G.	(7,7,9)
Good	G.	(5,7,7)
Medium good	M.G.	(5,5,7)
Medium	M.	(3,5,5)
Medium poor	M.P.	(3,3,5)
Poor	P.	(1,3,3)
Very poor	V.P.	(1,1,3)
Extremely poor	E.P.	(1,1,1)

Step 2: Extending the matrix. In this step, fuzzy ideal $\tilde{A}\left(ID\right)$ and fuzzy anti-ideal $\tilde{A}\left(AI\right)$ solutions are added to the initial decision matrix. Fuzzy ideal $\tilde{A}\left(ID\right)$ solution is the best alternative where-

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as fuzzy anti-ideal $\tilde{A}(AI)$ solution is the worst alternative. The fuzzy ideal $\tilde{A}(ID)$ and the fuzzy anti-ideal $\tilde{A}(AI)$ solutions are determined using Eq. (12) and Eq. (13), respectively:

$$\tilde{A}(ID) = \min_i \tilde{x}_{ij} \ \ if \ j \in C \ and \ \ \max_i \tilde{x}_{ij} \ \ if \ j \in B \tag{12}$$

$$\tilde{A}(AI) = \max_i \tilde{x}_{ij} \ \ if \ j \in C \ and \ \ \min_i \tilde{x}_{ij} \ \ if \ j \in B \tag{13}$$

In the equations, *C* represents cost-type criteria, which are desired to be minimized. On the contrary, *B* represents benefit-type criteria, which are desired to be maximized. The fuzzy extended decision matrix $(\tilde{X})$ is shown below (Eq. (14)):

$$\tilde{X} = \begin{array}{c} \\ \tilde{A}(AI) \\ \tilde{A}_1 \\ \tilde{A}_2 \\ \tilde{A}_3 \\ \tilde{A}_4 \\ \cdots \\ \tilde{A}_m \\ \tilde{A}(ID) \end{array} \begin{array}{c} \begin{array}{ccccccc} \tilde{C}_1 & \tilde{C}_2 & \cdots & & & \cdots & \tilde{C}_n \end{array} \\ \begin{bmatrix} \tilde{x}_{ai1} & \tilde{x}_{ai2} & \tilde{x}_{ai3} & \tilde{x}_{ai4} & \tilde{x}_{ai5} & \cdots & \tilde{x}_{ain} \\ \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \tilde{x}_{14} & \tilde{x}_{15} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \tilde{x}_{24} & \tilde{x}_{25} & \cdots & \tilde{x}_{2n} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \tilde{x}_{34} & \tilde{x}_{35} & \cdots & \tilde{x}_{3n} \\ \tilde{x}_{41} & \tilde{x}_{42} & \tilde{x}_{43} & \tilde{x}_{44} & \tilde{x}_{45} & \cdots & \tilde{x}_{4n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & \tilde{x}_{m4} & \tilde{x}_{m5} & \cdots & \tilde{x}_{mn} \\ \tilde{x}_{id1} & \tilde{x}_{id2} & \tilde{x}_{id3} & \tilde{x}_{id4} & \tilde{x}_{id5} & \cdots & \tilde{x}_{idn} \end{bmatrix} \end{array} \tag{14}$$

Step 3: Normalizing the fuzzy matrix $(\tilde{X})$: Normalized values of the elements in the fuzzy extended matrix $(\tilde{X})$ are calculated using Eq. (15) and Eq. (16) for benefit and cost criteria, respectively. All elements x_{id}^l, x_{id}^m, x_{id}^u, x_{ij}^l, x_{ij}^m, and x_{ij}^u belong to $(\tilde{X})$.

$$\tilde{n}_{ij} = \left(n_{ij}^l,\ n_{ij}^m,\ n_{ij}^u\right) = \left(\frac{x_{ij}^l}{x_{id}^u},\ \frac{x_{ij}^m}{x_{id}^u},\ \frac{x_{ij}^u}{x_{id}^u}\right) \ if \ J \in B \tag{15}$$

$$\tilde{n}_{ij} = \left(n_{ij}^l,\ n_{ij}^m,\ n_{ij}^u\right) = \left(\frac{x_{id}^l}{x_{ij}^u},\ \frac{x_{id}^l}{x_{ij}^m},\ \frac{x_{id}^l}{x_{ij}^l}\right) \ if \ J \in C \tag{16}$$

The fuzzy matrix that consists of elements computed using Eq. (15) and (16) is called normalized fuzzy matrix $(\tilde{N})$.

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Step 4: Establishing weighted matrix $\left(\tilde{V}\right)$. The elements of matrix $\left(\tilde{V}\right)$ are calculated using Eq. (17):

$$\tilde{v}_{ij} = \left(v_{ij}^{l}, v_{ij}^{m}, v_{ij}^{u}\right) = \tilde{n}_{ij} \otimes \tilde{w}_{j} = \left(n_{ij}^{l} * w_{j}^{l}, n_{ij}^{m} * w_{j}^{m}, n_{ij}^{u} * w_{j}^{u}\right) \tag{17}$$

$\tilde{w}_j$ value stands for fuzzy criteria weights that calculated by another method or determined by decision makers.

Step 5: Computing utility degrees. $\tilde{S}_i$ represents the sum of $\tilde{v}_{ij}$ values and $\tilde{K}_i$ notations represent the utility degrees of alternatives. These values are calculated using Eq. (18), (19) and (20).

$$\tilde{S}_i = \sum\nolimits_{i=1}^{n} \tilde{v}_{ij} \tag{18}$$

$$\tilde{K}_i^{+} = \frac{\tilde{S}_i}{\tilde{S}_{id}} = \left(\frac{s_i^l}{s_{id}^u}, \frac{s_i^m}{s_{id}^m}, \frac{s_i^u}{s_{id}^l}\right) \tag{19}$$

$$\tilde{K}_i^{-} = \frac{\tilde{S}_i}{\tilde{S}_{ai}} = \left(\frac{s_i^l}{s_{ai}^u}, \frac{s_i^m}{s_{ai}^m}, \frac{s_i^u}{s_{ai}^l}\right) \tag{20}$$

Step 6: Determining $\tilde{T}_i$. This matrix is obtained by summing $\tilde{K}_i^{+}$ and $\tilde{K}_i^{-}$ (Eq. (21)):

$$\tilde{T}_i = \tilde{t}_i = \left(t_i^l, t_i^m, t_i^u\right) = \left(k_i^{-l} + k_i^{+l}, k_i^{-m} + k_i^{+m}, k_i^{-u} + k_i^{+u}\right) \tag{21}$$

Step 7: Formulating the utility function: $f\left(\tilde{K}_i^{+}\right)$ and $f\left(\tilde{K}_i^{-}\right)$ are used for the utility function with respect to the ideal and anti-ideal solution, respectively (Eq. (24)-(25)). Here, $\tilde{D}$ is a fuzzy number and shown in Eq. (22). The crisp number df_c is calculated using Eq. (23).

$$\tilde{D} = \max_{i} \tilde{t}_{ij} = \left(d^l, d^m, d^u\right) \tag{22}$$

$$df_c = \frac{l + \left(4 * m\right) + u}{6} \tag{23}$$

$$f\left(\tilde{K}_i^{+}\right) = \frac{\tilde{K}_i^{-}}{df_c} = \left(\frac{k_i^{-l}}{df_c}, \frac{k_i^{-m}}{df_c}, \frac{k_i^{-u}}{df_c}\right) \tag{24}$$

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$$f\left(\tilde{K}_i^-\right)=\frac{\tilde{K}_i^+}{df_c}=\left(\frac{k_i^{+l}}{df_c},\frac{k_i^{+m}}{df_c},\frac{k_i^{+u}}{df_c}\right) \tag{25}$$

Step 8: Obtaining utility functions. The crisp values of K_i^+, K_i^-, $f\left(K_i^+\right)$, and $f\left(K_i^-\right)$ are calculated by defuzzification Eq. (23) are used to compute utility function of the alternatives (Eq. (26)).

$$f\left(K_i\right)=\frac{K_i^+ + K_i^-}{1+\frac{1-f\left(K_i^+\right)}{f\left(K_i^+\right)}+\frac{1-f\left(K_i^-\right)}{f\left(K_i^-\right)}} \tag{26}$$

Step 9: Ranking the alternatives. Creating a ranking of alternatives using final scores of alternatives subject to utility functions $f(K_i)$. The best alternative is the one with the highest score and vice versa.

The steps of proposed hybrid of fuzzy SWARA and fuzzy MARCOS approach is illustrated on a flowchart in Figure 2.

CASE STUDY

This study includes an analysis of health tourism destinations in Turkey in terms of sustainability. Turkey has a promising rise in health tourism. While the number of health tourists hosted by Turkey in 2012 was approximately 217,000 and this number exceeded 660,000 in 2019 (USHAŞ, 2020). Despite the increasing number of tourists, it is necessary to consider the sustainability of the resources. Turkey's share in health tourism is quite small compared to the Asian market. Advertisements to be shown on channels in other countries can be used to increase the number of health tourists and health tourism income. It is desired to choose the site that may stand out in Turkey's health tourism promotion campaign. Six alternative health tourism sites are listed (A1, A2, A3, A4, A5, A6). The weights of the criteria used in evaluating these alternatives were made with fuzzy SWARA, and the evaluation of the alternatives was made with fuzzy MARCOS. The summary of the study is included in Figure 3.

The ten criteria used in the evaluation of these sites were determined by the opinions of five experts and reviewing the studies in the literature. These criteria are shown in Table 3.

The opinions of five experts who have experience in the sector were asked. As the first two steps of the fuzzy SWARA method, each expert has listed the criteria in order of importance and compares consecutive alternatives using linguistic scale in Table 1. The orders of importance and comparative importance of average values given by experts are displayed in Table 4.

SOLUTIONS AND RECOMMENDATIONS

By applying the steps of fuzzy SWARA, the relative fuzzy weights of the criteria are calculated for each expert. As an example, the calculations for E3 are shown in Table 5.

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Figure 2. Flowchart of the proposed hybrid of fuzzy SWARA and fuzzy MARCOS approach

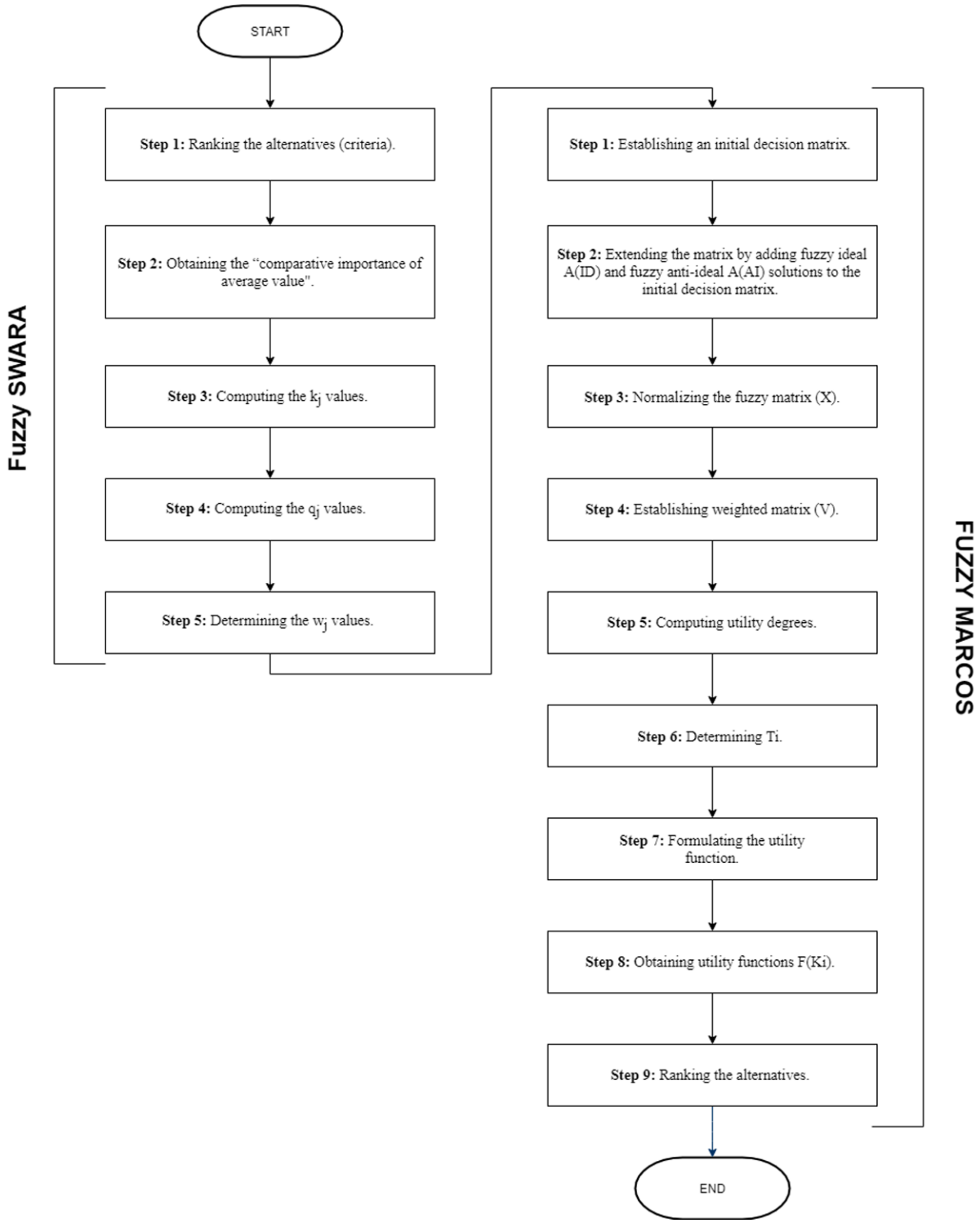

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Figure 3. The summary of the study

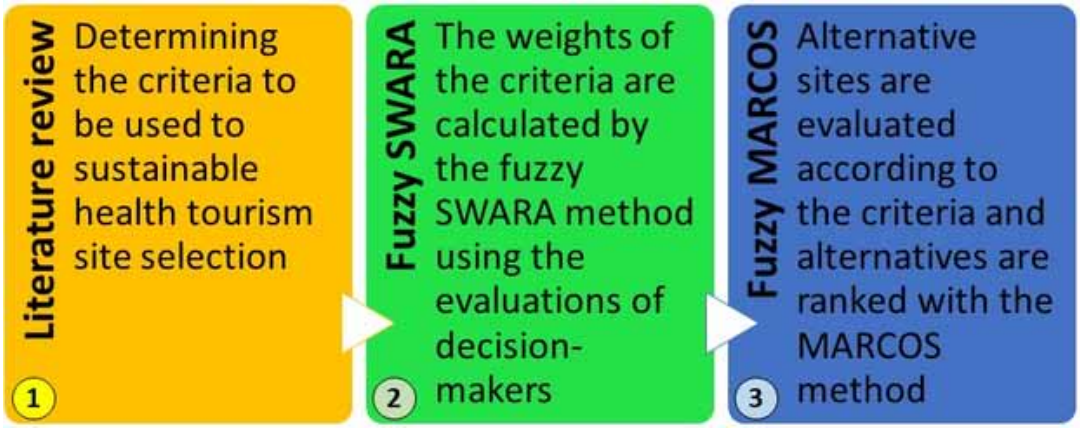

Table 3. Scale of linguistic responses for rating criteria (Zarbakhshnia et al., 2018)

Criteria Code	Criteria Type	Criteria	Reference
C1	Benefit	Staff quality	Nilashi et al. (2019)
C2	Benefit	Medical treatments diversity	Ghasemi et al. (2021)
C3	Cost	Expected cost	Levary (2011)
C4	Benefit	Quality of water (lakes, rivers, sea)	Tanguay et al. (2013)
C5	Benefit	Air quality	Choi and Sirakaya (2006)
C6	Benefit	Accommodation opportunities	Perkumienė et al. (2019)
C7	Benefit	Specialists with qualifications	Ghasemi et al. (2021)
C8	Benefit	Using modern medical eqiupments	Goodarzi et al. (2014)
C9	Cost	Waste production	Lozano-Oyola et al. (2012)
C10	Benefit	Environmental management system	Castellani and Sala (2010)

Table 4. The orders of importance and comparative importance of average values

No	E1		E2		E3		E4		E5	
1	C3	-	C1	-	C7	-	C3	-	C4	-
2	C7	L.I.	C2	M.L.	C3	M.L.	C5	M.L.	C3	V.L.
3	C8	V.L.	C6	L.I.	C6	L.I.	C7	L.I.	C6	Mod.L.
4	C1	L.I.	C3	L.I.	C10	L.I.	C2	L.I.	C10	L.I.
5	C2	L.I.	C10	M.L.	C1	M.L.	C8	M.L.	C2	M.L.
6	C4	L.I.	C9	L.I.	C9	L.I.	C9	L.I.	C1	Mod.L.
7	C9	Mod. L.	C4	Mod. L.	C8	Mod.L.	C4	V.L.	C5	Mod.L.
8	C6	Mod. L.	C7	Mod. L.	C2	Mod.L.	C1	Mod.L.	C9	M.L.
9	C10	V. L.	C5	M.L.	C5	Mod.L.	C10	M.L.	C8	Mod.L.
10	C5	L. I.	C8	V.L.	C4	M.L.	C6	V.L.	C7	V.L.

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Table 5. The fuzzy SWARA steps for E3

No	Criteria	$\tilde{s}_j$	$\tilde{k}_j$	$\tilde{q}_j$	$\tilde{w}_j$
1	C7	-	(1.0000, 1.0000, 1.0000)	(1.0000, 1.0000, 1.0000)	(0.2677, 0.2987, 0.3377)
2	C3	$\left(\frac{2}{9},\frac{1}{4},\frac{2}{7}\right)$	(1.2222, 1.2500, 1.2857)	(0.7778, 0.8000, 0.8182)	(0.2082, 0.2390, 0.2763)
3	C6	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$	(1.4000, 1.5000, 1.6667)	(0.4667, 0.5333, 0.5844)	(0.1249, 0.1593, 0.1974)
4	C10	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$	(1.4000, 1.5000, 1.6667)	(0.2800, 0.3556, 0.4174)	(0.0750, 0.1062, 0.1410)
5	C1	$\left(\frac{2}{9},\frac{1}{4},\frac{2}{7}\right)$	(1.2222, 1.2500, 1.2857)	(0.2178, 0.2844, 0.3415)	(0.0583, 0.0850, 0.1153)
6	C9	$\left(\frac{2}{5},\frac{1}{2},\frac{2}{3}\right)$	(1.4000, 1.5000, 1.6667)	(0.1307, 0.1896, 0.2440)	(0.0350, 0.0566, 0.0824)
7	C8	$\left(\frac{2}{3},1,\frac{3}{2}\right)$	(1.6667, 2.0000, 2.5000)	(0.0523, 0.0948, 0.1464)	(0.0140, 0.0283, 0.0494)
8	C2	$\left(\frac{2}{3},1,\frac{3}{2}\right)$	(1.6667, 2.0000, 2.5000)	(0.0209, 0.0474, 0.0878)	(0.0056, 0.0142, 0.0297)
9	C5	$\left(\frac{2}{3},1,\frac{3}{2}\right)$	(1.6667, 2.0000, 2.5000)	(0.0084, 0.0237, 0.0527)	(0.0022, 0.0071, 0.0178)
10	C4	$\left(\frac{2}{9},\frac{1}{4},\frac{2}{7}\right)$	(1.2222, 1.2500, 1.2857)	(0.0065, 0.0190, 0.0431)	(0.0017, 0.0057, 0.0146)

Subsequently, relative fuzzy weights are aggregated using Eq. (11) to compute the final fuzzy weights of the criteria. The final fuzzy weights of the criteria are included in Table 6.

After the fuzzy SWARA steps, the fuzzy MARCOS method is applied for the second part of the methodology. As the first step of MARCOS method, the initial fuzzy decision matrix is created in which six alternatives are evaluated according to ten criteria. Eq. (11) is used to aggregate the triangular fuzzy number equivalents of the evaluations made by the experts using the scale in Table 2. The initial fuzzy matrix created by the evaluations of E1 is given in Table 7. The aggregated initial matrix is extended with fuzzy ideal $\tilde{A}(ID)$ and fuzzy anti-ideal $\tilde{A}(AI)$ solutions in order to establish extended fuzzy matrix (Table 8).

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Table 6. The final fuzzy weights of evaluation criteria

Criteria	Final Fuzzy Weights
C1	(0.0096, 0.1096, 0.3363)
C2	(0.0056, 0.0997, 0.2751)
C3	(0.0743, 0.2460, 0.3844)
C4	(0.0017, 0.0962, 0.4154)
C5	(0.0018, 0.0545, 0.2680)
C6	(0.0043, 0.0952, 0.1974)
C7	(0.0014, 0.1383, 0.3377)
C8	(0.0020, 0.0582, 0.2136)
C9	(0.0051, 0.0412, 0.0824)
C10	(0.0031, 0.0611, 0.1410)

Table 7. The initial fuzzy matrix for E1

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	VG (7, 7, 9)	M (3, 5, 5)	MG (5, 5, 7)	P (1, 3, 3)	M (3, 5, 5)	MG (5, 5, 7)	G (5, 7, 7)	MP (3, 3, 5)	EG (7, 9, 9)	M (3, 5, 5)
A2	G (5, 7, 7)	M (3, 5, 5)	G (5, 7, 7)	MP (3, 3, 5)	MP (3, 3, 5)	MG (5, 5, 7)	G (5, 7, 7)	MP (3, 3, 5)	VG (7, 7, 9)	MG (5, 5, 7)
A3	MP (3, 3, 5)	MP (3, 3, 5)	MG (5, 5, 7)	G (5, 7, 7)	P (1, 3, 3)	MG (5, 5, 7)	G (5, 7, 7)	MP (3, 3, 5)	G (5, 7, 7)	M (3, 5, 5)
A4	P (1, 3, 3)	MP (3, 3, 5)	MG (5, 5, 7)	MG (5, 5, 7)	MP (3, 3, 5)	G (5, 7, 7)	MP (3, 3, 5)	M (3, 5, 5)	G (5, 7, 7)	M (3, 5, 5)
A5	MP (3, 3, 5)	MG (5, 5, 7)	M (3, 5, 5)	P (1, 3, 3)	M (3, 5, 5)	VG (7, 7, 9)	G (5, 7, 7)	M (3, 5, 5)	VG (7, 7, 9)	MG (5, 5, 7)
A6	P (1, 3, 3)	P (1, 3, 3)	M (3, 5, 5)	MP (3, 3, 5)	MP (3, 3, 5)	G (5, 7, 7)	G (5, 7, 7)	MG (5, 5, 7)	G (5, 7, 7)	M (3, 5, 5)

Table 8. The aggregated initial fuzzy matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	(3, 6.2, 9)	(1, 5.4, 9)	(1, 4.2, 7)	(1, 5.4, 7)	(3, 5.4, 7)	(1, 5.4, 9)	(3, 5.4, 7)	(1, 4.2, 7)	(1, 5.8, 9)	(1, 5.8, 9)
A2	(3, 6.2, 7)	(1, 4.6, 9)	(1, 3.8, 7)	(3, 5.8, 9)	(3, 5.4, 9)	(1, 5, 7)	(1, 5.4, 7)	(1, 4.6, 7)	(1, 5, 9)	(1, 5.4, 9)
A3	(3, 5, 9)	(1, 5.4, 9)	(3, 3.4, 7)	(3, 5, 7)	(1, 4.6, 9)	(3, 3.8, 7)	(1, 5.4, 7)	(1, 3.4, 7)	(1, 5.4, 9)	(3, 5.4, 9)
A4	(1, 5, 7)	(1, 4.6, 9)	(1, 5.4, 9)	(1, 5, 7)	(1, 4.6, 9)	(1, 5, 7)	(1, 4.2, 7)	(1, 5.4, 7)	(1, 5, 7)	(3, 5, 7)
A5	(1, 4.2, 7)	(3, 5, 9)	(3, 5, 7)	(1, 4.6, 7)	(1, 5, 7)	(1, 3.4, 9)	(3, 4.6, 7)	(1, 4.2, 7)	(3, 6.2, 9)	(1, 4.6, 9)
A6	(1, 5.8, 9)	(1, 4.6, 7)	(1, 4.6, 7)	(1, 5, 7)	(1, 2.6, 5)	(1, 4.2, 7)	(1, 3.8, 7)	(1, 3.8, 7)	(3, 6.2, 7)	(3, 5, 7)

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In the second step of MARCOS, the extended fuzzy matrix is established by adding fuzzy ideal $\tilde{A}(ID)$ and fuzzy anti-ideal $\tilde{A}(AI)$ solutions to the aggregated initial fuzzy matrix. All criteria except C3 and C9 are benefit-type criteria. Eq. (12) and Eq. (13) are used to determine fuzzy ideal $\tilde{A}(ID)$ and fuzzy anti-ideal $\tilde{A}(AI)$ solutions, respectively. To compare fuzzy numbers, crisp values are taken into account using Eq. (23). The extended fuzzy decision matrix $(\tilde{X})$ is shown in Table 9.

Table 9. The extended fuzzy decision matrix $(\tilde{X})$

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	Benefit	**Benefit**	**Cost**	**Benefit**	**Benefit**	**Benefit**	**Benefit**	**Benefit**	**Cost**	**Benefit**
$\tilde{A}(AI)$	(1, 4.2, 7)	(1, 4.6, 7)	(1, 5.4,9)	(1, 4.6, 7)	(1, 2.6, 5)	(1, 3.4, 9)	(1, 3.8, 7)	(1, 3.4, 7)	(3, 6.2, 9)	(1, 4.6, 9)
A1	(3, 6.2, 9)	(1, 5.4, 9)	(1, 4.2, 7)	(1, 5.4, 7)	(3, 5.4, 7)	(1, 5.4, 9)	(3, 5.4, 7)	(1, 4.2, 7)	(1, 5.8, 9)	(1, 5.8, 9)
A2	(3, 6.2, 7)	(1, 4.6, 9)	(1, 3.8, 7)	(3, 5.8, 9)	(3, 5.4, 9)	(1, 5, 7)	(1, 5.4, 7)	(1, 4.6, 7)	(1, 5, 9)	(1, 5.4, 9)
A3	(3, 5, 9)	(1, 5.4, 9)	(3, 3.4, 7)	(3, 5, 7)	(1, 4.6, 9)	(3, 3.8, 7)	(1, 5.4, 7)	(1, 3.4, 7)	(1, 5.4, 9)	(3, 5.4, 9)
A4	(1, 5, 7)	(1, 4.6, 9)	(1, 5.4, 9)	(1, 5, 7)	(1, 4.6, 9)	(1, 5, 7)	(1, 4.2, 7)	(1, 5.4, 7)	(1, 5, 7)	(3, 5, 7)
A5	(1, 4.2, 7)	(3, 5, 9)	(3, 5, 7)	(1, 4.6, 7)	(1, 5, 7)	(1, 3.4, 9)	(3, 4.6, 7)	(1, 4.2, 7)	(3, 6.2, 9)	(1, 4.6, 9)
A6	(1, 5.8, 9)	(1, 4.6, 7)	(1, 4.6, 7)	(1, 5, 7)	(1, 2.6, 5)	(1, 4.2, 7)	(1, 3.8, 7)	(1, 3.8, 7)	(3, 6.2, 7)	(3, 5, 7)
$\tilde{A}(ID)$	(3, 6.2, 9)	(3, 5, 9)	(1, 3.8, 7)	(3, 5.8, 9)	(3, 5.4, 9)	(1, 5.4, 9)	(3, 5.4, 7)	(1, 5.4, 7)	(1, 5, 7)	(3, 5.4, 9)

In the normalization process (the third step), Eq. (15) is used for benefit-type criteria elements and Eq. (16) is used for cost-type criteria elements and the normalized fuzzy matrix $(\tilde{N})$ is created with these values. In Step 4, the weighted fuzzy matrix $(\tilde{V})$ is calculated by multiplying the fuzzy weights obtained from the fuzzy SWARA method in Table 6, and the elements in the normalized fuzzy matrix $(\tilde{N})$. The weighted fuzzy matrix $(\tilde{V})$ is given in Table 10.

The utility degrees of alternatives $(\tilde{K}_i)$ are calculated using Eq. (18)-(20) in the Step 5. The utility functions with respect to the ideal and anti-ideal solution $f(\tilde{K}_i^+)$ and $f(\tilde{K}_i^-)$ are computed at the end of the Step 6 and Step 7. The fuzzy values calculated in Step 5 and Step 8 are included in Table 11.

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Table 10. The weighted fuzzy matrix $\left(\tilde{V}\right)$

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
$\tilde{A}(AI)$	(0.0011, 0.0511, 0.2615)	(0.0006, 0.0510, 0.2140)	(0.0083, 0.0456, 0.3844)	(0.0002, 0.0491, 0.3231)	(0.0002, 0.0157, 0.1489)	(0.0005, 0.0360, 0.1974)	(0.0002, 0.0751, 0.3377)	(0.0003, 0.0283, 0.2136)	(0.0006, 0.0067, 0.0275)	(0.0003, 0.0312, 0.1410)
A1	(0.0032, 0.0755, 0.3363)	(0.0006, 0.0598, 0.2751)	(0.0106, 0.0586, 0.3844)	(0.0002, 0.0577, 0.3231)	(0.0006, 0.0327, 0.2084)	(0.0005, 0.0571, 0.1974)	(0.0006, 0.1067, 0.3377)	(0.0003, 0.0349, 0.2136)	(0.0006, 0.0071, 0.0824)	(0.0003, 0.0394, 0.1410)
A2	(0.0032, 0.0755, 0.2615)	(0.0006, 0.0510, 0.2751)	(0.0106, 0.0647, 0.3844)	(0.0006, 0.0620, 0.4154)	(0.0006, 0.0327, 0.2680)	(0.0005, 0.0529, 0.1535)	(0.0002, 0.1067, 0.3377)	(0.0003, 0.0382, 0.2136)	(0.0006, 0.0082, 0.0824)	(0.0003, 0.0366, 0.1410)
A3	(0.0032, 0.0609, 0.3363)	(0.0006, 0.0598, 0.2751)	(0.0106, 0.0724, 0.1281)	(0.0006, 0.0534, 0.3231)	(0.0002, 0.0279, 0.2680)	(0.0014, 0.0402, 0.1535)	(0.0002, 0.1067, 0.3377)	(0.0003, 0.0283, 0.2136)	(0.0006, 0.0076, 0.0824)	(0.0010, 0.0366, 0.1410)
A4	(0.0011, 0.0609, 0.2615)	(0.0006, 0.0510, 0.2751)	(0.0083, 0.0456, 0.3844)	(0.0002, 0.0534, 0.3231)	(0.0002, 0.0279, 0.2680)	(0.0005, 0.0529, 0.1535)	(0.0002, 0.0830, 0.3377)	(0.0003, 0.0449, 0.2136)	(0.0007, 0.0082, 0.0824)	(0.0010, 0.0339, 0.1097)
A5	(0.0011, 0.0511, 0.2615)	(0.0019, 0.0554, 0.2751)	(0.0106, 0.0492, 0.1281)	(0.0002, 0.0491, 0.3231)	(0.0002, 0.0303, 0.2084)	(0.0005, 0.0360, 0.1974)	(0.0006, 0.0909, 0.3377)	(0.0003, 0.0349, 0.2136)	(0.0006, 0.0067, 0.0275)	(0.0003, 0.0312, 0.1410)
A6	(0.0011, 0.0706, 0.3363)	(0.0006, 0.0510, 0.2140)	(0.0106 0.0535, 0.3844)	(0.0002, 0.0534, 0.3231)	(0.0002, 0.0157, 0.1489)	(0.0005, 0.0444, 0.1535)	(0.0002, 0.0751, 0.3377)	(0.0003, 0.0316, 0.2136)	(0.0007, 0.0067, 0.0275)	(0.0010, 0.0339, 0.1097)
$\tilde{A}(ID)$	(0.0032, 0.0755, 0.3363)	(0.0019, 0.0554, 0.2751)	(0.0106, 0.0647, 0.3844)	(0.0006, 0.0620, 0.4154)	(0.0006, 0.0327, 0.2680)	(0.0005, 0.0571, 0.1974)	(0.0006, 0.1067, 0.3377)	(0.0003, 0.0449, 0.2136)	(0.0007, 0.0082, 0.0824)	(0.0010, 0.0366, 0.1410)

Eq. (23) is used for defuzzification of the fuzzy values in Table 11. Finally, the utility function $f(K_i)$ values are determined for each alternative using Eq. (26). All crisp values and final scores of alternatives are shown in Table 12.

Table 11. The fuzzy calculations of the fuzzy MARCOS method

	$\tilde{K}_i^+$	$\tilde{K}_i^-$	$f\left(\tilde{K}_i^+\right)$	$f\left(\tilde{K}_i^-\right)$
A1	(0.0066, 0.9735, 124.8234)	(0.0078, 1.3586, 204.4313)	(0.0001, 0.0238, 3.5763)	(0.0001, 0.0170, 2.1836)
A2	(0.0066, 0.9719, 126.4850)	(0.0078, 1.3563, 207.1527)	(0.0001, 0.0237, 3.6239)	(0.0001, 0.0170, 2.2127)
A3	(0.0071, 0.9079, 112.8075)	(0.0083, 1.2670, 184.7522)	(0.0001, 0.0222, 3.2320)	(0.0001, 0.0159, 1.9734)
A4	(0.0049, 0.8487, 120.3102)	(0.0058, 1.1844, 197.0398)	(0.0001, 0.0207, 3.4469)	(0.0001, 0.0148, 2.1047)
A5	(0.0061, 0.7994, 105.5490)	(0.0072, 1.1156, 172.8644)	(0.0001, 0.0195, 3.0240)	(0.0001, 0.0140, 1.8464)
A6	(0.0058, 0.8015, 112.2975)	(0.0069, 1.1185, 183.9170)	(0.0001, 0.0196, 3.2174)	(0.0001, 0.0140, 1.9645)

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Table 12. The results of the fuzzy MARCOS method

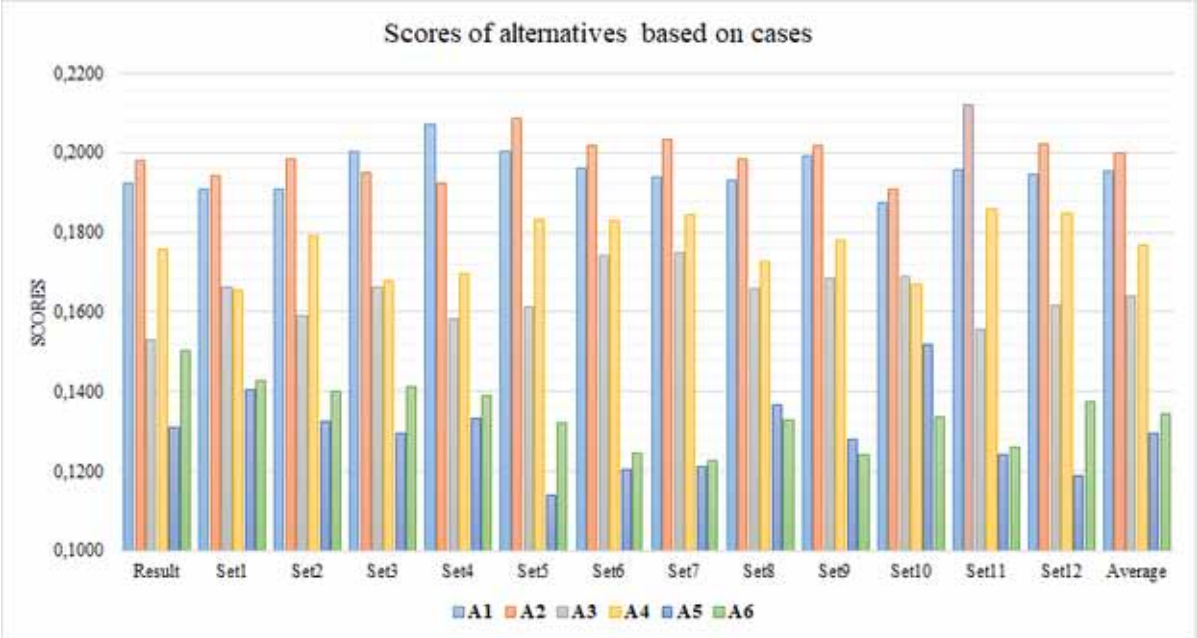
Alternative	K_i^+	K_i^-	$f(K_i^+)$	$f(K_i^-)$	K_i	Normalized K_i
A1	21.4540	34.9789	0.6119	0.3753	17.1076	0,1923
A2	21.7298	35.4309	0.6198	0.3801	17.6203	0,1980
A3	19.4077	31.6381	0.5535	0.3395	13.6042	0,1529
A4	20.6183	33.6306	0.5883	0.3607	15.6237	0,1756
A5	18.1255	29.5557	0.5170	0.3171	11.6641	0,1311
A6	19.2515	31.3996	0.5493	0.3368	13.3651	0,1502

The alternative with the highest K_i score is the best alternative, which is A2. Alternate ranking is found by ordering K_i values from the highest to the lowest. Alternative ranking with this approach occurs as follows:

$A2 > A1 > A4 > A3 > A6 > A5$.

In the sensitivity analysis, 12 cases are created by differing the weights of criteria. The fuzzy MARCOS method is applied by changing weights. Although the ranking of alternatives changes based on normalized scores in some cases, it is shown that the average ranking maintains in Figure 4.

Figure 4. The scores of alternatives based on cases



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FUTURE RESEARCH DIRECTIONS

In future studies, to improve the fuzzy linguistics in the expression of uncertainty, other extensions of fuzzy sets (intuitionistic, hesitant, Pythagorean, neutrosophic etc.) should be considered. The number of criteria can be increased in order to expand the scope of the cases. In addition, the study can be enlarged by adding agility and technological dimensions to the perspective of sustainability in health tourism. The results can be compared by calculating the criteria weights calculated with the fuzzy SWARA method with AHP, ANP or different methods. The weights of criteria containing real data such as number of tourists and price can be included in the study using objective weighting methods such as Entropy.

CONCLUSION

The sustainable tourism concept has gained importance in terms of permanent and environmentally friendly tourism. Health tourism is also transforming in accordance with this concept. The orientation towards sustainable healthcare sites ensures the conservation and continued use of resources. Since the correct use of resources and making the right settlements is a complex problem, MCDM methods can be used in the selection of these sites. The aim of this study is to evaluate sustainable tourism regions according to criteria. For this purpose, the hybrid fuzzy SWARA and fuzzy MARCOS method is proposed to the literature.

In this study, six health tourism destinations in Turkey have been evaluated in terms of sustainability. The literature was searched for the evaluation criteria and the opinions of five experts were used. The fuzzy SWARA method was performed to calculate the weights of the ten criteria, and the fuzzy MARCOS method was applied to rank the alternatives. According to the results, the ranking of the alternatives was A2> A1> A4> A3> A6> A5. In order to control the ranking, 12 different cases were created and the variation of the results was observed. Although the average ranking of alternative sites did not change, the ranking changed in some cases. This situation showed that the ranking is sensitive to the change of criteria weights.

This study has shown that the proposed methodology including fuzzy SWARA and fuzzy MARCOS can be used in the selection of sustainable health tourism areas. For further benefits, it is thought to be suitable for the solution of decision-making problems in other decision making areas.

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KEY TERMS AND DEFINITIONS

Criteria: Considered attribute, characteristics.

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Decision Maker: The person whose opinion is consulted, who is in the company or in the same sector, expert.

Fuzzy Logic: A logic that includes partial belonging to a set.

Linguistic Variable: The verbal scale elements that people usually use when evaluating.

Multi-Criteria Decision Making: A concept that deciding based on many conflicting alternatives.

Sustainability: A phenomenon about preserving the use of resources, not harming, the balance between use and resource.

Tourist: A person who travels for a specific time and purpose, traveler.

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Chapter 5
A Fuzzy Approach for Minimizing Machine Rental Cost for a Specially-Structured Three-Stages Flow-Shop Scheduling Problem in a Fuzzy Environment

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ABSTRACT

Scheduling problem is a problem concerned with searching for an optimal or near optimal schedule subject to a number of constraints. Neuromas approaches have been investigated to solve the problem of scheduling with an aim to determine the sequence of processing jobs on a given set of machines. Its importance and relevance to industry has been promoted. Scheduling requires a variety of activities to accomplish a particular purpose within time and budget. Flow shop is the most classical study manufacturing environment in scheduling literature. This chapter deals with a special structured n-job of three machine flow shop scheduling with piecewise quadratic fuzzy processing times. One of the best approximate intervals, namely close interval approximation of PQFNs, is introduced. A fuzzy approach with the help of a heuristic algorithm is developed for minimizing the rental cost of machines under the specified rental cost. A numerical example is given to demonstrate the computational efficiency of the proposed algorithm.

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1. INTRODUCTION

Scheduling problem is a problem concerned with searching for an optimal or near optimal schedule subject to a number of constraints. Neuromas approaches have been investigated to solve the problem of scheduling with an aim to determine the sequence of processing jobs on a given set of machines. It's important and relevance to industry has promoted. Scheduling requires a variety of activities to accomplish a particular purpose with time and budget. Flow shop is the most classical studies manufacturing environment in scheduling literature. One of the earliest results in flow shop scheduling theory is an algorithm given by Johnson's (1954) for scheduling jobs in a two, three machine flow shop to minimize the time at which all jobs are completed. Gupta (1975) gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. The work developed by Ignall and Scharge (1965), Bagga (1969), Szware (1977), Maggu and Das (1981), Singh et al. (2005), Gupta (1988) have an important consideration. Narain and Bagga (2005) studied a problem for obtaining a sequence gives minimum possible rental cost while minimize total elapsed time constrained by pre- defined rental policy. Schulz et al. (2020) studied a hybrid flow shop scheduling problem with variable discrete production speed level. Gheisariha et al. (2021) solved flexible flow shop scheduling problem with sequence based setup time, transportation time, and probable network using an enhanced multi- objective harmony search algorithm and Gaussian mutation.

In many scientific areas, such as system analysis and operators research, a model has to be setup-using data, which is only approximately known. Fuzzy sets theory, introduced by Zadeh (1965) makes this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers by use of a fuzzification principle. In spite of having a vast decision making experience, the decision maker cannot always articulate the goals precisely. Decision-making in a fuzzy environment, developed by Bellman and Zadeh (1970) improved and a great help in the management decision problems. Zimmermann (1978) proposed the fuzzy set theory and its applications. Petrovic and Song (2006) studied the job sequence problem in a two- machine flow shop with the processing times of the jobs are characterized by triangular fuzzy set. Saracoglu and Suer (2018) proposed a methodology for multi- product parallel multistage cellular manufacturing company, where they applied on a case study from a shoe manufacturing plant for producing the product based on their due dates. Pang et al. (2020) introduced the flow shop scheduling problem and hybrid flow shop scheduling together aim to find the best scheduling strategy for making production plants. Shao et al. (2020) investigated a distributed fuzzy blocking flow- shop scheduling problem with processing times characterized by fuzzy numbers aim to minimize the fuzzy makespan among all factors.

In this paper, a special structured *n*- job of three machines flow shop scheduling with piecewise quadratic fuzzy processing times. (PFN) is studied. A heuristic approach for obtaining the sequence of the jobs, which minimize the rental cost of the machines, is proposed.

The rest of the paper is outlined as below:

Section 2 Introduces some preliminaries related to piecewise quadratic fuzzy numbers.
Section 3 Introduces notation needed in the problem formulation.
Section 4 Formulates the mathematical model for the three-stage flow shop-scheduling problem.
Section 5 Proposes a heuristic approach for obtaining the sequence of the jobs, which minimize the rental cost of the machines.

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Section 6 Presents a numerical example for illustration to the proposed approach.
Section 7 Discussion of the results
Section 8 Paper is summarized with future directions.

2. PRELIMINARIES

In order to easily discuss the problem, it recalls basic rules and findings related to fuzzy numbers, piecewise quadratic fuzzy numbers, close interval approximation and its arithmetic operations.

Definition 1(Zadeh, 1965). Fuzzy number: A fuzzy number $\tilde{A}$ is a fuzzy set with a membership function defined as $\pi_{\tilde{A}}(\mathrm{x}):\Re\to[0,1]$, and satisfies:

1. $\tilde{\mathrm{A}}$ is fuzzy convex, i.e., $\pi_{\tilde{A}}(\delta x+(1-\delta)y)\geq\min\{\pi_{\tilde{A}}(x),\pi_{\tilde{A}}(y)\}$; $\forall x,y\in\Re$; $0\leq\delta\leq1$;
2. $\tilde{\mathrm{A}}$ is normal, i.e., $\exists x_0\in\Re$ for which $\pi_{\tilde{A}}(\mathrm{x}_0)=1$;
3. $\mathrm{Supp}(\tilde{A})=\{\mathrm{x}\in\Re:\ \pi_{\tilde{A}}(\mathrm{x})>0\}$ is the support of $\tilde{\mathrm{A}}$
4. $\pi_{\tilde{A}}(\mathrm{x})$ is an upper semi- continuous (i. e., for each $\alpha\in(0,1)$, the α- cut set $\tilde{A}_\alpha=\{x\in\Re:\ \pi_{\tilde{A}}\geq\alpha\}$ is closed.

Definition 2 (Jain, 2010). **A piecewise quadratic fuzzy number** (PQFN) is denoted by $\tilde{\mathrm{A}}_{\mathrm{PQ}}=(\mathrm{a}_1,\mathrm{a}_2,\mathrm{a}_3,\mathrm{a}_4,\mathrm{a}_5)$, where $\mathrm{a}_1\leq\mathrm{a}_2\leq\mathrm{a}_3\leq\mathrm{a}_4\leq\mathrm{a}_5$ are real numbers, and its membership function $\mu_{\tilde{\mathrm{a}}_{\mathrm{PQ}}}$ is given by (see,Fig. 1)

$$\mu_{\tilde{\mathrm{A}}_{\mathrm{PQF}}}=\begin{cases}0, & \mathrm{x}<\mathrm{a}_1;\\ \frac{1}{2}\frac{1}{(\mathrm{a}_2-\mathrm{a}_1)^2}(\mathrm{x}-\mathrm{a}_1)^2, & \mathrm{a}_1\leq\mathrm{x}\leq\mathrm{a}_2;\\ \frac{1}{2}\frac{1}{(\mathrm{a}_3-\mathrm{a}_2)^2}(\mathrm{x}-\mathrm{a}_3)^2+1, & \mathrm{a}_2\leq\mathrm{x}\leq\mathrm{a}_3;\\ \frac{1}{2}\frac{1}{(\mathrm{a}_4-\mathrm{a}_3)^2}(\mathrm{x}-\mathrm{a}_3)^2+1, & \mathrm{a}_3\leq\mathrm{x}\leq\mathrm{a}_4;\\ \frac{1}{2}\frac{1}{(a_5-a_4)^2}(x-a_5)^2, & a_4\leq x\leq a_5;\\ 0, & \mathrm{x}>\mathrm{a}_5.\end{cases}$$

The interval of confidence at level α for the PQFN is defined as

Definition 3. (Jain, 2010). Let $\tilde{\mathrm{A}}_{\mathrm{PQ}}=(\mathrm{a}_1,\mathrm{a}_2,\mathrm{a}_3,\mathrm{a}_4,\mathrm{a}_5)$ and $\tilde{\mathrm{B}}_{\mathrm{PQ}}=(\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3,\mathrm{b}_4,\mathrm{b}_5)$ be two piecewise quadratic fuzzy numbers. The arithmetic operations on $\tilde{\mathrm{A}}_{\mathrm{PQ}}$ and $\tilde{\mathrm{B}}_{\mathrm{PQ}}$ are:

(i) **Addition**: $\tilde{\mathrm{A}}_{\mathrm{PQ}}(+)\tilde{\mathrm{B}}_{\mathrm{PQ}}=(\mathrm{a}_1+\mathrm{b}_1,\mathrm{a}_2+\mathrm{b}_2,\mathrm{a}_3+\mathrm{b}_3,\mathrm{a}_4+\mathrm{b}_4,\mathrm{a}_5+\mathrm{b}_5)$.

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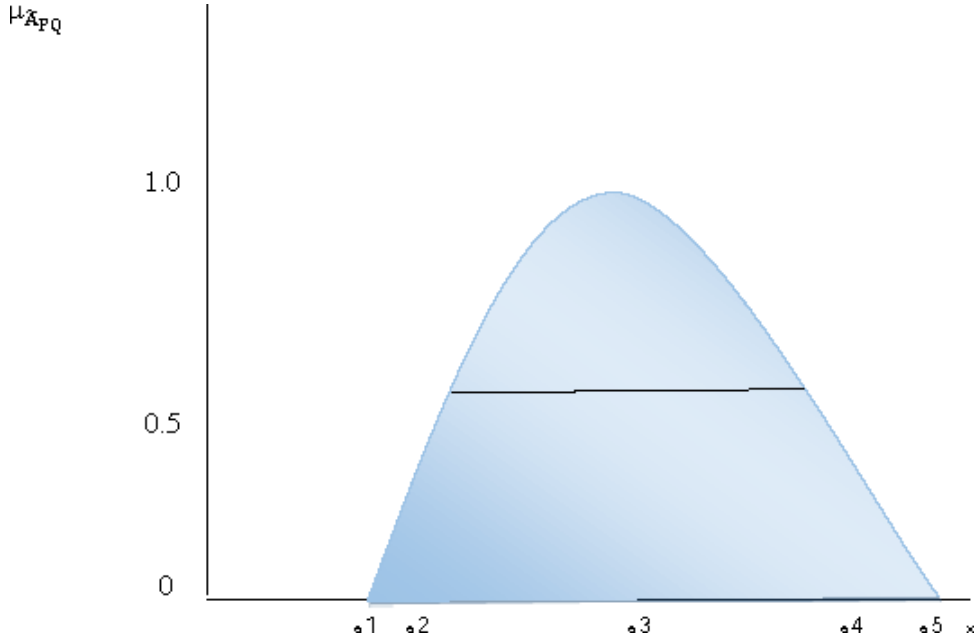
(ii) **Subtraction**: $\tilde{A}_{PQ}(-)\tilde{B}_{PQ} = (a_1 + b_5, a_2 + b_4, a_3 + b_3, a_4 + b_2, a_5 + b_1)$

(iii) **Scalar multiplication**:

$$k\tilde{A}_{PQ} = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5), & k > 0, \\ (ka_5, ka_4, ka_3, ka_2, ka_1), & k < 0. \end{cases}$$

Figure 1. Graphical representation of a Piecewise Quadratic Fuzzy Number (PQFN)



Definition 4. (Jain, 2010). **An interval approximation** $[A] = [a_\alpha^-, a_\alpha^+]$ of a PQFN $\tilde{A}$ is called closed interval approximation if:

$$a_\alpha^- = \inf\{x \in \mathbb{R} : \mu_{\tilde{A}} \geq 0.5\}, \text{ and } a_\alpha^+ = \sup\{x \in \mathbb{R} : \mu_{\tilde{A}} \geq 0.5\}.$$

Definition 5. Associated ordinary number (Jain, 2010). If $[A] = [a_\alpha^-, a_\alpha^+]$ is the close interval approximation of PQFN, the Associated ordinary number of [A] is defined as $\hat{A} = \frac{a_\alpha^- + a_\alpha^+}{2}$

Definition 6. (Jain, 2010). Let $[A] = [a_\alpha^-, a_\alpha^+]$, and $[B] = [b_\alpha^-, b_\alpha^+]$ be two interval approximations of PQFN. Then the arithmetic operations are:

1. Addition: $[A](+)[B] = [a_\alpha^- + b_\alpha^-,\ a_\alpha^+ + b_\alpha^+]$,

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2. Subtraction: $[\mathrm{A}](-)[\mathrm{B}]=\left[a_\alpha^- - b_\alpha^+,\ a_\alpha^+ - b_\alpha^-\right]$,
3. Scalar multiplication: $\alpha[\mathrm{A}]=\begin{cases}\left[\alpha a_\alpha^-,\ \alpha a_\alpha^+\right], & \alpha>0\\ \left[\alpha a_\alpha^+,\ \alpha a_\alpha^-\right], & \alpha<0.\end{cases}$
4. Multiplication: [A](×)[B] $\left[\frac{a_\alpha^+ b_\alpha^- + a_\alpha^- b_\alpha^+}{2},\ \frac{a_\alpha^- b_\alpha^- + a_\alpha^+ b_\alpha^+}{2}\right]$,
5. Division: [A](÷)[B] $\begin{cases}\left[2\left(\frac{a_\alpha^-}{b_\alpha^- + b_\alpha^+}\right), 2\left(\frac{a_\alpha^+}{b_\alpha^- + b_\alpha^+}\right)\right], [\mathrm{B}]>0, b_\alpha^- + b_\alpha^+ \neq 0\\ \left[2\left(\frac{a_\alpha^+}{b_\alpha^- + b_\alpha^+}\right), 2\left(\frac{a_\alpha^-}{b_\alpha^- + b_\alpha^+}\right)\right], [\mathrm{B}]<0, b_\alpha^- + b_\alpha^+ \neq 0\end{cases}$
6. The order relation**s:**
   - $[\mathrm{A}](\lesssim)[\mathrm{B}]$ if $a_\alpha^- \le b_\alpha^-$ and $a_\alpha^+ \le b_\alpha^+$ or $a_\alpha^- + a_\alpha^+ \le b_\alpha^- + b_\alpha^+$.
   - [A] is preferred to [B] if and only if $a_\alpha^- \ge b_\alpha^-,\ a_\alpha^+ \ge b_\alpha^+$.

It is noted that $\mathrm{P}(\mathbb{R}) \subset \mathrm{F}(\mathbb{R})$, where $\mathrm{F}(\mathbb{R})$, and $\mathrm{P}(\mathbb{R})$ are the sets of all PQFNs and close in interval approximation of PQFN; respectively.

Notation

In this section, some of natation needed in the paper are used:

S: Sequence of jobs, $i=\overline{1,n}$
S_k: Sequence obtained by applying Johnson's procedure, k=1,2,…,n
M_j: Machine j, j=1,2,3
M: Minimum makespan
$\tilde{a}_{ij}^P$: Piecewise quadratic fuzzy processing time of i^{th} job on machine M_j, j=1,2,3
$\left[a_{ij}^{PQ}\right]$: Close interval approximation of PQF processing time of i^{th} job of sequence S_k on machine M_j, j=1,2,3
$\hat{a}_{ij}$: Associated ordinary processing time of i^{th} job on machine M_j, j=1,2,3
$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M_j
$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k
$U_j(S_k)$: Utilization time for which machine M_j is required for sequence S_k of jobs.
$R(S_k)$: Total rental cost for the sequence S_k of all the machines
C: Rental cost of j^{th} machine
$CT(S_k)$: Total completion time of the jobs for sequence S_k

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3. PROBLEM FORMULATION

Before introducing the problem formulation, let us introduce the rental cost definition

3.1. Rental Cost

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required (i.e., the 1st machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on the 1st machine and the 3rd machine is taken on rent when the 2nd job is completed on 2nd machine.

Let some job $i, i = \overline{1,n}$ is to be processed on three machines M_j, j=1,2,3 under the specific rental policy *L*. Let $\tilde{a}_{ij}^{PQ}$ be the PQF processing time of i^{th} job on j^{th} machine, $\left[a_{ij}^{PQ}\right]$ be the close interval approximation corresponding to $\tilde{a}_{ij}^{PQ}$, and $\hat{a}_{ij}$ be the associated ordinary number. Let $S_{ij}, i = \overline{1,n}; j = 1,2,3.$ Evaluate the associated ordinary number of the processing times on three machines M_1, M_2 and M_3 such that either $\hat{a}_{j2} \le \hat{a}_{i1}$ or $\hat{a}_{j2} \le \hat{a}_{i3}$; $\forall i, j$. Our purpose is to find the sequence $\{S_k\}$ of the jobs, which minimize the rental cost of the machines.

Table 1. Mathematical model for the problem piecewise quadratic fuzzy number matrix form

Jobs	Machine M_1	Machine M_2	Machine M_3
i	$\tilde{a}_{i1}^{PQ}$	$\tilde{a}_{i2}^{PQ}$	$\tilde{a}_{i3}^{PQ}$
1	$\tilde{a}_{11}^{PQ}$	$\tilde{a}_{12}^{PQ}$	$\tilde{a}_{13}^{PQ}$
2	$\tilde{a}_{21}^{PQ}$	$\tilde{a}_{22}^{PQ}$	$\tilde{a}_{23}^{PQ}$
…	…	…	…
n	$\tilde{a}_{n1}^{PQ}$	$\tilde{a}_{n2}^{PQ}$	$\tilde{a}_{n3}^{PQ}$

Mathematically, the problem can be formulated as follows

$$\min \tilde{R}\left(S_k\right) = \sum_{i=1}^{n} \tilde{a}_{i1}^{PQ} \times C_1 + U_2\left(S_k\right) \times C_2 + U_3\left(S_k\right) \times C_3$$

Rental policy *L*.

Based of the close interval approximation of piecewise quadratic fuzzy number, problem (10) can be rewritten as follows

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$$\min \mathrm{R}\left(S_k\right)=\sum_{i=1}^{n}\left[\left(a_{i1}\right)_{\alpha}^{-},\left(a_{i1}\right)_{\alpha}^{+}\right]\times C_1+U_2\left(S_k\right)\times C_2+U_3\left(S_k\right)\times C_3$$

Rental policy *L.*

Table1 can be rewritten in the sense of close interval approximation of piecewise quadratic fuzzy number as follows:

Table 2. Mathematical model for the problem with close interval approximation matrix form

Jobs	Machine M_1	Machine M_2	Machine M_3
i	$\left[\left(a_{i1}\right)_{\alpha}^{-},\left(a_{i1}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{i2}\right)_{\alpha}^{-},\left(a_{i2}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{i3}\right)_{\alpha}^{-},\left(a_{i3}\right)_{\alpha}^{+}\right]$
1	$\left[\left(a_{11}\right)_{\alpha}^{-},\left(a_{11}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{12}\right)_{\alpha}^{-},\left(a_{12}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{13}\right)_{\alpha}^{-},\left(a_{13}\right)_{\alpha}^{+}\right]$
2	$\left[\left(a_{21}\right)_{\alpha}^{-},\left(a_{21}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{22}\right)_{\alpha}^{-},\left(a_{22}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{23}\right)_{\alpha}^{-},\left(a_{23}\right)_{\alpha}^{+}\right]$
…	…	…	…
n	$\left[\left(a_{n1}\right)_{\alpha}^{-},\left(a_{n1}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{n2}\right)_{\alpha}^{-},\left(a_{n2}\right)_{\alpha}^{+}\right]$	$\left[\left(a_{n3}\right)_{\alpha}^{-},\left(a_{n3}\right)_{\alpha}^{+}\right]$

Table 3. Mathematical model for the problem with the associated ordinary matrix form

Jobs	Machine M_1	Machine M_2	Machine M_3
i	$\frac{\left(a_{i1}\right)_{\alpha}^{-}+\left(a_{i1}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{i2}\right)_{\alpha}^{-}+\left(a_{i2}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{i3}\right)_{\alpha}^{-}+\left(a_{i3}\right)_{\alpha}^{+}}{2}$
1	$\frac{\left(a_{11}\right)_{\alpha}^{-}+\left(a_{11}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{12}\right)_{\alpha}^{-}+\left(a_{12}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{13}\right)_{\alpha}^{-}+\left(a_{13}\right)_{\alpha}^{+}}{2}$
2	$\frac{\left(a_{21}\right)_{\alpha}^{-}+\left(a_{21}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{22}\right)_{\alpha}^{-}+\left(a_{22}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{23}\right)_{\alpha}^{-}+\left(a_{23}\right)_{\alpha}^{+}}{2}$
…	…	…	…
n	$\frac{\left(a_{n1}\right)_{\alpha}^{-}+\left(a_{n1}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{n2}\right)_{\alpha}^{-}+\left(a_{n2}\right)_{\alpha}^{+}}{2}$	$\frac{\left(a_{n3}\right)_{\alpha}^{-}+\left(a_{n3}\right)_{\alpha}^{+}}{2}$

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4. SOLUTION PROCEDURE

In this section, a heuristic approach to minimize the utilization time and hence rental cost of special structured three stages flows hop scheduling with PQF processing time, where the makespan not taking in our consideration are illustrated as in the following steps:

Step 1: Find the associated ordinary number $\hat{a}_{ij}$, $i=\overline{1,2}; j=1,2,3$ of the PQF processing times for all jobs on the three machines M_1, M_2, and M_3.

Step 2: Check the condition either $\hat{a}_{j2} \le \hat{a}_{i1}$ or $\hat{a}_{j2} \le \hat{a}_{i3};\ \forall i, j$ i.e.,

$$\max\left\{\hat{a}_{i1}\right\} \ge \min\left\{\hat{a}_{j2}\right\} \text{ or } \max\left\{\hat{a}_{i3}\right\} \ge \min\left\{\hat{a}_{j2}\right\};\ \forall i, j.$$

If this condition is satisfied then go to step 3, otherwise the data in nit in the standard form.

Step 3: Introduce the two fictitious machines H_1 and H_2 with processing times H_1^i and H_2^i as

$$H_1^i = \hat{a}_{i1} + \hat{a}_{i2}, H_2^i = \hat{a}_{i2} + \hat{a}_{i3};\ \forall i.$$

Step 4: Apply Johnson (1954) algorithm on machines H_1 and H_2, to obtain the sequence S_1

Step 5: Put the 2nd, 3rd, …, n^{th} jobs of the sequence S_1 in the 1st position and all other jobs of S_1 in the same order to get other sequences, namely $S_2, S_3, \ldots, S_{n-1}$.

Step 6: Compute all of $\sum_{i=1}^{n}\hat{a}_{i1}$, $U_2(S_k)$, $U_3(S_k)$, and hence

$$R(S_k) = \sum_{i=1}^{n}\hat{a}_{ij} \times C_1 + U_2(S_k) \times C_2 + U_3(S_k) \times C_3$$

that is for all possible sequences $S_k, k=\overline{1,n}$.

Step 7: Find $\min\left\{R(S_k)\right\}, k=\overline{1,n}$, and assume that it is the minimum for the sequence S_q, and then the sequence S_q becomes the optimal sequence with rental cost $R(S_q)$.

5. NUMERICAL EXAMPLE

Consider 5 jobs, 3 machine flow shop problem with Pythagorean fuzzy processing time given as in the table 3. The rental cost per unit time for machines M_1, M_2, and M_3 are 4, 2, and 3 units; respectively under the rental policy *L*.

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Step1: Calculate the associated ordinary number for each PQF processing time.

Table 4. Piecewise quadratic fuzzy number processing times for machines

Jobs	Machine M_1	Machine M_2	Machine M_3
i	$\tilde{a}_{i1}^{PQ}$	$\tilde{a}_{i2}^{PQ}$	$\tilde{a}_{i3}^{PQ}$
1	(6,7,8,9,10)	(5,6,7,8,10)	(2,3,4,5,7)
2	(11,12,13,14,17)	(4,5,6,7,9)	(3,4,5,6,8)
3	(7,8,10,12,14)	(3,4,5,6,9)	(5,6,7,8,9)
4	(9,10,11,12,14)	(3,5,6,7,9)	(10,11,12,13,15)
5	(7,9,10,11,13)	(2,3,4,8,10)	(5,8,9,10,11)

Table 5. Piecewise quadratic fuzzy number processing times for machines

Jobs	Machine M_1	Machine M_2	Machine M_3
i	$\left[a_{i1}^{PQ}\right]$	$\left[a_{i2}^{PQ}\right]$	$\left[a_{i3}^{PQ}\right]$
1	[7,9]	[6,8]	[3,5]
2	[12,14]	[5,7]	[4,6]
3	[8,12]	[4,6]	[6,8]
4	[10,12]	[5,7]	[11,13]
5	[9,11]	[5,8]	[8,10]

Table 6. Defuzzified piecewise quadratic fuzzy number processing times for machines

Jobs	Machine M_1	Machine M_2	Machine M_3
i	$\hat{a}_{i1}$	$\hat{a}_{i2}$	$\hat{a}_{i3}$
1	8	7	4
2	13	6	5
3	10	5	7
4	11	6	12
5	10	6.5	9

Step3: The processing times H_1^i and H_2^i of the machines H_1 and H_2 are as follows

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Table 7. The associated ordinary of the processing times for the two fictitious machines

Jobs	H_1	H_2
1	15	11
2	19	11
3	15	12
4	17	18
5	16.5	15.5

By using Johnson's (1954) procedure, the sequence with minimum makespan is

S_1: 2 – 4 – 5 – 1 – 3.

The others feasible sequences correspond to minimum rental cost are

S_2: 4 – 2 – 5 – 1 – 3;

S_3: 1 – 2 – 4 – 5 – 3;

S_4: 3 – 2 – 4 – 5 – 1.

The following table illustrates the in- out for the sequence S_1 as

Table 8. The in- out flow for the sequence S_1in the PQFNs form

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In- out	In- out	In- out
2	(11,12,13,14,17)	(15,17,19,21,26)	(18,21,24,27,34)
4	(19,22,24,26,31)	(18,22,25,28,35)	(28,32,36,39,49)
5	(26,31,34,37,44)	(20,27,31,36,45)	(33,40,45,49,60)
1	(32,38,42,46,54)	(25,27,31,36,45)	(35,43,49,54,67)\
3	(39,46,52,58,68)	(25,37,43,50,64)	(40,49,56,62,76)

Table 9. The in- out flow for the sequence S_1 in close interval approximation

Jobs	Machine M_1	Machine M_2	Machine M_3
i	In- out	In- out	In- out
2	[12,14]	[17,21]	[21,27]
4	[22,26]	[22,28]	[32,39]
5	[31,37]	[27,36]	[40,49]
1	[38,46]	[33,44]	[43,54]
3	[46,58]	[37,50]	[49,62]

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For S_1, we get

The completion time for S_1 is $\widetilde{CT}^{PQ}(\mathrm{S}_1)$ = (40, 49, 56, 62, 76); [$CT(S_1)$]= [49, 62]; and $\widehat{CT}(\mathrm{S}_1)$ = 55.5 units.

The utilization time for Machine M_2 is $\tilde{U}_2^{PQ}(S_1)$ = (11, 23, 30, 38, 53);[$U_2(S_1)$]= [23, 38]; and $\widehat{U}_2(S_1)$ =30.5 unis.

The utilization time for Machine M_3 is $\tilde{U}_3^{PQ}(S_1)$ = (14, 28, 37, 45, 61); [$U_3(S_1)$]= 28, 45]; and $\widehat{U}_3(S_1)$ = 36.5 unis.

$$\tilde{R}^{PQ}(S_1)=\sum_{i=1}^{5}\tilde{a}_{i1}\times C_1+U_2(S_1)\times C_2+U_3(S_1)\times C_3=(572,726,831,935,1145),$$

[$R(S_1)$]=[726,935], and $\widehat{R}(S_1)$ =830.5 units

In similar way, we can have for the sequences as follows:

For S_2, we have

$\widetilde{CT}^{PQ}(\mathrm{S}_2)$ = (37,47,54,61,73), [$CT(S_2)$]=47,61], and $\widehat{CT}(\mathrm{S}_2)$ =54 units.

$\tilde{U}_2^{PQ}(S_2)$ =(12,23,30,38,52), [$U_2(S_2)$]=23,38, and $\widehat{U}_2(S_2)$ =30.5 unis.

$\tilde{U}_3^{PQ}(S_2)$ =(14,28,37,45,61), [$U_3(S_2)$]=[28,45], and $\widehat{U}_2(S_2)$ =36.5 unis.

$$\tilde{R}^{PQ}(S_2)=\sum_{i=1}^{5}\tilde{a}_{i1}^{PQ}\times C_1+\tilde{U}_2^{PQ}(S_2)\times C_2+\tilde{U}_3^{PQ}(S_2)\times C_3=(582,718,769,927,1131),$$

[$R(S_2)$]=718,927], and $\widehat{R}(S_1)$ =822.5 units

For S_3, we have

$\widetilde{CT}^{PQ}(\mathrm{S}_3)$ =(36,45,52,59,70), [$CT(S_3)$]=[45.59], and $\widehat{CT}(\mathrm{S}_3)$ =52 units.

$\tilde{U}_2^{PQ}(S_3)$ =(13,23,30,38,49), [$U_2(S_3)$]=[23,38], and $\widehat{U}_2(S_3)$ =30.5 unis.

$\tilde{U}_3^{PQ}(S_3)$ =(16,28,37,46,59), [$U_3(S_3)$]=[28,46], and $\widehat{U}_2(S_3)$ =37 unis.

$$\tilde{R}^{PQ}(S_3)=\sum_{i=1}^{5}\tilde{a}_{i1}^{PQ}\times C_1+\tilde{U}_2^{PQ}(S_3)\times C_2+\tilde{U}_3^{PQ}(S_3)\times C_3=(562,686,791,896,1075),$$

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$[R(S_3)]$=686,896], and $\widehat{R}\left(S_3\right)$=791 units

For S_4, we have

$\widetilde{CT}^{PQ}\left(\mathrm{S}_4\right)$=(35,44,52,60,73), $[CT(S_4)]$=44,60] and $\widehat{CT}\left(\mathrm{S}_4\right)$=52 units.

$\tilde{U}_2^{PQ}\left(S_4\right)$=(10,11,30,40,54), $[U_2(S_4)]$=[11,40], and $\widehat{U}_2\left(S_4\right)$=25.5 unis.

$\tilde{U}_3^{PQ}\left(S_4\right)$=(12,26,37,48,63), $[U_3(S_4)]$=26,48], and $\widehat{U}_2\left(S_4\right)$=37 unis.

$$\tilde{R}^{PQ}\left(S_4\right)=\sum_{i=1}^{5}\tilde{a}_{i1}^{PQ}\times C_1+\tilde{U}_2^{PQ}\left(S_4\right)\times C_2+\tilde{U}_3^{PQ}\left(S_4\right)\times C_3=\left(560,672,823,956,1161\right),$$

$[R(S_4)]$=[672,956], and $\widehat{R}\left(S_4\right)$=814 units

For S_5, we have $\widehat{R}\left(S_5\right)$=2066.6257 Units. Thus

$$\tilde{R}^{PQ}\left(S_3\right)=\sum_{i=1}^{5}\tilde{a}_{i1}^{PQ}\times C_1+\tilde{U}_2^{PQ}\left(S_3\right)\times C_2+\tilde{U}_3^{PQ}\left(S_3\right)\times C_3=\left(562,686,791,896,1075\right),$$

$[R(S_5)]$=[686,896], and $\widehat{R}\left(S_5\right)$=791 units is the minimum rental cost and the sequence S_3: 1–2–4–5–3 is the optimal sequence subject to the minimum rental cost with irrespective of total elapsed time.

Table 10. Comparision of proposed study with existing relevant literature

Authors	Research Title	Piecewise quadratic fuzzy numbers as fuzzy data	The optimal sequence	Rental cost with irrespective of total elapsed time
Gupta et al. 2012	Specially Structured Three Stage Flowshop Scheduling to Minimize the Rental Cost	No	Yes	Yes
Gupta et al., 2015	3-Stage Specially Structured Flow Shop Scheduling to Minimize the Rental Cost Including Transportation Time, Job Weightage and Job Block Criteria	No	Yes	Yes
Proposed study	A fuzzy approach for minimizing machines rental cost for a specially structured three- stages flow- shop scheduling problem in a fuzzy environment	Yes	Yes	Yes

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6. DISCUSSION

In this section, the proposed study is compared with some existing relevant literature to carve out the advantages aspect of the proposed study.

7. CONCLUSION

In this paper a fuzzy approach with cooperate with heuristics approach for minimizing the rental cost of the machines for a specially structured three stage flow shop scheduling has discussed with respect irrespective of total elapsed time, where the processing times are of piecewise quadratic fuzzy numbers instead of fuzzy numbers. The advantage of this approach is more flexible where it allows the decision maker to choose the targets he/she is willing, useful for potential study in widespread fluctuations. This study may be extended using other types of fuzzy numbers, chaos numbers and so on.

DATA AVAILABILITY

The data used to support the findings of this study are available from the corresponding author upon request.

CONFLICTS OF INTEREST

The author declares no conflicts of interest.

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Chapter 6
Application of Fractional Calculus on the Crisp and Uncertain Inventory Control Problem

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ABSTRACT

This chapter explores the possible application of fractional calculus in the field of operation research, more specifically inventory control problem. The sense of memory can be implemented in a dynamical system with the mathematical manipulation through fractional calculus. In this chapter, some recently published papers on generalized lot-sizing models described by fractional differential equation in crisp as well as uncertain environments are reviewed. The intuitional applicability, obstacles, and challenges for studying the inventory management problems under fractional differential equation (in Riemann-Liouville and Caputo approaches) are discussed in this chapter.

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1. INTRODUCTION

The journey of fractional calculus started with the famous conversation between L'Hospital and Liebnitz. Liebnitz's response to the question of L'Hospital on the existence of derivative of $\frac{1}{2}$ order was "an apparent paradox from which one day useful consequences will be drawn". The theory of fractional calculus grew very slowly through the last the centuries. However, the worldwide fervourfor fractional calculus (FC) as well as fractional order system (FOS) has been apparently exponential in the most recent decades because of its exactness on portraying the dynamical nature associated with different physical procedures as a general rule. As of late, this idea has been prominently utilized for the demonstrating in the broad area of AppliedMathematics, Physical Science, Technology and Management replacing the Newtonian calculus (Agila et al., 2016; Agrawal et al., 2004; Kilbas et al., 2006; Machado & Mata, 2015; Miller & Ross, 1993; Podlubny, 1999). Differential equation is one of the frequently used mathematical tools to describe the variability of a dynamic state over time. If the integer order derivative is replaced by the fractional counterpart, the differential equation is called the fractional differential equation (FDE). In reality has replaced the differential equation of integer order proving its smartness for describing dynamical system in a complicated situation. Several Studies and findings (Abbasbandy, 2007; Arikoglu & Ozkol, 2009; Bhrawy et al., 2013; Duan et al., 2013; Hajipour et al., 2019; Mainardi et al., 2007) are carried out to establishing that fact which ultimately enriched the world of technology and innovation it's applications. The physical meaning of fractional calculus seems to be little abstract. However, several studies proved the exactness of fractional calculus against the conventional integer order calculus on the fitting of real data from memory involved systems. Thus, interpretation of memory is regarded as one of the physical meaning of the fractional calculus. The Riemann-Liouville fractional integration and derivative and Caputo fractional derivative gained the popularity in this context. Here, in this present article, we fix our focuses on the exploration of the possible implementation of fractional differential equations on the inventory control problems in both crisp and uncertain environments. Thus, the literature review in the next section will be limited into some specific keywords, namely theory of fractional differential equations in uncertainty, crisp fractional inventory models and fuzzy fractional inventory models.

The rest of this article is decorated as the following: Brief literature reviewing about some particular keywords has been carried out in the section 2. Some basic theory of the fractional calculus is presented in the section 3. The section 4 is involved in the comparison of fractional and integer order system to describe the inventory models. The section 5 presents the comparisons between Riemann-Liouville and Caputo derivative to describe the inventory models with different assumptions. Several other obstacles and challenges are described in section 6. Finally, concluding remarks are made in the section 7.

2. LITERATURE REVIEW

2.1 Fuzzy Set Theory

Fuzzy set theory is one of the important areas where uncertainty can expressed in an graceful manner. In the crisp set theory if we give a graded membership inputs with every elements (the graded mem-

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bership vale belongs to 0 to 1) then it becomes fuzzy set theory concept. If the gradation is 1 then it is strictly belong to the set, if gradation is 0 then it not belongs to the set. Rest of cases happen for different behavior of the problem setting. After the first paper of L.A. Zadeh there is several theoretical and practical development happen among the industry and academic research. The fuzzy number ideology establish and play a significant role in modeling real situation. The different type of fuzzy number such as pentagonal fuzzy number (Chakraborty et al., 2019), heptagonal dense fuzzy number (Maity et al., 2020), interval valued fuzzy number (Mondal, 2016), hesitant fuzzy number (Ghorui et al., 2021), lock fuzzy number (Maity et al., 2019), type- 2 fuzzy number (Tudu et al., 2021), Gaussian fuzzy number (Rahaman, Mondal, Algehyne et al, 2021) is exists in present time.

2.2. Fuzzy Fractional Differential Equation

When, the fractional differential equation carrying fuzzy uncertainty in terms of its coefficients and initial or boundary conditions, it is called the fuzzy fractional differential equation. The fractional differential equation under uncertainty was first considered by Agarwal *et al* (Agarwal et al., 2010). In their study, they took Riemann-Liouville sense of fractional differentiability. Subsequently, Caputo differentiability was used by Hoa *et al* (Hoa et al., 2018) to describe an initial valued fuzzy fractional differential equation. Lupulescu *et al* (Lupulescu et al., 2017) contributed an interval analogue of the generalized Hukuhara (gH) fractional differentiability. The theory of Laplace transformation is a very useful tool associated with the theory of differential equations. So, the generalized (fuzzy) fractional Laplace transformation is introduced and improved to solve initial value problems in the recent studies (Allahviranloo & Ahmadi, 2010; Salahshour & Allahviranloo, 2013; Salahshour et al., 2012).

2.3. Crisp Fractional Inventory Models

Inventory control problem is a well-recognized theory in the research domain of Operation Research. The dynamical behaviors involved in the inventory control problems are generally described by the differential equation of integer order. But, the classical EOQ model was made to be more generalized by Das and Roy (Das & Roy, 2014) by incorporating the theory of fractional calculus in the place of the integer calculus. Later, Das and Roy (Das & Roy, 2015) extended their work assuming the demand to be a linear function of time. In another study, Das and Roy (Das & Roy, 2017) hinted for the possible optimization techniques for the generalized fractional EPQ model. Pakhira et al. (Pakhira et al., 2018a) added a literature of the fractional inventory model claiming the sense of fractional calculus to be representative of the memory of the system. In the banner of the memory motivated study, they contributed several articles considering linear demand rate with and without shortage (Pakhira et al., 2018b; Pakhira et al., 2019a), linear demand with salvage value (Pakhira et al., 2018c), quadratic demand rate with and without shortage (Pakhira et al., 2019b; Pakhira et al., 2019c), price dependent demand rate (Pakhira et al., 2019d; Pakhira et al., 2019e).Rahaman et al. (Rahaman, Mondal, & Shaikh, 2020) incorporated the deterioration factor in the study of fractional EPQ model with constant demand and production rates and solve the optimization problem using the rational approximation of the generalized Mittag-Leffer function. Later, Rahaman et al. (Rahaman, Mondal, Shaikh et al, 2020) added another article on the soft computing technique to solve an EPQ model with price and stock dependent demand, stock dependent production rate and constant deterioration in a memory sensitive decision-making scenario via Artificial Bee Colony Optimization.

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2.4. Fuzzy Fractional Inventory Models

Fuzzy is one of the very useful mathematical tools to trace the real-world uncertainty. So, the literature of the fuzzy inventory models is made much enriched justly. In this context, a very worthy documentation on the study of fuzzy inventory modelling was credited by Shakarian et al. (Shekarian et al., 2017). Fuzzy differential approach to solve the fuzzy inventory control problems explores the beauty of fuzzy valued calculus (Pakhira et al., 2019f). Pakhira et al. (Rahaman, Mondal, & Alam, 2020) discussed the memory effect on the fuzzy fractional EOQ model with no shortage. The fuzzy fractional differential equation under Riemann-Liouville derivative is utilized by Rahaman et al. (Rahaman, Mondal, Alam et al, 2021) to manifest the productive phase of an EPQ model incorporating the sense of memory and uncertainty together. The senses of memory, learning through repetition of decision-making job and uncertainty are combined in a recent article by Rahaman et al. (Chakraborty et al., 2019) to demonstrate an EOQ model under fuzzy fractional differential equation approach and extension principle.

2.5. Motivation

After brief reviewing of the existing literature, our observations are the followings:

(i) The potentiality of the research direction with memory study of inventory model is highly rich.
(ii) However, the current literature related to this domain is in the immature phase with little initiation.
(iii) The proper explanation and analysis on the application of fractional calculus on the operation research is not clear yet.

On this circumstance, the current article makes approach to analyze the prosperity of fractional calculus in describing the inventory control problem. Here, advantages and disadvantages, challenges and obstacles with the fractional calculus due to Riemann-Liouville and Caputo's approaches are pointed out.

3. PRELIMINARIES

This present section depicts the theory of fractional calculus very briefly focusing the Riemann-Liouville and Caputo definitions of fractional derivative and integration.

Definition 3.1(Miller & Ross, 1993; Podlubny, 1999) The left sided Riemann- Liouville integral ${}^{RL}I_x^{\alpha} f\left(x\right)$ of the function f of order $\alpha(0<\alpha\leq 1$, a real number) is given by

$${}^{RL}I_x^{\alpha} f\left(x\right) = \frac{1}{\Gamma\left(\alpha\right)} \int_0^x \left(x-v\right)^{\alpha-1} f\left(v\right) dv$$

Now, the semi group properties of the Riemann-Liouville integral give the following result:

$$\frac{d}{dx}\{{}^{RL}I_x^{1-\alpha}({}^{RL}I_x^{\alpha} f\left(x\right))\} = \frac{d}{dx}\{{}^{RL}I_x^{1} f\left(x\right)\} = f\left(x\right)$$

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This result gives a strong motivation of establishing the definition of Riemann-Liouville derivative in terms of the Riemann-Liouville fractional integral.

Definition 3.2(Miller & Ross, 1993; Podlubny, 1999) The left-sided Riemann-Liouville derivative ${}^{RL}D_x^{\alpha} f\left(x\right)$ of f of order $\alpha(0<\alpha\leq 1)$ is

$$ {}^{RL}D_x^{\alpha} f\left(x\right) = \frac{1}{\Gamma\left(1-\alpha\right)} \frac{d}{dx} \int_0^x \left(x-v\right)^{-\alpha} f\left(v\right) dv \,, $$

Interchanging the order of integration and differentiation in the above definition of Riemann-Liouville fractional derivative, the definition of Caputo fractional derivative is obtained.

Definition 3.3 (Miller & Ross, 1993; Podlubny, 1999) The left-sided Caputo fractional derivative ${}^{C}D_x^{\alpha} f\left(x\right)$ of the function f of order $\alpha(0<\alpha\leq 1)$ is given by

$$ {}_0^C D_x^{\alpha} f\left(x\right) = \frac{1}{\Gamma\left(1-\alpha\right)} \int_0^x \left(x-v\right)^{-\alpha} \frac{df}{dv} dv $$

3.1 Laplace Transformation of Fractional Derivative

The Laplace transformation plays a vital role in the theory of fractional calculus. This subsection presents two theorems regarding the Laplace transformation of Riemann-Liouville and Caputo derivative.

Theorem 3.1 (Miller & Ross, 1993; Podlubny, 1999) The Laplace transformation of the Caputo fractional derivative ${}^{C}D_x^{\alpha} f\left(x\right)$ is given by

$$ L\left[{}^{C}D_x^{\alpha} f\left(x\right); s\right] = s^{\alpha} F\left(s\right) - s^{\alpha-1} f\left(0\right), $$

Where, $F(s)$ is the Laplace transform of $f(x)$.

Theorem 2.1 (Miller & Ross, 1993; Podlubny, 1999) The Laplace transformation of the Riemann-Liouville fractional derivative ${}^{RL}D_x^{\alpha} f\left(x\right)$ is given by

$$ L\left[{}^{RL}D_x^{\alpha} f\left(x\right); s\right] = s^{\alpha} F\left(s\right) - {}^{RL}D_x^{\alpha-1} f\left(0\right), $$

Where, $F(s)$ is the Laplace transform of $f(x)$.

3.2 Important Functions Related to the Theory

The well-known Gamma and Beta functions have much contribution in the theory of fractional calculus. However, the most interesting function is the Mittag-Leffler function which is considered to be the fractional generalization of the popular exponential function of the conventional integer order calculus.

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Mittag-Leffler Function

(a) The two parameters Mittag-Leffler function is denoted by $E_{\alpha,\beta}\left(x\right)$ and is defined by

$$E_{\alpha,\beta}\left(x\right)=\sum_{k=0}^{\infty}\frac{x^{k}}{\Gamma\left(\alpha k+\beta\right)}, \text{ where } \alpha,\beta>0$$

(b) The one parameter Mittag-Leffler function is denoted by $E_{\alpha}\left(x\right)$ and is defined by

$$E_{\alpha}\left(x\right)=\sum_{i=0}^{\infty}\frac{x^{k}}{\Gamma\left(\alpha k+1\right)}, \text{ where } \alpha>0$$

4. FRACTIONAL ORDER CALCULUS TO DEFINE AND DESCRIBE INVENTORY CONTROL MODEL

4.1. Classical EOQ and EPQ Model

The classical economic order quantity (EOQ) model with the deterministic demand is given by

$$\begin{cases}\dfrac{dI\left(t\right)}{dt}=-D,\ \ when\ \ 0\le t\le T\\ \qquad I\left(0\right)=Q,I\left(T\right)=0\end{cases} \tag{4.1}$$

Here, $I(t)$ is the inventory level at any time t, D is the deterministic demand and Q is the order quantity. Then, the cost minimization problem is given by

$$\begin{cases}Min\dfrac{C_{h}DT}{2}+\dfrac{C_{o}}{T}\\ \qquad Q=DT\\ \qquad T>0\end{cases} \tag{4.2}$$

In the system given by the equation (4.2), C_o denotes the ordering cost per cycle and C_h is the holding cost per unit quantity per unit time.

The economic order quantity model was modified to develop the classical economic production quantity model (EPQ) with the finite production rate K and deterministic demand which is governed by the set of differential equations as the following:

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$$\begin{cases} \dfrac{dI\left(t\right)}{dt} = K - D, \ when \ 0 \le t \le T_1 \\ \dfrac{dI\left(t\right)}{dt} = -D, \ when \ T_1 \le t \le T \\ I\left(0\right) = 0 = I\left(T\right) \end{cases} \tag{4.3}$$

Then, the equivalent optimization problem is

$$\begin{cases} Min \dfrac{c_0}{T} + \dfrac{c_h}{2}\left(1 - \dfrac{D}{K}\right)DT \\ 0 < T_1 < T \end{cases} \tag{4.4}$$

When $K \to \infty$, the system (4.4) reduces to (4.2). This shows the exactness of the extension from the EOQ model to EPQ model.

4.2. Motivation of Fractional Calculus for Generalization of the Inventory Models

Suppose, being inspired by L'Hospital and Liebnitz curiosity on the existence of derivative of $\frac{1}{2}$ order,a mathematician is motivated to replace the first order differentiation $\frac{dI\left(t\right)}{dt}$ in (4.1) and (4.3). Now, the question is on the justification of these types of approaches. Here, some logic in the favour of considering the fractional generalization of EOQ and EPQ models are presented. The traditional integer order differential equation failed to describe the memory affected system. However, where the involvement of mankind exists in a dynamical system, the memory plays a crucial role on that system. In real world of manufacturing and retailing process, the demand of the customers may depend on the memory related to the previous experience with the same or similar types of the product. Again, production of items may depend on the memory related to the previous demand pattern. Thus, for both the EOQ and EPQ model, the memory issue is highly expected criteria to define more realistic problem. And in this context the fractional calculus fulfills the shortages of integer order calculus to deal with the memory effect.

4.3. Deduction of Fractional Differential Equation

The memory concerned inventory model can be described by the following differential equation:

$$\frac{dI\left(t\right)}{dt} = \int_0^t k\left(t - z\right)\theta\left(K, D\right)dz \tag{4.5}$$

Where, $k(t–z)$ is the iterated kernel which represents memory in model and is taken to be

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$$k\left(t-z\right)=\frac{\left(t-z\right)^{\alpha-2}}{\Gamma\left(\alpha-1\right)}.$$

Then, from the equation (4.5), we get

$$\frac{dI\left(t\right)}{dt}=\int_0^t\frac{\left(t-z\right)^{\alpha-2}}{\Gamma\left(\alpha-1\right)}\theta\left(K,D\right)dz={}^{RL}I_t^{\alpha-1}\left\{\theta\left(K,D\right)\right\}$$

This gives,

$${}^{C}D_t^{\alpha}I\left(t\right)={}^{RL}I_t^{1-\alpha}\left[\frac{dI\left(t\right)}{dt}\right]=\theta\left(K,D\right)$$

i.e., $${}^{C}D_t^{\alpha}\mathrm{I}\left(t\right)=\theta\left(K,D\right) \tag{4.6}$$

The equation (4.6) is the Caputo fractional differential equation equivalent to the equation (4.5).

4.4. Fractional Order EOQ and EPQ Model

Therefore, the fractional analogue (Caputo approach) of the equation (4.1) is given by

$$\begin{cases}{}^{C}D_t^{\alpha}I\left(t\right)=-D,\ when\ 0\le t\le T\\ \quad I\left(0\right)=Q,\ I\left(T\right)=0\end{cases} \tag{4.7}$$

Using Laplace transformation and inverse Laplace transformation subsequently on the equation (4.7) we get,

$$\mathrm{I}\left(t\right)=Q-\frac{Dt^{\alpha}}{\Gamma\left(\alpha+1\right)} \tag{4.8}$$

Again, $I(T)=0$ provides $Q=\frac{DT^{\alpha}}{\Gamma\left(\alpha+1\right)}$ and thus the equation (4.8) can be rewritten as

$$\mathrm{I}\left(t\right)=\frac{D\left(T^{\alpha}-t^{\alpha}\right)}{\Gamma\left(\alpha+1\right)},when\,0\le t\le T \tag{4.9}$$

Total holding cost,

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$$HC_{\alpha,\beta}\left(T\right)=\frac{C_h}{\Gamma\left(\beta\right)}\int_0^T\left(T-x\right)^{\beta-1}\frac{D\left(T^\alpha-x^\alpha\right)}{\Gamma\left(\alpha+1\right)}dx=\left[\frac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)}-\frac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]C_hDT^{\alpha+\beta} \tag{4.10}$$

Total average profit,

$$TAC_{\alpha,\beta}\left(T\right)=\left[\frac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)}-\frac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]C_hDT^{\alpha+\beta-1}+\frac{C_o}{T} \tag{4.11}$$

Then, the optimization problem equivalent to the fractional EOQ model is given by

$$\begin{cases} Min\left[\dfrac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)}-\dfrac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]C_hDT^{\alpha+\beta-1}+\dfrac{C_o}{T} \\ \text{Subject to}\quad Q=\dfrac{DT^\alpha}{\Gamma\left(\alpha+1\right)} \\ T>0 \end{cases} \tag{4.12}$$

Again, the fractional analogue (Caputo approach) of (4.3) is given by

$$\begin{cases} {}^CD_t^\alpha I\left(t\right)=K-D,\ when\ 0\le t\le T_1 \\ {}^CD_t^\alpha I\left(t\right)=-D,\ when\ T_1\le t\le T \\ I\left(0\right)=0,\ I\left(T\right)=0 \end{cases} \tag{4.13}$$

Also, the equivalent optimization problem is

$$\begin{cases} Min\dfrac{c_h}{\Gamma\left(\beta\right)T}\left(A_1+A_2\right)+\dfrac{c_0}{T} \\ where\ A_1=\displaystyle\int_0^{T_1}\left(T-x\right)^{\beta-1}\left(K-D\right)\frac{x^\alpha}{\Gamma\left(\alpha+1\right)}dx \\ A_{2=}\displaystyle\int_{T_1}^T\left(T-x\right)^{\beta-1}\frac{D}{\Gamma\left(\alpha+1\right)}\left[T^\alpha-x^\alpha\right]dx \\ 0<T_1<T \end{cases} \tag{4.14}$$

Remark 4.1. when, $\alpha,\beta\to1$, in (4.12) and (4.14) the optimization problems for the fractional models reduce to the corresponding cases of integer order EOQ and EPQ model respectively. This observation

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makes us to conclude that the fractional order modelling is theoretical generalization as the fractional calculus (better to say arbitrary order calculus) includes the integer order calculus also.

4.5. Fractional Order EOQ and EPQ Model Under Uncertainty

Therefore, the fractional analogue (Caputo approach) with fuzzy uncertain demand of the equation (4.1) is given by

$$\begin{cases} {}^{C}D_{t}^{\alpha}\tilde{I}\left(t\right) = -\tilde{D},\ when\ 0 \le t \le T \\ \tilde{I}\left(0\right) = \tilde{Q},\ \tilde{I}\left(T\right) = 0 \end{cases} \tag{4.15}$$

In the equation (4.15), $\tilde{\mathrm{I}}\left(t\right), \tilde{D}$, and $\tilde{Q}$ represent the fuzzy stock level, demand rate and lot size respectively. To solve the system given by the equation (4.15), fuzzy extension principle may be used (Chakraborty et al., 2019). Another approach deal with the system may be the fuzzy fractional differential equation approach. In this technique, the fuzzy valued variables and parameters are reorganized in its parametric represent with different level of aspirations as the following:

$$\tilde{I}\left(t\right) = \left[\underline{I}\left(t;r\right), \overline{I}\left(t;r\right)\right], \tilde{D} = \left[\underline{D}\left(r\right), \overline{D}\left(r\right)\right], \tilde{Q} = \left[\underline{Q}\left(r\right), \overline{Q}\left(r\right)\right],$$

where $r \in [0,1]$.

Using fuzzy Laplace transformation of the system (4.15),

$$\mathbb{L}\left[{}^{C}D_{t}^{\alpha}\tilde{I}\left(t\right)\right] = \mathbb{L}\left[-\tilde{D}\right] \tag{4.16}$$

Then, using the generalized Hukuhara fractional derivative of second type (Salahshour & Allahviranloo, 2013; Salahshour et al., 2012) and inverse Laplace transformation subsequently on the equation (4.16) we get,

$$\begin{cases} \underline{I}\left(t;r\right) = \underline{Q}\left(r\right) - \dfrac{\underline{D}\left(r\right)t^{\alpha}}{\Gamma\left(\alpha+1\right)} \\ \overline{I}\left(t;r\right) = \overline{Q}\left(r\right) - \dfrac{\overline{D}\left(r\right)t^{\alpha}}{\Gamma\left(\alpha+1\right)} \end{cases} \tag{4.17}$$

Again, $\tilde{I}\left(T\right) = 0$ provides

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$$\begin{cases} \underline{Q}\left(r\right) = \dfrac{\underline{D}\left(r\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} \\ \bar{Q}\left(r\right) = \dfrac{\bar{D}\left(r\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} \end{cases} \tag{4.18}$$

and thus, the equation (4.18) can be rewritten as

$$\begin{cases} \underline{I}\left(t;r\right) = \dfrac{\underline{D}\left(r\right)\left(T^{\alpha}-t^{\alpha}\right)}{\Gamma\left(\alpha+1\right)} \\ \bar{I}\left(t;r\right) = \dfrac{\bar{D}\left(r\right)\left(T^{\alpha}-t^{\alpha}\right)}{\Gamma\left(\alpha+1\right)} \end{cases} \tag{4.19}$$

Let, the fuzzy holding cost and set up cost are given by

$$\widetilde{C_h} = \left[\underline{C_h}\left(r\right), \overline{C_h}\left(r\right)\right] \text{ and } \widetilde{C_0} = \left[\underline{C_0}\left(r\right), \overline{C_0}\left(r\right)\right]$$

Total holding cost,

$$\widetilde{HC}_{\alpha,\beta}\left(T\right) = \left[\underline{HC}_{\alpha,\beta}\left(T;r\right), \overline{HC}_{\alpha,\beta}\left(T;r\right)\right]$$

where

$$\begin{cases} HCa,\beta(T;r) = \frac{C_h(r)}{\Gamma(\beta)}\int(T-x)^{\beta-1}\dfrac{\underline{D}(r)(T^a-x^a)}{\Gamma(a+1)}\,dx \\ \qquad\qquad = \left[\dfrac{1}{\Gamma(a+1)\Gamma(\beta+1)} - \dfrac{1}{\Gamma(a+\beta+1)}\right]\underline{C_h}(r)\underline{D}(r)T^{a+\beta} \\ HC_{a,\beta}(T;r) = \dfrac{C_h(r)}{\Gamma(\beta)}\int(T-x)^{\beta-1}\dfrac{\overline{D}(r)(T^a-x^a)}{\Gamma(a+1)}\,dx \\ \qquad\qquad = \left[\dfrac{1}{\Gamma(a+1)\Gamma(\beta+1)} - \dfrac{1}{\Gamma(a+\beta+1)}\right]\overline{C_h}(r)\overline{D}(r)T^{a+\beta} \end{cases} \tag{4.20}$$

Total average fuzzy profit,

$$\widetilde{TAC}_{\alpha,\beta}\left(T\right) = \left[\underline{TAC}_{\alpha,\beta}\left(T;r\right), \overline{TAC}_{\alpha,\beta}\left(T;r\right)\right]$$

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where,

$$\begin{cases} \underline{TAC}_{\alpha,\beta}\left(T;r\right) = \left[\dfrac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)} - \dfrac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]\underline{C_h}\left(r\right)\underline{D}\left(r\right)T^{\alpha+\beta-1} + \dfrac{\underline{C_o}\left(r\right)}{T} \\ \overline{TAC}_{\alpha,\beta}\left(T;r\right) = \left[\dfrac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)} - \dfrac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]\overline{C_h}\left(r\right)\bar{D}\left(r\right)T^{\alpha+\beta-1} + \dfrac{\overline{C_o}\left(r\right)}{T} \end{cases} \tag{4.21}$$

Then, the optimization problem equivalent to the fuzzy fractional EOQ model is given by

$$\begin{cases} Min\left\{\left[\dfrac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)} - \dfrac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]\underline{C_h}\left(r\right)\underline{D}\left(r\right)T^{\alpha+\beta-1} + \dfrac{\underline{C_o}\left(r\right)}{T}\right\} \\ Min\left\{\left[\dfrac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)} - \dfrac{1}{\Gamma\left(\alpha+\beta+1\right)}\right]\overline{C_h}\left(r\right)\bar{D}\left(r\right)T^{\alpha+\beta-1} + \dfrac{\overline{C_o}\left(r\right)}{T}\right\} \\ \text{Subject to} \quad \underline{Q}\left(r\right) = \dfrac{\underline{D}\left(r\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} \\ \qquad\qquad\quad \bar{Q}\left(r\right) = \dfrac{\bar{D}\left(r\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} \\ \qquad\qquad\quad T > 0 \end{cases} \tag{4.22}$$

Again, the fuzzy fractional analogue (Caputo approach) of (4.3) with fuzzy demand and production rate is given by

$$\begin{cases} {}^{C}D_t^{\alpha}\tilde{I}\left(t\right) = \tilde{K} - \tilde{D}, \ when \ 0 \le t \le T_1 \\ {}^{C}D_t^{\alpha}\tilde{I}\left(t\right) = -\tilde{D}, \ when \ T_1 \le t \le T \\ \tilde{I}\left(0\right) = 0, \ \tilde{I}\left(T\right) = 0 \end{cases} \tag{4.23}$$

In the similar approach corresponding to the fuzzy fractional EOQ model, the equivalent optimization model can be deduced from the equation (4.23).

Remark 4.2. For r=1, the fuzzy fractional inventory model is reduced to the crisp fractional inventory model. Again, when, $\alpha,\beta\rightarrow 1$, the optimization problems for the fractional models reduce to the corresponding cases of integer order EOQ and EPQ model respectively.Thus, fuzzy fractional model can be viewed as the most generalized one in this context.

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5. FRACTIONAL EOQ MODEL WITH PRICE AND TIME SENSITIVE DEMAND

5.1 Crisp Model

Consider the demand in the system (4.7) be a function of selling price and time. In general, the low selling price can boost up the demand rate. Also, after starting a new retailing business, the demand rate gradually as time forwards through the communication between retailer and consumer. So, we consider demand as $D(t,p)=a–bp+ct$. Then, the system given by the equation (4.7) transforms to the following system

$$\begin{cases} {}^{C}D_{t}^{\alpha} I\left(t\right) = -\left\{a - bp + ct\right\}, \ when \ 0 \le t \le T \\ I\left(0\right) = Q, \ I\left(T\right) = 0 \end{cases} \tag{5.1}$$

Taking the Laplace transformation on the fractional differential equation in the system given by the equation (5.1),

$$s^{\alpha}\mathbf{I}\left(s\right) - s^{\alpha-1}I\left(0\right) = -\frac{\left(a-bp\right)}{s} - \frac{c}{s^{2}},$$

where $\mathbf{I}(s)$ is the Laplace transformation of $I(t)$.

$$\text{i.e., } \mathbf{I}\left(s\right) = \frac{Q}{s} - \frac{\left(a-bp\right)}{s^{\alpha+1}} - \frac{c}{s^{\alpha+2}} \tag{5.2}$$

Taking the inverse Laplace transformation of (5.2),

$$I\left(t\right) = Q - \frac{\left(a-bp\right)t^{\alpha}}{\Gamma\left(\alpha+1\right)} - \frac{ct^{\alpha+1}}{\Gamma\left(\alpha+2\right)} \tag{5.3}$$

Now, using the condition $I(T)=0$, we get

$$Q = \frac{\left(a-bp\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} + \frac{cT^{\alpha+1}}{\Gamma\left(\alpha+2\right)} \tag{5.4}$$

Then, from (5.4) and (5.3),

$$I\left(t\right) = \frac{\left(a-bp\right)\left(T^{\alpha}-t^{\alpha}\right)}{\Gamma\left(\alpha+1\right)} + \frac{c\left(T^{\alpha+1}-t^{\alpha+1}\right)}{\Gamma\left(\alpha+2\right)}, \text{ for } 0 \le t \le T \tag{5.5}$$

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Total holding cost,

$$HC_{\alpha,\beta}\left(T\right)=\frac{C_h}{\Gamma\left(\beta\right)}\int_0^T\left(T-x\right)^{\beta-1}I\left(x\right)dx$$

$$=\frac{C_h}{\Gamma\left(\beta\right)}\int_0^T\left(T-x\right)^{\beta-1}\left\{\frac{\left(a-bp\right)\left(T^\alpha-x^\alpha\right)}{\Gamma\left(\alpha+1\right)}+\frac{c\left(T^{\alpha+1}-x^{\alpha+1}\right)}{\Gamma\left(\alpha+2\right)}\right\}dx$$

$$=\frac{C_h}{\Gamma\left(\beta\right)}\left[\frac{\left(a-bp\right)T^\alpha}{\Gamma\left(\alpha+1\right)}\int_0^T\left(T-x\right)^{\beta-1}dx-\frac{\left(a-bp\right)}{\Gamma\left(\alpha+1\right)}\int_0^T\left(T-x\right)^{\beta-1}x^\alpha dx\right.$$
$$\left.+\frac{cT^{\alpha+1}}{\Gamma\left(\alpha+2\right)}\int_0^T\left(T-x\right)^{\beta-1}dx-\frac{c}{\Gamma\left(\alpha+2\right)}\int_0^T\left(T-x\right)^{\beta-1}x^{\alpha+1}dx\right]$$

$$=\frac{C_h}{\Gamma\left(\beta\right)}\left[\frac{\left(a-bp\right)T^{\alpha+\beta}}{\beta\Gamma\left(\alpha+1\right)}-\frac{\left(a-bp\right)T^{\alpha+\beta}}{\Gamma\left(\alpha+1\right)}\int_0^1\left(1-x\right)^{\beta-1}x^\alpha dx+\frac{cT^{\alpha+\beta+1}}{\beta\Gamma\left(\alpha+2\right)}-\frac{cT^{\alpha+\beta+1}}{\Gamma\left(\alpha+2\right)}\int_0^1\left(1-x\right)^{\beta-1}x^{\alpha+1}dx\right]$$

$$=\frac{C_h}{\Gamma\left(\beta\right)}\left[\frac{\left(a-bp\right)T^{\alpha+\beta}}{\beta\Gamma\left(\alpha+1\right)}-\frac{\left(a-bp\right)T^{\alpha+\beta}B\left(\alpha+1,\beta\right)}{\Gamma\left(\alpha+1\right)}+\frac{cT^{\alpha+\beta+1}}{\beta\Gamma\left(\alpha+2\right)}-\frac{cT^{\alpha+\beta+1}B\left(\alpha+2,\beta\right)}{\Gamma\left(\alpha+2\right)}\right]$$

$$=C_h\left[\left(a-bp\right)T^{\alpha+\beta}\left\{\frac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)}-\frac{1}{\Gamma\left(\alpha+\beta+1\right)}\right\}+cT^{\alpha+\beta+1}\left\{\frac{1}{\Gamma\left(\alpha+2\right)\Gamma\left(\beta+1\right)}-\frac{1}{\Gamma\left(\alpha+\beta+2\right)}\right\}\right] \tag{5.6}$$

Total sales revenue

$$SR_{\alpha,\beta}=\frac{p}{\Gamma\left(\beta\right)}\int_0^T\left(T-x\right)^{\beta-1}\left(a-bp+cx\right)dx$$

$$=\frac{p}{\Gamma\left(\beta\right)}\left\{\left(a-bp\right)\int_0^T\left(T-x\right)^{\beta-1}dx+c\int_0^T\left(T-x\right)^{\beta-1}xdx\right\}$$

$$=\frac{p}{\Gamma\left(\beta\right)}\left\{\frac{\left(a-bp\right)T^\beta}{\beta}+cT^{\beta+1}\int_0^1\left(1-x\right)^{\beta-1}xdx\right\}$$

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$$= \frac{p}{\Gamma(\beta)}\left\{\frac{(a-bp)T^{\beta}}{\beta} + cT^{\beta+1}B(2,\beta)\right\}$$

$$= p\left\{\frac{(a-bp)T^{\beta}}{\Gamma(\beta+1)} + \frac{cT^{\beta+1}}{\Gamma(\beta+2)}\right\} \tag{5.7}$$

Total profit

$$TP_{\alpha,\beta} = SR_{\alpha,\beta} - \left(HC_{\alpha,\beta}(T) + c_0\right) \tag{5.8}$$

And then the total average profit

$$TAP_{\alpha,\beta} = \frac{SR_{\alpha,\beta} - \left(HC_{\alpha,\beta}(T) + c_0\right)}{T}$$

$$= p\left\{\frac{(a-bp)T^{\beta-1}}{\Gamma(\beta+1)} + \frac{cT^{\beta}}{\Gamma(\beta+2)}\right\} - C_h\left[(a-bp)T^{\alpha+\beta-1}\left\{\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\right\}\right. \\ \left. -cT^{\alpha+\beta}\left\{\frac{1}{\Gamma(\alpha+2)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+2)}\right\}\right] - \frac{C_o}{T} \tag{5.9}$$

$$\left\{\begin{array}{l} \max\left\{p\left\{\begin{array}{l}\frac{(a-bp)T^{\beta-1}}{\Gamma(\beta+1)} \\ +\frac{cT^{\beta}}{\Gamma(\beta+2)}\end{array}\right\} - C_h\left[(a-bp)T^{\alpha+\beta-1}\left\{\begin{array}{l}\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} \\ -\frac{1}{\Gamma(\alpha+\beta+1)}\end{array}\right\} + cT^{\alpha+\beta}\left\{\begin{array}{l}\frac{1}{\Gamma(\alpha+2)\Gamma(\beta+1)} \\ -\frac{1}{\Gamma(\alpha+\beta+2)}\end{array}\right\}\right] - \frac{C_o}{T}\right\} \\ \text{Subject to } T > 0 \\ Q = \frac{(a-bp)T^{\alpha}}{\Gamma(\alpha+1)} + \frac{cT^{\alpha+1}}{\Gamma(\alpha+2)} \end{array}\right. \tag{5.10}$$

5.2 Fuzzy Version of the Optimization Model

In the fuzzy version of the model, we consider only the selling price to be triangular fuzzy number. Other variables are taken to be crisp.

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5.2.1 Using Extension Principle

Using the Extension Principle on the objective function, we get the following form of the fuzzy objective function

Considering the *r*-level of the system (5.11)

So, the profit maximization model can be written as

$$\begin{cases} \max\left\{\underline{p}(r)\left\{\dfrac{(a-b\overline{p}(r))T^{\beta-1}}{\Gamma(\beta+1)}+\dfrac{cT^{\beta}}{\Gamma(\beta+2)}\right\}-C_{h}\left[(a-b\underline{p}(r))T^{a+\beta-1}\left\{\dfrac{1}{\Gamma(a+1)\Gamma(\beta+1)}-\dfrac{1}{\Gamma(a+\beta+1)}\right\}-cT^{a+\beta}\left\{\dfrac{1}{\Gamma(a+2)\Gamma(\beta+1)}-\dfrac{1}{\Gamma(a+\beta+2)}\right\}\right]-\dfrac{C_{0}}{T}\right\} \\ \max\left\{p(r)\left\{\dfrac{(a-b\underline{p}(r))T^{\beta-1}}{\Gamma(\beta+1)}+\dfrac{cT^{\beta}}{\Gamma(\beta+2)}\right\}-C_{h}\left[(a-b\overline{p}(r))T^{a+\beta-1}\left\{\dfrac{1}{\Gamma(a+1)\Gamma(\beta+1)}-\dfrac{1}{\Gamma(a+\beta+1)}\right\}-cT^{a+\beta}\left\{\dfrac{1}{\Gamma(a+2)\Gamma(\beta+1)}-\dfrac{1}{\Gamma(a+\beta+2)}\right\}\right]-\dfrac{C_{0}}{T}\right\} \\ \text{Subject to } T>0 \\ \underline{Q}(r)=\dfrac{(a-b\overline{p}(r))T^{a}}{\Gamma(a+1)}+\dfrac{cT^{a+1}}{\Gamma(a+2)} \qquad \overline{Q}(r)=\dfrac{(a-b\underline{p}(r))T^{a}}{\Gamma(a+1)}+\dfrac{cT^{a+1}}{\Gamma(a+2)} \end{cases} \tag{5.12}$$

5.2.2 Fuzzy Fractional Differential Equation Approach

The fuzzy fractional differential equation to describe the fuzzy uncertain demand is given by

$$\begin{cases} {}^{C}D_{t}^{\alpha}\tilde{I}\left(t\right)=-\left(a-b\tilde{p}+ct\right), \ when \ 0\le t\le T \\ \tilde{I}\left(0\right)=\tilde{Q}, \ \tilde{I}\left(T\right)=0 \end{cases} \tag{5.13}$$

Taking the fuzzy Laplace transformation on the fractional differential equation in the system given by the equation (5.13),

$$\left\{-s^{\alpha-1}\tilde{I}\left(0\right)\right\}\ominus_{h}\left\{-s^{\alpha}\mathbb{L}\left(\tilde{I}\left(t\right)\right)\right\}=-\mathbb{L}\left(a-b\tilde{p}+ct\right) \tag{5.14}$$

Then, using the gH differentiability of second type from the equation we get,

$$\begin{cases} s^{\alpha}\underline{\mathbf{I}}\left(s;r\right)-s^{\alpha-1}\underline{I}\left(0;r\right)=-\dfrac{\left(a-b\overline{p}\left(r\right)\right)}{s}-\dfrac{c}{s^{2}} \\ s^{\alpha}\overline{\mathbf{I}}\left(s;r\right)-s^{\alpha-1}\overline{I}\left(0;r\right)=-\dfrac{\left(a-b\underline{p}\left(r\right)\right)}{s}-\dfrac{c}{s^{2}} \end{cases} \tag{5.15}$$

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Taking the inverse Laplace transformations of (5.15),

$$\begin{cases} \underline{I}\left(t;r\right) = \underline{Q}\left(r\right) - \dfrac{\left(a - b\overline{p}\left(r\right)\right)t^{\alpha}}{\Gamma\left(\alpha+1\right)} - \dfrac{ct^{\alpha+1}}{\Gamma\left(\alpha+2\right)} \\ \overline{I}\left(t;r\right) = \overline{Q}\left(r\right) - \dfrac{\left(a - b\underline{p}\left(r\right)\right)t^{\alpha}}{\Gamma\left(\alpha+1\right)} - \dfrac{ct^{\alpha+1}}{\Gamma\left(\alpha+2\right)} \end{cases} \tag{5.16}$$

Now, using the condition $\tilde{I}\left(T\right) = 0$, we get

$$\begin{cases} \underline{Q}\left(r\right) = \dfrac{\left(a - b\overline{p}\left(r\right)\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} + \dfrac{cT^{\alpha+1}}{\Gamma\left(\alpha+2\right)} \\ \overline{Q}\left(r\right) = \dfrac{\left(a - b\underline{p}\left(r\right)\right)T^{\alpha}}{\Gamma\left(\alpha+1\right)} + \dfrac{cT^{\alpha+1}}{\Gamma\left(\alpha+2\right)} \end{cases} \tag{5.17}$$

Then, from (5.16) and (5.17),

$$\begin{cases} \underline{I}\left(t;r\right) = \dfrac{\left(a - b\overline{p}\left(r\right)\right)\left(T^{\alpha} - t^{\alpha}\right)}{\Gamma\left(\alpha+1\right)} + \dfrac{c\left(T^{\alpha+1} - t^{\alpha+1}\right)}{\Gamma\left(\alpha+2\right)} \\ \overline{I}\left(t;r\right) = \dfrac{\left(a - b\underline{p}\left(r\right)\right)\left(T^{\alpha} - t^{\alpha}\right)}{\Gamma\left(\alpha+1\right)} + \dfrac{c\left(T^{\alpha+1} - t^{\alpha+1}\right)}{\Gamma\left(\alpha+2\right)} \end{cases} \tag{5.18}$$

Total fuzzy holding cost,

$$\widetilde{HC}_{\alpha,\beta}\left(T\right) = \left[\underline{HC_{\alpha,\beta}\left(T;r\right)}, \overline{HC_{\alpha,\beta}\left(T;r\right)}\right]$$

where,

$$\underline{HC_{\alpha,\beta}\left(T;r\right)} = C_{h}\left[\left(a - b\overline{p}\left(r\right)\right)T^{\alpha+\beta}\left\{\begin{matrix} \dfrac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)} \\ -\dfrac{1}{\Gamma\left(\alpha+\beta+1\right)} \end{matrix}\right\} + cT^{\alpha+\beta+1}\left\{\begin{matrix} \dfrac{1}{\Gamma\left(\alpha+2\right)\Gamma\left(\beta+1\right)} \\ -\dfrac{1}{\Gamma\left(\alpha+\beta+2\right)} \end{matrix}\right\}\right] \tag{5.18}$$

And

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$$\overline{HC_{\alpha,\beta}\left(T;r\right)} = C_h\left[\left(a-b\underline{p}\left(r\right)\right)T^{\alpha+\beta}\left\{\begin{matrix}\frac{1}{\Gamma\left(\alpha+1\right)\Gamma\left(\beta+1\right)}\\ -\frac{1}{\Gamma\left(\alpha+\beta+1\right)}\end{matrix}\right\}+cT^{\alpha+\beta+1}\left\{\begin{matrix}\frac{1}{\Gamma\left(\alpha+2\right)\Gamma\left(\beta+1\right)}\\ -\frac{1}{\Gamma\left(\alpha+\beta+2\right)}\end{matrix}\right\}\right] \tag{5.19}$$

Total sales revenue

$$\widetilde{SR}_{\alpha,\beta} = \left[\underline{SR_{\alpha,\beta}\left(T;r\right)}, \overline{SR_{\alpha,\beta}\left(T;r\right)}\right],$$

where

$$\underline{SR_{\alpha,\beta}\left(T;r\right)} = \underline{p}\left(r\right)\left\{\frac{\left(a-b\overline{p}\left(r\right)\right)T^{\beta-1}}{\Gamma\left(\beta+1\right)}+\frac{cT^{\beta}}{\Gamma\left(\beta+2\right)}\right\} \tag{5.20}$$

And

$$\overline{SR_{\alpha,\beta}\left(T;r\right)} = \overline{p}\left(r\right)\left\{\frac{\left(a-b\underline{p}\left(r\right)\right)T^{\beta-1}}{\Gamma\left(\beta+1\right)}+\frac{cT^{\beta}}{\Gamma\left(\beta+2\right)}\right\} \tag{5.21}$$

Therefore, the profit maximization model can be written as

$$\begin{cases}\max\left\{p(r)\left\{\begin{matrix}\frac{(a-bp(r))T^{\beta-1}}{\Gamma(\beta+1)}\\ +\frac{cT^{\beta}}{\Gamma(\beta+2)}\end{matrix}\right\}-C_h\left[(a-bp(r))T^{a+\beta-1}\left\{\begin{matrix}\frac{1}{\Gamma(a+1)\Gamma(\beta+1)}\\ -\frac{1}{\Gamma(a+\beta+1)}\end{matrix}\right\}-cT^{a+\beta}\left\{\begin{matrix}\frac{1}{\Gamma(a+2)\Gamma(\beta+1)}\\ -\frac{1}{\Gamma(a+\beta+2)}\end{matrix}\right\}\right]-\frac{C_0}{T}\right\}\\ \max\left\{p(r)\left\{\begin{matrix}\frac{(a-bp(r))T^{\beta-1}}{\Gamma(\beta+1)}\\ +\frac{cT^{\beta}}{\Gamma(\beta+2)}\end{matrix}\right\}-C_h\left[(a-bp(r))T^{a+\beta-1}\left\{\begin{matrix}\frac{1}{\Gamma(a+1)\Gamma(\beta+1)}\\ -\frac{1}{\Gamma(a+\beta+1)}\end{matrix}\right\}-cT^{a+\beta}\left\{\begin{matrix}\frac{1}{\Gamma(a+2)\Gamma(\beta+1)}\\ -\frac{1}{\Gamma(a+\beta+2)}\end{matrix}\right\}\right]-\frac{C_0}{T}\right\}\\ \text{Subject to } T>0\\ Q(r)=\frac{(a-bp(r))T^{a}}{\Gamma(a+1)}+\frac{cT^{a+1}}{\Gamma(a+2)} \qquad Q(r)=\frac{(a-bp(r))T^{a}}{\Gamma(a+1)}+\frac{cT^{a+1}}{\Gamma(a+2)}\end{cases} \tag{5.22}$$

Remark 5.1. From the discussion, it is perceived that the optimization problems corresponding two different approaches coincides for the fuzzification of the proposed model in this section.

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5.3 Numerical Results

For numerical simulation of the fractional crisp model, we take the following values of the involved constants and parameters:

a=30; b=0.6; c=0.5; p=20; C_h=2.5; C_0=500

Then, using LINGO18 software we have done the numerical optimization of the concerned problem. At first, the integral memory index is fixed to be 1 and varying the differential memory index from 1 to 0.1 (smaller values of memory indices represent stronger senses of memory), the memory sensitivity of the optimal solution has been explored in the table 1. In the table 2, the sensitivity of the optimal solution with respect to the integral memory index is observed letting the differential memory index to be fixed at the value 1. The table 3 is obtained by varying both the memory indices.

Table 1. Differential memory sensitivity of optimal solution

Differential memory index (α)	T	**Q**	$TAP_{\alpha,\beta}$
1	6.414	68.006	186.94
0.9	24.000	286.739	259.93
0.8	24.000	227.452	332.44
0.7	24.000	179.650	383.65
0.6	24.000	141.263	418.96
0.5	24.000	110.558	442.58
0.4	24.000	86.104	457.73
0.3	24.000	66.715	466.85
0.2	24.000	51.412	471.79
0.1	24.000	39.392	473.89

Table 2. Integral memory sensitivity of optimal solution

Integral memory index (β)	T	**Q**	$TAP_{\alpha,\beta}$
1	6.414	68.006	186.94
0.9	6.022	63.258	158.32
0.8	5.695	59.361	132.05
0.7	5.460	56.591	107.80
0.6	5.354	55.357	85.33
0.5	5.456	56.543	64.50
0.4	24.000	360.000	40.87
0.3	24.000	360.000	29.11
0.2	24.000	360.000	19.21
0.1	24.000	360.000	10.96

From the representation of three tables, it is perceived that the most preferable situation in the profit maximization objective is obtained at α=0.1 and β=1. The data from the table 1, table 2 and table 3 are plotted in the figure 1, figure 2 and figure 3 respectively.

We consider the fuzzy versions for the memory less EOQ model (for α=1, β=1) and the memory effected model which provides best profit in crisp environment (for α=0.1, β=1) for numerical solution. Here, the selling price is taken as the triangular fuzzy number $\tilde{p} = \langle 15, 20, 25 \rangle$. Then, the table 4 describes the results for two selected fuzzy models.

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Table 3. Memory (both differential and integral) sensitivity of optimal solution

Differential memory index (α)	**Integral memory index (β)**	T	**Q**	$TAP_{\alpha,\beta}$
1	1	6.414	68.006	186.94
0.9	0.9	24.000	286.739	205.70
0.8	0.8	24.000	227.452	197.38
0.7	0.7	24.000	179.650	166.92
0.6	0.6	24.000	141.263	130.93
0.5	0.5	24.000	110.558	97.11
0.4	0.4	10.018	46.561	69.01
0.3	0.3	10.964	41.282	46.25
0.2	0.2	24.000	51.412	27.57
0.1	0.1	24.000	39.392	14.16

Table 4. Numerical results for the fuzzy models

Memory indices	r	T	$\underline{Q(r)}$	$\overline{Q(r)}$	$\underline{TAP_{\alpha,\beta}(T;r)}$	$\overline{TAC_{\alpha,\beta}(T;r)}$	Q_{def}	$TAP_{\alpha,\beta def}$
α=1	0.3	5.522	61.461	63.117	127.95	163.47		
	0.2	5.427	56.204	62.716	91.46	233.17	59.50	162.74
β=1	0.1	5.337	51.154	62.362	56.52	303.88		
α=0.1	0.3	24.00	41.362	42.149	413.95	450.15		
	0.2	24.00	39.392	42.543	377.10	521.89	40.97	449.99
β=1	0.1	24.00	37.422	42.937	341.75	595.13		

Figure 1. Total average profit vs Differential memory index

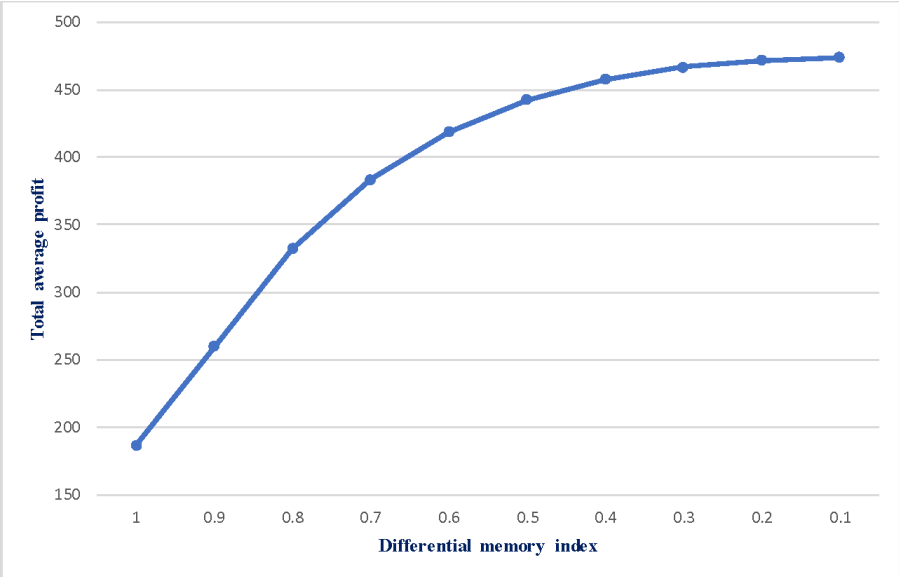

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Figure 2. Total average profit vs. integral memory index

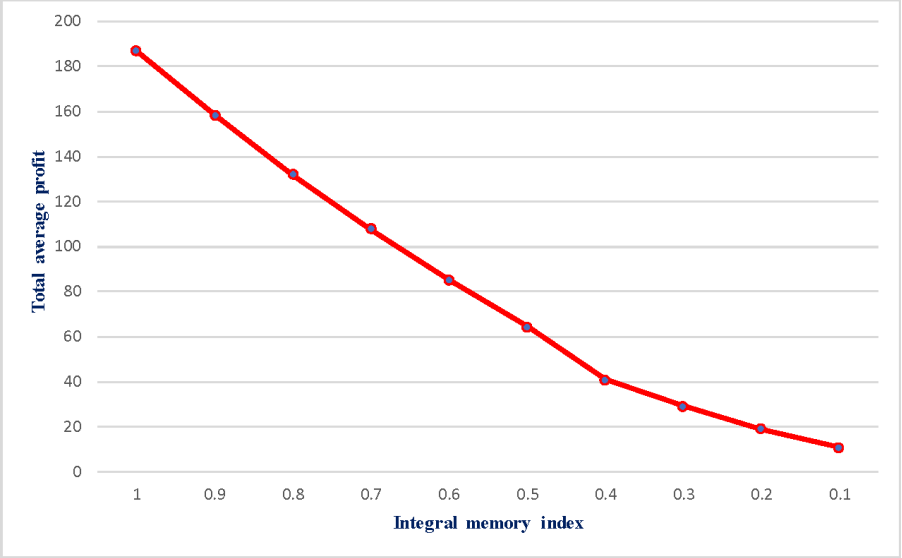
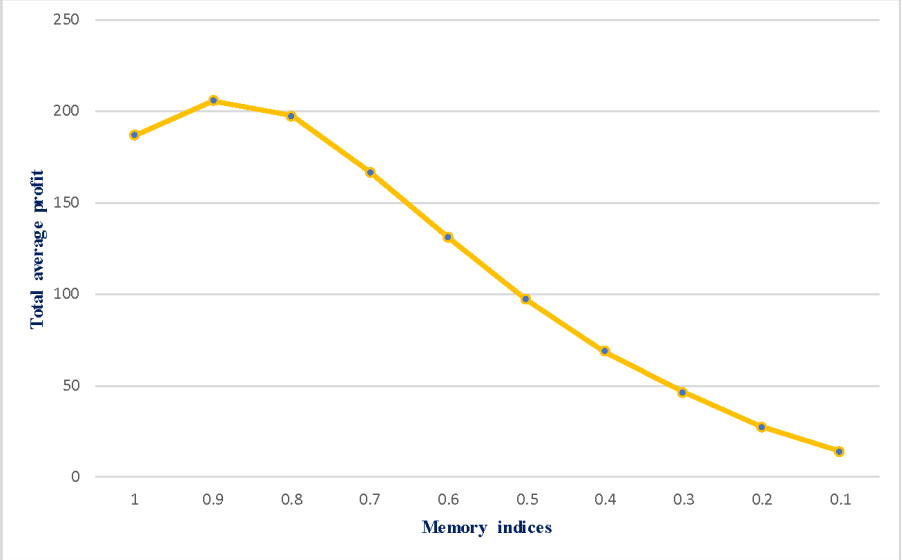


Figure 3. Total average profit vs. memory indices



In the numerical optimization, it is noticed that there is no feasible solution for the aspiration level greater than 0.3. So, we take the results corresponding to the aspiration levels 0.1 to 0.3. To compare with the corresponding crisp results, we use the following defuzzification formula:

$$Q_{def} = \frac{\left(\underline{Q(0.1)} + \underline{Q(0.2)} + \underline{Q(0.3)}\right) + \left(\overline{Q(0.1)} + \overline{Q(0.2)} + \overline{Q(0.3)}\right)}{6}$$

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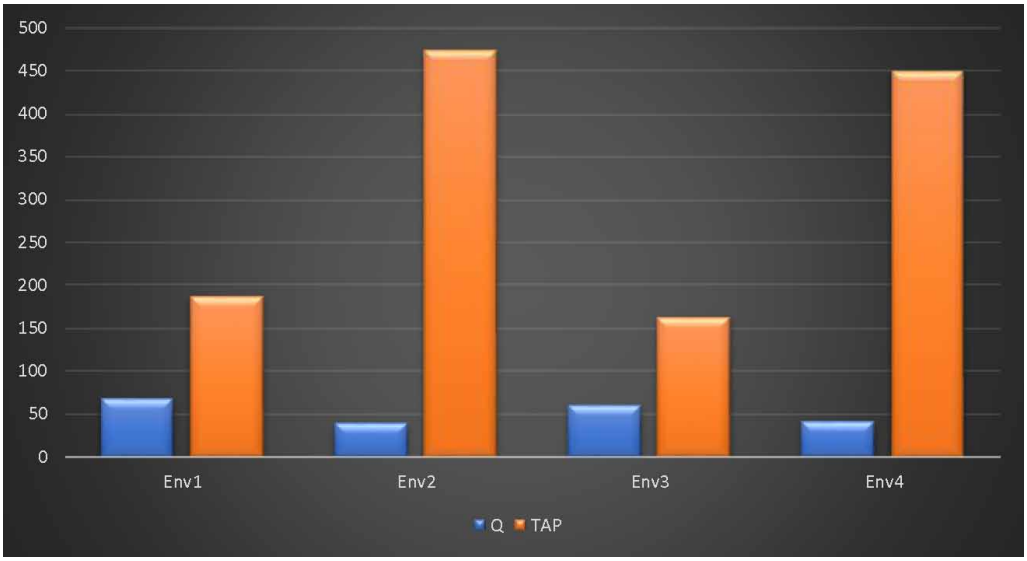
and

$$TAP_{\alpha,\beta def} = \frac{\left(\underline{TAP_{\alpha,\beta}\left(T;0.1\right)} + \underline{TAP_{\alpha,\beta}\left(T;0.2\right)} + \underline{TAP_{\alpha,\beta}\left(T;0.3\right)}\right) + \left(\overline{TAC_{\alpha,\beta}\left(T;0.1\right)} + \overline{TAC_{\alpha,\beta}\left(T;0.2\right)} + \overline{TAC_{\alpha,\beta}\left(T;0.3\right)}\right)}{6}$$

The comparison of the optimal lot sizes and the total average profits for the two selected fuzzy models along with their crisp counterparts is displayed in the figure 4.

Figure 4. Total average profits and lot-sizes in different environments



In the figure 4, Env 1, Env2, Env3 and Env 4 respectively represents the phenomenon namely, crisp-memoryless, crisp-higher memory, fuzzy-memoryless and fuzzy-higher memory. A very practical intuition is reflected in the figure 4 that is, the system memory has a positive impact on the profit maximization objective whereas the uncertainty may causes hindrance in the retailer's goal with lower profit than that of the crisp situation.

6. COMPARISON BETWEEN RIEMANN-LIOUVILLE AND CAPUTO DERIVATIVE

The previous section presents the Caputo fractional derivative inspired model as an equivalent of the integer order model with memory sensitivity. Now, the question arises whether the same type of demonstration is possible through Riemann-Liouville fractional derivative. The Laplace transformation of Riemann-Liouville fractional derivative shows the involvement of fractional initial state to describe a model. In reality the physical meaning of fractional initial state is not clear. This is one of the major

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drawbacks of the Riemann-Liouville definition to deal with the inventory control theory where the initial conditions are generally given in terms of integer order. On the other hand, the Caputo fractional derivative carrying initial state of integer order like integer order derivative in the presentation of Laplace transformation. This makes the Caputo definition smarter to handle the inventory problems in comparison to the Riemann-Liouville definition.

Actually, the relation between two definitions of fractional derivative is given by

$$ {}^{RL}D_t^{\alpha} I\left(t\right) = {}^{C}D_t^{\alpha} I\left(t\right) + \frac{I\left(0\right)t^{-\alpha}}{\Gamma\left(\alpha-1\right)} \tag{6.1} $$

Now, if the zero initial condition is considered, then $I(0)=0$ and from the equation (6.1) the two definitions become equivalent.

Also, then,

$$ L\left[{}^{RL}D_t^{\alpha} I\left(t\right);s\right] = L[{}^{C}D_t^{\alpha} I\left(t\right);s] $$

i.e., $s^{\alpha} L\left[I\left(t\right)\right] - {}^{RL}D_t^{\alpha-1} I\left(0\right) = s^{\alpha} L\left[I\left(t\right)\right] - s^{\alpha-1} I\left(0\right)$

i.e., $ {}^{RL}D_t^{\alpha-1} I\left(0\right) = 0$

Thus, when the zero initial condition of integer order is given, then fractional initial condition also becomes zero. This result creates an opportunity to deal the inventory model with zero inventory level initially through Riemann-Liouville fractional derivative. Thus, the following conclusions can be made:

(a) The EOQ model where the inventory cycle starts with some nonzero quantity of inventory level cannot be described by the Riemann-Liouville fractional derivative.
(b) The production phase of an EPQ model starts with the zero level of inventory initially. So, the cost minimization or profit maximization problems in the production phase only can be described by the Riemann-Liouville fractional derivative (see Rahaman et al. (Rahaman, Mondal, & Alam, 2020)).

The productive phases of memory affected inventory model can be described by

$$ {}^{RL}D_t^{\alpha} \mathrm{I}\left(t\right) = K - D, when\, 0 \le t \le T_1 $$

This gives

$$ I\left(t\right) = \frac{\left(K-D\right)t^{\alpha}}{\Gamma\left(\alpha+1\right)},\ when\ \ 0 \le t \le T_1 $$

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The Holding cost during production phase is

$$HC_{\alpha,\beta}\left(T\right)=\frac{C_h}{\Gamma\left(\beta\right)}\int_0^{T_1}\left(T_1-x\right)^{\beta-1}\frac{\left(K-D\right)x^{\alpha}}{\Gamma\left(\alpha+1\right)}dx$$

$$=\frac{C_h\left(K-D\right)}{\Gamma\left(\beta\right)}\frac{T_1^{\alpha+\beta}\mathrm{B}\left(\alpha+1,\beta\right)}{\Gamma\left(\alpha+1\right)}$$

$$=\frac{C_h\left(K-D\right)T_1^{\alpha+\beta}}{\Gamma\left(\alpha+\beta+1\right)}$$

So, the cost minimization problem of production phase of a memory concerned EPQ model is

$$\begin{cases} Min\dfrac{C_h\left(K-D\right)T_1^{\alpha+\beta-1}}{\Gamma\left(\alpha+\beta+1\right)}+\dfrac{C_o}{T_1} \\ \text{Subject to } T_1>0 \end{cases}$$

(c) However, the non-productive phase of EPQ model also faced difficulty to be analysed by Riemann-Liouville fractional calculus.
(d) The discussion in this section establishes the superiority of Caputo fractional derivative over the Riemann-Liouville fractional derivative.

7. OBSTACLES AND CHALLENGES

The Caputo fractional derivative is smart enough to describe the simple EOQ and EPQ model where demand and production rate are considered to be constant, linear or quadratic function of time. However, in more realistic problems involving the assumptions like deterioration or stock dependent demand etc. makes a lot of complexity. For example, if the deterioration is considered in the memory effected EOQ model then (4.7) takes the form

$$\begin{cases} {}^{C}D_t^{\alpha}I\left(t\right)+\varepsilon I\left(t\right)=-D,\ when\ 0\le t\le T \\ I\left(0\right)=Q,\ I\left(T\right)=0 \end{cases} \tag{7.1}$$

Here, ε is the rate of deterioration.
Taking the Laplace transform and inverse Laplace transformation of the equation (7.1) we get,

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$$I\left(t\right)=QE_{\alpha}\left(-\varepsilon t^{\alpha}\right)-\frac{D}{\varepsilon}\left\{1-E_{\alpha}\left(-\varepsilon t^{\alpha}\right)\right\},\text{ when }0\leq t\leq T \tag{7.2}$$

Also, $I(T)=0$ gives

$$Q=\frac{D}{\varepsilon}\left\{\left(E_{\alpha}\left(-\varepsilon T^{\alpha}\right)\right)^{-1}-1\right\} \tag{7.3}$$

And then equation (7.2) takes the form

$$I\left(t\right)=\frac{D}{\varepsilon}\left\{E_{\alpha}\left(\varepsilon T^{\alpha}\right)E_{\alpha}\left(-\varepsilon t^{\alpha}\right)-1\right\} \tag{7.4}$$

The holding cost,

$$HC_{\alpha,\beta}\left(T\right)=\frac{c_{h}}{\Gamma\left(\beta\right)}\left[\int_{0}^{T}\left(T-x\right)^{\beta-1}\frac{D}{\varepsilon}\left\{E_{\alpha}\left(\varepsilon T^{\alpha}\right)E_{\alpha}\left(-\varepsilon x^{\alpha}\right)-1\right\}dx\right] \tag{7.5}$$

Total average cost

$$TAC_{\alpha,\beta}\left(T\right)=\frac{HC_{\alpha,\beta}\left(T\right)+c_{o}}{T} \tag{7.6}$$

So, the equivalent of optimization problem is given by

$$\begin{cases} Min\dfrac{HC_{\alpha,\beta}\left(T\right)+c_{o}}{T} \\ HC_{\alpha,\beta}\left(T\right)=\dfrac{c_{h}}{\Gamma\left(\beta\right)}\left[\displaystyle\int_{0}^{T}\left(T-x\right)^{\beta-1}\frac{D}{\varepsilon}\left\{E_{\alpha}\left(\varepsilon T^{\alpha}\right)E_{\alpha}\left(-\varepsilon x^{\alpha}\right)-1\right\}dx\right] \\ T>0 \end{cases} \tag{7.7}$$

The involvement of the Mittag Leffler function in the integrand makes difficulty for the theoretical optimization of these types of problems. The approximation techniques of Mittag-Leffler function (see Rahaman et al. (Rahaman, Mondal, & Shaikh, 2020)) which is given in terms of infinite series may be one type partial solution toward the problem. Also, soft computing techniques for numerical optimization can be a useful tool (see Rahaman et al. (Rahaman, Mondal, Shaikh et al, 2020)). However, developing a theoretical optimization technique is great challenges to these types of problems.

Also, the fuzzy consideration for the nonlinear type of demand and production rates faces hindrances while solving in the differential equation approach. If the situation is assumed to be memory motivated,

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then handling such situation in terms of the fuzzy fractional differential equation will be more complicated to compute.

8. CONCLUSION

This present article has been carried out a brief investigation on the fruitfulness of application of fractional calculus on the inventory problems. Major findings of this study are the followings:

(i) The application of fractional calculus on the inventory control problem has much justification.
(ii) However, the literature is not so enriched because of the complexity of analysis in fractional order systems.
(iii) This particular research domain is a matter of concern with many challenges but having a lot of prosperity.

In future, this particular research domain can be enriched with the innovation of the optimization techniques and analysis for fractional calculus.

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Chapter 7
A Hybrid Multiple-Attribute Decision-Making Model Under Complex Q-Rung Orthopair Fuzzy Hamy Mean Aggregation Operators

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ABSTRACT

The concept of complex q-rung orthopair fuzzy sets (Cq-ROFSs) serves as an extension of the current complex orthopair fuzzy sets, including complex intuitionistic fuzzy sets and complex Pythagorean fuzzy sets. Hamy mean (HM) aggregation operator is a useful tool to model the interrelationship between attributes. In this contribution, the traditional HM is generalized to aggregate complex q-rung orthopair fuzzy information, and a multi-attribute decision-making (MADM) approach is proposed to solve decision-making problems with complex q-rung orthopair fuzzy information. Firstly, the complex q-rung orthopair fuzzy Hamy mean (Cq-ROFHM) operator and the complex q-rung orthopair fuzzy dual Hamy mean (Cq-ROFDHM) operator along with its weighted forms are proposed. Moreover, to deal with MADM problems, a novel approach is developed based on the proposed HM operators under complex q-rung orthopair fuzzy framework. Lastly, a numerical model is described for the selection of enterprise resource planning (ERP) systems to demonstrate the use of the generated technique and exhibit its adequacy.

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INTRODUCTION

One of the most common and influential research fields in decision science is MADM. It indicates that there is a set of alternatives that decision makers (DMs) need to assess with multiple attributes. The objective of MADM is to select from a finite number of alternatives the optimal one. It is the vital branch of the decision-making theory which has been extensively used in human activities. To describe the uncertainties, Zadeh (1965) put forward the concept of fuzzy sets (FSs) which has the membership degree (MD) only. Later on, to describe the fuzzy information, Atanassov (1986) introduced intuitionistic fuzzy sets (IFSs) which have the MD as well as non-membership degree (NMD), satisfies the condition that sum of MD and NMD would be less than or equal to 1. In recent decades, Pythagorean fuzzy sets (PFSs) (Yager, 2013) have proved to be very useful for describing uncertainty and complexity of the MADM problems. The PFS is expressed by the functions of membership and non-membership, which fulfils the condition that sum of squares of them is not greater than 1. The PFS is more useful to solve the MADM problems than IFSs, i.e., if MD=0.7 and NMD=0.6 then $0.7^2+0.6^2=0.85<1$, it is Pythagorean fuzzy number but not an intuitionistic fuzzy number. After that Fermatean fuzzy sets (FFSs) were proposed by Senapati and Yager (2020). But FFSs have some drawbacks. To overcome these drawbacks, Yager (2016) set forward the concept of q-rung orthopair fuzzy sets (q-ROFSs) to express complex fuzzy information and have widely been applied to the MADM problems. In q-ROFSs, each element satisfies the condition that the sum of the qth power of membership function and non-membership function is not greater to 1. The q-ROFSs provide more flexibility, stronger model ability and larger freedom for decision makers in expressing their preferences on alternatives. To choose the optimal alternative in generalized fuzzy environment, several decision-making approaches (Akram et al., 2021a,b,c,d; Akram et al., 2020; Akram et al., 2019; Akram and Naz, 2019; Akram et al., 2018; Krishankumar et al., 2017,2018,2019; Naz et al., 2021) have been introduced by researchers. Naz et al. (2018) proposed the concepts of Pythagorean fuzzy relations along with its application in MADM. To deal with the MADM problems, Naz and Akram (2019) developed a new decision-making approach based on graph theory in which the decision information is expressed by hesitant fuzzy elements.

If DMs change the codomain of FSs from [0,1] to unit disc, then to tackle certain type of problems, Ramot et al. (2002) design the idea of complex fuzzy sets (CFSs) expressed by complex valued mapping with codomain as unit circle in the complex plane. Moreover, to represent the complex valued NMD, Alkouri and Salleh (2012) generalized the idea of CFS to complex intuitionistic fuzzy sets (CIFSs) and proposed the concept of CIF relations and a distance measure in CIF circumstances. Further, Rani and Garg (2020) set forward the CIF robust averaging-geometric aggregation operators (AOs), explored certain series of distance measures between the two CIFSs (Rani and Garg, 2017), Archimedean t-norm and t-conorm based generalized CIF Bonferroni mean AOs (Garg and Rani, 2020), exponential, logarithmic, and compensative generalized AOs with CIF information (Garg and Rani, 2019), CIF power AOs (Garg and Rani, 2018) and presented their applications in the field of decision-making. The idea of q-ROFS deals with only one aspect at a time that sometimes causes data loss. In real life, however, we experience complex natural phenomena wherein measuring the second dimension of the expression of the MD and NMD becomes essential. Complete facts are projected into a collection by creating the second dimension, which prevents any loss of information. The amplitude term represents the degree to which an object belongs in a Cq-ROFS and the phase terms are usually related to periodicity. The CFS and traditional q-ROFS theories are differentiated by certain phase terms. Liu et al. (2020) put forward a novel, powerful and efficient tool to describe uncertain phenomena called Cq-ROFSs and developed

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the Cq-ROF weighted averaging operator and Cq-ROF weighted geometric operator. Garg et al. (2020) presented different types of operators such as power averaging, power weighted averaging, power hybrid averaging, power geometric, power weighted geometric, and power hybrid geometric operator in the context of Cq-ROFSs. Liu et al. (2017) introduced the concept of Cq-RO linguistic (Cq-ROL) sets and developed operators like the Cq-ROL Heronian mean, Cq-ROL weighted Heronian mean, Cq-ROL geometric Heronian mean, and Cq-ROL weighted geometric Heronian mean operator. The newly proposed Cq-ROFSs are incredibly versatile and efficient and, as opposed to many current fuzzy set theories, can clearly articulate the decision-making opinions of experts in a complex state. Compared to some current models in the literature, the Table 1 represents the superiority and versatility of the Cq-ROFS model.

A lot of research has been conducted over the last decades on the aggregation information. The aggregation operator refers to a function, which can combine a fixed number of inputs into one output. Information AO plays a key role in the process of decision-making, particularly in MADM. The Hamy mean (HM) operator, originally proposed by Hara et al. (1998) has an ability to capture the interrelationships between the arguments of the data. Inspired by the HM operator, Li et al. (2018) presented the Dombi HM operator and developed the intuitionistic fuzzy Dombi HM and the weighted intuitionistic fuzzy Dombi HM operator. Further, Wu et al. (2018) extended the idea of Dombi HM operator to interval-valued intuitionistic fuzzy circumstances. Generalizing HM operator to PFS, Li et al. (2018) developed some Pythagorean fuzzy HM operators to choose optimal green supplier. Utilizing the ideas of 2-tuple linguistic set (2TLS) and PFS, Deng et al. (2018) put forward 2-tuple linguistic Pythagorean fuzzy set (2TLPFS), and extended the HM operator to the 2-tuple linguistic Pythagorean fuzzy data. Wang et al. (2019) developed the q-ROFHM and the q-ROFDHM operator for aggregating the q-ROF information, and developed a model to deal with MADM problems with q-ROFS based on HM. Wei et al. (2019) and Rong et al. (2020) enlarged the scope of HM, DHM, weighted HM and weighted DHM operators to the dual hesitant Pythagorean fuzzy and hesitant fuzzy linguistic information, respectively.

Table 1. Comparison of Cq-ROFS model with extant models in literature

Model	MD	NMD	Hesitation	Periodicity	Represents two-dimensional information
FS (Zadeh, 1965)	√	×	×	×	×
Interval-valued FS (Zadeh, 1975)	√	×	×	×	×
q-ROFS ($q \geq 1$) (Yager, 2016)	√	√	√	×	×
Interval-valued q-ROFS (Joshi et al., 2018)	√	√	√	×	×
CFS (Ramot et.al., 2002)	√	×	×	√	√
Interval-valued CFS (Greenfield et al., 2016)	√	×	×	√	√
Cq-ROFS ($q \geq 1$) (Liu et al., 2020)	√	√	√	√	√

Cq-ROFSs theory has been broadly applied to many domains and all the above methods, however, are inadequate to aggregate these Cq-ROF numbers (Cq-ROFNs) based on the HM and DHM. So, an important subject is how to quantify such Cq-ROFNs on the bases of the HM and DHM. In this article, we

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design some C*q*-ROF aggregation operators based on the traditional HM and DHM to solve this problem. The remainder of the paper is composed as follows. Section 2 introduces some basic concepts related to *q*-ROFSs and C*q*-ROFSs. In Section 3, we propose the C*q*-ROFHM and C*q*-ROFWHM operators based on the HM and further, the C*q*-ROFDHM and C*q*-ROFWDHM operators based on the DHM. Section 4, presents MADM models based on the C*q*-ROFWHM and C*q*-ROFWDHM operators with C*q*-ROFNs. A numerical illustration for the selection of ERP systems with C*q*-ROFNs is presented in Section 5 to illustrate the approach proposed in this paper. At long last, we summarize the paper in Section 6.

2 PRELIMINARIES

To facilitate the next sections, the basic concepts of *q*-ROFSs, C*q*-ROFSs and HM operator are given in this section.

Definition 2.1 (Yager, 2016) *Let L be a fixed set. A q-ROFS* Ω *in L is expressed as:*

$$\Omega = \{\langle s, \zeta_\Omega(s), \delta_\Omega(s)\rangle \mid s \in L\},$$

characterized by a membership ζ_Ω and a non-membership function δ_Ω, where:

$$\zeta_\Omega : L \to [0,1], \quad s \in L \to \zeta_\Omega(s) \in [0,1]$$

$$\delta_\Omega : L \to [0,1], \quad s \in L \to \delta_\Omega(s) \in [0,1]$$

such that $\zeta_\Omega^q(s) + \delta_\Omega^q(s) \le 1, q \ge 1$ for all $s \in L$. Moreover, for all $s \in L$,

$$\pi_\Omega(s) = \sqrt[q]{\zeta_\Omega^q(s) + \delta_\Omega^q(s) - \zeta_\Omega^q(s)\delta_\Omega^q(s)}$$

is called the degree of indeterminacy of s in Ω.

Simply, $\beta = (\zeta_\beta, \delta_\beta)$ is a *q*-ROFN, where

$$\zeta_\beta, \delta_\beta \in [0,1], \zeta_\beta^q + \delta_\beta^q \le 1 \text{ and } \pi_\beta = \sqrt[q]{\zeta_\beta^q + \delta_\beta^q - \zeta_\beta^q \delta_\beta^q}.$$

Definition 2.2 (Liu et al., 2020) *A Cq-ROFS* $\Im$ *is defined as:*

$$\Im = \left\{\left(s, \hbar_\Im(s), \ell_\Im(s)\right) : s \in L\right\},$$

where $\hbar_\Im, \ell_\Im : L \to \{c : c \in C, |c| \le 1\}$ are termed as complex-valued membership and non-membership functions, respectively and are defined as:

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$$\hbar_{\mathfrak{J}}(s)=\zeta_{\mathfrak{J}}(s)e^{i2\pi\omega_{\zeta_{\mathfrak{J}}}(s)};\ell_{\mathfrak{J}}(s)=\delta_{\mathfrak{J}}(s)e^{i2\pi\omega_{\delta_{\mathfrak{J}}}(s)},$$

where

$$0\le\zeta_{\mathfrak{J}}(s),\delta_{\mathfrak{J}}(s),\zeta_{\mathfrak{J}}^{q}(s)+\delta_{\mathfrak{J}}^{q}(s)\le 1 \text{ and } 0\le\omega_{\zeta_{\mathfrak{J}}}^{q}(s),\omega_{\delta_{\mathfrak{J}}}^{q}(s),\omega_{\zeta_{\mathfrak{J}}}^{q}(s)+\omega_{\delta_{\mathfrak{J}}}^{q}(s)\le 1.$$

Further,

$$\pi_{\mathfrak{J}}(s)=(1-(\zeta_{\mathfrak{J}}^{q}(s)+\delta_{\mathfrak{J}}^{q}(s)))^{\frac{1}{q}} \text{ and } \omega_{\pi_{\mathfrak{J}}}(s)=(1-(\omega_{\zeta_{\mathfrak{J}}}^{q}(s)+\omega_{\delta_{\mathfrak{J}}}^{q}(s)))^{\frac{1}{q}}$$

are complex hesitancy degree of *s*. For simplicity, the pair $\aleph=\left(\left(\zeta,\omega_{\zeta}\right),\left(\delta,\omega_{\delta}\right)\right)$ is called the Cq-ROF number (C*q*-ROFN), where

$$0\le\zeta,\delta,\zeta^{q}+\delta^{q}\le 1, \text{ and } 0\le\omega_{\zeta},\omega_{\delta},\omega_{\zeta}^{q}+\omega_{\delta}^{q}\le 1.$$

Example 2.1 *Consider a fixed set L contains just one element s,*

$$\zeta_{\mathfrak{J}}(s)=0.5,\omega_{\zeta_{\mathfrak{J}}}(s)=0.7,\delta_{\mathfrak{J}}(s)=0.8,\omega_{\delta_{\mathfrak{J}}}(s)=0.6.$$

Then

$$\aleph=\left\{\left(s,(0.5,0.7),(0.8,0.6)\right)\right\}$$

is a C3-ROFN and denoted by $\aleph=\left((0.5,0.7),(0.8,0.6)\right)$ *for simplicity.*

Comparison of restrictions of CIFNs, CPFNs, and C*q*-ROFNs is shown in Figure 1.

Definition 2.3 (Garg et al., 2020) *Let*

$$\aleph=\left(\left(\zeta_{\aleph},\omega_{\zeta_{\aleph}}\right),\left(\delta_{\aleph},\omega_{\delta_{\aleph}}\right)\right),\aleph_1=\left(\left(\zeta_{\aleph_1},\omega_{\zeta_{\aleph_1}}\right),\left(\delta_{\aleph_1},\omega_{\delta_{\aleph_1}}\right)\right)$$

and $\aleph_2=\left(\left(\zeta_{\aleph_2},\omega_{\zeta_{\aleph_2}}\right),\left(\delta_{\aleph_2},\omega_{\delta_{\aleph_2}}\right)\right)$ *be three Cq-ROFNs, then*

1. $\aleph_1\oplus\aleph_2=\left(\left(\sqrt[q]{\zeta_{\aleph_1}^{q}+\zeta_{\aleph_2}^{q}-\zeta_{\aleph_1}^{q}\zeta_{\aleph_2}^{q}},\sqrt[q]{\omega_{\zeta_{\aleph_1}}^{q}+\omega_{\zeta_{\aleph_2}}^{q}-\omega_{\zeta_{\aleph_1}}^{q}\omega_{\zeta_{\aleph_2}}^{q}}\right),\left(\delta_{\aleph_1}\delta_{\aleph_2},\omega_{\delta_{\aleph_1}}\omega_{\delta_{\aleph_2}}\right)\right);$
2. $\aleph_1\otimes\aleph_2=\left(\left(\zeta_{\aleph_1}\zeta_{\aleph_2},\omega_{\zeta_{\aleph_1}}\omega_{\zeta_{\aleph_2}}\right),\left(\sqrt[q]{\delta_{\aleph_1}^{q}+\delta_{\aleph_2}^{q}-\delta_{\aleph_1}^{q}\delta_{\aleph_2}^{q}},\sqrt[q]{\omega_{\delta_{\aleph_1}}^{q}+\omega_{\delta_{\aleph_2}}^{q}-\omega_{\delta_{\aleph_1}}^{q}\omega_{\delta_{\aleph_2}}^{q}}\right)\right);$
3. $\lambda\aleph=\left(\left(\sqrt[q]{1-(1-\zeta_{\aleph}^{q})^{\lambda}},\sqrt[q]{1-(1-\omega_{\zeta_{\aleph}}^{q})^{\lambda}}\right),\left((\delta_{\aleph})^{\lambda},(\omega_{\delta_{\aleph}})^{\lambda}\right)\right),\ \lambda>0;$

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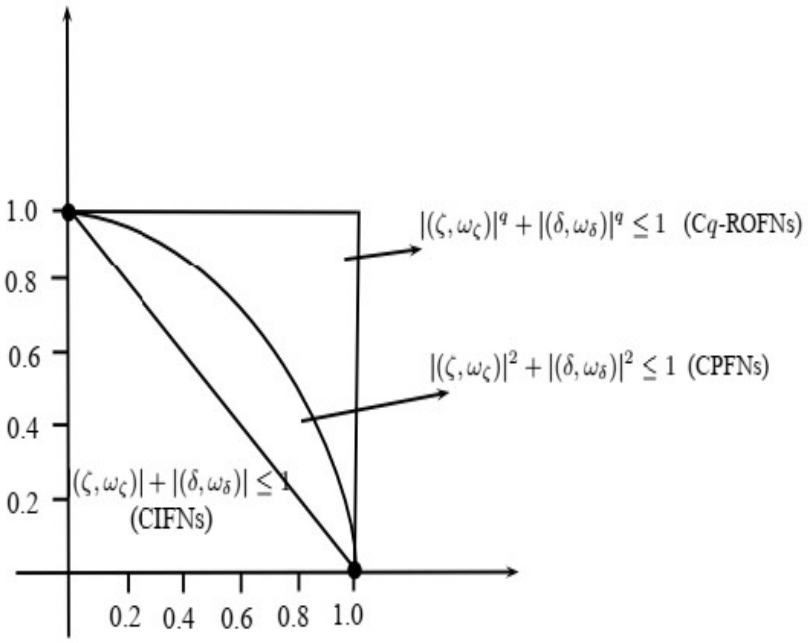
4. $\aleph^{\lambda}=\left(\left((\zeta_{\aleph})^{\lambda},(\omega_{\zeta_{\aleph}})^{\lambda}\right),\left(\sqrt[q]{1-(1-\delta_{\aleph}^{q})^{\lambda}},\sqrt[q]{1-(1-\omega_{\delta_{\aleph}}^{q})^{\lambda}}\right)\right),\ \lambda>0.$

Figure 1. Comparison of restrictions of CIFNs, CPFNs, and Cq-ROFNs



Definition 2.4 (Garg et al., 2020) *Let* $\aleph=\left(\left(\zeta,\omega_{\zeta}\right),\left(\delta,\omega_{\delta}\right)\right)$ *be a Cq-ROFN. Then the score function S and the accuracy function* $\beth$ *of a Cq-ROFN* $\aleph$, *can be given as*

$$S(\aleph)=\frac{1}{4}\left(\left(1+\zeta^{q}-\delta^{q}\right)+\left(1+\omega_{\zeta}^{q}-\omega_{\delta}^{q}\right)\right),S(\aleph)\in[0,1]$$

and $\beth(\aleph)=\frac{1}{2}\left(\zeta^{q}+\delta^{q}+\omega_{\zeta}^{q}+\omega_{\delta}^{q}\right),\beth(\aleph)\in[0,1]$, *respectively.*

Definition 2.5 (Garg et al., 2020) *Let*

$$\aleph_1=\left(\left(\zeta_1,\omega_{\zeta_1}\right),\left(\delta_1,\omega_{\delta_1}\right)\right)\text{ and }\aleph_2=\left(\left(\zeta_2,\omega_{\zeta_2}\right),\left(\delta_2,\omega_{\delta_2}\right)\right)$$

be two Cq-ROFNs,

1. if $S(\aleph_1)>S(\aleph_2)$, then $\aleph_1\succ\aleph_2$;
2. if $S(\aleph_1)=S(\aleph_2)$, then

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- if $\beth(\aleph_1) > \beth(\aleph_2)$, then $\aleph_1 \succ \aleph_2$;
- if $\beth(\aleph_1) = \beth(\aleph_2)$, then $\aleph_1 \sim \aleph_2$.

Definition 2.6 *Let* $\aleph_j \left(j = 1,2,\ldots,n\right)$ *be a set of non-negative real numbers. Some HM aggregation operators are defined as follows:*

1. Hamy mean (Hara et al., 1998):

$$HM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) = \frac{\sum_{1\le j_1<\ldots<j_t\le n}\left(\prod_{k=1}^{t}\aleph_{j_k}\right)^{\frac{1}{t}}}{C_n^t};$$

2. Weighted Hamy mean (Hara et al., 1998):

$$WHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) = \frac{\sum_{1\le j_1<\ldots<j_t\le n}\left(\prod_{k=1}^{t}\left(\aleph_{j_k}\right)^{w_{j_k}}\right)^{\frac{1}{t}}}{C_n^t};$$

3. Dual Hamy mean (Wu et al., 2018):

$$DHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) = \left(\prod_{1\le j_1<\ldots<j_t\le n}\left(\frac{\sum_{k=1}^{t}\aleph_{j_k}}{t}\right)\right)^{\frac{1}{C_n^t}};$$

4. Weighted dual Hamy mean (Wu et al., 2018):

$$WDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) = \left(\prod_{1\le j_1<\ldots<j_t\le n}\left(\frac{\sum_{k=1}^{t}\omega_{j_k}\aleph_{j_k}}{t}\right)\right)^{\frac{1}{C_n^t}}.$$

where t is a parameter and t=1,2,…,n, $j_1 j_2,\ldots,j_t$ are t integer values taken from the set {1,2,…,n} of k integer values, C_n^t denotes the binomial coefficient, and $C_n^t = n!/\left(t!\left(n-t\right)!\right)$.

3 SOME C*q*-ROFHM AGGREGATION OPERATORS

As the C*q*-ROFS is a powerful tool to express the values in decision-making process, in this section, we extend the HM and DHM operators to C*q*-ROFSs and develop the C*q*-ROFHM and C*q*-ROFDHM operators for aggregating C*q*-ROFNs, and study its desirable properties.

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3.1 C*q*-ROFHM Aggregation Operator

This subsection generalizes the HM operator with C*q*-ROFSs to develop the C*q*-ROFHM operator for aggregating C*q*-ROFNs, and study its desirable properties.

Definition 3.1 *Let*

$$\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k}\right), \left(\delta_k, \omega_{\delta_k}\right)\right)\left(k = 1, 2, \ldots, n\right)$$

be a collection of Cq-ROFNs. The Cq-ROFHM operator is a mapping $T^n \to T$ such that

$$Cq - ROFHM^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) = \frac{\oplus_{1 \le j_1 < \ldots < j_t \le n}\left(\otimes_{k=1}^{t} \aleph_{j_k}\right)^{\frac{1}{t}}}{C_n^t}. \tag{3.1}$$

Theorem 3.1 *The aggregated value by utilizing Cq-ROFHM operator is also a Cq-ROFN, where*

$$Cq - ROFHM^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) = \frac{\oplus_{1 \le j_1 < \ldots < j_t \le n}\left(\otimes_{k=1}^{t} \aleph_{j_k}\right)^{\frac{1}{t}}}{C_n^t} \tag{3.2}$$

$$= \left(\begin{array}{c} \sqrt[q]{1 - \prod\limits_{1 \le j_1 < \ldots < j_t \le n}\left(1 - \left(\prod\limits_{k=1}^{t} \zeta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1 - \prod\limits_{1 \le j_1 < \ldots < j_t \le n}\left(1 - \left(\prod\limits_{k=1}^{t} \omega_{\zeta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}, \\ \left(\prod\limits_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod\limits_{k=1}^{t}\left(1 - \left(\delta_k\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}, \left(\prod\limits_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod\limits_{k=1}^{t}\left(1 - \left(\omega_{\delta_k}\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}} \end{array} \right).$$

Proof. Utilizing Definition 2.3, we get

$$\otimes_{k=1}^{t} \aleph_{j_k} = \left(\left(\prod_{k=1}^{t} \zeta_k, \prod_{k=1}^{t} \omega_{\zeta_k}\right), \left(\sqrt[q]{1 - \prod_{k=1}^{t}\left(1 - \left(\delta_k\right)^q\right)}, \sqrt[q]{1 - \prod_{k=1}^{t}\left(1 - \left(\omega_{\delta_k}\right)^q\right)}\right)\right).$$

Thus,

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$$\left(\otimes_{k=1}^{t}\aleph_{j_k}\right)^{\frac{1}{t}}=\left(\left(\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{1}{t}},\left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{1}{t}}\right),\left(\sqrt[q]{1-\prod_{k=1}^{t}(1-(\delta_k)^q)^{\frac{1}{t}}},\sqrt[q]{1-\prod_{k=1}^{t}(1-(\omega_{\delta_k})^q)^{\frac{1}{t}}}\right)\right).$$

Therefore,

$$\oplus_{1\le j_1<\ldots<j_t\le n}\left(\otimes_{k=1}^{t}\aleph_{j_k}\right)^{\frac{1}{t}}$$

$$=\left(\begin{array}{c}\left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{q}{t}}\right)},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{q}{t}}\right)}\right),\\ \left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-(\delta_k)^q\right)^{\frac{1}{t}}},\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-(\omega_{\delta_k})^q\right)^{\frac{1}{t}}}\right)\end{array}\right).$$

Furthermore,

$$Cq-ROFHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)$$

$$=\left(\begin{array}{c}\left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\right.\\ \left.\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}(1-(\delta_k)^q)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}(1-(\omega_{\delta_k})^q)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}\right)\end{array}\right).$$

Example 3.1 *Let*

$$\aleph_1=\left(\left(0.4,0.3\right),\left(0.5,0.7\right)\right),\aleph_2=\left(\left(0.3,0.6\right),\left(0.4,0.2\right)\right),$$
$$\aleph_3=\left(\left(0.7,0.4\right),\left(0.6,0.8\right)\right)\text{ and }\aleph_4=\left(\left(0.2,0.8\right),\left(0.7,0.1\right)\right)$$

be four Cq-ROFNs, and suppose t=3, q=4; then utilizing *Eq. (3.2)*, we have

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$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},$$

$$=\sqrt[4]{1-\left(\begin{array}{l}\left(1-\left(0.4\times0.3\times0.7\right)^{\frac{4}{3}}\right)\times\left(1-\left(0.4\times0.3\times0.2\right)^{\frac{4}{3}}\right)\\ \times\left(1-\left(0.4\times0.7\times0.2\right)^{\frac{4}{3}}\right)\times\left(1-\left(0.3\times0.7\times0.2\right)^{\frac{4}{3}}\right)\end{array}\right)^{\frac{1}{4}}}=0.3727$$

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},$$

$$=\sqrt[4]{1-\left(\begin{array}{l}\left(1-\left(0.3\times0.6\times0.4\right)^{\frac{4}{3}}\right)\times\left(1-\left(0.3\times0.6\times0.8\right)^{\frac{4}{3}}\right)\\ \times\left(1-\left(0.3\times0.4\times0.8\right)^{\frac{4}{3}}\right)\times\left(1-\left(0.6\times0.4\times0.8\right)^{\frac{4}{3}}\right)\end{array}\right)^{\frac{1}{4}}}=0.5030$$

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\delta_k\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},$$

$$=\left(\sqrt[4]{\left(\begin{array}{l}\left(1-\left(\left(1-0.5^4\right)\times\left(1-0.4^4\right)\times\left(1-0.6^4\right)\right)^{\frac{1}{3}}\right)\times\left(1-\left(\left(1-0.5^4\right)\times\left(1-0.4^4\right)\times\left(1-0.7^4\right)\right)^{\frac{1}{3}}\right)\\ \times\left(1-\left(\left(1-0.5^4\right)\times\left(1-0.6^4\right)\times\left(1-0.7^4\right)\right)^{\frac{1}{3}}\right)\times\left(1-\left(\left(1-0.4^4\right)\times\left(1-0.6^4\right)\times\left(1-0.7^4\right)\right)^{\frac{1}{3}}\right)\end{array}\right)}\right)^{\frac{1}{4}}=0.5744$$

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\delta_k}\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}$$

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$$= \left(\sqrt[4]{\left(\begin{array}{l} (1-\left(\left(1-0.7^4\right) \times \left(1-0.2^4\right) \times \left(1-0.8^4\right))^{\frac{1}{3}} \right) \times (1-\left(\left(1-0.7^4\right) \times \left(1-0.2^4\right) \times \left(1-0.1^4\right))^{\frac{1}{3}} \right) \\ \times (1-\left(\left(1-0.7^4\right) \times \left(1-0.8^4\right) \times \left(1-0.1^4\right))^{\frac{1}{3}} \right) \times (1-\left(\left(1-0.2^4\right) \times \left(1-0.8^4\right) \times \left(1-0.1^4\right))^{\frac{1}{3}} \right) \end{array} \right)} \right)^{\frac{1}{4}} = 0.6140$$

Therefore, the overall evaluation value of $\aleph_1, \aleph_2, \aleph_3$ and $\aleph_4$ by utilizing Cq-ROFHM operator is:

$$Cq - ROFHM^{(t)}\left(\aleph_1, \aleph_2, \aleph_3, \aleph_4\right) = \left(\left(0.3727, 0.5030\right), \left(0.5744, 0.6140\right)\right).$$

Further, some desirable properties of the Cq-ROFHM operator are investigated as follows:

Property 3.1 *(Idempotency) If all*

$$\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k}\right), \left(\delta_k, \omega_{\delta_k}\right)\right)\left(k = 1, 2, \ldots, n\right)$$

are equal, for all k, then

$$Cq - ROFHM^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) = \aleph.$$

Proof.

$$Cq - ROFHM^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) = \frac{\oplus_{1 \le j_1 < \ldots < j_t \le n} \left(\otimes_{k=1}^{t} \aleph\right)^{\frac{1}{t}}}{C_n^t}$$

$$= \left(\begin{array}{l} \left(\sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} \zeta\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} \omega_{\zeta}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}} \right) \\ \left(\left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} (1 - \delta^q)^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} (1 - \omega_{\delta}^q)^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}} \right) \end{array} \right)$$

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$$= \left(\left(\sqrt[q]{1-\left(\left(1-(\zeta^t)^{\frac{q}{t}} \right)^{C_n^t} \right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1-\left((1-(\omega_\zeta^t)^{\frac{q}{t}} \right)^{C_n^t})^{\frac{1}{C_n^t}}} \right), \left(\left(\sqrt[q]{1-((1-\delta^q)^t)^{\frac{1}{t}}} \right)^{C_n^t} \right)^{\frac{1}{C_n^t}}, \left(\left(\sqrt[q]{1-((1-\omega_\delta^q)^t)^{\frac{1}{t}}} \right)^{C_n^t} \right)^{\frac{1}{C_n^t}} \right)$$

$$= \left(\left(\zeta, \omega_\zeta \right), \left(\delta, \omega_\delta \right) \right) = \aleph.$$

Property 3.2 *(Monotonicity) Let*

$$\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k} \right), \left(\delta_k, \omega_{\delta_k} \right) \right), \aleph'_k = \left(\left(\zeta'_k, \omega_{\zeta'_k} \right), \left(\delta'_k, \omega_{\delta'_k} \right) \right) \left(k = 1, 2, \ldots, n \right)$$

be two sets of Cq-ROFNs, if $\aleph_k \le \aleph'_k$, *for all k, then*

$$Cq - ROFHM^{(t)} \left(\aleph_1, \aleph_2, \ldots, \aleph_n \right) \le Cq - ROFHM^{(t)} \left(\aleph'_1, \aleph'_2, \ldots, \aleph'_n \right).$$

Proof. Let

$$\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k} \right), \left(\delta_k, \omega_{\delta_k} \right) \right), \aleph'_k = \left(\left(\zeta'_k, \omega_{\zeta'_k} \right), \left(\delta'_k, \omega_{\delta'_k} \right) \right), k = 1, 2, \ldots, n$$

be two sets of C*q*-ROFNs, let

$$\zeta = \left(\sqrt[q]{1-\prod_{1 \le j_1 < \ldots < j_t \le n} \left(1-\left(\prod_{k=1}^{t} \zeta_k \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1-\prod_{1 \le j_1 < \ldots < j_t \le n} \left(1-\left(\prod_{k=1}^{t} \omega_{\zeta_k} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}} \right),$$

$$\delta = \left(\left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1-\prod_{k=1}^{t} (1-(\delta_k)^q)^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1-\prod_{k=1}^{t} (1-(\omega_{\delta_k})^q)^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}} \right)$$

given that $\zeta_k \le \zeta'_k$ and $\omega_{\zeta_k} \le \omega_{\zeta'_k}$, then

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$$\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{q}{t}} \le \left(\prod_{k=1}^{t}\zeta'_k\right)^{\frac{q}{t}},\ \left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{q}{t}} \le \left(\prod_{k=1}^{t}\omega_{\zeta'_k}\right)^{\frac{q}{t}}.$$

Moreover,

$$\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}} \ge \prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta'_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}},$$

$$\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}} \ge \prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta'_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}.$$

Furthermore,

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}} \le \sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\zeta'_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},$$

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}} \le \sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\zeta'_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}.$$

Therefore $\zeta \le \zeta'$ and $\omega_\zeta \le \omega_{\zeta'}$. Similarly, we can show that $\delta \le \delta'$ and $\omega_\delta \le \omega_{\delta'}$. Hence

$$Cq-ROFHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) \le Cq-ROFHM^{(t)}\left(\aleph'_1,\aleph'_2,\ldots,\aleph'_n\right).$$

Property 3.3 *(Boundedness) Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)\left(k=1,2,\ldots,n\right)$ *be a collection of Cq-ROFNs, and let* $\aleph^- = min_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$ *and* $\aleph^+ = max_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$, *then*

$$\aleph^- \le Cq-ROFHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) \le \aleph^+.$$

From Property 3.1,

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$$Cq-ROFHM^{(t)}\left(\aleph_1^-,\aleph_2^-,\ldots,\aleph_n^-\right)=\aleph^-,$$

$$Cq-ROFHM^{(t)}\left(\aleph_1^+,\aleph_2^+,\ldots,\aleph_n^+\right)=\aleph^+.$$

From Property 3.2,

$$\aleph^-\le Cq-ROFHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)\le\aleph^+.$$

3.2 C*q*-ROFWHM Aggregation Operator

The C*q*-ROFHM operator does not consider the significance of the aggregated contentions. In any case the attribute's weights play a key role in the aggregation process in numerous real practical circumstances, particularly in MADM problems. To overcome C*q*-ROFHM's limitation, we put forward the C*q*-ROFWHM operator as follows:

Definition 3.2 *Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)\left(k=1,2,\ldots,n\right)$ *be a collection of Cq-ROFNs. The Cq-ROFWHM operator is a mapping* $T^n\to T$ *such that*

$$Cq-ROFWHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)=\frac{\underset{1\le j_1<\ldots<j_t\le n}{\oplus}\left(\underset{k=1}{\overset{t}{\otimes}}\left(\aleph_{j_k}\right)^{w_{jk}}\right)^{\frac{1}{t}}}{C_n^t}\tag{3.3}$$

Theorem 3.2 *The aggregated value by utilizing Cq-ROFWHM operator is also a Cq-ROFN, where*

$$Cq-ROFWHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)=\tag{3.4}$$

$$\left(\begin{array}{c}\left(\sqrt[q]{1-\underset{1\le j_1<\ldots<j_t\le n}{\prod}\left(1-\left(\underset{k=1}{\overset{t}{\prod}}\left(\zeta_k\right)^{w_{jk}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\ \sqrt[q]{1-\underset{1\le j_1<\ldots<j_t\le n}{\prod}\left(1-\left(\underset{k=1}{\overset{t}{\prod}}\left(\omega_{\zeta_k}\right)^{w_{jk}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\right),\\ \left(\left(\underset{1\le j_1<\ldots<j_t\le n}{\prod}\sqrt[q]{1-\underset{k=1}{\overset{t}{\prod}}\left(\left(1-\left(\delta_k\right)^q\right)^{w_{jk}}\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\ \left(\underset{1\le j_1<\ldots<j_t\le n}{\prod}\sqrt[q]{1-\underset{k=1}{\overset{t}{\prod}}\left(\left(1-\left(\omega_{\delta_k}\right)^q\right)^{w_{jk}}\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}\right)\end{array}\right).$$

Proof. Utilizing Definition 2.3, we get

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$$(\aleph_{j_k})^{w_{jk}} = \left(\left((\zeta_k)^{w_{jk}}, (\omega_{\zeta_k})^{w_{jk}} \right), \left(\sqrt[q]{1-(1-(\delta_k)^q)^{w_{jk}}}, \sqrt[q]{1-(1-(\omega_{\delta_k})^q)^{w_{jk}}} \right) \right).$$

Then,

$$\otimes_{k=1}^{t}(\aleph_{j_k})^{w_{jk}} = \left(\left(\prod_{k=1}^{t}(\zeta_k)^{w_{jk}}, \prod_{k=1}^{t}(\omega_{\zeta_k})^{w_{jk}} \right), \left(\sqrt[q]{1-\prod_{k=1}^{t}(1-(\delta_k)^q)^{w_{jk}}}, \sqrt[q]{1-\prod_{k=1}^{t}(1-(\omega_{\delta_k})^q)^{w_{jk}}} \right) \right).$$

Thus,

$$\left(\otimes_{k=1}^{t}\left(\aleph_{j_k}\right) \right)^{w_{jk}} \Bigg)^{\frac{1}{t}} = \left(\begin{array}{c} \left(\left(\prod_{k=1}^{t}(\zeta_k)^{w_{jk}} \right)^{\frac{1}{t}}, \left(\prod_{k=1}^{t}(\omega_{\zeta_k})^{w_{jk}} \right)^{\frac{1}{t}} \right), \\ \left(\sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\delta_k)^q)^{w_{jk}} \right)^{\frac{1}{t}}}, \sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\omega_{\delta_k})^q)^{w_{jk}} \right)^{\frac{1}{t}}} \right) \end{array} \right).$$

Therefore,

$$\oplus_{1\le j_1<\ldots<j_t\le n}\left(\otimes_{k=1}^{t}\left(\aleph_{j_k}\right)^{w_{jk}} \right)^{\frac{1}{t}} =$$

$$\left(\begin{array}{c} \left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\zeta_k)^{w_{jk}} \right)^{\frac{q}{t}} \right)}, \sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\zeta_k})^{w_{jk}} \right)^{\frac{q}{t}} \right)} \right), \\ \left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\delta_k)^q)^{w_{jk}} \right)^{\frac{1}{t}}}, \prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\omega_{\delta_k})^q)^{w_{jk}} \right)^{\frac{1}{t}}} \right) \end{array} \right).$$

Furthermore,

$$Cq-ROFWHM_{w}^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)$$

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$$= \left(\left(\sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\zeta_k)^{w_{jk}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\omega_{\zeta_k})^{w_{jk}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}} \right) \right.$$
$$\left. \left(\left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\delta_k)^q)^{w_{jk}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\omega_{\delta_k})^q)^{w_{jk}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}} \right) \right).$$

Example 3.2 *Let*

$\aleph_1 = \big((0.4, 0.3), (0.5, 0.7)\big), \aleph_2 = \big((0.3, 0.6), (0.4, 0.2)\big),$
$\aleph_3 = \big((0.7, 0.4), (0.6, 0.8)\big)$ and $\aleph_4 = \big((0.2, 0.8), (0.7, 0.1)\big)$

be four Cq-ROFNs, and suppose t=3, q=4, w=$(0.14, 0.27, 0.34, 0.25)^{T}$; then utilizing *Eq. (3.4)*, we have

$$\sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\zeta_k)^{w_{jk}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}},$$

$$= \sqrt[4]{1 - \left(\begin{array}{l} (1 - (0.4^{0.14} \times 0.3^{0.27} \times 0.7^{0.34})^{\frac{4}{3}}) \times (1 - (0.4^{0.14} \times 0.3^{0.27} \times 0.2^{0.25})^{\frac{4}{3}}) \\ \times (1 - (0.4^{0.14} \times 0.7^{0.34} \times 0.2^{0.25})^{\frac{4}{3}}) \times (1 - (0.3^{0.27} \times 0.7^{0.34} \times 0.2^{0.25})^{\frac{4}{3}}) \end{array} \right)^{\frac{1}{4}}} = 0.7872$$

$$\sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\omega_{\zeta_k})^{w_{jk}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}},$$

$$= \sqrt[4]{1 - \left(\begin{array}{l} (1 - (0.3^{0.14} \times 0.6^{0.27} \times 0.4^{0.34})^{\frac{4}{3}}) \times (1 - (0.3^{0.14} \times 0.6^{0.27} \times 0.8^{0.25})^{\frac{4}{3}}) \\ \times (1 - (0.3^{0.14} \times 0.4^{0.34} \times 0.8^{0.25})^{\frac{4}{3}}) \times (1 - (0.6^{0.27} \times 0.4^{0.34} \times 0.8^{0.25})^{\frac{4}{3}}) \end{array} \right)^{\frac{1}{4}}} = 0.8483$$

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$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\delta_k)^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}=$$

$$\sqrt[4]{\begin{pmatrix}(1-((1-0.5^4)^{0.14}\times(1-0.4^4)^{0.27}\times\left(1-0.6^4\right)^{0.34})^{\frac{1}{3}}\\ \times(1-((1-0.5^4)^{0.14}\times(1-0.4^4)^{0.27}\times\left(1-0.7^4\right)^{0.25})^{\frac{1}{3}}\\ \times(1-((1-0.5^4)^{0.14}\times(1-0.6^4)^{0.34}\times\left(1-0.7^4\right)^{0.25})^{\frac{1}{3}}\\ \times(1-((1-0.4^4)^{0.27}\times(1-0.6^4)^{0.34}\times\left(1-0.7^4\right)^{0.25})^{\frac{1}{3}}\end{pmatrix}^{\frac{1}{4}}}=0.3757$$

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\omega_{\delta_k})^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}$$

$$\sqrt[4]{\begin{pmatrix}(1-((1-0.7^4)^{0.14}\times(1-0.2^4)^{0.27}\times\left(1-0.8^4\right)^{0.34})^{\frac{1}{3}}\\ \times(1-((1-0.7^4)^{0.14}\times(1-0.2^4)^{0.27}\times\left(1-0.1^4\right)^{0.25})^{\frac{1}{3}}\\ \times(1-((1-0.7^4)^{0.14}\times(1-0.8^4)^{0.34}\times\left(1-0.1^4\right)^{0.25})^{\frac{1}{3}}\\ \times(1-((1-0.2^4)^{0.27}\times(1-0.8^4)^{0.34}\times\left(1-0.1^4\right)^{0.25})^{\frac{1}{3}}\end{pmatrix}^{\frac{1}{4}}}=0.3958$$

Therefore, the overall evaluation value of $\aleph_1,\aleph_2,\aleph_3$ and $\aleph_4$ by utilizing Cq-ROFWHM operator is:

$$Cq-ROFWHM_w^{(t)}\left(\aleph_1,\aleph_2,\aleph_3,\aleph_4\right)=\left(\left(0.7872,0.8483\right),\left(0.3757,0.3958\right)\right).$$

Property 3.4 *(Monotonicity) Let*

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$$\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k}\right), \left(\delta_k, \omega_{\delta_k}\right)\right), \aleph'_k = \left(\left(\zeta'_k, \omega_{\zeta'_k}\right), \left(\delta'_k, \omega_{\delta'_k}\right)\right) (k = 1, 2, \ldots, n)$$

be two sets of Cq-ROFNs, if $\aleph_k \leq \aleph'_k$, *for all k, then*

$$Cq - ROFWHM_w^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) \leq Cq - ROFWHM_w^{(t)}\left(\aleph'_1, \aleph'_2, \ldots, \aleph'_n\right).$$

Proof. Let

$$\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k}\right), \left(\delta_k, \omega_{\delta_k}\right)\right), \aleph'_k = \left(\left(\zeta'_k, \omega_{\zeta'_k}\right), \left(\delta'_k, \omega_{\delta'_k}\right)\right), k = 1, 2, \cdots, n$$

be two sets of C*q*-ROFNs; let

$$\zeta = \left(\sqrt[q]{1 - \prod_{1 \leq j_1 < \ldots < j_t \leq n} \left(1 - \left(\prod_{k=1}^{t} (\zeta_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1 - \prod_{1 \leq j_1 < \ldots < j_t \leq n} \left(1 - \left(\prod_{k=1}^{t} (\omega_{\zeta_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}} \right),$$

$$\delta = \left(\left(\prod_{1 \leq j_1 < \ldots < j_t \leq n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\delta_k)^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}, \left(\prod_{1 \leq j_1 < \ldots < j_t \leq n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\omega_{\delta_k})^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}} \right)$$

given that $\zeta_k \leq \zeta'_k$ and $\omega_{\zeta_k} \leq \omega_{\zeta'_k}$, then

$$\left(\prod_{k=1}^{t} (\zeta_k)^{w_{j_k}}\right)^{\frac{q}{t}} \leq \left(\prod_{k=1}^{t} (\zeta'_k)^{w_{j_k}}\right)^{\frac{q}{t}}, \left(\prod_{k=1}^{t} (\omega_{\zeta_k})^{w_{j_k}}\right)^{\frac{q}{t}} \leq \left(\prod_{k=1}^{t} (\omega_{\zeta'_k})^{w_{j_k}}\right)^{\frac{q}{t}}.$$

Moreover,

$$\prod_{1 \leq j_1 < \ldots < j_t \leq n} \left(1 - \left(\prod_{k=1}^{t} (\zeta_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}} \geq \prod_{1 \leq j_1 < \ldots < j_t \leq n} \left(1 - \left(\prod_{k=1}^{t} (\zeta'_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}},$$

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$$\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\zeta_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}\ge\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\zeta'_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}.$$

Furthermore,

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\zeta_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\le\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\zeta'_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},$$

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\zeta_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\le\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\zeta'_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}.$$

Therefore, $\zeta\le\zeta'$ and $\omega_\zeta\le\omega_{\zeta'}$. Similarly, we can show that $\delta\le\delta'$ and $\omega_\delta\le\omega_{\delta'}$. Hence

$$Cq-ROFWHM_w^{(t)}(\aleph_1,\aleph_2,\ldots,\aleph_n)\le Cq-ROFWHM_w^{(t)}(\aleph'_1,\aleph'_2,\ldots,\aleph'_n).$$

Property 3.5 *(Boundedness) Let*

$$\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)(k=1,2,\ldots,n)$$

be a collection of Cq-ROFNs, and let

$$\aleph^-=min_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right),\aleph^+=max_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right),$$

then

$$\aleph^-\le Cq-ROFWHM_w^{(t)}(\aleph_1,\aleph_2,\ldots,\aleph_n)\le\aleph^+.$$

Proof. From the Theorem 3.2, we get

$$Cq-ROFWHM_w^{(t)}(\aleph_1^-,\aleph_2^-,\ldots,\aleph_n^-)=\frac{\oplus_{1\le j_1<\ldots<j_t\le n}\left(\otimes_{k=1}^{t}\left(\min\aleph_{j_k}\right)^{w_{j_k}}\right)^{\frac{1}{t}}}{C_n^t}$$

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$$= \left(\begin{array}{c} \left(\sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\min \zeta_k)^{w_{j_k}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\min \omega_{\zeta_k})^{w_{j_k}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}} \right), \\ \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\max \delta_k)^q)^{w_{j_k}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\max \omega_{\delta_k})^q)^{w_{j_k}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}} \end{array} \right).$$

$$Cq - ROFWHM_w^{(t)}\left(\aleph_1^+, \aleph_2^+, \ldots, \aleph_n^+\right) = \frac{\oplus_{1 \le j_1 < \ldots < j_t \le n} \left(\otimes_{k=1}^{t} (\max \aleph_{j_k})^{w_{j_k}} \right)^{\frac{1}{t}}}{C_n^t}$$

$$= \left(\begin{array}{c} \left(\sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} (\max \zeta_k)^{w_{j_k}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1 - \prod_{1 \le j_1 < \ldots < j_t \le n} \left(1 - \left(\prod_{k=1}^{t} \max \omega_{\zeta_k})^{w_{j_k}} \right)^{\frac{q}{t}} \right)^{\frac{1}{C_n^t}}} \right), \\ \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\min \delta_k)^q)^{w_{j_k}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \left(\prod_{1 \le j_1 < \ldots < j_t \le n} \sqrt[q]{1 - \prod_{k=1}^{t} ((1 - (\min \omega_{\delta_k})^q)^{w_{j_k}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}} \end{array} \right).$$

From Property 3.4,

$$\aleph^- \le Cq - ROFWHM_w^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) \le \aleph^+.$$

Obviously, C*q*-ROFWHM operator loses the idempotency property.

3.3 C*q*-ROFDHM Aggregation Operator

This subsection generalizes the DHM operator with C*q*-ROFSs to develop the C*q*-ROFHM operator for aggregating C*q*-ROFNs, and study its desirable properties.

Definition 3.3 *Let* $\aleph_k = \left(\left(\zeta_k, \omega_{\zeta_k}\right), \left(\delta_k, \omega_{\delta_k}\right)\right)\left(k = 1, 2, \ldots, n\right)$ *be a collection of Cq-ROFNs. The Cq-ROFDHM operator is a mapping* $T^n \to T$ *such that*

$$Cq - ROFDHM^{(t)}\left(\aleph_1, \aleph_2, \ldots, \aleph_n\right) = \left(\otimes_{1 \le j_1 < \ldots < j_t \le n} \left(\frac{\oplus_{k=1}^{t} \aleph_{j_k}}{t} \right) \right)^{\frac{1}{C_n^t}}. \quad (3.5)$$

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Theorem 3.3 *The aggregated value by utilizing Cq-ROFDHM operator is also a Cq-ROFN, where*

$$Cq-ROFDHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) \tag{3.6}$$

$$=\left(\begin{array}{c}\left(\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\zeta_k\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\zeta_k}\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}\right),\\ \left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t<n}\left(1-\left(\prod_{k=1}^{t}\delta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t<n}\left(1-\left(\prod_{k=1}^{t}\omega_{\delta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\right)\end{array}\right).$$

Proof. Utilizing Definition 2.3, we get

$$\oplus_{k=1}^{t}\aleph_{j_k}=\left(\left(\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\zeta_k\right)^q\right)},\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\zeta_k}\right)^q\right)}\right),\left(\prod_{k=1}^{t}\delta_k,\prod_{k=1}^{t}\omega_{\delta_k}\right)\right).$$

Thus,

$$\left(\frac{\oplus_{k=1}^{t}\aleph_{j_k}}{t}\right)=\left(\left(\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\zeta_k\right)^q\right)^{\frac{1}{t}}},\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\zeta_k}\right)^q\right)^{\frac{1}{t}}}\right),\left(\left(\prod_{k=1}^{t}\delta_k\right)^{\frac{1}{t}},\left(\prod_{k=1}^{t}\omega_{\delta_k}\right)^{\frac{1}{t}}\right)\right).$$

Therefore,

$$\otimes_{1\le j_1<\ldots<j_t\le n}\left(\frac{\oplus_{k=1}^{t}\aleph_{j_k}}{t}\right)$$

$$=\left(\begin{array}{c}\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\zeta_k\right)^q\right)^{\frac{1}{t}}},\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\zeta_k}\right)^q\right)^{\frac{1}{t}}}\right),\\ \left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\delta_k\right)^{\frac{q}{t}}\right)},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\delta_k}\right)^{\frac{q}{t}}\right)}\right)\end{array}\right).$$

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Furthermore,

$$Cq-ROFDHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)$$

$$=\left(\begin{array}{c}\left(\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\zeta_k\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\zeta_k}\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}\right),\\ \left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\delta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\delta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\right)\end{array}\right).$$

Example 3.3 *Let*

$$\aleph_1=\left(\left(0.4,0.3\right),\left(0.5,0.7\right)\right),\aleph_2=\left(\left(0.3,0.6\right),\left(0.4,0.2\right)\right),$$
$$\aleph_3=\left(\left(0.7,0.4\right),\left(0.6,0.8\right)\right)\text{ and }\aleph_4=\left(\left(0.2,0.8\right),\left(0.7,0.1\right)\right)$$

be four Cq-ROFNs, and suppose t=3, q=4; then utilizing *Eq. (3.6)*, we have

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\zeta_k\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}$$

$$=\left(\sqrt[4]{\begin{array}{l}\left(1-\left(1-\left(0.4\right)^4\right)\times\left(1-\left(0.3\right)^4\right)\times\left(1-\left(0.7\right)^4\right)^{\frac{1}{3}}\right)\times\left(1-\left(1-\left(0.4\right)^4\right)\times\left(1-\left(0.3\right)^4\right)\times\left(1-\left(0.2\right)^4\right)^{\frac{1}{3}}\right)\\ \times\left(1-\left(1-\left(0.4\right)^4\right)\times\left(1-\left(0.7\right)^4\right)\times\left(1-\left(0.2\right)^4\right)^{\frac{1}{3}}\right)\times\left(1-\left(1-\left(0.3\right)^4\right)\times\left(1-\left(0.7\right)^4\right)\times\left(1-\left(0.2\right)^4\right)^{\frac{1}{3}}\right)\end{array}}\right)^{\frac{1}{4}}$$

=0.4686

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}\left(1-\left(\omega_{\zeta_k}\right)^q\right)^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}$$

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$$= \left(\sqrt[4]{\left(\begin{array}{l} (1-((1-(0.3)^4)\times(1-(0.6)^4)\times\left(1-(0.4)^4\right)^{\frac{1}{3}})\times(1-((1-(0.3)^4)\times(1-(0.6)^4)\times\left(1-(0.8)^4\right)^{\frac{1}{3}}) \\ \times(1-((1-(0.3)^4)\times(1-(0.4)^4)\times\left(1-(0.8)^4\right)^{\frac{1}{3}})\times(1-((1-(0.6)^4)\times(1-(0.4)^4)\times\left(1-(0.8)^4\right)^{\frac{1}{3}}) \end{array} \right)} \right)^{\frac{1}{4}}$$

$$=0.5969$$

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\delta_k\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}$$

$$= \sqrt[4]{1-\left(\begin{array}{l} (1-\left(0.5\times0.4\times0.6\right)^{\frac{4}{3}})\times(1-\left(0.5\times0.4\times0.7\right)^{\frac{4}{3}}) \\ \times(1-\left(0.5\times0.6\times0.7\right)^{\frac{4}{3}})\times(1-\left(0.4\times0.6\times0.7\right)^{\frac{4}{3}}) \end{array} \right)^{\frac{1}{4}}} = 0.5431$$

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\omega_{\delta_k}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}$$

$$= \sqrt[4]{1-\left(\begin{array}{l} (1-\left(0.7\times0.2\times0.8\right)^{\frac{4}{3}})\times(1-\left(0.7\times0.2\times0.1\right)^{\frac{4}{3}}) \\ \times(1-\left(0.7\times0.8\times0.1\right)^{\frac{4}{3}})\times(1-\left(0.2\times0.8\times0.1\right)^{\frac{4}{3}}) \end{array} \right)^{\frac{1}{4}}} = 0.3693$$

Therefore, the overall evaluation value of $\aleph_1, \aleph_2, \aleph_3$ and $\aleph_4$ by utilizing Cq-ROFDHM operator is:

$$Cq-ROFDHM^{(t)}\left(\aleph_1,\aleph_2,\aleph_3,\aleph_4\right)=\left(\left(0.4686,0.5969\right),\left(0.5431,0.3693\right)\right).$$

Property 3.6 *(Idempotency) If all* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)\left(k=1,2,\ldots,n\right)$ *are equal, for all k, then*

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$$Cq-ROFDHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)=\aleph.$$

Property 3.7 *(Monotonicity) Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$,

$\aleph'_k=\left(\left(\zeta'_k,\omega_{\zeta'_k}\right),\left(\delta'_k,\omega_{\delta'_k}\right)\right)\left(k=1,2,\ldots,n\right)$

be two sets of Cq-ROFNs, and if $\aleph_k\le\aleph'_k$, *for all k, then*

$$Cq-ROFDHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)\le Cq-ROFDHM^{(t)}\left(\aleph'_1,\aleph'_2,\ldots,\aleph'_n\right).$$

Property 3.8 *(Boundedness) Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)\left(k=1,2,...,n\right)$ *be a set of Cq-ROFNs, and let* $\aleph^-=min_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$ *and* $\aleph^+=max_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$, *then*

$$\aleph^-\le Cq-ROFDHM^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)\le\aleph^+.$$

3.4 C*q*-ROFWDHM Aggregation Operator

The C*q*-ROFDHM operator does not consider the significance of the aggregated contentions. In any case the attributes weights play a key role in the aggregation process in numerous real practical circumstances, particularly in MADM problems. To overcome C*q*-ROFDHM's limitation, we propose the C*q*-ROFWDHM operator as follows:

Definition 3.4 *Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)\left(k=1,2,\ldots,n\right)$ *be a collection of Cq-ROFNs. The Cq-ROFWDHM operator is a mapping* $T^n\to T$ *such that*

$$Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)=\left(\otimes_{1\le j_1<\ldots<j_t\le n}\left(\frac{\oplus_{k=1}^{t}\omega_{j_k}\aleph_{j_k}}{t}\right)\right)^{\frac{1}{C_n^t}}. \tag{3.7}$$

Theorem 3.4 *The aggregated value by utilizing Cq-ROFWDHM operator is also a Cq-ROFN, where*

$$Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right) \tag{3.8}$$

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$$= \left(\begin{array}{c} \left(\prod\limits_{1\le j_1<\ldots<j_t\le n} \sqrt[q]{1-\prod\limits_{k=1}^{t}((1-(\zeta_k)^q)^{\omega_{jk}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \left(\prod\limits_{1\le j_1<\ldots<j_t\le n} \sqrt[q]{1-\prod\limits_{k=1}^{t}((1-(\omega_{\zeta_k})^q)^{\omega_{jk}})^{\frac{1}{t}}} \right)^{\frac{1}{C_n^t}}, \\ \left(\sqrt[q]{1-\prod\limits_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod\limits_{k=1}^{t}(\delta_k)^{w_{jk}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}, \sqrt[q]{1-\prod\limits_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod\limits_{k=1}^{t}(\omega_{\delta_k})^{w_{jk}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}} \right) \end{array} \right).$$

Proof. Utilizing Definition 2.3, we get

$$\omega_{j_k}\aleph_{j_k} = \left(\left(\sqrt[q]{1-(1-(\zeta_k)^q)^{\omega_{jk}}}, \sqrt[q]{1-(1-(\omega_{\zeta_k})^q)^{\omega_{jk}}} \right), \left((\delta_k)^{w_{jk}}, (\omega_{\delta_k})^{w_{jk}} \right) \right).$$

Then,

$$\oplus_{k=1}^{t}\aleph_{j_k} = \left(\left(\sqrt[q]{1-\prod_{k=1}^{t}(1-\left(\zeta_k\right)^q)}, \sqrt[q]{1-\prod_{k=1}^{t}(1-\left(\omega_{\zeta_k}\right)^q)} \right), \left(\prod_{k=1}^{t}\delta_k, \prod_{k=1}^{t}\omega_{\delta_k} \right) \right).$$

Thus,

$$\left(\frac{\oplus_{k=1}^{t}\omega_{j_k}\aleph_{j_k}}{t} \right) =$$

$$\left(\left(\sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\zeta_k)^q)^{w_{jk}} \right)^{\frac{1}{t}}}, \sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\omega_{\zeta_k})^q)^{w_{jk}} \right)^{\frac{1}{t}}} \right), \left(\left(\prod_{k=1}^{t}(\delta_k)^{\omega_{jk}} \right)^{\frac{1}{t}}, \left(\prod_{k=1}^{t}(\omega_{\delta_k})^{\omega_{jk}} \right)^{\frac{1}{t}} \right) \right).$$

Therefore,

$$\otimes_{1\le j_1<\ldots<j_t\le n}\left(\frac{\oplus_{k=1}^{t}\omega_{j_k}\aleph_{j_k}}{t} \right) =$$

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$$\left(\begin{array}{c}\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\zeta_k)^q)^{\omega_{j_k}}\right)^{\frac{1}{t}}},\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\left(\prod_{k=1}^{t}(1-(\omega_{\zeta_k})^q)^{\omega_{j_k}}\right)^{\frac{1}{t}}}\right),\\ \left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\delta_k)^{\omega_{j_k}}\right)^{\frac{q}{t}}\right)},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\delta_k})^{\omega_{j_k}}\right)^{\frac{q}{t}}\right)}\right)\end{array}\right).$$

Furthermore,

$$Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)$$

$$=\left(\begin{array}{c}\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\zeta_k)^q)^{\omega_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\omega_{\zeta_k})^q)^{\omega_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\\ \left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\delta_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)}\right)^{\frac{1}{C_n^t}},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\delta_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)}\right)^{\frac{1}{C_n^t}}\end{array}\right).$$

Example 3.4 *Let*

$$\aleph_1=\left(\left(0.4,0.3\right),\left(0.5,0.7\right)\right),\aleph_2=\left(\left(0.3,0.6\right),\left(0.4,0.2\right)\right),$$
$$\aleph_3=\left(\left(0.7,0.4\right),\left(0.6,0.8\right)\right)\text{ and }\aleph_4=\left(\left(0.2,0.8\right),\left(0.7,0.1\right)\right)$$

be four Cq-ROFNs, and suppose t=3. q=4, w=$(0.14, 0.27, 0.34, 0.25)^{T}$; then utilizing *Eq. (3.8)*, we have

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\zeta_k)^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},$$

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$$\left(\sqrt[4]{\begin{array}{l}(1-((1-(0.4)^4)^{0.14}\times(1-(0.3)^4)^{0.27}\times(1-\left(0.7\right)^4)^{0.34})^{\frac{1}{3}}\right)\\ \times(1-((1-(0.4)^4)^{0.14}\times(1-(0.3)^4)^{0.27}\times(1-\left(0.2\right)^4)^{0.25})^{\frac{1}{3}}\right)\\ \times(1-((1-(0.4)^4)^{0.14}\times(1-(0.7)^4)^{0.34}\times(1-\left(0.2\right)^4)^{0.25})^{\frac{1}{3}}\right)\\ \times(1-((1-(0.3)^4)^{0.27}\times(1-(0.7)^4)^{0.34}\times(1-\left(0.2\right)^4)^{0.25})^{\frac{1}{3}}\right)\end{array}}\right)^{\frac{1}{4}}=0.3102$$

$$\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\omega_{\zeta_k})^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}=$$

$$\left(\sqrt[4]{\begin{array}{l}(1-((1-(0.3)^4)^{0.14}\times(1-(0.6)^4)^{0.27}\times(1-\left(0.4\right)^4)^{0.34})^{\frac{1}{3}}\right)\\ \times(1-((1-(0.3)^4)^{0.14}\times(1-(0.6)^4)^{0.27}\times(1-\left(0.8\right)^4)^{0.25})^{\frac{1}{3}}\right)\\ \times(1-((1-(0.3)^4)^{0.14}\times(1-(0.4)^4)^{0.34}\times(1-\left(0.8\right)^4)^{0.25})^{\frac{1}{3}}\right)\\ \times(1-((1-(0.6)^4)^{0.27}\times(1-(0.4)^4)^{0.34}\times(1-\left(0.8\right)^4)^{0.25})^{\frac{1}{3}}\right)\end{array}}\right)^{\frac{1}{4}}=0.3931$$

$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\delta_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},$$

$$=\sqrt[4]{1-\left(\begin{array}{l}(1-\left(0.5^{0.14}\times0.4^{0.27}\times0.6^{0.34}\right)^{\frac{4}{3}})\times(1-\left(0.5^{0.14}\times0.4^{0.27}\times0.7^{0.25}\right)^{\frac{4}{3}})\\ \times(1-\left(0.5^{0.14}\times0.6^{0.34}\times0.7^{0.25}\right)^{\frac{4}{3}})\times(1-\left(0.4^{0.27}\times0.6^{0.34}\times0.7^{0.25}\right)^{\frac{4}{3}})\end{array}\right)^{\frac{1}{4}}}=0.8608$$

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$$\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\delta_k})^{w_{jk}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},$$

$$=\sqrt[4]{1-\left(\begin{array}{l}(1-\left(0.7^{0.14}\times 0.2^{0.27}\times 0.8^{0.34}\right)^{\frac{4}{3}})\times(1-\left(0.7^{0.14}\times 0.2^{0.27}\times 0.1^{0.25}\right)^{\frac{4}{3}})\\ \times(1-\left(0.7^{0.14}\times 0.8^{0.34}\times 0.1^{0.25}\right)^{\frac{4}{3}})\times(1-\left(0.2^{0.27}\times 0.8^{0.34}\times 0.1^{0.25}\right)^{\frac{4}{3}})\end{array}\right)^{\frac{1}{4}}}=0.7645$$

Therefore, the overall evaluation value of $\aleph_1,\aleph_2,\aleph_3$ and $\aleph_4$ by utilizing Cq-ROFWDHM operator is:

$$Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\aleph_3,\aleph_4\right)=\left(\left(0.3102,0.3931\right)\right),\left(\left(0.8608,0.7645\right)\right).$$

Property 3.9 *(Monotonicity) Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$,

$\aleph'_k=\left(\left(\zeta'_k,\omega_{\zeta'_k}\right)\left(\delta'_k,\omega_{\delta'_k}\right)\right)\left(k=1,2,\ldots,n\right)$

be two sets of Cq-ROFNs, and if $\aleph_k\le\aleph'_k$*, for all k, then*

$$Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)\le Cq-ROFWDHM_w^{(t)}\left(\aleph'_1,\aleph'_2,\ldots,\aleph'_n\right).$$

Property 3.10 *(Boundedness) Let* $\aleph_k=\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)\left(k=1,2,\ldots,n\right)$ *be a set of Cq-ROFNs, and let* $\aleph^-=min_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$ *and* $\aleph^+=max_k\left(\left(\zeta_k,\omega_{\zeta_k}\right),\left(\delta_k,\omega_{\delta_k}\right)\right)$*, then*

$$\aleph^-\le Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)\le\aleph^+.$$

Obviously, the Cq-ROFWDHM operator loses the idempotency property.

4 AN APPROACH TO MADM PROBLEMS WITH COMPLEX Q-RUNG ORTHOPAIR FUZZY INFORMATION

In this section, based on the Cq-ROFWHM (Cq-ROFWDHM) operator, we propose two MADM methods with Cq-ROFNs. Let $A=\left\{\urcorner_1,\urcorner_2,\ldots,\urcorner_m\right\}$ and $Q=\left\{\varpi_1,\varpi_2,\ldots,\varpi_n\right\}$ be the discrete set of alterna-

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tives and the set of attributes, respectively. $w=(w_1,w_2,\ldots,w_n)^T$ is the weighting vector of the attribute $\varpi_k\left(k=1,2,\ldots,n\right)$, where $w_k\in[0,1]$, $\sum_{k=1}^{n} w_k=1$. Assume that

$$T=(\aleph_{jk})_{m\times n}=(\left(\zeta_{jk},\omega_{\zeta_{jk}}\right),\left(\delta_{jk},\omega_{\delta_{jk}}\right))_{m\times n}$$

is the Cq-ROF decision matrix, where ζ_{jk} and $\omega_{\zeta_{jk}}$ represent the real and imaginary degree that the alternative $\urcorner_j$ satisfies the decision makers's attribute ϖ_k, δ_{jk} and $\omega_{\delta_{jk}}$ represent the real and imaginary degree that the alternative $\urcorner_j$ does not satisfy the decision maker's attribute ϖ_k, $\zeta_{jk},\omega_{\zeta_{jk}}\subset\left[0,1\right]$, $\delta_{jk},\omega_{\delta_{jk}}\subset\left[0,1\right]$, $(\zeta_{jk})^q+(\delta_{jk})^q\le 1$ and $(\omega_{\zeta_{jk}})^q+(\omega_{\delta_{jk}})^q\le 1$, j=1,2,…,m, k=1,2,…,n.

The following is where we apply the Cq-ROFWHM (Cq-ROFWDHM) operator with Cq-ROFNs to the MADM problems.

Step 1: Aggregate all evaluation values $\aleph_{jk}=\left(\left(\zeta_{jk},\omega_{\zeta_{jk}}\right),\left(\delta_{jk},\omega_{\delta_{jk}}\right)\right)$ of the alternative $\urcorner_j\left(j=1,2,\ldots,m\right)$ on all attributes ϖ_k(k=1,2,…,n) into the overall evaluation

$$\aleph_j=\left(\left(\zeta_j,\omega_{\zeta_j}\right),\left(\delta_j,\omega_{\delta_j}\right)\right)$$

based on the Cq-ROFWHM operator

$$\aleph_j=Cq-ROFWHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)$$

$$\left(\begin{array}{l}\left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\left(\zeta_k\right)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}\left(\omega_{\zeta_k}\right)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\right),\\ \left(\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\delta_k)^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\omega_{\delta_k})^q)^{w_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}}\right)\end{array}\right)$$

or

$$\aleph_j=Cq-ROFWDHM_w^{(t)}\left(\aleph_1,\aleph_2,\ldots,\aleph_n\right)$$

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$$\left(\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\zeta_k)^q)^{\omega_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\left(\prod_{1\le j_1<\ldots<j_t\le n}\sqrt[q]{1-\prod_{k=1}^{t}((1-(\omega_{\zeta_k})^q)^{\omega_{j_k}})^{\frac{1}{t}}}\right)^{\frac{1}{C_n^t}},\right.$$
$$\left.\left(\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\delta_k)^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}},\sqrt[q]{1-\prod_{1\le j_1<\ldots<j_t\le n}\left(1-\left(\prod_{k=1}^{t}(\omega_{\delta_k})^{w_{j_k}}\right)^{\frac{q}{t}}\right)^{\frac{1}{C_n^t}}}\right)\right)$$

Step 2: Determine the scores $S(\aleph_j)(j=1,2,\ldots,m)$ of aggregated C*q*-ROFNs $\aleph_j(j=1,2,\ldots,m)$ in order to rank all the alternatives $\urcorner_j(j=1,2,\ldots,m)$ and then choose the optimal one(s), where

$$S(\aleph_j)=\frac{1}{4}\left(\left(1+\zeta_j^q-\delta_j^q\right)+\left(1+\omega_{\zeta_j}^q-\omega_{\delta_j}^q\right)\right). \tag{4.1}$$

If two scores $S(\aleph_j)$ and $S(\aleph_k)$ are same, then we determine the accuracy degrees $\beth(\aleph_j)$ and $\beth(\aleph_k)$ of the overall C*q*-ROFNs $\aleph_j$ and $\aleph_k$, respectively, where

$$\beth(\aleph_j)=\frac{1}{2}\left(\zeta_j^q+\delta_j^q+\omega_{\zeta_j}^q+\omega_{\delta_j}^q\right). \tag{4.2}$$

and then rank the alternatives $\urcorner_j$ and $\urcorner_k$ according to the accuracy degrees $\beth(\aleph_j)$ and $\beth(\aleph_k)$.

Step 3: Rank all the feasible alternatives $\urcorner_j(j=1,2,\ldots,m)$ according to $S(\aleph_j)$ and $\beth(\aleph_j)(j=1,2,\ldots,m)$ and select the optimal one(s).

Step 4: End.

5 ILLUSTRATIVE EXAMPLE AND COMPARATIVE ANALYSIS

In this section, an illustrative model is presented to exhibit the usage procedure of our proposed technique. At that point, the proposed C*q*-ROFWHM and C*q*-ROFWDHM operators are compared.

5.1 Illustrative Example

In this subsection, a practical case concerning the selection of ERP system is presented to exhibit the usage procedure of the proposed MADM model.

Market competitions are warming up increasingly more in the context of the worldwide economy. The competitive market environment forces enterprises to respond intelligently to market changes. It is only by improving management efficiency, shortening the supply period, reducing production costs and

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decreasing inventory storage that enterprises can maintain their dominance in the market. Today, electronic-commerce (e-commerce) has become a new business mode that can provide a smarter model of competition for businesses to compete in the market. At the heart of e-commerce, the ERP system is the back-up support for e-commerce. Because the implementation of the ERP project is complicated system engineering, many companies will face increasing risks during the implementation of the ERP. The low success rate and the high risks of implementation are the main obstacles to the implementation of an ERP. To reduce the risk of implementation and to ensure the success of ERP implementation, the contraction of a set of implementation methodology is necessarily. The selection of the ERP is an important issue in the industrial fields. Assume an organization plans to implement the ERP system (Liao et al., 2007). Firstly, a project team is selected consisting of CIO and three experienced representatives from user departments. To collect all possible information on ERP systems, the project team select five possible ERP systems $\neg_j \left(j = 1, 2, \ldots, 5 \right)$ as candidates. Further, to assess the alternatives, the project team chooses four attributes:

1. Strategic ability ϖ_1;
2. Function and technology ϖ_2;
3. Seller's reputation ϖ_3;
4. Seller's capacity ϖ_4.

The weight vector of the attributes is w=(0.14, 0.27, 0.34, 0.25)^T. Suppose the experts evaluate the five potential ERP systems $\neg_j \left(j = 1, 2, \ldots, 5 \right)$ according to the four attributes, based on Cq-ROFNs and the corresponding decision matrix is constructed in Table 2.

Table 2. Cq-ROF decision matrix

	ϖ**1**	ϖ**2**	ϖ**3**	ϖ**4**
$\neg_1$	((0.7,0.5), (0.3,0.6))	((0.3,0.4), (0.5,0.3))	((0.6,0.8), (0.4,0.6))	((0.3,0.3), (0.2,0.3))
$\neg_2$	((0.4,0.7), 0.8,0.1))	((0.3,0.8), (0.3,0.5))	((0.5,0.4), (0.7,0.5))	((0.6,0.7), (0.7,0.2))
$\neg_3$	((0.8,0.2), (0.6,0.7))	((0.8,0.6), (0.4,0.3))	((0.3,0.8), (0.3,0.5))	((0.5,0.5), (0.3,0.8))
$\neg_4$	((0.3,0.5), (0.6,0.7))	((0.4,0.7), (0.5, 0.6))	((0.6,0.5), (0.5,0.4))	((0.3,0.7), (0.8,0.4))
$\neg_5$	((0.8,0.5), (0.6,0.4))	((0.4,0.6), (0.5,0.2))	((0.8,0.4), (0.1,0.8))	((0.4,0.2), (0.5,0.7))

5.2 Decision-Making Process

The Cq-ROFWHM and Cq-ROFWDHM operators are used to solve the MADM problem with Cq-ROFNs in terms of choosing the most desirable ERP systems, resulting in the following computing steps:

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5.2.1 Decision-Making Process based on the Cq-ROFWHM Operator

Step 1: Utilize the Cq-ROFWHM operator in Eq. (3.4) to aggregate the Cq-ROF assessment values $\aleph_{jk}$ of alternative $\urcorner_j$ on all attributes ϖ_k(k=1,2,3,4) into the overall evaluation value $\aleph_j$ of the alternative $\urcorner_j\left(j=1,2,3,4,5\right)$ (suppose q = 3 and t = 1). The overall evaluation values of alternatives $\urcorner_j\left(j=1,2,3,4,5\right)$ are shown as follows:

$$\aleph_1=\left(\left(0.9176,0.8637\right),\left(0.2066,0.2674\right)\right),\aleph_2=\left(\left(0.8324,0.9129\right),\left(0.3809,0.1661\right)\right),$$

$$\aleph_3=\left(\left(0.9197,0.8696\right),\left(0.2405,0.3472\right)\right),\aleph_4=\left(\left(0.8075,0.8901\right),\left(0.3785,0.3237\right)\right),$$

$$\aleph_5=\left(\left(0.9035,0.8258\right),\left(0.2205,0.2948\right)\right).$$

Step 2: Calculate the score function $S\left(\aleph_j\right)$ of overall evaluation value $\aleph_j\left(j=1,2,3,4,5\right)$ utilizing Eq. (4.1).

$$S\left(\aleph_1\right)=0.8472,\ S\left(\aleph_2\right)=0.8194,\ S\left(\aleph_3\right)=0.8449,\ S\left(\aleph_4\right)=0.7859,\ S\left(\aleph_5\right)=0.8161.$$

Step 3: Rank all alternatives $\urcorner_j\left(j=1,2,3,4,5\right)$ on the basis of the score index. The ranking of all alternatives can be obtained as:

$$\urcorner_1\succ\urcorner_3\succ\urcorner_2\succ\urcorner_5\succ\urcorner_4$$

where the symbol "$\succ$" means "the most preferable". Therefore, the first ERP system $\urcorner_1$ is the best choice.

Step 4: End.

5.2.2 Decision-Making Process based on the Cq-ROFWDHM Operator

Step 1: Utilize the Cq-ROFWDHM operator in Eq. (3.8) to aggregate the Cq-ROF assessment values $\aleph_{jk}$ of alternative $\urcorner_j$ on all attributes ϖ_k(k=1,2,3,4) into the overall evaluation value $\aleph_j$ of the alternative $\urcorner_j\left(j=1,2,3,4,5\right)$ (suppose q = 3 and t = 1). The overall evaluation values of alternatives $\urcorner_j\left(j=1,2,3,4,5\right)$ are shown as follows:

$$\aleph_1=\left(\left(0.3808,0.2977\right),\left(0.7835,0.8374\right)\right),\aleph_2=\left(\left(0.2738,0.4096\right),\left(0.9080,0.7637\right)\right),$$

$$\aleph_3=\left(\left(0.4334,0.2989\right),\left(0.8173,0.8934\right)\right),\aleph_4=\left(\left(0.2401,0.3790\right),\left(0.8947,0.8683\right)\right),$$

$$\aleph_5=(\left(0.3672,0.2474\right),\left(0.8351,0.8773\right)$$

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Step 2: Calculate the score function $S(\aleph_j)$ of overall evaluation value $\aleph_j\ (j=1,2,3,4,5)$ utilizing Eq. (4.1).

$$S(\aleph_1)=0.2534,\ S(\aleph_2)=0.2238,\ S(\aleph_3)=0.2123,\ S(\aleph_4)=0.1743,\ S(\aleph_5)=0.2017.$$

Step 3: Rank all alternatives $\daleth_j\ (j=1,2,3,4,5)$ on the basis of the score index. The ranking of all alternatives can be obtained as:

$$\daleth_1 \succ \daleth_2 \succ \daleth_3 \succ \daleth_5 \succ \daleth_4.$$

Thus, the best alternative is $\daleth_1$.

Step 4: End.

The overall evaluation values, the score functions, and the ranking of alternatives $\daleth_j$ (j=1,2,3,4,5) for t=1,2,3 and 4 are shown in Tables 3-6.

Table 3. Overall evaluation values by Cq-ROFWHM and Cq-ROFWDHM operators

	Overall evaluation value by C*q*-ROFWHM operator		**Overall evaluation value by C*q*-ROFWDHM operator**	
Alternative (t=1)	**Overall evaluation value**	**Score function**	**Overall evaluation value**	**Score function**
$\daleth_1$	((0.9176,0.8637), (0.2066,0.2674))	0.8472	((0.3808,0.2977), (0.7835,0.8374))	0.2534
$\daleth_2$	((0.8324,0.9129), (0.3909,0.1661))	0.8194	((0.2738,0.4096), (0.9080,0.7637))	0.2238
$\daleth_3$	((0.9197,0.8696), (0.2405,0.3472))	0.8449	((0.4334,0.2989), (0.8173,0.8934))	0.2123
$\daleth_4$	((0.8075,0.8901), (0.3785,0.3237))	0.7859	((0.2401,0.3790), (0.8947,0.8683))	0.1743
$\daleth_5$	((0.9035,0.8258), (0.2205,0.2948))	0.8161	((0.3672,0.2474), (0.8351,0.8773))	0.2017
Ranking	$\daleth_1 \succ \daleth_3 \succ \daleth_2 \succ \daleth_5 \succ \daleth_4$		$\daleth_1 \succ \daleth_2 \succ \daleth_3 \succ \daleth_5 \succ \daleth_4$	

5.3 Influence of the Parameters on the Results

As the parameters q and t play significant role in the ranking results. Based on Cq-ROFWHM and Cq-ROFWDHM operators, the impact of parameters q and t on the score functions and ranking results is investigated in the following subsection. To describe the impact of parameters q and t on the score functions and the results of the ranking, we explain the effects of the following two aspects:

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(1) The fixed value is given to $t(t = 2)$ and the effect of the q parameter on the alternative ranking results is examined.
(2) The fixed value is given to q (q = 3) and the effect of the t parameter on the alternative ranking results is examined.

Details are in Tables 7-10. As found in Tables 7-10, the score functions and ranking results for the different parameters q and t can be different based on the proposed MADM approach related to Cq-ROFWHM and Cq-ROFWDHM operators. However, the optimal alternative is $\urcorner_3$ or $\urcorner_1$.

Table 4. Overall evaluation values by Cq-ROFWHM and Cq-ROFWDHM operators

	Overall evaluation value by Cq-ROFWHM operator		**Overall evaluation value by Cq-ROFWDHM operator**	
Alternative (t=2)	**Overall evaluation value**	**Score function**	**Overall evaluation value**	**Score function**
$\urcorner_1$	((0.8875,0.8453), (0.2355,0.2970))	0.8160	((0.4460,0.3459), (0.7719,0.8136))	0.2829
$\urcorner_2$	((0.8216,0.8969), (0.4316,0.2412))	0.7955	((0.2982,0.4362), (0.8845,0.7551))	0.2468
$\urcorner_3$	((0.9040,0.8616), (0.2503,0.3911))	0.8257	((0.4514,0.3963), (0.7842,0.8650))	0.2562
$\urcorner_4$	((0.8029,0.8837), (0.3966,0.3360))	0.7769	((0.2715,0.4002), (0.8799,0.8438))	0.2005
$\urcorner_5$	((0.8772,0.8021), (0.2892,0.3861))	0.7773	((0.4147,0.2848), (0.7889,0.8561))	0.2440
Ranking	$\urcorner_3 \succ \urcorner_1 \succ \urcorner_2 \succ \urcorner_5 \succ \urcorner_4$		$\urcorner_1 \succ \urcorner_3 \succ \urcorner_2 \succ \urcorner_5 \succ \urcorner_4$	

Table 5. Overall evaluation values by Cq-ROFWHM and Cq-ROFWDHM operators

	Overall evaluation value by Cq-ROFWHM operator		**Overall evaluation value by Cq-ROFWDHM operator**	
Alternative (t=3)	**Overall evaluation value**	**Score function**	**Overall evaluation value**	**Score function**
$\urcorner_1$	((0.8728,0.8381), (0.2478,0.3114))	0.8021	((0.4683,0.3813), (0.7675,0.8071))	0.2950
$\urcorner_2$	((0.8174,0.8870), (0.4374,0.2695))	0.7852	((0.3081,0.4449), (0.8742,0.7521))	0.2560
$\urcorner_3$	((0.8984,0.8592), (0.2544,0.4047))	0.8192	((0.4572,0.4210), (0.7766,0.8518))	0.2709
$\urcorner_4$	((0.8012,0.8805), (0.4071,0.3404))	0.7725	((0.2897,0.4062), (0.8741,0.8367))	0.2094
$\urcorner_5$	((0.8678,0.7930), (0.2932,0.4288))	0.7620	((0.4383,0.2942), (0.7598,0.8428))	0.2681
Ranking	$\urcorner_3 \succ \urcorner_1 \succ \urcorner_2 \succ \urcorner_4 \succ \urcorner_5$		$\urcorner_1 \succ \urcorner_3 \succ \urcorner_5 \succ \urcorner_2 \succ \urcorner_4$	

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Table 6. Overall evaluation values by Cq-ROFWHM and Cq-ROFWDHM operators

	Overall evaluation value by C*q*-ROFWHM operator		**Overall evaluation value by C*q*-ROFWDHM operator**	
Alternative (*t*=4)	**Overall evaluation value**	**Score function**	**Overall evaluation value**	**Score function**
$\neg_1$	((0.8661,0.8350), (0.2518,0.3182))	0.7959	((0.4821,0.4108), (0.7654,0.8043))	0.3032
$\neg_2$	((0.8153,0.8801), (0.4398,0.2745))	0.7795	((0.3116,0.4492), (0.8679,0.7506))	0.2611
$\neg_3$	((0.8962,0.8580), (0.2565,0.4124))	0.8161	((0.4602,0.4352), (0.7731,0.8465))	0.2778
$\neg_4$	((0.8004,0.8785), (0.4142,0.3422))	0.7699	((0.3016,0.4086), (0.8715,0.8335))	0.2137
$\neg_5$	((0.8642,0.7888), (0.2948,0.4417))	0.7561	((0.4487,0.2995), (0.7381,0.8336))	0.2840
Ranking	$\neg_3 \succ \neg_1 \succ \neg_2 \succ \neg_4 \succ \neg_5$		$\neg_1 \succ \neg_5 \succ \neg_3 \succ \neg_2 \succ \neg_4$	

Table 7. Score functions by Cq-ROFWHM and Cq-ROFWDHM operators (t = 2 and q∈[1.10])

	Collective decision matrix by C*q*-ROFWHM					**Collective decision matrix by C*q*-ROFWDHM**				
	$S(\neg_1)$	$S(\neg_2)$	$S(\neg_3)$	$S(\neg_4)$	$S(\neg_5)$	$S(\neg_1)$	$S(\neg_2)$	$S(\neg_3)$	$S(\neg_4)$	$S(\neg_5)$
q = 1	0.8722	0.8484	0.8648	0.8294	0.8424	0.2045	0.1833	0.1992	0.1531	0.1767
q = 2	0.8519	0.8290	0.8544	0.8105	0.8155	0.2399	0.2098	0.2223	0.1685	0.2056
q = 3	0.8160	0.7955	0.8257	0.7769	0.7773	0.2829	0.2468	0.2562	0.2005	0.2440
q = 4	0.7796	0.7616	0.7941	0.7422	0.7407	0.3219	0.2819	0.2892	0.2345	0.2798
q = 5	0.7464	0.7307	0.7638	0.7102	0.7081	0.3548	0.3125	0.3187	0.2664	0.3107
q = 6	0.7171	0.7034	0.7359	0.6819	0.6799	0.3819	0.3385	0.3442	0.2951	0.3370
q = 7	0.6916	0.6796	0.7108	0.6573	0.6558	0.4040	0.3603	0.3658	0.3205	0.3591
q = 8	0.6695	0.6589	0.6884	0.6361	0.6352	0.4218	0.3786	0.3842	0.3427	0.3778
q = 9	0.6503	0.6409	0.6684	0.6178	0.6177	0.4362	0.3939	0.3997	0.3620	0.3936
q= 10	0.6338	0.6254	0.6507	0.6022	0.6028	0.4478	0.4068	0.4128	0.3788	0.4070

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Table 8. Ranking results by Cq-ROFWHM and Cq-ROFWDHM operators (t = 2 and q∈[1,10])

	Ranking results by C*q*-ROFWHM	**Ranking results by C*q*-ROFWDHM**
q = 1	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 2	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 3	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 4	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 5	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 6	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 7	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 8	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q = 9	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
q= 10	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	$\gamma_1 \succ \gamma_3 \succ \gamma_5 \succ \gamma_2 \succ \gamma_4$

Table 9. Score functions by Cq-ROFWHM and Cq-ROFWDHM operators (q = 3 and t = 1, 2, 3, 4)

	Collective decision matrix by C*q*-ROFWHM					**Collective decision matrix by C*q*-ROFWDHM**				
	$S(\gamma_1)$	$S(\gamma_2)$	$S(\gamma_3)$	$S(\gamma_4)$	$S(\gamma_5)$	$S(\gamma_1)$	$S(\gamma_2)$	$S(\gamma_3)$	$S(\gamma_4)$	$S(\gamma_5)$
t = 1	0.8472	0.8194	0.8449	0.7859	0.8161	0.2534	0.2238	0.2123	0.1743	0.2017
t = 2	0.8160	0.7955	0.8257	0.7769	0.7773	0.2829	0.2468	0.2562	0.2005	0.2440
t = 3	0.8021	0.7852	0.8192	0.7725	0.7620	0.2950	0.2560	0.2709	0.2094	0.2681
t = 4	0.7959	0.7795	0.8161	0.7699	0.7561	0.3032	0.2611	0.2778	0.2137	0.2840

5.4 Comparative Analysis

To show the superiority and advantage of our developed method, we will compare the developed method with some existing methods in literature with C*q*-ROFSs. Liu et al. (2020) proposed the DM approach on the basis of the C*q*-ROFWA and C*q*-ROFWG operators. Darko and Liang (2020) set forward the DM method based on Hamacher weighted averaging operator with *q*-ROFS. Riaz et al. (2020) extended

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the Einstein weighted average and geometric operator with q-ROFS. Further, we compare the proposed approaches with complex intuitionistic fuzzy WHM (CIFWHM) operator, complex intuitionistic fuzzy WDHM (CIFWDHM) operator, complex Pythagorean fuzzy WHM (CPFWHM) operator and complex Pythagorean fuzzy WDHM (CPFWDHM) operator. We used these approaches to the illustrative example above in Cq-ROF context and compared the decision results with the developed approach for Cq-ROFSs in this paper. The results of these approaches are summarized in Table 11.

Table 10. Ranking results by Cq-ROFWHM and Cq-ROFWDHM operators (q = 3 and t = 1, 2, 3, 4)

	Ranking results by Cq-ROFWHM	**Ranking results by Cq-ROFWDHM**
t = 1	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	$\gamma_1 \succ \gamma_2 \succ \gamma_3 \succ \gamma_5 \succ \gamma_4$
t = 2	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
t = 3	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_3 \succ \gamma_5 \succ \gamma_2 \succ \gamma_4$
t = 4	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$	$\gamma_1 \succ \gamma_5 \succ \gamma_3 \succ \gamma_2 \succ \gamma_4$

Table 11. Scores and ranking values by WA and WG operators

	Scores and ranking values by WA aggregation operator		**Scores and ranking values by WG aggregation operator**	
Methods	**Score of alternatives**	**Ranking**	**Score of alternatives**	**Ranking**
Liu et al. (2020) (λ=1, q=3)	0.6286 0.5543 0.6117 0.5118 0.5699	$\gamma_1 \succ \gamma_3 \succ \gamma_5 \succ \gamma_2 \succ \gamma_4$	0.5270 0.4807 0.5273 0.4685 0.4576	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$
Riaz et al. (2020) (λ=2, q=3)	0.6156 0.5479 0.6034 0.5080 0.5614	$\gamma_1 \succ \gamma_3 \succ \gamma_5 \succ \gamma_2 \succ \gamma_4$	0.5337 0.4864 0.5361 0.4738 0.4658	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$
Darko and Liang. (2020) (λ=3, q=3)	0.6077 0.5443 0.5985 0.5058 0.5563	$\gamma_1 \succ \gamma_3 \succ \gamma_5 \succ \gamma_2 \succ \gamma_4$	0.5373 0.4898 0.5409 0.4768 0.4709	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_4 \succ \gamma_5$
CIFWHM (t=2,q=1)	0.8722 0.8484 0.8648 0.8294 0.8424	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	0.2045 0.1833 0.1992 0.1531 0.1767	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
CPFWHM (t=2,q=2)	0.8519 0.8290 0.8544 0.8105 0.8155	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	0.2399 0.2098 0.2223 0.1685 0.2056	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$
Our developed method (t=2,q=3)	0.8160 0.7955 0.8257 0.7769 0.7773	$\gamma_3 \succ \gamma_1 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$	0.2829 0.2468 0.2562 0.2005 0.2440	$\gamma_1 \succ \gamma_3 \succ \gamma_2 \succ \gamma_5 \succ \gamma_4$

From the above analysis, the ranking results of the methods above are obviously slightly different; however the optimal alternative is γ_1 or γ_3. This verifies that the developed Cq-ROFWHM and Cq-ROFWDHM operators are reasonable and useful for MADM problems with Cq-ROFNs.

(1) The downside of Liu et al.'s (2020) operator is that it does not recognize the interrelation to arguments. It presumes, in other words, that all attributes are separate, which is to some extent incorrect. In illustrative example, to select the most desirable ERP system, we have to consider not only

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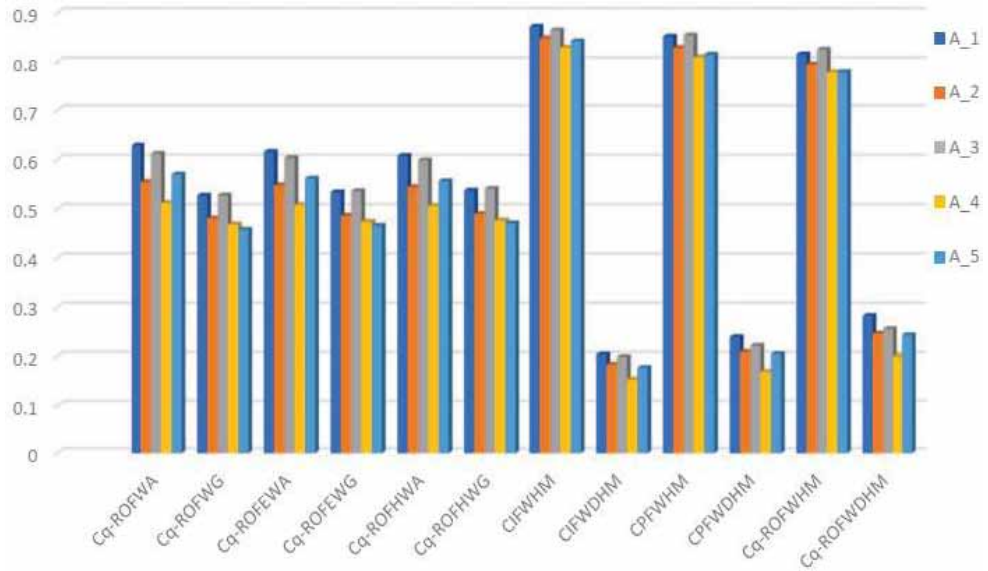
the attribute values of each alternative but also the correlation between these attributes. Thus, the approaches, C*q*-ROFWA and C*q*-ROFWG, are unsuitable for this problem. As our approach can easily capture variable correlations, it is more realistic to tackle this problem than the approaches of (Liu et al., 2020).

(2) The ranking results of proposed methods with (Darko and Liang 2020) and (Riaz et al., 2020) are slightly different. (Darko and Liang 2020) and (Riaz et al., 2020) are based on *q*-ROFSs. As *q*-ROFS is a special case of C*q*-ROFS. That is, when imaginary membership and imaginary non-membership are zero, then C*q*-ROFS is reduced to *q*-ROFS. Evidently, C*q*-ROFS is more and more general, and contains more information in the MADM process. Moreover, (Darko and Liang 2020) and (Riaz et al., 2020) do not reflect the interrelationship between attributes. Interrelation between attributes is very crucial almost in all MADM problems. Therefore, our developed method provides more general and powerful information in MADM problems.

(3) CIFS is the special case of C*q*-ROFS (when $q = 1$). Comparing with the CIFWHM and CIFWDHM operators, we obtained slightly different ranking results as the proposed method but the optimal choice is γ_4 or γ_3. These operators can only aggregate the complex intuitionistic fuzzy numbers (CIFNs) and CIFNs must satisfy the conditions, i.e, $0 \leq \zeta + \delta \leq 1$ and $0 \leq \omega_\zeta + \omega_\delta \leq 1$. In this example, most of the evaluation does not meet these conditions. So, application range of these operators is limited and not suitable for this example of decision-making.

(4) Comparing with the CPFWHM and CPFWDHM operators, we obtained the same ranking results as the proposed method but these operators can only aggregate the complex Pythagorean fuzzy numbers satisfying the conditions $0 \leq \zeta^2 + \delta^2 \leq 1$ and $0 \leq \omega_\zeta^2 + \omega_\delta^2 \leq 1$. The range of CPFS is wider than CIFS but limited than Cq-ROFS as it is the special case of Cq-ROFS (when q = 2). Thus, proposed method is more generalized.

Figure 2. Comparison with some other approaches



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In Figure 2, we make a graphical comparison among the ranking results of the proposed methods with some other approaches. From this bar graph, an overview of the ranking results of different approaches can be interpreted and analyzed. It can be seen that above techniques have different characteristics to the changes in the criteria input values and the ranking results are consistent which shows the validity of the proposed methods.

5.5 Advantages of the Proposed Work

Different aggregation operators have different functions, and the decision maker can choose suitable aggregation operators according to the real decision-making environment. Our method therefore has some benefits and superiorities when compared to other methods. The merits of our developed approach are summarized as follows:

- Cq-ROFHM operator uses Cq-ROFS as the information representation and Cq-ROFS can provide more comprehensive assessment details as it combines the excellent aspects of the Cq-ROFS. Moreover, more general decision-making circumstances can be accessibly represented by the Cq-ROFSs of the established method.
- The present analysis precedes the fact that the aggregation operators in IFS, PFS and q-ROFS are the particular cases of the given model, since the presented method can also be used to solve MADM problems under the existing techniques considering zero-phase terms with $q = 1$, 2, and q, respectively, while the existing models are not capable of solving the MADM problems in the presented environment.
- It is based on HM so that the attribute interrelationship can be interpreted and it can be widely applied in various cases by assigning preference values with different parameter t. Thus, Cq-ROFHM operator has an outstanding ability to describe the interaction amongst multi-input parameters and to apply them efficiently in the information fusion process.
- Due to the existence of the t parameter, Cq-ROFWHM and Cq-ROFWDHM operators can provide a more flexible and robust information fusion than other operators, as these operators become algebraic and Bonferroni mean operators, respectively, for $t = 1, 2$. We have used HM and DHM with more generalized fields in this model, i.e., Cq-ROFSs, which gave the more and more prevalent results.

6 CONCLUSION

The Cq-ROFS is an effective model that cannot only depict the complex ambiguous evaluation information, but also minimize the loss of information in MADM problems. Inspired by this idea, some novel Cq-ROFHM aggregation operators, such as Cq-ROFHM, Cq-ROFWHM, Cq-ROFDHM and Cq-ROFWDHM operator have been put forward for aggregating Cq-ROFNs. Subsequently, a novel approach to MADM based on Cq-ROFWHM operator and Cq-ROFWDHM operator has been proposed in Cq-ROF circumstances. Further, a numerical example associated with the ERP system selection has been presented to demonstrate the validity of the proposed notions in decision-making. Comparison analysis has been conducted and the superiorities have been illustrated. We have seen approximately that the q and t parameters of aggregation operators do also influence the alternatives ranks. The Cq-ROFWHM

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and C*q*-ROFWDHM operators have been distinguished from other operators, not only because the operators accommodate the C*q*-ROFNs, but also because the interdependent phenomena are considered among the attributes. In future, Hamy mean and dual Hamy mean operators will be extended to: (1) Cubic *q*-rung orthopair fuzzy sets; (2) Complex cubic *q*-rung orthopair fuzzy sets; and (3) Linguistic cubic *q*-rung orthopair fuzzy sets. Further, we can expand such aggregation operators' implementations to other domains.

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Chapter 8
Some New Distance and Similarity Algorithms for Pythagorean Fuzzy Sets With Application in Decision-Making Problems

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ABSTRACT

This chapter introduces some new distance and similarity measures for PFSs which resolve the inaccuracies observed in the existing measures. The objectives of this work are to introduce a new distance and similarity measures for PFSs, encapsulate the measures in an algorithm, and explore their applicability in some decision-making problems. Numerical experiments are supplied to ascertain the superiority of the new distance and similarity measures over the existing measures. Some decision-making problems such as students' career placement and pattern recognition represented in Pythagorean fuzzy pairs are addressed using the new distance and similarity measures coded with JAVA programming language.

1 INTRODUCTION

Decision-making is part and parcel of everyday life. In decision-making, identification and selection of choices based on the preferences of decision-makers are considered using some relevant assessment criteria. The prominent bottleneck of decision-making is the uncertainty of some data or variables. To cope with such uncertainty, Zadeh (1965) initiated the idea of fuzzy sets to model the uncertainty in human knowledge. It suffices to state that fuzzy models are not reliable because it considered only the

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membership degree (MD) of the concerned data. Atanassov (1986) initiated intuitionistic fuzzy sets (IFSs) to amend the setback of the ordinary fuzzy sets, to better model real-life problems. IFS constituents membership degree (MD) μ and non-membership degree (NMD) ν with the possibility of the existent of hesitation margin (HM) π where their aggregate is one and $\mu + \nu \leq 1$. Because of the appropriateness of IFSs, it has been used to solved many practical decision-making problems (De et al., 2001; Ejegwa et al., 2014; Ejegwa and Onyeke, 2021; Szmidt and Kacprzyk, 2001). The ideas of distance and similarity measures have been instrumental in solving many real-life problems in intuitionistic fuzzy domain. Burillo and Bustince (1996) initiated the concept of distances for IFSs and interval-valued fuzzy sets. Hatzimichailidis et al. (2012) proposed a novel distance measure between IFSs and applied it to pattern recognition. Boran and Akay (2014) presented a biparametric approach of similarity measure between IFSs and applied it to pattern recognition. A novel similarity measure of IFSs based on transformation techniques with pattern recognition application have been discussed (Chen and Chang, 2015). The diagnosis of viral hepatitis based on distance measure of IFSs was explained in (Ejegwa and Modom, 2015). Ejegwa and Onyeke (2018) discussed an object-oriented approach based on some distance measures and applied them to competency based test evaluation. A medical diagnostic reasoning using similarity measure for IFSs was explicated in (Szmidt and Kacprzyk, 2004a,b). Wang and Xin (2005) proposed a new distance measure between IFSs and its weighted version with application to the solution of pattern recognition problem, and Yen et al. (2013) proposed a new method of similarity measure for pattern recognition. An elaborate exposition on distance and similarity measures with some applications were discussed in (Szmidt, 2014; Szmidt and Kacprzyk, 2000).

Albeit important the notion of IFSs, there are cases where the sum of MD and NMD is more than one. For instance, if $\mu = \frac{\sqrt{3}}{2}$ and $\nu = \frac{1}{2}$, then IFS is handicapped to model such a problem. The shortcoming in IFSs naturally led to the introduction of intuitionistic fuzzy set of the second type (IFSST) (Atanassov, 1999), which is referred to Pythagorean fuzzy sets (PFSs) (Yager, 2013; Yager and Abbasov, 2013). PFS proposed a new tool to deal with vagueness considering MD μ and NMD ν satisfying the conditions, $\mu + \nu \geq 1$ and $\mu^2 + \nu^2 \leq 1$. As a generalized set, PFS has close relationship with IFS. PFS can be used to characterize the uncertain information more accurately than IFS. Some modal operators have been defined on PFSs (Ejegwa, 2019a). Because of the flexibility of the notion of PFSs, it has been used to addressed some real-life problems (Ejegwa, 2020d; Ejegwa et al., 2020a,b; Ejegwa and Jana, 2021; Ejegwa et al., 2021). A method for tackling multi-attribute decision-making with interval-valued Pythagorean fuzzy linguistic information was presented in (Du, et al.). Linguistic PFSs was proposed by Garg (2018) and applied to multi-attribute decision-making. Liang and Xu (2017) presented a new extension of TOPSIS method for multiple criteria decision-making with hesitant PFSs. Some aggregation operators using Einstein operator, Einstein t-norm and Einstein t-conorm defined in Pythagorean fuzzy environment for decision-making were discussed in (Garg, 2016, 2017). Distance measure is a soft computing device use to find the distance between two arbitrary PFSs akin to metric function. On the other hand, similarity measure is the dual of distance measure. Distance and similarity measures have been utilized in resolving many real-life problems in Pythagorean fuzzy domain. Zhang and Xu (2014) initiated the study on distance measure in Pythagorean fuzzy context by proposing a distance measure applied in the extension of TOPSIS to multiple criteria decision making. Li and Zeng (2018) introduced a new distance measure between PFSs with real-life applications. Some distance and similarity measures between PFSs have been introduced and characterized (Ejegwa, 2020a). Many studies have

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been carried out on Pythagorean fuzzy distance/similarity measures with applications to personnel appointment (Ejegwa, 2019b), pattern recognition (Ejegwa, 2020b,c; Ejegwa and Awolola, 2021) and multiple criteria group decision-making (Zeng et al., 2018).

The notion of distance and similarity measures between IFSs/PFSs studied in (Burillo and Bustince, 1996; Ejegwa, 2020a,b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014) are observed to have some inaccuracies due to the omission of Pythagorean fuzzy indexes, which needed to be strengthened to enhance reliability of the outputs. Thus, the motivation of this study is to introduce new distance and similarity measures between PFSs with better performance indexes compare to the existing measures. The specific objectives of the chapter are to; (i) explore some existing distance and similarity measures between PFSs, (ii) propose new distance and similarity measures between PFSs, (iii) present comparison of the new distance and similarity measures for PFSs with the existing measures, (iv) write an algorithm for the proposed measures, and (v) apply the distance and similarity algorithm in students' career placement and pattern recognition coded with JAVA programming language. The chapter is thus outlined; Section 2 presents some mathematical background of PFSs and discusses some existing distance and similarity measures for PFSs (Burillo and Bustince, 1996; Ejegwa, 2020 a,b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014), Section 3 introduces the new distance and similarity measures between PFSs with numerical comparison, Section 4 discusses the applications of the proposed distance and similarity measures in students' career placement and pattern recognition using an algorithm coded with JAVA programming language, and Section 5 draws conclusion with recommendation for future studies.

2 PRELIMINARIES

This section presents some mathematical background of PFSs and discusses some distance and similarity measures for PFSs (see Burillo and Bustince, 1996; Ejegwa, 2020a,b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014).

2.1 Pythagorean Fuzzy Sets

Myriad of works have been done on the mathematical background of IFSs and PFSs (see Atanassov, 1986, 1999; Yager, 2013; Yager and Abbasov, 2013). Here, some basic concepts of PFSs are presented to be used in the subsequent sections. Let us assume that X is a non-empty set throughout this chapter.

Definition 2.1. (Yager, 2013) A Pythagorean fuzzy sets A of X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | \, x \in X\}, \tag{1}$$

where the functions μ_A, ν_A: $X \to [0,1]$ define MD and NMD of $x \in X$ for

For a PFS

$$0 \leq \mu_A^2(x) + v_A^2(x) \leq 1. \;\; A \; in \; X, \pi_A(x) \in [0,1] = \sqrt{1 - \mu_A^2(x) - v_A^2(x)}$$

is the PFS index or hesitation margin of A.

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Definition 2.2. (Yager and Abbasov, 2013) Let A and B in X be PFSs, then for all $x \in X$ we have

(i) $A = B$ iff $\mu_A(x) = \mu_B(x)$ $\nu_A(x) = \nu_B(x)$.
(ii) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$.
(iii) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$.
(iv) $A \cup B = \{\langle x, \max[\mu_A(x),\mu_B(x)],\min[\nu_A(x),\nu_B(x)]\rangle | x \in X\}$.
(v) $A \cap B = \{\langle x, \min[\mu_A(x),\mu_B(x)],\max[\nu_A(x),\nu_B(x)]\rangle | x \in X\}$.

Definition 2.3. (Ejegwa, 2019a) Pythagorean fuzzy values (PFVs) or Pythagorean fuzzy pairs (PFPs) are characterized by the form $\langle x,y\rangle$ such that $x^2 + y^2 \leq 1$ where $x,y \in [0,1]$. PFVs evaluate the PFS for which the components (x and y) are interpreted as MD and NMD.

2.2 Measures for Pythagorean Fuzzy Sets

Measure in terms of distance and similarity measures are soft computing tools use in the applications of PFSs. Distance and similarity measures for PFSs are presented as follow.

2.2.1 Distance for Pythagorean Fuzzy Sets

The concept of distance measure between PFSs has been studied in literature. What follows is the definition of distance between PFSs:

Definition 2.4. (Ejegwa, 2020a) If A and B are PFSs of X, then the distance D between A and B is a function D: $PFS \times PFS \rightarrow [0,1]$ which satisfies

(i) $0 \leq D(A,B) \leq 1$ (boundedness)
(ii) $D(A,B) = 0$ if and only if $A = B$ (separability)
(iii) $D(A,B) = D(B,A)$ (symmetric)
(iv) $D(A,C) \leq D(A,B) + D(B,C)$, where C is also a PFS of X (triangle inequality).

When $D(A,B)$ approaches 0, it shows that A and B are more close or related with better performance index. Again, if $D(A,B)$ approaches 1 then A and B are not related or close. Let $X = \{x_1,...,x_n\}$ then for any two PFSs A and B of X, we present the following distances between PFSs.

Burillo and Bustince Distance Measures

By studying the distances between fuzzy sets presented in (Diamond and Kloeden, 1994), Burillo and Bustince (1996) introduced the following distances:

Hamming distance;

$$D_1(A,B) = \frac{\sum_i^n 1\left[\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|v_A(x_i) - v_B(x_i)\right|\right]}{2}. \quad (2)$$

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Euclidean distance;

$$D_2\left(A,B\right)=\sqrt{\frac{\sum_{i=1}^{n}\left[\left(\mu_A(x_i)-\mu_B(x_i)\right)^2+\left(v_A(x_i)-v_B(x_i)\right)^2\right]}{2}}. \tag{3}$$

normalised Hamming distance;

$$\begin{aligned} D_3\left(A,B\right)&=\frac{D_1\left(A,B\right)}{n}\\ \Rightarrow D_3\left(A,B\right)&=\sqrt{\frac{\sum_{i=1}^{n}\left[\left|\mu_A\left(x_i\right)-\mu_B\left(x_i\right)\right|+\left|v_A\left(x_i\right)-v_B\left(x_i\right)\right|\right]}{2n}}. \end{aligned} \tag{4}$$

normalised Euclidean distance;

$$\begin{aligned} D_4\left(A,B\right)&=\frac{D_2\left(A,B\right)}{\sqrt{n}}\\ \Rightarrow D_4\left(A,B\right)&=\sqrt{\frac{\sum_{i=1}^{n}\left[\left(\mu_A(x_i)-\mu_B(x_i)\right)^2+\left(v_A(x_i)-v_B(x_i)\right)^2\right]}{2n}}. \end{aligned} \tag{5}$$

In Eqs. (2–5), it is observed that the denominator signifies the number of parameters involved in the computation like in (Diamond and Kloeden, 1994), where the denominator is unity because fuzzy set considers only membership function. However, the approaches in (Burillo and Bustince, 1996) cannot produce reliable results because hesitation margin is omitted.

Szmidt and Kacprzyk Distance Measures

Because of the limitation in (Burillo and Bustince, 1996), Szmidt and Kacprzyk (2000) proposed the following distances incorporating hesitation margin:

Hamming distance;

$$D_5\left(A,B\right)=\frac{\sum_{i=1}^{n}\left[\left|\mu_A(x_i)-\mu_B(x_i)\right|+\left|v_A(x_i)-v_B(x_i)\right|+\left|\pi_A(x_i)-\pi_B(x_i)\right|\right]}{2} \tag{6}$$

Euclidean distance;

$$D_6\left(A,B\right)=\sqrt{\frac{T}{2}}, \tag{7}$$

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where

$$T = \sum_{i=1}^{n}\left[\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + \left(\pi_A(x_i) - \pi_B(x_i)\right)^2\right]$$

normalised Hamming distance;

$$\begin{aligned} D_7\left(A,B\right) &= \frac{D_5\left(A,B\right)}{n} \\ \Rightarrow D_7\left(A,B\right) &= \frac{\sum_{i=1}^{n}\left[\left|\mu_A\left(x_i\right) - \mu_B\left(x_i\right)\right| + \left|v_A\left(x_i\right) - v_B\left(x_i\right)\right| + \left|\pi_A\left(x_i\right) - \pi_B\left(x_i\right)\right|\right]}{2n} \end{aligned} \tag{8}$$

normalised Euclidean distance;

$$D_8\left(A,B\right) = \frac{D_6\left(A,B\right)}{\sqrt{n}} \Rightarrow D_8\left(A,B\right) = \sqrt{\frac{T}{2n}}, \tag{9}$$

where

$$T = \sum_{i=1}^{n}\left[\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + \left(\pi_A(x_i) - \pi_B(x_i)\right)^2\right]$$

Although the distances presented by Szmidt and Kacprzyk (2000) capture the three parameters of PFSs, a cognizance is not taken with regards to the number of parameters in the denominator as seen in (Burillo and Bustince, 1996).

Zhang and Xu Distance Measure

In Zhang and Xu (2014), a distance measure for PFSs was proposed as shown as follows:

$$D_9\left(A,B\right) = \frac{\sum_{i=1}^{n}\left[\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| + \left|v_A^2(x_i) - v_B^2(x_i)\right| + \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|\right]}{2} \tag{10}$$

Modified Zhang and Xu Distance Measure

In Ejegwa (2020b), a distance measure for PFSs was proposed which normalized the distance measure introduced by Zhang and Xu (2014). The distance is given by

$D_{10}\left(A,B\right) = \frac{D_9\left(A,B\right)}{n}$, i.e.

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$$D_{10}\left(A,B\right)=\frac{\sum_{i=1}^{n}\left[\left|\mu_A^2(x_i)-\mu_B^2(x_i)\right|+\left|v_A^2(x_i)-v_B^2(x_i)\right|+\pi_A^2(x_i)-\pi_B^2(x_i)\right]}{2n} \tag{11}$$

Eqs. (10,11) capture the three parameters of PFSs, but do not take cognizance of the number of parameters in the denominator.

2.2.2 Similarity for Pythagorean Fuzzy Sets

The notion of similarity measure is the dual of the concept of distance measure. Before considering similarities of PFSs, we first define the idea of similarity in Pythagorean fuzzy context.

Definition 2.5. (Ejegwa, 2020a) Suppose A and B are PFSs of X, then the similarity measure S between A and B is a function S: $PFS \times PFS \rightarrow [0,1]$ which satisfies

(i) $0 \leq S(A,B) \leq 1$ (boundedness)
(ii) $S(A,B) = 1$ if and only if $A = B$ (separability)
(iii) $S(A,B) = S(B,A)$ (symmetric)
(iv) $S(A,C) \leq S(A,B) + S(B,C)$, where C is also a PFS of X (triangle inequality).

When $S(A,B)$ approaches 1, it shows that A and B are more close or related with better performance index. Again, if $S(A,B)$ approaches 0 then A and B are not related or close. Let $X = \{x_1,...,x_n\}$ then for any two PFSs A and B of X, we present the following similarities between PFSs as the dual of the distance measures in Eqs. (6-11).

$$S_1\left(A,B\right)=\frac{2-\sum_{i=1}^{n}\left[\left|\mu_A(x_i)-\mu_B(x_i)\right|+\left|v_A(x_i)-v_B(x_i)\right|+\left|\pi_A(x_i)-\pi_B(x_i)\right|\right]}{2} \tag{12}$$

$$S_2\left(A,B\right)=\frac{2-\sqrt{2T}}{2}, \tag{13}$$

where

$$T=\sum_{i=1}^{n}\left[\left(\mu_A(x_i)-\mu_B(x_i)\right)^2+\left(v_A(x_i)-v_B(x_i)\right)^2+\left(\pi_A(x_i)-\pi_B(x_i)\right)^2\right].$$

$$S_3\left(A,B\right)=\frac{2n-\sum_{i=1}^{n}\left[\left|\mu_A(x_i)-\mu_B(x_i)\right|+\left|v_A(x_i)-v_B(x_i)\right|+\left|\pi_A(x_i)-\pi_B(x_i)\right|\right]}{2n} \tag{14}$$

$$S_4\left(A,B\right)=\frac{2n-\sqrt{2nT}}{2n} \tag{15}$$

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where

$$T = \sum_{i=1}^{n}\left[\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(v_A(x_i) - v_B(x_i)\right)^2 + \left(\pi_A(x_i) - \pi_B(x_i)\right)^2\right]$$

$$S_5\left(A,B\right) = \frac{2 - \sum_{i=1}^{n}\left[\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| + \left|v_A^2(x_i)v_B^2(x_i)\right| + \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|\right]}{2} \tag{16}$$

$$S_6\left(A,B\right) = \frac{2n - \sum_{i=1}^{n}\left[\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| + \left|v_A^2(x_i)v_B^2(x_i)\right| + \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|\right]}{2n} \tag{17}$$

3 NEW DISTANCE AND SIMILARITY BETWEEN PFSS

Now, we present new distance measure of PFSs and its dual measure, respectively.

Definition 3.1. Let $X = \{x_1,...,x_n\}$. Then for any two PFSs A and B of X, the distance D(A,B) is given by

$$D\left(A,B\right) = \frac{\sum_{i=1}^{n}\left[\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| + \left|v_A^2(x_i)v_B^2(x_i)\right| + \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|\right]}{3n} \tag{18}$$

Eqs. (18) captures the three parameters of PFSs and also takes cognizance of the number of parameters in the denominator. In a quick succession, the similarity S(A,B) is

$$S\left(A,B\right) = \frac{3n - \sum_{i=1}^{n}\left[\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| + \left|v_A^2(x_i)v_B^2(x_i)\right| + \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|\right]}{3n} \tag{19}$$

since similarity and distance are dual of each other.

Proposition 3.2. *Let A and B be PFSs in X. If* D(A,B) *is a distance measure between PFSs A and B, then* S(A,B) = 1 − D(A,B) *is a similarity measure of A and B. Again, If* S(A,B) *is a similarity measure between PFSs A and B, then* D(A,B) = 1 − S(A,B) *is a distance measure of A and B.*

Proposition 3.3. *Let A and B are PFSs in X. Then*

(i) D(A,B) = D($\overline{A}$,$\overline{B}$)
(ii) S(A,B) = S($\overline{A}$,$\overline{B}$).

Theorem 3.4. *Let A and B be PFSs in X, then the function* D(A,B) *satisfies*

(i) $0 \leq$ D(A,B) ≤ 1,
(ii) D(A,B) = 0 *iff* $A = B$,

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(iii) $D(A,B) = D(B,A)$.

Proof. The proof of (i) is straightforward. To proof (ii), we suppose that $D(A,B) = 0$. Then

$$\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| = 0, \left|v_A^2(x_i) - v_B^2(x_i)\right| = 0, \text{ and } \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right| = 0,$$

and thus,

$$\mu_A^2(x_i) = \mu_B^2(x_i), v_A^2(x_i) = v_B^2(x_i) \text{ and } \pi_A^2(x_i) = \pi_B^2(x_i),$$

hence $A = B$. The converse is easy so we omit. Because

$$D(A,B) = \frac{\sum_{i=1}^{n}\left[\left|\mu_A^2(x_i) - \mu_B^2(x_i)\right| + \left|v_A^2(x_i) - v_B^2(x_i)\right| + \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|\right]}{3n}$$
$$= \frac{\sum_{i=1}^{n}\left[\left|\mu_B^2(x_i) - \mu_A^2(x_i)\right| + \left|v_B^2(x_i) - v_A^2(x_i)\right| + \left|\pi_B^2(x_i) - \pi_A^2(x_i)\right|\right]}{3n},$$

we say $D(A,B) = D(B,A)$, and so (iii) holds. Hence, the function $D(A,B)$ is a distance measure of A and B.

Corollary 3.5. *Let A and B be PFSs in X, then the function* $S(A,B)$ *satisfies*

(i) $0 \le S(A,B) \le 1$,
(ii) $S(A,B) = 1$ *iff* $A = B$,
(iii) $S(A,B) = S(B,A)$.

Theorem 3.6. *If A, B and C are PFSs in X, and* $A \subseteq B \subseteq C$, *then*

(i) $D(A,C) \ge D(A,B)$,
(ii) $D(A,C) \ge D(B,C)$,
(iii) $D(A,C) \ge \max[D(A,B), D(B,C)]$.

Proof. Because $A \subseteq B \subseteq C$, it is easy to see that

$$\left|\mu_A^2(x_i) - \mu_C^2(x_i)\right| \ge \left|\mu_A^2(x_i) - \mu_B^2(x_i)\right|,$$

$$\left|v_A^2(x_i) - v_C^2(x_i)\right| \ge \left|v_A^2(x_i) - v_B^2(x_i)\right|,$$

and

$$\left|\pi_A^2(x_i) - \pi_C^2(x_i)\right| \ge \left|\pi_A^2(x_i) - \pi_B^2(x_i)\right|.$$

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Thus

$$\left|\mu_A^2(x_i)-\mu_C^2(x_i)\right|+\left|\nu_A^2(x_i)-\nu_C^2(x_i)\right|+\left|\pi_A^2(x_i)-\pi_C^2(x_i)\right|$$

$$\geq\left|\mu_A^2(x_i)-\mu_B^2(x_i)\right|+\left|\nu_A^2(x_i)-\nu_B^2(x_i)\right|+\left|\pi_A^2(x_i)-\pi_B^2(x_i)\right|,$$

implying that $D(A,C) \geq D(A,B)$, which proves (i). Using the same argument, (ii) is proved, that is, $D(A,C) \geq D(B,C)$. Synthesizing (i) and (ii), we have $D(A,C) \geq \max[D(A,B),D(B,C)]$, which proves (iii).

Corollary 3.7. *If A, B and C are PFSs in X, and $A \subseteq B \subseteq C$, then*

(i) $S(A,C) \leq S(A,B)$,
(ii) $S(A,C) \leq S(B,C)$,
(iii) $S(A,C) \leq \max[S(A,B),S(B,C)]$.

Theorem 3.8. *Suppose A, B and C are PFSs in X, then the distance and similarity functions of the PFSs satisfy triangle inequality.*

Proof. The distance functions of PFSs *A*, *B* and *C* satisfy triangle inequality if $D(A,C) \leq D(A,B) + D(B,C)$. Suppose

$$\begin{aligned} D(A,C) &= \max\left(\left|\mu_A^2(x_i)-\mu_C^2(x_i)\right|+\left|v_A^2(x_i)-v_C^2(x_i)\right|+\left|\pi_A^2(x_i)-\pi_C^2(x_i)\right|\right) \\ &= \left|\mu_A^2(x_k)-\mu_C^2(x_k)\right|+\left|v_A^2(x_k)-v_C^2(x_k)\right|+\left|\pi_A^2(x_k)-\pi_C^2(x_k)\right| \end{aligned}$$

for some fixed k, $1 \leq k \leq n$, that is, the maximum is attained at k. Then

$$|\mu^2{}_A(x_k)-\mu^2{}_C(x_k)| \leq |\mu^2{}_A(x_k)-\mu^2{}_B(x_k)|+|\mu_B{}^2(x_k)-\mu^2{}_C(x_k)|,$$

$$\left|\nu_A^2(x_k)-\nu_C^2(x_k)\right| \leq \left|\nu_A^2(x_k)-\nu_B^2(x_k)\right|+\left|\nu_B^2(x_k)-\nu_C^2(x_k)\right|,$$

$$\left|\pi_A^2(x_k)-\pi_C^2(x_k)\right| \leq \left|\pi_A^2(x_k)-\pi_B^2(x_k)\right|+\left|\pi_B^2(x_k)-\pi_C^2(x_k)\right|,$$

and

$$D(A,B)=\left|\mu_A^2(x_k)-\mu_B^2(x_k)\right|+\left|\nu_A^2(x_k)-\nu_B^2(x_k)\right|+\left|\pi_A^2(x_k)-\pi_B^2(x_k)\right|,$$

$$D(B,C)=\left|\mu_B^2(x_k)-\mu_C^2(x_k)\right|+\left|\nu_B^2(x_k)-\nu_C^2(x_k)\right|+\left|\pi_B^2(x_k)-\pi_C^2(x_k)\right|.$$

Hence, $D(A,C) \leq D(A,B) + D(B,C)$. The proof that the similarity functions of PFSs *A*, *B* and *C* satisfy triangle inequality is similar.

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3.1 Comparison of the Measures

The measures in (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014) and the proposed measures are applied to find the distances and similarities between some PFSs to enhance comparison.

Example 3.9. Let A, B and C be PFSs in $X = \{x_1, x_2, x_3\}$ defined as

$A = \{\langle x_1,0.6,0.2\rangle,\langle x_2,0.4,0.6\rangle,\langle x_3,0.5,0.3\rangle\}$,

$B = \{\langle x_1,0.8,0.1\rangle,\langle x_2,0.7,0.3\rangle,\langle x_3,0.6,0.1\rangle\}$,

$C = \{\langle x_1,0.9,0.2\rangle,\langle x_2,0.8,0.2\rangle,\langle x_3,0.7,0.3\rangle\}$.

Using Eqs. (6-11) and Eq. (18), we get the following distances in Table 1.

Table 1. Distances

Methods	(A,B)	(A,C)	(B,C)
$\mathcal{D}_5$	0.7232	0.9894	0.5662
$\mathcal{D}_6$	0.3974	0.5671	0.2826
$\mathcal{D}_7$	0.2411	0.3298	0.1887
$\mathcal{D}_8$	0.2294	0.3274	0.1632
$\mathcal{D}_9$	0.7200	1.1700	0.5600
$\mathcal{D}_{10}$	0.2400	0.3900	0.1867
$\mathcal{D}$	**0.1600**	**0.2600**	**0.1244**

From Table 1, it is seen that the new method of finding distance between PFSs satisfied the conditions in Definition 2.4 unlike the method in (Zhang and Xu, 2014). The new method gives the better performance index compare to the methods in (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014).

Similarly, using Eqs. (12-17) and Eq. (19), we obtain the following similarities in Table 2.

From Table 2, it is seen that the new approach of finding similarity between PFSs satisfied the conditions in Definition 2.5 unlike S_9. The new approach yields better performance index compare to the dual of the methods in (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014).

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Table 2. Similarities

Methods	(A,B)	(A,C)	(B,C)
$\mathcal{S}_5$	0.2768	0.0106	0.4338
$\mathcal{S}_6$	0.6026	0.4329	0.7174
$\mathcal{S}_7$	0.7589	0.6702	0.8113
$\mathcal{S}_8$	0.7706	0.6726	0.8368
$\mathcal{S}_9$	0.2800	−0.1700	0.4400
$\mathcal{S}_{10}$	0.7600	0.6100	0.8133
$\mathcal{S}$	**0.8400**	**0.7400**	**0.8756**

Example 3.10. Let A, B and C be PFSs in $X = \{x_1, x_2, x_3, x_4, x_5\}$ defined as

$A = \{<x_1,0.6,0.4>, <x_2,0.5,0.7>, <x_3,0.8,0.4>, <x_5,0.7,0.2>\}$,

$B = \{<x_1,0.7,0.3>, <x_3,0.4,0.7>, <x_4,0.9,0.2>\}$,

$C = \{<x_2,0.6,0.4>, <x_3,0.7,0.3>, <x_4,0.5,0.4>\}$.

These PFSs can be rewritten as

A= $\{<x_1,0.6,0.4>, <x_2,0.5,0.7>, <x_3,0.8,0.4>, <x_4,0,1>, <x_5,0.7,0.2>\}$,

B= $\{<x_1,0.7,0.3>, <x_2,0,1>, <x_3,0.4,0.7>, <x_4,0.9,0.2>, <x_5,0,1>\}$,

C= $\{<x_1,0,1>, <x_2,0.6,0.4>, <x_3,0.7,0.3>, <x_4,0.5,0.4>, <x_5,0,1>\}$.

Using Eqs. (6-11) and Eq. (18), we obtain the following distances in Table 3.

From Table 3, it is seen that the new method of finding distance between PFSs satisfied the conditions in Definition 2.4 unlike D_5, D_6 and D_9. The new method gives better performance index compare to the methods in (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014).

Similarly, using Eqs. (12-17) and Eq. (19), we get the following similarities in Table 4.

From Table 4, it is seen that the new approach of finding similarity between PFSs satisfied the conditions in Definition 2.5 unlike S_5, S_6 and S_9. The new approach yields better performance index compare to the dual of the methods in (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014).

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Table 3. Distances

Methods	(A,B)	(A,C)	(B,C)
$\mathcal{D}_5$	3.3360	3.4652	2.8391
$\mathcal{D}_6$	1.4305	1.4481	1.2646
$\mathcal{D}_7$	0.6672	0.6930	0.5678
$\mathcal{D}_8$	0.6398	0.6476	0.5655
$\mathcal{D}_9$	2.8900	3.1900	2.7100
$\mathcal{D}_{10}$	0.5780	0.6380	0.5420
$\mathcal{D}$	**0.4053**	**0.4253**	**0.3613**

In conclusion, the new distance and similarity measures for PFSs yield more reliable and reasonable results compare to the hitherto methods discussed in (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014), and as such we consider the applicability of the new methods of measuring distance and similarity in the next section.

Table 4. Similarities

Methods	(A, B)	(A,C)	(B,C)
$\mathcal{S}_5$	-2.3360	-2.4652	-1.8391
$\mathcal{S}_6$	-0.4305	-0.4481	-0.2646
$\mathcal{S}_7$	0.3328	0.3070	0.4322
$\mathcal{S}_8$	0.3602	0.3524	0.4345
$\mathcal{S}_9$	-1.8900	-2.1900	-1.7100
$\mathcal{S}_{10}$	0.4220	0.3620	0.4580
$\mathcal{S}$	**0.5947**	**0.5747**	**0.6387**

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4 APPLICATIONS IN DECISION-MAKING PROBLEMS

This section discusses some decision-making problems. For instance, suppose there are n alternatives represented as PFPs P_i for $i = 1,...,n$, considered in a feature space X. Assume there is a sample alternative represented as PFP Q to be associated with any of P_i. The value

$$D(P_i,Q) = \wedge[D(P_1,Q),...,D(P_n, Q)], \tag{20}$$

where $D(P_i,Q)$ states the distance between P_i and Q decides the decision-making. Similarly, The value

$$S(P_i,Q) = \vee[S(P_1,Q),...,S(P_n,Q)], \tag{21}$$

where $S(P_i,Q)$ states the similarity between P_i and Q determines the decision-making.. If Eqs. (20) and (21) exist, then we assert that the alternative Q is associated with any of such P_i for $i = 1,...,n$.

Now, we consider cases of students' career placement and pattern recognition to show the application of the new methods in decision-making problems.

4.1 Applicative Example in Students' Career Placement

Here, we consider career placement based on academic performance, which is the prominent factor to be assessed in admission process leading to a career development. Supposing in a typical career placement scenario, there are four high school students $S = \{S_1, S_2, S_3, S_4\}$ who are competing to be admitted to read course C in a particular university. Assume the subjects requirement to read course C is the set $R = \{r_1, r_2, r_3, r_4, r_5\}$, where r_1 = English studies, r_2 = Maths, r_3 = Physics, r_4 = Chemistry, and r_5 = Biology. After the students S took a multi-choice based qualification exams marked over % on the required subjects R to ascertain their suitability, their scores are represented by PFVs as follow:

$S_1 = \{<r_1,0.6,0.3>, <r_2,0.5,0.4>, <r_3,0.6,0.2>,<r_4,0.5,0.3>,<r_5,0.5,0.5>\}$,

$S_2 = \{<r_1,0.5,0.3>,<,r_2,0.6,0.2>,<r_3,0.5,0.3>,<r_4,0.4,0.5>,<r_5,0.7,0.2>\}$,

$S_3 = \{<r_1,0.7,0.1>,<r_2,0.6,0.3>,<r_3,0.7,0.1>,<r_4,0.5,0.4>,<r_5,0.4,0.5>\}$,

$S_4 = \{<r_1,0.9,0.2>,<r_2,1.0,0.0>,<r_3,0.5,0.3>,<r_4,0.8,0.2>,<r_5,0.4,0.4>\}$.

The university required scores for admission to read C is a Pythagorean fuzzy scores in each of the required subjects as follows:

$C = \{<r_1,0.8,0.1>,<r_2,0.7,0.2>,<r_3,0.9,0.0>,<r_4,0.6,0.3>,<r_5,0.8,0.1>\}$.

The admission process is determined by computing the distance/similarity using the new methods between each of the students S_i, for $i = 1,2,3,4$ and the university scores on the required subjects for C. Then, any student whose distance/similarity values with the course C is the smallest/greatest, is suitable to be admitted to study C.

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By incorporating Eqs. (18) and (19), we obtain the following algorithm for the computing of distance/similarity between S_i, $i = 1,...,4$ and C.

4.1.1 Algorithm for Computing Distance and Similarity Measures between Students and Course

PRE:

mu[rk] is membership degrees of students (Si), nuSi[rk] is non-membership degrees of Si, piSi[rk] is hesitation margins of Si where $i = 1,...,4$; muC[rk] is membership degrees of courses (C), nuC[rk] is non-membership degrees of C, piC[rk] is hesitation margins of C; $R = \{r_k\}$, $k = 1,...,5$, $n = 5$; dist is the new method of distance measure and simi is the new method of similarity measure.

POST: this algorithm calculates the distances and similarities between Si and C.

STEPS:

i Set the value for n
ii Initialize values for Si
iii Initialize values for C
iv Repeat for r = 1 to n

```
            Set sum = math.abs((math.pow(muSi[rk],2.0)-math.pow(muC[rk],2.0)))
+
                                math.abs((math.pow(nuSi[rk],2.0)-math.
pow(nuC[rk],2.0)))+
                                math.abs((math.pow(piSi[rk],2.0)-math.
pow(piC[rk],2.0)))
        End for
```

v Set dist = sum/(3*n)
vi Set simi = 1- dist
vii Exit

After coding using JAVA programming language, we obtain the following distance and similarity values;

$D(S_1,C) = 0.1960$, $D(S_2,C) = 0.3867$, $D(S_3,C) = 0.5453$, $D(S_4,C) = 0.8160$.

Also,

$S(S_1,C) = 0.8040$, $S(S_2,C) = 0.6133$, $S(S_3,C) = 0.4547$, $S(S_4,C) = 0.1840$.

From the results in both distance and similarity cases using Eqs. (20) and (21), the course C is suitable to be read by student S_1.

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4.2 Applicative Example in Pattern Recognition

Assume we have four patterns $C = \{C_1, C_2, C_3, C_4\}$ represented as PFVs in a given feature space $X = \{x_1, x_2, x_3\}$ as follow:

$C_1 = \{<x_1,1.0, 0.0>,<x_2,0.8,0.0>,<x_3,0.7,0.1>\}$,

$C_2 = \{<x_1,0.6, 0.2>,<x_2,0.8,0.0>,<x_3,1.0,0.0>\}$,

$C_3 = \{<x_1,1.0, 0.0>,<x_2,0.75,0.32>,<x_3,0.55,0.4>\}$,

$C_4 = \{<x_1,0.8,0.1>,<x_2,1.0,0.0>,<x_3,0.9,0.0>\}$.

Suppose there is an unclassified pattern P captured by PFVs in the same given feature space X as

$P = \{<x_1,0.5,0.3>,<x_2,0.6,0.2>,<x_3,0.8,0.1>\}$.

Then, the aim is to associate P with any of C_i, for $i = 1,2,3,4$ by computing the distance/similarity among P and C_i with the aid of the new methods.

Using Eqs. (18) and (19), we obtain the following algorithm for computing the distance/similarity between C_i, $i = 1,...,4$ and P.

4.2.1 Algorithm for Computing Distance and Similarity Measures between Patterns

PRE:

muCi[xk] is membership degrees of patterns (Ci), nuCi[xk] is non-membership degrees of Ci, piCi[xk] is hesitation margins of Ci where $i = 1,...,4$; muP[xk] is membership degrees of unclassified pattern (P), nuP[xk] is non-membership degrees of P, piP[xk] is hesitation margins of P; $X = \{x_k\}$, $k = 1,...,3$, $n = 3$; dist is the new method of distance measure and simi is the new method of similarity measure.

POST: this algorithm calculates the distances and similarities between Ci and P.

STEPS:

i Set the value for n

ii Initialize values for Ci

iii Initialize values for P

iv Repeat for x = 1 to n

```
      Set sum = math.abs((math.pow(muCi[xk],2.0)-math.pow(muP[xk],2.0)))+
  math.abs((math.pow(nuCi[xk],2.0)-math.pow(nuP[xk],2.0)))+
  math.abs((math.pow(piCi[xk],2.0)-math.pow(piP[xk],2.0)))
End for
```

v Set dist = sum/(3*n)

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vi Set simi = 1- dist
vii Exit

After coding using JAVA programming language, the following distance and similarity values are obtained;

$D(C_1,P) = 0.2622$, $D(C_2,P) = 0.4289$, $D(C_3,P) = 0.7294$, $D(C_4,P) = 0.9961$.

Also,

$S(C_1,P) = 0.7378$, $S(C_2,P) = 0.5711$, $S(C_3,P) = 0.2706$, $S(C_4,P) = 0.0039$.

From the results in both distance and similarity cases using Eqs. (20) and (21), the unclassified pattern P can be classified with the pattern C_1.

5 CONCLUSION

Pythagorean fuzzy set as a soft computing tool is applicable in diverse areas of applications using several measures like distance and similarity. This chapter has proposed new distance and similarity between PFSs using object-oriented approach to enhance performance and efficiency. Some set of examples were considered to ascertain the superiority of the new distance and similarity measures over the existing approaches (Ejegwa, 2020b; Szmidt, 2014; Szmidt and Kacprzyk, 2000; Zhang and Xu, 2014). By comparison, the new distance and similarity measures between PFSs have better accuracy indexes and minimize time of computation because they are based on an algorithmic approach implemented/run by a programming language. In terms of application, the new distance and similarity measures were applied to determine students' career placement and pattern recognition.

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Chapter 9
Fuzzy Decision Making on the Impacts of Corporate Environmental Responsibility and Citizen Science Approaches Towards Green Globe

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ABSTRACT

A green environment is one of the sustainability goals, and all nations are striving hard to make the environment free from pollution. The overwhelming corporate sectors should devise strategies towards promoting and preserving the environment from getting degraded by manufacturing activities. To control the environmental degradation, the industries should devise strategies to attain the goal of the green globe through the sub-goals such as livelihood sustainability, community progression, energy conservation, consumer compatibility, and resources revamp. This chapter explores the possible ways of handling the environmental mishaps, and it throws light on the possible contributions of corporate environmental responsibility and citizen science that involves the participatory approaches of the citizens as a whole. To make a decision on 16 proposed strategies, the hybrid decision-making method on multi goals cum strategies is proposed, and the integrated approach of FUCOM (full consistency method fuzzy additive ration assessment) and fuzzy ARAS (fuzzy additive ratio assessment) is applied.

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INTRODUCTION

Nature is the beautiful gift to the mankind and it is the responsibility of the populace of this universe to conserve it for the future generation. The resources are created for utilizing and channelizing into productive entities but presently in this era of technology the natural resources are exploited to greater extent without restoration. The voices are raised in favour of environmental conservation but the actions towards the mitigation of environmental calamities are not significant as the stability of the ecosystem is intervened by several factors and majority of the contribution is made by the human race. It is high time for the citizens of each nation and as the whole world to act immediately to avoid the environmental disasters. The reflection of environmental catastrophe is witnessed in these days and the futuristic predictions on environmental degradation has elevated the environmental concerns amidst many environmentalists which has drawn the attention of several volunteers and also this has paved the way for the conceptualization of citizen science. (Davi et al.,2017)

The environment has to be protected from the hurdles and obstacles that interludes the progression of the bio diversity and the sustenance of species. Many living organisms are becoming exotic and endangered and the changing environmental setup are becoming a threat to many species. (Blaney.,2016) A special focus has to be laid towards the conservatism but on profound examination, it is indeed a herculean task for the governments and non-governmental organizations, but to accomplish such kind of tasks, the volunteers and environmental well-wishers are made to involve in collection and analysis of data pertaining to environmental protection and monitoring and it is collectively called as citizen science. The observational framework for environment in United Kingdom has stated the dynamics of citizen science in vivid manner. (Loiselle et al 2016) Participation in environmental conservation is quite natural but collecting data and making analysis by the volunteers is a new concept and that is the hidden agenda of citizen science.

1.1 Need of Citizen Science

Is citizen science the need of the hour? Is it essential to involve volunteers in monitoring the environmental happenings? Will it bring a greater change or effectiveness to the environment? The answer is certainly yes. It is the prime responsibility of every citizen of the world to exercise environmental protecting activities to conserve and transfer the natural resources to the utility of the future generation (Newman et al. 2017).The natural resources are being depleted erratically without any control limits. In the name of modern technological era and infrastructural development the beautiful nature is being disturbed and degraded. The natural resources are classified into renewable and non-renewable resources. Some of the resources are refurbished very easily but not all, let us take water, the elixir of life. (Wei et al ., 2016). The water resources are bountiful but the quality of water is declining at recent times due to abdominal human activities and it is very blatantly visualized. The unhealthy practices of mankind has destroyed the purity of water. The oxygen content in the water is being decreased and it has made the resource unfit for the living organisms to survive. The mankind activities have also disturbed the quality of air. The emission of pollutants has initiated and intensified the disruption in the ozone layer and the radiations of UV rays have almost triggered global warming to greater extent (Wei et al ., 2016).The industrial sectors are the major contributors to all kinds of pollution. The findings and the inventions that make the life of human comfortable have indeed made the nature discomfort and it cannot be denied.

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The environmental costing though inadvertently taken up by the industrial sectors are the namesake indicators to make them escape form the environmental regulation imposed by the government in view of protecting the environment (Aceves et al .,2015). Plastic usage is yet another threat to our nature as the life span of plastic in is infinite, though research is taking place in discovering a new alternatives for plastic, the high costs of such products are not consumed by all as the problem of affordability exists at all levels (Ballard.,2016). One sector of companies have started working on projects of producing bio-friendly products, eco-conscious items, biodegradable products and so on, but these kinds of products are also expensive and therefore the reach of such kinds of products to the people is not upto the expected levels(Burgess et al.,2016). The environmental sustainable measures are taken but inspite of these remedial measures, the existence of environmental problems is perpetual and this is indeed a threat to our nature. The environmental conserving activities are less than environmental degrading activities which should be taken care of, and this is to be taken as the prime responsibility of every nation. The nature is quite common to all men and it is shared by all and also the responsibility of conserving it must be shared by all the people and that is the underlying principle of citizen science.

1.2 Approaches of Citizen Science

Citizen science involves people of all ages and people of all groups of diverse backgrounds, it takes place completely under voluntary grounds, the gathered community of people frame research questions, collect data and perform the analysis on the collected data and present inference and actionize the remedial measures towards the conservation of the environment. The group comprises of the participation of professional scientists, environmentalists to certain extent for adding strength and enhancing the effectiveness of the dynamism of the group. These tasks are taken up as projects and it is scaled to minor or major based on the nature of the project undertaken. The project centrals around any kind of organisms and the intention is to conserve the extinctions of species especially the rare living organisms (Donough et al.,2016).

The projects are taken deliberately across the world and it is governed by the respective environmental conserving organizations that is established and being functioning in every nation. Citizen science do not aim in collecting and analysing the data alone but rather it lays a suitable platform for creating public awareness and also to promote environmental concerns amidst the public through various quality initiatives. The participative approach in such kind of projects are adding glow to these kind of voluntary initiatives, in general the voluntary and cooperatives endeavours end in great success and citizen science approaches are certainly beneficial to the community as whole. The entire process of completing the project demands meticulous planning and implementation also it demands exercising suitable strategies of involving the volunteers and professionals to maximum extent for carrying out the project in a more convincing manner. The projects undertaken are varied in nature and it encompasses so many aspects and the project completion also requires systematic approaches and tactics to handle the mishaps that take place at times of project execution (Latapí et al.,2019).

The citizen science do not demand all the volunteers to work together always, the individual contribution may help and it may get added to the collaborative work. It is very easy for any individual to become a citizen scientist. It is very easy to render one's participation towards environmental conservation without making any kind of physical movements of many miles. The technology has made it very easier and simple, enrolling as citizen scientist is very simple as one gets attuned to any of the websites that support it. As per the interest of the volunteer, he or she may enrol and contribute towards any of

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the dimensions of the ecology, for instance if one is very delightful in knowing or contributing towards the examination or interrogation of the effect of climate change on the growth of the plants of a specific species, after beginning an account in the respective website, the interest shall be registered and the observations of the species pertaining to the impact of the climate change can be made periodically, age is certainly not a limit or restriction for the eligibility or qualifying checks for the citizen scientists. The use of smart technology has eased the process of enrolling and contributing towards the data collection, data sharing, systematic analysis and inferences drawing. Will the uploading of the information surely make a great difference in the minds of the people? It can be a million dollar questions that pricks the minds of the readers. But it is making so, it is not just a single upload rather it is a collective approach which will definitely make considerate changes in the mind set of the people. The data pool is filled with these kinds of uploads and information sharing, at many a times the social media platforms are also used and people are encouraged to join in such portals and the people are constantly motivated to take up such initiatives. Also the data collected serves the purpose of undertaking intensive study on the environment.

1.3 Corporate Environmental Responsibilities (CER) and Citizen Science

The ascending rate of Corporatization is creating major impacts on the economic and environmental setting of the universe. The corporates are the monarchs of the global markets and act as authoritarians in deciding the destiny of the business run. The production sectors are greatly influenced by the ramifications caused in the world economy, on the other hand these production sectors are also contributing to the surge of environmental complications. Is it because of the laxity of the production sectors or the legislations? On deep examination it is the individual laxity that has put the entire globe under intensive care (Tempels et al.,2017). The health of the environment is mainly curbed by pollutants and resource exploitation. The whole business world is drenched into the nuances of the corporate sector. New trends of business portfolio is getting emerged and it is being nourished by the existing elements of the production scenario which are influenced by the external environmental factors. The pandemic situation has also created a huge impact on the business channels. The production sectors are functioning on the principle of turning materials to products and this process will get ceased if the materials are getting exhausted at rapid rate. Is it not the responsibility of the users of these materials to restore back or carry out any kind of replacement activities? The industrial sectors take part in increasing the economic growth of the nation at the same time it has the social responsibility towards building green nation by preventing the globe from environmental hazards. The company is formulating several strategies in enhancing and extending its business but to combat the environmental mishaps at the same time, these industrial giants must formulate environmental strategies to contribute towards promoting green globe. One of the core responsibilities of these corporate sectors is to safeguard the environment and conserve it for the future generation. The accumulation of profit with environmental degradation will certainly not benefit the community and henceforth these corporate sectors shall encourage and support citizen science and as a next step these corporate sectors could channelize the initiatives of citizen science towards the accomplishment of their responsibilities. The collaborative endeavours pertaining to the environmental conservation will brighten the green signals towards green globe (Murata et al.,2018).

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1.4 Instances of Citizen Science

There are many noteworthy projects to illustrate the efficacy and the significance of citizen science such as many projects covering the following investigation.

- Freshwater death of turtles in Australia
- Air quality monitoring
- Patterns of Bird migration
- Quality check of water
- The Impact of light pollution
- Effect of climate change on plant growths
- Pollution and health

These are some instances of citizen science, on profoundly inspecting the dynamics and modalities of such projects, the involvement of citizens and their collaborative contribution together with the scientist ends in creating a kind of awareness to the public. The people are made to involve in such environmental related activities and this itself is an indication for setting things right to promote the growth of the environment.

The citizen science projects are undertaken and suitable toolkits are also prepared to make the processes involved in cognitive science more systematic and logic in all its way through. The trends in citizen science is subjected change and it is duly influenced by the dynamic aspects. The advanced technology are also employed in exploring the environmental happenings and the wide spatial coverage also takes place. The geographical disparities are also taken into consideration and wide range of data is also collected through modern techniques, also another added advantage of citizen science, the individuals involvement who are also excelled in one or other ways and all the added potentials are synchronized together for a global outcome. Many of the citizen projects involves the usage of observatory to address many environmental problems.

1.5 Collaborative Impacts of CER and Citizen Science on Environmental Harmony

As a part of this universe, the human species is designated as the supremacy of all creations by the scripture and in the code of religion. It is the mankind who should shoulder such an ultimate responsibility of preserving the nature from the mishaps of their actions though intentional or not so. The environment is subjected to degradation by the natural settings and it is quite inevitable, but the man-made disasters or the catastrophes that devastate many lives should be mitigated, controlled and extinguished for ever and ever (Latapí, 2017). The predictions and forecasting of the state of natural resources are quite alarming and it threatens out sustainability and it puts our survival into a pool of questions. Amidst such an inconvincible situation the initiatives of citizen science is really a helping hand for all kinds of people as it facilitate not just group participatory approaches rather it paves way for collaborative endeavours and involvement of public. It works on the principle of public professional to public. It increases the participation of the people; it expands the environmental concerns and also it strengthens the environmental endeavours. The participation of the volunteers do not just enable them in collecting data, it also facilitate the union of oneself with the nature and it instigates their social and environmental responsibilities. The

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oneness with the environment is very essential and each individual must feel that their responsibility towards conserving nature is not a single activity that must be taken up once in their life, rather it is a never ending process and it continues till the last breathe of an individual (Edmundas et al.,2011)

The joint ventures will certainly create green paths but how will it take place, it is essential to formulate suitable strategies to attain the goal of green globe, but how ? the green globe is the embodiment of Livelihood sustainability, Community progression, Energy conservation, Consumer compatibility and Resources revamp, these embodiments are considered as the sub-goals which should be reached with suitable choice of the strategies and this leads into the origin of the decision making.

The process of decision making is piled up with many interconnected sub processes, amidst which the two prime process are determination of criterion weight and ranking of the chosen alternatives. Decision making process in general consists of the following steps involved, selection of the alternatives and the criteria; construction of the initial decision making matrix; normalization of the matrix, criterion weight calculation, computation of weighted normalized matrix, finding the final score, ranking alternatives based on the score. The decision making methods available in literature used are ample and also the efficiency of these methods are compared with one another to make sensitivity analysis.

The Decision making process is based on criteria and it is termed as Multi criteria Decision Making (MCDM) and it is divided into Multi Attribute Decision Making (MADM) and Multi Objective Decision Making (MODM). The MADM methods encircle around discrete space and MODM methods enfold around continuous space (Edmundas et al.,2011).

MCDM methods that are widely applied ranges from simple to intricate methodology. These methods are sometimes direct in approach whereas others do not. The validity of the proposed MCDM methods depends on the feasibility in computing the optimal solutions. The decision matrix in MCDM methods may be deterministic or fuzzy in nature, but in the existing scenario the relation between alternatives and criteria cannot be represented as discrete values. Also MCDM methods involves the expert's opinion may be a single or many experts. Hwang and Yoon presented the grouping of MCDM methods. The most common MCDM methods are Weighted Sum method (Fishburn,1967), Weighted Product Method (Bridgman, Miller and Starr, 1922), Analytic Hierarchy Process (Saaty,1986), Revised Analytic Hierarchy Process (Belton and Gear, 1983), ELECTRE (Benayoun, et al.,1966), TOPSIS (Hwang and Yoon, 1981), PROMTHEE (Brans and P. Vicke, 1982), SMART (Edwards et al., 1994), VIKOR (Opricovic, 1998), MOORA (Zavadskas et al., 2002), COPRAS (Zavadskas et al.,1994), ARAS (Zavadskas et al.,1987), DEMATEL ((Fontela & Gabus, 1976), EDAS (Mehdi et al., 2015), Best-Worst method (Jafar Rezaei, 2020), MARCOS (Stevic et al.,2020), SWARA (Dragisa et al.,2015). In addition to the above described methods, the representation of data in MCDM approaches are also drawing new attention at recent times, Riaz et al has used the representations of linear Diophantine fuzzy sets (Riaz et al.,2019), Spherical Linear Diophantine Fuzzy Sets (Riaz et al.,2020a), m-polar neutrosophic soft mapping (Riaz and Hashmi.,2021), Q-rung Orthopair fuzzy soft sets (Riaz et al.,2020b), Pythagorean fuzzy soft Topology (Riaz et al.,2020c), Bipolar Picture fuzzy Operators (Riaz et al.,2020d) in MCDM.

There are many methods to calculate criterion weight and one of the newly initiated method is FUCOM by Dragan Pamuca (Dragan et al 2018). The FUCOM method is highly advantageous over the existing methods of weight calculation such as AHP and BWM in the context of smaller number of pairwise comparison, consistency and weight calculation FUCOM methods are applied in various decision making settings such as selection of optimal materials, man power, ranking and evaluation of alternatives (Fazlollahtabar et al.,2019). The method of FUCOM is also used in combination with other MCDM methods such as SWARA, MARCOS, WASPAS, ARAS (Feizi et al.,2021,). These hybrid method are

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discussed in classical sense and some in fuzzy environment. The integrated method of FUCOM and Fuzzy ARAS is discussed to make decisions on strategies adoption by the corporate sectors to attain the goals of green globe. Also FUCOM-ARAS (Hoan & Ha.,2021) has been discussed but the concept of fuzzy employed in ARAS in combination with FUCOM has not been explored by the researchers so far and this research work is the new initiative in this regard.

Table 1. Acronyms

ELECTRE	Elimination Et Choice Translating Reality
TOPSIS	Technique For Order Of Preference By Similarity To Ideal Solution
SMART	Simple Multi-Attribute Rating Technique
VIKOR	VIekriterijumsko KOmpromisno Rangiranje
MOORA	Multi-Objective Optimization By Ratio Analysis
COPRAS	Complex Proportional Assessment
ARAS	Additive Ration Assessment
DEMATEL	Decision Making Trial And Evaluation Laboratory
EDAS	Evaluation Based On Distance From Average Solution
MARCOS	Measurement Alternatives And Ranking According To The Compromise Solution
SWARA	Stepwise Weight Assessment Ratio Analysis
FUCOM	Full Consistency Method
AHP	Analytical Hierarchy Process
BWM	Best Worst Method

The following sections of the chapter consists of the methodology and its application towards making decisions on strategies towards green globe.

METHODOLOGY

Decision Making on Multi Goals cum Strategies using FUCOM-FARAS

Step 1:

The alternatives and the criteria are initially determined based on the requisite of the decision making on the strategic plan of the business firm. The decision making process involves two major parts one is determination of significance of the goals and the other is the ranking of the strategies.

Step 2:

Let G = (G1,G2,..Gm) be the set of goals and they are ranked based on its significance and its essentials, for instance $G_{(1)} > G_{j(2)} > \ldots > Gj_{(l),}$ where l denotes the ordering of the goals.

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Step 3:

The pairwise comparison of the goals are made to compute the value of weight coefficient $\pi_{\frac{l}{l+1}} = \frac{s_l}{s_{l+1}}$ and it must satisfy the property of transitivity, $\pi_{\frac{l}{l+1}} \otimes \pi_{\frac{l+1}{l+2}} = \pi_{\frac{l}{l+2}}$

Step 4:

A nonlinear programming model is formulated as follows and it is solved using Lingo software to calculate the goal significances.

Min ρ

Subject to

$$\left| \frac{s_{h(l)}}{s_{h(l+1)}} - \pi_{\frac{l}{l+1}} \right| \leq \rho$$

$$\left| \frac{s_{h(l)}}{s_{h(l+2)}} - \pi_{\frac{l}{l+1}} \otimes \pi_{\frac{l+1}{l+2}} \right| \leq \rho$$

$$\sum_{h=1}^{m} w_h = 1 \text{ for all h}$$

$w_h \geq 0$, for all h

Step 5:

Formulate the initial decision making matrix comprising of the strategies and the goals. The values in the matrix comprises of linguistic variables and are quantified using triangular fuzzy numbers of the form (p,q,r) with the membership function defined as

$$\begin{cases} \frac{x-p}{q-p}, & x \in [p,q] \\ \frac{s-x}{s-r}, & x \in [q,r] \\ 0 & x \langle p \text{ and } x \rangle r \end{cases}$$

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Step 6:

The goals are to be maximized and the optimal value x_{0j} is considered to be the maximum. For making the calculation easier, the triangular fuzzy values are defuzzified using the method of centroid (p+q+r)/3.

Step 7:

The decision matrix is normalized (N) by dividing each entry with its column sum and the weighted normalized matrix (W) is obtained.

$$n_{ij} = \frac{x_{ij}}{\sum_{i=0}^{n} x_{ij}}, \; w_{ij} = s_{ij} * x_{ij}, \; j = 1,..m$$

Step 8:

The optimality value Oi of each alternatives are calculated and the degree of utility Ui of the alternatives are determined. The ranking is made on the final scores.

$$Oi = \sum_{j} w_{ij}$$

DMMGS USING FUCOM-FARAS

The goals and the strategies of the decision making problem is defined as follows

Table 2. Goals

Goal: Green Globe	
Sub Goals: G1: Livelihood sustainability G2:Community progression G3: Energy conservation G4: Consumer compatibility G5: Resources revamp	**Strategies:** S1:Efficient utilization of Resources S2:Optimal Techniques of waste management S3:Green Technologies of production S4:Resource substitution and restoration S5:Environmental accounting S6:Reflecting social responsibility towards resource consumption S7:Using alternative raw inputs S8:Strict adherence to environmental regulations S9:Imposing green initiatives in company's premises S10:Company and public collaborative endeavours S11:Reinforcing individual's environmental concern S12:Computerized Data collection on environmental changes S13:Simulation of green ingenuities S14:Strengthening of networks with environmentalists S15:Environmental consultancy prior to materialization S16:Promotion of Green research

The prime goal of green globe shall be attained by achieving these sub-goals as the goal green globe encompasses these sub goals.

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The significance of the sub goals are determined by the following steps

The ranking of the sub goals by three decision makers D1, D2 & D3.

The pair wise comparison of the sub goals

	D1	D2	D3
G1	3	1	1
G2	5	3	5
G3	1	4	2
G4	4	5	3
G5	2	2	4

The formulation of the non-linear programming problem is

	D1	D2	D3
G1	2	1	1
G2	3	2	3
G3	1	3	2
G4	4	4	2
G5	2	3	3

Min ρ

Subject to

$$\left|\frac{s_3}{s_1}-2\right|\le\rho,\left|\frac{s_1}{s_5}-1\right|\le\rho,\left|\frac{s_5}{s_2}-1.5\right|\le\rho,\left|\frac{s_2}{s_4}-1.34\right|\le\rho,\left|\frac{s_3}{s_5}-2\right|\le\rho,\left|\frac{s_5}{s_4}-2.01\right|\le\rho$$

$$\sum_{h=1}^{5} w_h = 1 \text{ for all h}$$

$w_h \ge 0$, for all h

By solving using Lingo software the significance of the sub goals are attained.

Table 3. Significance of the sub goals

s_1	s_2	s_3	s_4	s_5
0.1936	0.1291	0.3873	0.0964	0.1936

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The linguistic decision making matrix representing the satisfaction degree of the strategies towards the sub goals is as follows

	C1	C2	C3	C4	C5
S1	VH	H	H	M	M
S2	H	H	M	M	L
S3	H	H	H	M	M
S4	M	M	H	M	VH
S5	M	L	H	M	VH
S6	H	H	M	M	M
S7	M	M	M	L	M
S8	H	M	M	M	M
S9	M	VH	M	M	M
S10	H	VH	M	M	M
S11	M	M	L	L	L
S12	L	L	M	M	M
S13	M	M	M	M	M
S14	M	M	M	M	M
S15	H	M	M	M	M
S16	M	M	M	H	M

Table 4. Quantification of Linguistic Variable

Triangular Quantification	Linguistic Variable For Criterion Satisfaction	Defuzzified value
(0,0,0.1)	Very Lowly Satisfied	0.033
(0,0.1,0.5)	Lowly Satisfied	0.2
(0.5,0.6,0.7)	Moderately Satisfied	0.6
(0.7,0.8,0.9)	Highly Satisfied	0.8
(0.9,1,1)	Very Highly Satisfied	0.97

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The Normalized matrix

0.090065	0.077369	0.08	0.066667	0.062893
0.07428	0.077369	0.06	0.066667	0.020964
0.07428	0.077369	0.08	0.066667	0.062893
0.05571	0.058027	0.08	0.066667	0.101677
0.05571	0.019342	0.08	0.066667	0.101677
0.07428	0.077369	0.06	0.066667	0.062893
0.05571	0.058027	0.06	0.022222	0.062893
0.07428	0.058027	0.06	0.066667	0.062893
0.05571	0.09381	0.06	0.066667	0.062893
0.07428	0.09381	0.06	0.066667	0.062893
0.05571	0.058027	0.02	0.022222	0.020964
0.01857	0.019342	0.06	0.066667	0.062893
0.05571	0.058027	0.06	0.066667	0.062893
0.05571	0.058027	0.06	0.066667	0.062893
0.07428	0.058027	0.06	0.066667	0.062893
0.05571	0.058027	0.06	0.088889	0.062893

The weighted normalized matrix

0.017437	0.009988	0.030984	0.006427	0.012176
0.014381	0.009988	0.023238	0.006427	0.004059
0.014381	0.009988	0.030984	0.006427	0.012176
0.010786	0.007491	0.030984	0.006427	0.019685
0.010786	0.002497	0.030984	0.006427	0.019685
0.014381	0.009988	0.023238	0.006427	0.012176
0.010786	0.007491	0.023238	0.002142	0.012176
0.014381	0.007491	0.023238	0.006427	0.012176
0.010786	0.012111	0.023238	0.006427	0.012176
0.014381	0.012111	0.023238	0.006427	0.012176
0.010786	0.007491	0.007746	0.002142	0.004059
0.003595	0.002497	0.023238	0.006427	0.012176
0.010786	0.007491	0.023238	0.006427	0.012176
0.010786	0.007491	0.023238	0.006427	0.012176
0.014381	0.007491	0.023238	0.006427	0.012176
0.010786	0.007491	0.023238	0.008569	0.012176

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The ranking of the strategies

Table 5. Ranking of the strategies

S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
1	14	3	2	4	6	13	8	7	5	16
S12	S13	S14	S15	S16						
15	11	11	8	10						

Comparative Analysis

Table 6 presents the ranking of the alternatives with respect to two methods FUCOM-ARAS and FUCOM-FARAS and the ranking results shows the consistency and the efficiency of the proposed method. Though the results obtained are same the representation of the data in fuzzy sense appears to be more realistic in making optimal decisions.

Table 6. Comparison of ranking

Factors	FUCOM-ARAS	FUCOM-FARAS
S1	1	1
S2	11	14
S3	3	3
S4	2	2
S5	4	4
S6	6	6
S7	12	13
S8	7	8
S9	8	7
S10	5	5
S11	15	16
S12	16	15
S13	12	11
S14	10	11
S15	13	8
S16	9	10

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DISCUSSION

It is very evident that the strategies S1, S4,S3,S5,S10 occupies the first five ranks and these strategies shall be implemented as the first step by the corporate sectors in association with the citizen science to attain the goal of green globe. Though many strategies are proposed, it is very significant to determine the most feasible and suitable strategies that shall pave the way of attaining the sub goals to reach the main goal. The proposed method helps in finding the significance of the sub goals and ranking of the strategies to make optimal decision towards conservation of our universe.

The research work encompasses the decision making problem on ranking the strategies and the new dimension of decision making approach of DMMGS is introduced and the method of FUCOM-FARAS is applied to make optimal decisions. The proposed approach of making decisions shall be extended and discussed with higher order of fuzzy numbers.

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Chapter 10
Characterization of Pythagorean Q-Fuzzy Ideal of Near-Ring

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ABSTRACT

Pythagorean fuzzy sets are an extension of the intuitionistic fuzzy sets, and they also overcome the limitations of the intuitionistic fuzzy sets. The theory of Pythagorean fuzzy sets possesses significant advantages in handling vagueness and complex uncertainty. Additionally, Pythagorean fuzzy information is used to simulate the ambiguous nature of subjective judgements and measure the fuzziness and impression more flexibly. This chapter presents some useful concepts of Pythagorean fuzzy sub-ring, upper, and lower cut seta. Also, the authors derive some useful properties of Pythagorean Q-fuzzy ideals of near-rings.

1 INTRODUCTION

Considering the imprecision in decision-making, Zadeh (1965) introduced the idea of fuzzy set which has a membership function, μ that assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. The notion of fuzzy sets generalizes classical sets theory by allowing intermediate situations between the whole and nothing. In a fuzzy set, a membership function is defined to describe the degree of membership of an element to a class. The membership value ranges from 0 to 1, where 0 shows that the element does not belong to a class, 1 means belongs, and other values indicate the degree of membership to a class. For fuzzy sets, the membership function replaced the characteristic function in crisp sets. Nasseri et. al. (2015) had done a lot research work in the field of fuzzy set theory and they also used some interesting method for ranking triangular fuzzy numbers and trapezoidal fuzzy number.

Albeit, the concept of fuzzy sets theory seems to be inconclusive because of the exclusion of nonmembership function and the disregard for the possibility of hesitation margin. The theory of Instuitionistic fuzzy set has been found to be more useful to deal with vagueness and uncertainty in decision situations than that of the fuzzy set. Atanassov (1986) critically studied these shortcomings and proposed a con-

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cept called intuitionistic fuzzy sets (IFSs). The construct (that is, IFSs) incorporates both membership function, μ and nonmembership function, ν with hesitation margin, π (that is, neither membership nor nonmembership functions), such that $\mu+\nu\leq 1$ and $\mu+\nu+\pi=1$. The notion of IFSs provides a flexible framework to elaborate uncertainty and vagueness. The idea of IFS seems to be resourceful in modeling many real-life situations like medical diagnosis, career determination, selection process, and multicriteria decision-making, among others. Adak et.al., (2011, 2012, 2013, 2014) have done of research work in the field IFSs.

There are situations where $\mu+\nu\geq 1$ unlike the cases capture in IFSs. This limitation in IFS naturally led to a construct, called Pythagorean fuzzy sets (PFSs). Pythagorean fuzzy set (PFS) proposed a new tool to deal with vagueness considering the membership grade, μ and nonmembership grade, ν satisfying the conditions $\mu+\nu\leq 1$ or $\mu+\nu\geq 1$, and also, it follows that $\mu^2+\nu^2+\pi^2=1$, where π is the Pythagorean fuzzy set index. The construct of PFSs can be used to characterize uncertain information more sufficiently and accurately than IFS. Garg (2016) presented an improved score function for the ranking order of interval-valued Pythagorean fuzzy sets (IVPFSs). Based on it, a Pythagorean fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) method by taking the preferences of the experts in the form of interval-valued Pythagorean fuzzy decision matrices was discussed.

Pythagorean fuzzy set has attracted great attentions of many researchers, and subsequently, the concept has been applied to many application areas such as decision-making, aggregation operators, and information measures.

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences and so on. This provides sufficient motivations to researchers to review various concepts and results from the real abstract algebra in the broader framework of fuzzy setting.

R.Biswas and S.Nanda (1994) introduced the concept of lower and upper approximation of a subgroup of a group. N.Kuroki (1997) gave the notion of rough ideal in a semigroup. B.Davvaz (2006, 2008) has introduced roughness in rings and the notion of rough prime ideals and rough primary ideals in rings. The authors V.Selvan and G.Senthil Kumar (2012) have studied the lower and upper approximations of ideals and fuzzy ideals in a semiring. The notion of rough intuitionistic fuzzy ideal in a semi group was given by Gosh et. al., (2012).

The notions of fuzzy subnear-ring and ideal were first introduced by Abou-Zaid in (2013). In 2001, Kyung Ho Kim and Young Bae Jun in their paper entitled “Normal fuzzy R-subgroups in nearrings” introduced the concept of a normal fuzzy R-subgroup in near-rings and explored some related properties. In 2005, Kuncham et. al., introduced fuzzy prime ideal of near-rings.

In this paper, we introduce and study the concept of pythagorean Q-fuzzy ideals of near-rings. The rest of the paper organized as follows. In Section 2, the preliminaries and some definitions are given and present some algebraic structures of pythagorean fuzzy sets. In Section 3, we studied the definition of rough pythagorean fuzzy sets and discussed some important properties of rough pythagorean fuzzy ideals of a near ring. Finally, a conclusion is made in Section 4.

2 PRELIMINARIES AND DEFINITIONS

To assemble this work self-sufficient, here briefly introduce a few definitions engaged in the remaining work.

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Definition 2.1 (Fuzzy set (FS)) A fuzzy set *A* in a universal set *X* is defined as $A = \{\langle x, \mu_A\left(x\right)\rangle \mid x \in X\}$ where μ_A: X→[0,1] is a mapping called the membership function of the fuzzy set *A*.

Definition 2.2 Let *X* be a fixed set. An intuitionistic fuzzy set (IFS) *I* in *X* is an expression having the form $I = \left\{\langle x, \mu_I\left(x\right), \nu_I\left(x\right)\rangle : x \in X\right\}$, where the functions $\mu_I(x)$ and $\nu_I(x)$ are the degree of membership and the degree of non-membership of the element $x \in X$ respectively. Also μ_I: $X \to [0,1]$, ν_I: $X \to [0,1]$ and $0 \le \mu_I\left(x\right) + \nu_I\left(x\right) \le 1$, for all $x \in X$.

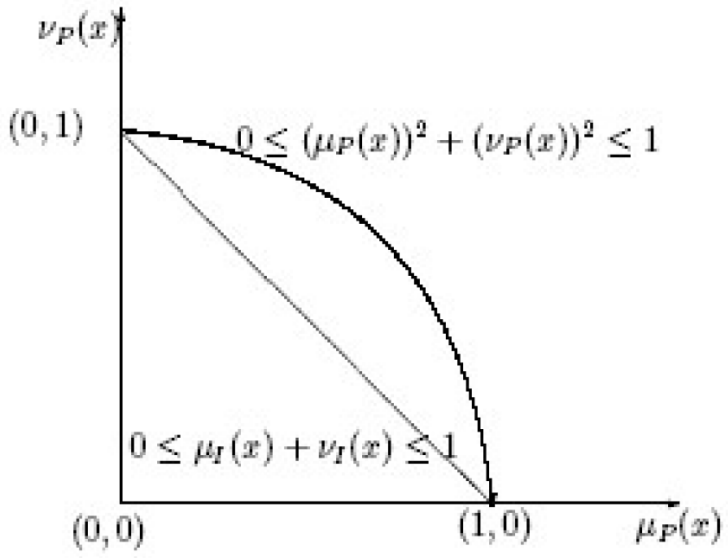
The degree of indeterminacy $\pi_I\left(x\right) = 1 - \mu_I\left(x\right) - \nu_I\left(x\right)$.

In practice, may be for some reason, the condition $0 \le \mu\left(x\right) + \nu\left(x\right) \le 1$, is not true. For instance, 0.4+0.7=1.1>1 but 0.42+0.72<1, or 0.5+0.7=1.2>1 but 0.52+0.72<1. To overcome this situation, in 2013 Yager[38] introduced the concept of the pythagorean fuzzy set.

Definition 2.3 A PFS P in a finite universe of discourse *X* is given by $P = \{\langle x, \mu_P\left(x\right), \nu_P\left(x\right)\rangle \mid x \in X\}$, where $\mu_P(x)$: $X \to [0,1]$ denotes the degree of membership and $\nu_P(x)$: $X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set A respectively with the condition that $0 \le (\mu_P\left(x\right))^2 + (\nu_P\left(x\right))^2 \le 1$.

The degree of indeterminacy $\pi_P\left(x\right) = \sqrt{1 - (\mu_P\left(x\right))^2 - (\nu_P\left(x\right))^2}$.

Figure 1. Intuitionistic fuzzy set vs. Pythagorean fuzzy set



2.1 Some Operations on Pythagorean Fuzzy Numbers

Here we introduce some some operations on pythagorean fuzzy numbers and pythagorean fuzzy sets those are used in the rest of the paper.

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Given three pythagorean fuzzy numbers (PFNs) $\alpha = \langle \mu, \nu \rangle, \alpha_1 = \langle \mu_1, \nu_1 \rangle$ and $\alpha_2 = \langle \mu_2, \nu_2 \rangle$. The basic operations can be defined as follows:

$\left(\mathrm{i}\right)\quad \bar{\alpha} = \langle \nu, \mu \rangle$

$\left(\mathrm{ii}\right)\quad \alpha_1 \vee \alpha_2 = \langle \max\left\{\mu_1, \mu_2\right\}, \min\left\{\nu_1, \nu_2\right\} \rangle$

$\left(\mathrm{iii}\right)\quad \alpha_1 \wedge \alpha_2 = \langle \min\left\{\mu_1, \mu_2\right\}, \max\left\{\nu_1, \nu_2\right\} \rangle$

$\left(\mathrm{iv}\right)\quad \alpha_1 \oplus \alpha_2 = \langle \sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \nu_1\nu_2 \rangle$

$\left(\mathrm{v}\right)\quad \alpha_1 \otimes \alpha_2 = \langle \mu_1\mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2\nu_2^2} \rangle$

$\left(\mathrm{vi}\right)\quad \lambda \cdot \alpha = \langle \sqrt{1-(1-\mu^2)^{\lambda}}, \nu^{\lambda} \rangle, \lambda > 0.$

$\left(\mathrm{vii}\right)\quad \alpha^{\lambda} = \langle \mu^{\lambda}, \sqrt{1-(1-\nu^2)^{\lambda}} \rangle, \lambda > 0.$

Definition 2.4 Let $\alpha_1 = \langle \mu_1, \nu_1 \rangle$ *and* $\alpha_2 = \langle \mu_2, \nu_2 \rangle$ be two PFNs; $S\left(\alpha_1\right) = \mu_1^2 - \nu_1^2$ and $S\left(\alpha_2\right) = \mu_2^2 - \nu_2^2$ be their score functions; $H\left(\alpha_1\right) = \mu_1^2 + \nu_1^2$ and $H\left(\alpha_2\right) = \mu_2^2 + \nu_2^2$ be the accuracy degrees of α_1 and α_2, then Yager and Abbasov [38] defined the following:

1. If $S(\alpha_1) < S(\alpha_2)$ then α_1 is smaller than α_2, that is $\alpha_1 < \alpha_2$;
2. If $S(\alpha_1) > S(\alpha_2)$ then $\alpha_1 > \alpha_2$;
3. If $S(\alpha_1) = S(\alpha_2)$ then $\alpha_1 = \alpha_2$;
4. If $H(\alpha_1) < H(\alpha_2)$ then $\alpha_1 < \alpha_2$;
5. If $H(\alpha_1) > H(\alpha_2)$ then $\alpha_1 > \alpha_2$;
6. If $H(\alpha_1) = H(\alpha_2)$ then α_1 and α_2 represent the same information that $\alpha_1 = \alpha_2$.

Definition 2.5 By a near-ring we mean a non-empty set R with two binary operations "+" and "." satisfying the following axioms:

(i) (R,+) is a group;
(ii) (R,.) is a semi-group;
(iii) x.(y+z) = x.y+x.z for all x,y.z∈R.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring". We denote xy instead of x…y. Note that x0=0 and x(-y)= -xy, but 0x=0 for x,y∈R.

Definition 2.6 An ideal I of a near-ring R is subset of R such that

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(i) (I,+) is a normal *s*ubgroup of (R,+);

(ii) $RI \subseteq I$;
(iii) (r+i)s – rs∈I and r,s∈R.

Note that if I satisfies (i) and (ii) then it is called a left ideal of R.

If I satisfies (i) and (ii) then it is called a right ideal of R.

Definition 2.7 A function μ_R×Q=[0.1] is called a Q-fuzzy set.

Definition 2.8 Let μ be a Q-fuzzy set in R and t∈[0,1], then an upper t-level cut of μ is defined by $U(\mu;t) = \left\{x \in R\mu(x,q) \geq t, q \in Q\right\}$

Definition 2.9 [11] Let μ be a Q-fuzzy set in R and t∈[0,1], then an lower t-level cut of μ is defined by $U(\mu;t) = \left\{x \in R\mu(x,q) \leq t, q \in Q\right\}$.

3 PYTHAGOREAN Q-FUZZY IDEALS OF NEAR-RINGS

Now we introduce the pythagorean Q- fuzzy *i*deal of near-rings as follows.

Definition 3.1 In pythagorean Q- fuzzy set P=(μ_P,ν_P) of a near-ring R is called an pythagorean Q-fuzzy subnear-ring of R if

(i) $\mu_P\left(x-y,q\right) \geq \mu_P\left(x,q\right) \wedge \mu_P\left(y,q\right)$ and $\nu_P\left(x-y,q\right) \leq \nu_P\left(x,q\right) \vee \nu_P\left(y,q\right)$

(ii) $\mu_P\left(xy,q\right) \geq \mu_P\left(x,q\right) \wedge \mu_P\left(y,q\right)$ and $\nu_P\left(xy,q\right) \leq \nu_P\left(x,q\right) \vee \nu_P\left(y,q\right)$

for all x,y∈R and q∈Q.

Definition 3.2 In P=(μ_P,ν_P) of a near-ring R is called a pythagorean Q fuzzy ideal of R if

(i) $\mu_P\left(x-y,q\right) \geq \mu_P\left(x,q\right) \wedge \mu_P\left(y,q\right)$ and $\nu_P\left(x-y,q\right) \leq \nu_P\left(x,q\right) \vee \nu_P\left(y,q\right)$

(ii) $\mu_P\left(y+x-y,q\right) = \mu_P\left(x,q\right)$ and $\nu_P\left(y+x-y,q\right) = \nu_P\left(x,q\right)$

(iii) $\mu_P\left(rx,q\right) \geq \mu_P\left(x,q\right)$ and $\nu_P\left(rx,q\right) \leq \nu_P\left(x,q\right)$

(iv) $\mu_P\left(\left(x+i\right)y-xy,q\right) \geq \mu_P\left(i,q\right)$ and $\nu_P\left(\left(x+i\right)y-xy,q\right) \leq \nu_P\left(i,q\right)$

for all *x,y,i*∈*R* and *q*∈*Q*.

If *P*=(μ_P,ν_P) satisfies (i), (ii) and (iii) then a is called a pythagorean Q- fuzzy ideal left ideal of R and if P=*(*μ_P,ν_P*)* satisfies (i), (ii) and (iv) then P is called a pythagorean Q- f*u*zzy ideal left ideal of R.

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Example 1 Let R=(a,b,c,d) be a non-empty set with two binary operations "+" and "." defined as follows.

Table 1.

+	a	b	c	d		.	a	b	c	d
a	a	b	c	d		**a**	a	a	a	a
b	b	a	d	c		**b**	a	a	a	a
c	c	d	b	a		**c**	a	a	a	a
d	d	c	a	b		**d**	a	a	b	b

Then (R,+,.) is a near-ring.

Define an pythagorean *Q*- fuzzy set, $P=(\mu_P,\nu_P)$ in R *a*s follows

$$\mu_P\left(a,q\right)=1,\ \mu_P\left(b,q\right)=.3,\ \mu_P\left(c,q\right)=0,\ \mu_P\left(d,q\right)=0\,,$$

$$\nu_P\left(a,q\right)=0,\ \nu_P\left(b,q\right)=.3,\ \nu_P\left(c,q\right)=1,\ \nu_P\left(d,q\right)=1\,,\text{ for all } q\in Q.$$

Then $P=(\mu_P,\nu_P)$ is a pythagorean Q- fuzzy ideal of a near- ring R.

Basic properties of a pythagorean Q fuzzy ideal of R are proved in the following theorem.

Theorem 3.1 If $P=(\mu_P,\nu_P)$ of a pythagorean Q *f*uzzy ideal of R *t*hen

(i) $\mu_P\left(0,q\right)\geq\mu_P\left(x,q\right)$ and $\nu_P\left(0,q\right)\leq\nu_P\left(x,q\right),$

(ii) $\mu_P\left(-x,q\right)=\mu_P\left(x,q\right)$ and $\nu_P\left(-x,q\right)=\nu_P\left(x,q\right)$

(iii) $\mu_P\left(x-y,q\right)\geq\mu_P\left(0,q\right)\Rightarrow\mu_P\left(x,q\right)=\mu_P\left(y,q\right)$

(iv) $\nu_P\left(x-y,q\right)\leq\nu_P\left(0,q\right)\Rightarrow\nu_P\left(x,q\right)=\nu_P\left(y,q\right)$

for all $x,y\in R$ and $q\in Q$.

Proof: Let $P=(\mu_P,\nu_P)$ be a pythagorean Q- fuzzy ideal of R.

(i) $\mu_P\left(0,q\right)=\mu_P\left(x-x,q\right)\geq\mu_P\left(x,q\right)\wedge\mu_P\left(x,q\right).$

Therefore, $\mu_P\left(0,q\right)\geq\mu_P\left(x,q\right)$ for all $x\in R$ and $q\in Q$.

Similarly, $\nu_P\left(0,q\right)\leq\nu_P\left(x,q\right)$ for all $x\in R$ and $q\in Q$.

(ii) $\mu_P\left(-x,q\right)=\mu_P\left(0-x,q\right)\geq\mu_P\left(0,q\right)\wedge\mu_P\left(x,q\right)=\mu_P\left(x,q\right).$

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Now, $\mu_P\left(x,q\right)=\mu_P\left(-\left(-x\right),q\right)\geq\mu_P\left(-x,q\right)\geq\mu_P\left(x,q\right)$.

Thus, we get $\mu_P\left(-x,q\right)=\mu_P\left(x,q\right)$ for all $x\in R$ and $q\in Q$.

Similarly, $\nu_P\left(-x,q\right)=\nu_P\left(x,q\right)$ for all $x\in R$ and $q\in Q$.

(iii) We have $\mu_P\left(x-y,q\right)\geq\mu_P\left(0,q\right)$. But, $\mu_P\left(0,q\right)\geq\mu_P\left(x-y,q\right)$.

Thus, $\mu_P\left(0,q\right)=\mu_P\left(x-y,q\right)$.

Now, consider $\mu_P\left(x,q\right)=\mu_P\left(x-y+y,q\right)$

$$=\mu_P\left(\left(x-y\right)+y,q\right)$$

$$\geq\mu_P\left(x-y,q\right)\wedge\mu_P\left(y,q\right)$$

$$=\mu_P\left(0,q\right)\wedge\mu_P\left(y,q\right)$$

$$=\mu_P\left(y,q\right)$$

Similarly, we can prove that $\mu_P\left(y,q\right)\geq\mu_P\left(x,q\right)$.

Hence $\mu_P\left(y,q\right)=\mu_P\left(x,q\right)$ for all $x\in R$ and $q\in Q$.

Theorem 3.2 If $\{P_i \mid i\in I\}$ is a family of pythagorean Q- fuzzy ideal of R then $\cap\{P_i \mid i\in I\}$ is an pythagorean Q- fuzzy ideal of R, where I is an index set.

Proof Let $x,y,a,r\in R$ and $q\in Q$. Then

(i) $\left(\cap\mu_{A_i}\right)\left(x-y,q\right)=\wedge\mu_{A_i}\left(x-y,q\right)\mid i\in I$

$$\geq(\wedge\mu_{A_i}\left(x,q\right)\mid i\in I)\wedge(\wedge\mu_{A_i}\left(y,q\right)\mid i\in I)$$

$$=(\cap\mu_{A_i}\left(x,q\right)\mid i\in I)\wedge(\cap\mu_{A_i}\left(y,q\right)\mid i\in I)$$

$\left(\cup\nu_{A_i}\right)\left(x-y,q\right)=\vee\nu_{A_i}\left(x-y,q\right)\mid i\in I$

$$\leq(\vee\nu_{A_i}\left(x,q\right)\mid i\in I)\vee(\vee\mu_{A_i}\left(y,q\right)\mid i\in I)$$

$$=(\cup\nu_{A_i}\left(x,q\right)\mid i\in I)\vee(\cup\nu_{A_i}\left(y,q\right)\mid i\in I)$$

(ii) $\left(\cap\mu_{A_i}\right)\left(y+x-y,q\right)=\wedge\mu_{A_i}\left(y+x-y,q\right)\mid i\in I$

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$= \wedge\mu_{A_i}\left(x,q\right) \mid i \in I$

$= (\cap\mu_{A_i}\left(x,q\right) \mid i \in I)$

$\left(\cup\nu_{A_i}\right)\left(x-y,q\right) = \vee\nu_{A_i}\left(x-y,q\right) \mid i \in I$

$\leq (\vee\nu_{A_i}\left(x,q\right) \mid i \in I) \vee (\vee\mu_{A_i}\left(y,q\right) \mid i \in I)$

$= (\cup\nu_{A_i}\left(x,q\right) \mid i \in I) \vee (\cup\nu_{A_i}\left(y,q\right) \mid i \in I).$

(iii) $\left(\cap\mu_{A_i}\right)\left(rx,q\right) = \wedge\mu_{A_i}\left(rx,q\right) \mid i \in I$

$\geq \wedge\mu_{A_i}\left(x,q\right) \mid i \in I$

$= (\cap\mu_{A_i}\left(x,q\right) \mid i \in I)$

$\left(\cup\nu_{A_i}\right)\left(rx,q\right) = \vee\nu_{A_i}\left(rx,q\right) \mid i \in I$

$\leq (\vee\nu_{A_i}\left(x,q\right) \mid i \in I)$

$= (\cup\nu_{A_i}\left(x,q\right) \mid i \in I)$

(iv) $\left(\cap\mu_{A_i}\right)\left(x+a\right)y - xy,q) = \wedge\mu_{A_i}\left(x+a\right)y - x,q) \mid i \in I$

$\geq \wedge\mu_{A_i}\left(x,q\right) \mid i \in I$

$= (\cap\mu_{A_i}\left(x,q\right) \mid i \in I)$

$\left(\cup\nu_{A_i}\right)\left(x+a\right)y - xy,q) = \vee\nu_{A_i}\left(x+a\right)y - xy,q) \mid i \in I$

$\leq (\vee\nu_{A_i}\left(x,q\right) \mid i \in I)$

$= (\cup\nu_{A_i}\left(x,q\right) \mid i \in I)$

for all $x,y,a,r \in R$ and $q \in Q$.

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Hence $\cap\{P_i \mid i \in I\}$ is a pythagorean Q- fuzzy ideal of R.

Definition 3.3 Let $P=(\mu_P, \nu_P)$ be a pythagorean fuzzy set. Then we define $\circ P$ and $\bullet P$ as $\circ P = \left(\mu_P, \mu_P^c\right)$ and $\bullet P = \left(\nu_P, \nu_P^c\right)$

Theorem 3.3 If $P=(\mu_P, \nu_P)$ is a pythagorean Q- fuzzy ideal of R *t*hen

(i) $\circ P = \left(\mu_P, \mu_P^c\right)$ is also a pythagorean Q- fuzzy ideal of R.

(ii) $\bullet P = \left(\nu_P^c, \nu_P\right)$ is also a pythagorean Q- fuzzy ideal of R.

Proof Let $x,y,a,r \in R$ and $q \in Q$.

(i) (a) $$\begin{aligned}\mu_P^c\left(x-y,q\right) &= 1-\mu_P\left(x-y,q\right) \le 1-\left(\mu_P\left(x,q\right)\wedge\mu_P\left(y,q\right)\right)\\ &= \left(1-\mu_P\left(x,q\right)\right)\vee\left(1-\mu\left(y,q\right)\right) = \mu_P^c\left(x,q\right)\vee\mu_P^c\left(y,q\right).\end{aligned}$$

(b) $\mu_P^c\left(y+x-y,q\right) = 1-\mu_P\left(y+x-y,q\right) = 1-\left(\mu_P\left(x,q\right) = \mu_P^c\left(x,q\right)\right).$

(c) $\mu_P^c\left(rx,q\right) = 1-\mu_P\left(rx,q\right) \le 1-\mu_P\left(x,q\right) = \mu_P^c\left(x,q\right).$

(d) $\mu_P^c\left(\left(x+a\right)y-xy,q\right) = 1-\mu_P\left(\left(x+a\right)y-xy,q\right) \le 1-\mu_P\left(a,q\right) = \mu_P^c\left(a,q\right)).$

(ii) (a) $$\begin{aligned}\nu_P^c\left(x-y,q\right) &= 1-\nu_P\left(x-y,q\right) \ge 1-\left(\nu_P\left(x,q\right)\vee\nu_P\left(y,q\right)\right)\\ &= \left(1-\nu_P\left(x,q\right)\right)\wedge\left(1-\nu\left(y,q\right)\right) = \nu_P^c\left(x,q\right)\wedge\nu_P^c\left(y,q\right).\end{aligned}$$

(b) $\nu_P^c\left(y+x-y,q\right) = 1-\nu_P\left(y+x-y,q\right) = 1-\left(\nu_P\left(x,q\right) = \nu_P^c\left(x,q\right)\right).$

(c) $\nu_P^c\left(rx,q\right) = 1-\nu_P\left(rx,q\right) \ge 1-\nu_P\left(x,q\right) = \nu_P^c\left(x,q\right).$

(d) $\nu_P^c\left(\left(x+a\right)y-xy,q\right) = 1-\nu_P\left(\left(x+a\right)y-xy,q\right) \le 1-\nu_P\left(a,q\right) = \nu_P^c\left(a,q\right)).$

Hence $\circ P = \left(\mu_P, \mu_P^c\right)$ and $\bullet P = \left(\nu_P^c, \nu_P\right)$ are a pythagorean Q-fuzzy ideal of R.

Definition 3.4 Let $P=(\mu_P, \nu_P)$ is a pythagorean Q- *f*uzzy ideal of R, we define $R\mu_P$ and $R\nu_P$ as

$$R\mu_P = \{x \in R \mid \mu_P\left(x,q\right) = \mu_P\left(0,q\right)\} \text{ and } R\nu_P = \{x \in R \mid \nu_P\left(x,q\right) = \nu_P\left(0,q\right)\}$$

Theorem 3.4 If a pythagorean Q- fuzzy ideal $P=(\mu_P, \nu_P)$ is an pythagorean Q- fuzzy ideal of R then the set $R\mu_P$ and $R\nu_P$ are ideals of R for all $q \in Q$.

Proof Let $P=(\mu_P, \nu_P)$ be an pythagorean Q- fuzzy ideal of R. Let $x,y \in R\mu_P$ and $q \in Q$. Th*e*n $\mu(x,q) = \mu(0,q)$, $\mu(y,q) = \mu(0,q)$.

(i) Now $0 \in R$ therefore $0 \in R\mu_P$. Hence $R\mu_P$ is a non-empty subset of R.

(ii) Since $P=(\mu_P, \nu_P)$ is an pythagorean Q- fuzzy ideal of R,

$$\mu_P(x-y,q) \ge \mu_P(x,q) \wedge \mu_P(y,q) = \mu_P(0,q).$$

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But $\mu_P(0,q) \geq \mu_P(x-y,q)$. Therefore $\mu_P(x-y,q) = \mu_P(0,q)$. Thus x – y∈R$\mu_P$ for all x,y∈R. Hence (Rμ_P,+) is a subgroup of (R,+).

(iii) again since P=(μ_P,ν_P) is an intuitionistic Q- fuzzy ideal of R,

$\mu(y+x-y,q) = \mu_P(x,q) = \mu_P(0,q)$.

Therefore y+x – y∈$R\mu_P$ for all x,y∈R. Hence (Rμ_P,+) is a normal subgroup of (R,+).

(iv) Since $\mu_P(r_x,q) \geq \mu_P(x,q)$ for all x∈R and q∈Q, therefore $\mu_P(r_x,q) \geq \mu_P(x,q)$. But $\mu_P(0,q) \geq \mu_P(r_x,q)$. Hence $\mu_P(r_x,q) = \mu_P(0,q)$ implies that r_x∈Rμ_P for all r∈R, x∈Rμ_P. Hence $R\mu_P$ is a left ideal of R.

(v) Next $\mu_P((u+i)v-uv,q) \geq \mu_P(i,q) = \mu_P(0,q)$ for all $u,v \in R, i \in R\mu_P$.

But $\mu_P(0,q) \geq \mu_P((u+i)v-uv,q)$. Therefore $\mu_P((u+i)v-uv,q) = \mu_P(0,q)$ for all $u,v \in R, i \in R\mu_P$. Hence $(u+i)v-uv \in R\mu_P$ for all u,v∈R, i∈Rμ_P.

Thus Rμ_P is a right ideal of R.

Theorem 3.5 If an P=(μP,νP) is a pythagorean Q- fuzzy ideal of R, then the set U(μ_P,t) and L(μ_P,t) are ideal of R for all $q \in Q, t \in I_m(\mu_P) \cap I_m(\nu_P)$.

Proof Let q∈Q, $t \in I_m(\mu_P) \cap I_m(\nu_P)$. We have

$U(\mu,t) = \{x \in R \mid \mu_P(x,q) \geq t, q \in Q\}$

$L(\mu,t) = \{x \in R \mid \mu_P(x,q) \geq t, q \in Q\}$

(i) Let x,y∈U(μ,t). Then μ_P(x,q)≥t and μ_P(y,q)≥t.

Since $\mu_P(x-y,q) \geq \mu_P(x,q) \wedge \mu_P(y,q) \geq t$, therefore x – y∈U($\mu$,t) for all x,y∈U($\mu$,t).
Hence (U(μ,t),+) is a subgroup of (R,+).

(i) Let x∈U(μ,t) and y∈R. Again since $\mu_P(y+x-y,q) = \mu_P(x,q) \geq t$,
(ii) therefore y+x – y∈U(μ,t) for all x∈U(μ,t) and y∈R.

Hence (U(μ,t),+) is a normal subgroup of (R,+).

(iii) Let x∈U(μ,t) and r∈R. Then $\mu_P(rx,q) \geq \mu_P(x,q) \geq t$ implies rx∈U(μ,t) for all x∈U(μ,t) and r∈R i.e, $RU(\mu,t) \subseteq U(\mu,t)$.

Hence U(μ,t) is a ideal of R.

(iv) Let i∈U(μ,t) and x,y∈R. Then $\mu_P((x+i)y-xy,q) \geq \mu_P(i,q) \geq t$.

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Therefore (x+i)y – x,y∈U(μ,t) and x,y∈R. Hence U(μ,t) is a right ideal of R.

Theorem 3.6 If P=(μ_P,ν_P) is a pythagorean Q- fuzzy ideal of R such that all non-empty ideal sets U(μ_P,t) and L(ν_p,t) are ideal of R, then P=(μ_P,ν_P) is a pythagorean Q- fuzzy ideal of R.

Proof Suppose P=(μ_P,ν_P) is a pythagorean Q- fuzzy set of R such that all non-empty level U(μ_P,t) and L(ν_p,t) are ideal of R.

(i) Let $t_0 = \mu_P(x,q) \wedge \mu_P(y,q)$ and $t_1 = \nu_P(x,q) \vee \nu_P(y,q)$ for all x,y∈R, q∈Q.
Then x,y∈U(μ_P,t_0) also x,y∈L(μ_P,t_1).
Therefore x – y∈U(μ_P,t_0) and x – y∈L(μ_P,t_1).
Thus $\mu_P(x-y,q) \geq t_0 = \mu_P(x,q) \wedge \mu_P(y,q)$ and $\mu_P(x-y,q) \leq t_1 = \mu_P(x,q) \vee \mu_P(y,q)$.

(ii) Let $t_0 = \mu_P(x,q)$ and t_1(x,q). Then x∈U(μ_P,t_0) also x∈L(ν_P,t_1) and y∈R.
Since U(μ_P,t) and L(μ_P,t) are ideal of R, they are normal.
Therefore y+x – y∈U(μ_P,t_0) and L(y+x – y∈(ν_P,t_1) for all x∈U(μ_P,t_0).
Now x∈L(μ_P,t_1) and y∈R.
$\Rightarrow \mu_P(y+x-y,q) \geq \mu_P(x,q)$ and $\nu_P(y+x-y,q) \leq \mu_P(x,q)$.

(iii) Let t_2=μ_P(x,q) and t_3=ν_P(x,q) for all x∈R and q∈Q.
Then x∈U(μ_P,t_2) and x∈L(ν_P,t_3).
Since U(μ_P,t_2) and L(ν_P,t_3) are left ideal of R, rx∈U(μ_P,t_2) and rx∈L(ν_P,t_3) for r∈R.
Hence $\mu_P(rx,q) \geq t_2 = \mu_P(x,q)$ and $\nu_P(rx,q) \leq t_3 = \nu_P(x,q)$.

(iv) Let t_2=μ_P(i,q) and t_3=ν_P(i,q) for x∈R and q∈Q.
Then i∈U(μ_P,t_2) and L(ν_P,t_3).
Since U(μ_P,t) is a right ideal of R.
Now, (x+i)y – xy∈U(μ_P,t2) for all x,y∈R and i∈U(μ_P,t_2) and (x+i)y – xy∈L(ν_P,t_3) for all x,y∈R and i∈U(ν_P,t_3).
Therefore, $\mu((x+i)y-xy,q) \geq t_2 = \mu_P(i,q)$ and $\nu_P((x+i)y-xy,q) \leq t_3 = \nu_P(i,q)$ for all x,y,i∈R and q∈Q.
Hence P=(μ_P,ν_P) is a pythagorean Q- fuzzy ideal of R.

CONCLUSION

In this chapter, the concept of Pythagorean fuzzy set are initiated. A relationship was shown netween pythagorean fuzzy set and intuitionistic fuzzy set. Also, studied the concept of PFS more expressly with relevant illustrations, where necessary. Here, the concept of pythagorean Q-fuzzy ideals of near-rings are focused. Also, derive some useful properties of pythagorean Q-fuzzy ideals of near-rings.

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Chapter 11
Fuzzy Integral Representation Associated With Spectral Measure

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ABSTRACT

One of the most ubiquitous constructions in functional analysis is the spectral measure. The aim of this chapter is to develop the possibility to use the fuzzy spectral measure to define spectral representation theorems. Firstly, the authors introduce the fuzzy spectral measure and examine some of its properties. Furthermore, they elucidate fuzzy Hilbert space, the fuzzy normal, bounded, and adjoint operator on it. Finally, they establish the main result of this study which states that for a given fuzzy integral representation of C(X), there is a unique fuzzy spectral measure E on the Borel subsets of X.

INTRODUCTION

Since the introduction of the fuzzy set by Zadeh in 1965, many research works have been dedicated to developing various aspects of the theory and its applications (Zadeh, 1965). Katsaras, in 1984, first introduced the concept of the fuzzy norm on a linear space (Katsaras, 1984). Later many authors like Felbin (1992), Cheng and Mordeson (1994) and Bag and Samanta (2015) have given different definitions of fuzzy normed spaces. In 2009, Goudarzi and Vaezpour introduced the definition of Fuzzy Hilbert

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space. Also, the papers by Hasankhani, Nazari & Saheli (2010) and Mukherjee & Bag (2011) give the connotation of fuzzy Hilbert spaces.

In 2017, Sudad M. Rasheed defined adjoint fuzzy linear operators, self-adjoint fuzzy linear operators acting on the fuzzy Hilbert space. Butnariu (1983) explained the foundations of the theory of fuzzy measures in his paper 'Additive fuzzy measures and integrals.' The notion of 'additive fuzzy measure' was introduced by Butnarju (1983) to deal with the concept of fuzzy integral for the extended real-valued functions. He constructed the idea similar to the construction of the classical theories, employing simple functions presented as pairs of simple functions. Our study is within the framework of fuzzy spectral measure and fuzzy Hilbert space.

In this chapter, we establish the fuzzy integral representation using fuzzy spectral measures and vice-versa. In a functional space like Hilbert space, the spectral theory is very important. The spectral theories support the study of spectral theorems. The fuzzy integral representation presented in this work helps to the detailed study of fuzzy pseudospectra, various linear operators and their spectra in terms of fuzzy spaces, topologies on vector spaces, operator theory, intermediate operatory theory, fuzzy Lebesgue normed linear spaces etc.

The organization of this chapter is as follows: Section 2 provides some preliminary results used in this chapter. In section 3, we discuss some properties of C(X). In Section 4, we introduce the idea of spectral measure in a fuzzy setting. Section 5 is devoted to establishing the fuzzy representation theorem and the unique fuzzy spectral measure corresponding to a representation.

PRELIMINARIES

Let X represent nonempty sets and A and B fuzzy sets on X. Then, it is characterized by a membership function in the sense of Zadeh. We shall define additive operations on fuzzy sets as

1. The *sum* of two fuzzy sets A and B in a set X, denoted by $A \bigoplus B$, is a fuzzy set in X defined by $(A \bigoplus B)(x) = \min(1, A(x) + B(x))$ for all $x \in X$
2. The *difference* of two fuzzy sets A and B in X, denoted by $A \Theta B$, is a fuzzy set in X defined by $(A \Theta B)(x) = \max(0, A(x) - B(x))$ for all $x \in X$
3. The *conjunction* of two fuzzy sets A and B, denoted by A&B, is a fuzzy set in X defined by $(A \& B)(x) = \max(0, A(x) + B(x) - 1)$ for all $x \in X$

Remark: If $A, B \in P(X)$, then $A \Sigma B = A \cup B$, $A \& B = A \cap B$ and $A \Theta B = A \setminus B$. Thus, ordinary topology and ordinary topological spaces become special cases of g- fuzzy topology and g- fuzzy topological spaces.

A fuzzy topological vector space is a fuzzy vector space that is also a fuzzy topological space such that the linear structure and the topological structure are vitally connected.

Definition: A fuzzy topology τ on a vector space X is said to be a fuzzy vector topology if

(a) the map $f: X \times X \to X$ defined by $(x, y) \to x + y$ is continuous

(b) the map $g: K \times X \to X$, $(k, x) \to k\,x$ is continuous when K has the usual topology and $X \times X$, $K \times X$ are given the product fuzzy topologies.

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A vector space X with a fuzzy vector topology τ, denoted by the pair (X, τ), is called *fuzzy topological vector space* (FTVS).

Definition: A fuzzy topological space (X, τ) is said to be *completely fuzzy regular* if for a point $p \in X$ and g-fuzzy closed set C of X not containing x, there exists a g-fuzzy continuous function $f: X \to [0,1]$ such that $f(x) = 0$ and $f(C) = \{1\}$

Definition: An *algebra* over **F** is a vector space X over a field **F** that also has a multiplication defined on it that makes X into a ring such that if $\alpha \in \mathbf{F}$ and $x, y \in X$

$\alpha(xy) = (\alpha x)\, y = x\, (\alpha y)$

In particular, when $\mathbf{F} = \mathbf{R}$, then X is real vector space

Definition: Let X be a real vector space. A function $N: X \times R \to [0, 1]$ is called a *fuzzy norm* on X if for all $x, y \in X$ and all $s, t \in R$,

(1) $N(x, t) = 0$ for $t \leq 0$
(2) $x = 0$ if and only if $N(x, t) = 1$ for all $t > 0$
(3) $N(cx, t) = N(x, t/|c|)$ if $c \neq 0$
(4) $N(x + y, s + t) \geq \min\{N(x, s), N(y,t)\}$
(5) $N(x,\times)$ is a non-decreasing function of R and $\lim_{t\to\infty} N(x,t) = 1$
(6) for $x \neq 0$, $N(x,\times)$ is continuous on R.

The pair (X, N) is called a *fuzzy normed vector space*.

Definition: Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to *be convergent* if there exists an $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, t) = 1$ for all $t > 0$.

In this case, x is called the *limit* of the sequence $\{x_n\}$, and we denote it as $\lim_{n\to\infty} x_n = x$ or $x_n \to x$.

Definition: Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is called called *Cauchy sequence* if $\lim_{n\to\infty} N(x_{n+p} - x_n, t) = 1$, for all $t > 0$ and for all $p > 0$.Every convergent sequence in a fuzzy normed vector space is Cauchy.

Definition: If each Cauchy sequence is convergent, then the fuzzy norm is complete, and the fuzzy normed vector space is called a *fuzzy Banach space*.

Definition: A mapping $f: X \to Y$ between fuzzy normed vector spaces X and Y is continuous at a point $x_0 \in X$ if for each sequence $\{x_n\}$ converging to x_0 in X, the sequence $\{f(x_n)\}$ converges to $f(x_0)$. If $f: X \to Y$ is continuous at each $x \in X$, then $f: X \to Y$ is said to be *continuous* on X

Definition: [20] if X is a linear space over the field of complex numbers. Let$\mu: X \times X \times \mathbf{C} \to I = [0, 1]$ satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$,

(1) $\mu(x+ y, z, |t| + |s|) \geq \min\{\mu(x, z, |t|, \mu(y, z, |s|\}$
(2) $\mu(x+ y, z, |st|) \leq \min\{\mu(x, x, |s|^2, \mu(y, y, |t|^2\}$
(3) $\mu(x, y, t) = \mu(y, x, t)$
(4) $\mu(\alpha x, y, t) = \mu(x, y, t/|\alpha|)$, $\alpha (\neq 0) \in \mathbf{C}$
(5) $\mu(x, y, t) = 0$ if and only if $x = y$ and $t \in C \setminus \mathbf{R}^+$

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(6) $\mu(x, y, t) = 1, t > 0$, if $x = 0$
(7) $\mu(x, y, .)$: $\mathbf{R} \to I = [0,1]$ is a monotonic non-decreasing function of R and $\lim_{t\to\infty} \mu(x, y, t) = 1$

Now we call μ to be the *fuzzy inner product function* on X, and (X,μ) is called an *inner product space*.

Example: Let X be a linear space over **C**. Let μ to be the *fuzzy inner product function* on X. Then,

$N(x, t) = \mu(y, y, |t|^2, t \in \mathbf{R},\ t > 0$ and 0 if $t \leq 0$ is a fuzzy norm on X.

Definition: Let X be algebra. The fuzzy normed space (X, N) is called a *fuzzy normed algebra* if N (xy, st) ≥ N(x, s)× N(y, t) for all x, y ∈ X and all positive real numbers s and t.

A complete fuzzy normed algebra is called a *fuzzy Banach algebra.*

Definition: The vector space X equipped with a fuzzy inner product is called a *fuzzy inner product space*. If the resulting fuzzy normed space is complete, it is called a *fuzzy Hilbert space*.

For fixed y, the inner product $\langle x, y\rangle$ is a linear function of x. For fixed x, it is a conjugate linear function of y. Such functions of two variables are sometimes called *sesquilinear*.

Let $(X, \|\cdot\|)$ be a fuzzy normed linear space. A subset A of X is complete if every Cauchy sequence in A converges in A.

Let X be a fuzzy inner product space. If $x_n \to x$ and $y_n \to y$, then $\langle x_n, y_n\rangle \to \langle x, y\rangle$ for all x, y ∈ X.

Let $(X, \|\cdot\|)$ and $(Y, \|\cdot\|)$ are fuzzy normed linear spaces. If T: X → Y be a fuzzy bounded linear operator, then

(i) $\|T\|$ is a fuzzy real number.
(ii) $\|T x\| \preccurlyeq \|T\|\|x\|, \forall x \in X$.

Definition: A mapping T: X → Y is *fuzzy continuous* at $x_0 \in X$ if and only if for all $x_n \subseteq X$, $x_n \to x_0$, we have $T(x_n) \to T(x_0)$.

If f is *fuzzy continuous* at every point of X, then f is said to be fuzzy continuous on X.

Definition: A fuzzy subset K of X is *fuzzy compact* if every open cover of K has a finite subcover.

In particular, if X is itself compact, then X is called a compact space.

A fuzzy inner product on X defines a fuzzy number $\| x \| = \sqrt{\langle x, x\rangle}$ for all x ∈ X (1)

The following inequality shows that the function $\| x \|$ is a well-defined fuzzy norm.

Lemma: A fuzzy inner product space X together with its corresponding norm $\|.\|$ satisfy the Schwarz inequality $|\langle x, x\rangle| \leq \| x \| \| y \|$ for all x, y ∈ X.

Definition: If $H \neq \{0\}$ is a fuzzy Hilbert space, then *B* (H) will denote the fuzzy Banach algebra of all bounded fuzzy` linear operators T on H, normed by

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$\| T \| = \sup \{\| T(x)\|: x \in H, \| x \| \leq 1\}$

Note: $B(H)$ is a fuzzy C*-algebra.

Theorem: If a fuzzy linear operator T defined on a fuzzy Hilbert space H satisfies

$\langle T x, y\rangle = \langle x, T y\rangle$, for all $x, y \in H$, then T is bounded.

Definition: An operator $T \in B(H)$ is said to be

(a) *normal* if it commutes with its (fuzzy) adjoint, that is $T^*T = T T^*$
(b) *self –adjoint* if $T^* = T$
(c) a *projection* if $T^2 = T$

Definition: If A is a fuzzy Banach algebra with identity and $a \in A$, the *spectrum* of a, denoted by $\sigma(a)$, is defined by $\sigma(a) = \{\alpha: a - \alpha \text{ is not invertible}\}$.

Definition: If A is a fuzzy Banach algebra with identity and $a \in A$, the *spectral radius* of a, denoted by r (a), is defined by $r(a) = \sup \{|\alpha|: \alpha \in \sigma(a)\}$. Because $\sigma(a) \neq \phi$ and is bounded, r(a) is well defined and finite; because $\sigma(a)$ is compact, this sup is attained.

A Borel measure on a fuzzy compact (locally compact) Hausdorff space X is a measure defined on the σ–algebra *C* of fuzzy sets of X.

Definition: A simple B - function is a pair $s = (\mathbf{A}, \mathbf{a})$ where $\mathbf{A} = \{A_1, A_2, \ldots, A_n\}$ is a finite fuzzy partition of X with $A_i \in \boldsymbol{C}$ and $\mathbf{a} = (a_1, a_2, \ldots, a_n) \in R^n$, $i = 1, 2, \ldots, n$

The simple function $s = (\mathbf{A}, \mathbf{a})$ is denoted by the formal symbol $s = \Sigma^n_{i=1} a_i . A_i$ and the class of non-negative simple B- functions over A is denoted by B_B^+

Let $(s_n)_{n \in N}$ be a sequence of simple B- functions .It is said to be increasing iff $s_{n+1} \geq s_n$ for all $n \in N$ and if all s_n are non-negative the given sequence is called non-negative.

If $(s_n(x))_{n \in N}$ converges to f(x) for any $x \in X$, then the sequence is said to converge to f, denoted by $s_n \rightarrow f$.

Let $f: X \rightarrow R^*_+ = (0, +\infty]$ be a function. We say that f is a *Borel function* with respect to B or *B- function* if and only if there is a nonnegative, increasing sequence of simple B- functions that converge to f.

If $f, g: X \rightarrow R^*$ are B- functions and $f + g$ exists then $f + g$ is a B- function

The difference of two nonnegative B- functions, if it exists, is also a B- function.

If $f,g: X \rightarrow R^*$ are B- functions then f .g is also a B- function.

Let *C* be the σ–algebra of fuzzy sets in X and F be the class of all real-valued B- functions on the Borel space (X, *C*)

Definition: A *set function* $\mu: M \rightarrow R_+^*$ is called a *fuzzy measure* if it satisfies the following properties:

1. $\mu(\Phi) = 0$
2. If $(A_n)_{n \in N}$ is a sequence in M, then $\mu(\Sigma_{n \in N} A_n) = \Sigma_1^{\infty} \mu(A_n)$.

Then, (X, *C*, μ) is called a *fuzzy measure space,* and members of *C* are called the *measurable sets* in X

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Definition: L^1 (m) is the collection of all complex B-functions f on X for which
$\int | f | dm < + \infty$. If f is a complex B-function, then | f | is also a B-function and hence the above integral is well defined. The members of L^1 (m) are called m-integrable functions.

Definition: Let (X, ***C***, m) be a measure space. We define the space L^2 (m) as the collection of all complex B-functions f such that

$\int | f |^2 dm < + \infty$.

SOME RESULTS ON C(X)

Definition: Let X be a completely regular space. The set of all fuzzy continuous functions from X into F is denoted by C(X).

We associate with each $f \in C(X)$ we define

$\|f\| = \sup \{|f(x)|: x \in X\}$.

Let X be completely fuzzy regular and C(X) be the set of all fuzzy continuous functions from X into the field F. If K is a fuzzy compact subset of X, define

$p_K(f) = \sup\{|f(x)|: x \in K\}$.

Then $\{p_K$: K fuzzy compact in X$\}$ is a family of semi- norms that makes C(X) into a fuzzy locally convex space. (1)

Definition: A topological space is a (topologically complete) fuzzy metrizable space if a (topologically complete) fuzzy metric induces the given topology.

Definition: A fuzzy locally convex space is metrizable if it has a topology that is defined by a norm.

Proposition: If X is fuzzy locally compact space, then C(X) is metrizable if and only if it is σ - compact. That is, X is the union of a sequence of fuzzy compact sets.

Definition: For a fuzzy LCS X, let X* denote the space of all fuzzy continuous linear functional f: X → F. Then X* is called *dual space* of X.

If $x^*, y^* \in X^*$ and $\alpha \in F$, then $(\alpha x^* + y^*)(x) = \alpha x^*(x) + y^*(x)$, x in X, defines an element $\alpha x^* + y^*$ in X*. Thus X* has a natural vector-space structure.

The dual space of a fuzzy topological vector space X is the vector space X* whose elements are continuous linear functionals on X. Note that addition and scalar multiplication are defined in X* as $(\Lambda_1 + \Lambda_2)(f) = \Lambda_1(f) + \Lambda_2(g)$ and $(\alpha\Lambda)(f) = \alpha.\ \Lambda(f)$. Λ is positive if $\Lambda(f) \geq 0$ whenever $f \geq 0$.With these operations X* is a vector space.

Let C(X) be a fuzzy locally convex space with the semi-norm given in (1). Then the space of all fuzzy continuous linear functional L: C(X) → F denoted by C(X)*. Then C(X)* is the *dual space* of the fuzzy locally convex space C(X).

If CX) is the algebra of fuzzy continuous functions from X into R, then C(X) has the property: If x, $y \in X$ and $x \neq y$, then there is an f in C(X) such that $f(x) \neq f(y)$.

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Then it is said to separate points of X.

Example: In the case of C(X), r (a) = sup {|α |: α ∈ σ (f) = f (X)}.

= sup {|f(x)|: x ∈ X} = ||*f*||

Definition: Let G is a region in **C** and X is a fuzzy Banach space. We say that f: G → **C** is analytic if f has a continuous derivative on G.

Definition: Let A be a fuzzy Banach algebra with identity and a ∈ A, then Hol (a) = all of the functions that are analytic in a neighbourhood of σ (a).

We can see that Hol (a) is algebra, but it is not a fuzzy Banach algebra
Let X be fuzzy compact. Then,

1. if f ∈ C(X), then σ (f) = f (X).
2. if g ∈ C(X) and f ∈ Hol (g), show that f (g) = f ° g

Proof: Hol (g) is the set of all functions that are analytic in a neighbourhood of σ (g) = g (X)

Here f ∈ Hol (g) implies f is analytic in the neighbourhood of σ (g) = g (X)
Then, f (g) (x) = f (g(x)) = (f ° g) (x)

∴ f (g) = f ° g

Definition: A fuzzy C*-algebra is a fuzzy Banach algebra A with an involution * such that for every a∈ A, $\|a^*a\|=\|a\|^2$

Example: If X is a fuzzy compact space, C*(X) is a C*-algebra where the involution is given by f*(x) = ¯f (x) (conjugate of f) for f in C(X) and x ∈ X

(It follows from the fact that $\|f^*(x)f(x)\|=\|f(x)\|^2$)

Definition: If A is a C*-algebra and a ∈ A, then a is *Hermitian* if a = a*. Re A denotes the Hermitian elements of A.

Definition: If A is a C*-algebra and a ∈ A then a is *positive* if a ∈ Re A and σ (a) ⊆ [0,∞)

Next, we define a fuzzy norm on C(X) y proving the following theorem.

Theorem 1: If X is a fuzzy compact space and (f g) (x) = f (x) g (x) whenever f, g ∈ C(X) and x∈ X. Define a function N: C(X) × R→ [0, 1] by

$$N(f, t) = \frac{t}{t+\|f\|} \text{ where } \|f\| = \sup \{úf(x)ú: x \in X\}.$$

Then N is a fuzzy norm and (C(X), N) is a fuzzy normed space

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Proof: N (f, t) = $\frac{t}{t+\|f\|}$, t > 0 and f ∈ C(X)

0 t ≤ 0 and f ∈ C(X)

1. N(f, t) = 0 for t ≤ 0
2. f = 0 if and only if N(f, t) = 1 for all t > 0
3. N(c f, t) = N(f, t/ |c|) if c ≠ 0
4. N(f + g, s + t) ≥ min{N(f, s), N(g,t)}
5. N(f,×) is a non-decreasing function of R and $\lim_{t\to\infty}$ N(f, t) = 1
6. For f ≠ 0, N (f,×) is continuous on R.

Thus N is a fuzzy norm on C(X), and hence the pair (C(X), N) is a fuzzy normed linear space. Every fuzzy normed space induces a fuzzy metric space.

Theorem 2: If X is a fuzzy compact space, then C(X) is a fuzzy Banach algebra if

(f g) (x) = f (x) g (x) whenever f, g ∈ C(X) and x∈ X.

Proof: Here, C(X) is algebra and (C(X), N) is a fuzzy normed vector space (By theorem).

Now consider N (f g, s t) = $\frac{st}{st+\|fg\|} = \frac{st}{st+\|f\|\|g\|}$

≥ $\frac{s}{s+\|f\|}.\frac{t}{t+\|g\|}$

≥ N (f, s) × N(g, t) for all f, g ∈ C(X) and all positive real numbers s

and t. Hence C(X) is a fuzzy normed algebra.

Next, we will show that C(X) is complete.

We have N (f, t) = $\frac{t}{t+\|f\|}$.

Let {f_n } be a Cauchy sequence in C(X). This means that $\lim_{n\to\infty}$ N ($f_{n+p} - f_n$, t) = 1, ∀ t > 0, ∀ p ∈ **N**. We have to show that {f_n} converges to some f in C(X). That is there exists an f ∈ C(X) such that $\lim_{n\to\infty}$ N ($f_n - f$, t) = 1 for all t > 0.

To prove there if an f ∈ C(X) such that $\lim_{n\to\infty}$ N ($f_n - f$, t) = 1

By definition, N ($f_{n+p} - f_n$, t) = $\frac{t}{t+\|f_{n+p}-f_n\|}$

Keeping n fixed, and let p→ ∞.

That is, $\lim_{p\to\infty}$ N ($f_{n+p} - f_n$, t) = $\lim_{p\to\infty}\frac{t}{t+\|f_{n+p}-f_n\|} = \frac{t}{t+\|f-f_n\|}$ = N (f - f_n, t).

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Hence $\lim_{n\to\infty} N(f_n - f, t) = 1$.

This establishes the completeness of C(X) as each Cauchy sequence in C(X) is convergent

Thus the fuzzy normed linear space C(X) is a fuzzy Banach space, and hence it is a fuzzy Banach algebra.

Remark: C(X) is abelian and has an identity (the constant, '1' function).

THE FUZZY SPECTRAL MEASURE

In this section, we introduce and study the fuzzy spectral measure E defined on the Borel subsets of X

Definition: Let *C* be the σ- algebra of fuzzy subsets in a set X, and let H be a fuzzy Hilbert space, a fuzzy *spectral measure* for (X, *C*, H) is a function E: $C \to B$ (H) with the following properties:

(a) For each Δ in *C*, E(Δ) is a self-adjoint projection

(b) $E(\Phi) = 0$ and $E(X) = 1$

(c) $E(\Delta_1 \& \Delta_2) = E(\Delta_1)\, E(\Delta_2)$ for Δ_1, Δ_2 in *C*

(d) If $(\Delta_n)_{n\in N}$ are pair wise disjoint sets in *C*, then $E(\Sigma_{n\in N} \Delta_n) = \Sigma_1^{\infty} E(\Delta_n)$.

Example: Let X be fuzzy compact set. Let *C* be the Borel subsets of X, μ = a fuzzy measure on *C* and $H = L^2(\mu)$. For Δ in *C* be, E (Δ) = multiplication by χ_Δ, the characteristic function of Δ. Then E is a fuzzy spectral measure for (X, *C*, H).

Definition: Let H is a fuzzy Hilbert space.

The *weak fuzzy operator topology* (WFOT) on *B* (H) if the fuzzy locally convex topology is defined by the semi-norms $\{p_{h,k}: h, k \in H\}$ where $p_{h,k}(A) = \langle Ah, k\rangle$.

The *strong fuzzy operator topology* (SFOT) on *B* (H) if the fuzzy locally convex topology is defined by the semi-norms $\{p_h: h \in H\}$ where $p_h(A) = \|Ah\|$.

Theorem 1: If E is a fuzzy spectral measure for (X, *C*, H), then $E_{g,h}(\Delta) \equiv \langle E(\Delta) g, h\rangle$ (g, h∈H) defines a countably additive fuzzy measure on with total variation $\leq \|g\| \|h\|$. Hence each $E_{g,h}$ is a regular Borel fuzzy measure.

Proof: Firstly, we prove $\mu = E_{g,h}$ is a countably additive fuzzy measure.

Let $(\Delta_n)_{n\in N}$ be pair-wise disjoint fuzzy sets in *C*. Then,

$$\mu(\Sigma_{n\in N} \Delta_n) = E_{g,h}(\Sigma_{n\in N} \Delta_n)$$

$$= \langle E(\Sigma_{n\in N} \Delta_n) g, h\rangle$$

$$= \langle \Sigma_1^{\infty} E(\Delta_n) g, h\rangle$$

$$= \Sigma_1^{\infty} \langle E(\Delta_n) g, h\rangle$$

$$= \Sigma_1^{\infty} E_{g,h}(\Delta_n)$$

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$= \Sigma_1^{\infty} \mu (\Delta_n)$

Thus, $\mu (\Sigma_{n\in N} \Delta_n) = \Sigma_1^{\infty} \mu (\Delta_n)$.

If $\Delta_1, \Delta_n..., \Delta_n$ are pair-wise disjoint sets in $\boldsymbol{C}$, let $\alpha_j \in \mathbf{C}$ such that $| \alpha_j | = 1$ and

$\langle E (\Delta_j) g, h\rangle = \alpha_j \langle E (\Delta_n)g, h\rangle$.

Now, $\Sigma_j |\mu (\Delta_j)| = \Sigma_j \alpha_j \langle E (\Delta_j) g, h\rangle$

$= \langle \Sigma_j E (\Delta_j) \alpha_j g, h\rangle$

$\leq \| \Sigma_j E (\Delta_j) \alpha_j g \| \| h \|$

But $\| \Sigma_j E (\Delta_j) \alpha_j g \|^2 = \Sigma_j \| E (\Sigma_{j=1}^{n} \Delta_j) g \|^2 = \Sigma_j \| E (\Delta_j) g \|^2 \leq \|g\|^2$

Hence $\Sigma_j |\mu (\Delta_j)| \leq \|g\| \|h\|$ and

$\|\mu \| \leq \|g\| \|h\|$

Theorem 2: $E_{g, g}$ is a positive fuzzy measure on C whose total variation is $\|g\|^2$
Proof:

$E_{g, g} (\Delta) = \langle E (\Delta)g, g\rangle = \| E(\Delta)g \|^2$, $g \in H$ so that $E_{g, g}$ is a positive fuzzy measure on C.

Total variation $= \| E_{g, g} \| = E_{g, g} (X) = \|g\|^2$

THE FUZZY REPRESENTATION

This section establishes the representation of B(X, C) using fuzzy spectral measure E and deduces the representation of C(X). Also, we prove the converse that each representation of C(X) corresponds to a unique fuzzy spectral measure E.

Definition: If E is a fuzzy spectral measure for (X, C, H) and ϕ: X→ **C** is a bounded C- measurable function, then there is a unique operator A in B (H) which is the integral of ϕ with respect E and is denoted by $\int \phi \, dE$.

Proposition: Let B(X, C) denote the set of bounded C – measurable functions ϕ: X→ **C** and let $\| \phi \| = \sup \{|\phi (x)|: x \in X\}$. Then B(X, C) is a fuzzy Banach algebra.

Proof: Since B(X, C) is the algebra of all bounded complex C – measurable functions on X with the fuzzy norm $\| \phi \| = \sup \{|\phi (x)|: x \in X\}$, B(X, C) is a complete normed fuzzy linear algebra and hence a fuzzy Banach algebra. B(X, C) has an involution $\phi^*(x) \equiv \phi (x)$, which makes it an abelian fuzzy C*-algebra.

Theorem: If E is a fuzzy spectral measure for (X, C, H) and ρ: B(X, C) → B (H) is defined by $\rho (\phi) = \grave{o} \, \phi \, dE$, then ρ is a representation of B(X, C).

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Proof: Letϕ, $\psi \in C(X)$. Let $\varepsilon > 0$ and choose a Borel partition $\{\Delta_1, \Delta_n..., \Delta_n\}$ of X such that $\sup\{| \omega (x) - \omega (x\not\in \Delta_k} |: x, x\not\in \Delta_k\} < \varepsilon$ for $\omega = \phi, \psi$ or $\phi\psi$ and for $1 \leq k \leq n$.

Hence, if $x_k \in \Delta_k$ $(1 \leq k \leq n)$,

$\| \int\omega \, dE - \Sigma_{k=1}^{n} \omega(x_k) E(\Delta_k) \| < \varepsilon$ for $\omega = \phi, \psi$ or $\phi\psi$.

The triangular inequality implies

$\| \int\phi\psi \, dE - (\int\phi \, dE) (\int\psi \, dE) \| \leq \varepsilon + \| \Sigma_{k=1}^{n} \phi(x_k) \psi(x_k) E(\Delta_k) - [\Sigma_{i=1}^{n} \phi(x_i) E(\Delta_i)] [\Sigma_{j=1}^{n} \psi(x_j) E(\Delta_j)] \| + \| [\Sigma_{i=1}^{n} \phi(x_i) E(\Delta_i)] [\Sigma_{j=1}^{n} \psi(x_j) E(\Delta_j)] - (\int\phi \, dE) (\int\psi \, dE) \|$

But $E(\Delta_i) E(\Delta_j) = E(\Delta_i \& \Delta_j)$ and $\{\Delta_1, \Delta_n..., \Delta_n\}$ is a partition. So the middle term in this sum is zero.

Hence $\| \int\phi\psi \, dE - (\int\omega \, dE) (\int\psi \, dE) \| \leq \varepsilon + \| [\Sigma_{i=1}^{n} \phi(x_i) E(\Delta_i)] [\Sigma_{j=1}^{n} \psi(x_j) E(\Delta_j) -$

$(\int\psi \, dE)] \| + \|[\Sigma_{i=1}^{n} \phi(x_i) E(\Delta_i) - (\int\phi \, dE)] [(\int\psi \, dE)] \|$

$\leq \varepsilon [1 + \|\phi\| + \|\psi\|]$

Since ε is arbitrary, $\int\phi\psi \, dE = (\int\phi \, dE) (\int\psi \, dE)$

We deduce the fuzzy integral representation of C(X) with respect to the fuzzy spectral measure E.

Corollary: If X is s fuzzy compact Hausdorff space and E is a Spectral measure defined on the Borel subsets of X, then $\rho: C(X) \to B(H)$ is defined by $\rho(u) = \int u \, dE$ a representation of C(X).

Remark: A representation$\rho: C(X) \to B(H)$ is a *-homomorphism with $\rho(1) = 1$. Also, $\|\rho\| = 1$. If $f \in C(X)$, then $f = g2$ where $g \in C(X)$, hence $\rho(f) = \rho(g2^{)} = \rho(g)^* \rho(g) \geq 0$. So ρ is a positive map. This is analogy with the fuzzy form of Riesz Representation theorem, that is $\rho(f) = \int f \, dE$ for some type of fuzzy measure E whose values are operators rather than scalars.

The following theorem is the main result of this section, and it states that the converse of the preceding corollary holds.

*Theorem***:** If $\rho: C(X) \to B(H)$ is a representation, there is a unique spectral measure E defined on the Borel subsets of X such that

$\rho(u) = \int u \, dE$ for every $u \in C(X)$.

(*Converse of the preceding corollary*)

Proof: We prove the theorem by extending ρ to a representation $\rho': B(X) \to B(H)$, where B(X) is the fuzzy C*-algebra of bounded Borel functions. The measure E of a Borel set Δ is then defined by $E(\Delta) = \rho'(\chi_\Delta)$.

Step I To prove that $\rho\not$*is a representation and* $\rho' | C(X) = \rho$

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If g, h ∈ H, then u → ⟨ρ(u) g, h⟩ is a fuzzy linear functional on C(X) with fuzzy norm ≤ ||*g*|| ||*h*||. Hence there is a unique fuzzy measure, $\mu_{g,h}$ in M(X) such that

$$\langle \rho(u)\, g, h\rangle = \int u\, d\mu_{g,h} \tag{1}$$

for all u ∈ C(X). Then the map (g, h) → $\mu_{g,h}$ is sesquilinear and $\|\mu g_{,hl}| \leq \|g\|\, \|h\|$.

Now fix ϕ in B(X) and define [g, h] = $\int \phi\, d\mu_{g,h}$. Then [., .] is sesquilinear form such that [g, h] ≤ ||*ϕ*|| ||g|| ||h||. Then there is a unique bounded operator A such that [g, h] = ⟨Ag, h⟩ and ||A||≤||*ϕ*||. Denote the operator A *b*y ||*ϕ*||ρ′(ϕ).

So ρ′: B(X) → B *(*H) is a well-defined function, ||ρ′(f)|| ≤ ||*ϕ*|| and for all g, h in H,

$$\langle \rho'(\phi)\, g, h\rangle = = \int \phi\, d\mu\, g,_{h} \tag{2}$$

C*laim:*ρ′: B(X) B (H) is a representation and ρ′| C(X) = ρ (3)

From (1) and (2) it follows that ρ′(u) = ρ(u) whenever u ∈ C(X)

If ϕ ∈ B(X) then ϕ correspond to the linear functional μ → $\int \phi\, d\mu$.

If ψ ∈ B(X), then ψμ ∈ M(X) whenever μ ∈ M(X)

In particular, if ψ ∈ C(X), then

ρ′(ϕψ) = WOT – lim ρ′(u_i ψ)

= WOT – lim ρ(u_i) ρ(ψ) = ρ′(ϕ) ρ′(ψ). That is, ρ′(ϕψ) = ρ′(ϕ) ρ′(ψ) whenever ϕ ∈ B(X) and ψ ∈ C(X).

Hence ρ′(u_i ψ) = ρ(u_i) ρ′(ψ) for any ψ ∈ B (X) and for all u_i.

Since ρ′(u_i) → ρ′(ϕ) (WOT) and ρ′(u_i ψ) → ρ′(ϕψ) (WOT), this implies that

ρ′(ϕψ) = ρ′(ϕ) ρ′(ψ) whenever ϕ, ψ ∈ B(X).

ρ′ is *linear* by (2).

To prove that ρ′(ϕ)* = ρ′($\bar{}$ϕ), let {u_i} be a net in C(X) obtained as in the preceding paragraph. If μ ∈ M(X), let $\bar{}$μ be the measure defined by $\bar{}$μ (Δ) = $\bar{\bar{}}$μ(Δ). Then ρ(u_i) → ρ′(ϕ) (WOT) implies ρ(u_i)* → ρ′(ϕ)* (WOT). But $\int \bar{}u_i\, d\mu = \int \bar{}\phi\, d\mu$ for every measureμ. Hence ρ($\bar{}u_i$) dμ → ρ′($\bar{}$ϕ). But ρ(u_i)* → ρ($\bar{}u_i$) since ρ is a *-homomorphism.

Thus, ρ′(ϕ)* = ρ′($\bar{}$ϕ) and ρ′ is a *representation*.

Step II To prove that E is a fuzzy spectral measure

Let E (Δ) ° ρ′(χ_Δ) for any Borel fuzzy subset Δ of X. We have to show that E is a fuzzy spectral measure. Since χ_Δ is Hermitian idempotent in B(X), E (Δ) is a projection (by (3)).

Since $\chi_\Phi = 0$ and $\chi_X = 1$, E (Φ) = 0 and E(X) = 1.

Also E(Δ_1& Δ_2) = ρ¢($\chi_{\Delta 1\,\&\,\Delta 2}$) = ρ′($\chi_{\Delta 1}\, \chi_{\Delta 2}$) = E($\Delta_1$) E($\Delta_2$) for $\Delta_{1,}$ Δ_2 in M . Now let (Δ_n) $_{n\in N}$ be a pair wise disjoint sets in M and put Δ_n = ($\Sigma^{\infty}_{k=n+1}$ Δ_k) then for h ∈ H,

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$\| E(\Sigma^{\infty}_{k=1} \Delta_k) h - \Sigma_{k=1}^{n} E(\Delta_k) h \| = \langle E(\Delta_n)h, E(\Delta_n) h \rangle$

$= \langle E(\Delta_n)h, h\rangle$

$= \langle \rho'(\chi_{\Delta n}) h, h\rangle$

$= \int (\chi_{\Delta n}) d\mu_{h, h}$

$= (\Sigma^{\infty}_{k=n+1} \mu_{h, h} (\Delta_k) \to o$ as $n \to \infty$

Thus, $E(\Sigma_{n\in N} \Delta_n) = \Sigma_1^{\infty} E(\Delta_n)$.

Therefore, E is a fuzzy spectral measure

Step III To prove $\rho(u) = \int u\, dE$

It remains to show that $\rho(u) = \int u\, dE$. It will be shown that $\rho'(\phi) = \int \phi\, dE$ for every ϕ in B(X). Fix ϕ in B(X) and $\varepsilon > 0$.

If $\{\Delta_1, \Delta_2, ..., \Delta_n\}$ is any Borel partition of X such that $\sup\{|\phi(x) - \phi(x')|: x, x' \in \Delta_k\} < \varepsilon$ for $1 \le k \le n$, then $\| \phi - \Sigma_{k=1}^{n} \phi(x_k) \chi_{\Delta k} \|_{\infty} < \varepsilon$ for every choice of x_k in Δ_k.

Since $\|\rho'\| = 1$, $\varepsilon > \| \rho'(\phi) - \Sigma_{k=1}^{n} \phi(x_k) E_{\Delta k} \|$ which implies that $\rho'(\phi) = \int \phi\, d E$ for any ϕ in B(X).

Step IV The uniqueness of E

Let E_1 and E_2 be two fuzzy spectral measures defined on the Borel subsets of X satisfying the conditions.

To prove $E_1(\Delta) = E_2(\Delta)$ for all $\Delta \in C$ we prove that $E_1(\phi) = E_2(\phi)$ for every ϕ in B(X).

$E_1(\Delta) = \rho'(\chi_{\Delta}) = \int \chi_{\Delta}\, dE_1 \le \int \phi\, dE_1 = \rho'(\phi) = \int \phi\, dE_2 = E_2(\Delta)$

Since ε is arbitrary, $E_1(\Delta) \le E_2(\Delta)$

By interchanging E_1 and E_2, we get $E_1(\Delta) \le E_2(\Delta)$

Thus, $E_1(\Delta) = E_2(\Delta)$ for all $\Delta \in C$

Hence E is a fuzzy spectral measure

CONCLUSION

We have seen that fuzzy spectral measure plays a significant role in the representation theorem. The theory developed in this work can prove more theorems that reveal the structure of fuzzy normal operators in the fuzzy setting. Therefore, there is scope for future work.

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Chapter 12
Tanimoto Similarity Coefficients Measuring Bipolar q-Rung Picture Fuzzy Information and Their Applications

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ABSTRACT

In this chapter, the concepts of bipolar q-rung picture fuzzy set and bipolar q-rung picture fuzzy number are introduced. Moreover, the fundamentals of bipolar q-rung picture fuzzy sets and bipolar q-rung picture fuzzy numbers are studied. Relatedly, some basic operations, aggregation operators, and relations on the bipolar q-rung picture fuzzy sets/numbers are derived. The intuitive definition of Tanimoto's similarity coefficient is proposed to measure the similarity between two bipolar q-rung picture fuzzy sets. Finally, it is shown that the proposed Tanimoto similarity measures can be used to deal with the problems of medical diagnosis and pattern recognition under the bipolar q-rung picture fuzzy environment.

INTRODUCTION

Decision-making is a process in which decision maker(s) is invited to participate in the evaluation of some given alternatives, and then the most suitable to be selected or all of them are ranked according to the evaluation information. As the decision making problems become more complex, it is now very difficult for decision-maker(s) to evaluate alternatives considering real values. In 1965, Zadeh (Zadeh, 1965) initiated the theory of fuzzy set (FSs), which provides the decision maker(s) with an efficient way to model the fuzzy information. Nevertheless, the FSs are incapable of expressing the non-membership degree of each element in the universe of discourse belonging to an FS. To deal with this imperfection of

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FSs, Atanassov (Atanassov, 1986) proposed the concept of intuitionistic fuzzy sets (IFSs) in which each intuitionistic fuzzy number (IFN) consists of membership degree and non-membership degree, and the sum of them is less than or equal to 1. In the following years, by improving membership degree and non-membership degree in the structure of IFS, many researchers discussed new types of IFSs, such as interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov, 1989; Broumi and Smarandache, 2014; Garg and Kumar 2019, 2020; Kamacı 2019), intuitionistic fuzzy multisets (IFMSs) (Ejegwa, 2015; Shinoj and John, 2012), Pythagorean fuzzy sets (PyFSs) (Yager, 2014; Garg, 2016, 2017), q-rung orthopair fuzzy sets (q-ROFSs) (Garg, 2020; Garg et al., 2020; Riaz et al., 2020a, 2020b, 2020c; Yager, 2017) and linear Diophantine fuzzy sets (LDFSs) (Riaz and Hashmi, 2019; Kamacı, 2021a). Moreover, the matrix representations of fuzzy sets and their extensions were also widely studied (Guleria and Bajaj, 2019; Kim and Roush, 1980; Padder and Murugadas, 2019; Kamacı et al., 2018a, 2018b; Petchimuthu et al., 2020; Petchimuthu and Kamacı, 2019, 2020). However, all these above theories failed to deal with abstinence, therefore, Cuong (Cuong, 2014) developed a new concept called picture fuzzy set (PFS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. Later, some aspects of this type of fuzzy sets were developed (Kamacı, 2020, 2021b, 2021c, 2021d; Kamacı et al., 2021a, 2021b; Khalil et al., 2019). In 2019, Mahmood et al. (Mahmood et al., 2019) defined the concept of spherical fuzzy sets (SFSs) in which each spherical fuzzy number (SFN) consists of membership degree, abstinence degree, and non-membership degree whose square sum is less than or equal to 1. Li et al. (Li et al., 2018) described the q-rung picture fuzzy sets (q-RPFSs) extending the SFSs, where $q \geq 1$. Especially, Mahmood et al. (Mahmood et al., 2019) named these sets as T-spherical fuzzy sets (TSFSs) for $q \in \mathbb{Z}^+$.The lax constraint of q-RPFS is that the sum of qth power of the membership degree, abstinence degree, and non-membership degree is equal to or less than 1. He et al. (He et al., 2019) described the q-rung picture fuzzy Dombi Hamy mean operators and a new q-rung picture fuzzy MAGDM method. Akram and Habib (Akram and Habib, 2020) and Sitara et al. (Sitara et al., 2021) studied q-rung picture fuzzy graph structures and their applications. Recently, q-rung picture fuzzy models have been interested in many researchers.

Bipolarity refers to the affinity of the human psyche in reasoning and settling on choices based on positive and negative impacts. Where positive information states what is satisfactory, possible, permissible, considered, or desired as being acceptable. Otherwise, negative statements express what is impossible, rejected, or forbidden. Negative inclinations compare to restrictions, since they indicate which esteems or articles must be dismissed (i.e., those that do not fulfill the requirements), while positive inclinations relate to wishes, as they specify which items are more alluring than others (i.e., fulfill client wishes) without dismissing those that don't meet the desires. Thus, Zhang (Zhang, 1994; Zhang, 1998) proposed the notion of BFSs, which is a generalization of the fuzzy sets. Inspired by the idea of BFSs, many papers were published on the hybridization of bipolarity with FSs, IFSs, PyFSs, and PFSs (Ezhilmaran and Sankar, 2015; Mandal, 2020; Naz et al., 2018). In 2020, Akalyedevi et al. (Akalyedevi et al., 2020) and Princy and Mohana (Princy and Mohana, 2020) proposed bipolar spherical fuzzy sets (BSFSs) as a combination of SFSs and BFSs.

The objectives of this chapter are: (1) to present the new idea that is bipolar q-rung picture fuzzy sets (Bq-RPFSs) (2) to characterize the bipolar q-rung picture fuzzy numbers (Bq-RPFNs) (3) to present score and accuracy functions for comparison of the Bq-RPFNs, (4) to describe bipolar q-rung picture fuzzy weighted average aggregation operators and bipolar q-rung picture fuzzy weighted geometric aggregation operator (5) to propose the Tanimoto similarity coefficient-based approaches providing the strength of the relationship between two Bq-RPFSs, and to give their applications.

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Background

In 1965, Zadeh (Zadeh, 1965) proposed the FSs as an effective means to model uncertain information. By offering the idea of FSs, he allocated membership grade functions to the elements of a set in the interval [0, 1]. In 1986, Atanassov (Atanassov, 1986) broadened the notion of FSs to IFSs in which each IFN consists of membership degree and non-membership degree such that the sum of them is less than or equal to 1. Nevertheless, the IFSs are incapable of expressing the abstinence degree of each element in the universe of discourse belonging to an IFS. To deal with the shortcoming of IFSs, Cuong (Cuong, 2014) developed the theory of PFSs in which each picture fuzzy number (PFN) consists of membership degree, non-membership degree, and abstinence degree such that the sum of them is less than or equal to 1. Fortifying the idea of PFS, Mahmood et al. (Mahmood et al., 2019) recommend SFSs, which somehow extended the space of membership, abstinence, and non-membership grades by proposing a new constraint that the sum of squares is less than or equal to 1. To further expand the SFSs, Li et al. (Li et al., 2018) described the q-RPFSs which somehow extended the space of membership, abstinence, and non-membership grade by introducing a new lax constraint that the sum of qth power ($q^3$1) of the membership degree, abstinence degree and non-membership degree is equal to or less than 1. Otherwise, in the later twentieth century, Zhang (Zhang, 1994; Zhang, 1998) proposed the bipolar approach to the FSs and thus introduced the fundamentals of BFSs. In the following years, bipolar types of IFSs, PFSs, SFSs were proposed and their characteristic properties were studied (Akalyedevi et al., 2020; Ezhilmaran and Sankar, 2015; Princy and Mohana, 2020).

Definition 1. (Zadeh, 1965) A fuzzy set (FS) $\mathcal{F}$ in $\mathfrak{H}$ is defined as ordered pairs

$$\mathcal{F} = \left\{\left(\mathfrak{h}, \alpha_{\mathcal{F}}\left(\mathfrak{h}\right)\right) : \ \mathfrak{h} \in \mathfrak{H}\right\} \tag{1}$$

where $\alpha_{\mathcal{F}} : \ \mathfrak{H} \to \left[0,1\right]$ is a membership function from $\mathfrak{H}$ to [0,1] and $\alpha_{\mathcal{F}}\left(\mathfrak{h}\right)$ represents the degree of membership of $\mathfrak{h}$ to the set $\mathcal{F}$.

Definition 2. (Zhang, 1994; Zhang, 1998) A bipolar fuzzy set (BFS) $\breve{\mathcal{F}}$ in $\mathfrak{H}$ is defined as

$$\breve{\mathcal{F}} = \left\{\left(\mathfrak{h}, \alpha^{+}_{\breve{\mathcal{F}}}\left(\mathfrak{h}\right), \alpha^{-}_{\breve{\mathcal{F}}}\left(\mathfrak{h}\right)\right) : \ \mathfrak{h} \in \mathfrak{H}\right\} \tag{2}$$

where $\alpha^{+}_{\breve{\mathcal{F}}} : \ \mathfrak{H} \to \left[0,1\right]$ and $\alpha^{-}_{\breve{\mathcal{F}}} : \ \mathfrak{H} \to \left[-1,0\right]$ are the membership function for positive information and the membership function for negative information, respectively. Also, $\alpha^{+}_{\breve{\mathcal{F}}}\left(\mathfrak{h}\right)$ and $\alpha^{-}_{\breve{\mathcal{F}}}\left(\mathfrak{h}\right)$ represent the degrees of positive membership and negative membership of $\mathfrak{h}$ to the set $\breve{\mathcal{F}}$, respectively.

Definition 3. (Atanassov, 1986) An intuitionistic fuzzy set (IFS) $\mathcal{I}$ in $\mathfrak{H}$ is defined as

$$\mathcal{I} = \left\{\left(\mathfrak{h}, \alpha_{\mathcal{I}}\left(\mathfrak{h}\right), \gamma_{\mathcal{I}}\left(\mathfrak{h}\right)\right) : \ \mathfrak{h} \in \mathfrak{H}\right\} \tag{3}$$

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where $\alpha_{\mathcal{I}}, \gamma_{\mathcal{I}}: \mathfrak{H} \to [0,1]$ are the functions of membership and non-membership, respectively. Also, $\alpha_{\mathcal{I}}(\mathfrak{h})$ and $\gamma_{\mathcal{I}}(\mathfrak{h})$ represent the degrees of membership and non-membership of $\mathfrak{h}$ to the set $\mathcal{I}$, with the condition

$$0 \le \alpha_{\mathcal{I}}(\mathfrak{h}) + \gamma_{\mathcal{I}}(\mathfrak{h}) \le 1$$

for every $\mathfrak{h} \in \mathfrak{H}$.

Definition 4. (Cuong, 2014) A picture fuzzy set (PFS) $\mathcal{P}$ in $\mathfrak{H}$ is defined as

$$\mathcal{P} = \left\{ \left(\mathfrak{h}, \alpha_{\mathcal{P}}(\mathfrak{h}), \beta_{\mathcal{P}}(\mathfrak{h}), \gamma_{\mathcal{P}}(\mathfrak{h}) \right) : \mathfrak{h} \in \mathfrak{H} \right\} \tag{4}$$

where $\alpha_{\mathcal{P}}, \beta_{\mathcal{P}}, \gamma_{\mathcal{P}} : \mathfrak{H} \to [0,1]$ are the functions of membership, abstinence, and non-membership, respectively. Also, $\alpha_{\mathcal{P}}(\mathfrak{h}), \beta_{\mathcal{P}}(\mathfrak{h})$, and $\gamma_{\mathcal{P}}(\mathfrak{h})$ represent the degrees of membership, abstinence, and non-membership of $\mathfrak{h}$ to the set $\mathcal{P}$, with the condition

$$0 \le \alpha_{\mathcal{P}}(\mathfrak{h}) + \beta_{\mathcal{P}}(\mathfrak{h}) + \gamma_{\mathcal{P}}(\mathfrak{h}) \le 1$$

for every $\mathfrak{h} \in \mathfrak{H}$.

Definition 5. (Li et al., 2018) A *q*-rung picture fuzzy set (q-RPFS) $\mathcal{Q}$ in $\mathfrak{H}$ is defined as

$$\mathcal{Q} = \left\{ \left(\mathfrak{h}, \alpha_{\mathcal{Q}}(\mathfrak{h}), \beta_{\mathcal{Q}}(\mathfrak{h}), \gamma_{\mathcal{Q}}(\mathfrak{h}) \right) : \mathfrak{h} \in \mathfrak{H} \right\} \tag{5}$$

where $\alpha_{\mathcal{Q}}, \beta_{\mathcal{Q}}, \gamma_{\mathcal{Q}}: \mathfrak{H} \to [0,1]$ are the functions of membership, abstinence, and non-membership, respectively. Also $\alpha_{\mathcal{Q}}(\mathfrak{h}), \beta_{\mathcal{Q}}(\mathfrak{h})$, and $\gamma_{\mathcal{Q}}(\mathfrak{h})$ represent the degrees of membership, abstinence, and non-membership of $\mathfrak{h}$ to the set $\mathcal{Q}$, with the condition

$$0 \le (\alpha_{\mathcal{Q}}(\mathfrak{h}))^q + \left(\beta_{\mathcal{Q}}(\mathfrak{h})\right)^q + \left(\gamma_{\mathcal{Q}}(\mathfrak{h})\right)^q \le 1$$

for every $\mathfrak{h} \in \mathfrak{H}$, where $q \ge 1$. Furthermore, the degree of refusal is

$$\delta_{\mathcal{Q}}(\mathfrak{h}) = \sqrt[q]{1 - (\alpha_{\mathcal{Q}}(\mathfrak{h}))^q - \left(\beta_{\mathcal{Q}}(\mathfrak{h})\right)^q - \left(\gamma_{\mathcal{Q}}(\mathfrak{h})\right)^q}$$

for every $\mathfrak{h} \in \mathfrak{H}$, where $q \ge 1$.

Especially, for $q \ge 1$ and $q \in \mathbb{Z}^+$, this set is said to be a *T*-spherical fuzzy set by Mahmood et al. (Mahmood et al., 2019).

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MAIN FOCUS OF THE CHAPTER

Bipolar q-Rung Picture Fuzzy Sets

The q-RPFSs based models may be satisfactory in conjunctures when human views concerning responses: yes, abstention, no and refusal are encountered. In some situations in the real-life, negative information should be categorized as well as positive information, so bipolarity may need to be dealt with. The pros and cons are the most obvious sample for this bipolarity model. For instance, a customer can vote as yes, abstention, and no for the positive aspects of the service quality of the restaurant, while can vote as yes, abstention and no for the negative aspects (counter-properties) of the service quality of the restaurant. The q-RPFSs cannot deal with such situations. Thus, in addition to the degrees of membership, abstinence, and non-membership of an element corresponding to a q-RPFS, the degrees of membership, abstinence, and non-membership of an element to some implicit counter-property should also be addressed. Therefore, the concept of bipolar *q*-rung picture fuzzy set is originated as follows:

From now on, |.| represents the absolute value.

Definition 6. A bipolar *q*-rung picture fuzzy set (Bq-RPFS) $\breve{\mathcal{Q}}$ in $\mathfrak{H}$ is defined as

$$\breve{\mathcal{Q}} = \left\{\left(\mathfrak{h}, \alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right) : \mathfrak{h} \in \mathfrak{H}\right\} \tag{6}$$

where $\alpha_{\breve{\mathcal{Q}}}^{+}, \beta_{\breve{\mathcal{Q}}}^{+}, \gamma_{\breve{\mathcal{Q}}}^{+} : \mathfrak{H} \to [0,1]$ are the functions of membership, abstinence, and non-membership for positive information, respectively and $\alpha_{\breve{\mathcal{Q}}}^{-}, \beta_{\breve{\mathcal{Q}}}^{-}, \gamma_{\breve{\mathcal{Q}}}^{-} : \mathfrak{H} \to [-1,0]$ are the functions of membership, abstinence, and non-membership for negative information, respectively. Also, $\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})$, and $\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})$ represent the degrees of membership, abstinence, and non-membership of $\mathfrak{h}$ to the set $\mathcal{Q}$ for positive information, and $\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})$, and $\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})$ represent the degrees of membership, abstinence and non-membership of $\mathfrak{h}$ to the set $\mathcal{Q}$ for negative information with the condition

$$0 \le \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})\right)^{q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})\right)^{q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})\right)^{q} \le 1$$

and

$$0 \le |\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})|^{q} + \left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right|^{q} + \left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right|^{q} \le 1$$

for every $\mathfrak{h} \in \mathfrak{H}$, where $q \ge 1$. Moreover, the degrees of refusal for positive information and negative information, denoted by $\delta_{\breve{\mathcal{Q}}}^{+}$ and $\delta_{\breve{\mathcal{Q}}}^{-}$, are described as follows, respectively:

$$\delta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) = \sqrt[q]{1 - \left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})\right)^{q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})\right)^{q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h})\right)^{q}\right)}$$

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and

$$\delta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) = -\sqrt[q]{1-(\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right|^{q} + \left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right|^{q} + \left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right|^{q})}$$

for every $\mathfrak{h} \in \mathfrak{H}$.

For Bq-RPFS

$$\breve{\mathcal{Q}} = \left\{\left(\mathfrak{h}, \alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right) : \mathfrak{h} \in \mathfrak{H}\right\},$$

the six components

$$\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h})\right)$$

are said to be a bipolar q-rung picture fuzzy number (Bq-RPFN) and each Bq-RPFN can be denoted by

$$\breve{\rho} = \left(\alpha^{+}, \beta^{+}, \gamma^{+}, \alpha^{-}, \beta^{-}, \gamma^{-}\right)$$

where

$$\alpha^{+}, \beta^{+}, \gamma^{+} : \mathfrak{H} \to [0,1] \text{ and } \alpha^{-}, \beta^{-}, \gamma^{-} : \mathfrak{H} \to [-1,0]$$

such that

$$0 \le \left(\alpha^{+}\right)^{q} + \left(\beta^{+}\right)^{q} + \left(\gamma^{+}\right)^{q} \le 1 \text{ and } 0 \le |\alpha^{-}|^{q} + \left|\beta^{-}\right|^{q} + \left|\gamma^{-}\right|^{q} \le 1$$

for $q \ge 1$.

Example 1. Let $\mathfrak{H} = \{\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3, \mathfrak{h}_4\}$ be a discourse universe set, then

$$\breve{\mathcal{Q}} = \begin{Bmatrix} (\mathfrak{h}_1, 0.6, 0.3, 0.5, -0.5, -0.4, -0.6), (\mathfrak{h}_2, 1, 0, 0, 0, -0.1, -0.8) \\ (\mathfrak{h}_3, 0.7, 0.7, 0.7, -0.8, -0.1, -0.3), (\mathfrak{h}_4, 0.5, 0.5, 0.7, 0, 0, -1) \end{Bmatrix}$$

is a Bq-RPFS in $\mathfrak{H}$ for q=3.

Some basic operations such as complement, intersection, union of the Bq-RPFSs are derived as follows:

Definition 7. Let $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ be two Bq-RPFSs in $\mathfrak{H}$.

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a) $\breve{\mathcal{Q}} \subseteq \breve{\mathcal{R}}$ iff
$\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \le \alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}),\ \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \le \beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}),\ \gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \ge \gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}),\ \alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \ge \alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}),\ \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \ge \beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})$ and
$\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \le \gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})$ for every $\mathfrak{h} \in \mathfrak{H}$.

b) $\breve{\mathcal{Q}} = \breve{\mathcal{R}}$ iff $\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) = \alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) = \beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) = \gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}), \alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) = \alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) = \beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})$ and $\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) = \gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})$ for every $\mathfrak{h} \in \mathfrak{H}$.

c) $$\breve{\mathcal{Q}} \cap \breve{\mathcal{R}} = \left\{ \left(\mathfrak{h}, \begin{pmatrix} \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \wedge \alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h})\right), \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \wedge \beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h})\right), \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \vee \gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h})\right) \\ \left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \vee \alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})\right), \left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \wedge \beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})\right), \left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \wedge \gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})\right) \end{pmatrix} \right) : \mathfrak{h} \in \mathfrak{H} \right\}.$$

d) $$\breve{\mathcal{Q}} \cup \breve{\mathcal{R}} = \left\{ \left(\mathfrak{h}, \begin{pmatrix} \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \vee \alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h})\right), \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \wedge \beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h})\right), \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}) \wedge \gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h})\right) \\ \left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \wedge \alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})\right), \left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \wedge \beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})\right), \left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \vee \gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h})\right) \end{pmatrix} \right) : \mathfrak{h} \in \mathfrak{H} \right\}.$$

e) $$\breve{\mathcal{Q}}^{c} = \left\{ \left(\mathfrak{h}, \gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}), \gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}), \alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}) \right) : \mathfrak{h} \in \mathfrak{H} \right\}.$$

Example 2. Consider the B-qRPFS $\breve{\mathcal{Q}}$ over $\mathfrak{H}$ given in Example 1. The complement of $\breve{\mathcal{Q}}$ is

$$\breve{\mathcal{Q}} = \left\{ \begin{matrix} \left(\mathfrak{h}_1, 0.5, 0.3, 0.6, -0.6, -0.4, -0.5\right), \left(\mathfrak{h}_2, 0, 0, 1, -0.8, -0.1, 0\right) \\ \left(\mathfrak{h}_3, 0.7, 0.7, 0.7, -0.3, -0.1, -0.8\right), \left(\mathfrak{h}_4, 0.7, 0.5, 0.5, -1, 0, 0\right) \end{matrix} \right\}$$

Also, assume that

$$\breve{\mathcal{R}} = \left\{ \begin{matrix} \left(\mathfrak{h}_1, 0.7, 0.4, 0.1, -0.6, -0.6, -0.2\right), \left(\mathfrak{h}_2, 1, 0, 0, -0.2, -0.4, -0.9\right) \\ \left(\mathfrak{h}_3, 0.8, 0.7, 0.2, -0.8, -0.1, -0.1\right), \left(\mathfrak{h}_4, 0.6, 0.7, 0.6, 0, 0, -1\right) \end{matrix} \right\}$$

and

$$\breve{\mathcal{S}} = \left\{ \begin{matrix} \left(\mathfrak{h}_1, 0.5, 0.2, 0, -0.4, -0.4, -0.4\right), \left(\mathfrak{h}_2, 0.9, 0.1, 0, -0.2, -0.3, -0.8\right) \\ \left(\mathfrak{h}_3, 0.4, 0.8, 0.4, -0.7, -0.2, -0.2\right), \left(\mathfrak{h}_4, 0.7, 0.6, 0.5, -0.1, 0, -0.9\right) \end{matrix} \right\}.$$

Then, $\breve{\mathcal{R}}$ and $\breve{\mathcal{S}}$ are two B-qRPFSs $\breve{\mathcal{Q}}$ in $\mathfrak{H}$. Obviously, $\breve{\mathcal{Q}} \subseteq \breve{\mathcal{R}}$. The intersection and union of Bq-RPFSs $\breve{\mathcal{Q}}$ and $\breve{\mathcal{S}}$ are respectively

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$$\breve{\mathcal{Q}} \cap \breve{\mathcal{R}} = \begin{Bmatrix} \left(\mathfrak{h}_1, 0.5, 0.2, 0.5, -0.4, -0.4, -0.6\right), \left(\mathfrak{h}_2, 0.9, 0, 0, 0, -0.3, -0.8\right) \\ \left(\mathfrak{h}_3, 0.4, 0.7, 0.4, -0.7, -0.2, -0.3\right), \left(\mathfrak{h}_4, 0.5, 0.5, 0.7, 0, 0, -1\right) \end{Bmatrix}$$

and

$$\breve{\mathcal{Q}} \cup \breve{\mathcal{R}} = \begin{Bmatrix} \left(\mathfrak{h}_1, 0.6, 0.2, 0.0, -0.5, -0.4, -0.4\right), \left(\mathfrak{h}_2, 1, 0, 0, -0.2, -0.3, -0.8\right) \\ \left(\mathfrak{h}_3, 0.7, 0.7, 0.4, -0.8, -0.2, -0.2\right), \left(\mathfrak{h}_4, 0.7, 0.5, 0.5, -0.1, 0, -0.9\right) \end{Bmatrix}.$$

Proposition 1. Let $\breve{\mathcal{Q}}, \breve{\mathcal{R}}$, and $\breve{\mathcal{S}}$ be three Bq-RPFSs in $\mathfrak{H}$. Then,

$\breve{\mathcal{Q}} \circledast \breve{\mathcal{R}}$ and $\breve{\mathcal{R}} \circledast \breve{\mathcal{S}}$ then $\breve{\mathcal{Q}} \circledast \breve{\mathcal{S}}$ for each $\circledast \in \{\subseteq, =\}$.

$\breve{\mathcal{Q}} \intercal \breve{\mathcal{R}} = \breve{\mathcal{R}} \intercal \breve{\mathcal{Q}}$ for each $\intercal \in \{\cap, \cup\}$.

$\breve{\mathcal{Q}} \intercal \left(\breve{\mathcal{R}} \intercal \breve{\mathcal{S}}\right) = \left(\breve{\mathcal{Q}} \intercal \breve{\mathcal{R}}\right) \intercal \breve{\mathcal{S}}$ for each $\intercal \in \{\cap, \cup\}$.

$\breve{\mathcal{Q}} \intercal \left(\breve{\mathcal{R}} \perp \breve{\mathcal{S}}\right) = \left(\breve{\mathcal{Q}} \intercal \breve{\mathcal{R}}\right) \perp \left(\breve{\mathcal{Q}} \intercal \breve{\mathcal{S}}\right)$ for each $\intercal, \perp \in \{\cap, \cup\}$.

$\left(\breve{\mathcal{Q}} \intercal \breve{\mathcal{R}}\right)^c = \breve{\mathcal{Q}}^c \perp \breve{\mathcal{R}}^c$ for each $\intercal, \perp \in \{\cap, \cup\}$ and $\intercal \neq \perp$.

Proof. The proofs are straightforward.

Several operational laws for Bq-RPFNs are proposed as follows:

Definition 8. Let $\breve{\rho}_1 = \left(\alpha_1^+, \beta_1^+, \gamma_1^+, \alpha_1^-, \beta_1^-, \gamma_1^-\right)$ and $\breve{\rho}_2 = \left(\alpha_2^+, \beta_2^+, \gamma_2^+, \alpha_2^-, \beta_2^-, \gamma_2^-\right)$ be two Bq-RPFNs in $\mathfrak{H}$ and ϖ>0. Then, the operations of Bq-RPFNs are proposed as

a) $$\breve{\rho}_1 \oplus \breve{\rho}_2 = \left\langle \begin{matrix} \sqrt[q]{\left(\alpha_1^+\right)^q + \left(\alpha_2^+\right)^q - \left(\alpha_1^+\right)^q \cdot \left(\alpha_2^+\right)^q}, \left(\beta_1^+ \cdot \beta_2^+\right), \left(\gamma_1^+ \cdot \gamma_2^+\right), \\ -\left(\alpha_1^- \cdot \alpha_2^-\right), -\left(\beta_1^- \cdot \beta_2^-\right), -\sqrt[q]{\left(-\gamma_1^-\right)^q + \left(-\gamma_2^-\right)^q - \left(-\gamma_1^-\right)^q \cdot \left(-\gamma_2^-\right)^q} \end{matrix} \right\rangle,$$

b) $$\breve{\rho}_1 \otimes \breve{\rho}_2 = \left\langle \begin{matrix} \left(\alpha_1^+ \cdot \alpha_2^+\right), \left(\beta_1^+ \cdot \beta_2^+\right), \sqrt[q]{\left(\gamma_1^+\right)^q + \left(\gamma_2^+\right)^q - \left(\gamma_1^+\right)^q \cdot \left(\gamma_2^+\right)^q}, \\ -\sqrt[q]{\left(-\alpha_1^-\right)^q + \left(-\alpha_2^-\right)^q - \left(-\alpha_1^-\right)^q \cdot \left(-\alpha_2^-\right)^q}, -\left(\beta_1^- \cdot \beta_2^-\right), -\left(\gamma_1^- \cdot \gamma_2^-\right)) \end{matrix} \right\rangle,$$

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c) $\varpi\breve{\rho}_1 = \left\langle \begin{array}{c} \sqrt[q]{1-\left(1-\left(\alpha_1^+\right)^q\right)^{\varpi}}, \left(\beta_1^+\right)^{\varpi}, \left(\gamma_1^+\right)^{\varpi}, \\ -\left(-\alpha_1^-\right)^{\varpi}, -\left(-\beta_1^-\right)^{\varpi}, -\sqrt[q]{1-\left(1-\left(-\gamma_1^-\right)^q\right)^{\varpi}} \end{array} \right\rangle,$

d) $\breve{\rho}_1^{\varpi} = \left\langle \begin{array}{c} \left(\alpha_1^+\right)^{\varpi}, \left(\beta_1^+\right)^{\varpi}, \sqrt[q]{1-\left(1-\left(\gamma_1^+\right)^q\right)^{\varpi}}, \\ -\sqrt[q]{1-\left(1-\left(-\alpha_1^-\right)^q\right)^{\varpi}}, -\left(-\beta_1^-\right)^{\varpi}, -\left(-\gamma_1^-\right)^{\varpi} \end{array} \right\rangle.$

Proposition 2. Let $\breve{\rho}, \breve{\rho}_1$, and $\breve{\rho}_2$ be the Bq-RPFNs in $\breve{\mathfrak{H}}$ and $\varpi, \varpi 1, \varpi 2 > 0$. Then,

1. $\breve{\rho}_1 \oplus \breve{\rho}_2 = \breve{\rho}_2 \oplus \breve{\rho}_1$.
2. $\breve{\rho}_1 \otimes \breve{\rho}_2 = \breve{\rho}_2 \otimes \breve{\rho}_1$.
3. $\varpi\left(\breve{\rho}_1 \oplus \breve{\rho}_2\right) = \varpi\breve{\rho}_1 \oplus \varpi\breve{\rho}_2$.
4. $\left(\breve{\rho}_1 \otimes \breve{\rho}_2\right)^{\varpi} = \breve{\rho}_1^{\varpi} \otimes \breve{\rho}_2^{\varpi}$.
5. $\varpi_1\breve{\rho} \oplus \varpi_2\breve{\rho} = \left(\varpi_1 + \varpi_2\right)\breve{\rho}$.
6. $\breve{\rho}^{\varpi_1} \otimes \breve{\rho}^{\varpi_2} = \breve{\rho}^{\left(\varpi_1 + \varpi_2\right)}$.

Proof. Let us prove the properties (1) and (4). Others can be demonstrated using similar techniques.

(1) $\breve{\rho}_1 \oplus \breve{\rho}_2 = \left\langle \begin{array}{c} \sqrt[q]{\left(\alpha_1^+\right)^q + \left(\alpha_2^+\right)^q - \left(\alpha_1^+\right)^q \cdot \left(\alpha_2^+\right)^q}, \left(\beta_1^+ \cdot \beta_2^+\right), \left(\gamma_1^+ \cdot \gamma_2^+\right), \\ -\left(\alpha_1^- \cdot \alpha_2^-\right), -\left(\beta_1^- \cdot \beta_2^-\right), -\sqrt[q]{\left(-\gamma_1^-\right)^q + \left(-\gamma_2^-\right)^q - \left(-\gamma_1^-\right)^q \cdot \left(-\gamma_2^-\right)^q} \end{array} \right\rangle$

$= \left\langle \begin{array}{c} \sqrt[q]{\left(\alpha_2^+\right)^q + \left(\alpha_1^+\right)^q - \left(\alpha_2^+\right)^q \cdot \left(\alpha_1^+\right)^q}, \left(\beta_2^+ \cdot \beta_1^+\right), \left(\gamma_2^+ \cdot \gamma_1^+\right), \\ -\left(\alpha_2^- \cdot \alpha_1^-\right), -\left(\beta_2^- \cdot \beta_1^-\right), -\sqrt[q]{\left(-\gamma_2^-\right)^q + \left(-\gamma_1^-\right)^q - \left(-\gamma_2^-\right)^q \cdot \left(-\gamma_1^-\right)^q} \end{array} \right\rangle$

$= \breve{\rho}_2 \oplus \breve{\rho}_1.$

(2) $\left(\breve{\rho}_1 \otimes \breve{\rho}_2\right)^{\varpi} = \left\langle \begin{array}{c} \left(\alpha_1^+ \cdot \alpha_2^+\right), \left(\beta_1^+ \cdot \beta_2^+\right), \sqrt[q]{\left(\gamma_1^+\right)^q + \left(\gamma_2^+\right)^q - \left(\gamma_1^+\right)^q \cdot \left(\gamma_2^+\right)^q}, \\ -\sqrt[q]{\left(-\alpha_1^-\right)^q + \left(-\alpha_2^-\right)^q - \left(-\alpha_1^-\right)^q \cdot \left(-\alpha_2^-\right)^q}, -\left(\beta_1^- \cdot \beta_2^-\right), -\left(\gamma_1^- \cdot \gamma_2^-\right) \end{array} \right\rangle^{\varpi}$

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$$= \left\langle \begin{array}{c} \left(\alpha_1^+ \cdot \alpha_2^+\right)^{\varpi}, \left(\beta_1^+ \cdot \beta_2^+\right)^{\varpi}, \sqrt[q]{1-\left(1-\left(\left(\gamma_1^+\right)^q + \left(\gamma_2^+\right)^q - \left(\gamma_1^+\right)^q \cdot \left(\gamma_2^+\right)^q\right)\right)^{\varpi}}, \\ -\sqrt[q]{1-\left(1-\left(\left(-\alpha_1^-\right)^q + \left(-\alpha_2^-\right)^q - \left(-\alpha_1^-\right)^q \cdot \left(-\alpha_2^-\right)^q\right)\right)^{\varpi}}, -\left(\beta_1^- \cdot \beta_2^-\right)^{\varpi}, -\left(\gamma_1^- \cdot \gamma_2^-\right)^{\varpi}) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{c} \left(\alpha_1^+\right)^{\varpi}, \left(\beta_1^+\right)^{\varpi}, \sqrt[q]{1-\left(1-\left(\gamma_1^+\right)^q\right)^{\varpi}}, \\ -\sqrt[q]{1-\left(1-\left(-\alpha_1^-\right)^q\right)^{\varpi}}), -\left(-\beta_1^-\right)^{\varpi}, -\left(-\gamma_1^-\right)^{\varpi} \end{array} \right\rangle$$

$$\otimes \left\langle \begin{array}{c} \left(\alpha_2^+\right)^{\varpi}, \left(\beta_2^+\right)^{\varpi}, \sqrt[q]{1-\left(1-\left(\gamma_2^+\right)^q\right)^{\varpi}}, \\ -\sqrt[q]{1-\left(1-\left(-\alpha_2^-\right)^q\right)^{\varpi}}), -\left(-\beta_2^-\right)^{\varpi}, -\left(-\gamma_2^-\right)^{\varpi} \end{array} \right\rangle$$

$= \breve{\rho}_1^{\varpi} \otimes \breve{\rho}_2^{\varpi}$.

The comparison rules for Bq-RPFNs are given as follows:

Definition 9. Let $\breve{\rho} = \left(\alpha^+, \beta^+, \gamma^+, \alpha^-, \beta^-, \gamma^-\right)$ be a Bq-RPFNs in $\mathfrak{H}$.

The score function of $\breve{\rho}$ is described as

$$\mathfrak{S}\left(\breve{\rho}\right) = \frac{1}{2}\left(\left(\alpha^+\right)^q - \left(\gamma^+\right)^q\right) - \left(\left|\alpha^-\right|^q - \left|\gamma^-\right|^q\right)) \tag{7}$$

and $\mathfrak{S}\left(\breve{\rho}\right) \in \left[-1,1\right]$.

The accuracy function of $\breve{\rho}$ is described as

$$\mathfrak{A}\left(\breve{\rho}\right) = \frac{1}{2}\left(\left(\left(\alpha^+\right)^q + \left(\beta^+\right)^q + \left(\gamma^+\right)^q\right) + \left(\left|\alpha^-\right|^q \mid + \left|\beta^-\right|^q + \left|\gamma^-\right|^q\right)\right) \tag{8}$$

and $\mathfrak{A}\left(\breve{\rho}\right) \in \left[0,1\right]$.

Based on these rules, for two Bq-RPFNs $\breve{\rho}_1$ and $\breve{\rho}_2$;

i. if $\mathfrak{S}\left(\breve{\rho}_1\right) > \mathfrak{S}\left(\breve{\rho}_2\right)$ then $\breve{\rho}_1 \succ \breve{\rho}_2$;
ii. if $\mathfrak{S}\left(\breve{\rho}_1\right) < \mathfrak{S}\left(\breve{\rho}_2\right)$ then $\breve{\rho}_1 \prec \breve{\rho}_2$;

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iii. if $\mathfrak{S}(\breve{\rho}_1)=\mathfrak{S}(\breve{\rho}_2)$ then $\breve{\rho}_1=\breve{\rho}_2$
   a. if $\mathfrak{A}(\breve{\rho}_1)>\mathfrak{A}(\breve{\rho}_2)$ then $\breve{\rho}_1\succ\breve{\rho}_2$;
   b. if $\mathfrak{A}(\breve{\rho}_1)<\mathfrak{A}(\breve{\rho}_2)$ then $\breve{\rho}_1\prec\breve{\rho}_2$;
   c. if $\mathfrak{A}(\breve{\rho}_1)=\mathfrak{A}(\breve{\rho}_2)$ then $\breve{\rho}_1\approx\breve{\rho}_2$.

Example 3. Let $\breve{\rho}_1=(0.3,0.9,0.7,-0.6,-0.3,-0.4)$ and $\breve{\rho}_2=(0.4,0.3,0.2,-0.4,-0.2,-0.7)$ be two Bq-RPFNs for *q*=4. Then, score values of Bq-RPFNs $\breve{\rho}_1$ and $\breve{\rho}_2$ are $\mathfrak{S}(\breve{\rho}_1)=-0.1679$ and $\mathfrak{S}(\breve{\rho}_2)=-0.0952$. So, $\breve{\rho}_1$ is superior to $\breve{\rho}_2$ (i.e., $\breve{\rho}_1\prec\breve{\rho}_2$).

Definition 10. Let $\breve{\rho}_i=\alpha_i^+,\beta_i^+,\gamma_i^+,\alpha_i^-,\beta_i^-,\gamma_i^-\,(i=1,2,\ldots n)$ be a collection of Bq-RPFNs.

1. The bipolar *q*-rung picture fuzzy weighted averaging (Bq-RPFWA) operator is described as below:

$$Bq-RPFWA(\breve{\rho}_1,\breve{\rho}_2,\ldots,\breve{\rho}_n)=\prod_{i=1}^{n}\varpi_i\breve{\rho}_i \tag{9}$$

where $\varpi=(\varpi_1,\varpi_2,\ldots,\varpi_n)^T$ and ϖ_i is the weight of $\breve{\rho}_i\,(i=1,2,\ldots n)$ such that $\varpi_i>0$ and $\sum_{i=1}^{n}\varpi_i=1$.

2. The bipolar q-rung picture fuzzy weighted geometric (Bq-RPFWG) operator is described as below:

$$Bq-RPFWG(\breve{\rho}_1,\breve{\rho}_2,\ldots,\breve{\rho}_n)=\prod_{i=1}^{n}\breve{\rho}_i^{\varpi_i} \tag{10}$$

where $\varpi=(\varpi_1,\varpi_2,\ldots,\varpi_n)^T$ and ϖ_i is the weight of $\breve{\rho}_i\,(i=1,2,\ldots n)$ such that $\varpi_i>0$ and $\sum_{i=1}^{n}\varpi_i=1$.

T**heorem 1.** Let $\breve{\rho}_i=(\alpha_i^+,\beta_i^+,\gamma_i^+,\alpha_i^-,\beta_i^-,\gamma_i^-)\,(i=1,2,\ldots n)$ be a collection of Bq-RPFNs.

1. Their aggregated value by using the Bq-RPFWA operator is also a Bq-RPFNs, and

$$Bq-RPFWA(\breve{\rho}_1,\breve{\rho}_2,\ldots,\breve{\rho}_n)=\left\langle\begin{array}{c}\sqrt[q]{1-\prod_{i=1}^{n}\left(1-(\alpha_i^+)^q\right)^{\varpi_i}},\prod_{i=1}^{n}(\beta_i^+)^{\varpi_i},\prod_{i=1}^{n}(\gamma_i^+)^{\varpi_i},\\ -\left(\prod_{i=1}^{n}(-\alpha_i^-)^{\varpi_i}\right),-\left(\prod_{i=1}^{n}(-\beta_i^-)^{\varpi_i}\right),-\sqrt[q]{1-\prod_{i=1}^{n}\left(1-(-\gamma_i^-)^q\right)^{\varpi_i}}\end{array}\right\rangle$$

where $\varpi=(\varpi_1,\varpi_2,\ldots,\varpi_n)^T$ and ϖ_i is the weight of $\breve{\rho}_i\,(i=1,2,\ldots n)$ such that $\varpi_i>0$ and $\sum_{i=1}^{n}\varpi_i=1$.

2. Their aggregated value by using the Bq-RPFWG operator is also a Bq-RPFNs, and

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$$Bq-RPFWG\left(\breve{\rho}_1,\breve{\rho}_2,\ldots,\breve{\rho}_n\right)=\left\langle \begin{array}{c} \prod_{i=1}^{n}\left(\alpha_i^{+}\right)^{\varpi_i},\prod_{i=1}^{n}\left(\beta_i^{+}\right)^{\varpi_i},\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(\gamma_i^{+}\right)^{q}\right)^{\varpi_i}}, \\ -\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(-\alpha_i^{-}\right)^{q}\right)^{\varpi_i}}\Big),-\left(\prod_{i=1}^{n}\left(-\beta_i^{-}\right)^{\varpi_i}\right),-\left(\prod_{i=1}^{n}\left(-\gamma_i^{-}\right)^{\varpi_i}\right) \end{array}\right\rangle$$

where $\varpi=\left(\varpi_1,\varpi_2,\ldots,\varpi_n\right)^T$ and ϖ_i is the weight of $\breve{\rho}_i\left(i=1,2,\ldots n\right)$ such that $\varpi_i>0$ and $\sum_{i=1}^{n}\varpi_i=1$.

Proof. They can be proved by utilizing the technique of mathematical induction, therefore omitted.

Tanimoto Similarity Measures for Bipolar q-Rung Picture Fuzzy Sets

The calculation of similarity between two structures is a matter of quantifying, by some appropriate measure, the similarity between their respective vectors. For this purpose, Tanimoto (Tanimoto, 1958) proposed the following similarity measure.

Let $X=(x_1,x_2,\ldots,x_m)$ and $Y=(y_1,y_2,\ldots,y_m)$ be two vectors of length m. Then, the Tanimoto similarity measure between these vectors is denoted and defined by

$$\mathfrak{T}_{sim}\left(X,Y\right)=\frac{X\cdot Y}{\|X\|^2+\|Y\|^2-X\cdot Y}=\frac{\sum_{j=1}^{m}x_jy_j}{\sum_{j=1}^{m}x_j^2+\sum_{j=1}^{m}y_j^2-\sum_{j=1}^{m}x_jy_j} \tag{11}$$

Note 1. In Eq. (11), assume that $x_j\neq0$ or $y_j\neq0$for at least one $j(j=1,2,\ldots,m)$.

Note that this similarity coefficient is also known as the Jaccard similarity coefficient (Jaccard, 1908).

A Tanimoto similarity measure and a weighted Tanimoto similarity measure for Bq-RPFSs can be described analogously to the Tanimoto similarity coefficient given in Eq. (11) as follows:

Definition 10. Assume that $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ are two Bq-RPFSs in $\mathfrak{H}=\left\{\mathfrak{h}_j:j=1,2,\ldots,m\right\}$, then the Tanimoto similarity measure (TSM) between $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ is denoted and defined as follows:

$$\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)=\frac{1}{m}\sum_{j=1}^{m}\frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\\+\left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}+\left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}+\left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right)\\+\left(\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right)\\-\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\\+\left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}+\left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}+\left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\end{array}\right)\end{array}\right)} \tag{12}$$

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Theorem 2. The TSMs between $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ fulfill the following axioms:

1. $0 \le \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right) \le 1$.
2. $\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right) = \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{R}},\breve{\mathcal{Q}}\right)$.
3. $\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right) = 1$ if $\breve{\mathcal{Q}} = \breve{\mathcal{R}}$.

Proof. Consider Eq. (12).

(1) It is known that $a^2+b^2 \ge ab$ for $a,b \in [0,1]$. Using this inequality, it is seen that $\xi \ge \eta$ where

$$\xi = \left(\begin{array}{c} \left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right) \\ + \left(\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right) \end{array} \right) \quad (13)$$

and

$$\eta = \left(\begin{array}{c} \left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} + \left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} + \left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} \end{array} \right) \quad (14)$$

Hence, ξ-$\eta>0$ (it is assumed that ξ-$\eta \ne 0$ (see Note 1)). Moreover, since

$$a^2 + b^2 - 2ab = \left(a-b\right)^2 \ge 0 \Rightarrow a^2 + b^2 - ab \ge ab$$

for a,$b \in [0,1]$, it is obtained that ξ-$\eta \ge \eta$ and so

$$0 \le \frac{\eta}{\xi - \eta} \le 1.$$

That is,

$$0 \le \frac{\left(\begin{array}{c} \left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} + \left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} + \left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} \end{array} \right)}{\left(\begin{array}{c} \left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right) \\ + \left(\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\alpha_{\breve{\mathcal{R}}^{c}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right) \\ - \left(\begin{array}{c} \left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q}\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{q} \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} + \left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} + \left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q}\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^{q} \end{array} \right) \end{array} \right)} \le 1$$

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for each $\mathfrak{h}_j \in \mathfrak{H}$. Then, it is obtained that

$$0 \le \sum_{j=1}^{m} \frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ +\left(\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ -\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)\end{array}\right)} \le m$$

and so

$$0 \le \frac{1}{m}\sum_{j=1}^{m} \frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ +\left(\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ -\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)\end{array}\right)} \le 1$$

Thus, it is deduced that $0 \le \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right) \le 1$.

(2) Further, on proving the condition two, $\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)$

$$= \frac{1}{m}\sum_{j=1}^{m} \frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ +\left(\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ -\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)\end{array}\right)}$$

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$$= \frac{1}{m}\sum_{j=1}^{m} \frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\\ +\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\\ +\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\\ -\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\\ +\left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)\end{array}\right)}$$

$$= \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{R}},\breve{\mathcal{Q}}\right).$$

(3) $\Rightarrow$: Assume that $\breve{\mathcal{Q}} = \breve{\mathcal{R}}$, then $\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{Q}}\right)$

$$= \frac{1}{m}\sum_{j=1}^{m} \frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\\ +\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\\ -\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q\\ +\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q\end{array}\right)\end{array}\right)}$$

$$= \frac{1}{m}\sum_{j=1}^{m} \frac{\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\\ +\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\\ -\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right)\end{array}\right)} = 1$$

(3) $\Leftarrow$: Suppose that $\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)$

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$$= \frac{1}{m}\sum_{j=1}^{m} \frac{\begin{pmatrix} \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q \end{pmatrix}}{\begin{pmatrix} \left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ + \left(\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ - \begin{pmatrix} \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q \end{pmatrix} \end{pmatrix}} = 1$$

$$\frac{\begin{pmatrix} \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q \end{pmatrix}}{\begin{pmatrix} \left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ + \left(\left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^{2q} + \left(\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q} + \left(\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right)^{2q}\right) \\ - \begin{pmatrix} \left(\alpha_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\alpha_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\beta_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\beta_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q + \left(\gamma_{\breve{\mathcal{Q}}}^{+}(\mathfrak{h}_j)\right)^q \left(\gamma_{\breve{\mathcal{R}}}^{+}(\mathfrak{h}_j)\right)^q \\ + \left|\alpha_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\alpha_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\beta_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\beta_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q + \left|\gamma_{\breve{\mathcal{Q}}}^{-}(\mathfrak{h}_j)\right|^q \left|\gamma_{\breve{\mathcal{R}}}^{-}(\mathfrak{h}_j)\right|^q \end{pmatrix} \end{pmatrix}} = 1$$

and so

$$\xi - \eta = \eta \Rightarrow \xi = 2\eta$$

(see Eqs. (13) and (14) for ξ and η). This requires that $\breve{\mathcal{Q}} = \breve{\mathcal{R}}$.

Thus, the proof is completed.

Definition 11. Assume that $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ are two Bq-RPFSs in $\mathfrak{H} = \{\mathfrak{h}_j : j = 1, 2, \ldots, m\}$, then the weighted Tanimoto similarity measure (WTSM) between $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ is denoted and defined as follows:

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$$\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)=\sum_{j=1}^{m}\varpi_j\frac{\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\\+\left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\end{array}\right)}{\left(\begin{array}{c}\left(\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right)\\+\left(\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}+\left(\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right)^{2q}\right)\\-\left(\begin{array}{c}\left(\alpha_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\left(\alpha_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^q+\left(\beta_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\left(\beta_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^q+\left(\gamma_{\breve{\mathcal{Q}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\left(\gamma_{\breve{\mathcal{R}}}^{+}\left(\mathfrak{h}_j\right)\right)^q\\+\left|\alpha_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\left|\alpha_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^q+\left|\beta_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\left|\beta_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^q+\left|\gamma_{\breve{\mathcal{Q}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\left|\gamma_{\breve{\mathcal{R}}}^{-}\left(\mathfrak{h}_j\right)\right|^q\end{array}\right)\end{array}\right)} \tag{15}$$

Theorem 3. The WTSMs between $\breve{\mathcal{Q}}$ and $\breve{\mathcal{R}}$ fulfill the following axioms:

1. $0\le\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)\le 1$.
2. $\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)=\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{\mathcal{R}},\breve{\mathcal{Q}}\right)$.
3. $\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{\mathcal{Q}},\breve{\mathcal{R}}\right)=1$ if $\breve{\mathcal{Q}}=\breve{\mathcal{R}}$.

Proof. They can prove similar to the proof of Theorem 2, so omitted.

APPLICATIONS

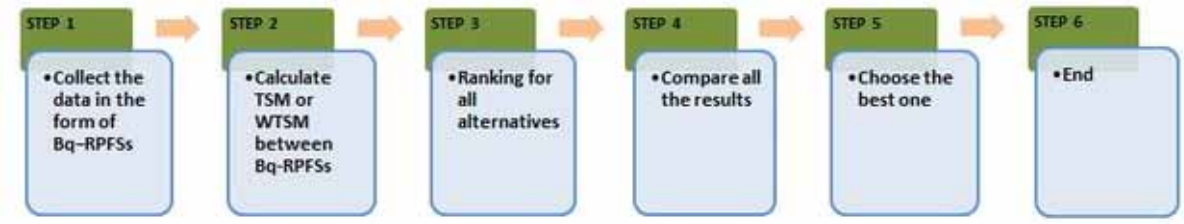
The similarity measure is an interesting tool in fuzzy mathematics that has been applied to many practical situations like medical diagnosis, pattern recognition, and multi-attribute decision making.

The following procedure is used for applications of TSM and WTSM based on Bq-RPFSs.

Algorithm

Step 1: Collect the data in the form of Bq−RPFSs.
Step 2: Calculate TSM or WTSM between Bq-RPFSs using Eqs. (11) and (14).
Step 3: Ranking for all alternatives.
Step 4: Compare all the results.
Step 5: Choose the best one.
Step 6: End.

Figure 1. Flow chart representation of Algorithm



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The flow chart of proposed Algorithm is given in Figure 1.

The applications of TSM and WTSM based on Bq-RPFSs are given as follows:

Example 4. (Medical Diagnosis)

Assume that a set of diseases $\breve{Q}_1$ – Malaria, $\breve{Q}_2$ – Typhoid, $\breve{Q}_3$ – Viral fever, $\breve{Q}_4$ – Diarrhea, and a patient $\breve{P}$ concerning the symptoms $\mathfrak{h}_1$ – HeadAche, $\mathfrak{h}_2$ – Temperature, $\mathfrak{h}_3$ – Cough are described as Bq-RPFSs for q=3, respectively as follows:

$$\breve{Q}_1 = \begin{Bmatrix} (\mathfrak{h}_1, 0.3, 0.1, 0.1, -0.3, -0.2, -0.1), \\ (\mathfrak{h}_2, 0.3, 0.1, 0.1, -0.4, -0.1, -0.1), \\ (\mathfrak{h}_3, 0.4, 0.1, 0.3, -0.3, -0.2, -0.1) \end{Bmatrix},$$

$$\breve{Q}_2 = \begin{Bmatrix} (\mathfrak{h}_1, 0.7, 0.2, 0.3, -0.5, -0.1, -0.1), \\ (\mathfrak{h}_2, 0.7, 0.1, 0.3, -0.4, -0.2, -0.2), \\ (\mathfrak{h}_3, 0.7, 0.2, 0.1, -0.6, -0.2, -0.2) \end{Bmatrix},$$

$$\breve{Q}_3 = \begin{Bmatrix} (\mathfrak{h}_1, 0.9, 0.2, 0.2, -0.8, -0.2, -0.1), \\ (\mathfrak{h}_2, 0.8, 0.2, 0.2, -0.6, -0.1, -0.2), \\ (\mathfrak{h}_3, 0.7, 0.1, 0.1, -0.9, -0.2, -0.1) \end{Bmatrix},$$

$$\breve{Q}_4 = \begin{Bmatrix} (\mathfrak{h}_1, 0.2, 0.2, 0.1, -0.3, -0.1, -0.2) \\ (\mathfrak{h}_2, 0.2, 0.1, 0.1, -0.3, -0.2, -0.2), \\ (\mathfrak{h}_3, 0.2, 0.1, 0.2, -0.3, -0.1, -0.2), \end{Bmatrix},$$

and

$$\breve{P} = \begin{Bmatrix} (\mathfrak{h}_1, 0.9, 0.2, 0.1, -0.6, -0.5, -0.2) \\ (\mathfrak{h}_2, 0.7, 0.2, 0.3, -0.8, -0.5, -0.1), \\ (\mathfrak{h}_3, 0.6, 0.5, 0.4, -0.6, -0.4, -0.3), \end{Bmatrix},$$

To diagnose the disease of the patient, calculate the similarity between the diseases $\breve{Q}_1$ – Malaria, $\breve{Q}_2$ – Typhoid, $\breve{Q}_3$ – Viral fever, $\breve{Q}_4$ – Diarrhea, and patient by using TSM. The TSM results are as follows.

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$$\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{Q}_1,\breve{P}\right)=0.1271944,\ \ \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{Q}_2,\breve{P}\right)=0.5989952,$$
$$\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{Q}_3,\breve{P}\right)=0.6609743,\ \ \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{Q}_4,\breve{P}\right)=0.0479006.$$

It is observed from the above results that the ranking order of the diseases is $\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$. It shows that the patient $\breve{P}$ is suffered from the disease $\breve{Q}_4$ – Diarrhea according to TSM. It is shown in Figure 2 visually. In case, 0.2, 0.3, and 0.5 are weights of symptoms $\mathfrak{h}_1, \mathfrak{h}_2$, and $\mathfrak{h}_3$ respectively, then calculate the similarity between the diseases $\breve{Q}_1$ – Malaria, $\breve{Q}_2$ – Typhoid, $\breve{Q}_3$ – Viral fever, $\breve{Q}_4$ – Diarrhea, and the patient $\breve{P}$ by using WTSM. The WTSM results are as follows.

$$\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{Q}_1,\breve{P}\right)=0.051095,\ \ \mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{Q}_2,\breve{P}\right)=0.2103037,$$
$$\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{Q}_3,\breve{P}\right)=0.1984579,\ \ \mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{Q}_4,\breve{P}\right)=0.018812.$$

As the weights of symptoms $\mathfrak{h}_1, \mathfrak{h}_2$ and $\mathfrak{h}_3$ are influenced in the similarity, the ranking order of the diseases is changed as $\breve{Q}_2 \succ \breve{Q}_3 \succ \breve{Q}_1 \succ \breve{Q}_4$ according to WTSM.

Figure 2. Pictorial form of Example 4 (Medical Diagnosis)

To analyze the effect of the parameter q on the TSM (Bq-RPFS) values, an experiment is performed by taking different values of q (q= 2, 2.5, 3, 3.5, 5, 10, 20) for Example 4. Based on these different parameters, TSM (Bq-RPFS) coefficients are computed for Example 4, and their results are summarized in Table 1. It has been observed that patient $\breve{P}$ is suffered from the disease $\breve{Q}_3$ – Diarrhea for any value of parameter q.

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Table 1. Results for different values of parameter q for Example 4 (Medical Diagnosis)

Value of parameter q	TSM (Bq-RPFS) values between Patient $\breve{P}$ and Disease $\breve{Q}_k$ (k=1,2,3,4)				Ranking
	$\breve{Q}_1$ – Malaria	$\breve{Q}_2$ – Typhoid	$\breve{Q}_3$ – Viral fever	$\breve{Q}_4$ – Diarrhea	
2	0.2568181	0.6883537	0.7504142	0.1364212	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$
2.5	0.1787867	0.6439083	0.7081516	0.0794424	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$
3	0.1271944	0.5989952	0.6609743	0.0479006	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$
3.5	0.0921758	0.5534646	0.6142499	0.0297195	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$
5	0.0382515	0.4228244	0.5028345	0.0080332	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$
10	0.0033577	0.1496576	0.3745515	0.000181	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$
20	0.0000506	0.0204586	0.3429673	0.0000002	$\breve{Q}_3 \succ \breve{Q}_2 \succ \breve{Q}_1 \succ \breve{Q}_4$

Example 5. Pattern Recognition

Assume that some known patterns $\breve{A}_1, \breve{A}_2, \breve{A}_3,$ and an unknown pattern $\breve{B}$ concerning the universe of discourse $\breve{X} = \{x_1, x_2, x_3, x_4\}$ are described as Bq-RPFSs for q=2, respectively as follows:

$$\breve{A}_1 = \left\{ \begin{array}{l} (x_1, 0.4, 0.3, 0.2, -0.3, -0.4, -0.1), (x_2, 0.3, 0.5, 0.3, -0.2, -0.4, -0.1), \\ (x_3, 0.2, 0.2, 0.2, -0.1, -0.1, -0.3), (x_4, 0.3, 0.5, 0.3, -0.4, -0.5, -0.3) \end{array} \right\},$$

$$\breve{A}_2 = \left\{ \begin{array}{l} (x_1, 0.8, 0.2, 0.1, -0.5, -0.4, -0.5), (x_2, 0.7, 0.1, 0.2, -0.4, -0.5, -0.3), \\ (x_3, 0.2, 0.2, 0.1, -0.3, -0.4, -0.7), (x_4, 0.2, 0.2, 0.1, -0.3, -0.8, -0.4) \end{array} \right\},$$

$$\breve{A}_3 = \left\{ \begin{array}{l} (x_1, 0.2, 0.5, 0.7, -0.1, -0.2, -0.1), (x_2, 0.6, 0.6, 0.4, -0.6, -0.6, -0.2), \\ (x_3, 0.3, 0.8, 0.2, -0.4, -0.5, -0.4), (x_4, 0.2, 0.1, 0.4, -0.4, -0.5, -0.5) \end{array} \right\}$$

and

$$\breve{B} = \left\{ \begin{array}{l} (x_1, 0.1, 0.1, 0.4, -0.3, -0.6, -0.3), (x_2, 0.1, 0.2, 0.7, -0.2, -0.5, -0.7), \\ (x_3, 0.1, 0.5, 0.3, -0.4, -0.4, -0.5), (x_4, 0.5, 0.3, 0.2, -0.4, -0.7, -0.2) \end{array} \right\}.$$

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To identify which one of the known patterns among $\breve{A}_1, \breve{A}_2,$ and $\breve{A}_3$ is closer to the unknown pattern $\breve{B}$, calculate the distance between unknown patterns $\breve{A}_1, \breve{A}_2, \breve{A}_3,$ and the unknown pattern $\breve{B}$ by using TSM. The TSM results are as follows.

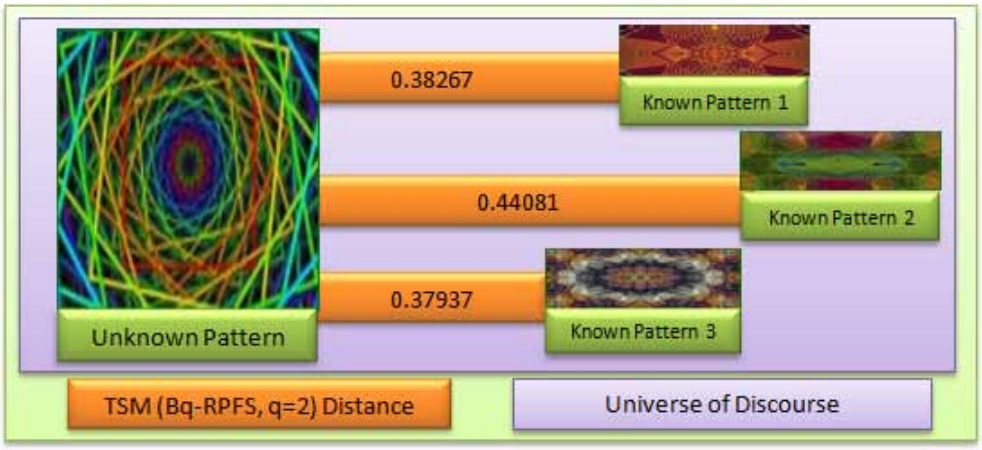
$$\mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{A}_1, \breve{B}\right) = 0.38267, \quad \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{A}_2, \breve{B}\right) = 0.44081, \quad \mathfrak{T}_{TSM}^{Bq-RPFS}\left(\breve{A}_3, \breve{B}\right) = 0.37937 .$$

It is observed from the above results that the ranking order of the diseases is $\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$. It shows that the known pattern $\breve{A}_3$ is closer with the unknown pattern $\breve{B}$ according to TSM. It is shown in Figure 3 visually. In case, 0.1, 0.3, 0.4, and 0.2 are weights of the elements of the universe of discourse x_1, x_2, x_3, and x_4 respectively, then calculate the distance between known patterns $\breve{A}_1, \breve{A}_2, \breve{A}_3,$ and the unknown pattern $\breve{B}$ by using WTSM. The WTSM results are as follows.

$$\mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{A}_1, \breve{B}\right) = 0.08230, \quad \mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{A}_2, \breve{B}\right) = 0.11853, \quad \mathfrak{T}_{WTSM}^{Bq-RPFS}\left(\breve{A}_3, \breve{B}\right) = 0.10870.$$

As the weights of elements in the universe of discourse are influenced in the similarity measure, the ranking order of the diseases is changed as $\breve{A}_2 \succ \breve{A}_3 \succ \breve{A}_1$ according to WTSM.

Figure 3. Pictorial form of Example 5 (Pattern Recognition)



To analyze the effect of the parameter q on the TSM (Bq-RPFS) values, an experiment is performed by taking different values of q (q= 2, 2.5, 3, 3.5, 5, 10, 20) for Example 5. Based on these different parameters, TSM (Bq-RPFS) coefficients are calculated for Example 5, and their outputs are presented in Table 2. It has been observed that the known pattern $\breve{A}_2$ is closer to the unknown pattern $\breve{B}$ for any value of parameter q.

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Table 2. Results for different values of parameter q for Example 5 (Pattern Recognition)

Value of parameter q	TSM (Bq-RPFS) values between Unknown Pattern $\breve{B}$ and Known Pattern $\breve{A}_k$ (k=1,2,3)			Ranking
	Known Pattern $\breve{A}_1$	Known Pattern $\breve{A}_2$	Known Pattern $\breve{A}_3$	
2	0.3826697	0.4408054	0.379374	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$
2.5	0.3039231	0.385562	0.2968464	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$
3	0.243456	0.3437177	0.2336991	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$
3.5	0.1957013	0.309071	0.1852028	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$
5	0.1028943	0.2285575	0.0952327	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$
10	0.0141394	0.0906422	0.0125311	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$
20	0.0003789	0.0187928	0.0003284	$\breve{A}_2 \succ \breve{A}_1 \succ \breve{A}_3$

Comparison Analysis

The picture fuzzy number and spherical fuzzy number (i.e., $\langle \alpha, \beta, \gamma \rangle$) can be fitted to the proposed Bq-RPFN as $\langle \alpha, \beta, \gamma, 0, 0, 0 \rangle$. This generalization allows comparisons of the proposed Bq-RPFS based TMS/WTMS with the present similarity measures: $WPFC^{1,2}$, $WPFCS^{1,2,3,4}$, $WPFCT^{1,2}$ of PFs (Wei, 2017) and S^S of SFSs (Khan et al., 2020). The intuitionistic fuzzy number, Pythagorean fuzzy number and q-rung orthopair fuzzy number (i.e., $\langle \alpha, 0, \gamma \rangle$) can be represented as Bq-RPFN (i.e., $\langle \alpha, 0, \gamma, 0, 0, 0 \rangle$). Thus, the outputs of the proposed Bq-RPFS based TMS/WTMS and some existing similarity measures: C_{IFS} of IFSs (Ye, 2011), $FCT^{1,2,3,4}$, $WPFC^{1,2}$, $WPFCS^{1,2,3,4}$, $WPFCT^{1,2,3,4}$ of PyFSs (Wei et al., 2018) and *q–ROFCot*[4], *q–ROFWCot*[4] of q-ROFSs (Wang et al., 2019) can be matched. The compared consequences are summarized in Table 3. From the compared consequences, it has been demonstrated that the most suitable result consistent with the proposed TSM/WTSM coincides with the present similarity measures and weighted similarity measures based on the IFSs, PyFSs, q-ROFSs, PFSs, SFSs and q-RPFSs.

Advantages of the proposed similarity measures: It is widely recognized that the present theories FSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, and q-ROFSs have been widely implemented in lots of different areas. In a few situations, as a unique case, for instance, the degree of membership, abstinence, and non-membership for positive information as well as negative information described as $\langle 0.3, 0.1, 0.1, -0.3, -0.2, -0.1 \rangle$. These present theories cannot fulfill such situations. An excellent ground to address such situations is the Bq-RPFSs, because the degrees of membership, abstinence, and non-membership for positive information as well as negative information are described on it. This demonstrates that the Bq-RPFS is greater widespread than the FSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, and q-ROFSs,

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and might explicit greater fuzzy data. In different words, the Bq-RPFSs can address the multi-criteria decision-making problems (MCDM) that cannot be dealt with through the theories of FSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, and q-ROFSs. It is obvious that FSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, and q-ROFSs are special instances of the Bq-RPFSs. It indicates that the Bq-RPFSs can be of greater power and effectiveness to address fuzzy and uncertain decision-making issues. Thus, the MCDM trouble with bipolar *q*-rung picture fuzzy information is greater powerful and appropriate for realistic medical and engineering applications.

As mentioned above, the space of Bq-RPFSs is larger than the ones of FSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, TSFSs, and q-RPFSs. While the present similarity measures based on FSs, BFSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, TSFSs, q-RPFSs and their bipolar hybridizations are inadequate to remedy the issues under the bipolar *q*-rung picture fuzzy environments, our proposed Bq-RPFS based TSM/WTSM may be used to solve the problems under these fuzzy environments.

Table 3. Comparison of proposed TSM/WTSM with existing similarity measures

Problem	Existing Approach		Proposed Approach	
	Similarity measure	Ranking	Similarity measure	Ranking
Table 1 (Wei, 2017)	$WPFC^{1,2}$	$A_4 \succ A_1 \succ A_3 \succ A_2$	$\mathfrak{T}_{WTSM}^{Bq-RPFS}$	$A_4 \succ A_1 \succ A_3 \succ A_2$
	$WPFCS^{1,2,3,4}$	$A_4 \succ A_1 \succ A_3 \succ A_2$		
	$WPFCT^{1,2}$	$A_4 \succ A_1 \succ A_3 \succ A_2$		
Example 5 (Khan et al., 2020)	S^S	$Q_2 \succ Q_1$		
			$\mathfrak{T}_{TSM}^{Bq-RPFS}$	$Q_2 \succ Q_1$
Table 1 (Khan et al., 2020)	S^S	$E_5 \succ E_4 \succ E_1 \succ E_2 \succ E_3$	$\mathfrak{T}_{WTSM}^{Bq-RPFS}$	$E_4 \succ E_2 \succ E_5 \succ E_1 \succ E_3$
Example 1 (Ye, 2011)	C_{IFS}	$C_3 \succ C_2 \succ C_1$	$\mathfrak{T}_{TSM}^{Bq-RPFS}$	$C_3 \succ C_2 \succ C_1$
Example 2 (Ye, 2011)	C_{IFS}	$Q_1 \succ Q_2 \succ Q_3 \succ Q_4 \succ Q_5$	$\mathfrak{T}_{TSM}^{Bq-RPFS}$	$Q_1 \succ Q_2 \succ Q_3 \succ Q_4 \succ Q_5$
Example 1 (Wei et al., 2018)	$PFCT^{1,2,3,4}$	$A_3 \succ A_1 \succ A_2$	$\mathfrak{T}_{TSM}^{Bq-RPFS}$	$A_3 \succ A_1 \succ A_2$
	$WPFC^{1,2}$	$A_3 \succ A_2 \succ A_1$	$\mathfrak{T}_{WTSM}^{Bq-RPFS}$	$A_3 \succ A_2 \succ A_1$
	$WPFCS^{1,2,3,4}$	$A_3 \succ A_2 \succ A_1$		
	$WPFCT^{1,2,3,4}$	$A_3 \succ A_2 \succ A_1$		
Example 2 (Wang et al., 2019)	$q-ROFCot^4$	$Y_3 \succ Y_2 \succ Y_1 \succ Y_5 \succ Y_4$	$\mathfrak{T}_{TSM}^{Bq-RPFS}$	$Y_3 \succ Y_2 \succ Y_1 \succ Y_5 \succ Y_4$
	$q-ROFWCot^4$	$Y_3 \succ Y_2 \succ Y_5 \succ Y_1 \succ Y_4$	$\mathfrak{T}_{WTSM}^{Bq-RPFS}$	$Y_3 \succ Y_2 \succ Y_5 \succ Y_1 \succ Y_4$

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Disadvantages of the proposed similarity measures: Our procedures cannot use to solve the problems based on the interval-valued, cubic, hesitant types of FSs, IFSs, PyFSs, q-ROFSs, PFSs, SFSs, TSFSs, and q-RPFSs. However, it is obvious that our similarity measure procedures can be improved in these directions.

FUTURE RESEARCH DIRECTIONS

This chapter focused mainly on the hybridization of q-PRFSs with bipolarity and their Tanimoto similarity coefficients. Many types of measures were proposed to measure $\boldsymbol{q}$-rung picture fuzzy information. In addition to these, various aggregation operators were created to fuse $\boldsymbol{q}$-rung picture fuzzy information. Each of these has its advantages in dealing with real-life problems. An interesting direction for future research is therefore required to investigate new aggregation operators and similarity-distance coefficients fusing/measuring the bipolar $\boldsymbol{q}$-rung picture fuzzy information. Further, the algebraic structures, topological structures and graphs of Bq-RPFSs could be developed and used in in networking problems, shortest path problems, communication, clustering, slicing, etc. This is another important direction/ impulse for future research.

CONCLUSION

One of the most significant research aspects is how to represent the information based on uncertainty and vague in human analysis. The picture fuzzy sets and spherical fuzzy sets are all excellent tools to handle fuzzy information when human opinions regarding responses not only yes or no but there are the refusal and abstinence too. The q-RPFS model is a more powerful tool in contrast with the picture and spherical fuzzy set models, as it provides a broad space of admissible triplets. In this chapter dedicated to bipolar hybridization of q-RPFSs, the Bq-RPFS, a powerful and effective tool for coping with uncertainty and vagueness, was proposed. Later, the operations and comparison rules for Bq-RPFSs/Bq-RPFNs were introduced. Some aggregation operators were proposed to fuse bipolar q-rung picture fuzzy information and their prominent characteristics were addressed. Besides, the Tanimoto similarity measures that can capture the interrelationship between Bq-RPFs were proposed and their desirable properties were studied. Thereafter, the proposed similarity measures were used to establish a new method dealing with the decision making problems. The validity of the proposed method was illustrated by solving the problems of medical diagnosis and pattern recognition. In conclusion, this chapter proposes a new extension of q-RPFSs so that this new approach can be useful in dealing with many real-life problems such as decision making, clustering analysis, networking problems, image segmentation-enhancement, signal systems, and shortest path problems.

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KEY TERMS AND DEFINITIONS

Bipolarity: Bipolarity refers to the tendency of the human psyche in reasoning and settling on choices based on positive and negative impacts. Where positive information reflects what is considered, satisfactory, permitted, possible, or desired as being acceptable. Otherwise, negative statements represent what is rejected, impossible or forbidden. Negative inclinations compare to restraints, since they indicate which esteems or articles must be dismissed, while positive inclinations relate to wishes, as they specify which items are more alluring than others without dismissing those that don't meet the desires.

Fuzzy Set: In mathematics, fuzzy sets (a.k.a. uncertain sets) are somewhat like sets whose elements have membership grades. The fuzzy set generalizes classical set, since the characteristic function of classical set is a special case of the membership function of fuzzy set, if the latter only take values 0 or 1. These sets can be used in a wide range of domains in which information is imprecise or incomplete.

Information Fusion: Fusion can be described as a process of merging information from multiple sources to produce the most specific and comprehensive unified data about an activity, entity or event. Information fusion involves the combination of information into a new set of information towards reducing redundancy and uncertainty.

Information Measure: The mathematical theory of information measures information with several quantities of information. Information measure is a system of measurement of information based on the

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probabilities of the events that convey information. It is a system of related measures that facilitates the quantification of some particular characteristic.

Intuitionistic Fuzzy Set: Intuitionistic fuzzy sets are sets whose elements have membership grades and non-membership grades. The intuitionistic fuzzy set generalizes fuzzy set, since the indicator function of fuzzy set is a special case of the membership function and non-membership function of intuitionistic fuzzy set.

Medical Diagnosis: Medical diagnosis is the process of determining which disease explains a person's symptoms and signs.

Pattern Recognition: Pattern recognition is the automated recognition of patterns and regularities in data. It is a technology that helps to recognize patterns, which are continuous and repetitive shapes.

Picture Fuzzy Set: Like intuitionistic fuzzy sets, picture fuzzy sets have the previous two functions: one for membership and another for non-membership. The major difference is that picture fuzzy sets have one more function: for abstinence. This value indicates that the grade of abstinence. This concept of having abstinence value can be particularly useful when one cannot be very confident on the membership or non-membership values for alternative/object.

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Chapter 13
Prediction of Capacity Utilization Rate for Turkey Using Adaptive Neuro-Fuzzy Inference System With Particle Swarm Optimization and Genetic Algorithm

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ABSTRACT

This chapter aims to propose prediction models to estimate Turkey's manufacturing sector's capacity utilization rate between 2008-2019 monthly basis using the adaptive neuro-fuzzy inference system (ANFIS) with genetic algorithm (GA) and particle swarm optimization (PSO) via determined indicators. The model's accuracy will be tested using some of the performance evaluation criteria, namely mean absolute error (MAE), root mean square error (RMSE), and the coefficient of determination (R2) values were used to compare the prediction ability. The coefficient of determination for GA-ANFIS, PSO-ANFIS, and ANFIS models are 0.9787, 0.9786, and 0.9679 in the training phase and 0.9591, 0.7677, and 0.7264 in the testing phase, respectively. The study results showed that the GA-ANFIS model showed better predictive ability with the least prediction error among other models. As a result, ANFIS, whose parameters are adjusted with GA, can predict the Turkish capacity utilization rate in the manufacturing industry with high accuracy.

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INTRODUCTION

The manufacturing industry's place in the country's economy is one of the most fundamental determinants of a country's development process. The shares of the sectors in the economy also change according to the development status of the countries. While the sectors based on natural resources such as agriculture have an important share in the economy with the industrial revolution, as the countries' development rate increased, the agricultural sector was replaced by the manufacturing sector.

It is crucial to determine the production capacity and production planning for the continuity of an enterprise. Production capacity is the amount of production a business creates using the production factors rationally in a specific time. Long-term demand forecasts are needed to determine this production capacity. Conditions that create uncertainty regarding future demand and production costs obtained by long-term demand forecasts make capacity planning activities difficult.

Capacity utilization rate (CUR), on the other hand, gives the ratio of the production amount realized in the production unit to the highest amount that can be produced in a certain period. Due to the high share of the manufacturing industry in the industrial sector, this sector was utilized in measuring capacity utilization. The latest published capacity utilization rate report for Turkey belongs to May 2021. The report's numerical data show that the manufacturing industry's capacity utilization rate decreased by 0.6 points compared to the previous month and fell to 75.3 percent in May 2021. In Figure 1, seasonally adjusted capacity rate and capacity utilization rate values for Turkey are given for the years 2013-2021.

Figure 1. CUR and Seasonally Adjusted CUR (%)
(Source: (CBTR, 2021))

According to Figure 1, the CUR decreased with a rapid acceleration between January and July of 2020. The declaration of the COVID-19 epidemic, which started in China in December 2019 as a pandemic by the World Health Organization (WHO) in March 2020, caused severe cuts in production, and the epidemic shook the manufacturing sector significantly. The effect of the epidemic on the manufacturing sector is seen in Figure 1.

Various methods are employed to find the CUR in the manufacturing sector in previous studies. These studies can be divided into two as statistical and artificial intelligence-based models. In addition, some studies conducted to investigate the manufacturing sector's capacity utilization rate with different

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methods such as (Ay, 2019; Dotsey & Stark, 2005; Eren Şenaras, 2016; Lima & Malgarini, 2016; Sen Rimo & Chai Tin, 2017; Sinan, 2020; Xu, 2019; Yu & Shen, 2020; Zia Ullah et al., 2017).

Previous studies on the capacity utilization rate have been summarized below. In these studies, prediction models were developed, and in others, indicators affecting CUR were evaluated.

Ay (2019) analyzed CUR and real sector confidence index data with the Granger Causality test. In this study, it is tested whether the use of capacity affects the price index. As a result of the research, a bidirectional causality relationship was found between the two variables. This shows that the variables have long-term interaction. (Ay, 2019).

Lima and Malgarini (2016) suggested estimating the output gap by examining the Brazilian manufacturing capacity utilization rate with the survey technique. System efficiency can be measured by observing the capacity utilization rate. If the output gap is determined well, an efficient decision support system will be created for the Brazilian business cycle. As a result, it is suggested that the survey technique can be used as a method to estimate the output gap with real-time GDP data set (Lima & Malgarini, 2016).

Sen Rimo and Chai Tin (2017) developed a stochastic discrete event simulation to determine the CUR and capacity gap in the production function for a production facility. The simulation is modeled on ARENA. Random variables were chosen as machine failure and product defect rate. At the end of the study, it was seen that the variables selected had an effect on CUR and capacity gap. Also, if the rate of defective products can reduce to zero, the CUR will improve by 73.56% (Sen Rimo & Chai Tin, 2017).

Deb (2014) tried to determine the impact of economic reforms on the Indian manufacturing sector. Production efficiency will increase by boost the CUR. Therefore, the focus of the study is the comparison of pre-reform and post-reform CUR. As stated by the regression analysis, the economic reforms increased CUR and positively affected the productivity increase in total manufacturing (Deb, 2014).

According to Ray et al. (2006) calculated CUR for the manufacturing sector for 1970-2001 in the USA. As a result of the study, it has been observed that there are significant differences in CUR over the years, both between industries and within industries. In addition, it has been shown that the spending constraint affects the CUR, especially in periods when interest rates are high (Ray et al., 2006)

Zhang (2020) examined the relationship between credit constraints and CUR and whether this relationship differs from manufacturing subsectors. In developing countries, the loan market is generally underdeveloped due to interest rates. Production decisions are related to their access to credit for the firms. For this reason, firms avoid additional costs even if they know that they will achieve optimum output since optimizing inputs creates additional costs for firms. In the study, capacity utilization scenarios between firms with and without credit restrictions were compared. Medium-high technology firms are more negatively affected by credit restrictions than low-tech firms, considering these results. In addition, it has been observed that the capacity utilization rate of high-tech firms with credit restrictions is more affected by labor productivity than capital productivity (Zhang, 2020).

Sinan (2020) applied a statistical test to examine the relationship between CUR and BIST-100 indexes in Turkey. In his study, the Granger causality test was established to show that companies' performances supply Turkish economic development. Consequently, it was observed that there was a long-term bidirectional causality relationship between variables (Sinan, 2020).

In previous studies, econometric analyzes have been mainly made to examine the relationship between CUR and inflation rates, such as Bahramian and Saliminezhad (2020), Belton and Cebula (2000), Dotsey and Stark (2005), and Nahuis (2003). The models developed in previous studies to predict CUR are mostly classical statistical models, as seen in Deb (2014), Morin and Stevens (2004). If the relationships between variables are not linear, the predictive performance of these models is not sufficient.

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Non-linear relationships between variables can be easily modeled using artificial intelligence-based models, machine learning, and deep learning algorithms. For this reason, considering the capacity utilization rate, an artificial intelligence-based CUR estimation model can be developed by using the variables associated with this rate. Furthermore, adaptive neuro-fuzzy inference system (ANFIS) models can quickly respond to changes in systems and, therefore, be an effective predictive tool for non-linear functions. The ANFIS method includes a fuzzy extraction system and artificial neural networks, so selecting the relevant parameters predicts performance effectively (Barak & Sadegh, 2016; Guleryuz, 2021a).

The ANFIS methodology has been preferred in many fields due to its effective prediction capability. Bektas Ekici and Aksoy (2011) proposed an ANFIS framework to predict building energy demand in a cold region. (Yang et al., 2020) aims to predict ground vibration using optimized ANFIS models. Guleryuz (2021) proposed a prediction model to determined industrial energy demand for Turkey using PSO-ANFIS. Boyacioglu and Avci (2010) estimated the stock market return via ANFIS. Estimation studies have been carried out in many areas using the ANFIS method.

The high share of the manufacturing industry in Turkey's economic CUR will be estimated in this study. CUR, which supplies knowledge about the economy's growth and demand, will be examined monthly between 2008 and 2019. The decrease in the capacity utilization rates will cause dispersion in industrial production. On the other hand, the increase in capacity utilization rates indicates that there will also be an increase in industrial production (Sinan, 2020). Thus, the economy will grow thanks to the increase in capacity utilization rate.

On the contrary, industrial production will decrease, and the economy will shrink. These increases in capacity utilization rates are significant advantages for economies. This chapter proposes estimation models to estimate Turkey's manufacturing sector's capacity utilization rate between 2008-2019 monthly basis using the ANFIS with particle swarm optimization (PSO) and genetic algorithm (GA) via determined indicators.

MAIN FOCUS OF THE CHAPTER

In this study, an estimation model was suggested for Turkey using the determinants affecting the CUR. The ANFIS method, a combination of artificial neural networks and fuzzy interface systems frequently used for prediction in the literature, was preferred. This study differs from other studies in two aspects. Initially, it is the first study for Turkey to estimate CUR with the ANFIS model. The second difference is that the ANFIS parameters, usually determined by the developer, were optimized with two different heuristic optimization methods, namely genetic algorithm and particle swarm optimization, and the results were compared. The continuing parts of this chapter as follows. In section 2, the data set is introduced, and theoretical information about the ANFIS, GA, and PSO is given. In section 3, the predictive findings of ANFIS, GA-ANFIS, PSO-ANFIS models are given, and the models' performances are compared. Section 4 is the conclusion of the study.

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MATERIAL AND METHOD

Data Collection Process

In this study, the capacity utilization rate in the manufacturing sector (CUR) in Turkey between 2008 –2019 is predicted using Industrial Production Index (IPI), USD to TRY Exchange Rate (USDTRY), the Employment Rate (ER), and the Economic Confidence Index (ECI) data. All data were obtained monthly basis from various data sources cover the years between 2008-2019. Table 1 shows the definition and source of the variables.

Table 1. Definition of the variables

Variable	Abbreviation	Source
Capacity Utilization Rate in the Manufacturing Sector	CUR	(TurkStat, 2020)
Industrial Production Index	IPI	(TurkStat, 2020)
USD to TRY Exchange Rate	USDTRY	(CentralBank, 2020)
Employment rate	ER	(TurkStat, 2020)
The Economic Confidence Index	ECI	(TurkStat, 2020)

CUR was stated as the output variable, while IPI, USDTRY, ER, and ECI variables were defined as inputs supposed to impact CUR, which will be tested. The statistics related to the dataset can be seen in Table 2.

Table 2. The descriptive statistics of data set

Statistics	Variables				
	CUR	IPI	USDTRY	ER	ECI
Max	82.20	129.99	7.25	48.35	113.95
Min	60.80	50.45	1.17	37.30	52.40
Mean	75.41	92.13	2.90	44.23	98.29
Std	4.01	19.32	1.61	2.56	11.31
Kurtosis	3.82	-0.95	0.05	-0.46	3.40
Skewness	-1.88	-0.10	1.12	-0.49	-1.76

The monthly changes of all variables between the years 2008-2019 are given in Figure 2.

Normalization is usually prepared for AI-based models prior to processing in the model. The purpose of normalization is to bring the values of the data set into a standard scale without disturbing the differences in value ranges. First, the variables were normalized between 0 and 1 (Guleryuz, 2021b). Then, the data set was split into two for training and testing phases at 70% and 30% with 104 and 44 observations, respectively.

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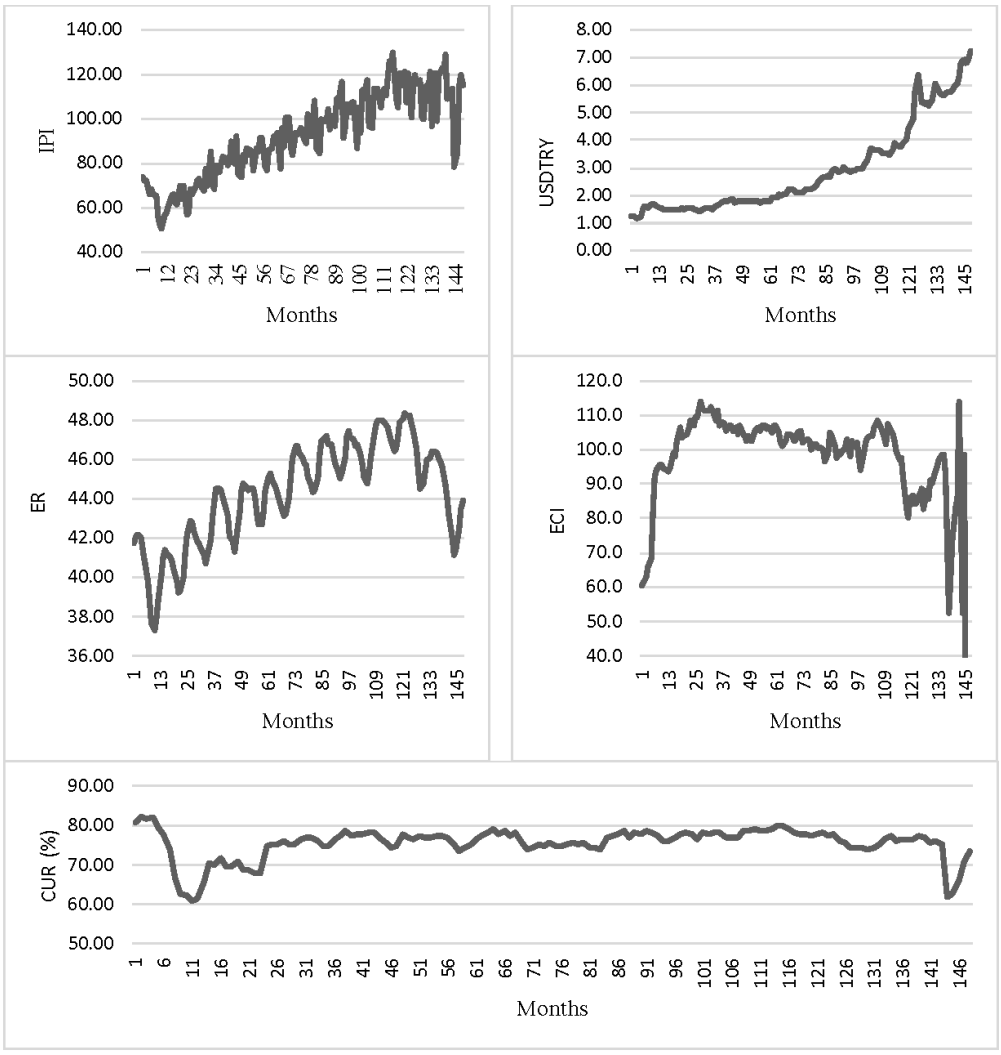
Figure 2. Change of variables between 2008-2019



Methods

Adaptive Neuro-Fuzzy Inference System (ANFIS)

Hybrid structures that combine different methods in solving complex problems can give successful results. Hybrid models are created using the complementary features of the models, taking into account the positive and negative features of each method. As a result, hybrid intelligent systems created by each method are effectively used to solve complex problems. The Adaptive Neuro-Fuzzy Inference System (ANFIS), first developed by Jang in 1991, is an example of such hybrid systems. ANFIS is a hybrid artificial intelligence technique developed by integrating the learning capability of ANNs and the best features of Fuzzy Inference Systems (FIS), such as the ease of providing expert knowledge and flexible computation methods. Since fuzzy systems do not have strong learning abilities like ANNs, they only

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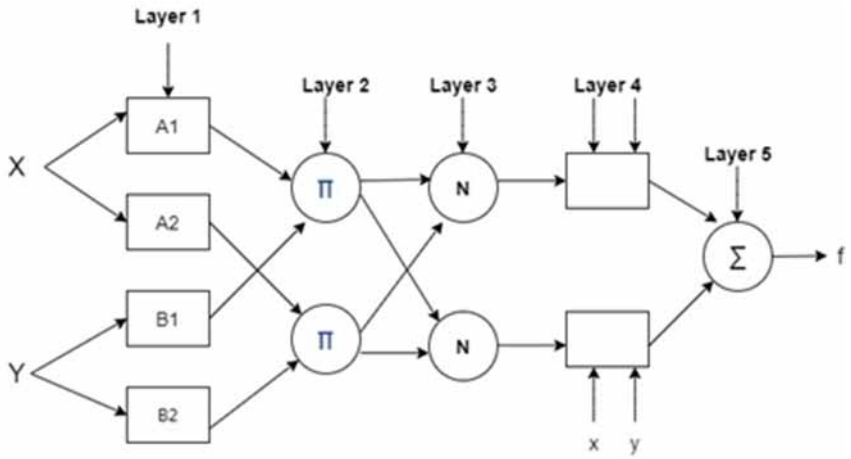
work on a rule-based basis and cannot connect input and output. However, the rule-based nature of fuzzy models enables better use of verbal information and interpretation of models. In addition, neural networks can effectively solve non-linear problems. The fuzzy logic system, like neural networks, converts non-linear input vectors to scalar outputs and can process linguistic expressions (Guleryuz, 2021a).

For this reason, using some techniques such as neural networks to increase the learning ability of fuzzy systems provides effective estimation results. The effective prediction models can be developed with ANFIS, which will be created by combining FIS and ANN. The ANFIS architecture is a network similar to the Takagi-Sugeno fuzzy inference system. It consists of five layers. The first and fourth layers contain an adaptive node, while the nodes in the other layers are fixed. Thus, five layers with different functions make up the ANFIS structure. These layers are fuzzy layer, product layer, normalized layer, defuzzification layer, and output layer (Pousinho et al., 2011). Figure 3 shows the basic ANFIS structure with two input variables and one output variable.

Figure 3. ANFIS basic structure with two input



Layer 1 is the fuzzification layer. It contains nodes that allow the input values to be transferred to the next layer. The output value is formed according to the input values and the membership degrees depending on the membership function. μA (x) and μB (x) indicate membership degrees. In layer 1, each node is defined by the function as in Eq. (1).

$$y_i^1 = \mu_{Ai(x)} \, (i=1,\ldots,n) \tag{1}$$

Layer 2 is known as the rule layer. The effect of a Sugeno-type fuzzy logic rule is seen at each node output. This layer can be represented mathematically as in Eq.(2).

$$y_i^2 = \mu_{Ai(x)}\mu_{Bi(y)=y} \, (\mathrm{i}=1, \ldots, \mathrm{n}) \tag{2}$$

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Where n is the number of nodes and yi2 is the output value of the 2nd layer. Layer 3 is called the normalization layer and has fixed nodes. The outputs of the second layer are input to this layer, and the firing power of a given rule is calculated with normalized values. The firing strength of their normalized form is known as $\overline{\mu}$ i, and the calculation of $\overline{\mu}$ i is given in Eq. (3).

$$y_i^3 = N_i = \frac{\overline{\mu}_i}{\sum_{i=1}^{n} \overline{\mu}_i} = \overline{\mu}\ (i{=}1,n) \tag{3}$$

Layer 4 is known as the defuzzification layer. In this layer, the firing levels of the normalized rules are multiplied by a linear function, and the fuzzy logic rules are recalculated. Eq. (4) performs a calculation of the refined results of a given rule at each node in the normalization layer.

$$y_i^4 = \overline{\mu}_i \left[p_i x_1 + q_i x_2 + r_i \right] = \overline{\mu}_i f_i \tag{4}$$

Where p, q, r show the variables and i represents the set of result parameters of the rule. Only one node in layer 5 and the output values of all nodes from layer 4 are collected in this node. Here, the output value of the ANFIS system is obtained with the output values of each node in the 4th layer via Eq. (5) (Pousinho et al., 2011).

$$y_i^5 = \sum_i \overline{\mu}_i f_i \tag{5}$$

ANFIS learning algorithm is a hybrid model that includes a backpropagation learning algorithm and the least-squares method. It aims to bring the error values closer to the minimum for all inputs at the end of the learning process. The learning process is completed in two steps. In the first step, input samples are produced, and the antecedent parameters are considered constant, and the best secondary parameters are determined by the least mean square method. In the second step, the inputs are reproduced, and the secondary parameters are supposed to be constant, and the antecedent parameters are changed with the gradient descent method. This process is repeated to ensure that the error value is minimal (Naji et al., 2016). Since ANFIS provides a rule-based structure for a problem, artificial neural networks can use expert opinions, ANFIS has better predictive ability than ANN. In addition, the ANFIS structure is more robust than other multi-layer perceptrons in terms of learning, parallel processing, structured information representation, and compatibility ability.

There are many different models of fuzzy inference systems in previous studies, and while these systems are similar to each other in general operations, they are separated by membership functions. According to the membership function, FIS has three groups: Mamdani, Tsukamoto, and Sugeno type inference systems (Blej & Azizi, 2016). The most widely used inference system is the Sugeno type. The Sugeno type inference system is preferable to other inference systems due to the easy optimization of parameters. Since this study includes parameter optimization, the Sugeno type inference system was preferred in this study.

The ANFIS structure has parameters that can be optimized in its two layers. For example, layer-1 has non-linear parameters, and linear parameters are in layer 4. By adjusting these parameters during

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the training phase, the predictive ability of the developed model is improved. For this reason, Particle Swarm Optimization and Genetic Algorithm methods were used for parameter optimization in this study.

Particle Swarm Optimization

Particle swarm optimization was designed in 1995 by psychologist and an electrical engineer, namely James Kennedy and Russell Eberhart, a population-based optimization algorithm form on the social behavior of animals living in swarms such as birds, ants, fish, and bees. Thanks to sharing information among individuals in a swarm, they follow the closest swarm element to reach rich food sources and update their speed and position according to this target (Guleryuz, 2021a).

In Particle Swarm Optimization, particles are the solution candidates. In order to find an optimum or near-optimal solution, the PSO initially designs particles, which is a solution candidate (Rezakazemi et al., 2017). The position of these particles is selected randomly under the constraints, and the initial swarm is determined. Each particle has position and velocity information. While the velocity information enables the particle to move from its current position to the next position, the position information also represents the solution. Each particle also has fitness values calculated by the fitness function. The fitness function evaluates the distance of each particle from the optimum solution. As the particle changes its position, the updated coordinates are sent to the fitness function, and the particle's fitness value, which is the distance from the optimum solution, is calculated. The position with the best fitness value of a particle is defined as pbest (individual best), the position with the best fitness value for the entire swarm is defined as gbest (global best). Particles memorize pbest and gbest in order to compare them with their past experiences in subsequent iterations. The equational summary of the PSO algorithm is listed below (Pousinho et al., 2011):

- For an n-dimensional position $\left(X_i^k = x_1^k, x_2^k, \ldots, x_n^k\right)$ and velocity $\left(vel_i^k = v_1^k, v_2^k, \ldots, v_n^k\right)$ vector ith particle begins with a random position and velocity.
- Particles have the positions and values of the objective function, the best position of particle i expressed: $\left(pbest_i^k = p_1^k, p_2^k, \ldots, p_n^k\right)$.
- The best position of the swarms: (Gbest = g1, g2, …, gn)

The PSO algorithm proceeds with the following equations:

$$vel_i^{k+1} = wt_k vel_i^k + c_1 r_1 \left(pbest_i^k - X_i^k\right) + c_2 r_2 \left(Gbest - X_i^k\right) \tag{6}$$

$$X_i^{k+1} = X_i^k + vel_i^{k+1} \tag{7}$$

Where vel_i^k represents the velocity of particle i in k iteration. wt_k is the inertia weight. X_i^k is the position of particle i in iteration k. $pbest_i^k$ represents the best position of particle i. Thus, Gbest is the global best of the swarm. c_1 and c_2 are learning factors. r_1 and r_2 are random variables that are uniform distributed between 0 and 1. The basic steps of PSO can be listed as follows (El-Telbany, 2007).

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1. The starting swarm is identified with random starting positions and speeds.
2. The fitness values of all individuals are calculated.
3. Find the local best (Pbest) for each particle from the current generation.
4. The global best (Gbest) is selected in the current generation
5. The velocities of all particles are updated using Eq. (6)
6. Using Eq. (7), each particle is moved to its new position.
7. Steps 2-6 are repeated until the convergence and stopping criterion is met.

Genetic Algorithm

A genetic algorithm, a class of evolutionary algorithms, is a scientific method inspired by biological processes. The term Genetic Algorithm (GA) was first used by Bagley in 1967 (Battle & Vose, 1991). Its basic principles were revealed and implemented by John Holland in 1975 (Mitchell, 1998). While creating GA, living life was simulated, and it was created by imitating the natural selection and mutation experienced in the transfer of hereditary characteristics in living life from generation to generation. The genetic algorithm is subjected to as many iterations as the number of generations. In each iteration, the coefficients (genes) that make up the solution set (chromosome) are converted to new values by subjecting them to various evolutionary processes (crossover, mutation), and the fitness function is calculated. These processes continue until they reach a certain number of generations or the desired fitness value (Jang, 1997). Genetic algorithms work according to probability rules and only need a fitness function. GA can be easily applied to problems where the solution space is huge, and the number of variables is vast. Genetic algorithms do not have to try all possible solutions, as the best-inherited values are passed on to the next generation thanks to the chromosomes.

Selecting parameters such as population size, crossover rate, mutation rate, generation gap, selection method, and function scaling play an active role in solving the problem while creating a genetic algorithm. Selecting a population size that is too small may result in local optimization while choosing it too large can increase the processing time and be costly. (Schaffer, J. B.,Konstan, J., And Riedl, 1999) stated that choosing the population size within a specific range (20 -30) positively affected the results in general. The crossover ratio is the process of creating new genes by crossing existing chromosomes. The crossover ratio indicates how much of a chromosome will be replaced. If this ratio is chosen large, some good chromosomes may deteriorate, and if it is chosen small, it may take a long time to reach the desired solution. Mutation, another critical parameter, changes a certain amount of bits in the chromosomes during transmission from generation to generation. If the mutation rate is significant, it will take a long time to find the current solution, and the algorithm starts to become random. Specifying the number of new chromosomes in each generation is called Generational Difference. If a high value is chosen, it may ignore possible reasonable solutions from the previous generation. The old generation is renewed in three different ways. In the “generational” method, all chromosomes are renewed through evolutionary processes. Good chromosomes are passed on to the next generation without being changed from the “Most Fit” method. In the “State of Balance” method, only a certain number of chromosomes are changed for each new generation. Function scaling is applied in two ways, linear and exponential scaling. Choosing the appropriate scaling affects the more successful resolution of the fitness function.

Working steps of the genetic algorithm:

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1. Initially, the function to be solved is identified, which is the fitness function.
2. Upper and lower limits are determined for the variables.
3. The initial population is created from random values.
4. The fitness value for the population is calculated.
5. It is checked whether the stopping condition (target generation number or target success value) is met.
6. If the condition is met, the algorithm is finished (Step 9).
7. Determine the new population by applying natural selection-crossover-mutation operations on chromosomes.
8. The fitness value for the population is calculated (Step 4).
9. Finish the algorithm.

In summary, in GA, all possible solutions are called chromosomes (binary bit sequence). In each generation, populations are kept, and the process is carried out. There can be more than one solution point within each population. The following population is created from the previous population by the selection, crossover, and mutation processes in each generation. In each generation, the best solutions are passed on to the next generation, making it better.

PSO-ANFIS and GA- ANFIS Models

In this study, the ANFIS model was combined with two different heuristic optimization methods. It is necessary to compare the performances of the developed hybrid models that were employed to estimate the CUR. For this reason, statistical metrics that were frequently used in previous studies were used. ANFIS model to forecast the CUR in the manufacturing sector is developed, relying on the Sugeno fuzzy inference system. It has four inputs: IPI, USDTRY, ER, and ECI, and the one output variable is CUR. All developed models are performed using Matlab 2020a.

Using PSO, ANFIS parameters have been optimized to increase the predictability of ANFIS. The Gaussian function, which many researchers suggest, has two parameters to optimize as a membership function. PSO optimization in the ANFIS model ensures closer relationships between output and inputs. PSO parameters such as population size, the maximum number of iterations, initial inertia weight, inertia weight damping ratio, and learning coefficients should be selected appropriately to develop PSO-ANFIS (Basser et al., 2015). GA-ANFIS technique is configured to estimate CUR. The flow chart of developed ANFIS-PSO models is shown in Figure 4.

GA parameters such as population size, the maximum number of generations, mutation percentage, crossover percentage, selection pressure, and mutation rate are obtained by trial (Rezakazemi et al., 2017). The flow chart of developed GA-ANFIS models is shown in Figure 5.

In the first step of the modeling, IPI, USDTRY, ER, ECI were used as input parameters, and CUR was determined as an output parameter. PSO-ANFIS and GA-ANFIS models were developed using MATLAB software. PSO and GA optimization in ANFIS allows outputs and inputs to have closer relationships, and the developed hybrid models may have more effective results in non-linear modeling relationships.

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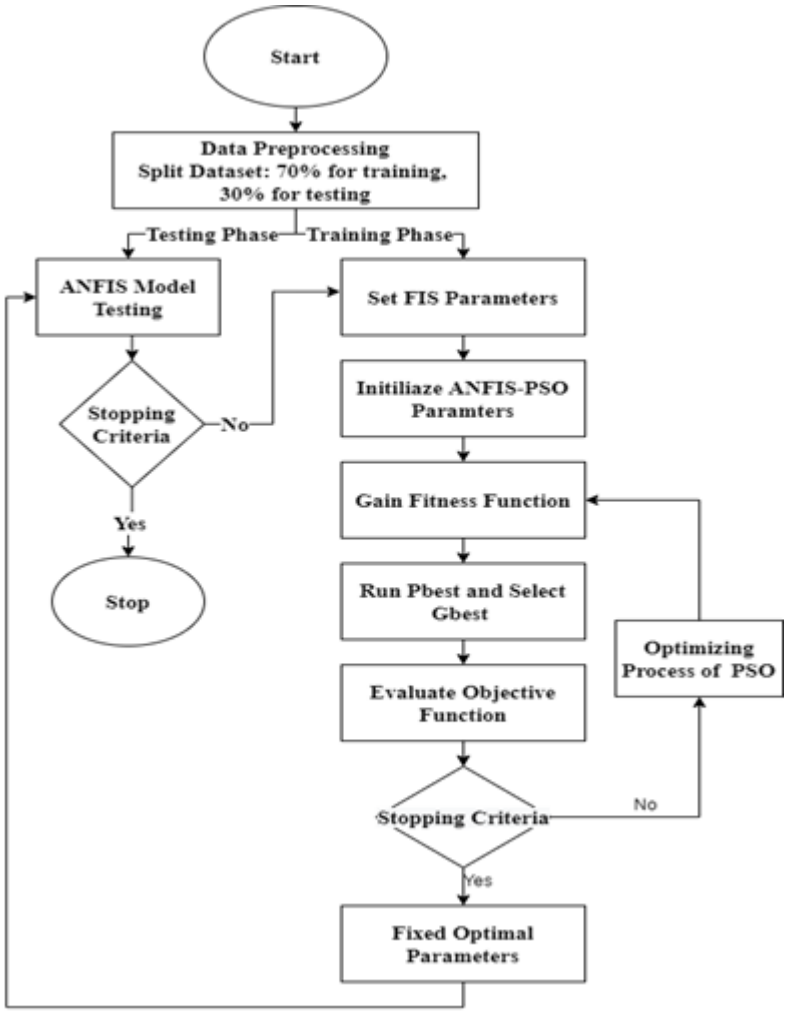
Figure 4. The flow chart of developed PSO-ANFIS



Performance Evaluation Criteria

The model's accuracy will be tested using some of the performance evaluation criteria in the literature. Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and the coefficient of determination (R^2) values was used to compare the forecasting ability of the ANFIS, PSO-ANFIS, GA-ANFIS methods to make meaningful interpretations. The equations of these metrics are given in Eq.(8) to Eq.(10), respectively (Guleryuz, 2021a).

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|CUR_i^{obs} - CUR_i^{pre}\right| \tag{8}$$

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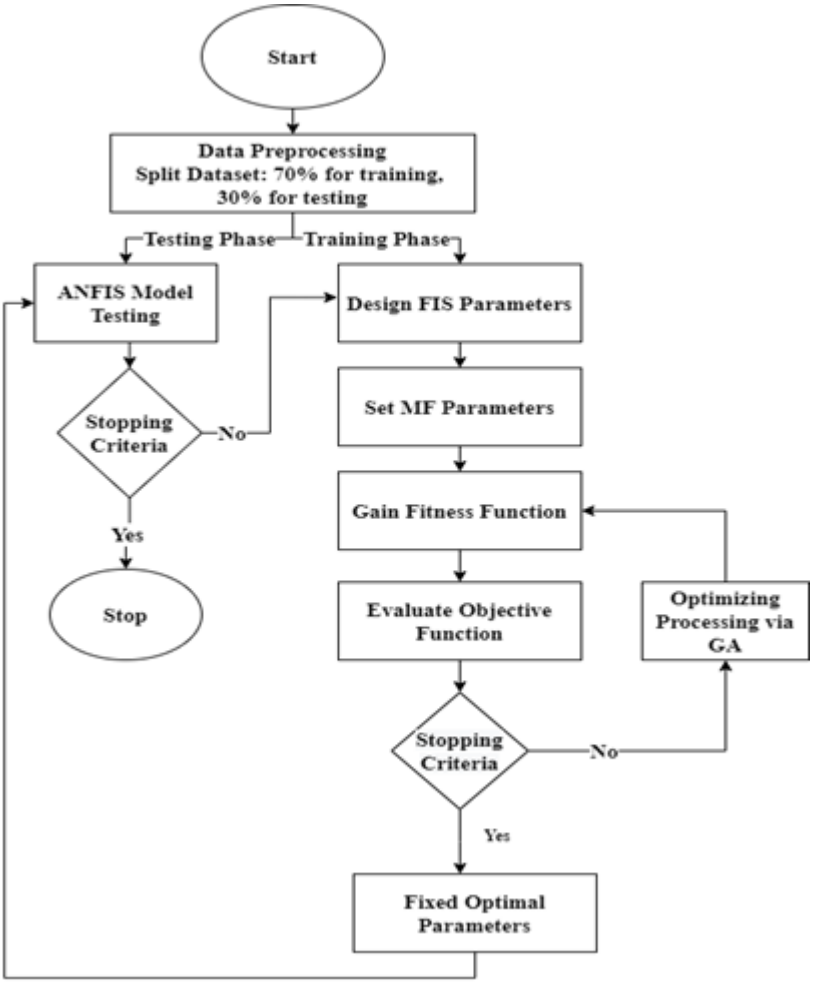
$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(CUR_i^{obs} - CUR_i^{pre})^2} \tag{9}$$

$$R^2 = \left(\frac{\sum_{i=1}^{n}(CUR_i^{obs} - \overline{CUR_i^{obs}})(CURP_i^{pre} - \overline{CUR_i^{pre}})}{\sqrt{\sum_{i=1}^{n}(CUR_i^{obs} - \overline{CUR_i^{obs}})^2 (CUR_i^{pre} - \overline{CUR_i^{pre}})^2}} \right)^2 \tag{10}$$

where the number of observed CUR is shown via n, CUR_i^{obs} is the actual value at time i and CUR_i^{pre} is the predicted value at time i.

Figure 5. The flow chart of developed GA-ANFIS



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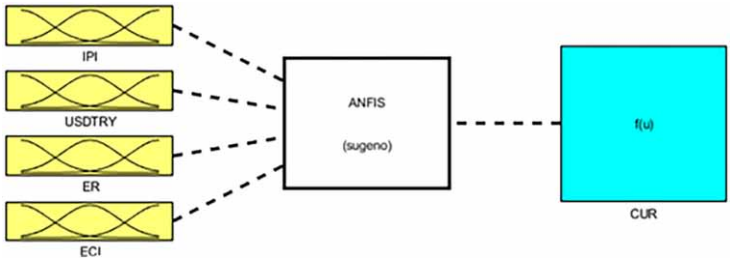
RESULTS

Manufacturing Industry Capacity Utilization Rate (CUR) is the actual capacity utilization of workplaces operating in the manufacturing industry according to their current physical capacities in the reference period. The economic growth of the countries is related to the CUR data. For this reason, economic and manufacturing capacity planning for the future can only be done with effective forecasting methods. In this section, the monthly data of the determinants that are thought to be effective in the CUR of Turkey between 2008-2019 and the results of the forecasting models will be compared.

ANFIS model to predict CUR is proposed based on the Sugeno fuzzy inference system. The model has four inputs variable: IPI, USDTRY, ER, and ECI, and the one output variable is CUR. The model was designed using MATLAB 2020a. Figure 6 represents the general ANFIS model framework.

Figure 6. Basic ANFIS model framework.



ANFIS model developed by using Sugeno-type structure. The function that uses the fuzzy c-means method known as "genfis3" in Matlab is used to specify the membership functions and the number of rules. The stopping criteria are 1000 iterations, the number of fuzzy rules 81, the input membership function type is Gaussian, the initial step size is 0.01.

ANFIS parameters were optimized using GA and PSO algorithms to increase the prediction accuracy of the ANFIS model. Thus, hybrid models were developed. In PSO-ANFIS, the ANFIS model activates the search space for PSO and searches for the best solution by comparing each solution of the PSO objective function. The difference between the observed values and the output of the developed prediction model can be minimized due to the PSO algorithm's feature of seeking the least error (Moayedi et al., 2020). PSO does not depend on the nature of the objective function and is a stable algorithm, so it reaches the optimal solution by adjusting the membership functions. If it is desired to adjust the ANFIS parameters with heuristic methods, the Gaussian membership function is recommended by many researchers since it has two tunable parameters. GA method was used to tune ANFIS parameters to broaden its estimation proficiency to make a meaningful comparison. All developed models use training data to find relationships between CUR and input variables. Then, the test dataset is required to check the accuracy of the generated models. The entire dataset was divided into two parts, 70% (104 data points) and 30% (44 data points), to achieve this purpose. Comparison results according to statistical performance metrics are shown in Table 3.

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Table 3. Performance metrics of the developed model

	Developed Models	MAE	RMSE	R^2	Rank
Training Phase	**ANFIS**	0.022701	0.034045	0.967820	3
	GA-ANFIS	**0.020017**	**0.027700**	**0.978700**	**1**
	PSO-ANFIS	0.020019	0.027800	0.978600	2
Testing Phase	**ANFIS**	0.061544	0.094443	0.726440	3
	GA-ANFIS	**0.029570**	**0.036516**	**0.959110**	**1**
	PSO-ANFIS	0.070169	0.087037	0.767670	2

According to the performance comparison results given in Table 3, it is seen that the PSO and GA algorithms improve the prediction performance of the ANFIS model. This is a clear indication that parameter optimization gives good results. For this reason, optimizing the parameters in the ANFIS model with these two methods may be beneficial for future studies. Since the coefficient of determination (R^2) values of the two-hybrid models were high in the test stages, the hybrid models are safe in terms of the prediction process. When the prediction performances of the developed models are examined, it is seen that GA-ANFIS and PSO-ANFIS models have better prediction performance during the training phase. If hybrid models are compared among themselves, they have almost the same accuracy in the training phase. However, in the control phase of the design of the model, it is seen that the GA-ANFIS model with the test data exhibits higher accuracy prediction performance. For this reason, the model that gives the optimum result for Turkey's CUR estimation between 2008-2019 is the GA-ANFIS model. The developed GA-ANFIS model can be used to estimate the capacity utilization rate in Turkey for the coming years. Comparative graphics of the evaluation results of the training and testing stages are given in Figure 7.

Scatterplots can give good ideas about future behavior, especially if you want to see the relationship between these two variables. If it is known how two-time series affect each other, predictions can be made. The coefficient of determination (R^2) during the test phase is 0.9591, 0.7676, and 0.7264 for the GA-ANFIS, PSO-ANFIS, and ANFIS models, respectively. These values show that GA-ANFIS has the best predictive accuracy. In Figure 8, scatter plots of these results are drawn. Thus, the relationship between predicted and actual values can be seen for all developed models.

FUTURE RESEARCH DIRECTIONS

In ANFIS, adjustable parameters consist of membership function parameters and result parameters. Parameter complexity directly affects results. Therefore, parameters in ANFIS architecture that are required in training and testing phases will make it difficult to calculate. Well-established optimization techniques (GA and PSO were used in this study) will be able to overcome this situation. In future studies, the ANFIS method can be combined with other heuristic optimization methods, and hybrid models can be compared with classical econometric models and machine learning methods.

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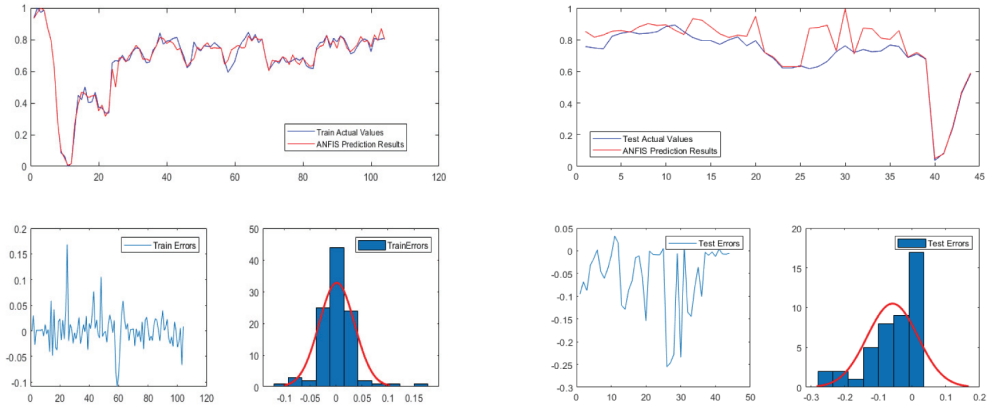
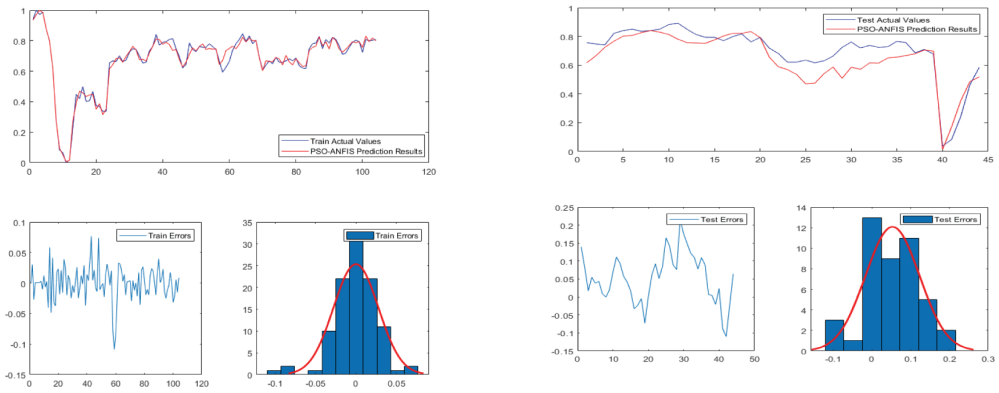
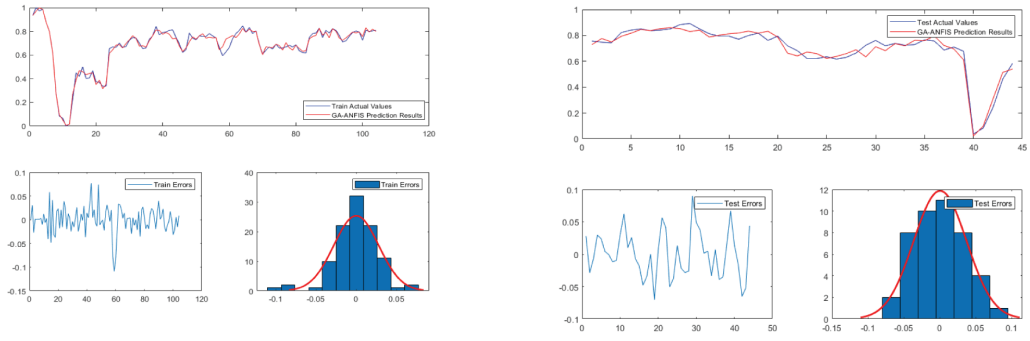
Figure 7. The evaluation results of the training and testing stages



ANFIS



PSO-ANFIS



GA-ANFIS

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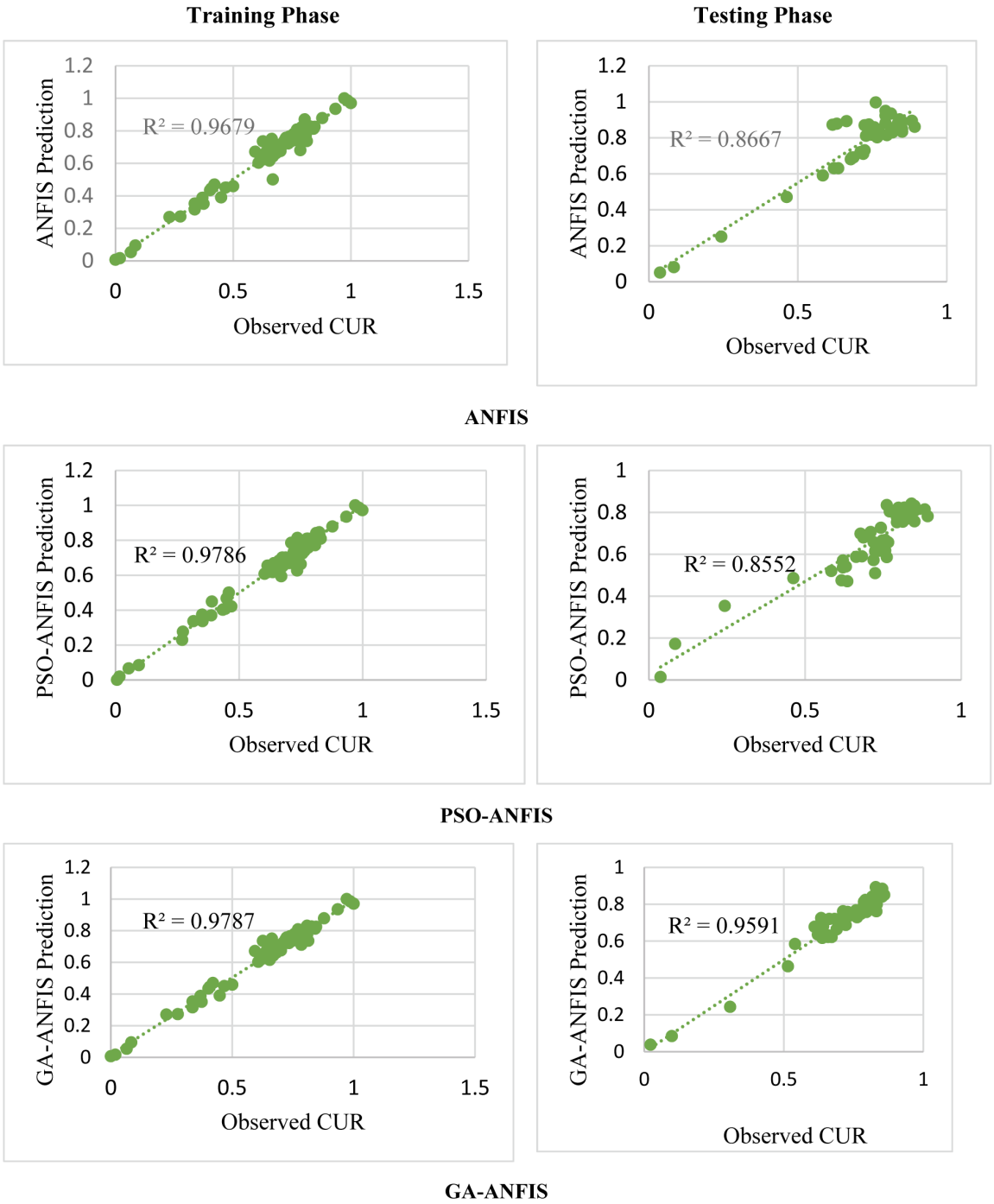
Figure 8. Scatter plots of the developed models



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CONCLUSION

The manufacturing industry constitutes a significant part of the Turkish economy. Therefore, many indicators such as industrial production index, economic confidence index, and capacity utilization rate in manufacturing show the status of this sector and ensure the traceability of production activities. Among these indicators, the utilization rate of the manufacturing industry capacity shows the actual capacity utilization for the manufacturing industry in general and its sub-sectors according to the physical capacities reported by the firms.

In this study, a prediction model is designed to enlighten the factors affecting Turkey's manufacturing sector's capacity utilization rate. The models are applied with artificial intelligence tools such as ANFIS, PSO-ANFIS, and GA-ANFIS. It has been observed that the capacity utilization rate in the manufacturing sector in Turkey, which varies depending on different economic parameters, has been estimated by the GA-ANFIS method accurately. Therefore, decision-makers can use these results as a source for future sectoral capacity plans.

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KEY TERMS AND DEFINITIONS

ANFIS: The adaptive neuro-fuzzy inference system is a hybrid artificial intelligence technique developed by integrating the learning capability of artificial neural networks and the best features of fuzzy inference systems.

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GA: A genetic algorithm, a class of evolutionary algorithms, is a scientific method inspired by biological processes.

GA-ANFIS: A hybrid algorithm which includes ANFIS and GA, GA is used to adjust the parameters of ANFIS.

PSO: Particle swarm optimization is a population-based optimization algorithm form on the social behavior of animals living in swarms such as birds, ants, fish, and bees.

PSO-ANFIS: A hybrid algorithm which includes ANFIS and PSO, PSO is used to adjust the parameters of ANFIS.

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Chapter 14
Fuzzy Soft Set and Its Engineering Applications

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ABSTRACT

Uncertainty is the difference between actual information and obtained information which can exists in two different forms: one is randomness, and another is impreciseness. The fuzzy soft sets are widely used in a variety of applications in the fields of medical science, automation industry, engineering, economics, share market, social science, game theory, operational research, and so on. Chemical engineering, electrical engineering, mechanical engineering, computer science engineering, and telecommunication engineering are prone to a variety of uncertainty in terms of system uncertainty and parameter uncertainty. Hence, with the application of fuzzy soft set, the uncertainty in the mentioned engineering types can be handled very well. In this chapter, fuzzy soft set is applied for handling the uncertainty in the mentioned engineering domains. The performance achieved by the fuzzy soft set-enabled computer science and engineering and biotechnology is found to be good towards the performance metrics like response time, throughput, resource utilization rate, and error rate.

INTRODUCTION

Uncertainty is the difference between actual information and obtained information which can exists in two different forms one is randomness and another is impreciseness. Probability theory was used exhaustively for handling uncertainty but it suffers from several drawbacks in terms of increased computational complexity, difficult to estimate the empirical possibilities of the event, not suitable to model mutual exclusive events, cannot handle linear interpolation between the random points, chances of false propagation of the belief is high, quality of predictions is low, and so on (Wang, and Matthies 2019). These drawbacks are overcome by an advanced and recently proposed mathematical tool which are widely used to handle the uncertainty and impreciseness in the dynamic computing environment popularly referred as soft set theory.

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Molodtsov invented soft set theory in the year 1990 to handle the real-world objects which are not properly defined and cause significant influence on decision making process (Arora, and Garg 2018). A fuzzy approach to the soft set is found to solve complex decision making problems especially in the field of engineering. Fuzzy soft set is defined as mapping from the set of parameters to power set of the universal set. It comes in different forms which include generalizes the fuzzy soft set, interval values fuzzy soft set, intuitionistic fuzzy soft set, fuzzy parameterized soft set, hybrid fuzzy soft set, and so on. The fuzzy soft sets are widely used in variety of applications in the field of medical science, automation industry, engineering, economics, share market, social science, game theory, operational research, and so on (Garg, and Arora 2020). Among all applications, engineering applications in the following type of engineering i.e. chemical engineering, electrical engineering, mechanical engineering, computer science engineering and telecommunication engineering are prone to variety of uncertainty in terms of system uncertainty and parameter uncertainty (Khan, Kumam, Liu, and Kumam 2020). Hence, with the application of the fuzzy soft set, the uncertainty in the mentioned engineering types can be handled very well.

BACKGROUND

The works carried out in literature for handling the uncertainties in engineering domains is discussed and following gaps are identified.

The uncertainty in computer science computing domain is addressed by making use of conventional fuzzy logic technique or probability theory (Tchernykh, Schwiegelsohn, Alexandrov, and Talbi 2015). But the conventional fuzzy logic works by forming rules that are static in nature which handles the uncertainty only up to some degree of uncertainty. The time consumed to formulate the knowledge database by forming the rules is too high.

Chemical engineering uncertainty is handled using finite discrete models, stochastic d design, transformation methods, and cubature rules (Mc Cabe, Smith, and Harriott, 2018). The degree up to which uncertainty is measured and handled is acute as the approaches followed are deterministic in nature and fails to solve the large state problems with high accuracy and speed of operation.

Mechanical engineering is prone several forms of uncertainty in terms of design, components, safety, operation, and maintenance (Vu, and Le 2019). The approaches followed to handle uncertainty in mechanical engineering are quasi Monte Carlo, intrinsic variables, nonlinear dynamic variables, linguistic variables and many more. But these approaches have inherent drawbacks in terms of poor degree of approximation, black box approach, poor knowledge formation, high probability of mechanical failures and so on.

The uncertainty in electrical engineering arises from several factors which include operator measurement errors, poor structure modelling, optimization process is slow, too much of power consumption and so on (Xu, Zhang, Jiang, Wang, Chen, Hu, and Chu 2019). The existing methods for handling uncertainty are approximate reasoning, contingency ranking, fuzzy logic, multi valued probability theory, and so on. The drawbacks observed are does not consider hidden uncertainty causing parameters, poor structuring of data, lesser accuracy is achieved in reasoning, accuracy gets affected due to distortion, and so on.

In biotechnology engineering the uncertainty arise due to federal circuit formation, input parameters variability, high volatility during computation, poor accuracy in bio chemical process, and many more (Haider, Haider and Kang 2018).. The commonly used approaches for handling uncertainty in biotechnology are soft set, Bayesian network, rule based systems, and belief functions. However, these approaches

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to suffer from poor efficiency in terms of poor diagnosis of the problem, not suitable for dynamically varying parameters, large amount of uncertainty gets ignored, not suitable for real-time computation errors, and so on.

The main drawbacks observed in the existing works carried out are parameter uncertainty and system uncertainty is not taken into consideration while making the performance decisions which results in poor performance, risk management factor in engineering domains is completely ignored, the time taken to solve large state space problem is more and is found to be exponential in nature and most of the approaches discussed above uses deterministic approach to solve engineering problems which fails to handle the dynamic nature of the computing domain.

MAIN FOCUS OF THE CHAPTER

The main focus of the chapter is to consider the potential engineering disciplines like computer science engineering, chemical engineering, mechanical engineering, electrical engineering, and biotech engineering. Highlight the sources of uncertainty in the above-mentioned engineering branches and issues/challenges faced by the scientists/developers in proposing performance improvement approaches by managing uncertainty challenges. The fuzzy soft set is found to be promising framework for handling the uncertainties in these engineering domains. A high-level architecture of the fuzzy soft set enabled engineering domains is provided and the benefits offered are also discussed. Generic performance analysis of the proposed fuzzy soft set approach in engineering domains is tabulated towards various performance metrics. It is found from the analysis that the fuzzy soft set framework is good in handling the uncertainty issues related to computer science and engineering discipline.

The novelty of the proposed chapter is the factors causing uncertainty in the engineering disciplines like computer science engineering, chemical engineering, mechanical engineering, electrical engineering, and biotech engineering are discussed as result the decision making accuracy is improved. The challenges faced in each of the engineering domains due to uncertainty are discussed in detail. A high level adaptable fuzzy enabled architecture for these engineering domains are designed to provide clear understanding of the benefits of fuzzy logic in solving engineering problems. By keeping the benchmark performance of fuzzy logic in engineering domains performance analysis is carried out to provide insight understanding about the performance achieved by the fuzzy set enabled frameworks in engineering disciplines.

In generic uncertainty is measured by computing the root mean square error of the results obtained from the system. In which x_i stands for the data read from the engineering domain dataset, IV is the input variable, μ is the mean value computed over the engineering dataset and n is the total number of rows present in the engineering dataset.

$$Uncertainty\left(IV\right)=\sqrt{\sum\frac{(x_i-\mu)2}{n*\left(n-1\right)}}$$

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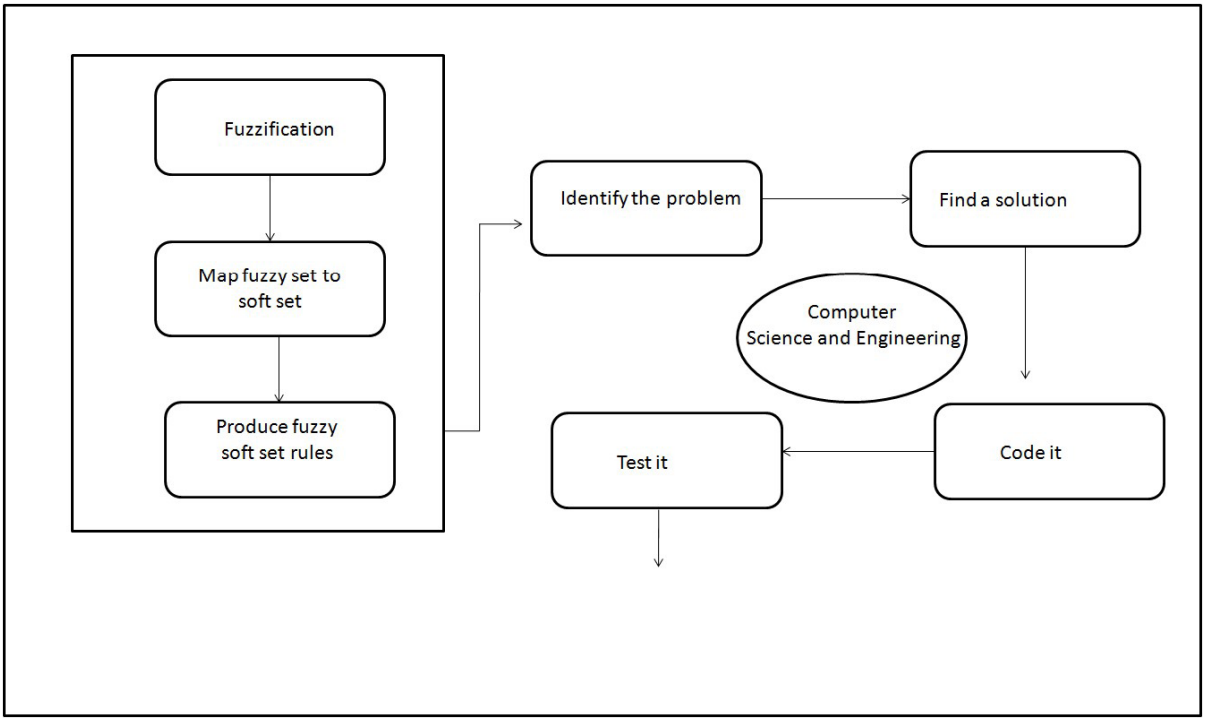
SOLUTIONS AND RECOMENDATIONS

For handling the uncertainties in the engineering domains, a novel architecture of the fuzzy soft set enabled approaches for engineering domains starting from problem identification to proposing of the solution is discussed along with their advantages.

1. Computer Science Engineering

Computer science discipline deals with the study of the algorithmic process which performs high intensity computation by applying the principles of mathematics, engineering, and artificial intelligence. Unlike other engineering disciplines, the computer science discipline deals with design, development and application of software systems. In computer science domain uncertainty prevails in many forms which include outcome uncertainty, effect uncertainty, state uncertainty, and agent uncertainty (Tchernykh, Schwiegelsohn, Alexandrov, and Talbi 2015). Outcome uncertainty occurs when the probability outcome is not known properly. Effect uncertainty when the possibility of occurrence of probable outcomes is not known. State uncertainty exists when the current state of the computing domain is not properly defined. Agent uncertainty occurs when one agent is not able to communicate with another agent. Hence, there is a need to develop uncertainty proof algorithms for solving complex large-scale problems by making precise decisions.

Figure 1. High-level view of the fuzzy soft set enabled computer science and engineering



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Challenges faced by the computer science scientists due to uncertainty are difficult to maintain numerical precision throughout calculation, inability to recognize the textual form of entailments, solutions for NP hard problems takes polynomial time, overhead performance bottleneck, adaptive scheduling schemes are prone to deadlocks, heterogeneous processors have incompatibility issues, network traffic control algorithms does not guarantee the less computational complexity, limited performance of distributed database systems, poor garbage collection mechanism, scalability problem as network size increases, overlapping of locks leads to livelock situation and so on (Hariri, Fredericks, and Bowers 2019). The uncertainty in computer science domain is mainly influenced by the combined uncertainty, sensitivity coefficient and standard uncertainty. In which U(IV) represent the uncertainty of input value, SCO stands for sensitivity coefficient, CU is the combined uncertainty and SCU is the standard uncertainty.

$$U\left(IV\right)=\sqrt{\sum\left(SCO*CU\right)2-\sum\left(SCU*SU\right)2}$$

Fuzzy soft sets are generally applied to solve decision making problems and critical diagnosis problems in computer science under uncertainty (Jana, and Pal 2018). Traditional soft set and crisp set has limited applications in computer science due to poor efficiency in achieving higher accuracy. In precise some of the specific applications of fuzzy soft sets in computer science domain include object recognition, reduction in the highly parameterized set, quantum computing, computer vision, software prototype development, matrix comparison, page ranking, control system design, medical diagnosis, and so on. A high-level view of the fuzzy soft set enabled computer science engineering domain is shown in Figure 1. During fuzzy soft set process the activities carried out are fuzzification, mapping fuzzy set to soft set and production of fuzzy soft set rules. Then the formulated fuzzy soft set rules are fed as input to computer science domain process which includes the activities like identification of the problem, finding a solution, implementation of the solution, and testing of the proposed solution.

The application of the fuzzy soft set enabled computer science engineering domain offers several advantages in improving the overall performance of the computer science process which includes efficient removal of noise from the input parameters, ease extraction of the knowledge, highly precise decision formulation, easy mitigation of the incomplete soft sets, exploits the benefits of soft intelligence in decision making, high computational efficiency, uncertainty is measured via indeterminacy, parameter reduction through two level optimization, scalable storage in terms of matrices, maximum reduction in cost through distributed clustering, and so on (Riaz, and Tehrim, 2019).

2. Chemical Engineering

Chemical engineering is one of the branches of engineering which focuses on the design and development of chemical plants to improve the rate of production. The engineers in chemical engineering develop novel economical processes which does conversion of the raw materials to the useful products (Mc Cabe, Smith, and Harriott, 2018). Even in chemical engineering process parameter uncertainty do exists which mainly arises due to improper numerical approximation. The parameter uncertainty exists when material properties are incorrect and algorithmic uncertainty arises when the numerical uncertainty is implemented in the form of finite computer models. Special attention need to be made when the uncertainty exists during the stages of chemical reaction control, modelling, synthesis and optimization of the chemical reaction. The parameters of the chemical process are sensitive towards uncertainties both

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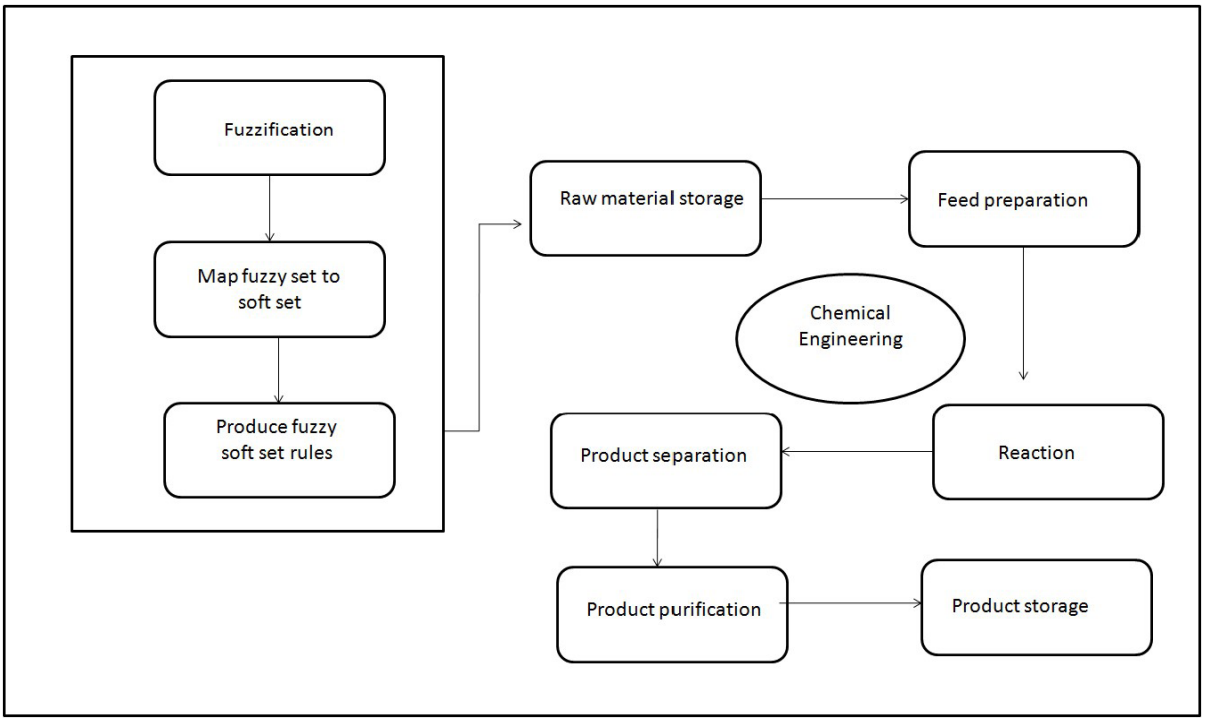
during development and testing of the chemical process in terms of variable of parameter inputs, stochastic format of the input variables and unpredictable nature of the output variables. If the uncertainty in chemical engineering is not handled properly then, it affects the widespread application of chemical engineering models in component level design and system level design (Maußner, and Freund, 2018). The chemical engineering is mostly affected by relative uncertainty which is influenced by the absolute uncertainty and magnitude of input value measurement. In which AU represent the absolute uncertainty, MM is the magnitude of measurement.

$$U\left(IV\right)=\frac{AU}{MM}*100$$

Figure 2. High-level view of the fuzzy soft set enabled chemical engineering



Challenges faced by the chemical engineers due to uncertainty are propagation of uncertainty from one chemical analysis model to another, too much sensitive to the effect of the outliers, statistical calibration of the results, not so robust chemical structure design, poor adoption of meta data models, improper handling of the large input and output parameters, increase chances of risks, polynomial increase in the evaluation time, increased computational cost, error occurrence probability is high due to lack of preciseness, huge impact on results due to external disturbances, low level chemical structure design, mismatch in demand and supply, work fluid selection quality is poor, inability to cope up with the changes in experimental data, and so on (Janet, Liu, Nandy, Duan, Yang, Lin, and Kulik, 2019).

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The application of fuzzy soft set to chemical engineering is beneficial in terms of modelling, parameter reduction, membership computation, development of expert system, successful application of variety of intersection operations, fast analysis of the hierarchical process, and so on. In specific some of the areas of chemical engineering where fuzzy soft set usage is found to be beneficial are thermodynamics, organic chemical analysis, inorganic chemical analysis, material quality assessment, simulation of chemical particles, preparation of carbon compounds, design of polymers, synthesis of chemical drugs, unparallelled chemical substance analysis, supramolecular particle formation, simulation of nucleic acids and enzymes, and so on (Yazdi, Korhan, and Daneshvar 2020). A high-level view of the fuzzy soft set enabled the chemical engineering domain is shown in Figure 2. Towards the fuzzy soft set, the activities performed are fuzzification, mapping of fuzzy set to soft set and production of fuzzy soft set rules. Towards chemical engineering process the activities carried out are raw material storage, feed preparation, reaction, product separation, product purification, and storage of the product formulated.

3. Mechanical Engineering

Mechanical engineering is the oldest engineering branch which mainly combines two subjects one is physics and another is mathematical principles which are applied to the design and development of the mechanical systems. Mechanical engineers use modern tools like computer aided design and manufacturing to design and develop the manufacturing plants. Uncertainty exists in every phase of mechanical engineering like design, development, construction, and testing (Vu, and Le 2019). Due to integral presence of uncertainty in design of mechanical components, only some percentage of machine component works fine and other percentage of machine component does not works fine which leads to the machine which does not satisfy the user requirements. One of the important requirements of handling uncertainty in mechanical engineering will enhance the safety, reliability, and computational efficiency of the system (Corotis 2015). Some of the challenges which arises due to the influence of uncertainty in mechanical engineering includes improper design of mechanical components artifacts, increase in operational price, huge gap between the theoretical analysis and implementation, low powered aerodynamic vehicles, decrease in industrial automation, poor efficiency in large project management, design limitations, decoupled design, poor acceleration of the development work, safe removal and joining of the mechanical components, high energy consumption, compression ratio of the engine is high, design of four stroke petrol engine is difficult, frequent fatigue failure of the devices, poor design of battery, highly stressed pistons, poor mechanical friction and lubrication, poor combustion chamber design, and so on (Deng, Bae, Denlinger and Miller 2020). The uncertainty in mechanical engineering is influenced by the factors causing stress f_i and force created by the cross sectional area A_i.

$$U\left(IV\right)=\frac{\sum_{i=1}^{i=n}f_i}{\sum_{i=1}^{i=n}A_i}$$

One of the promising approaches for handling uncertainty in the mechanical engineering domain is fuzzy soft set. To be precise fuzzy soft set are useful in the some of the areas of mechanical engineering which includes design of less complexity machine components, good assembly of machine components, ease development mechanical drawing toolkits, fastening of fluid power manufacturing process, con-

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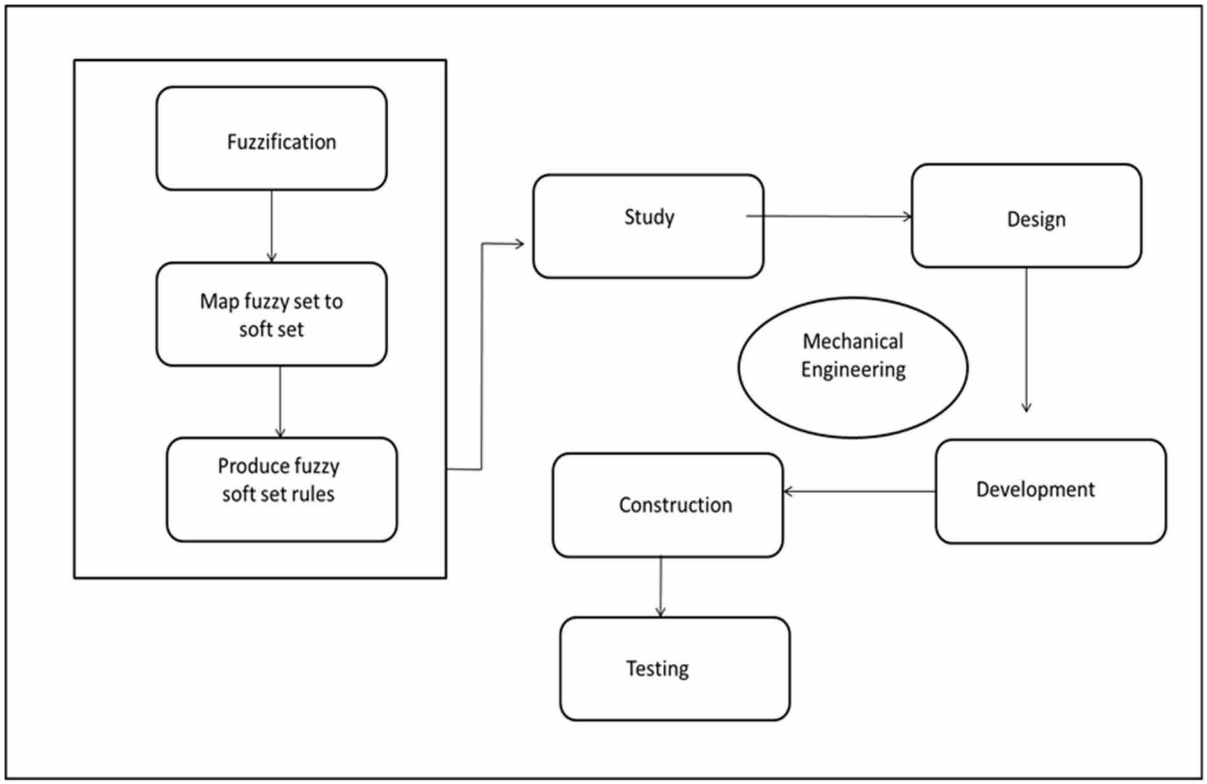
trolled movement of mechanical components, proper management of combustion in internal combustion engine, custom development of machineries, accurate overseeing of oil mining and refinement process, systematic pattern evaluation during exploration phase of the design, proper management of ventilation and head management issues, and so on (Khan, Phiangsungnoen and Kumam 2020). A high-level view of the fuzzy soft set enabled mechanical engineering is shown in Figure 3. The fuzzy soft set process mainly consists of three steps i.e. fuzzification, mapping fuzzy set to soft set and production of fuzzy soft set rules. Meanwhile, the mechanical engineering process begins with the study of the domain, design of the prototype, prototype development, construction, and testing (Surjadi, Gao, Du, Li, Xiong, Fang, and Lu 2019).

Figure 3. High-level view of the fuzzy soft set enabled mechanical engineering



4. Electrical Engineering

Electrical engineering is one of the branches of engineering that is concerned with the literature study, design, and application of electromagnetic systems. Electrical engineering nowadays overlaps with many other forms of engineering like telecommunication engineering, signal processing, and electronics engineering. Some of the frequently occurring forms of uncertainty in electrical engineering are measurement errors, poorly structured electrical models, beta distribution problem due to frequent solar radiation,

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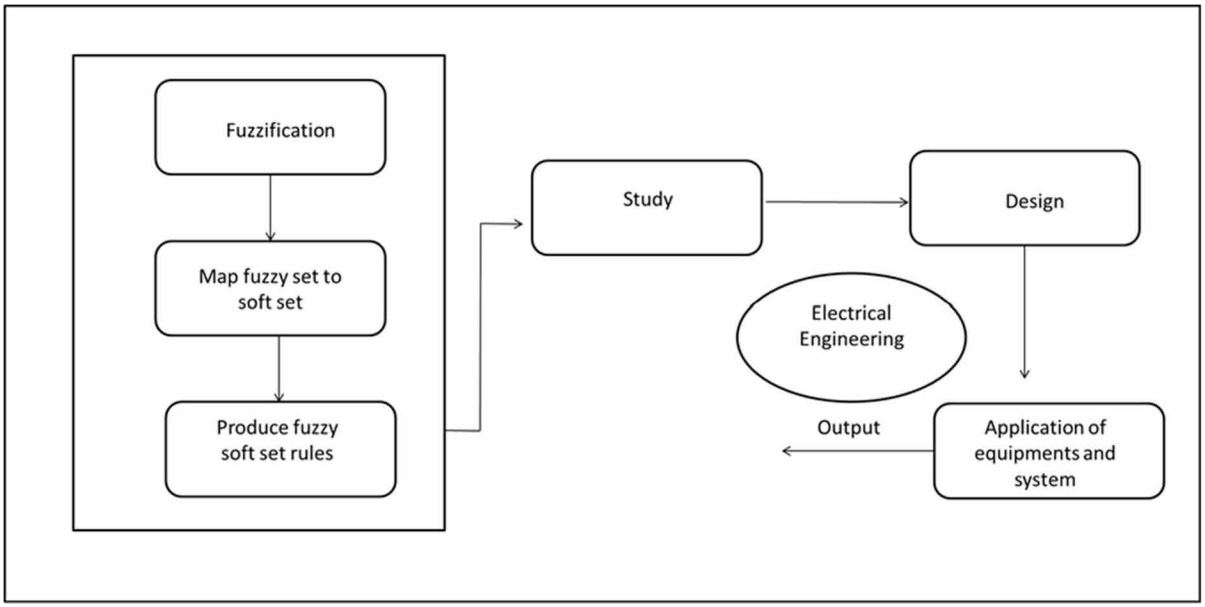
voltage instability, poor management of large-scale grids, unable to plan varying power supply properly for computation intensive applications, and so on (Xu, Zhang, Jiang, Wang, Chen, Hu, and Chu 2019).

The challenges faced in electrical engineering domain due to uncertainty are handling of fluctuating power, poor quality of power, probability of cyber attack is high, lower speed of operation, bottleneck situation, unreliable production of products, catastrophic failure of nodes, resource depletion, interference of high frequency electrical noise, mismatch of supply and demand, not optimized resource utilization, carbon emissions, increased circuit complexity, poor efficiency of sewage systems, high impact on environment, poor quality bridges construction, inability to properly monitor the large infrastructure based projects, increased proximity of electrical wires, isolated traffic control schemes, defective work rate is high, delay during construction, gap between the real-time projects and actual implementation, and so on (Massrur, Niknam, and Fotuhi-Firuzabad 2018). A high level view of the fuzzy soft set enabled electrical engineering is shown in Figure 4. As a part of fuzzy set, the activities performed are fuzzification, mapping of fuzzy set to soft set and production of fuzzy rules. This is fed as input to the electrical engineering process consisting of activities like study, design, and application of equipments and systems.

Figure 4. High-level view of the fuzzy soft set enabled electrical engineering



The uncertainty in electrical engineering is influenced by the combined uncertainty of the input parameter values ipv_1 and resource parameter values rpv_n.

$$U(IV) = \sqrt{\frac{\delta(ipv_1)}{|ipv_1|} + \frac{\delta(rpv_n)}{|rpv_n|}}$$

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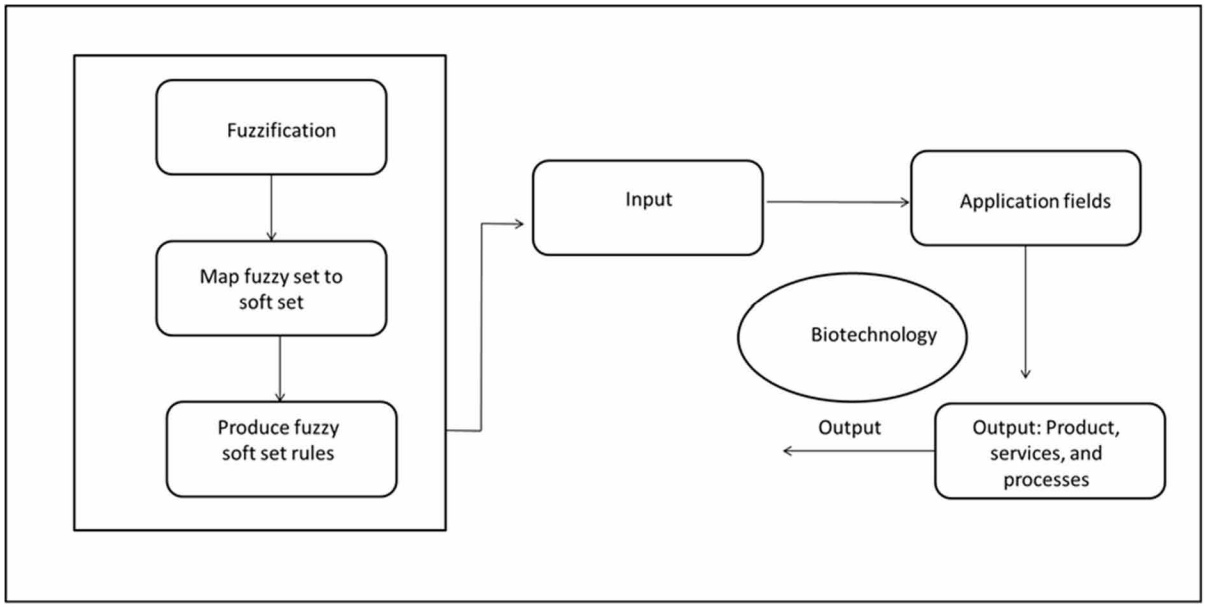
The application of fuzzy soft set based electrical engineering is found to be good in several aspects which includes design of voltmeter, design of parts of oscilloscope, ease conduction of high frequency experiments, mass production of silicon chips at high rate, maintenance of power grids, avoiding power surge, design of airlines, collecting sensory feedback with less interruption, designing electronic components at microscopic level, transmission of power, control of motor movements, wiring for building, satellite communications, establishing transport network, lift management, higher order ventilation systems, substation monitoring based GSM system development, and so on (Athira, John and Garg 2019).

5. Biotechnology

Biotechnology branch is a sub-branch of chemical engineering which studies microorganism, and cell functioning of the living organisms which helps in solving the problems in the field of medicine, engineering, and similar related applications. Modern applications of biotechnology also include the use of genetic algorithms and tissue culture algorithms to produce highly valued products. Biotechnology is being applied in variety of fields like waste management, agriculture, medicine, food processing, energy management system, plant tissue culture, modified food production, environmental sustainability, and so on. The field of biotechnology is classified into three categories i.e. ancient, classical, and modern. Uncertainty so exists in biotechnology field also in the form of poor regulatory decision making, high marketing risks, making large investment, side effects generation, premature decline of the life cycle, systems get obsolete and so on (Haider, Haider and Kang 2018).

Figure 5. High-level view of the fuzzy soft set enabled Biotechnology



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Biotechnology branch faces several challenges due to uncertainty which includes production safety management, environmental impact, destruction of micro-organisms, soil gets san tailings, disruption of the supply chain, harmonizing biological standards, waste of clean energy, multilayered coordination, triggering not targeted organism are high, genetically modified organisms are not good, tailored made food production rate is less, poor generic process optimization, irregular handling of genetic processes, and so on. The uncertainty in biotechnology is measured by considering the volume of the acid solution U_{AS}, volume of the base solution V_{AS} and cumulative concentration of the base solution V_{BS}.

$$U\left(IV\right)=\frac{U_{AS}*V_{AS}}{V_{BS}}$$

A high-level view of the fuzzy soft set enabled biotechnology is shown in Figure 5. The fuzzy soft set undergoes the stages like fuzzification, mapping fuzzy set to soft set and production of fuzzy soft set rules. These rules are fed as input to the biotechnology process which consists of three steps likes reading the input, applying it to the application fields and generation of output in terms of product, services, and processes (Frohling and Hiete 2020).

The application of fuzzy soft set based biotechnology is found to be good in several aspects which includes fingerprinting of DNA, early identification of diseases, extraction of DNA structure, purifying the DNAs from cells, development of biodegradable products, genetically modification of crops, determining the usage of pesticides, reducing toxic substances in waste water management, production of regenerative medicines, development of prognostic systems, electrophoresis of DNA, exploration of bacteria features, identifying anti-parallel nature of DNA, spread of virus and diseases, fermentation of food, manipulation of genetically related information, reinforcement of biological macromolecules, genetic testing of forensics, and so on (Sarala and Deepa 2020).

6. Performance Analysis of The Use of Fuzzy Soft Set in Handling Uncertainty in Various Kinds of Engineering Domains

The performance achieved by the use of the fuzzy soft set in handling uncertainty in various kinds of engineering domains towards the performance metrics like response time, throughput, resource utilization, and error rate is given in Table 1 (Shanthi, Kalyani and Devie 2020). It is observed from the Table 1 that the performance achieved by the engineering domains like computer science and biotechnology is high after the application of the fuzzy soft set for handling the parameter and system uncertainties among the five engineering domains considered for evaluation purpose i.e. computer science, chemical engineering, mechanical engineering, electrical engineering, and biotechnology. Potential advantages offered by fuzzy soft set in computer science and biotechnology branches are soft decision making ability is very high, uncertainty is handled efficiently by applying rough approximation theory, missing and unclear information is managed by drawing soft boundaries, size of the input parameters get reduced through multiple levels of parameterization, exact representation of real-world scenario through fuzzy soft set formulation, and so on.

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Table 1. Performance analysis of the use of fuzzy soft set in handling uncertainty in various kinds of engineering domains

Engineering domains	Response time	Throughput	Resource utilization rate	Error rate
Computer science engineering	Low	High	Medium	Low
Chemical engineering	High	Medium	Low	Medium
Mechanical Engineering	High	High	Medium	Medium
Electrical engineering	Low	Low	Medium	Medium
Biotechnology	Low	High	High	Low

FUTURE RESEARCH DIRECTIONS

In the chapter a generic representation of the fuzzy soft set enabled engineering domains are discussed. However customized representation of the fuzzy soft set in addressing the specific performance issues in particular engineering domains can be developed. Novel architecture and algorithms to support the customized approach fuzzy soft set in handling the uncertainty issues in uncertainty prone engineering domains like biotechnology and computer science and engineering can be proposed.

CONCLUSION

This chapter gives a brief introduction to the fuzzy soft set and its applications in different engineering domains. Some of the potential engineering domains include computer science and engineering, chemical engineering, mechanical engineering electrical engineering, and biotechnology. After applying fuzzy soft set theory to the above-mentioned engineering domains, their performance is analyzed towards the performance metrics like response time, throughput, resource utilization rate, and error rate. Among the five engineering domains considered by applying the fuzzy soft set theory to handle uncertainty the performance of the engineering domains i.e. Computer science and engineering and Biotechnology is found to be good.

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Chapter 15

An Intuitionistic Fuzzy Entropy Measure and Its Application in Multi-Attribute Decision Making With Incomplete Weights Information

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ABSTRACT

Fuzzy sets, as well as their extension intuitionistic fuzzy sets (IFS), are more effective and appealing tools for expressing quantitative complexity throughout the decision-making process, and they have gotten greater attention from researchers in recent years for new study directions. Keeping the advantages of IFS, this chapter proposes a novel information measure to measure the fuzziness of IFS known as entropy measure (EM). The various notable features of the proposed EM are also presented. EM is a very useful tool to determine the attribute's weight during the multi-attribute decision-making (MADM) process. Therefore, in this chapter, the authors propose a new MADM approach by using the proposed EM where the attribute's weights are absolutely unknown. Finally, the proposed MADM methodology is applied to solve the real-life MADM examples. Comparative studies are also given to show the advantages of the proposed MADM methodology. The proposed MADM methodology can overcome the weaknesses of the existing MADM approaches.

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INTRODUCTION

In the real world, every day, humans like as engineers, doctors, lawyers, scientists, or HR managers etc. face the various types of problems to complete their task successfully. In these problems to complete the task, the main important problem is to take the appropriate decision about the task. Appropriate decisions are taken to make the task easy and to find the optimal results, and it is an integral part of daily life. Decision-making is the most difficult problem for humans. Indeed, in ordinary life, "to do or not to do" is one of the foremost riddles that a person faces before jumping to action. In the real-world, for solving decision-making problems, the first main difficulty for decision maker(s) is to choose the environment to the give the assessments for performance towards the attributes of the alternatives. This aspect makes the problem more complex and uncertain for decision-makers, and can not provide their assessments in the form of crisp numbers. To remove such types of difficulties of the decision-makers Zadeh (Zadeh, 1965) introduced the concept of the fuzzy set (FS), and after that extensions of it like as intuitionistic FS (IFS) (Atanassov, 1986) have been proposed. Under the IFSs environment, the various authors (Garg & Kumar, 2019; C.-Y. Wang & Chen, 2017; Xu, 2007; Zeng et al., 2019) solve the decision-making (DM) issues.

In DM problems, multi-attribute decision-making (MADM) problems are usually use in daily-life. During the MADM process, attribute's weights directly affect the ranking orders (ROs) of alternatives. Due to the complexity of decision time, DMks cannot assign the weights for attributes. To manage this, entropy measure (EM) is an effective tool which measure the uncertainty of any data set. In last decade, EMs for the IFSs are introduced by the many researchers (Garg et al., 2017; Garg & Kaur, 2018; Liu & Ren, 2014; Szmidt & Kacprzyk, 2001; Verma & Sharma, 2013; J.-Q. Wang & Wang, 2012; Wei et al., 2012; Xu, 2005). Garg & Kaur (2018) EM is based on the two parameters R and S where either $R>1$, $0<S<1$ or $S>1$, $0<R<1$. During the MADM process, these parameters directly affect the ROs of the alternatives. In some cases, we get the different-different ROs of the alternatives as mentioned in ***Example 1*** and ***Example 2***. Therefore, there is a need of a new entropy measure that gives a unique ROs of the alternatives.

Keeping the disadvantages of the Garg & Kaur (2018) EM, in this chapter, a new EM to depict the uncertainty of the IFS has been proposed. The legality and validity of the proposed EM has been described by proofing some certain features of EM. The primary goal of defining the EM is to measure the uncertainty of IFS and to find out the weights of attribute's during the MADM process when they are absolutely unknown. Afterwards, based on the proposed EM, we establish a new MADM framework in the IFSs environment. Real-life illustrative examples are used to evaluate the developed MADM framework, and ROs are compared to current strategies to show the proposed strategy's benefits. The proposed MADM methodology can tackle the shortcomings of the MADM methods discussed in (Garg, 2019), (Garg & Kaur, 2018) and (G. Wang et al., 2018) which include the inability to classify the ROs of alternatives in certain scenarios.

This chapter is sorted out as: - A literature review about the IFS is given in Section "**LITERATURE REVIEW**". Section "**PRELIMINARIES**" reviews basic concepts of IVIFSs theory. In section "**PROPOSED ENTROPY MEASURE FOR IFS"**, an EM has been introduced for the IFSs, and defined some properties and axiom definitions. Section "**PROPOSED MADM APPROACH"** describes the proposed MADM method. In Section "**ILLUSTRATIVE EXAMPLE"**, an example has been created to demonstrate the proposed MADM technique. Section "**ADVANTAGES OF THE PROPOSED MADM APPROACH"** lists the benefits of the proposed MADM process. Section "**VALIDITY TEST**

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OF THE PROPOSED MADM METHOD" show the validity and feasibility of the proposed MADM method. Finally, Section **CONCLUSION** brings the chapter to a close.

LITERATURE REVIEW

Recently, intuitionistic fuzzy theory widely used in different discipline like as artifical intelligence, computation intelligence, decision-making (DM) etc. In MADM, Szmidt & Kacprzyk (2001) introduced the EM to quantify the uncertainty of the IFSs. The EM by using the cosine and cotangent functions for IFSs was introduced by Wei et al (2012), and Wang & Wang (2012) respectively. Exponential function-based EM for IFSs was presented by Verma & Sharma (2013). Furthermore, none of the existing Ems of the IFSs contain the degree of hesitancy. Other than these, a few shortcomings of the above existing EM of IFS were investigated by Liu & Ren (2014) and developed a new EM by containing the IFS hesitance degree. Garg et al. (2017) proposed generalized EM of degree β and order α. Garg & Kaur (2018) highlights the weakness of the existing EMs and proposed anew (R, S)-Norm novel EM for IFS.

G. Wang et al. (2018) defined the knowledge measure for IFS and MADM method based on it. Garg & Kumar (2018b) introduced the possibility degree measure for IFS and MADM. Kumar & Chen (2021) presented the improved Einstein aggregation operator for aggregating IFNs. Garg (2019) defined the Hamacher AO for aggregating IFNs. Li et al. (2017) defined the modified VIKOR method for IFSs. Zou et al. (2020) proposed a MADM method based on an improved geometric operator of IFVs. Joshi & Kumar (2018a) defined the parametric entropy based MADM method for IFS environment. Joshi & Kumar (2018b) defined the MADM method based on the R-S norm entropy. Garg & Kumar (2018a) defined the similarity measure for IFSs based on the set pair analysis (SPA) theory. Kumar & Garg, (2017) defined the TOPSIS method for IFSs based on the SPA theory.

PRELIMINARIES

Definition 2.1 (Atanassov, 1986). An IFS A in the universe of discourdse X is represented by

$$A = \left\{ \left\langle x_t, \tau_A\left(x_t\right), \theta_A\left(x_t\right) \right\rangle \mid x_t \in X \right\},$$

where $\tau_A\left(x_t\right)$ and $\theta_A(x_t)$ denote the membership grade and the non-membership grade of element x_t belonging to the IFS A, respectively, $x_t \in X$,

$$0 \le \tau_A\left(x_t\right) \le 1,\ 0 \le \theta\left(x_t\right) \le 1 \ \text{ and } \ 0 \le \tau_A\left(x_t\right) + \theta\left(x_t\right) \le 1.$$

The hesitant degree $\pi_A(x_t)$ of element x_t belonging to the IFS A is defined as

$$\pi_A\left(x_t\right) = 1 - \tau_A\left(x_t\right) - \theta_A\left(x_t\right),$$

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where $0 \leq \pi_A(x_t) \leq 1$ and $x_t \in X$.

In (Garg & Kumar, 2017), Xu called the the pair $\left\langle \tau_A\left(x_t\right),\ \theta_A\left(x_t\right)\right\rangle$ in the IFS

$$A = \left\{x_t, \tau_A\left(x_t\right), \theta_A\left(x_t\right) | x_t \in X\right\}$$

an IFN, where $0 \leq \tau_A\left(x_t\right) \leq 1$, $0 \leq \theta_A(x_t) \leq 1$, $0 \leq \tau_A\left(x_t\right) + \theta_A\left(x_t\right) \leq 1$ and $x_t \in X$.

Definition 2.2 (Garg & Kumar, 2018b). Let $\gamma = \left\langle \tau, \theta \right\rangle$ be an IFV, where $0 \leq \tau \leq 1$, $0 \leq \theta \leq 1$ and $0 \leq \tau + \theta \leq 1$. The score value $S(\gamma)$ of the IFV $\gamma = \left\langle \tau, \theta \right\rangle$ obtained by the score function S is defined as

$$S\left(\gamma\right) = \tau - \theta, \tag{1}$$

where $S(\gamma) \in [-1,1]$.

Definition 2.3 (Hong & Choi, 2000). Let $\gamma = \left\langle \tau, \theta \right\rangle$ be an IFV, where $0 \leq \tau \leq 1$, $0 \leq \theta \leq 1$ and $0 \leq \tau + \theta \leq 1$. The accuracy value $H(\gamma)$ of the IFV $\gamma = \left\langle \tau, \theta \right\rangle$ is defined as

$$H\left(\gamma\right) = \tau + \theta, \tag{2}$$

where $H(\gamma) \in [0,1]$.

Definition 2.4. For any two IFVs $\gamma 1$ and $\gamma 2$

(i) If $S(\gamma_1) > S(\gamma_2)$, then $\gamma_1 \succ \gamma_2$.
(ii) If $S(\gamma_1) < S(\gamma_2)\ S\left(\gamma_1\right) < S\left(\gamma_2\right)$, then $\gamma_1 \prec \gamma_2$.
(iii) If $S(\gamma_1) = S(\gamma_2)$, then
  a) If $H(\gamma_1) > H(\gamma_2)$, then $\gamma_1 \succ \gamma_2$.
  b) If $H(\gamma_1) < H(\gamma_2)$, then $\gamma_1 \prec \gamma_2$.
  c) If $H(\gamma_1) = H(\gamma_2)$, then $\gamma_1 = \gamma_2$.

Definition 2.5. For the IFS $A = \left\{\left\langle x_t, \tau_A\left(x_t\right), \theta_A\left(x_t\right)\right\rangle | x_t \in X\right\}$, Garg & Kaur (2018) define the following entropy measure:

$$E'\left(A\right) = \frac{R \times S}{n\left(R - S\right)} \sum_{t=1}^{n} \left[\left(\tau_A^S\left(x_t\right) + \theta_A^S\left(x_t\right) + \pi_A^S\left(x_t\right)\right)^{\frac{1}{S}} - \left(\tau_A^R\left(x_t\right) + \theta_A^R\left(x_t\right) + \pi_A^R\left(x_t\right)\right)^{\frac{1}{R}}\right],$$

where, R and S are two parameters which are either $R>1$, $0<S<1$ or $S>1$, $0<R<1$.

Since EM, Garg & Kaur (2018) EM is based on the two parameters R and S where either $R>1$, $0<S<1$ or $S>1$, $0<R<1$. During the MADM process, these parameters directly affect the ROs of the alternatives.

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In some cases, we did not get the unique ROs of the alternatives as mentioned in ***Example 1*** and ***Example 2***. Therefore, there is a need of a new entropy measure that gives a unique ROs of the alternatives.

PROPOSED ENTROPY MEASURE FOR IFS

This segment introduced the EM for IFS. Let $\varphi(X)$ is the gathering of all IFS.

Definition 3.1. (Garg & Kaur, 2018) If $A \in \varphi(X)$, then EM $E: \varphi(X) \to [0,1]$ fulfils the subsequent properties:

(P1) $E(A) = 0 \Leftrightarrow \tau_A(x_t) = 1 \text{ or } \theta_A(x_t) = 1$
(P2) $E(A) = 1 \Leftrightarrow, \pi_A(x_t) = 1 \quad \forall x_t \in X$;
(P3) $E(A) = E(A^c)$;
(P4) If $A_1 \leq A_2$ then $E(A_1) \leq E(A_2)$.

Definition 3.2. For a IFS $A = \left\{\left\langle x_t, \tau_A(x_t), \theta_A(x_t)\right\rangle \mid x_t \in X\right\}$, we define the following entropy measure:

$$E(A) = \frac{1}{3n}\sum_{t=1}^{n}\left(\sqrt{\tau_A(x_t)\theta_A(x_t)} + 2\pi_A(x_t) + \sqrt{\left(1-\tau_A(x_t)\right)\left(1-\theta_A(x_t)\right)}\right), \tag{4}$$

where $q \geq 1$.

Theorem 3.1. The EM $E(A)$ for IFS A defined in Eq. (4) satisfies (P1)–(P4) properties as given in Definition **3.1**.

Proof. Let $A = \left\{\left\langle x_t, \tau_A(x_t), \theta_A(x_t)\right\rangle \mid x_t \in X\right\}$ be a IFS.

(P1) We have

$E(A) = 0$

$$\Leftrightarrow \frac{1}{3n}\sum_{t=1}^{n}\left(\sqrt{\tau_A(x_t)\theta_A(x_t)} + 2\pi_A(x_t) + \sqrt{\left(1-\tau_A(x_t)\right)\left(1-\theta_A(x_t)\right)}\right) = 0$$

$$\Leftrightarrow \sqrt{\tau_A(x_t)\theta_A(x_t)} = 0, \pi_A(x_t) = 0 \text{ and } \sqrt{\left(1-\tau_A(x_t)\right)\left(1-\theta_A(x_t)\right)} = 0$$

$$\Leftrightarrow \pi_A(x_t) = 0, \tau_A(x_t) = 0 \text{ or } \pi_A(x_t) = 0, \theta_A(x_t) = 0$$

(P2) Since $\sqrt{xy}$ achieves its maximum value $\frac{x+y}{2}$ when $x=y$, therefore, we have

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$E(A) = 1$

$$\Leftrightarrow \frac{1}{3n}\sum_{t=1}^{n}(\sqrt{\tau_A(x_t)\theta_A(x_t)}+2\pi_A(x_t)+\sqrt{(1-\tau_A(x_t))(1-\theta_A(x_t))}=1$$

$$\Leftrightarrow \sum_{t=1}^{n}(\sqrt{\tau_A(x_t)\theta_A(x_t)}+2\pi_A(x_t)+\sqrt{(1-\tau_A(x_t))(1-\theta_A(x_t))}=3n$$

$$\Leftrightarrow n\theta_A(x_t)+2n\pi_A(x_t)+n(1-\tau_A(x_t))=3n$$

$\Leftrightarrow 2n\pi_A(x_t) = 2n$

$\Leftrightarrow \pi_A(x_t) = 1.$

(P3) Since $A^c = \{\langle x_t, \theta_A(x_t), \tau_A(x_t)\rangle \mid x_t \in X\}$, therefore we have

$$E(A) = \frac{1}{3n}\sum_{t=1}^{n}(\sqrt{\tau_A(x_t)\theta_A(x_t)}+2\pi_A(x_t)+\sqrt{(1-\tau_A(x_t))(1-\theta_A(x_t))}$$

$$= \frac{1}{3n}\sum_{t=1}^{n}(\sqrt{\theta_A(x_t)\tau_A(x_t)}+2\pi_A(x_t)+\sqrt{(1-\theta_A(x_t))(1-\tau_A(x_t))}$$

$= E(A^c)$

(P4) Construct the function $f(x,y) = \sqrt{xy} + 2(1-x-y) + \sqrt{(1-x)(1-y)}$, where $x,y \in [0,1]$ and $x+y \le 1$. Now we'll show that when $x \le y$, the function $f(x,y)$ increases with x and decreases with y. The partial derivatives of $f(x,y)$ in terms of x and y can be calculated as follows:

$$\frac{\partial f(x,y)}{\partial x} = \frac{1}{2}\sqrt{\frac{y}{x}} - \frac{1}{2}\sqrt{\frac{1-y}{1-x}} - 2$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{1}{2}\sqrt{\frac{x}{y}} - \frac{1}{2}\sqrt{\frac{1-x}{1-y}} - 2$$

Since $\frac{\partial f(x,y)}{\partial x} \le 0, \frac{\partial f(x,y)}{\partial x} \le 0,$ when $x \le y$ therefore, $f(x,y)$ is increasing with x and decreasing with y. Thus,

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$$f\left(\tau_1\left(x_t\right),\theta_1\left(x_t\right)\right)\leq f\left(\tau_2\left(x_t\right),\theta_2\left(x_t\right)\right)$$

when $\tau_2\left(x_t\right)\leq\theta_2\left(x_t\right)$ and $\tau_1\left(x_t\right)\leq\tau_2\left(x_t\right),\theta_1\left(x_t\right)\geq\theta_2\left(x_t\right)$.

Similarly, $\frac{\partial f\left(x,y\right)}{\partial x}\leq 0,\frac{\partial f\left(x,y\right)}{\partial x}\geq 0$, when $x\geq y$ therefore, $f(x,y)$ is decreasing with x and increasing with y. Thus,

$$f\left(\tau_1\left(x_t\right),\theta_1\left(x_t\right)\right)\leq f\left(\tau_2\left(x_t\right),\theta_2\left(x_t\right)\right)$$

when $\tau_2\left(x_t\right)\leq\theta_2\left(x_t\right)$ and $\tau_1\left(x_t\right)\geq\tau_2\left(x_t\right),\theta_1\left(x_t\right)\leq\theta_2\left(x_t\right)$.

Therefore, If $A_1\preceq A_2$ then

$$\frac{1}{3n}\sum_{t=1}^{n}f(\tau_1\left(x_t\right),\theta_1\left(x_t\right))\leq\frac{1}{3n}\sum_{t=1}^{n}f(\tau_2\left(x_t\right),\theta_2\left(x_t\right)).$$

Hence $E(A_1)\leq E(A_2)$.

PROPOSED MADM APPROACH

Adopt 'm' alternatives $A=\{A_1,A_2,\ldots,A_m\}$ and n different attributes $G=\{G_1,G_2,\ldots,G_n\}$. DMk evaluate the alternatives A_k towards the attribute G_t by using the q-ROFV $\tilde{\gamma}_{kt}=\left\langle\tilde{\tau}_{kt},\tilde{\theta}_{kt}\right\rangle$ to construct the DMx $\tilde{R}=\left(\tilde{\gamma}_{kt}\right)_{m\times n}$, where k is the +ve integer from 1 to m, and t is also +ve integer from 1 to n. The constructed DMx is shown as follows:

$$\tilde{R}=\begin{array}{c}\\A_1\\A_2\\\vdots\\A_m\end{array}\begin{array}{c}\begin{array}{cccc}G_1 & G_2 & \ldots & G_n\end{array}\\\begin{pmatrix}\tilde{\gamma}_{11} & \tilde{\gamma}_{12} & \ldots & \tilde{\gamma}_{1n}\\\tilde{\gamma}_{21} & \tilde{\gamma}_{22} & \ldots & \tilde{\gamma}_{2n}\\\vdots & \vdots & \ddots & \vdots\\\tilde{\gamma}_{m1} & \tilde{\gamma}_{m2} & \ldots & \tilde{\gamma}_{mn}\end{pmatrix}\end{array}$$

This MADM proposed technique consist of the following steps:

Step 1: By applying the Eq. (5) convert the DMx $\tilde{R}=\left(\tilde{\gamma}_{kt}\right)_{m\times n}$ into the normalized DMx $R=\left(\gamma_{kt}\right)_{m\times n}=\left(\left\langle\tilde{\tau}_{kt},\tilde{\theta}_{kt}\right\rangle\right)_{m\times n}$ as follows:

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$$\gamma_{kt} = \begin{cases} \left\langle \tilde{\tau}_{kt}, \tilde{\theta}_{kt} \right\rangle, & \text{if } G_t \text{ is a benefit type attribute} \\ \left\langle \tilde{\theta}_{kt}, \tilde{\tau}_{kt} \right\rangle, & \text{if } G_t \text{ is a cost type attribute} \end{cases} \tag{5}$$

Step 2: Obtain the EM matrix $D_E = \left(E\left(\gamma_{kt}\right)\right)_{m\times n}$ by calculating the EM for each q-ROFV given in the normalized DMx $R = \left(\gamma_{kt}\right)_{m\times n}$ *where*

$$E\left(\gamma_{kt}\right) = \frac{1}{3}\left(\sqrt{\tau_{kt}\theta_{kt}} + 2\pi_{kt} + \sqrt{\left(1-\tau_{kt}\right)\left(1-\theta_{kt}\right)}\right).$$

Step 3: Compute the weight ω_t of the attribute G_t, t=1,2,…,n, as:

$$\omega_t = \frac{1-e_t}{n-\sum_{t=n}^{n} e_t} \tag{6}$$

where $e_t = \frac{1}{m}\sum_{k=1}^{m} E\left(\gamma_{kt}\right)$ and

$$E\left(\gamma_{kt}\right) = \frac{1}{3}\left(\sqrt{\tau_{kt}\theta_{kt}} + 2\pi_{kt} + \sqrt{\left(1-\tau_{kt}\right)\left(1-\theta_{kt}\right)}\right)$$

is the EM for $\gamma_{kt} = \left\langle \tau_{kt}, \theta_{kt} \right\rangle$.

Step 4: Calculate each alternative A_k's overall output $\xi(A_k)$ where $\xi(A_k)$ is defined as follows:

$$\xi\left(A_k\right) = \sum_{t=1}^{n} w_t \times \left(1 - E\left(\gamma_{kt}\right)\right) \tag{7}$$

Step 5: Calculate the RO of the alternatives $A_1, A_2, \ldots, A_m$ corresponding to the descending value of, $\xi(A_k)$ where k=1,2,…,m

ILLUSTRATIVE EXAMPLE

In this section, we adopt an illustrative example from (Garg, 2019) to illustrate the proposed MADM method.

Example 1. A multinational company board has been decided to invest money in one of the four possible Asian markets, i.e., the Southern market (A_1), the Eastern market (A_2), the Northern market (A_3) and the local market (A_4). To achieve the target, four attributes are considered, i.e., the growth (G_1), the risk (G_2), the social–political impact (G_3) and the environmental impact (G_4). To solve this problem, the

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DMk choose the IFVs envirnoment to evaluates the attibutes of the four alternatives and get a DMx $\tilde{R}=\left(\tilde{\gamma}_{kt}\right)_{4\times4}$, shown as follows:

$$\tilde{R}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} 0.6,0.3 & 0.3,0.5 & 0.1,0.8 & 0.3,0.6 \\ 0.4,0.1 & 0.7,0.2 & 0.2,0.5 & 0.7,0.1 \\ 0.6,0.3 & 0.3,0.6 & 0.3,0.6 & 0.5,0.2 \\ 0.6,0.3 & 0.5,0.2 & 0.1,0.8 & 0.3,0.6 \end{pmatrix} \end{array}.$$

We apply the proposed method to deal with this example, shown as follows:

Step 1: Because the attributes G_1 and G_4 are benefit type attributes and because the attributes G_2 and G_3 are cost type attributes, by using Eq. (23), we get the normalized DMx $R=\left(\gamma_{kt}\right)_{4\times4}$, shown as follows:

$$\tilde{R}=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array}\begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} 0.6,0.3 & 0.5,0.3 & 0.8,0.1 & 0.3,0.6 \\ 0.4,0.1 & 0.2,0.7 & 0.5,0.2 & 0.7,0.1 \\ 0.6,0.3 & 0.6,0.3 & 0.6,0.3 & 0.5,0.2 \\ 0.6,0.3 & 0.2,0.5 & 0.8,0.1 & 0.3,0.6 \end{pmatrix} \end{array}$$

Step 2: The calculated values of the EM matrix $D_E=\left(E\left(\gamma_{kt}\right)\right)_{4\times4}$ by using the proposed EM given in Eq. (4) are given as:

$$D_E=\begin{pmatrix} 0.3845 & 0.4596 & 0.3024 & 0.3845 \\ 0.6449 & 0.3547 & 0.5162 & 0.3947 \\ 0.3845 & 0.3845 & 0.3845 & 0.5162 \\ 0.3845 & 0.5162 & 0.3024 & 0.3845 \end{pmatrix}$$

Step 3: By using the Eq. (6), calculate the attribute's weight ωt of the attribute G_t, where t=1,2,3,4 as ω_1=0.2367, ω_2=0.2457, ω_3=0.2682, and ω_4=0.2494.

Step 4: Calculate the overall performance $Q(A_k)$ by using the Eq. (7) for the alternatives A_k(k=1,2,3,4) and obtained the results as: $Q(A_1)$=0.6191, $Q(A_2)$=0.5253, $Q(A_3)$=0.5827, $Q(A_4)$=0.6052.

Step 5: Since $\xi(A_1)>\xi(A_4)>\xi(A_3)>\xi(A_2)$ therefore, $A_1 \succ A_4 \succ A_3 \succ A_2$ and hence, A_1 is the best alternative.

The experimental findings for several MADM approaches are compared in the following. Garg's MADM method (Garg, 2019) using the IFHIWA operator, Wang et al.'s MADM method (G. Wang et

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Table 1. A comparison of the ROs of the alternatives obtained by different MADM methods for Example 1

MADM methods	ROs
Garg's MADM method (Garg, 2019) using the IFHIWA operator	$A_1 \succ A_4 \succ A_3 \succ A_2$
Garg's MADM method (Garg, 2019) using the IFHIOWA operator	$A_3 \succ A_1 \succ A_2 \succ A_4$
Garg and Kaur's MADM method (Garg & Kaur, 2018) using Approach I ($R = 0.9$ and $S = 2.0$)	$A_3 \succ A_1 \succ A_2 \succ A_4$
Garg and Kaur's MADM method (Garg & Kaur, 2018) using Approach I ($R = 0.9$ and $S = 1.2$)	$A_1 \succ A_3 \succ A_2 \succ A_4$
Wang et al.'s MADM method (G. Wang et al., 2018)	$A_1 \succ A_3 \succ A_2 \succ A_4$
Kumar and Chen's MADM method (Kumar & Chen, 2021)	$A_1 \succ A_4 \succ A_3 \succ A_2$
The proposed MADM method	$A_1 \succ A_4 \succ A_3 \succ A_2$

al., 2018), Kumar and Chen's MADM (Kumar & Chen, 2021) and the proposed MADM methodology provide the same RO "$A_1 \succ A_4 \succ A_3 \succ A_2$", whereas Garg's MADM method (Garg, 2019) using the IFHIOWA operator yields the RO "$A_3 \succ A_1 \succ A_2 \succ A_4$". As a result, we can't get the unique RO using Garg's MADM technique (Garg, 2019). By using Approach I (R = 0.9 and S = 1.2) of Garg and Kaur's MADM method (Garg & Kaur, 2018) gets the RO "$A_1 \succ A_3 \succ A_2 \succ A_4$". While, by using Approach I (R = 0.9 and S = 2.0) of Garg and Kaur's MADM method (Garg & Kaur, 2018) gets the RO "$A_3 \succ A_1 \succ A_2 \succ A_4$". The values of parameters R and S are not fixed in Garg and Kaur's MADM technique (Garg & Kaur, 2018). The values of parameters R and S are either R>1, 0<S<1 or S>1, 0<R<1. When the values of parameters R and S will vary then the RO of the alternatives can vary. The proposed MADM approach acquires the unique RO of the alternative, which can overcome the disadvantages of the existing MADM methods (Garg, 2019) and (Garg & Kaur, 2018).

ADVANTAGES OF THE PROPOSED MADM APPROACH

In the following, we consider a MADM example to show that the proposed MADM method can overcome the shortcomings of Garg's MADM method (Garg, 2019), Garg and Kaur's MADM method, and Wang et al.'s MADM method (G. Wang et al., 2018), which have the drawbacks that they cannot distinguish the ROs of alternatives or cannot find the unique ROs of alternatives in some situations.

Example 2. Let A_1, A_2 and A_3 be three alternatives and let G_1, G_2, G_3 and G_4 be four attributes. Assume that the DMk evaluates the alternatives towards the attributes by using IFVs to get a DMx $\tilde{R} = \left(\tilde{\gamma}_{kt}\right)_{3\times 4}$, shown as follows:

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$$\tilde{R} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.4 \rangle & \langle 0.3, 0.5 \rangle & \langle 1, 0 \rangle & \langle 0.1, 0.8 \rangle \\ \langle 0.5, 0.2 \rangle & \langle 1, 0 \rangle & \langle 0.4, 0.5 \rangle & \langle 0.6, 0.3 \rangle \\ \langle 0.3, 0.6 \rangle & \langle 0.4, 0.6 \rangle & \langle 0.5, 0.2 \rangle & \langle 1, 0 \rangle \end{pmatrix} \end{array}$$

We apply the proposed MADM method to deal with this example, shown as follows:

Step 1: Since all the attributes are benefit type attributes, the normalization process is not needed to be performed by the proposed MADM method. Therefore, we have the normalized DMx $R = \left(\gamma_{kt}\right)_{3\times4} = \left(\tilde{\gamma}_{kt}\right)_{3\times4}$.

Step 2: The calculated values of the EM matrix $D_E = \left(E\left(\gamma_{kt}\right)\right)_{4\times4}$ by using the proposed EM given in Eq. (4) are given as:

Step 3: By using the Eq. (6), calculate the attribute's weight ωt of the attribute G_t, where t=1,2,3,4 as $\xi(A_1)$=0.6882, $\xi(A_2)$=0.6908, $\xi(A_3)$=0.7063.

Step 4: Calculate the overall performance $\xi(A_k)$ by using the Eq. () for the alternatives A_k(k=1,2,3) and obtained the results as: $\xi(A_1)$=0.6882, $\xi(A_2)$=0.6908, $\xi(A_3)$=0.7063.

Step 5: Since $\xi(A_3)>\xi(A_2)>\xi(A_1)$ therefore $A_3 \succ A_2 \succ A_1$, and hence, A_3 is the best alternative.

The experimental results for several MADM approaches are compared in the following section. From Table 2, we can see that Wang et al.'s MADM method (G. Wang et al., 2018), Garg's MADM method (Garg, 2019) using the IFHIWA operator and the IFHIOWA operator get the same RO "A1=A2=A3". In this scenario, they have the shortcoming of being unable to identify the RO among the alternatives A1, A2 and A_3 in this situation. In the Garg's MADM method (Garg, 2019) Step 4 **provide** the complexity to the DMk to choose the operator, the operator is not fixed to aggregate the information. DMk can choose either IFHIWA or IFHIOWA operator depending on the scenario, although selecting the suitable operator can be difficult at those times. By using Approach I (R = 0.9 and S = 2.0) of Garg and Kaur's MADM method (Garg & Kaur, 2018) gets the RO "$A_2 \succ A_3 \succ A_1$". While, by using Approach I (R = 0.9 and S = 1.2) of Garg and Kaur's MADM method (Garg & Kaur, 2018), Kumar and Chen' MADM method (Kumar & Chen, 2021) and proposed MADM get the same RO "$A_3 \succ A_2 \succ A_1$". The proposed MADM method can overcome the drawbacks of Garg's MADM method (Garg, 2019) and Wang et al's MADM method (G. Wang et al., 2018) which are unable to distinguish the RO among the alternatives A1, A2 and A3 in this situation. In Garg and Kaur's MADM method (Garg & Kaur, 2018), it is very difficult to choose the values of the parameters R and S according to the situation which create a complexity to the DMk. While the proposed MADM method is very easy to understand and obtain the RO of the alternative.

VALIDITY TEST OF THE PROPOSED MADM METHOD

In order to assess the validity of DM approaches, (X. Wang & Triantaphyllou, 2008) developed following test criteria.

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Table 2. A comparison of the ROs of the alternatives obtained by different MADM methods for Example 2

MADM methods	Ros
Garg's MADM method (Garg, 2019) using the IFHIWA operator	$A_1=A_2=A_3$
Garg's MADM method (Garg, 2019) using the IFHIOWA operator	$A_1=A_2=A_3$
Garg and Kaur's MADM method (Garg & Kaur, 2018) using Approach I (R = 0.9 and S = 1.2)	$A_3 \succ A_2 \succ A_1$
Garg and Kaur's MADM method (Garg & Kaur, 2018) using Approach I (R = 0.9 and S = 2.0)	$A_2 \succ A_3 \succ A_1$
Wang et al's MADM method (G. Wang et al., 2018)	$A_1=A_2=A_3$
Kumar and Chen's MADM method (Kumar & Chen, 2021)	$A_3 \succ A_2 \succ A_1$
The proposed MADM method	$A_3 \succ A_2 \succ A_1$

Test criterion 1: "A successful DM approach does not adjust the index of the best alternative by replacing a non-optimal alternative with a worse alternative without changing each decision attribute's corresponding significance."

Test criterion 2: "To achieve an efficient DM process transitive properties must be satisfied".

Test criterion 3: "When we have decomposed a DM problem into the sub-DM problems and the same DM approach is used to rank alternatives on subproblems, the cumulative ranking of alternatives must be equal to the ranking of un-composed DM issue."

Validity of the developed MADM approach is tried by utilizing these criterions on the ***Example 1*** as follows:

Validity test by test criterion 1: Under test criterion 1, we change the non-optimal alternative A_2 by the more terrible self-assertive $A_2^{'}$ of ***Example 1***'s DMx $\tilde{R}$ and summarized in new DMx $\tilde{R}^{'}$ as follows:

$$\tilde{R}' = \begin{array}{c} \\ A_1 \\ A_2' \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \end{array} \\ \begin{pmatrix} \langle 0.6, 0.3 \rangle & \langle 0.3, 0.5 \rangle & \langle 0.1, 0.8 \rangle & \langle 0.3, 0.6 \rangle \\ \langle 0.3, 0.3 \rangle & \langle 0.5, 0.3 \rangle & \langle 0.2, 0.6 \rangle & \langle 0.3, 0.6 \rangle \\ \langle 0.6, 0.3 \rangle & \langle 0.3, 0.6 \rangle & \langle 0.3, 0.6 \rangle & \langle 0.3, 0.6 \rangle \\ \langle 0.6, 0.3 \rangle & \langle 0.5, 0.2 \rangle & \langle 0.1, 0.8 \rangle & \langle 0.3, 0.6 \rangle \end{pmatrix} \end{array}$$

Now, by applying the proposed MADM method on the above DMx $\tilde{R}$, we get the ROs of the alternatives as $A_1 \succ A_4 \succ A_3 \succ A_2$. Hence, the most excellent alternative remains A_1 and it approve the test criterion 1.

Validity test by test criteria 2 and 3: In order to analyse the proposed MADM solution under test criteria 2 and 3, we decomposed the original MADM problem into three sub-MADM problems comprising the alternatives "A_1,A_2,A_4", "A_1,A_3,A_4", and "A_2,A_3,A_4". If we utilized the proposed MADM approach to these sub-MADM problems, then we get the ROs of alternatives positioned as $A_1 \succ A_4 \succ A_2$, $A_1 \succ A_4 \succ A_3$ and $A_4 \succ A_3 \succ A_2$ respectively. After combining the rankings, we get the ranking of these sub-problems

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as $A_1 \succ A_4 \succ A_3 \succ A_2$. This is identical to the undecomposed MADM problem rating and demonstrates the transitive property. Therefore, the established MADM method meets the test criteria 2 and 3 conditions.

CONCLUSION

IFS is the most commonly and powerful tool to express the uncertainty of the data. Therefore, whole objective of this chapter works under the IFSs environment for handling the MADM issues. For this, we have constructed an entropy measure (EM) for measuring the fuzziness of the IFSs which will be helpful for the DMk to measure the uncertainty and obtain the attribute weight of the MADM issues. Attribute's weight acting a fundamental role through the whole MADM process and directly affect the ROs of alternatives. Therefore, based on the proposed EM, a MADM method under the IFSs environment and utilized to handle the realistic cases to exhibit the efficiency of the developed MADM approach. Based on the computed results, it can be decided that the proposed framework of MADM can resolve the shortcomings of current MADM techniques given in (Garg, 2019) and (Garg & Kaur 2018). The proposed MADM method gives a practical strategy to dealing with MADM concerns in the context of IFSs. We will create additional applications in other disciplines in the future, such as reliability analysis, engineering, military, medical diagnostics, pattern recognition, and so on.

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Chapter 16
Evaluation of the Global Competitiveness Index (GCI) by Multi-Criteria Decision-Making Methods Based on Intuitionistic Fuzzy Sets: Comparative Analysis

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ABSTRACT

Fuzzy set theory proposed by Zadeh is introduced to cope with the imprecision and uncertainty. Intuitionistic fuzzy sets (IFS) introduced by Atanassov are a generalization of fuzzy sets characterized by a membership function and a non-membership function. Various multi-criteria decision-making (MCDM) methods based on IFSs have been proposed. Global competitiveness index (GCI) provides information about the competitiveness of the countries based on data collected by the World Economic Forum in a report. Competitiveness of the country is the economic strength necessary for an economy to upgrade its economic prosperity and standard of living. In the report, three main criteria—factor, efficiency, and innovation—and related 12 sub-criteria are covered for the evaluation of the countries. In this chapter, MCDM methods of TOPSIS, WASPAS, and COPRAS based on IFSs are applied to data obtained from the GCI report, and ranking of the countries is obtained to investigate relationship.

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INTRODUCTION

Multiple criteria decision making (MCDM) is the most well-known branch of decision making. In the MCDM problems, an appropriate alternative among a finite number of feasible alternatives in the presence of multiple, generally conflicting criteria are chosen. MCDM problems are classified into two categories: multi-objective decision making (MODM) and multi-attribute decision making (MADM), depends on the domain of the alternatives, i.e. continuous or discrete. In the MADM, problems with discrete domain whose number of alternatives has been predetermined in order to select/prioritize/rank a finite number of alternatives are handled. On the other hand, problems with decision variables that are determined in a continuous/integer domain with either an infinitive or a large number of alternatives to satisfy the decision maker's constraints and preference priorities are considered in the MODM (Rao, 2013). A typical MCDM method includes *i*) determining the number of criteria for the problem, *ii*) collecting the suitable data/information in which the preferences of DMs (i.e., construction of the preferences), *iii*) constructing a set of possible alternatives for attaining the goal (i.e., evaluation of the alternatives), *iv*) choosing a proper method for evaluating and ranking the alternatives (i.e., determination of the top alternative) (Tzeng and Huang, 2011). Various methods have been proposed in the field of MCDM, such as Analytic Hierarchy Process-AHP (Saaty, 1980), Technique for Order of Preference by Similarity to Ideal Solution-TOPSIS (Hwang and Yoon, 1981), COmplex PRoportional ASsessment-COPRAS (Zavadskas et al., 1994), VlseKriijumsko Optimizacijo I Kompromisno Resenje-VIKOR (Opricovic, 1998), Multi-Objective Optimization Method by Ratio Analysis-MOORA (Brauers and Zavadskas, 2006), MultiMOORA (Brauers and Zavadskas, 2010), Additive Ratio ASsessment-ARAS (Zavadskas and Turskis, 2010), Weighted Aggregated Sum Product ASsessment-WASPAS (Zavadskas et al., 2012), Evaluation based on Distance from Average Solution-EDAS (Ghorabaee et al., 2015), COmbinative Distance-based ASsessment-CODAS (Ghorabaee et al., 2016), etc.

Fuzzy set theory (FST) which was first introduced by Zadeh (1965) is a valuable tool to strengthen the comprehensiveness and reasonableness of the decision making process. Since the introduction of FST, some extensions have been developed, such as type-2 fuzzy sets, interval type-2 fuzzy sets, type-n fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, neutrosophic fuzzy sets, and hesitant fuzzy sets. Judgments and preferences of decision makers are affected by uncertainty, therefore the use of precise and crisp numbers in linguistic evaluations may not always be suitable for MCDM methods. Then, various MCDM methods based on fuzzy set theory and its extensions or generalizations have been proposed by several authors for selection, ordering and classification of the alternatives. In the literature, FMCDM methods have been comprehensively studied by various researchers and applied to different fields. Among them, intuitionistic fuzzy sets (IFS) based MCDM methods have attracted great attention by researchers and many studies have been conducted.

The Global Competitiveness Index (GCI) evaluates the performance of about 140 countries worldwide considering 12 pillars of competitiveness. The data handled in the evaluation are compiled and/or collected annually by the World Economic Forum. The GCI aims to measure national competitiveness defined as the set of institutions, policies and factors which indicates the productivity level of the countries. It helps policymakers to understand the complex and multifaceted nature of the development challenge and to set better policies based on public-private cooperation, as well as to act in a way that restores confidence in the possibilities for continued economic progress (GCI Report 2017-2018).

In this chapter, three well-known MCDM methods, TOPSIS, WASPAS and COPRAS, based on IFSs are applied to the data from the GCI report in order to rank the countries for examining competitiveness

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of them. TOPSIS, COPRAS, and WASPAS are preferred among other MCDM approaches because they are suitable for the problems including a great number of criteria and alternatives and are easy-to-understand methods. They are easily programmable due to their simple computational steps and also have comparative consistency in ranking the alternatives.

BACKGROUND

The fuzzy set theory first proposed by Zadeh (1965) is introduced to cope with the imprecision and uncertainty in decision making processes by using linguistic terms and membership degrees. In the study, a fuzzy set is explained as a class of objects with membership degrees and basic definitions on fuzzy sets are given. Intuitionistic Fuzzy set (IFS) introduced by Atanassov (1986) is a generalization of fuzzy set characterized by a membership function and a non-membership function.

In recent years, various multi-criteria decision making methods (MCDMs) based on fuzzy sets, type-2 fuzzy sets, hesitant fuzzy sets, and intuitionistic fuzzy sets have been proposed. MCDMs such as TOPSIS, ELECTRE, WASPAS, EDAS, CODAS, COPRAS based on IFSs have been introduced and applied to the supplier selection, personnel selection, facility location selection, and etc.

Atanassov et al. (2005) dealt with the elements of intuitionistic fuzzy sets with a membership degree and a non-membership degree whose sum is less than or equal to 1. Intuitionistic fuzzy interpretations of multi-person and multi-criteria decision-making processes are discussed in the study. Lin et al. (2007) presented a method dealing with MCDM problems based on intuitionistic fuzzy sets. This method allows the availability and non-achievement of each alternative according to the set of criteria expressed by intuitionistic fuzzy sets. Liu and Wang (2007) presented new methods for solving MCDM problem under intuitionistic fuzzy environment. A set of score functions for MCDM problem based on intuitionistic fuzzy point operators are defined. Boran et al. (2009) proposed the TOPSIS method based on intuitionistic fuzzy sets to select the appropriate supplier in a group decision-making environment. The intuitionistic fuzzy weighted average (IFWA) operator was used in the study. Boran et al. (2012) applied intuitionistic fuzzy TOPSIS method to evaluate renewable energy technologies for electricity generation in Turkey. Pei and Zheng (2012) proposed a new approach to solve MCDM problems in the intuitionistic fuzzy environment. This approach is based on a new ranking method for intuitionistic fuzzy sets. Hajiagha et al. (2013) introduced an algorithmic scheme for the COPRAS-IVIF method, which is an extended form of the COPRAS method used for group decision-making problems based on interval-valued intuitionistic fuzzy sets. This method is examined on two numerical examples. Joshi and Kumar (2014) proposed the intuitionistic fuzzy TOPSIS method based on distance measure and intuitionistic fuzzy entropy. Büyüközkan and Güleryüz (2016) proposed an integrated intuitionistic fuzzy group decision making model that includes IF-TOPSIS, IF-AHP and intuitionistic Fuzzy Analytical Technique. IF-AHP is used to determine the weights of the criteria, while the IF-TOPSIS method is used for ranking. Zavadskas et al. (2014) proposed the interval-valued intuitionistic fuzzy WASPAS (WASPAS-IVIF) method, which can be applied in uncertain decision-making environments. This method has been applied to the problems of reconstruction decisions of orphaned buildings and sorting of investment instruments. The results obtained were compared with the rankings obtained from TOPSIS-IVIF, COPRAS-IVIF and IFOWA methods. Wang and Chen (2017) presented a MCDM method based on interval-valued intuitionistic fuzzy sets (IVIFS). Mishra et al. (2018) used the intuitionistic fuzzy WASPAS (IF-WASPAS) method to compare the performance of cellular mobile telephony service providers in the Madhya Pradesh region

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in India. Shen et al. (2018) proposed a distance measure between intuitionistic fuzzy sets (IFS) and proved some of their properties. An intuitionistic fuzzy TOPSIS approach based on the proposed distance measure has been developed and applied to the credit risk assessment of potential strategic partners and compared it with other existing methods. Stanujkić and Karabašević (2018) introduced an extension of the WASPAS method based on intuitionistic fuzzy numbers and applied to the evaluation of a website. In the method, Hamming distance is used to rank the alternatives. Memari et al. (2019) presented the intuitionistic fuzzy TOPSIS method for selecting a sustainable supplier based on 9 criteria and 30 sub-criteria for an automotive spare parts manufacturer. Schitea et al. (2019) proposed WASPAS, COPRAS and EDAS methods based on intuitionistic fuzzy sets, in order to choose the best place for hydrogen mobility collection site selection in Romania. They compared the performance of the proposed methods with the intuitionistic fuzzy TOPSIS method. Cavallaro et al. (2019) introduced a modified intuitionistic fuzzy TOPSIS method to evaluate concentrated solar energy (CSP) technologies.

INTUITIONISTIC FUZZY SETS

Intuitionistic fuzzy set (IFS) introduced by Atanassov (1986) is a generalization of a fuzzy set. Let $X=\{x_1,x_2,\ldots,x_n\}$ be a universe of discourse set, an IFS A in X is defined by,

$$A=\left\{\left\langle x,\mu_A(\mathrm{x}),\nu_A(x)\right\rangle \middle| \, x\in X\right\} \tag{1}$$

where; μ_A: X→[0,1] represents the membership degree and ν_A: X→[0,1] represents the non-membership degree of the element x∈X to A with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$.

For any IFS A and $x\in X$,

$$\pi_A(\mathrm{x})=1-\mu_A(x)-\nu_A(x) \tag{2}$$

is called the hesitation degree of x to A and $\pi_A(x)$ expresses the lack of knowledge on whether x belongs to A or not. It is obvious that $0\le\pi_A(x)\le 1$ for every x (Atanassov, 1986; Atanassov, 1999).

Arithmetic Operations on Intuitionistic Fuzzy Sets

Let A and B are two intuitionistic fuzzy sets defined by

$A=\left\{\left\langle x,\mu_A(\mathrm{x}),\nu_A(x)\right\rangle \middle| \, x\in X\right\}$ and $B=\left\{\left\langle x,\mu_B(\mathrm{x}),\nu_B(x)\right\rangle \middle| \, x\in X\right\}$, respectively.

Equality: $A=B$ *iff* $\mu_A(x)=\mu_B(\mathrm{x}),\ \mu_A(x),\nu_A(x)=\nu_B(\mathrm{x})$ (3)

Complement: $A^C=\left\langle\nu_A(x),\mu_A(x)\right\rangle$ (4)

Summation: $A\oplus B=\left\langle\mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x),\nu_A(x)\nu_B(\mathrm{x})\right\rangle$ (5)

Multiplication: $A\otimes B=\left\langle\mu_A(x).\mu_B(x),\nu_A(x)+\nu_B(\mathrm{x})-\nu_A(x).\nu_B(\mathrm{x})\right\rangle$ (6)

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Multiplication by a scalar *k*: $\lambda \otimes A = \left\langle 1-(1-\mu_A(x))^{\lambda}, (\nu_A(x))^{\lambda} \right\rangle$ (7)

Exponentiation: $A^{\lambda} = \left\langle (\mu_A(x))^{\lambda}, 1-(1-\nu_A(x))^{\lambda} \right\rangle$ (8)

Score function: $S_{\tilde{A}} = \mu_A(x) - \nu_A(x),\ S_{\tilde{A}} \in [-1,1]$ (9)

Intuitionistic Fuzzy Weighted Arithmetic Mean (IFWA) operator of IFSs*:*

$$IFWA = \left\langle 1-\prod_{j=1}^{n}(1-\mu_{A_i}(x_j))^{w_j}, \prod_{j=1}^{n}(\nu_{A_i}(x_j))^{w_j} \right\rangle \quad (10)$$

where: w_j denote weight of *j*th element of collection, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Intuitionistic Fuzzy Weighted Geometric (IFWG) operator of IFSs:

$$IFWG = \left\langle \prod_{j=1}^{n}(\mu_{A_i}(x_j))^{w_j}, 1-\prod_{j=1}^{n}(1-\nu_{A_i}(x_j))^{w_j} \right\rangle \quad (11)$$

where: w_j denotes the weight and of *j*th element of collection, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ (Atanassov, 1986; Atanassov, 1999; Szmidt, 2014; Stanujkić and Karabašević, 2018).

Distances Between the Intuitionistic Fuzzy Sets

The most popular distances between intuitionistic fuzzy sets *A* and *B* in $X = \{x_1, x_2, \ldots, x_n\}$ are defined as follows:

The Hamming distance:

$$d_{H-IFS}(A,\mathrm{B}) = \sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|) \quad (12)$$

The normalized Hamming distance:

$$d_{NH-IFS}(A,\mathrm{B}) = \frac{1}{n}\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|) \quad (13)$$

The Euclidean distance:

$$d_{E-IFS}(A,\mathrm{B}) = \sqrt{\sum_{i=1}^{n}(\mu_A(x_i)-\mu_B(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2} \quad (14)$$

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The normalized Euclidean distance:

$$d_{NE-IFS}(A,\mathrm{B}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mu_A(x_i)-\mu_B(x_i))^2 + (\nu_A(x_i)-\nu_B(x_i))^2 + (\pi_A(x_i)-\pi_B(x_i))^2} \tag{15}$$

It is considered that; $0 \le d_{H\text{-}IFS}(A,B) \le 2n$, $0 \le d_{NH\text{-}IFS}(A,B) \le 2$, $0 \le d_{E-IFS}(A,B) \le \sqrt{2n}$, and $0 \le d_{NE-IFS}(A,B) \le \sqrt{2}$ (Szmidt and Kacprzyk, 2000; Szmidt and Kacprzyk, 2001).

MULTI-CRITERIA DECISION MAKING METHODS BASED ON INTUITIONISTIC FUZZY SETS

Fuzzy sets and their extensions have been used in multi-criteria decision making methods to better reflect uncertainties or linguistic expressions in decision makers' evaluations. MCDM methods have been extended to various types of fuzzy sets, i.e. Type-2 fuzzy sets, Intuitionistic fuzzy sets, Pythagorean fuzzy sets, and etc. In this section, multi-criteria decision making methods based on intuitionistic fuzzy sets, IF-TOPSIS, IF-COPRAS and IF-WASPAS, are investigated in detailed.

Intuitionistic Fuzzy TOPSIS Method

The TOPSIS is a well-known classical MCDM method first introduced by Hwang and Yoon (1981). The aim of the TOPSIS method is to rank the alternatives from the best to the worst, taking into account the shortest distance to the positive ideal solution as well as the farthest distance to the negative ideal solution. Various extensions of the Intuitionistic fuzzy TOPSIS method are introduced by several researchers such as Boran et al. (2009), Boran et al. (2012), Shen et al. (2018), Cavallaro et al. (2019), Rouyendegh et al. (2020).

Intuitionistic fuzzy TOPSIS procedures consist of the following steps:

Step 1: Construct the intuitionistic fuzzy decision matrix $D = \left[\left(\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j)\right)\right]$, i=1,2,…,m; j=1,2,…,n as follows:

$$D = \begin{bmatrix} \left\langle \mu_{A_1}(x_1), \nu_{A_1}(x_1), \pi_{A_1}(x_1) \right\rangle & \cdots & \left\langle \mu_{A_1}(x_n), \nu_{A_1}(x_n), \pi_{A_1}(x_n) \right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle \mu_{A_m}(x_1), \nu_{A_m}(x_1), \pi_{A_m}(x_1) \right\rangle & \cdots & \left\langle \mu_{A_m}(x_n), \nu_{A_m}(x_n), \pi_{A_m}(x_n) \right\rangle \end{bmatrix} \tag{16}$$

where; i represents alternatives (i=1,2,…,m) and j indicates criteria (j=1,2,…,n).

Step 2: Construct the intuitionistic fuzzy weighted decision matrix $D' = \left[\left(\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j)\right)\right]$, i=1,2,…,m; j=1,2,…,n, as follows:

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$$D' = \begin{bmatrix} \langle \mu_{A_1.W}(x_1), \nu_{A_1.W}(x_1), \pi_{A_1.W}(x_1) \rangle & \cdots & \langle \mu_{A_1.W}(x_n), \nu_{A_1.W}(x_n), \pi_{A_1.W}(x_n) \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{A_m.W}(x_1), \nu_{A_m.W}(x_1), \pi_{A_m.W}(x_1) \rangle & \cdots & \langle \mu_{A_m.W}(x_n), \nu_{A_m.W}(x_n), \pi_{A_m.W}(x_n) \rangle \end{bmatrix} \quad (17)$$

where:

$$\pi_{A_i.W(x)} = 1 - \nu_{A_i}(x) - \nu_W(x) - \mu_{A_i}(x).\mu_W(x) + \nu_{A_i}(x).\nu_W(x)$$

is computed by considering a set of grades of importance $W = \langle \mu_W(x), \nu_W(x) \rangle$.

Step 3: Determine the intuitionistic fuzzy positive ideal solution (A^*) and intuitionistic fuzzy negative ideal solution (A^-) as follows:

$$A^* = (\tilde{r}_1^*, \tilde{r}_2^*, \ldots, \tilde{r}_n^*),\ \tilde{r}_j^* = \left(\mu^*_{A_i.W}(x_j), \nu^*_{A_i.W}(x_j), \pi^*_{A_i.W}(x_j) \right) \quad (18)$$

$$A^- = (\tilde{r}_1^-, \tilde{r}_2^-, \ldots, \tilde{r}_n^-),\ \tilde{r}_j^- = \left(\mu^-_{A_i.W}(x_j), \nu^-_{A_i.W}(x_j), \pi^-_{A_i.W}(x_j) \right) \quad (19)$$

Where: B and C indicate the benefit criteria and cost criteria,

$$\mu^*_{A_i.W}(x_j) = \begin{cases} \max_i \{\mu_{A_i.W}(x_j)\}, j \in B \\ \min_i \{\mu_{A_i.W}(x_j)\}, j \in C \end{cases}$$

$$\nu^*_{A_i.W}(x_j) = \begin{cases} \min_i \{\nu_{A_i.W}(x_j)\}, j \in B \\ \max_i \{\nu_{A_i.W}(x_j)\}, j \in C \end{cases}$$

$$\mu^-_{A_i.W}(x_j) = \begin{cases} \min_i \{\mu_{A_i.W}(x_j)\}, j \in B \\ \max_i \{\mu_{A_i.W}(x_j)\}, j \in C \end{cases}$$

$$\nu^-_{A_i.W}(x_j) = \begin{cases} \max_i \{\nu_{A_i.W}(x_j)\}, j \in B \\ \min_i \{\nu_{A_i.W}(x_j)\}, j \in C \end{cases}$$

Step 4: Calculate the separation measures S_i^* and S_i^- of each alternative from A^* and A^- by using normalized Euclidean distance, respectively by following:

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$$S^{*}=\sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_{A_i.\mathrm{W}}(x_j)-\mu_{A^{*}\mathrm{W}}(x_j)\right)^{2}+\left(\nu_{A_i.\mathrm{W}}(x_j)-\nu_{A^{*}\mathrm{W}}(x_j)\right)^{2}+\left(\pi_{A_i.\mathrm{W}}(x_j)-\pi_{A^{*}\mathrm{W}}(x_j)\right)^{2}\right]} \tag{20}$$

$$S^{-}=\sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_{A_i.\mathrm{W}}(x_j)-\mu_{A^{-}\mathrm{W}}(x_j)\right)^{2}+\left(\nu_{A_i.\mathrm{W}}(x_j)-\nu_{A^{-}\mathrm{W}}(x_j)\right)^{2}+\left(\pi_{A_i.\mathrm{W}}(x_j)-\pi_{A^{-}\mathrm{W}}(x_j)\pi\right)^{2}\right]} \tag{21}$$

Step 5: Calculate the relative closeness coefficient of the alternatives as follows:

$$C_{i^{*}}=\frac{S_{i^{-}}}{S_{i^{*}}+S_{i^{-}}} \tag{22}$$

where; $0\leq C_{i^{*}}\leq 1$ (i=0,1,2,...,m).

Step 6: Rank the alternatives according to descending order of $C_{i^{*}}$ (Boran et al., 2012, Cavallaro et al., 2019).

Intuitionistic Fuzzy COPRAS Method

The COPRAS method introduced by Zavadskas and Kaklauskas (1996) can handle both qualitative and quantitative criteria in the evaluation and ranking of the alternatives in terms of criteria. It is very simple to use and therefore calculations can be made in a shorter time than other MCDM methods. Hajiagha et al. (2013) proposed COPRAS method based on interval-valued intiutionistic fuzzy sets.

In this section, Hajiagha et al. (2013)'s algorithm is performed by applying intiutionistic fuzzy numbers instead of interval-valued intiutionistic fuzzy numbers.

Intuitionistic fuzzy COPRAS algorithm includes the following steps:

Step 1: Construct the intuitionistic fuzzy decision matrix $D=\left[\left(\mu_{A_i}(x_j),\nu_{A_i}(x_j),\pi_{A_i}(x_j)\right)\right]$, i=1,2,…,m; j=1,2,…,n.

Step 2: Construct the intuitionistic fuzzy weighted decision matrix $D'=\left[\left(\mu_{A_i}(x_j),\nu_{A_i}(x_j),\pi_{A_i}(x_j)\right)\right]$, i=1,2,…,m; j=1,2,…,n.

Step 3: Calculate the index $\tilde{P}_i$ with regards to the benefit criteria for each alternative:

$$\tilde{P}_i=\sum_{j\in B}x'_{ij} \tag{23}$$

Step 4: Calculate the index $\tilde{R}_i$ with regards to the cost criteria for each alternative:

$$\tilde{R}_i=\sum_{j\in C}x'_{ij} \tag{24}$$

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Step 5: Determine the minimal value of $\tilde{R}_i$ with respect to the score function.

Step 6: Calculate the relative weight (Q_i) of each alternative by using score function of $s(\tilde{R}_i)$ and $s(\tilde{P}_i)$ as follows:

$$Q_i = s(\tilde{P}_i) + \frac{s(\tilde{R}_{\min})\sum_{i=1}^{m} s(\tilde{R}_i)}{s(\tilde{R}_i)\sum_{i=1}^{m} \frac{s(\tilde{R}_{\min})}{s(\tilde{R}_i)}}. \tag{25}$$

or

$$Q_i = s(\tilde{P}_i) + \frac{\sum_{i=1}^{m} s(\tilde{R}_i)}{s(\tilde{R}_i)\sum_{i=1}^{m} \frac{1}{s(\tilde{R}_i)}}. \tag{26}$$

where: $s(\tilde{R}_i)$ and $s(\tilde{P}_i)$ indicate the score function of $\tilde{R}_i$ and $\tilde{P}_i$, respectively.

Step 7: The utility degree of the each alternative is defined as follows:

$$N_i = \frac{Q_i}{Q_{\max}} 100\% \tag{27}$$

Step 8: Determine the ranking of the alternatives in descending order of Q_i or N_i (Hajiagha et al., 2013; Schitea et al., 2019).

Intuitionistic Fuzzy WASPAS Method

The WASPAS method proposed by Zavadskas et al. (2012) integrates weighted sum model (WSM) and weighted product model (WPM). Zavadskas et al. (2014) proposed WASPAS method based on interval-valued intiutionistic fuzzy sets. In the intuitionistic fuzzy WASPAS method, normalization is not necessary because the values of IFN belong to interval [0,1]. In this section, Zavadskas et al. (2014)'s algorithm is carried out by using intiutionistic fuzzy numbers instead of interval-valued intiutionistic fuzzy numbers.

Intuitionistic fuzzy WASPAS algorithm consists of the following steps:

Step 1: Construct the intuitionistic fuzzy decision matrix $D = \left[\left(\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j)\right)\right]$, i=1,2,…,m; j=1,2,…,n.

Step 2: Construct the intuitionistic fuzzy weighted decision matrix $D' = \left[\left(\mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j)\right)\right]$, i=1,2,…,m; j=1,2,…,n.

Step 3: Calculate the sum total relative importance of the alternatives based on the WSM approach by using IFWA operator given in Eq. (10) as follows:

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$$Q_i^{WSM} = \sum_{j=1}^{n} w_j x'_{ij} \tag{28}$$

Step 4: Calculate the product total relative importance of the alternatives based on the WPM approach by using IFWG operator given in Eq. (11) as follows:

$$Q_i^{WPM} = \prod_{j=1}^{n} {x'_{ij}}^{w_j} \tag{29}$$

Step 5: Calculate the overall importance of each alternative Q_{i*} as follows:

$$Q_i = 0.5 \otimes (Q_i^{WSM} \oplus Q_i^{WPM}) \tag{30}$$

Step 6: The score function is calculated for Q_{i*} values and the alternatives are ranked in descending order of Q_{i*} (Hajiagha et al., 2013, Stanujkić and Karabašević, 2018; Schitea et al., 2019).

THE GLOBAL COMPETITIVENESS REPORT

Competitiveness is defined as the set of institutions, policies, and factors that determine the level of productivity of a country. Productivity level determines prosperity level of an economy which can reach. The productivity level also identifies the rate of return from investments in an economy, which are the main drivers of growth rates. A more competitive economy means an economy that is likely to grow faster over time. There are numerous and complex determinants for competitiveness. For this aim, the World Economic Forum (WEF) is presented Global Competitiveness Index (GCI) to provide information on countries' competitiveness on the basis of data collected in a report. The competitiveness of a country is defined as the economic power necessary for an economy to raise its economic prosperity and standard of living. The GCI is calculated using three indicators of "Factor", "Efficiency" and "Innovation" indicators to assess countries. These indicators are grouped into 12 pillars as illustrated in Figure 1. These pillars are arranged in three subindexes: basic requirements, efficiency enhancers, and innovation & sophistication factors. Although the 12 pillars of competitiveness are defined separately, they are not independent, they tend not only to relate to each other but also to reinforce each other (GCI Report, 2017-2018). Readers can reach the calculation of the GCI of the countries from the link http://www3.weforum.org/docs/GCR2017-2018/05FullReport/TheGlobalCompetitivenessReport2017%E2%80%932018.pdf.

The 12 pillars of competitiveness are explained as follows:

Institutions: The institutional environment creates the framework interaction between individuals, firms, and governments to constitute income and wealth in the economy. It has a strong influence on competitiveness and growth, as it affects investment decisions and production organization.

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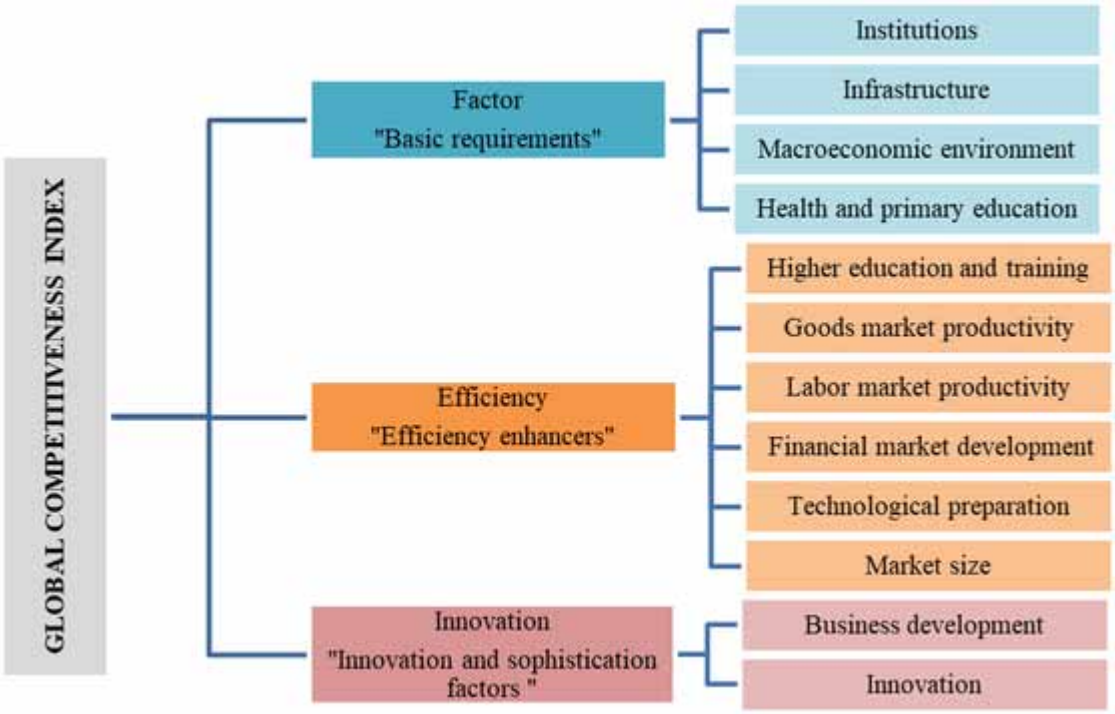
Figure 1. The 12 pillars of competitiveness



Infrastructure: Comprehensive and efficient infrastructure is a key factor of competitiveness. Since it determines the location of economic activity and the types of activity or sector that can develop in a particular economy, it is crucial to provide efficient functioning of the economy. The quality and availability of infrastructure networks affect economic growth remarkably while reducing income inequality and poverty.

Macroeconomic stability: The stable macroeconomic environment is crucial for businesses. Macroeconomic disorder harms the economy. If inflation gets out of control, firms cannot make informed decisions. When the government has to pay high interest for its past debts, it will not be able to provide services efficiently. The economy cannot grow without stable macro environment.

Health and primary education: A healthy workforce is critical for competitiveness and productivity of a country. Workers who are sick become less productive because they cannot realize their potential. Poor health, resulting from low investment in healthcare services, leads to significant costs for the business, as sick workers will often be unable to come to work or will be able to work at lower productivity levels.

Higher education and training: Quality higher education and training is essential for economies that want to expand the value chain beyond simple manufacturing processes and products. Today, due to globalizing economies, well-educated employees who can quickly adapt to the changing environments of economies are needed. In order to ensure that the skills of the employees are adapted to the changing needs of the developing economy, personnel training that includes vocational and continuous on-the-job training should be emphasized.

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Goods market efficiency: Countries with efficient goods markets can produce the right mix of products and services under supply and demand conditions, and ensure that these goods can be bought and sold in the most efficient way in the economy. Both domestic and foreign market competition is important in increasing market efficiency and thus business productivity.

Labor market efficiency: The efficiency and flexibility of the labor market is important as it ensures that employees are assigned to their most productive use in the economy and inducements are given to make the best effort in their jobs. Efficient labor markets ensure a relationship between employees' efforts and their inducements, and also the best possible equality between women and men in the work environment.

Financial market sophistication: An efficient financial sector is necessary to allocate resources to their most efficient uses, both saved by the citizens of a nation and entering the economy from abroad. It channels resources into entrepreneurship or investment projects which have the highest expected rates of return rather than those that are politically connected.

Technological readiness: Technology has become an increasingly important part for companies to compete and develop in globalizing world. Information and communication technologies (ICT) have become "general-purpose technology" as they provide effective infrastructure for critical spreads and commercial transactions to other economic sectors. ICT access and use (including the existence of an ICT-friendly regulatory framework) are key components of the overall level of technological readiness of economies.

Market size: The size of the market affects productivity, as large markets allow firms to benefit from economies of scale. In the age of globalization, international markets have become the substitutes of domestic markets, especially for small countries. Trade in general is that it has a positive impact on growth, especially for countries with small domestic markets.

Business sophistication: Business sophistication is suitable for higher efficiency in the production of goods and services in order to increase productivity for the competitiveness of the countries. It concerns both the quality of a country's overall business networks and the quality of individual firms' operations and strategies.

Innovation: Technological innovations are especially important for competitiveness of the economies because living standards can only be extended by technological innovations on the long view.

NUMERICAL EXAMPLE

The Global Competitiveness Index (GCI) provides information about competitiveness of the countries based on data collected by the World Economic Forum (WEF) in a report. In the report, three main criteria "Factor, Efficiency and Innovation" and related twelve sub-criteria are covered for the evaluation of the countries. The criteria and sub-criteria discussed in the GCI report are given in Table 1. All of the criteria discussed in this study are taken as benefit criteria (GCI Report, 2017-2018).

In this study, three MCDM methods, TOPSIS, WASPAS and COPRAS, based on intuitionistic fuzzy sets (IFSs) are applied to data of the Global Competitiveness Index (GCI) 2017-2018 edition and ranking of the countries is obtained. Ranking results of the considered IFSs based MCDM methods are compared with the GCI report and relationship between them are investigated. Dataset for 2017-2018 edition of GCI report to be used is shown in Table 4. Readers can reach the whole dataset from the link http://reports.weforum.org/global-competitiveness-index-2017-2018/downloads/. In the GCI report, since the

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evaluation scores for the countries are between 1 and 7, the linguistic variables related to these scores are determined as given in Table 2. Using these intervals, dataset given in Table A.1 is converted into linguistic variables and shown in Table 4.

Linguistic variables (VH, H, MH, M, ML, L, VL) in Table 2 are converted into corresponding intuitionistic fuzzy numbers as shown in Table 3 (Cavallaro et al., 2019).

Table 1. Criteria and sub-criteria in Report of Global Competitiveness Index

Criteria	Sub-criteria
Factor	C1: Institutions
	C2: Infrastructure
	C3: Macroeconomic environment
	C4: Health and primary education
Efficiency	C5: Higher education and traininig
	C6: Goods market productivity
	C7: Labor market productivity
	C8: Financial market development
	C9: Technological preparation
	C10: Market size.
Innovation	C11: Business development
	C12: Innovation

Table 2. Linguistic variables corresponding to evaluation scores for GCI Dataset

Evaluation score interval	Linguistic variable
(1.000, 1.8571)	Very Low (VL)
(1.8571, 2.714)	Low (L)
(2.714, 3.5714)	Medium Low (ML)
(3.5714, 4.4285)	Medium (M)
(4.4285, 5.2857)	Medium High (MH)
(5.2857, 6.1428)	High (H)
(6.1428, 7.0000)	Very High (VH)

A part of dataset for 2017-2018 edition of the GCI report is shown in Table 4. Then, the GCI data are converted into linguistic variables and part of them is given in Table 5. The intuitionistic fuzzy decision matrix constructed with the intuitionistic fuzzy numbers given in Table 3, is shown in Table 6. Then, intuitionistic fuzzy TOPSIS (IF-TOPSIS), intuitionistic fuzzy WASPAS (IF-WASPAS), and intuitionistic fuzzy COPRAS (IF-COPRAS) methods are applied to dataset of the GCI 2017-2018 edition. In these methods, weights of the criteria are considered as equal, i.e. 1/12. Carrying out all calculations, arithmetic operations of summation, multiplication, multiplication by a scalar k, exponentiation, score function,

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intuitionistic fuzzy weighted arithmetic mean (IFWA) operator and intuitionistic fuzzy weighted geometric (IFWG) operator, and normalized Euclidean distance, on intuitionistic fuzzy numbers which are given in previous chapters are used. Normalization procedure is not necessary because the intuitionistic fuzzy numbers in the intuitionistic fuzzy decision matrix are in interval [0,1].

For IF-TOPSIS method, at first intuitionistic fuzzy weighted decision matrix is constituted (Table 7). Then, intuitionistic fuzzy positive ideal solution (A^*) and intuitionistic fuzzy negative ideal solution (A^-) are found utilizing Eq (18) and Eq (19), respectively (Table 8). Separation measures (S_i^* and S_i^-) of each alternative are calculated using normalized Euclidean distance via Eq (20) and Eq (21), respectively. Finally, alternatives are ranked according to descending order of relative closeness coefficients (C_{i*}) of the each alternative which are computed using Eq (22) (Table 9).

For IF-COPRAS method, at first intuitionistic fuzzy weighted decision matrix is constructed (Table 7). Then, index $\tilde{P}_i$ for benefit criteria are calculated for each alternative by Eq (23) (Table 10). Since all criteria are considered as benefit criteria, index $\tilde{R}_i$ for cost criteria could not calculated. The relative weights (Q_i) and utility degrees (N_i) of each alternative are computed by Eq (26) and Eq (27), respectively. At final, alternatives are ranked according to descending order of Q_i or N_i(Table 10).

For IF-WASPAS method, the intuitionistic fuzzy weighted decision matrix is constituted (Table 7). The sum total relative importance based on the WSM approach (Q_i^{WSM}) by using IFWA operator and the product total relative importance based on the WPM approach (Q_i^{WPM}) by using IFWG operator are computed for each alternative using Eq (28) and Eq (29), respectively. Finally, alternatives are ranked with respect to descending order of overall importance (Q_i) which are computed using Eq (30) (Table 11).

Since the tables regarding the calculations take up a lot of space, not all of them can be given here. For more detailed information on Tables, see Günter (2019). Ranking results of the MCDM methods based on intuitionistic fuzzy sets (IF-MCDM) discussed are given in Table 12 - Table 15.

Table 3. Linguistic variables and related intuitionistic fuzzy numbers (IFSs)

	Intuitionistic fuzzy number		
Linguistic variables	μ	ν	π
Very high (VH)	0.80	0.10	0.10
High (H)	0.70	0.20	0.10
Medium high (MH)	0.60	0.30	0.10
Medium (M)	0.50	0.40	0.10
Medium low (ML)	0.40	0.50	0.10
Low (L)	0.25	0.60	0.15
Very low (VL)	0.10	0.75	0.15

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Table 4. A part of Dataset for 2017-2018 edition of the GCI report

2017-2018	Criteria											
COUNTRY	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
Albania	3.878	3.562	4.595	6.237	4.768	4.432	3.964	3.809	4.071	2.990	3.902	3.199
Algeria	3.629	3.562	4.634	5.766	3.952	3.639	3.266	3.057	3.358	4.778	3.317	2.941
Argentina	3.277	3.852	3.377	5.889	4.996	3.436	3.292	3.103	4.321	4.878	3.822	3.302
Armenia	4.058	3.853	4.135	5.995	4.421	4.699	4.401	3.878	4.086	2.788	3.986	3.308
Australia	5.352	5.267	5.674	6.523	5.877	4.876	4.685	5.453	5.718	5.133	4.854	4.515
Austria	5.151	5.725	5.523	6.402	5.677	4.893	4.487	4.577	5.971	4.589	5.577	5.026
Azerbaijan	4.649	4.540	4.796	5.718	4.463	4.797	5.007	3.843	4.554	3.973	4.439	3.997
Bahrain	5.040	5.067	3.977	6.224	4.986	4.981	4.552	4.305	5.568	3.307	4.494	3.604
Bangladesh	3.390	2.918	4.903	5.220	3.102	4.115	3.601	3.602	2.764	4.719	3.704	2.848
Belgium	5.017	5.419	4.867	6.627	5.820	5.183	4.469	4.682	5.936	4.788	5.418	4.951
Benin	3.526	2.308	3.938	4.692	3.130	3.665	4.412	3.362	2.389	2.659	3.303	3.027
Bhutan	4.800	3.644	4.583	5.420	4.005	4.164	4.730	4.010	3.212	1.935	3.833	3.235
Bosnia and Herzegovina	3.093	3.303	4.817	5.973	3.981	3.701	3.494	3.497	4.270	3.147	3.467	2.716
Botswana	4.364	3.639	6.094	4.829	3.839	4.214	4.517	4.041	3.627	2.955	3.691	3.189
Brazil	3.354	4.114	3.443	5.411	4.205	3.791	3.682	3.700	4.568	5.692	4.119	3.209
Brunei Darussalam	4.429	4.315	5.147	6.315	4.471	4.344	4.441	3.748	4.488	2.890	3.692	3.230
Bulgaria	3.481	4.061	5.723	5.801	4.618	4.322	4.255	4.144	5.135	3.924	3.819	3.317
Burundi	3.198	2.123	3.587	4.785	2.616	3.744	4.257	2.805	2.112	1.780	3.270	2.753
Cambodia	3.393	3.139	4.637	5.261	2.883	4.173	4.424	4.089	3.423	3.381	3.577	2.908
Cameroon	3.480	2.254	4.445	4.772	3.521	3.936	4.138	3.620	2.634	3.399	3.516	3.246
Canada	5.431	5.702	5.127	6.602	5.770	5.151	5.429	5.436	5.880	5.435	4.979	4.669
Cape Verde	3.936	3.502	4.136	5.849	4.057	4.007	3.674	3.213	3.796	1.556	3.444	3.077
Chile	4.535	4.783	5.384	5.816	5.254	4.652	4.417	4.922	5.212	4.541	4.261	3.456
China	4.419	4.657	5.999	6.212	4.781	4.549	4.548	4.229	4.183	7.000	4.514	4.136
Colombia	3.211	3.770	4.827	5.526	4.504	4.033	3.985	4.636	4.344	4.758	4.066	3.268
Congo. Democratic Rep.	3.203	2.332	3.454	4.254	2.748	3.591	4.340	3.043	2.498	3.215	3.178	2.812
Costa Rica	4.248	4.248	4.546	6.245	5.132	4.379	4.219	4.445	4.932	3.453	4.499	3.655
Croatia	3.455	4.647	4.849	6.130	4.545	4.043	3.767	3.647	5.039	3.617	3.793	2.937
Cyprus	4.180	5.107	4.192	6.212	4.858	4.896	4.530	3.439	5.521	2.899	4.154	3.416
Czech Republic	4.165	4.605	6.232	6.402	5.249	4.660	4.488	4.799	5.497	4.494	4.609	3.871
Denmark	5.458	5.511	6.222	6.408	5.972	5.114	5.195	4.873	6.093	4.289	5.418	5.134
Dominican Republic	3.048	3.300	5.100	5.073	3.934	3.907	3.617	3.569	3.710	3.887	3.779	2.781
Ecuador	3.053	4.118	4.345	5.907	4.251	3.651	3.408	3.337	3.603	3.920	3.594	2.897

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Table 5. A part of linguistic variables for the GCI data

2017-2018	Criteria											
COUNTRIES	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**	**C9**	**C10**	**C11**	**C12**
Albania	M	ML	MH	VH	MH	MH	M	M	M	ML	M	ML
Algeria	M	ML	MH	H	M	M	ML	ML	ML	MH	ML	ML
Argentina	ML	M	ML	H	MH	ML	ML	ML	M	MH	M	ML
Armenia	M	M	M	H	M	MH	M	M	M	ML	M	ML
Australia	H	MH	H	VH	H	MH	MH	H	H	MH	MH	MH
Austria	MH	H	H	VH	H	MH	MH	MH	H	MH	H	MH
Azerbaijan	MH	MH	MH	H	MH	MH	MH	M	MH	M	MH	M
Bahrain	MH	MH	M	VH	MH	MH	MH	M	H	ML	MH	M
Bangladesh	ML	ML	MH	MH	ML	M	M	M	ML	MH	M	ML
Belgium	MH	H	MH	VH	H	MH	MH	MH	H	MH	H	MH
Benin	ML	L	M	MH	ML	M	M	ML	L	L	ML	ML
Bhutan	MH	M	MH	H	M	M	MH	M	ML	L	M	ML
Bosnia and Herzegovina	ML	ML	MH	H	M	M	ML	ML	M	ML	ML	ML
Botswana	M	M	H	MH	M	M	MH	M	M	ML	M	ML
Brazil	ML	M	ML	H	M	M	M	M	MH	H	M	ML
Brunei Darussalam	MH	M	MH	VH	MH	M	MH	M	MH	ML	M	ML
Bulgaria	ML	M	H	H	MH	M	M	M	MH	M	M	ML
Burundi	ML	L	M	MH	L	M	M	ML	L	VL	ML	ML
Cambodia	ML	ML	MH	MH	ML	M	M	M	ML	ML	M	ML
Cameroon	ML	L	MH	MH	ML	M	M	M	L	ML	ML	ML
Canada	H	H	MH	VH	H	MH	H	H	H	H	MH	MH
Cape Verde	M	ML	M	H	M	M	M	ML	M	VL	ML	ML
Chad	L	L	M	M	L	ML	M	ML	L	ML	ML	L
Chile	MH	MH	H	H	MH	MH	M	MH	MH	MH	M	ML
China	M	MH	H	VH	MH	MH	MH	M	M	VH	MH	M
Colombia	ML	M	MH	H	MH	M	M	MH	M	MH	M	ML
Crotia	ML	MH	MH	H	MH	M	M	M	MH	M	M	ML
Cyprus	M	MH	M	VH	MH	MH	MH	ML	H	ML	M	ML
Czech Republic	M	MH	VH	VH	MH	MH	MH	MH	H	MH	MH	M
Denmark	H	H	VH	VH	H	MH	MH	MH	H	M	H	MH
Dominican Republic	ML	ML	MH	MH	M	M	M	ML	M	M	M	ML
Ecuador	ML	M	M	H	M	M	ML	ML	M	M	M	ML
Egypt	M	M	L	H	M	M	ML	M	ML	MH	M	ML
El Salvador	ML	M	MH	H	ML	M	ML	M	ML	ML	M	L
Estonia	MH	MH	H	VH	H	MH	MH	MH	H	ML	M	M

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Table 6. A part of intuitionistic fuzzy decision matrix

	Criteria					
COUNTRY	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**
Albania	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)
Algeria	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)
Argentina	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.4, 0.5, 0.1)
Armenia	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)
Australia	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
Austria	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
Azerbajian	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)
Bahrain	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.8, 0.1, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)
Bangladesh	(0.4, 0.5, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
Belgium	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.8, 0.1, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
Benin	(0.4, 0.5, 0.1)	(0.25, 0.6, 0.15)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)
Bhutan	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)
COUNTRY	**C7**	**C8**	**C9**	**C10**	**C11**	**C12**
Albania	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)
Algeria	(0.4, 0.5, 0.1)	(0.4, 0.5, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)	(0.4, 0.5, 0.1)	(0.4, 0.5, 0.1)
Argentina	(0.4, 0.5, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)
Armenia	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)
Australia	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)
Austria	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
Azerbajian	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)
Bahrain	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.7, 0.2, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)
Bangladesh	(0.5, 0.4, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)
Belgium	(0.6, 0.3, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)
Benin	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.25, 0.6, 0.15)	(0.25, 0.6, 0.15)	(0.4, 0.5, 0.1)	(0.4, 0.5, 0.1)
Bhutan	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)	(0.25, 0.6, 0.15)	(0.5, 0.4, 0.1)	(0.4, 0.5, 0.1)

SOLUTIONS AND RECOMMENDATIONS

Spearman's rho correlation coefficients are calculated using the ranking results of countries in Table 12-Table 15 and are shown in Table 16.

According to Table 16, since all correlation coefficients are greater than 0.90, it has been observed that the ranking results obtained from different MCDM methods based on intuitionistic fuzzy sets are compatible with the ranking results given in the GCI report.

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Table 7. A part of intuitionistic fuzzy weighted decision matrix

	Criteria					
COUNTRY	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**
Albania	(0.056, 0.926,0.017)	(0.056, 0.926,0.017)	(0.056, 0.926,0.017)	(0.095,0.874,0.030)	(0.056, 0.926,0.017)	(0.074,0.905,0.022)
Algeria	(0.056,0.926,0.017)	(0.042,0.944,0.014)	(0.056, 0.926,0.017)	(0.095,0.874,0.030)	(0.056, 0.926,0.017)	(0.056, 0.926,0.017)
Argentina	(0.042,0.944,0.014)	(0.056, 0.926,0.017)	(0.042,0.944,0.014)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.042,0.944,0.014)
Armenia	(0.056, 0.926,0.017)	(0.074,0.905,0.022)	(0.126,0.825,0.049)	(0.126,0.825,0.049)	(0.074,0.905,0.022)	(0.074,0.905,0.022)
Australia	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.126,0.825,0.049)	(0.095,0.874,0.030)	(0.074,0.905,0.022)
Austria	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.095,0.874,0.030)	(0.126,0.825,0.049)	(0.095,0.874,0.030)	(0.074,0.905,0.022)
Azerbajian	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.074,0.905,0.022)
Bahrain	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)	(0.126,0.825,0.049)	(0.074,0.905,0.022)	(0.074,0.905,0.022)
Bangladesh	(0.042,0.944,0.014)	(0.042,0.944,0.014)	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.042,0.944,0.014)	(0.056, 0.926,0.017)
Belgium	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.126,0.825,0.049)	(0.095,0.874,0.030)	(0.074,0.905,0.022)
Benin	(0.042,0.944,0.014)	(0.024,0.958,0.018)	(0.056, 0.926,0.017)	(0.074,0.905,0.022)	(0.042,0.944,0.014)	(0.056, 0.926,0.017)
Bhutan	(0.042,0.944,0.014)	(0.024,0.958,0.018)	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.042,0.944,0.014)	(0.056, 0.926,0.017)
COUNTRY	**C7**	**C8**	**C9**	**C10**	**C11**	**C12**
Albania	(0.056, 0.926,0.017)	(0.056, 0.926,0.017)	(0.056,0.926,0.017)	(0.042,0.944,0.014)	(0.056, 0.926,0.017)	(0.042,0.944,0.014)
Algeria	(0.056, 0.926,0.017)	(0.042,0.944,0.014)	(0.056,0.926,0.017)	(0.009,0.976,0.015)	(0.042,0.944,0.014)	(0.042,0.944,0.014)
Argentina	(0.042,0.944,0.014)	(0.042,0.944,0.014)	(0.056,0.926,0.017)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)	(0.042,0.944,0.014)
Armenia	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)
Australia	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.074,0.905,0.022)
Austria	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)
Azerbajian	(0.074,0.905,0.022)	(0.056, 0.926,0.017)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)
Bahrain	(0.074,0.905,0.022)	(0.056, 0.926,0.017)	(0.095,0.874,0.030)	(0.042,0.944,0.014)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)
Bangladesh	(0.056, 0.926,0.017)	(0.056, 0.926,0.017)	(0.042,0.944,0.014)	(0.074,0.905,0.022)	(0.056, 0.926,0.017)	(0.042,0.944,0.014)
Belgium	(0.074,0.905,0.022)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)	(0.095,0.874,0.030)	(0.074,0.905,0.022)
Benin	(0.056, 0.926,0.017)	(0.042,0.944,0.014)	(0.024,0.958,0.018)	(0.024,0.958,0.018)	(0.042,0.944,0.014)	(0.042,0.944,0.014)
Bhutan	(0.056, 0.926,0.017)	(0.056, 0.926,0.017)	(0.024,0.958,0.018)	(0.042,0.944,0.014)	(0.042,0.944,0.014)	(0.042,0.944,0.014)

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Table 8. Intuitionistic fuzzy positive ideal solution (A^*) and Intuitionistic fuzzy negative ideal solution (A^-) for the criteria

Criteria	C1	C2	C3	C4	C5	C6
A^*	(0.126,0.825,0.049)	(0.126,0.825,0.049)	(0.126,0.825,0.049)	(0.126,0.825,0.049)	(0.126,0.825,0.049)	(0.095,0.874,0.030)
A^-	(0.024,0.958,0.018)	(0.009,0.976,0.015)	(0.024,0.958,0.018)	(0.042,0.944,0.014)	(0.024,0.958,0.018)	(0.042,0.944,0.014)
Criteria	**C7**	**C8**	**C9**	**C10**	**C11**	**C12**
A^*	(0.095,0.874,0.030)	(0.095,0.874,0.030)	(0.126,0.825,0.049)	(0.126,0.825,0.049)	(0.095,0.874,0.030)	(0.095,0.874,0.030)
A^-	(0.042,0.944,0.014)	(0.024,0.958,0.018)	(0.024,0.958,0.018)	(0.009,0.976,0.015)	(0.024,0.958,0.018)	(0.024,0.958,0.018)

Table 9. A part of separation measures (S_i^*, S_i^-) and relative closeness coefficients (C_{i^*})

COUNTRY	S_i^*	S_i^-	C_{i^*}	Rank
Albania	0.0703	0.0452	0.3915	66
Algeria	0.0752	0.0365	0.3267	89
Argentina	0.0752	0.0379	0.3347	86
Armenia	0.0732	0.0358	0.3283	84
Australia	0.0408	0.0722	0.6391	21
Austria	0.0408	0.0715	0.6369	19
Azerbajian	0.0582	0.0494	0.4588	44
Bahrain	0.0593	0.0555	0.4835	39
Bangladesh	0.0774	0.0342	0.3062	95
Belgium	0.0437	0.0695	0.6136	25
Benin	0.0890	0.0203	0.1855	126
Bhutan	0.0733	0.0385	0.3442	83
Bosnia and Herzegovina	0.0786	0.0321	0.2899	102
Botswana	0.0714	0.0392	0.3545	74
Brazil	0.0705	0.0439	0.3836	71
Brunei Darussalem	0.0645	0.0498	0.4360	50
Bulgaria	0.0667	0.0447	0.4012	56

FUTURE RESEARCH DIRECTIONS

In future studies, it may be suggested to obtain results with MOORA, ELECTRE, EDAS, CODAS, and GRA methods based on intuitionistic fuzzy sets, and compare them with the results of this study. In addition, different ranking methods can be developed for intuitionistic fuzzy sets and can be used at the solution stage.

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Table 10. A part of $\tilde{P}_i$ index, relative weights (Q_i) and utility degrees (N_i)

COUNTRY	$\tilde{P}_i$ for μ	$\tilde{P}_i$ for v	Q_i	N_i (%)	Rank
Albania	0.5415	0.3507	0.0954	33.6297	61
Algeria	0.4943	0.4024	0.046	16.2001	91
Argentina	0.4943	0.4024	0.046	16.2001	92
Armenia	0.5152	0.3826	0.0663	23.3676	82
Australia	0.6651	0.2312	0.2169	76.4751	18
Austria	0.6651	0.2312	0.2169	76.4751	19
Azerbajian	0.5871	0.3117	0.1377	48.5414	42
Bahrain	0.5969	0.2968	0.15	52.8912	35
Bangladesh	0.4898	0.4085	0.0406	14.3284	96
Belgium	0.657	0.2391	0.2089	73.6425	23
Benin	0.414	0.4743	-0.0302	-10.6302	124
Bhutan	0.5168	0.3772	0.0698	24.6058	78
Bosnia and Herzegovina	0.4769	0.4198	0.0285	10.0529	103
Botswana	0.5241	0.3735	0.0753	26.5393	73
Brazil	0.5282	0.3679	0.0802	28.2652	70
Brunei Darussalem	0.5649	0.3281	0.1184	41.733	47
Bulgaria	0.5439	0.3525	0.0957	33.7299	59

Table 11. A part of total relative importances of Q_i^{WSM}, Q_i^{WPM} and overall importance (Q_i)

COUNTRY	Q_i^{WSM} for μ	Q_i^{WSM} for v	Q_i^{WPM} for μ	Q_i^{WPM} for v	Q_i	Rank
Albania	0.7775	0.1346	0.2787	0.5998	0.3153	63
Algeria	0.7340	0.1708	0.2343	0.6560	0.2141	90
Argentina	0.7340	0.1708	0.2343	0.6560	0.2141	91
Armenia	0.7590	0.1515	0.2591	0.6272	0.2692	79
Australia	0.8846	0.0565	0.4359	0.4190	0.5910	19
Austria	0.8846	0.0565	0.4359	0.4190	0.5910	18
Azerbajian	0.8268	0.0994	0.3409	0.5312	0.4324	39
Bahrain	0.8287	0.0961	0.3432	0.5244	0.4401	36
Bangladesh	0.7331	0.1724	0.2336	0.6581	0.2109	93
Belgium	0.8789	0.0604	0.4250	0.4314	0.5747	22
Benin	0.6419	0.2329	0.1610	0.7324	0.0389	124
Bhutan	0.7533	0.1513	0.2529	0.6269	0.2628	82
Bosnia and Herzegovina	0.7167	0.1849	0.2186	0.6753	0.1761	103
Botswana	0.7672	0.1450	0.2677	0.6168	0.2880	73
Brazil	0.7678	0.1438	0.2682	0.6149	0.2905	70
Brunei Darussalem	0.8001	0.1175	0.3053	0.5686	0.3689	48
Bulgaria	0.7835	0.1317	0.2857	0.5944	0.3270	58

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Table 12. Ranking of the countries by IF-TOPSIS, IF-COPRAS, IF-WASPAS, and GCI

	GCI	IF-TOPSIS		IF -COPRAS		IF-WASPAS	
COUNTRIES	**Rank**	CC_i	**Rank**	Q_i	**Rank**	**Rank**	CC_i
Switzerland	1	0.7775	1	0.2837	1	0.7173	1
United States	2	0.7534	3	0.2742	3	0.7015	3
Singapore	3	0.7326	4	0.2613	4	0.6731	5
Holland	4	0.7314	5	0.2613	5	0.6731	4
Germany	5	0.7278	6	0.2508	7	0.6559	7
Hong Kong. SAR	6	0.7672	2	0.2776	2	0.7036	2
Sweden	7	0.7227	7	0.2546	6	0.6585	6
United Kingdom	8	0.6980	9	0.2437	9	0.6409	9
Japan	9	0.6776	10	0.2383	10	0.6261	10
Finland	10	0.6993	8	0.2494	8	0.644	8
Norway	11	0.6609	13	0.229	15	0.6101	15
Denmark	12	0.6633	12	0.2309	13	0.6106	13
New Zealand	13	0.6568	15	0.2343	11	0.6232	11
Canada	14	0.6759	11	0.2323	12	0.6227	12
Taiwan. China	15	0.6522	16	0.2213	17	0.5943	17
Israel	16	0.6190	21	0.2155	20	0.5789	20
United Arab Emirates	17	0.6607	14	0.2290	16	0.6101	14
Austria	18	0.6369	19	0.2169	19	0.591	18
Luxemburg	19	0.6411	17	0.2306	14	0.598	16
Belgium	20	0.6136	24	0.2089	23	0.5747	22
Australia	21	0.6391	18	0.2169	18	0.591	19
France	22	0.6190	22	0.2028	25	0.5589	25
Malaysia	23	0.5801	27	0.1921	28	0.5412	26
Ireland	24	0.6167	23	0.2110	22	0.5754	21
Qatar	25	0.6043	25	0.2028	26	0.5589	24
Korea Rep.	26	0.6330	20	0.2116	21	0.5641	23
China	27	0.5546	29	0.1797	29	0.4976	29
Iceland	28	0.6040	26	0.2078	24	0.5394	27
Estonia	29	0.5429	30	0.1760	30	0.4928	30
Saudi Arabia	30	0.4961	36	0.1478	37	0.4512	33
Czech Republic	31	0.5771	28	0.1928	27	0.5299	28
Thailand	32	0.5051	33	0.1500	36	0.4401	37
Chile	33	0.4871	38	0.1419	38	0.4336	38
Spain	34	0.5375	31	0.1678	31	0.4767	31
Azerbajian	35	0.4588	44	0.1377	42	0.4324	39
Indonesia	36	0.4994	35	0.1503	34	0.4525	32
Malta	37	0.5060	32	0.1610	32	0.4503	34

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Table 13. Ranking of the countries by IF-TOPSIS, IF-COPRAS, IF-WASPAS, and GCI (continued)

	GCI	IF-TOPSIS		IF -COPRAS		IF-WASPAS	
COUNTRIES	**Rank**	CC_i	**Rank**	Q_i	**Rank**	**Rank**	CC_i
Russian Federation	38	0.4439	46	0.1123	49	0.3631	50
Poland	39	0.4671	43	0.1325	44	0.4041	44
India	40	0.4747	42	0.1351	43	0.4057	43
Lithuania	41	0.5000	34	0.1511	33	0.4418	35
Portugal	42	0.4788	40	0.1378	41	0.4073	40
Italy	43	0.4908	37	0.1393	40	0.4072	41
Bahrain	44	0.4835	39	0.1500	35	0.4401	36
Mauritius	45	0.4265	50	0.1175	48	0.3792	46
Brunei Darussalem	46	0.4360	49	0.1184	47	0.3689	48
Costa Rika	47	0.4459	45	0.1247	45	0.3866	45
Slovenia	48	0.4771	41	0.1393	39	0.4072	42
Bulgaria	49	0.4012	58	0.0957	59	0.327	58
Panama	50	0.4213	52	0.1106	52	0.3631	52
Mexico	51	0.4094	54	0.0957	58	0.327	59
Kuwait	52	0.3807	68	0.0854	67	0.3085	65
Turkey	53	0.4380	48	0.1123	51	0.3631	51
Latvia	54	0.4071	55	0.1041	53	0.3451	54
Vietnam	55	0.3746	71	0.0823	69	0.3063	68
Philippines	56	0.3912	63	0.0889	65	0.3089	64
Kazakhstan	57	0.3815	67	0.0859	66	0.3067	66
Rwanda	58	0.3777	69	0.0952	62	0.3043	69
Slovak Republic	59	0.4225	51	0.1123	50	0.3631	49
Hungary	60	0.4012	74	0.0753	75	0.288	75
South Africa	61	0.3964	60	0.1032	54	0.3594	53
Oman	62	0.4071	56	0.1012	55	0.3432	55
Botswana	63	0.3545	75	0.0753	73	0.288	73
Cyprus	64	0.4419	47	0.1228	46	0.3707	47
Jordan	65	0.3853	64	0.0977	56	0.343	56
Colombia	66	0.3968	59	0.0927	63	0.3249	61
Georgia	67	0.3454	79	0.0753	74	0.288	72
Romania	68	0.3827	66	0.0841	68	0.3065	67
Iran. Islamic Rep.	69	0.3752	70	0.0771	71	0.2882	71
Jamaica	70	0.3523	77	0.0753	76	0.288	74
Morocco	71	0.3542	76	0.0735	77	0.2877	76
Peru	72	0.4053	57	0.0957	60	0.327	60
Armenia	73	0.3283	87	0.0663	82	0.2692	79
Crotia	74	0.3957	61	0.0927	64	0.3249	62
Albania	75	0.3915	62	0.0954	61	0.3153	63

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Table 14. Ranking of the countries by IF-TOPSIS, IF-COPRAS, IF-WASPAS, and GCI (continued)

	GCI	IF-TOPSIS		IF -COPRAS		IF-WASPAS	
COUNTRIES	**Rank**	CC_i	**Rank**	Q_i	**Rank**	**Rank**	CC_i
Uruguay	76	0.4119	53	0.0974	57	0.3271	57
Montenegro	77	0.3453	80	0.0680	81	0.2624	83
Serbia	78	0.3410	83	0.0609	86	0.2511	86
Tajikistan	79	0.3055	97	0.0516	88	0.2321	87
Brazil	80	0.3836	65	0.0802	70	0.2905	70
Ukraine	81	0.3430	82	0.0609	85	0.2511	85
Bhutan	82	0.3442	81	0.0698	78	0.2628	82
Trinidad and Tobago	83	0.3459	78	0.0681	80	0.2695	78
Guatemala	84	0.3353	84	0.0650	83	0.2672	80
Sri Lanka	85	0.3684	73	0.0692	79	0.2601	84
Algeria	86	0.3267	88	0.046	91	0.2141	90
Greece	87	0.3737	72	0.0769	72	0.281	77
Nepal	88	0.3152	92	0.0415	95	0.1915	99
Moldova	89	0.3092	95	0.0379	99	0.1821	102
Namibia	90	0.2969	99	0.0463	90	0.2291	89
Kenya	91	0.3177	91	0.0612	84	0.2665	81
Argentina	92	0.3347	85	0.046	92	0.2141	91
Nicaragua	93	0.2819	104	0.0204	109	0.1508	110
Cambodia	94	0.2670	111	0.023	106	0.1726	104
Tunisia	95	0.3211	90	0.0496	89	0.2316	88
Honduras	96	0.3111	93	0.0383	98	0.1955	97
Ecuador	97	0.3105	94	0.0421	94	0.2129	92
Lao PDR	98	0.2702	107	0.0309	102	0.1915	100
Bangladesh	99	0.3062	96	0.0406	96	0.2109	93
Egypt	100	0.3255	89	0.0438	93	0.2066	95
Mongolia	101	0.2946	101	0.0363	100	0.1949	98
Kyrgyz Republic	102	0.2710	105	0.0183	111	0.15	111
Bosnia and Herzegovina	103	0.2899	103	0.0285	103	0.1761	103
Dominician Republic	104	0.2922	102	0.0387	97	0.2103	94
Lebanon	105	0.2946	100	0.0342	101	0.1874	101
Senegal	106	0.2642	112	0.0209	107	0.1719	105
Seychelles	107	0.3299	86	0.0533	87	0.203	96
Ethiophia	108	0.2531	115	0.0064	115	0.1217	115
El Salvador	109	0.2976	98	0.0283	104	0.1695	106
Cape Verde	110	0.2672	109	0.0183	110	0.1285	112
Ghana	111	0.2678	108	0.0208	108	0.1651	108
Paraguay	112	0.2385	120	-0.0019	118	0.1029	117
Tanzania	113	0.2538	114	0.0043	116	0.1208	116

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Table 15. Ranking of the countries by IF-TOPSIS, IF-COPRAS, IF-WASPAS, and GCI (continued)

	GCI	IF-TOPSIS		IF -COPRAS		IF-WASPAS	
COUNTRIES	**Rank**	CC_i	**Rank**	Q_i	**Rank**	$Q_{i(m\text{-}v)}$	**Rank**
Uganda	114	0.2671	110	0.0249	105	0.1666	107
Pakistan	115	0.2708	106	0.0129	112	0.1529	109
Cameroon	116	0.2390	119	-0.0019	119	0.1029	118
Gambia. The	117	0.2417	118	0.0064	114	0.1008	120
Zambia	118	0.2308	121	0.0024	117	0.1264	114
Guinea	119	0.2422	117	-0.004	120	0.1019	119
Benin	120	0.1855	125	-0.0302	124	0.0389	124
Madagascar	121	0.1999	122	-0.0212	121	0.064	122
Swaziland	122	0.1924	124	-0.0324	125	0.0377	125
Mali	123	0.1968	123	-0.0231	123	0.0753	121
Zimbabve	124	0.1816	126	-0.0389	126	0.0257	126
Nigeria	125	0.2588	113	0.0066	113	0.1282	113
Congo Democratic Rep.	126	0.1666	129	-0.0412	127	0.0245	127
Venezuela	127	0.2511	116	-0.022	122	0.0391	123
Haiti	128	0.1687	128	-0.0846	135	-0.1169	136
Burundi	129	0.1712	127	-0.0484	128	-0.0236	130
Sierra Leone	130	0.1308	137	-0.0697	134	-0.0502	132
Lesotho	131	0.1665	130	-0.0507	130	-0.0063	129
Malawi	132	0.1597	131	-0.065	131	-0.0698	134
Mauritania	133	0.1581	133	-0.0869	136	-0.0972	135
Liberia	134	0.1507	135	-0.0695	132	-0.0629	133
Chad	135	0.1557	134	-0.0697	133	-0.0502	131
Mozambique	136	0.1593	132	-0.0505	129	-0.0006	128
Yemen	137	0.1390	136	-0.0966	137	-0.1281	137

Table 16. Spearman's rho correlation coefficients between ranking results of the GCI and Intuitionistic fuzzy MCDM methods

	GCI	IF-TOPSIS	IF-COPRAS	IF-WASPAS
GCI	1.000	0.984**	0.986**	0.987**
IF-TOPSIS	0.984**	1.000	0.997**	0.995**
IF-COPRAS	0.986**	0.997**	1.000	0.999**
IF-WASPAS	0.987**	0.995**	0.999**	1.000
** Correlation is significant at the 0.01 level (2-tailed).				

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CONCLUSION

The ranking results of the IF-MCDM methods are compared with each other and also GCI 2017-2018 edition. According to the ranking results of IF-TOPSIS, the best 3 countries in terms of competitiveness are Switzerland, Hong Kong SAR and United States, while the worst 3 countries are Liberia, Yemen and Sierra Leone. Turkey is ranked 48th among the considered 137 countries. To the ranking results of IF-COPRAS, the best 3 countries in terms of competitiveness are Switzerland, Hong Kong SAR and United States, while the worst 3 countries are Haiti, Mauritania and Yemen. Turkey is ranked 51th among the considered 137 countries. With respect to the ranking results of IF-WASPAS, the best 3 countries in terms of competitiveness are Switzerland, Hong Kong SAR and United States, while the worst 3 countries are Mauritania, Haiti and Yemen. Turkey is ranked 51th among the considered 137 countries. It can be concluded that the ranking results of the IF-MCDM methods are generally compatible with the rankings given in the 2017-2018 period GCI report.

Among the best 10 countries in terms of competitiveness, six European countries, namely Switzerland, the Netherlands, Germany, Sweden, the United Kingdom, Finland, are the top performers. The other four countries, i.e. the United States of America, Singapore, Hong Kong SAR, and Japan, are powerful trade-driving countries. These are generally high-income countries and most of these are dominant in logistics, trading, innovation, technology, etc. globally. The worst 10 countries in terms of competitiveness are generally low income and lower-middle income countries, i.e., Congo Democratic Repuclic, Haiti, Burundi, Sierra Leone, Lesotho, Malawi, Mauritania, Liberia, Chad, Mozambique, Yemen. These are faced with either wars conflicts, natural disasters, poverty or political unrest.

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KEY TERMS AND DEFINITIONS

Alternative: A set of limited course of actions or choices to be evaluated.

Competitiveness: It is defined as the set of institutions, policies, and factors that determine the level of productivity of a country.

Complex Proportional Assessment (COPRAS): One of the multi-criteria decision-making methods used for ranking.

Criteria: A set of indicators to be used to evaluate or classify any problem.

Global Competitiveness Index (GCI): Providing information on countries' competitiveness on the basis of data collected by the World Economic Forum (WEF).

Intuitionistic Fuzzy Set: A special case of fuzzy sets which has membership degree and non-membership degree.

Multi-Criteria Decision Making: Decision making considering several criteria and alternatives.

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS): One of the multi-criteria decision-making methods used for ranking.

Weighted Aggregated Sum Product Assessment (WASPAS): One of the multi-criteria decision-making methods used for ranking.

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Chapter 17
On Prime Hesitant Fuzzy Ideals in Semigroups

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ABSTRACT

Theory of hesitant fuzzy sets is an important tool in algebraic systems, fuzzy sets theory, partially ordered sets, data mining, etc. The aim of this chapter is to apply prime hesitant fuzzy ideal for dealing with several kinds of theories in semigroups. The concepts of prime hesitant fuzzy, weakly prime hesitant fuzzy, completely prime hesitant fuzzy, and weakly completely prime hesitant fuzzy ideals on semigroups have been introduced. The authors deduce some fascinating properties and characterizations for these hesitant fuzzy ideals. Finally, they obtain sufficient conditions of a prime hesitant fuzzy (weakly prime hesitant fuzzy) in order to rephrase a completely prime hesitant fuzzy (weakly completely prime hesitant fuzzy) ideal in a semigroup.

INTRODUCTION

In 1998, Zhang (Zhang, 1998) defined the fuzzy prime ideals and prime fuzzy ideals of rings and discussed the operations on prime fuzzy ideals. In 1999, Dutta and Biswas (Dutta & Biswas, 1998) characterized fuzzy semiprime ideals in the ring of integers $\mathbb{Z}$. This concept has been studied further in (Martinez, 1999). In 2001, Jun and Xin (Jun & Xin, 2001) investigated relationships between fuzzy prime ideals and invertible fuzzy ideals in commutative *BCK*-algebras. In 2006, Davvaz (Davvaz, 2006) introduced the notion of the new sort of fuzzy subnear-ring (ideal and prime ideal) of a near-ring. In 2007, Zhan and Dudek (Zhan & Dudek, 2007) investigated the same fuzzy ideal in some class of hemirings and proved that a fuzzy subset f of a hemiring R is a prime fuzzy left (right) h-ideal of R if and only if f is two-valued, $f(0)=1$, and the set of all x in R such that $f(x)=1$ is a prime (left) right h-ideal of R. In 2008, Kazanci and Davvaz (Kazanci & Davvaz, 2008) introduced and studied the notations of rough prime (primary) ideals and rough fuzzy prime (primary) ideals in rings. In 2009, Kedukodi et al. (Kedukodi et al., 2009) introduced the ideas of equiprime fuzzy ideals, 3-prime fuzzy ideals and c-prime fuzzy ideals of nearrings. In 2010, Kumbhojkar (Kumbhojkar, 2010) introduced and studied the notation of

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prime fuzzy *h*-ideals of hemirings. In 2012, Navarro et al. (Navarro et al., 2012) introduced the notion of prime fuzzy ideals over non commutative rings. In 2014, Abdullah (Abdullah, 2014) defined the *n*-dimensional prime (*a,b*)-fuzzy ideals in hemirings. In 2001, Xie (Xie, 2001) introduced the ideas of prime, quasi-prime and weakly quasi-prime fuzzy left ideals of semigroups and characterized strongly semisimple semigroups in terms of quasi-prime fuzzy left ideals. In 2006, Xiao and Zhang (Xiao & Zhang, 2006) for example introduced and examined the notions of rough prime ideals and rough fuzzy prime ideals in semigroups. In 2007, Kehayopulu (Kehayopulu, 2007) investigated the same fuzzy ideal in some class of ordered semigroups and proved the fuzzy ideal of an ordered semigroup is prime if and only if it is both semiprime and weakly prime and that in commutative ordered semigroups the prime and weakly prime fuzzy ideals. In 2008, Xie and Tang (Xie & Tang, 2008) gave the concepts of weakly prime fuzzy ideals, completely prime fuzzy ideals, completely semiprime fuzzy ideals and weakly completely prime fuzzy ideals of ordered semigroups. In 2009, Kazanci and Yamak (Kazanci & Yamak, 2009) investigated relationships between different definitions of semiprime fuzzy ideals of fuzzy semigroups. In 2010, Khan and Shabir (Khan & Shabir, 2010) proved that an ordered semigroup is regular if and only if every intuitionistic fuzzy left (respectively, right) ideal of *S* is idempotent. In particular, they characterized of different classes of ordered semigroups by using intuitionistic fuzzy ideals. In 2011, Tang (Tang, 2011) defined and examined completely semiprime fuzzy ideals of ordered semigroups. In particular, he characterized an ordered semigroup that is a semilattice of simple ordered semigroups in terms of completely semiprime fuzzy ideals of ordered semigroups. In 2012, Rezvi and Mehmood (Rezvi & Mehmood, 2012) studied prime and semiprime fuzzy bi-ideals and proved that in case of the totally ordered set of fuzzy bi-ideals of semigroups, the concept of irreducible prime and strongly irreducible prime coincides. In 2013, Kar et al. (Kar et al., 2013) proved that if μ is an (i-v) fuzzy ideal of a semigroup S, then μ is an (i-v) prime fuzzy ideal of S *i*f and only if μc i^s an (i–v) fuzzy m-system of S. *In* 2016, Wang and Zhan (Wang & Zhan, 2016) studied rough semigroups, rough ideals, rough prime ideals, rough fuzzy semigroups, rough fuzzy ideals and rough fuzzy prime ideals according to the definitions of rough sets and rough fuzzy sets. In 2017, Sarkar and Kar (Sarkar & Kar, 2017) first studied (i-v) left (right) primary fuzzy ideals on semigroups. The theory of prime fuzzy sets on semigroups has been recently developed (Kim, 2009; Shabir et al., 2010), and Γ-semigroups (Chinram, 2009). In 2020, Yiarayong (Yiarayong, 2020) introduced the concept of hesitant fuzzy bi-ideals and hesitant fuzzy interior ideals in ternary semigroups. and investigated some of their interesting properties.

In this present study, we have investigated the concept of prime hesitant fuzzy, weakly prime hesitant fuzzy, completely prime hesitant fuzzy and weakly completely prime hesitant fuzzy ideals on semigroups. The aim of this manuscript is to apply prime hesitant fuzzy ideal for dealing with several kinds of theories in semigroups. We deduce some fascinating properties and characterizations for these hesitant fuzzy ideals. Finally, we obtain sufficient conditions of a prime hesitant fuzzy (weakly prime hesitant fuzzy) in order to be rephrased a completely prime hesitant fuzzy (weakly completely prime hesitant fuzzy) ideal in a semigroup.

HESITANT FUZZY SETS

In this section, we discuss some of the basic definitions and some of operations and notions on hesitant fuzzy sets are discussed.

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Definition 2.1. Let *S* be a reference set, a hesitant fuzzy set on *S* is in terms of a function that when applied to *S* returns a subset of [0,1].

For each a hesitant fuzzy set $\mathcal{H}$ on a semigroup *S*, we use the notations $\mathcal{H}_x := \mathcal{H}(x)$ for all $x \in S$.

The **hesitant fuzzy product** (Jun et al., 2015) of two hesitant fuzzy sets $\mathcal{H}$ and $\mathcal{F}$ on a semigroup *S* is defined to be a hesitant fuzzy set $\mathcal{H} \odot \mathcal{F}$ on *S* which is given by

$$(\mathcal{H} \odot \mathcal{F})_x = \begin{cases} \bigcup_{x=yz} \mathcal{H}_y \cap \mathcal{F}_z; & \exists y, z \in S \text{ such that } x = yz \\ \varnothing; & \text{otherwise.} \end{cases}$$

The following may be noted from the above definitions.

Theorem 2.2. Let $\mathcal{H}, \mathcal{F}$ and $\mathcal{G}$ be any hesitant fuzzy sets on a semigroup *S*. Then, $\mathcal{H} \odot (\mathcal{F} \cup \mathcal{G}) = \mathcal{H} \odot \mathcal{F} \cup \mathcal{H} \odot \mathcal{G}$.

Proof. Let *x* be an element of *S*. Then we have,

$$\left[\mathcal{H} \odot (\mathcal{F} \cup \mathcal{G})\right]_x = \bigcup_{x=yz} \mathcal{H}_y \cap (\mathcal{F} \cup \mathcal{G})_z$$

$$= \bigcup_{x=yz} \mathcal{H}_y \cap (\mathcal{F}_z \cup \mathcal{G}_z)$$

$$= \bigcup_{x=yz} (\mathcal{H}_y \cap \mathcal{F}_z) \cup (\mathcal{H}_y \cap \mathcal{G}_z)$$

$$= \bigcup_{x=yz} (\mathcal{H}_y \cap \mathcal{F}_z) \cup \bigcup_{x=yz} (\mathcal{H}_y \cap \mathcal{G}_z)$$

$$= \mathcal{H} \odot \mathcal{F} \cup \mathcal{H} \odot \mathcal{G},$$

which implies that $\mathcal{H} \odot (\mathcal{F} \cup \mathcal{G}) = \mathcal{H} \odot \mathcal{F} \cup \mathcal{H} \odot \mathcal{G}$.

Let *A* be a subset of a semigroup *S* and $\alpha \subseteq [0,1]$. Define the hesitant fuzzy set $\alpha\mathcal{C}_A$ on *S* as follows:

$$(\alpha\mathcal{C}_A)_x = \begin{cases} \alpha; & \text{if } x \in A \\ \varnothing; & \text{otherwise.} \end{cases}$$

Theorem 2.3. Let *A* and *B* be any subsets of a semigroup *S*. Then for any $\varnothing \neq \alpha \subseteq [0,1]$ the following statements hold:

1. $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B = \alpha\mathcal{C}_{AB}$.
2. $\alpha\mathcal{C}_A \cap \alpha\mathcal{C}_B = \alpha\mathcal{C}_{A \cap B}$.

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Proof. 1. Let x be an element in S such that $x \in AB$ Then, there exist $a \in A$ and $b \in B$ such that $x=ab$. Thus we have

$$\left(\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B\right)_x = \bigcup_{x=yz} \left(\alpha\mathcal{C}_A\right)_y \cap \left(\alpha\mathcal{C}_B\right)_z$$

$$= \bigcup_{x=yz} \alpha \cap \alpha$$

$$= \bigcup_{x=yz} \alpha$$

$$= \alpha$$

$$= \left(\alpha\mathcal{C}_{AB}\right)_\alpha .$$

Suppose that $x \notin AB$. Then $x \neq ab$ for any $a \in A$ and $b \in B$. If $x=yz$ for some $x,y \in S,$ then $y \notin A$ or $z \notin B$. It follows that,

$$\left(\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B\right)_x = \bigcup_{x=cd} \left(\alpha\mathcal{C}_A\right)_c \cap \left(\alpha\mathcal{C}_B\right)_d = \varnothing = \left(\alpha\mathcal{C}_{AB}\right)_x .$$

Now If $x \neq yz$ for all $x,y \in S,$ then $\left(\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B\right)_x = \varnothing = \left(\alpha\mathcal{C}_{AB}\right)_x .$ In any case, we have $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B = \alpha\mathcal{C}_{AB}.$

2. Let x be an element in S such that $x \in A \cap B.$ Then, we have $x \in A$ and $x \in B.$ Thus we have $\left(\alpha\mathcal{C}_A \bigcap \alpha\mathcal{C}_B\right)_x = \left(\alpha\mathcal{C}_A\right)_x \cap \left(\alpha\mathcal{C}_B\right)_x$

$$= \alpha \cap \alpha$$

$$= \alpha$$

$$= \left(\alpha\mathcal{C}_{A\cap B}\right)_x .$$

If $x \notin A \cap B,$ then $x \notin A$ or $x \notin B,$ which implies that

$$\left(\alpha\mathcal{C}_A \bigcap \alpha\mathcal{C}_B\right)_x = \left(\alpha\mathcal{C}_A\right)_x \cap \left(\alpha\mathcal{C}_B\right)_x = \varnothing = \left(\alpha\mathcal{C}_{A\cap B}\right)_x .$$

In any case, we have $\alpha\mathcal{C}_A \bigcap \alpha\mathcal{C}_B = \alpha\mathcal{C}_{A\cap B}.$

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Let $\mathcal{H}$ and $\mathcal{F}$ be two hesitant fuzzy sets on a semigroups *S*. Then, $\mathcal{H}$ is called the **subset** of $\mathcal{F}$, denoted by $\mathcal{H} \prec \mathcal{F}$ if $\mathcal{H}_x \subseteq \mathcal{F}_x$ for all $x \in S$.

Theorem 2.4. Let *A* and *B* be any subsets of a semigroup *S*. Then for any $\varnothing \neq \alpha \subseteq [0,1]$ the following statements hold:

1. If $A \subseteq B$, then $\alpha\mathcal{C}_A \prec \alpha\mathcal{C}_B$.
2. $\alpha\mathcal{C}_A \bigcup \alpha\mathcal{C}_B = \alpha\mathcal{C}_{A\cup B}$.

Proof. 1. Let *x* be an element in *S*. If $(\alpha\mathcal{C}_A)_x = \varnothing$, then it is clear that $\alpha\mathcal{C}_A \prec \alpha\mathcal{C}_B$. Otherwise, let *x* be an element *A*. Then we have $x \in B$, that is, $(\alpha\mathcal{C}_A)_x = \alpha = (\alpha\mathcal{C}_B)_x$. Therefore we obtain that $\alpha\mathcal{C}_A \prec \alpha\mathcal{C}_B$.

2. It is obvious.

Let *x* be an element of a semigroup *S* and $\alpha \subseteq [0,1]$. Define the hesitant fuzzy set x_α on S as follows:

$$(x_\alpha)_y = \begin{cases} \alpha \ ; \text{if } \ y = x \\ \varnothing \ ; \text{otherwise.} \end{cases}$$

Then x_α is called a h**esitant fuzzy point** on S.

Theorem 2.5. If *A* is a subset of a semigroup *S*, then $(\alpha\mathcal{C}_A)_x = \bigcup_{x\in A} x_\alpha$.

Proof. It is obvious.

Let $\mathcal{H}$ be a fuzzy set on a semigroup *S*. If $\mathcal{H}_x = [0,1]$ for all $x \in S$, then it is easy to see that $\mathcal{H}$ is a hesitant fuzzy set on *S*. We denote such type of hesitant fuzzy set $\mathcal{H}$ by Ψ. It is obvious that $[0,1]\mathcal{C}_S = \Psi$.

Theorem 2.6. Let *A* be any subset of a semigroup *S*. If $\varnothing \neq \alpha \subseteq [0,1]$, then the following properties hold.

1. $\Psi \odot \alpha\mathcal{C}_A = \alpha\mathcal{C}_{SA}$.
2. $\alpha\mathcal{C}_A \odot \Psi = \alpha\mathcal{C}_{AS}$.
3. $\Psi \odot \alpha\mathcal{C}_A \odot \Psi = \alpha\mathcal{C}_{SAS}$.

Proof. The proof is similar to the proof of Theorem 2.3(1).

Recall that a hesitant fuzzy set $\mathcal{H}$ on a semigroup *S* is called a **hesitant fuzzy left (resp., right) ideal** on *S* if

$$\mathcal{H}_{xy} \supseteq \mathcal{H}_y \left(\mathcal{H}_{xy} \supseteq \mathcal{H}_x\right)$$

for all $x,y \in S$. If a hesitant fuzzy set $\mathcal{H}$ on *S* is both a hesitant fuzzy left ideal and a hesitant fuzzy right ideal on *S*, we say that $\mathcal{H}$ is a **hesitant fuzzy two-sided ideal** (or a **fuzzy ideal**) on *S*.

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Theorem 2.7. Let *A* be any subset of a semigroup *S*. Then for any $\varnothing \neq \alpha \subseteq [0,1]$ the following statements hold:

1. Then, *A* is a left ideal of *S* if and only if $\alpha\mathcal{C}_A$ is a hesitant fuzzy left ideal on *S*.
2. Then, *A* is a right ideal of *S* if and only if $\alpha\mathcal{C}_A$ is a hesitant fuzzy right ideal on *S*.
3. Then, *A* is a ideal of *S* if and only if $\alpha\mathcal{C}_A$ is a hesitant fuzzy ideal on *S*.

Proof. 1. Suppose that *A* is a left ideal of *S*. Then we have, $SA \subseteq A$. By Theorem 2.6, $\Psi \odot \alpha\mathcal{C}_A = \alpha\mathcal{C}_{SA} \prec \alpha\mathcal{C}_A$. Consequently, $\alpha\mathcal{C}_A$ is a hesitant fuzzy left ideal on *S*.

Conversely, assume that $\alpha\mathcal{C}_A$ is a hesitant fuzzy left ideal on *S*. Let *x* be an element in *S* such that $x \in SA$. Then, by using Theorem 2.6, $(\alpha\mathcal{C}_A)_x \succ (\Psi \odot \alpha\mathcal{C}_A)_x = (\alpha\mathcal{C}_{SA})_x = \alpha$, which implies that $x \in A$. Consequently, *A* is a left ideal of *S*.

2-3. It can be proved similarly as 1.

Let $\mathcal{H}$ be a hesitant fuzzy set on semigroup *S* and let $\langle\mathcal{H}\rangle$ denote the intersection of all hesitant fuzzy ideals on *S* which contain $\mathcal{H}$. Then $\langle\mathcal{H}\rangle$ is a hesitant fuzzy ideal on *S,* called the **hesitant fuzzy ideal generated** by $\mathcal{H}$.

Theorem 2.8. Let x_α be a hesitant fuzzy point set on a semigroup S for all $\alpha \subseteq [0,1]$. Then the following statements hold:

1. The hesitant fuzzy ideal generated by x_α denoted by $\langle x_\alpha \rangle$, is

$$\langle x_\alpha \rangle_y = \begin{cases} \alpha \ ;\text{if } \ y \in \langle x \rangle \\ \varnothing \ ;\text{otherwise} \end{cases}$$

for any element *y* of *S,* where $\langle x \rangle$ is an ideal of *S* generated by *x*.

2. The hesitant fuzzy ideal generated by $\Psi \odot x_\alpha \odot \Psi$ denoted by $\langle \Psi \odot x_\alpha \odot \Psi \rangle$, is

$$\langle \Psi \odot x_\alpha \odot \Psi \rangle_y = \begin{cases} \alpha \ ;\text{if } \ y \in SxS \\ \varnothing \ ;\text{otherwise} \end{cases}$$

for any element *y* of *S*.

3. For all $\beta \subseteq [0,1]$ and hesitant fuzzy point y_β on $S, x_\alpha \odot y_\beta = (xy)_{\alpha\cap\beta}$.
4. $\langle x_\alpha \rangle = x_\alpha \cup (x_\alpha \odot \Psi) \cup (\Psi \odot x_\alpha) \cup (\Psi \odot x_\alpha \odot \Psi)$.
5. $\langle x_\alpha \rangle^3 = \Psi \odot x_\alpha \odot \Psi$.

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Proof. 1. Let *x* be any element of *S*. Then by Theorem 2.7, $\langle x_\alpha \rangle$ is a hesitant fuzzy ideal on *S*. Next, let $\mathcal{H}$ be a hesitant fuzzy ideal on *S* containing $x_{\alpha.}$ Then we have, $\langle x_\alpha \rangle \prec \langle \mathcal{H} \rangle = \mathcal{H},$ which implies that $\langle x_\alpha \rangle$ is a hesitant fuzzy ideal generated by $x_{\alpha.}$

2. It can be proved similarly as 1.
3. Let *s* be an element of *S* such that *s*=*xy*. Then we have,

$$\left(x_\alpha \odot y_\beta\right)_s = \bigcup_{s=ab} \left(x_\alpha\right)_a \cap \left(y_\beta\right)_b$$

$$= \alpha \cap \beta$$

$$= \left(\left(xy\right)_{\alpha\cap\beta}\right)_s .$$

If $s \neq xy$ then

$$\left(x_\alpha \odot y_\beta\right)_s = \bigcup_{s=ab} \left(x_\alpha\right)_a \cap \left(y_\beta\right)_b = \varnothing = \left(\left(xy\right)_{\alpha\cap\beta}\right)_s .$$

In any case, we have $x_\alpha \odot y_\beta = \left(xy\right)_{\alpha\cap\beta}$.

4. Let *y* be an element of *S* such that $y \in \langle x \rangle$. Then we have, $y \in x \cup Sx \cup xS \cup SxS,$ which means that

$$\left[x_\alpha \cup \left(x_\alpha \odot \Psi\right) \cup \left(\Psi \odot x_\alpha\right) \cup \left(\Psi \odot x_\alpha \odot \Psi\right)\right]_y = \left(\alpha \mathcal{C}_{x\cup xS\cup Sx\cup SxS}\right)_y = \alpha = \left\langle x_\alpha \right\rangle_y .$$

If $y \notin \langle x \rangle,$ then

$$\left[x_\alpha \cup \left(x_\alpha \odot \Psi\right) \cup \left(\Psi \odot x_\alpha\right) \cup \left(\Psi \odot x_\alpha \odot \Psi\right)\right]_y = \left(\alpha \mathcal{C}_{x\cup xS\cup Sx\cup SxS}\right)_y = \varnothing = \left\langle x_\alpha \right\rangle_y .$$

In any case, we have $\langle x_\alpha \rangle = x_\alpha \cup \left(x_\alpha \odot \Psi\right) \cup \left(\Psi \odot x_\alpha\right) \cup \left(\Psi \odot x_\alpha \odot \Psi\right).$

5. It can be proved similarly as 4.

PRIME HESITANT FUZZY IDEALS

In this section, we initiate the concepts of prime hesitant fuzzy and weakly prime hesitant fuzzy ideals on semigroups and analyze them in details. We characterize prime hesitant fuzzy ideals on a semigroup.

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Finally, we prove that if $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S,* then $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S* if $\varnothing \neq \mathcal{U}(\mathcal{H},\alpha)$ for all $\varnothing \neq \alpha \subseteq [0,1]$.

Definition 3.1. A hesitant fuzzy ideal $\mathcal{H}$ on a semigroup *S* is called **prime hesitant fuzzy** if for any two hesitant fuzzy ideals $\mathcal{F}$ and $\mathcal{G}$ on $S, \mathcal{F} \odot \mathcal{G} \prec \mathcal{H}$ implies that $\mathcal{F} \prec \mathcal{H}$ or $\mathcal{G} \prec \mathcal{H}$.

The following theorem is the main characterization of prime ideal in semigroups which is very much useful to continue the further study regarding this.

Theorem 3.2. Let *S* be a semigroup. Then the following conditions are equivalent.

1. *P* is a prime ideal of *S*.
2. $[0,1]\mathcal{C}_P$ is a prime hesitant fuzzy ideal on *S*.

Proof. First assume that *P* is a prime ideal of *S*. Then by Theorem 2.7 (3), $[0,1]\mathcal{C}_P$ is a hesitant fuzzy ideal on *S*. Let $\mathcal{F}$ and $\mathcal{G}$ be any hesitant fuzzy ideals on *S* such that $\mathcal{F} \odot \mathcal{G} \prec [0,1]\mathcal{C}_P$. If $\mathcal{F} \not\prec [0,1]\mathcal{C}_P$, then there exists a hesitant fuzzy point $x_\alpha \prec \mathcal{F}\left(\varnothing \neq \alpha \subseteq [0,1]\right)$ such that $x_\alpha \not\prec [0,1]\mathcal{C}_P$. It is easy to see that, $x_\alpha \odot y_\beta \prec \mathcal{F} \odot \mathcal{G} \prec [0,1]\mathcal{C}_P$ for all $y_\beta \prec \mathcal{G}\left(\varnothing \neq \alpha \subseteq [0,1]\right)$. Then, by Theorem 2.8 (3),

$$\left\langle x_\alpha \odot y_\beta \right\rangle_z = \begin{cases} \alpha \cap \beta \ ; \text{if } \ z \in \langle x \rangle \langle y \rangle \\ \varnothing \ ; \text{otherwise,} \end{cases}$$

which implies that $\langle x \rangle \langle y \rangle \subseteq P$. By assumption, it follows that $\langle x \rangle \subseteq P$ or $\langle y \rangle \subseteq P$. Since $x_\alpha \not\prec [0,1]\mathcal{C}_P$, we have $x \notin P$, which means that $\langle x \rangle \not\subseteq P$. Therefore, $\langle y \rangle \subseteq P$ and hence $y_\alpha \prec [0,1]\mathcal{C}_P$. Thus by Theorem 2.5, $\mathcal{G} = \bigcup_{y_\alpha \in \mathcal{G}} y_\alpha \prec [0,1]\mathcal{C}_P$. Consequently, $[0,1]\mathcal{C}_P$ is a prime hesitant fuzzy ideal on *S*.

Conversely, assume that $[0,1]\mathcal{C}_P$ is a prime hesitant fuzzy ideal on *S*. Let *A* and *B* be any ideals of *S* such that $AB \subseteq P$. Then by Theorem 2.7 (3), $[0,1]\mathcal{C}_A$ and $[0,1]\mathcal{C}_B$ are hesitant fuzzy ideals on *S* and

$$[0,1]\mathcal{C}_A \odot [0,1]\mathcal{C}_B = [0,1]\mathcal{C}_{AB} \prec [0,1]\mathcal{C}_P.$$

By assumption,

$$[0,1]\mathcal{C}_A \prec [0,1]\mathcal{C}_P \text{ or } [0,1]\mathcal{C}_B \prec [0,1]\mathcal{C}_P,$$

which implies that $A \subseteq P$ or $B \subseteq P$. Consequently, *P* is a prime ideal of *S*.

For each a hesitant fuzzy set $\mathcal{H}$ on a semigroup *S* and a subset α of [0,1], the set $\mathcal{U}(\mathcal{H},\alpha) = \{x \in S : \mathcal{H}_x \supseteq \alpha\}$, is called the **hesitant level set** of $\mathcal{H}$.

Theorem 3.3. If $\mathcal{H}$ is a prime hesitant fuzzy ideal on a semigroup *S,* then each hesitant level subset $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S* for all $\varnothing \neq \alpha \subseteq [0,1]$.

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Proof. Clearly, for any $\varnothing \neq \alpha \subseteq [0,1]$, $\mathcal{U}(\mathcal{H},\alpha)$ is an ideal of *S*. Let *A* and *B* be any ideals of *S* such that $AB \subseteq \mathcal{U}(\mathcal{H},\alpha)$. Then by Theorem 2.7 (3), $\alpha\mathcal{C}_A$ and $\alpha\mathcal{C}_B$ are hesitant fuzzy ideals on *S*. Thus by Theorem 2. 3 (1), $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B = \alpha\mathcal{C}_{AB}$. In fact, if $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B = \varnothing$ for all $x \in S$, then the desired result holds trivially. If $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \neq \varnothing$ for some $x \in S$, then there exist elements s_1 and s_2 in *S* such that $x=s_1s_2$ and $(\alpha\mathcal{C}_A)_{s_1} \cap (\alpha\mathcal{C}_B)_{s_2} \neq \varnothing$. Therefore, $s_1 \in A$ and $s_2 \in B$ and hence $s_1s_2 \in AB \subseteq \mathcal{U}(\mathcal{H},\alpha)$. It is easy to see that, $\mathcal{H}_x \supseteq \alpha$, which implies that

$$(\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B)_x = \bigcup_{x=ab} (\alpha\mathcal{C}_A)_a \cap (\alpha\mathcal{C}_B)_b$$

$$\subseteq \alpha$$

$$\subseteq \mathcal{H}_x,$$

i.e., $\$\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$. Since $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S*, we have $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$. If $\alpha\mathcal{C}_A \prec \mathcal{H}$, then for any element *a* of *A*, $(\alpha\mathcal{C}_A)_a = \alpha \subseteq \mathcal{H}_a$. Hence $A \subseteq \mathcal{U}(\mathcal{H},\alpha)$. Similarly, if $\alpha\mathcal{C}_B \prec \mathcal{H}$, then $B \subseteq \mathcal{U}(\mathcal{H},\alpha)$. Consequently, $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S*.

The following result is an immediate consequence of Theorem 3.3.

Theorem 3.4. Let $\mathcal{H}$ be a non constant hesitant fuzzy ideal on a semigroup *S* such that $\mathcal{H}_x = [0,1]$ for some $x \in S$. Then the following properties hold.

1. If $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S*, then $|\mathrm{Im}\,\mathcal{H}| = 2$.
2. If $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S*, then $\mathcal{U}(\mathcal{H},\psi) := \{x \in S : \mathcal{H}_x = [0,1]\}$ is a prime ideal of *S*.

Proof. 1. Let *x* and *y* be any elements of *S* such that $\mathcal{H}_x \neq [0,1]$ and $\mathcal{H}_y \neq [0,1]$. In the following discussion, *x* and *y* are kept fixed. Define the hesitant fuzzy sets $\mathcal{F}$ and $\mathcal{G}$ on *S* as follows:

$$\mathcal{F}_a = \begin{cases} [0,1] \text{ ;if } a \in x \cup xS \cup Sx \cup SxS \\ \varnothing \text{ ;otherwise,} \end{cases}$$

and $\mathcal{G}_a = \mathcal{H}_x$ for all $a \in S$. Then it is obvious that, $\mathcal{F}$ and $\mathcal{G}$ are hesitant fuzzy ideals of *S*. If $\mathcal{F}_a = \varnothing$, then $\mathcal{F}_a \cap \mathcal{G}_b = \varnothing \subseteq \mathcal{H}_{ab}$. If $\mathcal{F}_a = [0,1]$, then $a=x$ or $a=s_1x$ or $a=xs_2$ or $a=s_3xs_4$ for some $s_1,s_2,s_3,s_4 \in S$. This implies that, $\mathcal{F}_a \cap \mathcal{G}_b = \mathcal{G}_b = \mathcal{H}_x = \mathcal{H}_a \subseteq \mathcal{H}_{ab}$ or

$$\mathcal{F}_a \cap \mathcal{G}_b = \mathcal{G}_b = \mathcal{H}_x \subseteq \mathcal{H}_{s_1x} = \mathcal{H}_a \subseteq \mathcal{H}_{ab}$$

or

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$$\mathcal{F}_a \cap \mathcal{G}_b = \mathcal{G}_b = \mathcal{H}_x = \mathcal{H}_{xs_2} = \mathcal{H}_a \subseteq \mathcal{H}_{ab}$$

or

$$\mathcal{F}_a \cap \mathcal{G}_b = \mathcal{G}_b = \mathcal{H}_x \subseteq \mathcal{H}_{s_3 x s_4} = \mathcal{H}_a \subseteq \mathcal{H}_a.$$

In any case, we have

$$\left(\mathcal{F} \odot \mathcal{G}\right)_a = \bigcup_{a=bc} \mathcal{F}_b \cap \mathcal{G}_c \subseteq \bigcup_{a=bc} \mathcal{F}\mathcal{H}_{bc} = \mathcal{H}_a,$$

which means that $\mathcal{F} \odot \mathcal{G} \prec \mathcal{H}$. Since $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S,* we have $\mathcal{F} \prec \mathcal{H}$ or $\mathcal{G} \prec \mathcal{H}$. But $\mathcal{F}_x = [0,1] \supset \mathcal{H}_x$ and hence $\mathcal{G} \prec \mathcal{H}$. Therefore $\mathcal{H}_x = \mathcal{G}_y \prec \mathcal{H}_y$. Similarly we can prove that $\mathcal{H}_y \prec \mathcal{H}_x$. It is obvious that, $\mathcal{H}_x = \mathcal{H}_y$. Consequently, $\left|\operatorname{Im} \mathcal{H}\right| = 2$.

2. It is easy to see that, $\mathcal{U}(\mathcal{H}, \psi)$ is an ideal of *S*. Let *A* and *B* be any ideals of *S* such that $AB \subseteq \mathcal{U}(\mathcal{H}, \psi)$. Then we have,

$$[0,1]\mathcal{C}_A \odot [0,1]\mathcal{C}_B = [0,1]\mathcal{C}_{AB} \prec [0,1]\mathcal{C}_{\mathcal{U}(\mathcal{H},\psi)} \prec \mathcal{H}.$$

Since $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S,* we have $[0,1]\mathcal{C}_A \prec \mathcal{H}$ or $[0,1]\mathcal{C}_B \prec \mathcal{H}$, which implies that $A \subseteq \mathcal{U}(\mathcal{H}, \psi)$ or $B \subseteq \mathcal{U}(\mathcal{H}, \psi)$. Consequently, $\mathcal{U}(\mathcal{H}, \psi)$ is a prime ideal of *S*.

Next, we have the following result:

Theorem 3.5. Let $\mathcal{H}$ and $\mathcal{F}$ be any hesitant fuzzy ideals on a semigroup *S*. Then, $\mathcal{H} \bigcap \mathcal{F}$ is a prime hesitant fuzzy ideal on *S* if and only if $\mathcal{H} \prec \mathcal{F}$ or $\mathcal{F} \prec \mathcal{H}$.

Proof. It is obvious.

Now we define weakly prime hesitant fuzzy ideal on semigroups as follows:

Definition 3.6. A hesitant fuzzy ideal $\mathcal{H}$ on a semigroup *S* is called **weakly prime hesitant fuzzy** if for all ideals *A* and *B* of *S* and for all $\varnothing \neq \alpha \subseteq [0,1], \alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$ implies that $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$.

In Theorem 3.7, we give several characterizations of weakly prime hesitant fuzzy ideals on semigroups.

Theorem 3.7. Let $\mathcal{H}$ be a hesitant fuzzy ideal on a semigroup *S*. Then the following statements are equivalent.

1. $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S*.
2. For any elements *x* and *y* in *S* and $\varnothing \neq \alpha \subseteq [0,1]$ if $\varnothing \neq \alpha \subseteq [0,1], x_\alpha \odot \Psi \odot y_\alpha \prec \mathcal{H}$, then $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$.
3. For any elements *x* and *y* in *S* and $\varnothing \neq \alpha \subseteq [0,1]$ if $\langle x_\alpha \rangle \odot \langle y_\alpha \rangle \prec \mathcal{H}$, then $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$.
4. For any right ideals *A* and *B* of *S* and $\varnothing \neq \alpha \subseteq [0,1]$ if $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$, then $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$.

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5. For any left ideals A and B of S and $\varnothing \neq \alpha \subseteq [0,1]$ if $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$, then $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$.
6. For any right ideal A and for any left ideal B of S and $\varnothing \neq \alpha \subseteq [0,1]$ if $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$, then $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$.

Proof. (⇒2) Assume that 1 holds. Let x and y be any elements of S and $\varnothing \neq \alpha \subseteq [0,1]$ such that $x_\alpha \odot \Psi \odot y_\alpha \prec \mathcal{H}$. Since SxS and SyS are ideals of S, we have $\alpha\mathcal{C}_{SxS}$ and $\alpha\mathcal{C}_{SyS}$ are hesitant fuzzy ideals on S. Then by Theorem 2.6,

$$\alpha\mathcal{C}_{SxS} \odot \alpha\mathcal{C}_{SyS} = (\Psi \odot \alpha\mathcal{C}_x \odot \Psi) \odot (\Psi \odot \alpha\mathcal{C}_y \odot \Psi)$$

$$= \Psi \odot (\alpha\mathcal{C}_x \odot (\Psi \odot \Psi) \odot \alpha\mathcal{C}_y) \odot \Psi$$

$$\prec \Psi \odot (\alpha\mathcal{C}_x \odot \Psi \odot \alpha\mathcal{C}_y) \odot \Psi$$

$$\prec \Psi \odot \mathcal{H} \odot \Psi$$

$$\prec \mathcal{H}.$$

By assumption,

$$\langle x_\alpha \rangle^3 = \Psi \odot \alpha\mathcal{C}_x \odot \Psi = \alpha\mathcal{C}_{SxS} \prec \mathcal{H} \text{ or } \langle y_\alpha \rangle^3 = \Psi \odot \alpha\mathcal{C}_y \odot \Psi = \alpha\mathcal{C}_{SyS} \prec \mathcal{H}.$$

Since $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on S, we have $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$.

(2⇒3) Assume that 2 holds. Let x and y be any elements of S and $\varnothing \neq \alpha \subseteq [0,1]$ such that $\langle x_\alpha \rangle \odot \langle y_\alpha \rangle \prec \mathcal{H}$. Then we have,

$$x_\alpha \odot \Psi \odot y_\alpha \prec \langle x_\alpha \rangle \odot \Psi \odot \langle y_\alpha \rangle \prec \langle x_\alpha \rangle \odot \langle y_\alpha \rangle \prec \mathcal{H}.$$

By assumption, $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$.

(3⇒4) Assume that 3 holds. Let A and B be any right ideals of S and $\varnothing \neq \alpha \subseteq [0,1]$. Then by Theorem 2.3 (2), $\alpha\mathcal{C}_A$ and $\alpha\mathcal{C}_B$ are hesitant fuzzy right ideals on S. If $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$ and $\alpha\mathcal{C}_A \not\prec \mathcal{H}$, then there exists an element x in A such that $x_\alpha \not\prec \mathcal{H}$. For any $y \in B$. by Theorem 2.3, we have

$$\langle x_\alpha \rangle \odot \langle y_\alpha \rangle = \alpha\mathcal{C}_{\langle x \rangle} \odot \alpha\mathcal{C}_{\langle y \rangle}$$

$$= \alpha\mathcal{C}_{\langle x \rangle\langle y \rangle}$$

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$$\prec \alpha\mathcal{C}_{(A\cup SA)(B\cup SB)}$$

$$= \alpha\mathcal{C}_{AB\cup SAB\cup ASB\cup SASB}$$

$$\prec \alpha\mathcal{C}_{AB\cup AB\cup AB\cup SAB}$$

$$= \alpha\mathcal{C}_{AB\cup SAB}$$

$$= \left(\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B\right)\cup\left(\Psi \odot \alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B\right)$$

$$= \mathcal{H}\cup\left(\Psi \odot \mathcal{H}\right)$$

$$\prec \mathcal{H}.$$

By assumption, $\langle x_\alpha \rangle \prec \mathcal{H}$ or $\langle y_\alpha \rangle \prec \mathcal{H}$. Since $x_\alpha \not\prec \mathcal{H}$, we have $y_\alpha \prec \langle y_\alpha \rangle \prec \mathcal{H}$. Hence by Theorem 2.5, $\alpha\mathcal{C}_B = \bigcup_{y\in B} y_\alpha \prec \mathcal{H}$.

(4⇒1), (5⇒1) and (6⇒1) are obvious.
(3⇒5) is similar to that of (3⇒4).

(3⇒6) Assume that 3 holds. Let *A* and *B* be any right ideal and any left ideal of *S,* respectively. Then by Theorem 2.7 (2), $\alpha\mathcal{C}_A$ and $\alpha\mathcal{C}_B$ is a hesitant fuzzy right ideal and hesitant fuzzy left ideal on *S,* respectively. If $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$ and $\alpha\mathcal{C}_A \not\prec \mathcal{H},$ then there exists an element *x* in *A* such that $x_\alpha \not\prec \mathcal{H}$. For any *y*∈*B,* by Theorem 2.3, we have

$$\langle x_\alpha \rangle \odot \langle y_\alpha \rangle = \alpha\mathcal{C}_{\langle x\rangle} \odot \alpha\mathcal{C}_{\langle y\rangle}$$

$$= \alpha\mathcal{C}_{\langle x\rangle\langle y\rangle}$$

$$\prec \alpha\mathcal{C}_{(A\cup AS)(B\cup SB)}$$

$$= \alpha\mathcal{C}_{AB\cup ASB\cup ASB\cup ASSB}$$

$$\prec \alpha\mathcal{C}_{AB\cup AB\cup AB\cup AB}$$

$$= \alpha\mathcal{C}_{AB}$$

$$= \alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B$$

$$\prec \mathcal{H}.$$

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By assumption, $\langle x_\alpha \rangle \prec \mathcal{H}$ or $\langle y_\alpha \rangle \prec \mathcal{H}$. Since $x_\alpha \not\prec \mathcal{H}$, we have $y_\alpha \prec \langle y_\alpha \rangle \prec \mathcal{H}$. Hence by Theorem 2.5, $\alpha C_B = \bigcup_{y \in B} y_\alpha pH$.

Next, we prove that if $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S,* then $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S* if $\varnothing \neq \mathcal{U}(\mathcal{H},\alpha)$ for all $\varnothing \neq \alpha \subseteq [0,1]$.

Theorem 3.8. Let $\mathcal{H}$ be a hesitant fuzzy ideal on a semigroup *S*. Then, $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S* if and only if the non empty hesitant level subset $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S* for all $\varnothing \neq \alpha \subseteq [0,1]$.

Proof. First assume that $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S*. Clearly, for any $\varnothing \neq \alpha \subseteq [0,1], \mathcal{U}(\mathcal{H},\alpha)$ is an ideal of *S*. Let *A* and *B* be any ideals of *S* such that $AB \subseteq \mathcal{U}(\mathcal{H},\alpha)$. Then by Theorem 2.7 (3), $\alpha\mathcal{C}_A$ and $\alpha\mathcal{C}_B$ are hesitant fuzzy ideals on *S*. Thus by Theorem 2.3 (1), $\alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B = \alpha\mathcal{C}_{AB} \prec \mathcal{H}$. By assumption, $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$. It is easy to see that, $A \subseteq \mathcal{U}(\mathcal{H},\alpha)$ or $B \subseteq \mathcal{U}(\mathcal{H},\alpha)$. Consequently, $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S*.

Conversely, assume that $\mathcal{U}(\mathcal{H},\alpha)$ is a prime ideal of *S* for all $\varnothing \neq \alpha \subseteq [0,1]$. Let *A* and *B* be any ideals of *S* such that $\alpha\mathcal{C}_{AB} = \alpha\mathcal{C}_A \odot \alpha\mathcal{C}_B \prec \mathcal{H}$. This implies that, $AB \subseteq \mathcal{U}(\mathcal{H},\alpha)$. By hypothesis, $A \subseteq \mathcal{U}(\mathcal{H},\alpha)$ or $B \subseteq \mathcal{U}(\mathcal{H},\alpha)$, which implies that $\alpha\mathcal{C}_A \prec \mathcal{H}$ or $\alpha\mathcal{C}_B \prec \mathcal{H}$. Consequently, $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S*.

COMPLETELY PRIME HESITANT FUZZY IDEALS

In this section, we introduce the notion of completely prime hesitant fuzzy, weakly completely prime hesitant fuzzy ideals on semigroups and investigate some of their related properties.

Definition 4.1. Let $\varnothing \neq \alpha, \beta \subseteq [0,1]$. A hesitant fuzzy ideal $\mathcal{H}$ on a semigroup *S* is called **completely prime hesitant fuzzy** if for any two hesitant fuzzy points x_α and y_β on $S, x_\alpha \odot y_\beta \prec \mathcal{H}$ implies that $x_\alpha \prec \mathcal{H}$ or $y_\beta \prec \mathcal{H}$.

In Theorem 4.2, we give several characterizations of completely prime hesitant fuzzy ideals on semigroups.

Theorem 4.3. Let $\mathcal{H}$ be a hesitant fuzzy ideal on a semigroup *S*. Then the following conditions are equivalent.

1. $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on *S*.
2. For any hesitant fuzzy sets $\mathcal{F}$ and $\mathcal{G}$ on $S, \mathcal{F} \odot \mathcal{G} \prec \mathcal{H}$ implies that $\mathcal{F} \prec \mathcal{H}$ or $\mathcal{G} \prec \mathcal{H}$.

Proof. First assume that $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on *S*. Let $\mathcal{F}$ and $\mathcal{G}$ be any hesitant fuzzy sets on *S* such that $\mathcal{F} \odot \mathcal{G} \prec \mathcal{H}$ and $\mathcal{F} \not\prec \mathcal{H}$. Then there exists a hesitant fuzzy point $x_\alpha \prec \mathcal{F}$ on *S* such that $x_\alpha \not\prec \mathcal{H}$. It is easy to see that,

$$x_\alpha \odot y_\beta = \bigcup_{x_\alpha \prec \mathcal{F}} x_\alpha \odot \bigcup_{y_\beta \prec \mathcal{G}} y_\beta = \mathcal{F} \odot \mathcal{G} \prec \mathcal{H},$$

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which means that $x_\alpha \prec \mathcal{H}$ or $y_\beta \prec \mathcal{H}$. Since $x_\alpha \not\prec \mathcal{H}$, we have $y_\alpha \prec \mathcal{H}$ for all $y_\alpha \prec \mathcal{G}$. Consequently, $\mathcal{G} = \bigcup_{y_\beta \prec \mathcal{G}} y_\beta \prec \mathcal{H}$.

It is clear that (2⇒1).

The following may be noted from the above definitions.

Theorem 4.4. If $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on a semigroup *S,* then the non empty hesitant level subset $\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S* for all $\varnothing \neq \alpha \subseteq [0,1]$.

Proof. Clearly, for any $\varnothing \neq \alpha, \beta \subseteq [0,1], \mathcal{U}(\mathcal{H},\alpha)$ is an ideal of *S.* Let *x* and *y* be any elements of *S* such that $xy \in \mathcal{U}(\mathcal{H},\alpha)$. Then we have, $\mathcal{H}_{xy} \supseteq \alpha$, which implies that

$$x_\alpha \odot y_\alpha = \alpha\mathcal{C}_{\{x\}} \odot \alpha\mathcal{C}_{\{y\}}$$

$$= \alpha\mathcal{C}_{\{x\}\{y\}}$$

$$= \alpha\mathcal{C}_{\{xy\}}$$

$$= (xy)_\alpha$$

$$\prec \mathcal{H}.$$

Since $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on *S,* we have $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$. It is easy to see that, $x \in \mathcal{U}(\mathcal{H},\alpha)$ or $y \in \mathcal{U}(\mathcal{H},\alpha)$. Consequently, $\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S.*

Next, we define weakly completely prime hesitant fuzzy ideal on semigroups as follows:

Definition 4.5. Let $\varnothing \neq \alpha \subseteq [0,1]$. A hesitant fuzzy ideal $\mathcal{H}$ on a semigroup *S* is called **weakly completely prime hesitant fuzzy** if for any two hesitant fuzzy points x_α and y_α on $S, x_\alpha \odot y_\alpha \prec \mathcal{H}$ implies that $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$.

Next, we prove that if $\mathcal{H}$ a weakly completely prime hesitant fuzzy ideal on *S,* then $\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S* if $\varnothing \neq \mathcal{U}(\mathcal{H},\alpha)$ for all $\varnothing \neq \alpha \subseteq [0,1]$.

Theorem 4.6. Let $\mathcal{H}$ be a hesitant fuzzy ideal on a semigroup *S.* Then, $\mathcal{H}$ is a weakly completely prime hesitant fuzzy ideal on *S* if and only if the non empty hesitant level subset $\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S* for all $\varnothing \neq \alpha \subseteq [0,1]$.

Proof. First assume that $\mathcal{H}$ is a weakly completely prime hesitant fuzzy ideal on *S.* Clearly, for any $\varnothing \neq \alpha \subseteq [0,1], \mathcal{U}(\mathcal{H},\alpha)$ is an ideal of *S.* Let *x* and *y* be any elements of *S* such that $xy \in \mathcal{U}(\mathcal{H},\alpha)$. Then we have, $\mathcal{H}_{xy} \supseteq \alpha$, which implies that $x_\alpha \odot y_\alpha = (xy)_\alpha \prec \mathcal{H}$. Since $\mathcal{H}$ is a weakly completely prime hesitant fuzzy ideal on *S,* we have $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$, which implies that $\mathcal{H}_x \supseteq \alpha$ or $\mathcal{H}_y \supseteq \alpha$. Therefore, $x \in \mathcal{U}(\mathcal{H},\alpha)$ or $y \in \mathcal{U}(\mathcal{H},\alpha)$ and hence $\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S.*

Conversely, assume that $\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S.* Let *x* and *y* be any elements of *S* such that $x_\alpha \odot y_\alpha \prec \mathcal{H}$. Then we have, $(xy)_\alpha \prec \mathcal{H}$, which implies that $xy \in \mathcal{U}(\mathcal{H},\alpha)$. Since

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$\mathcal{U}(\mathcal{H},\alpha)$ is a completely prime ideal of *S,* we have $x \in \mathcal{U}(\mathcal{H},\alpha)$ or $y \in \mathcal{U}(\mathcal{H},\alpha)$. This implies that, $\mathcal{H}_x \supseteq \alpha$ or $\mathcal{H}_y \supseteq \alpha$. Therefore, $x_\alpha \prec \mathcal{H}$ or $y_\alpha \prec \mathcal{H}$ and hence $\mathcal{H}$ is a weakly completely prime hesitant fuzzy ideal on *S.*

The following result is an immediate consequence of Definitions 3.1, 3.6, 4.1 and 4.5.

Theorem 4.7. Let $\mathcal{H}$ be a hesitant fuzzy ideal on a semigroup *S.* Then the following statements hold:

1. lf $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on *S,* then $\mathcal{H}$ is a prime hesitant fuzzy ideal and a weakly completely prime hesitant fuzzy ideal on *S.*
2. lf $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S,* then $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S.*
3. lf $\mathcal{H}$ is a weakly completely prime hesitant fuzzy ideal on *S,* then $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S.*

Next we assert that under some conditions completely prime hesitant fuzzy (weakly completely prime hesitant fuzzy) ideals are prime hesitant fuzzy (weakly prime hesitant fuzzy).

Theorem 4.8. Let $\mathcal{H}$ be a hesitant fuzzy ideal on a commutative semigroup *S.* Then the following statements hold:

1. $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on *S* if and only if $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S.*
2. $\mathcal{H}$ is a weakly completely prime hesitant fuzzy ideal on *S* if and only if $\mathcal{H}$ is a weakly prime hesitant fuzzy ideal on *S.*

Proof. 1. First assume that $\mathcal{H}$ is a prime hesitant fuzzy ideal on *S.* Let *x* and *y* be any elements of *S* such that $x_\alpha \odot y_\beta \prec \mathcal{H}$. Since *S* is a commutative semigroup, we have

$$\langle x_\alpha \rangle \odot \langle y_\beta \rangle = \alpha \mathcal{C}_{\langle x \rangle} \odot \beta \mathcal{C}_{\langle y \rangle}$$

$$\prec (\alpha \cap \beta)\mathcal{C}_{\langle xy \rangle}$$

$$= \langle (xy)_{\alpha \cap \beta} \rangle$$

$$= \langle x_\alpha \odot y_\beta \rangle$$

$$\prec \langle \mathcal{H} \rangle$$

$$= \mathcal{H}.$$

By assumption, $x_\alpha \prec \langle x_\alpha \rangle \prec \mathcal{H}$ or $y_\beta \prec \langle y_\beta \rangle \prec \mathcal{H}$. Therefore, $\mathcal{H}$ is a completely prime hesitant fuzzy ideal on S.

The proof of 2 is similar to that of 1.

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CONCLUSION

In this paper, we proposed the concept of prime hesitant fuzzy, weakly prime hesitant fuzzy, completely prime hesitant fuzzy and weakly completely prime hesitant fuzzy ideals on semigroups. Our goal in this paper is to extend the concept of fuzzy prime and interval-valued prime fuzzy ideals given by Kim (Kim, 2009) and Kar et al. (Kar et al., 2013) to the context of prime hesitant fuzzy ideals. The aim of this manuscript is to apply prime hesitant fuzzy ideal for dealing with several kinds of theories in semigroups. We deduce some fascinating properties and characterizations for these hesitant fuzzy ideals.

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Chapter 18
An Introduction to Picture Fuzzy Graph and Its Application to Select Best Routes in an Airlines Network

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ABSTRACT

A picture fuzzy set is an extension of intuitionistic fuzzy set. In this set, a new parameter called neutral value of an object is incorporated. Based on this set, the picture fuzzy graph is defined and investigated by several scientists. Picture fuzzy graph is a new type of fuzzy graph, and it is used for solving many real life problems. In this chapter, the concept of picture fuzzy set is presented. It also introduced the picture fuzzy graph and investigated some useful properties of picture fuzzy graph. Some fundamental terms like degree of a vertex, order and size of picture fuzzy graph, strong picture fuzzy graph, complete picture fuzzy graph, independent strong and independent weak picture fuzzy edge, bipartite picture fuzzy graph, complement of a picture fuzzy graph, path, strength of a path, connectedness, homomorphism, isomorphism, automorphism, Cartesian product, composition, etc. are defined and presented some properties. An application is provided to select best airline route among multiple paths.

INTRODUCTION

A picture fuzzy set (PFS) is a generalization of intuitionistic fuzzy set (IFS). Intutionistic fuzzy graph (IFG) have lot of applications in reality. Picture fuzzy graph (PFG) is a new type of fuzzy graph which is a very useful tool for solving real world problems, so the study regarding PFG is clearly important.

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In this chapter, the idea of PFS is implemented and some useful properties of PFG are presented. Some fundamental terms like, complete PFG, strong PFG, degree of a node, order and size of PFG, independent strong and independent weak picture fuzzy edge, bipartite PFG, complement of a PFG, path, strength of a path, connectedness, homomorphism, isomorphism, automorphism, cartesian product and composition of PFGs etc. are defined. An application of PFG is discussed to find best possible route of Airline between two given cities.

Rosenfeld (1975) first defined the fuzzy graph (FG) considering fuzzy relations on fuzzy sets. IFG is an extension of FG. A PFS is a generalization of IFS (Atanassov, (1986)). Picture fuzzy models gives more flexibility, compatibility and precision to the system as compared to the IF model.

The concept of PFS is introduced by Coung (2013), which is an extenion of IFS. Coung added an extra components that determine the degree of neutral membership in addition to IFS. IFS give the degree of membership and non-membership of an element, while PFS give the degree of positive, neutral and non-membership of an element. These memberships are almost independent to each other, but their sum is less than or equal to one. PFS have been applied in different fields, like engineering, computer science, chemistry, mathematics, medicine, economics, etc. Some aspects of PFS and operators on PFS have been studied in (Cuong (2014) and Dutta & Ganju, (2018)).

After Rosenfeld (1975) the FG theory is going on with its different branches, such as - fuzzy threshold graph (Samanta & Pal (2011)), highly irregular interval valued FG (Rashmanlou & Pal (2013a)), balanced interval-valued FG (Rashmanlou & Pal (2013b)), fuzzy planar graphs (Samanta, Pal & Pal (2014) and Samanta & Pal (2015a)), m-step fuzzy compitition graphs (Samanta, Akram & Pal (2015)), etc. Fuzzy colouring of FG have been studied by Samanta *et al.*(Samanta, Pramanik &Pal (2015b)). Pramanik *et al.* have studied some problems on interval-valud fuzzy graph (Pramanik et al. (2016), (2017), (2016), (2016), (2018)). Voskoglouand Pramanik (2020) have discussed and characterized several fuzzy graph theoretic structure and fuzzy hypergraphs.

Sahoo& Pal (2015) have studied intuitionistic fuzzy (*IF*) competition graph. Akram& Davvaz (2012) defined strong IFG. They have also studied IF hypergraphs with applications (Akram& Dudek, (2013)). Also, in 2014, Akram and Alshehri (2014) introduced IF cycles and IF trees and IF planar graphs (Alshehri & Akram (2014). Balanced IFG is discussed by Karunambigai *et al.*(Karunambigai, Akram, Sivasankar & Palanivel (2013)). Sahoo & Pal (2015) have been presented the idea of IF competition graph. Also, Sahoo *et al.* discussed IF tolerance graph and its application (Sahoo & Pal (2017)), various type of products on IFG (Sahoo & Pal (2015a) and (2015b)).

PFG is a new dimension of graph theory, which is an extension of IFG. Graph theory, basically fuzzy graph theory and IFG theory have lot of applications in real life. There are several problems those are not solvable neither by fuzzy graph theory nor by IFG theory. These type of problems can be solved efficiently by using PFG. Motivated from that point, we consider PFG for this research study and have presented some definitions related to PFG and obtained significant results. An application of PFG is also given in this article.

BASIC CONCEPT OF FUZZY GRAPH

IFG is a generalization of fuzzy graph. The definition of IFG is given below:

Definition 1 An IFG is of the form $G=(T,\sigma,\mu)$ where $\sigma=\left(\sigma_1,\sigma_2\right), \mu=\left(\mu_1,\mu_2\right)$ and

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(i) $T=\{t_0,t_1,\ldots,t_n\}$ such that $\sigma_1: T\to[0,1]$ and $\sigma_2: T\to[0,1]$, denote the degree of membership and non-membership of the node $t_i\in T$ respectively and $0\le\sigma_1(t_i)+\sigma_2(t_i)\le 1$ for every $t_i\in T$(i=1,2,…,n).

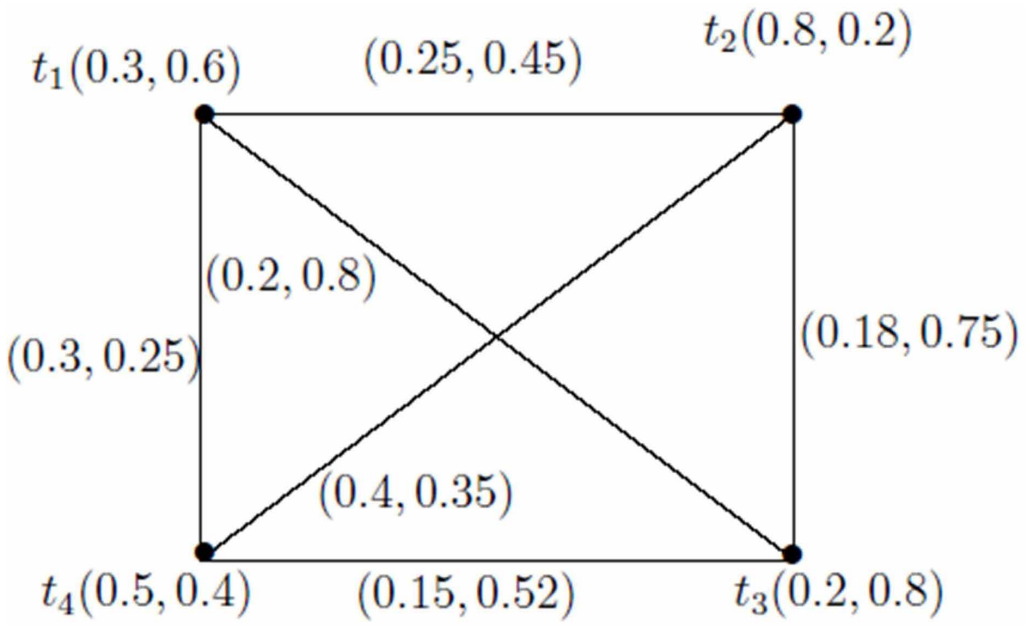
(ii) $\mu_1: T\times T\to[0,1]$ and $\mu_2: T\times T\to[0,1]$, where $\mu_1(t_i,t_j)$ and $\mu_2(t_i,t_j)$ denote the the degree of membership and non-membership value of the edge (t_i,t_j) respectively such that

$$\mu_1(t_i,t_j)\le\min\{\sigma_1(t_i),\sigma_1(t_j)\}\text{ and }\mu_2(t_i,t_j)\le\max\{\sigma_2(t_i),\sigma_2(t_j)\},$$

$0\le\mu_1(t_i,t_j)+\mu_2(t_i,t_j)\le 1$ for every (t_i,t_j).

An IFG is shown in Figure 1.

Figure 1. An IFG



PFS is an extension of IFS. The definition of PFS is presented below.

Definition 2 A PFSB on an universal set T is of the form

$$B=\{(t,\mu_B(t),\eta_B(t),\nu_B(t)):t\in T\},$$

where $\mu_B(t)\in[0,1]$ is called the positive membership degree of t in B, $\eta_B(x)\in[0,1]$ is called the neutral membership degree of t in B, $\nu_B(t)\in[0,1]$ is called the negative membership degree of t in B and they satisfies $0\le\mu_B(t)+\eta_B(t)+\nu_B(t)\le 1$ for all t∈T.

Now $1-\{\mu_B(t)+\eta_B(t)+\nu_B(t)\}$ is said to be the refusal degree of t in B.

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PICTURE FUZZY GRAPH

Here, we introduce a new dimension of graph theory, namely PFG. Some definitions and properties are introduced in this section. First we define PFG.

Definition 3 A PFG is of the form $G=(T,E,M_T,M_E)$, where $T=\{t_1,t_2,\ldots,t_n\}$ be the set of nodes, $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$ are two picture fuzzy sets such that

(i) μ_1: T→[0,1], η_1: T→[0,1] and v_1: T→[0,1] denotes the degree of positive, neutral and negative membership functions of a node $t_i \in T$ respectively and $0 \le \mu_1(t_i)+\eta_1(t_i)+\nu_1(t_i) \le 1$ for every $t_i \in T$, for i=1,2,…,n.

(ii) μ_2: T×T→[0,1], η_2: T×T→[0,1] and v_2: T×T→[0,1], where $\mu_2(t_i,t_j)$, $\eta_2(t_i,t_j)$ and $\nu_2(t_i,t_j)$ respectively denote the degree of positive, neutral and negative membership value of the edge (t_i,t_j), such that

$$\mu_2(t_i,t_j) \le \min\{\mu_1(t_i),\mu_1(t_j)\},$$

$$\eta_2(t_i,t_j) \le \min\{\eta_1(t_i),\eta_1(t_j)\},$$

$$\nu_2(t_i,t_j) \le \max\{\nu_1(t_i),\nu_1(t_j)\}$$

and

$$0 \le \mu_2(t_i,t_j)+\eta_2(t_i,t_j)+\nu_2(t_i,t_j) \le 1$$

for every $(t_i,t_j) \in T \times T$, for $i=1,2,\ldots,n$.

A PFG is depicted in Figure2.

Throughout the paper we denote a $PFGG=(T,E,M_T,M_E)$, where $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$.

Definition 4 A PFGG=(T,E,M_T,M_E), where $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$ is called a complete PFG if

$$\mu_2(t_i,t_j) = min\{\mu_1(t_i),\mu_1(t_j)\},$$

$$\eta_2(t_i,t_j) = min\{\eta_1(t_i),\eta_1(t_j)\},$$

$$\nu_2(t_i,t_j) = max\{\nu_1(t_i),\nu_1(t_j)\}$$

for all $t_i,t_j \in T$.

The complete PFG is shown in Figure 3.

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Figure 2. A PFG

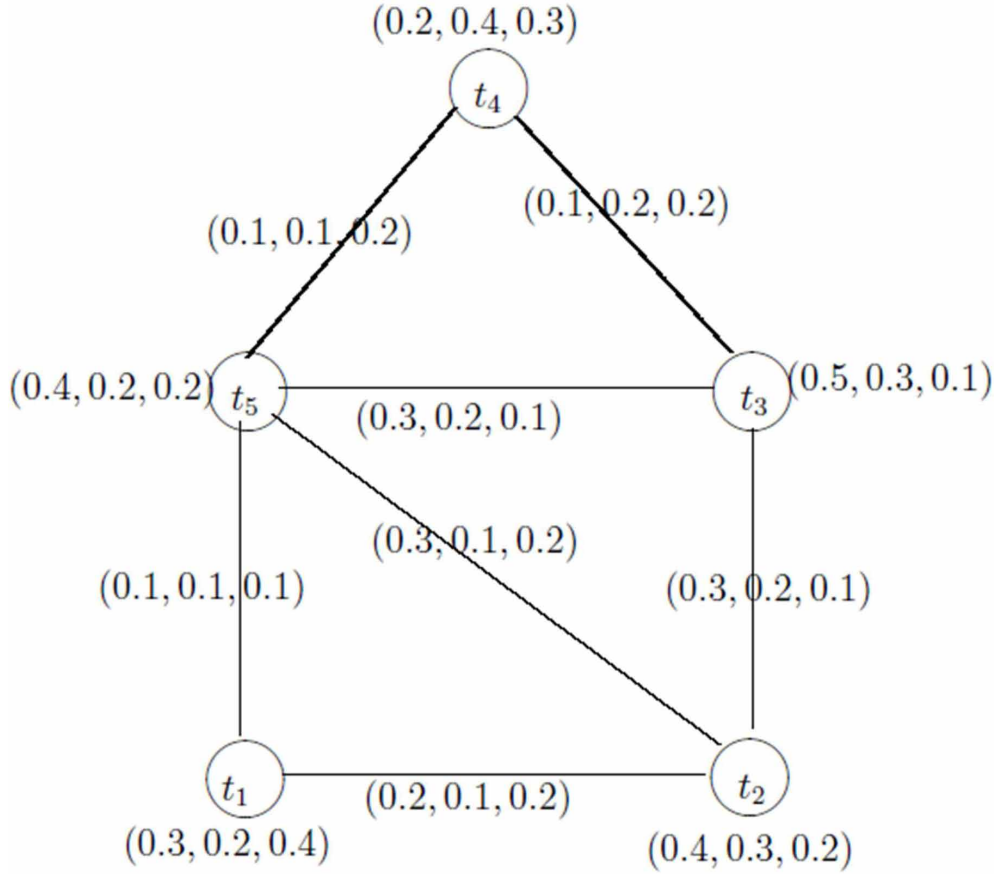
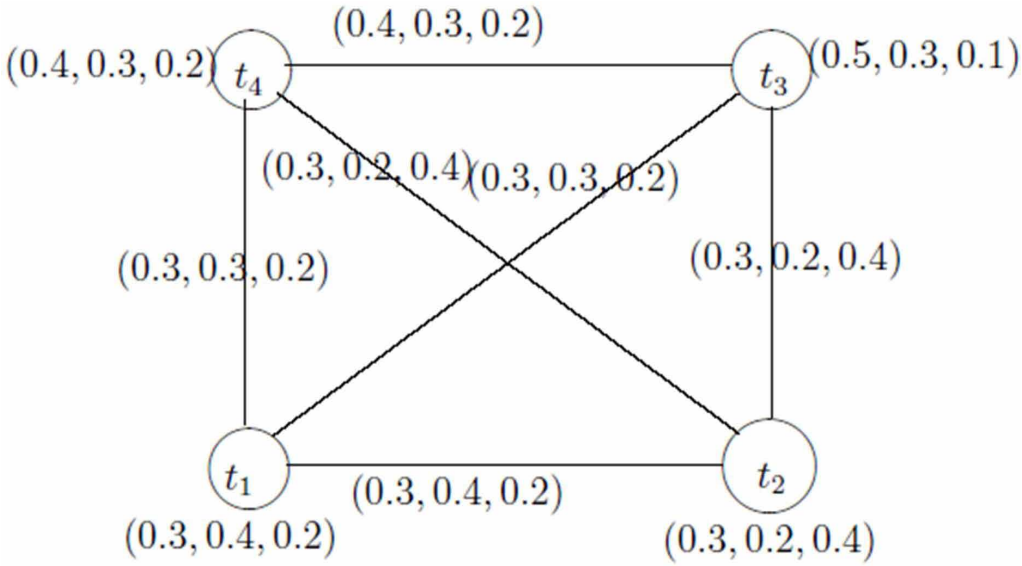


Figure 3. A complete PFG



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Definition 5 A P*FGG*=*(T,E,M*$_T$*,M*$_E$*)*, where $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$ is called a strong PFG if

$$\mu_2\left(t_i,t_j\right)=\min\left\{\mu_1\left(t_i\right),\mu_1\left(t_j\right)\right\},$$

$$\eta_2\left(t_i,t_j\right)=\min\left\{\eta_1\left(t_i\right),\eta_1\left(t_j\right)\right\},$$

$$\nu_2\left(t_i,t_j\right)=\max\left\{\nu_1\left(t_i\right),\nu_1\left(t_j\right)\right\}$$

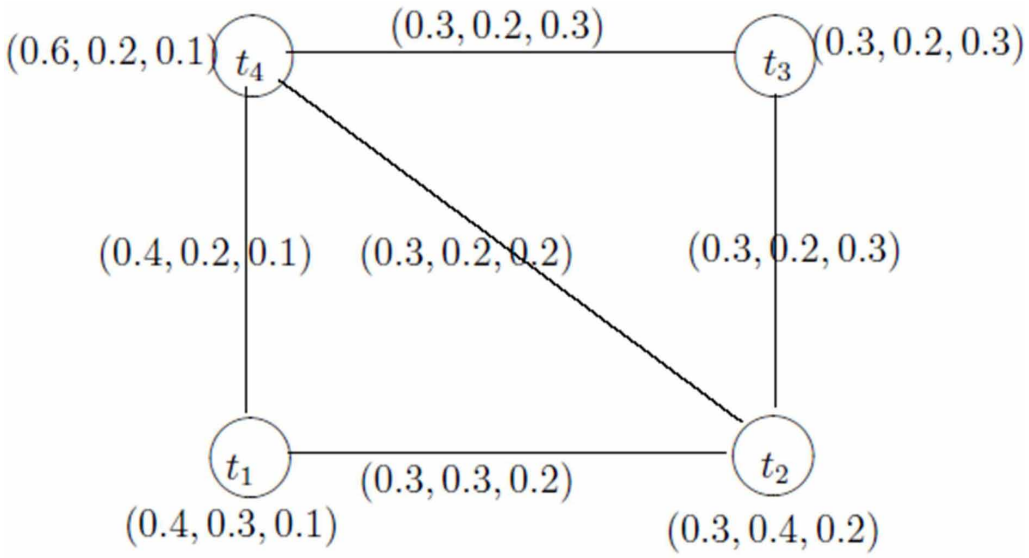
for all $(t_i,t_j)\in E$.

Definition 6 Let $G=(T,E,M_T,M_E)$ be a PFG. The degree of any node t of the PFG is denoted by $d(t)$ and is defined by $d\left(t\right)=\left(d_\mu,d_\eta,d_\nu\right)$, where

$$d_\mu\left(t\right)=\sum_{s\neq t}\mu_2\left(s,t\right),\ d_\eta\left(t\right)=\sum_{s\neq t}\eta_2\left(s,t\right),\ d_\nu\left(t\right)=\sum_{s\neq t}\nu_2\left(s,t\right).$$

For the PFG in Figure 4, $d(t_1)=(0.7,0.5,0.3)$, $d(t_2)=0.9,0.7,0.7)$, $d(t_3)=(0.6,0.4,0.6)$, $d(t_4)=(1.0,0.6,0.6)$.

Figure 4. A strong PFG



Definition 7 The order of a *PFGG*=(T,E,M_T,M_E) is denoted by $O(G)$ and is defined by $O\left(G\right)=\left(O_\mu\left(G\right),O_\eta\left(G\right),O_\nu\left(G\right)\right)$, where

$$O_\mu\left(G\right)=\sum_{v\in T}\mu_1\left(t\right),\ O_\eta\left(G\right)=\sum_{t\in T}\eta_1\left(t\right),\ O_\nu\left(G\right)=\sum_{t\in T}\nu_1\left(t\right).$$

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The value of O_{μ}(G), O_{η}(G) and $O_{\nu}(G)$ lies between 0 and 3, and their sum lies between 0 and 9.

For the PFGG in Figure 4, the order of G is O(G)=(1.6,1.1,0.7).

Definition 8 The size of a $PFGG=(T,E,M_T,M_E)$ is denoted by $S(D)$ and is defined by $S(G)=\left(S_{\mu}(G),S_{\eta}(G),S_{\nu}(G)\right)$, where

$$S_{\mu}(G)=\sum_{s\neq t}\mu_2(s,t),\ S_{\eta}(G)=\sum_{s\neq t}\eta_2(s,t),\ S_{\nu}(G)=\sum_{s\neq t}\nu_2(s,t).$$

The size of the PFG in Figure 4 is given by $S(G)$=(1.6,1.1,1.1).

Definition 9 Let $G=(T,E,M_T,M_E)$ be a PFG. An edge (s,t), $s,t\in T$ is called independent strong if

$$\frac{1}{2}min\left\{\mu_1(s),\mu_1(t)\right\}<\mu_2(s,t),$$

$$\frac{1}{2}max\left\{\eta_1(s),\eta_1(t)\right\}>\eta_2(s,t)$$

and

$$\frac{1}{2}max\left\{\nu_1(s),\nu_1(t)\right\}>\nu_2(s,t),$$

otherwise it is called an weak edge.

In the PFG in Figure 5, the edges (t_1,t_2) and (t_2,t_3) are independent strong, but the edge (t_1,t_3) is not independent strong. The edge (t_1,t_3) is an weak edge.

Definition 10 A $PFGG=(T,E,M_T,M_E)$ is called a bipartite PFG if the node set T can be decomposed into two non-empty sets T_1 and T_2 such that $\mu_2(t_i,t_j)=0$, $\eta_2(t_i,t_j)=0$, $\nu_2(t_i,t_j)=0$ if $t_i,t_j\in T_1$ or $t_i,t_j\in T_2$.

Definition 11 A bipartite $PFGG=(T,E,M_T,M_E)$ is said to be a complete bipartite PFG if

$$\mu_2(t_i,t_j)=min\left\{\mu_1(t_i),\mu_1(t_j)\right\},$$

$$\eta_2(t_i,t_j)=min\left\{\eta_1(t_i),\eta_1(t_j)\right\},$$

$$\nu_2(t_i,t_j)=max\left\{\nu_1(t_i),\nu_1(t_j)\right\}$$

for all $t_i\in T_1$ and $t_j\in T_2$.

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Figure 5. A PFG

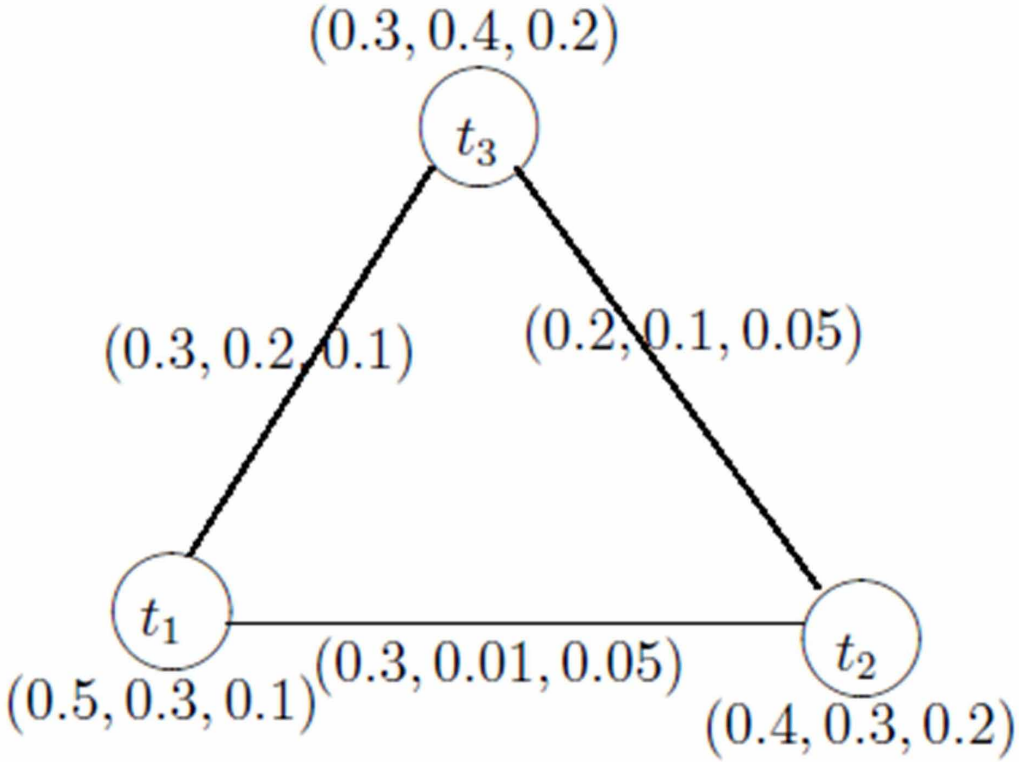

Figure 6. A bipartite PFG

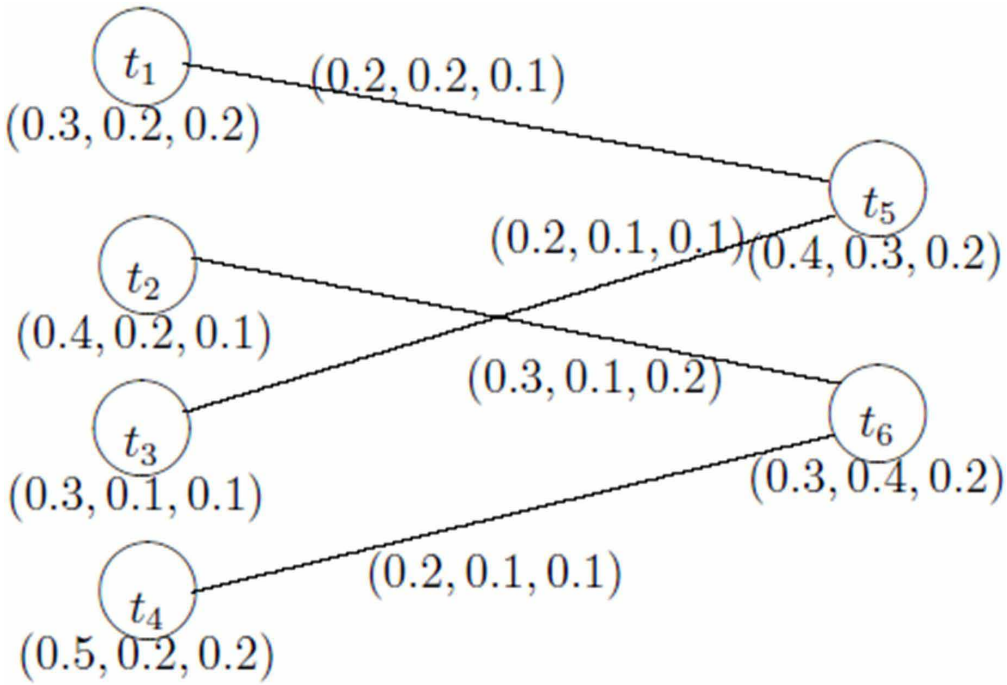

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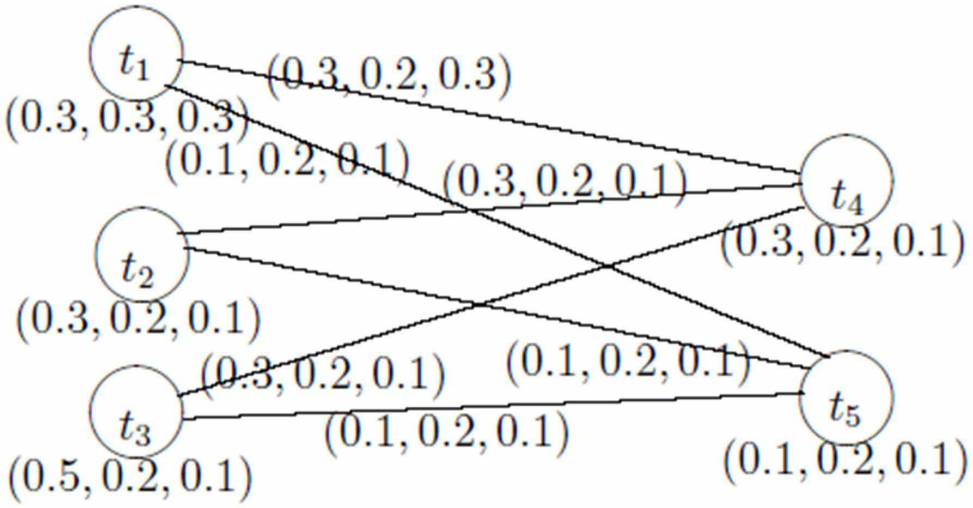
Figure 7. A complete bipartite PFG



Definition 12 A path in a PFG is a sequence of nodes $t_1,t_2,\ldots,t_n$ which satisfies one of the following conditions

(i) $\mu_2(t_i,t_{i+1})>0,\ \eta_2(t_i,t_{i+1})=0,\ \nu_2(t_i,t_{i+1})=0$ for all i.

(ii) $\mu_2(t_i,t_{i+1})>0,\ \eta_2(t_i,t_{i+1})=0,\ \nu_2(t_i,t_{i+1})>0$ for all i.

(iii) $\mu_2(t_i,t_{i+1})>0,\ \eta_2(t_i,t_{i+1})>0,\ \nu_2(t_i,t_{i+1})=0$ for all i.

(iv) $\mu_2(t_i,t_{i+1})>0,\ \eta_2(t_i,t_{i+1})>0,\ \nu_2(t_i,t_{i+1})>0$ for all i.

Now we discussed about strength of picture fuzzy path PFP. For a *PFPP=t*1*t*2…*tn*, the *μ-strength, η-strength a*nd v-*strength i*s defined by $\min\{\mu_2(t_i,t_{i+1})\}$, $\max\{\eta_2(t_i,t_{i+1})\}$ and $\max\{\nu_2(t_i,t_{i+1})\}$ respectively, for all i=1,2,…,n–1, and they are denoted by S_μ(P), S_η(P) and $S_\nu(P)$ respectively. Using the different types of strength, the various types of connectedness have presented. The μ-strength connectedness between the nodes t_i and t_j is defined by $CONN_{\mu(G)}=\max\{S_\mu\}$. Similarly, the η-strength and v-strength connectedness between the nodes t_i and t_j is defined by $CONN_{\eta(G)}=\min\{S_\eta\}$ and $CONN_{\nu(G)}=\min\{S_\nu\}$ respectively.

Definition 13 *A* PFGG *is called a connected* PFG *if there exists a* PFP *between any pair of nodes.*

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HOMOMORPHISM AND ISOMORPHISM OF PFG

In this section, we define homomorphism and isomorphism of PFG and also describes some properties of homomorphism and isomorphism of PFG. Also, using these definitions and properties later we proposed some lemmas and theorems.

Definition 14 A homomorphism between two PFGs $G_1 = \left(T_1, E_1, M_{T_1}, M_{E_1}\right)$ and $G_2 = \left(T_2, E_2, M_{T_2}, M_{E_2}\right)$ is a mapping ζ: $T_1 \to T_2$ that satisfy conditions below:

(i) $\mu_{T_1}\left(t_i\right) \le \mu_{T_2}\left(\zeta\left(t_i\right)\right), \eta_{T_1}\left(t_i\right) \le \eta_{T_2}\left(\zeta\left(t_i\right)\right), \nu_{T_1}\left(t_i\right) \ge \nu_{T_2}\left(\zeta\left(t_i\right)\right)$ for all $t_i \in T_1$.

(ii) $\mu_{E_1}\left(t_i, t_j\right) \le \mu_{E_2}\left(\zeta\left(t_i\right), \zeta\left(t_j\right)\right), \eta_{E_1}\left(t_i, t_j\right) \le \eta_{E_2}\left(\zeta\left(t_i\right), \zeta\left(t_j\right)\right), \nu_{E_1}\left(t_i, t_j\right)\left(t_i, t_j\right) \ge \nu_{E_2}\left(\zeta\left(t_i\right), \zeta\left(t_j\right)\right)$ for all $(t_i, t_j) \in E_1$.

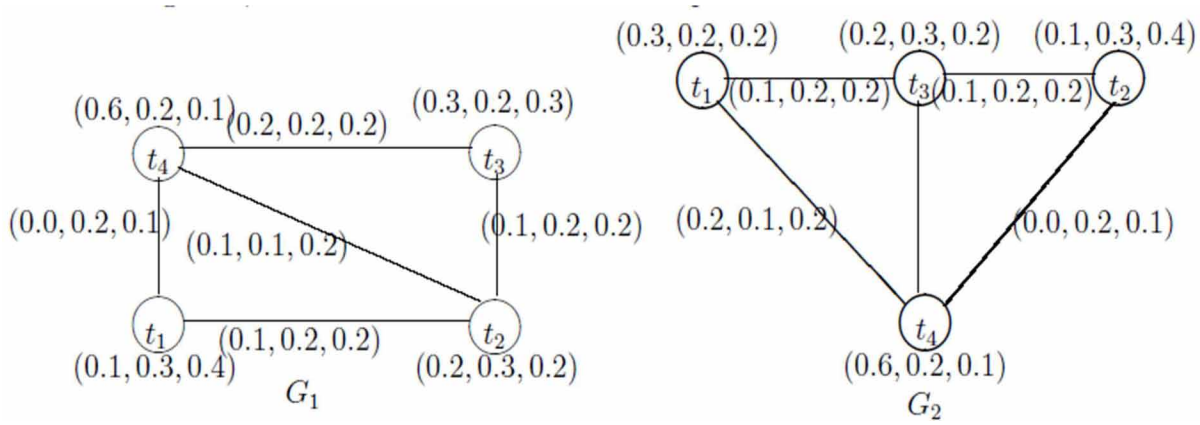
Definition 15 An isomorphism between two PFGs $G_1 = \left(T_1, E_1, M_{T_1}, M_{E_1}\right)$ and $G_2 = \left(T_2, E_2, M_{T_2}, M_{E_2}\right)$ is a mapping ζ: $T_1 \to T_2$ that satisfy the condition below:

(i) $\mu_{T_1}\left(t_i\right) = \mu_{T_2}\left(\zeta\left(t_i\right)\right),\ \eta_{T_1}\left(t_i\right) = \eta_{T_2}\left(\zeta\left(t_i\right)\right),\ \nu_{T_1}\left(t_i\right) = \nu_{T_2}\left(\zeta\left(t_i\right)\right)$ for all $t_i \in T_1$.

(ii) $\mu_{E_1}\left(t_i, t_j\right) = \mu_{E_2}\left(\zeta\left(t_i\right), \zeta\left(t_j\right)\right),\ \eta_{E_1}\left(t_i, t_j\right) = \eta_{E_2}\left(\zeta\left(t_i\right), \zeta\left(t_j\right)\right),\ \nu_{E_1}\left(t_i, t_j\right) = \nu_{E_2}\left(\zeta\left(t_i\right), \zeta\left(t_j\right)\right)$ for all $(t_i, t_j) \in E_1$.

In Figure 8, the PFGs G_1 and G_2 are isomorphic.

Figure 8. Two isomorphic PFG



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Definition 16 Let G_1 and G_2 be two PFGs. A weak isomorphism ζ: $G_1 \to G_2$ is a bijective mapping ζ: $T_1 \to T_2$ which satisfies the following conditions:

(i) ζ is a homomorphism.

(ii) $\mu_{T_1}(t_i) = \mu_{T_2}(\zeta(t_i))$, $\eta_{T_1}(t_i) = \eta_{T_2}(\zeta(t_i))$, $\nu_{T_1}(t_i) = \nu_{T_2}(\zeta(t_i))$ for all $t_i \in T_1$.

Definition 17 Let G_1 and G_2 be two PFGs. A co-weak isomorphism ζ: $G_1 \to G_2$ is a bijection ζ: $T_1 \to T_2$ that satisfies the conditions below:

(i) f is a homomorphism.

(ii) $\mu_{E_1}(t_i, t_j) = \mu_{E_2}(\zeta(t_i), \zeta(t_j))$, $\eta_{E_1}(t_i, t_j) = \eta_{E_2}(\zeta(t_i), \zeta(t_j))$, $\nu_{E_1}(t_i, t_j) = \nu_{E_2}(\zeta(t_i), \zeta(t_j))$ for all $(t_i, t_j) \in E_1$.

Definition 18 An automorphism of a $PFGG=(T,E,M_T,M_E)$ is an isomorphism of $G=(T,E,M_T,M_E)$ onto itself.

COMPLEMENT OF PICUTE FUZZY GRAPH

In this part, the definition of complement of a PFG, strong PFG and some related results are provided.

Definition 19 The complement of a $PFGG=(T,E,M_T,M_E)$ is a $PFG\bar{G} = (\bar{T}, \bar{E}, \bar{M}_T, \bar{M}_E)$, where

(i) $\bar{T} = T$

(ii) $\bar{\mu}_1(t_i) = \mu_1(t_i)$, $\bar{\eta}_1(t_i) = \eta_1(t_i)$, $\bar{\nu}_1(t_i) = \nu_1(t_i)$ for all $t_i \in T$.

(iii) $\bar{\mu}_2(t_i, t_j) = \min\{\mu_1(t_i), \mu_1(t_j)\} - \mu_2(t_i, t_j)$,
$\bar{\eta}_2(t_i, t_j) = \min\{\eta_1(t_i), \eta_1(t_j)\} - \eta_2(t_i, t_j)$

and $\bar{\nu}_2(t_i, t_j) = \max\{\nu_1(t_i), \nu_1(t_j)\} - \nu_2(t_i, t_j)$, for all $(t_i, t_j) \in T \times T$.

A PFGG and the complement of G, i.e., $\bar{G}$ are shown in Figure 9.

Definition 20 The complement of a strong PFG G on G^* is a strong $PFG\bar{G}$ on G^*, where

(i) $\bar{T} = T$

(ii) $\bar{\mu}_1(t_i) = \mu_1(t_i)$, $\bar{\eta}_1(t_i) = \eta_1(t_i)$, $\bar{\nu}_1(t_i) = \nu_1(t_i)$ for all $t_i \in T$.

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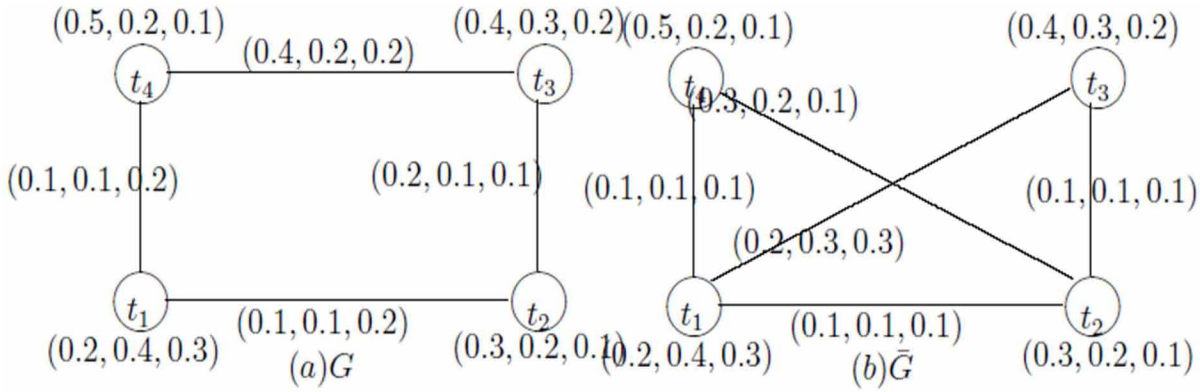
$$\text{(iii)}\quad \bar{\mu}_2\left(t_i,t_j\right)=\begin{cases}0 & \text{if}\mu_2\left(t_i,t_j\right)>0\\ \min\left\{\mu_1\left(t_i\right),\mu_1\left(t_j\right)\right\} & \text{if}\mu_2\left(t_i,t_j\right)=0\end{cases}$$

$$\bar{\eta}_2\left(t_i,t_j\right)=\begin{cases}0 & \text{if}\eta_2\left(t_i,t_j\right)>0\\ \min\left\{\eta_1\left(t_i\right),\eta_1\left(t_j\right)\right\} & \text{if}\eta_2\left(t_i,t_j\right)=0\end{cases}$$

$$\bar{\nu}_2\left(t_i,t_j\right)=\begin{cases}0 & \text{if}\nu_2\left(t_i,t_j\right)>0\\ \max\left\{\nu_1\left(t_i\right),\nu_1\left(t_j\right)\right\} & \text{if}\nu_2\left(t_i,t_j\right)=0\end{cases}$$

Figure 9. A PFG and its complement



Note 1: Let $G=(T,E,M_T,M_E)$ be a strong PFG with node set T and edge set E, $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$.

Then it follows from the above definition that $\bar{\bar{G}}$ is also a PFG given by $\bar{\bar{G}}=\left(\bar{\bar{T}},\bar{\bar{E}},\bar{\bar{M}}_T,\bar{\bar{M}}_E\right)$, where $\bar{\bar{T}}=T$ and

$$\bar{\bar{\mu}}_2\left(t_i,t_j\right)=\min\left\{\mu_1\left(t_i\right),\mu_1\left(t_j\right)\right\},$$

$$\bar{\bar{\eta}}_2\left(t_i,t_j\right)=\min\left\{\eta_1\left(t_i\right),\eta_1\left(t_j\right)\right\}$$

and

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$$\overline{\overline{\nu}}_2\left(t_i,t_j\right)=\max\left\{\nu_1\left(t_i\right),\nu_1\left(t_j\right)\right\},$$

for all $(t_i,t_j)\in T\times T$..

Thus $\overline{\overline{\mu}}_2=\mu_2,\overline{\overline{\eta}}_2=\eta_2,\overline{\overline{\nu}}_2=\nu_2$ on T.

Theorem 1 *For any strong* PFGG, $\overline{G}$ *is a strong* PFG.

Proof. Let $G=(T,E,M_T,M_E)$ be a strong PFG. Then the complement $\overline{G}$ of G is given by $\overline{G}=\left(\overline{T},\overline{E},\overline{M}_T,\overline{M}_E\right)$, where

$$\overline{T}=T,\ \overline{\mu}_1\left(t_i\right)=\mu_1\left(t_i\right),\ \overline{\eta}_1\left(t_i\right)=\eta_1\left(t_i\right),\ \overline{\nu}_1\left(t_i\right)=\nu_1\left(t_i\right)$$

for all $t_i\in T$ and

$$\overline{\mu}_2\left(t_i,t_j\right)=\begin{cases}0 & \text{if}\mu_2\left(t_i,t_j\right)>0\\ \min\left\{\mu_1\left(t_i\right),\mu_1\left(t_j\right)\right\} & \text{if}\mu_2\left(t_i,t_j\right)=0\end{cases}$$

$$\overline{\eta}_2\left(t_i,t_j\right)=\begin{cases}0 & \text{if}\eta_2\left(t_i,t_j\right)>0\\ \min\left\{\eta_1\left(t_i\right),\eta_1\left(t_j\right)\right\} & \text{if}\eta_2\left(t_i,t_j\right)=0\end{cases}$$

$$\overline{\nu}_2\left(t_i,t_j\right)=\begin{cases}0 & \text{if}\nu_2\left(t_i,t_j\right)>0\\ \max\left\{\nu_1\left(t_i\right),\nu_1\left(t_j\right)\right\} & \text{if}\nu_2\left(t_i,t_j\right)=0\end{cases}$$

Thus from the definition of strong PFG, it follows that $\overline{G}$ is a strong PFG.

Theorem 2 *Let* G *be a*, PFG *then* $G=\overline{\overline{G}}$.

Proof. Let the PFG be $G=(T,E,M_T,M_E)$. Then $\overline{G}=\left(\overline{T},\overline{E},\overline{M}_T,\overline{M}_E\right)$ and $\overline{\overline{G}}=\left(\overline{\overline{T}},\overline{\overline{E}},\overline{\overline{M}}_T,\overline{\overline{M}}_E\right)$. Then by the definition of complement of PFG we have,

(i) $\overline{\overline{T}}=\overline{T}=T$.

(ii) $\overline{\overline{\mu}}_1\left(t_i\right)=\overline{\mu}_1\left(t_i\right)=\mu_1\left(t_i\right),\ \overline{\overline{\eta}}_1\left(t_i\right)=\overline{\eta}_1\left(t_i\right)=\eta_1\left(t_i\right),\ \overline{\overline{\nu}}_1\left(t_i\right)=\overline{\nu}_1\left(t_i\right)=\nu_1\left(t_i\right)$ for all $t_i\in T$, and

(iii) $\overline{\overline{\mu}}_2\left(t_i,t_j\right)=\min\left\{\overline{\mu}_1\left(t_i\right),\overline{\mu}_1\left(t_j\right)\right\}-\overline{\mu}_2\left(t_i,t_j\right)$

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$$= \min\left\{\bar{\mu}_1(t_i), \bar{\mu}_1(t_j)\right\} - \left(\min\left\{\mu_1(t_i), \mu_1(t_j)\right\} - \mu_2(t_i, t_j)\right)$$

$$= \mu_2(t_i, t_j), since, \bar{\mu}_1(t_i) = \mu_1(t_i), forall t_i \in T$$

Therefore, $\bar{\bar{\mu}}_2 = \mu_2$.

Similarly, $\bar{\bar{\eta}}_2 = \eta_2$ and $\bar{\bar{\nu}}_2 = \nu_2$. Hence, $G = \bar{\bar{G}}$.

Definition 21 *A strong* PFGG *is called self complementary if* G *is isomorphic to* $\bar{G}$, *i.e.,* $G \simeq \bar{G}$

Theorem 3 *Any complete* PFGG *is self complementary.*

Proof. Let $G=(T,E,M_T,M_E)$ be a complete PFG, where $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$. Then

$$\mu_2(t_i, t_j) = \min\left\{\mu_1(t_i), \mu_1(t_j)\right\},$$

$$\eta_2(t_i, t_j) = \min\left\{\eta_1(t_i), \eta_1(t_j)\right\},$$

$$\nu_2(t_i, t_j) = \max\left\{\nu_1(t_i), \nu_1(t_j)\right\}$$

for all $t_i, t_j \in T$. Then G is a strong PFG and also $G \simeq \bar{G}$ under the identity mapping I: $T \to T$. Hence, G is a self complementary.

Theorem 4 Let G_1 and G_2 be two strong PFGs then G_1 and G_2 are isomorphic if and only if $\bar{G}_1$ and $\bar{G}_2$ are strong PFGs.

Proof. Let us assume that the PFGs G_1 and G_2 are isomorphic. Then there exist a bijection ζ: $T_1 \to T_2$ such that

$$\mu_{T_1}(t_i) = \mu_{T_2}\left(\zeta(t_i)\right),\ \eta_{T_1}(t_i) = \eta_{T_2}\left(\zeta(t_i)\right),\ \nu_{T_1}(t_i) = \nu_{T_2}\left(\zeta(t_i)\right)$$

for all $t_i \in T_1$, and

$$\mu_{E_1}(t_i, t_j) = \mu_{E_2}\left(\zeta(t_i), \zeta(t_j)\right),\ \eta_{E_1}(t_i, t_j) = \eta_{E_2}\left(\zeta(t_i), \zeta(t_j)\right),\ \nu_{E_1}(t_i, t_j) = \nu_{E_2}\left(\zeta(t_i), \zeta(t_j)\right)$$

for all $(t_i, t_j) \in E_1$.

Now by the definition of complement of strong PFG, for all $(t_i, t_j) \in E_1$ we have,

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$$\begin{aligned}\bar{\mu}_2\left(t_i,t_j\right) &= \min\left\{\mu_{T_1}\left(t_i\right),\mu_{T_1}\left(t_j\right)\right\}\\ &= \min\left\{\mu_{T_2}\left(\zeta\left(t_i\right)\right),\mu_{T_2}\left(\zeta\left(t_j\right)\right)\right\}\\ &= \bar{\mu}_{E_2}(\zeta\left(t_i\right),\zeta\left(t_j\right).\end{aligned}$$

$$\begin{aligned}\bar{\eta}_2\left(t_i,t_j\right) &= \min\left\{\eta_{T_1}\left(t_i\right),\eta_{T_1}\left(t_j\right)\right\}\\ &= \min\left\{\eta_{T_2}\left(\zeta\left(t_i\right)\right),\eta_{T_2}\left(\zeta\left(t_j\right)\right)\right\}\\ &= \bar{\eta}_{E_2}(\zeta\left(t_i\right),\zeta\left(t_j\right).\end{aligned}$$

And

$$\begin{aligned}\bar{\nu}_2\left(t_i,t_j\right) &= \max\left\{\nu_{T_1}\left(t_i\right),\nu_{T_1}\left(t_j\right)\right\}\\ &= \max\left\{\nu_{T_2}\left(\zeta\left(t_i\right)\right),\nu_{T_2}\left(\zeta\left(t_j\right)\right)\right\}\\ &= \bar{\nu}_{E_2}(\zeta\left(t_i\right),\zeta\left(t_j\right).\end{aligned}$$

This implies that G_1 and G_2 are isomorphic, i.e., $\bar{G}_1 \simeq \bar{G}_2$.

The converse part is straight forward.

Theorem 5 *The complement of a complete* PFGG *is a complete* PFG.

Proof. Let $G=(T,E,M_T,M_E)$ be a complete PFG where $M_T=(\mu_1,\eta_1,\nu_1)$ and $M_E=(\mu_2,\eta_2,\nu_2)$, is called a complete PFG if

$$\mu_2\left(t_i,t_j\right) = \min\left\{\mu_1\left(t_i\right),\mu_1\left(t_j\right)\right\},$$

$$\eta_2\left(t_i,t_j\right) = \min\left\{\eta_1\left(t_i\right),\eta_1\left(t_j\right)\right\},$$

$$\nu_2\left(t_i,t_j\right) = \max\left\{\nu_1\left(t_i\right),\nu_1\left(t_j\right)\right\}$$

for all $t_i,t_j \in T$.

Now the complement of G is $\bar{G} = \left(\bar{T},\bar{E},\bar{M}_T,\bar{M}_E\right)$, where

$$\bar{T} = T,\ \bar{\mu}_1\left(t_i\right) = \mu_1\left(t_i\right),\ \bar{\eta}_1\left(t_i\right) = \eta_1\left(t_i\right),\ \bar{\nu}_1\left(t_i\right) = \nu_1\left(t_i\right).$$

Again since, G is a complete PFG so,

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$$\bar{\mu}_2\left(t_i,t_j\right)=\min\left\{\mu_1\left(t_i\right),\mu_1\left(t_j\right)\right\},$$

$$\bar{\eta}_2\left(t_i,t_j\right)=\min\left\{\eta_1\left(t_i\right),\eta_1\left(t_j\right)\right\},$$

$$\bar{\nu}_2\left(t_i,t_j\right)=\max\left\{\nu_1\left(t_i\right),\nu_1\left(t_j\right)\right\}$$

for all $t_i,t_j\in T$. This implies that $\bar{G}$ is a complete PFG.

CARTESIAN PRODUCT OF PICTURE FUZZY GRAPHS

The Cartesian product of two PFGs is defined in this section. Also, the definition of the degree of a node in Cartesian product is given. Some results regarding Cartesian product of PFGs have also presented here.

Definition 22 The Cartesian product of two PFGs

$$G'=\left(T',E',M_{T'},M_{E'}\right) \text{ and } G''=\left(T'',E'',M_{T''},M_{E''}\right),$$

where,

$$M_{T'}=\left(\mu_{1'},\eta_{1'},\nu_{1'}\right),\ M_{T''}=\left(\mu_{1''},\eta_{1''},\nu_{1''}\right),\ M_{E'}=\left(\mu_{2'},\eta_{2'},\nu_{2'}\right),\ M_{E''}=\left(\mu_{2''},\eta_{2''},\nu_{2''}\right)$$

are defined as a

$$PFGG=G'\times G''=\left(T,E,M_{T'}\times M_{T''},M_{E'}\times M_{E''}\right),$$

where, $T=T'\times T''$ and

$$E=\left\{\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right):r_1=r_2,\left(h_1,h_2\right)\in E''\,or\,h_1=h_2,\left(r_1,r_2\right)\in E'\right\}$$

with

$$\left(\mu_{1'}\times\mu_{1''}\right)\left(r_1,h_1\right)=\min\left\{\mu_{1'}\left(r_1\right),\mu_{1''}\left(h_1\right)\right\}$$

$$\left(\eta_{1'}\times\eta_{1''}\right)\left(r_1,h_1\right)=\min\left\{\eta_{1'}\left(r_1\right),\eta_{1''}\left(h_1\right)\right\}$$

$$\left(\nu_{1'}\times\nu_{1''}\right)\left(r_1,h_1\right)=\max\left\{\nu_{1'}\left(r_1\right),\nu_{1''}\left(h_1\right)\right\}$$

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for all $(r_1,h_1)\in T'\times T''$ and

$$(\mu_{2'}\times\mu_{2''})\big((r_1,h_1),(r_2,h_2)\big)=\begin{cases}\min\{\mu_{1'}(r_1),\mu_{2''}(h_1,h_2)\} & \text{if} r_1=r_2,(h_1,h_2)\in E''\\ \min\{\mu_{2'}(r_1,r_2),\mu_{1''}(h_1)\} & \text{if} h_1=h_2,(r_1,r_2)\in E'\end{cases}$$

$$(\eta_{2'}\times\eta_{2''})\big((r_1,h_1),(r_2,h_2)\big)=\begin{cases}\min\{\eta_{1'}(r_1),\eta_{2''}(h_1,h_2)\} & \text{if} r_1=r_2,(h_1,h_2)\in E''\\ \min\{\eta_{2'}(r_1,r_2),\eta_{1''}(h_1)\} & \text{if} h_1=h_2,(r_1,r_2)\in E'\end{cases}$$

$$(\nu_{2'}\times\nu_{2''})\big((r_1,h_1),(r_2,h_2)\big)=\begin{cases}\max\{\nu_{1'}(r_1),\nu_{2''}(h_1,h_2)\} & \text{if} r_1=r_2,(h_1,h_2)\in E''\\ \max\{\nu_{2'}(r_1,r_2),\nu_{1''}(h_1)\} & \text{if} h_1=h_2,(r_1,r_2)\in E'\end{cases}$$

Definition 23 Let $G=G'\times G''=\big(T,E,M_{T'}\times M_{T''},M_{E'}\times M_{E''}\big)$ be the Cartesian product of two PFGs G' and G''. Then the degree of the node (r_1,h_1) in T is denoted by

$$d_{G'\times G''}(r_1,h_1)=\big(d_{1G'\times G''}(r_1,h_1),d_{2G'\times G''}(r_1,h_1),d_{3G'\times G''}(r_1,h_1)\big)$$

and is defined by

$$\begin{aligned}d_{1G'\times G''}(r_1,h_1)&=\sum_{((r_1,h_1),(r_2,h_2))\in E}(\mu_{2'}\times\mu_{2''})\big((r_1,h_1),(r_2,h_2)\big)\\&=\sum_{r_1=r_2,(h_1,h_2))\in E''}\min\{\mu_{1'}(r_1),\mu_{2''}(h_1,h_2)\}+\sum_{h_1=h_2,(r_1,r_2))\in E'}\min\{\mu_{2'}(r_1,r_2),\mu_{1''}(h_1)\}.\end{aligned}$$

$$\begin{aligned}d_{2G'\times G''}(r_1,h_1)&=\sum_{((r_1,h_1),(r_2,h_2))\in E}(\eta_{2'}\times\eta_{2''})\big((r_1,h_1),(r_2,h_2)\big)\\&=\sum_{r_1=r_2,(h_1,h_2))\in E''}\min\{\eta_{1'}(r_1),\eta_{2''}(h_1,h_2)\}+\sum_{h_1=h_2,(r_1,r_2))\in E'}\min\{\eta_{2'}(r_1,r_2),\eta_{1''}(h_1)\}.\end{aligned}$$

$$\begin{aligned}d_{3G'\times G''}(r_1,h_1)&=\sum_{((r_1,h_1),(r_2,h_2))\in E}(\nu_{2'}\times\nu_{2''})\big((r_1,h_1),(r_2,h_2)\big)\\&=\sum_{r_1=r_2,(h_1,h_2))\in E''}\max\{\nu_{1'}(r_1),\nu_{2''}(h_1,h_2)\}+\sum_{h_1=h_2,(r_1,r_2))\in E'}\max\{\nu_{2'}(r_1,r_2),\nu_{1''}(h_1)\}.\end{aligned}$$

Theorem 6 Let G' and G'' be two PFGs. If

$$\mu_{1'}\ge\mu_{2''},\ \eta_{1'}\ge\eta_{2''},\ \nu_{1'}\le\nu_{2''},\ \mu_{1''}\ge\mu_{2'},\ \eta_{1''}\ge\eta_{2'},\ \nu_{1''}\le\nu_{2'}$$

then $d_{G'\times G''}(r_1,h_1)=d_{G'}(r_1)+d_{G''}(h_1)$.

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Proof. From the definition of degree of a node in Cartesian product of PFGs, we get,

$$\begin{aligned} d_{1G'\times G''}(r_1,h_1) &= \sum_{((r_1,h_1),(r_2,h_2))\in E}(\mu_{2'}\times\mu_{2''})\big((r_1,h_1),(r_2,h_2)\big) \\ &= \sum_{r_1=r_2,(h_1,h_2))\in E''}\min\{\mu_{1'}(r_1),\mu_{2''}(h_1,h_2)\} \\ &\quad +\sum_{h_1=h_2,(r_1,r_2))\in E'}\min\{\mu_{2'}(r_1,r_2),\mu_{1''}(h_1)\} \\ &= \sum_{(h_1,h_2))\in E''}\mu_{2''}(h_1,h_2)+\sum_{(r_1,r_2))\in E'}\mu_{2'}(r_1,r_2) \end{aligned}$$

since, $\mu_{1'}\geq\mu_{2''}$, $\mu_{1''}\geq\mu_{2'}$.

$$=\sum_{(r_1,r_2))\in E'}\mu_{2'}(r_1,r_2)+\sum_{(h_1,h_2))\in E''}\mu_{2''}(h_1,h_2)=d_{1G'}(r_1)+d_{1G''}(h_1).$$

$$\begin{aligned} d_{2G'\times G''}(r_1,h_1) &= \sum_{((r_1,h_1),(r_2,h_2))\in E}(\eta_{2'}\times\eta_{2''})\big((r_1,h_1),(r_2,h_2)\big) \\ &= \sum_{r_1=r_2,(h_1,h_2))\in E''}\min\{\eta_{1'}(r_1),\eta_{2''}(h_1,h_2)\} \\ &\quad +\sum_{h_1=h_2,(r_1,r_2))\in E'}\min\{\eta_{2'}(r_1,r_2),\eta_{1''}(h_1)\} \\ &= \sum_{(h_1,h_2))\in E''}\eta_{2''}(h_1,h_2)+\sum_{(r_1,r_2))\in E'}\eta_{2'}(r_1,r_2) \end{aligned}$$

since, $\eta_{1'}\geq\eta_{2''}$, $\eta_{1''}\geq\eta_{2'}$.

$$=\sum_{(r_1,r_2))\in E'}\eta_{2'}(r_1,r_2)+\sum_{(h_1,h_2))\in E''}\eta_{2''}(h_1,h_2)=d_{2G'}(r_1)+d_{2G''}(h_1).$$

$$\begin{aligned} d_{3G'\times G''}(r_1,h_1) &= \sum_{((r_1,h_1),(r_2,h_2))\in E}(\nu_{2'}\times\nu_{2''})\big((r_1,h_1),(r_2,h_2)\big) \\ &= \sum_{r_1=r_2,(h_1,h_2))\in E''}\max\{\nu_{1'}(r_1),\nu_{2''}(h_1,h_2)\} \\ &\quad +\sum_{h_1=h_2,(r_1,r_2))\in E'}\max\{\nu_{2'}(r_1,r_2),\nu_{1''}(h_1)\} \\ &= \sum_{(h_1,h_2))\in E''}\nu_{2''}(h_1,h_2)+\sum_{(r_1,r_2))\in E'}\nu_{2'}(r_1,r_2) \end{aligned}$$

since, $\nu_{1'}\leq\nu_{2''}$, $\nu_{1''}\leq\nu_{2'}$.

$$=\sum_{(r_1,r_2))\in E'}\nu_{2'}(r_1,r_2)+\sum_{(h_1,h_2))\in E''}\nu_{2''}(h_1,h_2)=d_{3G'}(r_1)+d_{3G''}(h_1).$$

Hence, $d_{G'\times G''}(r_1,h_1)=d_{G'}(r_1)+d_{G''}(h_1)$.

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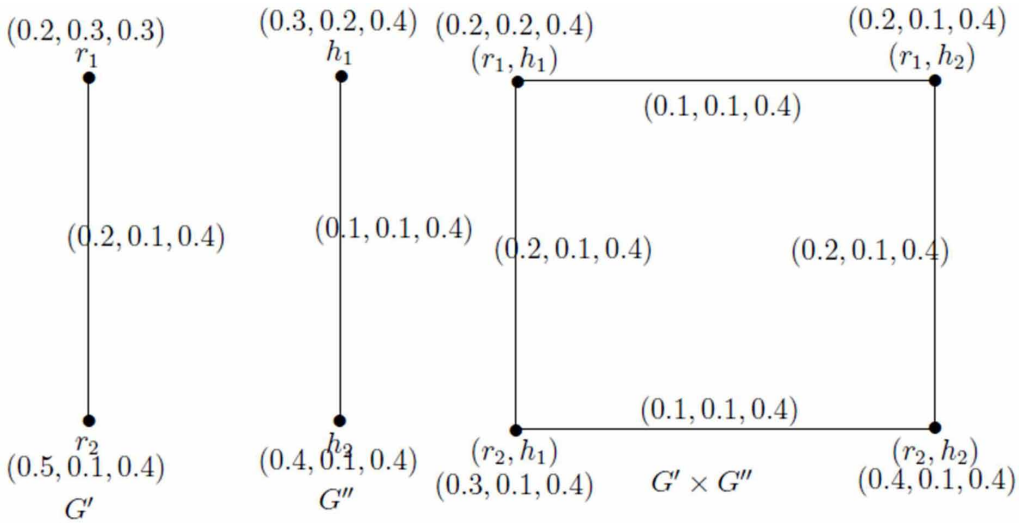
Figure 10. Cartesian product of two PFGs



EXAMPLE 1. Let us consider two PFGs G' and G''. The Cartesian product $G'\times G''$ of G' and G'' is shown in Figure 10. For these PFGs,

$\mu_{1'} \geq \mu_{2''},\ \eta_{1'} \geq \eta_{2''},\ \nu_{1'} \leq \nu_{2''},\ \mu_{1''} \geq \mu_{2'},\ \eta_{1''} \geq \eta_{2'},\ \nu_{1''} \leq \nu_{2'}$

by Theorem 6 we get,

$$d_{1G'\times G''}(r_1, h_1) = d_{1G'}(r_1) + d_{1G''}(h_1)$$

$$= 0.2 + 0.1 = 0.3$$

$$d_{2G'\times G''}(r_1, h_1) = d_{2G'}(r_1) + d_{2G''}(h_1)$$

$$= 0.1 + 0.1 = 0.2$$

$$d_{3G'\times G''}(r_1, h_1) = d_{3G'}(r_1) + d_{3G''}(h_1)$$

$$= 0.4 + 0.4 = 0.8$$

Therefore, $d_{G'\times G''}(r_1, h_1) = (0.3, 0.2, 0.8)$. Similarly, we can compute the degree of all other nodes in the Cartesian product $G'\times G''$.

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COMPOSITION ON PICTURE FUZZY GRAPHS

In the present section, the composition of two PFGs is introduced. Also, the degree of a node in composition of PFGs is calculated. Some results regarding composition of PFGs are presented in this section.

Definition 24 The composition of two PFGs

$G' = (T', E', M_{T'}, M_{E'})$ and $G'' = (T'', E'', M_{T''}, M_{E''})$,

where,

$M_{T'} = (\mu_{1'}, \eta_{1'}, \nu_{1'})$, $M_{T''} = (\mu_{1''}, \eta_{1''}, \nu_{1''})$, $M_{E'} = (\mu_{2'}, \eta_{2'}, \nu_{2'})$, $M_{E''} = (\mu_{2''}, \eta_{2''}, \nu_{2''})$

is defined as a

$PFGG = G'[G''] = (T, E, M_{T'} \circ M_{T''}, M_{E'} \circ M_{E''})$,

where, $T = T' \times T''$ and

$$E = \{((r_1, h_1), (r_2, h_2)) : r_1 = r_2, (h_1, h_2) \in E'' or h_1 = h_2, (r_1, r_2) \in E'\} \cup \{((r_1, h_1), (r_2, h_2)) : h_1 = h_2, (r_1, r_2) \in E'\}$$

with

$$(\mu_{1'} \bullet \mu_{1''})(r_1, h_1) = \min\{\mu_{1'}(r_1), \mu_{1''}(h_1)\}$$

$$(\eta_{1'} \bullet \eta_{1''})(r_1, h_1) = \min\{\eta_{1'}(r_1), \eta_{1''}(h_1)\}$$

$$(\nu_{1'} \bullet \nu_{1''})(r_1, h_1) = \max\{\nu_{1'}(r_1), \nu_{1''}(h_1)\}$$

for all $(r_1, h_1) \in T' \times T''$ and

$$(\mu_{2'} \bullet \mu_{2''})((r_1, h_1), (r_2, h_2)) = \begin{cases} \min\{\mu_{1'}(r_1), \mu_{2''}(h_1, h_2)\} & \text{if } r_1 = r_2, (h_1, h_2) \in E'' \\ \min\{\mu_{2'}(r_1, r_2), \mu_{1''}(h_1)\} & \text{if } h_1 = h_2, (r_1, r_2) \in E' \\ \min\{\mu_{2'}(r_1, r_2), \mu_{1'}(h_1), \mu_{1''}(h_2)\} & \text{if } h_1 \neq h_2, (r_1, r_2) \in E' \end{cases}$$

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$$\left(\eta_{2'}\bullet\eta_{2''}\right)\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)=\begin{cases}\min\left\{\eta_{1'}\left(r_1\right),\eta_{2''}\left(h_1,h_2\right)\right\} & \text{if } r_1=r_2,\left(h_1,h_2\right)\in E''\\ \min\left\{\eta_{2'}\left(r_1,r_2\right),\eta_{1''}\left(h_1\right)\right\} & \text{if } h_1=h_2,\left(r_1,r_2\right)\in E'\\ \min\left\{\eta_{2'}\left(r_1,r_2\right),\eta_{1'}\left(h_1\right),\eta_{1''}\left(h_2\right)\right\} & \text{if } h_1\neq h_2,\left(r_1,r_2\right)\in E'\end{cases}$$

$$\left(\nu_{2'}\bullet\nu_{2''}\right)\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)=\begin{cases}\max\left\{\nu_{1'}\left(r_1\right),\nu_{2''}\left(h_1,h_2\right)\right\} & \text{if } r_1=r_2,\left(h_1,h_2\right)\in E''\\ \max\left\{\nu_{1'}\left(r_1,r_2\right),\nu_{1''}\left(h_1\right)\right\} & \text{if } h_1=h_2,\left(r_1,r_2\right)\in E'\\ \max\left\{\nu_{1'}\left(r_1,r_2\right),\nu_{1'}\left(h_1\right),\nu_{1''}\left(h_2\right)\right\} & \text{if } h_1\neq h_2,\left(r_1,r_2\right)\in E'\end{cases}$$

Definition 25 Let $G=G'\left[G''\right]=\left(T,E,M_{T'}\bullet M_{T''},M_{E'}\bullet M_{E''}\right)$ be the composition of two PFGs G' and G''. Then the degree of the node (r_1,h_1) in T is denoted by

$$d_{G'\left[G''\right]}\left(r_1,h_1\right)=\left(d_{1G'\left[G''\right]}\left(r_1,h_1\right),d_{2G'\left[G''\right]}\left(r_1,h_1\right),d_{3G'\left[G''\right]}\left(r_1,h_1\right)\right)$$

and is defined by

$$\begin{aligned}d_{1G'\left[G''\right]}\left(r_1,h_1\right)&=\sum_{\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)\in E}\left(\mu_{2'}\bullet\mu_{2''}\right)\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)\\&=\sum_{r_1=r_2,\left(h_1,h_2\right)\in E''}\min\left\{\mu_{1'}\left(r_1\right),\mu_{2''}\left(h_1,h_2\right)\right\}\\&+\sum_{h_1=h_2,\left(r_1,r_2\right)\in E'}\min\left\{\mu_{2'}\left(r_1,r_2\right),\mu_{1''}\left(h_1\right)\right\}\\&+\sum_{h_1\neq h_2,\left(r_1,r_2\right)\in E'}\min\left\{\mu_{2'}\left(r_1,r_2\right),\mu_{1'}\left(h_1\right),\mu_{1''}\left(h_2\right)\right\}\end{aligned}$$

$$\begin{aligned}d_{2G'\left[G''\right]}\left(r_1,h_1\right)&=\sum_{\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)\in E}\left(\eta_{2'}\bullet\mu_{2''}\right)\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)\\&=\sum_{r_1=r_2,\left(h_1,h_2\right)\in E''}\min\left\{\eta_{1'}\left(r_1\right),\mu_{2''}\left(h_1,h_2\right)\right\}\\&+\sum_{h_1=h_2,\left(r_1,r_2\right)\in E'}\min\left\{\eta_{2'}\left(r_1,r_2\right),\mu_{1''}\left(h_1\right)\right\}\\&+\sum_{h_1\neq h_2,\left(r_1,r_2\right)\in E'}\min\left\{\eta_{2'}\left(r_1,r_2\right),\eta_{1'}\left(h_1\right),\eta_{1''}\left(h_2\right)\right\}\end{aligned}$$

$$\begin{aligned}d_{3G'\left[G''\right]}\left(r_1,h_1\right)&=\sum_{\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)\in E}\left(\nu_{2'}\bullet\nu_{2''}\right)\left(\left(r_1,h_1\right),\left(r_2,h_2\right)\right)\\&=\sum_{r_1=r_2,\left(h_1,h_2\right)\in E''}\max\left\{\nu_{1'}\left(r_1\right),\nu_{2''}\left(h_1,h_2\right)\right\}\\&+\sum_{h_1=h_2,\left(r_1,r_2\right)\in E'}\max\left\{\nu_{2'}\left(r_1,r_2\right),\nu_{1''}\left(h_1\right)\right\}\\&+\sum_{h_1\neq h_2,\left(r_1,r_2\right)\in E'}\max\left\{\nu_{2'}\left(r_1,r_2\right),\nu_{1'}\left(h_1\right),\nu_{1''}\left(h_2\right)\right\}\end{aligned}$$

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Theorem 7 Let G' and G'' be two PFGs. If

$$\mu_{1'} \geq \mu_{2''},\ \eta_{1'} \geq \eta_{2''},\ \nu_{1'} \leq \nu_{2''},\ \mu_{1''} \geq \mu_{2'},\ \eta_{1''} \geq \eta_{2'},\ \nu_{1''} \leq \nu_{2'}$$

then $d_{G'[G'']}(r_1,h_1) = |T''| d_{G'}(r_1) + d_{G''}(h_1)$.

Proof. From the definition of degree of a node in composition of PFGs, we get,

$$\begin{aligned} d_{1G'[G'']}(r_1,h_1) &= \sum_{((r_1,h_1),(r_2,h_2))\in E} (\mu_{2'} \bullet \mu_{2''})\big((r_1,h_1),(r_2,h_2)\big) = \sum_{r_1=r_2,(h_1,h_2))\in E''} \min\{\mu_{1'}(r_1), \mu_{2''}(h_1,h_2)\} \\ &+ \sum_{h_1=h_2,(r_1,r_2))\in E'} \min\{\mu_{2'}(r_1,r_2), \mu_{1''}(h_1)\} + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \min\{\mu_{2'}(r_1,r_2), \mu_{1'}(h_1), \mu_{1''}(h_2)\} \\ &= \sum_{(h_1,h_2))\in E''} \mu_{2''}(h_1,h_2) + \sum_{h_1=h_2,(r_1,r_2))\in E'} \mu_{2'}(r_1,r_2) + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \mu_{2'}(r_1,r_2) \end{aligned}$$

since, $\mu_{1'} \geq \mu_{2''},\ \mu_{1''} \geq \mu_{2'}$.

$$\begin{aligned} &= \sum_{h_1=h_2,(r_1,r_2))\in E'} \mu_{2'}(r_1,r_2) + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \mu_{2'}(r_1,r_2) \\ &+ \sum_{(h_1,h_2))\in E''} \mu_{2''}(h_1,h_2) = |T''| d_{1G'}(r_1) + d_{1G''}(h_1) \end{aligned}$$

.

$$\begin{aligned} d_{2G'[G'']}(r_1,h_1) &= \sum_{((r_1,h_1),(r_2,h_2))\in E} (\eta u_{2'} \bullet \eta_{2''})\big((r_1,h_1),(r_2,h_2)\big) \\ &= \sum_{r_1=r_2,(h_1,h_2))\in E''} \min\{\eta h_{1'}(r_1), \mu_{2''}(h_1,h_2)\} \\ &+ \sum_{h_1=h_2,(r_1,r_2))\in E'} \min\{\eta_{2'}(r_1,r_2), \eta_{1''}(h_1)\} + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \min\{\eta_{2'}(r_1,r_2), \eta_{1'}(h_1), \eta_{1''}(h_2)\} \\ &= \sum_{(h_1,h_2))\in E''} \eta_{2''}(h_1,h_2) + \sum_{h_1=h_2,(r_1,r_2))\in E'} \eta_{2'}(r_1,r_2) + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \eta_{2'}(r_1,r_2) \end{aligned}$$

since, $\eta_{1'} \geq \eta_{2''},\ \eta_{1''} \geq \eta_{2'}$.

$$= \sum_{h_1=h_2,(r_1,r_2))\in E'} \eta_{2'}(r_1,r_2) + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \eta_{2'}(r_1,r_2) + \sum_{(h_1,h_2))\in E''} \eta_{2''}(h_1,h_2) = |T''| d_{2G'}(r_1) + d_{2G''}(h_1)$$

$$\begin{aligned} d_{3G'[G'']}(r_1,h_1) &= \sum_{((r_1,h_1),(r_2,h_2))\in E} (\nu_{2'} \bullet \nu_{2''})\big((r_1,h_1),(r_2,h_2)\big) \\ &= \sum_{r_1=r_2,(h_1,h_2))\in E''} \max\{\nu_{1'}(r_1), \nu_{2''}(h_1,h_2)\} + \sum_{h_1=h_2,(r_1,r_2))\in E'} \max\{\nu_{2'}(r_1,r_2), \nu_{1''}(h_1)\} \\ &+ \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \max\{\nu_{2'}(r_1,r_2), \nu_{1'}(h_1), \nu_{1''}(h_2)\} \\ &= \sum_{(h_1,h_2))\in E''} \nu_{2''}(h_1,h_2) + \sum_{h_1=h_2,(r_1,r_2))\in E'} \nu_{2'}(r_1,r_2) + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \nu_{2'}(r_1,r_2) \end{aligned}$$

since, $\nu_{1'} \leq \nu_{2''},\ \nu_{1''} \leq \nu_{2'}$.

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$$= \sum_{h_1=h_2,(r_1,r_2))\in E'} \nu_{2'}(r_1,r_2) + \sum_{h_1\neq h_2,(r_1,r_2))\in E'} \nu_{2'}(r_1,r_2) + \sum_{(h_1,h_2))\in E''} \nu_{2''}(h_1,h_2) = |T''| d_{3G'}(r_1) + d_{3G''}(h_1)$$

Hence, $d_{G'[G'']}(r_1,h_1) = |T''| d_{G'}(r_1) + d_{G''}(h_1)$.

EXAMPLE 2. In this example, we consider two PFGs G' and G'' and their composition $G' \bullet G''$ (see Figure 11). For these PFG,

$$\mu_{1'} \geq \mu_{2''},\ \eta_{1'} \geq \eta_{2''},\ \nu_{1'} \leq \nu_{2''},\ \mu_{1''} \geq \mu_{2'},\ \eta_{1''} \geq \eta_{2'},\ \nu_{1''} \leq \nu_{2'}$$

by Theorem 7,

$$d_{1G'[G'']}(r_1,h_1) = |T''| d_{1G'}(r_1) + d_{1G''}(h_1)$$

$$= 2 \times 0.2 + 0.1 = 0.5$$

$$d_{2G'[G'']}(r_1,h_1) = |T''| d_{2G'}(r_1) + d_{2G''}(h_1)$$

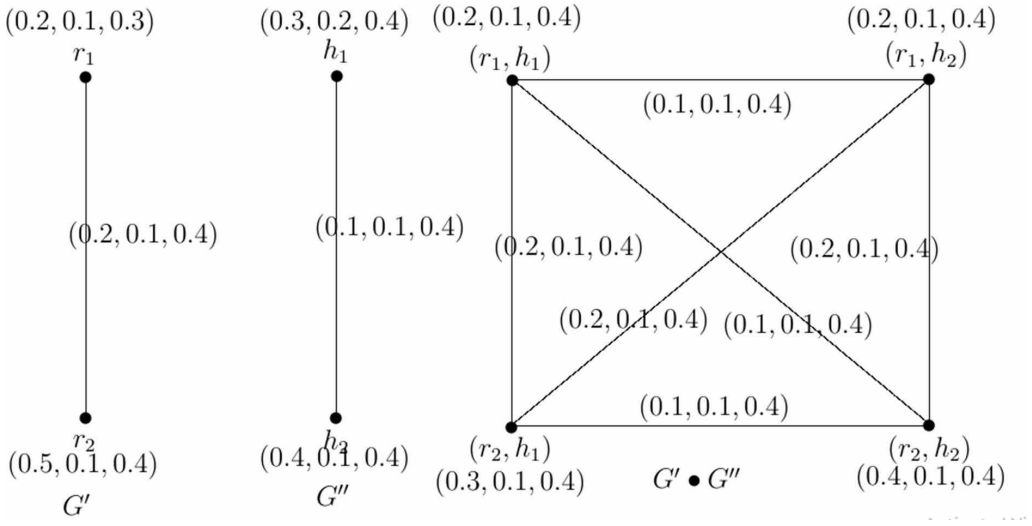
$$= 2 \times 0.1 + 0.1 = 0.3$$

$$d_{3G'[G'']}(r_1,h_1) = |T''| d_{3G'}(r_1) + d_{3G''}(h_1)$$

$$= 2 \times 0.4 + 0.4 = 1.2$$

Thus, $d_{G'[G'']}(r_1,h_1) = (0.5, 0.3, 1.2)$. Similarly, we can compute the degree of all nodes in $G'[G'']$.

Figure 11. Composition of two PFGs



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AN APPLICATION OF PICTURE FUZZY GRAPH

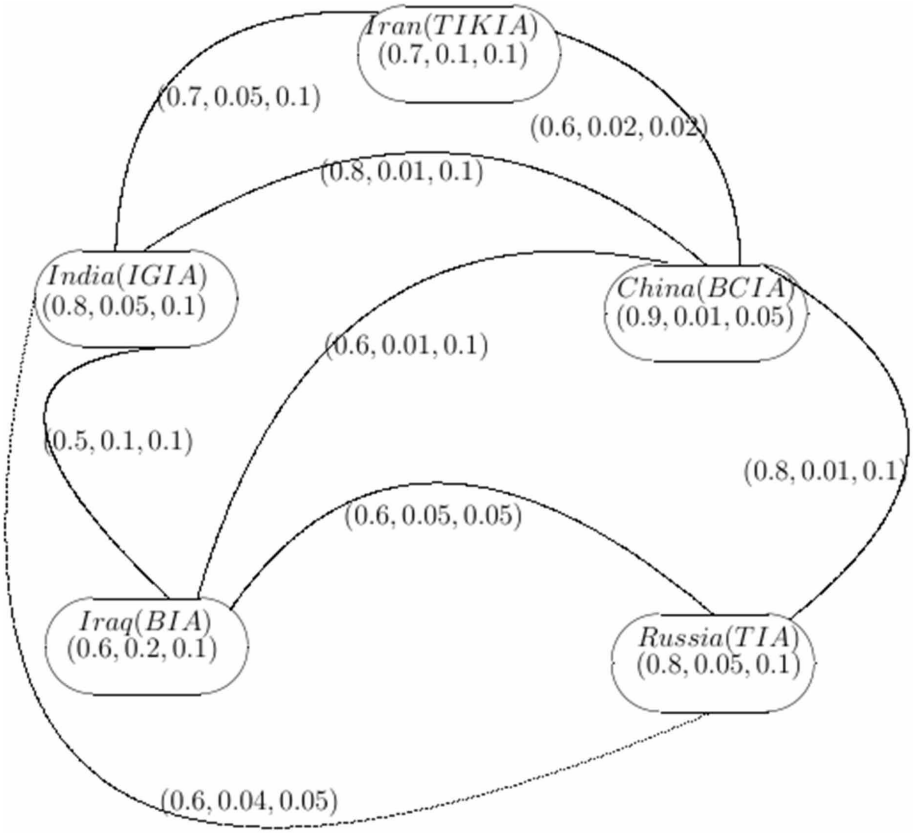
Many problems are availabe in our daily lifewhichneither solve by fuzzy graph theory nor by intuitionistic fuzzy graph theory. These type of problems can be solved by using PFG. An applicaion is presented here which is solved by picture fuzzy graph:

Let us consider five countries say, China, India, Iraq, Iran and Russia. Suppose anybody want to travel between any two countries by flight. We assued the following:

1. To travel India, Indra Gandhi International Airport (IGIA) is used.
2. To travel Iran, Tehran Imam Khomeini International Airport (TIKIA) is used.
3. To travel China, Beijing Capital International Airport (BCIA) is used.
4. To travel Russia, Tnukovo International Airport (TIA) is used.
5. To travel Iraq, Baghdad International Airport (BIA) is used.

The aim of the flight operators is to give the high quality facility for passengers. Air traffic controllers have to make sure that the flights must depart and arrive at right time. This task is possible by planning efficient routes for the flight. A picture fuzzy airline network among the five countries is shown in Figure 12. where countries are represented by nodes and flights between countries are represented by edges.

Figure 12. A picture fuzzy airline network



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Further we assumes that:

1. Positive membership degree of the nodes indicates that the strength of the airline system of the country.
2. Neutral membership degree of the nodes indicates the degree of undetermined part of theailine system.
3. Negative membership degree of each nodes indicates the degree of defect of the system.
4. Positive membership degree of each edge indicates the safety of the flight.
5. Neutral membership degree of every edge shows that the uncertain situations during a flight such as mechanical error, weather conditions, sabotage, etc.
6. Negative membership degree of every edge indicates the defects of the flight.

For example, the membership value of the edge (*India*(*IGIA*), *Iran(TIKIA*)) is (0.7, 0.05, 0.1). It indicates that the flight chosen for this journey is 70% safe, 5% depends on uncertain situation and 10% unsafe. Suppose anyone want to go from India(IGIA) to China(BCIA) by direct flight but due to personal problems or weather conditions or technical problems, a passenger missed his direct flight between these two countries. Therefore, if a passenger needs to move urgently from India(IGIA) to China(BCIA) then he/she has to choose alternative route, as there are alternative routes between these countries. There are many alternate routes between India(IGIA) and China(BCIA), and these are listed below:

R_1: India(IGIA) → Iran(TIKIA) → China(BCIA).
R_2: India(IGIA) → Iraq(BIA) →China(BCIA).
R_3: India(IGIA) → Russia(TIA) → China(BCIA).
R_4: India(IGIA) → Iraq(BIA) → Russia(TIA) China(BCIA).

Now, the problem is to find the most suitable route by finding the strength of these routes(paths). The μ-strength of the paths R_1, R_2, R_3, R_4 are respectively $S_\mu(R_1)=0.6$, $S_\mu(R_2)=0.5$, $S_\mu(R_3)=0.6$, $S_\mu(R_4)=0.5$. The η-strength of the same pathsR_1, R_2, R_3, R_4 are respectively $S_\eta(R_1)=0.06$, $S_\eta(R_2)=0.1$, $S_\eta(R_3)=0.04$, $S_\eta(R_4)=0.1$. The v-strengthare respectively $S_v(R_1)=0.1$, $S_v(R_2)=0.1$, $S_v(R_3)=0.1$, $S_v(R_4)=0.1$. By the definition of μ-strength it is obvious that more μ-strengthmeans more strong path. If the μ-strength of two or more paths are same then we can check the v-strength of the paths and the path with minimum v-strength is the suitable path among the paths. If the μ-strength and the v-strength of two or more paths are same then η-strength of the path play vital role and the path with minimum η-strength is the suitable path among the paths. In this example, $S_\mu(R_1)=S_\mu(R_3)=0.6$ and also $S_v(R_1)=S_v(R_3)=0.1$, but $S_\eta(R_1)>S_\eta(R_3)$, so the path R_3 is the best paths among other alternative paths. Hence, best chosen path from India (IGIA) to China (BCIA) is India(IGIA) → Russia(TIA) → China(BCIA).

SUMMARY

Fuzzy graph theory is a very large tool to solve a many real life problems, but it has some limitations. The intuitionistic fuzzy graphs overcome some limitations of fuzzy graph theory and recently, picture fuzzy graphs overcome more limitations. Still now some types of problems cannot be addressed properly by these graphs, so need further investigations. In this chapter, fundamentals terms and product of PFGsare

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included which helps to the beginners. Also, an application to solve a daily life problem is incorporated to illustrate the use of PFGs.

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Chapter 19
Separation Axioms and Connectedness on Fuzzy Parameterized Fuzzy Soft Topological Spaces

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ABSTRACT

The fuzzy parameterized fuzzy soft (FPFS) set structure was defined in 2010 as a different combination of fuzzy sets and soft sets. In 2016, the topological structure on this structure was defined. By using this set structure and topological concepts, different studies have been made and continue to be done on many decision-making problems and daily life problems. In this chapter, the authors define and investigate some properties of fuzzy parameterized fuzzy soft (FPFS) Ti (i=0,1,2,3,4) spaces using fuzzy parameterized fuzzy soft points. They also discuss some hereditary properties of these spaces. Further, they introduce fuzzy parameterized fuzzy soft connected spaces. Moreover, they explore some basic properties of these spaces.

1. INTRODUCTION

Classical mathematics is insufficient to model a complicated system when knowledge and information are imperfect. The probability theory has historically been employed to deal with uncertainty. However, while this technique is appropriate for representing certain information with certain bounds, the probability theory cannot describe all of the conceivable difficulties of the incomplete aspects. To overcome these uncertainties, certain theories were provided, such as the theory of fuzzy set (Zadeh, 1965), intu-

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itionistic fuzzy set (Atanassov, 1999), rough set (Pawlak, 1982), soft set (Molodtsov, 1999), i.e. all of which may be used as mathematical tools for dealing with uncertainties. To handle uncertain issues with sharp boundaries, the fuzzy set theory has developed. Zadeh (Zadeh, 1965) extended the usual definition of a set with the introduction of the fuzzy set in 1965. The fuzzy set theory has been effectively applied to a variety of real-world situations in an ambiguous and hazy situation. In 1968, Chang (Chang, 1968) described fuzzy topology and implemented a variety of topological concepts in a fuzzy setting.

Molodtsov (Molodtsov, 1999) pioneered the notion of soft set theory as a new mathematical tool for dealing with uncertainty that is free of the challenges mentioned above. Following the publication of soft set operations (Maji et. al., 2003), the characteristics and applications of soft set theory have been intensively researched (Ali et.al, 2009; Cagman, 2014; Sezgin&Atagün, 2011). Soft sets and information systems were explored by Xiao et al. (Xiao et.al., 2005) and Pei and Miao (Pei&Miao, 2005). They demonstrated that soft sets constitute a type of special information system. Many fascinating applications of soft set theory have been extended in recent years by incorporating the principles of fuzzy sets (Cagman el.al., 2011; Dizman&Ozturk, 2021; Khan&Taouti, 2021; Mohanty&Tripathy, 2021; Ozturk, 2020; Roy&Maji, 2007; Yolcu&Ozturk, 2021). The operations of soft sets are redefined in order to create soft set theory, and a uni-int decision making procedure is developed employing these new operations (Cagman&Enginoglu, 2014). Shabir and Naz (Shabir&Naz, 2011) introduced the research of soft topological spaces in 2011 and proposed several concepts about soft topological spaces. Min (Min, 2011) investigates some of the characteristics of these soft separation axioms. Kandil et al. introduced and explored the features of soft operations such as semi open soft set, pre-open soft set, α-open soft set and β-open soft set (Kandil et.al., 2014).

Researchers have combined fuzzy set and soft set to broaden the spaces and tackle increasingly difficult issues. In this approach, several intriguing applications of soft set theory have been extended. As the first hybrid of a fuzzy set with a soft set, Maji et al. presented the fuzzy soft set (Maji et.al., 2001). Many studies are carried out every year using fuzzy and fuzzy soft set structures (Zararsız, 2015; Riaz&Tehrim, 2019; Enginoglu et.al, 2019; Riaz&Hashmi, 2019; Garg et.al, 2021; Riaz et.al., 2021; Ayub et.al. 2021). Many concepts such as separation axioms, compactness, connectedness on fuzzy soft topological structure have been studied by various researchers (Abd El-latif, 2015; Kandil et.al., 2015; Karatas et.al., 2015; Mahanta&Das, 2012). Cagman and colleagues proposed a second combination of fuzzy set and soft set, called fuzzy parametrized soft set (as shortly FPS set) (Cagman et.al., 2010). Recently, many studies have been done on FPS set structure (Alkhazaleh et.al., 2011; Enginoglu&Cagman, 2020; Zhu&Zhan, 2016; Riaz&Raza, 2020). Zorlutuna and Atmaca present the topological structure of fuzzy parameterized fuzzy soft sets (shortly FPFS) (Zorlutuna&Atmaca, 2016). They also explored the basic properties of the notion of quasi-coincidence for fuzzy parametrized fuzzy soft sets. Riaz and Hashmi (Riaz&Hashmi, 2017) investigated the idea of fuzzifying soft sets, known as fuzzy parameterized fuzzy soft sets (FPFS-sets), as well as certain results that are true in crisp set theory but not in FPFS-set theory. In the sense of FPFS-topological spaces, they added closure, interior, frontier and exterior. They also mention certain quasi-coincidence and Q-neighborhood properties for the FPFS-sets.

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2. PRELIMINARIES

Throughout this paper U denotes initial universe, E denotes the set of all possible parameters which are attributes, characteristic or properties of the objects in U and the set of all subsets of U will be denoted by $P(U)$.

Definition 2.1 (Molodtsov, 1999) A pair (H,E) is called a soft set over U if it is a mapping defined by H: $E \to P(U)$. In the other words, a soft set is a parameterized family of subsets of the set U. For each $\alpha \in E$, the set $H(\alpha)$ may be considered as the set of α-elements of the soft set (H,E).

Definition 2.2 (Zadeh, 1965) Let U be an initial universe. A fuzzy set Λ in U, $\Lambda = \{(u, \mu_\Lambda(u)) : u \in U\}$, where μ_Λ: $U \to [0,1]$ is the membership function of the fuzzy set Λ. The set of all fuzzy sets over U will be denoted by $FP(U)$.

Definition 2.3 (Maji et.al., 2001) Let U be an initial universe, E be a set of parameters and $FP(U)$ be the set of all fuzzy sets in U. Then a pair (h,E) is called a fuzzy soft set over U, where h: $E \to FP(U)$."

Definition 2.4 (Cagman et.al., 2010) "Let U be an initial universe, E be the set of all parameters and Λ be a fuzzy set over U. A fuzzy parameterized fuzzy soft (FPFS) set H_Λ on the universe U is defined as follows;

$$H_\Lambda = \left\{(\mu_\Lambda(\alpha)/\alpha, \lambda_\Lambda(\alpha)) : \alpha \in E, \lambda_\Lambda(\alpha) \in FP(U); \mu_\Lambda(\alpha), \lambda_\Lambda(\alpha)(u) \in [0,1], u \in U\right\}$$

where λ_Λ: $E \to FP(U)$ is called approximate function of $H\Lambda$ such that $\lambda_\Lambda(\alpha)=0$ if $\mu_\Lambda(\alpha)=0$. The family of all FPFS sets over U denoted by $FPFS(U,E)$.

Definition 2.5 (Cagman et.al., 2010)

i) Let $H_\Lambda \in FPFS(U,E)$. If $\lambda_\Lambda(\alpha) = \varnothing$ for all $\alpha \in E$, then $H\Lambda$ is called an Λ-empty FPFS-set, denoted by $H_{\varnothing_\Lambda}$. If $\Lambda = \varnothing$, then Λ-empty FPFS-set is called empty FPFS-set and denoted by $H_\varnothing$.

ii) If $\mu_\Lambda(\alpha)=1$ and $\lambda_\Lambda(\alpha)=U$ for all $\alpha \in E$, then H_Λ is called absolute FPFS-set and denoted by $H_{\tilde{E}}$.

Definition 2.6 (Cagman et.al., 2010) Let $H_\Lambda \in FPFS(U,E)$. Then the complement of H_Λ is denoted by $(H_\Lambda)^c$ and defined as

$$\mu_{\Lambda^c}(\alpha) = 1 - \mu_\Lambda(\alpha) \text{ and } \lambda_{\Lambda^c}(\alpha) = 1 - \lambda_\Lambda(\alpha)(u)$$

for all $\alpha \in E$ and $u \in U$.

Definition 2.7 (Cagman et.al., 2010) Let $H_{\Lambda_1}, H_{\Lambda_2} \in FPFS(U,E)$. Then H_Λ is an *FPFS*-subset of H_{Λ_2}, denoted by $H_{\Lambda_1} \widetilde{\subseteq} H_{\Lambda_2}$, if

1. $\mu_{\Lambda_1}(\alpha) \le \mu_{\Lambda_2}(\alpha)$
2. $\lambda_{\Lambda_1}(\alpha) \subseteq \lambda_{\Lambda_2}(\alpha), \forall \alpha \in E$.

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Definition 2.8 (Zorlutuna&Atmaca, 2016) The FPFS set $H_\Lambda \in FPFS(U,E)$ is called FPFS point if Λ is a fuzzy point in E and $\lambda_\Lambda(\alpha)$ is a fuzzy point in U for $\alpha \in \text{supp}_\Lambda$. If $\Lambda=\{\alpha\}$, $\mu_\Lambda(\alpha)=\beta \in [0,1]$ and $\lambda_\Lambda(\alpha)$ (u) = x$\in$[0,1], *t*hen we denote this FPFS point by $\alpha_{u_x}^{\beta}$.

Definition 2.9 (Cagman et.al., 2010) Let $H_{\Lambda_1}, H_{\Lambda_2} \in FPFS(U,E)$.

1. The union of FPFS-sets $H_{\Lambda_1}, H_{\Lambda_2}$, written as $H_{\Lambda_1} \widetilde{\cup} H_{\Lambda_2}$, is defined by

$$\mu_{\Lambda_1 \cup \Lambda_2}(\alpha) = \max\{\mu_{\Lambda_1}(\alpha), \mu_{\Lambda_2}(\alpha)\},\ \lambda_{\Lambda_1 \cup \Lambda_2}(\alpha) = \{\lambda_{\Lambda_1}(\alpha) \cup \lambda_{\Lambda_2}(\alpha)\},\ \forall \alpha \in E.$$

2. The intersection of FPFS-sets $H_{\Lambda_1}, H_{\Lambda_2}$, written as $H_{\Lambda_1} \widetilde{\cap} H_{\Lambda_2}$, is defined by

$$\mu_{\Lambda_1 \cap \Lambda_2}(\alpha) = \min\{\mu_{\Lambda_1}(\alpha), \mu_{\Lambda_2}(\alpha)\},\ \lambda_{\Lambda_1 \cap \Lambda_2}(\alpha) = \{\lambda_{\Lambda_1}(\alpha) \cap \lambda_{\Lambda_2}(\alpha)\},\ \forall \alpha \in E.$$

Definition 2.10 (Zorlutuna&Atmaca, 2016) Let $FPFS(U,E)$ and $FPFS(Y,S)$ be families of all FPFS sets over U and Y, respectively. Let u: $U \to Y$ and v: $E \to S$ be two function. Then a mapping $\tilde{f}_{uv} : FPFS(U,E) \to FPFS(Y,S)$ is defined as;

(1) for $H_\Lambda \in FPFS(U,E)$, the image of H_Λ under the $\tilde{f}_{uv}$ (*briefly* $\tilde{f}$) is the FPFS set G_Υ over Y defined by the approximate function, $\forall s \in S$ and

$$g_\Upsilon(s)(y) = \begin{cases} \bigcup_{x \in u^{-1}(y)} \left(\bigcup_{\alpha \in v^{-1}(s) \cap \limsup p\Lambda} \tilde{f}_\Lambda(\alpha) \right)(x), & \text{if } u^{-1}(y) \neq \varnothing \text{ and } v^{-1}(s) \cap \limsup p\Lambda \neq \varnothing; \\ H_\varnothing, & \text{otherwise} \end{cases}$$

where $v(\Lambda)=\Upsilon$ is fuzzy set in S.

(2) For $G_\Upsilon \in FPFS(Y,S)$, then the pre-image of G_Υ under the $\tilde{f}$ is the FPFS set H_Λ over U defined by the approximation function, $\forall \alpha \in E.\ f_\Lambda(\alpha)(x) = g_\Upsilon(v(\alpha))(u(x))$ where $\Lambda=v^{-1}(\Upsilon)$ is fuzzy set in E.

If u and v is injective, then the FPFS mapping $\tilde{f}$ is said to be injective. If u and v is surjective, then the FPFS mapping $\tilde{f}$ is said to be surjective. The FPFS mapping $\tilde{f}$ is called constant, if u and v are constant.

Theorem 2.1 (Zorlutuna&Atmaca, 2016) Let U and Y crisp sets

$$H_{\Lambda_1}, H_{\Lambda_2} \in FPFS(U,E), G_{\Upsilon_1}, G_{\Upsilon_2} \in FPFS(Y,S),\ \forall i \in J,$$

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where J is an index set. Let $\tilde{f}: FPFS(U,E) \to FPFS(Y,S)$ be a FPFS mapping. Then following conditions are hold.

1. If $H_{\Lambda_1} \tilde{\subseteq} H_{\Lambda_2}$ then $\tilde{f}(H_{\Lambda_1}) \tilde{\subseteq} \tilde{f}(H_{\Lambda_2})$,
2. If $G_{\Upsilon_1} \tilde{\subseteq} G_{\Upsilon_2}$ then $\tilde{f}^{-1}(G_{\Upsilon_1}) \tilde{\subseteq} \tilde{f}^{-1}(G_{\Upsilon_2})$,
3. $H_\Lambda \tilde{\subseteq} \tilde{f}^{-1}(\tilde{f}(H_\Lambda))$ the equality holds if $\tilde{f}$ is injective.
4. $\tilde{f}(\tilde{f}^{-1}(G_\Upsilon)) \tilde{\subseteq} G_\Upsilon$, the equality holds if $\tilde{f}$ is surjective.
5. $\tilde{f}(\bigcup_{i\in J} H_{\Lambda_i}) = \bigcup_{i\in J} \tilde{f}(H_{\Lambda_i})$,
6. $\tilde{f}(\bigcap_{i\in J} H_{\Lambda_i}) \tilde{\subseteq} \bigcap_{i\in J} \tilde{f}(H_{\Lambda_i})$ the equality holds if $\tilde{f}$ is injective,
7. $\tilde{f}^{-1}(\bigcup_{i\in J} G_{\Upsilon_i}) = \bigcup_{i\in J} \tilde{f}^{-1}(G_{\Upsilon_i})$,
8. $\tilde{f}^{-1}(\bigcap_{i\in J} G_{\Upsilon_i}) = \bigcap_{i\in J} \tilde{f}^{-1}(G_{\Upsilon_i})$
9. $\left(\tilde{f}^{-1}(G_\Upsilon)\right)^c = \tilde{f}^{-1}(G_\Upsilon{}^c)$,
10. $\tilde{f}^{-1}(G_{\tilde{S}}) = H_{\tilde{E}}$,
11. $\tilde{f}^{-1}(G_\varnothing) = H_\varnothing$,
12. $\tilde{f}(H_{\tilde{E}}) \tilde{\subseteq} G_{\tilde{S}}$ the equality holds if $\tilde{f}$ is surjective,
13. $\tilde{f}(H_\varnothing) = G_\varnothing$

Definition 2.10 (Zorlutuna&Atmaca, 2016) A FPFS topological space is a pair $(U,\tilde{\tau})$ where U is a non-empty set and $\tilde{\tau}$ is a family of FPFS sets over U satisfying the following properties:

1. $H_\varnothing, H_{\tilde{E}} \in \tilde{\tau}$,
2. If $H_{\Lambda_1}, H_{\Lambda_2} \in \tilde{\tau}$, then $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} \in \tilde{\tau}$,
3. If $H_{\Lambda_i} \in \tilde{\tau}$, $\forall i \in I$, then $\bigcup_{i\in I} H_{\Lambda_i} \in \tilde{\tau}$

Then $\tilde{\tau}$ is called a FPFS-topology on U. Every member of $\tilde{\tau}$ is called FPFS-open set in $(U,\tilde{\tau})$. H_Λ is called a FPFS-closed set in $(U,\tilde{\tau})$ if $(H_\Lambda)^c \in \tilde{\tau}$.

Definition 2.11 (Zorlutuna&Atmaca, 2016) Let $(U,\tilde{\tau})$ be a FPFS topological space. If $\tilde{\tau} = \{H_\varnothing, H_{\tilde{E}}\}$, then $\tilde{\tau}$ is called a FPFS indiscrete topology on U and if $\tilde{\tau} = FPFS(U,E)$, then $\tilde{\tau}$ is called a FPFS discrete topology on U.

Definition 2.12 (Riaz&Hashmi, 2017) Let $(U,\tilde{\tau})$ be a FPFS topological space over U and H_Λ be a FPFS set over U. Then the FPFS topology $\tilde{\tau}_V = \{H_\Lambda \tilde{\cap} G_\Upsilon : G_\Upsilon \in \tilde{\tau}\}$ is called FPFS subspace topology and $(V,\tilde{\tau}_V)$ is called a FPFS subspace of $(U,\tilde{\tau})$.

Definition 2.13 (Zorlutuna&Atmaca, 2016) Let $(U,\tilde{\tau})$ be a FPFS topological space and $H_\Lambda \in FPFS(U,E)$. The FPFS closure of H_Λ, denoted by $\bar{H}_\Lambda$, is the intersection of all FPFS closed supersets of H_Λ.

Theorem 2.2 (Zorlutuna&Atmaca, 2016) Let $(U,\tilde{\tau})$ be a FPFS topological space and $H_{\Lambda_1}, H_{\Lambda_2} \in FPFS(U,E)$. Then,

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1. $\bar{H}_{\varnothing} = H_{\varnothing}$ and $\bar{H}_{\tilde{E}} = H_{\tilde{E}}$,
2. $H_{\Lambda} \tilde{\subseteq} \bar{H}_{\Lambda}$,
3. $\bar{\bar{H}}_{\Lambda} = \bar{H}_{\Lambda}$,
4. If $H_{\Lambda_1} \tilde{\subseteq} H_{\Lambda_2}$ then $\bar{H}_{\Lambda_1} \tilde{\subseteq} \bar{H}_{\Lambda_2}$,
5. H_{Λ} is a FPFS closed set if and only if $H_{\Lambda} = \bar{H}_{\Lambda}$,
6. $\overline{H_{\Lambda_1} \tilde{\cup} H_{\Lambda_2}} = \bar{H}_{\Lambda_1} \tilde{\cup} \bar{H}_{\Lambda_2}$.

Definition 2.14 (Zorlutuna&Atmaca, 2016) Let $(U, \tilde{\tau})$ be a FPFS topological space and $H_{\Lambda} \in FPFS(U,E)$. The FPFS interior of H_{Λ}, denoted by $H_{\Lambda}{}^{o}$, is the largest FPFS open set contained in H_{Λ}.

3. FPFS SEPARATION AXIOMS

Definition 3.1 Let $(U, \tilde{\tau})$ be an FPFS topological space and for every $\alpha_{u_x}^{\beta}, \alpha_{u_y}^{\gamma}$ be FPFS points over U such that $\alpha_{u_x}^{\beta} \neq \alpha_{u_y}^{\gamma}$ If there exist at least one FPFS open set H_{Λ_1} or H_{Λ_2} such that

$$\alpha_{u_x}^{\beta} \in H_{\Lambda_1} \text{ and } \alpha_{u_x}^{\beta} \widetilde{\cap} H_{\Lambda_2} = H_{\varnothing}$$

or

$$\alpha_{u_y}^{\gamma} \in H_{\Lambda_1} \text{ and } \alpha_{u_y}^{\gamma} \widetilde{\cap} H_{\Lambda_2} = H_{\varnothing}$$

then $(U, \tilde{\tau})$ is called an FPFS T_0-space.

Definition 3.2 Let $(U, \tilde{\tau})$ be an FPFS topological space and for every $\alpha_{u_x}^{\beta}, \alpha_{u_y}^{\gamma}$ be FPFS points over U such that $\alpha_{u_x}^{\beta} \neq \alpha_{u_y}^{\gamma}$ If there exist at least one FPFS open set H_{Λ_1} or H_{Λ_2} such that

$$\alpha_{u_x}^{\beta} \in H_{\Lambda_1} \text{ and } \alpha_{u_x}^{\beta} \widetilde{\cap} H_{\Lambda_2} = H_{\varnothing}$$

and

$$\alpha_{u_y}^{\gamma} \in H_{\Lambda_1} \text{ and } \alpha_{u_y}^{\gamma} \widetilde{\cap} H_{\Lambda_2} = H_{\varnothing}$$

then $(U, \tilde{\tau})$ is called an FPFS T_1-space.

Definition 3.3 Let $(U, \tilde{\tau})$ be an FPFS topological space and for every $\alpha_{u_x}^{\beta}, \alpha_{u_y}^{\gamma}$ be FPFS points over U such that $\alpha_{u_x}^{\beta} \neq \alpha_{u_y}^{\gamma}$ If there exist at least one FPFS open set H_{Λ_1} or H_{Λ_2} such that

$$\alpha_{u_x}^{\beta} \in H_{\Lambda_1}, \alpha_{u_y}^{\gamma} \in H_{\Lambda_2} \text{ and } H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$$

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then $(U,\tilde{\tau})$ is called an FPFS T_2-space.

Example 3.1 Let $U=\{u_1,u_2\}$ be a universal set, $E=\{\alpha_1,\alpha_2\}$ be a parameters set and $(\alpha_1)^{0.1}_{u_{1(0.4)}}$, $(\alpha_2)^{0.2}_{u_{1(0.5)}}$, $(\alpha_1)^{0.3}_{u_{2(0.3)}}$, and $(\alpha_2)^{0.4}_{u_{2(0.4)}}$ be FPFS points. Then the family

$$\tilde{\tau}=\{H_{\varnothing},H_{\tilde{E}},H_{\Lambda_1},H_{\Lambda_2},H_{\Lambda_3},H_{\Lambda_4},H_{\Lambda_5},H_{\Lambda_6},H_{\Lambda_7},H_{\Lambda_8}\},$$

where

$$H_{\Lambda_1}=\{(\alpha_1)^{0.1}_{u_{1(0.4)}}\},$$

$$H_{\Lambda_2}=\{(\alpha_2)^{0.2}_{u_{1(0.5)}}\},$$

$$H_{\Lambda_3}=\{(\alpha_1)^{0.3}_{u_{2(0.3)}}\},$$

$$H_{\Lambda_4}=H_{\Lambda_1}\tilde{\cup}H_{\Lambda_2},$$

$$H_{\Lambda_5}=H_{\Lambda_1}\tilde{\cup}H_{\Lambda_3},$$

$$H_{\Lambda_6}=H_{\Lambda_2}\tilde{\cup}H_{\Lambda_3},$$

$$H_{\Lambda_7}=H_{\Lambda_1}\tilde{\cup}H_{\Lambda_2}\tilde{\cup}H_{\Lambda_3},$$

$$H_{\Lambda_8}=\{(\alpha_1)^{0.1}_{u_{1(0.4)}},(\alpha_2)^{0.2}_{u_{1(0.5)}},(\alpha_1)^{0.3}_{u_{2(0.3)}},(\alpha_2)^{0.4}_{u_{2(0.4)}}\}$$

is a FPFS topology over U. Thus $(U,\tilde{\tau})$ is a FPFS topological space over U. Also $(U,\tilde{\tau})$ is a FPFS T_0-space but not a FPFS T_1-space because for FPFS points $(\alpha_1)^{0.1}_{u_{1(0.4)}}$ and $(\alpha_2)^{0.4}_{u_{2(0.4)}}$.

Example 3.2 Let $U=\{u_1,u_2\}$ be a universal set, $E=\{\alpha_1,\alpha_2\}$ be a parameters set and $(\alpha_1)^{0.1}_{u_{1(0.4)}}$, $(\alpha_2)^{0.2}_{u_{1(0.5)}}$, $(\alpha_1)^{0.3}_{u_{2(0.3)}}$, and $(\alpha_2)^{0.4}_{u_{2(0.4)}}$ be FPFS points. Then the family

$$\tilde{\tau}=\{H_{\varnothing},H_{\tilde{E}},H_{\Lambda_1},H_{\Lambda_2},H_{\Lambda_3},...,H_{\Lambda_{15}}\},$$

where

$$H_{\Lambda_1}=\{(\alpha_1)^{0.1}_{u_{1(0.4)}}\},$$

$$H_{\Lambda_2}=\{(\alpha_2)^{0.2}_{u_{1(0.5)}}\},$$

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$H_{\Lambda_3} = \{(\alpha_1)^{0.3}_{u_{2(0.3)}}\}$,

$H_{\Lambda_4} = \{(\alpha_2)^{0.4}_{u_{2(0.4)}}\}$,

$H_{\Lambda_5} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_2}$,

$H_{\Lambda_6} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_3}$,

$H_{\Lambda_7} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_4}$,

$H_{\Lambda_8} = H_{\Lambda_2} \tilde{\cup} H_{\Lambda_3}$,

$H_{\Lambda_9} = H_{\Lambda_2} \tilde{\cup} H_{\Lambda_4}$,

$H_{\Lambda_{10}} = H_{\Lambda_3} \tilde{\cup} H_{\Lambda_4}$

$H_{\Lambda_{11}} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_2} \tilde{\cup} H_{\Lambda_3}$,

$H_{\Lambda_{12}} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_2} \tilde{\cup} H_{\Lambda_4}$,

$H_{\Lambda_{13}} = H_{\Lambda_2} \tilde{\cup} H_{\Lambda_3} \tilde{\cup} H_{\Lambda_4}$,

$H_{\Lambda_{14}} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_3} \tilde{\cup} H_{\Lambda_4}$,

$H_{\Lambda_{15}} = \{(\alpha_1)^{0.1}_{u_{1(0.4)}}, (\alpha_2)^{0.2}_{u_{1(0.5)}}, (\alpha_1)^{0.3}_{u_{2(0.3)}}, (\alpha_2)^{0.4}_{u_{2(0.4)}}\}$

is a FPFS topology over *U*. Thus $(U, \tilde{\tau})$ is a FPFS topological space over *U*. Also $(U, \tilde{\tau})$ is a FPFS T_2-space.

Theorem 3.1 A FPFS subspace of a FPFS T_0-space is FPFS T_0-space.

Proof. Let $(V, \tilde{\tau}_V)$ be a FPFS subspace of a FPFS T_0-space $(U, \tilde{\tau})$ and let $\alpha^{\beta}_{u_x}, \alpha^{\gamma}_{u_y}$ be two distinct FPFS points of *V*. Then these FPFS points are also in *U*⇒∃ a FPFS open set H_Λ containing one FPFS point but not the other . Thus $H^V_\Lambda \tilde{\cap} H_\Lambda$ is a FPFS open set in $(V, \tilde{\tau}_V)$ containing one FPFS point but not the other.

Theorem 3.2 Let $(U, \tilde{\tau})$ be an FPFS topological space over *U*. Then $(U, \tilde{\tau})$ is an FPFS T_1-space if and only if each FPFS point is a FPFS closed set.

Proof. Suppose that $(U, \tilde{\tau})$ is an FPFS T_1-space and $\alpha^{\beta}_{u_x}$ be an arbitrary *IFH* point over *U*. We should show that $\left(\alpha^{\beta}_{u_x}\right)^c$ is an FPFS open set. Let $\alpha^{\gamma}_{u_y} \in \left(\alpha^{\beta}_{u_x}\right)^c$, then $\alpha^{\beta}_{u_x}$ and $\alpha^{\gamma}_{u_y}$ are disjoint FPFS points. Thus $\gamma \neq \beta$ or x≠*y*. Since $(U, \tilde{\tau})$ is an FPFS T*1*spsace, there exists FPFS open sets $H_{\Lambda_1}, H_{\Lambda_2}$ such that

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$\alpha_{u_x}^{\beta} \in H_{\Lambda_1}$ and $\alpha_{u_x}^{\beta} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$

and

$\alpha_{u_y}^{\gamma} \in H_{\Lambda_1}$ and $\alpha_{u_y}^{\gamma} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$

Then $\alpha_{u_x}^{\beta} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$. We have $\alpha_{u_y}^{\gamma} \in H_{\Lambda_2} \tilde{\subseteq} \left(\alpha_{u_x}^{\beta}\right)^c$. Therefore, $\left(\alpha_{u_x}^{\beta}\right)^c$ is an FPFS open set then $\alpha_{u_x}^{\beta}$ is an FPFS closed set.

Conversely, let each FPFS point $\alpha_{u_x}^{\beta}$ is an FPFS closed set. Then $\left(\alpha_{u_x}^{\beta}\right)^c$ is an FPFS open set. Suppose that $\alpha_{u_x}^{\beta} \tilde{\cap} \alpha_{u_y}^{\gamma} = H_{\varnothing}$, then $\alpha_{u_x}^{\beta} \tilde{\cap} \left(\alpha_{u_x}^{\beta}\right)^c = H_{\varnothing}$ and $\alpha_{u_y}^{\gamma} \in \left(\alpha_{u_x}^{\beta}\right)^c$. So $(U, \tilde{\tau})$ is an FPFS T_1-space.

Theorem 6 Let $(U, \tilde{\tau})$ be an FPFS topological space over *U*. $(U, \tilde{\tau})$ is an FPFS T_2-space if and only if for disjoint FPFS points $\alpha_{u_x}^{\beta}$ and $\alpha_{u_y}^{\gamma}$, there exist an FPFS open set H_{Λ} containing $\alpha_{u_x}^{\beta}$ but not $\alpha_{u_y}^{\gamma}$ such that $\alpha_{u_y}^{\gamma} \neq \bar{H}_{\Lambda}$.

Proof. Let $(U, \tilde{\tau})$ be an FPFS T_2-space and $\alpha_{u_x}^{\beta}, \alpha_{u_y}^{\gamma}$ be two FPFS point over *U*. Then there exist disjoint FPFS open sets H_{Λ_1} and H_{Λ_2} such that $\alpha_{u_x}^{\beta} \in H_{\Lambda_1}$ and $\alpha_{u_y}^{\gamma} \in H_{\Lambda_2}$. Since $\alpha_{u_x}^{\beta} \tilde{\cap} \alpha_{u_y}^{\gamma} = H_{\varnothing}$ and $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}, \alpha_{u_y}^{\gamma} \notin H_{\Lambda_1}$. It implies that $\alpha_{u_y}^{\gamma} \notin \bar{H}_{\Lambda_1}$.

Conversly, suppose that for distinct FPFS points $\alpha_{u_x}^{\beta}$ and $\alpha_{u_y}^{\gamma}$ there exist an FPFS open set H_{Λ_1} containing $\alpha_{u_x}^{\beta}$ but not $\alpha_{u_y}^{\gamma}$ such that $\alpha_{u_y}^{\gamma} \notin \bar{H}_{\Lambda}$. Then $\alpha_{u_y}^{\gamma} \in \left(H_{\Lambda}\right)^c$, i.e. H_{Λ} and $\left(\bar{H}_{\Lambda}\right)^c$ are disjoint FPFS open sets containing $\alpha_{u_x}^{\beta}$ and $\alpha_{u_y}^{\gamma}$, respectively.

Theorem 7 Let $(U, \tilde{\tau})$ be an FPFS T_1-space for every FPFS point $\alpha_{u_x}^{\beta} \in H_{\Lambda_1} \in \tilde{\tau}$. If there exist an FPFS open set H_{Λ_2} in $(U, \tilde{\tau})$ such that

$\alpha_{u_x}^{\beta} \notin H_{\Lambda_2} \tilde{\subseteq} \bar{H}_{\Lambda_2} \tilde{\subseteq} H_{\Lambda_1}$

then $(U, \tilde{\tau})$ is an FPFS T_2-space.

Proof. Suppose that $\alpha_{u_x}^{\beta} \tilde{\cap} \alpha_{u_y}^{\gamma} = H_{\varnothing}$. Since $(U, \tilde{\tau})$ is an FPFS T_1-space, $\alpha_{u_x}^{\beta}$ and $\alpha_{u_y}^{\gamma}$ are FPFS closed sets in $(U, \tilde{\tau})$. Thus $\alpha_{u_x}^{\beta} \in \left(\alpha_{u_y}^{\gamma}\right)^c \in \tilde{\tau}$. Then there exist an FPFS open set $H_{\Lambda_2} \in \tilde{\tau}$ such that

$\alpha_{u_x}^{\beta} \in H_{\Lambda_2} \tilde{\subseteq} \bar{H}_{\Lambda_2} \tilde{\subseteq} \left(\alpha_{u_y}^{\gamma}\right)^c$

So, we have $\alpha_{u_y}^{\gamma} \tilde{\in} \left(H_{\Lambda_2}\right)^c, \alpha_{u_x}^{\beta} \in H_{\Lambda_2}$ and $H_{\Lambda_2} \tilde{\cap} \left(\bar{H}_{\Lambda_2}\right)^c = H_{\varnothing}$, i.e. $(U, \tilde{\tau})$ is an FPFS T_2-space.

Definition 25 Let $(U, \tilde{\tau})$ be an FPFS topological space over *U*, G_{Υ} be an FPFS closed set in $(U, \tilde{\tau})$ and $\alpha_{u_x}^{\beta} \tilde{\cap} G_{\Upsilon} = H_{\varnothing}$. If there exist FPFS open sets H_{Λ_1} and H_{Λ_2} such that $\alpha_{u_x}^{\beta} \tilde{\in} H_{\Lambda_1}, G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_2}$ and

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$H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$, then $(U,\tilde{\tau})$ is called an FPFS regular space. $(U,\tilde{\tau})$ is said to be an FPFS T_3-space if it is an FPFS regular and FPFS T_1-space.

Theorem 8 Let $(U,\tilde{\tau})$ be an FPFS topological space over *U*. $(U,\tilde{\tau})$ is an FPFS T_3-space if and only if for every $\alpha_{u_x}^{\beta} \tilde{\in} H_{\Lambda_1} \in \tilde{\tau}$, there exists $H_{\Lambda_2} \in \tilde{\tau}$ such that

$$\alpha_{u_x}^{\beta} \in H_{\Lambda_2} \tilde{\subseteq} \bar{H}_{\Lambda_2} \tilde{\subseteq} H_{\Lambda_1}.$$

Proof. Let $(U,\tilde{\tau})$ be an FPFS T_3-space and $\alpha_{u_x}^{\beta} \in H_{\Lambda_1} \in \tilde{\tau}$. Since $(U,\tilde{\tau})$ is an FPFS T_3-space for the FPFS point $\alpha_{u_x}^{\beta}$ and FPFS closed set $(H_{\Lambda_1})^c$, there exist FPFS open set $H_{\Lambda_2}, H_{\Lambda_3}$ such that $\alpha_{u_x}^{\beta} \in H_{\Lambda_2}, (H_{\Lambda_1})^c \tilde{\subseteq} (H_{\Lambda_3})$ and $(H_{\Lambda_2}) \widetilde{\cap} (H_{\Lambda_3}) = H_{\varnothing}$. Thus, we have $\alpha_{u_x}^{\beta} \in H_{\Lambda_2} \tilde{\subseteq} (H_{\Lambda_3})^c \tilde{\subseteq} H_{\Lambda_1}$. Since $(H_{\Lambda_3})^c$ is an FPFS closed set, so $\bar{H}_{\Lambda_2} \tilde{\subseteq} (H_{\Lambda_3})^c$.

Conversely, suppose that $\alpha_{u_x}^{\beta} \tilde{\cap} G_{\Upsilon} = H_{\varnothing}$ and G_{Υ} is an FPFS closed set in $(U,\tilde{\tau})$ Thus $\alpha_{u_x}^{\beta} \in (G_{\Upsilon})^c$ and from the condition of the theorem, we have $\alpha_{u_x}^{\beta} \in H_{\Lambda} \tilde{\subseteq} \bar{H}_{\Lambda} \tilde{\subseteq} (G_{\Upsilon})^c$. Then $\alpha_{u_x}^{\beta} \in H_{\Lambda}, G_{\Upsilon} \tilde{\subseteq} \left(\bar{H}_{\Lambda}\right)^c$ and $H_{\Lambda} \tilde{\cap} \bar{H}_{\Lambda} = H_{\varnothing}$ are satisfied, i.e. $(U,\tilde{\tau})$ is an FPFS T_3-space.

Definition 26 Let $(U,\tilde{\tau})$ be an FPFS topological space over *U*. If for every two non empty FPFS closed sets $G_{\Upsilon_1}, G_{\Upsilon_2}$ such that $G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2} = H_{\varnothing}$, there exists FPFS open sets $H_{\Lambda_1}, H_{\Lambda_2}$ such that $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$ and $G_{\Upsilon_1} \tilde{\subseteq} H_{\Lambda_1}, G_{\Upsilon_2} \tilde{\subseteq} H_{\Lambda_2}$ then $(U,\tilde{\tau})$ is called an FPFS normal space. $(U,\tilde{\tau})$ is said to be an FPFS T_4-space if it is an FPFS normal and FPFS T_1-space.

Theorem 9 Let $(U,\tilde{\tau})$ be an FPFS topological space over *U*. Then $(U,\tilde{\tau})$ is an FPFS T_4-space if and only if for each FPFS closed set G_{Υ} and FPFS open set H_{Λ_1} with $G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_1}$, there exist an FPFS open set G_{Υ_2} such that

$$G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_2} \tilde{\subseteq} \bar{H}_{\Lambda_2} \tilde{\subseteq} G_{\Upsilon_1}.$$

Proof. Let $(U,\tilde{\tau})$ be an FPFS T_4-space, G_{Υ} be an FPFS closed set in $(U,\tilde{\tau})$ and $G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_1} \in \tilde{\tau}$. Then $(H_{\Lambda_1})^c$ is an FPFS closed set and $G_{\Upsilon} \tilde{\cap} (H_{\Lambda_1})^c = H_{\varnothing}$. Since $(U,\tilde{\tau})$ is an FPFS T_4-space, there exists FPFS open sets $H_{\Lambda_2}, H_{\Lambda_3}$ such that $G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_2}$, $(H_{\Lambda_1})^c \tilde{\subseteq} H_{\Lambda_3}$ and $H_{\Lambda_2} \tilde{\cap} H_{\Lambda_3} = H_{\varnothing}$. This implies that

$$G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_2} \tilde{\subseteq} (H_{\Lambda_3})^c \tilde{\subseteq} H_{\Lambda_1}.$$

$(H_{\Lambda_3})^c$ an FPFS closed set and $\bar{H}_{\Lambda_2} \widetilde{\subseteq} (H_{\Lambda_3})^c$ is satisfied. Thus, $G_{\Upsilon} \tilde{\subseteq} H_{\Lambda_2} \tilde{\subseteq} \bar{H}_{\Lambda_2} \tilde{\subseteq} H_{\Lambda_1}$ is obtained.

Conversly, suppose that $G_{\Upsilon_1}, G_{\Upsilon_2}$ be two non empty disjoint FPFS closed sets in $(U,\tilde{\tau})$. Then $G_{\Upsilon_1} \widetilde{\subseteq} (G_{\Upsilon_2})^c$. From the condition of theorem, the exist an FPFS open set H_{Λ} such that

$$G_{\Upsilon_1} \tilde{\subseteq} H_{\Lambda} \tilde{\subseteq} \bar{H}_{\Lambda} \tilde{\subseteq} (G_{\Upsilon_2})^c.$$

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Therefore H_Λ, $(\bar{H}_\Lambda)^c$ are FPFS open sets and $G_\Upsilon \tilde{\subseteq} H_\Lambda, G_{\Upsilon_2} \tilde{\subseteq} (\bar{H}_\Lambda)^c$ and $H_\Lambda \tilde{\cap} (\bar{H}_\Lambda)^c = H_\varnothing$ are obtained. So $(U,\tilde{\tau})$ is an FPFS T_4-space.

Theorem 10 Let $(U,\tilde{\tau})$ be an FPFS topological space and H_Λ be an FPFS set over *U*. If $(U,\tilde{\tau})$ is an FPFS T_i-space, then the FPFS topological subspace $\left(V,\tilde{\tau}_V\right)$ is an FPFS T_i-space for *i*=0,1,2,3.

Proof. Let $\alpha^\beta_{u_x}, \alpha^\gamma_{u_y} \in \left(V,\tilde{\tau}_V\right)$ such that $\alpha^\beta_{u_x} \tilde{\cap} \alpha^\gamma_{u_y} = H_\varnothing$. Hence there exist FPFS open sets $H_{\Lambda_1}, H_{\Lambda_2}$ satisfying the conditions of FPFS T_i-space such that $\alpha^\beta_{u_x} \in H_{\Lambda_1}$ and $\alpha^\gamma_{u_y} \in H_{\Lambda_2}$. Then $\alpha^\beta_{u_x} \in H_{\Lambda_1} \tilde{\cap} H_\Lambda$ and $\alpha^\gamma_{u_y} \in H_{\Lambda_2} \tilde{\cap} H_\Lambda$. Also, FPFS open sets $H_{\Lambda_1} \tilde{\cap} H_\Lambda, H_{\Lambda_2} \tilde{\cap} H_\Lambda$ in $\tilde{\tau}_V$ satisfying the conditions of FPFS T_i-space for *i*=0,1,2,3.

Theorem 11 Let $(U,\tilde{\tau})$ be an FPFS topological space over *U*. If $(U,\tilde{\tau})$ is *IFH* T_4-space and G_Υ is an FPFS closed set in $(U,\tilde{\tau})$ then $\left(G_\Upsilon,\tilde{\tau}_{G_\Upsilon}\right)$ is an FPFS T_4-space.

Proof. Let $(U,\tilde{\tau})$ be an FPFS T_4-space and G_Υ be an FPFS closed set in $(U,\tilde{\tau})$ Let G_{Υ_1} and G_{Υ_2} be two FPFS closed set in $\left(G_\Upsilon,\tilde{\tau}_{G_\Upsilon}\right)$ such that $G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2} = H_\varnothing$ When G_Υ is an FPFS closed set in $(U,\tilde{\tau})$ (F_1,Υ_1) and (F_2,Υ_2) are FPFS closed sets in $(U,\tilde{\tau})$ Since $(U,\tilde{\tau})$ is an FPFS T_4-space, there exist FPFS open sets $H_{\Lambda_1}, H_{\Lambda_2}$ such that $G_{\Upsilon_1} \tilde{\subseteq} H_{\Lambda_1}, G_{\Upsilon_2} \tilde{\subseteq} H_{\Lambda_2}$ and $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_\varnothing$ Then $G_{\Upsilon_1} \tilde{\subseteq} H_{\Lambda_1} \tilde{\cap} G_\Upsilon, G_{\Upsilon_2} \tilde{\subseteq} H_{\Lambda_2} \tilde{\cap} G_\Upsilon$ and $\left[H_{\Lambda_1} \tilde{\cap} G_\Upsilon\right] \tilde{\cap} \left[H_{\Lambda_2} \tilde{\cap} G_\Upsilon\right] = H_\varnothing$. This implies that $\left(G_\Upsilon,\tilde{\tau}_{G_\Upsilon}\right)$ is FPFS T_4-space.

4. FPFS CONNECTEDNESS

Definition 4.1 Let $(U,\tilde{\tau})$ be an FPFS topological space and $H_\Lambda \in FPFS(U,E)$. If H_Λ is FPFS open and FPFS closed, then it is called FPFS clopen set. In case $H_\Lambda \neq H_\varnothing$ and $H_\Lambda \neq H_{\tilde{E}}$, H_Λ is called FPFS proper set.

Definition 4.2 Let $(U,\tilde{\tau})$ be an FPFS topological space and $H_\Lambda \in FPFS(U,E)$. If there are two FPFS proper open sets G_{Υ_1} and G_{Υ_2} such that $H_\Lambda \tilde{\subseteq} G_{\Upsilon_1} \tilde{\cup} G_{\Upsilon_2}$ and $G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2} = H_\varnothing$, then H_Λ is called FPFS disconnect set. If there does not exist such two FPFS proper open sets, then the FPFS H_Λ is called FPFS connected set. If we take $H_{\tilde{E}}$ instead of H_Λ, then the $(U,\tilde{\tau})$ is called FPFS disconnected space.

Example 4.1

i. The discrete FPFS topological space with more than one member is always disconnected.
ii. The indiscrete FPFS topological space is always connected.

Theorem 4.1 Let $(U,\tilde{\tau})$ be an FPFS topological space. $(U,\tilde{\tau})$ FPFS connected if and only if there does not exist a FPFS clopen set H_Λ in $(U,\tilde{\tau})$.

Proof. ($\Rightarrow$): Let $(U,\tilde{\tau})$ be a FPFS connected space. Suppose that there exist a FPFS proper clopen set H_Λ in $(U,\tilde{\tau})$ such that $H_\Lambda \tilde{\cup} (H_\Lambda)^c = H_{\tilde{E}}$ and $H_\Lambda \tilde{\cap} (H_\Lambda)^c = H_\varnothing$. It is a contradiction.

($\Rightarrow$): It is clear.

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Theorem 4.2 Let $(U,\tilde{\tau})$ be a FPFS connected space and $\tilde{\rho} \tilde{\subseteq} \tilde{\tau}$. Then, $(U,\tilde{\rho})$ is a FPFS connected topological space.

Proof: Straightforward.

Theorem 4.3 Let $(U,\tilde{\tau})$ and $(Y,\tilde{\rho})$ be two FPFS topological spaces, $H_{\Lambda} \in FPFS(U,E)$ and $\tilde{f}:(U,\tilde{\tau}) \to (Y,\tilde{\rho})$ be FPFS continuous function. If H_{Λ} is a FPFS connected set, then $\tilde{f}(H_{\Lambda})$ is a FPFS connected set.

Proof. Suppose that $\tilde{f}(H_{\Lambda})$ is a FPFS disconnected set. Then, there exist two FPFS proper open set G_{Υ_1} and G_{Υ_2} in *Y* such that $\tilde{f}(H_{\Lambda}) \tilde{\subseteq} G_{\Upsilon_1} \tilde{\cup} G_{\Upsilon_2}$ and $G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2} = G_{\varnothing}$. Therefore

$$H_{\Lambda} \tilde{\subseteq} \tilde{f}^{-1}(\tilde{f}(H_{\Lambda})) \tilde{\subseteq} \tilde{f}^{-1}(G_{\Upsilon_1} \tilde{\cup} G_{\Upsilon_2}) = \tilde{f}^{-1}(G_{\Upsilon_1}) \tilde{\cup} \tilde{f}^{-1}(G_{\Upsilon_2})$$

and

$$\tilde{f}^{-1}(G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2}) = \tilde{f}^{-1}(G_{\Upsilon_1}) \tilde{\cap} \tilde{f}^{-1}(G_{\Upsilon_2}) = \tilde{f}^{-1}(G_{\varnothing}) = H_{\varnothing}.$$

So, we have a contradiction and this complete the proof.

Theorem 4.4 Let $(U,\tilde{\tau})$ and $(Y,\tilde{\rho})$ be two FPFS topological spaces and $\tilde{f}:(U,\tilde{\tau}) \to (Y,\tilde{\rho})$ be FPFS continuous and surjective function. If $(U,\tilde{\tau})$ is FPFS connected space, then $(Y,\tilde{\rho})$ is also FPFS connected space.

Proof. Suppose that $(Y,\tilde{\rho})$ is a FPFS disconnected space. Thereforei there exist two fuzzy proper open sets G_{Υ_1} and G_{Υ_2} in *Y* such that $G_{\Upsilon_1} \tilde{\cup} G_{\Upsilon_2} = G_{\tilde{F}}$ ($G_{\tilde{F}}$ is FPFS absolute set on (*Y,F*)), $G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2} = G_{\varnothing}$. Then $\tilde{f}^{-1}(G_{\Upsilon_1}) \tilde{\cup} \tilde{f}^{-1}(G_{\Upsilon_2}) = H_{\tilde{E}}$ and $\tilde{f}^{-1}(G_{\Upsilon_1}) \tilde{\cap} \tilde{f}^{-1}(G_{\Upsilon_2}) = H_{\varnothing}$. So we have a contradictionand this complete this proof.

Definition 4.3 Let $(U,\tilde{\tau})$ be a FPFS topological space. If there exist $H_{\Lambda_1}, H_{\Lambda_2} \in FPFS(U,E)$ which are proper, such that $\bar{H}_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$ and $H_{\Lambda_1} \tilde{\cap} \bar{H}_{\Lambda_2} = H_{\varnothing}$ then the FPFS sets H_{Λ_1} and H_{Λ_2} are called FPFS separated sets.

Theorem 4.5 Let $(U,\tilde{\tau})$ be a FPFS topological space, H_{Λ_1} and H_{Λ_2} be two FPFS open sets. If $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$, then H_{Λ_1} and H_{Λ_2} are FPFS separated sets.

Proof. Let $H_{\Lambda_1}, H_{\Lambda_2} \in \tilde{\tau}$ and $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$. Then, $H_{\Lambda_1}{}^c \tilde{\cup} H_{\Lambda_2}{}^c = H_{\tilde{E}}$. Therefore, $H_{\Lambda_1} \tilde{\subseteq} H_{\Lambda_2}{}^c$ and $H_{\Lambda_2} \tilde{\subseteq} H_{\Lambda_1}{}^c$, $H_{\Lambda_1}{}^c$ and $H_{\Lambda_2}{}^c$ are FPFS closed sets. We have

$$\bar{H}_{\Lambda_1} \tilde{\cap} \overline{\left(H_{\Lambda_2}{}^c\right)} = H_{\Lambda_2}{}^c \text{ and } \bar{H}_{\Lambda_2} \tilde{\subseteq} \overline{\left(H_{\Lambda_1}{}^c\right)} = H_{\Lambda_1}{}^c$$

So,

$$\bar{H}_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing} \text{ and } H_{\Lambda_1} \tilde{\cap} \bar{H}_{\Lambda_2} = H_{\varnothing}.$$

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Theorem 4.6 Let $(U,\tilde{\tau})$ be a FPFS topological space, H_{Λ_1} and H_{Λ_2} be two FPFS closed sets. If $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$, then H_{Λ_1} and H_{Λ_2} are FPFS separated sets.

Proof. Let $H_{\Lambda_1}{}^c, H_{\Lambda_2}{}^c \in \tilde{\tau}$ and $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}, \bar{H}_{\Lambda_1} = H_{\Lambda_1}$ and $\bar{H}_{\Lambda_2} = H_{\Lambda_2}$. Therefore we have

$$H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = \bar{H}_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$$

and

$$H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\Lambda_1} \tilde{\cap} \bar{H}_{\Lambda_2} = H_{\varnothing}$$

So, H_{Λ_1} and H_{Λ_2} are FPFS separated sets.

Theorem 4.7 Let $(U,\tilde{\tau})$ be a FPFS topological space and $H_\Lambda \in FPFS(U,E)$. H_Λ is PFFS connected if and only if $H_{\tilde{E}}$ cannot be written as union of FPFS separated set.

Proof. ($\Rightarrow$) If $H_\Lambda = H_{\varnothing}$, then it is FPFS connected set. Thus, let $H_\Lambda \neq H_{\varnothing}$. Assume that H_Λ can be written as union of FPFS sets G_{Υ_1} and G_{Υ_2}. But this situation contradicts by hypothesis.

($\Leftarrow$) It is clear.

Theorem 4.8 A FPFS topological space $(U,\tilde{\tau})$ is connected if and only if $H_{\tilde{E}}$ cannot be written as union of FPFS separated sets.

Proof. Suppose that $H_{\tilde{E}}$ can be written as union of FPFS separated sets H_{Λ_1} and H_{Λ_2}. Then, $H_{\tilde{E}} = H_{\Lambda_1} \tilde{\cup} H_{\Lambda_2}$, $\bar{H}_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$ and $H_{\Lambda_1} \tilde{\cap} \bar{H}_{\Lambda_2} = H_{\varnothing}$. Therefore, we have $H_{\Lambda_1} \tilde{\cap} H_{\Lambda_2} = H_{\varnothing}$, $H_{\Lambda_1} = H_{\Lambda_2}{}^c$ and $H_{\Lambda_2} = H_{\Lambda_1}{}^c$. Thus,

$$\begin{aligned}
\bar{H}_{\Lambda_1} &= \bar{H}_{\Lambda_1} \tilde{\cap} H_{\tilde{E}} \\
&= \bar{H}_{\Lambda_1} \tilde{\cap} (H_{\Lambda_1} \tilde{\cup} H_{\Lambda_2}) \\
&= (\bar{H}_{\Lambda_1} \tilde{\cap} H_{\Lambda_1}) \tilde{\cup} (\bar{H}_{\Lambda_1} \tilde{\cap} H_{\Lambda_2}) \\
&= H_{\Lambda_1}.
\end{aligned}$$

So, H_{Λ_1} is a FPFS closed set. Similarly, H_{Λ_2} is also a FPFS closed set. From $H_{\Lambda_1} = H_{\Lambda_2}{}^c$ and $H_{\Lambda_2} = H_{\Lambda_1}{}^c$, H_{Λ_1} and H_{Λ_2} are FPFS open sets. This is a contradiction.

($\Leftarrow$): Suppose that $(U,\tilde{\tau})$ is not a FPFS connected space. Then there exists a FPFS proper clopen set H_{Λ_1}. But this contradict by hypothesis.

Theorem 4.9 Let $(U,\tilde{\tau})$ be a FPFS topological space and $H_\Lambda \in FPFS(U,E)$ be a FPFS open connected set. If $H_{\Lambda_1} \tilde{\subseteq} H_{\Lambda_2} \tilde{\subseteq} \bar{H}_{\Lambda_1}$, then H_{Λ_2} is a FPFS connected set.

Proof. Suppose that H_{Λ_2} is a FPFS disconnected set. Then, there exist two FPFS proper sets G_{Υ_1} and G_{Υ_2} such that $G_{\Upsilon_1} \tilde{\cap} G_{\Upsilon_2} = H_{\varnothing}$ and $H_{\Lambda_2} \tilde{\subseteq} G_{\Upsilon_1} \tilde{\cup} G_{\Upsilon_2}$. Therefore

$$H_{\Lambda_1} = (H_{\Lambda_1} \tilde{\cap} G_{\Upsilon_1}) \tilde{\cup} (H_{\Lambda_1} \tilde{\cap} G_{\Upsilon_2})$$

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and

$$(H_{\Lambda_1} \tilde{\cap} G_{\Upsilon_1}) \tilde{\cap} (H_{\Lambda_1} \tilde{\cap} G_{\Upsilon_2}) = H_{\varnothing}.$$

But it is contradiction. Thus H_{Λ_2} is a FPFS connected set.

CONCLUSION

The concept of FPFS sets and FPFS topology is critical in the investigation of potential applications in classical and non-classical logic. Using soft point, the authors introduced and investigated FPFS separation axioms such as FPFS T_i-spaces (i=0,1,2), FPFS regular, FPFS T_3-space, FPFS normal, and FPFS T_4-space. These FPFS separation axioms would be important for the development of soft topology theory in order to tackle intricate issues involving uncertainties in economics, engineering, medicine, the environment, and in general man-machine systems of many sorts. Furthermore, they defined FPFS connected topological spaces and investigated their different features. These findings contribute to the expansion of the FPFS topological spaces.

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Chapter 20
Probability and Fuzziness: A Case Study

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ABSTRACT

From the beginning of the theory of fuzzy sets, it has been a matter of concern for many researchers in the field to find an appropriate connection between probability and possibility. One such effort can be found in the randomness-fuzziness consistency principle where a fuzzy number of the type [α,β,γ] is expressed as a combination of two probability laws. The main intention of this work is to show that a fuzzy number of the type [α,β,β] or [β,β,γ] can be expressed by one probability law with the help of superimposition of uniformly fuzzy intervals.

1. INTRODUCTION

The notion of fuzzy sets is an extension of the most fundamental property of sets. The concept of fuzzy sets were introduced by (Zadeh, 1965, pp.338-353) to represent or manipulate data and information which possesses uncertainties of non statistical nature. It was specifically designed for representing mathematically the uncertainties and vagueness which are intrinsic to many problems. It provides a means for representing uncertainties which cannot be dealt with probability. Fuzziness is a type of imprecision that arrived from the concept of grouping of elements into classes having no sharp boundaries. It has made inroads into all branches of studies. It can be found in many application areas in both theoretical and practical field of studies to arts and humanities, from computer science to health sciences, from physical sciences to life sciences and so on. In the beginning fuzziness and randomness were considered as two independent concepts. Since the inception of the theory of fuzzy sets, the method of finding a connection between probability and possibility has received attention as can be seen in the works of (Zadeh,1978; Klir,1993; Dubois & Prade,1993). These are known as Probability- Possibility Consistency Principle in the literature of fuzzy set theory. But all these attempts remained unclear and incomplete as in these consistency principles possibility space is defined in the same probability space. Many years

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later (Baruah, 2010, pp.37-48) found a probability- possibility consistency principle named as "The Randomness-Fuzziness Consistency Principle" with the help of a set operation called superimposition. In that principle it was stated that two probability spaces are required to define a possibility space. In order to show this a normal fuzzy number of the type $[\alpha,\beta,\gamma]$ is taken into consideration where the membership function explaining a fuzzy variable which takes a particular value is either the distribution function of a random event or the complementary distribution function of another random event.

Here in order to establish the result, the superimposition of sets and the Randomness-Fuzziness Consistency Principle are applied. Before proceeding further, let us have a look at the preliminaries and definitions of some of terms used here with special reference to superimposition of sets and the Randomness-Fuzziness Consistency Principle in the following subsection.

2. DEFINITIONS AND PRELIMINARIES

In this section some basic definitions and results are pt forward for better understanding of the underlying concept.

2.1 Crisp Set

A classical or conventional or crisp set contains objects that satisfy some precise properties. For example

A= numbers between 3 and 5
$= \{x \in R / 3 \leq x \leq 5\}$

$$\mu_A(x) = \begin{cases} 1, 3 \leq x \leq 5 \\ 0, otherwise \end{cases}$$

2.2 Fuzzy Set

Fuzzy sets are sets whose elements have degrees of membership. Unlike classical sets where an element either belongs or does not belong or does not belong to the set, in a fuzzy set there is a gradual assessment of the membership of elements in a set. In other words, fuzzy set contains objects having imprecise properties with varying degree. To be more precise if X is an universe of discourse and x is any element of X, then the fuzzy set, A defined on X may be defined as an ordered pair

$A = \{\mu_A(x), x; \forall x \in X\}$

where μ_A: X→[0,1], $\mu_A(x)$ is the membership function or degree of membership of x in A or degree of belongingness of x in A or degree of possessing some imprecise property represented by A.

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2.3 Fuzzy Numbers

Fuzzy numbers have been conceptualized by different authors in different ways. Some important works worth mentioning can be seen from (Rodabaugh, 1982; Dobois and Prade, 1978, 1981; Nahmias, 1978; Hutton,1975) and several other authors. A fuzzy number is an extension of regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own degree of memberships lying in the interval [0,1].

2.4 Fuzzy Interval

Fuzzy intervals are defined as fuzzy sets on the real line. Fuzzy intervals are usually defined through their membership functions. A membership function from the base set to a real number between 0 and 1.In other words, each elements of the said interval is connected to asset of possible values, where each possible value is has its own degree of membership between 0 and 1. Fuzzy numbers in particular can also be considered as fuzzy intervals.

2.5 Superimposition of Sets

Superimposition of set is a set operation defined by (Baruah, 1999,pp-25 – 31) and later used successfully in recognizing periodic patterns(Mahanta. This operation of *superimposition* is defined as follows:

Let us consider set P is superimposed over the set Q, then

$$P(S)Q = (P - Q) \bigcup (P \bigcap Q)^{(2)} \bigcup (Q - P)$$

Where S represent the operation of superimposition and $(P \cap Q)^{(2)}$ represents the element of $(P \cap Q)$ occurring twice, provided that $(P \cap Q)$ is not void.

2.6 Order Statistic

In statistical science, order statistic is a very important concept. Let a set of random vector $X=(X_1,X_2,X_3,\ldots,X_n)$ has been defined as a family of X_k, k=1,2,3,……..,n with every X_k inducing a sub-σ field so that X is measurable. Let $x_1,x_2,x_3,\ldots,x_n$ be a particular realization of X, and let X(k) realize the value x(k) where $x_{(1)}, x_{(2)}, x_{(3)}, \ldots x_{(n)}$ are the ordered values of $x_1,x_2,x_3,\ldots,x_n$ in increasing order of magnitudes.

2.7 The Glivenko-Cantelli Theorem

The Glivenk- Cantelli Theorem is defined in the following way.

Let

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$$F_n(x)=\begin{cases}0, x<x_1\\ \dfrac{r}{n}, \quad x_{(r)}\le x\le x_{(r+1)}, r=1,2,3,.....,n\\ 1, x\ge x_{(n)}\end{cases}$$

Where $x_1,x_2,x_3,\ldots,x_n$ are the realization of random variable X, and $x_{(1)},x_{(2)},x_{(3)},\ldots x_{(n)}$ are the values of $x_1,x_2,x_3,\ldots,x_n$ arranged in ascending order.

F(x) is random here with

$$Prob[F_n(x)=\frac{r}{n}]=\frac{n!}{r!(n-r)!}((F(x))^r(1-(F(x))^{n-r}$$

Where F(x) is the underlying theoretical distribution function, $F_n(x)$ being an empirical distribution function.In such a case Glivenko- Cantelli Theorem on order statistics states that

$$Prob[\sup|F_n(x)-F(x)|\to 0]=1$$

This means that $F_n(x)\to F(x)$ uniformly in x with probability 1.

2.8 Randomness Fuzziness Consistency Principle

Defining operation of superimposition of sets and using the Glivenko- Cantelli theorem (Loevee, 1977) on order statistic, (Baruah, 2010, 2011a, 2011b, 2011c, 2012) has established a theorem that uncovers the missing link between fuzziness and randomness.

According to the Randomness-Fuzziness Consistency Principle defined by (Baruah, 2010, 2011) every normal law of fuzziness can be expressed in terms of two laws of randomness and therefore two laws of probability. Thus for a normal fuzzy number N= [α, β, γ] with membership function

$$\mu_N(x)=\begin{cases}\psi_1(x), & \alpha\le x\le\beta\\ \psi_2(x), & \beta\le x\le\gamma\\ 0, & otherwise\end{cases}$$

two probability laws concerned with. One probability law is defined in the intervals [α, β] and another independent probability law is defined in the interval [β, γ]. The two probability laws defined on these two intervals are necessary and sufficient to define the normal fuzzy number [α, β, γ]. For the probability law defined in the interval [α, β], we have $\psi_1(x)$ as the distribution function and for the other independent probability law defined in the interval [β, γ], we have a complementary distribution function $\psi_2(x)$.

From Randomness-Fuzziness Consistency Principle, it is seen that two laws of randomness are required to define a law of fuzziness (Baruah, 2010, 2011). That is to say, In other words, two laws of random-

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ness defined on two disjoint spaces can construct a fuzzy membership function, for which however one needs to look into the matter through the applications of Glivenko-Canteli's theorem of order statistics on superimposed uniformly fuzzy intervals. This theory was later applied in finding the entropy of fuzzy numbers as can be seen in (Dhar, 2013a, 2013b ; Dhar & Baruah, 2013a, 2013b).

Here attempts are made to find a relationship between probability and fuzzy number expressed in terms of an interval. Proceeding in the similar manner, it is expected to show that for a fuzzy number N=[α,β,β] with membership function defined below

$$\mu_N(x)=\begin{cases}\psi(x), & \text{if } \alpha \le x \le \beta \\ 0, & \text{otherwise}\end{cases}$$

where $\psi(\alpha)=0$ and $\psi(\beta)=1$, with non-decreasing $\psi(x)$ in $\alpha \le x \le \beta$ and then we claim that $\psi(x)$ has to be a probability distribution function in the interval $[\alpha,\beta]$. In other words, every interval which is a probability space is basically nothing but a fuzzy number. That is to prove that a normal law of fuzziness can lead to one law of randomness. It can be inferred from the above theorem that law of randomness from a law of fuzziness or a law of fuzziness from a law of randomness can be easily established. Superimposition of sets as defined by (Baruah, 1999,pp.25-31) plays a crucial role in establishing the result claimed to be here.

3. CONSTRUCTION OF FUZZY NUMBER [α,β,β] USING SUPERIMPOSITION

From the Randomness-Fuzziness Consistency principle, one needs two laws of randomness, one in [α, β] and the other in the interval [β, γ] for constructing a normal fuzzy number of the type N= [α, β, γ]. Proceeding in the same way it is intended to express an interval of the type [α, β] which can be defined in the form [α,β,β] as a normal fuzzy number having a well defined membership function. In order to construct a fuzzy number of the type [α,β,β], it is essential to consider the operation of superimposition on intervals having the same end point. Before proceeding further, a brief discussion about the superimposition of intervals of the type [a_1,b] and [a_2,b] are made in the following:

Let us now see what happens when we superimpose intervals of the type [a_1,b] and [a_2,b]

The operation of superimposition of two real intervals [a1,b] and [a2,b] would look like

[a_1,b] (S) [a_2,b]

$$=\left[a_1,a_2\right]\cup\left[a_2,b\right]^{(2)},a_1<a_2<b$$

$a_1 \ldots a_2 \ldots b$

$$=\left[a_2,a_1\right]\cup\left[a_1,b\right]^{(2)},a_2<a_1<b$$

$a_2 \ldots a_1 \ldots b$

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Thus it is obtained that

$$[a_1,b]\ (S)[a_2,b]=\left[a_{(1)},a_{(2)}\right]\cup\left[a_{(2)},b\right]^{(2)}$$

Where $a_{(1)}=\min(a_1,a_2)$ and $a_{(2)}=\max(a_1,a_2)$

Here $a_1<a_2<b$ and $a_2<a_1<b$ are the two possibilities and it is assumed without loss of generality that $\left[a_1,b\right]\cap\left[a_2,b\right]$ is not void.

Now the operation of superimposition of three real intervals $[a_1,b]$, $[a_2,b]$ and $[a_3,b]$ would give the following result

$[a_1,b]\ (S)\ [a_2,b]\ (S)\ [a_3,b]$

$$=\left[a_1,a_2\right]\cup\left[a_2,a_3\right]^{(2)}\cup\left[a_3,b\right]^{(3)},a_1<a_2<a_3<b$$

$$=\left[a_2,a_1\right]\cup\left[a_1,a_3\right]^{(2)}\cup\left[a_3,b\right]^{(3)},a_2<a_1<a_3<b$$

$$=\left[a_3,a_1\right]\cup\left[a_1,a_2\right]^{(2)}\cup\left[a_2,b\right]^{(3)},a_3<a_1<a_2<b$$

$$=\left[a_3,a_2\right]\cup\left[a_2,a_1\right]^{(2)}\cup\left[a_1,b\right]^{(3)},a_3<a_2<a_1<b$$

$$=\left[a_1,a_3\right]\cup\left[a_3,a_2\right]^{(2)}\cup\left[a_2,b\right]^{(3)},a_1<a_3<a_2<b$$

$$=\left[a_2,a_3\right]\cup\left[a_3,a_1\right]^{(2)}\cup\left[a_1,b\right]^{(3)},a_2<a_3<a_1<b$$

Here $\left[a_1,b\right]\cap\left[a_2,b\right]\cap\left[a_3,b\right]$ is not void.

As before, if three intervals are superimposed then the result can be expressed as

$[a_1,b]\ (S)\ [a_2,b]\ (S)\ [a_3,b]$

$$=\left[a_{(1)},a_{(2)}\right]\cup\left[a_{(2)},a_{(3)}\right]^{(2)}\cup\left[a_{(3)},b\right]^{(3)}$$

where

$$a_{(1)}=\min(a_1,a_2)$$

$$a_{(2)}=\max(a_1,a_2)$$

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$a_{(3)} = \max(a_1,a_2,a_3)$

Here $a_1<a_2<a_3<b$, $a_2<a_1<a_3<b$, $a_3<a_1<a_2<b$, $a_3<a_2<a_1<b$, $a_1<a_3<a_2<b$ and $a_2<a_3<a_1<b$ are the six different possibilities.

Now if $[a_1,b]$, $[a_2,b]$, $[a_3,b]$, …, $[a_n,b]$ be n intervals with non void

$$\left[a_1,b\right]\cap\left[a_2,b\right]\cap\left[a_3,b\right]\cap,...,\cap\left[a_n,b\right],$$

then we have the following

$[a_1,b]$ (S) $[a_2,b]$ (S) $[a_3,b]$ (S), …, (S) $[a_n,b]$

$$=[a_{(1)},a_{(2)}]^{(1)}\cup[a_{(2)},a_{(3)}]^{(2)}\cup,...,\cup[a_{(n)},b]^{(n)}$$

Where $a_{(1)},a_{(2)},\ldots,a_{(n)}$ are values of $a_1, a_2,\ldots, a_n$ arranged in ascending order.

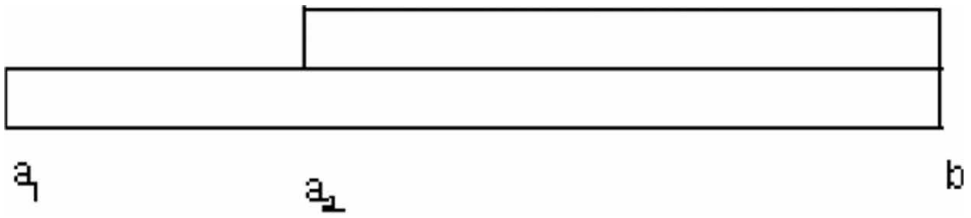
3.1 Superimposition of Two Fuzzy Intervals with Constant Level of Partial Presence

In this section, the superimposition of sets with some constant level of partial presence is taken into consideration. Here fuzzy intervals which are defined as fuzzy sets on the real line will be considered for discussion.

If $\left[a_1,b\right]^{\frac{1}{2}}$ and $\left[a_2,b\right]^{\frac{1}{2}}$ be two fuzzy intervals both with a constant level of partial presence $\frac{1}{2}$ everywhere is considered, then it can be obtained that

$$\left[a_1,b\right]^{\frac{1}{2}}(S)\left[a_2,b\right]^{\frac{1}{2}}=[a_{(1)},a_{(2)}]^{\frac{1}{2}}\cup[a_{(2)},b]^{1}$$

Figure 1. Superimposition of $\left[a_1,b\right]^{\frac{1}{2}}$ and $\left[a_2,b\right]^{\frac{1}{2}}$



Here it is pre assumed that $\left[a_1,b\right]^{\frac{1}{3}}\cap\left[a_2,b\right]^{\frac{1}{3}}$ is not void.

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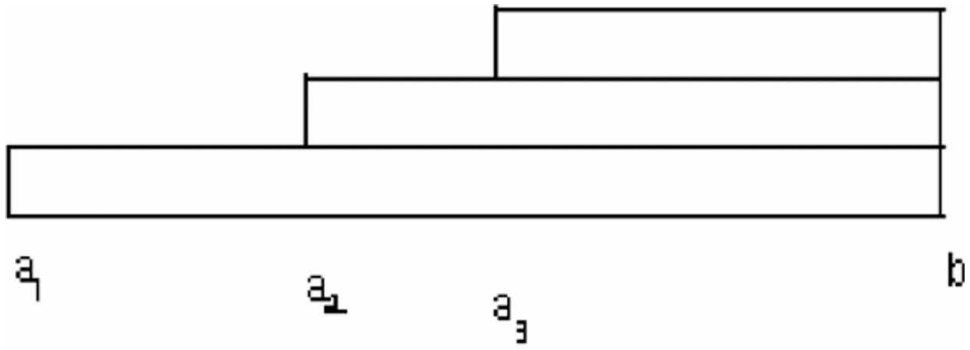
3.2 Superimposition of Three Fuzzy Intervals With Constant Level of Partial Presence

For three intervals $[a_1,b]^{\frac{1}{3}},[a_2,b]^{\frac{1}{3}},[a_3,b]^{\frac{1}{3}}$ all with a constant level of partial presence $\frac{1}{3}$ everywhere in the interval is considered, it can be seen that

$$[a_1,b]^{\frac{1}{3}}(S)[a_2,b]^{\frac{1}{3}}(S)[a_3,b]^{\frac{1}{3}}=[a_{(1)},a_{(2)}]^{\frac{1}{3}}\cup[a_{(2)},a_{(3)}]^{\frac{2}{3}}\cup[a_{(3)},b]^{1}$$

where $a_{(1)},a_{(2)},a_{(3)}$ being values of a_1,a_2,a_3 arranged in increasing order of magnitude. Here it is pre assumed that $[a_1,b]^{\frac{1}{3}}\cap[a_2,b]^{\frac{1}{3}}\cap[a_3,b]^{\frac{1}{3}}$ is not void.

Figure 2. Superimposition of $[a_1,b]^{\frac{1}{3}},[a_2,b]^{\frac{1}{3}}$ and $[a_3,b]^{\frac{1}{3}}$



Similarly as many as fuzzy intervals with constant level of partial presence can be considered. In the next section n number of fuzzy intervals with constant level of partial presence is taken into consideration to reach at the desired goal.

3.3 Superimposition of n Number of Fuzzy Intervals with Constant Level of Partial Presence

For 'n' number of fuzzy intervals $[x_1,b]^{\frac{1}{n}},[x_2,b]^{\frac{1}{n}},[x_3,b]^{\frac{1}{n}},...,[x_n,b]^{\frac{1}{n}}$ all with elements having constant level of partial presence $\frac{1}{n}$ everywhere and one is superimposed over the other then it can be seen that,

$$[x_1,b]^{\frac{1}{n}}(S)[x_2,b]^{\frac{1}{n}}(S)[x_3,b]^{\frac{1}{n}}(S)[x_4,b]^{\frac{1}{n}}.....(S)[x_n,b]^{\frac{1}{n}}$$

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$$=\left[x_{(1)},x_{(2)}\right]^{\frac{1}{n}}\cup\left[x_{(2)},x_{(3)}\right]^{\frac{2}{n}}\cup\left[x_{(3)},x_{(4)}\right]^{\frac{3}{n}}\cup.....\cup\left[x_{(n-1)},x_{(n)}\right]^{\frac{n-1}{n}}\cup\left[x_{(n)},b\right]^{1}$$

where for example $\left[x_{(n-1)},x_{(n)}\right]^{\frac{n-1}{n}}$ represents the interval $[x_{(n-1)},x_{(n)}]$ with level of partial presence $\frac{n-1}{n}$ everywhere for all elements in the entire interval, $x_{(1)},x_{(2)},x_{(3)},\ldots,x_{(n)}$ being values of $x_1,x_2,x_3,\ldots,x_n$ arranged in increasing order of magnitudes.

Thus for fuzzy intervals $\left[x_1,b\right]^{\frac{1}{n}},\left[x_2,b\right]^{\frac{1}{n}},\left[x_3,b\right]^{\frac{1}{n}}.....\left[x_n,b\right]^{\frac{1}{n}}$ all with uniform membership $\frac{1}{n}$, the values of the membership of the superimposed fuzzy intervals are

$$\frac{1}{n},\frac{2}{n},\frac{3}{n},.....\frac{n-1}{n},1$$

These values of membership would suggest that they can define an empirical probability distribution function.

Thus for realization of the values of $x_{(1)},x_{(2)},x_{(3)},\ldots,x_{(n)}$ in increasing order, it can defined that

$$\psi\left(x\right)=\begin{cases}0, & if\ x<x_{(1)}\\ \dfrac{r-1}{n}, & if\ x_{(r-1)}\le x\le x_{(r)},\ \text{if } r=2,3,\ldots,n\\ 0, & \text{otherwise}\end{cases}$$

From the above discussion, it can be said that the existence of one density $\frac{d\psi\left(x\right)}{dx}$ is sufficient condition to construct a fuzzy number of the type $[\alpha,\beta,\beta]$. In other words, the membership function explaining a fuzzy variable which takes a particular value is the distribution function of a random event. Proceeding in the same way, it is desired to find the condition for fuzzy number of the type $[\beta,\beta,\alpha]$. Now in order to construct a fuzzy number $[\beta,\beta,\gamma]$, the following cases are taken into consideration.

4. CONSTRUCTION OF FUZZY NUMBER $[\beta,\beta,\gamma]$

Considering two intervals of the type $[b,c_1]$ and $[b,c_2]$ and if superimposed one over the other and proceeding similarly as above, it can be obtained that

$[b,c_1]$ (S) $[b,c_2]$

$=\left[b,c_1\right]^{(2)}\cup\left[c_1,c_2\right]$ if $b<c_1<c_2$

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$= [b, c_2]^{(2)} \cup [c_2, c_1]$ if b<c_2<c_1

Thus it can be obtained that

$[b,c_1]\ (S)[b,c_2] = [b, c_{(1)}] \cup [c_{(1)}, c_{(2)}]^{(2)}$

where $c_{(1)}$=min(b,c_1) and $c_{(2)}$=max(c_1,c_2)

Here $b,c_1<c_2$ and $b<c_2<c_1$ are the two possibilities and it is assumed without loss of generality that $[b, c_1] \cap [b, c_2]$ is not void.

Now the operation of superimposition of three real intervals $[b,c_1]$, $[b,c_2]$, and $[b,c_3]$ would give the following result

$[b,c_1]$ (S) $[b,c_2]$ (S) $[b,c_3]$

$= [b, c_1]^{(3)} \cup [c_1, c_2]^{(2)} \cup [c_2, c_3], b < c_1 < c_2 < c_3$

$= [b, c_1]^{(3)} \cup [c_1, c_3]^{(2)} \cup [c_3, c_2]^{(3)}, b < c_1 < c_3 < c_2$

$= [b, c_3]^{(3)} \cup [c_3, c_1]^{(2)} \cup [c_1, c_2], b < c_3 < c_1 < c_2$

$= [b, c_3]^{(3)} \cup [c_3, c_2]^{(2)} \cup [c_2, c_1], b < c_3 < c_2 < c_1$

$= [b, c_2]^{(3)} \cup [c_2, c_1]^{(2)} \cup [c_1, c_3], b < c_2 < c_1 < c_3$

$= [b, c_2]^{(3)} \cup [c_2, c_3]^{(2)} \cup [c_3, c_1], b < c_2 < c_3 < c_1$

Here $[b, c_1] \cap [b, c_2] \cap [b, c_3]$ is not void.

As before, if three intervals are superimposed then the result can be expressed as

$[b,c_1]$ (S) $[b,c_2]$ (S) $[b,c_3]$

$= \left[b, c_{(1)} \right] \cup \left[c_{(1)}, c_{(2)} \right]^{(2)} \cup \left[c_{(2)}, c_{(3)} \right]^{(3)}$

where

$c_{(1)} = \min(c_1, c_2)$

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$c_{(2)} = \max(c_1, c_2)$

$c_{(3)} = \max(c_1, c_2, c_3)$

Here $b<c_1<c_2<c_3$, $b<c_1<c_3<c_2$, $b<c_3<c_1<c_2$, $b<c_3<c_2<c_1$, $b<c_2<c_1<c_3$ and $b<c_2<c_3<c_1$ are the six different possibilities.

Now if $[b,c_1]$, $[b,c_2]$, $[b,c_3]$, ……, $[b,c_n]$ be n such intervals then we have the following

$[b,c_1]$ (S) $[b,c_2]$ (S) $[b,c_3]$ (S) …… (S) $[b,c_n]$

$$=\left[b, c_{(1)}\right]^{(n)} \cup \left[c_{(1),} c_{(2)}\right]^{(n-1)} \cup \left[c_{(2)}, c_{(3)}\right]^{(n-2)} \cup .....\cup [c_{(n-1)}), c_{(n)}]^{(1)}$$

$c_{(1)}$, $c_{(2)}$, …, $c_{(n)}$ are the values of c_1, c_2, c_3, …, c_n arranged in ascending order of magnitude.

Here it is assumed that $\left[b, c_1\right] \cap \left[b, c_2\right] \cap \left[b, c_3\right] \cap ..... \cap \left[b, c_n\right]$ is not void.

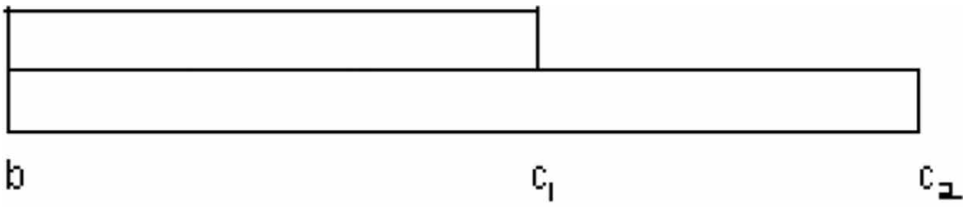
4.1 Superimposition of Two Fuzzy Intervals $[b,c_1]$, $[b,c_2]$ With Constant Level of Partial Presence

For two intervals $\left[b, c_1\right]^{\frac{1}{2}}$ and $\left[b, c_2\right]^{\frac{1}{2}}$ all with a constant level of partial presence $\frac{1}{2}$ everywhere, we shall have

$$\left[b, c_1\right]^{\frac{1}{2}} (S) \left[b, c_2\right]^{\frac{1}{2}} = \left[b, c_{(1)}\right]^{\frac{1}{2}} \cup \left[c_{(1)}, c_{(2),}\right]^{1}$$

Here it is pre assumed that $\left[b, c_1\right]^{\frac{1}{2}} \cap \left[b, c_2\right]^{\frac{1}{2}}$ is not void.

Figure 3. Superimposition of $\left[b, c_1\right]^{\frac{1}{2}}$ and $\left[b, c_2\right]^{\frac{1}{2}}$



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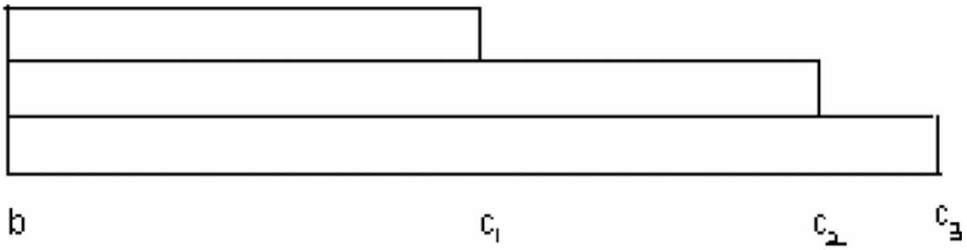
4.2 Superimposition of Three Fuzzy Intervals [b,c_1], [b,c_2] and [b,c_3] with Constant Level of Partial Presence

Now if three intervals $\left[b,c_1\right]^{\frac{1}{3}},\left[b,c_2\right]^{\frac{1}{3}},\left[b,c_3\right]^{\frac{1}{3}}$ with constant level of partial presence $\frac{1}{3}$ everywhere are superimposed one over the other then it can be obtained that

$$\left[b,c_1\right]^{\frac{1}{3}}(S)\left[b,c_2\right]^{\frac{1}{3}}(S)\left[b,c_3\right]^{\frac{1}{3}}=\left[b,c_1\right]^{\left(\frac{3}{3}\right)}\cup\left[c_{1,},c_2\right]^{\left(\frac{2}{3}\right)}\cup\left[c_{2,}c_3\right]^{\frac{1}{3}}$$

Figure 4. Superimposition of $\left[b,c_1\right]^{\frac{1}{3}},\left[b,c_2\right]^{\frac{1}{3}}$, and $\left[b,c_3\right]^{\frac{1}{3}}$



Here it is pre assumed that $\left[b,c_1\right]^{\frac{1}{3}}\cap\left[b,c_2\right]^{\frac{1}{3}}\cap\left[b,c_3\right]^{\frac{1}{3}}$ is not void.

Similarly for n intervals $\left[b,c_1\right]^{\frac{1}{n}},\left[b,c_2\right]^{\frac{1}{n}},\left[b,c_3\right]^{\frac{1}{n}},.....,\left[b,c_n\right]^{\frac{1}{n}}$ with constant level of partial presence $\frac{1}{n}$ everywhere we would have

$$\left[b,c_1\right]^{\frac{1}{n}}(S)\left[b,c_2\right]^{\frac{1}{n}}(S)\left[b,c_3\right]^{\frac{1}{n}}(S).....(S)\left[b,c_n\right]^{\frac{1}{n}}$$
$$=\left[b,c_1\right]^{\left(\frac{n}{n}\right)}\cup\left[c_1,c_2\right]^{\left(\frac{n-1}{n}\right)}\cup\left[c_2,c_3\right]^{\frac{n-2}{n}}\cup.....\cup\left[c_{n-2},c_{n-1}\right]^{\frac{2}{n}}\cup\left[c_{n-1},c_n\right]^{\frac{1}{n}}$$

We now consider a probability space (Ω,A,Π) where Ω is a real interval [α,β]. Let $y_1,y_2,y_3,\ldots,y_n$ be realizations in [α,β]. So for n intervals $\left[b,y_1\right]^{\frac{1}{n}},\left[b,y_2\right]^{\frac{1}{n}},\left[b,y_3\right]^{\frac{1}{n}},...,\left[b,y_n\right]^{\frac{1}{n}}$ all with elements with constant level of partial presence $\frac{1}{n}$ everywhere, we shall have,

$$\left[b,y_1\right]^{\frac{1}{n}}(S)\left[b,y_2\right]^{\frac{1}{n}}(S)\left[b,y_3\right]^{\frac{1}{n}}(S).....(S)\left[b,y_n\right]^{\frac{1}{n}}$$

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$$\left[b, y_{(1)}\right]^{\left(\frac{n}{n}\right)} \cup \left[y_{(1)}, y_{(2)}\right]^{\left(\frac{n-1}{n}\right)} \cup \left[y_{(2)}, y_{(3)}\right]^{\frac{n-2}{n}} \cup \ldots\ldots\ldots \cup \left[y_{(n-2)}, y_{(n-1)}\right]^{\frac{2}{n}} \cup \left[y_{(n-1)}, y_{(n)}\right]^{\frac{1}{n}}$$

Thus for fuzzy intervals $\left[b, y_1\right]^{\frac{1}{n}}, \left[b, y_2\right]^{\frac{1}{n}}, \ldots\ldots \left[b, y_n\right]^{\frac{1}{n}}$ all with uniform membership $\frac{1}{n}$, the values of the membership of the superimposed fuzzy intervals are

$$\frac{n}{n}, \frac{n-1}{n}, \ldots\ldots\ldots\ldots, \frac{2}{n}, \frac{1}{n}$$

Finally it is observed that the membership function of the superimposed fuzzy intervals $\left[x_1, b\right]^{\frac{1}{n}}, \left[x_2, b\right]^{\frac{1}{n}}, \left[x_3, b\right]^{\frac{1}{n}} \ldots\ldots \left[x_n, b\right]^{\frac{1}{n}}$ all with uniform membership $\frac{1}{n}$, are

$$\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots\ldots \frac{n-1}{n}, 1$$

representing the fuzzy number $A=[\alpha,\beta,\beta]$ and the membership functions of the superimposed fuzzy intervals $\left[b, y_1\right]^{\frac{1}{n}}, \left[b, y_2\right]^{\frac{1}{n}}, \ldots\ldots, \left[b, y_n\right]^{\frac{1}{n}}$ all with uniform membership $\frac{1}{n}$, are

$$\frac{n}{n}, \frac{n-1}{n}, \ldots\ldots\ldots\ldots, \frac{2}{n}, \frac{1}{n}$$

corresponding to the fuzzy number $B=[\beta,\beta,\gamma]$.

From the above two results it can be stated that the fuzzy number $N=[\alpha,\beta,\gamma]$ can be expressed as the union of two fuzzy numbers of the type $A=[\alpha,\beta,\beta]$ and $B=[\beta,\beta,\gamma]$.

For illustration purposes, the following numerical example is cited.

Numerical example: Let us consider a fuzzy number N= [1,2, 4] which we would like to express as the union of two numbers A=[1,2,2] and B=[2,2,4]

$${}^{\alpha}A = \left[1+\alpha, 2-0.\alpha\right] \text{ and } {}^{\alpha}B = \left[2+0.\alpha, 4-2.\alpha\right]$$

$${}^{\alpha}A \cup B = {}^{\alpha}A \cup {}^{\alpha}B = 1+\alpha, 2\rbrack \cup \lbrack 2.\alpha, 4-2.\alpha]$$

Then we get the following Alpha cut of the union of two fuzzy sets is equal to the union of the alpha cuts of the individual fuzzy sets.

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Table 1.

α	αA	αB	${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$
0	[1,2]	[2,4]	[1,4]
0.2	[1.2,2]	[2,3.6]	[1.2, 3.6]
0.3	[1.3,2]	[2, 3.4]	[1.3, 3.4]
0.6	[1.6, 2]	[2,2.8]	[1.6, 2.8]
0.8	[1.8,2]	[2, 2.4]	[1.8, 2.4]
1	[2,2]	[2, 2]	[2,2]

5. CONCLUSION

The main intention of this work was to establish the fact that probability can be related to possibility with the help of superimposition of fuzzy intervals and also with the result of the Randomness-Fuzziness Consistency Principle. In the process it is observed that every interval that is a probability space is basically a fuzzy number is established with the help of superimposition of uniform fuzzy intervals in which the end point of an interval and the beginning point of the other interval considered is kept common.. Here it is seen that probability is actually nothing but the fuzzy interval $[\alpha,\beta]$ which can also be expressed as fuzzy number $[\alpha,\beta,\beta]$. Thus it can be concluded here that the theory of probability could be considered as a special case of the theory of normal fuzziness.

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Chapter 21
Intuitionistic Fuzzy Sets in Large-Scale Software Requirement Prioritization

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ABSTRACT

Requirements prioritization (RP) is a crucial process which aims to evaluate candidate software requirements to be implemented in the next release of a software system. RP is often applied iteratively, according to various criteria, by multiple stakeholders, who may have different roles and needs. The problems encountered during the RP of large sets of requirements can be addressed by following a systematic large-scale decision-making (LSDM) process. The aim of this chapter is to propose an LSDM RP process which applies multi-criteria evaluation based on intuitionistic fuzzy sets (IFSs). IFSs are suitable to consider stakeholders hesitation for criteria importance and requirements ratings. The proposed process is supported by a recommender system which provides suggestions to stakeholders and also guide them to collaboratively reach consensus for the final RP. The proposed process is tested using an illustrative example. The results are promising since they demonstrate that the suggested process can effectively support multiple stakeholders to prioritize large requirements sets.

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INTRODUCTION

In this section the background of the areas related to this chapter is presented, focusing on the techniques required for the proposed approach.

Requirements Engineering

Requirements engineering has been identified as one of the most crucial processes in software design and development as it addresses the critical problem of designing the right software for the customer (Aurum & Wohlin, 2005). Poorly organized and expressed requirements, weak relation of requirements to stakeholder needs and unnecessary or unrealistic expectations are among the main reasons why software projects fail (Dick et al., 2017).

Requirements engineering deals with the identification of the objectives and the required functions of a software project, their conversion into services and constraints and their implementation through the assignment of tasks to persons. It is a systematic and disciplined approach to the specification and management of requirements that requires understanding the needs of the stakeholders, prioritization, reaching consensus, identifying and documenting the requirements according to standards and systematically managing them to minimize the risk that a system does not meet the goals (Aurum & Wohlin, 2005).

In large software projects with many requirements, it is not feasible to implement all of the requirements because of resources limitations, such as budget, time and staff. Therefore, the development process delivers requirements in successive releases, where each release contains an incremental number of requirements from all the candidate requirements. The selection of the most important requirements to be developed and delivered first in the early releases is important to meet stakeholders' demands. The requirements that are less important are left for latter releases (Hujainah et al., 2018).

Requirements engineering encompasses certain core activities such as elicitation, prioritization, analysis, negotiation, documentation (or specification), validation and management of requirements. In literature, various approaches exist, where the activities are presented separately, while others group or merge some activities (Bourque et al., 2014; Pohl & Rupp, 2015; Pressman, 2015; Sommerville, 2011).

The RP is one of the key software engineering activities as it helps determine the most important requirements among a pool of requirements and define the order of implementation (Achimugu et al., 2014). The RP process assists stakeholders in selecting the most important requirements of a system and facilitates release planning, and balancing of available resources, budget and time. Additionally, it supports balancing business benefit against the implementation cost, selecting a subset of the requirements that still satisfy the customer and resolving disagreements between stakeholders (Berander & Andrews, 2005).

The RP is an iterative activity which aims to rank the candidate requirements under various criteria. The most frequently used criteria for prioritizing requirements are (Hujainah et al., 2018):

- **Importance:** This is a multi-sided criterion as it varies according to the stakeholders' perspective, knowledge and role.
- **Cost:** The implementation cost is one of the most important criteria from the organization's perspective, since the budget restricts the available resources.
- **Business value:** Alignment with business goals and business value in the long run is of high importance for the RP.

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- Other criteria include value, dependency, risk, effort, penalty, software goals etc.

For the purpose of the RP numerous techniques have been developed aiming to increase the accuracy, the efficiency and the reliability and raise the limitations, since each technique comes with specific restrictions and drawbacks based on the project context (Bukhsh et al., 2020). The most popular techniques could be categorized based on the following approaches:

- **AHP:** The RP techniques based on the AHP (Analytic Hierarchy Process) approach follow a pairwise comparison of the criteria and the candidate features. It is considered the most accurate technique but it suffers from bad scalability and large computational complexity (Achimugu et al., 2014).
- **Fuzzy logic:** The RP techniques based on fuzzy logic aim to include stakeholders' uncertainty (Achimugu et al., 2014).
- **Advanced data processing:** Many recent techniques include data mining and machine learning practices to improve effectiveness and automation (Bukhsh et al., 2020).
- **Numerical assignment:** These techniques aim to group requirements into different categories.
- **Cost – Value:** These RP techniques focus to identify the requirements value against their implementation cost.

Large Scale Decision Making

Software engineering is a group-intensive task where stakeholders often have to make joint decisions, for example regarding the requirements that should be implemented in the next software release or regarding the evaluation of a defined set of requirements (Felfernig et al., 2018b). Large software projects have numerous candidate requirements prioritized under multiple criteria and involve multiple stakeholders. Many decisions have to be made concerning the requirements that will be finally released. In this context the requirements engineering is classified as a decision-rich complex problem solving activity (Aurum & Wohlin, 2005).

A decision-making event is defined as LSDM when the alternatives provided are to be judged or assessed under several criteria/attributes and the decision process is implemented as a dynamic process aimed to select the final decision and increasing the support shown by participants or (Ding et al., 2020).

The emergence of global markets and the growth of social media and social networks has enabled the participation of multiple people in a decision-making problem and becomes an important sub-area of the research on decision-making and consensus building. The main limitations and challenges in LSDM are the development of scalable and distributed decision support architectures, the identification and management of participants behaviors, the monitoring of the progress of the process and the implementation of adequate strategies for weighting and aggregating the decision information (Palomares, 2018).

The level of agreement among the participants of a decision-making event on an accepted opinion is measured using consensus measures, most of them are classified into two types:

- ***Consensus measures based on distances to the collective preference:*** To measure agreement in this type of consensus measures, firstly, the collective preference is computed by aggregating all individual preferences of experts with an aggregation operator. The individual consensus degrees are obtained by calculating the distances between each individual preference and the collective

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preference, using a distance metric. The level of consensus is determined by aggregating the individual CDs. For example the group consensus level is defined as (Xu et al., 2015):

$$GCI = \frac{1}{I}\sum_{i=1}^{I} CI\left(R^i\right)$$

where $CI(R^i)$ is the consensus level between a DM and the group decision of all stakeholders. $CI(R^i)$ is given by the equation:

$$CI\left(R^i\right) = 1 - \frac{1}{M \times N} d\left(R^i, R^c\right)$$

where $d(R^i, R^c)$ is the Manhattan distance and M, N are the number of alternatives and attributes respectively.

- ***Consensus measures based on pairwise similarities (or distances):*** In this type, the consensus measures are based on the similarity between participants (Chiclana et al., 2013). The degree of similarity between experts' opinions is computed using an appropriate distance metric for each pair of experts. The similarity values are then aggregated to obtain the consensus degrees. For a group with m members, this process implies calculating similarities between all the $m(m-1)/2$ different pairs of them. The consensus matrix is given by (Palomares et al., 2014):

$$cm^{lk} = \frac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} sm_{ij}^{lk}}{m\left(m-1\right)/2}$$

where $m(m-1)/2$ is the number of different pairs of experts and sm_{ij}^{lk} is the similarity degree between experts in their assessments, which is obtained by means of a similarity function. The CD at the level of pairs of alternatives is $cp^{lk}=cm^{lk}$, at the level of alternatives is:

$$ca^{l} = \frac{\sum_{k=1,k\neq l}^{n} cp^{lk}}{n-1}$$

and the overall CD is:

$$cr = \frac{\sum_{l=1}^{n} ca^{l}}{n}$$

In case the predefined consensus threshold is reached, then the group moves to the selection process, otherwise the participants are required to further discuss and modify their preferences. The CRP is repeated until the consensus threshold or a number of maximum discussion rounds is reached.

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If the initial consensus degree (CD) does not meet a predefined threshold, a procedure is required to improve the consensus level. An important issue in this procedure is to identify the stakeholders who should revise their preferences towards increasing the CD. Since the stakeholders involved in LSDM are diverse, there might exist important conflict between their preferences and to keep the final decision as beneficial to them as possible, some stakeholders may try leaving their initial preferences unchanged (Ding et al., 2020). The stakeholders who contribute the least to the consensus level or present negative behaviors are treated as non-cooperative participants (Palomares et al., 2014). The detection of non-cooperative stakeholders is a key aspect for improving the CD. Most non-cooperative behavior detection approaches at individual or cluster level, are based on distance measures. One of the most widely used and effective approach to reduce the negative influence of non-cooperative stakeholders in LSDM problems is to implement a penalizing mechanism on the decision weights. The rationale behind weight updating and penalizing schemes is to reduce the importance of the detected non-cooperative stakeholders (Ding et al., 2020).

The stakeholders who revise their preferences and move towards the collective preference are characterized cooperative. On the other hand, the stakeholders who do not revise their preferences, or move further away from the group preference, are characterized non-cooperative and their weights are penalized. If a weight punishment parameter β is assumed, then the updated penalized weight wi of noncooperative stakeholders after the CRP round T can be defined by the equation (Liu et al., 2019):

$$w_{i_(T+1)} = w_{i_T} \times \beta$$

and the updated weight w_j of cooperative stakeholders is:

$$w_{j_(T+1)} = \sum_{i=1}^{k-q} \frac{(1-\beta) w_{i_T}}{k-q} + w_{j_T}$$

where k is the total number of stakeholders and q is the number of non-cooperative stakeholders.

Fuzzy Logic

Fuzzy Sets

A classical set is a collection of elements that are defined in a universe of discourse such as $A = \{x | x>10, x \in U = \{0,1,2,\ldots,20\}\}$. In this set, each element either belongs or does not belong, which can be described by the following membership function:

$$x_A(x) = \begin{cases} 1, & iff \ x \in A \\ 0, & iff \ x \notin A \end{cases}$$

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where iff is ***if and only if***. In this function the limit of the membership and non-membership is clear and ***crisp***. Crisp means that the answer is "yes" or "no", rather than "a little bit less or a little bit more" and that the statement is either "true or false", which in mathematics may be defined as either 0 or 1.

However, in reality there are values whose membership in a set is not clear, such as the age of a person and the set of young people or the temperature and the set of low temperatures. The fuzzy logic of the gradual transition from non-membership to membership using a membership function in the interval [0,1] was introduced in 1965 by Zadeh. The fuzzy logic has been applied to various application areas such as decision support systems, artificial intelligence and control systems (Chaira, 2019).

Let X be a space of points and $x \in X$. A fuzzy set is characterized by a membership function $f_a(x)$ which associates with each point in X a real number in the interval [0,1], with the value of $f_a(x)$ at x representing the grade of membership of x in A (Zadeh, 1965). If the membership function is restricted to the values 0 and 1, then the fuzzy set corresponds to a crisp set, thus a fuzzy set is a generalization of a crisp set. A fuzzy set is written in the form $A = \{x, f_a(x) | \ x \in X\}$.

Intuitionistic Fuzzy Sets

In 1986 the concept of fuzzy set was generalized by Atanassov by proposing the IFSs, which introduce additional degrees of freedom. Each element of an IFS is described by using both the membership and non-membership of a value to a set, while their sum does not exceed 1.

Let X be a space of points and $x \in X$. An IFS is characterized by a membership function $\mu_a(x)$ and a non-membership function $v_a(x)$ which associate with each point in X real numbers in the interval [0,1], such that:

$$0 \le \mu_a(x) + v_a(x) \le 1$$

with the values of $\mu_a(x)$ and $v_a(x)$representing the grade of membership and the grade of non-membership of x in A respectively.

If $\pi_a(x) = 1 - \mu_a(x) - v_a(x)$, then $\pi_a(x)$ is the degree of indeterminacy of the element (Atanassov, 1986). An IFS is written in the form:

$$A = \{x, \mu_a(x), v_a(x) \mid x \in X \ \ and \ \ 0 \le \mu_a(x) + v_a(x) \le 1\}$$

The degree of indeterminacy is also defined as uncertainty (Atanassov, 2012), intuitionistic fuzzy index (Szmidt, 2014) and hesitation degree that represents the lack of knowledge or personal error while assigning the membership degree (Chaira, 2019).

Making use of the IFSs instead of fuzzy sets implies the introduction of additional degrees of freedom (non-memberships and hesitation margins) into the set description. Such a generalization of fuzzy sets gives an additional possibility to represent imprecise knowledge which may lead to describing many real problems in a more adequate way (Szmidt, 2014).

Taking into account that for each element x belonging to an intuitionistic fuzzy set A, the values of membership, non-membership and the intuitionistic fuzzy index sum up to one:

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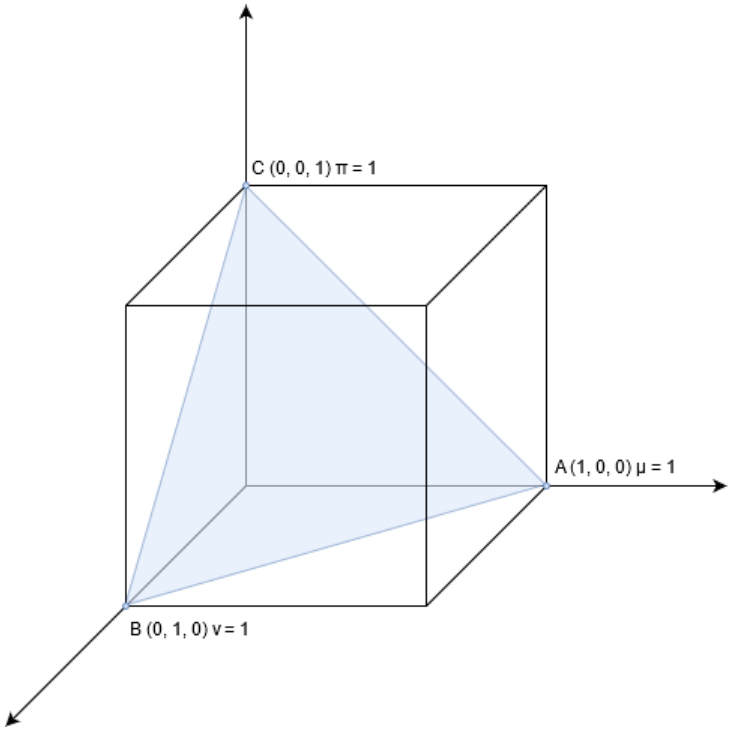
$$\mu_a(x)+\nu_a(x)+\pi_a(x)=1$$

and that each of the membership, non-membership, and the intuitionistic fuzzy index belongs to [0,1], any element belonging to an IFS can be represented by the surface of the ABC triangle inside a unit cube as shown in Figure 1:

Figure 1. Geometrical representation of IFS



Each point belonging to the ABC triangle is described via three coordinates (μ,ν,π). Points A and B represent crisp elements. Point A (1,0,0) represents elements fully belonging to an intuitionistic fuzzy set as μ=1. Point B(0,1,0) represents elements fully not belonging to an intuitionistic fuzzy set as v=1. Point C(0,0,1) represents elements which it is not possible at all to say if they belong or not to an intuitionistic fuzzy set (intuitionistic fuzzy index π=1) (Szmidt & Kacprzyk, 2000)

Intuitionistic Fuzzy Sets Distance and Similarity

Similarity and ***dissimilarity a***re important because they are used by techniques, such as clustering, nearest neighbor classification, and anomaly detection. In many cases, the initial data set is not needed once these similarities or dissimilarities have been computed. Such approaches can be viewed as transforming the data to a similarity space and then performing the analysis. The ***similarity*** between two objects is a numerical measure of the degree to which the two objects are alike. Consequently, similarities are higher

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for pairs of objects that are more alike. Similarities are usually nonnegative and are often between 0 (no similarity) and 1 (complete similarity). The di***ssimilarity b***etween two objects is a numerical measure of the degree to which the two objects are different. Dissimilarities are lower for more similar pairs of objects. Frequently, the term di***stance i***s used as a synonym for dissimilarity. Distances, have some well-known properties. More specifically, if d (x, y) is the distance between two points, x and y, then the following properties hold (Tan et al., 2014):

- Positivity:
  - $d(x,y) \geq 0$ for all x and y
  - $d(x,y)=0$ only if $x=y$
- Symmetry: $d(x,y)=d(y,x)$ for all x and y
- Triangle Inequality: $d(x,z) \leq d(x,y)+d(y,z)$ for all points x, y and z

The ***Euclidean distance*** between two points, x and y, in one-, two-, three-, or higher dimensional space, is given by the equation:

$$d\left(x,y\right)=\sqrt{\sum_{k=1}^{n}\left(x_k-y_k\right)^2}$$

where n is the number of dimensions and x_k, y_k are the k^{th} attributes of x and y respectively. The Euclidean distance measure is generalized by the ***Minkowski distance*** metric given by the equation:

$$d\left(x,y\right)=\left(\sum_{k=1}^{n}\left|x_k-y_k\right|^r\right)^{1/r}$$

where r is a parameter. Some common values of r are:

- For $r=1$, the equation gives the ***City Block*** (Manhattan, taxicab, L1 norm) distance. A common example is the Hamming distance, which is the number of bits that are different between two objects that have only binary attributes.
- For $r=2$, the equation gives the ***Euclidean distance*** (L2 norm)
- For $r=\infty$, the equation gives the ***Supremum distance*** (Lmax or $L\infty$ norm) distance. This is the maximum difference between any attribute of the objects.

Given the importance of the distance and similarity for clustering, nearest neighbor classification, and anomaly detection techniques, several measures have been proposed for IFSs. More specifically, initially the Hamming and Euclidean distance of IFSs were proposed (Szmidt & Kacprzyk, 2000):

- Hamming distance:

$$d_{Hamming}\left(A,B\right)=\frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_A\left(x_i\right)-\mu_B\left(x_i\right)\right|+\left|\nu_A\left(x_i\right)-\nu_B\left(x_i\right)\right|+\left|\pi_A\left(x_i\right)-\pi_B\left(x_i\right)\right|\right)$$

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- Euclidean distance:

$$d_{Euclidean}(A,B)=\sqrt{\frac{1}{2}\sum_{i=1}^{n}\left(\mu_A(x_i)-\mu_B(x_i)\right)^2+\left(\nu_A(x_i)-\nu_B(x_i)\right)^2+\left(\pi_A(x_i)-\pi_B(x_i)\right)^2}$$

Later a new similarity measure for IFSs using a geometrical interpretation was proposed (Szmidt & Kacprzyk, 2004):

- Szmidt & Kacprzyk distance:

$$d_{SK}(A,B)=\frac{1}{m}\sum_{i=1}^{m}\left(\frac{\left|\mu_1(x_i)-\mu_2(x_i)\right|+\left|\nu_1(x_i)-\nu_2(x_i)\right|+\left|\pi_1(x_i)-\pi_2(x_i)\right|}{\left|\mu_1(x_i)-\nu_2(x_i)\right|+\left|\nu_1(x_i)-\mu_2(x_i)\right|+\left|\pi_1(x_i)-\pi_2(x_i)\right|}\right)$$

The IFS Hausdorff distance, which is the maximum distance of a set to the nearest point in the other set, is given by the equation (Szmidt & Kacprzyk, 2011).

- Hausdorff distance:

$$d_{Hausdorff}(A,B)=\max\left\{\left|\mu_A(x_i)-\mu_B(x_i)\right|,\left|\nu_A(x_i)-\nu_B(x_i)\right|,\left|\pi_A(x_i)-\pi_B(x_i)\right|\right\}$$

In literature, several other distance and similarity measures are found (Chaira, 2019). However, most distance measures, have not thoroughly evaluated the intuitionistic fuzzy information, because the cross-evaluation between the degrees of membership and non-membership has not been considered. These measures do not appear to be effective in complex decision-making problems. A new distance measure called H-max of IFSs, where the cross-evaluation is considered, is proposed by (Ngan et al., 2018). More specifically for two IFSs A and B on the universe $X=\{x_1,x_2,\ldots,x_m\}$, which are defined by μ_1, ν_1 and μ_2, ν_2, where μ_1, μ_2 and ν_1, ν_2 are their membership and non-membership degrees respectively, the pairwise distance is defined as follows:

$$d_{Hm}(A,B)$$
$$=\frac{1}{3m}\sum_{i=1}^{m}\left(\left|\mu_1(x_i)-\mu_2(x_i)\right|+\left|\nu_1(x_i)-\nu_2(x_i)\right|+\left|\max\left\{\mu_1(x_i),\nu_2(x_i)\right\}-\max\left\{\mu_2(x_i),\nu_1(x_i)\right\}\right|\right)$$

The use of the H-max distance measure was demonstrated in a medical diagnosis process, where the degree of relation between patient symptoms and symptom characteristics is used to calculate the patient disease. The H-max distance measure was applied on various real-world datasets using a train and test cross-validation process and was evaluated using the Mean absolute error (MAE) metric. This distance measure produced the best results in most datasets (Ngan et al., 2018).

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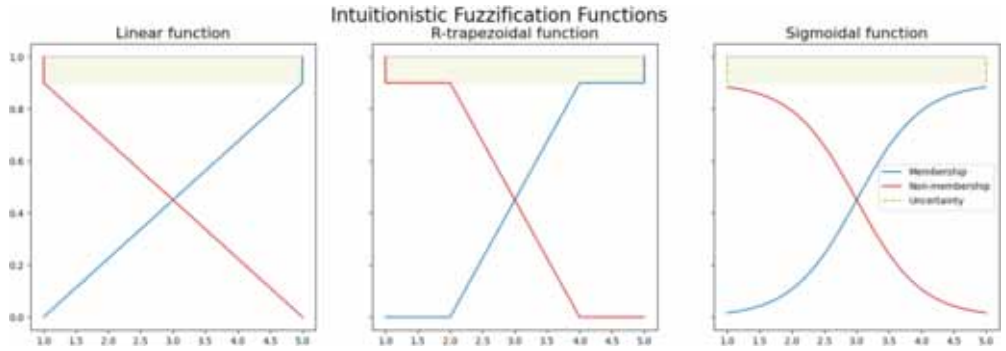
Intuitionistic Fuzzification - Defuzzification

Fuzzification is the process of converting crisp data to fuzzy. If uncertainty exists because of imprecision or vagueness, then the variable is fuzzy and can be represented by a membership function. The term intuitionistic fuzzification refers to formulating membership and non-membership functions of an IFS. Various types of intuitionistic fuzzification functions where introduced such as triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal, S-shaped, Z-shaped functions, which are useful in modeling real world situations in intuitionistic fuzzy environment (Radhika & Rangasamy, 2016).

Based on the concept of (Radhika & Rangasamy, 2016), various intuitionistic fuzzification functions are used in this chapter: linear, r-trapezoidal and sigmoidal, whose membership and non-membership form is shown in Figure 2:

Figure 2. Intuitionistic fuzzification functions form



The intuitionistic fuzzification functions include the variables a and d, which define the range of the fuzzified values, the variables b and c, which define the lower and upper support limit of the trapezoidal function and the variable π, which is the uncertainty degree and defines the level of uncertainty. The membership, non-membership and uncertainty degree are given by the following equations:

- Linear

$$\mu_A(x) = \begin{cases} 1, & x \geq d \\ (1-\pi)\left(\dfrac{x-a}{d-a}\right), & a < x < d \\ 0, & x \leq a \end{cases}$$

$$\nu_A(x) = \begin{cases} 0, & x \geq d \\ (1-\pi)\left(\dfrac{d-x}{d-a}\right), & a < x < d \\ 1, & x \leq a \end{cases}$$

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$$\pi_a(x) = 1 - \mu_a(x) - \nu_a(x)$$

- R-trapezoidal-1

$$\mu_A(x) = \begin{cases} 1, & x \geq d \\ 1-\pi, & c \leq x < d \\ (1-\pi)\left(\dfrac{x-a}{c-b}\right), & b < x < c \\ 0, & a < x \leq b \\ 0, & x \leq a \end{cases}$$

$$\nu_A(x) = \begin{cases} 0, & x \geq d \\ 0, & c \leq x < d \\ (1-\pi)\left(\dfrac{c-x}{c-b}\right), & b < x < c \\ 1-\pi, & a < x \leq b \\ 1, & x \leq a \end{cases}$$

$$\pi_a(x) = 1 - \mu_a(x) - \nu_a(x)$$

- R-trapezoidal-2

$$\mu_A(x) = \begin{cases} 1-\pi, & c \leq x \\ (1-\pi)\left(\dfrac{x-c}{c-b}\right), & b < x < c \\ 0, & x \leq b \end{cases}$$

$$\nu_A(x) = \begin{cases} 0, & c \leq x \\ (1-\pi)\left(\dfrac{c-x}{c-b}\right), & b < x < c \\ 1-\pi, & x \leq b \end{cases}$$

$$\pi_a(x) = 1 - \mu_a(x) - \nu_a(x)$$

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- Sigmoidal

$$\mu_A(x) = \begin{cases} \frac{1-\pi}{1+e^{(-2(x-2))}} \end{cases}$$

$$\nu_A(x) = \begin{cases} 1-\pi-\frac{1-\pi}{1+e^{(-2(x-2))}} \end{cases}$$

$$\pi_a(x) = 1-\mu_a(x)-\nu_a(x)$$

The intuitionistic defuzzification process refers to conversion of Intuitionistic Fuzzy Values (IFV) to crisp data. Based on the concept of the recommendation function by (Son & Thong, 2015), the following equation is used in this chapter to convert an IFV $A=(\mu_A,\nu_A,\pi_a)$, where μ_A i*s* the membership, ν_A i*s* the non-membership and ,π_a is the uncertainty degree, to the crisp value cA:

$$c_A = \mu_A + \mu_A(1-\mu_A-\nu_A)$$

Intuitionistic Fuzzy Sets in Requirements Prioritization

The RP methods using fuzzy logic techniques can handle vague, incomplete and inaccurate information and deal with stakeholders' uncertainty. The main advantage of the RP methods based on fuzzy sets techniques is the ability to preserve vague information, which means better accuracy in the final results. In literature, there is a number of techniques that implement the Fuzzy AHP, which is an extension of the AHP. In these techniques, pairwise comparisons of requirements and hierarchy criteria, is applied in linguistic terms expressed in triangular or trapezoidal functions instead of numerical values (Achimugu et al., 2014).

The application of IFSs in RP techniques is useful in order to take into account the uncertainty and lack of knowledge of the stakeholders. IFSs incorporate membership and non-membership scores, but also take into account vagueness during ranking candidate requirements. While prioritizing, some requirements remain unranked or are assigned the same rank with other requirements, because stakeholders do not have enough knowledge to evaluate them precisely. Stakeholders indeterminacy and hesitation to determine different ranks for all the requirements under multiple criteria prioritization is mapped to IFSs and rankings with lower vagueness are taken into account more considerably in the final priority list (Gerogiannis et al., 2018).

The IFSs are also applied to quantify the asymmetry of satisfaction and dissatisfaction when these two factors are considered as prioritization criteria of features. Since some features may generate satisfaction to users, if included in the next software release, but do not create the same value of dissatisfaction, if they are not included, there is an asymmetry which may introduce additional hesitation and uncertainty to stakeholders. The final priority list is derived with respect to the asymmetric criteria of satisfaction and dissatisfaction by calculating objective weights. The technique assumes the larger the hesitation associated with each stakeholder ranking, the smaller will be the weight of that ranking in the calculation of the final priorities (Gerogiannis et al., 2020). IFSs are also applied in more generalized decision support making processes as well (Ding et al., 2019; Liu et al., 2019).

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Recommender Systems

An RS is defined as "a specific type of advice-giving or decision support system that guides users in a personalized way to interesting or useful objects in a large space of possible options" (Felfernig et al., 2018a). Instead of navigating through all available items, the RS provide suggestions to help individual users or groups discover items of interest. The interest of a user is typically expressed in the form of numerical value ratings. The two primary models of RS are the prediction of a missing rating value and the determination of the top-k items to recommend to a user (Aggarwal, 2016).

The most common approaches used in RS are (Felfernig et al., 2018a):

- ***Collaborative Filtering:*** In collaborative filtering, recommendations are provided taking into account the preferences of similar users. The ratings provided specifies the degree of interest of a user for the items. The algorithm determines the k-nearest neighbors of a user, based on ratings similarity, and calculates a prediction rating.
- ***Content-Based Filtering:*** In content-based filtering the algorithm determines items that are similar to those positively rated by a user, based on the content or the category of an item.
- ***Constraint-Based Recommendation:*** Constraint-based recommendation relies on a set of rules and a set of items. Depending on the user search criteria and constraints the algorithm identifies the items that match the user's preferences.
- ***Critiquing-Based Recommendation:*** In critiquing-based RS the use is navigated through available items in critiquing cycles, where a reference item is presented to the user and the user either accepts the recommended item or searches for different solutions by specifying critiques.
- ***Hybrid Recommendation:*** Hybrid RS combine two or more approaches.

Collaborative filtering, which is used in the application of this chapter, is based on the idea of word-of-mouth promotion where opinions of relatives and friends play a major role when taking a decision. In recommendation algorithms, family members and friends are replaced by nearest neighbors who are users with preferences similar to the ones of the current user. In collaborative filtering, a user×item rating specifies to which extent a user likes an item. The rating predictions are determined based on the preferences of the ***k-nearest neighbors*** according to a similarity measure. The similarity between a user and another user can be determined, for example, using the Pearson correlation coefficient for the set of items that have been rated by both users. The information about the set of k-users with a rating behavior similar to that of the current user (k-nearest neighbors) is the basis for predicting the rating for an item that has not been rated yet. A challenge when determining the similarity between users is the sparsity of the rating matrix. Users typically provide ratings for only a very small subset of the offered items. One approach is to take into account the number of commonly rated items as a correlation significance (Felfernig et al., 2018a).

The integration of IFSs in RS was proposed in a medical diagnosis problem, based upon the observation that medical data are often imprecise, incomplete and vague so that using the standalone IFS and RS methods may not improve the accuracy of diagnosis (Son & Thong, 2015). The proposed process applied the collaborative filtering technique using an intuitionistic fuzzy similarity degree to predict the membership and non-membership value of a patient's disease, with the following equations (Son & Thong, 2015):

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$$\mu_{iD}^{P_u}\left(D_j\right)=\frac{\sum_{v=1}^{n} sim\left(P_u,P_v\right)\times\mu_{iD}^{P_v}\left(D_j\right)}{\sum_{v=1}^{n} sim\left(P_u,P_v\right)}$$

$$\nu_{iD}^{P_u}\left(D_j\right)=\frac{\sum_{v=1}^{n} sim\left(P_u,P_v\right)\times\nu_{iD}^{P_v}\left(D_j\right)}{\sum_{v=1}^{n} sim\left(P_u,P_v\right)}$$

A content-based RS for recommending a candidate to a voter in an election was proposed in (Dyczkowski & Stachowiak, 2012). To model incomplete knowledge the process integrated IFSs, while the similarity between candidates and voters was calculated using the Hausdorff distance.

PROPOSED LARGE SCALE REQUIREMENTS PRIORITIZATION PROCESS

In this section the proposed approach is presented in the context of RP of a large software system, rated under multiple criteria by multiple stakeholders. The presentation of the proposed approach is accompanied by a running example, which aims to demonstrate the techniques and the corresponding calculations. To keep the calculations simple, the running example includes 5 stakeholders (St1, St2, St3, St4, St5), who rate 3 candidate requirements (Req1, Req2, Req3), under 3 rating criteria (Cr1, Cr2, Cr3). The proposed process is simulated using an illustrative artificial dataset in the next Section.

The proposed approach, consists of the following phases:

- **Requirements prioritization problem definition:** During this phase the details of the problem are defined, such as the number of stakeholders, the stakeholders groups, the candidate requirements, the rating criteria, the prioritization classes and the requirements prioritization expert knowledge matrix.
- **Requirements recommendation:** The stakeholders are rating the candidate requirements. However, not all stakeholders are interested or have knowledge and expertise to rate all alternatives. During the requirements recommendations phase, a list of candidate requirements, that are predicted to be of higher interest, is presented to the stakeholders.
- **Consensus measurement and requirements ranking:** During this phase, the CD is calculated. If the threshold is reached the final ranking of the requirements is presented and the process finishes, otherwise the process may return to the requirements recommendation phase.

Requirements Prioritization Problem Definition

The first phase is the requirements prioritization problem definition. In this phase the organization experts or the project manager define the details of the RP problem, such as the candidate requirements, the rating criteria, the prioritization classes and the importance of each criterion for each priority class. The stakeholders are numerous as well as the number of candidate requirements. Each candidate requirement is evaluated according to multiple criteria and the event is classified as a multi-expert multi-criteria LSDM event (Ding et al., 2020).

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The prioritization set of classes could be a simple scheme such as "High", "Medium" and "Low", the release plan (Release 1, Release 2, etc) or other prioritization technique classes. In case only one class is defined, then the result of the prioritization process will be a decreasing priority ranking list.

Afterwards, the organization experts or the project manager are rating the importance of each criteria for each prioritization class using a numeric scale (e.g., 1-5), in order to create the ***Criteria-Priority Classes Relation Matrix (CPCRM)***. For positive criteria (e.g., business value, customer satisfaction etc.), higher values express that experts would like to classify requirements that are rated high with regards to these criteria, in the specific priority class. On the other hand, for negative criteria (e.g. cost, risk), lower values indicate that the experts would like to classify requirements that are rated low in the specific priority class. The numeric scale ratings are then converted to IFVs, which represent the membership and non-membership of the criteria to each prioritization class, as well as the degree of uncertainty, using an intuitionistic fuzzification function. In case the organization experts or the project manager are familiar with the intuitionistic fuzzy logic concepts, the membership and non-membership of the criteria to each prioritization class, as well as the degree of uncertainty is defined directly using IFVs.

Let $G=\{g_1,g_2,\ldots,g_O\}$ be the stakeholders groups set, where $O\geq 1$.
Let $S=\{s_1,s_2,\ldots,s_I\}$ be the stakeholders set, where $I\geq 2$.
Let $W=\{w_1,w_2,\ldots,w_I\}$ be the stakeholders' weights set.
Let $F=\{f_1,f_2,\ldots,f_M\}$ be the candidate requirements set, where $M\geq 2$.
Let $C=\{c_1,c_2,\ldots,c_N\}$ be the rating criteria set of each requirement, where $N\geq 1$.
Let $P=\{p_1,p_2,\ldots,p_L\}$ be the priority classes, where $L\geq 1$.

Let $\mathrm{M}_{\mathrm{CP_num}}$ be the numeric CPCRM.

$$M_{CP_num}=\begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1L} \\ k_{21} & k_{22} & \cdots & k_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ k_{N1} & k_{N2} & \cdots & k_{NL} \end{pmatrix}=\left(k_{ij}\right)\in\mathbb{R}^{\mathrm{N\times L}} \tag{1}$$

The numeric values of the matrix $\mathrm{M}_{\mathrm{CP_num}}$ are converted to IFVs using an intuitionistic fuzzification function, where $\mu_A(x)$, $v_A(x)$ and $\pi_A(x)$ are the membership, non-membership and uncertainty degree functions of each element respectively. The values of μ and v represent the membership and non-membership of the importance of each criterion to each prioritization class, while π represents the level of uncertainty of the experts. The intuitionistic fuzzification function could be chosen among various functions, such as linear, trapezoidal or sigmoidal (Chaira, 2019). The function and the parameterπ are defined by the organization experts or the project manager.

Let M_{CP} be the IFVs CPCRM, where each rating criteria for the priority class is described by two numbers: membership μ and non-membership v (the value of π is $\pi=1-\mu-v$):

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$$M_{CP}=\begin{pmatrix} \mu_{11},\nu_{11} & \mu_{12},\nu_{12} & \cdots & \mu_{1L},\nu_{1L} \\ \mu_{21},\nu_{21} & \mu_{22},\nu_{22} & \cdots & \mu_{2L},\nu_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{N1},\nu_{N1} & \mu_{N2},\nu_{N2} & \cdots & \mu_{NL},\nu_{NL} \end{pmatrix}=\left(\mu_{ij},\nu_{ij}\right)\in\mathbb{R}^{\mathrm{N\times L}} \tag{2}$$

Example 1

Consider an RP problem where 5 stakeholders (St1, St2, St3, St4, St5) are rating 3 candidate requirements (Req1, Req2, Req3), under 3 rating criteria (Cr1, Cr2, Cr3).

The project manager defines that the prioritization classes are 4 (Release1, Release2, Release3, Release4). Then the project manager rates the importance of each criterion for each prioritization class in the CPCRM using numerical values as shown in Table 1:

Table 1. Numeric CPCRM

	Release1	**Release2**	**Release3**	**Release4**
Customer Satisfaction (C1)	5	4	3	2
Business Value (C2)	5	3	2	1
Implementation Duration (C3)	1	2	3	4

The project manager defines that the following ***r-trapezoidal*** intuitionistic fuzzification function is used***:***

$$\mu_A(x)=\begin{cases} 1-\pi, & c\le x \\ (1-\pi)\left(\dfrac{x-c}{c-b}\right), & b<x<c \\ 0, & x\le b \end{cases} \tag{3}$$

$$\nu_A(x)=\begin{cases} 0, & c\le x \\ (1-\pi)\left(\dfrac{c-x}{c-b}\right), & b<x<c \\ 1-\pi, & x\le b \end{cases} \tag{4}$$

$$\pi_a(x)=1-\mu_a(x)-\nu_a(x) \tag{5}$$

where b=1.5, c=4.5, π=0.1. The intuitionistic fuzzification function is depicted in the plot of Figure 3:

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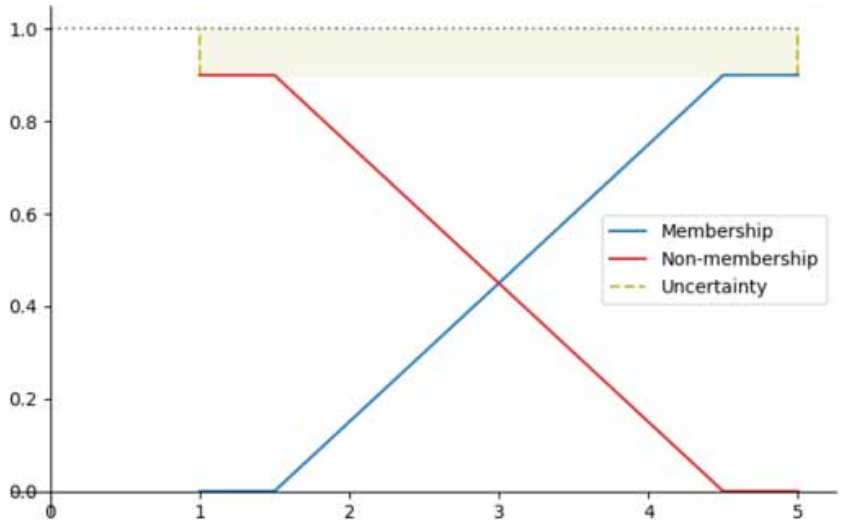
Figure 3. R-trapezoidal intuitionistic fuzzification function



The numeric CPCRM is converted to the IFVs CPCRM shown in Table 2 using the intuitionistic fuzzification equations ***(3)***, ***(4)*** and ***(5)***:

Table 2. Intutionistic Fuzzy Values CPCRM

	Release1	**Release2**	**Release3**	**Release4**
Customer Satisfaction (C1)	0.9, 0	0.75, 0.15	0.45, 0.45	0.15, 0.75
Business Value (C2)	0.9, 0	0.45, 0.45	0.15, 0.75	0, 0.9
Implementation Duration (C3)	0, 0.9	0.15, 0.75	0.45, 0.45	0.75, 0.15

Requirements Ratings Recommendations Phase

The second phase is the requirements ratings recommendations phase. During an RP process, where the stakeholders and the alternatives are numerous, not all stakeholders are interested or have the knowledge and expertise to rate all alternatives. Thus, personalized recommendations for requirements that attract stakeholders' interest assist in providing more accurate ratings. The requirements ratings recommendations phase applies the collaborative filtering approach to propose the top requirements according to the stakeholders' preferences. The ratings already provided by the stakeholders are used to calculate similar stakeholders and make recommendations to a target stakeholder.

Let $R_{SCF_{num}}$ be the ratings matrix, where each stakeholders' rating for the alternatives criteria is described by a rating in a continuous scale e.g. in a 1-5 scale, where 1 means that the requirement performs badly and 5 that the requirement performs excellent with regards to the specific criteria. In case a stakeholder has not provided a rating, the value is marked as Not Available (NA):

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$$R_{SCF_{num}} = \begin{pmatrix} r_{11}^1 & \cdots & r_{1N}^1 & r_{11}^2 \cdots & r_{1N}^2 \cdots & r_{11}^M \cdots r_{1N}^M \\ r_{21}^1 & \cdots & r_{2N}^1 & r_{21}^2 \cdots & r_{2N}^2 \cdots & r_{21}^M \cdots r_{2N}^M \\ \vdots & \vdots & \vdots & \vdots \vdots & \vdots \vdots & \vdots \ddots \vdots \\ r_{I1}^1 & \cdots & r_{IN}^r & r_{I1}^2 \cdots & r_{IN}^2 \cdots & r_{I1}^M \cdots r_{IN}^M \end{pmatrix} = (r_{ij}^k) \in \mathbb{R}^{I\times(N\times M)}$$

Each numerical value is converted to IFV using an intuitionistic fuzzification function in the same approach as described in section. Let R_{SCF} be the IFVs ratings matrix resulting from the $R_{SCF_{num}}$ matrix:

$$R_{SCF} = \begin{pmatrix} \mu_{11,}^1 v_{11}^1 & \cdots & \mu_{1N,}^1 v_{1N}^1 & \mu_{11,}^2 v_{11}^2 & \cdots & \mu_{1N,}^2 v_{1N}^2 & \cdots & \mu_{11,}^M v_{11}^M & \cdots & \mu_{1N,}^M v_{1N}^M \\ \mu_{21,}^1 v_{21}^1 & \cdots & \mu_{2N,}^1 v_{2N}^1 & \mu_{21,}^2 v_{21}^2 & \cdots & \mu_{2N,}^2 v_{2N}^2 & \cdots & \mu_{21,}^M v_{21}^M & \cdots & \mu_{2N,}^M v_{2N}^M \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mu_{I1,}^1 v_{I1}^1 & \cdots & \mu_{IN,}^1 v_{IN}^1 & \mu_{I1,}^2 v_{I1}^2 & \cdots & \mu_{IN,}^2 v_{IN}^2 & \cdots & \mu_{I1,}^M v_{I1}^M & \cdots & \mu_{IN,}^M v_{IN}^M \end{pmatrix} = (\mu_{ij,}^k v_{ij}^k) \in \mathbb{R}^{I\times(N\times M)} \quad (7)$$

Let A and B be two IFSs on the universe $X = \{x_1, x_2, \ldots, x_M\}$which are defined by μ_1, ν_1 and μ_2, ν_2, where μ_1, μ_2 and ν_1, ν_2 are their membership and non-membership degrees respectively. Then the H-max pairwise distance is defined as follows (Ngan et al., 2018):

$$d_{Hm}(A,B) = \frac{1}{3(\mathrm{N}\times\mathrm{M})} \sum_{i=1}^{(\mathrm{N}\times\mathrm{M})} \left(\left|\mu_1(x_i) - \mu_2(x_i)\right| + \left|\nu_1(x_i) - \nu_2(x_i)\right| + \left|\max\left\{\mu_1(x_i), \nu_2(x_i)\right\} - \max\left\{\mu_2(x_i), \nu_1(x_i)\right\}\right| \right) \quad (8)$$

where $0 \le d_{Hm}(A,B) \le 1$.

The stakeholders' pairwise distance matrix DM_{SS} is calculated using the H-max distance equation ***(8)*** to the R_{SCF} matrix, by taking into account common rated values only:

$$DM_{SS} = \begin{pmatrix} 0 & d_{12} & \cdots & d_{1I} \\ d_{21} & 0 & \cdots & d_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ d_{I1} & d_{I2} & \cdots & 0 \end{pmatrix} = \left(d_{ij}\right) \in \mathbb{R}^{\mathrm{I}\times\mathrm{I}} \quad (9)$$

where $d_{ij} = d_{ji}$.

The pairwise similarity is defined as:

$$sim_{Hm}(A,B) = 1 - d_{Hm}(A,B) \quad (10)$$

Thus, the stakeholders' pairwise similarity matrix SM_{SS} is:

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$$SM_{SS} = \begin{pmatrix} 1 & sim_{12} & \cdots & sim_{1I} \\ sim_{21} & 1 & \cdots & sim_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ sim_{I1} & sim_{I2} & \cdots & 1 \end{pmatrix} = \left(s_{ij}\right) \in \mathbb{R}^{I\times I} \tag{11}$$

where $s_{ij}=s_{ji}$.

In order to determine the top-*k* most similar stakeholders, the pairwise distance and pairwise similarity matrix of all stakeholders is calculated. Only the commonly rated requirements are taken into account for the calculation. The top-k most similar stakeholders are used to calculate the predicted rating of a requirement.

The equations to predict the membership and non-membership of a missing rating of stakeholder S_i for requirement F_m, criteria C_n are (Son & Thong, 2015):

$$\mu_{in}^{m} = \frac{\sum_{j=1}^{k} sim_{Hm}\left(s_i, s_j\right) \times \mu_j}{\sum_{j=1}^{k} sim_{Hm}\left(s_i, s_j\right)} \tag{12}$$

$$\nu_{in}^{m} = \frac{\sum_{j=1}^{k} sim_{Hm}\left(s_i, s_j\right) \times \nu_j}{\sum_{j=1}^{k} sim_{Hm}\left(s_i, s_j\right)} \tag{13}$$

The recommended rating value of missing ratings for stakeholder S_i for requirement F_m, criteria C_n is calculated with the equation (Son & Thong, 2015):

$$r_{in}^{m} = \mu_{in}^{m} + \mu_{in}^{m}\left(1 - \mu_{in}^{m} - \nu_{in}^{m}\right) \tag{14}$$

The recommended rating r_{in}^{m} given by equation ***(14)*** belongs to [0,1]. In case a numerical ratings range of values [*a,d*] is used, it is required to de-normalize the recommended rating using the following equation:

$$r_norm_{in}^{m} = r_{in}^{m}\left(d - a\right) + a \tag{15}$$

Example 2

Consider that the 5 stakeholders of Example 1 have assigned the numerical values to the candidate requirements under the rating criteria according to the following Table 3:

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Table 3. Numeric ratings matrix

	Req1			Req2			Req3		
	C1	**C2**	**C3**	**C1**	**C2**	**C3**	**C1**	**C2**	**C3**
St1	5	5	1	2	2	1	4	3	3
St2	4	5	2	4	3	2	1	2	1
St3	5	5	2	NA	NA	NA	2	3	3
St4	NA	NA	NA	4	5	4	2	1	2
St5	NA	NA	NA	1	2	1	NA	NA	NA

The numeric ratings matrix is converted to the IFVs ratings matrix shown in Table 4 using equations ***(3)***, ***(4)*** and ***(5)***:

Table 4. IFVs ratings matrix

	Req1			Req2			Req3		
	C1	**C2**	**C3**	**C1**	**C2**	**C3**	**C1**	**C2**	**C3**
St1	0.9, 0	0.9, 0	0, 0.9	0.15, 0.75	0.15, 0.75	0, 0.9	0.75, 0.15	0.45, 0.45	0.45, 0.45
St2	0.75, 0.15	0.9, 0	0.15, 0.75	0.75, 0.15	0.45, 0.45	0.15, 0.75	0, 0.9	0.15, 0.75	0, 0.9
St3	0.9, 0	0.9, 0	0.15, 0.75	NA	NA	NA	0.15, 0.75	0.45, 0.45	0.45, 0.45
St4	NA	NA	NA	0.75, 0.15	0.9, 0	0.75, 0.15	0.15, 0.75	0, 0.9	0.15, 0.75
St5	NA	NA	NA	0, 0.9	0.15, 0.75	0, 0.9	NA	NA	NA

The stakeholders' pairwise distance and similarity are calculated using equations ***(8)*** and ***(10)***. More specifically the distance between St1 and St2 is:

$$d_{Hm}\left(St1, St2\right) = \frac{1}{3\left(\mathrm{N}\times\mathrm{M}\right)}$$

$$\sum_{i=1}^{(\mathrm{N}\times\mathrm{M})}\left(\left|\mu_1\left(x_i\right)-\mu_2\left(x_i\right)\right|+\left|\nu_1\left(x_i\right)-\nu_2\left(x_i\right)\right|+\left|\max\left\{\mu_1\left(x_i\right),\nu_2\left(x_i\right)\right\}-\max\left\{\mu_2\left(x_i\right),\nu_1\left(x_i\right)\right\}\right|\right)$$

$$=\frac{1}{3\left(3\times3\right)}\left(\left(\left|0.9-0.75\right|+\left|0-0.15\right|+\left|\max\left\{0.9,0.15\right\}-\max\left\{0.75,0\right\}\right|\right)\right.$$

$$+\left(\left|0.9-0.9\right|+\left|0-0\right|+\left|\max\left\{0.9,0\right\}-\max\left\{0.9,0\right\}\right|\right)$$

$$\left.+\left(\left|0-0.15\right|+\left|0.9-0.75\right|+\left|\max\left\{0,0.75\right\}-\max\left\{0.15,0.9\right\}\right|\right)\right.$$

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$$+\left(\left|0.15-0.75\right|+\left|0.75-0.15\right|+\left|\max\left\{0.15,0.15\right\}-\max\left\{0.75,0.75\right\}\right|\right)$$

$$+\left(\left|0.15-0.45\right|+\left|0.75-0.45\right|+\left|\max\left\{0.15,0.45\right\}-\max\left\{0.45,0.75\right\}\right|\right)$$

$$+\left(\left|0-0.15\right|+\left|0.9-0.75\right|+\left|\max\left\{0,0.75\right\}-\max\left\{0.15,0.9\right\}\right|\right)$$

$$+\left(\left|0.75-0\right|+\left|0.15-0.9\right|+\left|\max\left\{0.75,0.9\right\}-\max\left\{0,0.15\right\}\right|\right)$$

$$+\left(\left|0.45-0.15\right|+\left|0.45-0.75\right|+\left|\max\left\{0.45,0.75\right\}-\max\left\{0.15,0.45\right\}\right|\right)$$

$$+\left(\left|0.45-0\right|+\left|0.45-0.9\right|+\left|\max\left\{0.45,0.9\right\}-\max\left\{0,0.45\right\}\right|\right))$$

$$=0.32$$

$$sim_{Hm}(St1, St2) = 1 - d_{Hm}(St1, St2) = 0.68$$

The pairwise distances of all stakeholders are calculated accordingly. The resulting distance matrix is shown in *Table 5* and the similarity matrix is shown in *Table 6* respectively:

Table 5. Distance matrix

	St1	St2	St3	St4	St5
St1					
St2	0.32				
St3	0.12	0.18			
St4	0.57	0.25	0.25		
St5	0.05	0.40	1	0.75	

Table 6. Similarity matrix

	St1	St2	St3	St4	St5
St1					
St2	0.68				
St3	0.88	0.82			
St4	0.43	0.75	0.75		
St5	0.95	0.60	0	0.25	

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In order to predict the missing values for Req1 (C1, C2, C3) of St4 we choose the top-k most similar stakeholders. From table 6**,** if we choose k=2, the 2 most similar stakeholders for $St4$ are $St2$ ($sim_{Hm}(St4,St2)$=0.75) and $St3$ ($sim_{Hm}(St4,St3)$=0.75). The predicted membership and non-membership for $C1$ are:

$$\mu_d = \frac{0.75\times 0.75 + 0.75\times 0.9}{0.75+0.75} = 0.82$$

$$\nu_d = \frac{0.75\times 0.15 + 0.75\times 0}{0.75+0.75} = 0.08$$

The predicted values for C2 and C3 are calculated accordingly. The results for Req1 (C1, C2, C3) of St4 are shown in *Table 7*:

Table 7. Predicted IFVs

	Req1			Req2			Req3		
	C1	**C2**	**C3**	**C1**	**C2**	**C3**	**C1**	**C2**	**C3**
St1	0.9, 0	0.9, 0	0, 0.9	0.15, 0.75	0.15, 0.75	0, 0.9	0.75, 0.15	0.45, 0.45	0.45, 0.45
St2	0.75, 0.15	0.9, 0	0.15, 0.75	0.75, 0.15	0.45, 0.45	0.15, 0.75	0, 0.9	0.15, 0.75	0, 0.9
St3	0.9, 0	0.9, 0	0.15, 0.75	NA	NA	NA	0.15, 0.75	0.45, 0.45	0.45, 0.45
St4	**0.82, 0.08**	**0.9, 0**	**0.15, 0.75**	0.75, 0.15	0.9, 0	0.75, 0.15	0.15, 0.75	0, 0.9	0.15, 0.75
St5	NA	NA	NA	0, 0.9	0.15, 0.75	0, 0.9	NA	NA	NA

Consensus Measurement and Requirements Ranking

The third phase is the consensus measurement and requirements ranking. If the consensus threshold is reached the final ranking of the requirements is calculated, otherwise the process may return to the requirements recommendation phase.

The consensus measurement includes both the group level and the overall CD. The level of agreement is calculated based on the pairwise similarities of the stakeholders (Palomares et al., 2014). The weight of each stakeholder pair (s_i,s_j), $i<j$ is $w_{ij}\in[0,1]$ satisfying $w_{ij}=\min(w_i,w_j)$ where w_i,w_j are the individual stakeholders' weights. The stakeholders are initially assigned equal weights $w_i=1$. The group CD, which consists of k stakeholders, is:

$$cd_o = \frac{\sum_{i=1}^{k-1}\sum_{j=i+1}^{k} w_{ij} sim_{Hm}\left(s_i, s_j\right)}{\sum_{i=1}^{k-1}\sum_{j=i+1}^{k} w_{ij}} \tag{16}$$

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The overall CD is:

$$cd = \frac{\sum_{o=1}^{O} cd_o}{O} \tag{17}$$

The overall CD is compared with the consensus threshold μ and if $cd \geq \mu$ then the process ends by calculating the final classification of the requirements, otherwise if $cd < \mu$ the process may return to the requirements recommendation phase. The process repeats for a predefined number of rounds Max*rounds,* in case consensus is not reached.

The stakeholders who revise their ratings and move towards the group preference are characterized cooperative. On the other hand, the stakeholders who do not revise the ratings, or move further away from the group preference, are characterized non-cooperative, thus their weights are penalized. The group preference is calculated as the average rating and then converted to IFVs using an intuitionistic fuzzification equation. The updated penalized weight of a noncooperative stakeholder s_i is calculated after round T as follows (Liu et al., 2019):

$$w_{i_(T+1)} = w_{i_T} \times \beta \tag{18}$$

and the updated weight of a cooperative stakeholder s_j is:

$$w_{j_(T+1)} = \sum_{i=1}^{k-q} \frac{(1-\beta) w_{i_T}}{q} + w_{j_T} \tag{19}$$

The degree $d(f_m, p_l)$ of relation between each requirement and each priority class is calculated by the weighted average of the pairwise similarity measure of equation ***(10)*** as follows (Ngan et al., 2018):

$$d(f_m, p_l) = \frac{1}{I} \sum_{i=1}^{I} w_i sim_{Hm}(s_i f_m, p_l) \tag{20}$$

To obtain the final classification of the requirements, each requirement is assigned to the priority class with the highest relation, because the similarity measure is used. In case a distance measure is used, then each requirement is assigned to the priority class with the lowest relation.

Example 3

Consider the Example 1 and the Example 2 and more specifically the IFV ratings and the similarity matrix of table 4 and table 6, respectively. In the first round the weight of each stakeholder is $w_{sti} = 1 \forall i \in \{1 \ldots 5\}$. The CD is:

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$$cd_1 = \frac{\sum_{i=1}^{4}\sum_{j=i+1}^{5} w_{ij} sim_{Hm}\left(s_i, s_j\right)}{\sum_{i=1}^{4}\sum_{j=i+1}^{5} w_{ij}}$$
$$= \frac{\left(0.68+0.88+0.43+0.95\right)+\left(0.82+0.75+0.6\right)+\left(0.75+0.0\right)+0.25}{10} = 0.611$$

If a threshold 0.75 is considered, then the CD is cd_1<0.75, thus the stakeholders are required to modify their preferences. Consider that stakeholder St5 provides the missing ratings for Req1 and Req3 and improves the ratings for Req2 according to recommended ratings from the recommender system. The updated ratings matrix is shown in *Table 8*:

Table 8. Updated IFVs ratings matrix

	Req1			**Req2**			**Req3**		
	C1	**C2**	**C3**	**C1**	**C2**	**C3**	**C1**	**C2**	**C3**
St1	0.9, 0	0.9, 0	0, 0.9	0.15, 0.75	0.15, 0.75	0, 0.9	0.75, 0.15	0.45, 0.45	0.45, 0.45
St2	0.75, 0.15	0.9, 0	0.15, 0.75	0.75, 0.15	0.45, 0.45	0.15, 0.75	0, 0.9	0.15, 0.75	0, 0.9
St3	0.9, 0	0.9, 0	0.15, 0.75	NA	NA	NA	0.15, 0.75	0.45, 0.45	0.45, 0.45
St4	NA	NA	NA	0.75, 0.15	0.9, 0	0.75, 0.15	0.15, 0.75	0, 0.9	0.15, 0.75
St5	**0.9, 0**	**0.9, 0**	**0.15, 0.75**	**0.45, 0.45**	**0.45, 0.45**	**0.15, 0.75**	**0.15, 0.75**	**0.15, 0.75**	**0.15, 0.75**

The pairwise distance of St5 with stakeholders St2, St3 and St4 decreases, thus St5 is characterized cooperative. The updated similarity matrix is shown in Table 9:

Table 9. Updated similarity matrix

	St1	**St2**	**St3**	**St4**	**St5**
St1					
St2	0.68				
St3	0.88	0.82			
St4	0.43	0.75	0.75		
St5	0.77	0.92	0.90	0.75	

The weight of each stakeholder remains w_{Sti}= 1∀i∈{1…5}. The updated CD is:

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$$cd_1 = \frac{\sum_{i=1}^{4}\sum_{j=i+1}^{5} w_{ij} sim_{Hm}\left(s_i, s_j\right)}{\sum_{i=1}^{4}\sum_{j=i+1}^{5} w_{ij}}$$
$$= \frac{\left(0.68+0.88+0.43+0.77\right)+\left(0.82+0.75+0.92\right)+\left(0.75+0.9\right)+0.75}{10} = 0.765$$

The consensus threshold is reached. The final classification of the requirements is calculated using equation ***(20)*** with the updated IFVs ratings matrix of table 8 and the IFVs CPCRM of table 2.

The degree of relation of Req1 and Release 1 is calculated as follows:

$$d(Req1, Release1) = \frac{1}{5}\sum_{(i=1)}^{5} \mathrm{w}_i sim_{Hm}(Sti_{\mathrm{Req1}}, Release1)$$
$$= \frac{1}{5}(sim_{Hm}(\{\langle 0.9, 0.0\rangle, \langle 0.9, 0.0\rangle, \langle 0.0, 0.9\rangle\}, \{\langle 0.9, 0.0\rangle, \langle 0.9, 0.0\rangle, \langle 0.0, 0.09\rangle\})$$
$$+sim_{Hm}\,0.75, 0.15, 0.90.0, 0.15, 0.75, 0.9, 0.0, 0.9, 0.0, 0.0, 0.09$$
$$+sim_{Hm}\,0.9, 0.0, 0.9, 0.0, 0.15, 0.75, 0.9, 0.0, 0.9, 0.0, 0.0, 0.09) + simHmNA, 0.9, 0.0, 0.9, 0.0, 0.0, 0.09)$$
$$+sim_{H}m(0.9, 0.0, 0.9, 0.0, 0.15, 0.75, 0.9, 0.0, 0.9, 0.0, 0.0, 0.09) = 151 + 0.9 + 0.95 + 0 + 0.95) = 0.76$$

accordingly. The final results are shown in *Table 10*:

Table 10. Requirements classification matrix

	Release1	**Release2**	**Release3**	**Release4**
Req1	**0.76**	0.64	0.4	0.2
Req2	0.52	**0.64**	0.56	0.44
Req3	0.48	0.71	**0.77**	0.7

Req1 is assigned to Release1, Req2 is assigned to Release2 and Req3 is assigned to Release3, as shown in Table 11:

Table 11. Final requirements classification

	Release1	**Release2**	**Release3**	**Release4**
Req1	✓			
Req2		✓		
Req3			✓	

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REQUIREMENTS PRIORITIZATION PROCESS SIMULATION

Large-scale software projects may include numerous requirements. For example, in one of the largest software development programmes in Norway, which has followed agile methods, it has been reported that about 800000 person-hours were used to develop around 300 epics, with a total of about 2500 user stories (Dingsøyr et al., 2018). To fully illustrate the scalability and flexibility of the proposed LSDM structured RP process, an illustrative artificial dataset is created and applied to a multi-criteria and multi-class prioritization process.

Requirements Prioritization Problem Definition

$G=\{g_1\}$ is the stakeholders groups set (O=1).
$S=\{s_1,s_2,\ldots,s_{200}\}$ is the stakeholders set (I=200).
$W=\{w_1,w_2,\ldots,w_{200}\}$ is the stakeholders' weights set, where $w_i=1\forall i\in\{1,\ldots,200\}$.
$F=\{f_1,f_2,\ldots,f_{100}\}$ is the candidate requirements set, M=200.
$C=\{Cr_1,Cr_2,Cr_3,Cr_4,Cr_5\}$ is the rating criteria set of each requirement (N = 5).
$P=\{Pr_1,Pr_2,Pr_3,Pr_4\}$ is the priority classes set (L=4).

The artificial dataset includes 200 stakeholders, who rate 200 candidate requirements under 5 different criteria: "Importance", "Benefit", "Business Value", "Risk" and "Implementation Duration". The goal of the RP problem is to classify the requirements according to 4 prioritization classes: "Must have", "Should have", "Could have" and "Won't have". Then the project manager rates the importance of each criterion for each prioritization class in the CPCRM using numerical values as shown in *Table 12*:

Table 12. Artificial dataset numeric CPCRM

	Must have (Pr1)	Should have (Pr2)	Could have (Pr3)	Won't have (Pr4)
Importance (Cr1)	5	4	3	2
Benefit (Cr2)	5	4	3	3
Business Value (Cr3)	5	3	2	1
Risk (Cr4)	1	1	2	3
Implementation Duration (Cr5)	1	2	3	4

The project manager defines that the following ***r-trapezoidal*** intuitionistic fuzzification function is used***:***

$$\mu_A(x)=\begin{cases}1-\pi, & c\le x\\ (1-\pi)\left(\dfrac{x-c}{c-b}\right), & b<x<c\\ 0, & x\le b\end{cases} \tag{21}$$

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$$v_A(x) = \begin{cases} 0, & c \le x \\ (1-\pi)\left(\dfrac{c-x}{c-b}\right), & b < x < c \\ 1-\pi, & x \le b \end{cases} \tag{22}$$

$$\pi_a(x) = 1 - \mu_a(x) - v_a(x) \tag{23}$$

where b=1.5, c=4.5, π=0.1. The intuitionistic fuzzification function is depicted in the plot of Figure 3.

The numeric CPCRM is converted to the IFVs CPCRM shown in Table 13 using the intuitionistic fuzzification equations ***(21)***, ***(22)*** and ***(23)***:

Table 13. Artificial dataset IFVs CPCRM

	Must have (Pr1)	**Should have (Pr2)**	**Could have (Pr3)**	**Won't have (Pr4)**
Importance (Cr1)	0.9, 0	0.75, 0.15	0.45, 0.45	0.15, 0.75
Benefit (Cr2)	0.9, 0	0.75, 0.15	0.45, 0.45	0.45, 0.45
Business Value (Cr3)	0.9, 0	0.45, 0.45	0.15, 0.75	0, 0.9
Risk (Cr4)	0, 0.9	0, 0.9	0.15, 0.75	0.45, 0.45
Implementation Duration (Cr5)	0, 0.9	0.15, 0.75	0.45, 0.45	0.75, 0.15

The artificial dataset includes about 60000 ratings in total, out of 200000 available items for rating (200 stakeholders, 200 requirements under 5 criteria). The rated items percentage is 29.9% (59762 out of 200000), thus the sparsity of the dataset is 70.1%.

Requirements Ratings Recommendations Phase

The stakeholders' pairwise distance and similarity are calculated using equations ***(8)*** and ***(10)***. The distance matrix is visualized using the heatmap of Figure 4:

In order to predict the missing values for St1, the top-10 most similar stakeholders are chosen and the predicted values are calculated using equations ***(12)***, ***(13)***, ***(14)*** and ***(15)***. The process suggests that the requirements Req_45, Req_21, Req_37, Req_49 and Req_5 will be rated the top-5 best values according to the rating criteria (higher values for "Importance", "Benefit", "Business Value" and lower values for "Risk" and "Implementation Duration"). More specifically the predicted values, as shown in *Table 14* are:

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Figure 4. Artificial dataset pairwise distances

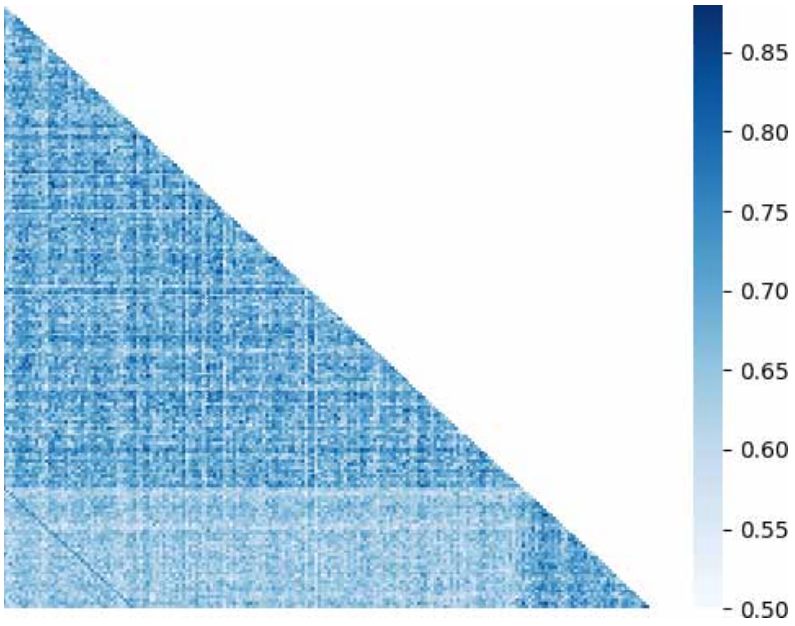

Table 14. Artificial dataset top-5 suggested requirements for St1

	Importance (Cr1)	Benefit (Cr2)	Business Value (Cr3)	Risk (Cr4)	Implementation Duration (Cr5)
Req_45	4.96	4.96	4.17	1	1
Req_21	4.96	4.65	4.96	1.22	1.31
Req_37	4.08	4.96	4.96	1	1
Req_49	4.96	4.83	4.96	1.79	1
Req_5	4.96	4.96	4.96	1.97	1

Consensus Measurement and Requirements Ranking

The proposed consensus measurement and requirements ranking technique is applied to the artificial dataset and the CD is cd=0.708. If a threshold 0.70 is considered, then the CD is cd>0.70, thus the consensus level is reached and the final classification of the requirements is depicted in the heatmap of Figure 5:

CONCLUSION

This chapter presents an LSDM-structured RP process which aims to support the critical task of evaluating the requirements of a large software project with multiple stakeholders. The advantages of the proposed RP approach could be enumerated to the following attributes:

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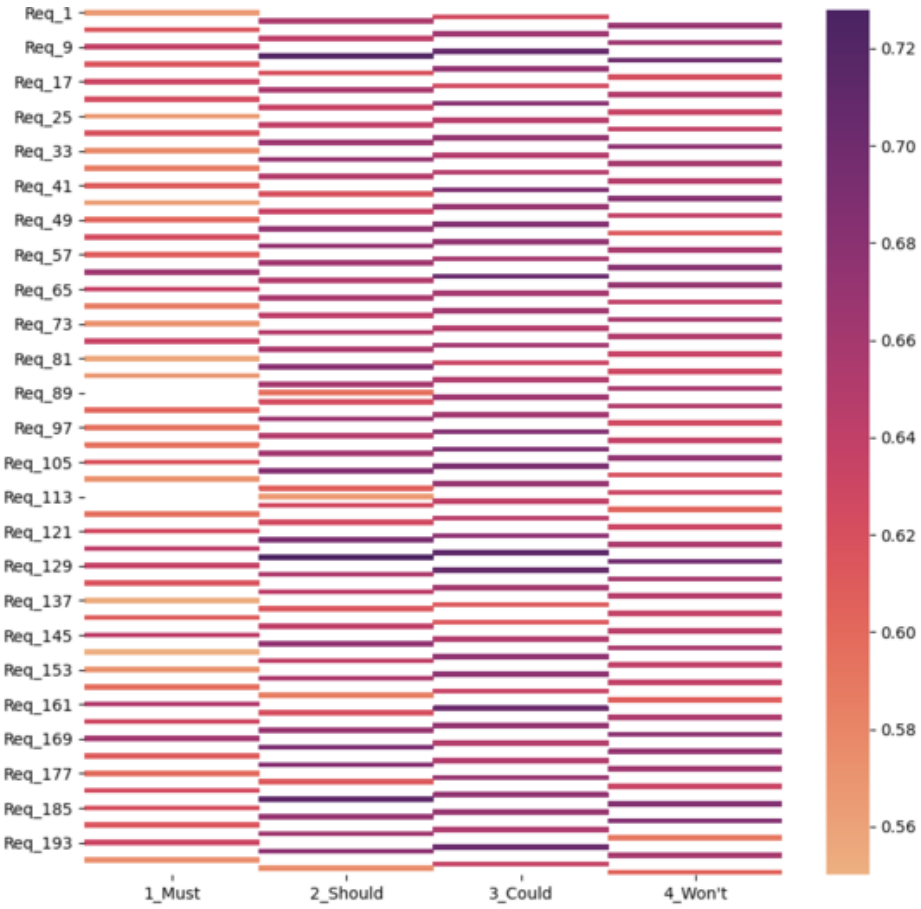
Figure 5. Artificial dataset requirements classification



- **Scalability:** It can be applied to a multi-criteria and multi-class RP event, with numerous stakeholders and numerous requirements.
- **Flexibility:** The technique is applicable using either a single criterion or multiple criteria. Furthermore, it may be utilized either as a requirements classification process with multiple classes or as a simple ranking technique, when using one prioritization class.
- **Ease of use:** The stakeholders express their preferences using a simple rating technique, such as the numeric scale from 1 to 5, while the underlying process converts the values to IFSs, which could be difficult or time consuming to use by non-familiar users. In case the stakeholders are familiar with the intuitionistic fuzzy logic concepts, the ratings could be directly mapped to IFVs.
- **Simple ranking updates:** The stakeholders may update their preferences easily, without having to reconsider all candidate requirements like in other pairwise comparison prioritization techniques. The ranking can be easily updated after inserting new requirements or after deleting existing ones.
- **Stakeholders' uncertainty:** The technique incorporates the use of IFSs, which are effective in handling the stakeholders' hesitancy and uncertainty.

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- **Implementation:** The core logic of the technique is implemented using the Python (van Rossum, 1995) programming language and is proved to be computationally lightweight even for large datasets (the artificial dataset includes about 60000 ratings in total, out of 200000 available items for rating, where 200 stakeholders rate 200 requirements under 5 criteria and the rated items percentage is 29.9%, with a dataset sparsity of 70.1%).

The proposed RP process has been presented in the context of requirements engineering and specifically for the purpose of RP in large software projects. However, the proposed process could be applied to various prioritization problems with multiple alternatives and multiple stakeholders who are rating the alternatives under multiple criteria. Furthermore, the research work of this chapter contributes to the literature dealing with LSDM problems in a fuzzy environment by incorporating the application of IFSs for personalized suggestions using recommender systems and for the alternatives classification and prioritization.

When compared with popular prioritizing techniques, based on their literature citations (Achimugu et al., 2014), it is derived that the proposed RP process can effectively handle large number of requirements rated under multiple criteria by a large number of stakeholders, while processes that utilize AHP or pairwise comparison techniques suffer from scalability and computational complexity. Furthermore, the proposed RP process reduces the time required to rate the alternatives by applying machine learning techniques and providing useful recommendations to the stakeholders. The proposed RP process is also capable of handling updates to the list of requirements by providing the ranking or the prioritization class of each requirement, which is an important limitation of most existing techniques.

The proposed large scale RP process could be further extended by applying various distance and similarity measures, by incorporating clustering algorithms and by integrating network analysis methods to identify stakeholders' connections and roles. To further evaluate the proposed RP process and especially the multi-criteria and multi-class features, the application of a real large scale multi-criteria and multi-class RP dataset is suggested along with the examination of various fuzzification functions. In order to improve the computational efficiency for large datasets, the application of dimensionality reduction methods could be further examined. Finally, the proposed RP process incorporates cooperative and non-cooperative behaviors management approaches combined with weight updates, which are not tested. Thus, the impact of the stakeholders' behavior management approaches to the group consensus level could be further examined.

A limitation of the proposed large scale RP process is the cold start problem of the recommender systems. More specifically, this problem is introduced in the beginning of the application of the proposed process when there is not enough data to provide relevant recommendations. This problem also appears when new requirements or new stakeholders are added to the process and there are not enough ratings, thus reducing the quality of the recommendations. The cold start problem could be further examined in a future work or addressed by existing recommender systems mitigation strategies (Son, 2014). Furthermore, another limitation of the proposed large scale RP process is the requirements dependencies. Usually, the dependencies have an impact in the prioritization and the planning of software projects, thus dependencies should be resolved in a separate software engineering activity or addressed in a future research work.

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KEY TERMS AND DEFINITIONS

Recommender System: A specific type of advice-giving or decision support system that guides users in a personalized way to interesting or useful objects in a large space of possible options (Felfernig et al., 2018a).

Requirements Engineering: The subset of systems engineering concerned with discovering, developing, tracing, analyzing, qualifying, communicating and managing requirements that define the system at successive levels of abstraction (Dick et al., 2017).

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Chapter 22
Comparison of Integrated Multi-Criteria Decision-Making Methods Based on Interval Type-2 Fuzzy Sets:
An Application to Life Quality Evaluation

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ABSTRACT

Multi-criteria decision making (MCDM) is a process of evaluating and ranking the various alternatives with respect to the criteria using subjective judgments of decision maker(s). Since the classical MCDM methods cannot deal with imprecise and uncertain information in the decision-making process, MCDM methods based on Type-1 fuzzy sets are suggested. Afterwards, Type-2 fuzzy set characterized by a fuzzy membership function was introduced as an extension to Type-1 fuzzy sets. Interval Type-2 fuzzy sets (IT2FSs) are the most widely used Type-2 fuzzy sets. Various MCDM methods based on IT2FSs have been proposed and applied on different fields. In this chapter, two integrated methods are introduced by using AHP, COPRAS, and VIKOR methods based on IT2FSs. The proposed integrated methods are carried out to evaluate life quality of 28 European Union countries and another six countries, Iceland, Macedonia, Montenegro, Serbia, Turkey, and Kosovo with respect to 30 subjective criteria.

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INTRODUCTION

Decision-making is defined as the decision-maker (DM) choosing the alternative that has the highest preference rating among different alternatives if there is a single criterion (Harris, 1998). However, decision-makers evaluate the alternatives based on multiple criteria in many problems. Such problems are called as Multi-Criteria Decision Making (MCDM) problems (Tzeng and Huang, 2013). There are a great number of MCDM methods: Analytic Hierarchy Process (AHP) (Saaty, 1980), Elimination Et Choice Translating Reality (ELECTRE) (Benayoun et al., 1966), Qualitative Flexible Multiple Criteria Method (QUALIFLEX) (Paelinck, 1976), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang amd Yoon, 1981), Complex Proportional Assessment (COPRAS) (Zavadskas and Kaklauskas, 1996), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Opricovic, 1998), Additive Ratio Assessment (ARAS) (Zavadskas et al., 2010), Weighted Aggregated Sum Product Assessment (WASPAS) (Zavadskas et al., 2012), and etc.

In decision-making process, if there is uncertain and inaccurate information, classical MCDM methods are insufficient. In such cases, it is more appropriate to use fuzzy MCDM methods. Since human thoughts and judgments usually contain uncertainty, numerical values are insufficient to express real life while evaluating, and evaluations may not be expressed in binary (classical) logic. In other words, linguistic variables that express human thoughts and judgments such as "very bad", "bad", "medium", "good", "very good" can be used in the decision-making process. Such linguistic evaluations, which are not included in classical logic, have become meaningful with the fuzzy set theory proposed by Zadeh (1965) (Chen, 2000; Şahin, 2016).

In fuzzy set theory, each element is assigned a degree of membership through the membership function. Fuzzy sets are insufficient in cases where the degree of full membership cannot be determined. To overcome this situation, interval Type-2 fuzzy sets (IT2Fs) were proposed by Zadeh (1975) (Özek, 2010). Fuzzy sets, also called Type-1 fuzzy sets (T1Fs), have definite-valued membership degrees, while IT2Fs have fuzzy-valued membership degrees (Mendel and John, 2002). The advantage of IT2Fs over T1Fs is that they can model uncertainties better and give better results in systems containing uncertainty (Mendel et al., 2006; Şahin, 2016). Hence, various methods based on IT2Fs are proposed to deal with MCDM problems in the literature.

Chen and Lee (2010) proposed the IT2FTOPSIS method for the problem of evaluating the recruitment process of a systems analysis engineer. Kahraman and Sarı (2012) used the IT2FTOPSIS method for environmental risk assessment. Chen et al. (2013) developed the IT2FQUALIFLEX method to solve a medical decision-making problem. Temur et al. (2014) proposed a multi-criteria approach for the selection of the most suitable logistic facility location using the IT2FTOPSIS method. Erdoğan and Kaya (2014) used the IT2FTOPSIS method to rank private universities in Istanbul. Chen (2014) developed the IT2F based ELECTRE method to solve a supplier selection problem. Ghorabaee et al. (2014) proposed a new method for extending COPRAS method for group decision-making with IT2F numbers, based on the centroid of fuzzy sets. Kılıç and Kaya (2015) proposed the IT2F based hybrid AHP-TOPSIS method to evaluate the investment projects of agencies developing in Turkey. Çebi and Otay (2015) suggested a comprehensive and systematic approach to the problem of multi-criteria and multi-stage facility location selection of a cement plant using the IT2FTOPSIS method. Ghorabaee (2016) developed a method based on IT2FVIKOR method for robot selection problem. Şahin and Pehlivan (2017) proposed an integration method of IT2FAHP and IT2FTOPSIS for life quality evaluation. Baykasoğlu and Gölcük (2017) was developed a new IT2F based MCDM model by integrated TOPSIS and Decision-Making

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Trial and Evaluation Laboratory (DEMATEL) and they implemented the proposed model in Strengths, Weaknesses, Opportunities, and Threats (SWOT)-based strategy selection problem. Najib et al. (2017) used the IT2FDEMATEL method for coastal erosion decision problem. Wu et al. (2018) suggested a new IT2F-TOPSIS model for large scale group decision making problems with social network information. Deveci et al. (2018) was developed a hybrid approach with TOPSIS and WASPAS method based on IT2FSs for car sharing station selection problem. Abdullah et al. (2018) suggested IT2FSAW method to establish a preference in ambulance location. Abdullah and Zulkifli (2019) proposed a modification fuzzy DEMATEL characterized by IT2FS for linguistic variables for developing casual relationship of knowledge management criteria. Wang et al. (2019) proposed an extended VIKOR method based on the signed area method to solve MADM problems under IT2FS environment. Bera et al. (2020) presented two novel MCDM techniques based on TOPSIS and MOORA methods in IT2FSs environment for supplier selection. Öztürk (2021) created MCDM framework with T2Fs, which are consisted of fuzzy AHP and fuzzy TOPSIS, for public transport services. Petrovic and Kankaras (2020) modified a model by using hybridized DEMATEL-AHP-TOPSIS and IT2Fs to select and evaluate the criteria for determination of air traffic control radar position. Sen et al. (2021) used AHP- ARAS method based on IT2Fs to select optimal wire electrical discharge machining parameters. Zhang et al. (2021) developed an IT2FTOPSIS approach with a utility function for subway station operational risk evaluation.

The aim of this study is to determine the ranking of countries in the "2012 European Quality of Life" study conducted by the European Agency for the Improvement of Living and Working Conditions (Eurofound, 2016) in terms of quality of life under certain criteria, via integrated MCDM methods based on interval type-2 fuzzy sets (IT2FSs) consisted of IT2FAHP, IT2FVIKOR, and IT2FCOPRAS. The IT2FAHP method is utilized to obtain the weights of criteria while the IT2FVIKOR and IT2FCOPRAS methods are used to rank the alternatives/countries. Since the IT2FAHP method has a powerful mechanism checking the consistency of pairwise comparison matrices, it reduces the bias in the decision-making process. However, the computation time increases considerably when too many criteria are involved in the IT2FAHP. Therefore, it is recommended that the criteria weights be obtained by FAHP and the ranking of alternatives by other MCDM methods (Kılıç and Kaya,2015; Şahin and Pehlivan, 2017; Öztürk, 2021).

IT2FAHP which is an extension of FAHP, is an extensively used in order to solve MCDM problems with high uncertainties. It uses pairwise comparison matrix to obtain the weights. Furthermore, this method has a useful mechanism to check the consistency of the fuzzy pairwise comparison matrices and this ensures considerable reduction of the bias in the decision-making process. However, when the number of criteria or alternatives is large, it takes a lot of pairwise comparisons and therefore a lot of computation time. To overcome this, two new methods are proposed integrating the IT2FAHP method with IT2FVIKOR and IT2FCOPRAS methods in this study. The IT2FVIKOR and IT2FCOPRAS methods do not require pairwise comparison and are easy to implement. Thus, an advantage is obtained in terms of both computation time and low bias. It is clear that other MCDM methods can also be used in this study. However, AHP, VIKOR, and COPRAS, which are the most widely used MCDM methods in recent years, are preferred in our study because they are easy to apply and have not been used in this field before.

The remainder of this study is organized as follows. Some main concepts, arithmetic operations and a ranking method for IT2 fuzzy sets are introduced in Section 2. IT2FAHP, IT2FCOPRAS, IT2FVIKOR and the integrated MCDM methods of IT2FAHP / IT2FCOPRAS and IT2FAHP / IT2FVIKOR are presented in Section 3. The applicability of the proposed integrated methods is demonstrated on a case study of evaluation of "Life Quality" in Section 4. Finally, conclusions are presented in Section 5.

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TYPE-2 FUZZY SETS AND INTERVAL TYPE-2 FUZZY SETS

A Type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse *X* which can be represented by a Type-2 membership function $\mu_{\tilde{\tilde{A}}}$ is expressed as follows:

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{\tilde{A}}}(x,u)) \mid \forall x \in X, \quad \forall u \in J_x \subseteq [0,1], \ 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1\} \tag{1}$$

Here, $0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1$, J_x is the basic membership function and *u*stands basic membership values.

Moreover, the Type-2 fuzzy set $\tilde{\tilde{A}}$ represented by:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u) / (x,u) \tag{2}$$

In Equation (2), $J_x \subseteq [0,1]$ and $\iint$ states union over all admissible *x* and *u*. For discrete universes of discourse $\int$ is replaced by $\sum$ (Chen and Lee, 2010; Mendel et al., 2006)

Let $\tilde{\tilde{A}}$ be a Type-2 fuzzy set in the universe of discourse *X* represented by the Type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called an "interval type-2 fuzzy set". An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of Type-2 fuzzy set and represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1 / (x,u) \tag{3}$$

where $J_x \subseteq [0,1]$ (Chen and Lee, 2010; Mendel et al., 2006).

The upper and lower membership functions of an interval type-2 fuzzy set are type-1 membership functions. The reference points and the heights of the upper and the lower membership functions of interval type-2 fuzzy sets are used to characterize interval type-2 fuzzy sets by (Lee and Chen, 2008).

$$\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)), \ (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L))) \tag{4}$$

where $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L$ and a_{i4}^L are the references points of the interval type-2 fuzzy set $\tilde{\tilde{A}}_i$, $H_j(\tilde{A}_i^U)$ denotes the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function $(\tilde{A}_i^U)$, $1 \le j \le 2$. $H_j(\tilde{A}_i^L)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function $(\tilde{A}_i^L)$, $1 \le j \le 2$. $H_1(\tilde{A}_i^U) \in [0,1]$, $H_2(\tilde{A}_i^U) \in [0,1]$, $H_1(\tilde{A}_i^L) \in [0,1]$, $H_2(\tilde{A}_i^L) \in [0,1]$ and $1 \le i \le n$. A trapezoidal type-2 fuzzy set is shown in Figure 1 (Chen and Lee, 2010; Mendel et al., 2006).

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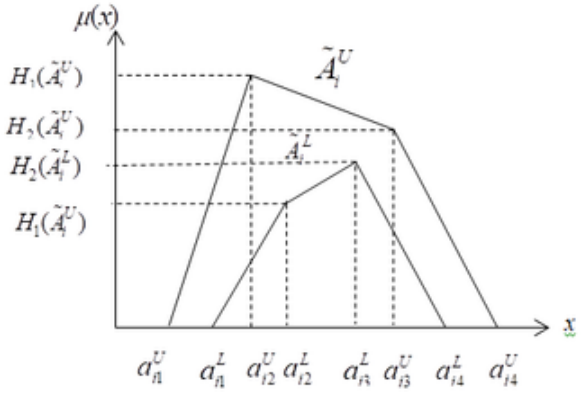
Figure 1. The membership functions of the trapezoidal IT2F set $\tilde{\tilde{A}}$



Arithmetic Operations on Interval Type-2 Fuzzy Sets

Arithmetic operations on trapezoidal interval type-2 fuzzy sets are given as follows (Chen, 2015; Lee and Chen, 2008):

Let $\tilde{\tilde{A}}_1$ and $\tilde{\tilde{A}}_2$ be trapezoidal interval type-2 fuzzy sets:

$$\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)),\ (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)))$$

and

$$\tilde{\tilde{A}}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)),\ (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L))).$$

Addition:

$$\begin{aligned}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_1^U, \tilde{A}_1^L) \\ &= ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \min(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)),\ \min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \min(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L)))).\end{aligned} \tag{5}$$

Subtraction:

$$\begin{aligned}\tilde{\tilde{A}}_1 - \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) - (\tilde{A}_1^U, \tilde{A}_1^L) \\ &= ((a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; \min(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)),\ \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; \min(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)))).\end{aligned} \tag{6}$$

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Multiplication:

$$\begin{aligned}\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_1^U, \tilde{A}_1^L) \\ &= ((a_{11}^U \times a_{21}^U, a_{12}^U \times, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \min(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)),\ \min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; \min(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L)))).\end{aligned} \tag{7}$$

Division:

$$\begin{aligned}\tilde{\tilde{A}}_1(:)\tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L)(:)(\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^U / a_{24}^U, a_{12}^U / a_{23}^U, a_{13}^U / a_{22}^U, a_{14}^U / a_{21}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)),\ \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), \\ &\quad (a_{11}^L / a_{21}^L, a_{12}^L / a_{22}^L, a_{13}^L / a_{23}^L, a_{14}^L / a_{24}^L; \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)), \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))).\end{aligned} \tag{8}$$

Multiplication by a non-negative real number k:

$$\begin{aligned}k \times \tilde{\tilde{A}}_1 = ((&k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \\ &(k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)).\end{aligned} \tag{9}$$

Division by non-negative real number k:

$$\begin{aligned}\frac{\tilde{\tilde{A}}_1}{k} = ((&\frac{1}{k}a_{11}^U, \frac{1}{k}\ a_{12}^U, \frac{1}{k}\ a_{13}^U, \frac{1}{k}a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \\ &(\frac{1}{k}a_{11}^L, \frac{1}{k}\ a_{12}^L, \frac{1}{k}a_{13}^L, \frac{1}{k}\ a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L))).\end{aligned} \tag{10}$$

Ranking Values of Trapezoidal IT2Fs

A concept of ranking values of trapezoidal interval type-2 fuzzy sets is introduced by Ghorabaee et al. (2014). Suppose that $\tilde{\tilde{A}}_i$ be an interval type-2 fuzzy set:

$$\tilde{\tilde{A}}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)),\ (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)))$$

In order to determine the centroid point C_{ix}^T and C_{iy}^T, ($T \in \{U,L\}$) of a trapezoidal interval type-2 fuzzy set $(\tilde{\tilde{A}}_i)$ as follows:

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$$C_{ix}^{T}=\frac{1}{3}\left(a_{i1}^{T}+a_{i2}^{T}+a_{i3}^{T}+a_{i4}^{T}-\frac{a_{i3}^{T}a_{i4}^{T}-a_{i1}^{T}a_{i2}^{T}}{(a_{i3}^{T}+a_{i4}^{T})-(a_{i1}^{T}+a_{i2}^{T})}\right),\ \ T\in\{U,L\} \tag{11}$$

$$C_{iy}^{T}=\frac{w^{T}}{3}\left(1+\frac{a_{i3}^{T}-a_{i2}^{T}}{(a_{i3}^{T}+a_{i4}^{T})-(a_{i1}^{T}+a_{i2}^{T})}\right),\ \ T\in\{U,L\} \tag{12}$$

where;

$$w^{T}=\frac{H_1(\tilde{A}_i^{T})+H_2(\tilde{A}_i^{T})}{2},\ T\in\{U,L\} \tag{13}$$

Suppose that $\tilde{\tilde{A}}_s$ and $\tilde{\tilde{A}}_t$ are two trapezoidal interval type-2 fuzzy sets:

$$\tilde{\tilde{A}}_s=(\tilde{A}_s^{U},\tilde{A}_s^{L})=((a_{s1}^{U},a_{s2}^{U},a_{s3}^{U},a_{s4}^{U};H_1(\tilde{A}_s^{U}),H_2(\tilde{A}_s^{U})),(a_{s1}^{L},a_{s2}^{L},a_{s3}^{L},a_{s4}^{L};H_1(\tilde{A}_s^{L}),H_2(\tilde{A}_s^{L})))$$

$$\tilde{\tilde{A}}_t=(\tilde{A}_t^{U},\tilde{A}_t^{L})=((a_{t1}^{U},a_{t2}^{U},a_{t3}^{U},a_{t4}^{U};H_1(\tilde{A}_t^{U}),H_2(\tilde{A}_t^{U})),(a_{t1}^{L},a_{t2}^{L},a_{t3}^{L},a_{t4}^{L};H_1(\tilde{A}_t^{L}),H_2(\tilde{A}_t^{L})))$$

Let,

$$E_x=\frac{(C_{tx}^{U}+C_{tx}^{L})-(C_{sx}^{U}+C_{sx}^{L})}{\left|C_{tx}^{U}+C_{tx}^{L}\right|+\left|C_{sx}^{U}+C_{sx}^{L}\right|} \tag{14}$$

$$E_x^{'}=\max\left(\left(1-\max\left(E_x,0\right)\right),0\right) \tag{15}$$

$$E_y=\frac{(C_{ty}^{U}+C_{ty}^{L})-(C_{sy}^{U}+C_{sy}^{L})}{C_{ty}^{U}+C_{ty}^{L}+C_{sy}^{U}+C_{sy}^{L}} \tag{16}$$

$$E_y^{'}=\max\left(\left(1-\max\left(E_y,0\right)\right),0\right) \tag{17}$$

$$M=\alpha E_x^{'}+(1-\alpha)E_y^{'} \tag{18}$$

where $E_x^{'}$ and $E_y^{'}$ are defined as the strength of $\tilde{\tilde{A}}_s$ over $\tilde{\tilde{A}}_t$ with respect to the domain and the membership function of them, respectively. They use α as trade-off factor between Ex and Ey. Then, the possibility degree of $\tilde{\tilde{A}}_s$ over $\tilde{\tilde{A}}_t$ is defined as:

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$$P(\tilde{\tilde{A}}_s \geq \tilde{\tilde{A}}_t) = \min(M,1) \tag{19}$$

If *n* trapezoidal interval type-2 fuzzy sets $\tilde{\tilde{A}}_1, \tilde{\tilde{A}}_2, ..., \tilde{\tilde{A}}_n$ need to be compared, the possibility degree matrix (*P*) can be obtained, shown as follows:

$$P = \begin{bmatrix} P(\tilde{\tilde{A}}_1 \geq \tilde{\tilde{A}}_1) & P(\tilde{\tilde{A}}_1 \geq \tilde{\tilde{A}}_2) & \cdots & P(\tilde{\tilde{A}}_1 \geq \tilde{\tilde{A}}_n) \\ P(\tilde{\tilde{A}}_2 \geq \tilde{\tilde{A}}_1) & P(\tilde{\tilde{A}}_2 \geq \tilde{\tilde{A}}_2) & \cdots & P(\tilde{\tilde{A}}_2 \geq \tilde{\tilde{A}}_n) \\ \vdots & \vdots & \vdots & \vdots \\ P(\tilde{\tilde{A}}_n \geq \tilde{\tilde{A}}_1) & P(\tilde{\tilde{A}}_n \geq \tilde{\tilde{A}}_2) & \cdots & P(\tilde{\tilde{A}}_n \geq \tilde{\tilde{A}}_n) \end{bmatrix} \tag{20}$$

and

$$RV(\tilde{\tilde{A}}_i) = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n} P(\tilde{\tilde{A}}_i \geq \tilde{\tilde{A}}_j) + \frac{n}{2} - 1\right). \tag{21}$$

INTEGRATED MULTI-CRITERIA DECISION-MAKING METHODS based on IT2FS

In this section, AHP method based on IT2FSs (IT2-FAHP), COPRAS method based on based on IT2FSs (IT2FCOPRAS), and VIKOR method based on based on IT2FSs (IT2FVIKOR) are first introduced. Then, integrated method of IT2FAHP / IT2FCOPRAS, and integrated method of IT2FAHP / IT2FVIKOR are presented (Şahin, 2016).

IT2FAHP Method

Analytical Hierarchy Process (AHP) is a variant of the additive weighting method developed by Saaty (1980). Decision-makers find it very difficult to accurately determine the importance weights of the criteria. The higher the number of criteria, the better results are obtained when the problem is converted into pairwise comparisons. AHP makes the criterion weighting problem more easily solvable by making pairwise comparisons between criteria. In the pairwise comparison process, AHP methods based on Type-2 fuzzy sets have been proposed to overcome some uncertainties. In this study, The AHP method based on IT2Fs proposed by Kahraman et al. (2014) is considered. The steps of IT2FAHP method are given as follows:

Step 1. A fuzzy pairwise comparison matrix is created using linguistic variables and their trapezoidal interval type-2 fuzzy scales for each criterion.

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$$\tilde{\tilde{A}} = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} 1 & \tilde{\tilde{a}}_{12} & \ldots & \tilde{\tilde{a}}_{1n} \\ 1/\tilde{\tilde{a}}_{12} & 1 & \ldots & \tilde{\tilde{a}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/\tilde{\tilde{a}}_{1n} & 1/\tilde{\tilde{a}}_{2n} & \ldots & 1 \end{bmatrix} \end{array} \tag{22}$$

Step 2. It is examined whether the fuzzy pairwise comparison matrix is consistent or not. For checking the consistency of the fuzzy pairwise comparison matrices, the DTriT or DTraT approaches proposed by Kahraman et al. (2014) can be used.

Step 3. The geometric mean for each row of the pairwise comparison matrix and fuzzy weight of *j*th criterion are calculated using the equations given below, respectively.

Calculate the geometric mean of each row and the fuzzy weights by normalization. The geometric mean of each row r_i is calculated as follows;

$$\tilde{\tilde{r}}_i = \left[\tilde{\tilde{a}}_{i1} \otimes \ldots \otimes \tilde{\tilde{a}}_{in}\right]^{1/n}, \quad i = 1, 2, \ldots, n \tag{23}$$

and

$$\tilde{\tilde{w}}_j = \tilde{\tilde{r}} \otimes \left[\tilde{\tilde{r}}_1 \otimes \ldots \otimes \tilde{\tilde{r}}_n\right]^{-1}, \quad j = 1, 2, \ldots, n \tag{24}$$

where:

$$\sqrt[n]{\tilde{\tilde{a}}_{ij}} = \left(\left(\sqrt[n]{\tilde{\tilde{a}}^U_{ij1}}, \sqrt[n]{\tilde{\tilde{a}}^U_{ij2}}, \sqrt[n]{\tilde{\tilde{a}}^U_{ij3}}, \sqrt[n]{\tilde{\tilde{a}}^U_{ij4}}; H_1^U(a_{ij}); H_2^U(a_{ij})\right)\right), \left(\left(\sqrt[n]{\tilde{\tilde{a}}^L_{ij1}}, \sqrt[n]{\tilde{\tilde{a}}^L_{ij2}}, \sqrt[n]{\tilde{\tilde{a}}^L_{ij3}}, \sqrt[n]{\tilde{\tilde{a}}^L_{ij4}}; H_1^L(a_{ij}); H_2^L(a_{ij})\right)\right) \tag{25}$$

IT2FCOPRAS Method

The COPRAS (Complex Proportional Assessment) method was first proposed by Zavadskas and Kaklauskas (1996). This method is based on choosing the best alternative by determining a ratio with a positive ideal solution and a ratio with a negative ideal solution (Yazdani et al., 2011). The COPRAS method based on Type-2 fuzzy sets, is proposed for situations where uncertainty is high.

Ghorabaee et al. (2014) proposed a new sorting method based on the center of gravity of fuzzy clusters to sort Type-2 fuzzy numbers. They used this sorting method for the COPRAS method in decision making with Type-2 fuzzy sets and applied it to the supplier selection problem. In this study, The COPRAS method based on interval Type-2 fuzzy sets developed by Ghorabaee et al. (2014) is discussed.

Step 1: The decision matrix $\tilde{\tilde{D}}$ and the weighting matrix $\tilde{\tilde{W}}$ are constructed by using linguistic terms and corresponding IT2Fs:

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$$\tilde{\tilde{D}} = \left[\tilde{\tilde{x}}_{ij}\right]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{x}}_{11} & \tilde{\tilde{x}}_{12} & \ldots & \tilde{\tilde{x}}_{1n} \\ \tilde{\tilde{x}}_{21} & \tilde{\tilde{x}}_{22} & \ldots & \tilde{\tilde{x}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{x}}_{m1} & \tilde{\tilde{x}}_{m2} & \ldots & \tilde{\tilde{x}}_{mn} \end{bmatrix} \end{array} \tag{26}$$

$$\tilde{\tilde{W}} = [\tilde{\tilde{w}}_j]_{1\times n} = \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{w}}_1 & \tilde{\tilde{w}}_2 & \ldots & \tilde{\tilde{w}}_n \end{bmatrix} \end{array} \tag{27}$$

where

$$\tilde{\tilde{x}}_{ij} = \left(\frac{\tilde{\tilde{x}}_{ij}^1 \oplus \tilde{\tilde{x}}_{ij}^2 \oplus \ldots \oplus \tilde{\tilde{x}}_{ij}^k}{k}\right),\ \tilde{\tilde{w}}_j = \left(\frac{\tilde{\tilde{w}}_j^1 \oplus \tilde{\tilde{w}}_j^2 \oplus \ldots \oplus \tilde{\tilde{w}}_j^k}{k}\right),\ i=1,2,\ldots,m;\ j=1,2,\ldots,n,$$

and k denotes the number of decision maker, $\tilde{\tilde{x}}_{ij}$ is the performance score of alternative i with respect to criterion j, and $\tilde{\tilde{w}}_j$ is the weight of the criterion j.

Step 2: The normalized matrix $N = \left[\tilde{\tilde{n}}_{ij}\right]_{m\times n}$ is constructed by normalizing the decision matrix $\tilde{\tilde{D}}$ through the following equations:

$$\tilde{\tilde{v}}_j = \left(\tilde{\tilde{X}}_{1j} \oplus \tilde{\tilde{X}}_{2j} \oplus \ldots \oplus \tilde{\tilde{X}}_{mj}\right) \tag{28}$$

$$\tilde{\tilde{n}}_{ij} = \tilde{\tilde{X}}_{ij} / \tilde{\tilde{v}}_j,\quad 1 \le i \le m, 1 \le j \le n\,. \tag{29}$$

Step 3: The weighted normalized decision matrix $E = \left[\tilde{\tilde{e}}_{ij}\right]_{n\times m}$ is determined by the following equation

$$\tilde{\tilde{e}}_{ij} = \tilde{\tilde{n}}_{ij} \otimes \tilde{\tilde{w}}_j \tag{30}$$

Step 4: The sum of weighted normalized values for both the beneficial criteria $\tilde{\tilde{S}}_{+i}$ and non-beneficial criteria $\tilde{\tilde{S}}_{-i}$ are calculated shown as follows:

$$\tilde{\tilde{S}}_{+i} = \left(\tilde{\tilde{e}}_{+i1} \oplus \tilde{\tilde{e}}_{+i2} \oplus \ldots \oplus \tilde{\tilde{e}}_{+in}\right) \tag{31}$$

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$$\tilde{\tilde{S}}_{-i} = \left(\tilde{\tilde{e}}_{-i1} \oplus \tilde{\tilde{e}}_{-i2} \oplus ... \oplus \tilde{\tilde{e}}_{-in}\right) \tag{32}$$

where $\tilde{\tilde{e}}_{+ij}$ and $\tilde{\tilde{e}}_{-ij}$ are the weighted normalized values for the benefit and cost criteria, respectively. The values of $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ give the degree of goals achieved by each alternative.

Step 5: The ranking values for both $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ ($1 \le i \le m$) are determined by using the ranking method suggested by Ghorabaee et al. (2014).

Step 6: The relative significances Q_i of the alternatives are determined as follows:

$$Q_i = Rank\left(\tilde{\tilde{S}}_{+i}\right) + \frac{Rank_{\min}\left(\tilde{\tilde{S}}_{-i}\right)\sum_{i=1}^{n} Rank\left(\tilde{\tilde{S}}_{-i}\right)}{Rank\left(\tilde{\tilde{S}}_{-i}\right)\sum_{i=1}^{n}\left(Rank_{\min}\left(\tilde{\tilde{S}}_{-i}\right) / Rank\left(\tilde{\tilde{S}}_{-i}\right)\right)} \tag{33}$$

where $Rank_{\min}\left(\tilde{\tilde{S}}_{-i}\right)$ denotes the minimum value of $Rank\left(\tilde{\tilde{S}}_{-i}\right)$.

Step 7: The degree of an alternative's utilities U_i are determined by comparing the relative significances of all alternatives with the most efficient one, as follows:

$$U_i = \left(\frac{Q_i}{Q_{\max}}\right) \times 100 \tag{34}$$

where $Q_{\max}$ is the maximum relative significance value. The larger the value of U_i, the more preference of the alternative A_i, $1 \le i \le m$.

IT2FVIKOR Method

The Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method was developed by Opricovic (1998) to solve MCDM problems with criteria that conflict with each other and cannot be measured with the same scale. The VIKOR method determines a compromised solution that provides maximum group benefit and minimum regret (Chen and Wang, 2009).

The VIKOR method based on Type-2 fuzzy sets has been developed to better model uncertainties in the decision-making process. In this study, VIKOR method based on Type-2 fuzzy sets developed by Ghorabaee (2016) is discussed.

Step 1: The decision matrix $\tilde{\tilde{D}}$ and the weighting matrix $\tilde{\tilde{W}}$ are constructed by using linguistic terms and corresponding IT2Fs by Equation (26) and Equation (27).

Step 2: The ideal ($\tilde{\tilde{X}}_j^*$) and the nadir ($\tilde{\tilde{X}}_j^o$) values are computed for all criteria. The ranking method is used in this step.

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$$\tilde{\tilde{X}}_j^* = \begin{cases} \max\limits_i \tilde{\tilde{X}}_{ij}, & c_j \in C_1 \\ \min\limits_i \tilde{\tilde{X}}_{ij}, & c_j \in C_2 \end{cases} \tag{35}$$

$$\tilde{\tilde{X}}_j^o = \begin{cases} \min\limits_i \tilde{\tilde{X}}_{ij}, & c_j \in C_1 \\ \max\limits_i \tilde{\tilde{X}}_{ij}, & c_j \in C_2 \end{cases} \tag{36}$$

where C_1 and C_2 denote benefit and cost criteria, respectively.

Step 3: The normalized fuzzy differences are calculated shown as follows:

$$\tilde{\tilde{d}}_{ij} = (\tilde{\tilde{X}}_j^* - \tilde{\tilde{X}}_{ij}) / \rho_b, \ c_j \in C_1 \tag{37}$$

$$\tilde{\tilde{d}}_{ij} = (\tilde{\tilde{X}}_{ij} - \tilde{\tilde{X}}_j^o) / \rho_h, \ c_j \in C_2 \tag{38}$$

where:

$$\rho_b = \frac{c_x^U(\tilde{\tilde{X}}_j^* - \tilde{\tilde{X}}_j^o) + c_x^L(\tilde{\tilde{X}}_j^* - \tilde{\tilde{X}}_j^o)}{2}$$

and

$$\rho_h = \frac{c_x^U(\tilde{\tilde{X}}_j^o - \tilde{\tilde{X}}_j^*) + c_x^L(\tilde{\tilde{X}}_j^o - \tilde{\tilde{X}}_j^*)}{2}.$$

Step 4: The $\tilde{\tilde{S}}_i$ and $\tilde{\tilde{R}}_i$ are determined for all alternatives, shown as follows:

$$\tilde{\tilde{S}}_i = (\tilde{\tilde{w}}_1 \otimes \tilde{\tilde{d}}_{i1}) \oplus (\tilde{\tilde{w}}_2 \otimes \tilde{\tilde{d}}_{i2}) \oplus \ldots \oplus (\tilde{\tilde{w}}_m \otimes \tilde{\tilde{d}}_{im}) \tag{39}$$

$$\tilde{\tilde{R}}_i = \max_j (\tilde{\tilde{w}}_j \otimes \tilde{\tilde{d}}_{ij}) \tag{40}$$

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Step 5: The best values of $\tilde{\tilde{S}}_i$ and $\tilde{\tilde{R}}_i$ are calculated, shown as follows:

$$\tilde{\tilde{S}}^* = \min_i \tilde{\tilde{S}}_i \tag{41}$$

$$\tilde{\tilde{R}}^* = \min_i \tilde{\tilde{R}}_i \tag{42}$$

Step 6: $\tilde{\tilde{Q}}_i$ is computed for all alternatives, shown as follows:

$$\tilde{\tilde{Q}}_i = v\left(\left(\tilde{\tilde{S}}_i - \tilde{\tilde{S}}^*\right)/\left(s^o - s_{i1}^{*U}\right)\right) \oplus (1-v)\left(\left(\tilde{\tilde{R}}_i - \tilde{\tilde{R}}^*\right)/\left(r^o - r_{i1}^{*U}\right)\right) \tag{43}$$

where $s^o = \max_i s_{i4}^U, r^o = \max_i r_{i4}^U$, ν is the weight of the strategy of maximum group utility and 1-ν is the weight of individual regret.

Step 7: The order of alternatives is obtained according to the increasing order of $\tilde{\tilde{Q}}_i$.

Integration of MCDM Methods Based on IT2FSs

IT2FAHP which is an extension of FAHP, is an extensively used in order to solve MCDM problems with high uncertainties. It uses pairwise comparison matrix to obtain the weights. Furthermore, this method has a useful mechanism to check the consistency of the fuzzy pairwise comparison matrices and this ensures considerable reduction of the bias in the decision-making process. However, when the number of criteria or alternatives is large, it takes a lot of pairwise comparisons and therefore a lot of computation time. To overcome this, two new methods are proposed integrating IT2FAHP method with IT2FVIKOR and IT2FCOPRAS methods in this study. IT2FAHP method is used to obtain criterion weights and IT2FVIKOR and IT2FCOPRAS methods are used to rank the alternatives. The IT2FVIKOR and IT2FCOPRAS methods do not require pairwise comparison and are easy to implement. Thus, an advantage is obtained in terms of both computation time and low bias.

The proposed integrated methods compose of three phases: (i) identification of criteria and alternatives (ii) computation of IT2F weights of the criteria via IT2FAHP, (iii) determination of the ranking of alternatives with IT2FVIKOR or IT2FCOPRAS. whose frameworks are given in Figure 2 and Figure 3, respectively.

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Figure 2. Framework of the proposed IT2FAHP / IT2FVIKOR method

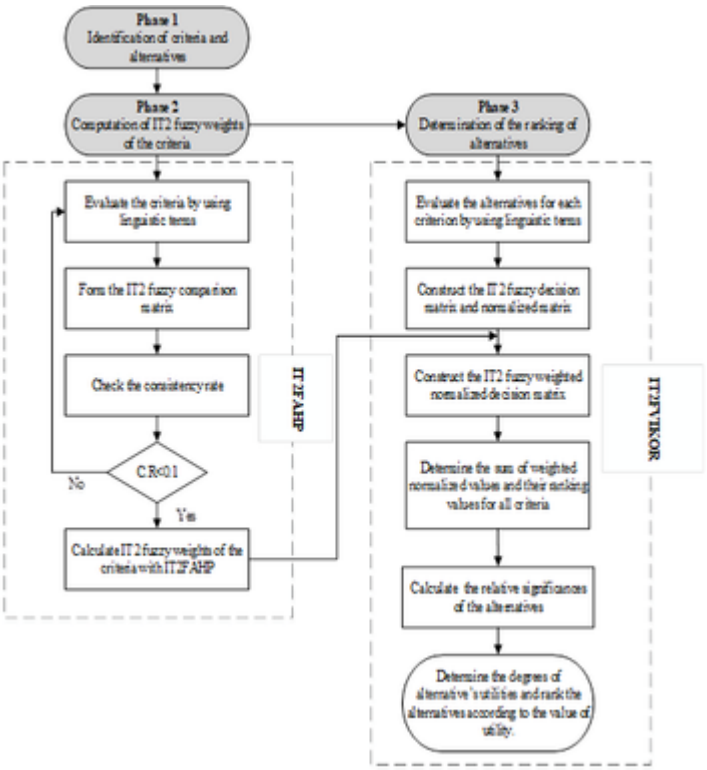
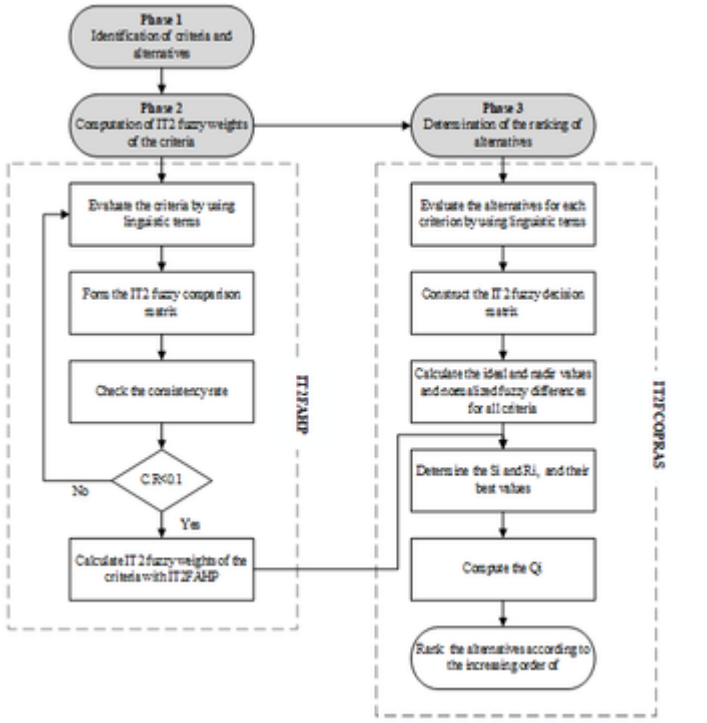


Figure 3. Framework of the proposed IT2FAHP / IT2FCOPRAS method



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AN APPLICATION TO EVALUATE LIFE QUALITY

In this section, an application is conducted to evaluate of life quality for 28 EU countries and other six countries: Iceland, Macedonia, Montenegro, Serbia, Turkey and Kosovo, with respect to 6 main criteria and related 30 sub-criteria via the integrated MCDM methods. The criteria and sub-criteria of the data taken from "European Quality of Life Surveys (EQLS) 2012" carried out by the Eurofound (2016) are given in Table 1. This survey examines the objective circumstances of European citizens' lives, how they feel about those circumstances and their life in general. It is also carried out every four years and data are published 2 years after the study is finished. Therefore, in this study, the data of 2012, which has the highest number of alternative/country evaluations and criteria evaluations can be reached, were discussed. In the evaluation process, 12 cost type criteria (C1, C2, C3, C4, C5, C15, C16, C17, C21, C22, C23, C26) and 18 benefit type criteria (C6, C7, C8, C9, C10, C11, C12, C13, C14, C18, C19, C20, C24, C25, C27, C28, C29, C30) are considered. For more information, see Eurofound (2016).

Table 1. Criteria and sub-criteria in the EQLS 2012

Main Criteria	Sub-criteria
Employment and Work-Life Balance	C1. Coming home from work too tired to do household jobs
	C2. Having difficult to fulfill family responsibilities because of time spent on job
	C3. Having difficult to concentrate at work because of family responsibilities
	C4. The possibility of losing your job in the next 6 months
	C5. If you lost your job, the possibility of finding a job of similar salary
Family and Social Life	C6. Face-to-face contact with children living outside the household
	C7. Face-to-face contact with mother and father living outside the household
	C8. Face-to-face contact with other relatives living outside the household
	C9. Face-to-face contact with friends and neighbors
	C10. Phone, internet or postal contact with children living outside the household.
	C11. Phone, internet or postal contact with mother and father living outside the household.
	C12. Phone, internet or postal contact with other relatives living outside the household.
	C13. Phone, internet or postal contact with friends and neighbors
Health	C14. Health status
	C15. Feeling particularly tense
	C16. Feeling lonely
	C17. Feeling downhearted and depressed
Social Exclusion and Community Involvement	C18. Attendance at religious services apart from weddings, funerals, christenings
	C19. Usage of internet other than for work
	C20. Taking part in sports or physical exercise
	C21. Feeling left out of society
	C22. Feeling that life is too complicated
	C23. Feeling that some people look down on me because of my job situation or income
	C24. Feeling close to people in the area where I live

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Table 1. Continued

Main Criteria	Sub-criteria
Standard of Living and Deprivation	C25. Financial situation of household compared to most people in the country
	C26. Struggling to meet household expense
Subjective Well-Being	C27. Be optimistic about the future
	C28. Feeling that what I do in life is worthwhile
	C29. Feeling free to decide how to live my life
	C30. Having time to do things I really enjoy

Calculation of Criteria Weights by IT2FAHP

The steps applied in calculating the weights of the criteria given in Table 1 using the IT2FAHP method are as follows:

Step 1: Each criterion is evaluated by an expert and a pairwise comparison matrix is created using the linguistic variables given in Table 2 and given in Table 3a & 3b.

Step 2: The consistency of the pairwise comparison matrix is examined. For this, first of all, instead of linguistic variables in the pairwise comparison matrix, their corresponding interval Type-2 fuzzy sets given in Table 2 are discussed. To check consistency ratio (CR), these interval Type-2 fuzzy numbers are converted into crisp ones by defuzzification method, i.e. DTriT or DTraT approaches, presented by Kahraman et al.(2014). Then, the consistency ratio (CR) of defuzzified pairwise matrix is calculated. Since the consistency ratio (CR) = 0.056 <0.10, it is concluded that the pairwise comparison matrix is consistent.

Step 3: Fuzzy weights are obtained by calculating the geometric mean for each row of the Type-2 fuzzy pairwise comparison matrix. Type-2 fuzzy weights obtained by using Equation (23) and Equation (24) for all criteria are given in Table 4.

Table 2. Linguistic terms for weights of criteria and their corresponding Interval Type 2 Fuzzy Sets (Kahraman et al., 2014)

Linguistic Terms	Triangular Interval Type 2 Fuzzy Sets
Exactly Equal (EE)	((1,1,1,1;1,1) (1,1,1,1;1,1))
Slightly Strong (SS)	((1,2,4,5;1,1) (1.2,2.2,3.8,4.8;0.8,0.8))
Fairly Strong (FS)	((3,4,6,7;1,1) (3.2,4.2,5.8,5.8;0.8,0.8))
Very Strong (VS)	((5,6,8,9;1,1) (5.2,6.2,7.8,8.8;0.8,0.8))
Absolutely Strong (AS)	((7,8,9,9;1,1) (7.2,8.2,8.9,9;0.8,0.8))

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Table 3a. Pairwise comparison matrix ($\tilde{\tilde{A}}$) via linguistic terms in Table 2

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
C1	EE	1/SS	EE	1/VS	1/VS	1/SS	EE	EE	EE	1/SS	EE	EE	EE	1/AS	1/SS
C2	SS	EE	SS	1/FS	1/FS	EE	SS	SS	SS	EE	SS	SS	SS	1/AS	EE
C3	EE	1/SS	EE	1/VS	1/VS	EE	EE	SS	SS	EE	EE	SS	SS	1/AS	EE
C4	VS	FS	VS	EE	EE	SS	SS	FS	FS	SS	SS	FS	FS	1/AS	FS
C5	VS	FS	VS	EE	EE	EE	SS	FS	FS	EE	SS	FS	FS	1/AS	FS
C6	SS	EE	EE	1/SS	EE	EE	SS	FS	FS	FS	FS	VS	VS	1/AS	EE
C7	EE	1/SS	EE	1/SS	1/SS	1/SS	EE	SS	SS	EE	SS	FS	FS	1/AS	EE
C8	EE	1/SS	1/SS	1/FS	1/FS	1/FS	1/SS	EE	EE	1/SS	1/SS	SS	SS	1/AS	1/SS
C9	EE	1/SS	1/SS	1/FS	1/FS	1/FS	1/SS	EE	EE	1/SS	1/SS	SS	SS	1/AS	1/SS
C10	SS	EE	EE	1/SS	EE	1/FS	EE	SS	SS	EE	SS	FS	FS	1/AS	EE
C11	EE	1/SS	EE	1/SS	1/SS	1/FS	1/SS	SS	SS	1/SS	EE	SS	SS	1/AS	EE
C12	EE	1/SS	1/SS	1/FS	1/FS	1/VS	1/FS	1/SS	1/SS	1/FS	1/SS	EE	EE	1/AS	1/SS
C13	EE	1/SS	1/SS	1/FS	1/FS	1/VS	1/FS	1/SS	1/SS	1/FS	1/SS	EE	EE	1/AS	1/SS
C14	AS	AS	AS	AS	AS	AS	AS	AS	AS	AS	AS	AS	AS	EE	VS
C15	SS	EE	EE	1/FS	1/FS	EE	EE	SS	SS	EE	EE	SS	SS	1/VS	EE
C16	SS	EE	EE	1/FS	1/FS	EE	EE	SS	SS	EE	EE	SS	SS	1/VS	EE
C17	SS	EE	EE	1/FS	1/FS	EE	EE	SS	SS	EE	EE	SS	SS	1/VS	EE
C18	1/SS	1/FS	1/SS	1/FS	1/FS	1/FS	1/FS	EE	EE	1/FS	1/FS	EE	EE	1/AS	1/FS
C19	1/VS	1/VS	1/VS	1/AS	1/AS	1/VS	1/VS	1/VS	1/VS	1/VS	1/VS	1/VS	1/VS	1/AS	1/VS
C20	FS	1/SS	1/SS	1/FS	1/FS	1/SS	1/SS	EE	EE	1/SS	1/SS	EE	EE	1/VS	1/SS
C21	VS	FS	SS	EE	EE	SS	FS	FS	FS	SS	FS	FS	FS	1/FS	EE
C22	EE	EE	EE	1/FS	1/FS	1/FS	1/SS	EE	EE	1/FS	1/SS	EE	EE	1/VS	1/SS
C23	VS	FS	FS	EE	EE	SS	SS	FS	FS	SS	SS	FS	FS	1/FS	SS
C24	EE	1/SS	1/SS	1/FS	1/FS	1/SS	1/SS	EE	EE	1/SS	1/SS	EE	EE	1/VS	1/FS
C25	FS	SS	SS	EE	EE	EE	SS	SS	SS	EE	SS	SS	SS	1/FS	EE
C26	FS	SS	SS	EE	EE	EE	SS	SS	SS	EE	SS	SS	SS	1/FS	EE
C27	EE	1/SS	EE	1/FS	1/FS	1/SS	1/SS	EE	EE	1/SS	1/SS	EE	EE	1/FS	1/SS
C28	1/SS	1/SS	1/SS	1/FS	1/FS	1/SS	EE	SS	SS	1/SS	EE	SS	SS	1/FS	EE
C29	EE	1/SS	1/SS	1/SS	1/SS	1/SS	1/SS	SS	SS	1/SS	1/SS	SS	SS	1/VS	1/SS
C30	SS	1/SS	EE	1/FS	1/FS	1/FS	1/SS	EE	EE	1/FS	1/SS	EE	EE	1/VS	EE

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Table 3b. Pairwise comparison matrix ($\tilde{\tilde{A}}$) via linguistic terms in Table 2

	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25	C26	C27	C28	C29	C30
C1	1/SS	1/SS	SS	VS	1/FS	1/VS	EE	1/VS	EE	1/FS	1/FS	EE	SS	EE	1/SS
C2	EE	EE	FS	VS	SS	1/FS	EE	1/FS	SS	1/SS	1/SS	SS	SS	SS	SS
C3	EE	EE	SS	VS	SS	1/SS	EE	1/FS	SS	1/SS	1/SS	EE	SS	SS	EE
C4	FS	FS	FS	AS	FS	EE	FS	EE	FS	EE	EE	FS	FS	SS	FS
C5	FS	FS	FS	AS	FS	EE	FS	EE	FS	EE	EE	FS	FS	SS	FS
C6	EE	EE	FS	VS	SS	1/SS	FS	1/SS	SS	EE	EE	SS	SS	SS	FS
C7	EE	EE	FS	VS	SS	1/FS	SS	1/SS	SS	1/SS	1/SS	SS	EE	SS	SS
C8	1/SS	1/SS	EE	VS	EE	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	1/SS	1/SS	EE
C9	1/SS	1/SS	EE	VS	EE	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	1/SS	1/SS	EE
C10	EE	EE	FS	VS	SS	1/SS	FS	1/SS	SS	EE	EE	SS	SS	SS	FS
C11	EE	EE	FS	VS	SS	1/FS	SS	1/SS	SS	1/SS	1/SS	SS	EE	SS	SS
C12	1/SS	1/SS	EE	VS	EE	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	1/SS	1/SS	EE
C13	1/SS	1/SS	EE	VS	EE	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	1/SS	1/SS	EE
C14	VS	VS	AS	AS	VS	FS	VS	FS	VS	FS	FS	FS	FS	VS	VS
C15	EE	EE	FS	VS	SS	EE	SS	1/SS	FS	EE	EE	SS	EE	SS	EE
C16	EE	EE	FS	VS	SS	EE	SS	1/SS	FS	EE	EE	SS	EE	SS	EE
C17	EE	EE	FS	VS	SS	EE	SS	1/SS	FS	EE	EE	SS	EE	SS	EE
C18	1/FS	1/FS	EE	VS	EE	1/VS	1/SS	1/FS	EE	1/FS	1/FS	EE	1/SS	1/SS	EE
C19	1/VS	1/VS	1/VS	EE	1/FS	1/VS	1/FS	1/AS	1/FS	1/VS	1/VS	1/VS	1/VS	1/VS	1/VS
C20	1/SS	1/SS	EE	FS	EE	1/VS	1/SS	1/VS	1/SS	1/FS	1/FS	1/SS	1/SS	1/SS	EE
C21	EE	EE	VS	VS	VS	EE	VS	EE	VS	FS	FS	FS	FS	FS	VS
C22	1/SS	1/SS	SS	FS	SS	1/VS	EE	1/FS	EE	1/FS	1/FS	EE	EE	EE	EE
C23	SS	SS	FS	AS	VS	EE	FS	EE	VS	FS	FS	FS	FS	FS	VS
C24	1/FS	1/FS	EE	FS	SS	1/VS	EE	1/VS	EE	1/FS	1/FS	EE	EE	EE	EE
C25	EE	EE	FS	VS	FS	1/FS	FS	1/FS	FS	EE	EE	SS	SS	SS	FS
C26	EE	EE	FS	VS	FS	1/FS	FS	1/FS	FS	EE	EE	SS	SS	SS	FS
C27	1/SS	1/SS	EE	VS	SS	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	EE	EE	SS
C28	EE	EE	SS	VS	SS	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	EE	EE	EE
C29	1/SS	1/SS	SS	VS	SS	1/FS	EE	1/FS	EE	1/SS	1/SS	EE	EE	EE	SS
C30	EE	EE	EE	VS	EE	1/VS	EE	1/VS	EE	1/FS	1/FS	1/SS	EE	1/SS	EE

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Table 4. Interval Type-2 Fuzzy Weights of Criteria

Criteria	Interval Type-2 Fuzzy Weights of Criteria
C1	((0.008,0.011,0.021,0.035;1,1) (0.008,0.012,0.020,0.031;0.8,0.8))
C2	((0.013,0.024,0.054,0.084;1,1) (0.015,0.026,0.049,0.077;0.8,0.8))
C3	((0.012,0.019,0.039,0.061;1,1) (0.013,0.021,0.036,0.055,0.8,0.8))
C4	((0.032,0.052,0.108,0.161;1,1) (0.036,0.056,0.100,0.148,0.8;0.8))
C5	((0.032,0.049,0.098,0.144;1,1) (0.035,0.053,0.092,0.133;0.8,0.8))
C6	((0.023,0.036,0.075,0.114;1,1) (0.025,0.039,0.070,0.105,0.8,0.8))
C7	((0.013,0.022,0.049,0.082;1,1) (0.015,0.024,0.045,0.073,0.8,0.8))
C8	((0.007,0.010,0.022,0.040;1,1) (0.007,0.011,0.020,0.035;0.8,0.8))
C9	((0.007,0.010,0.022,0.040;1,1) (0.007,0.011,0.020,0.035;0.8,0.8))
C10	((0.018,0.029,0.061,0.093;1,1) (0.020,0.031,0.056,0.085;0.8,0.8))
C11	((0.011,0.018,0.043,0.073;1,1) (0.012,0.020,0.039,0.065;0.8,0.8))
C12	((0.007,0.009,0.020,0.036;1,1) (0.007,0.010,0.018,0.032;0.8,0.8))
C13	((0.007,0.009,0.020,0.036;1,1) (0.007,0.010,0.018,0.032;0.8,0.8))
C14	((0.023,0.035,0.079,0.141;1,1) (0.025,0.037,0.071,0.123;0.8,0.8))
C15	((0.017,0.026,0.051,0.075;1,1) (0.018,0.028,0.048,0.069;0.8,0.8))
C16	((0.017,0.026,0.051,0.075;1,1) (0.018,0.028,0.048,0.069;0.8,0.8))
C17	((0.017,0.026,0.051,0.075;1,1) (0.018,0.028,0.048,0.069;0.8,0.8))
C18	((0.007,0.009,0.019,0.031;1,1) (0.007,0.010,0.017,0.028;0.8,0.8))
C19	((0.002,0.003,0.006,0.009;1,1) (0.002,0.003,0.005,0.008;0.8,0.8))
C20	((0.006,0.008,0.019,0.036;1,1) (0.006,0.009,0.017,0.031;0.8,0.8))
C21	((0.040,0.062,0.122,0.176;1,1) (0.044,0.066,0.114,0.164;0.8,0.8))
C22	((0.009,0.013,0.025,0.039;1,1) (0.010,0.014,0.023,0.035;0.8,0.8))
C23	((0.032,0.053,0.115,0.177;1,1) (0.035,0.057,0.107,0.162;0.8,0.8))
C24	((0.007,0.010,0.019,0.032;1,1) (0.008,0.011,0.018,0.028;0.8,0.8))
C25	((0.020,0.034,0.073,0.108;1,1) (0.023,0.038,0.068,0.100;0.8,0.8))
C26	((0.020,0.035,0.074,0.110;1,1) (0.023,0.038,0.069,0.101;0.8,0.8))
C27	((0.008,0.012,0.025,0.043;1,1) (0.009,0.013,0.023,0.038;0.8,0.8))
C28	((0.010,0.015,0.032,0.053;1,1) (0.011,0.016,0.029,0.047;0.8,0.8))
C29	((0.008,0.013,0.031,0.059;1,1) (0.009,0.014,0.028,0.051;0.8,0.8))
C30	((0.009,0.012,0.023,0.037;1,1) (0.010,0.013,0.022,0.033;0.8,0.8))

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Ranking the Countries by IT2FCOPRAS

The steps applied in ranking the countries using the IT2FCOPRAS method are as follows:

Step 1: A fuzzy decision matrix is created by taking into account the linguistic evaluations of each country with respect to each criterion using data from the European Quality of Life Survey 2012 (Eurofound, 2016). These linguistic evaluations are converted into their corresponding IT2FSs given in Table 5 for each criterion, separately. For detailed information, see Şahin (2016). The weights given in Table 4 calculated by the IT2-FAHP method are used as the importance weight of the criteria.

Step 2: Using the Equation (28) and Equation (29), the normalized decision matrix, $N = \left[\tilde{\tilde{n}}_{ij} \right]_{30\times34}$, is formed.

Step 3: The weighted normalized decision matrix, $E = \left[\tilde{\tilde{e}}_{ij} \right]_{30\times34}$, is formed by Equation (30).

Step 4: The sum of the weighted normalized values for the benefit criteria and cost criteria is calculated from Equation (31) and Equation (32), and are presented in Table 6 and Table 7, respectively.

Step 5: The ranking values for $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ $(1 \le i \le m)$ are determined using the ranking method proposed by Ghorabaee et al. (2014) and are presented in Table 6 and Table 7, respectively.

Step 6: The relative importance (Q_i) of the countries is calculated from Equation (33) and given in Table 8.

Step 7: Quantitative importance values for each country (U_i) are calculated from Equation (34) and given in Table 8. Denmark, the country with the greatest value, is identified as the best country in terms of quality of life.

Table 5. Linguistic terms and corresponding IT2FSs for the evaluation of alternatives (Abdullah & Najib, 2014)

Linguistic Terms	Interval Type-2 Fuzzy Sets
Very Poor (VP)	((0,0.1,0.1,0.1;1;1) (0,0.1,0.1,0.05;0.9;0.9))
Poor (P)	((0.2,0.3,0.3,0.4;1;1) (0.25,0.3,0.3,0.35;0.9;0.9))
Medium (M)	((0.4,0.5,0.5,0.6;1;1) (0.45,0.5,0.5,0.55;0.9;0.9))
Good (G)	((0.6,0.7,0.7,0.8;1;1) (0.65,0.7,0.7,0.75;0.9;0.9))
Very Good (VG)	((0.8,0.9,0.9,1;1;1) (0.85,0.9,0.9,0.95;0.9;0.9))

According to the integrated IT2FAHP / IT2FVIKOR method, the best three countries in terms of quality of life are Sweden, Iceland and Denmark, while the worst three countries are Bulgaria, Greece and Latvia. Turkey ranked 25th among 34 countries in terms of quality of life.

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Table 6. Sum of the weighted normalized values ($\tilde{\tilde{S}}_{+i}$) for benefit type criteria

Country	($\tilde{\tilde{S}}_{+i}$)	Ranking for $\tilde{\tilde{S}}_{+i}$
Iceland	((0.005,0.011,0.023,0.053;1,1) (0.007,0.012,0.021,0.040;0.8,0.8))	14
Montenegro	((0.005,0.011,0.023,0.053;1,1) (0.007,0.012,0.021,0.040;0.8,0.8))	29
Macedonia	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.021,0.039;0.8,0.8))	18
Serbia	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	31
Turkey	((0.005,0.010,0.022,0.050;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	22
Kosovo	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.039;0.8,0.8))	5
Austria	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.039;0.8,0.8))	25
Belgium	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	16
Bulgaria	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	8
Cyprus	((0.005,0.011,0.023,0.052;1,1) (0.006,0.012,0.021,0.040;0.8,0.8))	10
Czech Republic	((0.004,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	2
Germany	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	15
Denmark	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	6
Estonia	((0.004,0.009,0.020,0.046;1,1) (0.006,0.010,0.018,0.034;0.8,0.8))	11
Greece	((0.005,0.010,0.021,0.049;1,1) (0.006,0.011,0.020,0.037;0.8,0.8))	4
Spain	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	26
Finland	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.039;0.8,0.8))	3
France	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	23
Croatia	((0.005,0.011,0.022,0.051;1,1) (0.006,0.011,0.021,0.039;0.8,0.8))	33
Hungary	((0.005,0.010,0.021,0.049;1,1) (0.006,0.011,0.020,0.037;0.8,0.8))	7
Ireland	((0.005,0.010,0.022,0.050;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	21
Italy	((0.005,0.010,0.022,0.050;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	27
Lithuania	((0.004,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.036;0.8,0.8))	1
Luxembourg	((0.005,0.010,0.022,0.049;1,1) (0.006,0.011,0.020,0.037;0.8,0.8))	19
Latvia	((0.004,0.010,0.021,0.047;1,1) (0.006,0.010,0.019,0.036;0.8,0.8))	28
Malta	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	34
Netherlands	((0.005,0.010,0.021,0.049;1,1) (0.006,0.011,0.020,0.037;0.8,0.8))	12
Poland	((0.005,0.010,0.022,0.050;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	24
Portugal	((0.004,0.010,0.020,0.047;1,1) (0.006,0.010,0.019,0.035;0.8,0.8))	9
Romania	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.037;0.8,0.8))	13
Sweden	((0.005,0.011,0.023,0.052;1,1) (0.006,0.011,0.021,0.039;0.8,0.8))	30
Slovenia	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	32
Slovakia	((0.005,0.010,0.022,0.051;1,1) (0.006,0.011,0.020,0.038;0.8,0.8))	17
United Kingdom	((0.005,0.010,0.021,0.048;1,1) (0.006,0.011,0.019,0.037;0.8,0.8))	20

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Table 7. Sum of the weighted normalized values ($\tilde{\tilde{S}}_{-i}$) for cost type criteria

Country	($\tilde{\tilde{S}}_{-i}$)	**Ranking for** $\tilde{\tilde{S}}_{-i}$
Iceland	((0.003,0.008,0.016,0.040;1,1) (0.004,0.008,0.015,0.027;0.8,0.8))	29
Montenegro	((0.004,0.010,0.022,0.055;1,1) (0.006,0.011,0.020,0.039;0.8,0.8))	8
Macedonia	((0.004,0.011,0.023,0.058;1,1) (0.006,0.012,0.021,0.042;0.8,0.8))	7
Serbia	((0.005,0.012,0.024,0.060;1,1) (0.007,0.013,0.022,0.044;0.8,0.8))	12
Turkey	((0.005,0.012,0.025,0.062;1,1) (0.007,0.013,0.023,0.045;0.8,0.8))	21
Kosovo	((0.005,0.012,0.025,0.062;1,1) (0.007,0.013,0.023,0.045;0.8,0.8))	27
Austria	((0.003,0.008,0.017,0.042;1,1) (0.004,0.009,0.016,0.029;0.8,0.8))	19
Belgium	((0.003,0.010,0.020,0.049;1,1) (0.005,0.010,0.018,0.035;0.8,0.8))	32
Bulgaria	((0.005,0.012,0.025,0.061;1,1) (0.007,0.013,0.023,0.044;0.8,0.8))	31
Cyprus	((0.005,0.013,0.026,0.065;1,1) (0.007,0.014,0.024,0.047;0.8,0.8))	14
Czech Republic	((0.004,0.011,0.023,0.058;1,1) (0.006,0.012,0.022,0.042;0.8,0.8))	25
Germany	((0.003,0.008,0.017,0.042;1,1) (0.004,0.009,0.016,0.029;0.8,0.8))	2
Denmark	((0.002,0.007,0.014,0.035;1,1) (0.003,0.008,0.013,0.024;0.8,0.8))	20
Estonia	((0.004,0.011,0.022,0.055;1,1) (0.006,0.011,0.020,0.039;0.8,0.8))	9
Greece	((0.005,0.013,0.026,0.065;1,1) (0.007,0.014,0.025,0.048;0.8,0.8))	1
Spain	((0.004,0.010,0.020,0.050;1,1) (0.005,0.010,0.019,0.035;0.8,0.8))	34
Finland	((0.003,0.008,0.017,0.044;1,1) (0.004,0.009,0.016,0.030;0.8,0.8))	10
France	((0.004,0.010,0.020,0.049;1,1) (0.005,0.011,0.019,0.035;0.8,0.8))	3
Croatia	((0.004,0.011,0.023,0.057;1,1) (0.006,0.012,0.021,0.041;0.8,0.8))	23
Hungary	((0.004,0.011,0.022,0.056;1,1) (0.006,0.012,0.021,0.040;0.8,0.8))	4
Ireland	((0.003,0.009,0.019,0.049;1,1) (0.005,0.010,0.018,0.034;0.8,0.8))	15
Italy	((0.004,0.010,0.020,0.051;1,1) (0.005,0.011,0.019,0.037;0.8,0.8))	33
Lithuania	((0.004,0.011,0.022,0.056;1,1) (0.006,0.012,0.021,0.040;0.8,0.8))	22
Luxembourg	((0.003,0.009,0.019,0.047;1,1) (0.005,0.010,0.017,0.033;0.8,0.8))	6
Latvia	((0.005,0.012,0.024,0.060;1,1) (0.006,0.013,0.022,0.043;0.8,0.8))	24
Malta	((0.004,0.010,0.021,0.053;1,1) (0.005,0.011,0.019,0.038;0.8,0.8))	30
Netherlands	((0.003,0.008,0.017,0.042;1,1) (0.004,0.009,0.015,0.029;0.8,0.8))	28
Poland	((0.004,0.011,0.023,0.058;1,1) (0.006,0.012,0.021,0.042;0.8,0.8))	26
Portugal	((0.004,0.010,0.020,0.050;1,1) (0.005,0.011,0.019,0.036;0.8,0.8))	17
Romania	((0.004,0.011,0.022,0.054;1,1) (0.006,0.012,0.020,0.039;0.8,0.8))	16
Sweden	((0.003,0.008,0.016,0.041;1,1) (0.004,0.009,0.015,0.028;0.8,0.8))	11
Slovenia	((0.003,0.010,0.020,0.050;1,1) (0.005,0.010,0.018,0.035;0.8,0.8))	5
Slovakia	((0.004,0.010,0.022,0.054;1,1) (0.006,0.011,0.020,0.039;0.8,0.8))	13
United Kingdom	((0.004,0.010,0.020,0.050;1,1) (0.005,0.011,0.019,0.036;0.8,0.8))	18

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Table 8. Q_i and U_i values and ranking for the countries via IT2FCOPRAS

Country	Q_i	U_i	Ranking
Iceland	0.0899	99.0225	2
Montenegro	0.0880	96.9530	17
Macedonia	0.0879	96.7540	21
Serbia	0.0878	96.6469	25
Turkey	0.0877	96.5494	28
Kosovo	0.0878	96.6356	27
Austria	0.0894	98.4512	5
Belgium	0.0882	97.1179	11
Bulgaria	0.0875	96.3811	32
Cyprus	0.0878	96.6376	26
Czech Republic	0.0875	96.3983	31
Germany	0.0892	98.2730	7
Denmark	0.0908	100.0000	1
Estonia	0.0874	96.2250	34
Greece	0.0876	96.4166	30
Spain	0.0881	97.0620	16
Finland	0.0893	98.2944	6
France	0.0882	97.1065	12
Croatia	0.0879	96.7680	20
Hungary	0.0878	96.6656	24
Ireland	0.0885	97.4525	9
Italy	0.0882	97.1050	13
Lithuania	0.0877	96.5288	29
Luxembourg	0.0887	97.6359	8
Latvia	0.0874	96.2739	33
Malta	0.0882	97.0759	14
Netherlands	0.0894	98.4830	4
Poland	0.0878	96.6802	23
Portugal	0.0879	96.8048	19
Romania	0.0878	96.6877	22
Sweden	0.0897	98.8248	3
Slovenia	0.0884	97.3391	10
Slovakia	0.0880	96.9317	18
United Kingdom	0.0881	97.0625	15

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Ranking the Countries by IT2FVIKOR

The steps applied in ranking the alternative countries using the IT2FVIKOR method are as follows:

Step 1: A fuzzy decision matrix is created by taking into account the linguistic evaluations of each country with respect to each criterion using data from the European Quality of Life Survey 2012 (Eurofound, 2016). These linguistic evaluations are converted into their corresponding IT2FSs given in Table 5 for each criterion, separately. For detailed information, see Şahin (2016). The weights calculated by the IT2FAHP method and given in Table 4 were used as the importance weight of the criteria.

Step 2: Ideal values ($\tilde{\tilde{X}}_j^*$) and nadir values ($\tilde{\tilde{X}}_j^o$) for all criteria are calculated by using the Equations (35)-(36), and the ranking method proposed by Ghorabaee et al. (2014), and given in Table 9.

Step 3: Normalized fuzzy differences are calculated from Equations (37) and (38).

Step 4: For all countries, $\tilde{\tilde{S}}_i$ and $\tilde{\tilde{R}}_i$ values are calculated from Equations (39) and (40).

Step 5: The best values of $\tilde{\tilde{S}}_i$ and $\tilde{\tilde{R}}_i$ are calculated using Equations (41) and (42).

Step 6: $\tilde{\tilde{Q}}_i$ value was calculated from Equation (43) for all countries and given in Table 8. The weight of the group utility strategy is taken as v=0.8. $\tilde{\tilde{Q}}_i$ values can be calculated for different weights of v. However, it requires too many calculations to calculate for all v values since the number of criteria in this study is very high. Therefore, v=0.8, which is widely used in the literature, is preferred in this study (Wang and Chang, 2005; Wang et al. (2019); Ghorabaee, 2016).

Step 7: The ranking of the countries according to their values is given in Table 10. From Table 10, Sweden, the country with the lowest value, is determined as the best country in terms of quality of life.

Table 9. Ideal values ($\tilde{\tilde{X}}_j^*$) and nadir values ($\tilde{\tilde{X}}_j^o$) for the criteria

Criteria	$\tilde{\tilde{X}}_j^*$	$\tilde{\tilde{X}}_j^o$
C1	((0.354,0.454,0.454,0.533;1,1) (0.393,0.454,0.454,0.483;0.9,0.9))	((0.605,0.705,0.705,0.795;1,1) (0.650,0.705,0.705,0.745;0.9,0.9))
C2	((0.208,0.308,0.308,0.367;1,1) (0.237,0.308,0.308,0.317;0.9,0.9))	((0.491,0.591,0.591,0.677;1,1) (0.534,0.591,0.591,0.627;0.9,0.9))
C3	((0.115,0.215,0.215,0.253;1,1) (0.134,0.215,0.215,0.203;0.9,0.9))	((0.377,0.477,0.477,0.551;1,1) (0.414,0.477,0.477,0.501;0.9,0.9))
C4	((0.105,0.205,0.205,0.241;1,1) (0.123,0.205,0.205,0.191;0.9,0.9))	((0.368,0.468,0.468,0.549;1,1) (0.409,0.468,0.468,0.499;0.9,0.9))
C5	((0.530,0.630,0.630,0.722;1,1) (0.576,0.630,0.630,0.672;0.9,0.9))	((0.182,0.282,0.282,0.337;1,1) (0.210,0.282,0.282,0.287;0.9,0.9))
C6	((0.734,0.834,0.834,0.934;1,1) (0.784,0.834,0.834,0.884;0.9,0.9))	((0.520,0.620,0.620,0.719;1,1) (0.570,0.620,0.620,0.669;0.9,0.9))
C7	((0.674,0.774,0.774,0.872;1,1) (0.723,0.774,0.774,0.822;0.9,0.9))	((0.404,0.504,0.504,0.597;1,1) (0.451,0.504,0.504,0.547;0.9,0.9))
C8	((0.613,0.713,0.713,0.812;1,1) (0.662,0.713,0.713,0.762;0.9,0.9))	((0.344,0.444,0.444,0.540;1,1) (0.392,0.444,0.444,0.490;0.9,0.9))

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Table 9. Continued

Criteria	$\tilde{\tilde{X}}_j^*$	$\tilde{\tilde{X}}_j^o$
C9	((0.738,0.838,0.838,0.938;1,1) (0.788,0.838,0.838,0.888;0.9,0.9))	((0.450,0.550,0.550,0.645;1,1) (0.497,0.550,0.550,0.595;0.9,0.9))
C10	((0.738,0.838,0.838,0.934;1,1) (0.786,0.838,0.838,0.884;0.9,0.9))	((0.390,0.490,0.490,0.553;1,1) (0.422,0.490,0.490,0.503;0.9,0.9))
C11	((0.680,0.780,0.780,0.873;1,1) (0.726,0.780,0.780,0.823;0.9,0.9))	((0.279,0.379,0.379,0.425;1,1) (0.302,0.379,0.379,0.375;0.9,0.9))
C12	((0.657,0.757,0.757,0.854;1,1) (0.705,0.757,0.757,0.804;0.9,0.9))	((0.348,0.448,0.448,0.520;1,1) (0.384,0.448,0.448,0.470;0.9,0.9))
C13	((0.682,0.782,0.782,0.880;1,1) (0.731,0.782,0.782,0.830;0.9,0.9))	((0.396,0.496,0.496,0.572;1,1) (0.434,0.496,0.496,0.522;0.9,0.9))
C14	((0.629,0.730,0.729,0.828;1,1) (0.679,0.729,0.729,0.778;0.9,0.9))	((0.456,0.556,0.556,0.652;1,1) (0.504,0.556,0.556,0.602;0.9,0.9))
C15	((0.172,0.272,0.272,0.326;1,1) (0.199,0.272,0.272,0.276;0.9,0.9))	((0.492,0.592,0.592,0.682;1,1) (0.537,0.592,0.592,0.632;0.9,0.9))
C16	((0.082,0.182,0.182,0.207;1,1) (0.094,0.182,0.182,0.157;0.9,0.9))	((0.293,0.393,0.393,0.456;1,1) (0.324,0.393,0.393,0.406;0.9,0.9))
C17	((0.090,0.190,0.189,0.218;1,1) (0.103,0.189,0.189,0.168;0.9,0.9))	((0.387,0.487,0.487,0.564;1,1) (0.426,0.487,0.487,0.514;0.9,0.9))
C18	((0.436,0.536,0.536,0.613;1,1) (0.474,0.536,0.536,0.563;0.9,0.9))	((0.099,0.199,0.199,0.230;1,1) (0.114,0.199,0.199,0.180;0.9,0.9))
C19	((0.709,0.809,0.809,0.901;1,1) (0.755,0.809,0.809,0.851;0.9,0.9))	((0.263,0.363,0.363,0.408;1,1) (0.285,0.363,0.363,0.358;0.9,0.9))
C20	((0.582,0.682,0.682,0.776;1,1) (0.629,0.682,0.682,0.726;0.9,0.9))	((0.124,0.224,0.224,0.254;1,1) (0.139,0.224,0.224,0.204;0.9,0.9))
C21	((0.094,0.194,0.194,0.221;1,1) (0.107,0.194,0.194,0.171;0.9,0.9))	((0.298,0.398,0.398,0.484;1,1) (0.341,0.398,0.398,0.434;0.9,0.9))
C22	((0.150,0.250,0.250,0.305;1,1) (0.177,0.250,0.250,0.255;0.9,0.9))	((0.526,0.626,0.626,0.721;1,1) (0.573,0.626,0.626,0.671;0.9,0.9))
C23	((0.117,0.217,0.217,0.246;1,1) (0.131,0.217,0.217,0.196;0.9,0.9))	((0.360,0.460,0.460,0.547;1,1) (0.404,0.460,0.460,0.497;0.9,0.9))
C24	((0.633,0.733,0.733,0.831;1,1) (0.682,0.733,0.733,0.781;0.9,0.9))	((0.490,0.590,0.590,0.684;1,1) (0.537,0.590,0.590,0.634;0.9,0.9))
C25	((0.480,0.580,0.580,0.678;1,1) (0.529,0.580,0.580,0.628;0.9,0.9))	((0.331,0.431,0.431,0.520;1,1) (0.376,0.431,0.431,0.470;0.9,0.9))
C26	((0.292,0.392,0.392,0.469;1,1) (0.330,0.392,0.392,0.419;0.9,0.9))	((0.663,0.763,0.763,0.862;1,1) (0.713,0.763,0.763,0.812;0.9,0.9))
C27	((0.632,0.732,0.732,0.830;1,1) (0.681,0.732,0.732,0.780;0.9,0.9))	((0.294,0.394,0.394,0.474;1,1) (0.334,0.394,0.394,0.424;0.9,0.9))
C28	((0.702,0.802,0.802,0.902;1,1) (0.752,0.802,0.802,0.852;0.9,0.9))	((0.464,0.564,0.564,0.659;1,1) (0.511,0.564,0.564,0.609;0.9,0.9))
C29	((0.684,0.784,0.784,0.882;1,1) (0.733,0.784,0.784,0.832;0.9,0.9))	((0.409,0.509,0.509,0.598;1,1) (0.453,0.509,0.509,0.548;0.9,0.9))
C30	((0.468,0.568,0.568,0.666;1,1) (0.517,0.568,0.568,0.616;0.9,0.9))	((0.281,0.381,0.381,0.468;1,1) (0.325,0.381,0.381,0.418;0.9,0.9))

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Table 10. $\tilde{\tilde{Q}}_i$ values and ranking for the countries via IT2FVIKOR

Countries	$\tilde{\tilde{Q}}_i$	Ranking
Iceland	((-0.625, -0.038,0.083,0.732;1,1) (-0.030, -0.025,0.069,0.104;0.8,0.8))	2
Montenegro	((-0.619, -0.018,0.125,0.813;1,1) (-0.008, -0.004,0.108,0.100;0.8,0.8))	8
Macedonia	((-0.613,0.003,0.170,0.890;1,1) (0.007,0.020,0.150,0.121;0.8,0.8))	20
Serbia	((-0.608,0.020,0.206,0.955;1,1) (0.019,0.038,0.183,0.141;0.8,0.8))	27
Turkey	((-0.607,0.017,0.204,0.934;1,1) (0.018,0.035,0.181,0.112;0.8,0.8))	25
Kosovo	((-0.608,0.023,0.213,0.972;1,1) (0.021,0.041,0.190,0.138;0.8,0.8))	29
Austria	((-0.623, -0.027,0.109,0.780;1,1) (-0.021, -0.013,0.093,0.100;0.8,0.8))	4
Belgium	((-0.613,0.002,0.169,0.887;1,1) (0.006,0.018,0.148,0.118;0.8,0.8))	18
Bulgaria	((-0.607,0.027,0.223,0.985;1,1) (0.025,0.046,0.198,0.136;0.8,0.8))	32
Cyprus	((-0.606,0.022,0.208,0.958;1,1) (0.021,0.040,0.186,0.158;0.8,0.8))	28
Czech Republic	((-0.607,0.018,0.208,0.942;1,1) (0.019,0.036,0.184,0.109;0.8,0.8))	26
Germany	((-0.618, -0.017,0.130,0.798;1,1) (-0.011, -0.002,0.113,0.089;0.8,0.8))	7
Denmark	((-0.623, -0.032,0.097,0.747;1,1) (-0.026, -0.019,0.082,0.113;0.8,0.8))	3
Estonia	((-0.609,0.022,0.214,0.979;1,1) (0.022,0.040,0.190,0.122;0.8,0.8))	31
Greece	((-0.601,0.031,0.230,0.960;1,1) (0.029,0.050,0.206,0.115;0.8,0.8))	33
Spain	((-0.611, -0.005,0.154,0.822;1,1) (0.001,0.011,0.135,0.090;0.8,0.8))	13
Finland	((-0.623, -0.024,0.117,0.801;1,1) (-0.018, -0.009,0.100,0.100;0.8,0.8))	6
France	((-0.617,0.001,0.170,0.890;1,1) (0.000,0.018,0.149,0.108;0.8,0.8))	17
Croatia	((-0.609,0.004,0.176,0.875;1,1) (0.008,0.021,0.155,0.109;0.8,0.8))	22
Hungary	((-0.619, -0.002,0.166,0.924;1,1) (0.001,0.014,0.145,0.105;0.8,0.8))	21
Ireland	((-0.617, -0.014,0.134,0.826;1,1) (-0.004,0.001,0.117,0.109;0.8,0.8))	11
Italy	((-0.613, -0.012,0.139,0.799;1,1) (-0.003,0.004,0.121,0.083;0.8,0.8))	10
Lithuania	((-0.613,0.019,0.213,1.012;1,1) (0.015,0.037,0.188,0.135;0.8,0.8))	30
Luxembourg	((-0.620, -0.012,0.141,0.839;1,1) (-0.009,0.004,0.123,0.107;0.8,0.8))	12
Latvia	((-0.611,0.025,0.228,1.040;1,1) (0.020,0.044,0.202,0.135;0.8,0.8))	34
Malta	((-0.615, -0.005,0.152,0.865;1,1) (0.002,0.010,0.133,0.118;0.8,0.8))	14
Netherlands	((-0.620, -0.019,0.128,0.817;1,1) (-0.014, -0.004,0.110,0.112;0.8,0.8))	9
Poland	((-0.611,0.014,0.195,0.942;1,1) (0.015,0.031,0.173,0.124;0.8,0.8))	24
Portugal	((-0.611,0.002,0.170,0.855;1,1) (0.006,0.018,0.150,0.084;0.8,0.8))	16
Romania	((-0.610,0.008,0.189,0.919;1,1) (0.011,0.026,0.167,0.118;0.8,0.8))	23
Sweden	((-0.620, -0.041,0.077,0.666;1,1) (-0.024, -0.029,0.064,0.033;0.8,0.8))	1
Slovenia	((-0.617, -0.021,0.120,0.772;1,1) (-0.010, -0.007,0.103,0.071;0.8,0.8))	5
Slovakia	((-0.611, -0.003,0.161,0.854;1,1) (0.003,0.013,0.141,0.107;0.8,0.8))	15
United Kingdom	((-0.613,0.002,0.170,0.889;1,1) (0.007,0.019,0.149,0.116;0.8,0.8))	19

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According to the IT2FAHP / IT2FCOPRAS method, the best three countries in terms of quality of life are Denmark, Iceland and Sweden, while the worst three countries are Bulgaria, Latvia and Estonia. Turkey ranked 28th among 34 countries in terms of quality of life.

SOLUTIONS AND RECOMMENDATIONS

Spearman's rho Correlation Coefficient is used to compare the ranking results of the methods considered in this chapter. In addition, results of the integrated IT2FAHP/ IT2F-TOPSIS method used in Şahin and Pehlivan (2017) are also included in the comparison. The correlation coefficients obtained are given in the Table 11.

Table 11. Correlation coefficients between the integrated MCDM methods based on IT2FSs

Methods	IT2FAHP/ IT2FCOPRAS	IT2FAHP / IT2FVIKOR	IT2FAHP / IT2FTOPSIS
IT2FAHP / IT2FCOPRAS	1	0.943	0.970
IT2FAHP / IT2FVIKOR	0.943	1	0.954
IT2FAHP / IT2FTOPSIS	0.970	0.954	1

* Correlation is significant at the 0.01 level (2-tailed)

From Table 11, ranking results of the proposed methods are consistent with each other and the integrated IT2FAHP/ IT2F-TOPSIS because all Spearman's rho correlation coefficients are statistically significant at the 0.01 level.

FUTURE RESEARCH DIRECTIONS

In future studies, IT2FSs can be adapted to different MCDM methods such as MOORA, ELECTRE, PROMETHEE, QUALIFLEX, ARAS, EDAS, SAW, and WASPAS for determining the criteria weights and ranking the alternatives. Thus, obtained results can be compared with the results given in this chapter. In addition, different ranking methods can be developed for Type-2 fuzzy sets for utilizing at the solution stage. The life quality evaluation problem discussed in this study can also be extended to different extensions of fuzzy sets such as intuitionistic, pythagorean, hesitant for further researches.

CONCLUSION

MCDM methods based on crisp sets, fuzzy sets or their extensions have been performed on real world problems with multiple and often conflicting criteria that take into account the different preferences or judgements of decision makers. In the literature, there are various MCDM methods such as AHP, TOPSIS, VIKOR, COPRAS, ARAS, EDAS, COPRAS, and etc. introduced by several researchers to solve problems in different fields. Since, the integration of two or more MCDM methods could give the

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better and more effective results than single MCDM method, integrated methods of AHP with VIKOR and AHP with COPRAS on the basis of interval type-2 fuzzy sets are proposed in this chapter.

In this study, integrated MCDM methods by using IT2FAHP / IT2FVIKOR and IT2FAHP / IT-2FCOPRAS methods are proposed to rank the life quality of 34 countries in terms of 30 criteria. The proposed methods use the IT2FAHP method, which has a mechanism that tests the consistency of paired comparison matrices in determining the criterion weights, which is the most important stage of MCDM problems. In this way, it makes it possible to significantly reduce the bias in the process of determining criteria weights while calculating criteria weights. However, the ITF-AHP method has a disadvantage in terms of computation time since it will require too many pairwise comparisons if the number of criteria or alternatives is high. Therefore, the use of IT2FVIKOR and IT2FCOPRAS methods, which do not require binary comparisons and are easy to apply, in the proposed methods for ranking the alternatives provide an advantage in terms of calculation time. Furthermore, IT2F sets used in this study are more successful than IT1F sets in overcoming fuzziness and uncertainties in real world problems. As a result, it can be said that the proposed methods provide consistent, unbiased and reliable results in a short computation time.

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Chapter 23
Some Results on Weighted Intuitionistic Fuzzy Soft Sets

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ABSTRACT

A weighted intuitionistic fuzzy soft set (WIFSS) is an important generalization of intuitionistic fuzzy soft set (IFSS). In this research work, the authors define the basic binary set operations, such as union, restricted union, restricted intersection, extended intersection, restricted difference, sum, AND-product, OR-product, and some concepts on weighted intuitionistic fuzzy soft set. They also present some properties of the operations and established some fundamental results including various De Morgan's type of results in weighted intuitionistic fuzzy soft set context.

1. INTRODUCTION

Over the years, man has been working tirelessly to find solutions to complicated problems we encountered in various facets of human endeavors such as environmental sciences, engineering, economics, medical sciences, social sciences, management sciences, etc. These types of problems are associated with various degrees of uncertainties and imprecision in them. In search of solution to these problems, many mathematical models were postulated to handle the problems of uncertainties and the imprecision. These theories among others include theory of probability (Prade and Dubois, 1980), theory of fuzzy sets (Zadeh, 1965), theory of interval mathematics (Atanassov, 1994), rough sets (Pawlak, 1982), vague sets (Gau and Buehrer, 1993) which we can consider as mathematical tools for dealing with uncertainties. But it was discovered that all these theories have their short comings in handling the level of uncertainties.

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The major deficiency associated with these theories is the inadequacies of the parameterization tools. To overcome these limitations, (Molodtsov, 1999) introduced the concept of soft set as a new mathematical tool for dealing with uncertainties and imprecision that is free from the difficulties that have bedeviled the standard existing mathematical models. Molodtsov pointed out the application of soft set in several directions. This theory has proven useful in many different fields such as decision making (Roy and Maji, 2007), data analysis (Zou and Xiao, 2008), forecasting and so on.

Research on soft sets has been progressing rapidly, since its introduction by Molodtsov in 1999 up to the present and several important results have been achieved in theory and practice. Maji *et al.* (2003) defined many algebraic operations in soft set theory and published a detailed theoretical study on soft sets. Ali *et al*. (2009) further presented and investigated some new algebraic operations for soft sets. Sezgin and Atagun (2011) proved that certain De Morgan's law holds in soft set theory with respect to different operations on soft sets and discuss the basic properties of operations on soft sets such as intersection, extended intersection, restricted union and restricted difference. Thereafter, it was observed that soft set can be combined with other mathematical models. Maji *et al.* (2001) were the first to present fuzzy soft set. Maji *et al*. (2003) established classical soft set to intuitionistic fuzzy soft sets, which were further discussed in (Maji *et al*., 2003) and (Yin *et al*., 2012). Since then many researchers have established various hybrid soft sets.

Other applications of soft set and its hybrids in decision making are found in (Cagman and Deli, 2012a, 2012b), (Deli and Cagman, 2015), (Deli, 2015) and (Deli *et al*., 2018).

Deli and Cagman (2016) defined some kinds of distance between two intuitionistic fuzzy set and applied it in medical diagnosis of disease.

Smarandache in 2018 further generalized Soft Set theory to HyperSoft Set by transforming the function F in to a multi-attribute function.

Since decision making is an integral part of our day-to-day activity however, not all decision parameters or attributes are of equal importance to the decision maker(s). As a generalization of intuitionistic fuzzy soft set theory, (Onyeozili and Balami, 2021) introduced weighted intuitionistic fuzzy soft set theory that enables the descriptions of the objective world more realistic, practical and accurate in some cases, making it very promising.

In this chapter, the researchers studied and defined the binary soft set operations in weighted intuitionistic fuzzy soft set introduced by (Onyeozili and Balami, 2021). The authors also presented the basic properties of the operations of weighted intuitionistic fuzzy soft set and various De Morgan's type of results, which has indeed showed that soft set and its hybrids have developed enough basic supporting tools where various kinds of set operations could be define. Finally, practical applications of weighted intuitionistic fuzzy soft set in multicriteria decision making problems were presented.

2. PRELIMINARY CONCEPTS

2.1. Fuzzy Set

Definition 2.1 (Zadeh, 1965). Let U be a universe. A **fuzzy set** X over U is a set defined by a function μX representing a mapping,

μX: U $\rightarrow$ [0,1]

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μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$ and represents the degree of u belonging to the fuzzy set X. Thus a fuzzy set X over U, can be represented as follows:

$$X = \left\{ \frac{u}{\mu_{\mathrm{X}}(u)} : u \in U, \mu_{\mathrm{X}}(u) \in [0,1] \right\} \text{ or}$$

$$X = \left\{ \frac{\mu_{\mathrm{X}}(u)}{u} : u \in U, \mu_{\mathrm{X}}(u) \in [0,1] \right\} \text{ or}$$

$$X = \left\{ u, \mu_{\mathrm{X}}(u) : u \in U, \mu_{\mathrm{X}}(u) \in [0,1] \right\}.$$

Example 2. 1. Let $U=\{h1,h2,h3,h4\}$. A fuzzy set X over U can be represented by

$$X = \left\{ \frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.2}, \frac{h_4}{0.7} \right\}.$$

2.2. Intuitionistic Fuzzy Set

Definition 2.1.1 (Atanassov, 1986). Let X be a nonempty set. An **intuitionistic fuzzy set** A in X is an object having the form

$$A = \left\{ x, \mu_{\mathrm{A}}(x), \lambda_{\mathrm{A}}(x) : x \in X \right\},$$

where the functions $\mu_A(x)$: X→[0,1] and $\lambda_A(x)$: X→[0,1] defined respectively, the degree of membership and degree of non-membership of the element x∈X to the set A, which is a subset of X and for every element $x \in X, 0 \le \mu_{\mathrm{A}}(x) + \lambda_{\mathrm{A}}(x) \le 1$.

Furthermore, we have $\pi_{\mathrm{A}}(x) = 1 - \mu_{\mathrm{A}}(x) - \lambda_{\mathrm{A}}(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. π_A(x) is the degree of indeterminacy of x∈X to the intuitionistic fuzzy set A and $\pi_A(x) \in [0,1]$, that is, π_A(x): X→[0,1] and $0 \le \pi_A \le 1$ *f*or every x∈X. π_A(x) *exp*resses the lack of knowledge of whether x belongs to intuitionistic fuzzy set A or not.

For instance, let A be an intuitionistic fuzzy set with μ_A(x)=0.55 and

$$\lambda_{\mathrm{A}}(x) = 0.25, \Rightarrow \pi_{\mathrm{A}}(x) = 1 - (0.55 + 0.25) = 0.2.$$

It can be interpreted as the degree that the object x belongs to intuitionistic fuzzy set A is 0.55, the degree that the object x does not belong to the intuitionistic fuzzy set A is 0.25 and the degree of hesitancy is 0.2.

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2.3. Basic Operations on Intuitionistic Fuzzy Set

Let $A=\{x,\mu_A(x),\lambda_A(x):x\in X\}$ and $B=\{x,\mu_B(x),\lambda_B(x):x\in X\}$ be two intuitionistic fuzzy sets over *X*.

(i) [**Inclusion**] $A\subseteq B\leftrightarrow \mu_A(x)\leq\mu_B(x)$ and $\lambda_A(x)\geq\lambda_B(x),\forall x\in X$.

(ii) [**Equality**] $A=B\leftrightarrow \mu_A(x)=\mu_B(x)$ and $\lambda_A(x)=\lambda_B(x),\forall x\in X$.

(iii) **[Complement]** $A^C=\{x,\lambda_A(x),\mu_A(x):x\in X\}$.

(iv) **[Union]** $A\cup B=\{x,max(\mu_A(x),\mu_B(x)),min(\lambda_A(x),\lambda_B(x)):x\in X\}$.

(v) **[Intersection]** $A\cap B=\{x,min(\mu_A(x),\mu_B(x)),max(\lambda_A(x),\lambda_B(x)):x\in X\}$.

2.4. Soft Set

We first recall some basic notions in soft set theory. Let *U* be an initial universe set, *E* be a set of parameters or attributes with respect to *U*, *P*(*U*) be the power set of *U* and $A\subseteq E$.

Definition 2.4.1 (Molodtsov, 1999). A pair (*F,A*) is called a **soft set** over *U*, where *F* is a mapping given by $F: A\rightarrow P(U)$. In other words, a soft set over *U* is a parameterized family of subsets of the universe *U*. For $x\in A$, $F(x)$ may be considered as the set of *x*-elements or as the set of *x*-approximate elements of the soft set (*F,A*). The soft set (*F,A*) can be represented as a set of ordered pairs as follows: $(F,A)=\{(x, F(x)), x\in A, F(x)\in P(U)\}$

Definition 2.4.2 (Maji *et al.,* 2003). Let (*F,A*) and (*G,B*) be two soft sets over *U*. Then

(i) (*F, A*) is said to be a **soft subset** of (*G,b*), denoted by

$$(F,A)\tilde{\subseteq}(G,B),\ \text{if } A\subseteq B \text{ and } F(x)\subseteq G(x),\quad \forall x\in A$$

(ii) (*F,A*) and (*G, B*) are said to be **soft equal**, denoted by

$(F,A)=(G,B)$ if $(F,A)\tilde{\subseteq}(G,B)$ and $(G,B)\tilde{\subseteq}(F,A)$.

Definition 2.4.3 (Ali *et al.,* 2009). Let (*F,A*) be a soft set over *U*. Then the **support** of (*F,A*) written **supp** (*F,A*) is defined as $\textbf{supp}(F,A)=\{x\in A:F(x)\neq\varnothing\}$.

(i) (*F,A*) is called a **non-null** soft set if supp $(F,A)\neq\phi$.

(ii) (*F,A*) is called a **relative null** soft set denoted by $\varnothing_A$ *if* $F(x)=\varnothing,\ \forall x\in A$

(iii) (*F, A*) is called a **relative whole** soft set, denoted by U_A if $F(x)=U, \forall x\in A$.

Definition 2.4.4 (Koyuncu and Tanay, 2016). Let (*F, A*) be a soft set over *U*. If $F(x)\neq\varnothing$ for all $x\in A$, then (*F, A*) is called a **non-empty soft set**.

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Definition 2.4.5 (Ali *et al.,* 2009). Let (*F,A*) and (*G, B*) be two soft sets over *U*. Then the **union** of (*F,A*) and (*G, B*), denoted by $\left(F, A\right)\widetilde{\cup}\left(G, B\right)$ is a soft set defined as

$\left(F, A\right)\widetilde{\cup}\left(G,B\right)=\left(H,C\right),$ *where* $C = A\cup B$ *and* $\forall x \in C$,

$$H\left(x\right)=\begin{cases}F\left(x\right), & \text{if } x \in A-B\\ G\left(x\right), & \text{if } x \in B-A\\ F\left(x\right)\cup G\left(x\right), & \text{if } x \in A\cap B\end{cases}$$

Definition 2.4.6 (Koyuncu and Tanay, 2016). Let (*F, A*) and (*G,B*) be two soft sets over *U*. Then the **restricted union** of (*F,A*) and (*G, B*), denoted by (Γ,*A*) $\widetilde{\cup}_R$ (*G,B*) is a soft set defined as;

(*F,A*) $\widetilde{\cup}_R$ (*G,B*) = (*H,C*), where $C = A\cap B \neq \varnothing$ and $\forall\, x \in C$, $H(x) = F(x) \subseteq G(x)$.

Definition 2.4.7 (Ali *et al.,* 2009). Let (*F,A*) and (*G,B*) be two soft sets over *U*. Then the **extended intersection** of (*F, A*) and (*G,B*), denoted by $\left(F, A\right)\widetilde{\cap}_E\left(G, B\right)$, is a soft set defined as $\left(F, A\right)\widetilde{\cap}_E\left(G, B\right)$ = (*H, C*) where $C = A \cup B$ and $\forall\, x \in C$,

$$H\left(x\right)=\begin{cases}F\left(x\right), & \text{if } x \in A-B\\ G\left(x\right), & \text{if } x \in B-A\\ F\left(x\right)\cap G\left(x\right), & \text{if } x \in A\cap B\end{cases}$$

Definition 2.4.8 (Ali *et al.,* 2009). Let (*F,A*) and (*G,B*) be two soft sets over *U*. Then the **restricted intersection** of (*F,A*) and (*G,B*) denoted by (*F,A*) ⋓ (*G, B*), is a soft set defined as (*F,A*) ⋓ (*G,B*) = (*H,C*) where $C = A\cap B$ and $\forall\, x \in C$, $H(x) = F(x) \cap G(x)$.

Definition 2.4.9 (Maji *et al.,* 2003). Let (*F,A*) and (*G,B*) be two soft sets over *U*. Then the **AND-produc**t or **AND-intersection** of (*F,A*) and (*G,B*) denoted by $\left(F, A\right)\tilde{\wedge}\left(G, B\right)$ is a soft set defined as $\left(F, A\right)\tilde{\wedge}\left(G,B\right)=\left(H,C\right),$ where $C=A\times B$ and $\forall(x,y)\in A\times B$, $H(x,y) = F(x) \cap G(y)$.

Definition 2.4.10 (Maji *et al.,* 2003). Let (*F,A*) and (*G,B*) be two soft sets over *U*. Then the **OR-product** or **OR-union** of (*F,A*) and (*G, B*), denoted by $\left(F, A\right)\tilde{\vee}\left(G, B\right)$ is a soft set defined as

$\left(F, A\right)\tilde{\vee}\left(G,B\right)=\left(H,C\right),$

where $C=A\times B$ and $\forall(x,y)\in A\times B$, $H(x, y) = F(x) \cup G(y)$.

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2.5 Fuzzy Soft Set

Let U be an initial universe set and E be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let $P(U)$ denotes the set of all fuzzy subsets of U, and $A \subseteq E$.

Definition 2.5.1 (Maji *et al.,* 2001). A pair (F,A) is called a **fuzzy soft set** over U, where F is a mapping given by $F: A \to P(U)$. In other words, a fuzzy soft set over U is a parameterized family of fuzzy subsets of the universe U. For $x \in A$, $F(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the fuzzy soft set (F,A). Therefore, a fuzzy soft set (F,A) over U can be represented by the set of ordered pairs

$$(F,A) = \left\{ \left(x, F(x) \right) : x \in A, F(x) \in P(U) \right\}.$$

Example 2.5.1. Suppose that $U=\{h1,h2,h3,h4,h5\}$ be a universe set and $E=\{a1,a2,a3,a4\}$ be a set of parameters

$$A = \{a_1, a_2, a_3\} \subseteq E,\ F(a_1) = \left\{ \frac{h_2}{0.8}, \frac{h_4}{0.6} \right\},$$

$F(a2)=U$ and $F(a_3) = \left\{ \frac{h_1}{0.3}, \frac{h_3}{0.4}, \frac{h_5}{0.9} \right\}$, then the fuzzy soft set (F,A) is written as

$$(F,A) = \left\{ \left(a_1, \left\{ \frac{h_2}{0.8}, \frac{h_4}{0.6} \right\} \right), (a_2, U), \left(a_3, \left\{ \frac{h_1}{0.3}, \frac{h_3}{0.4}, \frac{h_5}{0.9} \right\} \right) \right\}.$$

2.6 Intuitionistic Fuzzy Soft Set

The following definitions are due to (Yin *et al.,* 2012).

Definition 2.6.1. Let U be an initial universe set, E a set of parameters, $I(U)$ denotes the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then a pair $\left(\widehat{F}, A \right)$ is called an **intuitionistic fuzzy soft set** over U, where $\widehat{F}$ is a mapping given by $\widehat{F} : A \to I(U)$.

In general, for every $e \in A$, $\widehat{F}(e)$ is an intuitionistic fuzzy set of U and it is called intuitionistic fuzzy value set of parameter e. Obviously, $\widehat{F}(e)$ can be written as an intuitionistic fuzzy set such that

$$\widehat{F}(e) = \left\{ x, \mu_{\widehat{F}(e)}(x), \lambda_{\widehat{F}(e)}(x) : x \in U \right\}.$$

Where $\mu_{\widehat{F}(e)}$ *and* $\lambda_{\widehat{F}(e)}$ are the membership and non-membership functions, respectively. The set of all intuitionistic fuzzy soft sets over U with parameters from E is called an intuitionistic fuzzy soft class and it is denoted by $I\widehat{F}(U,E)$.

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Definition 2.6.2. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over U. We say that $\left(\widehat{F},A\right)$ is an **intuitionistic fuzzy soft subset** of $\left(\widehat{G},B\right)$ and written as $\left(\widehat{F},A\right)\tilde{\subseteq}\left(\widehat{G},B\right)$ if,

(i) $A\subseteq B$,

(ii) For any $e\in A$, $\widehat{F}\left(e\right)\subseteq\widehat{G}\left(e\right)$, that is, for all $x\in U$ and

$e\in A,\mu_{\widehat{F}(e)}\left(x\right)\leq\mu_{\widehat{G}(e)}\left(x\right)$ and $\lambda_{\widehat{F}(e)}\left(x\right)\geq\lambda_{\widehat{G}(e)}\left(x\right)$.

Definition 2.6.3. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over U. Then

$\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$

are said to be **intuitionistic fuzzy soft equal**, denoted by $\left(\widehat{F},A\right)=\left(\widehat{G},B\right)$ if

$\left(\widehat{F},A\right)\tilde{\subseteq}\left(\widehat{G},B\right)$ and $\left(\widehat{G},B\right)\tilde{\subseteq}\left(\widehat{F},A\right)$.

Definition 2.6.4. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over U. Then, the **Union** of $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ is written as $\left(\widehat{F},A\right)\tilde{\cup}\left(\widehat{G},B\right)$ and is defined as

$\forall e\in A,\mu_{\widehat{F}'(e)}=\lambda_{\widehat{F}(e)}$ and $\lambda_{\widehat{F}'(e)}=\mu_{\widehat{F}(e)}$,

where $C=A\cup B$ and $\forall e\in C$,

$$\widehat{H}\left(e\right)=\begin{cases}\widehat{F}\left(e\right), & if\ e\in A/B\\ \widehat{G}\left(e\right), & if\ e\in B/A\\ \widehat{F}\left(e\right)\cup\widehat{G}\left(e\right), & if\ e\in A\cap B\end{cases}.$$

Definition 2.6.5. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over U, such that $A\cap B\neq\varnothing$. The **restricted intersection** of $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ is defined to be the intuitionistic fuzzy soft set $\left(\widehat{H},C\right)$, where

$C=A\cap B$ and $\widehat{H}\left(e\right)=\widehat{F}\left(e\right)\cap\widehat{G}\left(e\right),\forall e\in C$.

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This is written as $\left(\widehat{F},A\right)\Cap\left(\widehat{G},B\right)=\left(\widehat{H},C\right)$.

Definition 2.6.6. Let *U* be an initial universe set, *E* be the universe set of parameters and $A\subset E$. The intuitionistic fuzzy soft set $\left(\widehat{F},A\right)$ is called a **relative null intuitionistic fuzzy soft set** with respect to the parameter set *A* denoted by $\varnothing_A$, if $\widehat{F}\left(e\right)=$ null intuitionistic fuzzy set of *U*, for all $e\in A$. The relative null intuitionistic fuzzy soft set $\varnothing_E$ with respect to the universe set of parameters *E* is called the **absolute null intuitionistic fuzzy soft set** over *U*.

Definition 2.6.7. Let *U* be an initial universe set, *E* be a universe set of parameters and $A\subseteq E$. The intuitionistic fuzzy soft set $\left(\widehat{F},A\right)$ is called a **relative whole intuitionistic fuzzy soft set** with respect to the parameter set *A* denoted by *UA*, if $\widehat{F}\left(e\right)=U$, for all $e\in A$. The relative whole intuitionistic fuzzy soft set *UE* with respect to the universe set of parameters *E* is called the **absolute intuitionistic fuzzy soft set** over *U*.

Definition 2.6.8. The r**elative complement** of an intuitionistic fuzzy soft set $\left(\widehat{F},A\right)$ over *U* is denoted by $\left(\widehat{F},A\right)^r$ and is defined by $\left(\widehat{F}^r,A\right)$, where

$$\forall e\in A,\mu_{\widehat{F}^r(e)}=\lambda_{\widehat{F}(e)}\ and\ \lambda_{\widehat{F}^r(e)}=\mu_{\widehat{F}(e)},$$

that is, $\widehat{F}^r\left(e\right)=\left(\lambda_{\widehat{F}(e)},\mu_{\widehat{F}(e)}\right)$. Clearly, $\left(\left(\left(\widehat{F},A\right)^r\right)^r=\left(\widehat{F},A\right)\right.$.

Definition 2.6.9. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over a common universe *U*, such that $A\cap B\neq\varnothing$. The **restricted difference of** $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ is denoted by $\left(\widehat{F},A\right)\sim_R\left(\widehat{G},B\right)$ and is defined as

$$\left(\widehat{F},A\right)\sim_R\left(\widehat{G},B\right)=\left(\widehat{K},P\right),$$

where

$$P=A\cap B\text{ and }\forall p\in P,\widehat{K}\left(p\right)=\widehat{F}\left(p\right)-\widehat{G}\left(p\right)$$

(the intuitionistic fuzzy difference of two intuitionistic fuzzy sets $\widehat{F}\left(p\right)$ and $\widehat{G}\left(p\right)$ is denoted by

$$\widehat{F}\left(p\right)-\widehat{G}\left(p\right)$$

and is defined as $\widehat{F}\left(p\right)-\widehat{G}\left(p\right)=\widehat{F}\left(p\right)\cap\widehat{G}^c\left(p\right)$.

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Definition 2.6.10. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over U. The **extended intersection of** $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ denoted by $\left(\widehat{F},A\right)\widetilde{\cap}_E\left(\widehat{G},B\right)$ is defined as

$$\left(\widehat{F},A\right)\widetilde{\cap}_E\left(\widehat{G},B\right)=\left(\widehat{H},C\right),C=A\cup B,$$

and

$$\forall e\in C,\widehat{H}\left(e\right)=\begin{cases}\widehat{F}\left(e\right), & if\ e\in A/B\\ \widehat{G}\left(e\right), & if\ e\in B/A\\ \widehat{F}\left(e\right)\cap\widehat{G}\left(e\right), & if\ e\in A\cap B\end{cases}.$$

Definition 2.6.11. Let $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ be two intuitionistic fuzzy soft sets over U, such that $A\cap B\neq\varnothing$. The **restricted union** of $\left(\widehat{F},A\right)$ and $\left(\widehat{G},B\right)$ denoted by $\left(\widehat{F},A\right)\widetilde{\cup}_R\left(\widehat{G},B\right)$ is defined by

$$\left(\widehat{F},A\right)\widetilde{\cup}_R\left(\widehat{G},B\right)=\left(\widehat{H},C\right)$$

where $C=A\cap B$ and $\widehat{H}\left(e\right)=\widehat{F}\left(e\right)\cup\widehat{G}\left(e\right),\forall e\in C$.

3. WEIGHTED INTUITIONISTIC FUZZY SOFT SET

Definition 3.1. A **weighted intuitionistic fuzzy soft set** is a triple $\widehat{F},A,w$, where $\widehat{F},A$ is an intuitionistic fuzzy soft set over the universe set U and w: $A\rightarrow[0,1]$ is a weight function specifying the weight $wj=w(aj)$, for each parameter $aj\in A$.

By definition, every intuitionistic fuzzy soft set can be seen or considered as a weighted intuitionistic fuzzy soft set. Obviously, the notion of weighted intuitionistic fuzzy soft set provides a mathematical tool for modeling and analyzing the decision making problems in which all the choice parameters may not be of equal importance. The differences between the significance of attributes are characterized by the weight function in a weighted intuitionistic fuzzy soft set.

Example 3.1. Let us consider the weighted intuitionistic fuzzy soft set $\widehat{F},A,w$ which describes the conditions of some states in a country that an investor Mr. X with a budget is considering to site his manufacturing industry.

Suppose that there are six states in the initial universe $U=\{S1,S2,S3,S4,S5,S6\}$ under consideration and that $A=\{a1,0.5,a2,0.6,a3,0.7,a4,0.9,a5,0.5\}$ is a set of decision parameters or attribute related to U with their respective weights attached. The $ai(i=1,2,3,4,5)$ stand for the parameters “peaceful”, “power supply”, “accessible”, “densely populated” and “good weather”, respectively. Suppose that

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$$\widehat{F}\left(a_{1,0.5}\right)=\left\{S_1,0.8,0.2,\ S_2,0.6,0.2,\ S_3,0.7,0.2,\ S_4,0.4,0.3,\ S_5,0.9,0.1,\ S_6,0.6,0.4\right\},$$

$$\widehat{F}\left(a_{2,0.6}\right)=\left\{S_1,0.7,0.2,\ S_2,0.8,0.1,\ S_3,0.5,0.3,\ S_4,0.6,0.4,\ S_5,0.8,0.2,\ S_6,0.4,0.3\right\},$$

$$\widehat{F}\left(a_{3,0.7}\right)=\left\{S_1,0.7,0.1,\ S_2,0.6,0.2,\ S_3,0.3,0.4,\ S_4,0.8,0.1,\ S_5,0.9,0.1,\ S_6,0.6,0.3\right\},$$

$$\widehat{F}\left(a_{4,0.9}\right)=\left\{S_1,0.6,0.3,\ S_2,0.8,0.1,\ S_3,0.9,0.1,\ S_4,0.8,0.2,\ S_5,0.7,0.1,\ S_6,0.8,0.2\right\},$$

$$\widehat{F}\left(a_{5,0.5}\right)=\left\{S_1,0.8,0.2,\ S_2,0.4,0.5,\ S_3,0.9,0.1,\ S_4,0.8,0.1,\ S_5,0.5,0.4,\ S_6,0.8,0.2\right\}.$$

The weighted intuitionistic fuzzy soft set $\widehat{F},A,w$ is a parameterized family $\left\{\widehat{F}\left(a_i\right),i=1,2,3,4,5\right\}$ of fuzzy sets on *U* and

$$\widehat{F},A,w=$$
$$\left\{\begin{array}{l}\textit{Peaceful states}=\left\{S_1,0.8,0.2,\ S_2,0.6,0.2,\ S_3,0.7,0.2,\ S_4,0.4,0.3,\ S_5,0.9,0.1,\ S_6,0.6,0.4\right\},\\ \textit{power supply states}=\left\{S_1,0.7,0.2,\ S_2,0.8,0.1,\ S_3,0.5,0.3,\ S_4,0.6,0.4,\ S_5,0.8,0.2,\ S_6,0.4,0.3\right\},\\ \textit{accessible states}=\left\{S_1,0.7,0.1,\ S_2,0.6,0.2,\ S_3,0.3,0.4,\ S_4,0.8,0.1,\ S_5,0.9,0.1,\ S_6,0.6,0.3\right\},\\ \textit{densely populated states}=\left\{S_1,0.6,0.3,\ S_2,0.8,0.1,\ S_3,0.9,0.1,\ S_4,0.8,0.2,\ S_5,0.7,0.1,\ S_6,0.8,0.2\right\},\\ \textit{states with good weather}=\left\{S_1,0.8,0.2,\ S_2,0.4,0.5,\ S_3,0.9,0.1,\ S_4,0.8,0.1,\ S_5,0.5,0.4,\ S_6,0.8,0.2\right\}\end{array}\right\}$$

Table 1 gives the tabular representation of weighted intuitionistic fuzzy soft set in Example 3.

Table 1. Tabular representation of weighted intuitionistic fuzzy soft set $\widehat{F},A,w$

U/*A*	$a_{1,0.5}$	$a_{2,0.6}$	$a_{3,0.7}$	$a_{4,0.9}$	$a_{5,0.5}$
S_1	0.8, 0.2	0.7, 0.2	0.7, 0.1	0.6, 0.3	0.8, 0.2
S_2	0.6, 0.2	0.8, 0.1	0.6, 0.2	0.8, 0.1	0.4, 0.5
S_3	0.7, 0.2	0.5, 0.3	0.3, 0.4	0.9, 0.1	0.9, 0.1
S_4	0.4, 0.3	0.6, 0.4	0.8, 0.1	0.8, 0.2	0.8, 0.1
S_5	0.9, 0.1	0.8, 0.2	0.9, 0.1	0.7, 0.1	0.5, 0.4
S_6	0.6, 0.4	0.4, 0.3	0.6, 0.3	0.8, 0.2	0.8, 0.2

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Definition 3.2. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1,w2 \in [0,1]$. We say that $\widehat{F}, A, w_1$ is a weighted intuitionistic fuzzy soft **subset** of $\widehat{G}, B, w_2$ and written as $\widehat{F}, A, w_1 \tilde{\subseteq} \widehat{G}, B, w_2$ if and only if

(i) $A \subseteq B$ and $w1 \leq w2$,

(ii) For every $e \in A$, there exists $w_1 \in [0,1]$ such that $\widehat{F}(e) \subseteq \widehat{G}(e)$, that is for all $x \in U$ and $e \in A$, there exists $w_1 \in [0,1]$ such that

$$\mu_{\widehat{F}(e)}(x) \leq \mu_{\widehat{G}(e)}(x) \text{ and } \lambda_{\widehat{F}(e)}(x) \geq \lambda_{\widehat{G}(e)}(x).$$

Definition 3.3. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1,w2 \in [0,1]$. Then $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ are said to be **weighted intuitionistic fuzzy soft equal** denoted by $\widehat{F}, A, w_1 = \widehat{G}, B, w_2$ if and only if

$$\widehat{F}, A, w_1 \tilde{\subseteq} \widehat{G}, B, w_2 \text{ and } \widehat{G}, B, w_2 \tilde{\subseteq} \widehat{F}, A, w_1.$$

Definition 3.4. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where

$\widehat{F}, A$ and $\widehat{G}, B$

are intuitionistic fuzzy soft set over U and $w1,w2 \in [0,1]$. Then the **union** of

$\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$

denoted by $\widehat{F}, A, w_1 \tilde{\cup} \widehat{G}, B, w_2$ and is defined as

$$\widehat{F}, A, w_1 \tilde{\cup} \widehat{G}, B, w_2 = \widehat{H}, C, w^*$$

and for every $e \in C = A \cup B$, there exists $w^* = \max(w1,w2) \in [0,1]$ such that

$$\widehat{H}(e) = \begin{cases} \widehat{F}(e), & \text{if } e \in A / B \\ \widehat{G}(e), & \text{if } e \in B / A \\ \widehat{F}(e) \cup \widehat{G}(e), & \text{if } e \in A \cap B \end{cases}.$$

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Definition 3.5. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1,w2 \in [0,1]$ such that $A \cap B \neq \varnothing$. The **restricted intersection** of

$$\widehat{F}, A, w_1 \text{ and } \widehat{G}, B, w_2$$

is defined to be the weighted intuitionistic fuzzy soft set $\widehat{H}, C, w^*$ and for all $e \in C = A \cap B$, there exists $w^* = \min(w1, w2) \in [0,1]$ such that

$$\widehat{H}(e) = \widehat{F}(e) \cap \widehat{G}(e).$$

This is written as $\widehat{F}, A, w_1 \Cap \widehat{G}, B, w_2 = \widehat{H}, C, w^*$

Definition 3.6. Let U be an initial universe set, E be the universe set of parameters and $A \subset E$. The weighted intuitionistic fuzzy soft set $\widehat{F}, A, w$ is called a **relative null weighted intuitionistic fuzzy soft set** with respect to the parameter set A denoted by $\varnothing_A$, if for all $e \in A$, there exists $w \in [0,1]$ such that $\widehat{F}(e) =$ null intuitionistic fuzzy set over U. The relative null weighted intuitionistic fuzzy soft set $\varnothing_E$ with respect to the universe set of parameters E is called the **absolute null weighted intuitionistic fuzzy soft set.**

Definition 3.7. Let U be an initial universe set, E be a universe set of parameters and $A \subseteq E$. The weighted intuitionistic fuzzy soft set $\widehat{F}, A, w$ is called a **relative whole weighted intuitionistic fuzzy soft set** with respect to the parameter set A denoted by UA, if for all $e \in A$, there exists $w \in [0,1]$ such that $\widehat{F}(e) = U$. The relative whole weighted intuitionistic fuzzy soft set UE with respect to the universe set of parameters E is called the **absolute weighted intuitionistic fuzzy soft set**

Definition 3.8. The r**elative complement** of a weighted intuitionistic fuzzy soft set $\widehat{F}, A, w$ where $\widehat{F}, A$ is an intuitionistic fuzzy soft set over U and $w \in [0,1]$ denoted by $\widehat{F}, A, w^r$ and is and is defined by $\left(\widehat{F}^r, A, w\right)$, and for all $e \in A$, there exists $w \in [0,1]$ such that

$$\mu_{\widehat{F}^r(e)} = \lambda_{\widehat{F}(e)} \text{ and } \lambda_{\widehat{F}^r(e)} = \mu_{\widehat{F}(e)},$$

that is, $\widehat{F}^r(e) = \left(\lambda_{\widehat{F}(e)}, \mu_{\widehat{F}(e)}\right)$. Clearly, $\left(\widehat{F}, A, w^r\right)^r = \widehat{F}, A, w$.

Definition 3.9. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1,w2 \in [0,1]$ such that $A \cap B \neq \varnothing$. The **restricted difference** of $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ is denoted by $\widehat{F}, A, w_1 \sim_R \widehat{G}, B, w_2$ and is defined as

$$\widehat{F}, A, w_1 \sim_R \widehat{G}, B, w_2 = \widehat{K}, P, w^*$$

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and for all $e \in C = A \cap B$, there exists $w^* = |w1 - w2| \in [0,1]$ such that

$$\widehat{K}(e) = \widehat{F}(e) - \widehat{G}(e)$$

(the weighted intuitionistic fuzzy difference of two weighted intuitionistic fuzzy set $\widehat{F}(e)$ and $\widehat{G}(e)$ is denoted by $\widehat{F}(e) - \widehat{G}(e)$ and is defined as

$$\widehat{F}(e) - \widehat{G}(e) = \widehat{F}(e) \cap \widehat{G}^{c}(e).$$

Definition 3.10. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$. Then the **extended intersection** of

$$\widehat{F}, A, w_1 \text{ and } \widehat{G}, B, w_2 \; \widehat{G}, B, w_2$$

denoted by $\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2$ and is defined as

$$\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2 = \widehat{H}, C, w^*$$

and for every $e \in C = A \cup B$, there exists $w^* = \min(w1, w2) \in [0,1]$ such that

$$\widehat{H}(e) = \begin{cases} \widehat{F}(e), & \text{if } e \in A / B \\ \widehat{G}(e), & \text{if } e \in B / A \\ \widehat{F}(e) \cap \widehat{G}(e), & \text{if } e \in A \cap B \end{cases}.$$

Definition 3.11. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$ such that $A \cap B \neq \varnothing$. The **restricted union** of $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ denoted by $\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2$ and is defined by

$$\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2 = \widehat{H}, C, w^*$$

and for all $e \in C = A \cap B$, there exists $w^* = \max(w1, w2) \in [0,1]$ such that $\widehat{H}(e) = \widehat{F}(e) \cup \widehat{G}(e)$.

Definition 3.12. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$ such that $A \cap B \neq \varnothing$. The

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restricted intersection of $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ is defined to be the weighted intuitionistic fuzzy soft set $\widehat{H}, C, w^*$ and for all $e \in C = A \cap B$, there exists $w^* = \min(w1, w2) \in [0,1]$ such that

$$\widehat{H}(e) = \widehat{F}(e) \cap \widehat{G}(e).$$

This is written as $\widehat{F}, A, w_1 \Cap \widehat{G}, B, w_2 = \widehat{H}, C, w^*$

Definition 3.13. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$ The **sum** of

$$\widehat{F}, A, w_1 \text{ and } \widehat{G}, B, w_2$$

denoted by $\widehat{F}, A, w_1 + \widehat{G}, B, w_2$ and is defined as

$$\widehat{F}, A, w_1 + \widehat{G}, B, w_2 = \widehat{H}, C, w^*$$

and for all $e \in C = A \cap B$, there exists $w^* = \min(w1, w2) \in [0,1]$ such that $\widehat{H}(e) = \widehat{F}(e) \cap \widehat{G}(e)$.

Definition 3.14. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$ Then the **AND-product** of

$$\widehat{F}, A, w_1 \text{ and } \widehat{G}, B, w_2$$

denoted by $\widehat{F}, A, w_1 \tilde{\wedge} \widehat{G}, B, w_2$ and is defined as

$$\widehat{F}, A, w_1 \tilde{\wedge} \widehat{G}, B, w_2 = \widehat{H}, C, w^*$$

and for all $(e1, e2) \in C = A \times B$, there exists $w^* = (w1, w2) \in [0,1]$ such that

$$\widehat{H}(e_1, e_2) = \widehat{F}(e_1) \cap \widehat{G}(e_2).$$

Definition 3.15. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F}, A$ and $\widehat{G}, B$ are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$ Then the **OR-product** of

$$\widehat{F}, A, w_1 \text{ and } \widehat{G}, B, w_2$$

denoted by $\widehat{F}, A, w_1 \tilde{\vee} \widehat{G}, B, w_2$ and is defined as

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$$\widehat{F}, A, w_1 \tilde{\vee} \widehat{G}, B, w_2 = \widehat{H}, C, w^*$$

and for all $(e1,e2) \in C = A \times B$, there exists $w^* = (w1, w2) \in [0,1]$ such that

$$\widehat{H}\left(e_1, e_2\right) = \widehat{F}\left(e_1\right) \cup \widehat{G}\left(e_2\right).$$

Example 3.2. Let us consider two weighted intuitionistic fuzzy soft set $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ which describes the conditions of some states in a country that an investor Mr. X with a budget is considering to site his manufacturing industry.

Suppose that there are six states in the initial universe $U = \{S1, S2, S3, S4, S5, S6\}$ under consideration and that

$$A = \left\{a_{1,0.5}, a_{2,0.6}, a_{3,0.7}, a_{4,0.9}, a_{5,0.5}\right\} \text{ and } B = \left\{b_{1,0.5}, b_{2,0.7}, b_{3,0.4}, b_{4,0.7}, b_{5,0.7}\right\}$$

are sets of attributes or decision parameters related to U with their respective weights attached. The $ai,w(i=1,2,3,4,5)$ stand for the parameters "peaceful", "power supply", "accessible", "densely populated" and "good weather", respectively and the $bi,w(i=1,2,3,4,5)$ stand for the parameters "peaceful", "good internal road network", "good soil", "good topography" and "densely populated", respectively. Suppose that

$$\widehat{F}\left(a_{1,0.5}\right) = \left\{S_1, 0.8, 0.2,\ S_2, 0.6, 0.2,\ S_3, 0.7, 0.2,\ S_4, 0.4, 0.3,\ S_5, 0.9, 0.1,\ S_6, 0.6, 0.4\right\},$$

$$\widehat{F}\left(a_{2,0.6}\right) = \left\{S_1, 0.7, 0.2,\ S_2, 0.8, 0.1,\ S_3, 0.5, 0.3,\ S_4, 0.6, 0.4,\ S_5, 0.8, 0.2,\ S_6, 0.4, 0.3\right\},$$

$$\widehat{F}\left(a_{3,0.7}\right) = \left\{S_1, 0.7, 0.1,\ S_2, 0.6, 0.2,\ S_3, 0.3, 0.4,\ S_4, 0.8, 0.1,\ S_5, 0.9, 0.1,\ S_6, 0.6, 0.3\right\},$$

$$\widehat{F}\left(a_{4,0.9}\right) = \left\{S_1, 0.6, 0.3,\ S_2, 0.8, 0.1,\ S_3, 0.9, 0.1,\ S_4, 0.8, 0.2,\ S_5, 0.7, 0.1,\ S_6, 0.8, 0.2\right\},$$

$$\widehat{F}\left(a_{5,0.5}\right) = \left\{S_1, 0.8, 0.2,\ S_2, 0.4, 0.5,\ S_3, 0.9, 0.1,\ S_4, 0.8, 0.1,\ S_5, 0.5, 0.4,\ S_6, 0.8, 0.2\right\}.$$

The weighted intuitionistic fuzzy soft set $\widehat{F}, A, w_1$ is a parameterized family

$$\left\{\widehat{F}\left(a_{i,w}\right), i = 1,2,3,4,5\right\}$$

of intuitionistic fuzzy sets on U and

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$$\widehat{G}\left(b_{1,0.5}\right)=\left\{S_1,0.8,0.3,\ S_2,0.7,0.3,\ S_3,0.8,0.1,\ S_4,0.5,0.3,\ S_5,0.8,0.2,\ S_6,0.6,0.5\right\},$$

$$\widehat{G}\left(b_{2,0.7}\right)=\left\{S_1,0.8,0.2,\ S_2,0.9,0.2,\ S_3,0.6,0.3,\ S_4,0.6,0.3,\ S_5,0.8,0.3,\ S_6,0.5,0.3\right\},$$

$$\widehat{G}\left(b_{3,0.4}\right)=\left\{S_1,0.8,0.2,\ S_2,0.7,0.2,\ S_3,0.4,0.3,\ S_4,0.8,0.5,\ S_5,0.9,0.2,\ S_6,0.6,0.3\right\},$$

$$\widehat{G}\left(b_{4,0.7}\right)=\left\{S_1,0.7,0.4,\ S_2,0.8,0.1,\ S_3,0.9,0.2,\ S_4,0.8,0.2,\ S_5,0.7,0.1,\ S_6,0.9,0.3\right\},$$

$$\widehat{G}\left(b_{5,0.7}\right)=\left\{S_1,0.8,0.2,\ S_2,0.4,0.6,\ S_3,0.9,0.5,\ S_4,0.8,0.1,\ S_5,0.6,0.7,\ S_6,0.9,0.3\right\}.$$

Similarly, the weighted intuitionistic fuzzy soft set $\widehat{G},B,w_2$ is a parameterized family {$G(bi,w)$, i=1,2,3,4,5} of intuitionistic fuzzy sets on U.

$$\widehat{F},A,w_1 \Cap \widehat{G},B,w_2=\widehat{H},C,w^*,$$

where

$$C=A\cap B=\left\{a_1=b_1=\textit{peaceful states},\ a_4=b_5=\textit{densely populated}\right\}=\left\{c_{1,0.5},c_{2,0.7}\right\},$$

$$\widehat{H}\left(c_{1,0.5}\right)=\left\{S_1,0.7,0.2,\ S_2,0.6,0.2,\ S_3,0.7,0.1,\ S_4,0.4,0.3,\ S_5,0.8,0.1,\ S_6,0.6,0.5\right\},$$

$$\widehat{H}\left(c_{2,0.7}\right)=\left\{S_1,0.6,0.2,\ S_2,0.4,0.1,\ S_3,0.9,0.1,\ S_4,0.8,0.2,\ S_5,0.6,0.1,\ S_6,0.8,0.2\right\}.$$

$$\widehat{F},A,w_1 \Cap \widehat{G},B,w_2=\widehat{H},C,w^*=$$
$$\left\{\begin{array}{l}\textit{Peaceful states}=\left\{S_1,0.7,0.2,\ S_2,0.6,0.2,\ S_3,0.7,0.1,\ S_4,0.4,0.3,\ S_5,0.8,0.1,\ S_6,0.6,0.5\right\},\\ \textit{densely populated states}=\left\{S_1,0.6,0.2,\ S_2,0.4,0.1,\ S_3,0.9,0.1,\ S_4,0.8,0.2,\ S_5,0.6,0.1,\ S_6,0.8,0.2\right\}\end{array}\right\}$$

Therefore, $\widehat{H},C,w^*$ is a weighted intuitionistic fuzzy soft set.

Theorem 3.1. Let $\widehat{F},A,w_1$ and $\widehat{G},B,w_2$ be two weighted intuitionistic fuzzy soft sets, where $\widehat{F},A$ and $\widehat{G},B$ are intuitionistic fuzzy soft set over U, $w1$: $A\rightarrow[0,1]$ and $w2$: $B\rightarrow[0,1]$ are weight functions specifying the wi=w(ai) and wj=w(bj), for each parameters $ai\in A$ and $bj\in B$. Then

(i) $\widehat{F},A,w_1 \Cap \widehat{G},B,w_2$ is a weighted intuitionistic fuzzy soft set, if it is non-null.

(ii) $\widehat{F},A,w_1\widetilde{\cap}_E\widehat{G},B,w_2$ is a weighted intuitionistic fuzzy soft set, if it is non-null.

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(iii) $\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2$ is a weighted intuitionistic fuzzy soft set, whenever it is non-null and if $\widehat{F}(e)$ and $G(e)$ are ordered by inclusion relation for all

$e \in supp\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2$.

(iv) $\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2$ is a weighted intuitionistic fuzzy soft set, if it is non-null and if A and B are disjoint.

(v) $\widehat{F}, A, w_1 \tilde{\wedge} \widehat{G}, B, w_2$ is a weighted intuitionistic fuzzy soft set, if it is non-null.

(vi) $\widehat{F}, A, w_1 \tilde{\vee} \widehat{G}, B, w_2$ is a weighted intuitionistic fuzzy soft set if it is non-null and if $\widehat{F}(e)$ and $G(e)$ are ordered by inclusion relation for all

$(e_1, e_2) \in supp\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2$.

(vii) $\widehat{F}, A, w_1 \tilde{+} \widehat{G}, B, w_2$ is a weighted intuitionistic fuzzy soft set, if it is non-null.

Proof:

(i) Let $\widehat{F}, A, w_1 \Subset \widehat{G}, B, w_2 = \widehat{H}, C, w^*$ where

$\widehat{H}(e) = \widehat{F}(e) \cap \widehat{G}(e)$ for all $e \in C = A \cap B \neq \varnothing$

there exists $w^*=\min(w_1,w_2)\in[0,1]$. By hypothesis, $\widehat{H}, C, w^*$ is a non-null weighted intuitionistic fuzzy soft set. If $e \in supp\widehat{H}, C$ and $w^*\in[0,1]$ then $\widehat{H}(e) = \widehat{F}(e) \cap \widehat{G}(e) \neq \varnothing$ and w^*: $C\rightarrow[0,1]$. It follows that $\widehat{F}(e) \neq \varnothing$ and $\widehat{G}(e) \neq \varnothing$ are both subsets of U, for all $e \in supp\widehat{H}, C$ there exists $w^*=\min(w_1,w_2)\in[0,1]$. Therefore, $\widehat{H}, C, w^*$ is a weighted intuitionistic fuzzy soft set, where $\widehat{H}, C$ is an intuitionistic fuzzy soft set over U.

(ii) Let $\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2 = \hat{J}, D, w^*$, where

$$\hat{J}(e) = \begin{cases} \widehat{F}(e), & if\ e \in A / B \\ \widehat{G}(e), & if\ e \in B / A \\ \widehat{F}(e) \cap \widehat{G}(e), & if\ e \in A \cap B \end{cases},$$

for all $e \in D = A \cup B \neq \varnothing$ there exists $w^*=\min(w_1,w_2)\in[0,1]$. By hypothesis $\hat{J}, D, w^*$ is a non-null weighted intuitionistic fuzzy soft set over U. If $e \in \hat{J}, D$ and $w^*\in[0,1]$ then $\hat{J}(e) = \widehat{F}(e) \cup \widehat{G}(e) \neq \varnothing$

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and w^*: $D \to [0,1]$. It follows that $\widehat{F}(e) \neq \varnothing$ and $\widehat{G}(e) \neq \varnothing$ are both subsets of U. Hence, $\hat{J}(e)$ is a subset of U for all $e \in supp\hat{J}, D$ and $w^*=\min(w_1,w_2) \in [0,1]$. Thus, $\hat{J}, D, w^*$ is a weighted intuitionistic fuzzy soft set.

(iii) Let $\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2 = \widehat{K}, C, w^*$, for every $e \in C = A \cap B \neq \varnothing$, there exists $w^*=\min(w_1,w_2) \in [0,1]$ such that $\widehat{K}(e) = \widehat{F}(e) \cup \widehat{G}(e)$. Then by the hypothesis $\widehat{K}, C, w^*$ is a non-null weighted intuitionistic fuzzy soft set and $\widehat{K}, C,$ is a non-null intuitionistic fuzzy soft set over U and w^*: $C \to [0,1]$. If $e \in supp\widehat{K}, C,$, then there exists $w^* \in [0,1]$ such that

$$\widehat{K}(e) = \widehat{F}(e) \cup \widehat{G}(e) \neq \varnothing .$$

Since $\widehat{F}(e)$ and $\widehat{G}(e)$ are ordered by inclusion relation for every $e \in supp\widehat{K}, C,$ there exists $w^*=\max(w1,w2) \in [0,1]$ such that

$$\widehat{F}(e) \cup \widehat{G}(e) = \widehat{F}(e) \text{ or } \widehat{F}(e) \cup \widehat{G}(e) = \widehat{G}(e).$$

Since $\widehat{F}(e) \neq \varnothing$ and $\widehat{G}(e) \neq \varnothing$ are intuitionistic fuzzy sets over U. Then $\widehat{K}(e)$ is an intuitionistic fuzzy set over U for every $e \in supp\widehat{K}, C,$ and $w^* \in [0,1]$. Therefore, $\widehat{K}, C, w^*$ is a weighted intuitionistic fuzzy soft set.

(iv) Let $\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2 = \hat{V}, C, w^*$, and for everywhere $e \in C = A \cup B$, there exists $w^*=\max(w_1,w_2) \in [0,1]$ and $A \cap B = \varnothing$ such that

$$\hat{V}(e) = \begin{cases} \widehat{F}(e), & if\ e \in A / B \\ \widehat{G}(e), & if\ e \in B / A \\ \widehat{F}(e) \cup \widehat{G}(e), & if\ e \in A \cap B \end{cases},$$

It follows that either $e \in A/B$ or $e \in B/A$ for all $e \in C$ and $w^*=\max(w1,w2) \in [0,1]$. If $e \in A/B$, there exists $w1 \in [0,1]$ such that $\hat{V}(e) = \widehat{F}(e)$ is an intuitionistic fuzzy set over U and If $e \in B/A$, there exists $w2 \in [0,1]$ such that $\hat{V}(e) = \widehat{G}(e)$ is an intuitionistic fuzzy set over U. Therefore, $\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2$ is a weighted intuitionistic fuzzy soft set.

(v) Let $\widehat{F}, A, w_1 \tilde{\wedge} \widehat{G}, B, w_2 = \hat{T}, C, w^*$ and for all $(e_1,e_2) \in C = A \times B$, there exists $w^*=(w_1,w_2) \in [0,1]$ such that $\hat{T}(e_1, e_2) = \widehat{F}(e_1) \cap \widehat{G}(e_2)$. Then by the hypothesis, $\hat{T}, C, w^*$ is a non-null weighted intu-

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itionistic fuzzy soft set and $\widehat{T},C$ is a non-null intuitionistic fuzzy soft set over U and w^*: $C\to[0,1]$ or $w^*\in[0,1]$. If $(e_1,e_2)\in supp\widehat{T},C$ then there exists $w^*\in[0,1]$ such that

$$\widehat{T}(e_1,e_2)=\widehat{F}(e)\cap\widehat{G}(e)\neq\varnothing.$$

It follows that $\widehat{F}(e_1)\neq\varnothing$ and $\widehat{G}(e_2)\neq\varnothing$ are both intuitionistic fuzzy sets over U for all

$$(e_1,e_2)\in supp\widehat{T},C.$$

Therefore, $\widehat{F},A,w_1\,\tilde{\wedge}\,\widehat{G},B,w_2$ is a weighted intuitionistic fuzzy soft set.

(vi) Let $\widehat{F},A,w_1\,\tilde{\vee}\,\widehat{G},B,w_2=\widehat{N},C,w^*$ and for all $(e_1,e_2)\in C=A\times B$, there exists $w^*=(w_1,w_2)\in[0,1]$ such that

$$\widehat{N}(e_1,e_2)=\widehat{F}(e_1)\cup\widehat{G}(e_2).$$

Then by the hypothesis, $\widehat{N},C,w^*$ is a non-null weighted intuitionistic fuzzy soft set and $\widehat{N},C$ is a non-null intuitionistic fuzzy soft set over U and w^*: $C\to[0,1]$ or $w^*\in[0,1]$. If

$$(e_1,e_2)\in supp\widehat{N},C$$

then there exists $w^*=(w1,w2)\in[0,1]$ such that

$$\widehat{N}(e_1,e_2)=\widehat{F}(e)\cup\widehat{G}(e)\neq\varnothing.$$

Since $\widehat{F}(e_1)$ and $\widehat{G}(e_2)$ are ordered by inclusion relation for all $(e_1,e_2)\in supp\widehat{N},C$ and $w^*\in[0,1]$,

$$\widehat{F}(e_1)\cup\widehat{G}(e_2)=\widehat{F}(e_1)\text{ or }\widehat{F}(e_1)\cup\widehat{G}(e_2)=\widehat{G}(e_2).$$

Since $\widehat{F}(e_1)\neq\varnothing$ and $\widehat{G}(e_2)\neq\varnothing$ are both intuitionistic fuzzy set over U for all

$$(e_1,e_2)\in supp\widehat{N},C$$

and $w^*\in[0,1]$. Therefore, $\widehat{F},A,w_1\,\tilde{\vee}\,\widehat{G},B,w_2$ is a weighted intuitionistic fuzzy soft set.

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(vii) Let $\widehat{F}, A, w_1 + \widehat{G}, B, w_2 = \widehat{H}, C, w^*$ and for all $e \in C = A \cap B$, there exists $w^* = \min(w_1, w_2) \in [0,1]$ such that $\widehat{H}(e) = \widehat{F}(e) \cap \widehat{G}(e)$. Then by the hypothesis, $\widehat{F}(e) \cap \widehat{G}(e) \neq \varnothing$. It follows that

$$\widehat{F}(e) \neq \varnothing \text{ and } \widehat{G}(e) \neq \varnothing$$

are both intuitionistic fuzzy sets over U for all $e \in supp\widehat{H}, C$ and $w^* \in [0,1]$. Therefore,

$$\widehat{F}, A, w_1 + \widehat{G}, B, w_2$$

is a weighted intuitionistic fuzzy soft set.

Theorem 3.2. Let $\widehat{F}, A, w$ be a weighted intuitionistic fuzzy soft set, where $\widehat{F}, A,$ is an intuitionistic fuzzy soft set over U and $w \in [0,1]$. Then,

(i) $\widehat{F}, A, w \widetilde{\cup} U_E = U_E$

(ii) $\widehat{F}, A, w \Cap U_E = \widehat{F}, A, w$

(iii) $\widehat{F}, A, w \widetilde{\cup} \widehat{F}, A, w = \widehat{F}, A, w$

(iv) $\widehat{F}, A, w \Cap \widehat{F}, A, w = \widehat{F}, A, w$.

Proof. The proof is trivial, hence omitted.

The following results can be easily deduced.

Theorem 3.3. Let $\widehat{F}, A, w_1, \widehat{G}, B, w_2$ and $\widehat{H}, C, w_3$ be weighted intuitionistic fuzzy soft sets, where

$$\widehat{F}, A, \widehat{G}, B \text{ and } \widehat{H}, C$$

are intuitionistic fuzzy soft set over U and $w1, w2, w3 \in [0,1]$. Then,

(i) $\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2 = \widehat{G}, B, w_2 \widetilde{\cup} \widehat{F}, A, w_1$.

(ii) $\left(\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2\right) \widetilde{\cup} \widehat{H}, C, w_3 = \widehat{F}, A, w_1 \widetilde{\cup} \left(\widehat{G}, B, w_2 \widetilde{\cup} \widehat{H}, C, w_3\right)$.

Theorem 3. 4. Let $\widehat{F}, A, w_1, \widehat{G}, B, w_2$ and $\widehat{H}, C, w_3$ be weighted intuitionistic fuzzy soft sets, where

$$\widehat{F}, A, \widehat{G}, B \text{ and } \widehat{H}, C$$

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are intuitionistic fuzzy soft set over U and $w1,w2,w3\in[0,1]$. Then,

(i) $\widehat{F},A,w_1\widetilde{\cap}_E\varnothing_E=\varnothing_E$.

(ii) $\widehat{F},A,w_1\widetilde{\cap}_E\widehat{F},A,w_1=\widehat{F},A,w_1$

(iii) $\widehat{F},A,w_1\widetilde{\cap}_E\widehat{G},B,w_2=\widehat{G},B,w_2\widetilde{\cap}_E\widehat{F},A,w$

(iv) $\left(\widehat{F},A,w_1\widetilde{\cap}_E\widehat{G},B,w_2\right)\widetilde{\cap}_E\widehat{H},C,w_3=\widehat{F},A,w_1\widetilde{\cap}_E\left(\widehat{G},B,w_2\widetilde{\cap}_E\widehat{H},C,w_3\right)$.

Proof: The proof is straight forward.

Theorem 3.5. Let $\widehat{F},A,w_1,\widehat{G},B,w_2$ and $\widehat{H},C,w_3$ be weighted intuitionistic fuzzy soft sets, where

$\widehat{F},A,\widehat{G},B$ and $\widehat{H},C$

are intuitionistic fuzzy soft set over U and $w1,w2,w3\in[0,1]$. Then,

(i) $\widehat{F},A,w_1\widetilde{\cup}_R\varnothing_E=\widehat{F},A,w_1$.

(ii) $\widehat{F},A,w_1\widetilde{\cup}_R\widehat{F},A,w_1=\widehat{F},A,w_1$

(iii) $\widehat{F},A,w_1\widetilde{\cup}_R\widehat{G},B,w_2=\widehat{G},B,w_2\widetilde{\cup}_R\widehat{F},A,w_1$

(iv) $\left(\widehat{F},A,w_1\widetilde{\cup}_R\widehat{G},B,w_2\right)\widetilde{\cup}_R\widehat{H},C,w_3=\widehat{F},A,w_1\widetilde{\cup}_R\left(\widehat{G},B,w_2\widetilde{\cup}_R\widehat{H},C,w_3\right)$.

Proof: The proof is straight forward.

Theorem 3.6. Let $\widehat{F},A,w_1,\widehat{G},B,w_2$ and $\widehat{H},C,w_3$ be weighted intuitionistic fuzzy soft sets, where

$\widehat{F},A,\widehat{G},B$ and $\widehat{H},C$

are intuitionistic fuzzy soft set over U and $w1,w2,w3\in[0,1]$. Then,

(i) $\widehat{F},A,w_1\Cap\widehat{G},B,w_2=\widehat{G},B,w_2\Cap\widehat{F},A,w_1$.

(ii) $\left(\widehat{F},A,w_1\Cap\widehat{G},B,w_2\right)\Cap\widehat{H},C,w_3=\widehat{F},A,w_1\Cap\left(\widehat{G},B,w\Cap\widehat{H},C,w_3\right)$.

Proof: The proof is straight forward.

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The following theorem shows that, the absorption law with respect to the operations union ($\widetilde{\cup}$) and restricted intersection $(\Cap)$ holds.

Theorem 3.7. Let $\widehat{F}, A, w$ and $\widehat{G}, B, w_2$ be weighted intuitionistic fuzzy soft sets, where

$\widehat{F}, A \text{ and } \widehat{G}, B$

are intuitionistic fuzzy soft set over U and $w1,w2\in[0,1]$. Then,

(i) $\left(\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2\right) \Cap \widehat{F}, A, w_1 = \widehat{F}, A, w_1$.

(ii) $\left(\widehat{F}, A, w_1 \Cap \widehat{G}, B, w_2\right) \widetilde{\cup} \widehat{F}, A, w_1 = \widehat{F}, A, w_1$.

Proof: (i) $\left(\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2\right) \Cap \widehat{F}, A, w_1 = \left(\widehat{H}, \left(A \cup B\right) \cap A\right)$.

For any $e\in A$, there exists $w1\in[0,1]$ with the following cases being considered.

Case 1: $e\in B$. Then $\widehat{H}\left(e\right) = \left(\widehat{F}\left(e\right) \cup G\left(e\right)\right) \cap \widehat{F}\left(e\right) = \widehat{F}\left(e\right)$.

Case 2: $e\notin B$. Then $\widehat{H}\left(e\right) = \widehat{F}\left(e\right) \cap \widehat{F}\left(e\right) = \widehat{F}\left(e\right)$.

Therefore, $\widehat{F}$ and $\widehat{H}$ are the same operators and so

$\left(\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2\right) \Cap \widehat{F}, A, w_1 = \widehat{F}, A, w_1$.

(ii) The proof of (ii) follows from (i).

The following theorem shows that, the absorption law with respect to the operations extended intersection $(\widetilde{\cap}_E)$ and restricted union $(\widetilde{\cup}_R)$ holds.

Theorem 3.8. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be weighted intuitionistic fuzzy soft sets, where

$\widehat{F}, A \text{ and } \widehat{G}, B$

are intuitionistic fuzzy soft set over U and $w1,w2\in[0,1]$. Then,

(i) $\left(\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2\right) \widetilde{\cup}_R \widehat{F}, A, w_1 = \widehat{F}, A, w_1$

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(ii) $\left(\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2\right) \widetilde{\cap}_E \widehat{F}, A, w_1 = \widehat{F}, A, w_1$.

Proof. The proof is straight forward and hence omitted.

The absorption laws with respect to the operation $\widetilde{\cap}_E$ and $\widetilde{\cup}$, $\Cap$ and $\widetilde{\cup}_R$ may not hold in general as shown in the following example.

4. DE MORGAN'S TYPES OF RESULTS ON WEIGHTED INTUITIONISTIC FUZZY SOFT SET

It is well known from standard set that De Morgan's Laws interrelate union and intersection via complements. In theorem 4.1 and theorem 4.2, we shall show that De Morgan's Laws interrelate union ($\widetilde{\cup}$) and extended intersection $\widetilde{\cap}_E$, restricted union ($\widetilde{\cup}_R$) and restricted intersection ($\Cap$) operations respectively.

Theorem 4.1. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be weighted intuitionistic fuzzy soft sets, where

$\widehat{F}, A$ *and* $\widehat{G}, B$

are intuitionistic fuzzy soft set over *U* and *w*1,*w*2∈[0,1]. Then,

(i) $\left(\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2\right)^r = \widehat{F}, A, {w_1}^r \widetilde{\cap}_E \widehat{G}, B, {w_2}^r$,

(ii) $\left(\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2\right)^r = \widehat{F}, A, {w_1}^r \widetilde{\cup} \widehat{G}, B, {w_2}^r$.

Proof:

(i) Suppose that $\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2 = \widehat{H}, C, w^*$, and for all $e \in C = A \cup B$, there exists *w**∈[0,1] then,

$$\widehat{H}(e) = \begin{cases} \widehat{F}(e), & \text{if } e \in A / B \\ \widehat{G}(e), & \text{if } e \in B / A \\ \widehat{F}(e) \cup \widehat{G}(e), & \text{if } e \in A \cap B \end{cases}.$$

Now,

$$\left(\widehat{F}, A, w_1 \widetilde{\cup} \widehat{G}, B, w_2\right)^r = \widehat{H}, C, w^{*r} = \left(\widehat{H}^r, C, w^*\right).$$

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For all $e \in C = A \cup B$, there exists $w^* \in [0,1]$ such that,

$$\widehat{H}^r(e) = \begin{cases} \widehat{F}^r(e), & if\ e \in A / B \\ \widehat{G}^r(e), & if\ e \in B / A \\ \widehat{F}^r(e) \cap \widehat{G}^r(e), & if\ e \in A \cap B \end{cases}.$$

Also, let

$$\widehat{F}, A, w_1^r \widetilde{\cap}_E \widehat{G}, B, w_2^r = \hat{T}, D, w^*$$

and for all $e \in D = A \cup B$, there exists $w^* \in [0,1]$ such that,

$$\hat{T}^r(e) = \begin{cases} \widehat{F}^r(e), & if\ e \in A / B \\ \widehat{G}^r(e), & if\ e \in B / A \\ \widehat{F}^r(e) \cap \widehat{G}^r(e), & if\ e \in A \cap B \end{cases}.$$

Clearly, $\widehat{H}^r(e) = \hat{T}(e)$. Hence, the result has been established.

(i) Let $\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2 = \widehat{W}, C, w^*$, and for all $e \in C = A \cup B$, there exists $w^* \in [0,1]$, such that,

$$\widehat{W}(e) = \begin{cases} \widehat{F}(e), & if\ e \in A / B \\ \widehat{G}(e), & if\ e \in B / A \\ \widehat{F}(e) \cap \widehat{G}(e), & if\ e \in A \cap B \end{cases}.$$

Now,

$$\left(\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2\right)^r = \widehat{W}, C, w^{*r} = \left(\widehat{W}^r, C, w^*\right)$$

and for all $e \in C$, there exists $w^* \in [0,1]$, such that

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$$\widehat{W}^r(e) = \left(\widehat{W}, C\right)^r = \begin{cases} \widehat{F}^r(e), & \text{if } e \in A / B \\ \widehat{G}^r(e), & \text{if } e \in B / A \cdot \\ \widehat{F}^r(e) \cup \widehat{G}^r(e), & \text{if } e \in A \cap B \end{cases}$$

Also,

$$\widehat{F}, A, w_1{}^r \widetilde{\cup} \widehat{G}, B, w_2{}^r = \left(\widehat{F}^r, A, w_1\right) \widetilde{\cup} \left(\widehat{G}^r, B, w_2\right) = \left(\widehat{M}, C, w^*\right),$$

(say), and for all $e \in C = A \cup B$, there exists $w^* \in [0,1]$ such that,

$$\widehat{M}(e) = \begin{cases} \widehat{F}^r(e), & \text{if } e \in A / B \\ \widehat{G}^r(e), & \text{if } e \in B / A \ . \\ \widehat{F}^r(e) \cup \widehat{G}^r(e), & \text{if } e \in A \cap B \end{cases}$$

Obviously, $\widehat{W}^r(e) = \widehat{M}(e)$. Hence,

$$\left(\widehat{F}, A, w_1 \widetilde{\cap}_E \widehat{G}, B, w_2\right)^r = \widehat{F}, A, w_1{}^r \widetilde{\cup} \widehat{G}, B, w_2{}^r .$$

Theorem 4.2. Let $\widehat{F}, A, w_1$ and $\widehat{G}, B, w_2$ be weighted intuitionistic fuzzy soft sets, where

$\widehat{F}, A$ *and* $\widehat{G}, B$

are intuitionistic fuzzy soft set over U and $w1, w2 \in [0,1]$. Then,

(i) $\left(\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2\right)^r = \widehat{F}, A, w_1{}^r \Cap \widehat{G}, B, w_2{}^r$ if $A \cap B \neq \varnothing$,

(ii) $\left(\widehat{F}, A, w_1 \Cap \widehat{G}, B, w_2\right)^r = \widehat{F}, A, w_1{}^r \widetilde{\cup}_R \widehat{G}, B, w_2{}^r$ if $A \cap B \neq \varnothing$.

Proof:

(i) $\widehat{F}, A, w_1 \widetilde{\cup}_R \widehat{G}, B, w_2 = \left(\widehat{H}, C\right)$, and for all $e \in C = A \cap B$, there exists $w^* \in [0,1]$ such that,

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$\widehat{H}(e)=\widehat{F}(e)\cup\widehat{G}(e)$.

Now, let

$$\left(\widehat{F},A,w_1\widetilde{\cup}_R\widehat{G},B,w_2\right)^r=\left(\widehat{H},C,w^*\right)^r=\left(\widehat{H}^r,C,w^*\right),$$

and for all $e\in C$, there exists $w^*\in[0,1]$ such that

$$\widehat{H}^r(e)=\left(\widehat{H}(e)\right)^r=\left(\widehat{F}(e)\cup\widehat{G}(e)\right)^r=\widehat{F}^r(e)\cap\widehat{G}^r(e).$$

Also, let

$$\widehat{F},A,w_1^r\Cap\widehat{G},B,w_2^r=\left(\hat{I},C,w^*\right),$$

and for all $e\in C=A\cap B\neq\varnothing$, there exists $w^*\in[0,1]$,

$$\hat{I}(e)=\widehat{F}^r(e)\cap\widehat{G}^r(e).$$

Since, $\widehat{H}^r(e)=\hat{I}(e)$. Therefore, the result holds.

(ii) Let $\widehat{F},A,w_1\Cap\widehat{G},B,w_2=H,C,w^*$, and for all $e\in C=A\cap B\neq\varnothing$, there exists $w^*\in[0,1]$ such that

$$\widehat{H}(e)=\widehat{F}(e)\cap\widehat{G}(e).$$

Now, $\left(\widehat{F},A,w_1\Cap\widehat{G},B,w_2\right)^r=\left(\widehat{H},C,w^*\right)^r=\left(\widehat{H}^r,C,w^*\right)$.

For all $e\in C$, there exists $w^*\in[0,1]$ such

$$\widehat{H}^r(e)=\left(\widehat{H}(e)\right)^r=\left(\widehat{F}(e)\cap\widehat{G}(e)\right)^r=\widehat{F}^r(e)\cup\widehat{G}^r(e).$$

Also,

$$\widehat{F},A,w_1^r\widetilde{\cup}_R\widehat{G},B,w_2^r=\left(\widehat{F}^r,A,w_1\right)\widetilde{\cup}_R\left(\widehat{G}^r,B,w_2\right)=\left(\hat{J},D,w^*\right),$$

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(say) and for all $e \in D = A \cap B$ such that $\hat{J}(e) = \hat{F}^r(e) \cup \hat{G}^r(e)$. Clearly, $\hat{H}^r(e) = \hat{J}(e)$.

Hence,

$$\left(\hat{F}, A, w_1 \Cap \hat{G}, B, w_2\right)^r = \hat{F}, A, w_1^r \tilde{\cup}_R \hat{G}, B, w_2^r \text{ if } A \cap B \neq \varnothing .$$

5. APPLICATION OF WEIGHTED INTUITIONISTIC FUZZY SOFT SET IN DECISION MAKING PROBLEMS

This section presents the various definitions of level soft sets and their decision rules.

5.1. Level Soft Set of a Weighted Intuitionistic Fuzzy Soft Set

Lin in 1996 defined a theory of mathematical analysis, namely the weighted soft set (W-soft sets). In line with Lin's style, Maji et al. (2002) defined the weighted table of soft set. A weighted table of soft set is presented by *dij*=*wj*×*hij* instead of 0 and 1 only, where *hij* are entries in the table of the soft set and *wj* are the weights of the attributes *ej*. The weighted choice value of an object *Oi* is *ci*, given by $ci=\sum jdij$.

Definition 5.1. Let $\sum = \hat{F}, A, w$ be a weighted intuitionistic fuzzy soft set, where $\hat{F}, A$ is an intuitionistic fuzzy soft set over *U* and *A* is a set of parameters. For *s,t*∈[0,1], the (*s,t*)-level **soft set** of Σ is a standard soft set $L(\sum; s, t) = \hat{F}_{(s,t)}, A_W$ such that for every *a*∈*A*∃∈[0,1] defined by

$$\hat{F}_{(s,t)}(a_w) = L\left(\hat{F}(a_w); s, t\right) = \left\{x \in U : \mu_{\hat{F}(a_w)}(x) \geq s \text{ and } \lambda_{\hat{F}(a_w)}(x) \leq t\right\}.$$

This definition is clearly an extension of level soft sets of fuzzy soft sets. That is, *s*∈[0,1] can be seen as a given **least threshold** on membership values and *t*∈[0,1] can be seen as a given **greatest threshold** on non-membership values. In a real-life application of weighted intuitionistic fuzzy soft set based decision making, normally the thresholds are chosen in advance by the decision maker(s) and represent their requirements on membership levels and non-membership levels, respectively. While *w*∈[0,1] is a fixed value imposed on each parameter by the decision maker(s).

Definition 5.2. Let $\sum = \hat{F}, A, w$ be a weighted intuitionistic fuzzy soft set, where $\hat{F}, A$ is an intuitionistic fuzzy soft set over *U* and *A* is a set of parameters. Let *η*: A→[0,1]×[0,1] be an intuitionistic fuzzy set in A which is called a **Threshold intuitionistic fuzzy set**. The level soft set of Σ with respect to *η* is a crisp soft set $L(\sum, \eta) = \hat{F}_\Sigma, A_w$ such that ∀a∈A*w*∃∈[0,1] defined by

$$\hat{F}_\eta(a_w) = L\left(\hat{F}(a_w); \eta(a_w)\right) = \left\{x \in U : \mu_{\hat{F}(a_w)}(x) \geq \mu_\eta(a_w) \text{ and } \lambda_{\hat{F}(a_w)}(x) \leq \lambda_\eta(a_w)\right\}.$$

Clearly, the level soft sets of weighted intuitionistic fuzzy soft sets with respect to an intuitionistic fuzzy set are extensions of the level soft set.

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Definition 5.3. (**The mid-level soft set** of a weighted intuitionistic fuzzy soft set). Let $\sum = \widehat{F}, A, w$ be a weighted intuitionistic fuzzy soft set, where $\widehat{F}, A$ is an intuitionistic fuzzy soft set over *U* and *A* is a set of parameters. Based on the weighted intuitionistic fuzzy soft set $\sum = \widehat{F}, A, w$, we can define an intuitionistic fuzzy set

*mid*Σ: $A \to [0,1] \times [0,1]$,

such that $a \in Aw \exists \in [0,1]$ defined by

$$\mu_{mid\Sigma}(a_w) = \frac{1}{|U|}\sum_{x\in U}\mu_{\widehat{F}(a_w)}(x) \text{ and } \lambda_{mid\Sigma}(a_w) = \frac{1}{|U|}\sum_{x\in U}\lambda_{\widehat{F}(a_w)}(x).$$

The intuitionistic fuzzy set *mid*Ω is called the **mid-threshold** of the weighted intuitionistic fuzzy soft set Σ. In addition, the level soft set of Σ with respect to the mid-threshold intuitionistic fuzzy set *mid*Σ, namely $L(\sum, mid\Sigma)$ is called the mid-level soft set of Σ and is represented simply by $L(\Sigma; mid)$. In what follows the mid-level decision rule will mean using the mid-threshold and considering the mid-level soft set in weighted intuitionistic fuzzy soft sets based decision making.

Definition 5.4. (The **Top-Top-level soft set** and **Bottom-Bottom-level soft set** of a weighted intuitionistic fuzzy soft set). Let $\sum = \widehat{F}, A, w$ be a weighted intuitionistic fuzzy soft set, where $\widehat{F}, A$ is an intuitionistic fuzzy soft set over *U* and *A* is a set of parameters. Based on the intuitionistic fuzzy soft set $\sum = \widehat{F}, A, w$.

Intuitionistic fuzzy set *toptop*Σ: $A \to [0,1] \times [0,1]$ such that $\forall a \in Aw \exists \in [0,1]$ defined by

$$\mu_{toptop\Sigma}(a_w) = \max_{x\in U}\mu_{\widehat{F}(a_w)}(x) \text{ and } \lambda_{toptop\Sigma}(a_w) = \max_{x\in U}\lambda_{\widehat{F}(a_w)}(x).$$

Also, intuitionistic fuzzy set *bottombottom*Ω: $A \to [0,1] \times [0,1]$ such that $\forall a \in Aw \exists \in [0,1]$ define by

$$\mu_{bottombottom\Sigma}(a_w) = \min_{x\in U}\mu_{\widehat{F}(a_w)}(x) \text{ and } \lambda_{bottombottom\Sigma}(a_w) = \min_{x\in U}\lambda_{\widehat{F}(a_w)}.$$

To illustrate the above definitions, we shall consider the **Example 5.1**.

The algorithm below itemized the steps to be taken in arriving at an accurate decision to select the best region for the rice cultivation.

Algorithm

(I) Input the weighted intuitionistic fuzzy soft set $\sum = \widehat{F}, A, w$.

(II) Input a threshold intuitionistic fuzzy set η: $A \to [0,1] \times [0,1]$ (or give a threshold value pair $(s,t) \in [0,1] \times [0,1]$; or choose a mid-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.

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(III) Compute the level soft set $L(\Sigma,\eta)$ with respect to the threshold intuitionistic fuzzy set η (or the (s,t)-level soft set $L(\Sigma;s.t)$; or the mid-level soft set $L(\Sigma;mid)$; or the top-top-level soft set $L(\Sigma;toptop)$ or the bottom-bottom-level soft set $L(\Sigma;bottombottom)$).

(IV) Present the level soft set $L(\Sigma;\eta)$ (or $L(\Sigma;s,t)$; $L(\Sigma;mid)$; or $L(\Sigma;topbottom)$; or $L(\Sigma;toptop)$; or $L(\Sigma;bottombottom)$) in tabular form and compute the choice value c_i of o_i, for all i.

(V) The optimal decision is to select o_k if $c_k=max_i c_i$.

(VI) If k has more than one value then any one of o_k may be chosen.

Example 5.1. Let us consider another weighted intuitionistic fuzzy soft set $\sum = \widehat{F}, A, w$ which describes some geographical regions in a country that a farmer Mr. Y is considering for rice farming.

Suppose that there are six regions in the initial universe $U=\{R1,R2,R3,R4,R5,R6\}$ under consideration and that

$$A = \left\{a_{1,0.9}, a_{2,0.6}, a_{3,0.6}, a_{4,0.7}, a_{5,0.5}\right\}$$

is a set of decision parameters or attribute related to U with their respective weights attached. The $ai=(i=1,2,3,4,5)$ stand for the parameters "plain areas", "soil type", "accessible", "high rainfall" and "temperature", respectively. Suppose that

$$\widehat{F}\left(a_{1,0.9}\right) = \left\{R_1, 0.9, 0.1,\ R_2, 0.5, 0.2,\ R_3, 0.7, 0.2,\ R_4, 0.5, 0.4,\ R_5, 0.9, 0.2,\ R_6, 0.6, 0.5\right\},$$

$$\widehat{F}\left(a_{2,0.6}\right) = \left\{R_1, 0.8, 0.2,\ R_2, 0.8, 0.1,\ R_3, 0.5, 0.3,\ R_4, 0.6, 0.5,\ R_5, 0.8, 0.2,\ R_6, 0.4, 0.3\right\},$$

$$\widehat{F}\left(a_{3,0.6}\right) = \left\{R_1, 0.7, 0.1,\ R_2, 0.6, 0.2,\ R_3, 0.3, 0.4,\ R_2, 0.7, 0.5,\ R_5, 0.9, 0.1,\ R_6, 0.6, 0.3\right\},$$

$$\widehat{F}\left(a_{4,0.8}\right) = \left\{R_1, 0.5, 0.3,\ R_2, 0.8, 0.1,\ R_3, 0.9, 0.1,\ R_4, 0.8, 0.2,\ R_5, 0.7, 0.1,\ R_6, 0.8, 0.2\right\},$$

$$\widehat{F}\left(a_{5,0.5}\right) = \left\{R_1, 0.8, 0.2,\ R_2, 0.4, 0.5,\ R_3, 0.9, 0.1,\ R_4, 0.8, 0.1,\ R_5, 0.5, 0.4,\ R_6, 0.8, 0.2\right\}.$$

The weighted intuitionistic fuzzy soft set $\sum = \widehat{F}, A, w$ is a parameterized family

$$\left\{\widehat{F}\left(a_i\right),\ i = 1,2,3,4,5\right\}$$

of fuzzy sets on U and

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$$\widehat{F},A,w=\left\{\begin{array}{l}Plain\ areas=\{R_1,0.9,0.1,\ R_2,0.5,0.2,\ R_3,0.7,0.2,\ R_4,0.5,0.4,\ R_5,0.9,0.2,\ R_6,0.6,0.5\},\\ soil\ type=\{R_1,0.8,0.2,\ R_2,0.8,0.1,\ R_3,0.5,0.3,\ R_4,0.6,0.5,\ R_5,0.8,0.2,\ R_6,0.4,0.3\},\\ accessibility=\{R_1,0.7,0.1,\ R_2,0.6,0.2,\ R_3,0.3,0.4,\ R_4,0.7,0.1,\ R_5,0.9,0.1,\ R_6,0.6,0.3\},\\ high\ rainfall=\{R_1,0.5,0.3,\ R_2,0.8,0.1,\ R_3,0.9,0.1,\ R_4,0.8,0.2,\ R_5,0.7,0.1,\ R_6,0.8,0.2\},\\ temperature=\{R_1,0.8,0.2,\ R_2,0.4,0.5,\ R_3,0.9,0.1,\ R_4,0.8,0.1,\ R_5,0.5,0.4,\ R_6,0.8,0.2\}\end{array}\right\}$$

Table 2 gives the tabular representation of weighted intuitionistic fuzzy soft set in Example 5.1.

Table 2. Tabular representation of weighted intuitionistic fuzzy soft set $\sum=\widehat{F},A,w$

U/A	$a_{1,0.9}$	$a_{2,0.6}$	$a_{3,0.6}$	$a_{4,0.8}$	$a_{5,0.5}$
R_1	0.9, 0.1	0.8, 0.2	0.7, 0.1	0.5, 0.3	0., 0.2
R_2	0.5, 0.2	0.8 0.1	0.6, 0.2	0.8, 0.1	0.4, 0.5
R_3	0.7, 0.2	0.5, 0.3	0.3, 0.4	0.9, 0.1	0.9, 0.1
R_4	0.5, 0.4	0.4, 0.4	0.7, 0.1	0.8, 0.2	0.8, 0.1
R_5	0.9, 0.2	0.8, 0.2	0.9, 0.1	0.7, 0.1	0.5, 0.4
R_6	0.6, 0.5	0.4, 0.3	0.6, 0.3	0.8, 0.2	0.8, 0.2

We now consider (*s,t*)-*level* soft set of Σ using **Example 5.1**.

Let us take *S*=0,7 and *t*=0.3, then we have the following:

$$L\left(\widehat{F}\left(a_{1,0.5}\right);0.7,0.3\right)=\{R_1,R_3,R_5\},$$

$$L\left(\widehat{F}\left(a_{2,0.6}\right);0.7,0.3\right)=\{R_1,R_2,R_5\},$$

$$L\left(\widehat{F}\left(a_{3,0.7}\right);0.7,0.3\right)=\{R_1,R_4,R_5\},$$

$$L\left(\widehat{F}\left(a_{4,0.9}\right);0.7,0.3\right)=\{R_2,R_3,R_4,R_5,R_6\},$$

$$L\left(\widehat{F}\left(a_{5,0.5}\right);0.7,0.3\right)=\{R_1,R_3,R_4,R_6\}.$$

Hence, the (0.7, 0.3)-level soft set $\sum=\widehat{F},A,w$ is a soft set

$$L\left(\sum;0.7,0.3\right)=\widehat{F}_{(0.7,0.3)},A,$$

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where the set- valued mapping $\widehat{F}_{(0.7,0.3)} : A \to P(U)$ is defined by

$$\widehat{F}_{(0.7,0.3)}\left(a_{i,w}\right) = L\left(\widehat{F}\left(a_{i,w}\right); 0.7, 0.3\right)$$

for *i*=1,2,3,4,5. Table 3 gives the tabular representation of the (0.7, 0.3)-*level* soft set *L*(Σ; 0.7, 0.3) with choice value.

Table 3. Tabular representation of the level soft set L(Σ; 0.7,0.3) with choice value

U/A	$a_{1,0.9}$	$a_{2,0.6}$	$a_{3,0.6}$	$a_{4,0.8}$	$a_{5,0.5}$	*ChoiceValue* (c_i)
R_1	1	1	1	0	1	2.6
R_2	0	1	0	1	0	1.4
R_3	1	0	0	1	1	2.2
R_4	0	0	1	1	1	1.9
R_5	1	1	1	1	0	2.9
R_6	0	0	0	1	1	1.3

From the Table **3,** it follows that, the maximum choice value is *c*5=2.9 and therefore the optimal decision is to select region ***R5***.

For mid-level soft sets, let us consider **Example 5.1** and $\sum = \widehat{F}, A, w$ with tabular representation in Table 2. It is clear that, the mid-threshold of $\widehat{F}, A, w$ is an intuitionistic fuzzy set

$$mid_{\widehat{F},A} = \left\{a_1, 0.68, 0.27,\ a_2, 0.63, 0.25,\ a_3, 0.63, 0.2,\ a_4, 0.75, 0.17,\ a_5, 0.7, 0.25\right\},$$

and the mid-level soft set of $\widehat{F}, A$ is a soft set $L\left(\widehat{F}, A; mid\right)$ with its tabular representation given by **Table 4.**

Table 4. Tabular representation of the mid-level soft set $L\left(\widehat{F}, A; mid\right)$ *with choice value*

U/A	$a_{1,0.9}$	$a_{2,0.6}$	$a_{3,0.6}$	$a_{4,0.8}$	$a_{5,0.5}$	*ChoiceValue* (c_i)
R_1	1	1	1	0	1	2.6
R_2	0	1	0	1	0	1.4
R_3	1	0	0	1	1	2.2
R_4	0	0	1	0	1	1.1
R_5	1	1	1	0	0	2.1
R_6	0	0	0	0	1	0.5

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From the Table 4, the maximum choice value is $c1=2.6$ and so the optimal decision is to select region $R1$.

For illustrative example of **top-top-level soft sets** and **bottom-bottom-level soft sets**, let us again consider the weighted intuitionistic fuzzy soft set $\sum = \widehat{F}, A, w$ with its tabular representation given by Table 2. It is obvious that the top-top-threshold of $\widehat{F}, A, w$ is an intuitionistic fuzzy set

$$toptop_{\widehat{F},A} = \{a_1, 0.9, 0.5,\ a_2, 0.8, 0.4,\ a_3, 0.9, 0.4,\ a_4, 0.9, 0.3,\ a_5, 0.9, 0.5\}$$

and the top-top-level soft set of $\widehat{F}, A$ is a soft set $L\left(\widehat{F}, A; toptop\right)$ with its tabular representation given by Table 2. It is clear that the bottom-bottom-threshold of $\widehat{F}, A, w$ is an intuitionistic fuzzy set

$$bottombottom_{\widehat{F},A} = \{a_1, 0.4, 0.1,\ a_2, 0.4, 0.1,\ a_3, 0.3, 0.1,\ a_4, 0.6, 0.1,\ a_5, 0.4, 0.1\},$$

and the bottom-bottom-level soft set of $\widehat{F}, A$ is a soft set $L\left(\widehat{F}, A; bottombottom\right)$ with its tabular representation given by Table 2.

From Table 5, it follows that the maximum choice value is $c5=2.1$. The optimum decision is to select region $R5$.

Table 5. Tabular representation of the top-top-level soft set L(∑;toptop) with choice value

U/A	$a_{1,0.9}$	$a_{2,0.6}$	$a_{3,0.6}$	$a_{4,0.8}$	$a_{5,0.5}$	*ChoiceValue* (c_i)
R_1	1	1	0	0	0	1.5
R_2	0	1	0	0	0	0.6
R_3	0	0	0	1	1	1.3
R_4	0	0	0	0	0	0.0
R_5	1	1	1	0	0	2.1
R_6	0	0	0	0	0	0.0

Table 6. Tabular representation of the bottom-bottom-level soft set L(∑;bottombottom) with choice value

U/A	$a_{1,0.9}$	$a_{2,0.6}$	$a_{3,0.6}$	$a_{4,0.8}$	$a_{5,0.5}$	*ChoiceValue* (c_i)
R_1	0	0	1	0	1	0.6
R_2	0	1	0	1	0	1.4
R_3	0	0	0	1	1	1.3
R_4	0	0	1	0	0	0.6
R_5	0	0	1	1	0	1.4
R_6	0	0	0	0	0	0.0

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From Table 6, it follows that the maximum choice value is $c5=c5=1.6$. The optimum decision is to select either region $R2$ or region $R5$.

Also, after employing all decision rules and imposing weights on the parameters using Example 5.1, we observed that in most cases the choice of the regions differs and this is due to the decision maker's preference (or the choice of the decision rules).

6. CONCLUSION

In this chapter, authors have defined the basic soft set operations, such as union, restricted union, restricted intersection, extended intersection, restricted difference, AND-product, OR-product and some notions on weighted intuitionistic fuzzy soft set. Some concepts such as soft subset, relative whole, relative complement and absolute weighted intuitionistic fuzzy soft set were defined. Basic properties of these operations were established and some results presented including various De Morgan's type of results.

The chapter also presented the definitions of the various level soft sets and their decision rules. Real life practical problems in a multicriteria decision making problems are solved.

This showed that, weighted intuitionistic fuzzy soft set could be studied both in theoretical and applications point of view.

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Chapter 24
A New Neutrosophic Multigranulation Model for Multi-Attribute Group Decision Making

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ABSTRACT

Neutrosophic theories and methods are suitable for handling a variety of uncertain information, especially in complex decision-making situations. Nowadays, how to study multi-attribute group decision-making (MAGDM) problems in neutrosophic environments is a vital task for scholars and practitioners. Granular computing is a popular mathematical framework for uncertain problems, and it has been proved as a useful tool for solving MAGDM problems. In this chapter, a new neutrosophic multigranulation model for MAGDM is proposed, and some theoretical models and decision-making algorithms are presented. Finally, some case studies and comparative analysis are performed to show the merit of the presented models and algorithms. In general, this chapter is conducive to neutrosophic decision-making communities and intelligent decision-making methods.

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1. INTRODUCTION

With the advent of big data era, complex decision making situations are common in numerous realistic applications (Qudrat-Ullah et al., 2008). In general, there are two major challenges in addressing practical complex decision making situations, the one is the information depiction, and another one is the information fusion and analysis. For the sake of coping with the above two challenges, plenty of scholars and practitioners have started their researches from the perspective of fuzzy MAGDM. In specific, the fuzzy context is intended to precisely depict complex decision making scenarios, whereas MAGDM acts as a powerful tool for handling information fusion and analysis issues (Liu & You, 2018). In light of the above two challenges, we review some significant development trends as follows:

- The review of the fuzzy context. Since Zadeh (Zadeh, 1965) initiated the notion of fuzzy sets, many generalized fuzzy sets along with their applications were successively explored over the past 50 years or more. Among the fuzzy set community, neutrosophic sets (NSs), initiated by Smarandache (Smarandache, 1998; Smarandache, 1999), play a vital role in both theoretical and application studies in the family of generalized fuzzy sets. In a NS, there exist three different components for describing the truth aspect, the indeterminacy aspect and the falsity aspect, which have enlarged the conceptual implication of classic fuzzy sets, thus NSs are conducive to combating the challenges of the impreciseness and uncertainty in various decision making information. Ever since the birth of neutrosophic theories and methods, single-valued neutrosophic sets (SVNSs) were later put forward to provide a formal mathematical expression for solving realistic complex problems via the notion of neutrosophy (Wang et al., 2005). Recently, many realistic complex problems have been efficiently addressed in the context of SVNSs, such as decision making (Abdel-Basset et al., 2019; Alias et al., 2018; Deli et al., 2015; Zhang et al., 2017), graph theories (Akram et al., 2018; Broumi & Smarandache, 2013; Broumi et al., 2019; Ishfaq et al., 2018; Sayed et al., 2018), etc.
- The review of MAGDM. The core of MAGDM lies in involving an expert group to solve complex decision making problems from the aspect of multiple attributes, and viable information fusion and analysis rules need to be designed in corresponding problem solving processes. There are plenty of classic MAGDM methods, and it is widely recognized that granular computing-based ones act as powerful methods in terms of granularity features in complex decision making situations. In the community of granular computing, the rough set theory is a significant representative which does well in providing solvable approximations. In this chapter, due to the feature of group decision making, we aim to utilize multigranulation rough set (MGRS) models (Qian et al., 2010; Sun & Ma, 2015; Sun et al., 2017; Zhan et al., 2019; Zhang, Li, & Liang, 2020; Zhang et al., 2019) to explore a new solution path for MAGDM.

In light of the above statements, several works have been done by combining SVNSs with MGRSs (Garg & Nancy, 2018; Liu, 2016; Ren et al., 2020; Zhang, Li, Kang et al, 2020; Zhang et al., 2016). However, in real world, decision makers are usually confronted with bounded rationality in practical decision making scenarios, and bounded rationality acts as a natural state that decision makers are limited to finite information processing abilities. Thus, exploring single-valued neutrosophic (SVN) MAGDM under bounded rationality is imperative. In this chapter, by utilizing the TODIM (an acronym in Portuguese for Interative Multi-criteria Decision Making) method (Gomes & Lima, 1992) to process decision

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making information under bounded rationality, we aim to propose a new neutrosophic multigranulation model under the context of SVNSs. Therefore, the contributions of this chapter lie in the following aspect:

(1) A new SVN MAGDM approach is presented via MGRSs and TODIM.
(2) The validity of the proposed decision making approach is verified by a steam turbine fault diagnosis (STFD) case study.

The arrangement of the chapter is shown as follows: we first revisit some preliminary knowledge on SVNSs and SVN MGRSs in Section 2. In Section 3, a SVN MAGDM approach is put forward by virtue of MGRSs and TODIM. Afterwards, the validity of the proposed decision making approach is verified by a STFD example in Section 4. At last, some conclusions and future study directions are illustrated in Section 5.

2. BASIC KNOWLEDGE

2.1 SVNSs

In this section, we mainly revisit some key notions of SVNSs and SVN MGRSs.

Definition 1 (Wang et al., 2005) Let U be a universe of discourse. A SVNS B on U is characterized by a truth-membership function $T_B(x)$, an indeterminacy-membership function $I_B(x)$, and a falsity-membership function $F_B(x)$. Then a SVNS B on U is provided by:

$$B=\{ < x,\left(T_B\left(x\right),I_B\left(x\right),F_B\left(x\right)\right) > \left| x \in U \right.\},$$

where $T_B(x)$, $I_B(x)$, $F_B(x)\rightarrow$int[0,1] for all $x\in U$ to the set A, then we have the following condition: $0\leq T_B(x)+I_B(x)+F_B(x) \leq 3$. The set of all SVNSs on U is denoted by $SVN(U)$ in this chapter.

Definition 2 (Wang et al., 2005) Suppose B is a SVNS. A SVN score function with regard to an element x is provided as the following mathematical expression:

$$s\left(x\right)=\{< x,T_B\left(x\right)+1-I_B\left(x\right)+1-F_B\left(x\right) > \left| x \in U \right.\},$$

for two SVNSs B and B', if $s\left(x\right) > s\left(x'\right)$, then $B \succ B'$; if $s\left(x\right) < s\left(x'\right)$, then $B \prec B'$; if $s\left(x\right) = s\left(x'\right)$, then $B \sim B'$.

Definition 3 (Wang et al., 2005) Let U be a universe of discourse, $\forall A,B\in SVN(U)$, then the following operations are presented as follows:

1. $A \oplus B = \{< x,T_A\left(x\right)+T_B\left(x\right)-T_A\left(x\right)T_B\left(x\right),I_A\left(x\right)I_B\left(x\right),F_A\left(x\right)F_B\left(x\right) > | \, x \in U\}$;

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2. $A \otimes B =$
$\left\{\left\langle x, T_A(x)T_B(x), I_A(x)+I_B(x)-I_A(x)I_B(x), F_A(x)+F_B(x)-F_A(x)F_B(x)\right\rangle \mid x \in U\right\}$;

3. $\lambda \cdot A = \left\{\left\langle x, 1-\left(1-T_A(x)\right)^{\lambda}, I_A(x)^{\lambda}, F_A(x)^{\lambda}\right\rangle \mid x \in U\right\}$;

4. $A^{\lambda} = \left\{\left\langle x, T_A(x)^{\lambda}, 1-\left(1-I_A(x)\right)^{\lambda}, 1-\left(1-F_A(x)\right)^{\lambda}\right\rangle \mid x \in U\right\}$;

5. $A \boxminus B = \{< x, \frac{T_A(x)-T_B(x)}{1-T_B(x)}, \frac{I_A(x)}{I_B(x)}, \frac{F_A(x)}{F_B(x)} > \mid x \in U\}$;

6. $A \boxdot B = \{< x, \frac{T_A(x)}{T_B(x)}, \frac{I_A(x)-I_B(x)}{1-I_B(x)}, \frac{F_A(x)-F_B(x)}{1-F_B(x)} > \mid x \in U\}$;

7. $A^c = \{< x, F_A(x), 1-I_A(x), T_A(x) > \mid x \in U\}$;

8. $A \bigcap B = \{< x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) > \mid x \in U\}$;

9. $A \bigcup B = \{< x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) > \mid x \in U\}$.

Definition 4 (Liu, 2016) Suppose A and B are two SVNSs. Then their normalized Euclidean distance is provided by:

$$D(A,B) = \sqrt{\frac{1}{3}\left\{\left(T_A(x)-T_B(x)\right)^2 + \left(I_A(x)-I_B(x)\right)^2 + \left(F_A(x)-F_B(x)\right)^2\right\}}.$$

2.2 SVN MGRS Models

In this section, we review optimistic and pessimistic MGRS models in the SVN environment. In specific, the optimistic MGRS model is based on the information fusion strategy of "seeking common ground while reserving differences", that is, only at least one granular space is needed to satisfy the inclusion condition under the target concept. Similarly, the pessimistic MGRS model is based on the strategy of "seeking common ground while reserving differences". In what follows, we first present the concept of SVN relations (SVNRs).

Definition 7 (Zhang et al., 2016) Suppose U and V are two an arbitrary universes of discourse. A SVNR R over $U \times V$ is provided as the following mathematical expression:

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$$R=\left\{\left\langle (x,y),\left(T_R(x,y),I_R(x,y),F_R(x,y)\right)\right\rangle \middle| (x,y)\in U\times V\right\},$$

where T, I, F: $U, V\rightarrow$int[0,1]. Similar with the notion of SVNSs, there also exists the restriction of $0\leq T_R(x)+I_R(x)+F_R(x)\leq 3$. Moreover, a set that includes all SVNRs over $U\times V$ is further named as $SVNR(U\times V)$.

Based on the above-mentioned SVNRs, the following definition of SVN MGRSs is given.

Definition 8 (Zhang et al., 2016) Suppose U and V are two an arbitrary universes of discourse, $R_i\in SVNR(U\times V)$ is a family of m *SVNRs* over $U\times V$. For any $B\in SVN(V)$, the optimistic SVN multigranulation rough lower and upper approximations of B are presented as follows:

$$\underline{\sum_{i=1}^{m}R_i}^{O}(B)=\{<x,T_{\underline{\sum_{i=1}^{m}R_i}^{O}(B)}(x),I_{\underline{\sum_{i=1}^{m}R_i}^{O}(B)}(x),F_{\underline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)>\left|x\in U\right.\};$$

$$\overline{\sum_{i=1}^{m}R_i}^{O}(B)=\{<x,T_{\overline{\sum_{i=1}^{m}R_i}^{O}(B)}(x),I_{\overline{\sum_{i=1}^{m}R_i}^{O}(B)}(x),F_{\overline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)>\left|x\in U\right.\},$$

where

$$T_{\underline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)=\vee_{i=1}^{m}\wedge_{y\in V}\left\{F_{R_i}(x,y)\vee T_B(y)\right\};$$

$$I_{\underline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)=\wedge_{i=1}^{m}\vee_{y\in V}\left\{\left(1-I_{R_i}(x,y)\right)\wedge I_B(y)\right\};$$

$$F_{\underline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)=\wedge_{i=1}^{m}\vee_{y\in V}\left\{T_{R_i}(x,y)\wedge F_B(y)\right\};$$

$$T_{\overline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)=\wedge_{i=1}^{m}\vee_{y\in V}\left\{T_{R_i}(x,y)\wedge T_B(y)\right\};$$

$$I_{\overline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)=\vee_{i=1}^{m}\wedge_{y\in V}\left\{I_{R_i}(x,y)\vee I_B(y)\right\};$$

$$F_{\overline{\sum_{i=1}^{m}R_i}^{O}(B)}(x)=\vee_{i=1}^{m}\wedge_{y\in V}\left\{F_{R_i}(x,y)\vee F_B(y)\right\},$$

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then the pair $(\underline{\sum_{i=1}^{m} R_i}^{O}(B), \overline{\sum_{i=1}^{m} R_i}^{O}(B))$ is named as an optimistic SVN MGRS over two universes of *B*. In a similar manner, the pessimistic SVN lower and upper approximations of *B* are presented as follows.

Definition 9 (Zhang et al., 2016) Suppose *U* and *V* are two an arbitrary universes of discourse, $R_i \in SVNR(U \times V)$ is a family of *m SVNRs* over $U \times V$. For any $B \in SVN(V)$, the pessimistic SVN multi-granulation rough lower and upper approximations of *B* are defined as follows:

$$\underline{\sum_{i=1}^{m} R_i}^{P}(B) = \{< x, T_{\underline{\sum_{i=1}^{m} R_i}^{P}(B)}(x), I_{\underline{\sum_{i=1}^{m} R_i}^{P}(B)}(x), F_{\underline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) > \mid x \in U\};$$

$$\overline{\sum_{i=1}^{m} R_i}^{P}(B) = \{< x, T_{\overline{\sum_{i=1}^{m} R_i}^{P}(B)}(x), I_{\overline{\sum_{i=1}^{m} R_i}^{P}(B)}(x), F_{\overline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) > \mid x \in U\},$$

where

$$T_{\underline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) = \wedge_{i=1}^{m} \wedge_{y \in V} \{F_{R_i}(x,y) \vee T_B(y)\};$$

$$I_{\underline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) = \vee_{i=1}^{m} \vee_{y \in V} \{(1 - I_{R_i}(x,y)) \wedge I_B(y)\};$$

$$F_{\underline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) = \vee_{i=1}^{m} \vee_{y \in V} \{T_{R_i}(x,y) \wedge F_B(y)\};$$

$$T_{\overline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) = \vee_{i=1}^{m} \vee_{y \in V} \{T_{R_i}(x,y) \wedge T_B(y)\};$$

$$I_{\overline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) = \wedge_{i=1}^{m} \wedge_{y \in V} \{I_{R_i}(x,y) \vee I_B(y)\};$$

$$F_{\overline{\sum_{i=1}^{m} R_i}^{P}(B)}(x) = \wedge_{i=1}^{m} \wedge_{y \in V} \{F_{R_i}(x,y) \vee F_B(y)\},$$

then the pair $(\underline{\sum_{i=1}^{m} R_i}^{P}(B), \overline{\sum_{i=1}^{m} R_i}^{P}(B))$ is named as a pessimistic *SVN* MGRS over two universes of *B*.

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3. SVN MAGDM

This section aims to explore an optimal granulation selection of SVN MGRSs under bounded rationality, and further give a corresponding MAGDM method based on SVN MGRSs and TODIM.

Suppose $U=\{x_1,x_2,\ldots,x_m\}$ is a set of alternatives, another universe $V=\{y_1,y_2,\ldots,y_n\}$ is a set of attributes, $R_k \in SVNR(U\times V)$ ($k=1,2,\ldots,f$) is a SVNR over $U\times V$, and $B\in SVN(V)$ is a standard assessment set. Thus, we obtain a SVN information system which is denoted by (U,V,R_k,B).

In the TODIM method, suppose $P=\{p_1,\ldots,p_i,\ldots,p_l\}$ and $C=\{c_1,\ldots,c_j,\ldots,c_s\}$ represent an alternative set and an attribute set under bounded rationality. In addition, $w=\{w_1,\ldots,w_j,\ldots,w_s\}^T$ is a set of weights for attributes with $w_k\in[0,1]$ and $\sum_{j=1}^{s} w_j = 1$. Moreover, decision makers need to determine the reference attribute in the weight set. The maximum weight w_r in the weight set is usually selected, and its corresponding attribute is c_r. From the above statement, the decision matrix in TODIM can be expressed as $B=\left(b_{ij}\right)_{l\times s}$, where b_{ij} represents the SVN assessment value of p_i with respect to c_j. The concrete decision making steps in light of the TODIM method (Gomes & Lima, 1992) are sum up below.

First, the relative weight value $w_{jr}=w_j/w_r$ of any attribute c_j to the reference attribute c_r is calculated.

Second, we need to calculate the dominance

$$\varphi\left(p_i,p_f\right)=\sum_{j=1}^{s}\phi_j\left(p_i,p_f\right)$$

of attribute p_i over other alternative p_f. When $b_{ij}>b_{fj}$,

$$\phi\left(p_i,p_f\right)=\sqrt{w_{jr}d\left(b_{ij}-b_{fj}\right)\Big/\left(\sum_{j=1}^{s}w_{jr}\right)},$$

it represents the alternative p_i is better than another alternative p_f. When $b_{ij}=b_{fj}$, $\phi(p_i,p_f)=0$, it represents the alternative pi is in line with another alternative p_f. When $b_{ij}<b_{fj}$,

$$\phi\left(p_i,p_f\right)=-\frac{\sqrt{\left(\sum_{j=1}^{s}w_{jr}\right)d\left(b_{ij}-b_{fj}\right)\Big/w_{jr}}}{\theta},$$

it represents the alternative p_i is not better than another alternative p_f. In addition, θ is usually selected based on the personal experience of decision makers. The larger the value of θ, the lower the degree of loss avoidance. A large number of experiments in the field of psychology have confirmed that the parameter θ usually takes 1 or 2.5.

Then, we need to calculate the overall dominance $\psi(p_i)$ of the alternative p_i as follows.

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$$\psi\left(p_i\right)=\frac{\sum_{f=1}^{l}\varphi\left(p_i,p_f\right)-\min_i\left\{\sum_{f=1}^{l}\varphi\left(p_i,p_f\right)\right\}}{\max_i\left\{\sum_{f=1}^{l}\varphi\left(p_i,p_f\right)\right\}-\min_i\left\{\sum_{f=1}^{l}\varphi\left(p_i,p_f\right)\right\}}.$$

Finally, we need to compare $\psi(p_i)$ according to the method of comparing different SVN numbers given in Definition 3. Then the larger the $\psi(p_i)$, the better the alternative pi.

According to the above decision making steps in light of the TODIM method, let the optimistic and pessimistic SVN multigranulation rough approximations

$$\underline{\sum_{i=1}^{m}R_i^O}(B),\ \overline{\sum_{i=1}^{m}R_i}^{O}(B),\ \underline{\sum_{i=1}^{m}R_i^P}(B),\ \overline{\sum_{i=1}^{m}R_i}^{P}(B)$$

in terms of (U, V, R_k, B) be four alternatives in the TODIM method, thus the set of alternatives is $P=\{p_1,p_2,p_3,p_4\}$, the best p_i is the optimal granularity of SVN MGRSs with bounded rationality. In addition, in the selected set, the optimal alternatives is $x^*=\max_{j=1}^{s}\{s(x_j)\}$.

In what follows, a MAGDM method based on SVN MGRSs and TODIM is established below.

Input: A SVN multigranulation information system (U, V, R_k, B);
Output: The optimal alternative.
Step 1: Calculate SVN multigranulation rough approximations

$$\underline{\sum_{i=1}^{m}R_i^O}(B),\ \overline{\sum_{i=1}^{m}R_i}^{O}(B),\ \underline{\sum_{i=1}^{m}R_i^P}(B),\ \overline{\sum_{i=1}^{m}R_i}^{P}(B);$$

Step 2: Determine the weight corresponding to the attribute set in the TODIM method and calculate the relative weight w_{jr};
Step 3: Calculate the dominance of alternative p_i over other alternatives p_f;
Step 4: Calculate the overall dominance of the alternative p_i;
Step 5: Determine the optimal granularity of SVN MGRSs under bounded rationality;
Step 6: Determine the optimal alternative x^* from the selected set.

4. AN ILLUSTRATIVE EXAMPLE

4.1 Case Descriptions

In the context of STFD, we suppose a set of ten fault patterns is denoted by $U=\{x_1,x_2,\ldots,x_{10}\}$, where $x_j(j=1,2,\ldots,10)$ stands for unbalance, pneumatic force couple, offset center, oil-membrane oscillation, radial impact friction of rotor, symbiosis looseness, damage of antithrust bearing, surge, looseness of

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bearing block, nonuniform bearing stiffness. Another universe $V=\{y_1,y_2,\ldots,y_9\}$ stands for a set of nine frequency ranges, where $y_k(k=1,2,\ldots,9)$ denotes some frequency ranges for different frequency spectrum: $C_1(0.01–0.39f)$, $C_2(0.4–0.49f)$, $C_3(0.5f)$, $C_4(0.51-0.99f)$, $C_5(f)$, $C_6(2f)$, $C_7(3-5f)$, C_8 (odd times of f), C_9 (high frequency $>5f$). Moreover, the fault testing sample is denoted by B. In order to solve STFD under the group decision making background, each mechanical engineer need to provide their preferences on the system fault knowledge base that are shown as the following Tables 1, 2 and 3.

$$B=\left\{\left\langle y_1,\langle 0,0,1\rangle\right\rangle,\left\langle y_2,\langle 0,0,1\rangle\right\rangle,\left\langle y_3,\langle 0.1,0,0.9\rangle\right\rangle,\left\langle y_4,\langle 0.9,0,0.1\rangle\right\rangle,\right.$$
$$\left.\left\langle y_5,\langle 0,0,1\rangle\right\rangle,\left\langle y_6,\langle 0,0,1\rangle\right\rangle,\left\langle y_7,\langle 0,0,1\rangle\right\rangle,\left\langle y_8,\langle 0,0,1\rangle\right\rangle,\left\langle y_9,\langle 0,0,1\rangle\right\rangle\right\}$$

Table 1. Knowledge of system fault given by engineer 1

R_1	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_1	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.85,0.15,0⟩	⟨0.04,0.02,0.94⟩	⟨0.04,0.003,0.93⟩	⟨0,0,1⟩	⟨0,0,1⟩n̄
x_2	⟨0,0,1⟩	⟨0.28,0.03,0.69⟩	⟨0.09,0.03,0.88⟩	⟨0.55,0.15,0.3⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.08,0.05,0.87⟩
x_3	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.3,0.28,0.42⟩	⟨0.4,0.22,0.38⟩	⟨0.08,0.05,0.87⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_4	⟨0.09,0.02,0.89⟩	⟨0.78,0.04,0.18⟩	⟨0,0,1⟩	⟨0.08,0.04,0.88⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_5	⟨0.09,0.03,0.88⟩	⟨0.09,0.02,0.89⟩	⟨0.08,0.04,0.88⟩	⟨0.09,0.03,0.88⟩	⟨0.18,0.04,0.78⟩	⟨0.08,0.05,0.87⟩	⟨0.08,0.05,0.87⟩	⟨0.08,0.04,0.88⟩	⟨0.08,0.04,0.88⟩
x_6	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.18,0.03,0.78⟩	⟨0.12,0.05,0.83⟩	⟨0.37,0.08,0.55⟩	⟨0,0,1⟩	⟨0.22,0.06,0.72⟩
x_7	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.08,0.04,0.88⟩	⟨0.86,0.07,0.07⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_8	⟨0,0,1⟩	⟨0.27,0.05,0.68⟩	⟨0.08,0.04,0.88⟩	⟨0.54,0.08,0.38⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_9	⟨0.85,0.08,0.07⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.08,0.04,0.88⟩	⟨0,0,1⟩
x_{10}	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.77,0.06,0.17⟩	⟨0.19,0.04,0.77⟩	⟨0,0,1⟩	⟨0,0,1⟩

Table 2. Knowledge of system fault given by engineer 2

R_2	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_1	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.86,0.14,0⟩	⟨0.06,0.01,0.93⟩	⟨0.05,0.02,0.93⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_2	⟨0,0,1⟩	⟨0.29,0.02,0.69⟩	⟨0.08,0.05,0.87⟩	⟨0.58,0.12,0.3⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.1,0.04,0.86⟩
x_3	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.32,0.28,0.4⟩	⟨0.42,0.2,0.38⟩	⟨0.09,0.06,0.85⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_4	⟨0.08,0.03,0.89⟩	⟨0.77,0.06,0.17⟩	⟨0,0,1⟩	⟨0.09,0.06,0.85⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_5	⟨0.08,0.04,0.88⟩	⟨0.09,0.03,0.88⟩	⟨0.09,0.06,0.85⟩	⟨0.08,0.03,0.89⟩	⟨0.19,0.02,0.79⟩	⟨0.06,0.07,0.87⟩	⟨0.08,0.04,0.88⟩	⟨0.08,0.05,0.87⟩	⟨0.08,0.06,0.86⟩
x_6	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.2,0.03,0.77⟩	⟨0.14,0.04,0.82⟩	⟨0.39,0.06,0.55⟩	⟨0,0,1⟩	⟨0.21,0.07,0.72⟩
x_7	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.09,0.05,0.86⟩	⟨0.88,0.06,0.06⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_8	⟨0,0,1⟩	⟨0.29,0.04,0.87⟩	⟨0.1,0.03,0.87⟩	⟨0.55,0.09,0.36⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_9	⟨0.86,0.07,0.07⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.09,0.03,0.88⟩	⟨0,0,1⟩
x_{10}	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.8,0.04,0.16⟩	⟨0.2,0.03,0.77⟩	⟨0,0,1⟩	⟨0,0,1⟩

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Table 3. Knowledge of system fault given by engineer 3

R_3	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
x_1	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.88,0.12,0⟩	⟨0.06,0.02,0.92⟩	⟨0.05,0.03,0.92⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_2	⟨0,0,1⟩	⟨0.28,0.04,0.68⟩	⟨0.06,0.07,0.87⟩	⟨0.6,0.1,0.3⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.08,0.06,0.86⟩
x_3	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.31,0.3,0.39⟩	⟨0.43,0.19,0.38⟩	⟨0.1,0.07,0.83⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_4	⟨0.8,0.4,0.88⟩	⟨0.79,0.06,0.15⟩	⟨0,0,1⟩	⟨0,0.04,0.86⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_5	⟨0.08,0.03,0.89⟩	⟨0.09,0.04,0.87⟩	⟨0.08,0.05,0.87⟩	⟨0.07,0.03,0.9⟩	⟨0.2,0.01,0.79⟩	⟨0.07,0.06,0.87⟩	⟨0.09,0.04,0.87⟩	⟨0.09,0.05,0.86⟩	⟨0.09,0.07,0.84⟩
x_6	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.19,0.05,0.76⟩	⟨0.13,0.05,0.82⟩	⟨0.4,0.06,0.54⟩	⟨0,0,1⟩	⟨0.22,0.08,0.7⟩
x_7	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.08,0.05,0.87⟩	⟨0.87,0.07,0.06⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_8	⟨0,0,1⟩	⟨0.28,0.06,0.66n̄⟩	⟨0.09,0.04,0.87⟩	⟨0.56,0.08,0.36⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩
x_9	⟨0.87,0.07,0.06⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.07,0.04,0.89⟩	⟨0,0,1⟩
x_{10}	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0,0,1⟩	⟨0.81,0.04,0.15⟩	⟨0.21,0.04,0.75⟩	⟨0,0,1⟩	⟨0,0,1⟩

4.2 Decision Making Processes

According to Definition 8 and Definition 9, we calculate lower and upper approximations of optimistic and pessimistic SVN MGRSs in terms of (U, V, R_i, B), respectively.

$$\underline{\sum_{i=1}^{3} R_i^O}(B) = \Big\{\big\langle x_1, \langle 0,0,0.85\rangle\big\rangle, \big\langle x_2, \langle 0.69,0,0.28\rangle\big\rangle, \big\langle x_3, \langle 0.38,0,0.4\rangle\big\rangle,$$
$$\big\langle x_4, \langle 0.18,0,0.77\rangle\big\rangle, \big\langle x_5, \langle 0.79,0,0.18\rangle\big\rangle, \big\langle x_6, \langle 0.55,0,0.37\rangle\big\rangle,$$
$$\big\langle x_7, \langle 0.88,0,1\rangle\big\rangle, \big\langle x_8, \langle 0.68,0,0.27\rangle\big\rangle, \big\langle x_9, \langle 0.07,0,0.85\rangle\big\rangle, \big\langle x_{10}, \langle 0.17,0,0.77\rangle\big\rangle\Big\},$$

$$\overline{\sum_{i=1}^{3} R_i}^{O}(B) = \Big\{\big\langle x_1, \langle 0,0.02,1\rangle\big\rangle, \big\langle x_2, \langle 0.55,0.04,0.3\rangle\big\rangle, \big\langle x_3, \langle 0,0.07,1\rangle\big\rangle,$$
$$\big\langle x_4, \langle 0.08,0.04,0.89\rangle\big\rangle, \big\langle x_5, \langle 0.08,0.02,0.88\rangle\big\rangle, \big\langle x_6, \langle 0,0.05,1\rangle\big\rangle,$$
$$\big\langle x_7, \langle 0.86,0.05,0.1\rangle\big\rangle, \big\langle x_8, \langle 0.54,0.04,0.38\rangle\big\rangle, \big\langle x_9, \langle 0,0.04,1\rangle\big\rangle, \big\langle x_{10}, \langle 0,0.04,1\rangle\big\rangle\Big\},$$

$$\underline{\sum_{i=1}^{3} R_i^P}(B) = \Big\{\big\langle x_1, \langle 0,0,0.88\rangle\big\rangle, \big\langle x_2, \langle 0.68,0,0.29\rangle\big\rangle, \big\langle x_3, \langle 0.38,0,0.43\rangle\big\rangle,$$
$$\big\langle x_4, \langle 0.15,0,0.79\rangle\big\rangle, \big\langle x_5, \langle 0.79,0,0.2\rangle\big\rangle, \big\langle x_6, \langle 0.54,0,0.4\rangle\big\rangle,$$
$$\big\langle x_7, \langle 0.86,0,0.1\rangle\big\rangle, \big\langle x_8, \langle 0.66,0,0.29\rangle\big\rangle, \big\langle x_9, \langle 0.06,0,0.87\rangle\big\rangle, \big\langle x_{10}, \langle 0.15,0,0.81\rangle\big\rangle\Big\},$$

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$$\overline{\sum_{i=1}^{3} R_i}^{P}(B) = \{\langle x_1, \langle 0, 0.01, 1\rangle\rangle, \langle x_2, \langle 0.6, 0.02, 0.3\rangle\rangle, \langle x_3, \langle 0, 0.05, 1\rangle\rangle,$$
$$\langle x_4, \langle 0.1, 0.02, 0.85\rangle\rangle, \langle x_5, \langle 0.09, 0.01, 0.85\rangle\rangle, \langle x_6, \langle 0, 0.03, 1\rangle\rangle,$$
$$\langle x_7, \langle 0.88, 0.04, 0.1\rangle\rangle, \langle x_8, \langle 0.56, 0.03, 0.36\rangle\rangle, \langle x_9, \langle 0, 0.03, 1\rangle\rangle, \langle x_{10}, \langle 0, 0.03, 1\rangle\rangle\}.$$

Suppose in the STFD, fault patterns have no clear differences, which mean they have the same weight, i.e., $w_{jr}=1$. Let bounded rationality alternatives set be $P=\{p_1,p_2,p_3,p_4\}$, the elements be

$$\underline{\sum_{i=1}^{3} R_i^O}(B),\ \overline{\sum_{i=1}^{3} R_i}^{O}(B),\ \underline{\sum_{i=1}^{3} R_i^P}(B),\ \overline{\sum_{i=1}^{3} R_i}^{P}(B).$$

Suppose the parameter $\theta=1$, then the constructed matrices are listed as follows:

$$\phi_1 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & 0.0935 & 0.0416 & 0.0932 \\ p_2 & -0.9347 & 0 & -0.8381 & -0.2403 \\ p_3 & -0.4162 & 0.0838 & 0 & 0.0834 \\ p_4 & -0.9316 & 0.0240 & -0.8338 & 0 \end{pmatrix},$$

$$\phi_2 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -1.0986 & -0.4273 & -1.0082 \\ p_2 & 0.1099 & 0 & 0.1023 & 0.0558 \\ p_3 & 0.0427 & -1.0233 & 0 & -0.9169 \\ p_4 & 0.1008 & -0.5576 & 0.0917 & 0 \end{pmatrix},$$

$$\phi_3 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -1.4863 & 0.0240 & -1.4832 \\ p_2 & 0.1486 & 0 & 0.1486 & 0.0340 \\ p_3 & -0.2403 & -1.4855 & 0 & -1.4825 \\ p_4 & 0.1483 & -0.3398 & 0.1482 & 0 \end{pmatrix},$$

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$$\phi_4 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & 0.2030 & 0.0416 & 0.2027 \\ p_2 & -2.0298 & 0 & -1.9939 & -0.3398 \\ p_3 & -0.4162 & 0.1994 & 0 & 0.1991 \\ p_4 & -2.0275 & 0.0340 & -1.9914 & 0 \end{pmatrix},$$

$$\phi_5 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -1.8760 & -0.4949 & -1.8699 \\ p_2 & 0.1876 & 0 & 0.1823 & 0 \\ p_3 & 0.0495 & -1.8231 & 0 & -1.8181 \\ p_4 & 0.1870 & 0 & 0.1818 & 0 \end{pmatrix},$$

$$\phi_6 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -2.0405 & 0.0340 & -2.0173 \\ p_2 & 0.2041 & 0 & 0.2036 & -0.5144 \\ p_3 & -0.3398 & -2.0362 & 0 & -2.0144 \\ p_4 & 0.2017 & 0.0514 & 0.2014 & 0 \end{pmatrix},$$

$$\phi_7 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & 0.2400 & 0.0340 & 0.2365 \\ p_2 & -2.3995 & 0 & -2.3827 & -0.4376 \\ p_3 & 0.0359 & -2.2409 & 0 & -2.2095 \\ p_4 & 0.2238 & -0.4563 & 0.2210 & 0 \end{pmatrix},$$

$$\phi_8 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -2.2689 & -0.3593 & -2.2378 \\ p_2 & 0.2269 & 0 & 0.2241 & 0.0456 \\ p_3 & 0.0359 & -2.2409 & 0 & -2.2095 \\ p_4 & 0.2238 & -0.4563 & 0.2210 & 0 \end{pmatrix},$$

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$$\phi_9 = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & 0.1911 & 0.0456 & 0.1908 \\ p_2 & -1.9106 & 0 & -1.8644 & 0.0404 \\ p_3 & -0.4563 & 0.1864 & 0 & 0.1863 \\ p_4 & -1.9083 & -0.4041 & -1.8629 & 0 \end{pmatrix},$$

$$\phi_{10} = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & 0.0558 & 0.0340 & 0.0481 \\ p_2 & -0.5576 & 0 & -0.5373 & -0.3593 \\ p_3 & -0.3398 & 0.0537 & 0 & 0.0508 \\ p_4 & -0.4806 & 0.0359 & -0.5081 & 0 \end{pmatrix}.$$

Based on the above results, we further calculate the dominance matrix of alternative p_i over the alternative p_f below:

$$\varphi = \begin{pmatrix} & p_1 & p_2 & p_3 & p_4 \\ p_1 & 0 & -7.9871 & -1.0267 & -7.8451 \\ p_2 & -6.9553 & 0 & -6.7555 & -1.7156 \\ p_3 & -2.4202 & -7.8474 & 0 & -7.6869 \\ p_4 & -6.8516 & -1.5686 & -6.7006 & 0 \end{pmatrix}.$$

Finally, the overall dominance degree of alternative pi under four different granularity levels is calculated as follows, i.e., $\psi(p_1)$=0.3866, $\psi(p_2)$=0.8922, $\psi(p_3)$=0, $\psi(p_4)$=1. The above results show the optimal granularity of SVN MGRSs under bounded rationality is p_4, the corresponding target set is $\overline{\sum_{i=1}^{3} R_i}^{P}(B)$. According to $\max_{j=1}^{s}\left\{s\left(x_j\right)\right\}$, we obtain the results $x^*=x_7$. That is, the vibration faults of the steam turbine generator unit are originated from damage of antithrust bearing (x_7) at first.

4.3 Discussions

Compared with the previous method established in Literature (Zhang et al., 2016), although the results obtained by the neutrosophic probabilistic model are not completely consistent with the previous methods, the optimal results are consistent. Thus, the vibration faults of the steam turbine generator unit are originated from the damage of antithrust bearing (x_7) at first. This also proves the validity and effectiveness of the presented method.

In general, we present some basic notions of SVNSs along with SVN MGRSs, then we put forward a new SVN MAGDM approach by using the idea of TODIM. The merit of the presented SVN MAGDM approach lies in the consideration of bounded rationality in the chapter. In specific, the optimistic and

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pessimistic versions of SVN multigranulation rough approximations own the limitation of extreme information fusion tactics. Thus, we need to put forward a new information fusion tactic that can fuse the different sources of SVN information. In this chapter, we use the TODIM method to combat the above-stated limitation, and the merit of the TODIM method lies in this method can address the complex decision making issues with bounded rationality, which is more in line with real-world decision making situations. In light of the above statement, we can sum up the merit of the presented theoretical and application models presented in this chapter, that is, we propose a new SVN information fusion and analysis rule by means of MGRSs and the TODIM method, which can well fit the cognitive ability of various decision makers.

5. CONCLUSION

In this chapter, we mainly study a new MAGDM method based on SVN MGRSs and TODIM. Firstly, we revisit optimistic and pessimistic SVN MGRSs. Then, we study the optimal granularity selection of SVN MGRSs under bounded rationality by virtue of the TODIM method. At last, to verify the effectiveness of the decision making approach, a case study in the background of STFD is explored in detail.

In future studies, taking into consideration of the significance of MAGDM issues in neutrosophic contexts, exploring valid and applicable decision making algorithms plays a key role in the area of intelligent decision making and knowledge discoveries. Thus, scholars are suggested to put forward efficient MAGDM methods from two perspectives, i.e., information depiction, information fusion and analysis. In specific, on one hand, it is also necessary to extend the presented MAGDM approach in more generalized neutrosophic environments, such as refined NSs. On the other hand, it is meaningful to explore more intelligent SVN MAGDM approaches via explainable artificial intelligence tools, such as game theories.

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Chapter 25
Rough Neutrosophic Multisets: Operations Over Rough Neutrosophic Multisets

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ABSTRACT

The effect of uncertainty in information raises human logical thinking due to many variabilities from expert decision making. The generalization of rough neutrosophic multisets theory tries to overcome this problem. The rough neutrosophic multisets allow the multiplicity of information represented by truth membership sequences, indeterminacy membership sequences, and falsity membership sequences. The lower and upper approximations belong to this triple membership sequence, making the information result more valuable since the relationship among this membership sequence makes sense. This chapter introduces the generalization of rough neutrosophic multisets with some operators and an illustrative example. The rough neutrosophic multisets basic operators, algebraic operators, and "AND" and "OR" operators are defined with some property propositions.

INTRODUCTION

This chapter deliberates the definition of Rough Neutrosophic Multisets (RNM) based on the equivalence relation of the universe of discourse by raising the lower and upper approximations for a single-valued neutrosophic multisets (SVNM) introduced by Ye & Ye (2014) in Pawlak's approximation space (Pawlak, 1982). Generally, rough set are deemed as a powerful tool in explaining imprecise information with a set of prediction levels (lower and upper approximation). While the lower approximation in Pawlak's

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approximation space encompasses element sets that certainly belongs to the object, the upper approximation on the other hand contains element sets that possibly belongs to the object. Moreover, the RNM allows an object's multiplicity element to occur more than once with the same or different value. A RNM is also a generalization of a hybrid method of a neutrosophic set by Smarandache (1999) and a rough neutrosophic set by Broumi et al. (2014). Nevertheless, these previous theories are unable to deal with multiplicity element in the sets of prediction level. Therefore, gaining the determination from the hybrid method based on the generalization theory, RNM will overcome the complex decision-making situation with the aim to provide the exact value of interpretation of uncertainty data (uncertainty data interpretation) that are not explored. Relatively, a pair of lower and upper approximation of neutrosophic multisets $\left(\underline{Nm}(A), \overline{Nm}(A)\right)$ is called rough neutrosophic multisets (RNM). The new notion of RNM theory is useful in solving uncertainty information by considering the same contingent (equivalence relation) of information.

This paper aims to introduce a new notion for uncertainty information set theory namely RNM with a number of operators and properties. The basic operations such as complement, union, intersection, contained, and equality is discussed using examples. Subsequently, the algebraic operations over RNM namely addition, multiplication, scalar multiplication, and power of rough neutrosophic multisets are defined with an illustrative example. Finally, the "AND" and "OR" operation for RNM is presented.

Background

There are various circumstances with uncertain information that classical mathematical tools may not successfully formulate in our real-life problems. For instance, the opinion about a "beauty" can be described as more beauty, as beauty as, beauty than, or less beauty as compared to other opinions. Consequently, this situation demarcates that there are several mathematical tools to deal with uncertainties, such as fuzzy set theory introduced by Zadeh (1965), an intuitionistic fuzzy set theory presented by Atanassov (1983), a rough set theory introduced by Pawlak (1982), and soft set theory initiated by Maji et al., (2003). Thus, for each of the uncertainty theory, different element set in an object is discussed and combined with a suitable feature set accordingly with the presiding situation. Therefore, the rough set theory will be able to deal with approximation sets that are difficult to be described with the available information. The boundary region of the group is categorized as lower and upper approximations. The lower approximation outlines the certainly belonging information while the upper approximation describes the possibly belonging information that slants towards vagueness in decisions. Meanwhile, a fuzzy set theory and intuitionistic fuzzy set theory describes the element set in terms of interval value from 0 to 1. These theories are more applicable to convert the varieties of opinion in an uncertain condition (Atanassov, 1983).

Nonetheless, these concepts are unable to deal with indeterminacy information whereby in an uncertain condition, it often has irresolute opinion among the decision-makers. Therefore, Smarandache (1999) developed a new concept known as neutrosophic set theory, which generalizes a classical set, probability set, fuzzy set theory, and intuitionistic fuzzy set theory, hence introducing an element set of indeterminacy in the universe of discourse. The neutrosophic set theory is represented by a truth membership degree, an indeterminacy degree, and a non-membership degree. Cohesively, this triple element set still embeds the natural properties of fuzzy set and intuitionistic fuzzy set. Meanwhile, the interval value [0,1] is used to represent the element set in the neutrosophic set theory.

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Evidentially, the hybridization of neutrosophic set theory with other uncertainty concept is proven to be highly beneficial in various fields. Broumi (2013) generalized the neutrosophic soft set by extending the generalized neutrosophic set and soft set theory. This generalization set theory overcomes problems involving the indeterminacy for the parameter set, known as the soft set. Meanwhile, Sweety & Arockiarani (2010) defined a neutrosophic rough set for an arbitrary neutrosophic relation R over $U\times U$. A pair (U,R) is termed as neutrosophic approximation space in rough set relation. Subsequently, Ali & Smarandache (2016) introduced a complex neutrosophic where the element set is characterized by a complex-valued truth membership function, complex-valued indeterminate membership function, and complex-valued falsehood membership function. Al-Quran & Hassan (2016) trailed on further to generalize the concept of a neutrosophic vague soft expert set and its operations.

Successively, Broumi et al. (2014) proposed a hybrid structure termed rough neutrosophic set, a generalization of neutrosophic set theory and rough set theory whereby all the related primary operators and algebraic operators of the rough neutrosophic set were introduced. This theory perceptively defines the lower and upper approximation for neutrosophic set in equivalence relation while generalizing the rough fuzzy set theory (Dubois & Prade, 1990) and rough intuitionistic fuzzy set theory (Rizvi et al., 2002). Broumi & Smarandache, (2015) introduced an interval neutrosophic rough set that combines interval-valued neutrosophic set and rough set and deliberated the roughness in an interval-valued neutrosophic set and some of its properties. Hence, with the introduction of rough neutrosophic set theory, applications in various areas were reviewed, such as in the medical organizations (Pramanik & Mondal, 2015), (Mondal & Pramanik, 2015), (Mondal et al., 2016).

Despite numerous hybrid generalizations and application of single value neutrosophic set, this theory is deemed not suitable for multiple or redundant situations for uncertainty conditions, for instance where various phases are required in the selection process of teachers. Therefore, Smarandache (2013) discussed the n-symbol-valued refined neutrosophic logic. In general, the truth membership degree T can be split into many types of truths: $T_1,T_2,\ldots,T_p$, indeterminacy degree I can split into many types of indeterminacies: $I_1,I_2,\ldots,I_r$. Meanwhile, falsity degree F can be split into many types of falsities: $F_1,F_2,\ldots,F_s$, where all $p,r,s\geq 1$ are integers, and $p+r+s=n$. All subcomponents T_j: I_k: F_l are symbols for $j\in\{1,2,\ldots,p\}$, $k\in\{1,2,\ldots,r\}$, and $l\in\{1,2,\ldots,s\}$. Using the same idea, the n-numerical-valued refined neutrosophic logic is introduced. At this juncture, all subcomponents T_j: I_k: F_l are not symbols, but subsets of [0,1] for all $j\in\{1,2,\ldots,p\}$, $k\in\{1,2,\ldots,r\}$, and $l\in\{1,2,\ldots,s\}$. This theory is called neutrosophic refined set theory and it is the generalization of fuzzy multisets (Miyamoto, 2001) and intuitionistic fuzzy multisets (Shinoj & John, 2012). Additionally, Broumi et al. (2014) introduced the neutrosophic refined relation with properties, while Deli et al. (2015) applied this theory for medical diagnosis.

Meanwhile, Ye & Ye (2014) introduced a single-valued neutrosophic multisets as a generalization of a neutrosophic refined set and presented a dice similarity measure for a single-valued neutrosophic multiset in solving the medical diagnosis problem. Correspondingly, Chatterjee et al. (2015) defined a single-valued neutrosophic multisets relation with properties and basic operations in detail. Broumi et al. (2014a) also further discoursed the neutrosophic multisets relation with properties and solved the medical diagnosis problem. Besides that, the hybrid method of uncertainty information involving neutrosophic multisets was effectively presented, for instance, the neutrosophic soft multiset theory (Broumi et al., 2014c) and n-valued refined neutrosophic set theory (Alkhazaleh, 2017).

Constructively, the discussion about neutrosophic multisets is more interesting as this theory allows multiple information to take part in problem-solving. Therefore, based on these fundamentals, the RNM is introduced by representing the lower and upper approximation for triple membership sequences (truth

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membership sequences, indeterminacy sequences, and falsity membership sequences) equivalence relation. Moreover, this theory is the generalization of rough neutrosophic set theory (Broumi et al., 2014), which overcomes the equivalence relation's incomplete and vagueness of information. The RNM differs from the neutrosophic rough multisets, whereby the neutrosophic multisets' approximation space is defined in rough equivalence relation. Likewise, it is a rough set in neutrosophic approximation space (Sweety & Arockiarani, 2010) and rough Attanasov intuitionistic fuzzy set theory (Luo & Xu, 2014).

ROUGH NEUTROSOPHIC MULTISETS

In this section, the primary definition of a rough neutrosophic multisets (RNM) is introduced by defining the lower and upper approximation for neutrosophic multisets. Literally, the RNM theory is derived based on the assumption that only the case of the same repeated numbers of components i=1,2,…,p for a truth membership sequences T_A^i, an indeterminacy membership sequences I_A^i, and a falsity membership sequences F_A^i, of neutrosophic multisets on the same equivalence relation, (U,R) is considered in formulation of illustrative.

Main Definition

Definition 1 Let U be a non-null set with the generic elements in U denoted by x_j and R be an equivalence relation on U.

Let

$$A=\left\{\left\langle\left(T_A^1\left(x_j\right),T_A^2\left(x_j\right),\ldots T_A^p\left(x_j\right)\right),\left(I_A^1\left(x_j\right),I_A^2\left(x_j\right),\ldots I_A^p\left(x_j\right)\right),\left(F_A^1\left(x_j\right),F_A^2\left(x_j\right),\ldots F_A^p\left(x_j\right)\right)\right\rangle\left(x_j\right)\in U,\ j=1,2,\ldots,q\right\}$$

be neutrosophic multisets in U with the truth-membership sequence $\left(T_A^1,T_A^2,\ldots T_A^p\right)$, indeterminacy-membership sequences $\left(I_A^1,I_A^2,\ldots I_A^p\right)$ and falsity-membership sequences $\left(F_A^1,F_A^2,\ldots F_A^p\right)$. The lower and the upper approximations of A in the approximation (U,R) denoted by $\underline{Nm}\left(A\right)$ and $\overline{Nm}\left(A\right)$ are respectively defined as follows:

$$\underline{Nm}\left(A\right)=\left\{\left\langle x_j,\begin{pmatrix}\left(T_{\underline{Nm}(A)}^1\left(x_j\right),T_{\underline{Nm}(A)}^2\left(x_j\right),\ldots,T_{\underline{Nm}(A)}^p\left(x_j\right)\right),\\ \left(I_{\underline{Nm}(A)}^1\left(x_j\right),I_{\underline{Nm}(A)}^2\left(x_j\right),\ldots,I_{\underline{Nm}(A)}^p\left(x_j\right)\right),\\ \left(F_{\underline{Nm}(A)}^1\left(x_j\right),F_{\underline{Nm}(A)}^2\left(x_j\right),\ldots,F_{\underline{Nm}(A)}^p\left(x_j\right)\right)\end{pmatrix}\right\rangle\middle|\boldsymbol{y}\in\left[\boldsymbol{x_j}\right]_{\boldsymbol{R}},\ \boldsymbol{x_j}\in\boldsymbol{U}\right\},$$

$$=\left\{x_j,\left(T_{\underline{Nm}(A)}^i\left(x_j\right),I_{\underline{Nm}(A)}^i\left(x_j\right),F_{\underline{Nm}(A)}^i\left(x_j\right)\right)\middle|y\in\left[x_j\right]_R,\ i,j\in\mathbb{Z}^+,\ x_j\in U\right\}$$

and

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$$\overline{Nm}(A)=\left\{\left\langle x_j,\begin{pmatrix}\left(T^1_{\overline{Nm}(A)}(x_j),T^2_{\overline{Nm}(A)}(x_j),\ldots,T^p_{\overline{Nm}(A)}(x_j)\right),\\ \left(I^1_{\overline{Nm}(A)}(x_j),I^2_{\overline{Nm}(A)}(x_j),\ldots,I^p_{\overline{Nm}(A)}(x_j)\right),\\ \left(F^1_{\overline{Nm}(A)}(x_j),F^2_{\overline{Nm}(A)}(x_j),\ldots,F^p_{\overline{Nm}(A)}(x_j)\right)\end{pmatrix}\right\rangle \middle| \boldsymbol{y}\in\left[\boldsymbol{x_j}\right]_{\boldsymbol{R}},\ \boldsymbol{x_j}\in\boldsymbol{U}\right\}$$

$$=\left\{x_j,\left(T^i_{\overline{Nm}(A)}(x_j),I^i_{\overline{Nm}(A)}(x_j),F^i_{\overline{Nm}(A)}(x_j),\right)\middle| y\in\left[x_j\right]_R,\ i,j\in\mathbb{Z}^+,\ x_j\in U\right\}$$

where j=1,2,…,q and i=1,2,…,p are positive integer,

$$T^i_{\underline{Nm}(A)}(x_j)=\wedge_{y\in[x_j]_R}T^i_A(y_j),$$

$$I^i_{\underline{Nm}(A)}(x_j)=\vee_{y\in[x_j]_R}I^i_A(y_j),$$

$$F^i_{\underline{Nm}(A)}(x_j)=\vee_{y\in[x_j]_R}F^i_A(y_j),$$

$$T^i_{\overline{Nm}(A)}(x_j)=\vee_{y\in[x_j]_R}T^i_A(y_j),$$

$$I^i_{\overline{Nm}(A)}(x_j)=\wedge_{y\in[x_j]_R}I^i_A(y_j),$$

$$F^i_{\overline{Nm}(A)}(x_j)=\wedge_{y\in[x_j]_R}F^i_A(y_j)$$

Here, $\wedge$ and $\vee$ denote "min" and "max'' operators, respectively, and $[x_j]_R$ is the equivalence class of the x_j. $T^i_A(y_j), I^i_A(y_j)$, and $F^i_A(y_j)$ are the membership sequences, indeterminacy sequences, and non-membership sequences of y with respect to A.

It is proven that the triple membership sequences for lower approximation,

$$T^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j)\in[0,1]$$

and

$$0\le T^i_{\underline{Nm}(A)}(x_j)+I^i_{\underline{Nm}(A)}(x_j)+F^i_{\underline{Nm}(A)}(x_j)\le 3.$$

Then, $\underline{Nm}(A)$ is a neutrosophic multisets. Similarly for triple membership sequences for upper approximation,

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$$T^i_{\overline{Nm}(A)}(x), I^i_{\overline{Nm}(A)}(x), F^i_{\overline{Nm}(A)}(x) \in [0,1]$$

and

$$0 \le T^i_{\overline{Nm}(A)}(x) + I^i_{\overline{Nm}(A)}(x) + F^i_{\overline{Nm}(A)}(x) \le 3 .$$

Then, $\overline{Nm}(A)$ is a neutrosophic multisets. It is also proven that $\underline{Nm}(A)$ and $\overline{Nm}(A)$ defined this way are neutrosophic multisets. For $x_j \in U$, the result obtains successively:

$$T^i_A(x_j) + I^i_A(x_j) + F^i_A(x_j) \le 3,$$

$$T^i_A(x_j) \le 3 - \left(I^i_A(x_j) + F^i_A(x_j)\right),$$

$$\max\left\{T^i_A(x_j) : x_j \in U\right\} \le \max\left\{3 - \left(I^i_A(x_j) + F^i_A(x_j)\right)\right\},$$

$$\max\left\{T^i_A(x_j) : x_j \in U\right\} \le 3 - \max\left\{\left(I^i_A(x_j) + F^i_A(x_j)\right) : x_j \in U\right\},$$

$$\max\left\{T^i_A(x_j) : x_j \in U\right\} + \min\left\{\left(I^i_A(x_j) + F^i_A(x_j)\right) : x_j \in U\right\} \le 3,$$

$$\max\left\{T^i_A(x_j) : x_j \in U\right\} + \min\left\{I^i_A(x_j) : x_j \in U\right\} + \min\left\{F^i_A(x_j) : x_j \in U\right\} \le 3,$$

$$T^i_{\underline{Nm}(A)}(x_j) + I^i_{\underline{Nm}(A)}(x_j) + F^i_{\underline{Nm}(A)}(x_j) \le 3 .$$

Hence, $\underline{Nm}(A)$ is a neutrosophic multisets. Similarly, it can be proven that $\overline{Nm}(A)$ is a neutrosophic multisets. Since $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are two neutrosophic multisets in U, thus the neutrosophic multisets mappings $\underline{Nm}, \overline{Nm} : Nm(U) \to Nm(U)$ are respectively referred to as lower and upper rough neutrosophic multisets approximation operators, and the pair of $\left(\underline{Nm}(A), \overline{Nm}(A)\right)$. It is called the rough neutrosophic multisets (RNM) in (U,R), respectively. Rough neutrosophic multisets (RNM) is denoted by:

$$\begin{aligned}\mathcal{R}NM(A) &= \left(\underline{Nm}(A), \overline{Nm}(A)\right) \\ &= \left\{ \left\langle x_j, \left(\left[T^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j)\right], \left[T^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j)\right] \right) \right\rangle \middle| y \in \left[x_j\right]_R,\ i, j \in \mathbb{Z}^+,\ x_j \in U \right\} \end{aligned} \quad (1)$$

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The truth membership sequences $\left[T^i_{\underline{Nm}(A)}(x_j), T^i_{\overline{Nm}(A)}(x_j)\right]$, indeterminate membership sequences $\left[I^i_{\underline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j)\right]$, and falsity membership sequences $\left[F^i_{\underline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j)\right]$ for lower and upper approximation of rough neutrosophic multisets may be in decreasing or increasing order.

From the definition 1 and as eqn. (1), rough neutrosophic multisets has constant membership on the equivalence classes of *U*. If $\underline{Nm}(A) = \overline{Nm}(A)$; i.e.,

$$T^i_{\underline{Nm}(A)}(x) = T^i_{\overline{Nm}(A)}(x),\ I^i_{\underline{Nm}(A)}(x) = I^i_{\overline{Nm}(A)}(x) \text{ and } F^i_{\underline{Nm}(A)}(x) = F^i_{\overline{Nm}(A)}(x),$$

then *Nm* is a definable neutrosophic multisets in the approximation (*U,R*). Otherwise, *Nm* is rough neutrosophic multisets. In the following, the illustrated example is employed to the above concept.

Example 1 Consider the set $U = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8\}$ representing by eight criteria of university candidates and *R* is the equivalence relation of its partition criteria of *U* given by $U/R = \{\{t_1,t_4\}, \{t_2,t_3,t_6\}, \{t_5\}, \{t_7,t_8\}\}$. The three phases ($i$=1,2,3) of university committee's decision were done during the selection process. The criteria met and possibly met by the university candidates are obtained by the neutrosophic set denotes the decision degree of suitable, degree of neutral, and degree of not suitable, which is given by:

Let $Nm(A)$={(⟨t_1, (0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)⟩, ⟨t_4, (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.3)⟩), ⟨t_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)⟩, ⟨t_7, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)⟩} be a neutrosophic multisets of *U*. Therefore, by Definition 1 and as Eqn. (1);

$$\underline{Nm}(A) = \left\{t, \wedge_{y\in[x]_R} T^i_A(y), \vee_{y\in[x]_R} I^i_A(y), \vee_{y\in[x]_R} F^i_A(y)\right\}$$

$$\underline{Nm}(A) = \left\{\{t_1,t_4\}, \{t_5\}\right\} = \{t_1,t_4,t_5\}$$

= {⟨t_1, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)⟩, ⟨t_4, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)⟩, ⟨t_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)⟩} and

$$\overline{Nm}(A) = \{t, \vee_{y\in[x]_R} T^i_A(y), \wedge_{y\in[x]_R} I^i_A(y), \wedge_{y\in[x]_R} F^i_A(y)\}$$

$$\overline{Nm}(A) = \{t_1,t_4,t_5,t_7,t_8\}$$

= {⟨t_1, (0.8, 0.4, 0.3), (0.4, 0.2, 0.1), (0.6, 0.2, 0.1)⟩, ⟨t_4, (0.8, 0.4, 0.3), (0.4, 0.2, 0.1), (0.6, 0.2, 0.1)⟩, ⟨t_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)⟩, ⟨t_7, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)⟩, ⟨t_8 (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)⟩}.

The pair of $\left(\underline{Nm}(A), \overline{Nm}(A)\right)$ is a rough neutrosophic multisets.

For another neutrosophic multisets;

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Let $Nm(B)$= {⟨t_1, (0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)⟩, ⟨t_4, (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.3)⟩, ⟨t_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)⟩}. Therefore, by Definition 1 and as Eqn. (1);

$$\underline{Nm}(B) = \{t_1, t_4, t_5\}$$

= {⟨t_1, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)⟩, ⟨t_4, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)⟩, ⟨t_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)⟩} and

$$\overline{Nm}(B) = \{t_1, t_4, t_5\}$$

= {⟨t_1, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)⟩, ⟨t_4 (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)⟩, ⟨t_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)⟩}

Obviously, $\underline{Nm}(B) = \overline{Nm}(B)$, where $Nm(B)$ is a definable neutrosophic multisets in the approximation space (U,R). Otherwise, neutrosophic multisets are RNM. A pair of RNM collects neutrosophic multisets with the boundary region notated by lower and upper approximation conditions. Relational tables and matrices can present the RNM to represent the fuzzy set theory. Since the triple $\left(T_A^i, I_A^i, F_A^i\right)$ has values within the interval [0,1], the elements of the rough neutrosophic matrix also have values within [0,1].

Operations Over Rough Neutrosophic Multisets

Let $Nm(A)$, $Nm(B)$, and $Nm(C)$ are elements set in the neutrosophic multisets. The further study of the operations between $Nm(A)$, $Nm(B)$, and $Nm(C)$ verifies that length (L); $L(x_j: Nm(A))= L(x_j: Nm(B))= L(x_j: Nm(C))$ is valid for every $x_j \in U$, if not, then use a sufðcient number of zeroes to ðll the truth-membership values, the indeterminacy-membership values, and falsity-membership values of the smaller-length sequences, respectively, so that the lengths of sequences are equal to facilitate computing.

Basic Operations Over Rough Neutrosophic Multisets (Complement, Union, Intersection, Contained, Equality)

Definition 2 (Complement) Let $RNM(A) = \left(\underline{Nm}(A), \overline{Nm}(A)\right)$ be a rough neutrosophic multisets in (U,R). The rough complement of $RNM(A)$ is denoted by

$$\sim RNM(A) = \left(\underline{Nm}(A)^C, \overline{Nm}(A)^C\right),$$

where $\underline{Nm}(A)^C$ and $\overline{Nm}(A)^C$ are the complements of a neutrosophic multisets of $\underline{Nm}(A)$ and $\overline{Nm}(A)$ respectively,

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$$\underline{Nm}(A)^c = \left\{ x_j, \left(F^i_{\underline{Nm}(A)}(x_j), 1 - I^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(A)}(x_j), \right) | y \in \left[x_j \right]_R,\ i, j \in \mathbb{Z}^+,\ x_j \in U \right\}$$

$$= \left\{ \left\langle x_j, \begin{pmatrix} \left(F^1_{\underline{Nm}(A)}(x_j), F^2_{\underline{Nm}(A)}(x_j), \ldots, F^p_{\underline{Nm}(A)}(x_j) \right), \\ \left(1 - I^1_{\underline{Nm}(A)}(x_j), 1 - I^2_{\underline{Nm}(A)}(x_j), \ldots, 1 - I^p_{\underline{Nm}(A)}(x_j) \right), \\ \left(T^1_{\underline{Nm}(A)}(x_j), T^2_{\underline{Nm}(A)}(x_j), \ldots, T^p_{\underline{Nm}(A)}(x_j) \right) \end{pmatrix} \right\rangle | y \in \left[x_j \right]_R,\ i, j \in \mathbb{Z}^+,\ x_j \in U \right\},$$

and

$$\overline{Nm}(A)^C = \left\{ x_j, \left(F^i_{\overline{Nm}(A)}(x_j), 1 - I^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(A)}(x_j), \right) | y \in \left[x_j \right]_R,\ i, j \in \mathbb{Z}^+,\ x_j \in U \right\}$$

$$= \left\{ \left\langle x_j, \begin{pmatrix} \left(F^1_{\overline{Nm}(A)}(x_j), F^2_{\overline{Nm}(A)}(x_j), \ldots, F^p_{\overline{Nm}(A)}(x_j) \right), \\ \left(1 - I^1_{\overline{Nm}(A)}(x_j), 1 - I^2_{\overline{Nm}(A)}(x_j), \ldots, 1 - I^p_{\overline{Nm}(A)}(x_j) \right), \\ \left(T^1_{\overline{Nm}(A)}(x_j), T^2_{\overline{Nm}(A)}(x_j), \ldots, T^p_{\overline{Nm}(A)}(x_j) \right) \end{pmatrix} \right\rangle | y \in \left[x_j \right]_R,\ i, j \in \mathbb{Z}^+,\ x_j \in U \right\} \tag{2}$$

where j=1,2,…,q and i=1,2,…,p are positive integers.

Example 2 Consider rough neutrosophic multisets, *RNM*(*A*) in the set $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ represented by seven university candidates. Whereas $y \in [x]_R$ is the equivalence relation of *RNM*(*A*) by considering the top three highest school examination results from ($[x]_R$: $x = \{x_1, x_2, x_4\}$). Here, the multiple phases of the university committee were done for (i=1,2,3) during the selection process. The decision variable of lower and upper approximation is denoted by the degree of good, degree of intermediate, and degree of bad.

Therefore, let

RNM(*A*)= {⟨x_1,[(0.6, 0.4, 0.4), (0.8, 0.4, 0.5), (0.4, 0.3, 0.5)], [(0.7, 0.3, 0.4), (0.7, 0.6, 0.5), (0.3, 0.2, 0.7)]⟩, ⟨x_2, [(0.4, 0.3, 0.3), (0.2, 0.4, 0.4), (0.7, 0.8, 0.4)], [(0.5, 0.3, 0.4), (0.3, 0.3, 0.5), (0.7, 0.1, 0.5)⟩, ⟨x_4, [(0.2, 0.5, 0.7), (1.0, 1.0, 0.0), (0.1, 0.5, 0.3)], [(0.7, 0.8, 0.0), (0.9, 0.2, 0.5), (0.2, 0.8, 0.5)⟩}

Then the complement of *RNM*(*A*) is defined as:

$$\sim RNM(A) = \left(\underline{Nm}(A)^C, \overline{Nm}(A)^C \right)$$

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=$\{\langle x_1,$ [(0.4,0.6,0.6), (0.5,0.6,0.8), (0.5,0.7,0.4], [(0.4,0.7,0.7), (0.5,0.4,0.7), (0.7,0.8,0.3)], $\langle x_2,$ [(0.3,0.7,0.4), (0.4,0.6,0.2), (0.4,0.2,0.7)], [(0.4,0.7,0.5), (0.5,0.7,0.3), (0.5,0.9,0.7)]$\rangle$, $\langle x_4,$ [(0.7,0.5,0.2), (0.0,0.0,1.0), (0.3,0.5,0.1)], [(0.0,0.2,0.7), (0.5,0.8,0.9), (0.5,0.2,0.2)]$\rangle\}$.

Definition 3 (Union) The union of two rough neutrosophic multisets, *RNM*(*A*) and *RNM*(*B*) over *U*, denoted by *RNM*(*A*)∪*RNM*(*B*), is a rough neutrosophic multisets *RNM*(*C*), where $RNM(C)=\left(\underline{Nm}(C),\overline{Nm}(C)\right)$. Here, $\underline{Nm}(C)$ and $\overline{Nm}(C)$ are the union of lower and upper approximation of *RNM*(*A*) and *RNM*(*B*).

$$\underline{Nm}(C)=\underline{Nm}(A)\cup\underline{Nm}(B)=\left\{\left\langle x_j,\left(\max\left\{T^i_{\underline{Nm}(A)}(x_j),T^i_{\underline{Nm}(B)}(x_j)\right\},\min\left\{I^i_{\underline{Nm}(A)}(x_j),I^i_{\underline{Nm}(B)}(x_j)\right\},\right.\right.$$
$$\left.\left.\min\left\{F^i_{\underline{Nm}(A)}(x_j),F^i_{\underline{Nm}(B)}(x_j)\right\}\right)\right\rangle : x_j\in U, i,j\in\mathbb{Z}^+\right\},$$

and

$$\overline{Nm}(C)=\overline{Nm}(A)\cup\overline{Nm}(B)=\left\{\left\langle x_j,\left\{\max\left\{T^i_{\overline{Nm}(A)}(x_j),T^i_{\overline{Nm}(B)}(x_j)\right\},\min\left\{I^i_{\overline{Nm}(A)}(x_j),I^i_{\overline{Nm}(B)}(x_j)\right\}\right.\right.\right.$$
$$\left.\left.\left.\min\left\{F^i_{\overline{Nm}(A)}(x_j),F^i_{\overline{Nm}(B)}(x_j)\right\}\right\}\right\rangle : x_j\in U, i,j\in\mathbb{Z}^+\right\} \quad (3)$$

Example 3 Consider *RNM*(*A*) (as in Example 2) and *RNM*(*B*) are two rough neutrosophic multisets, given by:

RNM(*B*)= $\{\langle x_1,$ [(0.6, 0.1, 0.2), (0.7, 0.2, 0.5), (0.7, 0.3, 0.5)], [(0.3, 0.3, 0.3), (0.8, 0.6, 0.5), (1.0, 0.2, 0.7)]$\rangle$, $\langle x_2,$ [(0.4, 0.4, 0.7), (0.3, 0.4, 0.4), (0.6, 0.5, 0.6)], [(0.7, 0.8, 0.4), (0.6, 0.2, 0.5), (0.6, 0.1, 0.5)]$\rangle$, $\langle x_3,$ [(0.3, 0.4, 0.5), (0.6, 0.4, 0.0), (1.0, 1.0, 0.0)], [(0.7, 0.2, 0.5), (0.1, 0.5, 0.3), (0.2, 0.8, 0.5)]$\rangle$, $\langle x_4,$ [(0.4, 0.5, 0.6), (0.7, 0.8, 0.2), (1.0, 1.0, 0.0), [(0.9, 0.2, 0.1), (0.6, 0.5, 0.3), (0.2, 0.2, 0.7)]$\rangle\}$

Then,

$$RNM(A)\cup RNM(B)=RNM(C)=\left(\underline{Nm}(C),\overline{Nm}(C)\right)$$

= $\{\langle x_1,$ [(0.6, 0.4, 0.4), (0.7, 0.2, 0.5), (0.4, 0.3, 0.5)], [(0.7, 0.3, 0.4), (0.7, 0.6, 0.5), (0.3, 0.2, 0.7)]$\rangle$, $\langle x_2,$ [(0.4, 0.3, 0.7), (0.2, 0.3, 0.4), (0.6, 0.5, 0.4), [(0.7, 0.8, 0.4), (0.3, 0.2, 0.5), (0.6, 0.1, 0.5)]$\rangle$, $\langle x_3,$ [(0.3, 0.4, 0.5), (0.6, 0.4, 0.0), (1.0, 1.0, 0.0)], [(0.7, 0.2, 0.5), (0.1, 0.5, 0.3), (0.2, 0.8, 0.5)]$\rangle$, $\langle x_4,$ [(0.4, 0.5, 0.7), (0.7, 0.8, 0.0), (1.0, 0.5, 0.0)], [(0.9, 0.8, 0.1), (0.6, 0.2, 0.3), (0.2, 0.2, 0.5)]$\rangle\}$.

Definition 4 (Intersection) The intersection of two rough neutrosophic multisets *RNM*(*A*) and *RNM*(*B*) over *U*, denoted by *RNM*(*A*)∩*RNM*(*B*) is a rough neutrosophic multiset *RNM*(*D*), where

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$RNM(D) = \left(\underline{Nm}(D), \overline{Nm}(D)\right)$. Here, $\underline{Nm}(D)$ and $\overline{Nm}(D)$ are the intersection of lower and upper approximation of *RNM*(*A*) and *RNM*(*B*), respectively.

$$\underline{Nm}(D) = \underline{Nm}(A) \cap \underline{Nm}(B) = \left\{\left\langle x_j, \left(\min\left\{T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j)\right\}, \max\left\{I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j)\right\}, \max\left\{F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j)\right\}\right)\right\rangle : x_j \in U, i, j \in \mathbb{Z}^+\right\},$$

and

$$\overline{Nm}(D) = \overline{Nm}(A) \cap \overline{Nm}(B) = \left\{\left\langle x_j, \left(\min\left\{T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j)\right\}, \max I\left\{{}^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j)\right\}, \max\left\{F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j)\right\}\right)\right\rangle : x_j \in U, i, j \in \mathbb{Z}^+\right\} \tag{4}$$

Example 4 Considering the *RNM*(*A*) in example 2 and *RNM*(*B*) in example 3, then

$$RNM(A) \cap RNM(B) = RNM(D) = \left(\underline{Nm}(D), \overline{Nm}(D)\right)$$

= {⟨x_1, [(0.6, 0.1, 0.2), (0.8, 0.4, 0.5), (0.7, 0.3, 0.5)], [(0.3, 0.3, 0.3), (0.8, 0.6, 0.5), (0.3, 0.2, 0.7)]⟩, ⟨x_2, [(0.4, 0.3, 0.3), (0.3, 0.4, 0.4), (0.7, 0.8, 0.6), [(0.5, 0.3, 0.4), (0.6, 0.5, 0.5), (0.7, 0.5, 0.5)]⟩, ⟨x_3, [(0.3, 0.4, 0.5), (0.6, 0.4, 0.0), (1.0, 1.0, 0.0)], [(0.7, 0.2, 0.5), (0.1, 0.5, 0.3), (0.2, 0.8, 0.5)]⟩, ⟨x_4, [(0.2, 0.5, 0.6), (1.0, 1.0, 0.2), (1.0, 1.0, 0.3)], [(0.7, 0.2, 0.0), (0.9, 0.5, 0.5), (0.2, 0.8, 0.7)]⟩}.

Definition 5 (Contained) The rough neutrosophic multisets *RNM*(*A*) is contained in the other rough neutrosophic multisets *RNM*(*B*) over *U*, denoted as *RNM*(*A*)∩*RNM*(*B*) iff $\underline{Nm}(A) \subseteq \underline{Nm}(B)$ and $\overline{Nm}(A) \subseteq \overline{Nm}(B)$, where

$$\underline{Nm}(A) \subseteq \underline{Nm}(B) = \left\{\left\langle x_j, \left(T^i_{\underline{Nm}(A)}(x_j) \le T^i_{\underline{Nm}(B)}(x_j), I^i_{\underline{Nm}(A)}(x_j) \ge I^i_{\underline{Nm}(B)}(x_j), F^i_{\underline{Nm}(A)}(x_j) \ge F^i_{\underline{Nm}(B)}(x_j)\right)\right\rangle : x_j \in U, i, j \in \mathbb{Z}^+\right\},$$

and

$$\overline{Nm}(A) \subseteq \overline{Nm}(B) = \left\{\left\langle x_j, \left(T^i_{\overline{Nm}(A)}(x_j) \le T^i_{\overline{Nm}(B)}(x_j), I^i_{\overline{Nm}(A)}(x_j) \ge I^i_{\overline{Nm}(B)}(x_j), F^i_{\overline{Nm}(A)}(x_j) \ge F^i_{\overline{Nm}(B)}(x_j)\right)\right\rangle : x_j \in U, i, j \in \mathbb{Z}^+\right\} \tag{5}$$

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Definition 6 (equality) The rough neutrosophic multisets $RNM(A)$ is equal to rough neutrosophic multisets $RNM(B)$ over U, denoted as $RNM(A)=RNM(B)$ iff $\underline{Nm}(A)=\underline{Nm}(B)$ and $\overline{Nm}(A)=\overline{Nm}(B)$, where

$$\underline{Nm}(A)=\underline{Nm}(B),=\left\{\left\langle x_j,\left(T^i_{\underline{Nm}(A)}(x_j)=T^i_{\underline{Nm}(B)}(x_j),I^i_{\underline{Nm}(A)}(x_j)=I^i_{\underline{Nm}(B)}(x_j),\right.\right.$$
$$\left.\left.F^i_{\underline{Nm}(A)}(x_j)=F^i_{\underline{Nm}(B)}(x_j)\right)\right\rangle : x_j\in U,i,j\in\mathbb{Z}^+\right\},$$

and

$$\overline{Nm}(A)=\overline{Nm}(B)=\left\{\left\langle x_j,\left(T^i_{\overline{Nm}(A)}(x_j)=T^i_{\overline{Nm}(B)}(x_j),I^i_{\overline{Nm}(A)}(x_j)=I^i_{\overline{Nm}(B)}(x_j),\right.\right.$$
$$\left.\left.F^i_{\overline{Nm}(A)}(x_j)=F^i_{\overline{Nm}(B)}(x_j)\right)\right\rangle : x_j\in U,i,j\in\mathbb{Z}^+\right\} \quad (6)$$

If $RNM(A)$, $RNM(B)$ and $RNM(C)$ are rough neutrosophic multisets in (U,R), then the following properties are derived directly from Definition 2-6.

Proposition 1

(1) Involution property
$\sim(\sim RNM(A)) = RNM(A)$

(2) Idempotent property
i) $RNM(A) \cup RNM(A) = RNM(A)$
ii) $RNM(A) \cap RNM(A) = RNM(A)$

(3) Commutative property
i) $RNM(A) \cup RNM(B) = RNM(B) \cup RNM(A)$
ii) $RNM(A) \cap RNM(B) = RNM(B) \cap RNM(A)$

(4) Associative property
i) $(RNM(A) \cup RNM(B)) \cup RNM(C) = RNM(A) \cup (RNM(B) \cup RNM(C))$
ii) $(RNM(A) \cap RNM(B)) \cap RNM(C) = RNM(A) \cap (RNM(B) \cap RNM(C))$

(5) Distributive property
i) $RNM(A) \cup (RNM(B) \cap RNM(C)) = (RNM(A) \cup RNM(B)) \cap (RNM(A) \cup RNM(C))$
ii) $RNM(A) \cap (RNM(B) \cup RNM(C)) = (RNM(A) \cap RNM(B)) \cup (RNM(A) \cap RNM(C))$.

Proof:

$$(1)\quad \sim\left(\sim RNM(A)\right)=\sim\left(\underline{Nm}(A)^C,\overline{Nm}(A)^C\right)=\left(\underline{Nm}(A),\overline{Nm}(A)\right)=RNM(A)$$

$$(2)\ \text{i)}\ RNM(A)\cup RNM(A)=(\underline{Nm}(A)\cup\underline{Nm}(A),\overline{Nm}(A)\cup\overline{Nm}(A))$$

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$$= \begin{bmatrix} \left(\max\left\{ T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(A)}(x_j) \right\}, \min\left\{ I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j) \right\}, \min\left\{ F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j) \right\} \right), \\ \left(\max\left\{ T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(A)}(x_j) \right\}, \min\left\{ I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j) \right\}, \min\left\{ F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j) \right\} \right) \end{bmatrix}$$

$$= \left[T^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j) \right], \left[T^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j) \right]$$
$$= \left(\underline{Nm}(A), \overline{Nm}(A) \right) = RNM(A).$$

ii) The proof is similar the proof of (2) i)

(3) (i) $RNM(A) \cup RNM(B) = (\underline{Nm}(A) \cup \underline{Nm}(B), \overline{Nm}(A) \cup \overline{Nm}(B))$

$$= \begin{bmatrix} \left(\max\left\{ T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j) \right\}, \min\left\{ I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j) \right\}, \min\left\{ F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j) \right\} \right), \\ \left(\max\left\{ T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j) \right\}, \min\left\{ I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j) \right\}, \min\left\{ F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j) \right\} \right) \end{bmatrix}$$

$$= \begin{bmatrix} \left(\max\left\{ T^i_{\underline{Nm}(B)}(x_j), T^i_{\underline{Nm}(A)}(x_j) \right\}, \min\left\{ I^i_{\underline{Nm}(B)}(x_j), I^i_{\underline{Nm}(A)}(x_j) \right\}, \min\left\{ F^i_{\underline{Nm}(B)}(x_j), F^i_{\underline{Nm}(A)}(x_j) \right\} \right), \\ \left(\max\left\{ T^i_{\overline{Nm}(B)}(x_j), T^i_{\overline{Nm}(A)}(x_j) \right\}, \min\left\{ I^i_{\overline{Nm}(B)}(x_j), I^i_{\overline{Nm}(A)}(x_j) \right\}, \min\left\{ F^i_{\overline{Nm}(B)}(x_j), F^i_{\overline{Nm}(A)}(x_j) \right\} \right) \end{bmatrix}$$

$$= (\underline{Nm}(B) \cup \underline{Nm}(A), \overline{Nm}(B) \cup \overline{Nm}(A)) = RNM(B) \cup RNM(A)$$

ii) The proof is similar the proof of (3) i)

(4) i) $(RNM(A) \cup RNM(B)) \cup RNM(C)$

$$= (\underline{Nm}(A) \cup \underline{Nm}(B), \overline{Nm}(A) \cup \overline{Nm}(B)) \cup \left(\underline{Nm}(C), \overline{Nm}(C) \right)$$

$$= \begin{bmatrix} \left(\max\{ T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j) \}, \min\{ I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j) \}, \min\{ F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j) \} \right), \\ \left(\max\{ T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j) \}, \min\{ I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j) \}, \min\{ F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j) \} \right) \end{bmatrix}$$

$$\cup \left(\underline{Nm}(C), \overline{Nm}(C) \right)$$

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$$=\left[\left(\max\left(\max\{T^i_{\underline{Nm}(A)}(x_j),T^i_{\underline{Nm}(B)}(x_j)\}\right),T^i_{\underline{Nm}(C)}(x_j)\right),\left(\min\left(\min\{I^i_{\underline{Nm}(A)}(x_j),I^i_{\underline{Nm}(B)}(x_j)\}\right),I^i_{\underline{Nm}(C)}(x_j)\right),\right.$$
$$\left.\left(\min\left(\min\{F^i_{\underline{Nm}(A)}(x_j),F^i_{\underline{Nm}(B)}(x_j)\}\right),F^i_{\underline{Nm}(C)}(x_j)\right)\right],\left[\left(\max\left(\max\{T^i_{\overline{Nm}(A)}(x_j),T^i_{\overline{Nm}(B)}(x_j)\}\right),T^i_{\overline{Nm}(C)}(x_j)\right),\right.$$
$$\left.\left(\min\left(\min\{I^i_{\overline{Nm}(A)}(x_j),I^i_{\overline{Nm}(B)}(x_j)\}\right),I^i_{\overline{Nm}(C)}(x_j)\right),\left(\min\left(\min\{F^i_{\overline{Nm}(A)}(x_j),F^i_{\overline{Nm}(B)}(x_j)\}\right),F^i_{\overline{Nm}(C)}(x_j)\right)\right]$$

$$=\left[\left(\max\{T^i_{\underline{Nm}(A)}(x_j),\left(\max\{T^i_{\underline{Nm}(B)}(x_j),T^i_{\underline{Nm}(C)}(x_j)\}\right)\}\right),\left(\min\{I^i_{\underline{Nm}(A)}(x_j),\left(\min\{I^i_{\underline{Nm}(B)}(x_j),I^i_{\underline{Nm}(C)}(x_j)\}\right)\}\right),\right.$$
$$\left.\left(\min\{F^i_{\underline{Nm}(A)}(x_j),\left(\min\{F^i_{\underline{Nm}(B)}(x_j),F^i_{\underline{Nm}(C)}(x_j)\}\right)\}\right)\right],\left[\left(\max\{T^i_{\overline{Nm}(A)}(x_j),\left(\max\{T^i_{\overline{Nm}(B)}(x_j),T^i_{\overline{Nm}(C)}(x_j)\}\right)\}\right),\right.$$
$$\left.\left(\min\{I^i_{\overline{Nm}(A)}(x_j),\left(\min\{I^i_{\overline{Nm}(B)}(x_j),I^i_{\overline{Nm}(C)}(x_j)\}\right)\}\right),\left(\min\{F^i_{\overline{Nm}(A)}(x_j),\left(\min\{F^i_{\overline{Nm}(B)}(x_j),F^i_{\overline{Nm}(C)}(x_j)\}\right)\}\right)\right]$$

$$=\left(\underline{Nm}(A),\overline{Nm}(A)\right)\cup(\underline{Nm}(B)\cup\underline{Nm}(C),\overline{Nm}(B)\cup\overline{Nm}(C))$$

$$= RNM(A) \cup (RNM(B) \cup RNM(C))$$

ii) The proof is similar the proof of (4) i)

(5) i) Let us consider three rough neutrosophic multisets derived from medical information to diagnose the diseases of patient A. Two different time (i=1,2) of symptoms was taken to make the result more accurate and two types of symptoms (x_1,x_2) are considered as the same relation to diagnosing the disease.

$$RNM(A)=\{x_1|\left[\left((0.8,0.6),(0.3,0.2),(0.4,0.2)\right)\right],\left[\left((0.8,0.2),(0.5,0.4),(0.6,0.5)\right)\right],$$
$$x_2|\left[\left((0.5,0.4),(0.4,0.4),(0.6,0.3)\right)\right],\left[\left((0.7,0.1),(0.3,0.4),(0.5,0.3)\right)\right]\}$$

$$RNM(B)=\{x_1|\left[\left((0.5,0.5),(0.4,0.3),(0.2,0.1)\right)\right],\left[\left((0.7,0.4),(0.4,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.6,0.5),(0.3,0.8),(0.4,0.4)\right)\right],\left[\left((0.6,0.3),(0.6,0.5),(0.3,0.4)\right)\right]\}$$

$$RNM(C)=\{x_1|\left[\left((0.8,0.7),(0.2,0.1),(0.3,0.2)\right)\right],\left[\left((0.4,0.5),(0.5,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.4,0.5),(0.7,0.4),(0.7,0.4)\right)\right],\left[\left((0.8,0.9),(0.4,0.7),(0.5,0.4)\right)\right]\}$$

Then check for A1 and A2.

Let A1: $RNM(A) \cup (RNM(B) \cap RNM(C))$

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$$RNM(B)\cap RNM(C)=$$
$$\{x_1|\left[\left((0.5,0.5),(0.4,0.3),(0.3,0.2)\right)\right],\left[\left((0.4,0.4),(0.5,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.4,0.5),(0.7,0.8),(0.7,0.4)\right)\right],\left[\left((0.6,0.3),(0.6,0.7),(0.5,0.4)\right)\right]\}$$

$$RNM(A)\cup(RNM(B)\cap RNM(C))=$$
$$\{x_1|\left[\left((0.8,0.6),(0.3,0.2),(0.3,0.2)\right)\right],\left[\left((0.8,0.4),(0.5,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.5,0.5),(0.4,0.4),(0.6,0.3)\right)\right],\left[\left((0.7,0.3),(0.3,0.4),(0.5,0.3)\right)\right]\}$$

Let A2: $(RNM(A) \cup RNM(B)) \cap (RNM(A) \cup RNM(C))$,

$$(RNM(A)\cup RNM(B))=$$
$$\{x_1|\left[\left((0.8,0.6),(0.3,0.2),(0.2,0.1)\right)\right],\left[\left((0.8,0.4),(0.4,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.6,0.5),(0.3,0.4),(0.4,0.3)\right)\right],\left[\left((0.7,0.3),(0.3,0.4),(0.3,0.3)\right)\right]\}$$

$$(RNM(A)\cup RNM(C))=$$
$$\{x_1|\left[\left((0.8,0.7),(0.2,0.1),(0.3,0.2)\right)\right],\left[\left((0.8,0.5),(0.5,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.5,0.5),(0.4,0.4),(0.6,0.3)\right)\right],\left[\left((0.8,0.9),(0.3,0.4),(0.5,0.3)\right)\right]\}$$

$$(RNM(A)\cup RNM(B))\cap(RNM(A)\cup RNM(C))=$$
$$\{x_1|\left[\left((0.8,0.6),(0.3,0.2),(0.3,0.2)\right)\right],\left[\left((0.8,0.4),(0.5,0.3),(0.4,0.4)\right)\right],$$
$$x_2|\left[\left((0.5,0.5),(0.4,0.4),(0.6,0.3)\right)\right],\left[\left((0.7,0.3),(0.3,0.4),(0.5,0.3)\right)\right]\}$$

Therefore A1=A2, and distributive property is true for rough neutrosophic multisets.

ii) The proof is similar the proof of (5) i)

Algebraic Operations over Rough Neutrosophic Multisets (Addition, Multiplication, Scalar Multiplication, and Power of Rough Neutrosophic Multisets)

Definition 7 (Addition). The addition of two rough neutrosophic multisets $RNM(A)$ and $RNM(B)$ in (U,R) is denoted by

$$RNM(A)\oplus RNM(B)=\underline{Nm}(A)+\underline{Nm}(B),\overline{Nm}(A)+\overline{Nm}(B).$$

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Here, the upper and lower approximation for truth membership sequence, indeterminacy membership sequence, and falsity membership sequence are defined as

$$\underline{Nm}(A)+\underline{Nm}(B)=\left\{\left\langle x_j,\left(T^i_{\underline{Nm}(A)}(x_j)+T^i_{\underline{Nm}(B)}(x_j)-T^i_{\underline{Nm}(A)}(x_j)\cdot T^i_{\underline{Nm}(B)}(x_j)\right),\right.$$
$$\left.\left(I^i_{\underline{Nm}(A)}(x_j)\cdot I^i_{\underline{Nm}(B)}(x_j)\right),\left(F^i_{\underline{Nm}(A)}(x_j)\cdot F^i_{\underline{Nm}(B)}(x_j)\right)\right\rangle : x_j\in U, i, j\in\mathbb{Z}^+\right\},$$

and

$$\overline{Nm}(A)+\overline{Nm}(B)=\left\{\left\langle x_j,\left(T^i_{\overline{Nm}(A)}(x_j)+T^i_{\overline{Nm}(B)}(x_j)-T^i_{\overline{Nm}(A)}(x_j)\cdot T^i_{\overline{Nm}(B)}(x_j)\right),\right.$$
$$\left.\left(I^i_{\overline{Nm}(A)}(x_j)\cdot I^i_{\overline{Nm}(B)}(x_j)\right),\left(F^i_{\overline{Nm}(A)}(x_j)\cdot F^i_{\overline{Nm}(B)}(x_j)\right)\right\rangle : x_j\in U, i, j\in\mathbb{Z}^+\right\} \quad (7)$$

Example 5 Let D = {Viral Fever (Vf)} be a disease and S = {Temperature (x_1), Cough (x_2)} be a set of symptoms. The relation between patient and symptoms $RNM(A)$, and between disease and symptoms $RNM(B)$ are considered in the same equivalence relation. For diagnosis, the patient A is kept under supervision for one day, and his symptoms pattern are monitored at the two-time inspection (morning: 6.00 am, noon: 12.00 pm). There are three specified degree of membership function for each time inspection which is for truth description of symptom, indeterminacy description symptom and falsity description symptom. The findings of the patient A can be summarized and represented with the RNM as follows:

$$RNM(A)=\left\{x_1|\left[\left((0.7,0.5),(0.3,0.9),(0.6,0.7)\right)\right],\left[\left((0.7,0.4),(0.5,0.3),(0.9,0.4)\right)\right],\right.$$
$$\left.x_2|\left[\left((0.4,0.3),(0.6,0.7),(0.7,0.5)\right)\right],\left[\left((0.8,0.2),(0.4,0.5),(0.6,0.3)\right)\right]\right\},$$

and

$$RNM(B)=\left\{x_1|\left[\left((0.6,0.2),(0.5,0.4),(0.7,0.9)\right)\right],\left[\left((0.6,0.3),(0.3,0.4),(0.5,0.5)\right)\right],\right.$$
$$\left.x_2|\left[\left((0.6,0.7),(0.4,0.9),(0.4,0.5)\right)\right],\left[\left((0.7,0.4),(0.5,0.4),(0.5,0.5)\right)\right]\right\}$$

Then, we have,

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$$RNM(A) \oplus RNM(B) = \underline{Nm}(A) + \underline{Nm}(B), \overline{Nm}(A) + \overline{Nm}(B)$$
$$= \left\{ x_1 | \left[\left((0.88, 0.6), (0.15, 0.36), (0.42, 0.63) \right) \right], \left[\left((0.88, 0.58), (0.15, 0.12), (0.45, 0.2) \right) \right], \right.$$
$$\left. x_2 | \left[\left((0.76, 0.79), (0.24, 0.63), (0.28, 0.25) \right) \right], \left[\left((0.94, 0.52), (0.2, 0.2), (0.3, 0.15) \right) \right] \right\}.$$

For further result, the addition operation can be used to present the relationship measure of the relation between patient and symptoms *RNM*(*A*), and between disease and symptoms *RNM*(*B*).

Definition 8 (Multiplication). The multiplication between two rough neutrosophic multisets *RNM*(*A*) and *RNM*(*B*) in (*U,R*) is denoted by

$$RNM(A) \otimes RNM(B) = < \underline{Nm}(A) \cdot \underline{Nm}(B), \overline{Nm}(A) \cdot \overline{Nm}(B) > .$$

Here, the upper and lower approximation for truth membership sequence, indeterminacy membership sequence, and falsity membership sequence are defined as

$$\underline{Nm}(A) \cdot \underline{Nm}(B) =$$
$$\left\{ \left\langle x_j, \left(T^i_{\underline{Nm}(A)}(x_j) \cdot T^i_{\underline{Nm}(B)}(x_j) \right), \left(I^i_{\underline{Nm}(A)}(x_j) + I^i_{\underline{Nm}(B)}(x_j) - I^i_{\underline{Nm}(A)}(x_j) \cdot I^i_{\underline{Nm}(B)}(x_j) \right), \right.\right.$$
$$\left.\left. \left(F^i_{\underline{Nm}(A)}(x_j) + F^i_{\underline{Nm}(B)}(x_j) - F^i_{\underline{Nm}(A)}(x_j) \cdot F^i_{\underline{Nm}(B)}(x_j) \right) \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}$$

and

$$\overline{Nm}(A) \cdot \overline{Nm}(B) =$$
$$\left\{ \left\langle x_j, \left(T^i_{\overline{Nm}(A)}(x_j) \cdot T^i_{\overline{Nm}(B)}(x_j) \right), \left(I^i_{\overline{Nm}(A)}(x_j) + I^i_{\overline{Nm}(B)}(x_j) - I^i_{\overline{Nm}(A)}(x_j) \cdot I^i_{\overline{Nm}(B)}(x_j) \right), \right.\right. \quad (8)$$
$$\left.\left. \left(F^i_{\overline{Nm}(A)}(x_j) + F^i_{\overline{Nm}(B)}(x_j) - F^i_{\overline{Nm}(A)}(x_j) \cdot F^i_{\overline{Nm}(B)}(x_j) \right) \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}$$

Example 6 Consider the two rough neutrosophic multisets *RNM*(*A*) and *RNM*(*B*) in Example 5. Then,

$$RNM(A) \otimes RNM(B) = \left\langle \underline{Nm}(A) \cdot \underline{Nm}(B), \overline{Nm}(A) \cdot \overline{Nm}(B) \right\rangle$$
$$= \left\{ x_1 | \left[\left((0.42, 0.1), (0.65, 0.94), (0.88, 0.97) \right) \right], \left[\left((0.42, 0.12), (0.65, 0.58), (0.95, 0.7) \right) \right], \right.$$
$$\left. x_2 | \left[\left((0.24, 0.21), (0.76, 0.97), (0.82, 0.75) \right) \right], \left[\left((0.56, 0.08), (0.7, 0.7), (0.8, 0.65) \right) \right] \right\}$$

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Definition 9 (Scalar). Consider the scalar multiplication of rough neutrosophic multisets *RNM*(*A*) in (*U,R*), denoted by λ(R*NM(*A)), for λ>0 is defined as

$$\lambda\left(RNM\left(A\right)\right)=$$
$$\left\{\left\langle x_j,\left(\begin{bmatrix}1-(1-T^i_{\underline{Nm}(A)}\left(x_j\right))^\lambda,\left(I^i_{\underline{Nm}(A)}\left(x_j\right)\right)^\lambda,\left(F^i_{\underline{Nm}(A)}\left(x_j\right)\right)^\lambda\end{bmatrix}\right\rangle : x_j\in U,i,j\in\mathbb{Z}^+,\lambda>0\right\} \quad (9)$$

Example 7 Consider the rough neutrosophic multisets *RNM*(*A*) in Example 5 and λ=0.2 . Then,

$$\lambda\left(RNM\left(A\right)\right)=$$
$$\left\{x_1|\left[\left(\left(0.21,0.13\right),\left(0.79,0.98\right),\left(0.90,0.93\right)\right)\right],\left[\left(\left(0.21,0.1\right),\left(0.87,0.79\right),\left(0.98,0.83\right)\right)\right],\quad .\right.$$
$$\left.x_2|\left[\left(\left(0.10,0.07\right),\left(0.90,0.93\right),\left(0.93,0.87\right)\right)\right],\left[\left(\left(0.28,0.04\right),\left(0.83,0.87\right),\left(0.90,\mathbf{0}.79\right)\right)\right]\right\}$$

Definition 10 (Power). Consider the power of rough neutrosophic multisets *RNM*(*A*) in (*U,R*), denoted by $(RNM(A))^\lambda$, for λ>0 is defined as

$$\left(RNM\left(A\right)\right)^\lambda=$$
$$\left\{\left\langle x_j,\left(\begin{bmatrix}\left(T^i_{\underline{Nm}(A)}\left(x_j\right)\right)^\lambda,1-(1-I^i_{\underline{Nm}(A)}\left(x_j\right))^\lambda,1-(1-F^i_{\underline{Nm}(A)}\left(x_j\right))^\lambda\end{bmatrix}\right\rangle : x_j\in U,i,j\in\mathbb{Z}^+,\lambda>0\right\} \quad (10)$$

Example 8 Consider the rough neutrosophic multisets *RNM*(*A*) in Example 5 and λ=0.5. Then,

$$\left(RNM\left(A\right)\right)^\lambda=$$
$$\left\{x_1|\left[\left(\left(0.84,0.71\right),\left(0.16,0.68\right),\left(0.37,0.45\right)\right)\right],\left[\left(\left(0.8\mathbf{4},0.63\right),\left(0.29,0.16\right),\left(0.68,0.22\right)\right)\right],.\right.$$
$$\left.x_2|\left[\left(\left(0.63,0.55\right),\left(0.37,0.45\right),\left(0.45,0.29\right)\right)\right],\left[\left(\left(0.89,0.45\right),\left(0.23,0.29\right),\left(0.37,\mathbf{0}.16\right)\right)\right]\right\}$$

If *RNM*(*A*) and *RNM*(*B*) are two rough neutrosophic multisets in (*U,R*), then the proposition below comes straightforwardly from Definitions 7 and 8.

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Proposition 2

i) $RNM(A) \oplus RNM(B) = RNM(B) \oplus RNM(A)$

ii) $RNM(A) \cdot RNM(B) = RNM(B) \cdot RNM(A)$

Proof:

i) $RNM(A) \oplus RNM(B) = \underline{Nm}(A) + \underline{Nm}(B), \overline{Nm}(A) + \overline{Nm}(B)$

$$= \left\{ \left\langle x, \begin{bmatrix} \left(T^i_{\underline{Nm}(A)}(x_j) + T^i_{\underline{Nm}(B)}(x_j) - T^i_{\underline{Nm}(A)}(x_j) \cdot T^i_{\underline{Nm}(B)}(x_j) \right), \\ \left(I^i_{\underline{Nm}(A)}(x_j) \cdot I^i_{\underline{Nm}(B)}(x_j) \right), \left(F^i_{\underline{Nm}(A)}(x_j) \cdot F^i_{\underline{Nm}(B)}(x_j) \right) \end{bmatrix} \right\rangle \right.$$

$$\left. \begin{bmatrix} \left(T^i_{\overline{Nm}(A)}(x_j) + T^i_{\overline{Nm}(B)}(x_j) - T^i_{\overline{Nm}(A)}(x_j) \cdot T^i_{\overline{Nm}(B)}(x_j) \right), \\ \left(I^i_{\overline{Nm}(A)}(x_j) \cdot I^i_{\overline{Nm}(B)}(x_j) \right), \left(F^i_{\overline{Nm}(A)}(x_j) \cdot F^i_{\overline{Nm}(B)}(x_j) \right) \end{bmatrix} \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}$$

$$= \left\{ \left\langle x_j, \begin{bmatrix} \left(T^i_{\underline{Nm}(B)}(x_j) + T^i_{\underline{Nm}(A)}(x_j) - T^i_{\underline{Nm}(B)}(x_j) \cdot T^i_{\underline{Nm}(A)}(x_j) \right), \\ \left(I^i_{\underline{Nm}(B)}(x_j) \cdot I^i_{\underline{Nm}(A)}(x_j) \right), \left(F^i_{\underline{Nm}(B)}(x_j) \cdot F^i_{\underline{Nm}(A)}(x_j) \right) \end{bmatrix} \right\rangle \right.$$

$$\left. \begin{bmatrix} \left(T^i_{\overline{Nm}(B)}(x_j) + T^i_{\overline{Nm}(A)}(x_j) - T^i_{\overline{Nm}(B)}(x_j) \cdot T^i_{\overline{Nm}(A)}(x_j) \right), \\ \left(I^i_{\overline{Nm}(B)}(x_j) \cdot I^i_{\overline{Nm}(A)}(x_j) \right), \left(F^i_{\overline{Nm}(B)}(x_j) \cdot F^i_{\overline{Nm}(A)}(x_j) \right) \end{bmatrix} \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}$$

$$= \underline{Nm}(B) + \underline{Nm}(A), \overline{Nm}(B) + \overline{Nm}(A) = RNM(B) \oplus RNM(A)$$

ii) $RNM(A) \otimes RNM(B) = \left\langle \underline{Nm}(A) \cdot \underline{Nm}(B), \overline{Nm}(A) \cdot \overline{Nm}(B) \right\rangle$

$$= \left\{ \left\langle x_j, \left[\left(T^i_{\underline{Nm}(A)}(x_j) \cdot T^i_{\underline{Nm}(B)}(x_j) \right), \left(I^i_{\underline{Nm}(A)}(x_j) + I^i_{\underline{Nm}(B)}(x_j) - I^i_{\underline{Nm}(A)}(x_j) \cdot I^i_{\underline{Nm}(B)}(x_j) \right), \left(F^i_{\underline{Nm}(A)}(x_j) \right. \right. \right. \right.$$
$$\left. + F^i_{\underline{Nm}(B)}(x_j) - F^i_{\underline{Nm}(A)}(x_j) \cdot F^i_{\underline{Nm}(B)}(x_j) \right) \Big], \Big[\left(T^i_{\overline{Nm}(A)}(x_j) \cdot T^i_{\overline{Nm}(B)}(x_j) \right), \left(I^i_{\overline{Nm}(A)}(x_j) + I^i_{\overline{Nm}(B)}(x_j) \right.$$
$$\left. \left. \left. \left. - I^i_{\overline{Nm}(A)}(x_j) \cdot I^i_{\overline{Nm}(B)}(x_j) \right), \left(F^i_{\overline{Nm}(A)}(x_j) + F^i_{\overline{Nm}(B)}(x_j) - F^i_{\overline{Nm}(A)}(x_j) \cdot F^i_{\overline{Nm}(B)}(x_j) \right) \right] \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}$$

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$$= \left\{ \left\langle x_j, \left[\left(T^i_{\underline{Nm}(B)}(x_j) \cdot T^i_{\underline{Nm}(A)}(x_j) \right), \left(I^i_{\underline{Nm}(B)}(x_j) + I^i_{\underline{Nm}(A)}(x_j) - I^i_{\underline{Nm}(B)}(x_j) \cdot I^i_{\underline{Nm}(A)}(x_j) \right), \left(F^i_{\underline{Nm}(B)}(x_j) \right. \right. \right.$$
$$\left. + F^i_{\underline{Nm}(A)}(x_j) - F^i_{\underline{Nm}(B)}(x_j) \cdot F^i_{\underline{Nm}(A)}(x_j) \right) \Big], \Big[\left(T^i_{\overline{Nm}(B)}(x_j) \cdot T^i_{\overline{Nm}(A)}(x_j) \right), \left(I^i_{\overline{Nm}(B)}(x_j) + I^i_{\overline{Nm}(A)}(x_j) \right.$$
$$\left. \left. \left. - I^i_{\overline{Nm}(B)}(x_j) \cdot I^i_{\overline{Nm}(A)}(x_j) \right), \left(F^i_{\overline{Nm}(B)}(x_j) + F^i_{\overline{Nm}(A)}(x_j) - F^i_{\overline{Nm}(B)}(x_j) \cdot F^i_{\overline{Nm}(A)}(x_j) \right) \right] \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}$$

$$\left\langle \underline{Nm}(B) \cdot \underline{Nm}(A), \overline{Nm}(B) \cdot \overline{Nm}(A) \right\rangle = RNM(B) \otimes RNM(A)$$

The "AND" and "OR" Operation

Definition 11 (AND) Let *RNM*(*A*) and *RNM*(*B*) be two rough neutrosophic multisets over (*U,R*). Then, the "AND" operation is denoted by $RNM(A) \wedge RNM(B)$, defined by

$$RNM(A) \wedge RNM(B) = \left(\left(\underline{Nm}(A) \wedge \underline{Nm}(B) \right), \left(\overline{Nm}(A) \wedge \overline{Nm}(B) \right) \right),$$

where $\underline{Nm}(A) \wedge \underline{Nm}(B)$ and $\overline{Nm}(A) \wedge \overline{Nm}(B)$ are the essential intersection of lower and upper approximation of *RNM*(*A*) and *RNM*(*B*), respectively.

$$\underline{Nm}(A) \wedge \underline{Nm}(B) = \underline{Nm}(A) \bigcap \underline{Nm}(B)$$
$$= \left\{ \left\langle x_j, \left(\min \left\{ T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j) \right\}, \max \left\{ I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j) \right\}, \right. \right.$$
$$\left. \left. \max \left\{ F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j) \right\} \right) \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \Big\},$$

and

$$\overline{Nm}(A) \wedge \overline{Nm}(B) = \overline{Nm}(A) \bigcap \overline{Nm}(B)$$
$$= \left\{ \left\langle x_j, \left(\min \left\{ T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j) \right\}, \max \left\{ I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j) \right\}, \right. \right. \quad (11)$$
$$\left. \left. \max \left\{ F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j) \right\} \right) \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \Big\}.$$

Definition 12 (OR) Let *RNM*(*A*) and *RNM*(*B*) be two rough neutrosophic multisets over (*U,R*). Then the "OR" operation on them is denoted by $RNM(A) \vee RNM(B)$ and is defined by

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$$RNM(A) \vee RNM(B) = \left(\left(\underline{Nm}(A) \vee \underline{Nm}(B) \right), \left(\overline{Nm}(A) \vee \overline{Nm}(B) \right) \right),$$

where $\underline{Nm}(A) \vee \underline{Nm}(B)$ and $\overline{Nm}(A) \vee \overline{Nm}(B)$ are the essential union of lower and upper approximation of *RNM*(*A*) and *RNM*(*B*), respectively.

$$\begin{aligned} \underline{Nm}(A) \vee \underline{Nm}(B) &= \underline{Nm}(A) \cup \underline{Nm}(B) \\ &= \left\{ \left\langle x_j, \left(\max\left\{ T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j) \right\}, \min\left\{ I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j) \right\}, \right. \right. \right. \\ &\quad \left. \left. \left. \min\left\{ F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j) \right\} \right) \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\}, \end{aligned}$$

and

$$\begin{aligned} \overline{Nm}(A) \vee \overline{Nm}(B) &= \overline{Nm}(A) \cup \overline{Nm}(B) \\ &= \left\{ \left\langle x_j, \left(\max\left\{ T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j) \right\}, \min\left\{ I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j) \right\}, \right. \right. \right. \\ &\quad \left. \left. \left. \min\left\{ F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j) \right\} \right) \right\rangle : x_j \in U, i, j \in \mathbb{Z}^+ \right\} \end{aligned} \tag{12}$$

If *RNM*(*A*) and *RNM*(*B*) are two rough neutrosophic multisets over (*U*,*R*), then the proposition below comes straightforwardly from Definitions 11 and 12.

Proposition 3

i) $\left(RNM(A) \wedge RNM(B) \right)^C = \left(RNM(A) \right)^C \vee \left(RNM(B) \right)^C$

ii) $\left(RNM(A) \vee RNM(B) \right)^C = \left(RNM(A) \right)^C \wedge \left(RNM(B) \right)^C$

Proof:

i) $\left(RNM(A) \wedge RNM(B) \right)^C = \left(\left(\underline{Nm}(A) \wedge \underline{Nm}(B) \right), \left(\overline{Nm}(A) \wedge \overline{Nm}(B) \right) \right)^C$

$$\left(\left(\underline{Nm}(A) \wedge \underline{Nm}(B) \right), \left(\overline{Nm}(A) \wedge \overline{Nm}(B) \right) \right)^C = \left(\left(\underline{Nm}(A) \cap \underline{Nm}(B) \right), \left(\overline{Nm}(A) \cap \overline{Nm}(B) \right) \right)^C$$

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$$= \left(\begin{bmatrix} \min\left\{T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j)\right\}, \max\left\{I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j)\right\}, \max\left\{F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j)\right\} \end{bmatrix}, \right.$$
$$\left. \begin{bmatrix} \min\left\{T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j)\right\}, \max\left\{I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j)\right\}, \max\left\{F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j)\right\} \end{bmatrix} \right)^C$$

$$= \left(\begin{bmatrix} \max\left\{F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j)\right\}, \min\left\{1-I^i_{\underline{Nm}(A)}(x_j), 1-I^i_{\underline{Nm}(B)}(x_j)\right\}, \min\left\{T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j)\right\} \end{bmatrix}, \right.$$
$$\left. \begin{bmatrix} \max\left\{F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j)\right\}, \min\left\{1-I^i_{\overline{Nm}(A)}(x_j), 1-I^i_{\overline{Nm}(B)}(x_j)\right\}, \min\left\{T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j)\right\} \end{bmatrix} \right)$$

$$= \left(\left(\underline{Nm}(A) \cup \underline{Nm}(B)\right)^C, \left(\overline{Nm}(A) \cup \overline{Nm}(B)\right)^C \right)$$
$$= \left(\left(\underline{Nm}(A) \vee \underline{Nm}(B)\right)^C, \left(\overline{Nm}(A) \vee \overline{Nm}(B)\right)^C \right)$$
$$= \left(RNM(A)\right)^C \vee \left(RNM(B)\right)^C$$

ii) $\left(RNM(A) \vee RNM(B)\right)^C = \left(\left(\underline{Nm}(A) \vee \underline{Nm}(B)\right), \left(\overline{Nm}(A) \vee \overline{Nm}(B)\right)\right)^C$

$$\left(\left(\underline{Nm}(A) \vee \underline{Nm}(B)\right), \left(\overline{Nm}(A) \vee \overline{Nm}(B)\right)\right)^C = \left(\left(\underline{Nm}(A) \cup \underline{Nm}(B)\right), \left(\overline{Nm}(A) \cup \overline{Nm}(B)\right)\right)^C$$

$$= \left(\begin{bmatrix} \max\left\{T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j)\right\}, \min\left\{I^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(B)}(x_j)\right\}, \min\left\{\left\{F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j)\right\}\right] \\ \begin{bmatrix} \max\left\{T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j)\right\}, \min\left\{I^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(B)}(x_j)\right\}, \min\left\{\left\{F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j)\right\}\right] \end{bmatrix} \right)^C$$

$$= \left(\begin{bmatrix} \min\left\{F^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(B)}(x_j)\right\}, \max\left\{1-I^i_{\underline{Nm}(A)}(x_j), 1-I^i_{\underline{Nm}(B)}(x_j)\right\}, \max\left\{\left\{T^i_{\underline{Nm}(A)}(x_j), T^i_{\underline{Nm}(B)}(x_j)\right\}\right] \\ \begin{bmatrix} \min\left\{F^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(B)}(x_j)\right\}, \max\left\{1-I^i_{\overline{Nm}(A)}(x_j), 1-I^i_{\overline{Nm}(B)}(x_j)\right\}, \max\left\{\left\{T^i_{\overline{Nm}(A)}(x_j), T^i_{\overline{Nm}(B)}(x_j)\right\}\right] \end{bmatrix} \right)$$

$$= \left(\left(\underline{Nm}(A) \cap \underline{Nm}(B)\right)^C, \left(\overline{Nm}(A) \cap \overline{Nm}(B)\right)^C \right)$$
$$= \left(\left(\underline{Nm}(A) \wedge \underline{Nm}(B)\right)^C, \left(\overline{Nm}(A) \wedge \overline{Nm}(B)\right)^C \right)$$
$$= \left(RNM(A)\right)^C \wedge \left(RNM(B)\right)^C$$

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DISCUSSION

Collectively, in a non-empty set U with generic elements in U denoted as x_j, a rough neutrosophic multisets will generalize the following sets whereby all the basic operators and properties are valid for each generalized set.

(1) Rough neutrosophic set (RNS) is obtained when i=1 for all elements in rough neutrosophic multisets, defined as follows:

$$= \left\{ x_j, \left(\left[T_{\underline{N}(A)}\left(x_j\right), I_{\underline{N}(A)}\left(x_j\right), F_{\underline{N}(A)}\left(x_j\right) \right], \left[T_{\overline{N}(A)}\left(x_j\right), I_{\overline{N}(A)}\left(x_j\right), F_{\overline{N}(A)}\left(x_j\right) \right] \right) \middle| y \in \left[x_j \right]_R, x_j \in U \right\}$$

(2) A rough intuitionistic fuzzy set (RIFS) is obtained when i=1 for elements T and F, where element I is omitted. RIFS is defined as follows:

$$= \left\{ x_j, \left(\left[T_{\underline{N}(A)}\left(x_j\right), F_{\underline{N}(A)}\left(x_j\right) \right], \left[T_{\overline{N}(A)}\left(x_j\right), F_{\overline{N}(A)}\left(x_j\right) \right] \right) \middle| y \in \left[x_j \right]_R, x_j \in U \right\}$$

(3) A rough fuzzy set (RFS) is obtained when i=1 for element T and element I, where F is omitted. Then, RFS is defined as follows:

$$= \left\{ x_j, \left(\left[T_{\underline{N}(A)}\left(x_j\right), T_{\overline{N}(A)}\left(x_j\right) \right] \right) \middle| y \in \left[x_j \right]_R, x_j \in U \right\}$$

FUTURE RESEARCH DIRECTIONS

Associatively, the rough neutrosophic multisets (RNM) is a generalization of rough neutrosophic set theory, rough intuitionistic fuzzy set theory, and rough fuzzy set theory as deliberated throughout this chapter. This theory defines the lower and upper approximation for neutrosophic multisets as an equivalence relation. Besides that, the RNM is formulated based on the hybrid set theory of neutrosophic multiset and rough sets. Therefore, this theory is deemed suitable in solving the multiple phases of criteria for uncertainty information, whereas the results yield was categorized as 'certainly belongs' and 'possibly belongs' relationship. In addition, the triple membership value in a neutrosophic environment provides more accurate results in solving the decision-making process. Furthermore, many applications are able to be represented by RNM, for instance, in medical diagnosis whereby, the data for patients are obtained at different time intervals (multiple) in equivalence relation (relation between patients and symptoms). Similarly, this can also be employed in the teacher's selection process, where various phases are attained for the candidate in equivalence relation (relation of criteria and candidate). Besides that, this can also be applied in the area of marketing alike on customers who want to buy a car with multiple test drives on the road in equivalence relation (relation between types of car and criteria). Contemplatively, the

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RNM will be able to assist decision-makers to deliver the best decision according to an authentic relationship for future research directions. Additionally, RNM theory is able to agitate more relationship concepts for important information such as aggregation operation, distance and similarity measure, and entropy to measure the fuzziness of the information. These concepts can be the grounds for exploration by researchers in the future.

CONCLUSION

Conclusively, this chapter represents the novel approach of the research by introducing rough neutrosophic multiset (RNM), a new mathematical theory for uncertainty conditions. Perceptibly, the essential operations over the RNM such as complement, union, intersection, contained, and equality was successfully defined. Besides that, involution property, idempotent property, commutative property, associative property, distributive property, and De Morgan's Law properties were satisfied by the RNM theory. Subsequently, the algebraic operations over RNM and multiplication, scalar multiplication, and power of RNM was detailed and deliberated in this chapter. Over and beyond, the "AND" and "OR" operations over RNM theory was also discussed where all the propositions were successfully proven. Effectively, the future directions obtained from the development of RNM was construed diligently and provide grounds for future exploration.

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KEY TERMS AND DEFINITIONS

Approximation: The nearest value for representing the result in some situations.

Falsity: The rejected value when the decision making is involved.

Indeterminate: The unsure about something, whether to give a chance or not.

Multisets: Redundant elements occur more than once and group together.

Neutrosophic: Term used to replace the philosophy of neutral condition in uncertainty information with a relationship of truth and falsity.

Truth: The acceptable value when the decision making has happened.

Uncertainty Information: The unbalance information according to decision-maker when involving human thinking.

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Chapter 26
A Novel Neutrosophic Interpretive Structural Modeling Approach: Hierarchical Visual Graphs of Indeterminate Causalities

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ABSTRACT

Interpretive structural modeling is of paramount importance in barrier/risk/challenge analysis as a hierarchical visual map by emphasizing the root cause of the problems. It asks expert opinions to evaluate the causal relationships of identified variables. However, in many cases experts cannot determine a relationship or can be doubtful about stating an idea. In the meantime, the neutrosophic cognitive mapping provides a step-by-step guidance in order to deal with the indeterminate relationships. Therefore, this study aims to propose a neutrosophic ISM approach and to implement it for the medical tourism services' barriers during the COVID-19 pandemic as an illustrative example. In order to do that, text mining was conducted to the medical tourism-related tweets written in English from January to December 2020 via RapidMiner software, and the barriers in medical tourism were identified. Next, the relationships between these barriers were examined via expert evaluations, and the proposed neutrosophic ISM was applied to construct a structural model.

INTRODUCTION

Interpretive Structural Modeling (ISM) is defined as a process of interpreting systems with well-defined factors. It models the relationships that appear in mind and transforms them into visual forms (Sage, 1977). The method summarizes the relationship between certain variables in defining a problem

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(Warfield, 1974). Thus, the hierarchical structure or the order between these variables is also modeled. ISM provides a decision support to present undesired factors / risks / barriers visually in a system in hierarchical graphics and clarifies the relationships between these obstacles. It is used to identify the root causes of a problem with a graph of directed relationships. For this purpose, the factors causing a problem are clarified and mutual relationships are shown based on their power of influence (Mandal & Deshmukh, 1994).

ISM requires expert opinions to evaluate the causal relationships of identified variables. However, in many cases experts cannot determine a relationship, or can be doubtful about stating an idea. This is a well-known problem which is not well-discussed in determining the relationships. The analysis of indeterminate concepts is one of the difficult way for individualts to provide mathematical expressions. Therefore, neutrosophy is introduced by Smarandache as an extension / combination of the fuzzy logic by including the indeterminacy (Smarandache., 1999). While, fuzzy logic only measures the existence or non-existence membership grades, the neutrosophic sets can attribute the concepts when the relations between indeterminate concepts (Kandasamy & Smarandache, 2003).

The ISM and cognitive mapping have common relationship matrices and similar prodecures. For instance, the connection matrix transforms to reachability matrix in ISM, while it is called adjacency matrix in cognitive mapping. Or as another example, the indirect relationships are detected via transitivity rules in ISM, while the neutrosophic cognitive maps implements the hidden pattern determination rules. Therefore, the well-known neutrosophic cognitive mapping steps (Kandasamy & Smarandache, 2003) can be applied to transform the traditional ISM to propose the neutrosophic ISM. The neutrosophic cognitive mapping is a fundamental technique by providing a step-by-step guidance to deal with the indeterminate relationships of ISM.

Hence, this study aims to propose a neutrosophic ISM approach, and to implement it for the medical tourism services' barriers during the COVID-19 Pandemic as an illustrative example. In order to that, text mining was conducted to the medical tourism related tweets written in English from January to December 2020 via RapidMiner software, and the barriers in medical tourism were identified. Next, the relationships between these barriers were examined via expert evaluations, and the proposed neutrosophic ISM was applied to construct a structural model by presenting how the barriers are related with a hierarchy between them. In addition, the relationships between the barriers of this study were evaluated by the owner of a Dental Hospital, an Eye Hospital partner, and an academician working on health tourism operations management. ISM studies are generally applied by referring to less than 6 industry experts and less than 3 academicians (Dwivedi et al., 2017). In addition, it is emphasized that at least 3 expert evaluations are sufficient for validation in studies using ISM (Yudatama et al., 2018).

In the following sections, the background section includes neutrosophic sets, hidden pattern determination procedure, medical tourism services network, and the related literature. Next, the steps of the proposed methodology are presented. Then, the barriers were identified via text mining, and these barriers were analyzed with neutrosophic Interpretative Structural Modeling. Findings, future research directions and conclusion are stated.

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BACKGROUND

Neutrosophic Sets

The analysis of indeterminate concepts is one of the difficult way for individualts to provide mathematical expressions. Therefore, neutrosophy is introduced by Smarandache as an extension / combination of the fuzzy logic by including the indeterminacy. While, fuzzy logic only measures the existence or non-existence membership grades, the neutrosophic sets can attribute the concepts when the relations between indeterminate concepts (Kandasamy & Smarandache, 2003).

A neutrosophic set $x=(T,I,F)$ denotes T as degree of *truth*, I as degree of *indeterminacy*, and F as degree of *falsity* (Smarandache, 1999).

Determining Hidden Patterns

A neutrosophic directed graph includes concepts as nodes, and causalities (i.e. indeterminates) as edges. The indeterminant relations are shown as arcs with dots. Let $C_1,C_2,\ldots,C_n$ denote n nodes as neutrosophic vectors from the neutrosophic vector space V. Therefore, a node C_i is represented by $(x_1,\ldots x_n)$ where x_k is 0 or 1or I (indeterminate). Here, $x_k=1$ means that the node C_k is in the on *state* and $x_k=0$ represents the node is in the *off state* and $x_k=I$ refers that the nodes state is *indeterminate* in this case. Let C_i and C_j denote the two nodes and the directed edge refers to the causality called *connections* or *relations.* Every edge is weighted with a number in the set $\{-1, 0, 1, I\}$. This weight is 0 if C_i does not have any effect on C_j, it is 1 if increase/decrease in C_i causes increase/decreases in C_j, the weight is -1 if increase/decrease in C_i causes decrease/increase in C_j. And the weight is I if the relation or effect of C_i on C_j is an indeterminate (Kandasamy & Smarandache, 2003).

Let the neutrosophic matrix $N(E)$ be defined as $N(E) = (e_{ij})$, where e_{ij} is the weight of the directed edge C_iC_j, $e_{ij}\in \{0, 1, -1, I\}$. $N(E)$ is called the neutrosophic adjacency matrix. Let $A=(a_1,a_2,\ldots,a_n)$ where $a_i \in \{0,1,I\}$. A is called the instantaneous *state neutrosophic* vector and it denotes the on- or off-indeterminate state position of the node at an instant; $a_i=0$ if a_i is off (no effect), $a_i=1$ if a_i is on (has effect), and $a_i=I$ if a_i is indeterminate (i.e. effect cannot be determined) for $i=1,2,\ldots n$. Let $\overline{C_1C_2},\overline{C_2C_3},\ldots,\overline{C_iC_j}$ be the edges of the neutrosophic graph. Then the edges become a directed cycle. A neutrosophic CM is called as cyclic if it has a directed cycle, acyclic otherwise, which means also a feedback. The feedback in the neutrosophic graphs refers to a dynamical system. Let $\overline{C_1C_2},\overline{C_2C_3},\ldots,\overline{C_{n-1}C_n}$ be a cycle, when C_i is switched on and if the causality flow through the edges of a cycle, and if it again trigger to cause C_i, we say that the dynamical system goes round and round. The equilibrium state for this dynamical system is called the hidden pattern. If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. For example, let us start the dynamical system by switching on C1. Let us assume that the neutrosophic CM settles down with C1 and C_n on, i.e. the state vector remains as (1, 0,…, 0, 1) this neutrosophic state vector (1,0, …, 0, 1) is called the fixed point. If the neutrosophic CM settles with a neutrosophic state vector repeating in the form $A_1\rightarrow A_2\rightarrow \ldots \rightarrow A_t\rightarrow A_{t+1}\rightarrow \ldots \rightarrow A_n\rightarrow A_t$ where A_i is the the vector passing into a dynamical system repeatedly, $1\leq i\leq n$, then this equilibrium is a limit cycle (Vasantha et al., 2019; Zafar & Wajid, 2019; Kandasamy & Smarandache, 2003).

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In order to determine the hidden patterns, when C_1 is switched on when an input is given as the vector $A_1 = (1, 0, 0,\ldots, 0)$, the data should pass through the neutrosophic matrix by multiplying A_1 by the matrix N(E). Let $A_1N(E) = (a_1, a_2,\ldots,a_n)$ with the threshold operation obtaining by replacing a_i by 1 if $a_i \geq k$ and a_i by 0 if $a_i < k$ (k: a *s*uitable positive integer) and a_i by I if a_i is not an integer. Then the resulting concept is updated. The concept C_1 is included in the updated vector by making the first co-ordinate as 1 in the resulting vector. Suppose $A_1N(E) \rightarrow A_2$, then consider $A_2N(E)$ and repeat the same procedure. This procedure is repeated till we get a limit cycle or a fixed point (Kalaichelvi & Gomathy, 2011; Banerjee, 2008).

These defined adjacency matrix and hidden pattern concepts correspond to the ISM's reachability matrix and indirect relationship identification procedures.

Reachability matrix is a form to develop a structural model when the contextual relations being modeled are transitive and multilevel. It consists of the ordering of elements of the system in such a way that much of the data required can be computed from supplied data, using the transitivity condition (Warfield, 1974).

Hierarchy means classifying the variables and creating a step by step order (Cambridge Dictionary, 2021).

Cognitive map (CM) is a visual model illustrating the factors of a concept/problem and understanding the cause and effect relationships between these factors (Swan, 1995).

Neutrosophic CMs are generalization of fuzzy CMs that are defining the notions of neutrosophic matrix and defining the neutrosophic relational maps (Kandasamy & Smarandache, 2003).

Therefore, this background knowledge will be used in following sections.

Medical Tourism Services Network

Medical tourism is emerging both as a business and as an academic research area that provides patients access to medical institutions for treatment/rehabilitation outside their country of residence. Since the transnational medical travels are interrupted due to the COVID-19 pandemic, the medical tourism, especially the mostly requested elective low-risk treatments like dentistry, laser eye surgery, esthetics and hair transplantation are almost come to a standstill.

Although the terms "medical tourism" and "health tourism" are often used interchangeably, health tourism is a broad concept including thermal tourism, spa tourism, medical tourism and elderly / disabled tourism. Medical tourism is especially preferred by the patients who want to combine treatment with a vacation. It is an area open to both sectoral and academic research (Lee & Fernando, 2015). There are numerous advantages such as accessing the high quality and reliable medical services, instant access to medical treatment without any queues or long waiting times, reaching to medical institutions with the latest technology and healthcare professionals, travelling to foreign cities and countries (Connell, 2006; Glinos and al., 2006; Kumar et al., 2012). Medical tourism is considered as an opportunity to make a new investment and to update the health services in especially developing countries (Health Tourism Development Council, 2012; Connell, 2006).

Before the COVID-19 pandemic having a worldwide effect in 2020, the share of tourism was 3.5 trillion dollars, the share of health tourism was 500 billion dollars, and every year 3-4% of the world population was traveling for this purpose. In the countries like Turkey, India, Malaysia, Singapore, Korea and Thailand, Brazil, Costa Rica, Cuba and Mexico have come into prominence in this service industry. Current in demand medical tourism are stem cell therapy (Unsworth et al., 2020), aesthetic surgery

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(Asher et al., 2020; Leto Barone et al., 2020; Raggio et al., 2020), fertility and IVF treatment (Andreou et al., 2020; Yang, 2020), organ transplantation (Lumenta, 2020; Hindi et al., 2020; Liman et al., 2020) and dental tourism (Akbar et al., 2020).

Medical tourism service includes medical tourists / patients, health institutions, assistant companies / intermediary firms, accommodation service providers, transfer and / or ambulance services, flight ticket reservation providers, travel agencies, translation offices, visa service providers, insurance companies. It is a supply chain structure that requires the participation of various parties (Karadayi-Usta and Serdarasan, 2020a). The medical tourism service network consists of a large number of businesses and people that plan medical tourism services, provide the necessary resources, provide related services, and manage information and financial flows between service providers and medical tourists (Ferrer & Medhekar, 2012). The must-have services demanded by medical tourists are treatment, accommodation and transfer, and medical tourists are asked whether they want optional tour service, flight ticket reservation, translation and visa services (Karadayı-Usta and Serdarasan, 2020b).

Literature Review

Current publications discussed the following issues during the pandemic period: difficulties faced by medical tourists (Sharma et al., 2020), factors affecting medical tourism preferences (Sevim & Sevim, 2019), factors motivating medical tourists and analysis of the risks they perceive (Ko, 2020), health tourist experience analysis based on dialogues (Brown et al., 2020), the relationship between digital media and medical tourism marketing (Alghizzawi et al., 2020), risks awaiting medical tourists as travel constraints, medical tourists perception for the hosting country, and the related factors affecting the travelling decision for treatment purposes (Khan et al., 2020; Cham et al., 2020), the behaviors of young medical tourists (Boguszewicz-Kreft et al., 2020), medical tourism for groups of nations (diaspora) outside the country (Mathijsen and Mathijsen, 2020), health travels that can turn into immigration (Murray and Khan, 2020), problems in importing products (Dalstrom et al., 2020), benefits of medical tourism to institutions (customer focus, job satisfaction, performance increase) (Park et al., 2020; Foster, 2020), internationalization of hospitals with dynamic behavior (Uner et al., 2020), one-to-one contact with the patient (Sunanta, 2020), training of healthcare professionals on globalization (Camargo-Plazas et al., 2020), service quality, satisfaction, loyalty and the effect of trust concepts on attracting foreign patients (Aljumah et al., 2020).

Medical tourism draws attention not only in terms of the opportunities that it offers, but also in terms of the obstacles encountered. The serious damages in the sector with this pandemic have brought the issue of sustainability of the development of medical tourism into the forefront. Correct identification of the points preventing the business continuity is important in terms of developing measures and action plans for the related problems.

The methods utilized for the barriers and risks studies in medical tourism are: determining the perceived risks and the hosting country image via questionnaires by factor analysis (Khan et al., 2020; Ko, 2020; Kim & Choi, 2019), understanding the difficulties encountered with thematic analysis as a result of interviews with a certain focus group (Jackson et al., 2019), retrospective review, determining the problems identified by individuals who have received medical tourism services before (Lee et al., 2018). These studies take the statements of real medical tourists into account. The pandemic period, in which individual interviews were restricted, triggered medical tourists to become more active on social media. Besides, there is a few study dealing with the statements of medical tourists on social media and

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conducting text mining. There are also papers emphasizing that text mining has become an important tool and that it is not used sufficiently within the scope of medical tourism (Kim & Lee, 2019; Moon & Hong, 2018; Hong & Moon, 2017). Therefore, within the scope of this study, text mining was used in identifying the medical tourism barriers during the pandemic period.

In addition, the ISM method was used in determining the quality factors in medical tourism services (Sadeh & Garkaz, 2019) and determining the factors affecting medical tourism (Aiwerioghene et al., 2019; Jain & Ajmera, 2018, Ranjan Debata et al., 2013). This technique is rarely encountered in medical tourism risks / barriers / challehnges / difficulties. For example, the degrading and racist difficulties faced by people traveling for IVF treatment were analyzed with ISM (Rodriguez, 2018), and ISM was also applied to reveal the causal relationships of tourists' satisfaction in spa tourism (Mi et al., 2019).

Hence, the literature review address that the text mining in factor identification, and that the ISM technique for the analysis are suitable for the analysis.

PROPOSED METHODOLOGY STEPS

Step 1: Identify the barriers affecting medical tourism services via text mining

Step 2: Conduct in-depth interviews with the experts in order to define the relationships between medical tourism barriers, i.e. generate a reachability/adjacency matrix. Ask them to determine the direct relationships. If there is a relation that cannot be determined, label the connection link as *I* with dots in graph referring to the indeterminacy.

Step 3: Apply the "hidden pattern determination" procedure. Select an indeterminate barrier as the on-state factor, identify a $(1\times n)$ state vector S including 1 in the place of on-state concept. Multiply the $(1\times n)$ vector S with the reachability/adjacency matrix which illustrates the opinions of the experts in order to find the hidden patterns.

Here $\bullet$ denotes the matrix multiplication. $S\bullet N\left(E\right)$ gives a $(1\times n)(n\times n) = (1\times n)$ result. Update this result according to the *limit cycle procedure* in order to obtain S_1. Next, repeat the same procedure, find the $S_1\bullet N\left(E\right)$ and update the previous S_1 to generate S_2. Then, repeat the same step until $S_{k-1} = S_k$ as a *fixed point*.

Step 4: Repeat the step 3 for the indeterminate barriers.

Step 5: Starting from the initial directed graph, the reachability/adjacency matrix is divided into different hierarchical levels. The table used in this process includes "accessibility set", "precursor set", "intersection" and "level" columns. In the *accessibility set*, there are both the variable itself and other interrelated variables. In the *predecessor set*, there are other variables that are directed towards the related variable and this variable itself. The *intersection* of these two sets is used as the next step in dividing the accessibility set into different levels. If the accessibility set and the intersection column take the same values, this variable is positioned at the *level 1*, that is, at the top of the hierarchy (Table 2). *Level 1* also represents the situation of being most affected by the other variables. Thus, the first iteration of the "separating into levels" process has been completed.

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For the remaining variables except for the variables assigned to the *level 1*, the second iteration continues, and these iterations continue until a final neutrosophic ISM is formed (Table 2).

Step 6: Discuss the causalities of the relations. As a result of all these steps, the neutrosophic ISM diagram is created, reviewed in terms of consistency, and necessary changes (Figure 2).

The following part will discuss the provided background for this study.

IDENTIFYING THE BARRIERS VIA TEXT MINING

Text mining is the exploration and extraction of interesting and important information from a free or unstructured text. It covers everything from the acquisition of information (from the document or website) to the classification and aggregation of text, as well as entities, relationships and event extractions. In this context, Natural Language Processing (NLP) is used. NLP is an attempt to extract meaning from a text, often using spoken language and grammatical structure. It makes use of dictionaries, grammar rules, ontology of entities and actions, synonyms and abbreviations (Kao & Poteet, 2007).

In order to extract a specialty via text mining, the following steps are applied: (*i*) feature definition (e.g. "size too large", " slow browser") to reveal a feature with text mining, (*ii*) determine views about the feature (e.g. "size too big" corresponding to the "size" attribute), (*iii*) determining the polarity of opinions (positive opinion: "this scanner is very good" or negative opinion: "this scanner is a complete disappointment"), (*iv*) sorting opinions according to their strength (eg "terrible" is a stronger expression than "bad") (Popescu & Etzioni, 2007).

There are two approaches to reveal the relationships. In the first approach, each potential relationship is represented indirectly as a vector of properties, where each property corresponds to a "word sequence" fixed to two entities that genarates the relationship. An association inference system is trained on a successor kernel. This kernel is also generalized so that words can be replaced with word classes. Thus, it becomes possible to use tagging, entity recognition or information fragmentation. The second approach focuses on "representation" and "the shortest dependency path" between the two entities in the dependency graph of the sentence. Since syntactic analysis is important in this method, its applicability is limited due to "syntactic parsing" (Bunescu & Mooney, 2007).

The current text mining applications in medical tourism literature focuses on: reviewing the newspapers (Kim & Lee, 2019), effects of being online in healthcare (Moon & Hong, 2018), analysis of keywords used on the internet (Hong & Moon, 2017), determination of risks encountered in care services (Nishikawa et al., 2014). The use of text mining in medical tourism risk studies is limited.

Within the scope of this study, Twitter texts written in English written between January and December 2020 on medical tourism with the use of RapidMiner software were analyzed. The tags in the text were converted to plain text, and typos were corrected using SpaCy software as a NLP tool. As a result of the application of text mining to the personal statements of real medical tourists, the medical tourism barriers encountered during the COVID-19 pandemic process have been identified as:

- B1. Medical tourism bans and restrictions brought by countries due to the COVID-19 outbreak
- B2. Disruptions in the medical supply systems of the countries (medical equipment shortage, prohibition / restriction of the import of medical equipment and drugs, etc.)
- B3. Restrictions on transportation between countries and decrease in international patient demand

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- B4. Operational and financial damage to many institutions (accommodation service providers, airline companies, travel agencies, etc.)
- B5. Disruptions in the financial flows of hospitals
- B6. Difficulty in finding qualified healthcare professionals, insufficient human resources
- B7. Complicated medical tourism operations (procedures) due to the pandemic (the problems in the process until the publication of the new regulation (Regulation on International Health Tourism and Tourist Health, 17/06/2020))
- B8. Wrong interventions to medical tourists
- B9. Unexpected fluctuations in exchange rates

While the obstacles and difficulties encountered in the medical tourism literature before the pandemic stand out as inconsistency in wages, fraud and misinformation (Mason & Wright, 2011), difficulties in adapting to the target country in social terms, the concern of life and property safety (Gan & Frederick, 2015), the points that cause anxiety and are perceived as obstacles have changed.

Visualizing the identified barriers and the structural relationship between them and revealing the root causes will be possible with Interpretative Structural Modeling. For this reason, in the following section, obstacles are examined on a structural basis, the relationships between them are analyzed with expert evaluations, and finally, how these risks affect each other is presented.

NEUTROSOPHIC ISM APPLICATION

ISM is a decision support method that presents undesired factors / risks / barriers visually in a system in hierarchical graphics and clarifies the relationships between these obstacles. It is used to identify the root causes of a problem with a graph of directed relationships. For this purpose, the factors causing a problem are clarified and mutual relationships are shown based on their power of influence (Mandal & Deshmukh, 1994). A systematic model of factors that are directly and indirectly related to each other is created (Saatçioğlu & Özmen, 2010). The method designs with various graphics, gives direction and order to the complex relationship between system elements (Singh et al., 2003). This method, which focused on both mathematical and philosophical researches, has also benefited in areas such as management studies or policy analysis (Diabat et al., 2012). However, in terms of taking expert opinions as a basis, the model created with ISM is shaped by the judgments of the individuals who express opinions for the relationships between variables. The level of knowledge and knowledge level of the person giving the opinion on the subject matter is also important.

A small number of expert opinions are sufficient in the ISM technique, because knowledge level and experience are more important than the number of experts (Sadeh & Garkaz, 2019). In addition, the method does not assign importance or weight to variables (Kannan & Haq, 2007).

The relationships between the barriers of this study were evaluated by the *owner* of a *Dental Hospital*, an *Eye Hospital partner*, and an *academician* working on health tourism operations management. ISM studies are generally applied by referring to less than 6 industry experts and less than 3 academicians (Dwivedi et al., 2017). In addition, it is emphasized that at least 3 expert evaluations are sufficient for validation in studies using ISM (Yudatama et al., 2018). The proposed ISM steps can be listed as follows (Yenradee & Dangton, 2000; Kannan & Haq, 2007):

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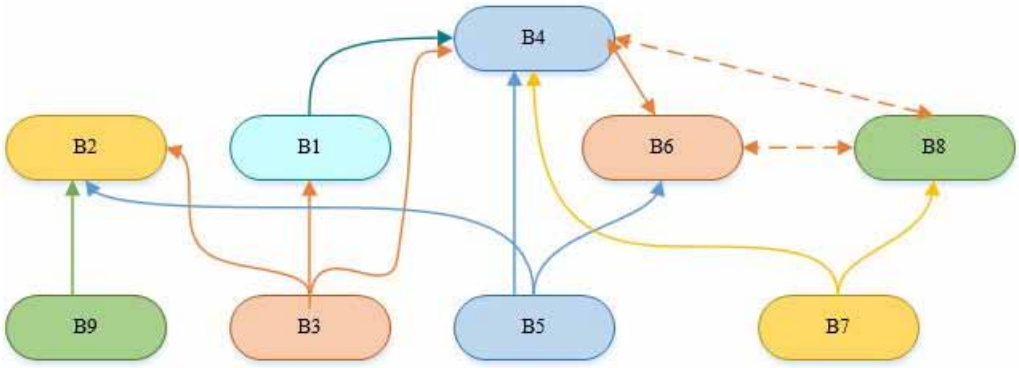
Step 1: Identify the barriers affecting medical tourism services via text mining

Step 2: Conduct in-depth interviews with the experts in order to define the relationships between medical tourism barriers, i.e. generate a reachability/adjacency matrix. Ask them to determine the direct relationships. If there is a relation that cannot be determined, label the connection link as *I* with dots in graph referring to the indeterminacy (see Table 1 and Figure 1).

Table 1. Neutrosophic reachability/adjacency matrix

Solving the barrier *i* is also the solution for the barrier *j*, or vice versa.	**B1**	**B2**	**B3**	**B4**	**B5**	**B6**	**B7**	**B8**	**B9**
B1. Medical tourism bans and restrictions	1	0	0	1	0	0	0	0	0
B2. Disruptions in the medical supply systems	0	1	0	0	0	0	0	0	0
B3. Restrictions on transportation between countries and decrease in international patient demand	1	1	1	1	0	0	0	0	0
B4. Operational and financial damage to institutions	0	0	0	1	0	1	0	*I*	0
B5. Disruptions in the financial flows of hospitals	0	1	0	1	1	1	0	0	0
B6. Difficulty in finding qualified healthcare professionals	0	0	0	1	0	1	0	*I*	0
B7. Complicated medical tourism procedures	0	0	0	1	0	0	1	1	0
B8. Wrong interventions to medical tourists	0	0	0	*I*	0	0	0	1	0
B9. Unexpected fluctuations in exchange rates	0	1	0	0	0	0	0	0	1

Figure 1. Initial directed neutrosophic graph with indeterminate causalities



Step 3: Apply the "hidden pattern determination" procedure. Select an indeterminate barrier as the on-state factor, identify a $(1\times n)$ state vector S including 1 in the place of on-state concept. Multiply the $(1\times n)$ vector S with the reachability/adjacency matrix which illustrates the opinions of the experts in order to find the hidden patterns.

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Here • denotes the matrix multiplication. $S \bullet N(E)$ gives a $(1\times n)(n\times n) = (1\times n)$ result. Update this result according to the *limit cycle procedure* in order to obtain S_1. Next, repeat the same procedure, find the $S_1 \bullet N(E)$ and update the previous S_1 to generate S_2. Then, repeat the same step until $S_{k-1} = S_k$ as a *fixed point*.

Let S(000100000) be the on-state vector. The effect of S on the dynamical system N is given by

$$S \bullet N = (0001010I0) \rightarrow (0001010I0) = S_1.$$

Here, the on state vector is updated to get S_1 from the $S \bullet N$ multiplication.

Next,

$$S_1 \bullet N = (000, I+2, 0, 203I0) \rightarrow (0001010I0) = S_2$$

The $S_1 \bullet N$ multiplication is updated by the limit cycle procedure. This is the threshold operation done by replacing $I+2$ and 2 by 1 since s_i values are greater than 1.

For example in case of $s_i <$ k, i.e. in case of -1 relationbetween concepts, the threshold operation updates the negative values by 0. Or in another case when s_i is not an integer, the value is updated by I.

Then,

$$S_2 \bullet N = (0I0, 2+I^2, 0203I) \rightarrow (0I01010I0) = S_3$$

$$S_3 \bullet N = (0I0, 2+I^2, 0203I0) \rightarrow (0I01010I0) = S_4 = S_3 \textit{fixed point}.$$

Here, one can result in that the barrier 4 is also in an indeterminate relation with B2.

Let S=(000001000) be the on-state vector. The effect of S on the dynamical system N is given by

$$S \bullet N = (0001010I0) \rightarrow (0001010I0) = S_1.$$

Since the multiplication gave the same result, we can say that the barrier 6 is also in an indeterminate relation with B2.

Let S=(000000010) be the on-state vector. The effect of S on the dynamical system N is given by

$$S \bullet N = (000I00010) \rightarrow (000I00010) = S_1.$$

Next,

$$S_1 \bullet N = (0002I0I0, I^2+1, 0) \rightarrow (000I0I010) = S_2$$

Then,

$$S_2 \bullet N = (0003I02I02I^2+1, 0) \rightarrow (000I0I010) = S_3 = S_2 \textit{fixed point.}$$

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Therefore, one can conclude that barrier 8 has indeterminate relationship with B4 and B6.

Step 4: Repeat the step 3 for the indeterminate barriers.

The same procedure is applied for all concepts. The neutrosophic reachability/adjacency matrix of this study is not change in the "hidden pattern determination" prodecure. Hence, Table 1 will be used in the following methology steps directly.

Step 5: Starting from the initial directed graph, the reachability/adjacency matrix is divided into different hierarchical levels. The table used in this process includes "accessibility set", "precursor set", "intersection" and "level" columns. In the *accessibility set*, there are both the variable itself and other interrelated variables. In the *predecessor set*, there are other variables that are directed towards the related variable and this variable itself. The *intersection* of these two sets is used as the next step in dividing the accessibility set into different levels. If the accessibility set and the intersection column take the same values, this variable is positioned at the *level 1*, that is, at the top of the hierarchy (Table 2). *Level 1* also represents the situation of being most affected by the other variables. Thus, the first iteration of the "separating into levels" process has been completed. For the remaining variables except for the variables assigned to the *level 1*, the second iteration continues, and these iterations continue until a final neutrosophic ISM is formed (Table 2).

Table 2. Dividing the neutrosophic reachability/adjacency matrix into hierarchical levels

	Accessibility Set	Precursor Set	Intersection	Level
Iteration 1				
B1	1, 3, 4	1, 3	1, 3	
B2	**2, 3, 5, 9**	2, 3, 5, 9	**2, 3, 5, 9**	**1**
B3	1, 2, 3, 4	3	3	
B4	**1, 3, 4, 5, 6, 7, 8**	1, 3, 4, 5, 6, 7, 8	**1, 3, 4, 5, 6, 7, 8**	**1**
B5	2, 4, 5, 6, 8	5	5	
B6	**4, 5, 6, 7, 8**	4, 5, 6, 7, 8	**4, 5, 6, 7, 8**	**1**
B7	4, 6, 7, 8	7	7	
B8	**4, 5, 6, 7, 8**	4, 5, 6, 7, 8	**4, 6, 7, 8**	**1**
B9	2, 9	9	9	
Iteration 2				
B1	**1, 3**	1, 3	**1, 3**	**2**
B3	1, 3	3	3	
B5	5	5	5	**2**
B7	**7**	7	**7**	**2**
B9	**9**	9	**9**	**2**
Iteration 3				
B3	**3**	3	**3**	**3**

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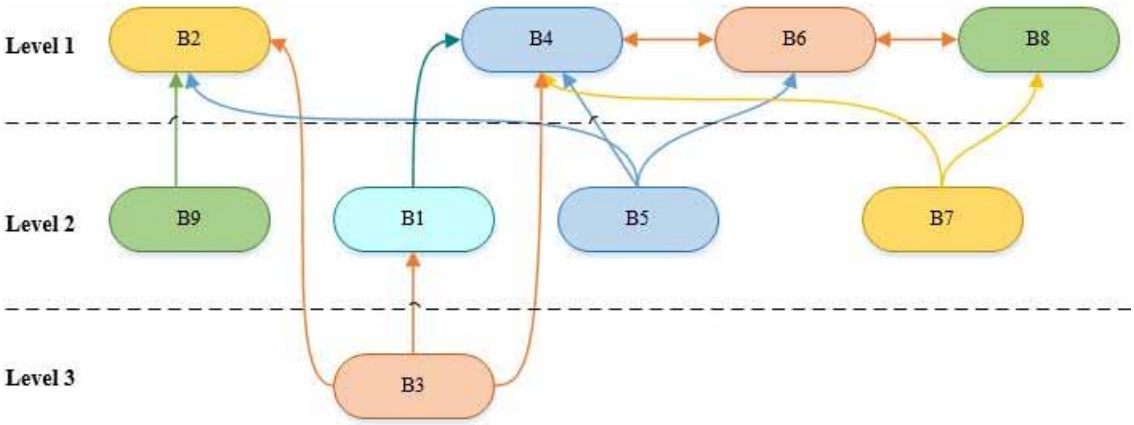
Step 6: Discuss the causalities of the relations. As a result of all these steps, the neutrosophic ISM diagram is created, reviewed in terms of consistency, and necessary changes (Figure 2).

Figure 2. Final ISM graph



FINDINGS

According to the results of the neutrosophic ISM analysis, the third level in the hierarchy, in other words, the first obstacle to be solved stands out as " B3. Restrictions on transportation between countries and decrease in international patient demand". All other obstacles except B5, B7 and B9 are affected by this barrier.

"B5. Disruptions in the financial flows of hospitals" is at the second level in the hierarchy, not affected by other obstacles defined in this study. "B2. Disruptions in the medical supply systems of the countries", "B4. Operational and financial damage to many institutions" and "B6. Difficulty in finding qualified healthcare professionals" will be overcomed by solving the B5 barrier. Moreover, "B9. Unexpected fluctuations in exchange rates" is not affected by other barriers too, and it is at the second level in the hierarchy, and basically has an impact on "B2. Disruptions in the medical supply systems of the countries", which is associated with import and export restrictions, in terms of creating price balances. Similarly, "B4. Operational and financial damage to many institutions" is decreased with the regulation of "B7. Complicated medical tourism procedures", which is not affected by other obstacles, and positioned at the second level in the hierarchy, while a decrease in "B6. Difficulty in finding qualified healthcare professionals" and "B8. Wrong interventions to medical tourists" can also be expected. The second level "B1. Medical tourism bans and restrictions" barrier is directly affected by "B3. Restrictions on transportation between countries and decrease in international patient demand". The B1 barrier also directly causes the damage (B4) of many institutions involved in medical tourism. Therefore, with the lifting of the restrictions on international circulation, it will be possible for medical tourism parties to continue their business.

At the first level, at the top of the hierarchy, there are "B2. Disruptions in the medical supply systems of the countries", which is vulnerable to other obstacles, "B4. Operational and financial damage to many institutions, " B6. Difficulty in finding qualified healthcare professionals " and "B8. Wrong interventions

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to medical tourists" barriers. B4, B6 and B8 affect each other mutually. Especially the damage to many institutions (B4) barrier is directly or indirectly affected by all other variables. In other words, it reflects the effect of changes in all other obstacles and in a sense expressesing the result of the model. Obstacles located at the top level of the hierarchy can be interpreted as a result of other barriers of the lower levels.

With the rapid spread of the Covid-19 outbreak all over the world and declared as a pandemic, the restrictions on transportation between countries (B3) stopped international patient mobility in terms of medical tourism, as well as directly affecting the health services, which are the main component of medical tourism, and disrupted the operations of hospitals (B5), and the medical functioning of tourism services has become uncertain and complex (B7). While the pandemic caused the greatest global economic recession in recent history, it caused fluctuations in exchange rates (B9) by bringing various problems in terms of medical tourism. These four obstacles can be described as the main obstacles that bring medical tourism to a standstill in the pandemic duration. It is necessary to create a suitable environment in order to overcome these obstacles and to continue the development of medical tourism.

DISCUSSION

In order to compare the proposed neutrosophic ISM approach with Fuzzy Cognitive Maps (FCM) and Neutrosophic Cognitive Maps (NCM) approaches, the preliminaries of those techniques must be taken into consideration in detail. All of them are based transitivity / reachability matrices, but ISM is different from those by providing a hierarchical causal structure. Thus, ISM is a powerful tool to identify the root causes (Jahangoshai Rezaee et al., 2019).

Some of the recent applications of the ISM studies interrelating FCM and NCM techniques are discussing causal relationships between delay factors (Jahangoshai Rezaee et al., 2019), critical success factors (Baykasoglu and Gölcük, 2017), and readiness assessment factors (Kalantari and Khoshalhan, 2018). ISM has an advantage in analyzing the negative factors such as risks, barriers, obstacles, challenges, disruptions, etc. over FCM and NCM due to presenting a hierarchical causal structure. The implications of the ISM technique are based on evaluating these negative variables.

FUTURE RESEARCH DIRECTIONS

In the future researches, published texts in different languages can be accessed by expanding the data sources. With text mining, real medical tourist experiences that have stated their medical tourism preferences in other social media accounts or personal blogs can be accessed and their opinions can be used as data.

CONCLUSION

Medical tourism, which has been developing rapidly in recent years, and it is an open research field in terms of the opportunities it offers, as well as the obstacles and difficulties encountered in its development. The coronavirus pandemics brought medical tourism to a halt with the closing of the borders by the countries, and attracted attention as an important risk factor affecting the medical tourism. Studies

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published during the pandemic period focuses on issues such as the difficulties and perceived barriers faced by medical tourists, travel restrictions and health travel, immigrations, and the problems experienced in the import of medicines and medical supplies needed by countries.

This study aims to identify the barriers experienced in medical tourism services that is interrupted due to the coronavirus pandemics and to analyze the relationships between these obstacles. For this purpose, text mining and neutrosophic Interpretative Structural Modeling analyzes were applied. Barriers and difficulties expressed by real medical tourists have been identified by text mining. The relationships between the identified barriers were evaluated through expert evaluations, and finally, how these obstacles affect each other was examined in a hierarchical structure.

Accordingly, with the announcement of the Covid-19 pandemic, restrictions on transportation between countries stopped international patient mobility in terms of medical tourism. While the outbreak directly affected the health services, which is the main component of medical tourism, and the functioning of the hospitals was disrupted. Parallel to these, the functioning of medical tourism services has become uncertain and complex. The fact that the pandemic caused the greatest global economic recession in recent history brought various financial and developmental negativities in terms of medical tourism. The main barriers that bring medical tourism to a halt during the pandemic process are restrictions on transportation (B3), disruption of hospitals (B5), complexity of medical tourism services (B7), and unexpected fluctuations in exchange rates (B9). It is necessary to create a suitable environment in order to overcome these barriers specified in time and to continue the development of medical tourism.

One of the contributions of the study was to determine the barriers that negatively affect the development of medical tourism under pandemic conditions using text mining. The other contribution is to propose a novel way of ISM to analyze the relationships between barriers with interpretive structural modeling and to reveal the root barriers.

The limitations of this paper is based on the defined barriers reflecting certain social media users in a certain period of time. It would not be correct to generalize the results of the study as it belongs to a special time duration. In addition, the neutrosophic ISM method is provided by a particular group of expert opinions with personal views and experiences of these individuals. Therefore, the results of the analysis can also be affected by the biased opinions of the selected experts. In order to overcome this constraint, the selection of experts was considered as an important point.

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KEY TERMS AND DEFINITIONS

Interpretive Structural Modeling (ISM): A process of interpreting systems with well-defined factors by modeling the relationships in visual forms.

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Medical Tourism: An opportunity fot the patients' access to medical institutions for treatment/ rehabilitation outside their country of residence.

Medical Tourism Service: The bundle of services including medical tourists / patients, health institutions, assistant companies / intermediary firms, accommodation service providers, transfer and / or ambulance services, flight ticket reservation providers, travel agencies, translation offices, visa service providers, insurance companies.

Neutrosophic Set: $x=(T,I,F)$ denotes T as degree of truth, I as degree of indeterminacy, and F as degree of falsity.

Neutrosophy: An extension / combination of the fuzzy logic by including the indeterminacy that is introduced by Smarandache in 1999.

Text Mining: The exploration and the extraction of interesting and important information from a free or unstructured text covering everything from the acquisition of information to the classification and aggregation of text, as well as entities, relationships and event extractions.

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Chapter 27
The Role of the Paper Packaging Industry in the Circular Economy:
The Causal Relationship Analysis via Neutrosophic Cognitive Maps

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ABSTRACT

The circular economy movement has paved the way for the sustainable supply chains with its nature-friendly structure via recycling and repeated use of products. Hence, this study aims to determine the factors affecting the paper packaging industry in the circular economy in order to provide a conceptual model of the cause-and-effect relationships through neutrosophic cognitive mapping. Paper packaging machinery manufacturer Windmöller & Hölscher's representatives have been in-depth interviewed to gather their opinions. Since there are some relations that cannot be determined (i.e., existence of indeterminacy), neutrosophy became a necessity in this chapter. Delphi technique is implemented to obtain a single neutrosophic adjacency matrix. As a result, the sustainable supply chain structure based on environmental awareness and recycling has been modeled visually, and the causal relationships were determined.

INTRODUCTION

Operations management studies have taken an important role in the sustainability field of study since the early 2000s. Especially as a new research topic, "circular economy" (CE) has made effective contributions, and it is clearly stated in the business reports that the CE movement has trillions of dollars' economic potential (Ellen MacArthur Foundation, 2021; European Union, 2021; McKinsey Quarterly,

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2017; Accenture, 2017). In addition to this economic attractiveness, the environmental potential of the movement is also accelerated the transition to the CE (Ellen MacArthur Foundation, 2017). This concept has gained worldwide popularity, has had a significant impact on business and politics. For example, many Fortune 500 multinational companies, including famous brands such as HP, H&M, Philips, Google, Unilever, Danone, and Nike, have adopted the circular economy (Agrawal et al., 2019).

The CE aims to create better business models for industry and environmental consequences for society. The basic idea is here to achieve a CE that reproduces and restores traditional business models, materials, products, and other resources by circulating them through a system (Ellen MacArthur Foundation, 2017). In this context, there are three operational strategies: (i) Reverse flow is the establishment of large-scale industrial systems capable of returning all materials, components and products to the system. Activities such as repair, remanufacturing, renovation and recycling are included. (ii) Circular design emphasizes the need to design products that are more durable, repairable and recyclable to enable circularity. It also draws attention to the need to standardize components and materials in order to facilitate modularity / adaptability of product design and reverse flow. (ii) Circular business model emphasizes the transition of companies to circular business models such as renting, receiving services or sharing economy (Agrawal et al., 2019).

The first step to make a CE possible is to recycle the products used and establish closed-loop systems. In the background of the CE movement, it contains awareness elements such as recycling, biodegradability, environmentally friendly production and environmental awareness.

Today, the environmental awareness has reached to the highest importance level, and hygiene sensitivity has increased due to the COVID-19 global pandemics. Therefore, the need for cleaning materials and disposable products has reached high levels too. In line with the efforts to raise environmental awareness about the damage caused by plastic materials to nature, the production of disposable products has started to be shifted to the paper industry. Due to COVID-19, the increase in the ordering home frequency with plastic bags, the widespread use of plastic gloves, increased the use of plastics in this period. For example, the consumption of paper packaging in Latin American countries has decreased, up to the data of 2015 (Barisauskas, 2020). This situation, which is introduced as "from the pandemics to the plastics epidemic" (Ak, 2020), started to meet the seasonal demand increase, environmental awareness and policies in a different way, and paved the way for the production of paper equivalents of plastic products. Examples include paper straws (Termokap, 2020), paper plates, paper bags (Milliyet, 2020), paper diaper packaging (Hijyen Ambalaj, 2020).

The environmental pollution of plastics which remains undissolved in nature for centuries, the harm to marine organisms that is detected in the sea as micro-plastic and nano-plastic (Greenpeace, 2019), and the transfer of these harmful substances to seawater, fish, land creatures and human beings, even to the placenta (BBC, 2020) has triggered the demand for paper packaging.

On a country basis, while sustainable environmental policies, laws and practices are on the agenda, companies organize their strategic plans with the least use of plastic. For example, Germany will ban single-use plastic bags by 2022 (CNN, 2020), Canada will ban similarly (NTV, 2020), the UK, some states of the USA and major cities of China will also impose planned bans (TRT, 2020). As a company example, the British retail industry giant named Morrison plans to be the first supermarket to remove all plastic bags in the checkout box (MatbaaHaber, 2020).

Paper packages are consumer products that we use frequently in daily life. There are flat and twisted handled paper bags, flour and tea paper bags, etc. In addition to being the most economical packaging type, it makes possible to obtain durable, thin and light products with less raw material usage. It is a

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highly preferred type of packaging for its ease of production and economy (Packaging Manufacturers Association, 2021). In addition, it is reported that the consumption and production of paper packaging has increased, especially with the acceleration of e-commerce and the export of paper packaging to abroad (Dünya, 2021; Bloomberght, 2021).

The aim of this study is to determine the factors determining the role of the paper packaging industry in CE through in-depth interviews with industry representatives and to model the cause-effect relationships of these factors through neutrosophic cognitive mapping. In this respect, paper packaging machinery manufacturer Windmöller & Hölscher officials were interwieved, and expert opinions were gathered for the analysis. As a result of the study, the sustainable supply chain structure was visually modeled and the connections of the triggering precursor variables were determined.

The advantage of using neutrosophic logic is the ability to distinguish between relative truth which is one or a few worlds only noted by 1, and absolute truth denoted by 1^+. Similarly, neutrosophic logic distinguishes between relative falsehood, noted by 0 and absolute falsehood noted by $^-0$ handled (Kandasamy & Smarandache, 2003). Thus, the outcomes of the methodology provide the intederminacy degree of the concepts under examination.

The following sections of the study include the supply chain structure of the paper packaging industry, the theoretical infrastructure of neutrodophic cognitive mapping, analysis and results sections.

BACKGROUND

Paper Packaging Industry and Supply Chain Structure

The most important supplier of the paper packaging industry is the paper manufacturer. In particular, companies producing recycled paper are in an important position in terms of sustainability. According to the Environmental Defense Fund and the Environmental Innovation Alliance, recycled paper reduces total energy consumption by 38%, greenhouse gas emissions by 40%, particulate emissions by 36%, waste water by 32% and solid waste by 46% (EDF, 2013). Recycled paper, including industrial waste and consumer waste, does not have any quality disadvantages (Lunati, 2007). There are also glue and ink suppliers.

Companies that manufacture paper packaging supply directly to sectors such as flour and tea that use paper packages in their products, or provide paper bags to the retail, textile, food sectors through wholesalers, and serve both domestic and foreign markets. Costs are related to raw material, energy and labor. There are difficulties in supplying raw materials in the sector, the decrease in supply or fluctuations in the exchange rate are effective. In particular, restrictions on scrap imports in some countries make difficult to access recycled paper raw materials.

By the paper packaging machinery manufacturer Windmöller & Hölscher company representatives' opinions and the literature (Bortolini et al., 2018; Cheng, 2019; Niero & Hauschild, 2017; Giray Resat & Unsal, 2019; Meherish et al., 2019; Yokokawa et al., 2018; O'Rourke, 2014; Sonneveld et al.2005), the concepts affecting the paper packaging industry are determined as follows:

1. Paper packaging production / investments
2. Domestic consumption of paper packaging
3. Paper packaging export

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4. Employment
5. The cost incurred by the manufacturer
6. Increasing business volume
7. Supplier coordination requirements
8. Increased need for recycling suppliers
9. Environmental awareness of recycling
10. Sustainable supply chain

These concepts will be analyzed in the following sections.

Neutrosophic Cognitive Maps

Cognitive map (CM) is a visual model illustrating the factors of a problem and understanding the cause and effect relationships between these factors (Swan, 1995). Cognitive mapping technique is an effective method to reveal the covered opinions of individuals with detailed directional relations about a pre-defined subject. CM types are basicly concept maps, semantic maps and cause-effect maps (Siau & Tan, 2005). Moreover, CM elements are defined as concepts, relationships, and power of relationships (Nadkarni and Nah, 2003). In addition, there are simple, weighted, fuzzy, and neutrosophic cognitive maps in the literature.

In the context of simple CMs, the relationships among concepts take values of +, -, and 0. A positive relationship indicates that the two concepts increase or decrease in the same direction; negative relationship shows that the two concepts increase / decrease in opposite directions; 0 indicates that there is no relationship between these two concepts (Eden, 2004). Expert opinions can be collected through questionnaires, data extraction from documents or interview (Axelrod, 1976). Especially in the analysis of complex systems, surveys and data analysis are used. As the number of criteria discussed in this study is relatively low, in-depth interviews were preferred.

Cognitive mapping process: Determining causal relationships, concepts, conceptual maps, coding concepts and structures, updating concept diagrams in line with expert opinions, creating, verifying and analyzing the cognitive map (Armstrong, 2005).

CM is drawn visually after determining the concepts and the relationships between these concepts. Nodes and directed arrows connecting these nodes are clarified, and indirect relationships are rendered visually. For example, if there is a relationship from concept x to concept y, and from concept y to concept z, this also means that there is a relationship from concept x to concept z. This operation is called "multiplication" and the following rules apply: a positive value multiplied by the corresponding value is equal to zero multiplied by zero, the product of two opposite signs is negative, the product of negative value and negative value is positive, multiplication has distributive property and is symmetric. Also, if more than two directed relationships start at the same point and end at the same point, the effect of the first point is added to the "total effect" on the second point and the operation is called the "addition" operation, the addition rules apply (Axelrod, 1976).

Visually represented CMs can be transformed into matrix form, called "valency matrix" or "adjacency matrix" (Özesmi and Özesmi, 2004). The value matrix is an $n \times n$ square matrix with concepts in rows and columns. The cells in the matrix contain the strength of the relationships (v_{ij}) between the related concepts (by taking these 1, -1 or 0 values). All the elements on the diagonal take the value 0.

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Fuzzy CMs are fuzzy signed directed graphs including feedbacks. A causal concept from C_i to C_j can be measured by the directed edge e_{ij} by stating how much C_i causes C_j. e_{ij} takes values in [-1,1] fuzzy interval. e_{ij}=0 means that there is no causality, e_{ij}>0 denotes a positive causality, and e_{ij}<0 refers to a negative causality. Here, the directed edge e_{ij} measures only the presence or absence of the impact, but indeterminacy of a relation between two concept are not handled (Kandasamy & Smarandache, 2003). In addition, indeterminacy refers to the situation of no relation can be determined between two concepts

Neutrosophic CMs are generalization of fuzzy CMs that are defining the notions of neutrosophic matrix and defining the neutrosophic relational maps (Kandasamy & Smarandache, 2003). In case of analysis including indeterminate concepts, one cannot provide a mathematical expression. Therefore, neutrosophy is introduced by Smarandache as an extension / combination of the fuzzy logic by including the indeterminacy. Fuzzy logic only measures the existence or non-existence membership grades, while the neutrosophic sets can attribute the concepts when the relations between indeterminate concepts (Kandasamy & Smarandache, 2003). In particular, there are criticisms focusing on traditional fuzzy CMs that are not sensitive to different initial conditions, i.e. the same result is obtained regardless of the starting vector (Asan & Kadaifci, 2020). Hence, this study prefers to apply neutrosophic CMs for any possible indeterminant expert opinion condition.

Fuzzy Cognitive Map (FCM) that was introduced by Bart Kosko have a major role when the concern is an unsupervised concept. It is a directed graph with concepts like policies, events etc. as nodes and causalities as edges. It represents causal relationship between concepts (Kosko, 1986; Kandasamy & Smarandache, 2003). When the nodes of the FCM are fuzzy sets then they are called as fuzzy nodes. FCMs with edge weights or causalities from the set {–1, 0, 1} are called simple FCMs. Consider the concepts $C_1, \ldots, C_n$ of the FCM. Suppose the directed graph is drawn using edge weight $e_{ij} \in \{0, 1, -1\}$. The matrix E be defined by $E = (e_{ij})$ where e_{ij} is the weight of the directed edge C_iC_j. E is called the adjacency matrix of the FCM, also known as the connection matrix of the FCM. It is important to note that all matrices associated with an FCM are always square matrices with diagonal entries as zero. Let $C_1, C_2, \ldots, C_n$ be the nodes of an FCM. $A = (a_1, a_2, \ldots, a_n)$ where $a_i \in \{0, 1\}$. A is called the instantaneous state vector and it denotes the on-off position of the node at an instant. $a_i = 0$ if a_i is off and $a_i = 1$ if ai is on for i = 1, 2, …, n (Kandasamy & Smarandache, 2003). FCM uses the equation $a_i^{(k+1)} = f(a_i^k + \sum_{i\neq 1}^{n}{}_{j=1}\left(a_j^k * w_{ji}\right))$ where the left side of the equation represents the value of concept a_i in the $(k+1)^{th}$ step. This value is obtained by multiplying the value of k^{th} step with the causality matrix, and then adding this this multiplication to this value in order to generate a threshold function that normalizes the values of the concept so that one can compare them. Moreover, by applying the previous equation for an appropriate number of iterations, a simulation process for a long-term behavior of a system can be observed. The simulation can be repeated many times with different initial vectors so that the effect of different initial states on the dynamic behavior of the system can be monitored. When the examined system reaches a fixed equilibrium point or oscillating behavior between certain lower and upper values, the simulation process is terminated, and the outputs are interpreted according to the values obtained as a result of the last iteration (Asan & Kadaifci, 2020).

A neutrosophic set $x=(T,I,F)$ denotes T as degree of *truth*, I as degree of *indeterminacy*, and F as degree of *falsity*. In a neutrosophic graph, an indeterminant relation is shown with dots as an arc. A neutrosophic CM is a neutrosophic directed graph with concepts as nodes, and causalities (i.e. indeterminates) as edges. Let $C_1, C_2, \ldots, C_n$ denote n nodes as neutrosophic vectors from the neutrosophic vector

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space *V*. Therefore, a node C_i is represented by $(x_1,\ldots x_n)$ where x_k is 0 or 1or I (indeterminate). Here, $x_k = 1$ means that the node C_k is in the on state and $x_k=0$ represents the node is in the off state and $x_k=I$ refers that the nodes state is an indeterminate in this case. Let C_i and C_j denote the two nodes and the directed edge refers to the causality called *connections* or *relations.* Every edge in the neutrosophic CM is weighted with a number in the set $\{-1, 0, 1, I\}$. Let e_{ij} be the weight of the directed edge C_iC_j, $e_{ij}\in\{-1, 0, 1, I\}$. $e_{ij} = 0$ if C_i does not have any effect on C_j, $e_{ij}=1$ if increase (or decrease) in C_i causes increase (or decreases) in C_j, $e_{ij}= -1$ if increase (or decrease) in C_i causes decrease (or increase) in C_j. $e_{ij}=I$ if the relation or effect of C_i on Cj is an indeterminate (Kandasamy & Smarandache, 2003).

A simple neutrosophic CM has an edge weight from $\{-1, 0, 1, I\}$. Let the neutrosophic matrix $N(E)$ be defined as $N(E) = (e_{ij})$, where e_{ij} is the weight of the directed edge C_iC_j, $e_{ij}\in \{0, 1, -1, I\}$. $N(E)$ is called the neutrosophic adjacency matrix of the neutrosophic CMs. Let $A=(a_1,a_2,\ldots,a_n)$ where $a_i \in \{0,1,I\}$. A is called the instantaneous state neutrosophic vector and it denotes the on- or off-indeterminate state position of the node at an instant; $a_i=0$ if ai is off (no effect), $a_i=1$ if a_i is on (has effect), and $a_i=I$ if aiis indeterminate (effect cannot be determined) for $i=1,2,\ldots n$. Let $\overline{C_1C_2}, \overline{C_2C_3},\ldots, \overline{C_iC_j}$ be the edges of the neutrosophic CMs. Then the edges become a directed cycle. A neutrosophic CM is called as cyclic if it has a directed cycle, acyclic otherwise. This is also a feedback. In case of a feedback in the neutrosophic CMs refers to a dynamical system. Let $\overline{C_1C_2}, \overline{C_2C_3},\ldots, \overline{C_{n-1}C_n}$ be a cycle, when C_i is switched on and if the causality flow through the edges of a cycle, and if it again trigger to cause C_i, we say that the dynamical system goes round and round. The equilibrium state for this dynamical system is called the hidden pattern. If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. For instance, let us start the dynamical system by switching on C_1. Let us assume that the neutrosophic CM settles down with C_1 and C_n on, i.e. the state vector remains as (1, 0,…, 0, 1) this neutrosophic state vector (1,0, …, 0, 1) is called the fixed point. If the neutrosophic CM settles with a neutrosophic state vector repeating in the form $A_1\rightarrow A_2\rightarrow \ldots \rightarrow A_t\rightarrow A_{t+1}\rightarrow \ldots \rightarrow A_n\rightarrow A_t$ where A_i is the the vector passing into a dynamical system repeatedly, $1\leq i\leq n$, then this equilibrium is a limit cycle. (Vasantha et al., 2019; Zafar & Wajid, 2019; Kandasamy & Smarandache, 2003).

In order to determine the hidden patterns, when C_1 is switched on when an input is given as the vector $A_1 = (1, 0, 0,\ldots, 0)$, the data should pass through the neutrosophic matrix by multiplying A_1 by the matrix N(E). Let A_1N(E) = $(a_1, a_2,\ldots,a_n)$ with the threshold operation obtaining by replacing a_i by 1 if $a_i \geq k$ and a_i by 0 if $a_i < k$ (k: a suitable positive integer) and a_i by I if a_i is not an integer. Then the resulting concept is updated. The concept C_1 is included in the updated vector by making the first coordinate as 1 in the resulting vector. Suppose $A_1N(E) \rightarrow A_2$, then consider $A_2N(E)$ and repeat the same procedure. This procedure is repeated till we get a limit cycle or a *fixed point* (Kalaichelvi & Gomathy, 2011; Banerjee, 2008).

METHODOLOGY STEPS

Step 1. Investigate and define the concepts

Step 2. Conduct in-depth interviews with the experts in order to define the relationships between concepts, i.e. in order to generate a neutrosophic adjacency matrix. Ask them to determine the direct relationships. If there is a relation that cannot be determined, label the connection link as I with dots *i*n graph referring to the indeterminacy.

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Step 3. Select an on-state concepts, identify a $(1\times n)$ state vector S including 1 in the place of on-state concept.

Step 4. Multiply the $(1\times n)$ vector S with the adjacency matrix $N(E)$ which illustrates the opinions of the experts in order to find the hidden patterns. Here $\bullet$ denotes the matrix multiplication. $S\bullet N\left(E\right)$ gives a $(1\times n)(n\times n) = (1\times n)$ result. Update this result according to the limit cycle procedure in order to obtain S_1. Next, repeat the same procedure, find $S_1\bullet N\left(E\right)$ and update the previous S_1 to generate S_2. Then, repeat the same step until $S_{k-1} = S_k$ as a *fixed point.*

Step 5. Repeat the step 4 for the other on-state concepts.

Step 6. Discuss the causalities of the relations.

The following part will discuss the provided background for this study's research question.

ANALYSIS

How the concepts of the paper packaging industry that defined in the "paper packaging industry supply chain" section affect each other in a positive or negative way is illustrated in the neutrosophic adjacency matrix (see Figure 1 and Table 1). Delphi technique is used to obtain a single neutrosophic adjacency matrix in this study. Based on in-depth interviews with sector representatives, "recycling and environmental awareness" triggers the consumption/production of the paper packages in both domestic and international markets. Paper package consumption in markets canalizes the related companies to increase their production capacities and to make new investments. These investments trigger employment, business volume and the need for coordination with suppliers, and the recycling vendor requirements in a positive way. However, the incurred costs' impacts cannot be determined and labeled as "indeterminant" (i.e. the dotted arcs) since some of the producers have a high financial budget, while some others are facing difficulty in payments. In particular, imported machinery, paper and glue costs are affected by the fluctuations in exchange rates.

Similarly, the causality between paper package consumption in the "domestic" and "international" market is not determined by the industry representatives, and the concept is labeled as "indeterminant". Alhough it is widely known that especially due to the pandemic period, the declined manufacturing in Europe, the increased need for single-use packages and the plastic bag usage prohibitions brought by the countries created a high demand from the European market; there is no strong evidence to talk about the causality. Therefore, the producers earning in foreign currency prefers to export the produced goods.

Let S=(0010000000) be the on-state vector. The effect of S on the dynamical system N is given by $S\bullet N = (1I01011100)\rightarrow(1I11011100) = S_1$. Here, the on state vector is updated to get $S_1$from the $S\bullet N$ multiplication.

Next, $S_1\bullet N = (I+1, I, 0, I+2, 0, I+2, I+2, I+2, 1, 3)\rightarrow (1I01011111) = S_2$

The $S_1\bullet N$ multiplication is updated by the limit cycle procedure. This is the threshold operation done by replacing $I+1$, $I+2$, 3 by 1 since s_i values are greater than 1.

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Figure 1. Neutrosophic cognitive map for the paper packaging industry's role in the circular economy

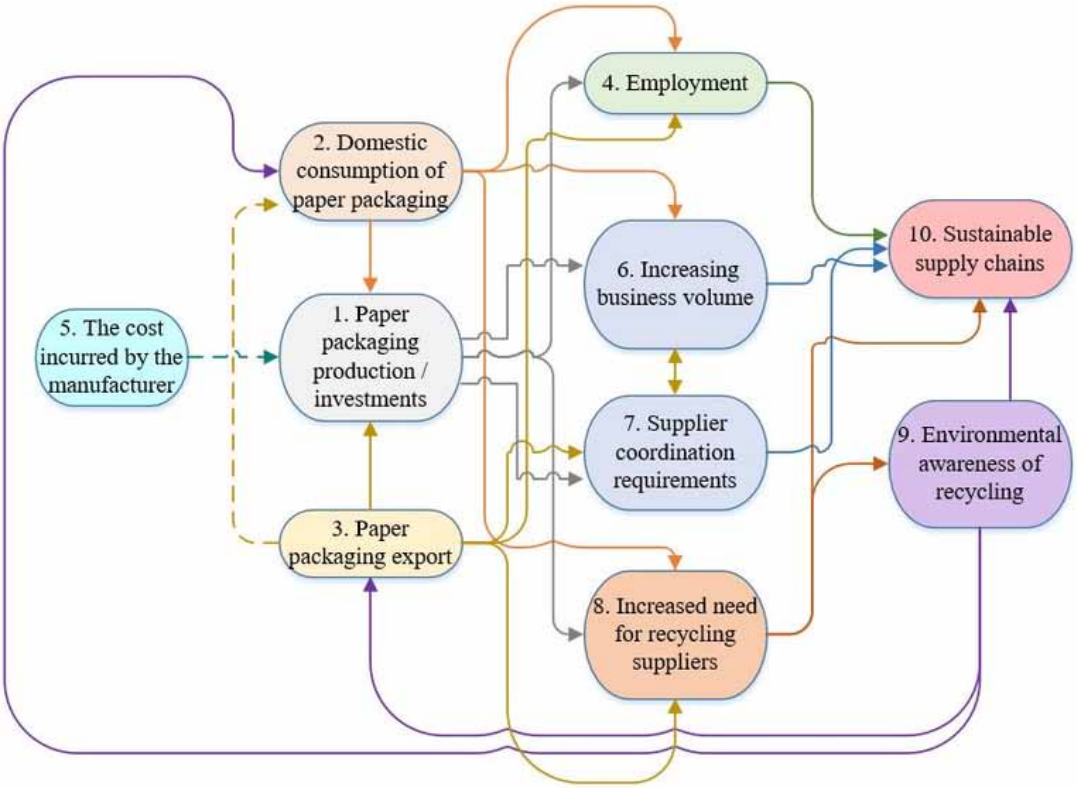

Table 1. Neutrosophic adjacency matrix

Concept #	1	2	3	4	5	6	7	8	9	10
1	0	0	0	1	0	1	1	1	0	0
2	1	0	0	1	0	1	1	1	0	0
3	1	*I*	0	1	0	1	1	1	0	0
4	0	0	0	0	0	0	0	0	0	1
5	*I*	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	0	0	1
8	0	0	0	0	0	0	0	0	1	1
9	0	1	1	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0

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For example in case of $s_i < k$, i.e. in case of -1 relationbetween concepts, the threshold operation updates the negative values by 0. Or in another case when s_i is not an integer, the value is updated by *I.* Then,

$$S_2 \bullet N = (I,1,1,I+1,0,I+1,I+1,I+1,1,4) \rightarrow (I111011111) = S_3$$

$$S_3 \bullet N = (2,I+1,1,I+2,0,I+2,I+2,I+2,1,4) \rightarrow (1111011111) = S_4$$

$$S_4 \bullet N = (2,I+1,1,I+2,0,I+2,I+2,I+2,1,4) \rightarrow (1111011111) = S_5 = S_4 \text{ } \textit{fixed point.}$$

The indeterminate relation of concept 3 "paper packaging export" has a positive relation with almost all of the other concepts, except concept 5 as "the cost incurred by the manufacturer".

For another indeterminate concept, let *S*=(000010000) be the on-state vector.

$$S \bullet N = (I000000000) \rightarrow (I000100000) = S_1 .$$

Here, the on state vector is updated to get S_1 from the $S \bullet N$ multiplication. Next,

$$S_1 \bullet N = (I,0,0,I,0,I,I,I,0,0) \rightarrow (I,0,0,I,0,I,I,I,0,0) = S_2$$

$$S_2 \bullet N = (000I0IIII3I) \rightarrow (000I0IIIII) = S_3$$

$$S_3 \bullet N = (0II0000I4I) \rightarrow (0II00000II) = S_4$$

$$S_4 \bullet N = (2I2II2I02I2I2I0I) \rightarrow (IIII0III0I) = S_5$$

$$S_5 \bullet N = (2II03I03I3I3II3I) \rightarrow (II0I0IIIII) = S_6 =$$

$$S_6 \bullet N = (III2I02I2I2II4I) \rightarrow (IIII0IIIII) = S_7$$

$$S_7 \bullet N = (2I2II3I03I3I3II4I) \rightarrow (IIII0IIIII) = S_8 = S_7 \text{ } \textit{fixed point.}$$

The indeterminate relation of concept 5 "the cost incurred by the manufacturer" has a positive relation with all of the other concepts.

The same procedure is applied for all concepts.

In addition, connection index value was calculated as $CI = \frac{2(n-1)}{n^2 - n} = \frac{2(9-1)}{9^2 - 9} = 0.22$, the acceptance value which is detailed in Table 2 is obtained (Çoban & Seçme, 2005; Kandasamy & Smarandache, 2003; Özesmi & Özesmi, 2004)

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Table 2. Acceptable connection index values

n	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**12**	**14**
CI	.67	.50	.40	.33	.29	.25	.22	.20	.18	.17	.15	.14

Findings of this analysis and discussion will be provided in the following section.

FINDINGS

This analysis steps are repeated to conclude these findings: Both domestic consumption and exports affect employment in a positive way. In order to produce more, companies need to work and hire human resources more. Similarly, the increase in manufacturing also increases the need for suppliers.

Employment is indispensable for sustainable supply chains and service networks. The increasing business volume with suppliers and the increasing need for recycling vendors also paves the way for a sustainable business environment. In addition, the increasing need for recycling vendors and environmental awareness directly affect sustainability.

"Recycling and environmental awareness" triggers the consumption of paper packaging both in the country and abroad. Paper package production and investments are directly affected by the combination of cost and consumption items. Subsequently, the need for employment, coordination with suppliers and a recycling supplier increases, followed by "recycling and environmental awareness". Finally, a suitable environment is provided for "sustainable supply chains".

FUTURE RESEARCH DIRECTIONS

The future studies may apply large-scale surveys covering more companies and nations to be consulted, comparative analyzes on countries basis can be performed, and structural equation models can be established with the available data. Moreover, in case of obtaining big data, data mining applications can be done with various classification algorithms.

CONCLUSION

The role of the paper packaging industry in the circular economy is at an important point in creating recycling and environmental awareness. For a sustainable world and business environment, awareness on the consumption of products that do not harm human health has reached to an extremely high level. The process of transition to cleaner, leaner, repetitive manufacturing environments makes a great progress thanks to the developing technology and opportunities.

The findings of this study indicates that investments and production have increased due to the increase in domestic and international paper consumption. Paper packaging consumption, on the other hand, increases thanks to environmental awareness and recycling, and directly affects employment in a positive way. At the same time, the need for cooperation and coordination with paper, glue and ink suppliers is increasing. There is a need for recycling vendors. Hence, environmental awareness is triggered again, brings along a big cycle. As a result of this cycle, where production and consumption are supported with

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environmental awareness, a suitable ground is obtained for sustainable supply chains. Doing the good and the right thing triggers the formation of environment friendly consumption, production, investment, business volume and profit, together with the bullwhip effect.

This study contributes theoretically and practically in terms of addressing how this sector can be modeled with the help of neutrosophic cognitive maps. It is made possible for an industry representative to visualize ideas by basing them on mathematical rules. In addition, the importance of recycling and environmental awareness was also revealed, and was determined as the first factor that should be evaluated.

Interviewing with only one firm's representatives, revealing the current situation of only one country can be shown as a limitation of the study. However, due to the high business volume in the sector and the fact that the selected company actively dominates the market, the selected company is thought to reflect the majority of the industry. In addition, it is clear that the company is sufficient to represent the global market in terms of its international structure, exporting machinery to multiple countries and employing employees from many nations.

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KEY TERMS AND DEFINITIONS

Adjacency Matrix: The matrix form of visually represented cognitive maps.

Cognitive Map: A visual model illustrating the factors of a concept/problem and understanding the cause-and-effect relationships between these factors.

Cognitive Mapping Process: Determining causal relationships, concepts, conceptual maps, coding concepts and structures, updating concept diagrams in line with expert opinions, creating, verifying, and analyzing the cognitive map.

Dynamical System: Neutrosophic cognitive maps with feedbacks.

Fixed Point: The case of the equilibrium state of a dynamical system is a unique state vector.

Fuzzy Cognitive Maps: Fuzzy signed directed graphs including feedbacks.

Hidden Pattern: The equilibrium state for a dynamical system.

Indeterminacy: The situation of no relation can be determined between two concepts.

Neutrosophic Cognitive Maps: The generalization of fuzzy cognitive maps that are defining the notions of neutrosophic matrix and defining the neutrosophic relational maps. It is a neutrosophic directed graph with concepts as nodes and causalities (i.e., indeterminates) as edges.

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Chapter 28
An Adaptive eLearning Sequence Based on Neutrosophic Logic

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ABSTRACT

In recent years, e-learning has become a revolutionary competitive method. Adapting the content according to learner knowledge is a current challenge in e-learning systems. Currently, most of the e-learning systems evaluate the learner's knowledge level according to crisp responses that are taken during the learning process. Therefore, one of the most significant challenges in e-learning is how to improve the course adaptation in order to achieve high-quality interaction for all learners. Adaptation is an efficient way to help learners to learn their learning activities in easy and a suitable ways. However, there are many factors that lead to uncertainty about the learner evaluation process. This chapter presents a novel approach to handle imprecision, vagueness, ambiguity, and inconsistency in the learner evaluation process to recommend the suitable learning material according to the learner's knowledge level.

INTRODUCTION

ELearning has been emerged to enhance the learning performance in schools, universities and everywhere. The fast development of the digital world supports people to improve their knowledge and learning. Therefore, people can interact with adaptive eLearning system to their interests and needs (Xu, et al., 2020). Recent researches in eLearning suggest adaptive eLearning system that can model learner's goals, needs and preferences, cognitive levels, knowledge degree and special requirements for effective e-evaluation (Benchoff, et al., 2018). The adaptive eLearning systems provide an environment in which learners can move forward by utilizing automatic processes. Such systems integrate techniques

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to consider learners' knowledge level and define their knowledge understanding to achieve their goals by following a probable sequential educational route (De Bra, 2017).

Traditional eLearning systems present the same learning materials to all learners without taking in consideration the learners' different intellect. Hence, to rectify this problem adaptive eLearning systems are suggested to present sequencing content suitable for learner's level (Goyal et al., 2015 a). Learners with different skill sets use the system when some of them are fast learners, others are slow. Some learners need to practice more exercises or examples according to their intellect) Radwan, 2014). Therefore, ELearning system should be based on the learner's level by collecting information about learner's to make it more reasonable. Adaptive eLearning system attempts to change the material presentation to fit the learner's intellectual level (Almohammadi & Hagras, 2013)

In recent years, adaptive eLearning method has become very innovative method due to computer and network technologies advancement (Makram et al., 2020). An interesting example is the development of adaptive eLearning environment in which learner responses to certain questions helps to determine the next level of the learning process)Goyal et al., 2012). Many studies indicate that adaptive eLearning create higher learner satisfaction levels, decrease learning times, and increase learner's achievement (Nafea et al., 2019).

ELearning organizations are working in a competitive medium as researches in eLearning are achieved in the assumption of complete information availability, while in the real environment this is impossible as many uncertainty aspects exist. This has resulted in emerging different methods to manage uncertainty. Various methods have been used to define uncertainty in learner's knowledge such as classical probability, Bayes probability and Dempster-Shafer theory. Classical and Bayesian theories depend on accurate statistical data. Dempster–Shafer theory allows expert to link the assignment of probabilities according to their knowledge, but it does not provide a way to determine these assignments. Therefore, the evaluation in these theories is implemented on the crisp response fundamental. Few researchers are dealing with the uncertainty of the learner's knowledge, in which the learner has the option of determining the percentage that he/she believes each answer to be correct in multiple-choice questions)Hosseini, & Kardan, 2011))Goyal, et al., 2015 b) (Goyal, 2017) (Megahed, & Mohammed, 2020).

Very few researches count uncertainty in determining the learner's knowledge level based on fuzzy logic and intuitionistic fuzzy logic (Goyal, et al., 2015 b) (Goyal, 2017) (Megahed, & Mohammed, 2020). Uncertainty handling for solving real life situations is one of the important issues discussed by artificial intelligence. This leads to emerging approaches such as fuzzy, intuitionistic fuzzy and a new approach called neutrosophic logic to give better attribute meaning (Radwan et al., 2016). Fuzzy theory was introduced by Professor Lotfi Zadeh which describes element attributes with a degree of membership in a set (Zadeh, 1965). Then Atanassov presented intuitionistic fuzzy set theory as an extension of the fuzzy sets which adds another dimension to fuzzy set which is the false value (Atanassov, 1983). Smarandache proposed a new approach to handle indeterminacy information called neutrosophic logic (Smarandache, 1999).

The adaptive eLearning system presents an adaptive eLearning sequence based on learner's knowledge and background. Learner's evaluation helps in finding the value of the knowledge acquired by students. Evaluation of learner's knowledge under uncertain conditions is not effective as most of adaptive eLearning systems depend on crisp answers to the tests taken during the learning process. There are neglected influences about evaluation process that participate to uncertainty such as selecting more likely correct solution or selecting a solution by guess (Goyal, et al., 2015 b) (Goyal, 2017) (Megahed, & Mohammed, 2020).

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In this chapter, Adaptive eLearning system based on neutrosophic logic is presented. This approach decides what and how much content should be presented to a learner according to his knowledge level. This approach decides the learner's score on the basis of test taken to study a particular topic. A test based on domain topic is provided to assess the learner knowledge level with respect to the total time taken by the learner to solve the test with the time recommended by the system. Neutrosophic logic is used to match the learning material to the learner in the form of text, examples and exercises. In which, learning material is having a certain percentage of membership, non-membership and indeterminacy instead of one particular user level. The main objective of this chapter is to show that such a suggested adapted eLearning system will enhance learner's knowledge and recognition; also it will help in evaluating learner's understanding.

This chapter is organized as following: The first section provides a brief review of eLearning, adaptive eLearning systems and its architecture. The second section discusses the previous work presented about adaptive eLearning systems. The third part discusses uncertainty approaches to understand the differences between them. The fourth section presents the proposed method for the learner's assessment under uncertainty and describes how this assessment can be used to provide a personalized learning path for the learner. Least but not last, solutions and recommendations are presented in section five. Finally, conclusion is made in section six.

BACKGROUND

eLEARNING

Most of the researches define learning as acquiring knowledge, remembering information, obtaining skills that can be used to understand reality. Learning process is defined as whereby knowledge is created including four-stage cycle which is perceiving, thinking, feeling, and acting (Sugarman, 1987). ELearning is a different method for teaching and learning (Govindasamy, 2001). ELearning is based on network and that gives the ability to real-time sharing information, delivery to the user via his/her device using internet technology, and the largest view of learning that increases the paradigms of education (Kulkarni et al., 2020).

ELearning is a kind of submission way to allow the concurrent and noncurrent interchange of resources over network. These systems allow communication and collaboration remotely by using electronic correspondence. ELearning provides a balance between the technology provider and the acceptance matter. Incorporating the novel technology Web 3.0 in the eLearning process is complicated, as eLearning 3.0 focuses on involving the learners in the learning process (Muniasamy, & Alasiry, 2020). Learners can access a lot of contents to learn various topics but finding the suitable content to learner is a weary process. Therefore, recommending the eLearning contents to a learner based on the tasks already done by the learner becomes an important task (Ennouamani, & Mahani, 2017).

TYPES OF eLEARNING

Most common types of eLearning are (Oye, & Salleh, 2012):

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A. Computer Managed Learning (CML)

In this type of learning, Knowledge repositories are used for computer-assisted teaching methods. These data include pieces of information that the participant must learn, as well as a variety of ranking criteria that allow the model to be adapted to the student's needs. Furthermore, academic institutions use computer-assisted learning systems to store and retrieve data, which helps with effective teaching. This may include information including lecture notes, training materials, grades, programmer details, and enrolment details, etc.

B. Computer Assisted Instruction (CAI)

Computer Assisted Instruction is another form of eLearning that combines computers with conventional instruction. This may refer to interactive tools or training software for students. To improve learning, computer-assisted training techniques use a mix of multimedia like text, images, audio, and video. Most academic institutions today, both online learning and traditional learning, use various forms of computer-assisted learning to support students' improvement of skills and expertise.

C. Synchronous Online Learning

This type of learning allows educators from all over the world to engage in a learning process at same time. Online chatting and video - conferencing are used in real-time synchronous online learning since these resources enable students and teachers to start answering questions and get answers while communicating with the other students. The rapid growth of online learning technology has facilitated this form of online learning. Nowadays, synchronous eLearning is thought to be extremely beneficial because it removes many of the traditional drawbacks of eLearning, such as social separation, as well as weak teacher-student and student-student relationships.

D. Asynchronous Online Learning

In this type of e learning, student groups study separately at multiple locations and times, with no real-time contact. Asynchronous eLearning offers to students more mobility, also Asynchronous eLearning perceived to become more student-centered than the synchronous alternatives. It enables learners to learn at their own pace. They really aren't expected to train at regular time periods like many students, and they may establish the own learning schedules.

E. Fixed eLearning

Fixed eLearning, "fixed" means that the material used during the education process does not differ from its traditional state, and all of the people who enrolled receive the same material. The materials are set in stone by the educators and do not respond to the needs of the students. For hundreds of years, this style of learning has always been the normal in traditional classes, but it isn't suitable in eLearning worlds. Since fixed eLearning doesn't really take advantage of the useful real-time data gathered from student inputs. Analyzing student's information and making improvements to the materials based on that data contributes to improved learning objectives for a class.

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F. Linear eLearning

The word "linear communication" refers to the transfer of data from transmitter to recipient without interruption. In case of this type of eLearning, it prevents two-way contact between students and teachers. While this form of eLearning has a place in education, it is becoming less important as time passes by.

G. Interactive Online Learning

Senders may become recipients and vice versa with this form of e learning, essentially allowing a two-way channel of communication between the involved parties. Teachers and students will modify their teaching methods based on the messages sent and received. As a consequence, interactive eLearning is much more common as it enables teachers and students to collaborate.

H. Individual Online Learning

This type of e learning means to the number of students who are actively involved in reaching the learning objectives, instead of the material's student-centeredness. It was used as traditional learning as students practice individual learning, they review the teaching content on their own (independently) and are required to fulfil the learning objectives. Since it relies on students studying independently, this way of learning is not suitable for improving communication skills and teamwork abilities in learners.

I. Collaborative Online Learning

This is a new type of e learning where a group of learners learns and achieves their learning goals together. In order to accomplish shared learning goals, students must cooperate and practice as teamwork. This is accomplished by the creation of successful classes, in which each student must consider the strengths and limitations of the others. The students' communication and collaborative abilities are improved as a result of this. Collaborative eLearning focuses on the principle that information is better built through a community of people who can communicate and learn from the others.

J. Adaptive eLearning system

Traditional eLearning systems ignore the variance of learners and present the same learning content to all. Adaptation is a proposed approach to overcome these issues as it considers learners diversity due to their preferences, knowledge, learning style and skills (Alshammari et al., 2014). Adaptive eLearning is a proposed way in which internet-enabled computers are used to adapt the presentation of learning content according to learner's needs automatically defined by the system based on learner's interaction to the system in the course. Therefore, eLearning system is a tool that plays vital role to provide adaptive eLearning environment (Sweta, 2021).

The researches in adaptive eLearning systems are directed toward learning platforms where learner's motivation, knowledge, learning styles and skills are increasingly taken into account. Adaptation is the capability of a system to change its attitude. The factors that affect learner focus on the idea of adaptive learning systems as an alternative to the traditional approach in the development of learning materials (Kulaglić et al., October). There are two kinds of adaptive learning system; the first one is the static that

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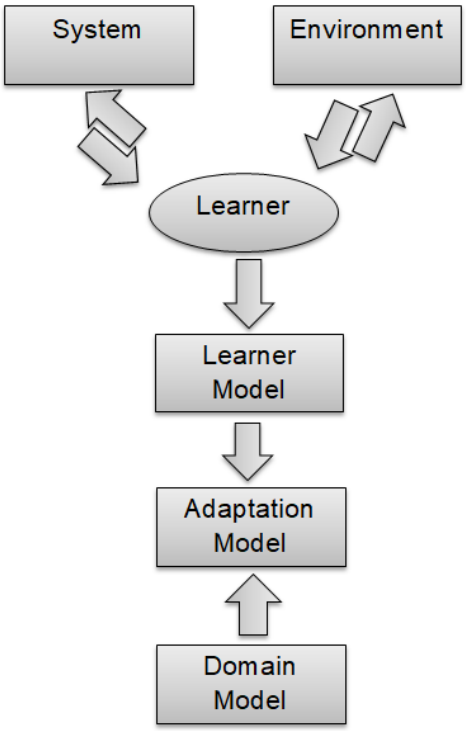
use the predefined learner's characteristics. It's made before the learning process begins. The second one is the dynamic one which requires a process of interaction between the learner and the system (Prasad, 2020).

The most vastly applied adaptation objective is the learning content as courses are presented in a special path to each learner taking into account learner's knowledge level. Some researchers have suggested the adaptation of recommended links and additional materials while using the system (Knott, 2020). In other words, the learning content, user interface, learning navigation, can be an adapted the learner's preferences. In addition, the learning path also can be adapted by referring the learner to external sources or skipping some contents. Also, the collaborative learning is a sort of adaptation by creating groups of learners with the same characteristics in order to allocate learner to the convenient set (Verbert et al., 2012) (Conati, et al., 2002) (Mitrovic et al., 2002).

ARCHITECTURE OF ADAPTIVE ELEARNING SYSTEMS

The Adaptive ELearning System introduce a set of approaches that are combined together to provide eLearning content to the learners with the objective of offering customized materials. These systems focus on adaptive contents which are generated to the learners considering the learners preferences (Kolekar et al., 2019). In this section, the main components of adaptive eLearning systems are presented as following (Figure 1):

Figure 1. Adaptive eLearning systems' components
Source: Ennouamani, & Mahani, 2017



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A. Learner Model

Learner model includes learner characteristics, representation, and learning style. Most Adaptive eLearning systems focus on learner knowledge, preferences and learning style. The most widely used model of learner model is knowledge; it uses learner's interactions to build the learner model. Different methods could be used to build this model such as Bayesain network, sequential pattern mining, association rules based on visited, time spent on learning material and test results (Alshammari et al., 2014, July). The learner model is defines as a component that provides a structured presentation based on learner's preferences. This component is the most significant part of the learning systems including objectives, knowledge background preferences, learning styles, cognitive styles, reasoning styles, background, skills, affection, needs, habits, motivation, and interests (Sweta, 2021).

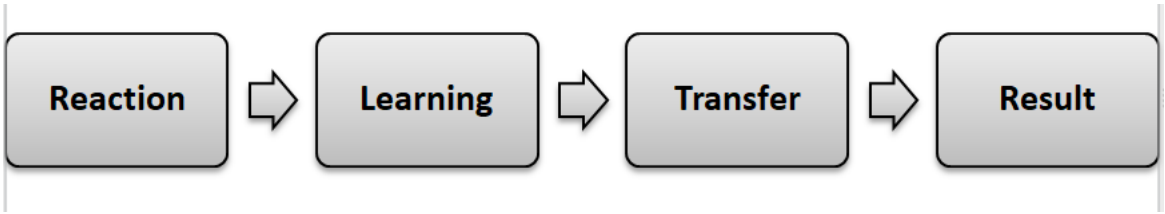
B. Adaptation Model

The adaption model defines the mode in which the learner's define learning content presentation. The adaptation model is not constant as it is updated according to the feedback mechanism of users' preferences and knowledge level acquired by the system (Mitrovic et al., 2002). The adaptation model acts as a link between the learner model and the domain model as it identifies the appropriate learning content, or sequence of learning content according to the learner's preferences and knowledge of learner. Previous researches suggested popular taxonomies for adaptive concept such as; adaptive presentation, adaptive navigation, adaptive content, these concepts cover different procedures such as introducing, updating, deleting and sorting layout (Oye & Salleh, 2012).

C. Domain Model

The domain model contains the learning content of the course to be learnt which is designed by to handle different ways of presenting the material content (Alshammari et al., 2014). The domain model can be defines as a set of structured knowledge related to a special field (Oye & Mazleena, 2012). A significant point in domain models is how to organize learning content in particular. Learning content includes learning objects which defined as a digital piece of learning content that addresses an identifiable topic and has the prospect l to be reused in different states (Alshammari et al., 2014) (Sweta, 2021) (Kulaglić, 2013).

Figure 2. Evaluation Methods of e learning



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EVALUATION METHODS OF ELEARNING

It's important to evaluate eLearning program through the following methods as shown in the following figure 2 (Prasad, 2020):

A. Reaction

By answering questions, this stage analyses how individuals responded to the practice. Each system must be evaluated at this level in order to develop for potential use. At this stage, learners' answers often decide how involved they would be in the next education. This can be done through feedbacks, interviews, and reports.

B. Learning

This type of evaluation assesses how learners' awareness, abilities, and attitudes have improved as a result of the training. At this step, the following methods can be used; exams,

C. Transfer

This phase explores post-training improvements in workplace behavior. This allows evaluating, if learned information and skills are being applied. This type of evaluation can be done through; Supervisory observation, 360-degree reviews, and Evaluations made as part of the scenario.

D. Results

This evaluation tries to measure return on investment ROI, quality, time, and exam results which are the factors that influence the learning success. It is considered effective if the return on investment improves after applying eLearning.

ROI = [(Gain from investment – Cost of investment)/Cost of investment] x100

eLEARNING ASSESSMENT

"Assessment is the systematic collection of information about student learning, using the time, knowledge, expertise, and resources available, in order to inform decisions that affect student learning" as defined by Walvoord (Walvoord, 2010).

E learning assessment is a major element with the learning approach, therefore, assessment, described as a method for ability to react in the learning experience intended, ideally, for more growth performance, as in the following figure:

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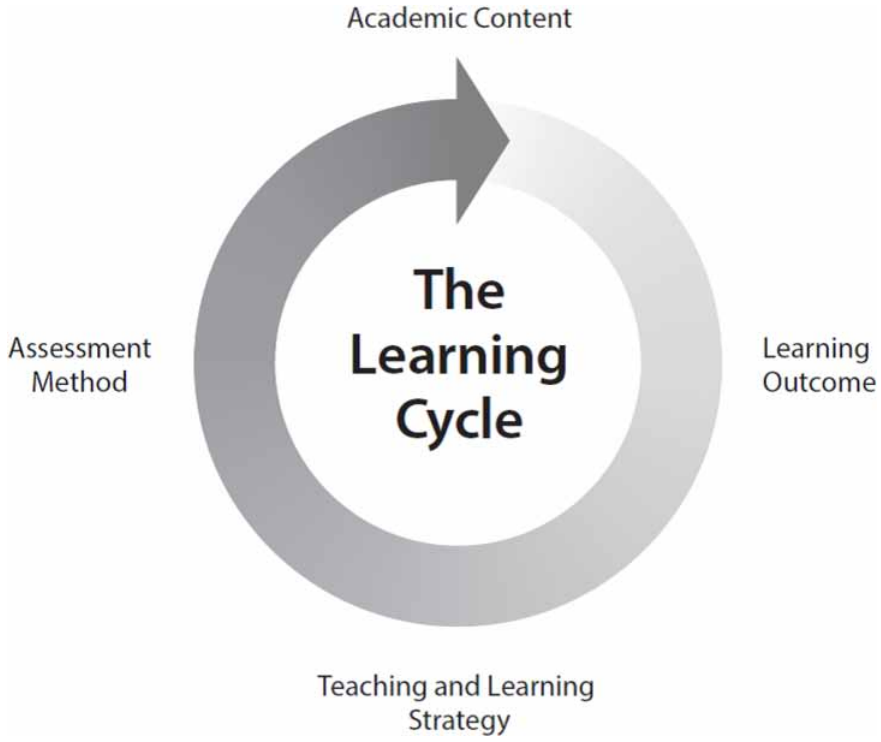
Figure 3. The Learning Cycle



As a result, assessment is an important part of teaching methods, and it must be adequately incorporated into the learning process as a method of teaching that both represents and adds to learning. Assessment, by definition of its collaborative nature, allows for the measurement both of students' learning performance in a wide range of ways.

Construction, cooperation, are parts of teaching, and design covers evaluation, organisation. Assessment has a significant impact on the quality of learners and teachers, and delivery. In other words, students do what they are rewarded for. As a result, it's critical to ensure that activities that promote truly meaningful learning are rewarded.

When creating assessments, it's critical to develop and organize for as much student feedback as possible. Good feedback clarifies what represents great result, facilitates self-evaluation and evaluation, supports teacher and student about learning, promotes positive motivating beliefs and self-esteem, offers opportunities to reduce the current and desired achievement, and can be used by teachers to help improve their teaching.

There are two main types of assessments used during eLearning classes: formative and summative. After the topic has been completed, formative assessments are given. After each question is solved, formative assessments provide feedback. A formative assessment's aim is to strengthen learning. At the end of the course, a summative evaluation is given. A summative assessment represents a final exam in that no feedback is given and the results are presented at the completion of the course (Knott, 2020).

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RELATED WORK

Adaptive eLearning systems are suggested to enhance the learning process, as these systems are able to present the most suitable learning content and to the learners. These provide learners to get learning content that satisfy their needs. Most of the approaches applied to adapt learning content based on the learner's characteristics especially knowledge level (Conati et al., 2002).

Different adaptive eLearning systems models are presented based on an agent and machine learning techniques approach. These models are inspired by intelligent methods to present adaptability to eLearning environments. A research proposed adaptive e-learning systems based on semantic web to present the suitable knowledge, content representation according to learner's learning styles and their knowledge levels (Mitrovic, 2002). A study is presented to focus on primary information to design an eLearning system that adapts to learners' preferences automatically. This adaptation by the model does not require the learner to perform any introductory action (Verbert, 2012).

Another study investigates machine learning techniques to be used in eLearning systems as to design adaptive learning content, which increases learners' understanding. The study analyzes the collected data to learner's needs using machine learning to adjust course content. Different machine learning models are used such as; Bayesian networks, decision tree, artificial neural networks, association rule mining and clustering to handle different issues like personalization, content delivery, and support tools (Kolekar, 2019).

Another work proposes a common approach to provide the learning contents with adaptive user interface based on learner's learning styles. The study defines rules which produce self-directing eLearning course with the adaptive contents. Also, the approach used in the work takes in consideration new learners by providing learning path as a user interface component on the system (Knott, 2020). Another study present an adaptive e-learning system based on a multi-agent approach to recommend a learning path to learners based on their knowledge level, and disabilities. The study takes into account three types of disabilities which are ring impairment, visual impairment, and dyslexia in order to increase system performance (Conati, 2002).

Another work tries to fill the gap between learning material adaptation according to learner's learning style and knowledge level. The study presents a novel adaptive approach based on learning style and knowledge level. Also, an evaluation of the adaptive eLearning approach according to learning style and knowledge level produces better learning gains (Adetunji & Ademola, 2014).

Modern approach in adaptive eLearning takes into account the state of learner's knowledge of a person in the learning process (Rasulova & Salieva, 2021). Fuzzy logic used as adaptive technique applied to eLearning system based on concept map to assess learner's knowledge. The concept map shows the hierarchical of learning materials, and captures the understanding of the learner as discussed in (Al Duhayyim & Newbury, 2019).

The previous researches showed that the lack in the development of adaptive eLearning systems that it didn't take into account the uncertainty of learners' knowledge. Only few researches take in consideration uncertainty in determining the learner's knowledge level. Novel studies suggested adaptive eLearning systems that take into account the learner's knowledge under uncertain conditions on the contrary of traditional adaptive eLearning systems that depend on crisp answers (Goyal, et al., 2015 b) (Goyal, 2017) (Megahed, & Mohammed, 2020). As most of the eLearning systems evaluate the learner's knowledge in terms of the learner's crisp answers to the tests that are taken throughout the learning process. Although, there are multiple factors such as successful assumption or more possible answer selection leads to

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uncertainty about the evaluation operation []Goyal, et al., 2015 b) (Radwan et al., 2016) (Goyal, 2017) (Megahed, & Mohammed, 2020).

Various studies present an adaptive eLearning and test sheet based on and learner level to perform a better adaptation (Makram et al., 2020) (Megahed, & Mohammed, 2020). The problem of fuzzy logic that it cannot handle imprecision, ambiguity, and inconsistence data, as it represents the uncertainty value in a crisp value between 0 and 1 (Radwan et al., 2016).

Another study explores the use of intuitionistic fuzzy approach to handle uncertain information about the learner in the assessment process to present adaptive sequence of learning concepts (Goyal et al., 2017). Intuitionistic fuzzy logic can handle vagueness and imprecision data but it cannot handle ambiguity, and inconsistence data. The membership in intuitionistic fuzzy logic of an element is a single value between 0and 1. However, it is not true that the value of non-membership of an element is equal to 1 minus the membership degree as there is a hesitation degree which cannot described by this approach (Radwan et al., 2016).

Therefore, in this chapter a novel approach is presented to handle imprecision, vagueness, ambiguity, and inconsistence in learner's evaluation process as to resent an adaptive learning material content according to learner's knowledge level. In the coming section, the different approaches that handle uncertainty will be discussed to be clear why neutrosophic logic is selected to handle the adaptive eLearning sequence.

UNCERTAINTY

Uncertainty means the lack of knowledge nonetheless the incomplete data cause. One of the important issues in artificial intelligence is handling uncertainty for solving real life problems [El Fazazi et al., 2021). Former researches presented different approaches that handle uncertainty by emulating human thinking, but these approaches are not enough to handle uncertainty as decision making includes undefined issues where it is not true or false. Decision makers need to define, distinguish, describe, and evaluate uncertainties types (Krendzelak, 2014,).

Therefore, novel approaches to handle uncertainty such as fuzzy logic, intuitionistic fuzzy logic, and neutrosophic logic are emerged to provide better value interpretations. Professor Lotfi Zadeh presents Fuzzy logic in 1965, which considers the degree of the membership of attribute in a set. Atanassov presented Intuitionistic fuzzy logic in 1983, as an extension of fuzzy logic theory. Smarandache proposed neutrosophic logic in 1999, as to handle indeterminacy of information.

A. Fuzzy Logic

The fuzzy set theory, unlike crisp set deals with objects belonging to a set with a membership between 0 and 1. In fuzzy set theory, each object $x \in U$ (Universe of discourse) is assigned a single membership value. A fuzzy set $A = \{< x, \mu A(x) > | x \in U\}$ in a universe of discourse U is characterized by a membership function, μA [15].

Fuzzy logic provides inferior representation of uncertain data, as it represents the membership in a crisp value between 0 and 1. The membership degree of a given element in a fuzzy logic does not express the equivalent degree of non-membership as the complement to 1.

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B. Intuitionistic Fuzzy Logic

Atanassov added to the fuzzy set definition a degree of non-membership which emerges the intuitionistic fuzzy set. The sum of membership and non-membership of an element is always less or equal to 1. An intuitionistic fuzzy set A = {< u, μA(u), vA(u) > |u ∈ U} in a universe of discourse U is characterized by a membership function μA, and a non-membership function vA [16].

Intuitionistic fuzzy logic cannot handle the impreciseness of human understanding as decision includes not only true or false information, but it also depends on indeterminate information, which is represented by an ignorance value between truth and falsehood.

C. Neutrosophic Logic

Smarandache proposed neutrosophic logic as an extension of fuzzy logic in which the variable x is described by triple values which are for the truth degree, f for the false degree and i for the indeterminacy degree. These degrees are separate from each other's as there is no limitation on their sum. For example, the proposition "Robots can talk" has not a fixed-valued; this proposition may be 50% true, 40% indeterminate and 30% false at a time; but at in a second time may change at 40% true, 50% indeterminate, and 60% false [17].

In this section, a brief review of the general concepts of neutrosophic set is presented (Alshammari & Qtaish, 2019) (Uusitalo et al., 2015) (Liu et al., 2012) (Motro, & Smets, 2012) (Ansari et al., 2011) (Ye, 2014) (Albeanu & Vlada, 2014) (Shee & Wang, 2008) (Peng et al., 2016):

Definition 1

If $N_1 = (t_1, i_1, f_1)$ is a single valued neutrosophic numbers, then:

$$0 <= t_1 + i_1 + f_1 <= 3 \tag{1}$$

Definition 2

If $N_1 = (t_1, i_1, f_1)$ and $N_2 = (t_2, i_2, f_2)$ are two single valued neutrosophic numbers, then the summation between N_1 and N_2 can be expressed as follows:

$$N_1 + N_2 = \left(t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2\right) \tag{2}$$

Definition 3

If N1 is a single-valued neutrosophic number, a score function is mapped N1 into the single crisp output as S(N) follows: Let $x = (T_x, I_x, F_x)$ be a single value of neutrosophic number, then the proposed score function S(x) is defined as:

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$$S\left(N\right)=\frac{2}{3}+\frac{Tx}{3}-\frac{lx}{3}-\frac{Fx}{3} \tag{3}$$

THE PROPOSED ADAPTIVE ELEARNING SYSTEM

Artificial intelligence discipline interest is handling unawareness value between true and false. For example, if the answer of a learner asked about certain statement then he will answer as true, false and indeterminacy is 0.5, 0.6 and 0.3 respectively. This can be appropriately handled by neutrosophic logic.

This section explains the proposed method for adaptive eLearning system according to learner's knowledge level as shown in Figure 2. Given below are the following steps to model the user.

Step 1. The learner selects a learning topic from the list that must be learned throughout the learning process, the learner has no previous records in his profile, and then the learner is free to select any concept from the list.

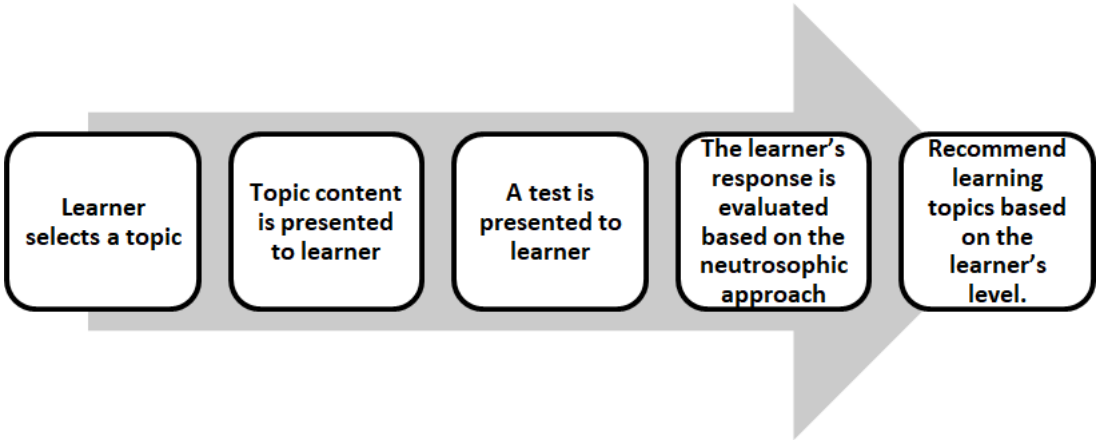
Step 2. The learning content related to the leaner selected topic is then presented to him.

Step 3. A test is presented to him after the learned content to assess his understanding degree.

Step 4. The learner's response is evaluated based on the neutrosophic approach. A multiple choice test is designed for each of the concepts in the knowledge domain. The learner has the option to set the correct, indeterminacy, and false percentage for each option of the multiple choice question. Neutrosophic approach incorporates the lack of the learner's knowledge about his answer.

Step 5. The learner's knowledge level is updated according to the value obtained on the test to recommend learning topics based on the learner's level. Next, the topics that the learner learned are removed from the list.

Figure 4. Steps of the Proposed Method



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A. Learner's Assessment through Neutrosophic Approach

An evaluation to learner's knowledge level for a domain concept needs a pretest to determine whether the learner knows or don't know or learned to a domain concept. Experts presented multiple choice questionnaire based on domain concepts to the learners. Learner has to select an option in a neutrosophic manner where learner has the option to set the correctness percentage for each option of the multiple choice question. Neutrosophic logic is used to assess the knowledge level and misconception of a learner for a domain concept. Let (X, T_a, I_a, F_a) are the variables which define X as set of questions, T_a- degree of true, I_a – degree of Indeterminacy, F_a- degree of false.

Where S(N) is the score of concept and T(N), I(N), F(N) are the degree of the learner's knowledge level and misconception of the concept respectively. T(N), I(N), F(N) are calculated as following, where C(N) is correctness percentage that the learner has assigned to the correct option of the multiple choice questionnaire, I(N) is the unknown percentage where F(N) is the incorrect percentage that the learner has assigned.

$$T\left(N\right) = \frac{C(N)}{100} \tag{4}$$

$$I(N) = \frac{I(N)}{100} \tag{5}$$

$$F(N) = \frac{F(N)}{100} \tag{6}$$

Let S(N) score of a learner is (0.8, 0.2, 0.2), then learner is 80% known to a domain concept, 20% undetermined to domain concept, and 20% unknown to domain concept. If the learner IF score is (0.4, 0.4, 0.6), then he is 40% known to a domain concept, 40% is hesitant degree where he is not sure about the domain concept to domain concept, 60% unknown to domain concept. Calculate weighted neutrosophic score for all question is calculated using (3). Then calculate the Score for each learner using equation (2).

B. Time Taken to Solve Questionnaire

Time also is an important factor to determine the learner status. If learner takes less time than expected to solve the questionnaire, this indicates that he is a fast learner. If he takes more time on solving the questionnaire, he is a slow learner. So learner's status can be identified by comparing the time taken by the learner with the time expected by the system. Total time is calculated with respect to time taken to each learner divided by total time expected for a particular learner. The learner's solving time for test is determined as follows:

Time suggest for n questions = 1000 seconds, Learner spent total = 500 seconds, then Percentage = 500/1000.

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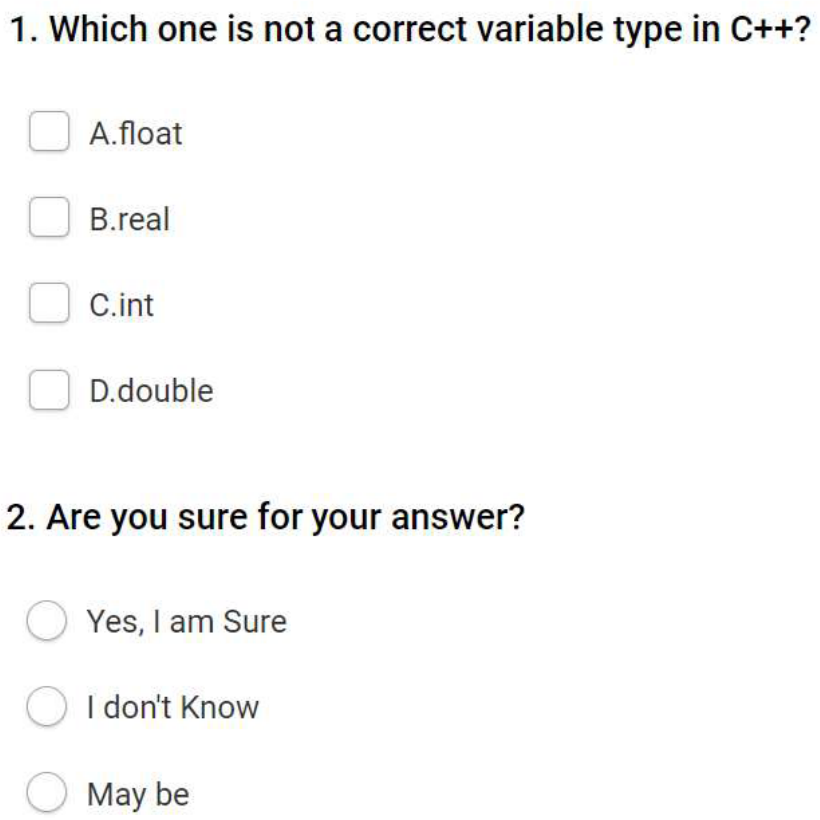
C. Learner's Classification

A weight is calculated for each question using equation (4), (5) and (6). Score is calculated for each question using equation (3). Where T is the membership value of the question, I is the Indeterminacy value of the question and F is the non-membership value of the question.

Figure 5. Sample of C++ Evaluation test



For example, If the score of the learner for question one, question two and question three is (0.7, 0.2, 0.1), (0.5, 0.6, 0.1), and (0.6, 0.3, 0.3) consequently. The score for the three questions consequently using equation (3) is 80%, 60%, 66%. Therefore the average of these three scores is (0.8+0.6+0.66)/3 =0.68. Then it is concluded that the learner score is 68%. Table 1 shows criteria for learner classification. We have one input which is the score and three outputs unknown, known, and learned for domain concepts.

Table 2 shows concepts that should be learned to each learner. The learner should learn seven concepts in this course which are A, B, C, D, E, F, and G, but some concepts if he has passed in it may lead him to escape the forward concept. For example, if the learner passes concept B with score above 71%, then he "learned" this concept so he will pass concept C automatically and he could learn concept D directly. Another example, if the learner study concept B and his score is between 46%-70% then he "Known" this concept so he will study concept C.

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Table 1. Criteria for Learner Classification

Score	Classification
0-45	Unknown
46-70	Known
71-100	Learned

Table 2. Concepts should be learned with Learner

Concept	Perquisites	Un Known	Known	Learned
A	----	A	B	B
B	----	B	C	D
C	B	G	D	D
D	C	D	E	F
E	D	G	F	F
F	B, C, D	G	PASS	PASS
G	-------			

SOLUTIONS AND RECOMMENDATIONS

The authors presented an adaptive eLearning sequence based on neutrosophic logic as to represent three components truth, indeterminacy, and falsity. Neutrosophic logic can express learner's knowledge including the correctness percentage of his answers. For example; a question like: "float is a correct variable type in C++", the answer will not only be true or false or not sure but also the leaner have to define a percentage for his answer such as 0.7 for true degree, 0.4 for false degree, and 0.2 for indeterminacy. In this case by using equation (3), the learner is sure of his answer by 64% which will affect the final result. These results will help to recommend the most probable sequence for the learner.

CONCLUSION

This chapter introduced E learning, and types of e learning, how to evaluate and asset eLearning focusing on the adaptive e-learning system and its architecture, the uncertainty and neutrosophic Logic. Also, this chapter proposed the methodology steps of adaptive learning system according to learner's knowledge level based on neutrosophic logic as follows; first step topic selection, second step is topic presentation, third step is test conduction, fourth step (the most important step) the learner evaluation based on the neutrosophic approach, and finally learning topics recommendation. An evaluation test is implemented to determine learner's knowledge after each concept has been learned. In this test each correctness percentage of each option is measured to clarify the differences between learner's successful guesses and probable answer. The results help to determine the suitable sequence of concepts to each learner. Future work of this research will explore the effect of other attributes which have an impact in determining the probable sequence of concepts for the learner. In addition, test sheet according to learner's style in

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customized surroundings is needed. The incorporation of neutrosophic logic and learning style model provides a suitable way of assessment that has an important role in enhancing learner recognition, and performance.

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Chapter 29
Application of Triangular Fuzzy Neutrosophic Number to an Effective Economic Order Quantity Model

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ABSTRACT

The advance payment of acquiring cost is mostly encouraged by both supplier and retailer in current situations. This chapter discusses the economic order quantity model with multiple payments of acquiring cost in advance at equal intervals. The current research examines two-warehouse economic order quantity model with imperfect items under neutrosophic environment by considering the proportion of defective product as triangular fuzzy neutrosophic number. An inventory model is framed, and the proposed model is formulated under triangular neutrosophic environment. The expected total profit and the optimal order quantity have been computed under both the crisp and the neutrosophic sense. De-neutrosophication of the proposed model has been done using expected value of triangular fuzzy neutrosophic number. The numerical illustration is analyzed in both crisp and neutrosophic situation along with the comparative studies. The sensitivity analysis is performed to show the effect of neutrosophication on defective rate along with the changes in parameters.

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INTRODUCTION

The production manager executes a much effective manufacturing process to avoid defective units to be manufactured. But still, substandard products are manufactured and supplied to the retailers. So that, according to Salameh and Jaber (2000), in every cycle the retailer received a lot of substandard items, and to rectify this issue, the retailer enforces a screening process once replenishing a lot. This paper focuses on the advance payment option that is the acquisition cost should be paid in advance as multiple equal payments instead of paying at the time of replenishment.

This research studies a two-warehouse EOQ model with imperfect quality items and the advance payment on acquiring cost under fuzzy environment and neutrosophic environment. Once a lot received, the screening process takes place and separates the flawed items from the inventory in both the warehouses and sold them at a lower price after the screening period in one shot. Since the demand can be satisfied during the screening period with perfect items, it is considered that the screening rate is always more than the demand rate. The retailer utilizes the advance payment on acquisition cost offered by the supplier with some price discount on prepaid items. In this advance payment option, the fraction of acquiring cost to be paid in advance with equal payments at multiple intervals. Since the defective proportion in every lot is unpredictable, the proposed study utilizes a fuzzy notion for an effective outcome. As an enlargement of the fuzzy set and intuitionistic fuzzy set, Smarandache established the neutrosophic set to attain more approximate outcomes for uncertain situations rather than the fuzzy set and intuitionistic fuzzy set. Thus, the present model is framed under the neutrosophic arena to reach the most approximate solution for unpredictable defective rate than the crisp model and fuzzy model.

The inventory model under fuzzy arena is also developed to compare the study with the neutrosophic model. Initially, the crisp EOQ model is obtained by presuming the proportion of defective items as uniform distribution then the fuzzy EOQ model is designed by presuming the proportion of substandard items as Triangular Fuzzy Number [TFN] and defuzzified by computing the expected value of TFN. Finally, the inventory model under neutrosophic sense is framed by considering the defective rate as Triangular Fuzzy Neutrosophic Number [TFNN] and de-neutrosophication of the proposed model has been performed by using expected value of TFNN. Finally, the model is illustrated with a numerical example and the sensitivity, as well as the comparative analysis, is discussed.

The main objective of the research is to maximize the retailer's expected profit for the EOQ model with substandard products under a double warehouse system. The proposed study scrutinizes under both fuzzy and neutrosophic arena to attain the approximate profit for the unpredictable parameter. This study motivates the retailer to pay the advance payment offered by the supplier who provides the price discount on the prepaid amount which leads to upticks in the retailer profit.

The elementary definitions of fuzzy set, intuitionistic fuzzy set, neutrosophic set, and the expected value of TFNN are provided in section 2. Followed by the notations and the assumptions, the crisp EOQ model is formulated in section 3. Further, the proposed crisp model is flourished by fuzzy model by presuming the substandard proportion as TFN, and also the neutrosophic EOQ model is framed by presuming the substandard proportion as TFNN in the same section. In section 4, the numerical computation and sensitivity analysis is performed with the comparative studies. Finally, the results of this research are given in section 5.

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LITERATURE REVIEW

In this section, some of the research works under various topics related to the proposed model are discussed. Rosenblatt and Lee (1986) perused the impact of the defective manufacturing process on optimal manufacturing duration of a cycle in a single machine - single production system. Duration from the starting time of the manufacturing process till the system turns uncontrollable was assumed to be exponential and also considered all the imperfect products are mended at a specific price. They modified the economic manufacturing duration of the classical EMQ formula and the result was found to be shorter than classical EMQ. Finally, they concluded that, if the rate of defectives decreases then the production run also will decrease otherwise the cost of defective items will increase. Also, Salameh and Jaber (2000) designed an inventory model with substandard items; they considered removing such items from inventory and sold them at a lower cost as one shot after investigation time. They also showed that the optimal order size increases when the average percentage of imperfect products increases. In the effect of this, Cardenas-Barron (2000) rectified an error present in Salameh and Jaber (2000) as well as Goyal and Cardenas-Barron (2002) given a practical approach to Salameh and Jaber (2000). In addition to this, Papachristos and Konstantaras (2006) dealt with the problem that occurred when there were no shortages in the inventory model with the percent of imperfect quantity and they pointed out that the proposed conditions of Salameh and Jaber (2000) cannot prevent the shortages. Khan M et al. (2011) used the work of Salameh and Jaber (2000) for confirming an inventory situation for defective products without an error-free screening procedure. They discussed two kinds of screening errors. The inspector classified perfect products as imperfect and imperfect products as perfect. All defectives and the products returned from the market are kept for sales at less cost in a batch. They signified the effect of assessment errors by comparing the analysis with Salameh and Jaber (2000) and concluded that, the annual profit decreased when the proportion of imperfect products increased. In the same year, Lin and Chen (2011) presented a model of EOQ with substandard products, misclassification errors, and shortages with quality discount. Subsequently, in 2013, Hsu J T and Hsu L F (2013) designed an EOQ model with defective products, screening errors, shortage backordering, and sales returned. They also developed a model with an order overlapping scheme to prevent shortages. In 2016, Zhou et al. (2016) discussed an EOQ model that incorporates substandard items, misclassification errors, shortages, and credit payment. They showed that the expected profit is concave and obtained a closed-form optimal solution. Sebatjane and Adetunji (2019) designed the EOQ model for growing products where the quality of the received items is faulty. They proposed the model with three growth functions. The same model is enlarged by Alfares and Afzal (2021) by considering with scarcity of products.

For uncertain situations, many researchers of inventory and production management areas used to adopt the perception of fuzzy sets which was established by Zadeh (1965). In 2002, an approach of the expected value operator of the fuzzy variable was conferred by Liu B and Liu Y K as well as they framed a fuzzy simulation to calculate the expected value. Subsequently, Wang and Tian (2010) proposed a paper to formulate the expected values and variance of various fuzzy variables. Some research works on the inventory model with substandard products under the notion of fuzziness are as follows. Liu and Zheng (2012) examined the EOQ model with defective products, shortages, and inspection errors under a fuzzy environment. They guessed that the fraction of imperfect items is a triangular fuzzy number and suggested that the expected total profit reduces when the fraction of defective upticks. Mahata and Goswami (2013) considered the inventory models with poor quality and shortage backordering under the fuzzy environment. They developed a fuzzy EOQ model along with the crisp decision variables and

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the fuzzy decision variables. They concluded that the change of percentage because of fuzziness leads to double percentage changes in total profit per year and the backordering quantity is closely similar to the changes in components, but the ordering quantity changes largely. Patro et al. (2019) constructed EOQ models for defective products with proportionate discounts under both crisp and fuzzy environments. They framed the crisp EOQ model, model with fuzzy defective proportion, the model with fuzzy defective proportion and fuzzy demand, the model with all fuzzy costs, fuzzy defective proportion and fuzzy demand, and finally, the model having screening errors. Currently, Rajeswari and Sugapriya (2020) designed the EOQ model with substandard products by presuming to restore the defective products as good ones under the fuzzy environment. Recently, Supakar and Mahato (2020) and Tahami and Fakhravar (2020) studied the fuzzy inventory model for uncertain parameters under various situations.

As the extension of Zadeh (1965), Atanassov (1986) elaborated the fuzzy set into Intuitionistic fuzzy sets. Liu and Yuan (2007) incorporate triangular fuzzy numbers and intuitionistic fuzzy sets and then designed the triangular intuitionistic fuzzy set. On the use of cloud-type Intuitionistic Fuzzy parameters, Maity et al. (2019) examined the EOQ model with shortage backordering under an intuitionistic dense fuzzy situation. Nagarajan et al. (2019) developed an optimization process Hidden Markov model under type-2 fuzzy variable. As a generalization of the intuitionistic fuzzy set, Smarandache (1998) established the neutrosophic sets and the notion of the neutrosophic environment with a different fuzzy neutrosophic parameter has been applied in various decision-making strategies as well as inventory management techniques. In the view of practicality, the singled valued neutrosophic set was established by Wang et al. (2010), as an exemplification neutrosophic set, and many researchers utilized this set in many decision-making problems. Biswas et al. (2014) framed the cosine similarity measures of trapezoidal fuzzy neutrosophic numbers to attain effective outcomes for the multi-attribute decision-making model. Subsequently, in 2016, they implement various operators to accumulate the triangular fuzzy neutrosophic number applied to solve the multi-attribute decision-making problem Biswas et al. (2016). Recently, the implementation of a neutrosophic environment to the various inventory management strategies is encouraged. As a result of this, Mullai et al. (2020) flourished the inventory models under the single-valued neutrosophic environment with the triangular fuzzy neutrosophic number and examined the graded mean integration of triangular fuzzy neutrosophic number. Pal and Chakrabothy (2020) designed the production inventory model of decaying products under the neutrosophic situation. They presumed the EPQ with ramp type demand and formulated the problem for good quality cost-effective EPQ under time discounting with scarcity.

For scrutinizing the remittance situation of payments, Taleizadeh (2014a) proposed an EOQ model for an item of evaporation with sectional backordering under the advance payment option. In the same year, Taleizadeh (2014b) presented a model for decaying products under multiple prepayments with scarcity and without scarcity. Finally, to amend the issues on storage capacity in a single warehouse, it is presumed that the present model was extended to the two-warehouse EOQ model. Jaggi et al. (2015) develop an EOQ model which coordinates the notion of double storage place, substandard products, and decaying products. In the same year, Jaggi et al. (2015) extended the above said two-warehouse model along with single level trade credit. Further, Mohanty et al. (2016) framed a double storehouse inventory problem for delayed decaying items with scarcity by considering the planning horizon as a random variable. Al-Amin Khan et al. (2019) implemented the advance payment strategy on the two-repository EOQ model for decaying items. They discussed the model with a price discount on the prepaid charge which raises the revenue. Rajeswari et al. (2021) studied the notion of prepayment strategy in a two-repository fuzzy EOQ model with substandard items under lockdown situations. They concluded that the use of

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an advance payment strategy helps the retailer to overcome the payment issue during the replenishment and the price discount received from the supplier on prepaid charge upticks the retailer's total profit.

PRELIMINARIES

Some Basic Definitions of Fuzzy Set and Intuitionistic Fuzzy Set

Fuzzy Set [Zadeh (1965)]: A fuzzy set $\tilde{\Delta}$ in the universal set *Y* is defined as a set of ordered pairs and it is expressed as

$$\tilde{\Delta} = \left(y', \mu_{\tilde{\Delta}} \left(y' \right) \right) : y' \in Y$$

in which $\mu_{\tilde{\Delta}} \left(y' \right)$ is a membership function of y' which assumes values in the range from 0 to 1 (ie.,) $\mu_{\tilde{\Delta}} \left(y' \right) \in \left[0,1 \right]$

Fuzzy Number [Zadeh (1965)]: A fuzzy number $\tilde{\Delta}$ is a subset of real line R, which has the membership function $\mu_{\tilde{\Delta}}$ fullfilling the given features:

(1) $\mu_{\tilde{\Delta}} \left(y' \right)$ is piecewise continuous in its domain.

(2) $\tilde{\Delta}$ is normal, i.e., there is a $y_0^{'} \in \tilde{\Delta}$, such that $\mu_{\tilde{\Delta}} \left(y_0^{'} \right) = 1$.

(3) $\tilde{\Delta}$ is convex, i.e., $\mu_{\tilde{\Delta}} \left(\epsilon\, y_1^{'} + \left(1 - \epsilon \right) y_2^{'} \right) \geq min\left(\mu_{\tilde{\Delta}} \left(y_1^{'} \right), \mu_{\tilde{\Delta}} \left(y_2^{'} \right) \right) \forall y_1^{'}, y_2^{'}$ in *Y*.

Triangular Fuzzy Number [Patro et al. (2019)]: A fuzzy number $\tilde{\Delta} = \left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3 \right)$ is considered as a triangular fuzzy number suppose it possesses the ensuing membership function

$$\mu_{\tilde{\Delta}} \left(y' \right) = \begin{cases} \dfrac{y' - \sigma_1}{\sigma_2 - \sigma_1}, & \sigma_1 \leq y' < \sigma_2 \\ 1, & y' = \sigma_2 \\ \dfrac{\sigma_3 - y'}{\sigma_3 - \sigma_2}, & \sigma_2 < y' \leq \sigma_3 \\ 0, & otherwise \end{cases}$$

Intuitionistic Fuzzy Set [Atanassov (1986)]: Let *Y* be a finite universe of discourse then an intuitionistic fuzzy set $\tilde{\mathcal{A}}$ is defined as

$$\tilde{\mathcal{A}} = \left\{ y, \varphi_{\tilde{\mathcal{A}}} \left(y \right), \psi_{\tilde{\mathcal{A}}} \left(y \right) \mid y \in Y \right\},$$

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where $\varphi_{\tilde{A}} : Y \to [0,1]$ and $\psi_{\tilde{A}} : Y \to [0,1]$ satisfying the condition $0 \le \varphi_{\tilde{A}}(y) + \psi_{\tilde{A}}(y) \le 1$. The number $\varphi_{\tilde{A}}(y)$ represents the degree of membership of $y \in Y$ in $\tilde{A}$ and the number $\psi_{\tilde{A}}(y)$ represents the degree of non-membership of $y \in Y$ in $\tilde{A}$. In addition with this, the hesitance degree of $y \in Y$ in $\tilde{A}$ is denoted as $\varepsilon_{\tilde{A}}(y)$ and is given as $\varepsilon_{\tilde{A}}(y) = 1 - \varphi_{\tilde{A}}(y) - \psi_{\tilde{A}}(y)$. For convenience, the intuitionistic fuzzy number is considered as $\tilde{A} = (\varphi_{\tilde{A}}(y), \psi_{\tilde{A}}(y))$.

Triangular Intuitionistic Fuzzy Set [Liu and Yuan (2007)]: A triangular intuitioninstic fuzzy set $\tilde{A}$ in a finite universe of discourse *Y* is defined as follows:

$$\tilde{A} = \{ y, \varphi_{\tilde{A}}(y), \psi_{\tilde{A}}(y) \mid y \in Y \}$$

where $\varphi_{\tilde{A}}(y)$ and $\psi_{\tilde{A}}(y)$ are the real subsets of [0,1] and $\varphi_{\tilde{A}}(y)$ and $\psi_{\tilde{A}}(y)$ are two triangular fuzzy numbers

$$\varphi_{\tilde{A}}(y) = (\varphi^1_{\tilde{A}}(y), \varphi^2_{\tilde{A}}(y), \varphi^3_{\tilde{A}}(y)) : Y \to [0,1]$$

and

$$\psi_{\tilde{A}}(y) = (\psi^1_{\tilde{A}}(y), \psi^2_{\tilde{A}}(y), \psi^3_{\tilde{A}}(y)) : Y \to [0,1]$$

with $0 \le \varphi^3_{\tilde{A}}(y) + \psi^3_{\tilde{A}}(y) \le 1, \ y \in Y$

Some Basic Definitions Of Neutrosophic Set

Neutrosophic Set [Smarandache (1998)]: Let *Y* be a space of points (objects) with generic element $y \in Y$. Then a neutrosophic set $\tilde{A}_N$ in *Y* is characterized by a truth membership function $\mathrm{T}_{\tilde{A}_N}$, an indeterminacy membership function $\mathrm{I}_{\tilde{A}_N}$ and a falsity membership function $\mathrm{F}_{\tilde{A}_N}$. The functions $\mathrm{T}_{\tilde{A}_N}$, $\mathrm{I}_{\tilde{A}_N}$, and $\mathrm{F}_{\tilde{A}_N}$ are real standard or nonstandard subsets of $]^-0,1^+[$ that is

$$\mathrm{T}_{\tilde{A}_N} : Y \to]^-0,1^+[; \ \mathrm{I}_{\tilde{A}_N} : Y \to]^-0,1^+[; \ \mathrm{F}_{\tilde{A}_N} : Y \to]^-0,1^+[.$$

$\mathrm{T}_{\tilde{A}_N}(\mathrm{y})$, $\mathrm{I}_{\tilde{A}_N}(\mathrm{y})$, and $\mathrm{F}_{\tilde{A}_N}(\mathrm{y})$ satisfy the relation

i.e. $^-0 \le \sup \mathrm{T}_{\tilde{A}_N}(\mathrm{y}) \le \sup \mathrm{I}_{\tilde{A}_N}(\mathrm{y}) \le \sup \mathrm{F}_{\tilde{A}_N}(\mathrm{y}) \le 3^+$

Single-valued Neutrosophic Set [Wang et al. (2010)]: Let *Y* be a space of points (objects) with generic element $y \in Y$. A single-valued neutrosophic set $\tilde{A}_N$ in *Y* is characterized by a truth membership function

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$\mathrm{T}_{\tilde{A}_N}(\mathrm{y})$, an indeterminacy membership function $\mathrm{I}_{\tilde{A}_N}(y)$ and a falsity membership function $\mathrm{F}_{\tilde{A}_N}(\mathrm{y})$ and is given by

$$\tilde{A}_N = \left\{ y, \mathrm{T}_{\tilde{A}_N}(y), \mathrm{I}_{\tilde{A}_N}(y), \mathrm{F}_{\tilde{A}_N}(y) \mid y \in Y \right\}.$$

Here $\mathrm{T}_{\tilde{A}_N}(y), \mathrm{I}_{\tilde{A}_N}(y), \mathrm{F}_{\tilde{A}_N}(y) \in [0,1]$ and the relation

$$0 \le \sup \mathrm{T}_{\tilde{A}_N}(y) \le \sup \mathrm{I}_{\tilde{A}_N}(y) \le \sup \mathrm{F}_{\tilde{A}_N}(y) \le 3$$

holds for all $y \in Y$

Fuzzy Neutrosophic Number [Biswas et al. (2014)]: A neutrosophic set $\tilde{A}_N$ in a universe of discourse Y is defined as

$$\tilde{A}_N = \left\{ \mathrm{T}_{\tilde{A}_N}(y), \mathrm{I}_{\tilde{A}_N}(y), \mathrm{F}_{\tilde{A}_N}(y) \mid y \in Y \right\}$$

where the truth membership function $\mathrm{T}_{\tilde{A}_N}(y)$, an indeterminacy membership function $\mathrm{I}_{\tilde{A}_N}(y)$ and a falsity membership function $\mathrm{F}_{\tilde{A}_N}(y)$ are the real subsets of [0,1]. Fuzzy neutrosophic number can be defined by extending a discrete set to a continuous set.

Let $\tilde{A}_N$ be a fuzzy neutrosophic number in the set of real numbers R. Then its truth membership function, indeterminacy membership function and falsity membership function are respectively written as follows:

$$\mathrm{T}_{\tilde{A}_N}(y) = \left\langle \begin{array}{cc} \mathrm{T}^L_{\tilde{A}_N}(y) & l_1 \le y \le l_2 \\ 1 & l_2 \le y \le l_3 \\ \mathrm{T}^U_{\tilde{A}_N}(y) & l_3 \le y \le l_4 \\ 0 & otherwise \end{array} \right\rangle$$

$$\mathrm{I}_{\tilde{A}_N}(y) = \left\langle \begin{array}{cc} \mathrm{I}^L_{\tilde{A}_N}(y) & m_1 \le y \le m_2 \\ 0 & m_2 \le y \le m_3 \\ \mathrm{I}^U_{\tilde{A}_N}(y) & m_3 \le y \le m_4 \\ 1 & otherwise \end{array} \right\rangle$$

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$$\mathrm{F}_{\tilde{A}_N}(y)=\left\langle\begin{array}{cc}\mathrm{F}^{L}_{\tilde{A}_N}(y) & n_1\le y\le n_2\\ 0 & n_2\le y\le n_3\\ \mathrm{F}^{U}_{\tilde{A}_N}(y) & n_3\le y\le n_4\\ 1 & otherwise\end{array}\right\rangle$$

where $0\le \sup \mathrm{T}_{\tilde{A}_N}(y)\le \sup \mathrm{I}_{\tilde{A}_N}(y)\le \sup \mathrm{F}_{\tilde{A}_N}(y)\le 3,\ \forall y\in Y$ and for all i=1,2,3,4; $l_i,m_i,n_i\in$R, such that $l_1\le l_2\le l_3\le l_4$; $m_1\le m_2\le m_3\le m_4$; $n_1\le n_2\le n_3\le n_4$. Here $\mathrm{T}^{L}_{\tilde{A}_N}(y),\mathrm{I}^{U}_{\tilde{A}_N}(y),\mathrm{F}^{U}_{\tilde{A}_N}(y)\in[0,1]$ are continuous monotonic increasing functions and $\mathrm{T}^{U}_{\tilde{A}_N}(y),\mathrm{I}^{L}_{\tilde{A}_N}(y),\mathrm{F}^{L}_{\tilde{A}_N}(y)\in[0,1]$ are continuous monotonic increasing functions.

Triangular Fuzzy Neutrosophic Number [Biswas et al. (2016)] A triangular fuzzy neutrosophic number (TFNN) $\tilde{A}_N$ in a universe of discourse Y is defined as follows:

$$\tilde{A}_N=(p_1,p_2,p_3),(q_1,q_2,q_3),(r_1,r_2,r_3)$$

where $p_1\le p_2\le p_3$, $q_1\le q_2\le q_3$ and $r_1\le r_2\le r_3$. Its truth member function is given as

$$\mathrm{T}_{\tilde{A}_N}(y)=\left\langle\begin{array}{cc}\dfrac{y-p_1}{p_2-p_1} & p_1\le y<p_2\\ 1 & y=p_2\\ \dfrac{p_3-y}{p_3-p_2} & p_2<y\le p_3\\ 0 & otherwise\end{array}\right\rangle$$

Its indeterminacy membership function is written as

$$\mathrm{I}_{\tilde{A}_N}(y)=\left\langle\begin{array}{cc}\dfrac{y-q_1}{q_2-q_1} & q_1\le y<q_2\\ 0 & y=q_2\\ \dfrac{q_3-y}{q_3-q_2} & q_2<y\le q_3\\ 1 & otherwise\end{array}\right\rangle$$

and its falsity membership function is written as

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$$\mathrm{F}_{\tilde{A}_N}(y)=\begin{cases}\dfrac{y-r_1}{r_2-r_1} & r_1\le y<r_2\\ 0 & y=r_2\\ \dfrac{r_3-y}{r_3-r_2} & r_2<y\le r_3\\ 1 & otherwise\end{cases}$$

Expected Value of Triangular Fuzzy Neutrosophic number [Biswas et al. (2014)]

Let $\tilde{A}_N=(p_1,p_2,p_3),(q_1,q_2,q_3),(r_1,r_2,r_3)$ be triangular fuzzy neutrosophic number (TFNN) in a universe of discourse *Y* then the expected value of TFNN can be written as follows:

$$\mathrm{E}\left[\tilde{A}_N\right]=\frac{p_1+2p_2+p_3+q_1+2q_2+q_3+r_1+2r_2+r_3}{12}. \quad (1)$$

MATHEMATICAL DEVELOPMENT OF THE EOQ MODEL

Notations

$\mathscr{D}$ Demand rate (unit/year)
$\mathscr{I}$ Order size per cycle
u Storage capacity of Owned Warehouse (OW)
K Constant ordering price
$\mathscr{a}$ Unit acquiring price
$\mathscr{i}$ Unit inspection price
S Price for selling good items per unit
S' Price for selling defective items per unit
ρ Screening rate(unit/year)
p' Proportion of flawed items in *I*
$\widetilde{p'}$ Fuzzy proportion of flawed item, where $\widetilde{p'}=\left(p_1^{'},p_2^{'},p_3^{'}\right)$
$\widetilde{p_{\mathcal{N}}}$ Neutrosophic proportion of substandard produts,
where $\widetilde{p_{\mathcal{N}}}=(p_{11},p_{12},p_{13}),(p_{21},p_{22},p_{23}),(p_{31},p_{32},p_{33})$
t Time duration for prepayment, $\tau>0$
$\mathscr{T}$ Cycle's length
$\mathscr{T}\mathrm{R}_1$ Investigation period of Rented Warehouse (RW)
$\mathscr{T}\mathrm{R}_2$ Time duration from the completion of screening process to the zero stock level of rented warehouse
$\mathscr{T}\mathrm{O}_1$ Screening period of owned warehouse
$\mathscr{T}\mathrm{O}_2$ Time duration in between screening period and starting point of demand of owned warehouse
$\mathscr{T}\mathrm{O}_3$ Time duration of satisfying demand from owned warehouse
m The number of advance payments paid before replenishing the order quantity

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$\hbar$r Holding price per unit item in RW
$\hbar$o Holding price per unit item in OW
ω' The proportion of the acquiring amount which should be paid in advance for every replenishment, where $0 \leq \omega \leq 1$
A Acquiring cost per cycle
P_d The rate of price discount

Assumptions

1. The rate of demand is stable.
2. Cent percent screening of products takes place and the demand proceeds simultaneously.
3. Rate of investigation is more than the demand rate, $\rho > \mathscr{D}$.
4. The Owned Warehouse (OW) has limited space which occupies y units and the Rented Warehouse (RW) has immeasurable space.
5. Holding price in RW is higher than the carrying cost in OW (i.e. $\hbar_r > \hbar_o$).
6. Advance payment option offered by the supplier. Under the condition that ω' proportion of acquiring amount P to be prepaid (ie., $\omega' A$) prior to the replenishment. Then the balance amount ($1-\omega')A$ is given during replenishment.

Crisp EOQ Model

At the beginning of every cycle $\mathscr{T}$, u items from a lot $\mathscr{I}$ are replenished in the owned warehouse and the rest of the items are stored in the rented warehouse and are depletes at a constant demand rate $\mathscr{D}$. Due to improper production process, weak maintenance, transportation damages, and so on, the received lot may contain some fraction of defective products say p'. So that the screening process takes place for all the received units in both the warehouse at the rate of ρ during the time $\left[0, \mathcal{T}_{R1}\right]$ for RW and $\left[0, \mathcal{T}_{O1}\right]$ for OW. It is also considered that the supplier should offer an advance payment option to the retailer in which a proportion $\omega'\left(\text{here, } 0 \leq \omega' \leq 1\right)$ of acquiring cost A, ie., $\omega' A$ to be paid in advance before the replenishment of every lot by m equal number of multiple payments at preceding cycle time and the remaining amount $\left(1-\omega'\right)A$ is used to pay at the time of replenishment with some price discount for prepaid quantities $\omega' A$ at some rate P_d. The time taken for prepayment belongs to the previous cycle length (ie.,) $t \in \left[0, \mathcal{T}\right]$. The defective products in every lot are separated and sold them at a lower cost after the time $\mathcal{T}_{R1}$ and $\mathcal{T}_{O1}$ from both the warehouses. At the end of the time $\mathcal{T}_{R2}$, the stock level in RW becomes zero then the products in OW depletes at the same demand rate throughout the time $\mathcal{T}_{O3}$.

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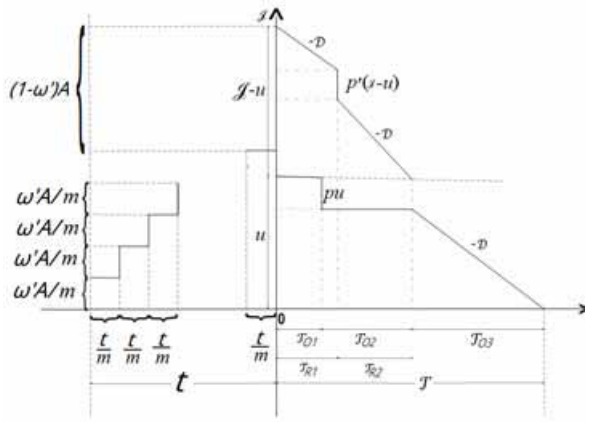
Figure 1. Inventory system for Two-warehouses with acquiring cost prepayments



Total Revenue

Let S and S' be the unit selling price for good items and defective items respectively, then the sales revenue from non-defective and defective products is obtained as follows:

$$R = S\mathcal{I} - \left(S - S'\right) p'\mathcal{I} . \tag{2}$$

The supplier should offer a price discount on the acquiring cost for prepaid quantities. Thus, the price rebate is given as

$$PR = \omega' a t \mathcal{I} P_d \left(\frac{m+1}{2m} \right). \tag{3}$$

The total revenue is the sum of the sales revenue as well as the price rebate, which is given as

$$TR = S\mathcal{I} - \left(S - S'\right) p'\mathcal{I} + \omega' a t \mathcal{I} P_d \left(\frac{m+1}{2m} \right). \tag{4}$$

Total Cost

The relevant costs consist of procurement cost, screening charge in both OW and RW, and carrying charge in both OW and RW. The procurement cost for every cycle has both ordering cost K and purchasing cost A, i.e.,

$$PC = K + A = K + a\mathcal{I} \tag{5}$$

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The screening charge in RW and OW are given as

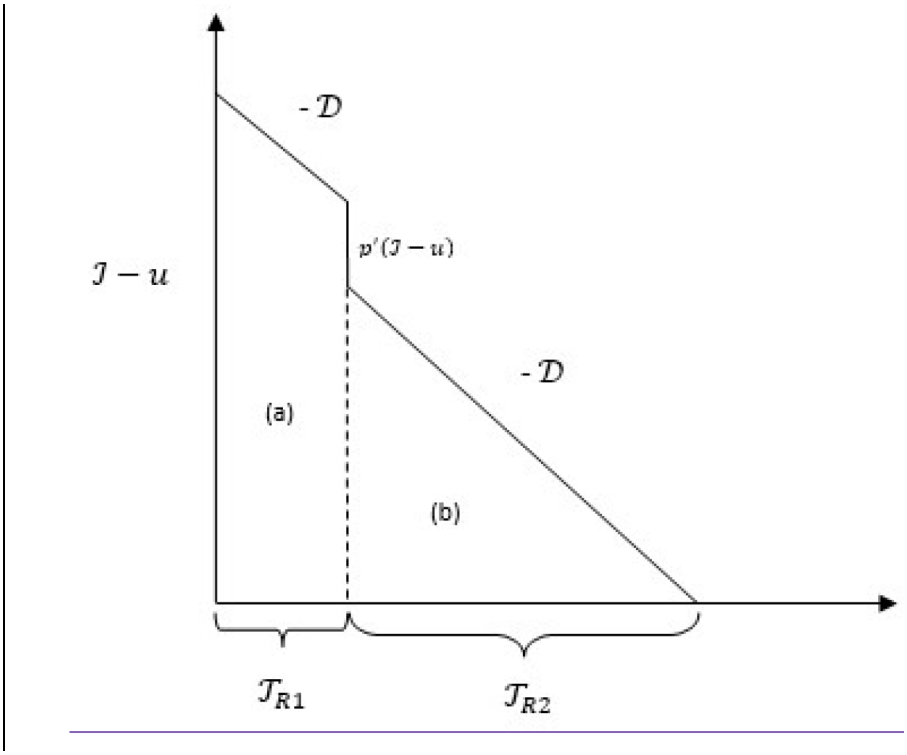
$$IC_R = \dot{\iota}(\mathcal{F} - \mathrm{u}) \tag{6}$$

$$IC_O = \dot{\iota}\mathrm{u} \tag{7}$$

The owned warehouse has limited space that occupies *u* units, whereas the rented warehouse has immeasurable space. It is considered that the holding price in RW is higher than the carrying cost in OW (i.e. $\hbar_r > \hbar_o$).

Thus, the carrying charge in RW per cycle is obtained as follows

Figure 2. Stock variation in both RW and OW



By adding the two parts (a) and (b) in Figure 2(A), the carrying charge in RW is derived as

$$(\mathrm{a}) \quad = \frac{h_r}{2}(\mathcal{I}-u)^2\left(\frac{2}{\rho}-\frac{\mathcal{D}}{\rho^2}\right)$$

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$$(\mathrm{b}) \quad = \frac{h_r}{2}(\mathcal{I}-u)^2\left(\frac{(1-p'-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right)$$

On adding (a) and (b)

$$HC_R = \frac{h_r}{2}\left(\mathcal{I}-u\right)^2\left(\frac{2}{\rho}-\frac{\mathcal{D}}{\rho^2}+\frac{(1-p'-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right). \tag{8}$$

Also, by summing the three parts (c), (d) and (e) in Figure 2(B), the carrying charge in OW per cycle is derived as

$$(\mathrm{c}) \quad = \frac{h_o u^2}{\rho}$$

$$(\mathrm{d}) \quad = h_o u\left(1-p'\right)\left(\frac{\left(\mathcal{I}-u\right)\left(1-p'-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}+\frac{\left(\mathcal{I}-2u\right)}{\rho}\right)$$

$$(e) \quad = \frac{h_o u^2\left(1-p'\right)^2}{2\mathcal{D}}$$

Adding (c), (d) and (e), it is obtained as

$$\begin{aligned} HC_O = \frac{h_o}{2}u^2\left(\frac{1}{\rho}-\frac{2\left(1-p'\right)}{\rho}-\frac{\left(1-p'\right)\left(1-p'-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}+\frac{\left(1-p'\right)^2}{2\mathcal{D}}\right) \\ +h_o\mathcal{I}u\left(\frac{\left(1-p'\right)}{\rho}+\frac{\left(1-p'\right)\left(1-p'-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}\right). \end{aligned} \tag{9}$$

The total cost per cycle is:

$$TC(\mathcal{J}) = PC + IC_R + IC_O + HC_R + HC_O \tag{10}$$

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$$TC(\mathcal{I}) = K + a\mathcal{I} + i\mathcal{I} + \frac{h_r}{2}(\mathcal{I}-u)^2\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-p'-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right)$$
$$+\frac{h_o}{2}u^2\left(\frac{1}{\rho} - \frac{2(1-p')}{\rho} - \frac{(1-p')\left(1-p'-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}} + \frac{(1-p')^2}{2\mathcal{D}}\right) + h_o\mathcal{I}u\left(\frac{(1-p')}{\rho} + \frac{(1-p')\left(1-p'-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}\right) \tag{11}$$

Expected Total Profit Per Unit Time

The expected total profit for each cycle is the difference between the total income per cycle and the total cost for each cycle and the total cycle time is $\mathcal{T} = \frac{\mathcal{I}(1-p')}{\mathcal{D}}$.

$$TP(\mathcal{I}) = TR(\mathcal{I}) - TC(\mathcal{I}). \tag{12}$$

The total profit per unit time is given as

$$TPU(\mathcal{I}) = \frac{TP(\mathcal{I})}{\mathcal{T}}.$$

Thus, the expected total profit per unit time is

$$E\left[TPU(\mathcal{I})\right] = \chi_1\mathcal{I} + \frac{\chi_2}{\mathcal{I}} + \chi_3 \tag{13}$$

Where

$$\chi_1 = \frac{-\mathcal{D}}{1-E[p']}\left(\frac{h_r}{2}\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E[p']-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right)\right) < 0;$$

$$\chi_2 = -\mathcal{D}\left[h_o u^2\left(\frac{1-E[p']}{2\mathcal{D}} - \frac{2}{\rho} - \frac{\left(1-E[p']-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}\right)\right.$$
$$\left.+\left(K + \frac{h_r u^2}{2}\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E[p']-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right) + \frac{h_o u^2}{\rho}\right)\frac{1}{1-E[p']}\right] < 0;$$

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$$\chi_3 = \mathcal{D}\left[S - S' - h_o u\left(\frac{1}{\rho} + \frac{\left(1 - E\left[p'\right] - \frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}} \right) + \left(S' + \omega' a t P_d \left(\frac{m+1}{2m} \right) - a - i + h_r u \left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1 - E\left[p'\right] - \frac{\mathcal{D}}{\rho})^2}{\mathcal{D}} \right) \right) \frac{1}{1 - E\left[p'\right]} \right]$$

Preposition 1

(a) $E[TPU(\mathcal{I})]$ is strictly concave.

(b) The optimal order quantity is

$$\mathcal{I}^* = \sqrt{\frac{\chi_2}{\chi_1}}\,. \tag{14}$$

Proof:

Differentiating eqn. (13) partially with respect to $\mathcal{I}$, it is obtained as

$$\frac{\partial E\left[TPU\left(\mathcal{I}\right)\right]}{\partial \mathcal{I}} = \chi_1 - \frac{\chi_2}{\mathcal{I}^2}\,.$$

Again, differentiating partially with respect to $\mathcal{I}$, it follows that

$$\frac{\partial^2 E\left[TPU\left(\mathcal{I}\right)\right]}{\partial \mathcal{I}^2} = \frac{2\chi_2}{\mathcal{I}^3}\,. \tag{15}$$

Since χ_2<0, the equation (15) is negative. Thus, the expected total profit for each unit item is strictly concave.

By setting $\frac{\partial E\left[TPU\left(\mathcal{I}\right)\right]}{\partial \mathcal{I}} = 0$, it follows that

$$\chi_1 - \frac{\chi_2}{\mathcal{I}^2} = 0 \;\Rightarrow -\frac{\chi_2}{\mathcal{I}^2} = -\chi_1 \;\Rightarrow \mathcal{I}^2 = \frac{\chi_2}{\chi_1} \;\Rightarrow \mathcal{I}^* = \sqrt{\frac{\chi_2}{\chi_1}}$$

Thus, the optimal order quantity is obtained as $\mathcal{I}^* = \sqrt{\frac{\chi_2}{\chi_1}}$

This completes the proof.

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EFFECT OF TFN TO THE PROPOSED EOQ MODEL

Expected total profit per unit time in a fuzzy sense is framed by presuming the proportion of defective items into a triangular fuzzy number, that is $\widetilde{p'} = \left(p_1^{'}, p_2^{'}, p_3^{'}\right)$.

Therefore, the expected total profit per unit time in fuzzy view, $E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right]$ is written as

$$E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right] = \widetilde{\chi_1}\mathcal{I} + \frac{\widetilde{\chi_2}}{\mathcal{I}} + \widetilde{\chi_3} \tag{16}$$

Where

$$\widetilde{\chi_1} = \frac{-\mathcal{D}}{1-E\left[\widetilde{p'}\right]}\left(\frac{h_r}{2}\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E\left[\widetilde{p'}\right] - \frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right)\right) < 0;$$

$$\widetilde{\chi_2} = -\mathcal{D}\left(h_o u^2\left(\frac{1-E\left[\widetilde{p'}\right]}{2\mathcal{D}} - \frac{2}{\rho} - \frac{\left(1-E\left[\widetilde{p'}\right] - \frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}\right) + \left(K + \frac{h_r u^2}{2}\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E\left[\widetilde{p'}\right] - \frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right) + \frac{h_o u^2}{\rho}\right)\frac{1}{1-E\left[\widetilde{p'}\right]}\right) < 0;$$

$$\widetilde{\chi_3} = \mathcal{D}\left[S - S' - h_o u\left(\frac{1}{\rho} + \frac{\left(1-E\left[\widetilde{p'}\right] - \frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}}\right) + \left(S' + \omega' a t P_d\left(\frac{m+1}{2m}\right) - a - i + h_r u\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E\left[\widetilde{p'}\right] - \frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right)\right)\frac{1}{1-E\left[\widetilde{p'}\right]}\right]$$

Preposition 2: If $\tilde{p}$ is a triangular fuzzy number, where $\widetilde{p'} = \left(p_1^{'}, p_2^{'}, p_3^{'}\right)$, and then

$$E\left[\widetilde{p'}\right] = \frac{p_1^{'} + 2p_2^{'} + p_3^{'}}{4}.$$

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Proof:

It is easy to prove the lemma by adopting a theorem from [13], let ξ be a continuous fuzzy variable which has membership function μ. If the expected value is present, and there is a point x_0 such that $\mu(x)$ *i*s increasing on (-¥,x_0) and decreasing on (x_0,+¥), then

$$E\left[\xi\right] = x_0 + \frac{1}{2}\int_{x_0}^{+\infty} \mu\left(x\right) - \frac{1}{2}\int_{-\infty}^{x_0} \mu\left(x\right)$$

Now, let $\widetilde{p'}$ be a TFN given as $\widetilde{p'} = \left(p_1^{'}, p_2^{'}, p_3^{'}\right)$, with membership function

$$\mu_{\widetilde{p'}}\left(x\right) = \begin{cases} 0, & x < p_1^{'} \\ \dfrac{x - p_1^{'}}{p_2^{'} - p_1^{'}}, & p_1^{'} \le x \le p_2^{'} \\ 1, & x = p_2^{'} \\ \dfrac{p_3^{'} - x}{p_3^{'} - p_2^{'}}, & p_2^{'} \le x \le p_3^{'} \\ 0, & x > p_3^{'} \end{cases}$$

where $p_1^{'} \le p_2^{'} \le p_3^{'}$ then it has an expected value which is defined as,

$$E\left[\widetilde{p'}\right] = p_3^{'} + \frac{1}{2}\int_{p_3}^{p_4} \frac{x - p_3^{'}}{p_2^{'} - p_3^{'}}\,dx - \frac{1}{2}\int_{p_1}^{p_2} \frac{x - p_1^{'}}{p_2^{'} - p_1^{'}}\,dx = \frac{p_1^{'} + 2p_2^{'} + p_3^{'}}{4}.$$

This completes the proof.

Preposition 3:

(a) $E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right]$ is strictly concave.

(b) The optimal order quantity is

$$\mathcal{I}^* = \sqrt{\frac{\widetilde{\chi_2}}{\widetilde{\chi_1}}}. \tag{17}$$

Proof:

Differentiating equation (16) partially with respect to $\mathcal{I}$, it is obtained as

$$\frac{\partial E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right]}{\partial \mathcal{I}} = \widetilde{\chi_1} - \frac{\widetilde{\chi_2}}{\mathcal{I}^2}.$$

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Again, differenstiating partially with respect to $\mathcal{I}$, it follows that

$$\frac{\partial^2 E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right]}{\partial \mathcal{I}^2} = \frac{2\widetilde{\chi_2}}{\mathcal{I}^3}. \tag{18}$$

Since $\widetilde{\chi_2} < 0$, the equation (18) is negative. Thus, the expected total profit for each unit item is strictly concave.

By setting $\frac{\partial E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right]}{\partial \mathcal{I}} = 0$, it follows that

$$\widetilde{\chi_1} - \frac{\widetilde{\chi_2}}{\mathcal{I}^2} = 0 \;\Rightarrow -\frac{\widetilde{\chi_2}}{\mathcal{I}^2} = -\widetilde{\chi_1} \;\Rightarrow \mathcal{I}^2 = \frac{\widetilde{\chi_2}}{\widetilde{\chi_1}} \;\Rightarrow \mathcal{I}^* = \sqrt{\frac{\widetilde{\chi_2}}{\widetilde{\chi_1}}}$$

Thus, the optimal order quantity is obtained as $\mathcal{I}^* = \sqrt{\frac{\widetilde{\chi_2}}{\widetilde{\chi_1}}}$.

This completes the proof.

IMPACT OF TFNN TO THE PROPOSED EOQ MODEL

In most of the classical EOQ models, some proportion of the defective products have been replenished by the supplier. Since the proportion of defective items is uncertain, this research scrutinizes the neutrosophic environment. The EOQ model under neutrosophic situation is framed by presuming the proportion of defective items into TFNN, that is $\widetilde{p_{\mathcal{N}}} = (p_{11},p_{12},p_{13}), (p_{21},p_{22},p_{23}), (p_{31},p_{32},p_{33})$.

Therefore, the expected total profit per unit time in neutrosophic sense, $E\left[\widetilde{TPU\left(\mathcal{I}\right)}\right]$ is written as

$$E\left[\widetilde{TPU_{Neu}\left(\mathcal{I}\right)}\right] = \widetilde{\chi_{Neu_1}}\mathcal{I} + \frac{\widetilde{\chi_{Neu_2}}}{\mathcal{I}} + \widetilde{\chi_{Neu_3}} \tag{19}$$

where

$$\widetilde{\chi_{Neu_1}} = \frac{-\mathcal{D}}{1 - E\left[\widetilde{p_{\mathcal{N}}}\right]}\left(\frac{h_r}{2}\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1 - E\left[\widetilde{p_{\mathcal{N}}}\right] - \frac{\mathcal{D}}{\rho})^2}{\mathcal{D}}\right)\right) < 0;$$

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$$\widetilde{\chi_{Neu_2}} = -\mathcal{D}\left(h_o u^2 \left(\frac{1-E\left[\widetilde{p_{\mathcal{N}}}\right]}{2\mathcal{D}} - \frac{2}{\rho} - \frac{\left(1-E\left[\widetilde{p_{\mathcal{N}}}\right]-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}} \right) + \left(K + \frac{h_r u^2}{2}\left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E\left[\widetilde{p_{\mathcal{N}}}\right]-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}} \right) + \frac{h_o u^2}{\rho} \right) \frac{1}{1-E\left[\widetilde{p_{\mathcal{N}}}\right]} \right) < 0;$$

$$\widetilde{\chi_{Neu_3}} = \mathcal{D}\left[S - S' - h_o u \left(\frac{1}{\rho} + \frac{\left(1-E\left[\widetilde{p_{\mathcal{N}}}\right]-\frac{\mathcal{D}}{\rho}\right)}{\mathcal{D}} \right) + \left(S' + \omega' a t P_d \left(\frac{m+1}{2m} \right) - a - i + h_r u \left(\frac{2}{\rho} - \frac{\mathcal{D}}{\rho^2} + \frac{(1-E\left[\widetilde{p_{\mathcal{N}}}\right]-\frac{\mathcal{D}}{\rho})^2}{\mathcal{D}} \right) \right) \frac{1}{1-E\left[\widetilde{p_{\mathcal{N}}}\right]} \right]$$

Preposition 4

If $\widetilde{p_{\mathcal{N}}} = (p_{11},p_{12},p_{13}), (p_{21},p_{22},p_{23}), (p_{31},p_{32},p_{33})$ is the TFNN, then the expected value $\widetilde{p_{\mathcal{N}}}$ is

$$\left[\widetilde{p_{\mathcal{N}}}\right] = \frac{p_{11}+2p_{12}+p_{13}+p_{21}+2p_{22}+p_{23}+p_{31}+2p_{32}+p_{33}}{12}.$$

Proof:

Using Equation (1), the expected value of the TFNN, $\widetilde{p_{\mathcal{N}}} = (p_{11},p_{12},p_{13}), (p_{21},p_{22},p_{23}), (p_{31},p_{32},p_{33})$ can be obtained as

$$E\left[\widetilde{p_{\mathcal{N}}}\right] = \frac{p_{11}+2p_{12}+p_{13}+p_{21}+2p_{22}+p_{23}+p_{31}+2p_{32}+p_{33}}{12} \tag{20}$$

Preposition 5

(a) $E\left[\widetilde{TPU_{Neu}(\mathcal{I})}\right]$ is strictly concave.

(b) The optimal order quantity is

$$\mathcal{I}^* = \sqrt{\frac{\widetilde{\chi_{Neu_2}}}{\widetilde{\chi_{Neu_1}}}}. \tag{21}$$

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Proof:

Using the proof of preposition 1 or the proof of preposition 2, it follows that the expected total profit for each unit item $E\left[\widetilde{TPU_{Neu}\left(\mathcal{I}\right)}\right]$ is strictly concave.

By setting $\frac{\partial E\left[\widetilde{TPU_{Neu}\left(\mathcal{I}\right)}\right]}{\partial \mathcal{I}} = 0$, it is derived as

$$\widetilde{\chi_{Neu_1}} - \frac{\widetilde{\chi_{Neu_2}}}{\mathcal{I}^2} = 0 \;\Rightarrow -\frac{\widetilde{\chi_{Neu_2}}}{\mathcal{I}^2} = -\widetilde{\chi_{Neu_1}} \;\Rightarrow \mathcal{I}^2 = \frac{\widetilde{\chi_{Neu_2}}}{\widetilde{\chi_{Neu_1}}} \;\Rightarrow \mathcal{I}^* = \sqrt{\frac{\widetilde{\chi_{Neu_2}}}{\widetilde{\chi_{Neu_1}}}}$$

Thus, the optimal order quantity is obtained as $\mathcal{I}^* = \sqrt{\frac{\widetilde{\chi_{Neu_2}}}{\widetilde{\chi_{Neu_1}}}}$.

This completes the proof.

NUMERICAL ANALYSIS

Analysis 1: Using some of the values of parameters from Salameh and Jaber (2000), the numerical illustration of the proposed EOQ crisp model is computed. The values of the parameters are $\mathcal{D}$=50,000 (unit/year), ρ=*1*,75,200 (unit/year), u=*5*00 units, S=*$*50 /unit, $S' = \$30$ /unit, $\mathcal{a}$=$25 /unit, $\hbar_o$=$5 / unit, $\hbar_r$=$7 /unit, i=%0.5 /unit, K=$100, m=5, t=0.20 years, $\omega' = 0.4$, P_d=0.1. The proportion of defective product p' is considered as uniform distribution with p.d.f

$$g\left(p'\right) = \begin{cases} \dfrac{1}{\delta} & 0 \le p' \le \delta \\ 0 & otherwise \end{cases}$$

and the expected value of p is given as

$$E\left[p'\right] = \int_0^{\delta} p' g\left(p'\right) dp' = \frac{\delta}{2}.$$

From equations (13) and (14), the expected total profit is $E[TPU(\mathcal{J})]$=$1,216,400 and the optimal order quantity is $\mathcal{J}^*$=1243.2. The analysis of changes in the proportion of defective items is discussed below.

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Table 1. Effect of the defective rate on optimal solution

δ	$E[p']$	**I***	$E[TPU(\mathbf{I})]$ **(USD)**
0.02	0.01	1234	1,216,400
0.04	0.02	1243.2	1,216,400
0.06	0.03	1252.6	1,216,500
0.08	0.04	1262.0	1,216,500
0.10	0.05	1271.5	1,216,500

Analysis 2: The numerical analysis for the proposed model under fuzzy sense has been developed by presuming the fraction of the flawed items is considered as a triangular fuzzy number instead of uniform distribution. Here, the proportion of the flawed items is given as TFN by considering "about 0.020, not more than 0.030, not less than 0.015", that is $\widetilde{p'} = (0.015, 0.020, 0.030)$.

From equations (16) and (17), the expected total profit is $\widetilde{E[TPU(\mathcal{I}^*)]} = \$1,216,500$ and the optimal order quantity is $\mathcal{I}^*$=1244.4. The impact of fuzziness of defective rate on the optimal solutions is examined and given in table 2.

Table 2. Effect of fuzziness of defective rate on optimal solutions

$\widetilde{p'}$	$E[\widetilde{p'}]$	**I***	$\widetilde{E[TPU(\mathcal{I}^*)]}$ **(USD)**
(0.011,0.016,0.026)	0.01725	1240.7	1,216,400
(0.013,0.018,0.028)	0.01925	1242.5	1,216,400
(0.015,0.020,0.030)	0.02125	1244.4	1,216,500
(0.017,0.022,0.032)	0.02325	1246.3	1,216,500
(0.019,0.024,0.034)	0.02525	1248.1	1,216,500

Analysis 3: The numerical analysis for the proposed model under neutrosophic environment has been developed by considering the proportion of the flawed items is considered as triangular fuzzy neutrosophic number (TFNN) instead of TFN or uniform distribution, that is $\widetilde{p_{\mathcal{N}}}$ = (0.0095,0.01,0.0107), (0.0195,0.02,0.0207), (0.0245,0.025,0.0257) and the remaining parameters are considered as the same.

From equations (19) and (21), the expected total profit is $\widetilde{E[TPU_{Neu}(\mathcal{I})]} = \$1,216,400$ and the optimal order quantity is $\mathcal{I}^*$=1241.7.

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Table 3. Effect of neutrosophication of the defective rate on optimal solution

$\widetilde{p_{\mathcal{N}}}$	$E\left[\widetilde{p_{\mathcal{N}}}\right]$	**I***	$E\left[\widetilde{TPU_{Neu}(\mathcal{I})}\right]$ **(USD)**
⟨(0.0055,0.006,0.0067), (0.0144,0.016,0.0167), (0.0205,0.021,0.0217)⟩	0.0144	1238.0	1,216,400
⟨(0.0075,0.008,0.0087), (0.0175,0.018,0.0278), (0.0225,0.023,0.0237)⟩	0.0164	1239.9	1,216,400
⟨(0.0095,0.01,0.0107), (0.0195,0.02,0.0207), (0.0245,0.025,0.0257)⟩	0.0184	1241.7	1,216,400
⟨(0.115,0.012,0.0127), (0.0215,0.022,0.0227), (0.0265,0.027,0.0277)⟩	0.0224	1243.6	1,216,400
⟨(0.0125,0.014,0.0147), (0.0235,0.024,0.0247), (0.0285,0.029,0.0297)⟩	0.0223	1245.4	1,216,500

The effect of the proportion of advance payment on acquiring cost, rate of price discount, the number of equal prepayments, and the carrying charge of RW on the expected total profit and the optimal order quantity under triangular fuzzy neutrosophic number are discussed below:

Table 4. Analysis of expected total profit on some parameter under neutrosophic arena

Parameters	**Parameter changes**	$\widetilde{E[TPU\left(\mathcal{I}^{*}\right)]}$ **(USD)**
ω'	0.2	1,213,400
	0.3	1,214,900
	0.4	1,216,400
	0.5	1,217,900
	0.6	1,219,400
P_d	0.06	1,214,000
	0.08	1,215,200
	0.1	1,216,400
	0.12	1,217,600
	0.14	1,218,800
m	3	1,217,000
	4	1,216,700
	5	1,216,400
	6	1,216,300
	7	1,216,200
$\hbar$r	6	1,217,700
	7	1,216,400
	8	1,215,200
	9	1,214,100
	10	1,212,900

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Table 1, Table 2, and Table 3 show that when the proportion of flawed items increases, the optimal order quantity also increases, and the expected total profit increases slightly for the proposed model under a crisp, fuzzy, and neutrosophic sense. From Table 4, it is noted that the fraction of acquiring cost paid in advance and the rate of price discount increase, the expected total profit increases also when the number of equal prepayments and the carrying charge of RW increase, the expected total profit decreases.

Comparative Studies

Analysis 1 shows that, by utilizing the advance payment option under the double storage economic order quantity model, the expected total profit of the proposed EOQ model is comparatively more than that of Salameh and Jaber (2000). From analysis 2, it is observed that the expected total profit under the fuzzy parameter is comparatively more than that under crisp and neutrosophic arena but the optimal order quantity of fuzzy EOQ model is high. So that, for the high order quantity, the fuzzy EOQ model attains more profit than crisp and neutrosophic models which is shown in Fig. 2. Also from analysis 3, it is observed that the expected total profit for the proposed model under both crisp and neutrosophic arena remains the same but the optimal order quantity for the model under neutrosophic environment is less than that under the crisp model. Since for minimum ordering quantity the same profit has been gained, the model under triangular fuzzy neutrosophic number is better when compared to the crisp model. Thus, from the above analysis under triangular fuzzy neutrosophic EOQ model is comparatively better than the model under crisp parameter and the model under fuzzy environment (Refer Fig. 3 and Fig. 4).

Figure 3. Comparison of Optimal Order Quantity for the proposed model

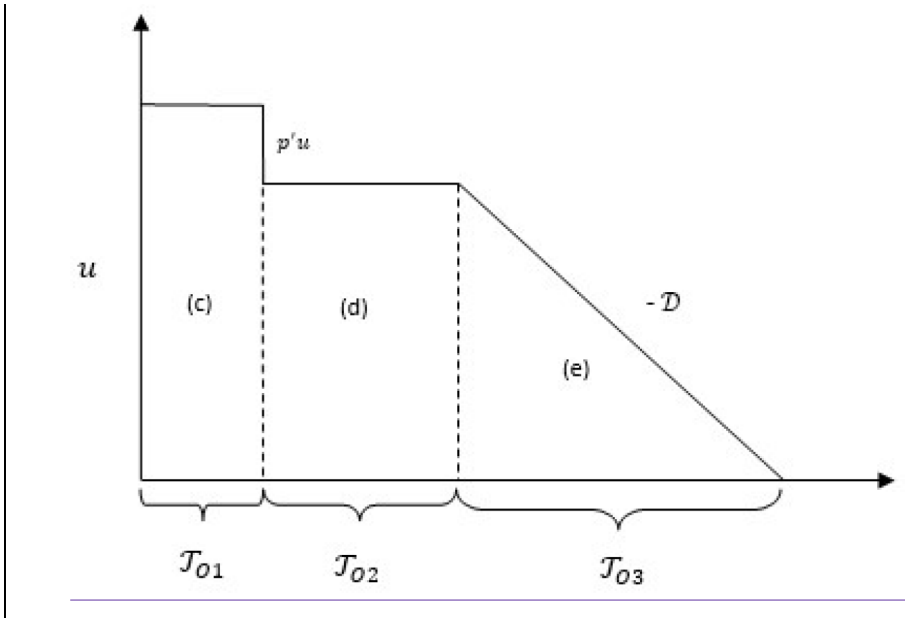

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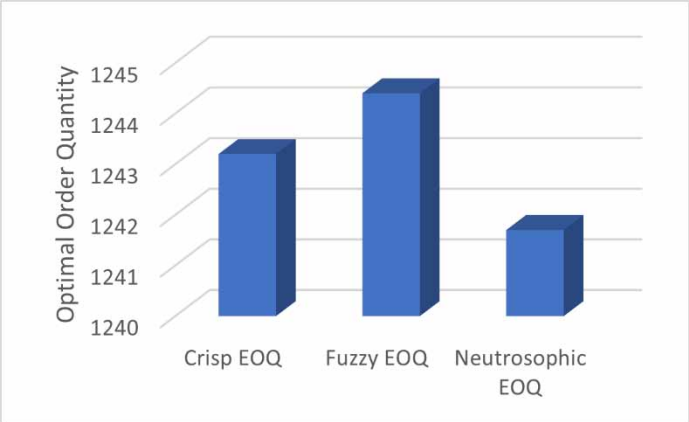
Figure 4. Comparison of Expected Total Profit for the proposed model



CONCLUSION

This research examines the concept of advance payment option on double storage EOQ model with substandard products under fuzzy situation and neutrosophic environment. As the aim of satisfying the customer with a standard product, the present research focused on screening of items once received a lot. At the end of the screening procedure, the defective units are withdrawn from both OW and RW and sold at a fewer price as one lot. The revenue from price rebate encourages the retailer to utilize the prepayment option on acquiring cost. On utilizing the advance payment strategy, this study helps to maximize the retailer's expected profit.

For uncertainty of the proportion of substandard units, to attain the most approximate outcomes, this research focuses on the concept of fuzziness as well as the neutrosophication of the proposed EOQ model. Thus, the EOQ model is framed under the crisp, fuzzy, and neutrosophic arena by considering the defective proportion as uniform distribution for the crisp model, as a TFN for fuzzy EOQ model, and as a TFNN for neutrosophic EOQ model. The defuzzification of the fuzzy EOQ model has been attained by calculating the expected value of the triangular fuzzy variable and the de-neutrosophication of the EOQ model under neutrosophic sense has been obtained by computing the expected value of the TFNN. The numerical results and the sensitivity analysis shows that the impact of crisp defective rate, triangular fuzzy defective proportion, the triangular neutrosophic defective rate on the optimal solution. Also, the effect of the proportion of acquiring amount to be paid in advance, rate of price rebate for the prepaid products, the number of equal multiple prepayments, and the unit carrying charge in RW on the optimal solution are analyzed under the neutrosophic environment. The expected profit of the crisp EOQ model with double storage and advance payment option is comparatively more than that of the model Salameh and Jaber (2000) and the model under the neutrosophic domain is better when compared to both crisp model and fuzzy EOQ model. In the future, the same model can be extended by presuming the various parameters as different Fuzzy Neutrosophic Numbers.

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Chapter 30
Improved MABAC Method for Multicriteria Group Decision Making With Trapezoidal Fuzzy Neutrosophic Numbers

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ABSTRACT

Many prestigious researchers have been exploring the issues of imprecision, inconsistency, and uncertain information in decision making, which are still challenges in developing a decision support system that is more feasible and efficient. This chapter proposes a new multicriteria group decision-making (MCGDM) strategy to overcome those issues. This strategy integrates the original MABAC method with trapezoidal fuzzy neutrosophic numbers (TrFNNs). First, the proposed method converts independent judgments from experts in the form of linguistic variables into TrFNNs and aggregates them using some aggregation operators. Next, it utilizes the functions of score and accuracy to rank the evaluated alternatives. By using the distance measure between the alternatives and the border approximation area, the proposed MABAC selects the best solution. Finally, the chapter illustrates an example of COVID-19 vaccine selection and a comparative analysis to show that the proposed MABAC has benefits to support indeterminate information and is more reasonable and practicable in handling MCGDM problems.

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1. INTRODUCTION

Zadeh (1965) first proposed the theory of fuzzy set to depict vague information by defining a membership function in which its membership value range between 0 and 1. Atanassov (1986) then initiated an Intuitionistic fuzzy set (IFS) in which each element of the universe of discourse is defined not only by membership function but also by non-membership function and the sum of these membership values is less than or equal one. In fact, it can handle incomplete and vague information but cannot deal with indeterminate and inconsistent information, where often occurs in real-case. Therefore, Smarandache (1998) originally proposed a neutrosophic set (NS) theory to generalize the concepts of FS and IFS. This theory is very victorious in overcoming cases and situations in uncertain, vagueness, indeterminate, and inconsistent information. It can be described by a truth-membership function, an indeterminacy-membership and a falsity-membership. Therefore, many scholars utilized and applied this theory to overcome their real-life research studies such as medical image processing (Akbulut et al., 2017; Guo & Sengur, 2015; Sayed et al., 2015; Zhao et al., 2016), medical diagnosis (Broumi et al., 2015; Elnazer et al., 2016; Jayanthi, 2016; Sayed & Hassanien, 2017; Zhang et al., 2018), investment (Irvanizam et al., 2020a; Liu et al., 2016; Zhang et al, 2016), sciences (Smarandache, 2017, 2018a), etc.

The theory of neutrosophic set has been intensively developed in decision making fields. Most extended areas are to provide novel concepts of neutrosophic sets and numbers in expressing vagueness, indeterminate, and inconsistent information. Some properties, operators, related theorems, and distance measurements between two neutrosophic numbers or sets have also been demonstrated and proved. Afterward, they were used to be applied for Multiple-criteria group decision-making problems.

Multiple-criteria group decision-making (MCGDM) is still a new approach of the decision-making process in evaluating numerous alternatives based on their corresponding criteria weight. It involves many decision-makers (DMs) assigning criterion values for all alternatives based on their different personal opinions. DMs even tend to choose linguistic terms to judge the criteria of alternatives. This makes the concept of linguistic terms can effectively be utilized to deal with indeterminate and inconsistent information. However, recent studies have demonstrated that by integrating MCGDM and a decision-making method can obtain more suitable, feasible, and reliable results.

In decision sciences, some prestigious researchers have proposed conventional decision-making methods such as TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) (Yoon and Hwang, 1980), VIKOR (VIšekriterijumsko KOmpromisno Rangiranje) (Opricovic & Tzeng, 2004), and MABAC (Multi-Attributive Border Approximation area Comparison) (Pamučar & Ćirović, 2015). The TOPSIS method selects the best alternative which is the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution, but it does not contemplate the relative greatness of these distances (Yoon et al., 1989). The VIKOR determines the best solution based on a particular closeness to the positive-ideal solution to find maximum profit without considering the decisions' risk (Cristóbal, 2011; Sanayei et al., 2010). Meanwhile, the MABAC calculates the distance between each alternative and the border approximation area (BAA). Additionally, the MABAC has a large number of valuable features such as: (1) it has stability in obtaining the computing results; (2) the calculating steps of the MABAC method are simple and easy to understand; (3) it considers the potential values of gains and losses; (4) it provides opportunities to integrate this model with other approaches (Wang et al., 2020). Hence, this method is a reliable and suitable instrument to obtain rational decision-making results.

There are existing studies related to MCGDM and decision-making under a neutrosophic environment (Abu-Faty et al., 2019; Akram et al., 2019, 2021; Awang et al., 2019; Bhowmik & Pal, 2009, 2010; Broumi

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et al., 2015, 2018; Broumi & Smarandache, 2013, 2014; Das et al, 2019; Deli et al., 2015; Irvanizam et al., 2018a, 2018b; Jana & Pal, 2019; Karaaslan & Deli, 2020; Liu & You, 2019; Naz & Akram, 2019; Naz et al., 2021; Smarandache, 2018b, 2020). Some of the studies proposed novel neutrosophic sets and applied them directly to decision-making cases for MCGDM by using their aggregation operators. Others extended neutrosophic numbers that were motivated by some FS and IFS characteristics.

Similarly, several relevant studies have extended MABAC using a fuzzy set approach in dealing with multiple-criteria decision making (Li, 2021; Pamučar et al., 2018a, 2018b; Mishra et al., 2020, 2021; Irvanizam et al., 2020b; Liu & Zhang, 2021; Şahin & Altun, 2020). As the authors described previously, this approach has a bit of limitation to depict indeterminate and inconsistent information. In addition, there are still very few studies regarding the extension of decision-making methods by integrating the neutrosophic concept with the MABAC method. Looking at the benefits of this MABAC and the ability of neutrosophic, proposed method candidates will have potential ideas to produce a reliable approach in overcoming complex MCGDM problems under indeterminate and inconsistent information.

The emphasis of this chapter will be on describing information in terms of trapezoidal fuzzy neutrosophic numbers (TrFNNs) as a tool that can be used to define linguistic term values in decision-making, and integrate it with the original MABAC method to determine an optimal solution. Afterward, the contributions and motivations of this chapter are as follows: (1) There are two motivations for using the TrFNNs. First, TrFNNs adopted by the concepts of Trapezoidal Fuzzy Number and Neutrosophic Sets (NSs) can effectively support vagueness, indeterminate, uncertain, and inconsistent information. In real-practical cases, the same assessments with the linguistic variable may interpret different meanings for different people. So, TrFNNs provide experts their own decisions to define the membership function, and it also can have an ability to better express and tackle inconsistent information. Second, TrFNNs are useful tools to handle incomplete and indeterminate information for MCGDM problems; (2) Compared with some other decision-making methods and considering the number of valuable features, the MABAC is a potential method to be enhanced. It can efficiently deal with the conflicting criteria and logically explain the decision-making theory with simple and systematic computational steps.

The leftover of this chapter is systematically organized as follows: Section 2 presents a brief overview of the concept of neutrosophic set such as TrFNNs, their aggregation operators, and the distance measure between two TrFNNs based on their centroids. Section 3 provides the computational steps of the proposed MABAC. The computational steps describe how the proposed method can determine the best solution and evaluate all alternatives through an illustrative example. Hence, the combination of TrFNNs and the original MABAC used to construct the proposed MABAC provides the description of gains and losses values in decision-making, which in turn is expected to apply to actual decision-making cases. Furthermore, in Section 4, the obtained results in terms of ranking order of alternatives will be analyzed and compared with results calculated by using aggregation operators to verify the reasonability and conformity of the proposed MABAC method. Finally, Section 5 summarizes some conclusions and addresses possible future works and studies.

2. BASIC DEFINITIONS AND CONCEPTS

Section 2 re-addresses briefly several definitions and basic concepts of NS, TrFNNs, and some TrFNNs aggregation operators. Moreover, this section also brings back the theory of how to measure the distance between two TrFNNs based on their centroid points.

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2.1. Neutrosophic Sets

Definition 1. (Smarandache, 2015) Let N be a universal set with an element in N denoted by n, then a neutrosophic set (NS) A in N is characterized by a truth-membership function (TMF) $T_A(n)$, an indeterminacy-membership function (IMF) $I_A(n)$, and a falsity-membership function (FMF) $F_A(n)$. Neutrosophic set A can be formulated by:

$$A = \left\{ n, T_A\left(n\right), I_A\left(n\right), F_A\left(n\right) : n \in N \right\},$$

where the functions $T_A\left(n\right), I_A\left(n\right), F_A\left(n\right) : N \to \left]0^-, 1^+\right[$, such that

$$T_A\left(n\right), I_A\left(n\right), F_A\left(n\right) \in \left]0^-, 1^+\right[\text{ and } 0^- \le T_A\left(n\right) + I_A\left(n\right) + F_A\left(n\right) \le 3^+.$$

Obviously, this definition is difficult to implement for real decision problems, particularly in the sciences and engineering sectors. Therefore, Wang et al. (2010) initiatively introduced the idea of a single-valued neutrosophic set (SVNS), which is also a sub-class of the NS, depicted its definition as follows.

Definition 2. (Smarandache, 2015) Let M be a universal set with an element in M denoted by m, then a single-valued neutrosophic set (SVNS) B in M is characterized by a TMF $T_B(m)$, an IMF $I_B(m)$, and a FMF $F_B(m)$. Single-valued neutrosophic set B can be depicted by:

$$B = \left\{ m, T_B\left(m\right), I_B\left(m\right), F_B\left(m\right) : m \in M \right\},$$

where the functions $T_B\left(m\right), I_B\left(m\right), F_B\left(m\right) : M \to \left[0,1\right]$, such that

$$T_B\left(m\right), I_B\left(m\right), F_B\left(m\right) \in \left[0,1\right] \text{ and } 0^- \le T_B\left(m\right) + I_B\left(m\right) + F_B\left(m\right) \le 3^+.$$

2.2. Trapezoidal Neutrosophic Sets

Definition 3. (Ye, 2015) Let Y be a universal set, then a trapezoidal neutrosophic set (TrNS) $\tilde{N}_z$ in Y is depicted as the following:

$$\tilde{N}_z = \left\{ y, T_z\left(y\right), I_z\left(y\right), F_z\left(y\right) : y \in Y \right\},$$

where $T_z\left(y\right) \subset \left[0,1\right], I_z\left(y\right) \subset \left[0,1\right]$ and $F_z\left(y\right) \subset \left[0,1\right]$ are three trapezoidal fuzzy numbers (TrFNs)

$$T_z\left(y\right) = \left(T_z^{a_1}\left(y\right), T_z^{a_2}\left(y\right), T_z^{a_3}\left(y\right), T_z^{a_4}\left(y\right)\right) : Y \to \left[0,1\right],$$

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$I_z\left(y\right)=\left(I_z^{b_1}\left(y\right),I_z^{b_2}\left(y\right),I_z^{b_3}\left(y\right),I_z^{b_4}\left(y\right)\right):Y\to\left[0,1\right]$, and

$F_z\left(y\right)=\left(F_z^{c_1}\left(y\right),F_z^{c_2}\left(y\right),F_z^{c_3}\left(y\right),F_z^{c_4}\left(y\right)\right):Y\to\left[0,1\right]$

with the condition $0\le T_z^{a_4}\left(y\right)+I_z^{b_4}\left(y\right)+F_z^{c_4}\left(y\right)\le 3$, $y\in Y$.

For more convenience, the three trapezoidal fuzzy numbers are simplified to

$T_z\left(y\right)=\left(T_z^{a_1},T_z^{a_2},T_z^{a_3},T_z^{a_4}\right),I_z\left(y\right)=\left(I_z^{b_1},I_z^{b_2},I_z^{b_3},I_z^{b_4}\right)$ and $F_z\left(y\right)=\left(F_z^{c_1},F_z^{c_2},F_z^{c_3},F_z^{c_4}\right)$.

Therefore, a trapezoidal fuzzy neutrosophic number is formed by

$n_i=\left(T_i^{a_1},T_i^{a_2},T_i^{a_3},T_i^{a_4}\right),\left(I_i^{b_1},I_i^{b_2},I_i^{b_3},I_i^{b_4}\right),\left(F_i^{c_1},F_i^{c_2},F_i^{c_3},F_i^{c_4}\right)$.

Definition 4. (Ye, 2015) Let

$n_A=\left(T_A^{a_1},T_A^{a_2},T_A^{a_3},T_A^{a_4}\right),\left(I_A^{b_1},I_A^{b_2},I_A^{b_3},I_A^{b_4}\right),\left(F_A^{c_1},F_A^{c_2},F_A^{c_3},F_A^{c_4}\right)$ and

$n_B=\left(T_B^{a_1},T_B^{a_2},T_B^{a_3},T_B^{a_4}\right),\left(I_B^{b_1},I_B^{b_2},I_B^{b_3},I_B^{b_4}\right),\left(F_B^{c_1},F_B^{c_2},F_B^{c_3},F_B^{c_4}\right)$

be two trapezoidal fuzzy neutrosophic numbers, then the following mathematical operations are satisfied:

(i) $n_A\oplus n_B=$
$\left\langle\begin{array}{l}(T_A^{a_1}+T_B^{a_1}-T_A^{a_1}T_B^{a_1},T_A^{a_2}+T_B^{a_2}-T_A^{a_2}T_B^{a_2},T_A^{a_3}+T_B^{a_3}-T_A^{a_3}T_B^{a_3},T_A^{a_4}+T_B^{a_4}-T_A^{a_4}T_B^{a_4}),;\\ (I_A^{b_1}I_B^{b_1},I_A^{b_2}I_B^{b_2},I_A^{b_3}I_B^{b_3},I_A^{b_4}I_B^{b_4}),(F_A^{b_1}F_B^{b_1},F_A^{b_2}F_B^{b_2},F_A^{b_3}F_B^{b_3},F_A^{b_4}F_B^{b_4})\end{array}\right\rangle$

(ii) $n_A\oplus n_B=$
$\left\langle\begin{array}{l}(T_A^{a_1}T_B^{a_1},T_A^{a_2}T_B^{a_2},T_A^{a_3}T_B^{a_3},T_A^{a_4}T_B^{a_4}),\\ (I_A^{a_1}+I_B^{a_1}-I_A^{a_1}I_B^{a_1},I_A^{a_2}+I_B^{a_2}-I_A^{a_2}I_B^{a_2},I_A^{a_3}+I_B^{a_3}-I_A^{a_3}I_B^{a_3},I_A^{a_4}+I_B^{a_4}-I_A^{a_4}I_B^{a_4}),\\ (F_A^{a_1}+F_B^{a_1}-F_A^{a_1}F_B^{a_1},F_A^{a_2}+F_B^{a_2}-F_A^{a_2}F_B^{a_2},F_A^{a_3}+F_B^{a_3}-F_A^{a_3}F_B^{a_3},T_A^{a_4}+T_B^{a_4}-T_A^{a_4}T_B^{a_4})\end{array}\right\rangle$;

(iii) $\varphi n_A=\left\langle\begin{array}{l}\left(1-\left(1-T_A^{a_1}\right)^{\varphi},1-\left(1-T_A^{a_2}\right)^{\varphi},1-\left(1-T_A^{a_3}\right)^{\varphi},1-\left(1-T_A^{a_4}\right)^{\varphi}\right),\\ \left(\left(I_A^{b_1}\right)^{\varphi},\left(I_A^{b_2}\right)^{\varphi},\left(I_A^{b_3}\right)^{\varphi},\left(I_A^{b_4}\right)^{\varphi}\right),\left(\left(F_A^{b_1}\right)^{\varphi},\left(F_A^{b_2}\right)^{\varphi},\left(F_A^{b_3}\right)^{\varphi},\left(F_A^{b_4}\right)^{\varphi}\right)\end{array}\right\rangle,\varphi>0;$

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$$\left(\left(T_A^{a_1}\right)^{\varphi},\left(T_A^{a_2}\right)^{\varphi},\left(T_A^{a_3}\right)^{\varphi},\left(T_A^{a_4}\right)^{\varphi}\right),$$

(iv) $n_A^{\varphi} = \left(1-\left(1-I_A^{a_1}\right)^{\varphi},1-\left(1-I_A^{a_2}\right)^{\varphi},1-\left(1-I_A^{a_3}\right)^{\varphi},1-\left(1-I_A^{a_4}\right)^{\varphi}\right)$, , φ>0

$$\left(1-\left(1-F_A^{a_1}\right)^{\varphi},1-\left(1-F_A^{a_2}\right)^{\varphi},1-\left(1-F_A^{a_3}\right)^{\varphi},1-\left(1-F_A^{a_4}\right)^{\varphi}\right)$$

Definition 5. (Ye, 2015) Let

$$n_A = \left(T_A^{a_1},T_A^{a_2},T_A^{a_3},T_A^{a_4}\right),\left(I_A^{b_1},I_A^{b_2},I_A^{b_3},I_A^{b_4}\right),\left(F_A^{c_1},F_A^{c_2},F_A^{c_3},F_A^{c_4}\right)$$

be a trapezoidal fuzzy neutrosophic number, then the functions of score $\mathcal{S}\left(n_A\right)$ and accuracy $\mathcal{H}\left(n_A\right)$ of n_A are defined as:

(i) $\mathcal{S}\left(n_A\right) = \frac{1}{3}\left(2+\frac{T_A^{a_1}+T_A^{a_2}+T_A^{a_3}+T_A^{a_4}}{4}-\frac{I_A^{b_1}+I_A^{b_2}+I_A^{b_3}+I_A^{b_4}}{4}-\frac{F_A^{c_1}+F_A^{c_2}+F_A^{c_3}+F_A^{c_4}}{4}\right)$, ;
$\mathcal{S}\left(n_A\right)\in\left[0,1\right]$

(ii) $\mathcal{H}\left(n_A\right) = \frac{T_A^{a_1}+T_A^{a_2}+T_A^{a_3}+T_A^{a_4}}{4}-\frac{F_A^{c_1}+F_A^{c_2}+F_A^{c_3}+F_A^{c_4}}{4}, \mathcal{H}\left(n_A\right)\in\left[-1,1\right]$

Definition 6. (Ye, 2015) Let

$n_A = \left(T_A^{a_1},T_A^{a_2},T_A^{a_3},T_A^{a_4}\right),\left(I_A^{b_1},I_A^{b_2},I_A^{b_3},I_A^{b_4}\right),\left(F_A^{c_1},F_A^{c_2},F_A^{c_3},F_A^{c_4}\right)$ and

$n_B = \left(T_B^{a_1},T_B^{a_2},T_B^{a_3},T_B^{a_4}\right),\left(I_B^{b_1},I_B^{b_2},I_B^{b_3},I_B^{b_4}\right),\left(F_B^{c_1},F_B^{c_2},F_B^{c_3},F_B^{c_4}\right)$

be two trapezoidal fuzzy neutrosophic numbers, $\mathcal{S}\left(n_A\right)$ and $\mathcal{S}\left(n_B\right)$ are the score functions of n_A and n_B respectively, and $\mathcal{H}\left(n_A\right)$ and $\mathcal{H}\left(n_B\right)$ are the accuracy functions of n_A and n_B respectively, then the following order conditions are satisfied:

i. If $\mathcal{S}\left(n_A\right) > \mathcal{S}\left(n_B\right)$ then n_A>n_B ;
ii. If $\mathcal{S}\left(n_A\right) < \mathcal{S}\left(n_B\right)$ then n_A<n_B;
iii. If $\mathcal{S}\left(n_A\right) = \mathcal{S}\left(n_B\right)$ and
   a. If $\mathcal{H}\left(n_A\right) > \mathcal{H}\left(n_B\right)$ then n_A>n_B ;
   b. If $\mathcal{H}\left(n_A\right) = \mathcal{H}\left(n_B\right)$ then n_A=n_B;

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2.3. Aggregation Operators of TrFNNs

There are two existing aggregation operators usually utilized by researchers for data aggregation in decision-making. Those operators are the weighted arithmetic averaging and the weighted geometric averaging operators. In this chapter, the authors apply the two aggregation operators of trapezoidal neutrosophic numbers first introduced by Ye (2015) to unite the TrFNNs information.

Definition 7. Let $\tilde{\mathcal{N}} = \{n_1, n_2, n_3, \ldots, n_m\}$ be a collection of *m* TrFNNs where each

$$n_k = \left(T_k^{a_1}, T_k^{a_2}, T_k^{a_3}, T_k^{a_4}\right), \left(I_k^{b_1}, I_k^{b_2}, I_k^{b_3}, I_k^{b_4}\right), \left(F_k^{c_1}, F_k^{c_2}, F_k^{c_3}, F_k^{c_4}\right)$$

and $\mathcal{W} = \left\{\omega_1, \omega_2, \ldots, \omega_m\right\}$ be a collection of corresponding weights of *m* TrFNNs where $0 \leq \omega_t \leq 1$ and $\sum_{t=1}^{m} \omega_t = 1$, then a trapezoidal fuzzy neutrosophic number weighted arithmetic averaging (TrFNNWA) operator are formulated as:

$$TrFNNWA_W(\tilde{N}) = \omega_1 n_1 \oplus \omega_2 n_2 \oplus \ldots \oplus \omega_m n_m$$
$$= \Big\langle \Big(1 - \prod_{(t=1)}^{m} \left(1 - T_t^{a_1}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - T_t^{a_2}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - T_t^{a_3}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - T_t^{a_4}\right)^{w_t}\Big),$$
$$\Big(\prod_{(t=1)}^{m} \left(I_t^{b_1}\right)^{w_t}, \prod_{(t=1)}^{m} \left(I_t^{b_2}\right)^{w_t}, \prod_{(t=1)}^{m} \left(I_t^{b_3}\right)^{w_t}, \prod_{(t=1)}^{m} \left(I_t^{b_4}\right)^{w_t}\Big), \Big(\prod_{(t=1)}^{m} \left(F_t^{c_1}\right)^{w_t}, \prod_{(t=1)}^{m} \left(F_t^{c_2}\right)^{w_t}, \prod_{(t=1)}^{m} \left(F_t^{c_3}\right)^{w_t}, \prod_{(t=1)}^{m} \left(F_t^{c_4}\right)^{w_t}\Big)\Big\rangle$$

Definition 8. Let $\tilde{\mathcal{N}} = \{n_1, n_2, n_3, \ldots, n_m\}$ be a collection of *m* TrFNNs where each

$$n_k = \left(T_k^{a_1}, T_k^{a_2}, T_k^{a_3}, T_k^{a_4}\right), \left(I_k^{b_1}, I_k^{b_2}, I_k^{b_3}, I_k^{b_4}\right), \left(F_k^{c_1}, F_k^{c_2}, F_k^{c_3}, F_k^{c_4}\right)$$

and $\mathcal{W} = \left\{\omega_1, \omega_2, \ldots, \omega_m\right\}$ be a collection of corresponding weights of *m* TrFNNs where $0 \leq \omega_t \leq 1$ and $\sum_{t=1}^{m} \omega_t = 1$, then a trapezoidal fuzzy neutrosophic number weighted geometric averaging (TrFNNWG) operator are formulated as:

$$TrFNNWG_W(\tilde{N}) = \omega_1 n_1 \oplus \omega_2 n_2 \oplus \ldots \oplus \omega_m n_m$$
$$= \Big\langle \Big(\prod_{(t=1)}^{m} \left(T_t^{a_1}\right)^{w_t}, \prod_{(t=1)}^{m} \left(T_t^{a_2}\right)^{w_t}, \prod_{(t=1)}^{m} \left(T_t^{a_3}\right)^{w_t}, \prod_{(t=1)}^{m} \left(T_t^{a_4}\right)^{w_t}\Big),$$
$$\Big(1 - \prod_{(t=1)}^{m} \left(1 - I_t^{b_1}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - I_t^{b_2}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - I_t^{b_3}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - I_t^{b_4}\right)^{w_t}\Big),$$
$$\Big(1 - \prod_{(t=1)}^{m} \left(1 - F_t^{c_1}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - F_t^{c_2}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - F_t^{c_3}\right)^{w_t}, 1 - \prod_{(t=1)}^{m} \left(1 - F_t^{c_4}\right)^{w_t}\Big)\Big\rangle$$

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2.4. The distance of two TrFNNs Based on Their Centroid Points

There are some distance measurement methods for TrFNNs such as Euclidean, Manhattan, Normalized Hamming, Minkowski, etc. However, measuring two arbitrary TrFNNs can also be used by using their centroid approaches. So, it encouraged Broumi et al. (2020) to propose a suitable tool to determine the distance between two TrFNNs based on the centroid. Due to its simplicity, significant accuracy, and correlation analysis, the authors utilize this distance measure method to determine the distance between two arbitrary TrFNNs in the MCGDM problem.

Definition 9. (Broumi et al., 2020) Let

$$n_A = \left(T_A^{a_1}, T_A^{a_2}, T_A^{a_3}, T_A^{a_4}\right), \left(I_A^{b_1}, I_A^{b_2}, I_A^{b_3}, I_A^{b_4}\right), \left(F_A^{c_1}, F_A^{c_2}, F_A^{c_3}, F_A^{c_4}\right) \text{ and}$$

$$n_B = \left(T_B^{a_1}, T_B^{a_2}, T_B^{a_3}, T_B^{a_4}\right), \left(I_B^{b_1}, I_B^{b_2}, I_B^{b_3}, I_B^{b_4}\right), \left(F_B^{c_1}, F_B^{c_2}, F_B^{c_3}, F_B^{c_4}\right)$$

be two TrFNNs, $\mathcal{O}_A = \left(x\left(n_A\right), y\left(n_A\right)\right)$ and $\mathcal{O}_B = \left(x\left(n_B\right), y\left(n_B\right)\right)$ are the centroids of n_A and n_B respectively, then the distance between n_A and n_B is calculated by:

$$D_c(n_A, n_B) = \frac{1}{9}\left\{\begin{array}{l}\left[\sum_{t=1}^{4} T_A^{a_t} + \sum_{t=1}^{4} I_A^{b_t} + \sum_{t=1}^{4} F_A^{c_t} - \sum_{t=1}^{4} T_B^{a_t} - \sum_{t=1}^{4} I_B^{b_t} - \sum_{t=1}^{4} I_B^{c_t}\right. \\ -\left(\frac{T_A^{a_4}T_A^{a_3} - T_A^{a_1}T_A^{a_2}}{\left(T_A^{a_4} + T_A^{a_3}\right) - \left(T_A^{a_1} + T_A^{a_2}\right)} - \frac{T_B^{a_4}T_B^{a_3} - T_B^{a_1}T_B^{a_2}}{\left(T_B^{a_4} + T_B^{a_3}\right) - \left(T_B^{a_1} + T_B^{a_2}\right)}\right) \\ -\left(\frac{I_A^{b_4}I_A^{b_3} - I_A^{b_1}I_A^{b_2}}{\left(I_A^{b_4} + I_A^{b_3}\right) - \left(I_A^{b_1} + I_A^{b_2}\right)} - \frac{I_B^{b_4}I_B^{b_3} - I_B^{b_1}I_B^{b_2}}{\left(I_B^{b_4} + I_B^{b_3}\right) - \left(I_B^{b_1} + I_B^{b_2}\right)}\right) \\ \left.-\left(\frac{F_A^{c_4}F_A^{(c_3)} - F_A^{c_1}F_A^{c_2}}{\left(F_A^{c_4} + F_A^{c_3}\right) - \left(F_A^{c_1} + F_A^{c_2}\right)} - \frac{F_B^{c_4}F_B^{c_3} - F_B^{c_1}F_B^{c_2}}{\left(F_B^{c_4} + F_B^{c_3}\right) - \left(F_B^{c_1} + F_B^{c_2}\right)}\right)\right]^2 \\ +\left[\left(\frac{T_A^{a_4}T_A^{a_3} - T_A^{a_1}T_A^{a_2}}{\left(T_A^{a_4} + T_A^{a_3}\right) - \left(T_A^{a_1} + T_A^{a_2}\right)} - \frac{T_B^{a_4}T_B^{a_3} - T_B^{a_1}T_B^{a_2}}{\left(T_B^{a_4} + T_B^{a_3}\right) - \left(T_B^{a_1} + T_B^{a_2}\right)}\right)\right. \\ +\left(\frac{I_A^{b_4}I_A^{b_3} - I_A^{b_1}I_A^{b_2}}{\left(I_A^{b_4} + I_A^{b_3}\right) - \left(I_A^{b_1} + I_A^{b_2}\right)} - \frac{I_B^{b_4}I_B^{b_3} - I_B^{b_1}I_B^{b_2}}{\left(I_B^{b_4} + I_B^{b_3}\right) - \left(I_B^{b_1} + I_B^{b_2}\right)}\right) \\ \left.+\left(\frac{F_A^{c_4}F_A^{c_3} - F_A^{c_1}F_A^{c_2}}{\left(F_A^{c_4} + F_A^{c_3}\right) - \left(F_A^{c_1} + F_A^{c_2}\right)} - \frac{F_B^{c_4}F_B^{c_3} - F_B^{c_1}F_B^{c_2}}{\left(F_B^{c_4} + F_B^{c_3}\right) - \left(F_B^{c_1} + F_B^{c_2}\right)}\right)\right]^2\end{array}\right.$$

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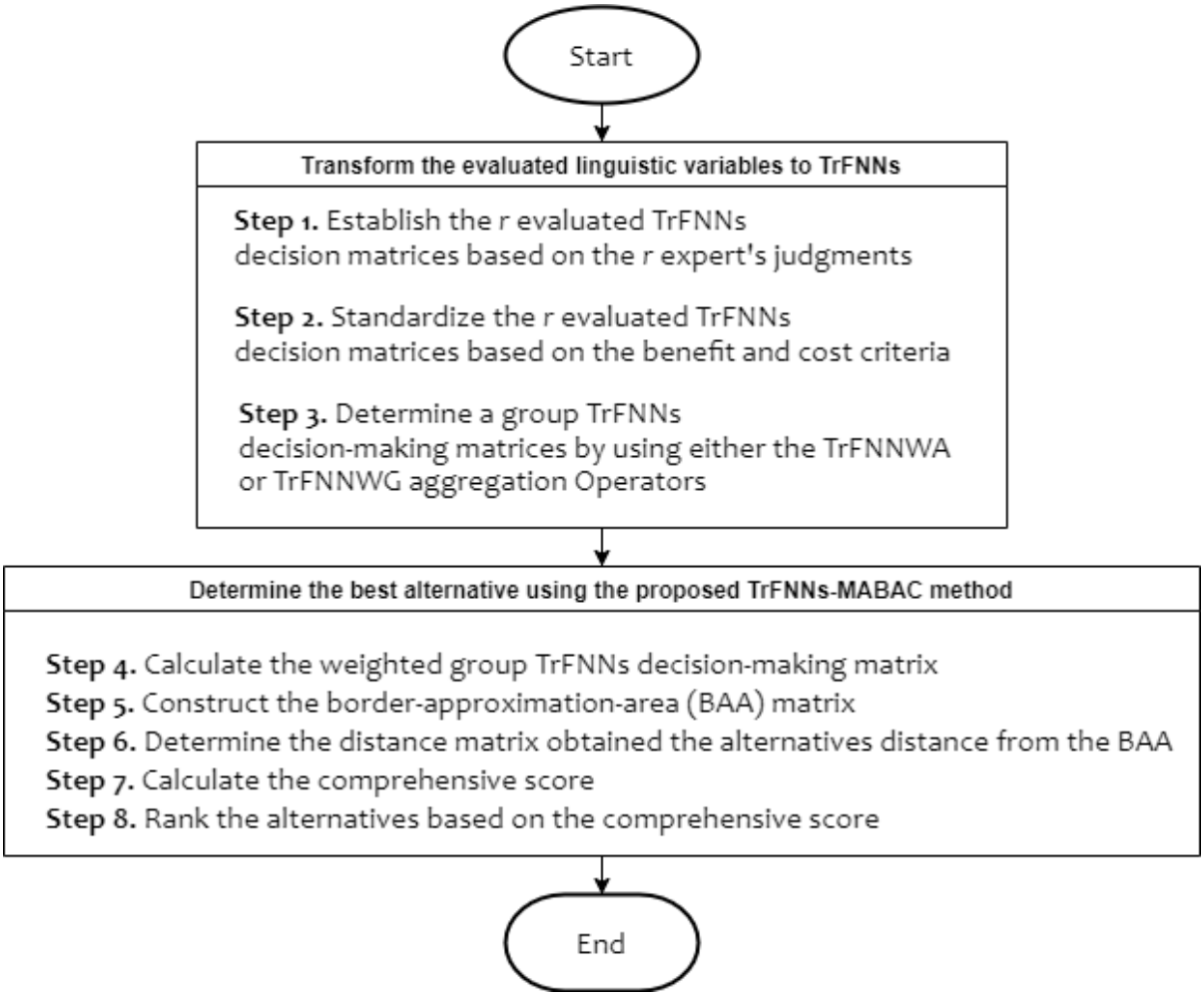
3. THE IMPROVED MABAC METHOD WITH TRFNNS FOR A MCGDM PROBLEM

This section presents a MCGDM methodology by combining the traditional MABAC with TrFNNs. Suppose there is a board of directors of a pharmaceutical company be a committee of z experts $\{CE_1,CE_2,CE_3,\ldots,CE_z\}$ in which the experts have judgement weights or the experts weight vector $\psi=(t_1,t_2,t_3,\ldots,t_z)^T$ that satisfies $t_r\in[0,1]$ and $\sum_{r=1}^{z} t_r = 1$. The experts will have responsibility to evaluate p alternatives $\left\{\mathcal{V}_1,\mathcal{V}_2,\ldots,\mathcal{V}_p\right\}$ and q criteria $\{C_1,C_2,\ldots,C_q\}$ with the criterion weight matrix $\mathcal{W}=\left\{\omega_1,\omega_2,\ldots,\omega_j\right\}$ that satisfies $\omega_j\in[0,1]$ and $\sum_{j=1}^{q}\omega_j = 1$. The algorithm of the improved MABAC with TrFNNs is designed as below and also depicted in Figure 1.

Figure 1. The flow chart of the proposed TrFNNs-MABAC method



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Algorithm:

Step 1. Establish the TrFNNs decision-matrix $\mathcal{N}^{(r)} = \left[n_{ij}^{r}\right]_{p\times q}$ according to the experts evaluation, in which $n_{ij}^{r} = T_{ij}^{r}, I_{ij}^{r}, F_{ij}^{r}$, with

$T_{ij}^{r} = \left(\left(T_{ij}^{a_1}\right)^{r}, \left(T_{ij}^{a_2}\right)^{r}, \left(T_{ij}^{a_3}\right)^{r}, \left(T_{ij}^{a_4}\right)^{r}\right) \in \left[0,1\right]$ is the TMF,

$I_{ij}^{r} = \left(\left(I_{ij}^{b_1}\right)^{r}, \left(I_{ij}^{b_2}\right)^{r}, \left(I_{ij}^{b_3}\right)^{r}, \left(I_{ij}^{b_4}\right)^{r}\right) \in \left[0,1\right]$ is the IMF,

$F_{ij}^{r} = \left(\left(F_{ij}^{c_1}\right)^{r}, \left(F_{ij}^{c_2}\right)^{r}, \left(F_{ij}^{c_3}\right)^{r}, \left(F_{ij}^{c_4}\right)^{r}\right) \in \left[0,1\right]$ is the FMF,

$0 \le \left(T_{ij}^{a_4}\right)^{r} + \left(I_{ij}^{b_4}\right)^{r} + \left(F_{ij}^{c_4}\right)^{r} \le 3$ i=1,2,3,…,p, j=1,2,3,…,q, and r=1,2,3,…,z.

Step 2. Standardize the TrFNNs decision-matrix $\mathcal{N}^{(r)}$ into $\mathcal{N}^{\prime(r)} = \left[n_{ij}^{\prime r}\right]_{p\times q}$ in which $n_{ij}^{\prime r} = T_{ij}^{\prime r}, I_{ij}^{\prime r}, F_{ij}^{\prime r}$ can be yielded by using Equation (1).

$$n_{ij}^{r} = \begin{cases} \left(\dfrac{(T_i j^{a_1})^r}{(T_m ax^{a_4})^r}, \dfrac{(T_i j^{a_2})^r}{(T_m ax^{a_4})^r}, \dfrac{(T_i j^{a_3})^r}{(T_m ax^{a_4})^r}, \dfrac{(T_i j^{a_4})^r}{(T_m ax^{a_4})^r}\right), \left(\dfrac{(I_i j^{b_1})^r}{(I_m ax^{b_4})^r}, \dfrac{(I_i j^{b_2})^r}{(I_m ax^{b_4})^r}, \dfrac{(I_i j^{b_3})^r}{(I_m ax^{b_4})^r}, \dfrac{(I_i j^{b_4})^r}{(I_m ax^{b_4})^r}\right), \\ \left(\dfrac{(F_i j^{c_1})^r}{(F_m ax^{c_4})^r}, \dfrac{(F_i j^{c_2})^r}{(F_m ax^{c_4})^r}, \dfrac{(F_i j^{c_3})^r}{(F_m ax^{c_4})^r}, \dfrac{(F_i j^{c_4})^r}{(F_m ax^{c_4})^r}\right), for\ \ C_{Benefit-type} \\ \left(1-\dfrac{(T_i j^{a_4})^r}{(T_m ax^{a_4})^r}, 1-\dfrac{(T_i j^{a_3})^r}{(T_m ax^{a_4})^r}, 1-\dfrac{(T_i j^{a_2})^r}{(T_m ax^{a_4})^r}, 1-\dfrac{(T_i j^{a_1})^r}{(T_m ax^{a_4})^r}\right), 1-\dfrac{(I_i j^{b_4})^r}{(I_m ax^{b_4})^r}\right), \\ \left(1-\dfrac{(F_i j^{c_4})^r}{(F_m ax^{c_4})^r}, 1-\dfrac{(F_i j^{c_3})^r}{(F_m ax^{c_4})^r}, 1-\dfrac{(F_i j^{c_2})^r}{(F_m ax^{c_4})^r}, 1-\dfrac{(F_i j^{c_1})^r}{(F_m ax^{c_4})^r}\right), for\ \ C_{\cos t-type} \end{cases} \quad (1)$$

Where

$\left(T_{\max}^{a_4}\right)^{r} = \max\left\{\max_{1\le i\le p}\left(T_{ij}^{a_4}\right)^{r}, \max_{1\le j\le q}\left(T_{ij}^{a_4}\right)^{r}\right\}$,

$\left(I_{\max}^{b_4}\right)^{r} = \max\left\{\max_{1\le i\le p}\left(I_{ij}^{b_4}\right)^{r}, \max_{1\le j\le q}\left(I_{ij}^{b_4}\right)^{r}\right\}$, and

$\left(F_{\max}^{c_4}\right)^{r} = \max\left\{\max_{1\le i\le p}\left(F_{ij}^{c_4}\right)^{r}, \max_{1\le j\le q}\left(F_{ij}^{c_4}\right)^{r}\right\}$.

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Step 3. Determine the group TrFNNs decision-making matrix in which is fused by using Equation (2) either for TrFNNWA operator or TrFNNWG operator, respectively.

$$g_{ij}=\begin{cases}\Big\langle(1-\prod_{(r=1)}^{z}(1-(T_{ij}^{a_1})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(T_{ij}^{a_2})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(T_{ij}^{a_3})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(T_{ij}^{a_4})^r)^{t_r}),\\ (\prod_{(r=1)}^{z}((I_{ij}^{b_1})^r)^{t_r},\prod_{(r=1)}^{z}((I_{ij}^{b_2})^r)^{t_r},\prod_{(r=1)}^{z}((I_{ij}^{b_3})^r)^{t_r},\prod_{(r=1)}^{z}?((I_{ij}^{b_4})^r)^{t_r}),\\ (\prod_{(r=1)}^{z}((F_{ij}^{c_1})^r)^{t_r},\prod_{(r=1)}^{z}((F_{ij}^{c_1})^r)^{t_r},\prod_{(r=1)}^{z}((F_{ij}^{c_1})^r)^{t_r})\Big\rangle,\ for\ \ TrFNNWA\ \ Operator\\ \Big\langle(\prod_{(r=1)}^{z}((T_{ij}^{a_1})^r)^{t_r},\prod_{(r=1)}^{z}((T_{ij}^{a_2})^r)^{t_r},\prod_{(r=1)}^{z}((T_{ij}^{a_3})^r)^{t_r},\prod_{(r=1)}^{z}((T_{ij}^{a_4})^r)^{t_r}),\\ (1-\prod_{(r=1)}^{z}(1-(I_{ij}^{b_1})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(I_{ij}^{b_2})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(I_{ij}^{b_3})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(I_{ij}^{b_4})^r)^{t_r}),\\ (1-\prod_{(r=1)}^{z}(1-(F_{ij}^{c_1})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(F_{ij}^{c_1})^r)^{t_r},1-\prod_{(r=1)}^{z}(1-(F_{ij}^{c_1})^r)^{t_r})\Big\rangle,\ for\ \ TrFNNWG\ \ Operator\end{cases}\tag{2}$$

Step 4. From the matrix G and utilizing Equation (3), calculate the weighted group TrFNNs matrix $L=\left[T_{ij}^{L},I_{ij}^{L},F_{ij}^{L}\right]_{p\times q}$ in which $L=\left[l_{ij}\right]_{p\times q}$ and $\sum_{j=1}^{q}w_j=1$, in such a way that l_{ij}=w_j*g_{ij}.

$$\begin{aligned}&l_ij=\Big\langle(1-(1-T_ij^{a_1})^{w_j},1-(1-T_ij^{a_2})^{w_j},1-(1-T_ij^{a_3})^{w_j},1-(1-T_ij^{a_4})^{\boldsymbol{w}_j}),\\&((I_ij^{b_1})^{w_j},(I_ij^{b_2})^{w_j},(I_ij^{b_3})^{w_j},(I_ij^{b_4})^{w_j}),\\&((F_ij^{c_1})^{w_j},(F_ij^{c_2})^{w_j},(F_ij^{c_3})^{w_j},(F_ij^{c_1})^{w_4})\Big\rangle\end{aligned}\tag{3}$$

Step 5. From the matrix G, construct the border-approximation-area (BAA) matrix $B=\left[T_{ij}^{B},I_{ij}^{B},F_{ij}^{B}\right]_{1\times q}$ in which $B=\left[b_j\right]_{1\times q}$ can be obtained from Equation (4), b_j is the BAA for criterion j, and m is a number of alternatives.

$$\begin{aligned}&b_j=\\&\Big\langle(1-\prod_{(i=1)}^{p}(1-T_ij^{a_1})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_2})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_3})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_4})^{1/m}),\\&(1-\prod_{(i=1)}^{p}(1-T_ij^{a_1})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_2})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_3})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_4})^{1/m}),\\&(1-\prod_{(i=1)}^{p}(1-T_ij^{a_1})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_2})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_3})^{1/m},1-\prod_{(i=1)}^{p}(1-T_ij^{a_4})^{1/m})\Big\rangle,\ for\ \forall\ \mathrm{j}=1,2,3,\ldots,\mathrm{q}\end{aligned}\tag{4}$$

Step 6. According to Definition 6 and 9, determine the distance matrix $\mathcal{D}=\left[\delta_{ij}\right]_{p\times q}$ in which element δ_{ij} is the alternatives' distance from BAA and yielded by using Equation (5).

$$\delta_{ij}=\begin{cases}D_C\left(l_{ij},b_j\right),\ if\ l_{ij}>b_j\\ 0,\ if\ l_{ij}=b_j\\ -D_C\left(l_{ij},b_j\right),\ if\ l_{ij}<b_j\end{cases}\tag{5}$$

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where $D_C(l_{ij},b_j)$ is the distance between the alternative l_{ij} and the BAA b_j based on their centroids.

Referring to the principle of original MABAC, it indicates that if δ_{ij} =0, the alternative $\mathcal{V}_i$ will be located to the border approximation area $\left(\mathcal{G}\right)$; if δ_{ij} >0 (δ_{ij} is positive), the alternative $\mathcal{V}_i$ will be located the upper approximation area $\left(\mathcal{G}^+\right)$; and if δ_{ij} <0 (δ_{ij} is negative), the alternative $\mathcal{V}_i$ will be located to the lower approximation area $\left(\mathcal{G}^-\right)$.

Due to the ideal alternative $\mathcal{V}_i^+$ and the non-ideal alternative $\mathcal{V}_i^-$ are spreading cover the upper approximation border $\left(\mathcal{G}^+\right)$ and the lower approximation border $\left(\mathcal{G}^-\right)$ respectively, in order to select the most suitable alternative, there should exist as many the criterion of the alternative as possible sticked on the upper approximation border $\left(\mathcal{G}^+\right)$.

Step 7. Calculate the final comprehensive score $\tilde{V}\left(\mathcal{V}_i\right)$ by using Equation (6).

$$\tilde{V}\left(\mathcal{V}_i\right)=\sum_{j=1}^{q}\delta_{ij}, \quad \left(\forall\, i=1,2,3,\ldots,p\right) \tag{6}$$

Step 8. From the scores of $\tilde{V}\left(\mathcal{V}_i\right)$, make a list of the alternatives based on descending order. In a selection problem for MCGDMs, the highest score $\tilde{V}\left(\mathcal{V}_i\right)$ indicates that the alternative $\mathcal{V}_i$ is the selected optimal alternative.

4. ILLUSTRATIVE EXAMPLE

This section demonstrates a case study for the MCGDM problem under a trapezoidal fuzzy neutrosophic (TrFN) environment to illustrate the practicable and the effectiveness of the improved MABAC approach. Assume there is a decision-making case in which a pharmaceutical company intends to order a corona virus disease 2019 (COVID-19) vaccine from four vaccine variants $\mathcal{V}=\left\{\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4\right\}$ available at a world-renowned vaccine company. The company considers the following criteria: efficacy and quality (Cr_1), safety (Cr_2), and cost (Cr_3). The four potential alternatives in $\mathcal{V}_i\left(i=1,2,3,4\right)$ will be assessed using MABAC algorithm with TrFNNs by four experts on the three criteria of Cr_j(j=1,2,3) with the criterion weight $\mathcal{W}=\left\{0.339,0.351,0.31\right\}$. Let a committee of the experts is CE= {CE_1, CE_2, CE_3, CE_4}, and each expert has been given respectively a weight in the vector ψ= (0.165, 0.335, 0.315, 0.185) T.

Section 3 depicts the algorithm of the improved MABAC method that consists of eight steps. Steps 1, 2, 3, and 4 are the how to perform a decision-making process until to yield the group TrFNNs matrix using TrFNNWA and TrFNNW operators, whereas Step 5 until Step 8 continues calculation procedures to evaluate the alternatives using the improved MABAC method, and later on, it names as the trapezoidal fuzzy neutrosophic numbers MABAC (TrFNNs-MABAC) method.

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Table 1. The linguistic variables under TrFNNs

Linguistic Variables	TrFNNs
Absolutely Low (AL)	< (0, 0, 0, 0), (0, 0, 0, 0), (0, 0, 0, 0)>
Very Low (VL)	< (0, 0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35)>
Low (L)	< (0.1, 0.15, 0.35, 0.4), (0.1, 0.15, 0.35, 0.4), (0.15, 0.2, 0.3, 0.45)>
Fairly Low (FL)	< (0.15, 0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55)>
Medium (M)	< (0.35, 0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75)>
Fairly High (FH)	< (0.55, 0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95)>
High(H)	< (0.65, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95)>
Very High (VH)	< (0.75, 0.85, 0.95, 1), (0.75, 0.85, 0.95, 1), (0.75, 0.85, 0.95, 1)>
Absolutey High (AH)	< (1, 1, 1, 1), (1, 1, 1, 1), (1, 1, 1, 1)>

4.1. MCGDM Strategy Based on TrFNNWA and TrFNNWG Operators

Section 4.1 demonstrates the computation steps of COVID-19 vaccine selection using the new MCGDM strategy as presented in the following.

Step 1. Assume that the director of the pharmaceutical company appoints experts to evaluate the potential COVID-19 vaccines (alternatives). The experts will consider this based on their professional assessments based on the linguistic values under TrFNNs shown in Table 1. The experts then give their reference scores as provided respectively in Tables 2-5. Those tables are the evaluated TrFNNs decision-matrices of the experts.

Table 2. The evaluated linguistic values of expert CE_1

Alternative\ Criterion	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	FH	VL	M
$\mathcal{V}_2$	M	H	FH
$\mathcal{V}_3$	VL	M	L
$\mathcal{V}_4$	VH	FH	H

Table 3. The evaluated linguistic values of expert CE_2

Alternative\ Criterion	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	H	L	VL
$\mathcal{V}_2$	FH	FH	M
$\mathcal{V}_3$	FL	FL	L
$\mathcal{V}_4$	H	H	H

Step 2. Due to all criteria are benefit-type criteria, so the experts do not need to normalize their TrFNNs decision-making matrices. The evaluated TrFNNs decision-making matrix for each expert can be seen in Table 6.

Step 3. The algorithm fuses the TrFNNS decision-makiing matrices both using TrFNNWA operator and TrFNNWG operator and the results show in Tables 7 and 8 respectively.

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Table 4. The evaluated linguistic values of expert CE_3

Alternative\ Criterion	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	FH	M	FL
$\mathcal{V}_2$	FL	VH	FH
$\mathcal{V}_3$	M	L	VL
$\mathcal{V}_4$	FH	VH	FH

Table 5. The evaluated linguistic values of expert CE_4

Alternative\ Criterion	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	FL	FL	M
$\mathcal{V}_2$	M	FH	M
$\mathcal{V}_3$	FL	M	FL
$\mathcal{V}_4$	FH	H	M

Table 6. The TrFNNs decision-making matrix CE_z for alternative $\mathcal{V}_i$ with corresponding to criterion Cr_j

Experts	Alternatives	Cr_1	Cr_2	Cr_3
CE_1	$\mathcal{V}_1$	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0,0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >
	$\mathcal{V}_2$	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >
	$\mathcal{V}_3$	< (0,0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >	< (0.1,0.15, 0.35, 0.4), (0.1, 0.15, 0.35, 0.4), (0.15, 0.2, 0.3, 0.45) >
	$\mathcal{V}_4$	< (0.75,0.85, 0.95, 1), (0.75, 0.85, 0.95, 1), (0.75, 0.85, 0.95, 1) >	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >
CE_2	$\mathcal{V}_1$	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >	< (0.1,0.15, 0.35, 0.4), (0.1, 0.15, 0.35, 0.4), (0.15, 0.2, 0.3, 0.45) >	< (0,0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35) >
	$\mathcal{V}_2$	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >
	$\mathcal{V}_3$	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >	< (0.1,0.15, 0.35, 0.4), (0.1, 0.15, 0.35, 0.4), (0.15, 0.2, 0.3, 0.45) >
	$\mathcal{V}_4$	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >

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Table 6. Continued

Experts	Alternatives	Cr_1	Cr_2	Cr_3
CE_3	$\mathcal{V}_1$	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >
	$\mathcal{V}_2$	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >	< (0.75,0.85, 0.95, 1), (0.75, 0.85, 0.95, 1), (0.75, 0.85, 0.95, 1) >	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >
	$\mathcal{V}_3$	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >	< (0.1,0.15, 0.35, 0.4), (0.1, 0.15, 0.35, 0.4), (0.15, 0.2, 0.3, 0.45) >	< (0,0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35), (0, 0.15, 0.25, 0.35) >
	$\mathcal{V}_4$	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.75,0.85, 0.95, 1), (0.75, 0.85, 0.95, 1), (0.75, 0.85, 0.95, 1) >	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >
CE_4	$\mathcal{V}_1$	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >
	$\mathcal{V}_2$	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >
	$\mathcal{V}_3$	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >	< (0.15,0.25, 0.35, 0.45), (0, 0.25, 0.35, 0.55), (0, 0.25, 0.35, 0.55) >
	$\mathcal{V}_4$	< (0.55,0.65, 0.75, 0.85), (0.45, 0.65, 0.75, 0.95), (0.45, 0.65, 0.75, 0.95) >	< (0.65,0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95), (0.55, 0.75, 0.85, 0.95) >	< (0.35,0.45, 0.55, 0.65), (0.25, 0.45, 0.55, 0.75), (0.25, 0.45, 0.55, 0.75) >

Table 7. The group TrFNNs matrix using TrNNWA operator

Alternatives	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	< (0.535,0.640, 0.748, 0.868), (0, 0.571, 0.680, 0.859), (0, 0.571, 0.679, 0.859) >	< (0.182,0.276, 0.407, 0.495), (0, 0.23, 0.382, 0.506), (0, 0.257, 0.362, 0.526) >	< (0.183,0.298, 0.400, 0.503), (0, 0.259, 0.366, 0.527), (0, 0.259, 0.366, 0.527) >
$\mathcal{V}_2$	< (0.375,0.479, 0.585, 0.696), (0, 0.423, 0.529, 0.736), (0, 0.423, 0.529, 0.736) >	< (0.641, 0.746, 0.861, 1), (0.546, 0.724, 0.825, 0.965), (0.546, 0.724, 0.825, 0.965) >	< (0.455, 0.557, 0.661, 0.767), (0.331, 0.537, 0.638, 0.840), (0.331, 0.537, 0.638, 0.840) >
$\mathcal{V}_3$	< (0.198,0.306, 0.407, 0.510), (0, 0.277, 0.382, 0.563), (0, 0.277, 0.382, 0.563) >	< (0.212, 0.3, 0.428, 0.517), (0, 0.261, 0.410, 0.554), (0, 0.286, 0.390, 0.575) >	< (0.079, 0.169, 0.32, 0.394), (0, 0.165, 0.315, 0.407), (0, 0.190, 0.2914, 0.4314) >
$\mathcal{V}_4$	< (0.625,0.728, 0.838, 1), (0.524, 0.713, 0.813, 0.958), (0.524, 0.713, 0.813, 0.958) >	< (0.672,0.775, 0.885, 1), (0.587, 0.762, 0.862, 0.965), (0.587, 0.762, 0.862, 0.965) >	< (0.575,0.678, 0.784, 0.899), (0.446, 0.652, 0.754, 0.909), (0.446, 0.652, 0.754, 0.909) >

Step 4. Based on Tables 7 and 8 with $\mathcal{W} = \{0.339, 0.351, 0.31\}$, the algorithm calculates the aggregated group TrFNNs decision-making matrix for the two TrFNNs aggregation operators by using Definition 7, so the aggregated group TrFNNs matrices for TrFNNWA and TrFNNWG are shown in Table 9 and Table 10 respectively.

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Table 8. The group TrFNNs matrix using TrNNWG operator

Alternatives	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	< (0.457,0.571, 0.679, 0.784), (0.425, 0.639, 0.748, 0.924), (0.425, 0.639, 0.748, 0.924) >	< (0,0.233, 0.381, 0.465), (0.118, 0.275, 0.407, 0.562), (0.135, 0.29, 0.392, 0.575) >	< (0,0.258, 0.366, 0.47), (0.095, 0.298, 0.4, 0.585), (0.095, 0.298, 0.4, 0.585) >
$\mathcal{V}_2$	< (0.311,0.422, 0.529, 0.633), (0.259, 0.478, 0.585, 0.824), (0.259, 0.478, 0.585, 0.824) >	< (0.623,0.724, 0.824, 0.911), (0.584, 0.746, 0.861, 1), (0.584, 0.746, 0.861, 1) >	< (0.434,0.536, 0.638, 0.739), (0.353, 0.557, 0.66, 0.884), (0.353, 0.557, 0.66, 0.884) >
$\mathcal{V}_3$	< (0,0.276, 0.381, 0.484), (0.086, 0.305, 0.407, 0.602), (0.086, 0.305, 0.407, 0.602) >	< (0.177,0.261, 0.409, 0.493), (0.125, 0.3, 0.428, 0.598), (0.14, 0.313, 0.414, 0.609) >	< (0,0.164, 0.314, 0.391), (0.051, 0.169, 0.32, 0.416), (0.078, 0.194, 0.294, 0.441) >
$\mathcal{V}_4$	< (0.612,0.712, 0.813, 0.906), (0.548, 0.728, 0.838, 1), (0.548, 0.728, 0.838, 1) >	< (0.661,0.761, 0.862, 0.947), (0.613, 0.775, 0.884, 1), (0.613, 0.775, 0.884, 1) >	< (0.549,0.652, 0.753, 0.855), (0.473, 0.678, 0.784, 0.932), (0.473, 0.678, 0.784, 0.932) >

Table 9. The aggregated group TrFNNs matrix using TrFNNWA operator

Alternatives	TrFNNWA
$\mathcal{V}_1$	< (0.317,0.426, 0.547, 0.673), (0, 0.321, 0.452, 0.607), (0, 0.332, 0.444, 0.615) >
$\mathcal{V}_2$	< (0.509,0.617, 0.737, 1), (0, 0.553, 0.658, 0.846), (0, 0.553, 0.658, 0.846) >
$\mathcal{V}_3$	< (0.166,0.262, 0.388, 0.478), (0, 0.229, 0.368, 0.504), (0, 0.248, 0.352, 0.52) >
$\mathcal{V}_4$	< (0.627,0.731, 0.842, 1), (0.518, 0.709, 0.81, 0.944), (0.518, 0.709, 0.81, 0.944) >

Table 10. The aggregated group TrFNNs matrix using TrFNNWG operator

Alternatives	TrFNNWG
$\mathcal{V}_1$	< (0,0.34, 0.47, 0.569), (0.221, 0.419, 0.538, 0.747), (0.225, 0.423, 0.534, 0.749) >
$\mathcal{V}_2$	< (0.453,0.561, 0.664, 0.761), (0.403, 0.596, 0.716, 1), (0.403, 0.596, 0.716, 1) >
$\mathcal{V}_3$	< (0,0.242, 0.381, 0.468), (0.084, 0.251, 0.374, 0.532), (0.098, 0.263, 0.362, 0.542) >
$\mathcal{V}_4$	< (0.618,0.717, 0.815, 0.906), (0.534, 0.715, 0.828, 1), (0.534, 0.715, 0.828, 1) >

The further steps are to calculate the score function value for all TrFNNs elements of those matrices as follows.

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For TrFNNWA approach:

$$\mathcal{S}(\mathcal{V}_1) = \frac{1}{3} * \left(2 + \frac{0.317 + 0.426 + 0.547, 0.673}{4} - \frac{0, 0.321 + 0.452 + 0.607}{4} - \frac{0 + 0.332 + 0.444 + 0.615}{4}\right) = 0.599$$

$$\mathcal{S}(\mathcal{V}_2) = \frac{1}{3} * \left(2 + \frac{0.509 + 0.617 + 0.737 + 1}{4} - \frac{0 + 0.553 + 0.658 + 0.846}{4} - \frac{0 + 0.553 + 0.658 + 0.846}{4}\right) = 0.562$$

$$\mathcal{S}(\mathcal{V}_3) = \frac{1}{3} * \left(2 + \frac{0.166 + 0.262 + 0.388 + 0.478}{4} - \frac{0 + 0.229 + 0.368 + 0.504}{4} - \frac{0 + 0.248 + 0.352 + 0.52}{4}\right) = 0.589$$

$$\mathcal{S}(\mathcal{V}_4) = \frac{1}{3} * \left(2 + \frac{0.627 + 0.731 + 0.842 + 1}{4} - \frac{0.518 + 0.709 + 0.81 + 0.944}{4} - \frac{0.518 + 0.709 + 0.81 + 0.944}{4}\right) = 0.436$$

For TrFNNWG approach:

$$\mathcal{S}(\mathcal{V}_1) = \frac{1}{3} * \left(2 + \frac{0 + 0.34 + 0.47 + 0.569}{4} + \frac{0.221 + 0.419 + 0.538 + 0.747}{4} + \frac{0.225, 0.423, 0.534, 0.749}{4}\right) = 0.460$$

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$$\mathcal{S}(\mathcal{V}_2)=\frac{1}{3}$$
$$*\left(2+\frac{0.453+0.561+0.664+0.761}{4}+\frac{0.403+0.596+0.716+1}{4}+\frac{0.403+0.596+0.716+1}{4}\right)$$
$$=0.417$$

$$\mathcal{S}(\mathcal{V}_3)=\frac{1}{3}$$
$$*\left(2+\frac{0+0.242+0.381+0.468}{4}+\frac{0.084+0.251+0.374+0.532}{4}+\frac{0.098+0.263+0.362+0.542}{4}\right)$$
$$=0.548$$

$$\mathcal{S}(\mathcal{V}_4)=\frac{1}{3}$$
$$*\left(2+\frac{0.618+0.717+0.815+0.906}{4}+\frac{0.534+0.715+0.828+1}{4}+\frac{0.534+0.715+0.828+1}{4}\right)$$
$$=0.408$$

Based on these scores function values and Definition 6, the results of the four alternatives' ranking order based on TrFNNWA and TrFNNWG are $\mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_2 \succ \mathcal{V}_4$ and $\mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_4$, respectively. The TrFNNWA approach recommends that the alternative $\mathcal{V}_1$ is the chosen alternative whereas the TrFNNWG approach advises that the alternative $\mathcal{V}_3$ is the best alternative.

4.2. Improved MABAC Method based on TrFNNs for MCGDM

Section 4.2 continues the computation procedures of the TrFNNs-MABAC method to illustrate the comprehensive computation steps in selecting the optimal solution using the proposed MCGDM strategy as continue presented in the following steps.

Step 1. This step is the same way that described in Step 1 in Section 4.1.

Step 2. Again, in this case, Cr_1, Cr_2, and Cr_3 are benefit-type criteria; hence, the experts do not require to standardize by normalizing the TrFNNs decision-making matrices. Table 6 shows the evaluated TrFNNs decision-making matrix for each expert.

Step 3. The TrFNNs-MABAC algorithm utilizes the TrFNNWA operator to fuses the TrFNNS decision-makiing matrix and the results show in Table 7.

Step 4. Based on Tables 7, the TrFNNs-MABAC algorithm enumerates the weight group TrFNNs decision-making matrix using Equation (3). The result is shown as Table 11.

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Table 11. The weighted group TrFNNs matrix using TrNNWA operator

Alternatives	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	< (0.217,0.278, 0.357, 0.476), (0, 0.836, 0.883, 0.952), (0, 0.836, 0.883, 0.952) >	< (0.069,0.109, 0.17, 0.217), (0, 0.593, 0.708, 0.783), (0, 0.614, 0.695, 0.794) >	< (0.062,0.107, 0.151, 0.201), (0, 0.647, 0.723, 0.813), (0, 0.647, 0.723, 0.813) >
$\mathcal{V}_2$	< (0.139,0.188, 0.245, 0.316), (0, 0.759, 0.815, 0.906), (0, 0.759, 0.815, 0.906) >	< (0.307,0.388, 0.507, 1), (0.805, 0.89, 0.933, 0.987), (0.805, 0.89, 0.933, 0.987) >	< (0.177,0.23, 0.293, 0.374), (0.7, 0.818, 0.865, 0.945), (0.7, 0.818, 0.865, 0.945) >
$\mathcal{V}_3$	< (0.068,0.11, 0.154, 0.203), (0, 0.662, 0.734, 0.832), (0, 0.662, 0.734, 0.832) >	< (0.081,0.119, 0.181, 0.229), (0, 0.618, 0.726, 0.809), (0, 0.639, 0.714, 0.82) >	< (0.026,0.058, 0.116, 0.149), (0, 0.559, 0.689, 0.748), (0, 0.586, 0.672, 0.762) >
$\mathcal{V}_4$	< (0.269,0.34, 0.441, 1), (0.812, 0.897, 0.935, 0.986), (0.812, 0.897, 0.935, 0.986) >	< (0.328,0.413, 0.538, 1), (0.826, 0.907, 0.948, 0.987), (0.826, 0.907, 0.948, 0.987) >	< (0.24,0.306, 0.389, 0.521), (0.771, 0.871, 0.913, 0.969), (0.771, 0.871, 0.913, 0.969) >

Step 5. From Table 7, the TrFNNs-MABAC algorithm establishes the BAA matrix using Equation (4). The BAA matrix can be seen as presented in Table 12.

Table 12. The TrFNNs BAA matrix

	Cr_1	Cr_2	Cr_3
BAA	< (0.455,0.566, 0.683, 1), (0, 0.467, 0.577, 0.764), (0, 0.467, 0.577, 0.764) >	< (0.475,0.587, 0.728, 1), (0, 0.428, 0.577, 0.715), (0, 0.448, 0.563, 0.728) >	< (0.354,0.463, 0.584, 0.709), (0, 0.349, 0.485, 0.636), (0, 0.362, 0.476, 0.645) >

Step 6. Utilizing Definition 6, Tables 11 and 12, and Equation (5), the TrFNNs-MABAC algorithm can enumerate the distance matrix between the group TrFNNs decision-making matrix and the BAA matrix, and Table 13 shows the result.

Table 13. The distance matrix between the matrix L and the matrix B

Alternative\Criterion	Cr_1	Cr_2	Cr_3
$\mathcal{V}_1$	0.3700	0.5037	0.3289
$\mathcal{V}_2$	0.4312	0.1110	0.0956
$\mathcal{V}_3$	0.4967	0.4892	0.3675
$\mathcal{V}_4$	0.1206	0.1054	0.0891

Step 7. Finally, the algorithm determines the final scores $\tilde{V}\left(V_i\right)$ by using Equation (6). Table 14 provides the final score results.

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Table 14. The final comprehensive score

	$\mathcal{V}_1$	$\mathcal{V}_2$	$\mathcal{V}_3$	$\mathcal{V}_4$
$\tilde{V}(V_i)$	1.2026	0.6377	1.3535	0.3152

Step 8. According to the results from Step 7, the algorithm can easily give a solution for the pharmaceutical company to precisely offer the best COVID-19 vaccine which is in which the alternatives' final score order is $\mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_4$.

4.3. Comparative Analysis of the Group Decision-Making Methods

As can be noticed from the above-demonstrated results and Table 15, the chosen alternative generated by the TrFNNWG and the improved MABAC methods is the same. Even the ranking order results provided by these two methods are also the same. However, the TrFNNWA has a slightly different in which the first and the second position of alternatives exchange with the results generated by the other two methods.

The first and second methods utilize TrFNNWA and TrFNNWG operators respectively to fuse the trapezoidal fuzzy neutrosophic (TrFN) information, and uses the functions of score and accuracy to generate the ranking order result. The score function value regarding to each alternative is not that much difference. The two aggregation operators not only recognize the importance of criteria but also consider the importance of criterion location order. Even it also ignores the highest and lowest scores and their impacts on the decision-making result. The third method integrates the MABAC method with TrFNNs in dealing with the MCGDM problem for the optimal solution selection. It focuses on evaluating the alternative through spreading the alternative position into the upper and lower approximation borders based on the alternative distance. Due to all positive distance values are located on the upper approximation border, an alternative has to have as many criteria of the alternative as possible to stuck on the upper approximation border. So, this alternative will be the preferred solution. These three methods have different perspectives in the decision-making under TrFN environment.

Table 15. The alternatives' ranking order based on TrFNNWA, TrFNNWG, and TrFNNs-MABAC methods

Methods	Ranking Order
TrFNNWA	$\mathcal{V}_1 \succ \mathcal{V}_3 \succ \mathcal{V}_2 \succ \mathcal{V}_4$
TrFNNWG	$\mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_4$
The proposed TrFNNs-MABAC	$\mathcal{V}_3 \succ \mathcal{V}_1 \succ \mathcal{V}_2 \succ \mathcal{V}_4$

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5. CONCLUSION AND FUTURE STUDIES

This chapter proposed a novel group decision-making method for the TrFN environment, which is an integration method between the original MABAC with the concept of TrFNNs. Firstly, the chapter recalled some basic definitions and related concepts, TrFNNs characteristics, and mathematical operations of TrFNNs, and discussed the functions of TrFNNs accuracy and score. Afterward, the chapter reviewed two existing TrFNNs aggregation operators and re-discussed the concepts of how to measure the alternative distance between two TrFNNs based on centroid points. Secondly, the chapter explained step by step the TrFNNs-MABAC method by integrating the traditional MABAC with TrFNNs and demonstrated in detail the computational steps of the improved TrFNNs-MABAC algorithm through an illustrative example for a COVID-19 vaccine selection. Eventually, the chapter provided a comparative analysis to emphasize the benefits of the improved method. The improved MABAC method confirmed functionality, reasonableness, effectiveness, and practicability for applications to MCGDM problems.

In recent years, the TrFNNs have become proper tools to overcome the impreciseness, uncertainty, and incompleteness of experts in real-life cases. However, this approach still has limitations, especially in performing its computational way when many decision-makers have to assess many criteria and alternatives. As a result, the complexity of its computational procedure would relatively be a little bit slow. The computation needs at least times where is the number of experts and is the number of alternatives to calculate the fusion value using the two aggregation TrFNNs operators. However, the calculation of how to determine the best alternative using the MABAC procedure is still fast and stable. Therefore, to overcome the limitation, this requires high computing performance technologies such as Hadoop data clustering (Gerakidis et al., 2021) and MapReduce (Bathla et al., 2018) to execute the overall aggregated value quickly in the process of MCGDM strategy.

The authors strongly recommend researchers implement this improved MABAC method as a computer code at some different platforms such as web, computer, and mobile in future studies. Other potential future works are to improve the traditional MABAC by using other neutrosophic approaches such as bipolar neutrosophic sets (Deli et al., 2015), possibility neutrosophic cubic sets (Xue et al., 2020), α-Discounting method (Smarandache, 2010, 2015), non-linear α-Discounting method (Smarandache, 2013a), and Interval α-Discounting method (Smarandache, 2013b).

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Chapter 31
Neutrosophic TOPSIS Method for Sustainable Supplier Selection in a Discount Market Chain

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ABSTRACT

Sustainable supplier selection is one of the most important decisions for companies in sustainable supply chain management. Therefore, sustainable supplier performance evaluation and selection are of great importance in terms of economic, environmental, and social aspects of sustainable development. Sustainable supplier selection problem can be considered as a multiple-criteria decision-making (MCDM) problem. In the MCDM problems, decision-makers evaluate conflicting criteria and alternatives according to their own preferences/judgments. One of the most well-known MCDM methods is the technique for order preference by similarity to an ideal solution (TOPSIS). Neutrosophic set (NS) is an extension of fuzzy set where each element of the universe has the degrees of truth, indeterminacy, and falsity. A single-valued neutrosophic set (SVNS) is a special case of the neutrosophic set. This chapter aims to evaluate sustainable supplier selection in a Turkish discount market chain using the single-valued neutrosophic TOPSIS method based on normalized Euclidean and Hamming distances.

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INTRODUCTION

Sustainability is one of the most vital elements that is integrated into the corporate strategy of many organizations. Developing models to guide decisions in achieving sustainability goals is one of the main challenges, as described in the Brundtland Commission report, which emphasizes the relationships between society, environment, and economic development. In this case, organizations will be increasingly dependent on the performance of their suppliers. Therefore, companies will not only have to monitor and evaluate the sustainability performance of their operations, but also disseminate this assessment to their suppliers and other stakeholders. Institutions that can protect their resources and realize their core competencies without harming the environment will be able to continue their strong sustainable activities against their competitors in the future.

In sustainable supply chain management, one of the most important strategic decisions of a firm regarding sustainability performance is the selection of sustainable suppliers and their performance evaluation (Chen et al., 2006; Chai et al., 2013; Luthra et al., 2017; Giannakis et al., 2020). Since sustainability has become a strategically central goal for many businesses, sustainable supplier selection can also be considered as a multi-criteria decision-making (MCDM) problem for them within the framework of strategic decision-making. Multi-criteria decision-making (MCDM) approaches such as AHP, ANP, SAW, VIKOR, TOPSIS, and many others can be easily applied to the sustainable supplier selection problem which contains many quantitative and/or qualitative criteria generally conflicting. In the MCDM methods, decision makers evaluate both criteria and alternatives according to their preferences/judgments. However, evaluations of decision makers cannot express by crisp (exact) numbers in real-life applications. In such cases, fuzzy sets have been adapted to MCDM methods in order to overcome this deficiency.

Fuzzy set (FS) theory is first proposed by Zadeh (1965) to deal with fuzzy information. In the FS, only one membership exists and it cannot express non-membership. Based on the FS, the intuitionistic fuzzy set (IFS) by adding a non-membership is introduced by Atanassov (1986). The FS and IFS can only handle incomplete information but not indeterminate information and inconsistent information. By adding an independent indeterminacy-membership based on the IFS, neutrosophic logic and neutrosophic sets (NSs) are first proposed by Smarandache (1998, 1999). The NS is a generalization of the IFS, and it is very helpful for decision makers to express their opinions more accurately in detail. A neutrosophic set is characterized by three independent degrees, i.e. truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F) is more capable to catch up on incomplete or unobtainable information. An important feature of NS is that every element of the universe has not only a certain degree of truth, but also a falsity degree and indeterminacy degree. Afterwards, Rivieccio (2008) pointed out that a NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity and it lies within $]0^-,1^+[$, the non-standard unit interval. Wang et al. (2005) presented interval neutrosophic sets (INSs) in which the truth-membership, indeterminacy-membership, and falsity membership are considered as interval numbers and demonstrated various properties of INSs.

In real engineering and scientific problems, direct application of the NS is difficult. To overcome this difficulty, single-valued neutrosophic set (SVNS) is proposed by Wang et al. (2010) which is a special case of the NS. The single-valued neutrosophic set is introduced as a generalization of the crisp sets, fuzzy sets, and intuitionistic fuzzy sets in order to handle the uncertainty, imprecise, incomplete, and inconsistent information. Due to the fact that SVNS can independently express truth-membership degree (T), indeterminacy-membership degree (I), falsity-membership degree (F), it can deal with incomplete,

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indeterminate, and inconsistent information. Later, Smarandache (2013) presented n-valued refined neutrosophic logic in which the truth value (T) is splited into types of sub-truths, $T_1,T_2,\ldots,T_p$, similarly indeterminacy value (I) is splited into types of sub-indeterminacies, $I_1,I_2,\ldots,I_p$, and the falsity value (F) is splited into sub-falsities, $F_1,F_2,\ldots,F_p$.

The aim of this study is to use the extension of TOPSIS method based on single-valued neutrosophic set (SVNS-TOPSIS) for a sustainable supplier selection problem in a well-known Turkish discount market chain. The essence of the TOPSIS, one of the most widely used MCDM methods, is to simultaneously select alternatives with the shortest distance to the positive ideal solution and the farthest distance to the negative ideal solution. In this context, the sustainable supplier selection problem is solved using the SVNS-TOPSIS method developed by Biswas et al. (2016). The originality of the SVNS-TOPSIS method, which uses only the Euclidean distance measure, is based on the solution of group multi-criteria decision making problems using information provided by experts in different fieldsHowever, the sustainable supplier selection problem discussed in this study is considered as a multi-criteria decision-making problem instead of a group multi-criteria decision-making problem, since it is possible for experts to decide with consensus. Therefore, the decision matrix based on the single-valued neutrophic set, which is formed as a result of the consensus of the experts, is taken into consideration in the evaluation of criteria and alternatives. In addition, in this study, it is suggested to use neutrosophic TOPSIS based not only on Euclidean distance but also on Hamming distance to eliminate the bias of TOPSIS.

The main contribution of this study is that the discussed SVNS-TOPSIS method can be easily applied to the sustainable supplier selection problem under different distance measures. Also, considered sustainable supplier selection problem is solved by the TOPSIS method with type-1 fuzzy and intuitionistic fuzzy extensions using normalized Euclidean and normalized Hamming distances in order to verify the stability and validity of the SVNS-TOPSIS method.

BACKGROUND

Sustainability is vital for the future of many organizations. In a sustainability focused supply chain, sustainable supplier selection is one of the most important strategic decisions for companies. Sustainable supplier performance evaluation and selection are of great importance in terms of sustainability issues that are economic, environmental, and social. Therefore, sustainable supplier selection problem can be considered as a multiple-criteria decision-making (MCDM) problem. In the supply chain management literature, many studies have applied various MCDM methods in sustainable supplier assessment and selection as a useful tool. Lu et al. (2007) used a multi-objective decision analysis to explore environmental principles applicable to green supplier assessment. Lee et al. (2009) used the fuzzy extended AHP method in a green supplier selection model, which takes into account the uncertainty of experts' views in evaluating the criteria. Bai and Sarkis (2010) used gray system theory and rough set theory to develop a sustainability framework for supplier selection processes. Büyüközkan and Çifçi (2011) proposed a fuzzy hybrid model that includes fuzzy DEMATEL, fuzzy ANP, and fuzzy PROMETHEE methods that take into account the shortcomings in individual judgments in the sustainable supplier selection problem. Erol et al. (2011) developed fuzzy entropy and fuzzy multi-criteria utility analysis for a sustainability performance model using an alert management system. Amindoust et al. (2012) developed a mathematical sustainable supplier ranking model, a fuzzy inference system, which takes into account the subjectivity of decision makers' perceptions. Govindan et al. (2013) applied the fuzzy TOPSIS method which

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uses triangular fuzzy numbers to measure the sustainability performance of a supplier based on a triple bottom line approach. Kannan et al. (2013) proposed an integrated fuzzy AHP / Multi-objective linear programming approach that assesses suppliers and allocates optimum order quantities according to various constraints in a green supply chain. Orji and Wei (2014) proposed a decision support tool including fuzzy DEMATEL and TOPSIS methods for sustainable supplier selection and applied it to a company for the manufacturing of gears in China. Chaharsooghi and Ashrafi (2014) presented an approach based on neofuzzy TOPSIS method for sustainable supplier performance evaluation and selection for the oil and petroleum industry in Iran. Govindan et al. (2015) conducted a systematic literature review to identify the most commonly used MCDM techniques in green supplier assessment. Hashemi et al. (2015) used the ANP / Gray Relational Analysis method in the selection of green suppliers which allows linguistic evaluations. Luthra et al. (2017) presented an evaluation framework using AHP and VIKOR methods to a sustainable supplier selection problem under various evaluation criteria. Mahdi Paydar et al. (2017) proposed an approach consisting of two main stages which combining the fuzzy MOORA method with failure mode and effects analysis (FMEA) in the first stage and mixed-integer linear programming in the second stage to sustainable supplier selection for the chemical industry in Iran. Zhou and Xu (2018) developed an integrated methodology including DEMATEL, ANP, and VIKOR algorithm and hybrid information aggregation for sustainable supplier selection and applied it to a retail enterprise which is a manufacturing and trading company in China. Lu et al. (2018) proposed an approach based on rough set theory and ELECTRE for sustainable supplier selection in a solar air conditioning manufacturer. Amindoust (2018) applied the weight restriction fuzzy DEA approach for a sustainable supplier selection of a company in the automotive industry. Sen et al. (2018) applied three MCDM approaches which are TOPSIS, MOORA, and GRA under the intuitionistic fuzzy environment for sustainable supplier selection. Rabbani et al. (2019) implemented a sustainable supplier selection model based on interval-valued fuzzy sets and possibilistic statistical reference point systems under uncertainty. Memari et al. (2019) suggested an approach based on the intuitionistic fuzzy TOPSIS method for sustainable supplier selection in the automotive spare part industry. Abdel-Baset et al. (2019) proposed an integrated MCDM approach including ANP and VIKOR methods under neutrosophic environment for sustainable supplier selection problem in importing field. Chen et al. (2019) proposed a rough fuzzy DEMATEL-ANP method to evaluate the sustainable value requirement of product service system. Li et al. (2019) proposed the rough cloud TOPSIS method to evaluate sustainable supplier selection in sustainable supply chain management practices. Bai et al. (2019) proposed a group decision support approach to select suppliers using social sustainability criteria. Stević et al. (2020) proposed the MARCOS method for sustainable supplier selection in the health care industry in Bosnia and Herzegovina.

In the last decades, many MCDM methods with extensions of fuzzy sets such as NS, INS, and SVNS have been presented in the literature as an effective way to solve decision-making problems. Chi and Liu (2013) proposed an extended TOPSIS method for interval neutrosophic set (INS) based MCDM problems in which the criteria weights are unknown and the criteria values take the form of INSs. The weights of criteria were determined based on the maximizing deviation method and alternatives were ranked using the INS-based TOPSIS method. Ye (2013) established the MCDM method using the correlation coefficient and weighted correlation coefficient of SVNSs and testified that the cosine similarity degree is a special case of the correlation coefficient in SVNS. Biswas et al. (2014a) extended the grey relational analysis (GRA) method for solving MCDM problems with SVNS. Biswas et al. (2014b) proposed a modified GRA method to determine ranking the alternatives for an MCDM problem with SVNSs. Ye (2014a) defined the Hamming and Euclidean distances between interval neutrosophic sets

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(INSs) for an MCDM method on the basis of similarity degree of INS. Ye (2014b) proposed an MCDM method on the basis of the single-valued neutrosophic cross entropy which is constituted in which ratings of the alternatives with respect to criteria are considered as SVNSs. Majumdar and Samant (2014) presented the notion of distance between two SVNSs, defined several similarity measures between them, investigated their characteristics, and also introduced a measure of the entropy of SVNS. Liu and Wang (2014) proposed a single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM) operator on the basis of Bonferroni mean, the weighted Bonferroni mean (WBM), and the normalized WBM. In this study, they also developed an approach to solve MCDM problems with SVNNs based on the SVNNWBM operator. Zhang et al. (2014) defined the interval neutrosophic weighted averaging (INWA) operator and the interval neutrosophic weighted geometric (INWG) operator and applied these operators to solve the multi-criteria group decision-making problems. Baušys et al. (2015) presented an extension of the COPRAS method on the basis of SVNS and applied it to a location selection problem for a liquefied natural gas terminal. Broumi et al. (2015) presented an approach for group multi-criteria decision making on the basis of n-valued interval neutrosophic sets and applied it to medical diagnosis problem. Şahin and Yiğider (2016) applied the TOPSIS method based on SVNSs to a multi-criteria group decision-making problem using single-valued neutrosophic weighted averaging (SVNWA) operator to aggregate individual evaluations of decision makers for a supplier selection problem. Biswas et al. (2016) proposed an extended TOPSIS method under SVNSs for multi-criteria group decision-making problems and used an SVNS-based weighted averaging operator to aggregate individual evaluations of decision makers. Zhang et al. (2016) proposed an outranking approach for MCDM problems with interval neutrosophic sets based on ELECTRE IV to select the best investment company. Şahin and Liu (2016) developed a maximizing deviation method for solving the MCDM problems based on the SVNS or INSs with incomplete weight information. Garg and Nancy (2018) presented logarithm operational laws with real number base λ for the single-valued neutrosophic numbers (SVNNs) and developed different weighted averaging and geometric aggregation operators to present an MCDM method. Sodenkamp et al. (2018) designed an algorithm on the basis of SVNSs to aggregate decision makers' judgments in multi-criteria group decision-making. Wang et al. (2019) constructed an MCDM method based on similarity measure considering interval neutrosophic sets (INSs) and introduced an improved cosine similarity measure between interval neutrosophic numbers for solving supplier selection problem. Luo et al. (2019) presented a new tangent similarity between SVNSs and a best-worst MCDM method based on the SVNSs is proposed. Nancy and Garg (2019) presented a TOPSIS Method based on improved divergence measure under SVNS environment.

SUSTAINABLE SUPPLY CHAIN MANAGEMENT

Sustainable supply chain management is defined by Seuring and Müller (2008) as the management of material, information, and capital flow obtained from customer and stakeholder requirements among companies throughout the supply chain, considering sustainable development dimensions, namely economic, environmental and social. These three dimensions taking into account various sub-dimensions depending on the nature of the problem have been frequently used in supplier sustainability performance evaluation. Another definition of sustainable supply chain management by Carter and Easton (2011) is the integration and realization of the companies' objectives based on economic, environmental, and social perspectives in the coordination of critical business processes to enhance their economic performance

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in the long term. Sustainable supply chain management is of interest to practitioners and researchers owing to increasing awareness of environmental and social issues recently (Yu et al., 2019). As suppliers must deliver the desired amount of quality material and raw materials on time, supplier selection is of great importance in supply chain management (Stević et al., 2020). Accordingly, the sustainable supplier selection problem is also of great importance in sustainable supply chain management.

Organizations have usually evaluated supplier performance regarding several criteria such as price, quality, flexibility, etc. Recently, due to environmental and social pressures, the purchasing process has become more complex, as sustainability factors have a crucial role in the long-term success of a supply chain. For this reason, many organizations have also taken into account three sustainability dimensions, namely economic, social, and environmental in order to measure the sustainability performance of suppliers (Govindan et al. 2013).

The economic dimension intends to maximize the income flow that can be produced with minimum assets or capital stock. The social dimension refers to social problems including human rights, education, and training. On the other hand, the environmental dimension concerns controlling the depletion and pollution generation of renewable and non-renewable resources. The most commonly used evaluation criteria for the economic, social, and environmental dimensions of sustainability in the literature can be summarized as follows (Govindan et al., 2013; Luthra et al., 2017; Gören, 2018; Yu et al., 2019):

Economic: Price/Cost, Quality, Productivity, Delivery reliability, Technology Capability, Service, Flexibility, the capacity of the supplier, lead time, Responsiveness, Long-term relationship - Continuity

Social: Occupational health and safety, Information disclosure, Employee right and welfare, Employment practices, Contractual stakeholders influence

Environmental: Pollution production, Resource consumption, Eco-design, Environmental management system, 3R (Reduce, Re-use, Recycle).

SINGLE-VALUED NEUTROSOPHIC SETS

Neutrosophic set (NS) is a part of neutrosophy which is a new branch of philosophy and studies the origin, nature, and scope of neutralities, besides their interactions with different ideational spectra. (Smarandache, 1999). Let X be a universe of discourse with an element of X denoted by x. A neutrosophic set $\mathcal{N} \subset X$, is characterized by a truth-membership function $T_{\mathcal{N}}\left(x\right)$, an indeterminacy-membership function $I_{\mathcal{N}}\left(x\right)$, and a falsity-membership function $F_{\mathcal{N}}\left(x\right)$ with

$$T_{\mathcal{N}}\left(x\right), I_{\mathcal{N}}\left(x\right), F_{\mathcal{N}}\left(x\right) \in \left[{}^{-}0,\ 1^{+}\right]$$

for $\forall x \in X$ The relation ${}^{-}0 \leq T_{\mathcal{N}}\left(x\right) + I_{\mathcal{N}}\left(x\right) + F_{\mathcal{N}}\left(x\right) \leq 3^{+}$ for $\forall x \in X$, is satisfied for a neutrosophic set $\mathcal{N}$.

Single-valued neutrosophic set (SVNS) is a special case of neutrosophic set. It can be used in real scientific and engineering applications. In this section, some of the basic definitions, operations, and properties are presented of SVNSs (Wang et al., 2010). Let the element of X be denoted by x, where X is a universe of discourse. A single-valued neutrosophic set $\tilde{\mathcal{N}} \subset X$, is defined by

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$$\tilde{\mathcal{N}} = \left\{\left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)\right\rangle,\ x \in X\right\} \tag{1}$$

where; $T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \in [0,1]$ with the relation $T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \in [0,1]$ for $\forall x \in X$. $T_{\tilde{\mathcal{N}}}(x)$ is truth-membership function, $I_{\tilde{\mathcal{N}}}(x)$ is an indeterminacy-membership function, and $F_{\tilde{\mathcal{N}}}(x)$ is falsity-membership function. A single-valued neutrosophic set $\tilde{\mathcal{N}}$ is written as

$$\tilde{\mathcal{N}} = \int_x \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}\right\rangle \Big| x,\ \forall x \in X$$

for continuous *X* and is written as $\tilde{\mathcal{N}} = \sum_x \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}\right\rangle \Big| x,\ \forall x \in X$, for discrete *X*.

Set of Operations for Single-Valued Neutrosophic Sets

Let $\tilde{A} = \left\langle T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\right\rangle$ and $\tilde{B} = \left\langle T_{\tilde{B}}(x), I_{\tilde{B}}(x), F_{\tilde{B}}(x)\right\rangle$ be any two SVNSs. The set of operations for SVNSs are defined for all $\forall x \in X$, as follows (Wang et al., 2010; Liu and Wang, 2014):

Summation: $\tilde{A} \oplus \tilde{B} = \left\langle T_{\tilde{A}}(x) + T_{\tilde{B}}(x) - T_{\tilde{A}}(x).T_{\tilde{B}}(x), I_{\tilde{A}}(x).I_{\tilde{B}}(x), F_{\tilde{A}}(x).F_{\tilde{B}}(x)\right\rangle$ (2)

Multiplication:
$\tilde{A} \otimes \tilde{B} = \left\langle T_{\tilde{A}}(x).T_{\tilde{B}}(x), I_{\tilde{A}}(x) + I_{\tilde{B}}(x) - I_{\tilde{A}}(x).I_{\tilde{B}}(x), F_{\tilde{A}}(x) + F_{\tilde{B}}(x) - F_{\tilde{A}}(x).F_{\tilde{B}}(x)\right\rangle$ (3)

Union: $\tilde{A} \cup \tilde{B} = \left\langle \max\left(T_{\tilde{A}}(x), T_{\tilde{B}}(x)\right), \min\left(I_{\tilde{A}}(x), I_{\tilde{B}}(x)\right), \min\left(F_{\tilde{A}}(x), F_{\tilde{B}}(x)\right)\right\rangle$ (4)

Intersection: $\tilde{A} \cap \tilde{B} = \left\langle \min\left(T_{\tilde{A}}(x), T_{\tilde{B}}(x)\right), \max\left(I_{\tilde{A}}(x), I_{\tilde{B}}(x)\right), \max\left(F_{\tilde{A}}(x), F_{\tilde{B}}(x)\right)\right\rangle$ (5)

Inclusion: $\tilde{A} \subseteq \tilde{B}$ if and only if $T_{\tilde{A}}(x) \le T_{\tilde{B}}(x), I_{\tilde{A}}(x) \ge I_{\tilde{B}}(x), F_{\tilde{A}}(x) \ge F_{\tilde{B}}(x)$ (6)

Equity: $\tilde{A} = \tilde{B}$ if and only if $\tilde{A} \subseteq \tilde{B}$ *and* $\tilde{B} \subseteq \tilde{A}$ (7)

Complement: $\tilde{A}^c = \left\{\left(x_1 \left| \left\langle F_{\tilde{A}}(x), 1 - I_{\tilde{A}}(x), T_{\tilde{A}}(x)\right\rangle\right.\right) \Big| x \in X\right\}$ (8)

Distance Between Two SVNSs

The Euclidean, normalized Euclidean, Hamming, and normalized Hamming distances of neutrosophic sets are calculated as follows (Majumdar and Samanta, 2014; Biswas et al. 2016).

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Let assume that $\tilde{A}$ and $\tilde{B}$ be the two SVNSs for $x_i \in X$, $i=1,2,\ldots,n$.

$$\tilde{A} = \left\{\left\langle T_{\tilde{A}}(x_1), I_{\tilde{A}}(x_1), F_{\tilde{A}}(x_1)\right\rangle, \left\langle T_{\tilde{A}}(x_2), I_{\tilde{A}}(x_2), F_{\tilde{A}}(x_2)\right\rangle, \ldots, \left\langle T_{\tilde{A}}(x_n), I_{\tilde{A}}(x_n), F_{\tilde{A}}(x_n)\right\rangle\right\}$$

$$\tilde{B} = \left\{\left\langle T_{\tilde{B}}(x_1), I_{\tilde{B}}(x_1), F_{\tilde{B}}(x_1)\right\rangle, \left\langle T_{\tilde{B}}(x_2), I_{\tilde{B}}(x_2), F_{\tilde{B}}(x_2)\right\rangle, \ldots, \left\langle T_{\tilde{B}}(x_n), I_{\tilde{B}}(x_n), F_{\tilde{B}}(x_n)\right\rangle\right\}$$

The Euclidean distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$D_{Eucl}\left(\tilde{A},\tilde{B}\right) = \sqrt{\sum_{i=1}^{n}\left\{\left(T_{\tilde{A}}(x_i) - T_{\tilde{B}}(x_i)\right)^2 + \left(I_{\tilde{A}}(x_i) - I_{\tilde{B}}(x_i)\right)^2 + \left(F_{\tilde{A}}(x_i) - F_{\tilde{B}}(x_i)\right)^2\right\}} \tag{9}$$

The normalized Euclidean distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$D_{NEucl}\left(\tilde{A},\tilde{B}\right) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}\left\{\left(T_{\tilde{A}}(x_i) - T_{\tilde{B}}(x_i)\right)^2 + \left(I_{\tilde{A}}(x_i) - I_{\tilde{B}}(x_i)\right)^2 + \left(F_{\tilde{A}}(x_i) - F_{\tilde{B}}(x_i)\right)^2\right\}} \tag{10}$$

The Hamming distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$D_{Hamm}\left(\tilde{A},\tilde{B}\right) = \sum_{i=1}^{n}\left\{\left|T_{\tilde{A}}(x_i) - T_{\tilde{B}}(x_i)\right| + \left|I_{\tilde{A}}(x_i) - I_{\tilde{B}}(x_i)\right| + \left|F_{\tilde{A}}(x_i) - F_{\tilde{B}}(x_i)\right|\right\} \tag{11}$$

The normalized Hamming distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$D_{NHamm}\left(\tilde{A},\tilde{B}\right) = \frac{1}{3n}\sum_{i=1}^{n}\left\{\left|T_{\tilde{A}}(x_i) - T_{\tilde{B}}(x_i)\right| + \left|I_{\tilde{A}}(x_i) - I_{\tilde{B}}(x_i)\right| + \left|F_{\tilde{A}}(x_i) - F_{\tilde{B}}(x_i)\right|\right\} \tag{12}$$

TOPSIS METHOD BASED ON SINGLE-VALUED NEUTROSOPHIC SETS

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) introduced by Hwang and Yoon (1981) is one of the most well-known MCDM techniques. In the method, a closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distances to both the positive-ideal solution (PIS) and negative-ideal solution (NIS) simultaneously. The basis of the TOPSIS method is that the chosen alternative should have the shortest distance to the PIS and the farthest distance to the NIS.

The main steps of the TOPSIS method based on single-valued neutrosophic sets are given in the following (Biswas et al., 2016):

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Step 1. Construction of the Decision Matrix with SVNSs

Decision matrix with SVNSs, $D_{\tilde{N}} = \left\langle d_{ij} \right\rangle_{m\times n} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m\times n}$, is constructed as follows:

$$D_{\tilde{N}} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & & C_n \end{array} \\ \begin{bmatrix} \left\langle T_{11}, I_{11}, F_{11} \right\rangle & \left\langle T_{12}, I_{12}, F_{12} \right\rangle & \cdots & \left\langle T_{1n}, I_{1n}, F_{1n} \right\rangle \\ \left\langle T_{21}, I_{21}, F_{21} \right\rangle & \left\langle T_{22}, I_{22}, F_{22} \right\rangle & \cdots & \left\langle T_{2n}, I_{2n}, F_{2n} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle T_{m1}, I_{m1}, F_{m1} \right\rangle & \left\langle T_{m2}, I_{m2}, F_{m2} \right\rangle & \cdots & \left\langle T_{mn}, I_{mn}, F_{mn} \right\rangle \end{bmatrix} \end{array} \tag{13}$$

where: T_{ij}, I_{ij} and F_{ij} denote the degree of truth-membership value, indeterminacy-membership value and falsity-membership value of alternative A_i with respect to criterion C_j, respectively which satisfies the following properties:

i) $0 \le T_{ij} \le 1;\ \ 0 \le I_{ij} \le 1;\ \ 0 \le F_{ij} \le 1.$

ii) $0 \le T_{ij} + I_{ij} + F_{ij} \le 3 \quad$ for $\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m.$

Step 2. Determination of the Criteria Weights

The weight vector of the criteria is defined by

$$W = \left[\left\langle w_1 \right\rangle, \left\langle w_2 \right\rangle, ..., \left\langle w_n \right\rangle\right] \tag{14}$$

For criterion C_j, weight $\left\langle w_j \right\rangle = \left\langle T_{w_j}, I_{w_j}, F_{w_j} \right\rangle$, is considered as the single-valued neutrosophic number.

Step 3. The Weighted Decision Matrix with SVNSs

The weighted decision matrix with SVNSs, $D^w = D_{\tilde{N}} \otimes W = \left\langle d_{ij}^{w_j} \right\rangle_{m\times n}$, is obtained by using the multiplication of two neutrosophic sets as follows:

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$$D^{w} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & & C_n \end{array} \\ \begin{bmatrix} \left\langle T_{11}^{w_1}, I_{11}^{w_1}, F_{11}^{w_1} \right\rangle & \left\langle T_{12}^{w_2}, I_{12}^{w_2}, F_{12}^{w_2} \right\rangle & \cdots & \left\langle T_{1n}^{w_n}, I_{1n}^{w_n}, F_{1n}^{w_n} \right\rangle \\ \left\langle T_{21}^{w_1}, I_{21}^{w_1}, F_{21}^{w_1} \right\rangle & \left\langle T_{22}^{w_2}, I_{22}^{w_2}, F_{22}^{w_2} \right\rangle & \cdots & \left\langle T_{2n}^{w_n}, I_{2n}^{w_n}, F_{2n}^{w_n} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle T_{m1}^{w_1}, I_{m1}^{w_1}, F_{m1}^{w_1} \right\rangle & \left\langle T_{m2}^{w_2}, I_{m2}^{w_2}, F_{m2}^{w_2} \right\rangle & \cdots & \left\langle T_{mn}^{w_n}, I_{mn}^{w_n}, F_{mn}^{w_n} \right\rangle \end{bmatrix} \end{array} \tag{15}$$

where;

$$d_{ij}^{w} = \left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle = \left\langle T_{ij}.T_{w_j}, I_{ij} + I_{w_j} - I_{ij}.I_{w_j}, F_{ij} + F_{w_j} - F_{ij}.F_{w_j} \right\rangle$$

Step 4. Determination of the Relative Neutrosophic Positive Ideal Solution (RNPIS) and the Relative Neutrosophic Negative Ideal Solution (RNNIS) for SVNSs

The relative neutrosophic positive ideal solution (RNPIS) indicated by $Q_{\tilde{N}}^{+} = \left[d_1^{w+}, d_2^{w+}, \ldots, d_n^{w+}\right]$ and the relative neutrosophic negative ideal solution (RNNIS) indicated by $Q_{\tilde{N}}^{-} = \left[d_1^{w-}, d_2^{w-}, \ldots, d_n^{w-}\right]$ are determined considering benefit-type (*B*) criteria and cost-type (*C*) criteria.

$d_j^{w+} = \left\langle T_j^{w+}, I_j^{w+}, F_j^{w+} \right\rangle \; j = 1, 2, \ldots, n$ are defined by following,

$$T_j^{w+} = \left\{ \left(\max_i \left\{ T_{ij}^{w_j} \right\} \middle| \, j \in B \right), \left(\min_i \left\{ T_{ij}^{w_j} \right\} \middle| \, j \in C \right) \right\} \tag{16}$$

$$I_j^{w+} = \left\{ \left(\min_i \left\{ I_{ij}^{w_j} \right\} \middle| \, j \in B \right), \left(\max_i \left\{ I_{ij}^{w_j} \right\} \middle| \, j \in C \right) \right\} \tag{17}$$

$$F_j^{w+} = \left\{ \left(\min_i \left\{ F_{ij}^{w_j} \right\} \middle| \, j \in B \right), \left(\max_i \left\{ F_{ij}^{w_j} \right\} \middle| \, j \in C \right) \right\} \tag{18}$$

and $d_j^{w-} = \left\langle T_j^{w-}, I_j^{w-}, F_j^{w-} \right\rangle, \;\; j = 1, 2, \ldots, n$ are defined by following

$$T_j^{w-} = \left\{ \left(\min_i \left\{ T_{ij}^{w_j} \right\} \middle| \, j \in B \right), \left(\max_i \left\{ T_{ij}^{w_j} \right\} \middle| \, j \in C \right) \right\} \tag{19}$$

$$I_j^{w-} = \left\{ \left(\max_i \left\{ I_{ij}^{w_j} \right\} \middle| \, j \in B \right), \left(\min_i \left\{ I_{ij}^{w_j} \right\} \middle| \, j \in C \right) \right\} \tag{20}$$

$$F_j^{w-} = \left\{ \left(\max_i \left\{ F_{ij}^{w_j} \right\} \middle| \, j \in B \right), \left(\min_i \left\{ F_{ij}^{w_j} \right\} \middle| \, j \in C \right) \right\} \tag{21}$$

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Step 5. Determination of the Distance Measure of Each Alternative from the RNPIS and the RNNIS for SVNSs

The normalized Euclidean distance and normalized Hamming distance between

$$d_{ij}^{w_j} = \left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle \text{ and } d_j^{w+} = \left\langle T_j^{w+}, I_j^{w+}, F_j^{w+} \right\rangle$$

are computed for each alternative by Eq (22) and Eq (23), respectively:

$$D_{NEucl}^{i+}\left(d_{ij}^{w_j}, d_j^{w+}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\left(T_{ij}^{wj}(x_j) - T_j^{w+}(x_j)\right)^2 + \left(I_{ij}^{wj}(x_j) - I_j^{w+}(x_j)\right)^2 + \left(F_{ij}^{wj}(x_j) - F_j^{w+}(x_j)\right)^2\right\}} \tag{22}$$

$$D_{NHamm}^{i+}\left(d_{ij}^{w_j}, d_j^{w+}\right) = \frac{1}{3n}\sum_{j=1}^{n}\left\{\left|T_{ij}^{wj}(x_j) - T_j^{w+}(x_j)\right| + \left|I_{ij}^{wj}(x_j) - I_j^{w+}(x_j)\right| + \left|F_{ij}^{wj}(x_j) - F_j^{w+}(x_j)\right|\right\} \tag{23}$$

Similarly, the normalized Euclidean distance and normalized Hamming distance between

$$d_{ij}^{w_j} = \left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle \text{ and } d_j^{w-} = \left\langle T_j^{w-}, I_j^{w-}, F_j^{w-} \right\rangle$$

are computed for each alternative by Eq (24) and Eq (25), respectively:

$$D_{NEucl}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\left(T_{ij}^{wj}(x_j) - T_j^{w-}(x_j)\right)^2 + \left(I_{ij}^{wj}(x_j) - I_j^{w-}(x_j)\right)^2 + \left(F_{ij}^{wj}(x_j) - F_j^{w-}(x_j)\right)^2\right\}} \tag{24}$$

$$D_{NHamm}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right) = \frac{1}{3n}\sum_{j=1}^{n}\left\{\left|T_{ij}^{wj}(x_j) - T_j^{w-}(x_j)\right| + \left|I_{ij}^{wj}(x_j) - I_j^{w-}(x_j)\right| + \left|F_{ij}^{wj}(x_j) - F_j^{w-}(x_j)\right|\right\} \tag{25}$$

Step 6. Determination of the Relative Closeness Coefficient to the Neutrosophic Ideal Solution for SVNSs

The relative closeness coefficient of each alternative is calculated as follows:

$$C_i^* = \frac{D_{Eu}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right)}{D_{Eu}^{i+}\left(d_{ij}^{w_j}, d_j^{w+}\right) + D_{Eu}^{i-}\left(d_{ij}^{w_j}, d_j^{w-}\right)}, 0 \le C_i^* \le 1 \tag{26}$$

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Step 7. Ranking the Alternatives

The alternatives are ranked in descending order of the C_i^* index. Alternative with the higher C_i^* is chosen as the best alternative.

NUMERICAL EXAMPLE

To demonstrate the applicability of the presented methodology of the study, the subject matter addressed as an example in this section is the problem of sustainable supplier selection in a well-known Turkish discount market chain.

The company, which is the widespread retail industry chain, operates in 81 provinces and all districts of Turkey. The company, which opened more than ten thousand markets by the end of 2020, continues to grow. The company, which is 100% domestic capital, is among the largest market chains in the world. In addition to being the most widespread chain in Turkey, it is among the leading companies in the retail sector with its large number of employees and many suppliers. The company chain market, which is among the 15 largest companies in Turkey, offers its customers quality food and consumption materials at the cheapest prices. Building on trust with all its stakeholders, especially its customers, employees and suppliers, with its 12 years of experience, the company continues to be the most widespread chain of the retail industry with its innovative and dynamic structure.

The company is the biggest customer of most of Turkey's largest suppliers. Among many suppliers, four sustainable suppliers are chosen by three purchasing managers (i.e. decision makers/experts). These four suppliers are chosen because they provide qualified products that meet the customers' expectations, especially by these suppliers. Three purchasing managers as a team are asked to rate the importance of nine sustainability criteria by Alikhani et al. (2019) shown in Table 1. Then, the experts are also asked to rate the four candidate suppliers concerning these sustainability criteria using linguistic evaluations.

The experts evaluate each supplier based on nine sustainability criteria using the linguistic terms shown in Table 2: "Very Good" (VG), "Good" (G), "Fair" (F), "Bad" (B), "Very Bad" (VB) and their corresponding single-valued neutrosophic numbers (SVNNs). The experts also evaluate the importance weight of each sustainability criterion by using SVNNs shown in Table 2.

Step 1. Construction of the Single-Valued Neutrosophic Decision Matrix

Experts evaluate criteria and suppliers by using the linguistic terms as depicted in Table 3. Then, linguistic terms are converted into SVNNs to construct a neutrosophic decision matrix shown in Table 4.

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Table 1. Sustainability criteria and their descriptions (Alikhani et al., 2019)

Criteria (Abbrevation)	Description
Cost/Price (C/P)	The various sources of cost mainly include the purchase cost of products, transportation cost, inventory cost, maintenance cost, buyer-supplier coordination, and controlling cost.
Quality (Q)	Quality products provision, consistent conformance to required specifications settled by the buyer, and quality stability in a long-term cooperation.
Technology Capabilities (TC)	The capability of suppliers to adjust themselves towards innovations, adopt advanced process technologies and react to technological turbulence promptly.
Green Design (GD)	The design of products in which issues such as reuse, recycle, recovery, reduced consumption of material and energy, and avoiding the use of hazardous materials are taken into consideration.
Environmental Management System (EMS)	Environmental standards and certifications like ISO 14000, planning, performing, monitoring, and controlling of environmental activities, and staff training for environmental objectives.
Environmental Competencies (EC)	Using cleaner production systems; Ability to change and amend manufacturing process into green and environmentally friendly products.
Interests and Rights of Employees (IRE)	This includes workers' pay, benefits and safe working condition, forced labor, working hours, compensation, and disciplinary practices.
Rights of Stakeholders (RS)	Stakeholder rights, including informing them about business activities, hearing them, identify and satisfy their requirements and respect them.
Social Management Commitment (SMC)	The commitment ensuring compliance and continuous improvement in delivering important social aspects such as child labor, health, safety, discrimination and etc.

Table 2. Linguistic terms for evaluation of sustainability criteria and suppliers with SVNNs

Linguistic terms	SVNNs
Very good/very important (VG/VI)	<0.90, 0.10, 0.10>
Good/important (G/I)	<0.80, 0.20, 0.15>
Fair/medium (F/M)	<0.50, 0.40, 0.45>
Bad/unimportant (B/UI)	<0.35, 0.60, 0.70>
Very bad/very unimportant (VB/VUI)	<0.10, 0.80, 0.90>

Table 3. Evaluation of alternatives and criteria weights given by experts

Suppliers	C/P	Q	TC	GD	EMS	EC	IRE	RS	SMC
S_1	VG	VG	G	B	VG	F	B	G	B
S_2	F	VG	VG	VG	VG	VG	G	G	G
S_3	G	G	F	F	F	F	B	G	B
S_4	F	G	VG	VG	VG	VG	G	G	G
Weights	VI	VI	M	M	M	M	VUI	UI	VUI

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Table 4. The neutrosophic decision matrix

Suppliers	***C/P***	***Q***	***TC***
S_1	<0.90, 0.10, 0.10>	<0.90, 0.10, 0.10>	<0.80, 0.20, 0.15>
S_2	<0.50, 0.40, 0.45>	<0.90, 0.10, 0.10>	<0.90, 0.10, 0.10>
S_3	<0.80, 0.20, 0.15>	<0.80, 0.20, 0.15>	<0.50, 0.40, 0.45>
S_4	<0.50, 0.40, 0.45>	<0.80, 0.20, 0.15>	<0.90, 0.10, 0.10>
Suppliers	***GD***	***EMS***	***EC***
S_1	<0.35, 0.60, 0.70>	<0.90, 0.10, 0.10>	<0.50, 0.40, 0.45>
S_2	<0.90, 0.10, 0.10>	<0.90, 0.10, 0.10>	<0.90, 0.10, 0.10>
S_3	<0.50, 0.40, 0.45>	<0.50, 0.40, 0.45>	<0.50, 0.40, 0.45>
S_4	<0.90, 0.10, 0.10>	<0.90, 0.10, 0.10>	<0.90, 0.10, 0.10>
Suppliers	***IR***	***RS***	***SMC***
S_1	<0.35, 0.60, 0.70>	<0.80, 0.20, 0.15>	<0.35, 0.60, 0.70>
S_2	<0.80, 0.20, 0.15>	<0.80, 0.20, 0.15>	<0.80, 0.20, 0.15>
S_3	<0.35, 0.60, 0.70>	<0.80, 0.20, 0.15>	<0.35, 0.60, 0.70>
S_4	<0.80, 0.20, 0.15>	<0.80, 0.20, 0.15>	<0.80, 0.20, 0.15>

Step 2. Determination of the Weights of Criteria

The criteria weight vector is determined by converting the linguistic term of each criterion given in the last row of Table 3 into its corresponding SVNN and illustrated as follows:

$$W = \begin{bmatrix} \langle 0.90, 0.10, 0.10 \rangle & \langle 0.90, 0.10, 0.10 \rangle & \langle 0.50, 0.40, 0.45 \rangle \\ \langle 0.50, 0.40, 0.45 \rangle & \langle 0.50, 0.40, 0.45 \rangle & \langle 0.50, 0.40, 0.45 \rangle \\ \langle 0.10, 0.80, 0.90 \rangle & \langle 0.35, 0.60, 0.70 \rangle & \langle 0.10, 0.80, 0.90 \rangle \end{bmatrix}$$

Step 3. The Weighted Single-Valued Neutrosophic Decision Matrix

After obtaining the ratings of criteria and suppliers, the weighted neutrosophic decision matrix shown in Table 5 is formed. For example, the element of the weighted decision matrix for the supplier S_1 with respect to criterion *C/P* is determined by using Eq. (15) which is the multiplication of two SVNNs, as follows:

$$\langle T_{11}^w, I_{11}^w, F_{11}^w \rangle =$$
$$\langle 0.90 \times 0.90, 0.10 + 0.10 - 0.10 \times 0.10, 0.10 + 0.10 - 0.10 \times 0.10 \rangle = \langle 0.81, 0.19, 0.19 \rangle$$

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Table 5. The weighted neutrosophic decision matrix

Suppliers	***C/P***	***Q***	***TC***
S_1	<0.81, 0.19, 0.19>	<0.81, 0.19, 0.19>	<0.40, 0.52, 0.53>
S_2	<0.45, 0.46, 0.51>	<0.81, 0.19, 0.19>	<0.45, 0.46, 0.51>
S_3	<0.72, 0.28, 0.24>	<0.72, 0.28, 0.24>	<0.25, 0.64, 0.70>
S_4	<0.45, 0.55, 0.51>	<0.72, 0.28, 0.24>	<0.45, 0.46, 0.51>
Suppliers	***GD***	***EMS***	***EC***
S_1	<0.18, 0.76, 0.84>	<0.45, 0.46, 0.51>	<0.25, 0.64, 0.70>
S_2	<0.45, 0.46, 0.51>	<0.45, 0.46, 0.51>	<0.45, 0.46, 0.51>
S_3	<0.25, 0.64, 0.70>	<0.25, 0.64, 0.70>	<0.25, 0.64, 0.70>
S_4	<0.45, 0.46, 0.51>	<0.45, 0.46, 0.51>	<0.45, 0.46, 0.51>
Suppliers	***IR***	***RS***	***SMC***
S_1	<0.04, 0.92, 0.97>	<0.28, 0.68, 0.75>	<0.04, 0.92, 0.97>
S_2	<0.08, 0.84, 0.92>	<0.28, 0.68, 0.75>	<0.08, 0.84, 0.92>
S_3	<0.04, 0.92, 0.97>	<0.28, 0.68, 0.75>	<0.04, 0.92, 0.97>
S_4	<0.08, 0.84, 0.92>	<0.28, 0.68, 0.75>	<0.08, 0.84, 0.92>

Step 4. Determination of the Relative Neutrosophic Positive Ideal Solution (RNPIS) and the Relative Neutrosophic Negative Ideal Solution (RNNIS)

The RNPIS is calculated from the weighted decision matrix on the basis of types of criteria, i.e., benefit type or cost type by using Eqs. (16-18) as

$$Q_{\tilde{N}}^{+}=\begin{bmatrix}\langle 0.45,0.55,0.51\rangle & \langle 0.81,0.19,0.19\rangle & \langle 0.45,0.46,0.51\rangle \\ \langle 0.45,0.46,0.51\rangle & \langle 0.45,0.46,0.51\rangle & \langle 0.45,0.46,0.51\rangle \\ \langle 0.08,0.84,0.92\rangle & \langle 0.28,0.68,0.75\rangle & \langle 0.08,0.84,0.92\rangle\end{bmatrix}$$

where, for instance, $d_1^{w+}=\left\langle T_1^{w+},I_1^{w+},F_1^{w+}\right\rangle=\left\langle 0.45,0.55,0.51\right\rangle$ is calculated for criterion *C/P* as follows:

$$T_1^{w+}=\min\left\{0.81,0.45,0.72,0.45\right\}=0.45$$

$$I_1^{w+}=\max\left\{0.19,0.46,0.28,0.55\right\}=0.55$$

$$F_1^{w+}=\max\left\{0.19,0.51,0.24,0.51\right\}=0.51$$

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In a like manner, the RNNIS is calculated from the weighted decision matrix on the basis of types of criteria, i.e., benefit type or cost type by using Eqs. (19-21) as

$$Q_{\tilde{N}}^{-}=\begin{bmatrix}\langle 0.81,0.19,0.19\rangle & \langle 0.72,0.28,0.24\rangle & \langle 0.25,0.64,0.70\rangle\\ \langle 0.18,0.76,0.84\rangle & \langle 0.25,0.64,0.70\rangle & \langle 0.25,0.64,0.70\rangle\\ \langle 0.04,0.92,0.97\rangle & \langle 0.28,0.68,0.75\rangle & \langle 0.04,0.92,0.97\rangle\end{bmatrix}$$

where, for instance, $d_1^{w-}=\left\langle T_1^{w-},I_1^{w-},F_1^{w-}\right\rangle=\langle 0.81,0.19,0.19\rangle$ is calculated for criterion *C/P* as follows:

$$T_1^{w-}=\max\left\{0.81,0.45,0.72,0.45\right\}=0.81$$

$$I_1^{w-}=\min\left\{0.19,0.46,0.28,0.55\right\}=0.19$$

$$F_1^{w-}=\min\left\{0.19,0.51,0.24,0.51\right\}=0.19$$

Step 5. Determination of the Distance Measure of Each Alternative from the RNPIS and the RNNIS and Relative Closeness Coefficient

The distances of each alternative from the RNPIS and RNNIS are calculated by using the normalized Euclidean distance measures defined in Eq. (22) and Eq. (24) and also normalized Hamming distance measures defined in Eq. (23) and Eq. (25). Then, the relative closeness coefficient is calculated via these distances using Eq. (26). The distance measures ($D_{NEucl}^{i+},D_{NEucl}^{i-}$) and ($D_{NHamm}^{i+},D_{NHamm}^{i-}$), and relative closeness coefficient (C_i^*) of each alternative are presented in Table 6.

Table 6. Distance measures and relative closeness coefficient of alternatives for SVNN based TOPSIS

Alternatives	D_{NEucl}^{i+}	D_{NEucl}^{i-}	C_i^*	**Ranking**
S_1	0.1691	0.0843	0.3326	3
S_2	0.0173	0.1872	0.9153	1
S_3	0.1607	0.0460	0.2225	4
S_4	0.0260	0.1909	0.8802	2
Alternatives	D_{NHamm}^{i+}	D_{NHamm}^{i-}	C_i^*	**Ranking**
S_1	0.1114	0.0456	0.2905	3
S_2	0.0033	0.1537	0.9787	1
S_3	0.1364	0.0206	0.1314	4
S_4	0.0083	0.1487	0.9469	2

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Step 6. Ranking the Alternatives

According to the relative closeness coefficient of each alternative supplier given in Table 6, the ranking order of four alternative suppliers is obtained as $S_2 \succ S_4 \succ S_1 \succ S_3$. Thus, S_2 is chosen as the best sustainable supplier for the Turkish discount market chain.

SOLUTIONS AND RECOMMENDATIONS

In this chapter, sustainable supplier selection is applied to a Turkish discount market chain using the TOPSIS method under the single-valued neutrosophic environment based on normalized Euclidean and Hamming distances. Rankings of four alternative suppliers depend on both distance measures are shown in Table 6. As seen from Table 6, the ranking of the suppliers is obtained as $S_2 \succ S_4 \succ S_1 \succ S_3$.. As a result, supplier S2 is determined as the best sustainable supplier to be selected for the company operating in the Turkish retail industry.

Furthermore, in order to demonstrate the rationality and effectiveness of the proposed method, two alternative TOPSIS methods based on type-1 fuzzy set (T1FS-TOPSIS) and intuitionistic fuzzy set (IFS-TOPSIS) using both normalized Euclidean and normalized Hamming distances are applied to the sustainable supplier selection problem. In Table 7, a comparison of the ranking results for the fuzzy extensions of the TOPSIS method is given.

Table 7. Comparison of ranking results for the fuzzy extensions of TOPSIS method

	T1FS-TOPSIS		**IFS-TOPSIS**		**SVNS-TOPSIS**	
Alternatives	C_i^*	**Ranking**	C_i^*	**Ranking**	C_i^*	**Ranking**
	Normalized Euclidean distance					
S_1	0.3922	3	0.3519	3	0.3326	3
S_2	0.4979	1	1.0000	1	0.9153	1
S_3	0.3751	4	0.2328	4	0.2225	4
S_4	0.4841	2	0.8238	2	0.8802	2
	Normalized Hamming distance					
S_1	0.4309	3	0.4079	3	0.2905	3
S_2	0.4939	1	1.0000	1	0.9787	1
S_3	0.4211	4	0.2963	4	0.1314	4
S_4	0.4854	2	0.7580	2	0.9469	2

Since all ranking results are the same as seen in Table 7, it can be said that the ranking results of the SVNS-TOPSIS methods with normalized Euclidean and normalized Hamming distances, are consistent with the methods of T1FS-TOPSIS and IFS-TOPSIS.

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FUTURE RESEARCH DIRECTIONS

In further studies, different extensions of fuzzy sets such as intuitionistic, pythagorean, hesitant can be adapted to other MCDM methods for the problem of sustainable supplier selection. Thus, a comparison can be made between the results obtained from this study and other methods. The sustainable supplier selection problem discussed in this study can also be formed as a group multi-criteria decision-making problem for further studies. Moreover, the TOPSIS method can be extended to n-valued refined neutrosophic TOPSIS for future research.

CONCLUSION

The problem of sustainable supplier selection, one of the most important strategic issues of organizations in supply chain management, has become more and more attractive for researchers for a long time. The sustainability performance of suppliers has been measured by taking into account the economic, social, and environmental dimensions, which are the three pillars of sustainability. Due to a finite number of decision criteria and alternatives in the sustainable supplier selection problem, it can be considered as a multi-criteria decision making (MCDM) problem. Thus, MCDM approaches can be considered as one of the most efficient ways to solve this problem.

In real-life applications of decision-making problems, fuzzy set theory and its extensions (e.g. type-2, intuitionistic, hesitant, neutrosophic, etc.) have been widely applied to MCDM methods which include linguistic evaluations of decision makers. Various fuzzy extensions have been introduced in the technique for order preference by similarity to an ideal solution (TOPSIS) which is one of the most used MCDM methods. A single-valued neutrosophic set (SVNS), a special case of the neutrosophic set, has also been applied to the TOPSIS method.

Despite the difficulty of direct application of NS in real engineering and scientific applications, SVNS is characterized by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree. The main advantage of SVNS is that it can be applied quite well to problems involving uncertainty, imprecise, incomplete, and inconsistent information in real science and engineering. Therefore, SVNS is adapted as an extension of the TOPSIS method for the sustainable supplier selection problem in this study.

In this chapter, sustainable supplier selection in a company, which is a discount retail chain in Turkey, is discussed by the SVNS-TOPSIS method using normalized Euclidean distance and normalized Hamming distance. In the evaluation framework, the criteria of sustainability dimensions are determined as cost/price (C/P), quality (Q), and technology capabilities (TC) for economic; green design, green design (GD), environmental management system (EMS), environmental competencies (EC) for environmental; interests and rights of employees (IRE), rights of stakeholders (RS), and social management commitment (SMC) for social. The linguistic evaluations, which are obtained as a result of the decision makers' consensus, for all criteria and alternatives are converted into single valued neutrosophic numbers. Then, ranking of the alternatives is obtained via the SVNS-TOPSIS method using distances of normalized Euclidean and normalized Hamming, in a comparable manner. It is seen that final rankings are achieved as the same for both distances.

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Another advantage of this chapter is that it enables a comparison between the TOPSIS methods based on the type-1 fuzzy set, intuitionistic fuzzy set, and single-valued neutrosophic set which use normalized Euclidean and normalized Hamming distances.

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KEY TERMS AND DEFINITIONS

Alternative: A set of limited course of actions or choices to be evaluated.

Criteria: A set of indicators to be used to evaluate or classify any problem.

Multi-Criteria Decision Making: A branch of operational research dealing with finding optimal results in complex scenarios including various indicators, conflicting objectives, and criteria.

Single-Valued Neutrosophic Set: A special case of neutrosophic sets and used in real scientific and engineering applications.

Supplier Selection: The process of selecting a supplier to acquire the necessary materials to support the outputs of organisations.

Sustainibility: The process of ensuring long-term availability of physical, natural, and social resources.

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS): One of the numerical methods of the multi-criteria decision making.

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Chapter 32
Inverse Single-Valued Neutrosophic Soft Set and Its Application

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ABSTRACT

Single-valued neutrosophic set (SVNS) can be viewed as an instance of the neutrosophic set (NS) proposed by Smarandache in 2005 to solve real decision-making problems that contain imperfect knowledge of the domain. Embedding the idea of SVNS with the soft set (SS), a new mathematical structure known as a single-valued neutrosophic soft set (SVNSS) is introduced to model the vague concept parametrically. In this chapter, the authors introduce a new soft model, namely inverse single-valued neutrosophic soft set (ISVNSS), that can be perceived as another mathematical model which is used to solve the purposes that are just opposite in the context of SVNSS. Also, they define different properties and operators associated with ISVNSSs. Finally, by using the ISVNSSs, an algorithm has been introduced that will serve the purpose of decision-makers to solve various real decision-making problems, and it will enable to unveil another type of uncertainty domain hidden in nature. This new approach surely provides more information to the decision-makers before they make any decisions on various attributes.

1. INTRODUCTION

Presently, researchers are facing such kinds of problems in their real-life that cannot be answered by using classical mathematics. In a classical or ordinary or crisp sense, a decision-makers decision in the cognitive domain is restricted within 0 or 1. So, it will unable to answer those questions whose value lies between 0 and 1 as it is the demand of the decision-makers. For example, it is difficult to explain the qualitative term “excellent” by using two-valued logic(0 0r 1) as there are different categories to measure excellent such as very excellent, average excellent, less excellent, and so on. We give another example, in the classical sense, the quality of a student can be determined by using the linguistic term ‘good’ or

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'bad. We assign 1 for good and 0 for bad. But, what value we assign for 'very good', 'relatively good', 'average', and 'satisfactory' etc. Like this, researchers give many instances that they are experienced in our day- to- day activity. To eradicate such issues to some extent Zadeh(Zadeh,1965) introduced the fuzzy set (FS) in 1965 and since then it can attract the attention of many researchers and mathematicians over the decades due to its novelty. It is a mathematical technique to model uncertainty. FS can be viewed as an extension of the classical or crisp sets. In FS, every object in the universe has a membership value that belongs to the interval [0,1]. For instance, suppose an individual having an IQ within 85 and 115 can be considered to be an intelligent person. So, here we consider "intelligent" as a fuzzy word. Therefore, we can assign membership values corresponding to some IQs of a group of individuals as (85,0), (100,0.5), (115,1) etc. Membership values decide the degree of intelligence of the group of individuals. As we know every theory has its inherent limitation like this FS cannot solve the purpose of incomplete information in data. For incomplete information, a decision-maker needs to assign a non-membership value along with a membership value to every member of a universe. To remove such an issue Atanassov(Atanassov,1986) introduced an intuitionistic fuzzy set(IFS) in 1986. An IFS is a direct extension of FS with the restriction that the sum of the membership and the non-membership degree does not exceed the unity where both the membership degrees belong to the interval [0,1]. If there is a situation where the non-membership value becomes zero then the IFS reduces to FS. To deal various types of uncertainties present in different fields of knowledge, some other extensions and applications of FSs are given in the literature proposed in the following: In 1989, the notion of interval-valued intuitionistic fuzzy set was given by Atanassov(Atanassov,1989). Bustince et al.(Bustince & Burillo, 1996) defined vague sets are intuitionistic fuzzy sets. In 2010, interval-valued fuzzy sets in soft computing was introduced by Bustice(Bustince, 2010). An application of interval-valued fuzzy soft was introduced by Chetia et al. (Chetia & Das, 2010). Cuong et al.(Cuong & Kreinovich, 2013) presented the picture fuzzy set for computational intelligence problems. Robust decision-making using intuitionistic fuzzy numbers were given by Das et al. (Das, Kar & Pal, 2017). In 2016, Garg(Garg, 2016) defined correlation coefficients between Pythagorean fuzzy sets. The linguistic Pythagorean fuzzy sets and its applications in multi-attribute decision-making process was initiated by Garg(Garg,2018). In 2014, Rodriguez et al. (Rodriguez, Martínez, Torra, Xu & Herrera, 2014) used hesitant fuzzy sets in future directions. The idea of the distance and similarity measures for dual hesitant fuzzy sets and their applications in pattern recognition was given by Su et al.(Su, Xu, Liu & Liu, 2015). The dual hesitant fuzzy aggregation operator in multiple attribute decision-making was introduced by Wang et al. (Wang, Zhao & Wei, 2014). In 2015, Zadeh(Zadeh, 2015) used fuzzy logic as a personal perspective. Fuzzy set theory and its various applications were given by Zimmermann(Zimmermann, 2010). Zhu et al.(Zhu, Xu & Xia, 2012) gave the notion of dual hesitant fuzzy sets.

From the above discussions, it is clear that the boundary of the FS and its extensions do not depend on parameters. But, sometimes a decision-maker needs to make a precise decisions based on parametric behavior in data. To handle the parametric data, Molodtsov (Molodtsov, 1999) introduced a soft set(SS) in 1999. A soft set is a generalized framework to model vagueness and uncertainty in a parametric manner. It is a mathematical tool for dealing various decision-making problems that we encounter in various forms in our daily life. Many researchers focused on soft set theory and develop different methods to handle various types of uncertainties present in the decision problem. A soft set over the universal set U is denoted by the ordered pair (F,A), where F: $A \rightarrow P(U)$. Due to the novelty of soft set, It has a rich potential for applications in several directions such as engineering, medical science, game theory, social science, economics, Riemann integration, etc. As compared to FS and its invariants, SS

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can be considered as the more general framework to model uncertainty or ambiguity in a parametric way. For example, suppose Mr. X wants to buy a house from a set of alternatives, and for this purpose he appoints a decision-maker. He/ she may consider the parameters for choosing a house as 'beautiful',' green surrounding', 'costly', 'comfortable', 'wooden' etc. The decision-makers will solve the purpose of Mr. X in the context of parameters where he/she can be able to choose the best option by using various decision-making methods. So, SS will give the approximate description of the house where he will select the best option by using mathematical modeling or aggregate operator or SWARA method, TOPSIS method, or Similarity measure. Due to the nature of the problem, there may be a requirement of a group of decision-makers to aggregate the alternatives. Some applications of SS in various decision-making problems are given in (Ali, 2011; Maji, Roy & Biswas, 2002; Yuksel, Dizman,Yildizdan & Sert, 2013). Pertaining the idea of a soft set with the fuzzy set, intuitionistic fuzzy set, interval-valued fuzzy set, interval-valued fuzzy set, etc. fuzzy soft set(FSS) and its application(Enginoglu, Memis & Karaaslan, 2019; Eraslan & Karaaslan, 2015; Roy & Maji, 2007; Xiao, Gong & Zou, 2009), intuitionistic fuzzy soft set(IFSS) and its application(Das & Kar, 2014; Garg & Arora, 2018; Zhang, 2012), interval-valued fuzzy soft set(IVFSS) and its application(Chetia & Das, 2010),interval-valued intuitionistic fuzzy soft set(IVIFSS) and its application(Jiang, Tang, Chen, Liu & Tang, 2010), and many more have been introduced for the generalization of the soft set. Also, Jana et al. (Jana, Pal & Wang, 2019) studied aggregation operator under bipolar fuzzy soft environment, Karaslan et al.(Karaslan & Karatas, 2015) shown a new approach to bipolar soft sets and their applications. Moreover, in (Garg & Arora, 2020), Garg et al. defined the TOPSIS method based on correlation coefficient for solving decision-making problems with intuitionistic fuzzy soft set information, the Maclaurin symmetric mean aggregation operators based on t-norm operations for the dual hesitant fuzzy soft set is proposed in(Garg & Arora, 2020), the generalized Maclaurin symmetric mean aggregation operators based on Archimedean t-norm of the intuitionistic fuzzy soft set information given in (Garg & Arora, 2021), the spherical fuzzy soft sets and its applications in decision-making problems introduced in(Perveen PA, Sunil, Babitha & Garg, 2019).

An idea or a statement or an axiom doesn't need to always address the fuzzy character. Fuzzy sets and their variants are viable to solve uncertain phenomena with their inherent constraints. Practically we find some phenomenon that is indeterministic or inconsistent. So, indeterminate or inconsistent phenomena cannot be solved by a fuzzy set and its invariants. To overcome such an issue, Smarandache (Smarandache, 1998) introduced the notion of neutrosophy in 1998. Neutrosophy is the study of neutralities present in various disciplines such as sociology, economics, biology, environmental science, etc. Neutrosophy can be considered as a branch of philosophy. In neutrosophy, an idea or a proposition, or a statement is denoted by the symbol $\langle A \rangle$, and then its negation is denoted by $\langle anti\ A \rangle$. To make the completeness of A we need $\langle not\ A \rangle$. Still there exists a knowledge gap that can be filled with the notation $\langle neut\ A \rangle$ that exhibits the notion which lies in between $\langle A \rangle$ and $\langle anti\ A \rangle$. For example, suppose, we consider an idea as "a set of convex polygons" and it is denoted by $\langle P \rangle$. Then, $\langle anti\ P \rangle$ represents a set of concave polygons. The $\langle not\ P \rangle$ denotes all types of polygons(except convex polygons) such as concave polygons, regular polygons, irregular polygons, equilateral polygons, etc., and $\langle neut\ P \rangle$ signifies those which are neither convex nor concave polygons. Therefore, the concept of 'neutrosophy' is more logical and more valid as compared to the other existing logical theories proposed in (Smarandache, 1998). Pertaining to the idea of neutrosophy, the neutrosophic set(Smarandache, 2005) was introduced, as a direct extension of an intuitionistic fuzzy set. In a neutrosophic set (NS), each object in

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the universe has three components namely truth-membership, falsity-membership, and indeterminacy-membership values and these values are independent of each other. As in NS, there is no restriction in considering the three membership values; the decision-makers can't determine the exact solution to the problem. To remove such issue, Wang et al. (Wang, Smarandache, Zhang & Sunderraman, 2010) introduced a single-valued neutrosophic set (SVNS) denoted by,

$$\left\{\left\langle x, T_A(x), I_A(x), F_A(x)\right\rangle : x \in U\right\},$$

with the restriction $T_A, I_A, F_A: U \to [0,1]$ such that

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3.$$

After the introduction of SVNS, it has a wide application in different decision-making problems. It is proved to be the better model as compared to the neutrosophic set to deal with scientific and engineering problems. Some significant works and the extensions associated to NSs are given in (Abdel Basset, Mohamed, Zhou & Hezam, 2017; Abdel Basset, Manogaran, Gamal & Smarandache,2019; Maji, 2012; Manna, Basu & Mondal, 2020). To avoid the technical issues, NS is always termed as SVNS i.e., SVNS contains in NS. Some novel works on SVNS given in (Broumi & Smarandache, 2015; Jana & Pal, 2019; Khalil, Cao, Azzam, Smarandache & Alharbi, 2020; Ye, 2015). In addition, linguistic single-valued neutrosophic power aggregation operators and their applications given in(Garg, 2019), decision-making analysis based on sine trigonometric operational laws for single-valued neutrosophic sets are presented in(Garg, 2020), novel neutrality aggregation operator based multi-attribute group decision-making method for single-valued neutrosophic numbers are discussed in (Garg, 2020), multiple attribute decision-making based on immediate probabilities aggregation operators for single-valued and interval neutrosophic sets are mentioned in(Garg, 2020), Algorithms for single-valued neutrosophic decision-making based on TOPSIS and clustering methods with new distance measure is studied in(Garg, 2020).

To get the insight into the present literature proposed in this paper we consider the following:

Let, $U=\{u_1,u_2,\ldots,u_n\}$ be a set of alternatives under consideration and $E=\{e_1,e_2,\ldots,e_n\}$ be a set of parameters where $A \subseteq E$. Then, a soft set is denoted by an ordered pair (F,A) where $F: E \to P(U)$.

That is soft set maps each object or alternative to the power set of U. Clearly, soft set is considered over the set of objects.

But, sometimes decision-maker has his/her own point of view i.e., he/she wants to consider another type of soft set which is defined over the set of parameters i.e., $F: U \to P(E)$.

Therefore, such a pair (F,U) is called the inverse soft set (ISS) discussed in (Cetkin, Aygunoglu & Aygun, 2016). So, ISS is a new method to describe the character of each object in U over the set of parameters E. After that, the new approaches of inverse soft rough sets and their applications in a decision-making problem is discussed in(Demirtas, Hussain & Dalkilic, 2020), the cardinality inverse soft matrix theory and its applications in multi-criteria group decision-making is proposed in(Kamaci, Saltik, Akiz & Atagun, 2018), design and analysis of inverse fuzzy soft set for triangular and trapezoidal membership function in decision-making presented in(Kavitha & Prabhakaran, 2021), the generalized

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inverse of fuzzy neutrosophic soft matrix given in(Uma, Murugadas & Sriram, 2017), the row-products of inverse soft matrices in multi-criteria decision-making in(Petchimuthu & Kamaci, 2019), the adjustable approaches to multi-criteria group decision-making based on inverse fuzzy soft matrices in(Petchimuthu & Kamaci, 2020). The concept of ISS has been extended by introducing an inverse fuzzy soft set (IFSS) (Khalil & Hassan, 2019) in a decision-making problem. So, the main motivation behind proposing this chapter is to extend the notion of IFSS by introducing an inverse single-valued neutrosophic soft set. A single-valued neutrosophic soft set is a hybrid structure that is formed by the mixture of SVNS and SS. The advantage of using SVNSS is that it is capable to handle the inconsistent and indeterminate concept with a wide range of domains to cover maximum possible decision-making problems and SVNSS can be used in various directions according to the choice of the decision-maker to identify the best alternatives. SVNSS is the more generalized parametric framework to model indeterminacy. Pertaining to the idea of the SVNSS another soft model called ISVNSS is introduced to describe all the alternatives in a universe over a set of parameters. Thus, ISVNSS act as an opposite operator of SVNSS.

The novelty of the proposed approach is that, unlike SVNSS, the concept of ISVNSS is a new soft model that is formed by exchanging the set of universe U and the set of parameters E and it can be applied comfortably without much arithmetic operation in decision-making problem to give the optimal solution in a simple way. Therefore, the ISVNSS provides another information domain where the decision-maker describes the abstract character of every object in U. In this approach, there is no need of any extra information provided by the decision-makers and thus it is independent of subjective information. So, the ISVNSS is a new approach of decision-making and thus it is viable to describe another opinion of a decision-maker in a sophisticated manner.

The rest of the chapter is organized in the following manner: section 2 gives some basic definitions with suitable examples. Section 3 focuses on inverse single-valued neutrosophic soft set and its various operations. Section 4 discusses an application of inverse single-valued neutrosophic soft set in decision-making. Section 5 discusses the conclusion and the future scope of the proposed study.

2. PRELIMINARIES

In this section, we give a brief literature insight that fulfills the objective of the paper:

Definition 2.1 (Molodtsov, 1999)

Let U be a set of universe and E denotes the set of parameters, then the ordered pair (F,E) is called a soft set where F: $E \rightarrow P(U)$ such that $F(x) = \varnothing$ if $x \notin E$.

Example 2.1.1

Suppose Mr. X wants to buy a car from a showroom and the available cars in the showroom denoted by $U=\{c_1,c_2,c_3,c_4,c_5\}$ and the criteria for selecting a car is denoted by the set of parameters $E=\{e_1,e_2,e_3,e_4\}$ where e_1 stands for style, e_2 stands for reliability, e_3 stands for fuel-economy and e_4 stands for cost.

Suppose, $F(e_1)=\{c_1,c_3,c_5\}$, $F(e_2)=\{c_1,c_4,c_5\}$, $F(e_3)=\{c_1,c_3\}$, $F(e_4)=\{c_2,c_5\}$.

Then, the soft set (F,E) can be represented in the tabular form given by:

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Table 1. Tabular representation of the soft set (F,E)

U	e_1	e_2	e_3	e_4
c_1	1	1	1	0
c_2	0	0	0	1
c_3	1	0	1	0
c_4	0	1	0	0
c_5	1	1	0	1

For more than two decision makers we have to construct two soft sets and their aggregate soft set will give the best alternative.

Definition 2.2 (Roy & Maji, 2007)

Let U be an initial universe and F^U denotes the collection of all fuzzy sets over U. Let E be the set of parameters and $A \subseteq E$. Then a fuzzy soft set Γ_A over U is denoted by the ordered pair as follows:

$$\Gamma_A = \left\{ \left(x, \gamma_A(x) \right) : x \in E, \gamma_A(x) \in F^U \right\},$$

where γ_A: $E \to FU$ such that $\gamma_A(x) = \varnothing$ if $x \notin A$.

Example 2.2.1

Suppose, $U=\{h_1,h_2,h_3,h_4\}$ is the set of houses under consideration and E be the set of parameters, where each parameter is a fuzzy word or a sentence. We consider the parameter set as, $E=\{e_1,e_2,e_3,e_4,e_5\}$, where e_1 stands for beautiful, e_2 stands for expensive, e_3 stands for comfortable, e_4 stands for cheap, and e_5 stands for wooden.

Now, the fuzzy soft set Γ_E can be defined as a collection of fuzzy approximations given below:

$$\Gamma_E = \left\{ \begin{array}{l} \textit{beautiful houses} = \left\{ h_1 / 0.4, h_2 / 0.4, h_3 / 0.8, h_4 / 0.7 \right\}, \\ \text{expensive houses} = \left\{ h_1 / 0.6, h_2 / 0.5, h_3 / 0.3, h_4 / 0.3 \right\}, \\ \textit{comfortable houses} = \left\{ h_1 / 0.5, h_2 / 0.8, h_3 / 0.3, h_4 / 0.9 \right\}, \\ \textit{cheap houses} = \left\{ h_1 / 0.0, h_2 / 0.6, h_3 / 0.7, h_4 / 0.3 \right\}, \\ \textit{wooden houses} = \left\{ h_1 / 0.6, h_2 / 0.8, h_3 / 0.6, h_4 / 0.4 \right\} \end{array} \right\}$$

Γ_E Can be represented in the form of a table given as:

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Table 2. Representation of fuzzy soft set Γ_E

U	Beautiful	Expensive	Comfortable	Cheap	Wooden
h_1	0.4	0.6	0.5	0.0	0.6
h_2	0.4	0.5	0.8	0.6	0.8
h_3	0.8	0.3	0.3	0.7	0.6
h_4	0.7	0.3	0.9	0.3	0.4

Definition 2.3 (Smarandache, 2005)

A neutrosophic set *A* over *U* (as an object) is defined as

$$\left\{\left\langle x, T_A(x), I_A(x), F_A(x)\right\rangle : x \in U\right\} = \left\{x / \left\langle T_A, I_A, F_A\right\rangle : x \in U\right\},$$

where $T_A, I_A, F_A: U \rightarrow]^-0,1^+[$ such that $^-0 \leq T_A(x), I_A(x), F_A(x) \leq 3^+$.

From a philosophical point of view, the interval $]^-0,1^+[$ is meaningful. But for real-life application, it is difficult to assign value. To eradicate such an issue, we use a single-valued neutrosophic set proposed by Wang et al. (Wang, Smarandache, Zhang & Sunderraman, 2010) where we consider the unit closed interval [0,1] with the condition $0 \leq T_A(x), I_A(x), F_A(x) \leq 3$.

Definition 2.4

Let *U* denote the universe of discourse and *E* be a set of parameters under consideration. Suppose $A \subseteq E$ and SVN^U denotes the set of all single-valued neutrosophic sets over *U*. Then the collection (*F,A*) is known as the soft single-valued neutrosophic set of *U*, where $F: A \rightarrow SVN^U$.

For an illustration we consider example **2.2.1,** and represent the single-valued neutrosophic soft set in the given tabular form:

Table 3. Representation of single-valued neutrosophic soft set

U	Beautiful	Expensive	Comfortable	cheap	wooden
h_1	<0.3,0.6,0.7>	<0.4,0.5,0.2>	<0.4,0.7,0.6>	<0.5,0.7,0.8>	<0.7,0.5,0.6>
h_2	<0.2,0.8,0.6>	<0.1,0.8,0.6>	<0.3,0.7,0.9>	<0.2,0.7,0.5>	<0.8,0.5,0.6>
h_3	<0.7,0.4,0.9>	<0.4,0.7,0.8>	<0.7,0.5,0.9>	<0.4,0.6,0.8>	<0.2,0.5,0.8>
h_4	<0.5,0.7,0.6>	<0.6,0.8,0.5>	<0.4,0.6,0.3>	<0.1,0.3,0.4>	<0.2,0.4,0.7>

Definition 2.5 (Cetkin, Aygunoglu & Aygun, 2016)

Let *P*(*E*) denote the set of all subsets of the parameter set *E*. Then a pair (*F,U*) is called an inverse soft set over *E* and $F: U \rightarrow P(E)$.

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Example 2.5.1

Continue to consider example **2.2.1**, we define the inverse soft set as follows:

$F(h_1)=\{e_1,e_3,e_4\}$, $F(h_2)=\{e_1,e_2,e_5\}$, $F(h_3)=\{e_2,e_3,e_4\}$, $F(h_1)=\{e_1,e_2,e_3\}$

Thus, we can view the inverse soft set (F,U) as a collection of approximation as follows:

$$(F,U)=\left\{\begin{array}{l} F\left(h_1\right)=\left\{beautiful, comfortable, cheap\right\}, F\left(h_2\right)=\left\{beautiful, \exp ensive, wooden\right\}, \\ F\left(h_3\right)=\left\{\exp ensive, comfortable, cheap\right\}, F\left(h_4\right)=\left\{beautiful, \exp ensive, comfortable\right\} \end{array}\right\}$$

Definition 2.6 (Khalil & Hassan, 2019)

Let μ_θ be a fuzzy parameter set over a parameter set E then we define μ_θ: $E\rightarrow[0,1]$. For $e\in E$, the membership value $\mu_\theta(e)$ specifies the degree to which e belongs to the fuzzy parameter set μ_θ. The set of all fuzzy parameter sets denoted by IE and it can be characterized by the set of ordered pairs of the form $\mu_\theta=\left\{\left(e,\mu_\theta\left(e\right)\right):e\in E\right\}$.

Example 2.6.1

Let E= {intelligent, diligent, punctual, courteous, self-discipline, responsible} be a set of qualities. We consider a fuzzy parameter set μ_θ of qualities are as follows: μ_θ(intelligent)=0.6, μ_θ(diligent)=0.4, μ_θ(punctual)=0.5, μ_θ(courteous)=0.8, μ_θ(self-discipline)=0.9, μ_θ(responsible)=0.6. Then, we represent the fuzzy parameter set as follows:

μ_θ= {(intelligent,0.6), (diligent,0.4), (punctual,0.5), (courteous,0.8), (self-discipline,0.9), (responsible,0.6)}

Definition 2.7 (Khalil & Hassan, 2019)

An ordered pair $\left(\tilde{\tilde{F}},U\right)$ is called an inverse fuzzy soft set over E, where $\tilde{\tilde{F}}:U\rightarrow I^E$.

Example 2.7.1

Considering example 2.2.1, we define the inverse fuzzy soft set as follows:

$$\tilde{\tilde{F}}\left(h_1\right)=\left\{e_1/0.6, e_2/0.5, e_3/0.0, e_4/0.6, e_5/1.0\right\}$$
$$\tilde{\tilde{F}}\left(h_2\right)=\left\{e_1/0.3, e_2/0.4, e_3/0.6, e_4/0.4, e_5/0.8\right\}$$
$$\tilde{\tilde{F}}\left(h_3\right)=\left\{e_1/0.7, e_2/0.6, e_3/1.0, e_4/0.5, e_5/0.5\right\}$$
$$\tilde{\tilde{F}}\left(h_4\right)=\left\{e_1/0.5, e_2/0.0, e_3/0.7, e_4/0.8, e_5/0.3\right\}$$

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Thus,

$$\left(\tilde{\tilde{F}},U\right)=\left\{\begin{array}{l}\tilde{\tilde{F}}\left(h_1\right)=\left\{e_1/0.6,e_2/0.5,e_3/0.0,e_4/0.6,e_5/1.0\right\},\\ \tilde{\tilde{F}}\left(h_2\right)=\left\{e_1/0.3,e_2/0.4,e_3/0.6,e_4/0.4,e_5/0.8\right\},\\ \tilde{\tilde{F}}\left(h_3\right)=\left\{e_1/0.7,e_2/0.6,e_3/1.0,e_4/0.5,e_5/0.5\right\},\\ \tilde{\tilde{F}}\left(h_4\right)=\left\{e_1/0.5,e_2/0.0,e_3/0.7,e_4/0.8,e_5/0.3\right\}\end{array}\right\}$$

3. INVERSE SINGLE-VALUED NEUTROSOPHIC SOFT SET

The notion of a neutrosophic set is initiated by smarandache. Later, Maji introduced neutrosophic soft set (Maji, 2012) as a combination of neutrosophic set and soft set. In neutrosophic soft set, the parameters are neutrosophic words or sentences or propositions or axioms, and so on. In (Broumi & Smarandache, 2015), Broumi et al. proposed single-valued soft expert sets and their application. In this section, first, we define a single-valued neutrosophic parameter set and study its properties. Then, with the help of a single-valued parameter set, we define inverse single-valued neutrosophic sets and perform some operations on them.

Definition 3.1

Let $\mu_\theta,\nu_\theta,\gamma_\theta$ respectively be the single-valued neutrosophic truth, indeterminacy, and falsity parameter set with the condition $\mu_\theta,\nu_\theta,\gamma_\theta$: $E\rightarrow[0,1]$ such that $\forall e\in E$, there exists a membership value ($\mu_\theta(e)$), indeterminacy membership value ($\nu_\theta(e)$) and the falsity membership value ($\gamma_\theta(e)$) such that $0\le\mu_\theta\left(e\right)+\nu_\theta\left(e\right)+\gamma_\theta\left(e\right)\le 3$. The set of all single-valued parameter set is denoted by the notation SVNE. A complete single-valued neutrosophic parameter set (SVN-parameter set) is defined by

$$SVN^E=\left\{\left(e,\mu_\theta\left(e\right),\nu_\theta\left(e\right),\gamma_\theta\left(e\right)\right):e\in E\right\}.$$

It is to be noted that all the parameter sets are independent to each other.

Let,

$$SVN_1{}^E=\left\{\left(e,\mu_{\theta_1}\left(e\right),\nu_{\theta_1}\left(e\right),\gamma_{\theta_1}\left(e\right)\right):e\in E\right\},$$

$$SVN_2{}^E=\left\{\left(e,\mu_{\theta_2}\left(e\right),\nu_{\theta_2}\left(e\right),\gamma_{\theta_2}\left(e\right)\right):e\in E\right\}\text{ and}$$

$$SVN_3{}^E=\left\{\left(e,\mu_{\theta_3}\left(e\right),\nu_{\theta_3}\left(e\right),\gamma_{\theta_3}\left(e\right)\right):e\in E\right\}$$

be three SVN-parameter sets over the common parameter set E, then *w*e have the following operations:

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(i) $SVN_1^E \oplus SVN_2^E =$
$\left\{\left\langle e, \mu_{\theta_1}(e)+\mu_{\theta_2}(e)-\mu_{\theta_1}(e).\mu_{\theta_2}(e), \nu_{\theta_1}(e)+\nu_{\theta_2}(e)-\nu_{\theta_1}(e).\nu_{\theta_2}(e), \gamma_{\theta_1}(e)+\gamma_{\theta_2}(e)-\gamma_{\theta_1}(e).\gamma_{\theta_2}(e)\right\rangle\right\}$

(ii) $SVN_1^E \otimes SVN_2^E = \left\{\left\langle e, \mu_{\theta_1}(e).\mu_{\theta_2}(e), \nu_{\theta_1}(e).\nu_{\theta_2}(e), \gamma_{\theta_1}(e).\gamma_{\theta_2}(e)\right\rangle\right\}$

(iii) $SVN_1^E \bigcup SVN_2^E = \left\{\left\langle e, \max\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), \max\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), \min\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right\rangle\right\}$

(iv) $SVN_1^E \bigcap SVN_2^E = \left\{\left\langle e, \min\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), \min\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), \max\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right\rangle\right\}$

(v) $\left(SVN_1^E\right)^C = \left\{\left\langle e, 1-\mu_{\theta_1}(e), 1-\nu_{\theta_1}(e), 1-\gamma_{\theta_1}(e)\right\rangle\right\}$

(vi) $\left(SVN_1^E \bigcup SVN_2^E\right)^c = SVN^c{}_1^E \bigcap SVN^c{}_2^E$

Proof: We have,

$\left(SVN_1^E \bigcup SVN_2^E\right)^c = \left\{\left\langle e, \max\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), \max\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), \min\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right\rangle\right\}^c$
$= \left\{\left\langle e, 1-\max\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), 1-\max\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), 1-\min\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right\rangle\right\}$
$= \left\{\left\langle e, \min\left(1-\mu_{\theta_1}(e), 1-\mu_{\theta_2}(e)\right), \min\left(1-\nu_{\theta_1}(e), 1-\nu_{\theta_2}(e)\right), \max\left(1-\gamma_{\theta_1}(e), 1-\gamma_{\theta_2}(e)\right)\right\rangle\right\}$
$= \left\{\left\langle e, 1-\mu_{\theta_1}(e), 1-\nu_{\theta_1}(e), 1-\gamma_{\theta_1}(e)\right\rangle\right\} \cap \left\{\left\langle e, 1-\mu_{\theta_2}(e), 1-\nu_{\theta_2}(e), 1-\gamma_{\theta_2}(e)\right\rangle\right\} = SVN^c{}_1^E \bigcap SVN^c{}_2^E$

Hence, proved.

(vii) $\left(SVN_1^E \bigcap SVN_2^E\right)^c = SVN^c{}_1^E \bigcup SVN^c{}_2^E$

Proof: It is similar to (vi).

(viii) $\left(SVN^c{}_1^E\right)^c = SVN_1^E$

Proof: we have,

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$$\left(SVN_1^{\,E}\right)^c = \left\{\left\langle e, 1-\mu_{\theta_1}(e), 1-\nu_{\theta_1}(e), 1-\gamma_{\theta_1}(e)\right\rangle\right\}$$

$$\text{So, } \left(\left(SVN_1^{\,E}\right)^c\right)^c = \left\{\left\langle e, 1-\left(1-\mu_{\theta_1}(e)\right), 1-\left(1-\nu_{\theta_1}(e)\right), 1-\left(1-\gamma_{\theta_1}(e)\right)\right\rangle\right\}$$

$$= \left\{\left\langle e, \mu_{\theta_1}(e), \nu_{\theta_1}(e), \gamma_{\theta_1}(e)\right\rangle\right\} = SVN_1^{\,E}$$

(ix) $SVN_1^{\,E} \subseteq SVN_2^{\,E}$ iff $\mu_{\theta_1}(e) \le \mu_{\theta_2}(e), \nu_{\theta_1}(e) \le \nu_{\theta_2}(e)$ and $\gamma_{\theta_1}(e) \ge \nu_{\theta_2}(e)$

(x) $SVN_1^{\,E} \cup SVN_2^{\,E}$ and $SVN_2^{\,E} \cup SVN_1^{\,E}$ (law of commutativity)

(xi) $SVN_1^{\,E} \cup \left(SVN_2^{\,E} \cup SVN_3^{\,E}\right) = \left(SVN_1^{\,E} \cup SVN_2^{\,E}\right) \cup SVN_3^{\,E}$ and

$SVN_1^{\,E} \cap \left(SVN_2^{\,E} \cap SVN_3^{\,E}\right) = \left(SVN_1^{\,E} \cap SVN_2^{\,E}\right) \cap SVN_3^{\,E}$ (law of associativity)

(xii) $SVN_1^{\,E} \cup \left(SVN_2^{\,E} \cap SVN_3^{\,E}\right) = \left(SVN_1^{\,E} \cup SVN_2^{\,E}\right) \cap \left(SVN_1^{\,E} \cup SVN_3^{\,E}\right)$ and

$SVN_1^{\,E} \cap \left(SVN_2^{\,E} \cup SVN_3^{\,E}\right) = \left(SVN_1^{\,E} \cap SVN_2^{\,E}\right) \cup \left(SVN_1^{\,E} \cap SVN_3^{\,E}\right)$ (law of distributivity)

Proof: $SVN_1^{\,E} \cup \left(SVN_2^{\,E} \cap SVN_3^{\,E}\right) =$

$$\left\langle \mu_{\theta_1}(e), \nu_{\theta_1}(e), \gamma_{\theta_1}(e)\right\rangle \cup \left\langle \min\left(\mu_{\theta_2}(e), \mu_{\theta_3}(e)\right), \min\left(\nu_{\theta_2}(e), \nu_{\theta_3}(e)\right), \max\left(\gamma_{\theta_2}(e), \gamma_{\theta_3}(e)\right)\right\rangle$$

$$= \left\langle \max\left(\mu_{\theta_1}(e), \min\left(\mu_{\theta_2}(e), \mu_{\theta_3}(e)\right)\right), \max\left(\nu_{\theta_1}(e), \min\left(\nu_{\theta_2}(e), \nu_{\theta_3}(e)\right)\right),\right.$$

$$\left.\min\left(\gamma_{\theta_1}(e), \max\left(\gamma_{\theta_2}(e), \gamma_{\theta_3}(e)\right)\right)\right\rangle = \left\langle \min\left(\max\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), \max\left(\mu_{\theta_1}(e), \mu_{\theta_3}(e)\right)\right),\right.$$

$$\left.\min\left(\max\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), \max\left(\nu_{\theta_2}(e), \nu_{\theta_3}(e)\right)\right), \max\left(\min\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right), \min\left(\gamma_{\theta_1}(e), \gamma_{\theta_3}(e)\right)\right)\right\rangle$$

$$= \left\langle \max\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), \max\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), \min\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right\rangle$$

$$\cap \left\langle \max\left(\mu_{\theta_1}(e), \mu_{\theta_{13}}(e)\right), \max\left(\nu_{\theta_1}(e), \nu_{\theta_3}(e)\right), \min\left(\gamma_{\theta_1}(e), \gamma_{\theta_3}(e)\right)\right\rangle$$

$$= \left(SVN_1^{\,E} \cup SVN_2^{\,E}\right) \cap \left(SVN_1^{\,E} \cup SVN_3^{\,E}\right)$$

Thus,

$$SVN_1^{\,E} \cup \left(SVN_2^{\,E} \cap SVN_3^{\,E}\right) = \left(SVN_1^{\,E} \cup SVN_2^{\,E}\right) \cap \left(SVN_1^{\,E} \cup SVN_3^{\,E}\right)$$

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Similarly, we can prove

$$SVN_1^E \cap \left(SVN_2^E \cup SVN_3^E\right) = \left(SVN_1^E \cap SVN_2^E\right) \cup \left(SVN_1^E \cap SVN_3^E\right)$$

(xiii) $SVN_1^E \cup \left(SVN_1^E \cap SVN_2^E\right) = SVN_1^E$ and $SVN_1^E \cap \left(SVN_1^E \cup SVN_2^E\right) = SVN_1^E$ (absorption laws)

Proof: we have,

$$SVN_1^E \cup \left(SVN_1^E \cap SVN_2^E\right) =$$

$$\left\langle e, \mu_{\theta_1}(e), \nu_{\theta_1}(e), \gamma_{\theta_1}(e)\right\rangle \cup \left\langle e, \min\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right), \min\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right), \max\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right\rangle$$
$$= \left\langle e, \max\left(\mu_{\theta_1}(e), \min\left(\mu_{\theta_1}(e), \mu_{\theta_2}(e)\right)\right), \max\left(\nu_{\theta_1}(e), \min\left(\nu_{\theta_1}(e), \nu_{\theta_2}(e)\right)\right),\right.$$
$$\left.\min\left(\gamma_{\theta_1}(e), \max\left(\gamma_{\theta_1}(e), \gamma_{\theta_2}(e)\right)\right)\right\rangle = \left\langle e, \mu_{\theta_1}(e), \nu_{\theta_1}(e), \gamma_{\theta_1}(e)\right\rangle = SVN_1^E$$

Similarly, $SVN_1^E \cap \left(SVN_1^E \cup SVN_2^E\right) = SVN_1^E$

(xiv) $SVN_1^E \subseteq SVN_2^E$ *and* $SVN_2^E \subseteq SVN_3^E \Leftrightarrow SVN_1^E \subseteq SVN_3^E$

Proof: $SVN_1^E \subseteq SVN_2^E$ *and* $SVN_2^E \subseteq SVN_3^E$

$$\Leftrightarrow \left(\mu_{\theta_1}(e) \le \mu_{\theta_2}(e), \nu_{\theta_1}(e) \le \nu_{\theta_2}(e), \gamma_{\theta_1}(e) \ge \gamma_{\theta_2}(e)\right)$$
$$\text{and } \left(\mu_{\theta_2}(e) \le \mu_{\theta_3}(e), \nu_{\theta_2}(e) \le \nu_{\theta_3}(e), \gamma_{\theta_2}(e) \ge \gamma_{\theta_3}(e)\right)$$
$$\Leftrightarrow \left(\mu_{\theta_1}(e) \le \mu_{\theta_3}(e), \nu_{\theta_1}(e) \le \nu_{\theta_3}(e), \gamma_{\theta_1}(e) \ge \gamma_{\theta_3}(e)\right) \Leftrightarrow SVN_1^E \subseteq SVN_3^E$$

(xv) $SVN_1^E \subseteq SVN_1^E$

(xvi) $SVN_1^E \subseteq SVN_2^E$ *and* $SVN_2^E \subseteq SVN_1^E \Leftrightarrow SVN_1^E = SVN_2^E$

Example 3.1.1

Let E= {e_1=cost, e_2=fuel-efficiency, e_3=engine capacity, e_4=comfort} be a set of parameters (criteria) to select a car from a showroom. Then we consider the SVN-parameter set as

$$SVN^E = \left\{\left\langle\left(e_1, 0.4, 0.5, 0.6\right), \left(e_2, 0.6, 0.7, 0.3\right), \left(e_3, 0.7, 0.8, 0.9\right), \left(e_4, 0.8, 0.7, 0.6\right)\right\rangle\right\}.$$

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A soft parameter set, fuzzy parameter set, intuitionistic fuzzy parameter set, interval-valued fuzzy parameter set are the instances of SVN-parameter set.

Definition 3.2

A pair $\left(\overset{\prec}{F},U\right)$ is called an inverse single-valued neutrosophic soft set(ISVNSS in short) over *E*, where $\overset{\prec}{F}$ is a mapping given by $\overset{\prec}{F}:U\to SVN^{E}$.

Example 3.2.1

Consider a set of families $U=\{f_1,f_2,f_3,f_4,f_5\}$ living in a society. The situation of the families can be determined by the four variables (parameters) that belong to the set

$E=\{e_1=$income, $e_2=$expenditure, $e_3=$education, $e_4=$occupation$\}$

Then the SVNSS $\left(\overset{\prec}{F},E\right)$ can then describe the "socio-economic" status of the families under the neutrosophic environment given by:

$$\overset{\prec}{F}\left(e_1\right)=\left\{f_1/\left\langle 0.3,0.6,0.9\right\rangle,f_2/\left\langle 0.2,0.5,0.8\right\rangle,f_3/\left\langle 0.5,0.6,0.8\right\rangle,f_4/\left\langle 0.6,0.7,0.8\right\rangle,f_5/\left\langle 0.0,0.7,0.8\right\rangle\right\}$$

$$\overset{\prec}{F}\left(e_2\right)=\left\{f_1/\left\langle 0.7,0.8,0.4\right\rangle,f_2/\left\langle 0.5,0.6,0.3\right\rangle,f_3/\left\langle 0.4,0.5,0.6\right\rangle,f_4/\left\langle 0.3,0.5,0.7\right\rangle,f_5/\left\langle 0.3,0.6,0.7\right\rangle\right\}$$

$$\overset{\prec}{F}\left(e_3\right)=\left\{f_1/\left\langle 0.3,0.4,0.5\right\rangle,f_2/\left\langle 0.7,0.8,0.9\right\rangle,f_3/\left\langle 0.3,0.2,0.4\right\rangle,f_4/\left\langle 0.8,0.7,0.6\right\rangle,f_5/\left\langle 0.4,0.7,0.8\right\rangle\right\}$$

$$\overset{\prec}{F}\left(e_4\right)=\left\{f_1/\left\langle 0.6,0.7,0.8\right\rangle,f_2/\left\langle 0.4,0.5,0.7\right\rangle,f_3/\left\langle 0.1,0.8,0.6\right\rangle,f_4/\left\langle 0.4,0.6,0.7\right\rangle,f_5/\left\langle 0.6,0.8,0.9\right\rangle\right\}$$

The tabular representation of SVNSS $\left(\overset{\prec}{F},E\right)$ is given below:

Table 4. Representation of SVNSS $\left(\overset{\prec}{F},E\right)$

U	Income (e_1)	Expenditure (e_2)	Education (e_3)	Occupation (e_4)
f_1	⟨0.3,0.6,0.9⟩	⟨0.7,0.8,0.4⟩	⟨0.3,0.4,0.5⟩	⟨0.6,0.7,0.8⟩
f_2	⟨0.2,0.5,0.8⟩	⟨0.5,0.6,0.3⟩	⟨0.7,0.8,0.9⟩	⟨0.4,0.5,0.7⟩
f_3	⟨0.5,0.6,0.8⟩	⟨0.4,0.5,0.6⟩	⟨0.3,0.2,0.4⟩	⟨0.1,0.8,0.6⟩
f_4	⟨0.6,0.7,0.8⟩	⟨0.3,0.5,0.7⟩	⟨0.8,0.7,0.6⟩	⟨0.4,0.6,0.7⟩
F_5	⟨0.0,0.7,0.8⟩	⟨0.3,0.6,0.7⟩	⟨0.4,0.7,0.8⟩	⟨0.6,0.8,0.9⟩

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Reference to the SVNSS $\left(\overset{\prec}{F},E\right)$ we defines another soft model known as ISVNSS $\left(\overset{\prec}{F},U\right)$ given by:

$$\overset{\prec}{F}\left(f_1\right)=\left\{e_1,e_3\right\},\overset{\prec}{F}\left(f_2\right)=\left\{e_3,e_4\right\},\overset{\prec}{F}\left(f_3\right)=\left\{e_1,e_2\right\},\overset{\prec}{F}\left(f_4\right)=\left\{e_2,e_3\right\},\overset{\prec}{F}\left(f_5\right)=\left\{e_2,e_4\right\}$$

Then the ISVNSS $\left(\overset{\prec}{F},U\right)$ can be viewed as a collection of approximation given as:

$$\left(\overset{\prec}{F},U\right)=\left\{\begin{array}{l}\overset{\prec}{F}\left(f_1\right)=\left\{income,education\right\},\\ \overset{\prec}{F}\left(f_2\right)=\left\{education,occupation\right\},\overset{\prec}{F}\left(f_3\right)=\left\{income,\exp enditure\right\},\\ \overset{\prec}{F}\left(f_4\right)=\left\{\exp enditure,education\right\},\overset{\prec}{F}\left(f_5\right)=\left\{\exp enditure,\text{occupation}\right\}\end{array}\right\}$$

So, from the above set-theoretic representation, it has been observed that by ISVNSS we describe the character of each object of the universe over the set of parameters. So, ISVNSS provided a new method to describe the essence of every object of the universe *U*.

Definition 3.3

Let $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ be two ISVNSS over a common parameter set *E*. Then, $\left(\overset{\prec}{F},U\right)$is called an ISVNS subset of $\left(\overset{\prec}{G},V\right)$ if $U\subseteq V$ and for every $u\in U$ there exist $v\in V$ such that $\overset{\prec}{F}\left(U\right)\subseteq\overset{\prec}{F}\left(V\right)$.

Example 3.3.1

Let $U=\{h_1,h_2\}$ and $V=\{h_1,h_2,h_3\}$ and they defined on $E=\{e_1,e_2,e_3,e_4\}$.

If we consider,

$$\begin{array}{l}\overset{\prec}{F}\left(h_1\right)=\left\langle e_1,\mathrm{e}_2\right\rangle\\ \overset{\prec}{F}\left(h_2\right)=\left\langle e_1,e_3,e_4\right\rangle\end{array}\quad\text{and}\quad\begin{array}{c}\overset{\prec}{G}\left(h_1\right)=\left\langle e_1,e_2,e_3\right\rangle\\ \overset{\prec}{G}\left(h_2\right)=\left\langle e_1,e_2,e_3,e_4\right\rangle\\ \overset{\prec}{G}\left(h_3\right)=\left\langle e_2,e_3\right\rangle\end{array}$$

Clearly, $U\subseteq V$.

Definition 3.4

The two ISVNSSs $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ over the common parameter set *E* are said to be equal iff

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$\left(\tilde{F},U\right)=\left(\tilde{G},V\right)\Rightarrow U=V.$

Definition 3.5

An ISVNSS $\left(\tilde{F},U\right)$ over *E* is said to be the null SVNSS if $\tilde{F}\left(u\right)=\varnothing,\forall u\in U$ and it is denoted by $SVN^{\varnothing E}$.

Definition 3.6

An ISVNSS $\left(\tilde{F},U\right)$ over *E* is said to be the absolute SVNSS if $\tilde{F}\left(u\right)=E,\forall u\in U$ and it is denoted bySVN^{UE}.

Remark:

(i) $SVN^{\varnothing E}\subseteq\left(\tilde{F},U\right)\subseteq SVN^{UE}$

(ii) $\left(SVN^{\varnothing E}\right)^{c}=SVN^{UE}\ and\ \left(SVN^{UE}\right)^{c}=SVN^{\varnothing E}$

Now we define some operations on ISVNSSs as follows:

Definition 3.7

The complement of an ISVNSS $\left(\tilde{F},U\right)$ is denoted by $\left(\tilde{F},U\right)^{c}$ and is defined as $\left(\tilde{F},U\right)^{c}=\left(\tilde{F}^{c},U\right)$ where $\tilde{F}^{c}:U\to P\left(E\right)$ is a mapping given by $\tilde{F}^{c}\left(u\right)=\left\{e:e\in E-\tilde{F}\left(u\right)\right\}$, for all $u\in U$.

Definition 3.8

Let $\left(\tilde{F},U\right)$ and $\left(\tilde{G},V\right)$ be two ISVNSS over a common parameter set *E*, then the operator AND between $\left(\tilde{F},U\right)$ and $\left(\tilde{G},V\right)$ is denoted and defined by $\left(\tilde{F},U\right)\wedge\left(\tilde{G},V\right)=\left(\tilde{H},U\times V\right)$, where

$\tilde{H}\left(u,v\right)=\tilde{F}\left(u\right)\cap\tilde{G}\left(v\right),\forall\left(u,v\right)\in U\times V$

Definition 3.9

Let $\left(\tilde{F},U\right)$ and $\left(\tilde{G},V\right)$ be two ISVNSS over a common parameter set *E*, then the operator OR between $\left(\tilde{F},U\right)$ and $\left(\tilde{G},V\right)$ is denoted and defined by $\left(\tilde{F},U\right)\vee\left(\tilde{G},V\right)=\left(\tilde{K},U\times V\right)$, where

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$$\overset{\prec}{K}(u,v)=\overset{\prec}{F}(u)\bigcup\overset{\prec}{G}(v),\forall(u,v)\in U\times V$$

Theorem 3.10

Let $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ be two ISVNSS over a common parameter set *E*, then

$$\left(\left(\overset{\prec}{F},U\right)\wedge\left(\overset{\prec}{G},V\right)\right)^{c}=\left(\overset{\prec}{F},U\right)^{c}\vee\left(\overset{\prec}{G},V\right)^{c}\text{ and }\left(\left(\overset{\prec}{F},U\right)\vee\left(\overset{\prec}{G},V\right)\right)^{c}=\left(\overset{\prec}{F},U\right)^{c}\wedge\left(\overset{\prec}{G},V\right)^{c}$$

Proof: It is straightforward.

Definition 3.11

Let $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ be two ISVNSS over a common parameter set *E* such that $U\bigcap V\neq\varnothing$. Then the union of $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ denoted and defined by $\left(\overset{\prec}{F},U\right)\tilde{\tilde{\cup}}\left(\overset{\prec}{G},\mathrm{V}\right)=\left(\overset{\prec}{M},W\right),W=U\cap V$ and for $w\in W$,

$$\overset{\prec}{M}(w)=\overset{\prec}{F}(w)\cup\overset{\prec}{G}(w)=\left\langle\max(\mu_{F}(e),\mu_{G}(e)),\min(\nu_{F}(e),\nu_{G}(e)),\min(\gamma_{F}(e),\gamma_{G}(e))\right\rangle$$

Definition 3.12

Let $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ be two ISVNSS over a common parameter set *E* such that $U\bigcap V\neq\varnothing$. Then the intersection of $\left(\overset{\prec}{F},U\right)$ and $\left(\overset{\prec}{G},V\right)$ denoted and defined by

$\left(\overset{\prec}{F},U\right)\tilde{\tilde{\cap}}\left(\overset{\prec}{G},V\right)=\left(\overset{\prec}{O},W\right),W=U\cap V$ and for $w\in W$,

$$\overset{\prec}{O}(w)=\overset{\prec}{F}(w)\cap\overset{\prec}{G}(w)=\left\langle\min(\mu_{F}(e),\mu_{G}(e)),\max(\nu_{F}(e),\nu_{G}(e)),\max(\gamma_{F}(e),\gamma_{G}(e))\right\rangle$$

4. APPLICATION OF ISVNSS IN DECISION MAKING

The problem of decision-making (DM) in an imprecise environment has been found paramount importance in recent years. DM is a problem-solving approach to find either an optimal solution or a satisfactory result. It is a process that is more or less rational or irrational. DM is regarded as a course of action among several possible alternatives or options. It is basically of two types, group decision making (GDM) and

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individual decision-making (IDM). In 2017, Baset et al.(Abdel-Basset, Mohamed, Zhou & Hezam, 2017) proposed multi-criteria GDM based on neutrosophic hierarchy process, in (Abdel-Basset, Manogaran, Gamal & Smarandache, 2019), Baset et al. introduced GDM framework based on TOPSIS approach, In 2015, Broumi et al. (Broumi & Smarandache, 2015) introduced single-valued neutrosophic soft expert sets in DM problems, in (Das, Roy, Kar, Kar & Pamučar, 2020), Das et al. proposed neutrosophic fuzzy set and its application in DM, Jana et al. (Jana & Pal, 2019) proposed a robust single-valued neutrosophic soft aggregate operator in multi-criteria DM method, Khalil et al.(Khalil, Cao, Azzam, Smarandache & Alharbi, 2020) used the single-valued neutrosophic fuzzy set in DM problem, Manna et al. (Manna, Basu & Mondal, 2020) proposed the soft set based VIKOR approach for DM under complex neutrosophic environment, Ye(Ye, 2015) extended the TOPSIS approach in GDM based on single-valued neutrosophic linguistic numbers. In 2019, Khalil et al. (Khalil & Hassan, 2019) introduced the inverse fuzzy soft set in DM. In this section, we propose a new model in a DM based on ISVNSS. For this, we need to define the following operators.

Definition 4.1

Let $\left(\overset{\prec}{F},U\right)$ be an inverse single-valued neutrosophic soft set (ISVNSS) such that $\overset{\prec}{F}:U\to SVN^{E}$ where

$$SVN^{E}=\left\{\left\langle e,\mu_{\theta}(e),\nu_{\theta}(e),\gamma_{\theta}(e)\right\rangle : e\in E\right\},$$

then the max-min-min decision ISVNSS of SVN^{E} is denoted and defined as

$$F^{+}_{SV_{p}(e_{i})}(u)=\left\{\left\langle \vee_{e\in E}\left[\overset{\prec}{F}(u)(e)\wedge\mu_{\theta}(e)\right],\wedge_{e\in E}\left[\overset{\prec}{F}(u)(e)\vee\nu_{\theta}(e)\right],\wedge_{e\in E}\left[\overset{\prec}{F}(u)(e)\vee\gamma_{\theta}(e)\right]\right\rangle : \forall u\in U\right\}$$

Definition 4.2

Let $\left(\overset{\prec}{F},U\right)$ be an inverse single-valued neutrosophic soft set (ISVNSS) such that $\overset{\prec}{F}:U\to SVN^{E}$ where

$$SVN^{E}=\left\{\left\langle e,\mu_{\theta}(e),\nu_{\theta}(e),\gamma_{\theta}(e)\right\rangle : e\in E\right\},$$

then the max-min-min decision ISVNSS of SVN^{E} is denoted and defined as

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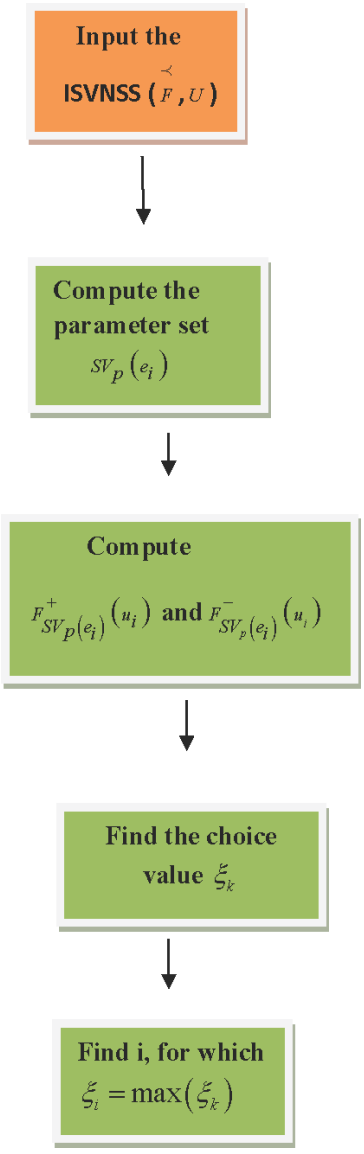
$$F^{-}_{SV_P}(e_i)(u)=\left\{\left\langle \wedge_{e\in E}\left[\left(1-\overset{\prec}{F}(u)(e)\right)\vee \mu_\theta(e)\right],\right.\right.$$
$$\left.\left.\vee_{e\in E}\left[\left(1-\overset{\prec}{F}(u)(e)\right)\wedge \nu_\theta(e)\right],\vee_{e\in E}\left[\left(1-\overset{\prec}{F}(u)(e)\right)\wedge \gamma_\theta(e)\right]\right\rangle : \forall u\in U\right\}$$

Now we construct an algorithm for a decision-making problem for selection of a suitable investing sector given in Fig 1:

Figure 1. Represents the flowchart of the proposed algorithm



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Algorithm:

Step 1: Input the inverse single-valued neutrosophic soft set $\left(\overset{\prec}{F}, U\right)$.

Step 2: Compute the SVN-parameter set

$$SV_p\left(e_i\right)=\left\langle \max\left(\mu_\theta\left(e_i\right)\right), \min\left(\nu_\theta\left(e_i\right)\right), \min\left(\gamma_\theta\left(e_i\right)\right)\right\rangle.$$

Step 3: Compute $F^{+}_{SV_p(e_i)}\left(u_i\right)$ and $F^{-}_{SV_p(e_i)}\left(u_i\right)$ defined in definition **4.1** and **4.2** respectively.

Step 4: Compute the choice value $\xi_k=\mu_{\overset{\prec}{F}^+}.\mu_{\overset{\prec}{F}^-}+\nu_{\overset{\prec}{F}^+}.\nu_{\overset{\prec}{F}^-}-\gamma_{\overset{\prec}{F}^+}.\gamma_{\overset{\prec}{F}^-}$ such that $0\leq\xi_k\leq1$, for all $u_k\in U$.

Step 5: The optimal decision is to select $u_i\in U$ if $\xi_i=\max(\xi_k)$

Step 6: If i has more than one value then any one of u may be chosen.

Example 4.3

Suppose Mr. X wants to invest his money into private sectors and for this, he has an option of five alternatives denoted by the set $U=\{s_1,s_2,s_3,s_4,s_5\}$ and the factors affecting investment is denoted and defined by the set of parameters given by:

$E=\{e_1$=economic growth, e_2=availability of finance, e_3=interest rates, e_4=state of technology$\}$

To choose the best alternatives to invest his money, we need the follow the steps discussed above.

Step1: We consider the table for inverse single-valued neutrosophic soft set given by:

Table 5. Representation of ISVNSS

U	Economic growth (e_1)	Availability of finance (e_2)	Interest rates (e_3)	State of technology (e_4)
s_1	⟨0.3,0.6,0.9⟩	⟨0.7,0.8,0.4⟩	⟨0.3,0.4,0.5⟩	⟨0.6,0.7,0.8⟩
s_2	⟨0.2,0.5,0.8⟩	⟨0.5,0.6,0.3⟩	⟨0.7,0.8,0.9⟩	⟨0.4,0.5,0.7⟩
s_3	⟨0.5,0.6,0.8⟩	⟨0.4,0.5,0.6⟩	⟨0.3,0.2,0.4⟩	⟨0.1,0.8,0.6⟩
s_4	⟨0.6,0.7,0.8⟩	⟨0.3,0.5,0.7⟩	⟨0.8,0.7,0.6⟩	⟨0.4,0.6,0.7⟩
s_5	⟨0.0,0.7,0.8⟩	⟨0.3,0.6,0.7⟩	⟨0.4,0.7,0.8⟩	⟨0.6,0.8,0.9⟩

Step2: We obtain the following parameter sets:

$SV_p(e_1)=\langle0.6,0.5,0.8\rangle$; $SV_p(e_2)=\langle0.7,0.5,0.3\rangle$; $SV_p(e_3)=\langle0.8,0.2,0.4\rangle$; $SV_p(e_4)=\langle0.6,0.5,0.6\rangle$

Step3: Using **Table 5** and parameter sets obtained in **step 2**, we calculate the values of the decision-making operators:

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$$F^{+}_{SV_p}(s_1) = \langle \vee\{0.3,0.7,0.3,0.6\}, \wedge\{0.6,0.8,0.4,0.7\}, \wedge\{0.9,0.4,0.5,0.8\}\rangle = \langle 0.7,0.4,0.4\rangle$$
$$F^{-}_{SV_p}(s_1) = \langle \wedge\{0.7,0.7,0.8,0.6\}, \vee\{0.4,0.2,0.2,0.3\}, \vee\{0.1,0.3,0.4,0.2\}\rangle = \langle 0.6,0.4,0.4\rangle$$

$$F^{+}_{SV_p}(s_2) = \langle \vee\{0.2,0.5,0.7,0.4\}, \wedge\{0.5,0.6,0.8,0.5\}, \wedge\{0.8,0.3,0.9,0.7\}\rangle = \langle 0.7,0.5,0.3\rangle$$
$$F^{-}_{SV_p}(s_2) = \langle \wedge\{0.8,0.7,0.8,0.6\}, \vee\{0.5,0.4,0.2,0.5\}, \vee\{0.2,0.3,0.1,0.3\}\rangle = \langle 0.6,0.5,0.3\rangle$$

$$F^{+}_{SV_p}(s_3) = \langle \vee\{0.5,0.4,0.3,0.1\}, \wedge\{0.6,0.5,0.2,0.8\}, \wedge\{0.8,0.6,0.4,0.6\}\rangle = \langle 0.5,0.2,0.4\rangle$$
$$F^{-}_{SV_p}(s_3) = \langle \wedge\{0.6,0.7,0.8,0.9\}, \vee\{0.4,0.5,0.2,0.2\}, \vee\{0.2,0.3,0.4,0.4\}\rangle = \langle 0.6,0.5,0.4\rangle$$

$$F^{+}_{SV_p}(s_4) = \langle \vee\{0.6,0.3,0.8,0.4\}, \wedge\{0.7,0.5,0.7,0.6\}, \wedge\{0.8,0.7,0.6,0.7\}\rangle = \langle 0.8,0.5,0.6\rangle$$
$$F^{-}_{SV_p}(s_4) = \langle \wedge\{0.6,0.7,0.8,0.6\}, \vee\{0.3,0.5,0.2,0.4\}, \vee\{0.2,0.3,0.4,0.3\}\rangle = \langle 0.6,0.5,0.4\rangle$$

$$F^{+}_{SV_p}(s_5) = \langle \vee\{0.0,0.3,0.4,0.6\}, \wedge\{0.7,0.6,0.7,0.8\}, \wedge\{0.8,0.7,0.8,0.9\}\rangle = \langle 0.6,0.6,0.7\rangle$$
$$F^{-}_{SV_p}(s_5) = \langle \wedge\{0.1,0.7,0.8,0.6\}, \vee\{0.3,0.4,0.2,0.2\}, \vee\{0.2,0.3,0.2,0.1\}\rangle = \langle 0.6,0.4,0.3\rangle$$

Step 4: Compute the choice values for each alternative given by:

ξ_1=0.42, ξ_2=0.58, ξ_3=0.24, ξ_4=0.49, and ξ_5=0.39

Step5: ξ_i=max{ξ_1,ξ_2,ξ_3,ξ_4,ξ_5}=0.58
Step6: As ξ_2 has the maximum choice value, hence the optimal choice for investment opportunity is s_2.

5. CONCLUSION AND SCOPE

In this chapter, a new soft model known as the inverse single-valued neutrosophic soft set has been proposed for decision-making problems. In this approach, there is no need to establish any other set except input information compared to the other existing single-valued neutrosophic soft decision-making algorithms. Also, we have introduced the single-valued parameter sets and perform different operations on them and study their properties. An algorithm has been introduced in solving real decision-making problems to get the best alternative. Finally, a suitable example is used to show the applicability of the inverse single-valued neutrosophic soft set. Further, in 2018, Smarandache(Smarandache, 2018) introduced a hypersoft set which can be viewed as a multiple objective functions. So, there is a huge scope to extend the present literature by using the notion of hypersoft set in various fields of knowledge that contain uncertainties. In addition, the author believe that, there will be many methods will be developed for decision-making by using the matrix representation of inverse single-valued neutrosophic soft set and

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for group decision-making problems, the researchers may concentrate to develop inverse Pythagorean single-valued neutrosophic soft set.

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Chapter 33
Neutrosophic Fuzzy Soft Matrix Theory and Its Application in Group Decision Making

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ABSTRACT

In 2020, Sujit et al. introduced the notion of the neutrosophic fuzzy set (NFS) to deal with uncertainty under a fuzzy environment where the fuzzy membership grade of every element of the field of the domain is associated with three independent neutrosophic components, namely truth, indeterminacy, and falsity membership grade. The main focus of the chapter is to introduce a neutrosophic fuzzy soft set (NFSS) to deal with uncertainty parametrically, and it gives the approximate solution to the problem. NFSS is a mixture of a neutrosophic set, fuzzy sets, and soft sets. So, it can be treated as a hybrid structure that gives more flexibility to solve multi-criteria decision-making problems under a fuzzy environment. NFSS evolved as an extension of fuzzy set, soft set, neutrosophic set, etc. Then the authors propose an algorithmic approach for group decision-making (GDM) problems using a neutrosophic fuzzy soft matrix (NFSM) and its related properties. Finally, an illustrative example shows the applicability of the proposed approach.

1. INTRODUCTION AND MOTIVATION

Classical or crisp notion gives the precise or deterministic value of an object to a set. But, sometimes due to the error in data collection, a wrong technique used for data collection, we don't know the exact value of the missing data i.e there is an information gap. Such incomplete, indeterministic or imprecise information leads to uncertainty, vagueness,or ambiguity and it cannot be described by the classical set. This leads to a fuzzy set (FS) (Zadeh, 1965) introduced by Zadeh in 1965.The Fuzzy set uses the membership degree $\mu_A(x) \in [0,1]$ to describe uncertainty. If $\mu_A(x)=0$ or 1, then there is no ambiguity arises and $\mu_A(x)$ *t*urned to a characteristic function. Due to complexity in some real-life problems, $\mu_A(x)$ can't be a crisp

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value, then we need to introduce an interval-valued fuzzy set (IVFS) (Gorzalczany, 1987) to remove such issue. In fuzzy set and interval-valued fuzzy set, there is no scope of considering the non-membership degree $\gamma_A(x)$ as we know there exist some fields of knowledge where we have to exhibit not only the membership value in support of evidence but also the non-membership value against the evidence. This leads to the introduction of the intuitionistic fuzzy set (IFS) (Atanassov, 1986). An IFS has been further extended by introducing an interval-valued intuitionistic fuzzy set (IVIFS) (Atanassov & Gargov, 1989). Some significant works related to fuzzy sets and their extensions are given in (Ashtiani,Haghighirad, Makui & Ali Montazer, 2009; De, Biswas & Roy,2001; Dengfeng & Chuntian, 2002; Garg, 2016; Mahmood, Ullah, Khan & Jan, 2019). In all types of fuzzy sets there exist difficulties to set the membership function in each particular case and it arises due to restriction given on membership function. Fuzzy set and its extensions are enabled to answer a question that is either true or false but it can't able to answer a question that is neither true nor false (unknown) i.e., there is a contradiction. For example, a set of 10 true/false type questions are given to a student and the response of the student is recorded as 5 are true, 3 are false and the remaining 2 are unknown. Similarly, we give many instances in real-life where it is not possible to answer the evidence. Such situations are managed by a neutrosophic set (NS)(Smarandache, 2005). In NS, each element of the universe is characterized by three membership degrees such as truth-membership, indeterminacy-membership, and falsity-membership degree. In the neutrosophic set, each membership degree belongs to the non-standard unit interval]-0,1+[. NS is a direct extension of the intuitionistic fuzzy set. After the introduction of NS, it has been progressed more rapidly as it provides a more general framework to explain uncertainty that contains indeterminacy and inconsistency. For the scientific and technical application we need standard unit interval and this problem is resolved by introducing a single-valued neutrosophic set(SVNS)(Wang, Smarandache, Zhang & Sunderraman, 2010) . So, SVNS can be treated as an instance of NS. For all practical application, we use SVNS instead of NS. NS has several applications among which some major contributions are proposed in (Huang, 2016; Patrascu, 2015; Ye, 2014; Ye, 2014). We find some instances in real life that contain both uncertainty and inconsistency and these can be handled by the combination of fuzzy set and neutrosophic set, which gives rise to the introduction of the neutrosophic fuzzy set(NFS)(Das, Roy, Kar, Kar & Pamučar, 2020). IN NFS, each membership degree is associated with neutrosophic components.

The Fuzzy sets and their extensions unable to describe the uncertainty parametrically. This gives rise to the introduction of the soft set (SS) (Molodtsov,1999) proposed by Molodtsov in 1999. The main advantage of using SS is that it is independent of membership degree. So, by using SS we can easily describe any kind of parametric data approximately. Many real-life applications exist which are based on SS. According to the level of uncertainty, SS has been generalized by introducing fuzzy soft set (Roy & Maji, 2007), intuitionistic fuzzy soft set(Çağman & Karataş, 2013), interval-valued fuzzy soft set(Yang, Lin, Yang, Li & Yu, 2009), bipolar soft set(Shabir & Naz, 2013), interval-valued intuitionistic fuzzy soft set(Jiang, Tang, Chen, Liu & Tang, 2010), neutrosophic soft set(Maji, 2012), fuzzy neutrosophic soft set(Arockiarani, 2014),etc.

Matrices play a key role in the broad area of science and engineering because of their concise representation and it is helpful for numerical calculation and can easily implemented through computer. Earlier classical matrix theory has been used to solve various problems that are deterministic. But classical matrix is unable to solve the real problems that contain uncertainty. To solve the purpose, soft matrices and their properties are proposed in (Çağman & Enginoğlu, 2010), Hashimoto(Hashimoto, 1983) introduced a canonical form of a transitive matrix, Thomson(Thomason,1977) proposed convergence of powers of the fuzzy matrix, Khan et al. (Khan & Pal, 2007) proposed intuitionistic fuzzy matrices,

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In (Khan & Pal, 2014), Khan et al. defined interval-valued intuitionistic fuzzy matrices, Pal (Pal, 2015) grounded interval-valued fuzzy matrices, Varol et al. (Varol, Cetkin & Aygun, 2019) proposed neutrosophic matrix, Priya et al.(Priya, Ramya, Sangeetha & Saranya, 2019) introduced the basic concept of fuzzy matrices, Raich et al.(Raich, Gawande & Triapath, 2011) proposed fuzzy matrix theory and its application recognizing the quality of effective teacher, Muthuraji et al.(Muthuraji, Sriram & Murugadas, 2016) introduced the decomposition of intuitionistic fuzzy matrices, Pal et al.(Pal & Mondal, 2019) defined bipolar fuzzy matrices, Gani et al.(Gani, Kannan & Manikandan, 2016) proposed inner product over fuzzy matrices, Dhar et al.(Dhar, Broumi & Smarandache, 2014) presents a note on square neutrosophic fuzzy matrices, Smarandache et al. (Smarandache, Broumi & Dhar, 2013) studied fuzzy soft matrix based on reference function.

By embedding the idea of a soft set, Faried et al.(Faried, Ali & Sakr, 2019) introduced vague soft matrix-based decision-making, Lavanya et al.(Lavanya & Akila, 2019) gave an application of fuzzy soft matrix in medical diagnosis, Yang et al.(Yang & Ji, 2011) defined fuzzy soft matrices and their applications, Basu et al.(Basu, Mahapatra & Mondal, 2012) introduced different types of matrices in fuzzy soft set theory, Warrier et al.(Warrier, Mathew & Alcantud, 2020) introduced fuzzy soft matrices on fuzzy soft multi-set and its applications in optimization problems, Borah et al.(Borah, Neog & Sut,2012) defined fuzzy soft matrix theory and its application in decision-making, Mao et al.(Mao, Yao & Wang, 2013) introduced group decision-making methods based on intuitionistic fuzzy soft matrices, Rathika et al.(Rathika & Subramanian, 2018) proposed intuitionistic fuzzy soft matrices, Zulquarnain et al.(Zulqarnain, Saeed, Ahamad, Abdal, Zafar & Aslam, 2020) present intuitionistic fuzzy soft matrices for disease diagnosis, Basu et al.(Basu, Mahapatra & Mondal, 2014) introduced interval-valued fuzzy soft matrices and their application, Singh et al.(Singh, Hooda & Malik, 2021)described geometric mean method for interval-valued fuzzy soft matrix, Zulqarnain et al.(Zulqarnain & Saeed, 2017) presented an application of interval-valued fuzzy soft matrix in decision making, Das et al.(Das, Kar & Pal, 2014) introduced multiple attribute group decision-making using an interval-valued intuitionistic fuzzy soft matrix, Chinnadurai et al.(Chinnadurai, Swaminathan, Bobin & Kathar Kani, 2019) proposed the generalized interval-valued intuitionistic fuzzy soft matrix and their application to multi-criteria decision-making, Deli et al.(Deli & Broumi, 2015) defined neutrosophic soft matrix and their operators, Das et al.(Das, Kumar, Kar & Pal, 2019) introduced group decision-making using a neutrosophic soft matrix, Basu et al.(Basu & Mondal, 2015) introduced neutrosophic soft matrix and its application in solving group decision-making problems from medical science, Dhar(Dhar, 2021) introduced neutrosophic soft matrix and its application in medical diagnosis, Kavitha et al.(Kavitha, Murugadas & Sriram, 2018) defined the power of fuzzy neutrosophic soft matrices, in(Kavitha,Murugadas & Sriram, 2018), Kavitha proposed minus order on fuzzy neutrosophic soft matrices, Sumathi et al.(Sumathi & Arockiaranii, 2015) defined fuzzy neutrosophic soft matrix model in decision-making, Karaslan et al.(Karaaslan & Hayat, 2018) introduced some new operators on single-valued neutrosophic matrices and their application in multi-criteria group decision-making, Deli et al.(Deli & Broumi, 2014) proposed neutrosophic soft sets and neutrosophic soft matrices based on decision-making etc. In addition, Ali et al.(Ali, Deli & Smarandache, 2016) introduced the theory of neutrosophic cubic sets and their application in pattern recognition, the bipolar neutrosophic soft sets and application in decision-making done in(Ali, Son, Deli & Tien, 2017), Deli et al.(Deli, Toktas & Broumi, 2014) presented the neutrosophic parameterized soft relations and their application, the npn-soft sets theory and their applications are proposed in (Deli, 2015), the neutrosophic soft relation and some properties studied in (Deli & Broumi, 2015), the interval-valued neutrosophic soft sets and its decision-making is initiated by Deli (Deli, 2017), the

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ivnpiv-neutrosophic soft sets and their decision-making based on similarity measure is done in (Deli, Eraslan & Çağman, 2018).

The neutrosophic fuzzy soft matrix(NFSM) is a matrix defined on a neutrosophic fuzzy soft set(NFSS) and the NFSS is a hybrid structure that is formed as a combination of soft set, fuzzy set, and neutrosophic set and it gives more flexibility in solving multi-criteria decision-making problems. The soft set is used to describe uncertainty parametrically, the fuzzy set is efficient to describe uncertainty by a membership degree, and the neutrosophic set is capable of handling inconsistency and indeterminacy by a triple membership degree, but NFSS proved to be more functional as it can handle both uncertain,inconsistent, and indeterminate information. The use of NFSS is quiet logical in the sense that, fuzzy soft set is capable to measure parametric uncertainty by a crisp value, but NFSS offer a higher concept of modeling the parametric inconsistent and indeterminate fuzzy information. For example, we know that diagnosis is a process to determine the nature of disease by observing a certain set of symptoms of a patient. Suppose a doctor wants to diagnose a patient for having a viral disease by observing twelve symptoms. After observation, the doctor founds that, four symptoms are positive, three are negative, and the remaining five vary with time, i.e., at time t*1* two symptoms can be detected, and at time t*2* the remaining three symptoms can be detected, i.e., they are inconsistent. These two time variation instances can be represented by neutrosophic numbers

$\left(\frac{4}{12},\frac{2}{12},\frac{3}{12}\right)$ and $\left(\frac{4}{12},\frac{3}{12},\frac{3}{12}\right)$ respectively.

There may be more time variations occur in some cases. Such issue can be resolved by associating membership grade with neutrosophic numbers to express the indeterminate and inconsistent information. Therefore, the hybrid structure called neutrosophic fuzzy soft set is the more generalized form to represent uncertainty, indeterminacy, and inconsistency parametrically. The main motivation behind introducing NFSM is that to represent the parametric data present in real life that contain both uncertainty and indeterminacy in a matrix form and formulate a model with an aid of choice matrix to solve group decision-making problems. Matrix representation of NFSS is easy to handle and perform several algebraic operations to solve group-decision making problems with the help of an algorithm. Moreover, NFSM is a generalization of a classical matrix, soft matrix, fuzzy soft matrix, intuitionistic fuzzy soft matrix, interval-valued fuzzy soft matrix, interval-valued intuitionistic fuzzy soft matrix, neutrosophic soft matrix, etc. So, NFSM plays an important role in both uncertain and inconsistent group decision-making problems with neutrosophic fuzzy parameters.

The rest of the chapter is organized in the following manner: In section 2, some of the basic concepts have been given. In section 3, neutrosophic fuzzy soft set and its associated properties are studied. In section 4, neutrosophic fuzzy soft matrices and their kinds with an aid of choice matrices are studied in details. In section 5, neutrosophic fuzzy sets based cross-entropy aggregation operator have been derived. In section 6, an algorithm based on NFSM and NFCM for GDM have been proposed. In section 7, to validate the proposed algorithm, an example is executed. In section 8, conclusion and the scope of the present work is given.

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2. PRELIMINARIES

Some basic definitions that are relevant to the subsequent sections are presented here.

Definition 2.1 (Zadeh, 1965)

Let S be the universal set and $x \in S$, then a fuzzy set $\overset{\prec}{A}$ in S is defined as

$$\overset{\prec}{A} = \left\{ \left(x, \mu_{\overset{\prec}{A}}(x) \right) : \mu_{\overset{\prec}{A}}(x) \in [0,1], x \in S \right\},$$

where $\mu_{\overset{\prec}{A}}(x)$ denotes the degree of belongingness of x in $\overset{\prec}{A}$. The main challenge in a fuzzy set is to set the membership function in each particular case.

Definition 2.2 (Molodtsov, 1999)

Let S be an initial universe and E be the set of parameters under consideration. Let $\overset{\prec}{A} \subseteq E$ and $P(S)$ denotes the power set of S. Then the pair $\left(F, \overset{\prec}{A} \right)$ is called the soft set over S, where $F : \overset{\prec}{A} \to P(S)$. The problem of setting membership function does not arise in soft set theory and so it is very convenient and easily applicable in practice. It gives the approximate description of parametric data.

Definition 2.3 (Smarandache, 2005)

Let S be the set of universe and $x \in S$. Then a neutrosophic set $\overset{\prec}{A}$ in S is characterized by the truth, indeterminacy, and the falsity membership-function are denoted by $T_{\overset{\prec}{A}}, I_{\overset{\prec}{A}}$ and $F_{\overset{\prec}{A}}$ respectively and defined by

$$\overset{\prec}{A} = \left\{ \left\langle x, T_{\overset{\prec}{A}}(x), I_{\overset{\prec}{A}}(x), F_{\overset{\prec}{A}}(x) \right\rangle : x \in S \right\}$$

with the condition

$$T_{\overset{\prec}{A}}, I_{\overset{\prec}{A}}, F_{\overset{\prec}{A}} : S \to \left] {}^{-}0, 1^{+} \right[\text{ and } {}^{-}0 \le \sup T_{\overset{\prec}{A}}(x) + \sup I_{\overset{\prec}{A}}(x) + \sup F_{\overset{\prec}{A}}(x) \le 3^{+}.$$

Here, $]^{-}0,1^{+}[$ represents the real non-standard interval as an extension of the real standard interval $[0,1]$. For practical implication, we use the notion of the single-valued neutrosophic set which is defined below.

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Definition 2.4 (Wang, Smarandache, Zhang & Sunderraman, 2010)

Let *S* be the set of universe and $x \in S$. Then a single-valued neutrosophic set (SVNS) $\overset{\prec}{A}$ in *S* is characterized by the truth, indeterminacy, and the falsity membership function are denoted by $T_{\overset{\prec}{A}}, I_{\overset{\prec}{A}}$, and $F_{\overset{\prec}{A}}$ respectively and defined by

$$\overset{\prec}{A} = \left\{ \left\langle x, T_{\overset{\prec}{A}}(x), I_{\overset{\prec}{A}}(x), F_{\overset{\prec}{A}}(x) \right\rangle : x \in S \right\}$$

with the condition

$$T_{\overset{\prec}{A}}, I_{\overset{\prec}{A}}, F_{\overset{\prec}{A}} : S \to [0,1] \text{ and } 0 \le \sup T_{\overset{\prec}{A}}(x) + \sup I_{\overset{\prec}{A}}(x) + \sup F_{\overset{\prec}{A}}(x) \le 3.$$

Definition 2.5 (Das, Roy, Kar, Kar & Pamučar, 2020)

Let *S* be a set of objects and

$$\overset{\prec}{A} = \left\{ \left(y, \mu_{\overset{\prec}{A}}(y) \right) : \mu_{\overset{\prec}{A}}(y) \in [0,1], y \in S \right\}$$

be a fuzzy set. Then a neutrosophic fuzzy set (NFS)

$$\overset{\prec\prec}{A} = \left\{ \left\langle y, \mu_{\overset{\prec\prec}{A}}(y), T_{\overset{\prec\prec}{A}}(y,\mu), I_{\overset{\prec\prec}{A}}(y,\mu), F_{\overset{\prec\prec}{A}}(y,\mu) \right\rangle : y \in S \right\},$$

where each membership value is expressed by truth, indeterminacy, and falsity membership function which are denoted by $T_{\overset{\prec\prec}{A}}(y,\mu), I_{\overset{\prec\prec}{A}}(y,\mu)$, and $F_{\overset{\prec\prec}{A}}(y,\mu)$ respectively with the condition

$$T_{\overset{\prec\prec}{A}}, I_{\overset{\prec\prec}{A}}, F_{\overset{\prec\prec}{A}} : S \to \left]^{-}0, 1^{+}\right[\text{ and } {}^{-}0 \le \sup T_{\overset{\prec\prec}{A}}(x) + \sup I_{\overset{\prec\prec}{A}}(x) + \sup F_{\overset{\prec\prec}{A}}(x) \le 3^{+}.$$

This notion is used when the membership function itself is inconsistent.

In scientific and engineering problems it is very difficult to apply neutrosophic fuzzy sets as the membership functions belong to the real standard or non-standard interval. Therefore, to eradicate such an issue, throughout the paper we use the notion of the single-valued neutrosophic fuzzy set(SVNFS) as an instance of neutrosophic fuzzy set. In SVNFS we use the real standard interval [0, 1] instead of $]^{-}0,1^{+}[$.

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3. NEUTROSOPHIC FUZZY SOFT SET

Definition 3.1

Let Y be the set of universe and E be the set of parameters. Let $NFS(Y)$ denotes the set of all neutrosophic fuzzy subset of Y, and $A \subseteq E$. Then a pair $(NF_{(A)},E)$ is called the neutrosophic fuzzy soft set (NFSS) over Y, where $NF_{(A)}$ is a mapping given by $NF_{(A)}: E \rightarrow NFS(Y)$.

Example 3.1.1

Let $Y=\{p_1,p_2,p_3,p_4,p_5\}$ be a set of five objects i.e. places under consideration and $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ be a set of parameters that influence the climate of a place, where e_1 stands for latitude, e_2 stands for elevation, e_3 stands for ocean currents, e_4 stands for topography, e_5 stands for vegetation, and e_6 stands for prevailing winds. Let $A=\{e_1,e_3,e_4,e_6\} \subseteq E$. Here the parameters associated with the objects are considered both uncertain and inconsistent. Uncertainty can be determined by the fuzzy set to some extent by membership degree but to deal with inconsistency we need to assign neutrosophic membership degrees corresponding to each fuzzy membership degree. Then, to describe the climatic condition of the places, we have the following neutrosophic fuzzy soft set:

$$(NF_{(A)},E)=$$
$$\left\{\begin{array}{l}\left(e_1,\left\{p_1/(0.45,0.6,0.35,0.55),p_3/(0.65,0.74,0.8,0.6),p_4/(0.2,0.4,0.55,0.6)\right\}\right),\\ \left(e_3,\{\varnothing\}\right),\left(e_4,\left\{p_1/(0.5,0.6,0.5,0.7),p_2/(0.3,0.5,0.42,0.56),p_5/(0.5,0.3,0.7,0.8)\right\}\right),\\ \left(e_6,\left\{p_3/(0.24,0.36,0.75,0.4),p_4/(0.6,0.12,0.34,0.45),p_5/(0.4,0.56,0.75,0.35)\right\}\right)\end{array}\right\}$$

Definition 3.2 Let $(NF_{(A)},E_1)$ and $(NF_{(B)},E_2)$ be two NFSSs over the common universe Y. Then $(NF_{(A)},E_1)$ is said to be neutrosophic fuzzy soft subset of $(NF_{(B)},E_2)$ if $E_1 \subseteq E_2$, and

$$\mu_{NF_{A(e)}}(y) \le \mu_{NF_{B(e)}}(y), T_{NF_{A(e)}}(y) \le T_{NF_{B(e)}}(y), I_{NF_{A(e)}}(y) \ge I_{NF_{B(e)}}(y), F_{NF_{A(e)}}(y) \ge F_{NF_{B(e)}}(y),$$
$\forall e \in E_1$

and $y \in Y$.

Definition 3.3 Two NFSSs $(NF_{(A)},E_1)$ and $(NF_{(B)},E_2)$ over the common universe Y are said to be equal if $(NF_{(A)},E_1)$ is said to be neutrosophic fuzzy soft subset of $(NF_{(B)},E_2)$ and $(NF_{(B)},E_2)$ is said to be neutrosophic fuzzy soft subset of $(NF_{(A)},E_1)$ and we denote it by $(NF_{(A)},E_1) = (NF_{(B)},E_2)$.

Definition 3.4 The complement of a NFSS $(NF_{(A)},E)$ is denoted by $(NF_{(A)},E)^c$ and is defined as $\left(NF_{(A)},E\right)^c = \left(NF^c{}_{(A)},\leftarrow E\right)$ where $NF^c{}_{(A)}: E \rightarrow NFS(Y)$ is a mapping given by

$$NF^c_{A(\neg e)}(y) = \left\langle \mu^c_{A(\neg e)}(y), T^c_{A(\neg e)}(y), I^c_{A(\neg e)}(y), F^c_{A(\neg e)}(y) \right\rangle.$$

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Definition 3.5 A NFSS ($NF_{(A)}$,E) over Y is said to be null or empty NFSS if

$$\mu_{NF_{A(e)}}(y)=0, T_{NF_{A(e)}}(y)=0, I_{NF_{A(e)}}(y)=0, F_{NF_{A(e)}}(y)=0 \text{ , } \forall e\in E \text{ and } \forall y\in Y,$$

and it is denoted by Φ.

Definition 3.6 Let ($NF_{(A)}$,E_1) and ($NF_{(B)}$,E_2) be two NFSSs over the common universe Y. Then their union is denoted and defined as

$$\left(NF_{(A)},E_1\right)\tilde{\cup}\left(NF_{(B)},E_2\right)=\left(NF_{(C)},E_3\right)$$

where $E_3=E_1\cup E_2$ and the fuzzy-membership, truth-membership, indeterminacy-membership, and the falsity-membership of ($NF_{(C)}$,E_3) are given as follows:

$$\mu_{NF_{C(e)}}(y)=\begin{cases}\mu_{NF_{A(e)}}(y), if\ e\in E_1-E_2\\ \mu_{NF_{B(e)}}(y), if\ e\in E_2-E_1\\ \max\left(\mu_{NF_{A(e)}}(y),\mu_{NF_{B(e)}}(y)\right), if\ e\in E_1\cap E_2\end{cases}$$

$$T_{NF_{C(e)}}(y)=\begin{cases}T_{NF_{A(e)}}(y), if\ e\in E_1-E_2\\ T_{NF_{B(e)}}(y), if\ e\in E_2-E_1\\ \max\left(T_{NF_{A(e)}}(y),T_{NF_{B(e)}}(y)\right), if\ e\in E_1\cap E_2\end{cases}$$

$$I_{NF_{C(e)}}(y)=\begin{cases}I_{NF_{A(e)}}(y), if\ e\in E_1-E_2\\ I_{NF_{B(e)}}(y), if\ e\in E_2-E_1\\ \dfrac{I_{NF_{A(e)}}(y)+\mathrm{I}_{NF_{B(e)}}(y)}{2}, if\ e\in E_1\cap E_2\end{cases}$$

$$F_{NF_{C(e)}}(y)=\begin{cases}F_{NF_{A(e)}}(y), if\ e\in E_1-E_2\\ F_{NF_{B(e)}}(y), if\ e\in E_2-E_1\\ \min\left(F_{NF_{A(e)}}(y),F_{NF_{B(e)}}(y)\right), if\ e\in E_1\cap E_2\end{cases}$$

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Definition 3.7 Let $(NF_{(A)},E_1)$ and $(NF_{(B)},E_2)$ be two NFSSs over the common universe Y. Then their intersection is denoted and defined as

$$\left(NF_{(A)},E_1\right)\tilde{\cap}\left(NF_{(B)},E_2\right)=\left(NF_{(C)},E_3\right)$$

where $E_3=E_1\cap E_2$ and the fuzzy-membership, truth-membership, indeterminacy-membership, and the falsity-membership of $(NF_{(C)},E_3)$ are given as follows:

$$\mu_{NF_{C(e)}}(y)=\min\left(\mu_{NF_{A(e)}}(y),\mu_{NF_{B(e)}}(y)\right),$$

$$T_{NF_{C(e)}}(y)=\min\left(T_{NF_{A(e)}}(y),T_{NF_{B(e)}}(y)\right),$$

$$I_{NF_{C(e)}}(y)=\frac{I_{NF_{A(e)}}(y)+\mathrm{I}_{NF_{B(e)}}(y)}{2},\text{ and}$$

$$F_{NF_{C(e)}}(y)=\max\left(F_{NF_{A(e)}}(y),F_{NF_{B(e)}}(y)\right)\forall e\in E_3.$$

Proposition 3.8

1. $\left(NF_{(A)},E\right)\tilde{\cup}\left(NF_{(A)},E\right)=\left(NF_{(A)},E\right)$
2. $\left(NF_{(A)},E\right)\tilde{\cup}\left(NF_{(B)},E\right)=\left(NF_{(B)},E\right)\tilde{\cup}\left(NF_{(A)},E\right)$
3. $\left(NF_{(A)},E\right)\tilde{\cap}\left(NF_{(A)},E\right)=\left(NF_{(A)},E\right)$
4. $\left(NF_{(A)},E\right)\tilde{\cup}\,\Phi=\left(NF_{(A)},E\right),\left(NF_{(A)},E\right)\tilde{\cup}\,\Phi=\Phi$
5. $\left[\left(NF_{(A)},E\right)^c\right]^c=\left(NF_{(A)},E\right)$
6. $\left(\left(NF_{(A)},E\right)\tilde{\cup}\left(NF_{(B)},E\right)\right)^c=\left(NF_{(A)},E\right)^c\tilde{\cap}\left(NF_{(B)},E\right)^c$

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7. $\left(\left(NF_{(A)},E\right)\tilde{\cap}\left(NF_{(B)},E\right)\right)^{c}=\left(NF_{(A)},E\right)^{c}\tilde{\cup}\left(NF_{(B)},E\right)^{c}$

Proof: The proofs of the propositions are obvious.

4. NEUTROSOPHIC FUZZY SOFT MATRICES AND OPERATIONS ON THEM WITH AN AID OF CHOICE MATRICES

In this section, we present some basic concepts of neutrosophic fuzzy soft matrices and discuss some basic operations on them with necessary examples:

Definition 4.1

Let $(NF_{(A)},E)$ be a NFSS over the universe of discourse U and E be the set of parameters such that $A\acute{I}E$. Then a subset of $U\times E$ is uniquely defined by the relation $\{(y,e)$: $e\in E$, $y\in NF_{(A)}(e)\}$ and is denoted by $R_A=(NF_{(A)},E)$. The relation R_A is characterized by the fuzzy membership function μ_A: $U\times E\to[0,1]$, truth fuzzy membership function $T_{\mu_A}:U\times E\to[0,1]$, indeterminacy fuzzy membership function $I_{\mu_A}:U\times E\to[0,1]$, and the falsity fuzzy membership function $F_{\mu_A}:U\times E\to[0,1]$. Then the representation of R_A is given by

$$R_A=\left\{\left\langle\mu_A(y,e),T_{\mu_A}(y,e),I_{\mu_A}(y,e),F_{\mu_A}(y,e)\right\rangle:\mu_A\in[0,1],0\le T_{\mu_A}+I_{\mu_A}+F_{\mu_A}\le 3,(y,e)\in U\times E\right\}$$

If we consider $U=\{y_1,y_2,\ldots,y_m\}$ and $E=\{e_1,e_2,\ldots,e_n\}$, then the matrix representation of R_A is given by:

$$R_A=\left(a_{ij}\right)_{m\times n}=\begin{pmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\vdots&\vdots&\ddots&\vdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{pmatrix},$$

Where $a_{ij}=\left(\mu_A(y_i,e_j),T_{\mu_A}(y_i,e_j),I_{\mu_A}(y_i,e_j),F_{\mu_A}(y_i,e_j)\right)$

The above matrix is called the neutrosophic fuzzy soft matrix (NFSM) corresponding to the neutrosophic fuzzy soft set (NFSS) $(NF_{(A)},E)$ over U.

For example, the matrix representation of NFSS described in example **3.1.1** is shown in Table 1 given by:

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Table 1. Matrix representation of NFSS ($NF_{(A)}$,E)

U/E	e_1	e_2	e_3	e_4	e_5	e_6
p_1	(0.45, 0.6, 0.35, 0.55)	0	0	(0.5, 0.6, 0.5, 0.7)	0	0
p_2	0	0	0	(0.3, 0.5, 0.42, 0.56)	0	0
p_3	(0.65, 0.74, 0.8, 0.6)	0	0	0	0	(0.24, 0.36, 0.75, 0.4)
p_4	(0.2, 0.4, 0.55, 0.6)	0	0	0	0	(0.6, 0.12, 0.34, 0.45)
p_5	0	0	0	(0.5, 0.3, 0.7, 0.8)	0	(0.4, 0.56, 0.75, 0.35)

A matrix whose rows and columns both indicate parameters is called a choice matrix. Pertaining to this idea we give the following definition.

Definition 4.2

Let ζ be a neutrosophic fuzzy choice matrix that is square and whose rows and columns both indicate parameters. Then the elements of ζ are denoted by $\zeta(i,j)$ and it is defined as $\zeta(i,j)= (1,1,0.5,0)$ when both the parameters at the i^{th} row and j^{th} column are choice parameters of the decision-maker

$=(0,0,0.5,1)$

when at least one of the i^{th} row or j^{th} column parameters be not under the choice of a decision-maker

E**xample 4.2.1**

Let $U= \{p_1,p_2,p_3,p_4\}$ be a set of plants under consideration from which Mr. X wants to select the best plant for gardening in a yard and the factors affecting the gardening is denoted by the set of parameters $E= \{e_1,e_2,e_3,e_4,e_5\}$ where e_1=environment, e_2=plant size, e_3=amount of shade, e_4=flower time, e_5=resistance to disease and parasites. Let $A_X= \{e_1,e_3,e_5\} \subseteq E$ and based on A, Mr. X wants to select the plant. Therefore, the neutrosophic fuzzy choice matrix(NFCM) for Mr. X is represented by

$$\zeta_x(i,j) = e_x \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \overset{\begin{matrix} e_x & & & & \end{matrix}}{\begin{pmatrix} (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) \end{pmatrix}}$$

In combined NFCM, rows indicate choice parameters of the single decision-maker and columns indicate the combined choice parameters of the remaining decision-makers. For example, suppose Mr. X and Mr. Y want to select a plant as per their combined opinion. Let the attributes preferred by Mr. Y is denoted by the set $A_Y= \{e_1,e_2,e_5\} \subseteq E$. Then, the combined neutrosophic fuzzy choice matrix is given below:

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$$\zeta_{(x,y)}(i,j) = e_x \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \overset{\begin{matrix} e_y \end{matrix}}{\begin{pmatrix} (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \end{pmatrix}}$$

The above-combined choice matrix can also be represented in its transpose form.

Suppose we are interested to form another combined matrix associated with three decision-makers. For this, let Mr. Z wants to select a plant together with Mr. X and Mr. Y based on the choice of attributes $A_Z = \{e_3, e_4, e_5\} \subseteq E$.

Then the combined choice matrix of Mr. X, Mr. Y, and Mr. Z will be of three different types given as below:

$$\zeta_{(x,y\wedge z)}(i,j) = e_x \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \overset{\begin{matrix} e_{y\wedge z} \end{matrix}}{\begin{pmatrix} (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \end{pmatrix}}$$

$$\zeta_{(y,x\wedge z)}(i,j) = e_y \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \overset{\begin{matrix} e_{x\wedge z} \end{matrix}}{\begin{pmatrix} (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) \end{pmatrix}}$$

$$\zeta_{(z,x\wedge y)}(i,j) = e_z \begin{matrix} \\ \\ \\ \\ \\ \end{matrix} \overset{\begin{matrix} e_{x\wedge y} \end{matrix}}{\begin{pmatrix} (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) \end{pmatrix}}$$

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Definition 4.3

Let $(N, F_A)_{m\times n} = [a_{ij}]_{m\times n}$ and $(N, F_B)_{m\times n} = [b_{ij}]_{m\times n}$ be two NFSMs. Then, they are said to be conformable for addition if they are of the same order and their sum is denoted by $(c_{ij})_{m\times n} = [a_{ij}]_{m\times n} \tilde{\oplus} [b_{ij}]_{m\times n}$, where $(c_{ij})_{m\times n}$ is also an NFSM and it is obtained as follows:

$$(c_{ij}) = \left(\max\left(\mu_{A_{ij}}, \mu_{B_{ij}}\right), \max\left(T_{\mu_{A_{ij}}}, T_{\mu_{B_{ij}}}\right), avg\left(I_{\mu_{A_{ij}}}, I_{\mu_{B_{ij}}}\right), \min\left(F_{\mu_{A_{ij}}}, F_{\mu_{B_{ij}}}\right)\right), \forall i, j.$$

Example 4.3.1

Let

$$[a_{ij}] = \begin{pmatrix} (0.3, 0.5, 0.45, 0.65) & (0.25, 0.65, 0.5, 0.4) & (0.12, 0.4, 0.6, 0.45) \\ (0.4, 0.25, 0.6, 0.55) & (0.35, 0.6, 0.75, 0.8) & (0.7, 0.4, 0.56, 0.75) \\ (0.2, 0.6, 0.55, 0.45) & (0.4, 0.5, 0.35, 0.55) & (0.4, 0.45, 0.5, 0.6) \end{pmatrix}$$

and

$$[b_{ij}] = \begin{pmatrix} (0.2, 0.6, 0.55, 0.35) & (0.34, 0.45, 0.6, 0.25) & (0.34, 0.56, 0.6, 0.4) \\ (0.7, 0.45, 0.57, 0.65) & (0.15, 0.56, 0.55, 0.38) & (0.35, 0.56, 0.6, 0.8) \\ (0.4, 0.5, 0.75, 0.85) & (0.6, 0.4, 0.55, 0.34) & (0.2, 0.6, 0.35, 0.45) \end{pmatrix}$$

be two NFMs. Then, their sum is given by

$$(c_{ij})_{m\times n} = (c_{ij}) = \begin{pmatrix} (0.3, 0.6, 0.5, 0.35) & (0.34, 0.65, 0.55, 0.25) & (0.34, 0.56, 0.6, 0.4) \\ (0.7, 0.45, 0.58, 0.55) & (0.35, 0.6, 0.65, 0.38) & (0.7, 0.56, 0.58, 0.75) \\ (0.4, 0.6, 0.65, 0.45) & (0.6, 0.5, 0.45, 0.34) & (0.4, 0.6, 0.42, 0.45) \end{pmatrix}$$

Definition 4.4

Let $(N, F_A)_{m\times n} = [a_{ij}]_{m\times n}$ and $(N, F_B)_{m\times n} = [b_{ij}]_{m\times n}$ be two NFSMs. Then, they are said to be conformable for subtraction if they are of the same order and their difference is denoted by $(c_{ij})_{m\times n} = [a_{ij}]_{m\times n} \tilde{o} [b_{ij}]_{m\times n}$, where $(c_{ij})_{m\times n}$ is also an NFSM and it is obtained as follows:

$$(c_{ij}) = \left(\min\left(\mu_{A_{ij}}, \mu_{B_{ij}}\right), \min\left(T_{\mu_{A_{ij}}}, T_{\mu_{B_{ij}}}\right), avg\left(I_{\mu_{A_{ij}}}, I_{\mu_{B_{ij}}}\right), \max\left(F_{\mu_{A_{ij}}}, F_{\mu_{B_{ij}}}\right)\right), \forall i, j.$$

Example 4.4.1

For the two NFSMs considered in example 3.2.1, their difference is given by

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$$\left(c_{ij}\right)=\begin{pmatrix} (0.2,0.5,0.5,0.65) & (0.25,0.45,0.55,0.4) & (0.12,0.4,0.6,0.45) \\ (0.4,0.25,0.58,0.65) & (0.15,0.56,0.65,0.8) & (0.35,0.4,0.58,0.8) \\ (0.2,0.5,0.65,0.85) & (0.4,0.4,0.45,0.55) & (0.2,0.45,0.42,0.6) \end{pmatrix}$$

Definition 4.5

The complement of an NFSM $\left(N,F_A\right)_{m\times n}=\left[a_{ij}\right]_{m\times n}$ is denoted by $\left(N^c,F_A\right)_{m\times n}=\left[a^c{}_{ij}\right]_{m\times n}$ where $\left[a^c{}_{ij}\right]_{m\times n}$ is the matrix representation of the NFSS $\left(NF_{(\neg A)},E\right)$ and is defined as

$$a^c{}_{ij}=\left(\mu^c_{a_{ij}},T^c_{\mu_{a_{ij}}},I^c_{\mu_{a_{ij}}},F^c_{\mu_{a_{ij}}}\right)=\left(1-\mu_{a_{ij}},1-T_{\mu_{a_{ij}}},1-I_{\mu_{a_{ij}}},1-F_{\mu_{a_{ij}}}\right)$$

Definition 4.6

The cardinal set of NFSS ($NF_{(A)}$,E) is denoted by ($\chi NF_{(A)}$,E) and is defined by

$$\left(\chi NF_{(A)},E\right)=\left\{\left\langle \mu_{\chi NF_{(A)}}(x),T_{\mu_{\chi NF_{(A)}}}(x),I_{\mu_{\chi NF_{(A)}}}(x),F_{\mu_{\chi NF_{(A)}}}(x)\right\rangle : x\in E\right\},$$

Where

$$\mu_{\chi NF_{(A)}}(x)=\sum_{x\in E,u\in U}\mu_{NF_{(A)}}(x)/\left|E_n\right|,$$

$$T_{\mu_{\chi NF_{(A)}}}(x)=\sum_{x\in E,u\in U}T_{\mu NF_{(A)}}(x)/\left|E_n\right|,$$

$$I_{\mu_{\chi NF_{(A)}}}(x)=\sum_{x\in E,u\in U}\left|0.5-I_{\mu NF_{(A)}}(x)\right|/\left|E_n\right|$$

and $F_{\mu_{\chi NF_{(A)}}}(x)=\sum_{x\in E,u\in U}F_{\mu NF_{(A)}}(x)/\left|E_n\right|$. E_n can be defined as

$$E_n=\left\{i:\mu_{NF_{(A)}}(x),T_{\mu_{NF_{(A)}}}(x),I_{\mu_{NF_{(A)}}}(x),F_{\mu_{NF_{(A)}}}(x)\neq 0,\ \forall x\in E\right\}.$$

Cardinal score of an NFSS ($NF_{(A)}$,E) corresponding to the cardinal set ($\chi NF_{(A)}$,E) is defined as

$$S\left(\chi NF_{(A)}\right)=\left(\mu_{\chi NF_{(A)}}(x)+T_{\mu_{\chi NF_{(A)}}}(x)+I_{\mu_{\chi NF_{(A)}}}(x)-F_{\mu_{\chi NF_{(A)}}}(x)\right)/\left|U\right|$$

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Definition 4.7

The product of NFSM (N,F_A) and the combined NFCM $\zeta(c)$ exists if the number of columns of NFSM is equal to the number of rows of combined NFCM. Their product is denoted by $\left\{\left(N,F_A\right)\otimes\zeta\left(c\right)\right\}$ and defined as

$$\left\{\left(N,F_A\right)\otimes\zeta\left(c\right)\right\}=\left\{\left[a_{ij}\right]_{m\times n}\otimes\left[b_{jk}\right]_{n\times p}\right\}=\left\{\left[c_{ik}\right]_{m\times p}\right\},$$

where

$$c_{ik}=\left(\max_{j=1}^{n}\left(\min\left\{\mu_{a_{ij}},\mu_{b_{jk}}\right\}\right),\max_{j=1}^{n}\left(\min\left\{T_{\mu_{a_{ij}}},T_{\mu_{b_{jk}}}\right\}\right),\max_{j=1}^{n}\left(\min\left\{I_{\mu_{a_{ij}}},I_{\mu_{b_{jk}}}\right\}\right),\min_{j=1}^{n}\left(\max\left\{F_{\mu_{a_{ij}}},F_{\mu_{b_{jk}}}\right\}\right)\right)$$

Example 4.7.1

Let us consider the NFSM and the NFCM of MR. X are respectively defined by

$(N,F_A)=$

$$\begin{pmatrix}(0.3,0.4,0.3,0.6) & (0.4,0.3,0.5,0.3) & (0.1,0.5,0.6,0.4) & (0.2,0.4,0.8,0.3) & (0.3,0.6,0.7,0.0)\\ (0.3,0.5,0.7,0.4) & (0.7,0.0,0.4,0.6) & (0.4,0.5,0.8,0.6) & (0.4,0.1,0.6,0.0) & (0.4,0.7,0.0,0.5)\\ (0.4,0.6,0.8,0.6) & (0.4,0.0,0.7,0.6) & (0.1,0.4,0.5,0.0) & (0.8,0.5,0.6,0.7) & (0.7,0.6,0.5,0.6)\\ (0.2,0.5,0.4,0.7) & (0.4,0.6,0.5,0.8) & (0.6,0.5,0.5,0.3) & (0.2,0.6,0.4,0.6) & (0.4,0.0,0.6,1.0)\\ (0.3,0.8,0.1,0.9) & (0.6,0.5,0.4,0.7) & (0.2,0.6,0.4,0.4) & (0.6,0.4,0.7,1.0) & (0.4,1.0,0.5,0.6)\end{pmatrix}$$

$$\zeta^{x}\left(c\right)=e_x\begin{matrix}\\ e_x\end{matrix}\begin{pmatrix}(1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0)\end{pmatrix}$$

Then, by definition **4.7**, their product is denoted by (NF,P) and defined as follows:

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$(NF,P)= \left(N,F_A\right)\otimes \zeta^{x}\ \left(c\right)=$

$$\begin{matrix} & & & e_x & \\ \end{matrix}$$

$$\begin{pmatrix} (0.3,0.6,0.5,0.0) & (0.0,0.0,0.5,1) & (0.3,0.6,0.5,0.0) & (0.0,0.0,0.5,1) & (0.3,0.6,0.5,0.0) \\ (0.4,0.7,0.5,0.4) & (0.0,0.0,0.5,1) & (0.4,0.7,0.5,0.4) & (0.0,0.0,0.5,1) & (0.4,0.7,0.5,0.4) \\ (0.7,0.6,0.5,0.0) & (0.0,0.0,0.5,1) & (0.7,0.6,0.5,0.0) & (0.0,0.0,0.5,1) & (0.7,0.6,0.5,0.0) \\ (0.6,0.5,0.5,0.3) & (0.0,0.0,0.5,1) & (0.6,0.5,0.5,0.3) & (0.0,0.0,0.5,1) & (0.6,0.5,0.5,0.3) \\ (0.4,1,0.5,0.4) & (0.0,0.0,0.5,1) & (0.4,1,0.5,0.4) & (0.0,0.0,0.5,1) & (0.4,1,0.5,0.4) \end{pmatrix}$$

Definition 4.8

Let $\left(N,F_A\right)_{m\times m}=\left[a_{ij}\right]_{m\times m}$ be a neutrosophic fuzzy soft square matrix then the trace of is denoted by $tr\left(N,F_A\right)_{m\times m}$ and it is defined as

$$tr\left(N,F_A\right)_{m\times m}=\max\left(\mu_{a_{ii}}\right)+\max\left(T_{\mu_{a_{ii}}}\right)+\max\left(I_{\mu_{a_{ii}}}\right)+\min\left(F_{\mu_{a_{ii}}}\right).$$

5. CROSS-ENTROPY OF NEUTROSOPHIC FUZZY SETS

Entropy and cross-entropy operators give more flexibility in the critical decision-making process as compared to weight operators. Uncertainty, along with inconsistency plays a major role in various real-life problems and these can be handled with a mathematical tool known as neutrosophic fuzzy set,and cross-entropy defined on neutrosophic fuzzy sets provides authentic results for the decision-makers. Earlier fuzzy entropy is used to determine a qualitative measure of uncertainty associated with each fuzzy variable. In (Zadeh, 1968), Zadeh introduced fuzzy entropy as a weighted shanon entropy. Researchers have given several definitions from several angles, such as De Luca and Termini(De Luca & Termini, 1972), Yagar(Yager, 2000), Kaufmann(Kaufmann, 1975), Pal and Pal(Pal & Pal, 1992). In (Li, 2015), Li introduced fuzzy cross-entropy to measure the divergence of fuzzy variables, Wang et al. (Wang, Yang & Qin, 2019) presented consistency between cross-entropy and distance measure in fuzzy sets. In(Peng, Wang, Wu, Zhang & Chen, 2015), Peng et al. discuss the fuzzy cross-entropy for intuitionistic hesitant fuzzy sets and their applications in multi-criteria decision-making, Qin et al.(Qin, Li & Ji, 2009) used fuzzy cross-entropy for portfolio selection, Shang et al.(Shang & Jiang, 1997) introduced a measure of fuzzy cross-entropy between fuzzy sets to measure discriminate information between them, Zhang et al.(Zhang & Jiang, 2008) defined vague cross-entropy between vague sets. Ye (Ye, 2011) extended the idea of fuzzy cross-entropy to interval-valued intuitionistic fuzzy sets, Gou et al.(Gou, Xu & Liao, 2017) proposed hesitant fuzzy linguistic entropy and cross-entropy, Patraseu(Patrascu,2015)discussed neutrosophic entropy and its five components, Ye(Ye, 2014) proposed single-valued neutrosophic cross-entropy for multi-criteria decision-making problems. The proposed study is based on neutrosophic fuzzy cross-entropy in group decision making(GDM) and it is given by,

Let

$$A=\left\{\left\langle \mu_A\left(x_i\right),T_{\mu_A}\left(x_i\right),I_{\mu_A}\left(x_i\right),F_{\mu_A}\left(x_i\right)\right\rangle : x_i\in X\right\},$$

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and

$$B=\left\{\left\langle \mu_B\left(x_i\right),T_{\mu_B}\left(x_i\right),I_{\mu_B}\left(x_i\right),F_{\mu_B}\left(x_i\right)\right\rangle : x_i\in X\right\}$$

be two NFSs and $X=\{x_1,x_2,\ldots,x_n\}$ be the set of universe where

$$\mu_A\left(x_i\right),T_{\mu_A}\left(x_i\right),I_{\mu_A}\left(x_i\right),F_{\mu_A}\left(x_i\right)\in[0,1],$$

and

$$\mu_B\left(x_i\right),T_{\mu_B}\left(x_i\right),I_{\mu_B}\left(x_i\right),F_{\mu_B}\left(x_i\right)\in[0,1].$$

Now, the amount of information discrimination of

$\mu_A(x_i)$ to $\mu_B(x_i)$, $T_{\mu_A}\left(x_i\right)$ to $T_{\mu_B}\left(x_i\right)$, $I_{\mu_A}\left(x_i\right)$ to $I_{\mu_B}\left(x_i\right)$, and $F_{\mu_A}\left(x_i\right)$ to $F_{\mu_B}\left(x_i\right)$

are respectively computed as follows:

$$E^{\mu}\left(A,B\right)=\sum_{i=1}^{n}\left[\mu_A\left(x_i\right)\log_2\frac{\mu_A\left(x_i\right)}{\frac{1}{2}\left(\mu_A\left(x_i\right)+\mu_B\left(x_i\right)\right)}+\left(1-\mu_A\left(x_i\right)\right)\log_2\frac{1-\mu_A\left(x_i\right)}{1-\frac{1}{2}\left(\mu_A\left(x_i\right)+\mu_B\left(x_i\right)\right)}\right]$$

$$E^{T_\mu}\left(A,B\right)=\sum_{i=1}^{n}\left[T_{\mu_A}\left(x_i\right)\log_2\frac{T_{\mu_A}\left(x_i\right)}{\frac{1}{2}\left(T_{\mu_A}\left(x_i\right)+T_{\mu_B}\left(x_i\right)\right)}+\left(1-T_{\mu_A}\left(x_i\right)\right)\log_2\frac{1-T_{\mu_A}\left(x_i\right)}{1-\frac{1}{2}\left(T_{\mu_A}\left(x_i\right)+T_{\mu_B}\left(x_i\right)\right)}\right]$$

$$E^{I_\mu}\left(A,B\right)=\sum_{i=1}^{n}\left[I_{\mu_A}\left(x_i\right)\log_2\frac{I_{\mu_A}\left(x_i\right)}{\frac{1}{2}\left(I_{\mu_A}\left(x_i\right)+I_{\mu_B}\left(x_i\right)\right)}+\left(1-I_{\mu_A}\left(x_i\right)\right)\log_2\frac{1-I_{\mu_A}\left(x_i\right)}{1-\frac{1}{2}\left(I_{\mu_A}\left(x_i\right)+I_{\mu_B}\left(x_i\right)\right)}\right]$$

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$$E^{F_\mu}(A,B)=\sum_{i=1}^{n}\left[F_{\mu_A}(x_i)\log_2\frac{F_{\mu_A}(x_i)}{\frac{1}{2}\left(F_{\mu_A}(x_i)+F_{\mu_B}(x_i)\right)}+\left(1-F_{\mu_A}(x_i)\right)\log_2\frac{1-F_{\mu_A}(x_i)}{1-\frac{1}{2}\left(F_{\mu_A}(x_i)+F_{\mu_B}(x_i)\right)}\right]$$

The neutrosophic fuzzy cross-entropy between A and B is evaluated as a sum of four membership cross-entropies given by,

$$E(A,B)=\sum_{i=1}^{n}\left[\mu_A(x_i)\log_2\frac{\mu_A(x_i)}{\frac{1}{2}\left(\mu_A(x_i)+\mu_B(x_i)\right)}+\left(1-\mu_A(x_i)\right)\log_2\frac{1-\mu_A(x_i)}{1-\frac{1}{2}\left(\mu_A(x_i)+\mu_B(x_i)\right)}\right]$$

$$+\sum_{i=1}^{n}\left[T_{\mu_A}(x_i)\log_2\frac{T_{\mu_A}(x_i)}{\frac{1}{2}\left(T_{\mu_A}(x_i)+T_{\mu_B}(x_i)\right)}+\left(1-T_{\mu_A}(x_i)\right)\log_2\frac{1-T_{\mu_A}(x_i)}{1-\frac{1}{2}\left(T_{\mu_A}(x_i)+T_{\mu_B}(x_i)\right)}\right]$$

$$+\sum_{i=1}^{n}\left[I_{\mu_A}(x_i)\log_2\frac{I_{\mu_A}(x_i)}{\frac{1}{2}\left(I_{\mu_A}(x_i)+I_{\mu_B}(x_i)\right)}+\left(1-I_{\mu_A}(x_i)\right)\log_2\frac{1-I_{\mu_A}(x_i)}{1-\frac{1}{2}\left(I_{\mu_A}(x_i)+I_{\mu_B}(x_i)\right)}\right]$$

$$+\sum_{i=1}^{n}\left[F_{\mu_A}(x_i)\log_2\frac{F_{\mu_A}(x_i)}{\frac{1}{2}\left(F_{\mu_A}(x_i)+F_{\mu_B}(x_i)\right)}+\left(1-F_{\mu_A}(x_i)\right)\log_2\frac{1-F_{\mu_A}(x_i)}{1-\frac{1}{2}\left(F_{\mu_A}(x_i)+F_{\mu_B}(x_i)\right)}\right] \quad (1)$$

$E(A,B)$ also represents a discrimination degree between A and B, where $E(A,B)\geq 0$.
$E(A,B)=0$ iff $\mu_A(x_i)=\mu_B(x_i)$, $T_{\mu_A}(x_i)=T_{\mu_B}(x_i)$, $I_{\mu_A}(x_i)=I_{\mu_B}(x_i)$, and $F_{\mu_A}(x_i)=F_{\mu_B}(x_i)$.

Since, $E(A,B)$ is not symmetric, so a symmetric discriminate informative measure for NFSs is obtained as

$$D(A,B)=E(A,B)+E(B,A) \quad (2)$$

By motivating the idea proposed by Ye (Ye,2014), and using equations (1) and (2), the weighted neutrosophic fuzzy cross-entropy $D_i(A^*,A_i)$ between an alternative A_i and the ideal alternative A^* is given by,

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$$D_i\left(A^*, A_i\right) = \sum_{j=1}^{n}\left[\log_2 \frac{1}{\frac{1}{2}\left(1+\mu_{ij}\right)} + \log_2 \frac{1}{\frac{1}{2}\left(1+T_{\mu_{ij}}\right)} + \log_2 \frac{1}{1-\frac{1}{2}\left(I_{\mu_{ij}}\right)} + \log_2 \frac{1}{1-\frac{1}{2}\left(F_{\mu_{ij}}\right)}\right]$$

$$+\sum_{j=1}^{n}\left[\mu_{ij}\log_2 \frac{\mu_{ij}}{\frac{1}{2}\left(1+\mu_{ij}\right)} + \left(1-\mu_{ij}\right)\log_2 \frac{1-\mu_{ij}}{1-\frac{1}{2}\left(1+\mu_{ij}\right)}\right] + \sum_{j=1}^{n}\left[T_{\mu_{ij}}\log_2 \frac{T_{\mu_{ij}}}{\frac{1}{2}\left(1+T_{\mu_{ij}}\right)} + \left(1-T_{\mu_{ij}}\right)\log_2 \frac{1-T_{\mu_{ij}}}{1-\frac{1}{2}\left(1+T_{\mu_{ij}}\right)}\right]$$

$$+\sum_{j=1}^{n}\left[I_{\mu_{ij}} + \left(1-I_{\mu_{ij}}\right)\log_2 \frac{1-I_{\mu_{ij}}}{1-\frac{1}{2}\left(I_{\mu_{ij}}\right)}\right] + \sum_{j=1}^{n}\left[F_{\mu_{ij}} + \left(1-F_{\mu_{ij}}\right)\log_2 \frac{1-F_{\mu_{ij}}}{1-\frac{1}{2}\left(F_{\mu_{ij}}\right)}\right]$$

Here, the smaller the value $D_i(A^*A_i)$, the better will be the alternative A_i and A_i is close to the ideal alternative A^*. The concept of ideal alternative is used in the decision-making environment for the identification of the best alternative in the decision set. Basically, the ideal alternative does not exist in the real-world but it provides a theoretical framework for evaluating the alternatives. The ideal alternative A^* in neutrosophic fuzzy concept is defined by

$$A^* = \left\langle \mu^*, T_\mu^*, I_\mu^*, F_\mu^* \right\rangle = \left\langle 1,1,0,0 \right\rangle$$

6. AN ALGORITHM FOR GDM USING NFSM AND COMBINED NFCM

The steps of the proposed algorithm are given below:

Step1: Input the neutrosophic fuzzy soft matrices of each decision-maker d_k, k=1,2,3,…,n.

Step2: Construct the combined neutrosophic fuzzy choice matrices according to the interest of each decision-maker d_k, k=1,2,3,…,n.

Step3: Find the product of the NFSM and the combined NFCM for each decision-maker d_k, k=1,2,3,…,n.

Step4: Aggregate the product of NFSMs.

Step5: Compute the neutrosophic fuzzy cross-entropy between the ideal alternative and the i^{th} alternative O_i, $\forall_I$ and rank the alternative.

Step6: Alternative(s) having the lowest neutrosophic fuzzy cross-entropy value is(are) selected as the most desirable one(s).

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7. AN ILLUSTRATIVE EXAMPLE

To demonstrate the application of the proposed algorithm, we cite an example about the selection of a teaching faculty. Suppose a university needs to appoint a teaching faculty in the department of mathematics and for this, a panel of three experts namely, Mr. Biden, Mr. Obama, and Mr. Trump has been appointed. The set of experts is denoted by $D=\{d_1,d_2,d_3\}$ and the set of candidates is denoted by $O=\{o_1,o_2,o_3\}$. The members of the set $D=\{d_1,d_2,d_3\}$ jointly want to select the best teaching faculty from the set $O=\{o_1,o_2,o_3\}$ with attributes denoted by the set $E=\{e_1,e_2,e_3,e_4,e_5\}$, where e_1=academic credentials and abilities, e_2=ability to communicate complex ideas, e_3=research skills, e_4=publications, e_5=experience, knowledge, and creativity.

Suppose, Mr. Biden is interested in $\{e_1,e_2\}$, Mr. Obama is interested in $\{e_1,e_3,e_4\}$, and Mr. Trump interested in $\{e_1,e_4,e_5\}$.

Opinions of Mr. Biden, Mr. Obama, and Mr. Trump are recorded by three NFSMs denoted by $R^{d_1}_{NFSM}, R^{d_2}_{NFSM}$, and $R^{d_3}_{NFSM}$ respectively which are given below:

$$R^{d_1}_{NFSM}=\begin{matrix}o_1\\o_2\\o_3\end{matrix}\begin{bmatrix}(0.4,0.5,0.6,0.3) & (0.5,0.4,0.6,0.7) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0)\\(0.6,0.7,0.8,0.3) & (0.4,0.5,0.6,0.3) & (0,0,0,\mathbf{0}) & (0,0,0,0) & (0,0,0,0)\\(0.8,0.3,0.5,0.7) & (0.4,0.6,0.5,0.7) & (0,\mathbf{0},0,0) & (0,0,0,0) & (0,0,0,0)\end{bmatrix}$$

$$R^{d_2}_{NFSM}=\begin{matrix}o_1\\o_2\\o_3\end{matrix}\begin{bmatrix}(0.6,0.3,0.8,0.4) & (0,0,0,0) & (0.7,0.8,0.5,0.6) & (0.5,0.6,0.7,0.3) & (0,0,0,0)\\(0.8,0.3,0.4,0.5) & (0,0,0,0) & (0.3,0.\mathbf{5},0.6,0.4) & (0.3,0.8,0.2,0.5) & (0,0,0,0)\\(0.7,0.5,0.6,0.3) & (0,0,0,0) & (0.7,0.2,0.3,0.4) & (0.3,0.5,0.6,0.5) & (0,0,0,0)\end{bmatrix}$$

$$R^{d_3}_{NFSM}=\begin{matrix}o_1\\o_2\\o_3\end{matrix}\begin{bmatrix}(0.3,0.5,0.6,0.4) & (0,0,0,0) & (0,0,0,0) & (0.7,0.2,0.3,0.5) & (0.6,0.1,0.2,0.4)\\(0.3,0.5,0.7,0.5) & (0,0,0,0) & (0,0,0,\mathbf{0}) & (0.8,0.1,0.4,0.6) & (0.7,0.8,0.4,0.5)\\(0.3,0.6,0.7,0.4) & (0,0,0,0) & (0,0,0,0) & (0.7,0.6,0.3,0.4) & (0.6,0.3,0.4,0.5)\end{bmatrix}$$

Neutrosophic fuzzy choice matrices and their combined choice matrices are given by

$$\zeta_{d_1}(i,j)=\begin{bmatrix}(1,1,0.5,0) & (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\(1,1,0.5,0) & (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\(0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\(0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\(\mathbf{0},0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\end{bmatrix}$$

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$$\zeta_{d_2}(i,j)=\begin{bmatrix}(1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (1,1,0.5,0) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (1,1,0.5,0) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (1,1,0.5,0) & (1,1,0.5,0) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\end{bmatrix}$$

$$\zeta_{d_3}(i,j)=\begin{bmatrix}(1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (1,1,0.5,0)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (1,1,0.5,0)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (1,1,0.5,0)\end{bmatrix}$$

$$\zeta_{(d_1,d_2\wedge d_3)}(i,j)=\begin{bmatrix}(1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\end{bmatrix}$$

$$\zeta_{(d_2,d_3\wedge d_1)}(i,j)=\begin{bmatrix}(1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\end{bmatrix}$$

$$\zeta_{(d_3,d_1\wedge d_2)}(i,j)=\begin{bmatrix}(1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1)\end{bmatrix}$$

Now we calculate the product matrices as follows:

$$P_{NFSM}^{d_1}=R_{NFSM}^{d_1}\otimes\zeta_{(d_1,d_2\wedge d_3)}(i,j)$$

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$$=\begin{bmatrix} (0.4,0.5,0.6,0.3) & (0.5,0.4,0.6,0.7) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) \\ (0.6,0.7,0.8,0.3) & (0.4,0.5,0.6,0.3) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) \\ (0.8,0.3,0.5,0.7) & (0.4,0.6,0.5,0.7) & (0,0,0,0) & (0,0,0,0) & (0,0,0,0) \end{bmatrix}$$

$$\otimes\begin{bmatrix} (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (1,1,0.5,0) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \end{bmatrix}$$

$$=\begin{bmatrix} (0.5,0.5,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.5,0.5,0.5,0.3) & (0.0,0.0,0.5,1) \\ (0.6,0.7,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.6,0.7,0.5,0.3) & (0.0,0.0,0.5,1) \\ (0.8,0.6,0.5,0.7) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.8,0.6,0.5,0.7) & (0.0,0.0,0.5,1) \end{bmatrix}$$

$$P_{NFSM}^{d_2} = R_{NFSM}^{d_2} \otimes \zeta_{(d_2,d_3\wedge d_1)}(i,j)$$

$$=\begin{bmatrix} (0.6,0.3,0.8,0.4) & (0,0,0,0) & (0.7,0.8,0.5,0.6) & (0.5,0.6,0.7,0.3) & (0,0,0,0) \\ (0.8,0.3,0.4,0.5) & (0,0,0,0) & (0.3,0.5,0.6,0.4) & (0.3,0.8,0.2,0.5) & (0,0,0,0) \\ (0.7,0.5,0.6,0.3) & (0,0,0,0) & (0.7,0.2,0.3,0.4) & (0.3,0.5,0.6,0.5) & (0,0,0,0) \end{bmatrix}$$

$$\otimes\begin{bmatrix} (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \end{bmatrix}$$

$$=\begin{bmatrix} (0.7,0.8,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.8,0.8,0.5,0.4) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.7,0.5,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \end{bmatrix}$$

$$P_{NFSM}^{d_3} = R_{NFSM}^{d_3} \otimes \zeta_{(d_3,d_1\wedge d_2)}(i,j)$$

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$$= \begin{bmatrix} (0.3,0.5,0.6,0.4) & (0,0,0,0) & (0,0,0,0) & (0.7,0.2,0.3,0.5) & (0.6,0.1,0.2,0.4) \\ (0.3,0.5,0.7,0.5) & (0,0,0,0) & (0,0,0,0) & (0.8,0.1,0.4,0.6) & (0.7,0.8,0.4,0.5) \\ (0.3,0.6,0.7,0.4) & (0,0,0,0) & (0,0,0,0) & (0.7,0.6,0.3,0.4) & (0.6,0.3,0.4,0.5) \end{bmatrix}$$

$$\otimes \begin{bmatrix} (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \\ (1,1,0.5,0) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) & (0,0,0.5,1) \end{bmatrix}$$

$$= \begin{bmatrix} (0.7,0.5,0.5,0.4) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.8,0.8,0.5,0.5) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.7,0.6,0.5,0.4) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \end{bmatrix}$$

Now, the aggregate of the product is given by,

$$R = P_{NFSM}^{d_1} \oplus P_{NFSM}^{d_2} \oplus P_{NFSM}^{d_3}$$

$$= \begin{bmatrix} (0.5,0.5,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.5,0.5,0.5,0.3) & (0.0,0.0,0.5,1) \\ (0.6,0.7,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.6,0.7,0.5,0.3) & (0.0,0.0,0.5,1) \\ (0.8,0.6,0.5,0.7) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.8,0.6,0.5,0.7) & (0.0,0.0,0.5,1) \end{bmatrix}$$

$$\oplus \begin{bmatrix} (0.7,0.8,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.8,0.8,0.5,0.4) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.7,0.5,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \end{bmatrix}$$

$$\oplus \begin{bmatrix} (0.7,0.5,0.5,0.4) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.8,0.8,0.5,0.5) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \\ (0.7,0.6,0.5,0.4) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) \end{bmatrix}$$

$$= \begin{matrix} o_1 \\ o_2 \\ o_3 \end{matrix} \begin{bmatrix} (0.7,0.8,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.5,0.5,0.5,0.3) & (0.0,0.0,0.5,1) \\ (0.8,0.8,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.6,0.7,0.5,0.3) & (0.0,0.0,0.5,1) \\ (0.8,0.6,0.5,0.3) & (0.0,0.0,0.5,1) & (0.0,0.0,0.5,1) & (0.8,0.6,0.5,0.7) & (0.0,0.0,0.5,1) \end{bmatrix}$$

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Now, the neutrosophic fuzzy cross-entropy value of each alternative are given by:

$D(o_1,o_1^*)$=23.3791, $D(o_2,o_2^*)$=18.9142 and $D(o_3,o_3^*)$=23.6044

According to the cross-entropy values, we rank the alternatives as $o_3 \succ o_1 \succ o_2$.

Since, o_2 has the least cross-entropy value, therefore the second candidate will be the selected as a teaching faculty with the agreement of all the experts in the panel.

8. CONCLUSION AND SCOPE

In this work, we first study the concept of neutrosophic fuzzy soft sets(NFSS) and their properties. Then, neutrosophic fuzzy soft matrix(NFSM) is introduced which can be viewed as a sophisticated framework of neutrosophic fuzzy soft set in the sense that it is enabled to do numerical calculations like addition, subtraction, multiplication,etc quite easily. Some properties related to NFSMs have been addressed. Also, Neutrosophic fuzzy cross-entropy is discussed with its property. Furthermore, an algorithm has been proposed for solving group decision-making problems. Finally, with the help of an example, it has been shown that the proposed algorithm can be applied successfully in real GDM problems. In 2018, soft set theory is extended by introducing hypersoft set theory proposed by Smarandache in (Smarandache, 2018). Hypersoft set(HSS) is used as a multi attribute function. After the invention of HSS, it becomes very popular among the researchers and they are trying to develop new theories associated to HSS to address the issues that cannot be solved by SS. Therefore, in future, there is a scope to develop a new matrix theory by extending the neutrosophic fuzzy soft matrix to neutrosophic fuzzy hypersoft matrix(NFHSM) to solve more critical GDM problems that exist in the real-world. NFHSM provides a lot of scope in solving several disease diagnosis-related problems as well. Also, NFSM can be applied in different fields such as, computer graphics, cryptography, signal processing, economics, robotics and animation etc.

CONFLICT OF INTEREST

There is no conflict of interest regarding the publication of the article.

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Chapter 34
Quadripartitioned Neutrosophic Soft Graphs

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ABSTRACT

The neutrosophic quadripartitioned soft model is a hybrid model by combining neutrosophic soft sets with quadripartitioned sets. This work concerns with the quadripartitioned neutrosophic soft graphs for treating neutrosophic soft information by employing the theory of quadripartitioned neutrosophic soft sets with graphs. Operations like Cartesian product, cross product, lexicographic product, and strong product of quadripartitioned neutrosophic soft graphs are established. The proposed concepts are explained with examples.

1 INTRODUCTION

The author Zadeh in 1975 invented the interval-valued fuzzy sets as the generalisation of fuzzy sets (Zadeh, 1975). To handle the uncertainty, it utilizes the values of intervals of numbers instead of numbers as the membership function. To reflect the grade of membership of fuzzy set A $[T_{AL}(x), T_{AU}(x)]$ with $0 \leq T_{AL}(x), T_{AU}(x) \leq 1$ is widely employed (Turksen, 1986). It is crucial to use interval-valued fuzzy sets in

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applications such as fuzzy control. The indeterminacy portion of the neutrosophic set is divided into two parts: 'Contradiction' (both true and false) and 'Unknown' (neither true nor false), that is $\mathcal{T}, \mathcal{C}, \mathcal{U}$ and $\mathcal{F}$ which defines a new set called 'quadripartitioned single valued neutrosophic set', introduced by (Chatterjee, 2016). This study is completely based on "Belnap's four valued logic" (Belnap, 1977) and Smarandache's "Four Numerical valued neutrosophic logic" (Smarandache, 2014). The efficiency of neutrosophic sets in holding incomplete data and handling uncertain information is the foundation for their current success in modelling natural occurrences. It is the base of neutrosophic logic, a multiple value logic that generalizes the fuzzy logic that carries with paradoxes, contradictions, antitheses, antinomies, invented by the author Smarandache in (Smarandache, 2014; Smarandache, 1999; Smarandache, 2010). The single-valued neutrosophic set is the generalization of intuitionistic fuzzy sets and is used expediently to deal with real-world problems, especially in decision support (Wang, 2010). The computation of belief in that element (truth), they disbelieve in that element (falsehood) and the indeterminacy part of that element with the sum of these three components are strictly less than 1. Neutrosophic set and related notions have shown applications in many different fields. In the definition of neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are defined completely independent with the sum of these values lies between 0 and 3 (Smarandache, 1998; Smarandache, 2010; Turksen 1986).

Thus, a single-valued neutrosophic set is a powerful general formal framework which generalizes the concept of fuzzy set and intuitionistic fuzzy set. Group decision-making is a commonly used tool in human activities, which determines the optimal alternative from a given finite set of alternatives using the evaluation information given by a group of decision-makers or experts. With the rapid development of society, group decision-making plays an increasingly important role when dealing with the decision-making problems

Molodtsov in (Molodstov, 1999) successfully proposed a completely new theory namely soft set theory by using classical sets in 1999 and after that, there has been a rapid development of interest in soft sets and their various applications (Broumi, 2016). This theory provides a parametrized point of view for uncertainty modelling and soft computing. Vague sets are considered as a particular case of context-dependent fuzzy sets. Neutrosophic soft rough graphs with applications are established in (Akram, 2018). Neutrosophic soft relations and neutrosophic refined relations with their properties are studied in (Broumi, 2014). Recently, neutrosophic soft graphs, neutrosophic graphs, co-neutrosophic graphs Single valued neutrosophic graphs are established in (Akram, 2017b; Broumi, 2016; Devaseelan 2018). An application of neutrosophic soft graph to decision making are studied in (Akram, 2017b). Motivated by the aforementioned works and by employing the concept of Quadripartitioned Single Valued Neutrosophic Soft Set (QSVNSS), this work depends on the quadripartitioned single valued neutrosophic soft graphs. The work presented here intends to overcome the lack of a mathematical approach towards indeterminate information and inconsistent information. This chapter deals with the quadripartitioned neutrosophic soft graphs and their applications. This chapter is useful for the researchers in mathematics, computer scientists and social scientists alike.The major contributions of this work are as follows:

- The operations on quadripartitioned neutrosophic soft graphs are established and this manuscript makes the first attempt in this domain. Some basic definitions regarding to it are explained with an example.

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- Results on the Cartesian product, cross product, lexicographic product and strong product of quadripartitioned neutrosophic soft graph are illustrated with examples.
- This work will generalize the existing work in the literature (Mary, 2021).

The chapter is organised as follows: The basic preliminaries and results are given in Section 2. The graph operations like Cartesian product, cross product, lexicographic product and strong product of quadripartitioned neutrosophic soft graphs are formulated and the results are proved.

2 PRELIMINARIES

In this section, the basic definitions are explained which are needed for the main results.

Definition 2.1 *(Samarandache, 1998) Let $\mathfrak{X}$ be the universe. A neutrosophic set Λ in $\mathfrak{X}$ is characterized by a truth membership function $\mathcal{T}_\Lambda$, an indeterminacy membership function $\mathcal{I}_\Lambda$ and a falsity membership function $\mathcal{F}_\Lambda$ where $\mathcal{T}_\Lambda$, $\mathcal{I}_\Lambda$ and $\mathcal{F}_\Lambda$ are real standard elements of* [0,1]. *It can be written as for each point p in $\mathfrak{X}$, $\mathcal{T}_\Lambda(p), \mathcal{F}_\Lambda(p), \mathcal{I}_\Lambda(p) \in [0,1]$. Also*

$$\Lambda = \langle p, \mathcal{T}_\Lambda(p), \mathcal{F}_\Lambda(p), \mathcal{I}_\Lambda(p) : p \in \mathfrak{X} \rangle \text{ and } 0 \leq \mathcal{T}_\Lambda(p) + \mathcal{I}_\Lambda(p) + \mathcal{F}_\Lambda(p) \leq 3.$$

Definition 2.2 (*Samarandache, 1998; Samarandache, 2005; Wang 2010)Let $\mathfrak{X}$ be a space of points (objects), with generic elements in $\mathfrak{X}$ denoted by p. A single-valued neutrosophic set Λ in $\mathfrak{X}$ is characterized by truth-membership function $\mathcal{T}_\Lambda(p)$, indeterminacy-membership function $\mathcal{I}_\Lambda(p)$ and falsity-membership-function $\mathcal{F}_\Lambda(p)$.*

For each point *p* in $\mathfrak{X}$, $\mathcal{T}_\lambda(p), \mathcal{I}_\lambda(p), \mathcal{F}_\lambda(p) \in [0,1]$,

$$\Lambda = \{p, \mathcal{T}_\Lambda(p), \mathcal{I}_\Lambda(p), \mathcal{F}_\Lambda(p)\} \text{ and } 0 \leq \mathcal{T}_\Lambda(p) + \mathcal{I}_\Lambda(p) + \mathcal{F}_\Lambda(p) \leq 3.$$

Definition 2.3 (*Samarandache, 1998; Samarandache 2015*) *A Neutrosophic set $\mathbb{A}$ is contained in another neutrosophic set $\mathbb{B}$, (i.e) $\mathbb{A} \subseteq \mathbb{B}$ if*

$$\forall p \in \mathfrak{X}, \mathcal{T}_\mathbb{A}(p) \leq \mathcal{T}_\mathbb{B}(p), \mathcal{I}_\mathbb{A}(p) \geq \mathcal{I}_\mathbb{B}(p) \text{ and } \mathcal{F}_\mathbb{A}(p) \geq \mathcal{F}_\mathbb{B}(p).$$

Definition 2.4 *(Akram, 2017a) A single-valued neutrosophic graph is defined as a pair $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ where*

(i) $\mathbb{V} = \{a_1, a_2, .., a_n\}$ such that $\mathcal{T}_1 : \mathbb{V} \to [0,1]$, $\mathcal{I}_1 : \mathbb{V} \to [0,1]$, and $\mathcal{F}_1 : \mathbb{V} \to [0,1]$, denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and

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$$0 \le \mathcal{T}_1(a) + \mathcal{I}_1(a) + \mathcal{F}_1(a) \le 3,$$

(ii) $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ where $\mathcal{T}_2 : \mathbb{E} \to [0,1]$, $\mathcal{I}_2 : \mathbb{E} \to [0,1]$, and $\mathcal{F}_2 : \mathbb{E} \to [0,1]$ are such that

$$\mathcal{T}_2(a_1a_2) \le \min\{\mathcal{T}_1(a_1), \mathcal{T}_1(a_2)\},$$

$$\mathcal{I}_2(a_1a_2) \le \min\{\mathcal{I}_1(a_1), \mathcal{I}_1(a_2)\},$$

$$\mathcal{F}_2(a_1a_2) \le \max\{\mathcal{F}_1(a_1), \mathcal{F}_1(a_2)\},$$

and

$$0 \le \mathcal{T}_2(a_1a_2) + \mathcal{I}_2(a_1a_2) + \mathcal{F}_2(a_1a_2) \le 3, \ \forall a_1a_2 \in \mathbb{E}.$$

For more details about the following definitions and results, see the article (Chatterjee, 2016)

Definition 2.5 *Let $\mathfrak{X}$ be a non-empty set. A quadripartitioned neutrosopohic set (QSVNS) Λ over $\mathfrak{X}$ characterizes each element p in $\mathfrak{X}$ by a truth membership function $\mathcal{T}_\Lambda$, a contradiction membership function $\mathcal{C}_\Lambda$, an ignorance membership function $\mathcal{U}_\Lambda$ and a false membership function $\mathcal{F}_\Lambda$ such that for each*

$$p \in \mathfrak{X}, \mathcal{T}_\Lambda, \mathcal{C}_\Lambda, \mathcal{U}_\Lambda, \mathcal{F}_\Lambda \in [0,1] \text{ and } 0 \le \mathcal{T}_\Lambda(p) + \mathcal{C}_\Lambda(p) + \mathcal{U}_\Lambda(p) + \mathcal{F}_\Lambda(p) \le 4.$$

Remark 2.6 *A QSVNS Λ, can be decomposed to yields two SVNS, say, Λ_t and Λ_b where the respective membership functions of both these sets are defined as*

$$\mathcal{T}_{\Lambda_t}(p) = \mathcal{T}_\Lambda(p) = \mathcal{T}_{\Lambda_b}(p)$$

$$\mathcal{I}_{\wedge_t}(p) = \mathcal{C}_\wedge(p), \mathcal{I}_{\wedge_b}(p) = \mathcal{U}_\wedge(p)$$

$$\mathcal{F}_{\Lambda_t}(p) = \mathcal{F}_\Lambda(p) = \mathcal{F}_{\Lambda_b}(p), \quad \forall p \in \mathfrak{X}.$$

In this respect to needs to be stated that while performing set-theoretic operations over these SVNS, behaviour of $\mathcal{I}_{\wedge_t}$ is treated similar to that of $\mathcal{T}_{\wedge_t}$ while the behaviour of $\mathcal{I}_{\wedge_b}$ is modelled in a way similar to that of $\mathcal{F}_{\wedge_b}$.

Definition 2.7 *A QSVNS is said to be an absolute QSVNS, denoted by Λ, if its is membership values are respectively defined as*

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$$\mathcal{T}_{\Lambda}\left(p\right)=1,\ \mathcal{C}_{\Lambda}\left(p\right)=1,\ \mathcal{U}_{\Lambda}\left(p\right)=0,\ \text{and}\ \mathcal{F}_{\Lambda}\left(p\right)=0.$$

Definition 2.8 *Consider two QSVNS* $\mathfrak{A}$ and $\mathfrak{B}$, *over* $\mathfrak{X}$. $\mathfrak{A}$ *is said to be contained in* $\mathfrak{B}$, *denoted by* $\mathfrak{A}\subseteq\mathfrak{B}$ *if, and only, if*

$$\mathcal{T}_{\mathfrak{A}}\left(p\right)\leq\mathcal{T}_{\mathfrak{B}}\left(p\right),\ \mathcal{C}_{\mathfrak{A}}\left(p\right)\leq\mathcal{C}_{\mathfrak{B}}\left(p\right),\ \mathcal{U}_{\mathfrak{A}}\left(p\right)\geq\mathcal{U}_{\mathfrak{B}}\left(p\right),\ \text{and}\ \mathcal{F}_{\mathfrak{A}}\left(p\right)\geq\mathcal{F}_{\mathfrak{B}}\left(p\right).$$

Definition 2.9 *The complement of a QSVNS* $\mathfrak{A}$, *is denoted by* $\mathfrak{A}^{c}$ *and is represented as*

$$\mathfrak{A}^{c}=\sum_{i=1}^{n}\langle\mathcal{F}_{\mathfrak{A}}\left(p_{i}\right),\mathcal{U}_{\mathfrak{A}}\left(p_{i}\right),\mathcal{C}_{\mathfrak{A}}\left(p_{i}\right),\mathcal{T}_{\mathfrak{A}}\left(p_{i}\right)\rangle,\quad\forall p_{i}\in\mathfrak{X}$$

i.e.

$$\mathcal{T}_{\mathfrak{A}}^{c}\left(p_{i}\right)=\mathcal{F}_{\mathfrak{A}}\left(p_{i}\right),\mathcal{C}_{\mathfrak{A}}^{c}\left(p_{i}\right)=\mathcal{U}_{\mathfrak{A}}\left(p_{i}\right)\mathcal{U}_{\mathfrak{A}}^{c}\left(p_{i}\right)=\mathcal{C}_{\mathfrak{A}}\left(p_{i}\right),\mathcal{F}_{\mathfrak{A}}^{c}\left(p_{i}\right)=\mathcal{T}_{\mathfrak{A}}\left(p_{i}\right),\quad\forall p_{i}\in\mathfrak{X}.$$

Definition 2.10 *The union of two QSVNS* $\mathfrak{A}$ and $\mathfrak{B}$ *is denoted by* $\mathfrak{A}\cup\mathfrak{B}$ *and is represented as*

$$\mathfrak{A}\cup\mathfrak{B}=\sum_{i=1}^{n}\langle\mathcal{T}_{\mathfrak{A}}\left(p_{i}\right)\vee\mathcal{T}_{\mathfrak{B}}\left(p_{i}\right),\mathcal{C}_{\mathfrak{A}}\left(p_{i}\right)\vee\mathcal{C}_{\mathfrak{B}}\left(p_{i}\right)$$

$$\mathcal{U}_{\mathfrak{A}}\left(p_{i}\right)\wedge\mathcal{U}_{\mathfrak{B}}\left(p_{i}\right),\mathcal{F}_{\mathfrak{A}}\left(p_{i}\right)\wedge\mathcal{F}_{\mathfrak{B}}\left(p_{i}\right)\rangle\ /\ \mathfrak{X}.$$

Definition 2.11 *The intersection of two QSVNS* $\mathfrak{A}$ and $\mathfrak{B}$ *is denoted by* $\mathfrak{A}\cap\mathfrak{B}$ *and is represented as*

$$\mathfrak{A}\cap\mathfrak{B}=\sum_{i=1}^{n}\langle\mathcal{T}_{\mathfrak{A}}\left(p_{i}\right)\wedge\mathcal{T}_{\mathfrak{B}}\left(p_{i}\right),\mathcal{C}_{\mathfrak{A}}\left(p_{i}\right)\wedge\mathcal{C}_{\mathfrak{B}}\left(p_{i}\right)$$

$$\mathcal{U}_{\mathfrak{A}}\left(p_{i}\right)\vee\mathcal{U}_{\mathfrak{B}}\left(p_{i}\right),\mathcal{F}_{\mathfrak{A}}\left(p_{i}\right)\vee\mathcal{F}_{\mathfrak{B}}\left(p_{i}\right)\rangle\ /\ \mathfrak{X}$$

Definition 2.12 *(Mary, 2021) Let* $\mathfrak{X}$ *is an initial universe set and E is a set of parameters. Consider a non empty set* Λ *and* $\Lambda\subseteq E$. *Let* $P\left(\mathfrak{X}\right)$ *denote the set of all quadripartitioned neutrosophic sets of* $\mathfrak{X}$. *The collection F,*Λ *is termed to be the quadripartitioned neutrosophic soft set over* $\mathfrak{X}$, *where F is mapping given by* $F:\Lambda\rightarrow P\left(\mathfrak{X}\right)$ *where*

$$\Lambda=\langle\left\langle p,\mathcal{T}_{\Lambda}\left(p\right),\mathcal{C}_{\Lambda}\left(p\right),\mathcal{U}_{\Lambda}\left(p\right),\mathcal{F}_{\Lambda}\left(p\right)\right\rangle p\in\mathfrak{X}\rangle$$

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where

$$\mathcal{T}_{\Lambda}, \mathcal{C}_{\Lambda}, \mathcal{U}_{\Lambda}, \mathcal{F}_{\Lambda} : \mathfrak{X} \to [0,1] \text{ and } 0 \leq \mathcal{T}_{\Lambda}(p) + \mathcal{C}_{\Lambda}(p) + \mathcal{U}_{\Lambda}(p) + \mathcal{F}_{\Lambda}(p) \leq 4.$$

Here $\mathcal{T}_{\lambda}(p)$ denotes the truth membership, $\mathcal{C}_{\lambda}(p)$ denotes the contradiction membership, $\mathcal{U}_{\lambda}(p)$ is ignorance membership and $\mathcal{F}_{\lambda}(p)$ denotes the false membership.

3 OPERATIONS ON QUADRIPARTITIONED NEUTROSOPHIC SOFT GRAPHS

In this section, the results on operations of Quadripartitioned neutrosophic soft graphs with example are established.

Let $\mathcal{U}$ be an initial universe and *P* be the set of all parameters. $P(\mathcal{U})$ denotes the set of all Quadripartitioned neutrosophic soft sets of $\mathcal{U}$. Let *A* be a subset of *P*. A pair (*F,A*) is called a quadripartitioned neutrosophic soft set over $\mathcal{U}$. Let *P*(*V*) denotes the set of all quadripartitioned neutrosophic sets of *V* and *P*(*E*) denotes the set of all quadripartitioned neutrosophic sets of *E*.

Definition 3.1 *A quadripartitioned neutrosophic soft graph G=(G*,J,K,R) is an ordered four tuple if it satisfies the following conditions:*

- $G^*=(V,E)$ is a simple graph,
- *R* is a non-empty set of parameters,
- (*J,R*) is a quadripartitioned neutrosophic soft set over *V*,
- (*K,R*) is a quadripartitioned neutrosophic soft set over *E*,
- (*J*(*e*),*K*(*e*)) is a quadripartitioned neutrosophic soft graph of G^*, that is,

$$\mathbb{T}_{K(e)}(ab) \leq \min\left\{\mathbb{T}_{J(e)}(a), \mathbb{T}_{J(e)}(b)\right\},$$

$$\mathbb{C}_{K(e)}(ab) \leq \min\left\{\mathbb{C}_{J(e)}(a), \mathbb{C}_{J(e)}(b)\right\},$$

$$\mathbb{U}_{K(e)}(ab) \leq \max\left\{\mathbb{U}_{J(e)}(a), \mathbb{U}_{J(e)}(b)\right\},$$

$$\mathbb{F}_{K(e)}(ab) \leq \max\left\{\mathbb{F}_{J(e)}(a), \mathbb{F}_{J(e)}(b)\right\}.$$

such that,

$$0 \leq \mathbb{T}_{K(e)}(ab) + \mathbb{C}_{K(e)}(ab) + \mathbb{U}_{K(e)}(ab) + \mathbb{F}_{K(e)}(ab) \leq 4,$$

$\forall e \in R$, $a,b \in V$.

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For the convenience, the quadripartitioned neutrosophic soft graph $(J(e),K(e))$ is denoted by $H(e)$.

Definition 3.2 *Let $G_1=(J_1,K_1,R)$ and $G_2=(J_2,K_2,S)$ be two quadripartitioned neutrosophic soft graphs of G^*. Then G_1 is quadripartitioned neutrosophic soft subgraph of G_2 if*

- $R \subseteq S$.
- $H_1(e)$ partial subgraph of $H_2(e)$ for all $e \in R$.

Example 3.1 *Consider a simple graph $G^*=(V,E)$ such that $V=\{v_1,v_2,v_3,v_4\}$ and $E=\{v_1v_2, v_1v_3, v_1v_4, v_2v_4, v_3v_4\}$.*

Let $R=\{e_1,e_2\}$ be a set of parameters and let (J,R) be a quadripartitioned neutrosophic soft set over V with neutrosophic approximation function $J: R \to \rho(\mathrm{V})$ defined by

$$J(e_1) =$$
$$v_1 = (0.4,0.3,0.6,0.5), v_2 = (0.3,0.3,0.3,0.4), v_3 = (0.5,0.4,0.4,0.5), v_4 = (0.8,0.5,0.7,0.4)$$

$$J(e_2) =$$
$$v_1 = (0.5,0.4,0.6,0.4), v_2 = (0.3,0.3,0.3,0.6), v_3 = (0.5,0.4,0.4,0.4), v_4 = (0.8,0.5,0.7,0.6).$$

Let (K,R) be a quadripartitioned neutrosophic soft set over E with quadripartitioned neutrosophic approximation function $K: R \to \rho(\mathrm{E})$ defined by

$$K(e_1) = \{(v_1v_2) = (0.3,0.2,0.5,0.3), (v_1v_3) = (0.4,0.3,0.3,0.4), (v_1v_4) = (0.3,0.2,0.5,0.4),\}$$

$$K(e_2) = \{(v_1v_2) = (0.4,0.3,0.5,0.5), (v_1v_3) = (0.2,0.2,0.5,0.3), (v_1v_4) = (0.4,0.3,0.6,0.5),\}$$

Clearly, $H(e_1)=(J(e_1),K(e_1))$ and $H(e_2)=(J(e_2),K(e_2))$ are quadripartitioned neutrosophic graphs corresponding to the parameters e_1 and e_2 respectively as shown in Figure 1.

Definition 3.3 *The quadripartitioned neutrosophic soft graphs $G_1=(G^*,J_1,K_1,A)$ is called spanning quadripartitioned neutrosophic soft subgraph of $G=(G^*,J,K,B)$ if*

$$B \subseteq A.$$

$$\mathbb{T}_{J_1(e)}(v) = \mathbb{T}_{J(e)}(v),\ \mathbb{C}_{J_1(e)}(v) = \mathbb{C}_{J(e)}(v)$$

$$\mathbb{U}_{J_1(e)}(v) = \mathbb{U}_{J(e)}(v) \mathbb{F}_{J_1(e)}(v) = \mathbb{F}_{J(e)}(v)$$

for all $e \in A$, $v \in V$.

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Figure 1. Quadripartitioned neutrosophic soft graph 1

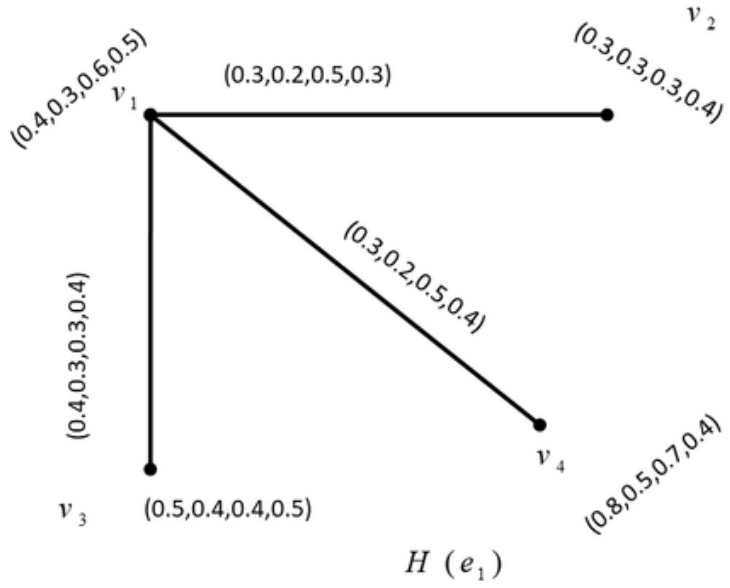
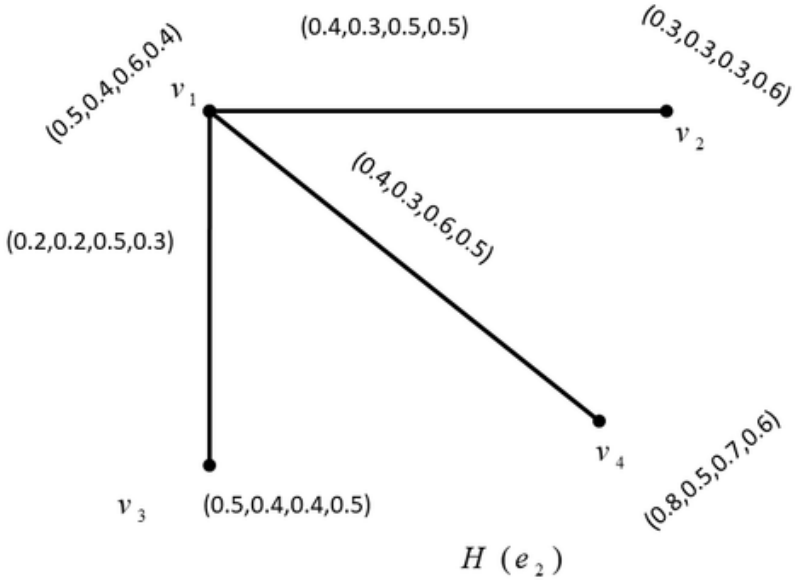


Figure 2. Quadripartitioned neutrosophic soft graph 2



Definition 3.4 *Let G_1=(J_1,K_1,R) and G_2=(J_2,K_2,S) be two quadripartitioned neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The Cartesian product of G_1 and G_2 is a quadripartitioned neutrosophic soft graph G=G_1×G_2=(J,K,R,S), where (J=J_1×J_2, R×S) is a quadripartitioned neutrosophic soft set over V=V_1×V_2, (K=K_1×K_2, R×S) is a quadripartitioned neutrsophic soft set over*

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$$E = \left\{\left((u,v_1),(u,v_2)\right) : u \in V_1, (v_1,v_2) \in E_2\right\} \cup \left\{\left((u_1,v),(u_2,v)\right) : v \in V_2, (u_1,u_2) \in E_1\right\}$$

and $(J,K,R\times S)$ are quadripartitioned neutrosophic soft graphs such that

(i) $\mathbb{T}_{J(a,b)}(u,v) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{T}_{J_2(b)}(v)$,

$\mathbb{C}_{J(a,b)}(u,v) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{C}_{J_2(b)}(v)$,

$\mathbb{U}_{J(a,b)}(u,v) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{J_2(b)}(v)$,

$\mathbb{F}_{J(a,b)}(u,v) = \mathbb{F}_{J_1(a)}(u) \vee \mathbb{F}_{J_2(b)}(v)$,

$\forall (u,v) \in V$, $(a,b) \in R \times S$.

(ii) $\mathbb{T}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{T}_{K_2(b)}(v_1,v_2)$,

$\mathbb{C}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{C}_{K_2(b)}(v_1,v_2)$,

$\mathbb{U}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{K_2(b)}(v_1,v_2)$,

$\mathbb{F}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{F}_{J_1(a)}(u) \vee \mathbb{F}_{K_2(b)}(v_1,v_2)$,

$\forall u \in V_1$, $(v_1,v_2) \in E_2$

(iii) $\mathbb{T}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{T}_{J_2(a)}(v) \wedge \mathbb{T}_{K_2(b)}(u_1,u_2)$,

$\mathbb{C}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{C}_{J_2(a)}(v) \wedge \mathbb{C}_{K_2(b)}(u_1,u_2)$,

$\mathbb{U}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{U}_{J_2(a)}(v) \vee \mathbb{U}_{K_2(b)}(u_1,u_2)$,

$\mathbb{F}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{F}_{J_2(a)}(v) \vee \mathbb{F}_{K_2(b)}(u_1,u_2)$,

$\forall v \in V_2$, $(u_1,u_2) \in E_1$.

Theorem 3.5 *The Cartesian product of two quadripartitioned neutrosophic soft graph is a quadripartitioned neutrosophic soft graph.*

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Proof. Let $G_1=(J_1,K_1,R)$ and $G_2=(J_2,K_2,S)$ be two quadripartitioned neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Let $G=G_1\times G_2=(J,K,R\times S)$ be the Cartesian product of G_1 and G_2.

We claim that $G=(J,K,R\times S)$ is a quadripartitioned neutrosophic soft graph and

$$(H, R\times S) = \left\{(J_1\times J_2)(a_i,b_j),(K_1\times K_2)(a_i,b_j)\right\} \forall a_i \in R, b_j \in S$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$ are quadripartitioned neutrosophic graphs of G.

Consider,

$$\mathbb{T}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) = \min\left\{\mathbb{T}_{J_1(a_i)}(u), \mathbb{T}_{K_2(b_j)}(v_1,v_2)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\leq \min\left\{\mathbb{T}_{J_1(a_i)}(u), \min\left\{\mathbb{T}_{J_2(b_j)}(v_1), \mathbb{T}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \min\left\{\min\left\{\mathbb{T}_{J_1(a_i)}(u), \mathbb{T}_{J_2(b_j)}(v_1)\right\}, \min\left\{\mathbb{T}_{J_1(a_i)}(u), \mathbb{T}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{T}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) \leq \min\left\{\left(\mathbb{T}_{J_1(a_i)}\times\mathbb{T}_{J_2(b_j)}\right)(u,v_1), \left(\mathbb{T}_{J_1(a_i)}\times\mathbb{T}_{J_2(b_j)}\right)(u,v_2)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) = \min\left\{\mathbb{C}_{J_1(a_i)}(u), \mathbb{C}_{K_2(b_j)}(v_1,v_2)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\leq \min\left\{\mathbb{C}_{J_1(a_i)}(u), \min\left\{\mathbb{C}_{J_2(b_j)}(v_1), \mathbb{C}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \min\left\{\min\left\{\mathbb{C}_{J_1(a_i)}(u), \mathbb{C}_{J_2(b_j)}(v_1)\right\}, \min\left\{\mathbb{C}_{J_1(a_i)}(u), \mathbb{C}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) \leq \min\left\{\left(\mathbb{C}_{J_1(a_i)}\times\mathbb{C}_{J_2(b_j)}\right)(u,v_1), \left(\mathbb{C}_{J_1(a_i)}\times\mathbb{C}_{J_2(b_j)}\right)(u,v_2)\right\}$$

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for i=1,2,…,m; j=1,2,…,n

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right)=\max\left\{\mathbb{U}_{J_1(a_i)}(u),\mathbb{U}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq\max\left\{\mathbb{U}_{J_1(a_i)}(u),\max\left\{\mathbb{U}_{J_2(b_j)}(v_1),\mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$=\max\left\{\max\left\{\mathbb{U}_{J_1(a_i)}(u),\mathbb{U}_{J_2(b_j)}(v_1)\right\},\max\left\{\mathbb{U}_{J_1(a_i)}(u),\mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right)\leq\max\left\{\left(\mathbb{U}_{J_1(a_i)}\times\mathbb{U}_{J_2(b_j)}\right)(u,v_1),\left(\mathbb{U}_{J_1(a_i)}\times\mathbb{U}_{J_2(b_j)}\right)(u,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right)=\max\left\{\mathbb{F}_{J_1(a_i)}(u),\mathbb{F}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq\max\left\{\mathbb{F}_{J_1(a_i)}(u),\max\left\{\mathbb{F}_{J_2(b_j)}(v_1),\mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$=\max\left\{\max\left\{\mathbb{F}_{J_1(a_i)}(u),\mathbb{F}_{J_2(b_j)}(v_1)\right\},\max\left\{\mathbb{F}_{J_1(a_i)}(u),\mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right)\leq\max\left\{\left(\mathbb{F}_{J_1(a_i)}\times\mathbb{F}_{J_2(b_j)}\right)(u,v_1),\left(\mathbb{F}_{J_1(a_i)}\times\mathbb{F}_{J_2(b_j)}\right)(u,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

Similarly,

$$\mathbb{T}_{K_{(a_i,b_j)}}\left((u_1,v),(u_2,v)\right)\leq\min\left\{\left(\mathbb{T}_{J_1(a_i)}\times\mathbb{T}_{J_2(b_j)}\right)(u_1,v),\left(\mathbb{T}_{J_1(a_i)}\times\mathbb{T}_{J_2(b_j)}\right)(u_2,v)\right\}$$

for i=1,2,…,m; j=1,2,…,n

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$$\mathbb{C}_{K_{(a_i,b_j)}}\left(\left(u_1,v\right),\left(u_2,v\right)\right)\le\min\left\{\left(\mathbb{C}_{J_1(a_i)}\times\mathbb{C}_{J_2(b_j)}\right)\left(u_1,v\right),\left(\mathbb{C}_{J_1(a_i)}\times\mathbb{C}_{J_2(b_j)}\right)\left(u_2,v\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{U}_{K_{(a_i,b_j)}}\left(\left(u_1,v\right),\left(u_2,v\right)\right)\le\max\left\{\left(\mathbb{U}_{J_1(a_i)}\times\mathbb{U}_{J_2(b_j)}\right)\left(u_1,v\right),\left(\mathbb{U}_{J_1(a_i)}\times\mathbb{U}_{J_2(b_j)}\right)\left(u_2,v\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{F}_{K_{(a_i,b_j)}}\left(\left(u_1,v\right),\left(u_2,v\right)\right)\le\max\left\{\left(\mathbb{F}_{J_1(a_i)}\times\mathbb{F}_{J_2(b_j)}\right)\left(u_1,v\right),\left(\mathbb{F}_{J_1(a_i)}\times\mathbb{F}_{J_2(b_j)}\right)\left(u_2,v\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

Hence G=(J,K,$R\times S$) is a quadripartitioned neutrosophic soft graph.

Definition 3.6 *The cross product of* G_1 *and* G_2 *is a quadripartitioned neutrosophic soft graphs of* $G=G_1\circledcirc G_2=\left(J,K,R\times S\right)$, *where* ($J$,$R\times S$) *is a quadripartitioned neutrosophic soft set over* $V=V_1\times V_2$, (K, $R\times S$) *is a quadripartitioned neutrsophic soft set over*

$$E=\left\{\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right):\left(u_1,u_2\right)\in E_1,\left(v_1,v_2\right)\in E_2\right\}$$

and (J,K,R×S) are quadripartitioned neutrosophic soft graphs such that

(i) $\mathbb{T}_{J(a,b)}\left(u,v\right)=\mathbb{T}_{J_1(a)}\left(u\right)\wedge\mathbb{T}_{J_2(b)}\left(v\right)$,

$$\mathbb{C}_{J(a,b)}\left(u,v\right)=\mathbb{C}_{J_1(a)}\left(u\right)\wedge\mathbb{C}_{J_2(b)}\left(v\right),$$

$$\mathbb{U}_{J(a,b)}\left(u,v\right)=\mathbb{U}_{J_1(a)}\left(u\right)\vee\mathbb{U}_{J_2(b)}\left(v\right),$$

$$\mathbb{F}_{J(a,b)}\left(u,v\right)=\mathbb{F}_{J_1(a)}\left(u\right)\vee\mathbb{F}_{J_2(b)}\left(v\right),$$

$\forall(u,v)\in V$, $(a,b)\in R\times S$.

(ii) $\mathbb{T}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{T}_{K_1(a)}\left(u_1,u_2\right)\wedge\mathbb{T}_{K_2(b)}\left(v_1,v_2\right)$,

$$\mathbb{C}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{C}_{K_1(a)}\left(u_1,u_2\right)\wedge\mathbb{C}_{K_2(b)}\left(v_1,v_2\right),$$

$$\mathbb{U}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{U}_{K_1(a)}\left(u_1,u_2\right)\vee\mathbb{U}_{K_2(b)}\left(v_1,v_2\right),$$

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$$\mathbb{F}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{F}_{K_1(a)}\left(u_1,u_2\right)\vee\mathbb{F}_{K_2(b)}\left(v_1,v_2\right),$$

$\forall(u_1,u_2)\in E_1$, $(v_1,v_2)\in E_2$.

$$H\left(a,b\right)=H_1\left(a\right)\circledcirc H_2\left(b\right)$$

for all $(a,b)\in R\times S$ are quadripartitioned neutrosophic graphs of G.

Theorem 3.7 *The cross product of two quadripartitioned neutrosophic soft graph is a quadripartitioned neutrosophic soft graph.*

Proof. Let $G_1=(J_1,K_1,R)$ and $G_2=(J_2,K_2,S)$ be two quadripartitioned neutrosophic soft graphs of $G_1^*=\left(V_1,E_1\right)$ and $G_2^*=\left(V_2,E_2\right)$, respectively. Let $G=G_1\circledcirc G_2=\left(J,K,R\times S\right)$ be the cross product of G_1 and G_2. We claim that $G=(J,K,R\times S)$ is a quadripartitioned neutrosophic soft graph and

$$\left(H,R\times S\right)=\left\{J_1\circledcirc J_2\left(a_i,b_j\right),K_1\circledcirc K_2\left(a_i,b_j\right)\right\}\forall a_i\in R,b_j\in S$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$ are quadripartitioned neutrosophic graphs of G.

Consider,

$$\mathbb{T}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\min\left\{\mathbb{T}_{K_1(a_i)}\left(u_1,u_2\right),\mathbb{T}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\leq\min\left\{\min\left\{\mathbb{T}_{J_1(a_i)}\left(u_1\right),\mathbb{T}_{J_1(a_i)}\left(u_2\right)\right\},\min\left\{\mathbb{T}_{J_2(b_j)}\left(v_1\right),\mathbb{T}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$=\min\left\{\min\left\{\mathbb{T}_{J_1(a_i)}\left(u_1\right),\mathbb{T}_{J_2(b_j)}\left(v_1\right)\right\},\min\left\{\mathbb{T}_{J_1(a_i)}\left(u_2\right),\mathbb{T}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$\mathbb{T}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)\leq\min\left\{\left(\mathbb{T}_{J_1(a_i)}\circledcirc\mathbb{T}_{J_2(b_j)}\right)\left(u_1,v_1\right),\left(\mathbb{T}_{J_1(a_i)}\circledcirc\mathbb{T}_{J_2(b_j)}\right)\left(u_2,v_2\right)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\min\left\{\mathbb{C}_{K_1(a_i)}\left(u_1,u_2\right),\mathbb{C}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

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$$\leq \min\left\{\min\left\{\mathbb{C}_{J_1(a_i)}(u_1), \mathbb{C}_{J_1(a_i)}(u_2)\right\}, \min\left\{\mathbb{C}_{J_2(b_j)}(v_1), \mathbb{C}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \min\left\{\min\left\{\mathbb{C}_{J_1(a_i)}(u_1), \mathbb{C}_{J_2(b_j)}(v_1)\right\}, \min\left\{\mathbb{C}_{J_1(a_i)}(u_2), \mathbb{C}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) \leq \min\left\{\left(\mathbb{C}_{J_1(a_i)} \circledcirc \mathbb{C}_{J_2(b_j)}\right)(u_1,v_1), \left(\mathbb{C}_{J_1(a_i)} \circledcirc \mathbb{C}_{J_2(b_j)}\right)(u_2,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) = \max\left\{\mathbb{U}_{K_1(a_i)}(u_1,u_2), \mathbb{U}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq \max\left\{\max\left\{\mathbb{U}_{J_1(a_i)}(u_1), \mathbb{U}_{J_1(a_i)}(u_2)\right\}, \max\left\{\mathbb{U}_{J_2(b_j)}(v_1), \mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \max\left\{\max\left\{\mathbb{U}_{J_1(a_i)}(u_1), \mathbb{U}_{J_2(b_j)}(v_1)\right\}, \max\left\{\mathbb{U}_{J_1(a_i)}(u_2), \mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) \leq \max\left\{\left(\mathbb{U}_{J_1(a_i)} \circledcirc \mathbb{U}_{J_2(b_j)}\right)(u_1,v_1), \left(\mathbb{U}_{J_1(a_i)} \circledcirc \mathbb{U}_{J_2(b_j)}\right)(u_2,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) = \max\left\{\mathbb{F}_{K_1(a_i)}(u_1,u_2), \mathbb{F}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq \max\left\{\max\left\{\mathbb{F}_{J_1(a_i)}(u_1), \mathbb{F}_{J_1(a_i)}(u_2)\right\}, \max\left\{\mathbb{F}_{J_2(b_j)}(v_1), \mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \max\left\{\max\left\{\mathbb{F}_{J_1(a_i)}(u_1), \mathbb{F}_{J_2(b_j)}(v_1)\right\}, \max\left\{\mathbb{F}_{J_1(a_i)}(u_2), \mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) \leq \max\left\{\left(\mathbb{F}_{J_1(a_i)} \circledcirc \mathbb{F}_{J_2(b_j)}\right)(u_1,v_1), \left(\mathbb{F}_{J_1(a_i)} \circledcirc \mathbb{F}_{J_2(b_j)}\right)(u_2,v_2)\right\}$$

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for i=1,2,…,m; j=1,2,…,n

Hence G=(J,K,R×S) is a quadripartitioned neutrosophic soft graph.

Definition 3.8 *The lexicographic product of* G_1 *and* G_2 *is a quadripartitioned neutrosophic soft graphs of* $G = G_1 \odot G_2 = (J, K, R \times S)$, *where* ($J$,$R$×$S$) *is a quadripartitioned neutrosophic soft set over* V=V_1×V_2, (K, R×S) *is a quadripartitioned neutrosophic soft set over*

$$E = \left\{\left((u,v_1),(u,v_2)\right) : u \in V_1, (v_1,v_2) \in E_2\right\} \cup \left\{\left((u_1,v_1),(u_2,v_2)\right) : (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\right\}$$

and (J,K,R×S) are quadripartitioned neutrosophic soft graphs such that

(i) $\mathbb{T}_{J(a,b)}(u,v) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{T}_{J_2(b)}(v)$,

$\mathbb{C}_{J(a,b)}(u,v) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{C}_{J_2(b)}(v)$,

$\mathbb{U}_{J(a,b)}(u,v) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{J_2(b)}(v)$,

$\mathbb{F}_{J(a,b)}(u,v) = \mathbb{F}_{J_1(a)}(u) \vee \mathbb{F}_{J_2(b)}(v)$,

$\forall (u,v) \in V$, $(a,b) \in R \times S$.

(ii) $\mathbb{T}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{T}_{K_2(b)}(v_1,v_2)$,

$\mathbb{C}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{C}_{K_2(b)}(v_1,v_2)$,

$\mathbb{U}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{K_2(b)}(v_1,v_2)$,

$\mathbb{F}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{F}_{J_1(a)}(u) \vee \mathbb{F}_{K_2(b)}(v_1,v_2)$,

$\forall u \in V_1$, $(v_1,v_2) \in E_2$.

(iii) $\mathbb{T}_{K(a,b)}\left((u_1,v_1),(u_2,v_2)\right) = \mathbb{T}_{K_1(a)}(u_1,u_2) \wedge \mathbb{T}_{K_2(b)}(v_1,v_2)$,

$\mathbb{C}_{K(a,b)}\left((u_1,v_1),(u_2,v_2)\right) = \mathbb{C}_{K_1(a)}(u_1,u_2) \wedge \mathbb{C}_{K_2(b)}(v_1,v_2)$,

$\mathbb{U}_{K(a,b)}\left((u_1,v_1),(u_2,v_2)\right) = \mathbb{U}_{K_1(a)}(u_1,u_2) \vee \mathbb{U}_{K_2(b)}(v_1,v_2)$,

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$$\mathbb{F}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{F}_{K_1(a)}\left(u_1,u_2\right)\vee\mathbb{F}_{K_2(b)}\left(v_1,v_2\right),$$

$\forall(u_1,u_2)\in E_1$, $(v_1,v_2)\in E_2$.

$$H\left(a,b\right)=H_1\left(a\right)\odot H_2\left(b\right)$$

for all $(a,b)\in R\times S$ are quadripartitioned neutrosophic graphs of G.

Theorem 3.9 *The lexicographic product of two quadripartitioned neutrosophic soft graphs is a quadripartitioned neutrosophic soft graph.*

Proof. Let $G_1=(J_1,K_1,R)$ and $G_2=(J_2,K_2,S)$ be two quadripartitioned neutrosophic soft graphs of $G_1^*=\left(V_1,E_1\right)$ and $G_2^*=\left(V_2,E_2\right)$, respectively. Let $G=G_1\odot G_2=\left(J,K,R\odot S\right)$ be the lexicographic product of G_1 and G_2.

Consider,

$$\mathbb{T}_{K_{(a_i,b_j)}}\left(\left(u,v_1\right),\left(u,v_2\right)\right)=\min\left\{\mathbb{T}_{J_1(a_i)}\left(u\right),\mathbb{T}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\leq\min\left\{\mathbb{T}_{J_1(a_i)}\left(u\right),\min\left\{\mathbb{T}_{J_2(b_j)}\left(v_1\right),\mathbb{T}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$=\min\left\{\min\left\{\mathbb{T}_{J_1(a_i)}\left(u\right),\mathbb{T}_{J_2(b_j)}\left(v_1\right)\right\},\min\left\{\mathbb{T}_{J_1(a_i)}\left(u\right),\mathbb{T}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$\mathbb{T}_{K_{(a_i,b_j)}}\left(\left(u,v_1\right),\left(u,v_2\right)\right)\leq\min\left\{\left(\mathbb{T}_{J_1(a_i)}\odot\mathbb{T}_{J_2(b_j)}\right)\left(u,v_1\right),\left(\mathbb{T}_{J_1(a_i)}\odot\mathbb{T}_{J_2(b_j)}\right)\left(u,v_2\right)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left(\left(u,v_1\right),\left(u,v_2\right)\right)=\min\left\{\mathbb{C}_{J_1(a_i)}\left(u\right),\mathbb{C}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

$$\leq\min\left\{\mathbb{C}_{J_1(a_i)}\left(u\right),\min\left\{\mathbb{C}_{J_2(b_j)}\left(v_1\right),\mathbb{C}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

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$$= \min\left\{\min\left\{\mathbb{C}_{J_1(a_i)}(u), \mathbb{C}_{J_2(b_j)}(v_1)\right\}, \min\left\{\mathbb{C}_{J_1(a_i)}(u), \mathbb{C}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) \le \min\left\{\left(\mathbb{C}_{J_1(a_i)} \odot \mathbb{C}_{J_2(b_j)}\right)(u,v_1), \left(\mathbb{C}_{J_1(a_i)} \odot \mathbb{C}_{J_2(b_j)}\right)(u,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) = \max\left\{\mathbb{U}_{J_1(a_i)}(u), \mathbb{U}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\le \max\left\{\mathbb{U}_{J_1(a_i)}(u), \max\left\{\mathbb{U}_{J_2(b_j)}(v_1), \mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \max\left\{\max\left\{\mathbb{U}_{J_1(a_i)}(u), \mathbb{U}_{J_2(b_j)}(v_1)\right\}, \max\left\{\mathbb{U}_{J_1(a_i)}(u), \mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) \le \max\left\{\left(\mathbb{U}_{J_1(a_i)} \odot \mathbb{U}_{J_2(b_j)}\right)(u,v_1), \left(\mathbb{U}_{J_1(a_i)} \odot \mathbb{U}_{J_2(b_j)}\right)(u,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) = \max\left\{\mathbb{F}_{J_1(a_i)}(u), \mathbb{F}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\le \max\left\{\mathbb{F}_{J_1(a_i)}(u), \max\left\{\mathbb{F}_{J_2(b_j)}(v_1), \mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \max\left\{\max\left\{\mathbb{F}_{J_1(a_i)}(u), \mathbb{F}_{J_2(b_j)}(v_1)\right\}, \max\left\{\mathbb{F}_{J_1(a_i)}(u), \mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u,v_1),(u,v_2)\right) \le \max\left\{\left(\mathbb{F}_{J_1(a_i)} \odot \mathbb{F}_{J_2(b_j)}\right)(u,v_1), \left(\mathbb{F}_{J_1(a_i)} \odot \mathbb{F}_{J_2(b_j)}\right)(u,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

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$$\mathbb{T}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\min\left\{\mathbb{T}_{K_1(a_i)}\left(u_1,u_2\right),\mathbb{T}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq \min\left\{\min\left\{\mathbb{T}_{J_1(a_i)}\left(u_1\right),\mathbb{T}_{J_1(a_i)}\left(u_2\right)\right\},\min\left\{\mathbb{T}_{J_2(b_j)}\left(v_1\right),\mathbb{T}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$=\min\left\{\min\left\{\mathbb{T}_{J_1(a_i)}\left(u_1\right),\mathbb{T}_{J_2(b_j)}\left(v_1\right)\right\},\min\left\{\mathbb{T}_{J_1(a_i)}\left(u_2\right),\mathbb{T}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$\mathbb{T}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)\leq\min\left\{\left(\mathbb{T}_{J_1(a_i)}\odot\mathbb{T}_{J_2(b_j)}\right)\left(u_1,v_1\right),\left(\mathbb{T}_{J_1(a_i)}\odot\mathbb{T}_{J_2(b_j)}\right)\left(u_2,v_2\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{C}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\min\left\{\mathbb{C}_{K_1(a_i)}\left(u_1,u_2\right),\mathbb{C}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq \min\left\{\min\left\{\mathbb{C}_{J_1(a_i)}\left(u_1\right),\mathbb{C}_{J_1(a_i)}\left(u_2\right)\right\},\min\left\{\mathbb{C}_{J_2(b_j)}\left(v_1\right),\mathbb{C}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$=\min\left\{\min\left\{\mathbb{C}_{J_1(a_i)}\left(u_1\right),\mathbb{C}_{J_2(b_j)}\left(v_1\right)\right\},\min\left\{\mathbb{C}_{J_1(a_i)}\left(u_2\right),\mathbb{C}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

$$\mathbb{C}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)\leq\min\left\{\left(\mathbb{C}_{J_1(a_i)}\odot\mathbb{C}_{J_2(b_j)}\right)\left(u_1,v_1\right),\left(\mathbb{C}_{J_1(a_i)}\odot\mathbb{C}_{J_2(b_j)}\right)\left(u_2,v_2\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{U}_{K_{(a_i,b_j)}}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\max\left\{\mathbb{U}_{K_1(a_i)}\left(u_1,u_2\right),\mathbb{U}_{K_2(b_j)}\left(v_1,v_2\right)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\leq \max\left\{\max\left\{\mathbb{U}_{J_1(a_i)}\left(u_1\right),\mathbb{U}_{J_1(a_i)}\left(u_2\right)\right\},\max\left\{\mathbb{U}_{J_2(b_j)}\left(v_1\right),\mathbb{U}_{J_2(b_j)}\left(v_2\right)\right\}\right\}$$

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$$= \max\left\{\max\left\{\mathbb{U}_{J_1(a_i)}(u_1), \mathbb{U}_{J_2(b_j)}(v_1)\right\}, \max\left\{\mathbb{U}_{J_1(a_i)}(u_2), \mathbb{U}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{U}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) \le \max\left\{\left(\mathbb{U}_{J_1(a_i)} \odot \mathbb{U}_{J_2(b_j)}\right)(u_1,v_1), \left(\mathbb{U}_{J_1(a_i)} \odot \mathbb{U}_{J_2(b_j)}\right)(u_2,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) = \max\left\{\mathbb{F}_{K_1(a_i)}(u_1,u_2), \mathbb{F}_{K_2(b_j)}(v_1,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

$$\le \max\left\{\max\left\{\mathbb{F}_{J_1(a_i)}(u_1), \mathbb{F}_{J_1(a_i)}(u_2)\right\}, \max\left\{\mathbb{F}_{J_2(b_j)}(v_1), \mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$= \max\left\{\max\left\{\mathbb{F}_{J_1(a_i)}(u_1), \mathbb{F}_{J_2(b_j)}(v_1)\right\}, \max\left\{\mathbb{F}_{J_1(a_i)}(u_2), \mathbb{F}_{J_2(b_j)}(v_2)\right\}\right\}$$

$$\mathbb{F}_{K_{(a_i,b_j)}}\left((u_1,v_1),(u_2,v_2)\right) \le \max\left\{\left(\mathbb{F}_{J_1(a_i)} \odot \mathbb{F}_{J_2(b_j)}\right)(u_1,v_1), \left(\mathbb{F}_{J_1(a_i)} \odot \mathbb{F}_{J_2(b_j)}\right)(u_2,v_2)\right\}$$

for i=1,2,…,m; j=1,2,…,n

Hence the Proof.

Definition 3.10 *The strong product of G_1 and G_2 is a quadripartitioned neutrosophic soft graphs of $G = G_1 \otimes G_2 = (J, K, R \times S)$, where (J,R×S) is a quadripartitioned neutrosophic soft set over V=V_1×V_2, (K, R×S) is a neutrsophic vague soft set over*

$$E = \left\{\left((u,v_1),(u,v_2)\right) : (u) \in V_1, (v_1,v_2) \in E_2\right\}$$
$$\cup\left\{\left((u_1,v),(u_2,v)\right) : (v) \in V_2, (u_1,u_2) \in E_1\right\} \cup \left\{\left((u_1,v_1),(u_2,v_2)\right) : (u_1,u_2) \in E_1, (v_1,v_2 \in E_2)\right\}$$

and (*J,K,R*×*S*) are quadripartitioned neutrosophic soft graphs such that

(i) $\mathbb{T}_{J(a,b)}(u,v) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{U}_{J_2(b)}(v)$,

$\mathbb{C}_{J(a,b)}(u,v) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{U}_{J_2(b)}(v)$,

$\mathbb{U}_{J(a,b)}(u,v) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{J_2(b)}(v)$,

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$$\mathbb{F}_{J(a,b)}(u,v)=\mathbb{F}_{J_1(a)}(u)\vee\mathbb{F}_{J_2(b)}(v),$$

$\forall(u,v)\in V$, $(a,b)\in R\times S$.

(ii) $\mathbb{T}_{K(a,b)}\big((u,v_1),(u,v_2)\big)=\mathbb{T}_{J_1(a)}(u)\wedge\mathbb{T}_{K_2(b)}(v_1,v_2),$

$$\mathbb{C}_{K(a,b)}\big((u,v_1),(u,v_2)\big)=\mathbb{C}_{J_1(a)}(u)\wedge\mathbb{C}_{K_2(b)}(v_1,v_2),$$

$$\mathbb{U}_{K(a,b)}\big((u,v_1),(u,v_2)\big)=\mathbb{U}_{J_1(a)}(u)\vee\mathbb{U}_{K_2(b)}(v_1,v_2),$$

$$\mathbb{F}_{K(a,b)}\big((u,v_1),(u,v_2)\big)=\mathbb{F}_{J_1(a)}(u)\vee\mathbb{F}_{K_2(b)}(v_1,v_2),\forall u\in V_1,(v_1,v_2)\in E_2.$$

(iii) $\mathbb{T}_{K(a,b)}\big((u_1,v),(u_2,v)\big)=\mathbb{T}_{J_2(b)}(v)\wedge\mathbb{T}_{K_2(a)}(u_1,u_2),$

$$\mathbb{C}_{K(a,b)}\big((u_1,v),(u_2,v)\big)=\mathbb{C}_{J_2(b)}(v)\wedge\mathbb{C}_{K_2(a)}(u_1,u_2),$$

$$\mathbb{U}_{K(a,b)}\big((u_1,v),(u_2,v)\big)=\mathbb{U}_{J_2(b)}(v)\vee\mathbb{U}_{K_2(a)}(u_1,u_2),$$

$$\mathbb{F}_{K(a,b)}\big((u_1,v),(u_2,v)\big)=\mathbb{F}_{J_2(b)}(v)\vee\mathbb{F}_{K_2(a)}(u_1,u_2),\forall v\in V_2,(u_1,u_2)\in E_1.$$

(iv) $\mathbb{T}_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big)=\mathbb{T}_{K_1(a)}(u_1,u_2)\wedge\mathbb{T}_{K_2(b)}(v_1,v_2),$

$$\mathbb{C}_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big)=\mathbb{C}_{K_1(a)}(u_1,u_2)\wedge\mathbb{C}_{K_2(b)}(v_1,v_2),$$

$$\mathbb{U}_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big)=\mathbb{U}_{K_1(a)}(u_1,u_2)\vee\mathbb{U}_{K_2(b)}(v_1,v_2),$$

$$\mathbb{F}_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big)=\mathbb{F}_{K_1(a)}(u_1,u_2)\vee\mathbb{F}_{K_2(b)}(v_1,v_2),\forall(u_1,u_2)\in E_1,(v_1,v_2)\in E_2.$$

$$H(a,b)=H_1(a)\otimes H_2(b)$$

for all $(a,b)\in R\times S$ are quadripartitioned neutrosophic graphs of G.

Theorem 3.11 *The strong product of two quadripartitioned neutrosophic soft graphs is a quadripartitioned neutrosophic soft graph.*

Definition 3.12 *The composition of G_1 and G_2 is a quadripartitioned neutrosophic soft graphs of $G=G_1[G_2]=(J,K,R\times S)$, where $(J,R\times S)$ is a quadripartitioned neutrosophic soft set over $V=V_1\times V_2$, $(K, R\times S)$ is a quadripartitioned neutrosophic soft set over*

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$$E = \left\{\left((u,v_1),(u,v_2)\right) : u \in V_1, (v_1,v_2) \in E_2\right\}$$
$$\cup \left\{\left((u_1,v),(u_2,v)\right) : v \in V_2, (u_1,u_2) \in E_1\right\} \cup \left\{\left((u_1,v_1),(u_2,v_2)\right) : (u_1,u_2) \in E_1, (v_1 \neq v_2)\right\}$$

and $(J,K,R\times S)$ are quadripartitioned neutrosophic soft graphs such that

(i) $\mathbb{T}_{J(a,b)}(u,v) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{T}_{J_2(b)}(v)$,

$\mathbb{C}_{J(a,b)}(u,v) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{C}_{J_2(b)}(v)$,

$\mathbb{U}_{J(a,b)}(u,v) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{J_2(b)}(v)$,

$\mathbb{F}_{J(a,b)}(u,v) = \mathbb{F}_{J_1(a)}(u) \vee \mathbb{U}_{J_2(b)}(v)$,

$\forall (u,v) \in V$, $(a,b) \in R\times S$.

(ii) $\mathbb{T}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{T}_{J_1(a)}(u) \wedge \mathbb{T}_{K_2(b)}(v_1,v_2)$,

$\mathbb{C}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{C}_{J_1(a)}(u) \wedge \mathbb{C}_{K_2(b)}(v_1,v_2)$,

$\mathbb{U}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{U}_{J_1(a)}(u) \vee \mathbb{U}_{K_2(b)}(v_1,v_2)$,

$\mathbb{F}_{K(a,b)}\left((u,v_1),(u,v_2)\right) = \mathbb{F}_{J_1(a)}(u) \vee \mathbb{F}_{K_2(b)}(v_1,v_2)$,

$\forall\ u \in V_1$, $(v_1,v_2) \in E_2$.

(iii) $\mathbb{T}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{T}_{J_2(b)}(v) \wedge \mathbb{T}_{K_2(a)}(u_1,u_2)$,

$\mathbb{C}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{C}_{J_2(b)}(v) \wedge \mathbb{C}_{K_2(a)}(u_1,u_2)$,

$\mathbb{U}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{U}_{J_2(b)}(v) \vee \mathbb{U}_{K_2(a)}(u_1,u_2)$,

$\mathbb{F}_{K(a,b)}\left((u_1,v),(u_2,v)\right) = \mathbb{F}_{J_2(b)}(v) \vee \mathbb{F}_{K_2(a)}(u_1,u_2)$,

$\forall\ v \in V_2$, $(u_1,u_2) \in E_1$.

(iv) $\mathbb{T}_{K(a,b)}\left((u_1,v_1),(u_2,v_2)\right) = \mathbb{T}_{K_1(a)}(u_1,u_2) \wedge \mathbb{T}_{J_2(a)}(v_1) \wedge \mathbb{T}_{J_2(b)}(v_2)$,

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$$\mathbb{C}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{C}_{K_1(a)}\left(u_1,u_2\right)\wedge\mathbb{C}_{J_2(a)}\left(v_1\right)\wedge\mathbb{C}_{J_2(b)}\left(v_2\right),$$

$$\mathbb{U}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{U}_{K_1(a)}\left(u_1,u_2\right)\vee\mathbb{U}_{J_2(a)}\left(v_1\right)\vee\mathbb{U}_{J_2(b)}\left(v_2\right),$$

$$\mathbb{F}_{K(a,b)}\left(\left(u_1,v_1\right),\left(u_2,v_2\right)\right)=\mathbb{F}_{K_1(a)}\left(u_1,u_2\right)\vee\mathbb{F}_{J_2(a)}\left(v_1\right)\vee\mathbb{F}_{J_2(b)}\left(v_2\right),$$

$\forall(u_1,u_2)\in E_1$, where $v_1\neq v_2$.

$H(a,b)=H_1(a)[H_2(b)]$

for all $(a,b)\in R\times S$ are quadripartitioned neutrosophic graphs of G.

Theorem 3.13 *The composition product of two quadripartitioned neutrosophic soft graphs is a quadripartitioned neutrosophic soft graph.*

DISCUSSION

Quadripartitioned neutrosophic soft sets provide a powerful tool to represent the information with uncertain data and have fruitful applications. In this work, the quadripartitioned neutrosophic soft sets and quadripartitioned neutrosophic soft graphs have been developed. The significant operations of interval-valued intuitionistic neutrosophic sets and its graphs are investigated. This helps the decision-makers more sufficient for taking their input best suit to their domain of reference. Hence, the proposed sets, graphs and their operations have enough capabilities to address the related dependability on the imprecise information.

CONCLUSION

In this work, quadripartitioned neutrosophic soft graphs have been developed. This helps the decision-makers more sufficient for taking their input best suit to their domain of reference. Operations like Cartesian product, cross product, lexicographic product and strong product of quadripartitioned neutrosophic soft graphs are established. Hence, the proposed graphs and their operations have enough capabilities to address the related dependability on the imprecise information. Further, the authors will aim to develop this research to the isomorphic properties of the proposed concepts in future.

COMPLIANCE WITH ETHICAL STANDARDS

Conflict of interest: The authors declare that they have no conflict of interest.

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Chapter 35
Priority Weighted Neutrosophic Refined Soft Set: A Multi-Criteria Decision-Making Approach

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ABSTRACT

This chapter devises a new concept of priority weighted neutrosophic refined soft set (PWNRSS) by combining neutrosophic refined sets and soft sets implemented with prioritized universal elements and weightage-imposed parameters. The concepts of PWNRS subset, PWNRS null set, and PWNRS universal set are defined. Based on the definitions of n-norm and n-co-norm, the theoretical operations of PWNRS sets such as union, intersection, and complement are defined. AND-product and OR-product between two priority-weighted neutrosophic refined soft sets are introduced. Furthermore, priority weights neutrosophic refined soft set is expanded to MCDM technique to handle decision-making issues. TOPSIS has been examined more thoroughly for the PWNRSS decision-making issue. This proposed method might be extremely valuable in large-scale decision-making situations. Numerical examples are also provided to demonstrate the methodologies' dependability and applicability.

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INTRODUCTION

The majority of real-time issues include some or other type of inaccurate information. Many researchers have offered various mathematical techniques to deal with these types of data. Fuzzy sets, intuitionistic fuzzy sets, soft sets, vague sets and so on are few such techniques. Only a partial information is handled by fuzzy sets with membership grade and intuitionistic fuzzy accounts both the truth-membership (and simply membership) and falsity-membership (or non-membership) values. Both the frame works, doesn't work with the ambiguous and conflicting information that remains in the belief system. As a result, developing and generalizing the fuzzy sets proved to be insufficient to address all sorts of uncertainty in real-world situations. As a consequence, several additional theories were needed.

Neutrosophy is a modern science that investigates the source, structure, and scale of indeterminacies, as well as their connections to other conceptual regimes. The notion of neutrosophy is made understandable by the application of a rule, an idea, an axiom, or a conceptual structure to an ambiguous, imprecise, and indeterminate situation. In addition, the neutrosophic set theory is used in a variety of areas, including topology, operations research, control theory, mechanics, and a variety of other subjects. The development of Single Valued Neutrosophic Sets, neutrosophic cubic sets, bipolar neutrosophic sets, and other related concepts has simplified neutrosophic theory. Hence Smarandache's neutrosophic theory has known to be one of the most important and adaptable techniques to cope with complex and uncertain conditions in physical difficulties. To be more specific, neutrosophic sets' Decision Making approaches are extremely significant since they have wide range of applications in various fields.

On other hand, soft sets as novel mathematical theory for dealing with uncertainty has been applied to several directions with the formulated notions of soft number, soft derivative, soft integral, etc. Many researchers worked on theoretical study of soft sets in detail and presented its applications in the decision making problems. Several researchers have incorporated various mathematical hybrid structures by generalizing and expanding classical soft set theory. The Neutrosophic soft sets are introduced to make the existing theories more practical in real-world physical and engineering issues. It is a very good tool by which one can manage vague data in a more effective way as compared to soft sets and other models of fuzzy sets. The classical neutrosophic logic is extended to n-valued refined neutrosophic logic, by refining each neutrosophic component in T,I,F to respectively $T_1,T_2,\ldots,T_m$, $I_1,I_2,\ldots,I_p$ and $F_1,F_2,\ldots,F_r$. Refined Neutrosophic Sets (RNS) are a generalization of a neutrosophic sets, intuitionistic fuzzy sets, fuzzy sets, intuitionistic fuzzy multi-sets and fuzzy multi-sets. Similarly, refined neutrosophic soft sets are a generalization of a neutrosophic soft sets, intuitionistic fuzzy soft sets, fuzzy soft sets, intuitionistic fuzzy soft multi-sets and fuzzy soft multi-sets. These sets are a powerful general formal framework that has been proposed to solve uncertain issues.

In this article a new hybridized structure named Priority weighted Neutrosophic Refined Soft Set (PWNRSS) is defined and its basic properties are discussed. Algebraic operations between two PWNRS set and some of their features are studied. Further the theory is extended to Priority Weighted Neutrosophic Refined Soft Decision Making (PWNRSDM) techniques. Numerical examples are provided to demonstrate how the technique may be effectively applied to the circumstances investigated.

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BACKGROUND

Lotfi L. A. Zadeh (1965) created fuzzy set and it is a collection of items with a grade of membership that ranges from zero to one. Krassimir Atanassov (1986) introduced the intuitionistic fuzzy set as an elaborate version of fuzzy set and that considered the non-membership degree along with membership grade. Both the works advanced the conventional notion of a classical set. Smarandache (1998) discussed on neutrosophy, neutrosophic probability, Set and logic and (2005), (2006), (2010) he proposed the neutrosophic set methodology, a mathematical strategy for dealing with concerns comprising imprecise, indeterminate and incompatible data. In neutrosophic representation membership, indeterminacy, and non-membership values are modeled by the neutrosophic considerations *T,I,F* within $^{-}]0,1[^{+}$, the non-standard unit interval.

Molodtsov (1999) proposed the remarkable work of Soft set theory. Since then, Soft set theory has been hybridized with other existing theories. To name a few, Alkhazaleh et al. (2011) proposed Possibility fuzzy soft sets. Babitha K V and Sunil J.J. (2011) studied Generalized intuitionistic fuzzy soft sets and its applications, Bashir (2012) studied Possibility intuitionistic fuzzy soft Sets. Maji P. K. (2013) formulated Neutrosophic soft set. Broumi. S (2013) developed Intuitionistic Neutrosophic Soft. Maji, P.K. (2015) further developed and studied weighted neutrosophic soft sets approach in a multi-criteria decision-making problem. Broumi. S, (2015) extended Neutrosophic soft relations and some of its properties. Sahin. M et al built Neutrosophic soft expert sets. Alkhazaleh S. (2016), (2017) implemented Time-neutrosophic soft set and 'n'-valued refined neutrosophic soft set theory. Ulucay V et al. (2018) gave Time-Neutrosophic Soft Expert Sets and applied in decision Making Problem. Hassan N et al. (2018) devised Q-neutrosophic soft expert set and applied in decision making. Saha, A., Broumi, S. (2020) worked in parameter reduction of neutrosophic soft sets and presented their applications.

On the flip side, Sebastian and Ramakrishnan (2011) introduced multi-fuzzy sets, Shinoj's. T.K, Sunil Jacob John (2012) initiated intuitionistic fuzzy multisets. Although, the fuzzy multi set, intuitionistic fuzzy multi set which are important tools to model problems involving uncertainty, they may not be sufficient for modeling of some problems containing inconsistent data. Thus, Smarandache (2013) introduced n-Valued refined neutrosophic set, Deli et al. (2015) gave some concepts of neutrosophic refined sets including its algebraic properties and distance measures. Then, Broumi and Smarandache (2014) developed the cosine similarity measure of neutrosophic refined sets. Ye and Smarandache (2016) proposed similarity measures between two refined single-valued neutrosophic sets and applied to multi-criteria decision-making. Smarandache (2018) also extended soft set to hypersoft Set, then to plithogenic hypersoft Set. Tooranloo et al. (2020) tried to present a neutrosophic electrode model on refined neutrosophic sets. Recently, neutrosophic refined sets were extended in the field of algebra. Abobala. M (2020) investigated the Classical Homomorphisms between Refined Neutrosophic Rings and Neutrosophic Rings. And also he proposed some Special Substructures of Refined Neutrosophic Rings in (2020). Adeleke. E.O, Agboola. A.A.A, and Smarandache, F (2020) proposed the idea of Refined Neutrosophic Rings I, Refined Neutrosophic Rings II .Agboola, A.A.A, F. Smarandache, M. Abobala, H. Sankari (2020) investigated the notion of Refined Neutrosophic Quotient Groups, n-Refined neutrosophic Rings, n-Refined neutrosophic vector spaces, and n-Refined neutrosophic modules. M. A. Ibrahim, A. A. A. Agboola, B. S. Badmus, and S. A. Akinleye (2020) imported refined neutrosophic vector spaces. M. Abobala (2020) investigated the ranking of refined neutrosophic matrices and their applications in refined neutrosophic algebraic equations.

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Nowadays the theory of neutrosophy is widely applied in many decision concerns. Some collections include: TOPSIS for single valued neutrosophic soft expert set based multi-attribute decision making problems by Pramanik S (2015), A novel extension of TOPSIS to MCGDM with Bipolar Neutrosophic soft topology studied by Tehrim (2019). Riaz M et al. worked on application of generalized fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018; A Mathematical Analysis. Muhammad Saqlain, et al. (2020) projected on Neutrosophic Soft Set with Generalized Fuzzy TOPSIS and application of Smart Phone Selection. Chakraborty, A., Mondal, S.P., Alam, S. (2021). Classified Trapezoidal Bipolar Neutrosophic Numbers, De-Bipolarization and Implementation in Cloud Service Based MCGDM Problem.

Motivated by the above said considerations, this chapter is designed as follows:

- A novel trend of Priority Weighted Neutrosophic Refined Soft Set (PWNRSS) is devised taking into the account that every element of the initial universe is dealt with a priority and the parameters are given a weightage.
- Some axiomatic characteristics such PWNRS subset, PWNRS null set, and PWNRS universal set are defined.
- By imposing the concepts of n-norm and n-conorm, the set theoretical laws such as union, intersection, and complement of PWNRSs are discussed and specific algebraic properties of these operations are analyzed.
- Further, a new decision making method using AND-product of PWNRSs called Priority Weighted Neutrosophic Refined Soft set Decision Making is discussed.
- Furthermore TOPSIS method is studied for PWNRSS.
- Enduringly, numerical examples that show the applicability of proposed techniques are presented.

PRIORITY WEIGHTED NEUTROSOPHIC REFINED SOFT SET (PWNRSS)

This introduces a new type of set called Priority Weighted Neutrosophic Refined Soft Set (PWNRSS) and further constructs a priority weighted neutrosophic soft decision-making method.

Definition 2.1

Let U be the initial universal set, E be the set of parameters, NRS(U) be the collection of all neutrosophic refined sub sets of U and $F(U)$ be a collection of all fuzzy subset of U. A priority weighted neutrosophic refined soft set (PWNRS-set) $P_{[\bar{\alpha},\bar{\beta}]}$ over U is a combination of triplets given by

$$P_{[\bar{\alpha},\bar{\beta}]} = \left\{ \left(e_k, \left\{ \frac{u_j}{p^N(e_k)(u_j)}, \bar{\alpha}(e_k)(u_j) \right\}, \bar{\beta}(e_k) \right) : e_k \in E \right\}$$

or a mapping defined by $P_{[\bar{\alpha},\bar{\beta}]} : E \to NRS(U) \times F(U) \times F(U)$, where, $i \in \Lambda_1$ and $k \in \Lambda_2$, p^N is a mapping given by P^N: $E \to NRS(U)$, where $N=1,2,\ldots,n$ and $\bar{\alpha}(e_k), \bar{\beta}(e_k)$ is a fuzzy set such that $\bar{\alpha} : E \to F(U)$ and $\bar{\beta} : E \to F(U)$. For each parameter

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$$e_k \in E, P^N(e_k) = \left\{ \left\langle u_j, T_{P^1(e_k)}(u_j), I_{P^1(e_k)}(u_j), F_{P^1(e_k)}(u_j) \right\rangle, \right.$$
$$\left. \left\langle u_j, T_{P^2(e_k)}(u_j), I_{P^2(e_k)}(u_j), F_{P^2(e_k)}(u_j) \right\rangle, ..., \left\langle u_j, T_{P^n(e_k)}(u_j), I_{P^n(e_k)}(u_j), F_{P^n(e_k)}(u_j) \right\rangle : u_j \in U \right\}$$

indicates neutrosophic refined valued set of the parameters e_k and where *T,I,F* are the truth, indeterminacy and falsity values respectively of the element $u_j \in U$. For each $u_j \in U$ and $e_k \in E$,

$$0 \le T_{P^N(e_k)}(u_j) + I_{P^N(e_k)}(u_j) + F_{P^N(e_k)}(u_j) \le 3.$$

$\bar{\alpha}(e_k)$ depicts the degree of priority given for the belongingness of elements of *U* in $P^N(e_k)$ and $\bar{\beta}(e_k)$ the weightage given to the parameters by the expertise. So

$$P_{[\bar{\alpha},\bar{\beta}]}(e_k) = \left\{ \left(\frac{u_1}{p^N(e_k)(u_1)}, \alpha(e_k)(u_1) \right) \left(\frac{u_2}{p^N(e_k)(u_2)}, \alpha(e_k)(u_2) \right), ..., \left(\frac{u_n}{p^N(e_k)(u_n)}, \alpha(e_k)(u_n) \right), \bar{\beta}(e_k) \right\}$$

Example 2.2

Let $U=\{u_1,u_2,u_3\}$ be a set of three restaurants. Let $E=\{e_1,e_2,e_3\}$ be a set of qualities where e_1=Taste, e_2= Variety, e_3 =Service and let $\bar{\alpha} : E \to F(U)$ and $\bar{\beta} : E \to F(U)$. We can define a function $P_{[\bar{\alpha},\bar{\beta}]} : E \to NRS(U) \times F(U) \times F(U)$ as follow:

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$$P_{[\bar{\alpha},\bar{\beta}]}=\left\{\begin{array}{l}P^N(e_1)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.4,0.1,0.5)},0.7\right),\left(\frac{u_2}{(0.6,0.2,0.4)},0.3\right),\left(\frac{u_3}{(0.3,0.4,0.7)},0.6\right)\right],0.8\right\}\\ \left\{\left[\left(\frac{u_1}{(0.7,0.3,0.4)},0.5\right),\left(\frac{u_2}{(0.4,0.6,0.1)},0.7\right),\left(\frac{u_3}{(0.6,0.2,0.8)},0.3\right)\right],0.7\right\}\\ \left\{\left[\left(\frac{u_1}{(0.5,0.6,0.7)},0.2\right),\left(\frac{u_2}{(0.4,0.2,0.6)},0.5\right),\left(\frac{u_3}{(0.5,0.4,0.3)},0.4\right)\right],0.6\right\}\end{array}\right\}\\ P^N(e_2)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.8,0.3,0.2)},0.5\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.4\right),\left(\frac{u_3}{(0.7,0.2,0.4)},0.3\right)\right],0.7\right\}\\ \left\{\left[\left(\frac{u_1}{(0.6,0.3,0.3)},0.6\right),\left(\frac{u_2}{(0.6,0.6,0.4)},0.6\right),\left(\frac{u_3}{(0.7,0.4,0.4)},0.7\right)\right],0.9\right\}\\ \left\{\left[\left(\frac{u_1}{(0.9,0.3,0.3)},0.7\right),\left(\frac{u_2}{(0.7,0.3,0.6)},0.3\right),\left(\frac{u_3}{(0.8,0.1,0.2)},0.5\right)\right],0.8\right\}\end{array}\right\}\\ P^N(e_3)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.3,0.2,0.7)},0.8\right),\left(\frac{u_2}{(0.9,0.5,0.7)},0.4\right),\left(\frac{u_3}{(0.7,0.5,0.4)},0.5\right)\right],0.6\right\}\\ \left\{\left[\left(\frac{u_1}{(0.5,0.6,0.3)},0.9\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.3\right),\left(\frac{u_3}{(0.6,0.2,0.5)},0.8\right)\right],0.9\right\}\\ \left\{\left[\left(\frac{u_1}{(0.8,0.2,0.3)},0.5\right),\left(\frac{u_2}{(0.6,0.7,0.4)},0.5\right),\left(\frac{u_3}{(0.5,0.3,0.4)},0.5\right)\right],0.8\right\}\end{array}\right\}\end{array}\right\}$$

The matrix representation of priority weighted neutrosophic refined soft set $P_{[\bar{\alpha},\bar{\beta}]}$ can be written as follows: for $m,n\in\Lambda$

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$$P_{[\bar{\alpha},\bar{\beta}]}=\left\{\begin{array}{cccccc} & u_1,\bar{\alpha} & u_2,\bar{\alpha} & u_3,\bar{\alpha} & \bar{\beta} \\ \left\{\begin{array}{l}\left(T_{P^1}(e_1),I_{P^1}(e_1),F_{P^1}(e_1)\right)\\ \left(T_{P^2}(e_1),I_{P^2}(e_1),F_{P^2}(e_1)\right)\\ \left(T_{P^3}(e_1),I_{P^3}(e_1),F_{P^3}(e_1)\right)\end{array}\right. & \begin{array}{c}\left(\langle 0.4,0.1,0.5\rangle,0.7\right)\\ \left(\langle 0.7,0.3,0.4\rangle,0.5\right)\\ \left(\langle 0.5,0.6,0.7\rangle,0.2\right)\end{array} & \begin{array}{c}\left(\langle 0.6,0.2,0.4\rangle,0.3\right)\\ \left(\langle 0.4,0.6,0.1\rangle,0.7\right)\\ \left(\langle 0.4,0.2,0.6\rangle,0.5\right)\end{array} & \begin{array}{c}\left(\langle 0.3,0.4,0.7\rangle,0.6\right)\\ \left(\langle 0.6,0.2,0.8\rangle,0.3\right)\\ \left(\langle 0.5,0.4,0.3\rangle,0.4\right)\end{array} & \begin{array}{c}(0.8)\\(0.7)\\(0.6)\end{array} \\ \left\{\begin{array}{l}\left(T_{P^1}(e_2),I_{P^1}(e_2),F_{P^1}(e_2)\right)\\ \left(T_{P^2}(e_2),I_{P^2}(e_2),F_{P^2}(e_2)\right)\\ \left(T_{P^3}(e_2),I_{P^3}(e_2),F_{P^3}(e_2)\right)\end{array}\right. & \begin{array}{c}\left(\langle 0.8,0.3,0.2\rangle,0.5\right)\\ \left(\langle 0.6,0.3,0.3\rangle,0.6\right)\\ \left(\langle 0.9,0.3,0.3\rangle,0.7\right)\end{array} & \begin{array}{c}\left(\langle 0.7,0.3,0.5\rangle,0.4\right)\\ \left(\langle 0.6,0.6,0.4\rangle,0.6\right)\\ \left(\langle 0.7,0.3,0.6\rangle,0.3\right)\end{array} & \begin{array}{c}\left(\langle 0.7,0.2,0.4\rangle,0.3\right)\\ \left(\langle 0.7,0.4,0.4\rangle,0.7\right)\\ \left(\langle 0.8,0.1,0.2\rangle,0.5\right)\end{array} & \begin{array}{c}(0.7)\\(0.9)\\(0.8)\end{array} \\ \left\{\begin{array}{l}\left(T_{P^1}(e_3),I_{P^1}(e_3),F_{P^1}(e_3)\right)\\ \left(T_{P^2}(e_3),I_{P^2}(e_3),F_{P^2}(e_3)\right)\\ \left(T_{P^3}(e_3),I_{P^3}(e_3),F_{P^3}(e_3)\right)\end{array}\right. & \begin{array}{c}\left(\langle 0.3,0.2,0.7\rangle,0.8\right)\\ \left(\langle 0.5,0.6,0.3\rangle,0.9\right)\\ \left(\langle 0.8,0.2,0.3\rangle,0.5\right)\end{array} & \begin{array}{c}\left(\langle 0.9,0.5,0.7\rangle,0.4\right)\\ \left(\langle 0.7,0.3,0.5\rangle,0.3\right)\\ \left(\langle 0.6,0.7,0.4\rangle,0.5\right)\end{array} & \begin{array}{c}\left(\langle 0.7,0.5,0.4\rangle,0.5\right)\\ \left(\langle 0.6,0.2,0.5\rangle,0.8\right)\\ \left(\langle 0.5,0.3,0.4\rangle,0.5\right)\end{array} & \begin{array}{c}(0.6)\\(0.9)\\(0.8)\end{array} \end{array}\right\}$$

where the m-th row vector shows $P^N(e_m)$ and n-th column vector shows u_n.

Definition 2.3

Let $P_{[\bar{\alpha},\bar{\beta}]},Q_{[\bar{\gamma},\bar{\delta}]}\in PWRN(U,E)$. Then, $P_{[\bar{\alpha},\bar{\beta}]}$ is said to be a priority weighted neutrosophic refined soft subset (PWNRS-subset) of $Q_{[\bar{\gamma},\bar{\delta}]}$, and denoted by $P_{[\bar{\alpha},\bar{\beta}]}\subseteq Q_{[\bar{\gamma},\bar{\delta}]}$ if

1. $\bar{\alpha}(e)$ and $\bar{\beta}(e)$ are a fuzzy subset of $\bar{\gamma}(e)$ and $\bar{\delta}(e)$, for all $e\in E$
2. P is a neutrosophic refined subset of Q.

Example 2.4

Let $U=\{u_1,u_2,u_3\}$ be a set of three broadband service, and let $E=\{e_1,e_2,e_3\}$ be a set of qualities where $e_1=$ Stability, $e_2=$ Security, $e_3=$ price. Let $P_{[\bar{\alpha},\bar{\beta}]}$ be a PWNRS-set define as follows:

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$$P_{[\bar{\alpha},\bar{\beta}]}=\left\{\begin{array}{l}P^N(e_1)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.4,0.7,0.6)},0.8\right),\left(\frac{u_2}{(0.8,0.6,0.8)},0.3\right),\left(\frac{u_3}{(0.4,0.5,0.8)},0.7\right)\right],0.8\right\}\\ \left\{\left[\left(\frac{u_1}{(0.7,0.4,0.6)},0.6\right),\left(\frac{u_2}{(0.5,0.7,0.5)},0.7\right),\left(\frac{u_3}{(0.7,0.4,0.8)},0.5\right)\right],0.7\right\}\\ \left\{\left[\left(\frac{u_1}{(0.5,0.6,0.7)},0.2\right),\left(\frac{u_2}{(0.6,0.5,0.8)},0.5\right),\left(\frac{u_3}{(0.6,0.5,0.7)},0.4\right)\right],0.6\right\}\end{array}\right\}\\ P^N(e_2)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.8,0.3,0.2)},0.5\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.4\right),\left(\frac{u_3}{(0.7,0.2,0.4)},0.3\right)\right],0.7\right\}\\ \left\{\left[\left(\frac{u_1}{(0.6,0.3,0.3)},0.6\right),\left(\frac{u_2}{(0.6,0.6,0.4)},0.6\right),\left(\frac{u_3}{(0.7,0.4,0.4)},0.7\right)\right],0.9\right\}\\ \left\{\left[\left(\frac{u_1}{(0.9,0.3,0.3)},0.7\right),\left(\frac{u_2}{(0.7,0.3,0.6)},0.3\right),\left(\frac{u_3}{(0.8,0.1,0.2)},0.5\right)\right],0.8\right\}\end{array}\right\}\\ P^N(e_3)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.3,0.2,0.7)},0.8\right),\left(\frac{u_2}{(0.9,0.5,0.7)},0.4\right),\left(\frac{u_3}{(0.7,0.5,0.4)},0.5\right)\right],0.6\right\}\\ \left\{\left[\left(\frac{u_1}{(0.5,0.6,0.3)},0.9\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.3\right),\left(\frac{u_3}{(0.6,0.2,0.5)},0.8\right)\right],0.9\right\}\\ \left\{\left[\left(\frac{u_1}{(0.8,0.2,0.3)},0.5\right),\left(\frac{u_2}{(0.6,0.7,0.4)},0.5\right),\left(\frac{u_3}{(0.5,0.3,0.4)},0.5\right)\right],0.8\right\}\end{array}\right\}\end{array}\right\}$$

Also we can define a function $Q_{[\gamma,\delta]:}\mathrm{E}\rightarrow N\mathrm{R}S(U)\times\mathrm{F}(U)$ as follows:

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$$Q_{[\bar{\gamma},\bar{\delta}]}=\left\{\begin{array}{l} Q^N(e_1)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.4,0.7,0.6)},0.8\right),\left(\frac{u_2}{(0.8,0.6,0.8)},0.3\right),\left(\frac{u_3}{(0.4,0.5,0.8)},0.7\right)\right],0.8\right\}\\ \left\{\left[\left(\frac{u_1}{(0.8,0.6,0.5)},0.7\right),\left(\frac{u_2}{(0.6,0.8,0.3)},0.8\right),\left(\frac{u_3}{(0.8,0.5,0.6)},0.5\right)\right],0.7\right\}\\ \left\{\left[\left(\frac{u_1}{(0.6,0.7,0.5)},0.4\right),\left(\frac{u_2}{(0.7,0.6,0.7)},0.7\right),\left(\frac{u_3}{(0.8,0.6,0.4)},0.6\right)\right],0.6\right\}\end{array}\right\}\\ Q^N(e_2)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.9,0.5,0.1)},0.6\right),\left(\frac{u_2}{(0.8,0.4,0.4)},0.5\right),\left(\frac{u_3}{(0.9,0.4,0.3)},0.5\right)\right],0.8\right\}\\ \left\{\left[\left(\frac{u_1}{(0.8,0.5,0.2)},0.7\right),\left(\frac{u_2}{(0.8,0.8,0.3)},0.8\right),\left(\frac{u_3}{(0.9,0.5,0.3)},0.8\right)\right],0.9\right\}\\ \left\{\left[\left(\frac{u_1}{(0.9,0.5,0.2)},0.8\right),\left(\frac{u_2}{(0.8,0.5,0.5)},0.4\right),\left(\frac{u_3}{(0.8,0.6,0.6)},0.8\right)\right],0.8\right\}\end{array}\right\}\\ Q^N(e_3)\left\{\begin{array}{l}\left\{\left[\left(\frac{u_1}{(0.4,0.3,0.6)},0.9\right),\left(\frac{u_2}{(0.9,0.6,0.6)},0.5\right),\left(\frac{u_3}{(0.8,0.6,0.3)},0.6\right)\right],0.7\right\}\\ \left\{\left[\left(\frac{u_1}{(0.6,0.7,0.2)},0.9\right),\left(\frac{u_2}{(0.8,0.5,0.4)},0.5\right),\left(\frac{u_3}{(0.7,0.3,0.2)},0.9\right)\right],0.9\right\}\\ \left\{\left[\left(\frac{u_1}{(0.9,0.3,0.1)},0.6\right),\left(\frac{u_2}{(0.7,0.8,0.2)},0.7\right),\left(\frac{u_3}{(0.6,0.5,0.3)},0.8\right)\right],0.9\right\}\end{array}\right\}\end{array}\right\}$$

It is clear that $P_{[\bar{\alpha},\bar{\beta}]}$ is PWNRS –subset of $Q_{[\bar{\gamma},\bar{\delta}]}$.

Definition 2.5

Let $P_{[\bar{\alpha},\bar{\beta}]},Q_{[\bar{\gamma},\bar{\delta}]}\in PWRN(U,E)$. Then, $P_{[\bar{\alpha},\bar{\beta}]}$ and $Q_{[\bar{\gamma},\bar{\delta}]}$ are called priority weighted neutrosophic refined soft equal set and denote by $P_{[\bar{\alpha},\bar{\beta}]}=Q_{[\bar{\gamma},\bar{\delta}]}$ if $P_{[\bar{\alpha},\bar{\beta}]}\subseteq Q_{[\bar{\gamma},\bar{\delta}]}$ and $P_{[\bar{\alpha},\bar{\beta}]}\supseteq Q_{[\bar{\gamma},\bar{\delta}]}$.

Definition 2.6

Let $P_{[\bar{\alpha},\bar{\beta}]}\in PWRN(U,E)$. Then, $P_{[\bar{\alpha},\bar{\beta}]}$ is said to be priority weighted neutrosophic refined soft null set, denoted by $\phi_{[\bar{\alpha},\bar{\beta}]}$, if $\forall e\in E$, $\phi_{[\bar{\alpha},\bar{\beta}]}:E\to NRS(U)\times F(U)\times F(U)$ such that

$$\phi_{[\bar{\alpha},\bar{\beta}]}(e)=\left\{\left[\frac{u}{\phi(e)(u)},\bar{\alpha}(e)(u):u\in U\right]\bar{\beta}(e)\right\}$$

where

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$\phi(e)=\{\langle u,0,0,1\rangle : u\in U\}$ and $\bar{\alpha}(e)=\{(u,0) : u\in U\}$ and $\bar{\beta}(e)=\{(u,0) : u\in U\}$.

Definition 2.7

Let $P_{[\bar{\alpha},\bar{\beta}]}\in PWNR(U,E)$. Then, $P_{[\bar{\alpha},\bar{\beta}]}$ is said to be priority weighted neutrosophic refined soft universal set, denoted by $U_{[\bar{\alpha},\bar{\beta}]}$, if $\forall e\in E$, $U_{[\bar{\alpha},\bar{\beta}]}: E\to NRS(U)\times F(U)$ such that

$$U^{N}{}_{[\bar{\alpha},\bar{\beta}]}(e)=\left\{\left[\frac{u}{U(e)(u)},\bar{\alpha}(e)(u) : u\in U\right]\bar{\beta}(e)\right\}$$

where

$U(e)=\{\langle u,1,1,0\rangle : u\in U\}$ and $\bar{\alpha}(e)=\{(u,1) : u\in U\}$ and $\bar{\beta}(e)=\{(u,1) : u\in U\}$.

Proposition 2.8

Let $P_{[\bar{\alpha},\bar{\beta}]}, Q_{[\bar{\gamma},\bar{\delta}]}$ and $H_{[\theta,\phi]}\in PWRN(U,E)$. Then,

1. $\phi_{[\bar{\alpha},\bar{\beta}]}\subseteq P_{[\bar{\alpha},\bar{\beta}]}$
2. $P_{[\bar{\alpha},\bar{\beta}]}\subseteq U_{[\bar{\alpha},\bar{\beta}]}$
3. $P_{[\bar{\alpha},\bar{\beta}]}\subseteq Q_{[\bar{\gamma},\bar{\delta}]}$ and $Q_{[\bar{\gamma},\bar{\delta}]}\subseteq H_{[\theta,\phi]}\Rightarrow P_{[\bar{\alpha},\bar{\beta}]}\subseteq H_{[\theta,\phi]}$

Proof . The proof follows from the Definitions (2.5) - (2.7)

Definition 2.9

Let $P_{[\bar{\alpha},\bar{\beta}]}\in PWRN(U,E)$, where

$$P^{N}{}_{[\bar{\alpha},\bar{\beta}]}(e_k)=\left\{\left(\left[P^{N}(e_k)(u_i),\bar{\alpha}(e_k)(u_i)\right],\bar{\beta}(e_k)\right) : e_k\in E, u_i\in U\right\}$$

$$P^{N}(e_k)=\left\{\left\langle u, T_{P^{N}(e_k)}(u_i), I_{P^{N}(e_k)}(u_i), F_{P^{N}(e_k)}(u_i)\right\rangle\right\}\forall e_k\in E, u_i\in U.$$

Then for $e_k\in E$ and $u_i\in U$,

1. $P^{T}{}_{[\bar{\alpha},\bar{\beta}]}$ is said to be truth- membership part of $P_{[\bar{\alpha},\bar{\beta}]}$

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$$P^{T}_{[\bar{\alpha},\bar{\beta}]}=\left\{\left(P^{T}_{kj}(e_k),\bar{\alpha}_{kj}(e_k)\right)\right\} \text{ and } P_{kj}{}^{T}(e_k)=\left\{\left(u_j,T_{P^{N}(e_k)}(u_i)\right)\right\},\bar{\alpha}_{kj}(e_k)=\left\{\left(u_j,\bar{\alpha}(e_k)(u_j)\right)\right\}$$

2. $P^{I}_{[\bar{\alpha},\bar{\beta}]}$ is said to be indeterminacy - membership part of $P_{[\bar{\alpha},\bar{\beta}]}$

$$P^{I}_{[\bar{\alpha},\bar{\beta}]}=\left\{\left(P^{I}_{kj}(e_k),\bar{\alpha}_{kj}(e_k)\right)\right\} \text{ and } P_{kj}{}^{I}(e_k)=\left\{\left(u_j,I_{P^{N}(e_k)}(u_i)\right)\right\},\bar{\alpha}_{kj}(e_k)=\left\{\left(u_j,\bar{\alpha}(e_k)(u_j)\right)\right\}$$

3. $P^{F}_{[\bar{\alpha},\bar{\beta}]}$ is said to be falsity- membership part of $P_{[\bar{\alpha},\bar{\beta}]}$

$$P^{F}_{[\bar{\alpha},\bar{\beta}]}=\left\{\left(P^{F}_{kj}(e_k),\bar{\alpha}_{kj}(e_k)\right)\right\} \text{ and } P_{kj}{}^{F}(e_k)=\left\{\left(u_j,F_{P^{N}(e_k)}(u_i)\right)\right\},\bar{\alpha}_{kj}(e_k)=\left\{\left(u_j,\bar{\alpha}(e_k)(u_j)\right)\right\}$$

We can write a PWNRS in form $P_{[\bar{\alpha},\bar{\beta}]}=\left(P^{T}_{[\bar{\alpha},\bar{\beta}]},P^{I}_{[\bar{\alpha},\bar{\beta}]},P^{F}_{[\bar{\alpha},\bar{\beta}]}\right)$.

A PWNRS can be expressed in matrix form.

Let us consider priority weighted neutrosophic refined soft set $P_{[\bar{\alpha},\bar{\beta}]}$ given in Example 2.4. Then

$$P^{T}_{[\bar{\alpha},\bar{\beta}]}=\begin{pmatrix}(0.4,0.8) & (0.8,0.3) & (0.4,0.7) & (0.8)\\ (0.7,0.6) & (0.5,0.7) & (0.7,0.5) & (0.7)\\ (0.5,0.2) & (0.6,0.5) & (0.6,0.4) & (0.6)\end{pmatrix}$$

$$P^{I}_{[\bar{\alpha},\bar{\beta}]}=\begin{pmatrix}(0.7,0.8) & (0.6,0.3) & (0.5,0.7) & (0.8)\\ (0.4,0.6) & (0.7,0.7) & (0.4,0.5) & (0.7)\\ (0.6,0.2) & (0.5,0.5) & (0.5,0.4) & (0.6)\end{pmatrix}$$

$$P^{F}_{[\bar{\alpha},\bar{\beta}]}=\begin{pmatrix}(0.6,0.8) & (0.8,0.3) & (0.8,0.7) & (0.8)\\ (0.6,0.6) & (0.5,0.7) & (0.8,0.5) & (0.7)\\ (0.7,0.2) & (0.8,0.5) & (0.7,0.4) & (0.6)\end{pmatrix}$$

Definition 2.10

Let $P_{[\bar{\alpha},\bar{\beta}]},Q_{[\bar{\gamma},\bar{\delta}]}\in PWNR(U,E)$. The union of two PWNRSs $P_{[\bar{\alpha},\bar{\beta}]}$ and $Q_{[\bar{\gamma},\bar{\delta}]}$ over U, denote by $P_{[\bar{\alpha},\bar{\beta}]}\cup Q_{[\bar{\gamma},\bar{\delta}]}$ is defined by

$$P_{[\bar{\alpha},\bar{\beta}]}\cup Q_{[\bar{\gamma},\bar{\delta}]}=\left\{\left(e_k,\left(\rho,\bar{\alpha}_{kj}(e_k)\oplus\bar{\gamma}_{kj}(e_k)\right),\left(\bar{\beta}_{kj}(e_k)\oplus\bar{\delta}_{kj}(e_k)\right)\right):e_k\in E\right\}$$

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where

$$\rho = \frac{u_j}{\left(P_{kj}^T(e_k) \oplus Q_{kj}^T(e_k), P_{kj}^I(e_k) \oplus Q_{kj}^I(e_k), P_{kj}^F(e_k) \oplus Q_{kj}^F(e_k)\right)}$$

and $\oplus$ represents n-co norm and $\otimes$ represents n-norm functions respectively.

Definition 2.11

Let $P_{[\bar{\alpha},\bar{\beta}]}, Q_{[\bar{\gamma},\bar{\delta}]} \in PWRN(U,E)$. The intersection of two PWNRSs $P_{[\bar{\alpha},\bar{\beta}]}$ and $Q_{[\bar{\gamma},\bar{\delta}]}$ over *U*, denote by $P_{[\bar{\alpha},\bar{\beta}]} \cap Q_{[\bar{\gamma},\bar{\delta}]}$ is defined by

$$P_{[\bar{\alpha},\bar{\beta}]} \cap Q_{[\bar{\gamma},\bar{\delta}]} = \left\{\left(e_k, \left(\theta, \bar{\alpha}_{kj}(e_k) \otimes \bar{\gamma}_{kj}(e_k)\right), \left(\bar{\beta}_{kj}(e_k) \otimes \bar{\delta}_{kj}(e_k)\right)\right) : e_k \in E\right\}$$

Where

$$\theta = \frac{u_j}{\left(P_{kj}^T(e_k) \otimes Q_{kj}^T(e_k), P_{kj}^I(e_k) \otimes Q_{kj}^I(e_k), P_{kj}^F(e_k) \otimes Q_{kj}^F(e_k)\right)}$$

and $\oplus$ represents n-conorm and $\otimes$ represents n-norm functions respectively.

Example 2.12

Let us consider the PWNRSs $P_{[\bar{\alpha},\bar{\beta}]}$ and $Q_{[\bar{\gamma},\bar{\delta}]}$. Defined as in Example 2.4. Let us suppose that n-norm is defined by $a \oplus b = \min\{a, b\}$ and the n-conorm is defined by $a+b=\max\{a,b\}$ for $a,b \in [0,1]$

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$$P_{[\bar{\alpha},\bar{\beta}]} \cup Q_{[\bar{\gamma},\bar{\delta}]} = \left\{ \begin{array}{l} \left(P^N \cup Q^N\right)(e_1) \left\{ \begin{array}{l} \left\{\left[\left(\frac{u_1}{(0.6,0.7,0.5)},0.8\right),\left(\frac{u_2}{(0.9,0.7,0.6)},0.3\right),\left(\frac{u_3}{(0.5,0.6,0.7)},0.8\right)\right],0.8\right\} \\ \left\{\left[\left(\frac{u_1}{(0.8,0.6,0.5)},0.7\right),\left(\frac{u_2}{(0.6,0.8,0.3)},0.8\right),\left(\frac{u_3}{(0.8,0.5,0.6)},0.5\right)\right],0.7\right\} \\ \left\{\left[\left(\frac{u_1}{(0.6,0.7,0.5)},0.4\right),\left(\frac{u_2}{(0.7,0.6,0.7)},0.7\right),\left(\frac{u_3}{(0.8,0.6,0.4)},0.6\right)\right],0.6\right\} \end{array} \right. \\ \left(P^N \cup Q^N\right)(e_2) \left\{ \begin{array}{l} \left\{\left[\left(\frac{u_1}{(0.9,0.5,0.1)},0.6\right),\left(\frac{u_2}{(0.8,0.4,0.4)},0.5\right),\left(\frac{u_3}{(0.9,0.4,0.3)},0.5\right)\right],0.8\right\} \\ \left\{\left[\left(\frac{u_1}{(0.8,0.5,0.2)},0.7\right),\left(\frac{u_2}{(0.8,0.8,0.3)},0.8\right),\left(\frac{u_3}{(0.9,0.5,0.3)},0.8\right)\right],0.9\right\} \\ \left\{\left[\left(\frac{u_1}{(0.9,0.5,0.2)},0.8\right),\left(\frac{u_2}{(0.8,0.5,0.5)},0.4\right),\left(\frac{u_3}{(0.8,0.6,0.6)},0.8\right)\right],0.8\right\} \end{array} \right. \\ \left(P^N \cup Q^N\right)(e_3) \left\{ \begin{array}{l} \left\{\left[\left(\frac{u_1}{(0.4,0.3,0.6)},0.9\right),\left(\frac{u_2}{(0.9,0.6,0.6)},0.5\right),\left(\frac{u_3}{(0.8,0.6,0.3)},0.6\right)\right],0.7\right\} \\ \left\{\left[\left(\frac{u_1}{(0.6,0.7,0.2)},0.9\right),\left(\frac{u_2}{(0.8,0.5,0.4)},0.5\right),\left(\frac{u_3}{(0.7,0.3,0.2)},0.9\right)\right],0.9\right\} \\ \left\{\left[\left(\frac{u_1}{(0.9,0.3,0.1)},0.6\right),\left(\frac{u_2}{(0.7,0.8,0.2)},0.7\right),\left(\frac{u_3}{(0.6,0.5,0.3)},0.8\right)\right],0.9\right\} \end{array} \right. \end{array} \right\}$$

And

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$$P_{[\bar{\alpha},\bar{\beta}]} \cap Q_{[\bar{\gamma},\bar{\delta}]} = \left\{ \begin{array}{l} (P_1 \cap Q_1)(e_1) \left\{ \begin{array}{l} \left\{ \left[\left(\frac{u_1}{(0.4,0.7,0.6)}, 0.8 \right), \left(\frac{u_2}{(0.8,0.6,0.8)}, 0.3 \right), \left(\frac{u_3}{(0.4,0.5,0.8)}, 0.7 \right) \right], 0.8 \right\} \\ \left\{ \left[\left(\frac{u_1}{(0.7,0.4,0.6)}, 0.6 \right), \left(\frac{u_2}{(0.5,0.7,0.5)}, 0.7 \right), \left(\frac{u_3}{(0.7,0.4,0.8)}, 0.5 \right) \right], 0.7 \right\} \\ \left\{ \left[\left(\frac{u_1}{(0.5,0.6,0.7)}, 0.2 \right), \left(\frac{u_2}{(0.6,0.5,0.8)}, 0.5 \right), \left(\frac{u_3}{(0.6,0.7,0.5)}, 0.4 \right) \right], 0.6 \right\} \end{array} \right\} \\ (P_1 \cap Q_1)(e_2) \left\{ \begin{array}{l} \left\{ \left[\left(\frac{u_1}{(0.8,0.3,0.2)}, 0.5 \right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.4 \right), \left(\frac{u_3}{(0.7,0.2,0.4)}, 0.3 \right) \right], 0.7 \right\} \\ \left\{ \left[\left(\frac{u_1}{(0.6,0.3,0.3)}, 0.6 \right), \left(\frac{u_2}{(0.6,0.6,0.4)}, 0.6 \right), \left(\frac{u_3}{(0.7,0.4,0.4)}, 0.7 \right) \right], 0.9 \right\} \\ \left\{ \left[\left(\frac{u_1}{(0.9,0.3,0.3)}, 0.7 \right), \left(\frac{u_2}{(0.7,0.3,0.6)}, 0.3 \right), \left(\frac{u_3}{(0.8,0.1,0.2)}, 0.5 \right) \right], 0.8 \right\} \end{array} \right\} \\ (P_1 \cap Q_1)(e_3) \left\{ \begin{array}{l} \left\{ \left[\left(\frac{u_1}{(0.3,0.2,0.7)}, 0.8 \right), \left(\frac{u_2}{(0.9,0.5,0.7)}, 0.4 \right), \left(\frac{u_3}{(0.7,0.5,0.4)}, 0.5 \right) \right], 0.6 \right\} \\ \left\{ \left[\left(\frac{u_1}{(0.5,0.6,0.3)}, 0.9 \right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.3 \right), \left(\frac{u_3}{(0.6,0.2,0.5)}, 0.8 \right) \right], 0.9 \right\} \\ \left\{ \left[\left(\frac{u_1}{(0.8,0.2,0.3)}, 0.5 \right), \left(\frac{u_2}{(0.6,0.7,0.4)}, 0.5 \right), \left(\frac{u_3}{(0.5,0.3,0.4)}, 0.5 \right) \right], 0.8 \right\} \end{array} \right\} \end{array} \right\}$$

Proposition 2.13

Let $P_{[\bar{\alpha},\bar{\beta}]}, Q_{[\bar{\gamma},\bar{\delta}]}$, and $H_{[\theta,\phi]} \in PWRN(U,E)$ .Then,

1. $P_{[\bar{\alpha},\bar{\beta}]} \cap \phi_{\bar{\alpha}} = \phi_{\bar{\alpha}}$ and $P_{[\bar{\alpha},\bar{\beta}]} \cap U_{\bar{\alpha}} = P_{[\bar{\alpha},\bar{\beta}]}$

2. $P_{[\bar{\alpha},\bar{\beta}]} \cup \phi = P_{[\bar{\alpha},\bar{\beta}]}$ and $P_{[\bar{\alpha},\bar{\beta}]} \cup U_{\bar{\alpha}} = U_{\bar{\alpha}}$

3. $P_{[\bar{\alpha},\bar{\beta}]} \cup \left(Q_{[\bar{\gamma},\bar{\delta}]} \cap H_{[\theta,\phi]} \right) = \left(P_{[\bar{\alpha},\bar{\beta}]} \cup Q_{[\bar{\gamma},\bar{\delta}]} \right) \cap H_{[\theta,\phi]}$ and

$P_{[\bar{\alpha},\bar{\beta}]} \cap \left(Q_{[\bar{\gamma},\bar{\delta}]} \cup H_{[\theta,\phi]} \right) = \left(P_{[\bar{\alpha},\bar{\beta}]} \cap Q_{[\bar{\gamma},\bar{\delta}]} \right) \cup H_{[\theta,\phi]}$

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4. $P_{[\bar{\alpha},\bar{\beta}]} \cap \left(Q_{[\bar{\gamma},\bar{\delta}]} \cup H_{[\theta,\phi]}\right) = \left(P_{[\bar{\alpha},\bar{\beta}]} \cap Q_{[\bar{\gamma},\bar{\delta}]}\right) \cup \left(P_{[\bar{\alpha},\bar{\beta}]} \cap H_{[\theta,\phi]}\right)$ and

$P_{[\bar{\alpha},\bar{\beta}]} \cup \left(Q_{[\bar{\gamma},\bar{\delta}]} \cap H_{[\theta,\phi]}\right) = \left(P_{[\bar{\alpha},\bar{\beta}]} \cup Q_{[\bar{\gamma},\bar{\delta}]}\right) \cap \left(P_{[\bar{\alpha},\bar{\beta}]} \cup H_{[\theta,\phi]}\right)$

Proof: The proof can be obtained from Definitions 2.10 and 2.11

Definition 2.14

Let $P_{[\bar{\alpha},\bar{\beta}]} \in PWNR(U,E)$. Complement of PWNRS $P_{[\bar{\alpha},\bar{\beta}]}$, denoted by $P^c_{[\bar{\alpha},\bar{\beta}]}$, is defined by

$$P^c_{[\bar{\alpha},\bar{\beta}]} = \left\{\left(e, \left\{\left(\frac{u_j}{\sim (p^N(e_k))}, \sim\left(\bar{\alpha}_{kj}(e_k)\left(u_j\right)\right)\right), \sim\left(\beta_{kj}(e_k)\right)\right\}\right), e \in E\right\}$$

Where,

$$\left(\sim\left(P^N{}_{kj}\right)(e_k)\right) = \left(\sim\left(P^T{}_{kj}\right)(e_k)\right), \left(\sim\left(P^I{}_{kj}\right)(e_k)\right), \left(\sim\left(P^F{}_{kj}\right)(e_k)\right) \forall k \in \Lambda_1, j \in \Lambda_2$$

Example 2.15

Let us consider the PWNRS $P_{[\bar{\alpha},\beta]}$ define in Example 2.4. Suppose that the negation is defined by

$$\left(\sim\left(P^T{}_{kj}\right)(e_k)\right) = \left(\left(P^F{}_{kj}\right)(e_k)\right), \left(\sim\left(P^F{}_{kj}\right)(e_k)\right) = \left(\left(P^T{}_{kj}\right)(e_k)\right), \left(\sim\left(P^I{}_{kj}\right)(e_k)\right) = 1 - \left(\left(P^I{}_{kj}\right)(e_k)\right)$$

and $\left(\sim \bar{\alpha}_{ij}(e_k)\right) = 1 - \bar{\alpha}_{ij}(e_k)$ and $\left(\sim \beta_{ij}(e_k)\right) = 1 - \beta_{ij}(e_k)$ respectively.

Then, $\sim P_{[\bar{\alpha},\bar{\beta}]}$ is defined as follow:

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$$\sim P_{[\bar{\alpha},\bar{\beta}]} = \left\{ \begin{array}{l} \sim P_1^N(e_1)\left\{ \begin{array}{l} \left\{\left[\left(\frac{u_1}{(0.6,0.3,0.4)},0.2\right),\left(\frac{u_2}{(0.8,0.6,0.8)},0.7\right),\left(\frac{u_3}{(0.8,0.5,0.4)},0.2\right)\right],0.2\right\} \\ \left\{\left[\left(\frac{u_1}{(0.6,0.6,0.7)},0.4\right),\left(\frac{u_2}{(0.5,0.3,0.5)},0.3\right),\left(\frac{u_3}{(0.8,0.6,0.7)},0.3\right)\right],0.3\right\} \\ \left\{\left[\left(\frac{u_1}{(0.7,0.4,0.5)},0.8\right),\left(\frac{u_2}{(0.8,0.6,0.6)},0.5\right),\left(\frac{u_3}{(0.7,0.5,0.6)},0.4\right)\right],0.4\right\} \end{array} \right\} \\ \sim P_1^N(e_2)\left\{ \begin{array}{l} \left\{\left[\left(\frac{u_1}{(0.2,0.7,0.8)},0.5\right),\left(\frac{u_2}{(0.5,0.7,0.7)},0.6\right),\left(\frac{u_3}{(0.4,0.8,0.7)},0.7\right)\right],0.3\right\} \\ \left\{\left[\left(\frac{u_1}{(0.3,0.7,0.6)},0.4\right),\left(\frac{u_2}{(0.4,0.4,0.6)},0.4\right),\left(\frac{u_3}{(0.4,0.6,0.7)},0.3\right)\right],0.1\right\} \\ \left\{\left[\left(\frac{u_1}{(0.3,0.7,0.9)},0.3\right),\left(\frac{u_2}{(0.6,0.7,0.7)},0.7\right),\left(\frac{u_3}{(0.2,0.9,0.8)},0.5\right)\right],0.2\right\} \end{array} \right\} \\ \sim P_1^N(e_3)\left\{ \begin{array}{l} \left\{\left[\left(\frac{u_1}{(0.7,0.8,0.3)},0.2\right),\left(\frac{u_2}{(0.7,0.5,0.9)},0.6\right),\left(\frac{u_3}{(0.4,0.5,0.7)},0.5\right)\right],0.4\right\} \\ \left\{\left[\left(\frac{u_1}{(0.3,0.4,0.5)},0.1\right),\left(\frac{u_2}{(0.5,0.7,0.7)},0.7\right),\left(\frac{u_3}{(0.5,0.8,0.6)},0.2\right)\right],0.1\right\} \\ \left\{\left[\left(\frac{u_1}{(0.3,0.8,0.8)},0.5\right),\left(\frac{u_2}{(0.4,0.3,0.6)},0.5\right),\left(\frac{u_3}{(0.4,0.7,0.5)},0.5\right)\right],0.2\right\} \end{array} \right\} \end{array} \right\}$$

Proposition 2.16

Let $P_{[\bar{\alpha},\bar{\beta}]} \in PWRN(U,E)$. Then

1. $\sim \phi_{[\bar{\alpha},\bar{\beta}]} = U_{[\bar{\alpha},\bar{\beta}]}$
2. $\sim U_{[\bar{\alpha},\bar{\beta}]} = \phi_{[\bar{\alpha},\bar{\beta}]}$
3. $\sim\left(\sim P_{[\bar{\alpha},\bar{\beta}]}\right) = P_{[\bar{\alpha},\bar{\beta}]}$

Proof. It is clear from Definition 2.14.

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Proposition 2.17

Let $P_{[\bar{\alpha},\bar{\beta}]}, Q_{[\bar{\gamma},\bar{\delta}]} \in PWRN(U,E)$. Then De Morgans law is valid.

1. $\sim\left(P_{[\bar{\alpha},\bar{\beta}]} \cup Q_{[\bar{\gamma},\bar{\delta}]}\right) = \sim P_{[\bar{\alpha},\bar{\beta}]} \cap \sim Q_{[\bar{\gamma},\bar{\delta}]}$

2. $\sim\left(P_{[\bar{\alpha},\bar{\beta}]} \cap Q_{[\bar{\gamma},\bar{\delta}]}\right) = \sim P_{[\bar{\alpha},\bar{\beta}]} \cup \sim Q_{[\bar{\gamma},\bar{\delta}]}$

Proof: 1. Let $i.j \in A, \sim\left(P_{[\bar{\alpha},\bar{\beta}]} \cup Q_{[\bar{\gamma},\bar{\delta}]}\right)$

$$=\sim\left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P_{kj}^T(e_k) \oplus Q_{kj}^T(e_k), P_{kj}^I(e_k) \oplus Q_{kj}^I(e_k), P_{kj}^F(e_k) \otimes Q_{kj}^F(e_k)\right)}, \bar{\alpha}_{kj}(e_k) \oplus \bar{\gamma}_{kj}(e_k)\right)\right.\right.$$

$$\left.\left.\bar{\beta}_{kj}(e_k) \oplus \bar{\delta}_{kj}(e_k)\right\}\right) : u_j \in U, e_k \in E\right\}$$

$$=\sim\left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P_{kj}^F(e_k) \otimes Q_{kj}^F(e_k), P_{kj}^I(e_k) \oplus Q_{kj}^I(e_k), P_{kj}^T(e_k) \oplus Q_{kj}^T(e_k)\right)}, \sim\left(\bar{\alpha}_{kj}(e_k) \oplus \bar{\gamma}_{kj}(e_k)\right)\right)\right.\right.$$

$$\left.\left.\sim\left(\bar{\beta}_{kj}(e_k) \oplus \bar{\delta}_{kj}(e_k)\right)\right\}\right) : u_j \in U, e_k \in E\right\}$$

$$=\sim\left\{\left(e_k, \left\{\left(\frac{u_j}{\left(P_{kj}^F(e_k) \otimes Q_{kj}^F(e_k), \sim\left(P_{kj}^I(e_k)\right) \oplus \sim\left(Q_{kj}^I(e_k)\right), P_{kj}^T(e_k) \oplus Q_{kj}^T(e_k)\right)}, \sim \bar{\alpha}_{kj}(e_k) \otimes \sim \bar{\gamma}_{kj}(e_k)\right)\right.\right.$$

$$\left.\left.\sim \bar{\beta}_{kj}(e_k) \otimes \sim \bar{\delta}_{kj}(e_k)\right\}\right) : u_j \in U, e_k \in E\right\}$$

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$$= \left(\left\{ \left(e_k, \left\{ \left(\frac{u_j}{\left(P_{kj}^F(e_k), \sim \left(P_{kj}^I(e_k) \right), P_{kj}^T(e_k) \right)}, \sim \left(\bar{\alpha}_{kj}(e_k) \right) \right), \sim \bar{\beta}_{kj}(e_k) \right\} \right) : u_j \in U, e_k \in E \right\} \right)$$

$$= \left(\left\{ \left(e_k, \left\{ \left(\frac{u_j}{\left(P_{kj}^T(e_k), \sim \left(P_{kj}^I(e_k) \right), P_{kj}^F(e_k) \right)}, \left(\bar{\alpha}_{kj}(e_k) \right) \right), \bar{\beta}_{kj}(e_k) \right\} \right) : u_j \in U, e_k \in E \right\} \right)^c$$

$$\cap \left(\left\{ \left(e_k, \left\{ \left(\frac{u_j}{\left(P_{kj}^T(e_k), \sim \left(P_{kj}^I(e_k) \right), P_{kj}^F(e_k) \right)}, \left(\bar{\gamma}_{kj}(e_k) \right) \right), \bar{\delta}_{kj}(e_k) \right\} \right) : u_j \in U, e_k \in E \right\} \right)^c$$

$$= \sim P^c_{[\bar{\alpha}, \bar{\beta}]} \cap \sim Q^c_{[\bar{\gamma}, \bar{\delta}]}$$

2. The techniques used to prove (2) are similar to those used for (1), therefore we skip this proof.

Definition 2.18

Let $P_{[\bar{\alpha}, \bar{\beta}]}, Q_{[\bar{\gamma}, \bar{\delta}]} \in PWRN(U, E)$. Then 'AND' product of PWNRS set $P_{[\bar{\alpha}, \bar{\beta}]}$ and $Q_{[\bar{\gamma}, \bar{\delta}]}$ denoted by $P_{[\bar{\alpha}, \bar{\beta}]} \wedge Q_{[\bar{\gamma}, \bar{\delta}]}$ id defined as follows:

$$P_{[\bar{\alpha}, \bar{\beta}]} \wedge Q_{[\bar{\gamma}, \bar{\delta}]} = \left\{ \left(\left(e_k, e_1 \right), P_{kj}^T(e_k) \wedge Q_{kj}^T(e_k), P_{kj}^I(e_k) \wedge Q_{kj}^I(e_k), P_{kj}^F(e_k) \wedge Q_{kj}^F(e_k), \right. \right.$$
$$\left. \left. \left(\bar{\alpha}_{kj}(e_k) \wedge \bar{\gamma}_{ij}(e_1) \right), \left(\bar{\beta}_{kj}(e_k) \wedge \bar{\delta}_{ij}(e_1) \right) \right) : (e_k, e_1) \in E \times E, j, k, l \in \wedge \right\}$$

Definition 2.19

Let $P_{[\bar{\alpha}, \bar{\beta}]}, Q_{[\bar{\gamma}, \bar{\delta}]} \in PWNR(U, E)$. Then 'OR' product of PWNRS set $P_{[\bar{\alpha}, \bar{\beta}]}$ and $Q_{[\bar{\gamma}, \bar{\delta}]}$ denoted by $P_{[\bar{\alpha}, \bar{\beta}]} \vee Q_{[\bar{\gamma}, \bar{\delta}]}$ id defined as follows:

$$P_{[\bar{\alpha}, \bar{\beta}]} \vee Q_{[\bar{\gamma}, \bar{\delta}]} = \left\{ \left(\left(e_k, e_1 \right), P_{kj}^T(e_k) \vee Q_{kj}^T(e_k), P_{kj}^I(e_k) \vee Q_{kj}^I(e_k), P_{kj}^F(e_k) \wedge g_{kj}^F(e_k), \right. \right.$$
$$\left. \left. \left(\bar{\alpha}_{kj}(e_k) \vee \bar{\gamma}_{ij}(e_1) \right), \left(\bar{\beta}_{kj}(e_k) \vee \bar{\delta}_{ij}(e_1) \right) \right) : (e_k, e_1) \in E \times E, j, k, l \in \wedge \right\}$$

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PWNRSS IN MULTI CRITERIA DECISION MAKING

Definition 2.20

Let

$$Q_{[\bar{\gamma},\bar{\delta}]}, L_{[\bar{\theta},\bar{\phi}]} \in PWRN(U,E),\ P_{[\bar{\alpha},\bar{\beta}]} = Q_{[\bar{\gamma},\bar{\delta}]} \wedge L_{[\bar{\theta},\bar{\phi}]} \text{ and } P^{T}_{[\bar{\alpha},\bar{\beta}]}, P^{I}_{[\bar{\alpha},\bar{\beta}]}, P^{F}_{[\bar{\alpha},\bar{\beta}]}$$

be the truth, indeterminacy and falsity matrices of $\wedge-$product matrix, respectively. Then, weighted matrices of $\left(P_{[\bar{\alpha},\bar{\beta}]}\right)^{T}, \left(P_{[\bar{\alpha},\bar{\beta}]}\right)^{I}, \left(P_{[\bar{\alpha},\bar{\beta}]}\right)^{F}$ denoted $\wedge^{T}, \wedge^{I}, \wedge^{F}$ are defined as follows:

$$\wedge^{T}(e_{kj}, u_r) = T_{Q_{[\bar{\gamma},\bar{\delta}]\wedge L[\bar{\theta},\bar{\phi}]}(e_{kj})}(u_r) + \left(\bar{\gamma}_{kr}(e_k) \wedge \theta_{jr}(e_j)\right) - T_{Q_{[\bar{\gamma},\bar{\delta}]\wedge L[\bar{\theta},\bar{\phi}]}(e_{kj})}(u_r) \times \left(\bar{\alpha}_{kr}(e_k) \wedge \left(\bar{\gamma}_{jr}(e_j)\right), \bar{\beta}(e_k)\right)$$

$$\wedge^{I}(e_{kj}, u_r) = I_{Q_{[\bar{\gamma},\bar{\delta}]\wedge L[\bar{\theta},\bar{\phi}]}(e_{kj})}(u_r) + \left(\bar{\gamma}_{kr}(e_k) \wedge \theta_{jr}(e_j)\right) - I_{Q_{[\bar{\gamma},\bar{\delta}]\wedge L[\bar{\theta},\bar{\phi}]}(e_{kj})}(u_r) \times \left(\bar{\alpha}_{kr}(e_k) \wedge \left(\bar{\gamma}_{jr}(e_j)\right), \bar{\beta}(e_k)\right)$$

$$\wedge^{F}(e_{kj}, u_r) = P_{Q_{[\bar{\gamma},\bar{\delta}]\wedge L[\bar{\theta},\bar{\phi}]}(e_{kj})}(u_r) \times \left(\bar{\gamma}_{kr}(e_k) \wedge \theta_{jr}(e_j)\right), \bar{\beta}(e_k)$$

Definition 2.21

Let $Q_{[\bar{\gamma},\bar{\delta}]}, L_{[\bar{\theta},\bar{\phi}]} \in PWRN(U,E),\ P_{[\bar{\alpha},\bar{\beta}]} = Q_{[\bar{\gamma},\bar{\delta}]} \wedge L_{[\bar{\theta},\bar{\phi}]}$, and let $\wedge^{T}, \wedge^{I}, \wedge^{F}$ be the weighted matrices of $\left(P_{[\bar{\alpha},\bar{\beta}]}\right)^{T}, \left(P_{[\bar{\alpha},\bar{\beta}]}\right)^{I}, \left(P_{[\bar{\alpha},\bar{\beta}]}\right)^{F}$ respectively. Then, in the weighted matrices $\wedge^{T}, \wedge^{I}, \wedge^{F}$ scores of $u_n \in U$ denoted by $s^{T}(u_n)$, $s^{I}(u_n)$, $s^{F}(u_n)$ are defined as follows:

$$s^{T}(u_n) = \sum_{k,j\in\wedge} \left(\pi^{T}_{kj}(u_n) \times \bar{\beta}(e_k)\right)$$

$$s^{I}(u_n) = \sum_{k,j\in\wedge} \left(\pi^{I}_{kj}(u_n) \times \bar{\beta}(e_k)\right)$$

$$s^{F}(u_n) = \sum_{k,j\in\wedge} \left(\pi^{F}_{kj}(u_n) \times \bar{\beta}(e_k)\right)$$

Where

$$\pi^{T}_{kj}(u_m) = \begin{cases} \wedge^{T}(e_{kJ}, u_n), \wedge^{T}(e_{kJ}, u_n) = \max\left\{\wedge^{T}(e_{kJ}, u_m) : u_m \in U\right\} \\ 0, \qquad otherwise \end{cases}$$

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$$\pi_{kj}^{I}(u_m)=\begin{cases}\wedge^{I}\left(e_{kJ},u_n\right),\wedge^{I}\left(e_{kJ},u_n\right)=\max\left\{\wedge^{I}\left(e_{kJ},u_m\right):u_m\in U\right\}\\0, \qquad otherwise\end{cases}$$

$$\pi_{kj}^{F}(u_m)=\begin{cases}\vee^{F}\left(e_{kJ},u_n\right),\vee^{F}\left(e_{kJ},u_n\right)=\max\left\{\vee^{F}\left(e_{kJ},u_m\right):u_m\in U\right\}\\0, \qquad otherwise\end{cases}$$

Definition 2.22

Let $s^T(u_n)$, $s^I(u_n)$ and $s^F(u_n)$ be scores of $u_n \in U$ in the weighted matrices $\wedge^T, \wedge^I, \wedge^F$. Then, decision score of $u_n \in U$, denoted by $ds(u_n)$, is defined by $ds(u_n)=s^T(u_n)+s^I(u_n)-s^F(u_n)$. Now, we construct a PWNRS –decision making method by the following algorithm:

Algorithm

Step 1: Input the PWNRSs,
Step 2: Construct the PWNRSs in matrix form,
Step 3: Construct the truth, indeterminacy and falsity matrices,
Step 4: Construct the weighted matrices $\wedge^T, \wedge^I, \wedge^F$.
Step 5: Compute score of $u_t \in U$, for each of the weighted matrices,
Step 6: Compute decision score, for all $u_t \in U$.
Step 7: The optimal decision is to select $u_t=\max ds(u_i)$

Example

Assume that $U=\{c_1,c_2,c_3\}$ is a set of three colleges and $E=\{e_1,e_2,e_3\}=$ {Academics, Sports, Fine Arts} is a set of parameters which is best college among the three, -represents priority given to the parameters by the college and $\overline{\beta}$ -represents the weightage given to the parameters by the experts.

Suppose that a committee wants to award a particular college. $P_{\left[\overline{\gamma},\overline{\delta}\right]}$

Step 1: Based on the choice parameters, PWNRSs $P_{\left[\overline{\alpha},\overline{\beta}\right]}$, constructed by three experts are as follows:

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$$P_{[\bar{\alpha},\bar{\beta}]}=\left\{\begin{array}{l} P_1\left\{\begin{array}{l} e_1=\left\{\left[\left(\frac{c_1}{(0.4,0.5,0.6)},0.7\right),\left(\frac{c_2}{(0.7,0.3,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.6,0.6)},0.5\right)\right],0.7\right\} \\ e_2=\left\{\left[\left(\frac{c_1}{(0.5,0.5,0.6)},0.5\right),\left(\frac{c_2}{(0.3,0.3,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.7,0.6)},0.5\right)\right],0.8\right\} \\ e_3=\left\{\left[\left(\frac{c_1}{(0.3,0.2,0.7)},0.6\right),\left(\frac{c_2}{(0.7,0.3,0.6)},0.2\right),\left(\frac{c_3}{(0.7,0.5,0.6)},0.5\right)\right],0.6\right\} \end{array}\right\} \\ P_2\left\{\begin{array}{l} e_1=\left\{\left[\left(\frac{c_1}{(0.4,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.8,0.4,0.5)},0.3\right),\left(\frac{c_3}{(0.3,0.5,0.5)},0.5\right)\right],0.7\right\} \\ e_2=\left\{\left[\left(\frac{c_1}{(0.5,0.5,0.6)},0.5\right),\left(\frac{c_2}{(0.3,0.3,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.7,0.6)},0.5\right)\right],0.8\right\} \\ e_3=\left\{\left[\left(\frac{c_1}{(0.3,0.2,0.7)},0.6\right),\left(\frac{c_2}{(0.7,0.3,0.6)},0.2\right),\left(\frac{c_3}{(0.7,0.5,0.6)},0.5\right)\right],0.6\right\} \end{array}\right\} \\ P_3\left\{\begin{array}{l} e_1=\left\{\left[\left(\frac{c_1}{(0.4,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.5,0.4,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.4,0.6)},0.3\right)\right],0.7\right\} \\ e_2=\left\{\left[\left(\frac{c_1}{(0.5,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.3,0.6,0.4)},0.4\right),\left(\frac{c_3}{(0.5,0.4,0.6)},0.3\right)\right],0.7\right\} \\ e_3=\left\{\left[\left(\frac{c_1}{(0.3,0.2,0.6)},0.5\right),\left(\frac{c_2}{(0.5,0.5,0.6)},0.4\right),\left(\frac{c_3}{(0.6,0.4,0.6)},0.3\right)\right],0.6\right\} \end{array}\right\} \end{array}\right\}$$

Step 2: Construct the PWNRSs in matrix form

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$$P_{[\bar{\alpha},\bar{\beta}]} = \begin{pmatrix} & u_1,\bar{\alpha} & u_2,\bar{\alpha} & u_3,\bar{\alpha} & \bar{\beta} \\ P_1(e_1) & \left(\langle 0.4,0.5,0.6\rangle, 0.7\right) & \left(\langle 0.7,0.3,0.5\rangle, 0.3\right) & \left(\langle 0.5,0.6,0.6\rangle, 0.5\right) & (0.7) \\ P_1(e_2) & \left(\langle 0.5,0.5,0.6\rangle, 0.5\right) & \left(\langle 0.3,0.3,0.5\rangle, 0.3\right) & \left(\langle 0.5,0.7,0.6\rangle, 0.5\right) & (0.8) \\ P_1(e_3) & \left(\langle 0.3,0.2,0.7\rangle, 0.6\right) & \left(\langle 0.7,0.3,0.6\rangle, 0.2\right) & \left(\langle 0.7,0.5,0.6\rangle, 0.6\right) & (0.6) \\ P_2(e_1) & \left(\langle 0.4,0.3,0.6\rangle, 0.5\right) & \left(\langle 0.8,0.4,0.5\rangle, 0.3\right) & \left(\langle 0.3,0.5,0.5\rangle, 0.5\right) & (0.7) \\ P_2(e_2) & \left(\langle 0.4,0.3,0.6\rangle, 0.5\right) & \left(\langle 0.3,0.6,0.4\rangle, 0.6\right) & \left(\langle 0.3,0.5,0.5\rangle, 0.7\right) & (0.8) \\ P_2(e_3) & \left(\langle 0.3,0.2,0.7\rangle, 0.5\right) & \left(\langle 0.8,0.5,0.6\rangle, 0.5\right) & \left(\langle 0.3,0.5,0.5\rangle, 0.5\right) & (0.6) \\ P_3(e_1) & \left(\langle 0.4,0.3,0.6\rangle, 0.5\right) & \left(\langle 0.5,0.4,0.5\rangle, 0.3\right) & \left(\langle 0.5,0.4,0.6\rangle, 0.3\right) & (0.7) \\ P_3(e_2) & \left(\langle 0.5,0.3,0.6\rangle, 0.5\right) & \left(\langle 0.3,0.6,0.4\rangle, 0.4\right) & \left(\langle 0.5,0.4,0.6\rangle, 0.3\right) & (0.7) \\ P_3(e_3) & \left(\langle 0.3,0.2,0.6\rangle, 0.5\right) & \left(\langle 0.5,0.5,0.6\rangle, 0.4\right) & \left(\langle 0.6,0.4,0.6\rangle, 0.3\right) & (0.6) \end{pmatrix}$$

Step 3: We construct matrices $P^{T}_{[\bar{\alpha},\bar{\beta}]}, P^{I}_{[\bar{\alpha},\bar{\beta}]}$ and $P^{F}_{[\bar{\alpha},\bar{\beta}]}$ as follows:

$$\begin{pmatrix} & u_1,\bar{\alpha} & u_2,\bar{\alpha} & u_3,\bar{\alpha} & \bar{\beta} \\ P_1(e_1) & (0.4,0.7) & (0.7,0.3) & (0.5,0.5) & (0.7) \\ P_1(e_2) & (0.5,0.5) & (0.3,0.3) & (0.5,0.5) & (0.8) \\ P_1(e_3) & (0.3,0.6) & (0.7,0.2) & (0.7,0.6) & (0.6) \\ P_2(e_1) & (0.4,0.5) & (0.8,0.3) & (0.3,0.5) & (0.7) \\ P_2(e_2) & (0.4,0.5) & (0.3,0.6) & (0.3,0.7) & (0.8) \\ P_2(e_3) & (0.3,0.5) & (0.8,0.5) & (0.3,0.5) & (0.6) \\ P_3(e_1) & (0.4,0.5) & (0.5,0.3) & (0.5,0.3) & (0.7) \\ P_3(e_2) & (0.5,0.5) & (0.3,0.4) & (0.5,0.3) & (0.7) \\ P_3(e_3) & (0.3,0.5) & (0.5,0.4) & (0.6,0.3) & (0.6) \end{pmatrix}$$

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	$u_1,\bar{\alpha}$	$u_2,\bar{\alpha}$	$u_3,\bar{\alpha}$	$\bar{\beta}$
$P_1(e_1)$	$(0.5,0.7)$	$(0.3,0.3)$	$(0.6,0.5)$	(0.7)
$P_1(e_2)$	$(0.5,0.5)$	$(0.3,0.3)$	$(0.7,0.5)$	(0.8)
$P_1(e_3)$	$(0.2,0.6)$	$(0.3,0.2)$	$(0.5,0.6)$	(0.6)
$P_2(e_1)$	$(0.3,0.5)$	$(0.4,0.3)$	$(0.5,0.5)$	(0.7)
$P_2(e_2)$	$(0.3,0.5)$	$(0.6,0.6)$	$(0.5,0.7)$	(0.8)
$P_2(e_3)$	$(0.2,0.5)$	$(0.5,0.5)$	$(0.5,0.5)$	(0.6)
$P_3(e_1)$	$(0.3,0.5)$	$(0.4,0.3)$	$(0.4,0.3)$	(0.7)
$P_3(e_2)$	$(0.3,0.5)$	$(0.6,0.4)$	$(0.4,0.3)$	(0.7)
$P_3(e_3)$	$(0.2,0.5)$	$(0.5,0.4)$	$(0.4,0.3)$	(0.6)

	$u_1,\bar{\alpha}$	$u_2,\bar{\alpha}$	$u_3,\bar{\alpha}$	$\bar{\beta}$
$P_1(e_1)$	$(0.6,0.7)$	$(0.5,0.3)$	$(0.6,0.5)$	(0.7)
$P_1(e_2)$	$(0.6,0.5)$	$(0.5,0.3)$	$(0.6,0.5)$	(0.8)
$P_1(e_3)$	$(0.7,0.6)$	$(0.6,0.2)$	$(0.6,0.6)$	(0.6)
$P_2(e_1)$	$(0.6,0.5)$	$(0.5,0.3)$	$(0.5,0.5)$	(0.7)
$P_2(e_2)$	$(0.6,0.5)$	$(0.4,0.6)$	$(0.5,0.7)$	(0.8)
$P_2(e_3)$	$(0.7,0.5)$	$(0.6,0.5)$	$(0.5,0.5)$	(0.6)
$P_3(e_1)$	$(0.6,0.5)$	$(0.5,0.3)$	$(0.6,0.3)$	(0.7)
$P_3(e_2)$	$(0.6,0.5)$	$(0.4,0.4)$	$(0.6,0.3)$	(0.7)
$P_3(e_3)$	$(0.6,0.5)$	$(0.6,0.4)$	$(0.6,0.3)$	(0.6)

Step 4: We obtain weighted matrices $\wedge^T, \wedge^I, \wedge^F$ by using Definition 2.21 as follws:

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$$\begin{pmatrix} \wedge^T & u_1,\bar{\alpha} & u_2,\bar{\alpha} & u_3,\bar{\alpha} & \bar{\beta} \\ P_1(e_1) & 0.82 & 0.79 & 0.75 & 0.7 \\ P_1(e_2) & 0.75 & 0.51 & 0.70 & 0.8 \\ P_1(e_3) & 0.79 & 0.79 & 0.88 & 0.6 \\ P_2(e_1) & 0.70 & 0.86 & 0.65 & 0.7 \\ P_2(e_2) & 0.70 & 0.72 & 0.79 & 0.8 \\ P_2(e_3) & 0.65 & 0.90 & 0.65 & 0.6 \\ P_3(e_1) & 0.70 & 0.65 & 0.65 & 0.7 \\ P_3(e_2) & 0.75 & 0.58 & 0.65 & 0.7 \\ P_3(e_3) & 0.65 & 0.70 & 0.72 & 0.6 \end{pmatrix}, \begin{pmatrix} \wedge^I & u_1,\bar{\alpha} & u_2,\bar{\alpha} & u_3,\bar{\alpha} & \bar{\beta} \\ P_1(e_1) & 0.85 & 0.51 & 0.80 & 0.7 \\ P_1(e_2) & 0.70 & 0.51 & 0.85 & 0.8 \\ P_1(e_3) & 0.76 & 0.51 & 0.80 & 0.6 \\ P_2(e_1) & 0.65 & 0.58 & 0.75 & 0.7 \\ P_2(e_2) & 0.65 & 0.84 & 0.85 & 0.8 \\ P_2(e_3) & 0.60 & 0.75 & 0.75 & 0.6 \\ P_3(e_1) & 0.65 & 0.58 & 0.58 & 0.7 \\ P_3(e_2) & 0.65 & 0.76 & 0.58 & 0.7 \\ P_3(e_3) & 0.60 & 0.70 & 0.58 & 0.6 \end{pmatrix},$$

$$\begin{pmatrix} \wedge^F & u_1,\bar{\alpha} & u_2,\bar{\alpha} & u_3,\bar{\alpha} & \bar{\beta} \\ P_1(e_1) & 0.42 & 0.15 & 0.15 & 0.7 \\ P_1(e_2) & 0.15 & 0.12 & 0.15 & 0.8 \\ P_1(e_3) & 0.49 & 0.18 & 0.24 & 0.6 \\ P_2(e_1) & 0.30 & 0.12 & 0.15 & 0.7 \\ P_2(e_2) & 0.15 & 0.24 & 0.21 & 0.8 \\ P_2(e_3) & 0.35 & 0.20 & 0.20 & 0.6 \\ P_3(e_1) & 0.30 & 0.09 & 0.09 & 0.7 \\ P_3(e_2) & 0.15 & 0.12 & 0.09 & 0.7 \\ P_3(e_3) & 0.30 & 0.12 & 0.12 & 0.6 \end{pmatrix}$$

Matrix $\left(P_{[\bar{\alpha},\bar{\beta}]}\right)^T, \left(P_{[\bar{\alpha},\bar{\beta}]}\right)^I, \left(P_{[\bar{\alpha},\bar{\beta}]}\right)^F$ from up to down, respectively

Step 5: For all $u \in U$, we find a scores by using Definition 2.22 as follows:

$s^T(u_1)=2.19$, $s^T(u_2)=1.045$, $s^T(u_3)=0.7$;

$s^I(u_1)=2.10$, $s^I(u_2)=0.952$, $s^I(u_3)=0.26$;

$s^F(u_1)=1.59$, $s^F(u_2)=2.815$, $s^F(u_3)=0.29$

Step 6: For all $u \in U$, we find a scores by using Definition 2.22 as follows:

$ds(u_1)= 2.19+1.045–0.7 =2.535$;

$ds(u_2)= 2.19+0.952–0.26 =2.792$;

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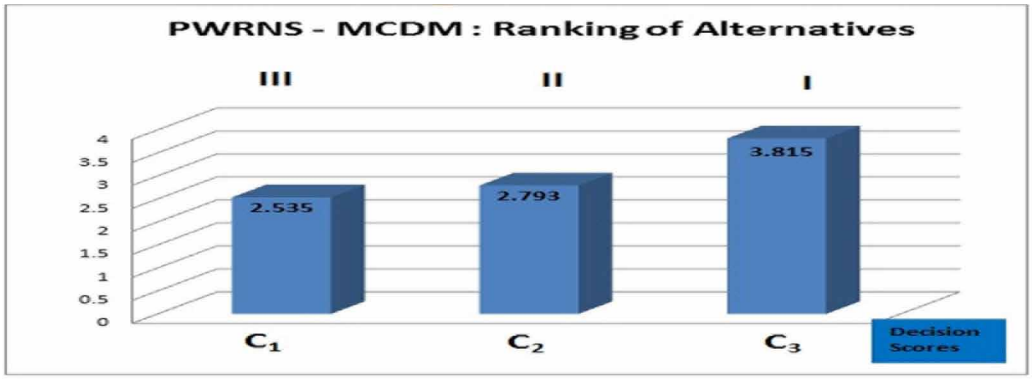
$ds(u_1)$= 1.59+2.815–0.29 =3.815

Step 7: Then the optimal selection of the committee is the college C_3.

Figure 1. Decision scores and ranking of alternatives



PRIORITY WEIGHTED NEUTROSOPHIC REFINED SOFT set: TOPSIS METHOD

A systematic approach to extend the TOPSIS to the prioritized neutrosophic refined environment has been proposed in this section. This method is very suitable for solving the group decision-making problem under the priority refined neutrosophic environment.

Step 1

Let us consider a group of r decision makers (D_1,D_2,D_3) each decision makers have k alternatives (A_1,A_2,… ,A_k) and q attributes (C_1,C_2,…,C_q). Then form the priority weighted decision matrix by using the given priority weighted neutrosophic refined soft sets. The priority weighted decision matrix is defined as follow:

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Decision Matrix

$$\begin{pmatrix} & u_1,\bar{\alpha} & u_2,\bar{\alpha} & \cdots & u_3,\bar{\alpha} & \bar{\beta} \\ D_1\begin{cases} A_1 \\ A_2 \\ \cdots \\ A_k \end{cases} & \begin{matrix} \left(\left\langle T_{11}^1, I_{11}^1, F_{11}^1\right\rangle, \alpha_{11}^1\right) \\ \left(\left\langle T_{11}^2, I_{11}^2, F_{11}^2\right\rangle, \alpha_{11}^2\right) \\ \cdots \\ \left(\left\langle T_{11}^k, I_{11}^k, F_{11}^k\right\rangle, \alpha_{11}^k\right) \end{matrix} & \begin{matrix} \left(\left\langle T_{12}^1, I_{12}^1, F_{12}^1\right\rangle, \alpha_{12}^1\right) \\ \left(\left\langle T_{12}^2, I_{12}^2, F_{12}^2\right\rangle, \alpha_{12}^2\right) \\ \cdots \\ \left(\left\langle T_{12}^k, I_{12}^k, F_{12}^k\right\rangle, \alpha_{12}^k\right) \end{matrix} & \cdots & \begin{matrix} \left(\left\langle T_{11q}^1, I_{1q}^1, F_{1q}^1\right\rangle, \alpha_{1q}^1\right) \\ \left(\left\langle T_{11q}^2, I_{1q}^2, F_{1q}^2\right\rangle, \alpha_{1q}^2\right) \\ \cdots \\ \left(\left\langle T_{11q}^k, I_{1q}^k, F_{1q}^k\right\rangle, \alpha_{1q}^k\right) \end{matrix} & \begin{matrix} \beta_{11} \\ \beta_{12} \\ \cdots \\ \beta_{1k} \end{matrix} \\ D_2\begin{cases} A_1 \\ A_2 \\ \cdots \\ A_k \end{cases} & \begin{matrix} \left(\left\langle T_{21}^1, I_{21}^1, F_{21}^1\right\rangle, \alpha_{21}^1\right) \\ \left(\left\langle T_{21}^2, I_{21}^2, F_{21}^2\right\rangle, \alpha_{21}^2\right) \\ \cdots \\ \left(\left\langle T_{21}^k, I_{21}^k, F_{21}^k\right\rangle, \alpha_{21}^k\right) \end{matrix} & \begin{matrix} \left(\left\langle T_{22}^1, I_{22}^1, F_{22}^1\right\rangle, \alpha_{22}^1\right) \\ \left(\left\langle T_{22}^2, I_{22}^2, F_{22}^2\right\rangle, \alpha_{22}^2\right) \\ \cdots \\ \left(\left\langle T_{22}^k, I_{22}^k, F_{22}^k\right\rangle, \alpha_{22}^k\right) \end{matrix} & \cdots & \begin{matrix} \left(\left\langle T_{2q}^1, I_{2q}^1, F_{2q}^1\right\rangle, \alpha_{1q}^1\right) \\ \left(\left\langle T_{2q}^2, I_{2q}^2, F_{2q}^2\right\rangle, \alpha_{2q}^2\right) \\ \cdots \\ \left(\left\langle T_{2q}^k, I_{2q}^k, F_{2q}^k\right\rangle, \alpha_{2q}^k\right) \end{matrix} & \begin{matrix} \beta_{21} \\ \beta_{22} \\ \cdots \\ \beta_{2k} \end{matrix} \\ \vdots \quad \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ D_3\begin{cases} A_1 \\ A_2 \\ \cdots \\ A_k \end{cases} & \begin{matrix} \left(\left\langle T_{r1}^1, I_{r1}^1, F_{r1}^1\right\rangle, \alpha_{r1}^1\right) \\ \left(\left\langle T_{r1}^2, I_{r1}^2, F_{r1}^2\right\rangle, \alpha_{r1}^2\right) \\ \cdots \\ \left(\left\langle T_{r1}^k, I_{r1}^k, F_{r1}^k\right\rangle, \alpha_{r1}^k\right) \end{matrix} & \begin{matrix} \left(\left\langle T_{r2}^1, I_{r2}^1, F_{r2}^1\right\rangle, \alpha_{r2}^1\right) \\ \left(\left\langle T_{r2}^2, I_{r2}^2, F_{r2}^2\right\rangle, \alpha_{r2}^2\right) \\ \cdots \\ \left(\left\langle T_{r2}^k, I_{r2}^k, F_{r2}^k\right\rangle, \alpha_{r2}^k\right) \end{matrix} & \cdots & \begin{matrix} \left(\left\langle T_{rq}^1, I_{rq}^1, F_{rq}^1\right\rangle, \alpha_{rq}^1\right) \\ \left(\left\langle T_{rq}^2, I_{rq}^2, F_{rq}^2\right\rangle, \alpha_{rq}^2\right) \\ \cdots \\ \left(\left\langle T_{rq}^k, I_{rq}^k, F_{rq}^k\right\rangle, \alpha_{rq}^k\right) \end{matrix} & \begin{matrix} \beta_{r1} \\ \beta_{r2} \\ \cdots \\ \beta_{rk} \end{matrix} \end{pmatrix}$$

Step 2

Priority Weights

Find the priority weights w_{pm}. Each $w_{pm} = \left(\left\langle T_{pm}, I_{pm}, F_{pm}\right\rangle, \alpha_{pm}\right), \left(\beta_{pm}\right)$ is defined as

$$w_{pm} = \left(\left\langle \max T_{pm}, \min I_{pm}, \min F_{pm}\right\rangle, \min \alpha_{pm}\right), \left(\max \beta_{pm}\right),$$

where m=1,2,…,r. If r decision makers we get r priority weights.

Crispfication of Priority Weights

The r decision makers have their own priority weights (w_{p1},w_{p2},…,w_{pr}). The equivalent crisp priority weight can be obtained using the following equation

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$$w_{pm}^{c}=\frac{1-\sqrt{\frac{\left(\left(1-T_{pm}\right)^{2}+\left(I_{pm}\right)^{2}+\left(F_{pm}\right)^{2}+\left(\alpha_{pm}\right)^{2}+\left(1-\beta_{pm}\right)^{2}\right)}{5}}}{\sum_{m=1}^{r}\left(1-\frac{\left(\left(1-T_{pm}\right)^{2}+\left(I_{pm}\right)^{2}+\left(F_{pm}\right)^{2}+\left(\alpha_{pm}\right)^{2}+\left(1-\beta_{pm}\right)^{2}\right)}{5}\right)} \tag{1}$$

and

$$w_{pm}^{c}\geq 0,\sum_{m=1}^{r}w_{pm}^{c}=1$$

Step 3

Construction of Aggregated Decision Matrix

The aggregated neutrosophic decision matrix can be constructed as follows:

Aggregated Decision Matrix

$$d_{mn}^{\sim}=\begin{pmatrix} & C_1,\alpha & C_2,\alpha & \dots & C_n,\alpha & \beta \\ D_1 & d_{11}^{\sim} & d_{12}^{\sim} & \dots & d_{1n}^{\sim} & \beta_1^{\sim} \\ D_2 & d_{21}^{\sim} & d_{22}^{\sim} & \dots & d_{2n}^{\sim} & \beta_2^{\sim} \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ D_r & d_{m1}^{\sim} & d_{m2}^{\sim} & \dots & d_{mn}^{\sim} & \beta_r^{\sim} \end{pmatrix}$$

$$d_{mn}^{\sim}=\begin{pmatrix} & C_1,\alpha & C_2,\alpha & \dots & C_q,\alpha & \beta \\ D_1 & \left(\left\langle T_{11}^{\sim},I_{11}^{\sim},F_{11}^{\sim}\right\rangle\alpha_{11}^{\sim}\right) & \left(\left\langle T_{12}^{\sim},I_{12}^{\sim},F_{12}^{\sim}\right\rangle\alpha_{12}^{\sim}\right) & \dots & \left(\left\langle T_{1q}^{\sim},I_{1q}^{\sim},F_{1q}^{\sim}\right\rangle\alpha_{1q}^{\sim}\right) & \beta_1^{\sim} \\ D_2 & \left(\left\langle T_{21}^{\sim},I_{21}^{\sim},F_{21}^{\sim}\right\rangle\alpha_{21}^{\sim}\right) & \left(\left\langle T_{22}^{\sim},I_{22}^{\sim},F_{22}^{\sim}\right\rangle\alpha_{22}^{\sim}\right) & \dots & \left(\left\langle T_{2q}^{\sim},I_{2q}^{\sim},F_{2q}^{\sim}\right\rangle\alpha_{2q}^{\sim}\right) & \beta_2^{\sim} \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ D_r & \left(\left\langle T_{r1}^{\sim},I_{r1}^{\sim},F_{r1}^{\sim}\right\rangle\alpha_{r1}^{\sim}\right) & \left(\left\langle T_{r2}^{\sim},I_{r2}^{\sim},F_{r2}^{\sim}\right\rangle\alpha_{r2}^{\sim}\right) & \dots & \left(\left\langle T_{rq}^{\sim},I_{rq}^{\sim},F_{rq}^{\sim}\right\rangle\alpha_{rq}^{\sim}\right) & \beta_r^{\sim} \end{pmatrix}$$

Where,

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$$d_{mn}^{\sim}=\left(\left\langle 1-\sum_{i=1}^{k}\left(1-T_{in}^{m}\right)^{w_m},1-\sum_{i=1}^{k}\left(1-I_{in}^{m}\right)^{w_m},1-\sum_{i=1}^{k}\left(1-F_{in}^{m}\right)^{w_m}\right\rangle,\right.$$
$$\left.1-\sum_{i=1}^{k}\left(1-\alpha_{in}^{m}\right)^{w_m},\beta_{m}^{\sim}=\left(1-\sum_{i=1}^{k}\left(1-\beta_{in}^{m}\right)^{w_m}\right),n=1,2,...q,m=1,2,...r\right. \tag{2}$$

Step 4

Description of Priority Matrix

In decision making situation, decision makers would not like to give equal importance to all attributes. Thus each DM would have different opinion regarding the weights of attribute. The priority matrix is selected by the given maximum priority value β in the each decision makers values suppose the decision have same priority value for different attribute then we use the next level of given maximum priority value α suppose they are also same we use remaining given values in the following order that is maximum truth value, maximum indeterminacy value, maximum falsity value.

Priority Matrix

$$\begin{pmatrix} & C_1,\alpha & C_2,\alpha & \dots & C_q,\alpha & \beta \\ D_1 & \left(\left\langle T_{11}^{'},I_{11}^{'},F_{11}^{'}\right\rangle\alpha_{11}^{'}\right) & \left(\left\langle T_{12}^{'},I_{12}^{'},F_{12}^{'}\right\rangle\alpha_{12}^{'}\right) & \dots & \left(\left\langle T_{1q}^{'},I_{1q}^{'},F_{1q}^{'}\right\rangle\alpha_{1q}^{'}\right) & \beta_1^{'} \\ D_2 & \left(\left\langle T_{21}^{'},I_{21}^{'},F_{21}^{'}\right\rangle\alpha_{21}^{'}\right) & \left(\left\langle T_{22}^{'},I_{22}^{'},F_{22}^{'}\right\rangle\alpha_{22}^{'}\right) & \dots & \left(\left\langle T_{2q}^{'},I_{2q}^{'},F_{2q}^{'}\right\rangle\alpha_{2q}^{'}\right) & \beta_2^{'} \\ \vdots & \vdots \quad \vdots \quad \dots & \vdots \quad \vdots & & & \\ D_r & \left(\left\langle T_{r1}^{'},I_{r1}^{'},F_{r1}^{'}\right\rangle\alpha_{r1}^{'}\right) & \left(\left\langle T_{r2}^{'},I_{r2}^{'},F_{r2}^{'}\right\rangle\alpha_{r2}^{'}\right) & \dots & \left(\left\langle T_{rq}^{'},I_{rq}^{'},F_{rq}^{'}\right\rangle\alpha_{rq}^{'}\right) & \beta_r^{'} \end{pmatrix}$$

The aggregated weights for all attributes are presented below:

$$w^{''}=\left(\left\langle\sum_{m=1}^{r}\left(T_{mn}\times w_m\right),\sum_{m=1}^{r}\left(I\times w_m\right),\sum_{m=1}^{r}\left(F_{mn}\times w_m\right)\right\rangle,\sum_{m=1}^{r}\left(\alpha_{mn}\times w_m\right)\right),\sum_{m=1}^{r}\left(\beta_{mn}\times w_m\right) \tag{3}$$

Where m=1,2,…,r and n=1,2,…,q

Step 5

Construction of Aggregated Priority Weighted Decision Matrix

The aggregated weighted decision matrix can be formed as:

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Aggregated Priority Weighted Decision Matrix

$$\begin{pmatrix} & C_1,\alpha & C_2,\alpha & \dots & C_q,\alpha & \beta \\ D_1 & \left(\left\langle T_{11}^w, I_{11}^w, F_{11}^w \right\rangle \alpha_{11}^w\right) & \left(\left\langle T_{12}^w, I_{12}^w, F_{12}^w \right\rangle \alpha_{12}^w\right) & \dots & \left(\left\langle T_{1q}^w, I_{1q}^w, F_{1q}^w \right\rangle \alpha_{1q}^w\right) & \beta_1^w \\ D_2 & \left(\left\langle T_{21}^w, I_{21}^w, F_{21}^w \right\rangle \alpha_{21}^w\right) & \left(\left\langle T_{22}^w, I_{22}^w, F_{22}^{\mathbf{w}} \right\rangle \alpha_{22}^w\right) & \dots & \left(\left\langle T_{2q}^w, I_{2q}^w, F_{2q}^w \right\rangle \alpha_{2q}^w\right) & \beta_2^w \\ \vdots & \vdots \quad \vdots \quad \dots & \vdots \quad \vdots & & & \\ D_r & \left(\left\langle T_{r1}^w, I_{r1}^w, F_{r1}^w \right\rangle \alpha_{r1}^w\right) & \left(\left\langle T_{r2}^w, I_{r2}^w, F_{r2}^w \right\rangle \alpha_{r2}^w\right) & \dots & \left(\left\langle T_{rq}^w, I_{rq}^w, F_{rq}^w \right\rangle \alpha_{rq}^w\right) & \beta_r^w \end{pmatrix}$$

Where

$$w^{''}d^w = \left(\left(\left\langle T_n^{''}, I_n^{''}, F_n^{''} \right\rangle \alpha_n^{''}\right), \left(\beta_n^{''}\right)\right) \otimes \left(\left(\left\langle T_{mn}^{\sim}, I_{mn}^{\sim}, F_{mn}^{\sim} \right\rangle, \alpha_{mn}^{\sim}\right), \left(\beta_m^{\sim}\right)\right) \tag{4}$$

$$w^{''}d^w = \left(\left(\left\langle T_n^{''}.T_{mn}^{\sim}, I_n^{''}.I_{mn}^{\sim}, F_n^{''}.F_{mn}^{\sim} \right\rangle, \alpha_n^{''} + \alpha_{mn}^{\sim} - \alpha_n^{''}.\alpha_{mn}^{\sim}\right), \beta_m^w = \left(\beta_n^{''}.\beta_m^{\sim}\right)\right)$$

Where m=1,2,…,r and n=1,2,…,q

Step 6

Relative Positive Ideal Solution (RPIS) and Relative Negative Ideal Solution (RNIS)

In this step, we find out relative positive ideal solution (RPIS) and the relative negative ideal solution (RNIS) for the above aggregated neutrosophic decision matrix.

Assume R^{w+} as the relative positive ideal solution and R^{w-} as relative negative ideal solution.

Next, R^{w+} and R^{w-} can be described by:

$$R^{w+} = \left\{\infty\left[\left(\left\langle T_1^{w+}, I_1^{w+}, F_1^{w+} \right\rangle, \alpha_1^{w+}\right), \left(\left\langle T_2^{w+}, I_2^{w+}, F_2^{w+} \right\rangle, \alpha_2^{w+}\right), ..., \left(\left\langle T_r^{w+}, I_r^{w+}, F_r^{w+} \right\rangle, \alpha_r^{w+}\right)\right], \left[\left(\beta^{w+}\right)\right]\right\} \tag{5}$$

$$R^{w-} = \left\{\infty\left[\left(\left\langle T_1^{w-}, I_1^{w-}, F_1^{w-} \right\rangle, \alpha_1^{w-}\right), \left(\left\langle T_2^{w-}, I_2^{w-}, F_2^{w-} \right\rangle, \alpha_2^{w-}\right), ..., \left(\left\langle T_r^{w-}, I_r^{w-}, F_r^{w-} \right\rangle, \alpha_r^{w-}\right)\right], \left[\left(\beta^{w-}\right)\right]\right\} \tag{6}$$

Where

$$\left\langle T_n^{w+}, I_n^{w+}, F_n^{w+} \right\rangle = \left(\left\langle \max_m\left(T_{mn}^w\right), \min_m\left(I_{mn}^w\right), \min_m\left(F_{mn}^w\right)\right\rangle, \min_m\left(\alpha_{mn}^w\right)\right), \left(\max_m\left(\beta_{mn}^w\right)\right)$$

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Or

$$\left\langle T_n^{w+}, I_n^{w+}, F_n^{w+} \right\rangle = \left(\left\langle \min_m \left(T_{mn}^w \right), \max_m \left(I_{mn}^w \right), \max_m \left(F_{mn}^w \right) \right\rangle, \max_m \left(\alpha_{mn}^w \right) \right), \left(\min_m \left(\beta_{mn}^w \right) \right)$$

And

$$\left\langle T_n^{w-}, I_n^{w-}, F_n^{w-} \right\rangle = \left(\left\langle \min_m \left(T_{mn}^w \right), \max_m \left(I_{mn}^w \right), \max_m \left(F_{mn}^w \right) \right\rangle, \max_m \left(\alpha_{mn}^w \right) \right), \left(\min_m \left(\beta_{mn}^w \right) \right)$$

Or

$$\left\langle T_n^{w-}, I_n^{w-}, F_n^{w-} \right\rangle = \left(\left\langle \max_m \left(T_{mn}^w \right), \min_m \left(I_{mn}^w \right), \min_m \left(F_{mn}^w \right) \right\rangle, \min_m \left(\alpha_{mn}^w \right) \right), \left(\max_m \left(\beta_{mn}^w \right) \right)$$

Step 7

Determination of Distances of Each Alternative from the RPIS and the RNIS

The normalized Euclidean distance between

$$E_{uk}^{+} = \sqrt{\frac{1}{5q} \sum_{n=1}^{q} \left\{ \left(T_{mn}^w - T_n^{w+} \right)^2 + \left(I_{mn}^w - I_n^{w+} \right)^2 + \left(F_{mn}^w - F_n^{w+} \right)^2 + \left(\alpha_{mn}^w - \alpha_n^{w+} \right)^2 + \left(\beta_{mn}^w - \beta_n^{w+} \right)^2 \right\}} \quad (7)$$

and

$$E_{uk}^{-} = \sqrt{\frac{1}{5q} \sum_{n=1}^{q} \left\{ \left(T_{mn}^w - T_n^{w-} \right)^2 + \left(I_{mn}^w - I_n^{w-} \right)^2 + \left(F_{mn}^w - F_n^{w-} \right)^2 + \left(\alpha_{mn}^w - \alpha_n^{w-} \right)^2 + \left(\beta_{mn}^w - \beta_n^{w-} \right)^2 \right\}} \quad (8)$$

Step 8

Calculation of Relative Closeness Coefficient

The relative closeness coefficient for each alternative Ak with respect to R^{w+} is defined as:

$$R_k = \frac{E_{u_k}^{+}}{E_{u_k}^{+} - E_{u_k}^{-}} \quad (9)$$

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Step 9

Ranking of Alternatives

The alternative, for which the closeness coefficient is least, has become the best alternative.

A NUMERICAL EXAMPLE 5

Assume that $U=\{C_1,C_2,C_3\}$ is a set of three colleges and $A=\{A_1,A_2,A_3\}$. $A=\{$Academics, Sports, Fine Arts$\}$ is a set of parameters which is best college among the three, α-represents priority given to the parameters by the college and β-represents the weightage given to the parameters by the three experts.

Suppose that a committee wants to award a particular college.

Step 1: The PWNRSs constructed by the experts are:

$$D_1=\left\{\begin{array}{l} A_1=\left\{\left[\left(\frac{c_1}{(0.4,0.5,0.6)},0.7\right),\left(\frac{c_2}{(0.7,0.3,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.6,0.6)},0.5\right)\right],0.7\right\} \\ A_2=\left\{\left[\left(\frac{c_1}{(0.5,0.5,0.6)},0.5\right),\left(\frac{c_2}{(0.3,0.3,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.7,0.6)},0.5\right)\right],0.8\right\} \\ A_3=\left\{\left[\left(\frac{c_1}{(0.3,0.2,0.7)},0.7\right),\left(\frac{c_2}{(0.7,0.3,0.6)},0.3\right),\left(\frac{c_3}{(0.7,0.5,0.6)},0.5\right)\right],0.6\right\} \end{array}\right\}$$

$$D_2=\left\{\begin{array}{l} A_1=\left\{\left[\left(\frac{c_1}{(0.4,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.8,0.4,0.5)},0.3\right),\left(\frac{c_3}{(0.3,0.5,0.5)},0.5\right)\right],0.7\right\} \\ A_2=\left\{\left[\left(\frac{c_1}{(0.4,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.3,0.6,0.4)},0.6\right),\left(\frac{c_3}{(0.3,0.5,0.5)},0.7\right)\right],0.8\right\} \\ A_3=\left\{\left[\left(\frac{c_1}{(0.3,0.2,0.7)},0.5\right),\left(\frac{c_2}{(0.8,0.5,0.6)},0.4\right),\left(\frac{c_3}{(0.3,0.5,0.5)},0.5\right)\right],0.6\right\} \end{array}\right\}$$

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$$D_3=\left\{\begin{array}{l} A_1=\left\{\left[\left(\frac{c_1}{(0.4,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.5,0.4,0.5)},0.3\right),\left(\frac{c_3}{(0.5,0.4,0.6)},0.3\right)\right],0.7\right\} \\ A_2=\left\{\left[\left(\frac{c_1}{(0.5,0.3,0.6)},0.5\right),\left(\frac{c_2}{(0.3,0.6,0.4)},0.4\right),\left(\frac{c_3}{(0.5,0.4,0.6)},0.3\right)\right],0.7\right\} \\ A_3=\left\{\left[\left(\frac{c_1}{(0.3,0.2,0.7)},0.5\right),\left(\frac{c_2}{(0.5,0.5,0.6)},0.4\right),\left(\frac{c_3}{(0.6,0.4,0.6)},0.3\right)\right],0.6\right\} \end{array}\right\}$$

The decision matrix

$$\left(\begin{array}{cccccc} & & c_1,\alpha & c_2,\alpha & c_3,\alpha & \beta \\ D_1 & A_1 & (\langle 0.4,0.5,0.6\rangle,0.7) & (\langle 0.7,0.3,0.5\rangle,0.3) & (\langle 0.5,0.6,0.6\rangle,0.5) & (0.7) \\ & A_2 & (\langle 0.5,0.5,0.6\rangle,0.5) & (\langle 0.3,0.3,0.5\rangle,0.3) & (\langle 0.5,0.7,0.6\rangle,0.5) & (0.8) \\ & A_3 & (\langle 0.3,0.2,0.7\rangle,0.7) & (\langle 0.7,0.3,0.6\rangle,0.3) & (\langle 0.7,0.5,0.6\rangle,0.5) & (0.6) \\ D_2 & A_1 & (\langle 0.4,0.3,0.6\rangle,0.5) & (\langle 0.8,0.4,0.5\rangle,0.3) & (\langle 0.3,0.5,0.5\rangle,0.5) & (0.7) \\ & A_2 & (\langle 0.4,0.3,0.6\rangle,0.5) & (\langle 0.3,0.6,0.4\rangle,0.6) & (\langle 0.3,0.5,0.5\rangle,0.7) & (0.8) \\ & A_3 & (\langle 0.3,0.2,0.7\rangle,0.5) & (\langle 0.8,0.5,0.6\rangle,0.5) & (\langle 0.3,0.5,0.5\rangle,0.5) & (0.6) \\ D_3 & A_1 & (\langle 0.4,0.3,0.6\rangle,0.5) & (\langle 0.5,0.4,0.5\rangle,0.3) & (\langle 0.5,0.4,0.6\rangle,0.3) & (0.7) \\ & A_2 & (\langle 0.5,0.3,0.6\rangle,0.5) & (\langle 0.3,0.6,0.4\rangle,0.4) & (\langle 0.5,0.4,0.6\rangle,0.3) & (0.7) \\ & A_3 & (\langle 0.3,0.2,0.7\rangle,0.5) & (\langle 0.5,0.5,0.6\rangle,0.4) & (\langle 0.6,0.4,0.6\rangle,0.3) & (0.6) \end{array}\right)$$

Step 2: The Priority weights w_{pi} of decision makers are

$$\{(\langle 0.7,0.2,0.5\rangle,0.3),(0.8)\},\{(\langle 0.8,0.2,0.4\rangle,0.3),(0.8)\},\{(\langle 0.6,0.2,0.4\rangle,0.3),(0.7)\}$$

Using the equation (1), the equivalent crisp priority weights are {0.331140, 0.339319, 0.329540}

Step 3: The aggregated decision matrix can be determined by using equation (2), and calculated as below:

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Aggregated Decision Matrix

$$\begin{pmatrix} & c_1,\alpha & c_2,\alpha & c_3,\alpha & \beta \\ D_1 & \left(\langle 0.404,0.404,0.636\rangle,0.68\right) & \left(\langle 0.569,0.305,0.537\rangle,0.305\right) & \left(\langle 0.569,0.603,0.603\rangle,0.503\right) & (0.8) \\ D_2 & \left(\langle 0.355,0.254,0.627\rangle,0.491\right) & \left(\langle 0.627,0.491,0.389\rangle,0.423\right) & \left(\langle 0.287,0.491,0.491\rangle,0.559\right) & (0.8) \\ D_3 & \left(\langle 0.407,0.275,0.668\rangle,0.506\right) & \left(\langle 0.439,0.506,0.506\rangle,0.374\right) & \left(\langle 0.539,0.407,0.605\rangle,0.308\right) & (0.7) \end{pmatrix}$$

Step 4: The priority matrix of attributes as described in algorithm (step 4), can be formed as: priority matrix

$$\begin{pmatrix} & c_1,\alpha & c_2,\alpha & c_3,\alpha & \beta \\ D_1 & \left(\langle 0.5,0.5,0.6\rangle,0.5\right) & \left(\langle 0.3,0.3,0.5\rangle,0.3\right) & \left(\langle 0.5,0.7,0.6\rangle,0.5\right) & (0.8) \\ D_2 & \left(\langle 0.4,0.3,0.6\rangle,0.5\right) & \left(\langle 0.3,0.6,0.4\rangle,0.6\right) & \left(\langle 0.3,0.5,0.5\rangle,0.7\right) & (0.8) \\ D_3 & \left(\langle 0.5,0.3,0.6\rangle,0.5\right) & \left(\langle 0.3,0.6,0.4\rangle,0.4\right) & \left(\langle 0.5,0.4,0.6\rangle,0.3\right) & (0.7) \end{pmatrix}$$

Using equation (3), the aggregated weights for all attributes are presented below:

$$w^{''} = \begin{Bmatrix} \left(\langle 0.46602,0.36619,0.60012\rangle,0.49995\right),\left(\langle 0.29997,0.50061,0.43307\rangle,0.4347\right), \\ \left(\langle 0.43209,0.53322,0.56601\rangle,0.76697\right) \end{Bmatrix}$$

Step 5: The aggregated priority weighted neutrosophic decision matrix can be formed as:
The aggregated priority weighted neutrosophic decision matrix

$$\begin{pmatrix} & c_1,\alpha & c_2,\alpha & c_3,\alpha & \beta \\ D_1 & \left(\langle 0.188,0.622,0.854\rangle,0.818\right) & \left(\langle 0.170,0.653,0.734\rangle,0.607\right) & \left(\langle 0.246,0.815,0.828\rangle,0.752\right) & (0.538) \\ D_2 & \left(\langle 0.165,0.622,0.854\rangle,0.746\right) & \left(\langle 0.188,0.786,0.654\rangle,0.674\right) & \left(\langle 0.124,0.762,0.779\rangle,0.780\right) & (0.533) \\ D_3 & \left(\langle 0.189,0.540,0.867\rangle,0.75\right) & \left(\langle 0.132,0.769,0.786\rangle,0.688\right) & \left(\langle 0.232,0.723,0.829\rangle,0.655\right) & (0.515) \end{pmatrix}$$

Step 6: *Calculation of RPIS and RNIS*, using the equations (5), (6):

$$R^{w+} = \left\{\left(\langle 0.189,0.540,0.854\rangle,0.746\right),\left(\langle 0.188,0.653,0.654\rangle,0.607\right),\left(\langle 0.246,0.723,0.779\rangle,0.655\right),\left(0.538\right)\right\}$$

And

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$$R^{w-} = \{(\langle 0.165, 0.622, 0.867\rangle, 0.818), (\langle 0.132, 0.769, 0.786\rangle, 0.688), (\langle 0.124, 0.815, 0.829\rangle, 0.780), (0.515)\}$$

Step 7: The normalized Euclidean distance from RPIS by using (7), is given as below:

$$E^{+}_{u_1} = 0.050929, E^{+}_{u_2} = 0.060714, E^{+}_{u_3} = 0.045385$$

The normalized Euclidean distance from RNIS by using (8), is given as below:

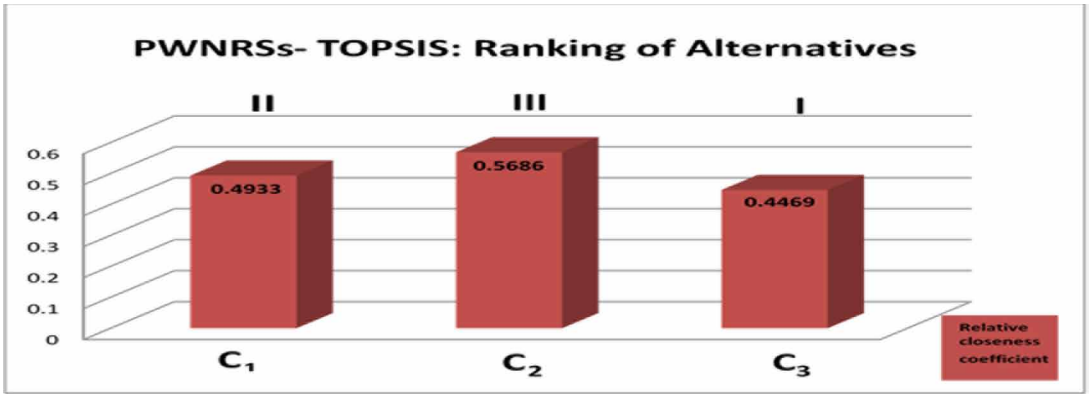
$$E^{-}_{u_1} = 0.052320, E^{-}_{u_2} = 0.046060, E^{-}_{u_3} = 0.056149$$

Step 8: The relative closeness coefficient, for each alternative are given by the following.

Alternative $R_k = \dfrac{E^{+}_{u_k}}{E^{+}_{u_k} - E^{-}_{u_k}}$

We observe the order of the values as $0.4933 < 0.4469 < 0.5686$ and hence C_3 is the best college.

Figure 2. The relative closeness coefficient value and Ranking of the Alternatives by TOPSIS



CONCLUSION

This chapter brings out the new notion of an extended structure of neutrosophic sets called priority weighted neutrosophic refined soft set. It deals with the set operations and some properties related to the proposed set. It also studies MCDM and TOPSIS method for priority weighted neutrosophic refined soft set. Further it presents constructive examples to show the applicability and utility of the proposed technique. In contrast to the existing theories this newly defined method can be considered to be efficient to solve the real time problems which include various indeterminate circumstances where refinement

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along with parameterization is essential. This paves a way for future research as it can hybridized with other existing dimensions of neutrosophic logics.

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Chapter 36
Neutrosophic Contra RW-Continuous Functions in Neutrosophic Topological Spaces

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ABSTRACT

The aim of this chapter is to introduce a new class of contra continuous functions, namely neutrosophic contra RW-continuous functions, in neutrosophic topological spaces. Further, the authors introduce neutrosophic almost RW-continuous function, neutrosophic almost contra RW-continuous function, and some of its properties have been discussed. Relationships with some other existing neutrosophic contra continuous functions have been analyzed. Furthermore, NRW-T0, NRW-T1, NRW-T2, and NRW-normal spaces have been introduced.

1. INTRODUCTION

L.A. Zedah (Zadeh, 1965) introduced the fuzzy sets and fuzzy topological spaces. The theory of fuzzy topological space was introduced and developed by Chang (Chang, 1968) in 1968. Since then it has been extended into various forms of topological spaces by many authors. The concept of "Intuitionistic fuzzy set" was first introduced by Atanassov (Atanassov, 1984) in 1983. After the introduction of the concepts of neutrosophic set by Smarandache (Smarandache, 2002a) in 1999, the idea of neutrosophic

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crisp sets and neutrosophic crisp topological spaces were introduced by A.A.Salama and Alblowi .S.A (Salama & Alblowi, 2012a).

In 2007, S.S. Benchalli, R.S. Wali (Benchalli & Wali, 2007) introduced the concept of RW-closed sets in topological spaces. The authors D. Savithiri, C.Janaki (Savithiri, 2019) introduced Neutrosophic RW-closed sets in neutrosophic topological spaces. As an extension, in this chapter the authors introduce neutrosophic contra RW-continuous functions. Several properties have been discussed and interrelations among the concepts are analyzed with concrete examples.

2. TERMINOLOGIES

In this section some important basic preliminaries, and in particular, the work of Smarandache (Smarandache, 2002a) and Salama (Salama & Alblowi, 2012a) have been recalled.

Definition 2.1:(Smarandache, 2002a) Let X be a non-empty fixed set a Neutrosophic set (NS for short) A is an object having the form $A = \left\langle x, \mu_A\left(x\right), \sigma_A\left(x\right), \gamma_A\left(x\right)\right\rangle$, x∈X where $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element x∈X to the set A.

Remark 2.2: (Smarandache, 2002a) For the sake of simplicity A neutrosophic set A = {x, $\mu_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$>; x∈X} can be identified to be an ordered triple < μ_A,σ_A,γ_A>.

Definition 2.3:(Salama & Alblowi, 2012a) The neutrosophic subsets 0_N and I_N in X are defined as follows:

0_N = {<x, (0, 0, 1)>; x ∈X} (2) I_N = {<x, (1, 0, 0)>; x ∈X} (1)

Definition 2.4: (Salama & Alblowi, 2012a) Let X be a nonempty set and neutrosophic sets A and B in the form A = {< x, ($\mu_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$)>, x∈X} and B = {< x, ($\mu_B(x)$,$\sigma_B(x)$,$\gamma_B(x)$)>, x∈X}.

$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \leq \sigma_B(x)$, and $\gamma_A(x) \geq \gamma_B(x)\ \forall\ x \in X$ (1)

$\bar{A}$ = {<x, $\gamma_A(x)$, $\sigma_A(x)$, $\mu_A(x)$ >; x∈ X} (2)

A = B iff A ⊆ B and B ⊆ A. (3)

A ∩ B = < x, min($\mu_A(x)$,$\mu_B(x)$), min($\sigma_A(x)$,$\sigma_B(x)$), max($\gamma_A(x)$, $\gamma_B(x)$) > (4)

A ∪ B = < x, max($\mu_A(x)$,$\mu_B(x)$), max ($\sigma_A(x)$,$\sigma_B(x)$), min ($\gamma_A(x)$, $\gamma_B(x)$) > (5)

Definition 2.5: (Salama & Alblowi, 2012a) Let {A_j: j∈J } be a arbitrary family of NS sets in X, then

$\cap A_j = < x, \wedge_{j\in J}\mu_{A_j}(x), \wedge_{j\in J}\sigma_{A_j}(x), \vee_{j\in J}\gamma_{A_j}(x)$ (1)

$\cup A_j = < x, \vee_{j\in J}\mu_{A_j}(x), \vee_{j\in J}\sigma_{A_j}(x), \wedge_{j\in J}\gamma_{A_j}(x)$ (2)

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Definition 2.6: (Salama & Alblowi, 2012a) A neutrosophic topology [NT for short] is a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

$0_N, I_N \in \tau$, (1)

$G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$, (2)

$\cup G_i \in \tau$ for every $\{G_i: i \in J\} \subseteq \tau$ (3)

In this case, the pair (X,τ) is called a neutrosophic topological space **[NTS for short].** The elements of τ are called neutrosophic open sets **[NOS for short].** The complement of A [$\bar{A}$ for short] of NOS is called a neutrosophic closed set **[NCS for short**] in X.

Definition 2.7: (Salama & Alblowi, 2012a) Let (X,τ) be NTS and $A = < x, \mu_A(x), \sigma_A(x), \gamma_A(x) >$ be a NS in X. Then the neutrosophic closure and neutrosophic interior of A are defined by

$NCl(A) = \cap \{K: K \text{ is a NCS in X and } A \subseteq K\}$

$NInt(A) = \cup \{G: G \text{ is a NOS in X and } G \subseteq A\}$

Definition 2.8:(Dhavaseelan et al., n.d.) Let X be a nonempty set. Whenever r,t,s, be real or non-standard subsets of $]0^-,1^+[$ then the neutrosophic set $x_{r,s,t}$ is termed as a neutrosophic point(In short NP) in X given by

$$x_{r,s,t} = \begin{cases} (r,s,t), & if\ x = x_p \\ (0,0,1), & if\ x \neq x_p \end{cases}$$

for $x_p \in X$ is termed as the support of $x_{r,s,t}$, wherein r indicates the degree of membership value, s indicates the degree of indeterminacy and t as the degree of non-membership value of $x_{r,s,t}$.

Definition 2.9: (Dhavaseelan et al., n.d.) Let A be a neutrosophic subset of a NTS X. Then A is said to be a

(i) neutrosophic regular open **(In short NRO)** set if A =NInt(NCl(A)) and regular closed **(In short NRC)** set if A = NCl(NInt(A).
(ii) neutrosophic regular semi open **(In short NRSO)** if there exists an NRO set U such that $U \subseteq A \subseteq NCl(A)$

Definition 2.10: (Savithiri, 2019) Let A be a neutrosophic set of a NTS X. Then A is said to be Neutrosophic Regular Weakly - Closed set **(shortly NRWCS)** of X if $NCl(A) \subseteq U$ whenever $A \subseteq U$ and U is an NRSO set.

The complement of NRW – closed set is known as neutrosophic RW open (shortly **NRWOS)** set.

Definition 2.11: (Dhavaseelan et al., n.d.; Savithiri, 2019) Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces. A function $f: (X,\tau) \to (Y,\sigma)$ is said to be a

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(i) neutrosophic continuous function if the inverse image of every neutrosophic open set in (Y,σ) is a neutrosophic open set in (X,τ).
(ii) neutrosophic RW-continuous **(shortly NRW - continuous)** if the inverse image of every neutrosophic closed set in (Y,σ) is neutrosophic RW- closed set in (X,τ).
(iii) neutrosophic RW-irresolute **(shortly NRW - irresolute)** if the inverse image of every neutrosophic RW- closed set in (Y,σ) is neutrosophic RW- closed set in (X,τ).
(iv) neutrosophic contra-continuous function if the inverse image of every neutrosophic open set in (Y,σ) is a neutrosophic closed set in (X,τ).
(v) generalized neutrosophic contra-continuous if the inverse image of every neutrosophic open set in (Y,σ) is a generalized neutrosophic closed set in (X,τ).
(vi) strongly generalized neutrosophic contra-continuous function if the inverse image of every generalized neutrosophic open set in (Y,σ) is a neutrosophic closed set in (X,τ).
(vii) neutrosophic almost continuous function if the inverse image of every neutrosophic regular open set in (Y,σ) is a neutrosophic open set in (X,τ).
(viii) neutrosophic almost contra continuous function if the inverse image of every neutrosophic regular open set in (Y,σ) is a neutrosophic closed set in (X,τ).
(ix) Let $f: (X,\tau) \to (Y,\sigma)$ be a function Then a neutrosophic graph of f is defined by $g(x) = (x,f(x))$, for all $x \in X$.

Definition 2.12: (Dhavaseelan et al., n.d.; Savithiri, 2021) A neutrosophic topological space (X,τ) is a

(i) neutrosophic locally indiscrete if every neutrosophic open set is neutrosophic closed.
(ii) NRWT1/2 space if every NRW-closed set is NC set.
(iii) neutrosophic-connected if X cannot be written as union of two disjoint nonempty NO sets.

3. NEUTROSOPHIC CONTRA RW-CONTINUOUS FUNCTIONS

Definition 3.1: Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces. A function $f: (X,\tau) \to (Y,\sigma)$ is said to be a

(i) neutrosophic contra RW-continuous if the inverse image of every neutrosophic open set in (Y,σ) is a neutrosophic RW-closed set in (X,τ).

Equivalently, if the inverse image of every neutrosophic closed set in (Y,σ) is a neutrosophic RW-open set in (X,τ).

(ii) neutrosophic contra RW-irresolute if the inverse image of every neutrosophic RW-open set in (Y,σ) is a neutrosophic RW-closed set in (X,τ).

Equivalently, if the inverse image of every neutrosophic RW-closed set in (Y,σ) is a neutrosophic RW-open set in (X,τ).

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(iii) strongly neutrosophic contra RW-continuous if the inverse image of every neutrosophic RW-open set in (Y,σ) is neutrosophic closed set in $(X.\tau)$.

Equivalently, if the inverse image of every neutrosophic RW-closed set in (Y,σ) is a neutrosophic open set in (X,τ).

Proposition 3.2: Let $f: (X,\tau) \to (Y,\sigma)$ be a bijection. Then f is a neutrosophic contra RW-continuous function if $NCl(f(A) \subseteq f(NRWInt(A))$ for every neutrosophic set A in (X,τ).

Proof: Let A be a neutrosophic closed set in (Y,σ). Then $NCl(A) = A$ and $f^{-1}(A)$ is a neutrosophic set in (X,τ). By hypothesis, $NCl(f(f^{-1}(A)) \subseteq f(NRWInt(f^{-1}(A))$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore $A = NCl(A) = NCl(f(f^{-1}(A)) \subseteq f(NRWInt(f^{-1}(A))$. Now $A \subseteq f(NRWInt(f^{-1}(A))$, $f^{-1}(A) \subseteq f^{-1}(f(NRWInt(f^{-1}(A))$ implies $NRWInt(f^{-1}(A) \subseteq f^{-1}(A)$ i.e., $f^{-1}(A)$ is NRW-open in (X,τ). Thus, f is a neutrosophic RW-continuous function.

Proposition 3.3: Let (X,τ) and (Y,σ) be any two topological spaces. If $f: (X,\tau) \to (Y,\sigma)$ be a neutrosophic contra-continuous function then it is a neutrosophic contra RW-continuous function.

Proof: Let A be a neutrosophic open set in (Y,σ). Since f is a neutrosophic contra-continuous function, $f^{-1}(A)$ is neutrosophic closed in (X,τ). Every NC set is an NRW-closed set implies that f is a neutrosophic contra RW-continuous function.

The converse of the proposition 3.3 need not be true as shown in the following example.

Example 3.4: Let $X = Y = \{a,b,c\}$. The neutrosophic open sets A and B are $A = x$, (0.4,0.4,0.4), (0.5,0.5,0.4), (0.4,0.4,0.4), $B = x$, (0.3,0.3,0.5), (0.6,0.4,0.4), (0.7,0.3,0.3). Then $\tau = \{0_N, A, 1_N\}$ and $\sigma = \{0_N, B, 1_N\}$ are neutrosophic topologies on X and Y respectively. Define $f: (X,\tau) \to (Y,\sigma)$ by f(a) =b, f(b) = a, f(c) = c. Then f is a neutrosophic contra RW-continuous function but it is not a neutrosophic contra-continuous function since $f^{-1}(B)$ is NRW-closed in X but it is not a neutrosophic closed set in X.

Proposition 3.5: Let (X,τ) and (Y,σ) be any two topological spaces If $f: X,\tau) \to (Y,\sigma)$ is

(i) a generalized neutrosophic contra-continuous then f is a neutrosophic contra RW-continuous function.

(ii) a strongly generalized neutrosophic contra-continuous then f is a strongly neutrosophic contra RW-continuous function.

Proof: Obvious.

The following example makes clear that the converse of the proposition 3.5 need not be true.

Example 3.6: Let $X = \{a,b,c\} = Y$. Consider the neutrosophic open sets A, B and C as follows: $A = x$, (0.3,0.2,0.4), (0.2,0.2,0.5), (0.2,0.2,0.5), $B = x$, (0.1,0.1,0.9), (0.1,0.1,0.9), (0.1,0.1,0.9) and $C = x$, (0.8,0.1,0.1), (0.7,0.1,0.1), (0.3,0.1,0.1). Then $\tau = \{0_N, A, B, 1_N\}$ and $\sigma = \{0_N, C, 1_N\}$ are neutrosophic topologies on X and Y respectively. Define $f: (X,\tau) \to (Y,\sigma)$ by f(a) =a, f(b) = c, f(c) = b. Then f is a neutrosophic contra RW-continuous function as well as strongly neutrosophic contra RW-continuous but it is neither a generalized neutrosophic contra-continuous nor a strongly generalized neutrosophic contra-continuous function.

Proposition 3.7: Let (X,τ) and (Y,σ) be any two topological spaces. If $f: (X,\tau) \to (Y,\sigma)$ be a strongly neutrosophic contra RW-continuous function then f is a neutrosophic contra RW-continuous function.

Proof: Let f be a strongly neutrosophic RW-continuous function and let A be a neutrosophic open set in (Y,σ). Since NO set is NRW-open set, A is a NRW-open set. By hypothesis, $f^{-1}(A)$ is neutrosophic closed hence NRW-closed in (X,τ). Thus f is a neutrosophic contra RW-continuous function.

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The converse of the proposition 3.7 need not be true as seen in the following example.

Example 3.8: Let X = Y= {a,b,c}. The neutrosophic open sets A and B are A = *x*, (0.4,0.4,0.4), (0.5,0.5,0.4), (0.4,0.4,0.4), B = *x*, (0.2,0.3,0.5), (0.3,0.4,0.7), (0.3,0.4,0.5). Then $\tau = \{0_N, A, 1_N\}$ and $\sigma = \{0_N, B, 1_N\}$ are neutrosophic topologies on X and Y respectively. Define an identity function f: $(X,\tau) \to (Y,\sigma)$. Then f is a neutrosophic contra RW-continuous function but it is not a strongly neutrosophic contra RW-continuous function.

Proposition 3.9: Let (X,τ) and (Y,σ) be any two topological spaces. If f: $(X,\tau) \to (Y,\sigma)$ be a strongly neutrosophic contra RW-continuous function then f is a neutrosophic contra RW-irresolute function.

Proof: Let A be a neutrosophic RW- open set in (Y,σ). Since f is strongly neutrosophic contra RW-continuous, $f^{-1}(A)$ is neutrosophic closed in (X,τ). Every NC set is NRW-closed hence $f^{-1}(A)$ is NRW-closed in (X,τ). Thus f is a neutrosophic contra RW-irresolute function.

The converse of the proposition 3.9 need not be true as seen in the following example.

Example 3.10: Let X = {a,b,c} = Y, $\tau = \{0_N, A, B, 1_N\}$ and $\sigma = \{0_N, C, 1_N\}$ where the neutrosophic open sets A, B and C as follows: A = *x*, (0.4,0.4,0.5), (0.5,0.5,0.5), (0.5,0.5,0.5), B = *x*, (0.6,0.6,0.4), (0.7,0.7,0.3), (0.5,0.5,0.5) and C = *x*, (0.4,0.4,0.4), (0.4,0.4,0.5), (0.5,0.5,0.5). Define f: $(X,\tau) \to (Y,\sigma)$ by f(a) =a, f(b) = c, f(c) = b. Then f is a neutrosophic contra RW-irresolute function but it is not a strongly neutrosophic contra RW-continuous function.

Proposition 3.11: Let (X,τ), (Y,σ), (Z,η) be any three neutrosophic topological spaces. Let f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\eta)$ be two functions.

(i) If f is neutrosophic contra RW- irresolute and g is neutrosophic contra RW-continuous function then, $g \circ f$: $(X,\tau) \to (Z,\eta)$ is NRW-continuous function.
(ii) If f is NRW-continuous function and g is strongly neutrosophic RW-continuous function then, $g \circ f$ is neutrosophic contra RW-irresolute function.
(iii) If f is NRW-irresolute function and g is neutrosophic contra RW-continuous then, $g \circ f$ is neutrosophic contra RW-continuous function.
(iv) If f is neutrosophic contra RW-irresolute function and g is neutrosophic RW-continuous then $g \circ f$ is neutrosophic contra RW-continuous function.

Proof: Obvious.

Proposition 3.12: Let (X,τ), (Y,σ), (Z,η) be neutrosophic topological spaces. Let f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\eta)$ be two functions and (Y,σ) be a NRWT1/2-space if f and g are neutrosophic contra RW–continuous functions, then $g \circ f$ is a neutrosophic RW-continuous function.

Proof: Let A be a neutrosophic open set in (Z,η). Since g is a neutrosophic contra RW-continuous function, $g^{-1}(A)$ is an NRW-closed set in (Y,σ). Since (Y,σ) is a NRWT1/2-space, $g^{-1}(A)$ is a neutrosophic closed set in (Y,σ). Since f is a neutrosophic contra RW-continuous functions, $f^{-1}(g^{-1}(A))$ is an NRW-open set in (X,τ). Hence $g \circ f$ is a neutrosophic RW-continuous function.

Proposition 3.13: Let (X,τ), (Y,σ), (Z,η) be any three neutrosophic topological spaces. Let f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\eta)$ be two functions.

(i) If f is neutrosophic contra RW- continuous function and g is strongly neutrosophic contra RW-continuous function then, $g \circ f$: $(X,\tau) \to (Z,\eta)$ is NRW-irresolute function.
(ii) If f is strongly neutrosophic contra RW-continuous function and g is neutrosophic contra RW-continuous function then, $g \circ f$ is neutrosophic -continuous function.

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(iii) If f is strongly neutrosophic RW-continuous function and g is neutrosophic contra RW-continuous then, $g \circ f$ is neutrosophic contra -continuous function.
(iv) If f is strongly neutrosophic contra RW-continuous function and g is neutrosophic RW-irresolute then $g \circ f$ is strongly neutrosophic contra RW-continuous function.

Proof: Obvious.

Proposition 3.14: Let (X,τ), (Y,σ), (Z,η) be neutrosophic topological spaces. Let $f: (X,\tau) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (Z,\eta)$ be two functions and (Y,σ) be a NRWT1/2-space. If f is a neutrosophic contra continuous function and g is a neutrosophic contra RW–irresolute function, then $g \circ f$ is a strongly neutrosophic RW–continuous function.

Proof: Let A be an NRW- open set in (Z,η). Since g is a neutrosophic contra RW-irresolute function, $g^{-1}(A)$ is an NRW-closed set in (Y,σ). Since (Y,σ) is a NRWT1/2-space, $g^{-1}(A)$ is a neutrosophic closed set in (Y,σ). Since f is a neutrosophic contra –continuous functions, $f^{-1}(g^{-1}(A))$ is a neutrosophic -open set in (X,τ). Hence $g \circ f$ is a strongly neutrosophic RW-continuous function.

Proposition 3.15: Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces. Let $f: (X,\tau) \to (Y,\sigma)$ be a function, then the following statements are equivalent.

(i) f is a neutrosophic contra RW-continuous function.
(ii) For each neutrosophic point $x_{r,s,t}$ of X and for each neutrosophic closed set B of (Y,σ) containing $f(x_{r,s,t})$, there exists an NRW-open set A of (X,τ) containing $x_{r,s,t}$ such that $A \subseteq f^{-1}(B)$.
(iii) For each neutrosophic point $x_{r,s,t}$ of X and for each neutrosophic closed set B of $(Y,\sigma$ containing f $(x_{r,s,t})$, there exists an NRW-open set A of (X,τ) containing $x_{r,s,t}$ such that $f(A) \subseteq B$.

Proof: (i) Þ (ii): Let f be a neutrosophic contra RW-continuous function. Let B be a neutrosophic closed set in (Y,σ) and $x_{r,s,t}$ a neutrosophic point of X such that $f(x_{r,s,t}) \in B$. Then $x_{r,s,t} \in f^{-1}(B) = NRWInt(f^{-1}(B))$. Let $A = NRWInt(f^{-1}(B))$, then A is a neutrosophic RW-open set and $A = NRWInt(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies that $A \subseteq f^{-1}(B)$.

(ii) Þ (iii): Let B be a neutrosophic closed set in (Y,σ) and let $x_{r,s,t}$ be a neutrosophic point in X, such that $f(x_{r,s,t}) \in B$. Then $x_{r,s,t} \in f^{-1}(B)$. By hypothesis, $f^{-1}(B)$ is an NRW-open set in (X,τ) and $A \subseteq f^{-1}(B)$ which implies that $f(A) \subseteq B$.

(iii) Þ(i): Let B be a neutrosophic closed set in (Y,σ) and let $x_{r,s,t}$ be a neutrosophic point in X such that $f(x_{r,s,t}) \in B$. Then $x_{r,s,t} \in f^{-1}(B)$. By hypothesis, there exists an NRW-open set A of (X,τ) such that $x_{r,s,t} \in A$ and $f(A) \subseteq B$. This implies that $x_{r,s,t} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Since A is an NRW-open set, $A = NRWInt(A) \subseteq NRWInt(f^{-1}(B))$. Hence $x_{r,s,t} \in NRWInt(f^{-1}(B))$, $f^{-1}(B) = \bigcup_{x_{r,s,t} \in f^{-1}(B)} x_{r,s,t} \subseteq NRWInt(f^{-1}(B))$.

Therefore $f^{-1}(B)$ is an NRW-open set in (X,τ). Thus f is a neutrosophic contra RW-continuous function.

Proposition 3.16: Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces. Let $f: (X,\tau) \to (Y,\sigma)$ be any function and g be a graph of f such that $g: X \to X \times Y$.

(i) If g is a neutrosophic contra RW-continuous function, then f is also a neutrosophic contra RW-continuous function.
(ii) If g is a strongly neutrosophic contra RW-continuous function, then f is also a strongly neutrosophic contra RW-continuous function.

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(iii) If g is a neutrosophic contra RW-irresolute function, then f is also a neutrosophic contra RW-irresolute function.

Proof: (i) Let A be a neutrosophic open set in (Y,σ). By definition, $f^{-1}(A) = I_N \cap f^{-1}(A) = g^{-1}(I_N \times A)$. Since g is a neutrosophic contra RW-continuous, $g^{-1}(I_N \times A)$ is an NRW-closed set in (X,τ). Now $f^{-1}(A)$ is an NRW-closed set in (X,τ). Thus f is a neutrosophic contra RW-continuous function.

Proof of (ii) and (iii) is as same as (i).

4. NEUTROSOPHIC ALMOST CONTRA RW-CONTINUOUS FUNCTION

Definition 4.1: Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces. A function $f: (X,\tau) \to (Y,\sigma)$ is said to be a

(i) neutrosophic almost RW-continuous if the inverse image of each neutrosophic regular open set in (Y,σ) is NRW-open in (X,τ).
(ii) neutrosophic almost contra RW-continuous if the inverse image of each neutrosophic regular open set in (Y,σ) is NRW-closed in (X,τ).

Proposition 4.2: Let $f: X \to Y$ be a function along with $g: X \to X \times Y$ be the graph function defined by $g(x) = (x, f(x))$ being each $x \in X$. If g is neutrosophic almost contra RW-continuous function, then f is also a neutrosophic almost contra RW-continuous function.

Proof: Let M be a neutrosophic regular closed set in (Y,σ). Then $X \times M$ is a neutrosophic regular closed set in $X \times Y$. Since g is a neutrosophic almost contra RW-continuous, $f^{-1}(M) = g^{-1}(X \times M)$ is an NRW-open in X which implies that f is a neutrosophic almost contra RW-continuous.

Definition 4.3: A neutrosophic topological space (X,τ) is a

(i) neutrosophic RW locally -indiscrete **(In short NRW locally -indiscrete)** if every NRW- open set is neutrosophic closed in (X,τ).
(ii) neutrosophic RW-connected **(In short NRW-connected)** if X cannot be expressed as union of two disjoint non-empty neutrosophic RW-open sets.

Proposition 4.4: If a function $f: X \to Y$ is a neutrosophic almost contra RW-continuous function and X is neutrosophic RW locally -indiscrete space, then f is neutrosophic almost -continuous function.

Proof: Let F be a neutrosophic regular closed set in Y. Since f is a neutrosophic almost contra RW-continuous function, $f^{-1}(F)$ is an NRW-open in X and X is neutrosophic RW- locally indiscrete space which implies $f^{-1}(F)$ is a neutrosophic closed set in X. Hence f is neutrosophic almost –continuous function.

Proposition 4.5: Let $f: (X,\tau) \to (Y,\sigma)$ be a function and $NRWO(X,\tau)$ is closed under arbitrary union. Then the following statements are equivalent.

(i) f is a neutrosophic almost contra RW-continuous function.
(ii) $f^{-1}(F)$ is an NRW-open set in X for every neutrosophic regular closed set F in Y.
(iii) For each $x \in X$ and each regular closed set F in Y containing f(x), there exists an NRW-open set U in X containing x such that $f(U) \subseteq F$.

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(iv) For each x ∈ X and each neutrosophic regular open set V in Y containing f(x), there exists an NRW-closed set K in X not containing x such that $f^{-1}(V) \subseteq K$.
(v) $f^{-1}(NInt(NCl(G)) \in NRWC(X)$ for every NO set G of Y.
(vi) $f^{-1}(NCl(NInt(H)) \in NRWO(X)$ for every NC set H of Y.

Proof: (i) Þ (ii): Let G be a NRC set in Y. Then Y-F is NRO set in Y and by hypothesis, $f^{-1}(Y-G) = X-f^{-1}(G)$ is NRW-closed in X which implies that $f^{-1}(G)$ is NRW-open set in X.

(ii) **Þ (i):** Let G be a NRO set in Y. Then Y-G is NRC set in Y. By (ii), $f^{-1}(Y-G) = X-f^{-1}(G)$ is NRW-open in X which implies that $f^{-1}(G)$ is NRW-closed set in X. Therefore f is a neutrosophic almost contra RW-continuous.
(iii) **Þ (iii):** Let H be a NRC in Y containing f(x). By assumption $f^{-1}(H)$ is NRW-open in X and $x \in f^{-1}(H)$. Take $U = f^{-1}(H)$ implies $f(U) \subseteq H$.
(iv) **Þ (ii):** Let F be a NRC in Y and $x \in f^{-1}(F)$. From (iii), there exists an NRW-open set U_x in X containing x such that $U_x \subseteq f^{-1}(F)$ which implies $f^{-1}(F) = \cup \{U_x, x \in f^{-1}(F)\}$. Hence $f^{-1}(F)$ is NRW-open.
(v) **Þ (iv):** Let R be any NRC-set in Y containing f(x). By (iii), there exists a NRW-open set U in X containing x such that $f(U) \subseteq Y-V$. Hence $U \subseteq f^{-1}(Y-V) \subseteq X-f^{-1}(V)$. Then $f^{-1}(V) \subseteq X-U$. Consider $X-U = K$, then there exists an NRW-closed set in X not containing x such that $f^{-1}(V) \subseteq K$.
(vi) **Þ (iii):** Let F be a NRC set in Y containing f(x). Then Y-F is an NRO set in Y not containing f(x). by hypothesis, there exists an NRW-closed set K in X not containing x such that $f^{-1}(Y-K) \subseteq K$. This implies $X-f^{-1}(F) \subseteq K$, hence $f(X-K) \subseteq F$. Take $X-K = U$, then U is an NRW-open set in x containing x such that $f(U) \subseteq F$.
(vii) **Þ (v):** Let G be a NO set in Y. Then NInt (NCl (G)) is a NRO set in Y which implies that Y-NInt(NCl(G)) is NRC set in Y. By assumption, $f^{-1}(Y - NInt (NCl (G)))$ is an NRW-open in X means that $f^{-1}(NInt (NCl (G)) \in NRWC(X)$.
(viii) **Þ (vi):** Let H be an NC set in Y, so Y-H is a NO set in Y. By (v), $f^{-1}(NInt(NCl(Y-H))$ is an NRW-closed set in X implies that $f^{-1}(NInt(NCl(Y-H))$ is NRW-closed in X, i.e., $f^{-1}(NInt(Y-NCl(H)) = f^{-1}(Y-NCl(NInt(H)) = X- f^{-1}(NCl(NInt(H))$ is NRW-closed. Thus $f^{-1}(NCl(NInt(H)) \in NRWO(X)$.
(ix) **Þ(v):** Let G be a NO set in Y. Then Y-G is NC set in Y which implies NCl(NInt(Y-G)) is NRC set. By (ii), $f^{-1}(NCl (NInt(Y-G))$ is NRW-open in X. Hence $f^{-1}(NInt(NCl(G))$ is NRW-closed in X.
(x) **Þ (v):** Let F be a NO subset of Y. since NInt(NCl(F)) is NRO, by (i), $f^{-1}(NInt(NCl(F)) \in NRWC(X)$.
(xi) **Þ(i):** Let V be NRO set of Y. Then V is NO set in Y. By hypothesis, $f^{-1}(NInt(NCl(V))$ is an NRW-closed set in X which implies that $f^{-1}(V)$ is NRW-closed set. Thus f is a neutrosophic almost contra RW-continuous function.

Proposition 4.6: Let f: X → Y, g: Y → Z and $g \circ f$: X → Z be a composition function. Then

(i) If f is a neutrosophic almost contra RW-continuous and g is a neutrosophic R-map, then $g \circ f$ is neutrosophic almost contra RW -continuous.
(ii) If f is a neutrosophic contra RW-continuous and g is a neutrosophic almost continuous, then $g \circ f$ is neutrosophic almost contra RW -continuous.
(iii) If f is a neutrosophic almost contra RW-continuous and g is a neutrosophic regular set connected, then $g \circ f$ is neutrosophic almost contra RW –continuous and neutrosophic almost RW- continuous.

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(iv) If f is a neutrosophic contra RW-continuous and g is a neutrosophic regular set connected, then $g \circ f$ is neutrosophic almost contra RW –continuous and neutrosophic almost RW-continuous.

Proof: Obvious.

Proposition 4.7: Every neutrosophic regular set connected map is neutrosophic almost contra RW-continuous and neutrosophic almost RW-continuous.

Proof: Let f: X → Y be a neutrosophic regular connected map and U be any NRO set in Y. By hypothesis, $f^{-1}(V)$ is neutrosophic clopen in X. Therefore $f^{-1}(V)$ is NRW-closed and NRW-open in X. Hence f is neutrosophic almost contra RW-continuous and neutrosophic almost RW-continuous.

The converse of the Proposition 4.7 need not be true as shown in the following example.

Example 4.8: Let X = Y = {a,b,c}, $\tau = \{0_N, A, 1_N\}$ and $\sigma = \{0_N, B, 1_N\}$. the NO sets A and B are A =*x*, (0.3,0.3,0.5), (0.4,0.4,0.6), (0.3,0.3,0.7); B = *x*, (0.3,0.2,0.5), (0.4,0.3,0.5), (0.4,0.2,0.6). Define an identity function f: X → Y, then f is a neutrosophic almost contra RW-continuous but it is not a neutrosophic regular set connected.

Proposition 4.9: If a function f: X → Y is neutrosophic almost contra RW-continuous, neutrosophic almost continuous and X is an NRWT1/2 space, then f is neutrosophic regular set connected.

Proof: Let V be a NRO set in Y. By hypothesis, $f^{-1}(V)$ is NRW-closed and NO in X. Since X is an NRWT1/2 space, $f^{-1}(V)$ is NC in X and hence $f^{-1}(V)$ is neutrosophic clopen. This shows that f is a neutrosophic regular set connected.

Proposition 4.10: If a function f: X → Y is neutrosophic almost contra RW-continuous and a neutrosophic R map, then f is neutrosophic regular set connected.

Proof: Let V be an NRO set in Y. By hypothesis, $f^{-1}(V)$ is NRW-closed and NRO in X. Since V is NRO, it is a NRSO set. So V ⊆ V and NCl(V) ⊆ V implies that V is neutrosophic clopen. Hence f is a neutrosophic regular set connected.

Proposition 4.11: If a surjection f: X → Y is a neutrosophic almost contra RW- continuous and X is a NRW-connected space then Y is neutrosophic connected.

Proof: Let f: X → Y is neutrosophic almost contra RW-continuous surjection with X is NRW-connected space. Assume that Y is not a neutrosophic connected space. Then there exists disjoint NO sets P and Q such that Y = P∪Q. Then P and Q are neutrosophic clopen in Y. By hypothesis, $f^{-1}(P)$ and $f^{-1}(Q)$ are NRW-open sets in X. Moreover $f^{-1}(P)$ and $f^{-1}(Q)$ are disjoint non-empty and $X = f^{-1}(P) \cup f^{-1}(Q)$ which is a contradiction to the fact that X is NRW-connected. Thus Y is a neutrosophic connected space.

Proposition 4.12: If a function g: X → Y is a neutrosophic almost contra RW- continuous and X is a NRW locally-indiscrete space then f is neutrosophic almost continuous.

Proof: Let R be a NRO set in Y. Since g is neutrosophic almost contra RW-continuous function, $g^{-1}(R)$ is an NRW-open set in X and X is NRW locally indiscrete implies that $g^{-1}(R)$ is a NC set in X. Hence g is neutrosophic almost continuous function.

Definition 4.13: A neutrosophic topological space (X,τ) is called as a

(i) neutrosophic RW- T_0 (Shortly $NRWT_0$) if for each pair of distinct neutrosophic points $x_{r,s,t}$ and $y_{r,s,t}$ in X, there exists NRW-open set U such that $x_{r,s,t} \in U$ and $y_{r,s,t} \notin U$ or $x_{r,s,t} \notin U$ and $y_{r,s,t} \in U$.

(ii) NRW-T_1 if for each pair of distinct neutrosophic points $x_{r,s,t}$ and $y_{r,s,t}$ in X, there exists NRW-open set U and V containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively such that $y_{r,s,t} \notin U$ and $x_{r,s,t} \notin V$.

(iii) NRW-T_2 if for each pair of distinct neutrosophic points $x_{r,s,t}$ and $y_{r,s,t}$ in X, there exists NRW-open set U and V containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively such that $U \cap V = 0_N$.

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(iv) NRW-normal if each pair of non-empty disjoint neutrosophic closed sets can be separated by disjoint NRW-open sets.

Proposition 4.14: If f: X → Y is a neutrosophic almost contra RW-continuous injection and Y is N-weakly Hausdorff space, then X is NRW-T_1.

Proof: Let Y be a N- weakly Hausdorff space. For any distinct neutrosophic points $x_{r,s,t}$ and $y_{r,s,t}$ in X, there exists NRC sets U and V in Y such that f($x_{r,s,t}$) ∈ U, f($y_{r,s,t}$) ∉ U, f($y_{r,s,t}$) ∈ V and f($x_{r,s,t}$) ∉ V. As f is neutrosophic almost contra RW-continuous, f^{-1}(U) and f^{-1}(V) are NRW-open subsets of X such that $x_{r,s,t} \in f^{-1}(U)$, $y_{r,s,t} \notin f^{-1}(U)$, $y_{r,s,t} \in f^{-1}(V)$ and $x_{r,s,t} \notin f^{-1}(V)$. Thus X is NRW-$T_1$.

Proposition 4.15: If h: X → Y is a neutrosophic almost contra RW-continuous injective mapping and Y is N-Ultra Hausdorff space, then X is NRW-T_2.

Proof: The neutrosophic points $x_{r,s,t}$ and $y_{r,s,t}$ be any two elements in X. Since h is an injective h($x_{r,s,t}$) ^{1} h($y_{r,s,t}$) and Y is N-Ultra Hausdorff space, there exists disjoint neutrosophic clopen sets U and V of Y containing h($x_{r,s,t}$) and h($y_{r,s,t}$) respectively. Hence $x_{r,s,t} \in h^{-1}(U)$ and $y_{r,s,t} \in h^{-1}(V)$, wherein h^{-1}(U) and h^{-1}(V) are disjoint NRW-open sets in X. Thus X is NRW-T_2.

Proposition 4.16: If Y is N-strongly normal and g: X → Y is a neutrosophic almost contra RW-continuous closed injection, then X is NRW-normal.

Proof: Suppose P and Q are disjoint NC sets of X. Let F is N-closed and injective then f(P) and f(Q) are disjoint NC sets in Y. Since Y is N-strongly normal, there exists NO sets U and V in Y so that g(P) ⊂ U and g(Q) ⊂ V and NCl(U) ∩ NCl(V) = 0_N. Since NCl(U) and NCl(V) are NRC sets, g is neutrosophic almost contra RW-continuous, g^{-1}(NCl(U)) and g^{-1}(NCl(V)) are NRW-open sets in X which implies P ⊆ g^{-1}(NCl(U)), Q ⊆ g^{-1}(NCl(V)) are disjoint, so X is NRW-normal.

Proposition 4.17: If f: X → Y is a neutrosophic almost contra RW-continuous, N-closed injection along with Y is N-Ultra normal, then X is NRW-normal.

Proof: Let P and Q be disjoint sets of X. Since f is N-closed and injective, f(P) and f(Q) are disjoint N-closed sets in Y. As Y is N-Ultra normal, there exist disjoint N-clopen sets u and V in Y such that f(P) ⊂ U and f(Q) ⊂ V implies P ⊂ f^{-1}(U) and Q ⊂ f^{-1}(V). Since f is neutrosophic almost contra RW-continuous injection, f^{-1}(U) and f^{-1}(V) are disjoint NRW-open sets in X. Therefore X is NRW-normal.

CONCLUSION

In this article, the authors have introduced and studied the concepts such as, Neutrosophic contra RW-continuous, Neutrosophic almost contra continuous, NRW-connected space, NRW-separation axioms and analyzed some of its properties. In future it can be extended to some new forms of continuous functions.

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CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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